# Vibration Analysis of Stylus Instrument for Random Surface Measurement

Y. Tian[1,3], X. Liu[2], D.G. Chetwynd[2], B. Shirinzadeh[3], D. Zhang[1]

[1]School of Mechanical Engineering, Tianjin University, Tianjin 300072, China

[2]School of Engineering, University of Warwick, Coventry CV4 7AL, UK

[3]Robotics and Mechatronics Research Laboratory, Department of Mechanical and Aerospace Engineering, Monash University, Clayton, VIC 3800, Australia

**Abstract**: This paper presents dynamic modeling and vibration analysis methodologies for investigating surface profile fidelity in stylus instruments, using a novel dynamic model that takes account of Hertzian contact stiffness between the stylus tip and surface. It briefly describes generation methods for modelling random surfaces and finite tips, which provide input for this nonlinear dynamic system. The influence of the damping ratio on the dynamic performance of the stylus instrument is examined and tends to confirm other recent work. The best combinations of signal fidelity and measurement speed occur with damping ratios considerably higher than conventionally used, mainly because the damping can suppress or delay tip flight. Residual vibrations associated with start-up transients are shown to have potentially serious effects on the signal fidelity. Hence, the responses of the stylus system to three typical velocity control signals are investigated. The normally-used step control signal can generate relatively large residual vibration, while slope control and, even better, cycloidal control signal can suppress the residual vibration and guarantee the signal fidelity.

**Keywords:** Dynamic modelling, Signal fidelity, Contact stiffness, Stylus instrument

## 1. Introduction

The stylus instrument is one of the key enabling techniques for surface metrology and characterisation. Its advantages include convenient utilization, low sensitivity to surface contaminants and vibration, and direct recording of the mechanical surface relevant to many tribological applications. In modern engineering and scientific fields, high scanning speed is always desirable to reduce the environmental effects and to improve the efficiency of surface metrology. Thus, dynamic characteristics of such instruments are an increasingly important factor for signal fidelity in surface characterisation. In order to guarantee measurement accuracy and signal fidelity, vibration within such devices is one of the key issues needing to be systematically explored to avoid the signal distortion, which may be due to the tip flight, residual vibration, impact and collision, etc.

Two ways in which the dynamic performance of stylus instruments affects signal fidelity are tip flight and surface damage due to inertial and damping forces. Tip flight refers to cases when the stylus tip fails to follow the profile of the measured surface during scanning process: the measured signal does not represent the actual profile of the real surface. This problem is reduced by using a stylus assembly with high natural frequency to enhance the tracking capability of its tip. Generally, the square of angular natural frequency is proportional to the stiffness and to the reciprocal of mass. For stylus instruments, the reduction of mass is limited by the geometric and functional requirements. For instance, the length of a probe lever arm might relate to workspace requirements and it must remain adequately rigid. Thus, the common method to improve the natural frequency of stylus instruments is to increase support stiffness. On the other hand, increased stiffness will increase the contact force at the measured surface, which might possibly damage the surface, distort the measured signal and reduce measurement accuracy. Thus, these two aspects must be traded off for signal fidelity improvement.

A few studies, mainly simulation, have been directed towards the signal fidelity of stylus surface measurement in recent years. Damir [1] presented the effects of stylus kinematics for several deterministic surfaces without considering the stylus tip radius. The point of separation of the stylus tip from the measured surface, maximum lift and path of the stylus after separation were established. McCool [2] assessed the combined effects of stylus tip radius and flight on the surface topography measurement. Simulations showed the magnitude of the distortion and the effects of the sample length and frequency on traced profiles. Song and Vorburger [3] described theoretical and experimental work on stylus flight and introduced different models of stylus flight in the profiling of sinusoidal, rectangular, triangular, and random surfaces. Pawlus [4] developed a model for predicting the flight distortion on a measured surface topography. Whitehouse [5, 6] made a major theoretical contribution to the dynamic aspects of scanning surface instruments. In terms of best measurement fidelity, he proposed that the optimal damping ratio for the mechanical pick-up of a stylus instrument be $\xi$=0.59. This was confirmed experimentally by Liu [7] using a modified stylus instrument to show that the minimal distortion on typical surfaces occurred with a damping ratio between 0.5 and 0.7. Tian [8] established a dynamic model for random surface measurement using a stylus instrument. The effects of the design parameters of the instrument on the signal fidelity and critical scanning speed were systematically investigated. The finite tip size effects on dynamic performance were also investigated.

None of these previous dynamic models has considered the Hertzian contact stiffness and damping effects between the stylus tip and the measured surface. The interaction between the tip and surface is only modelled as kinematic pairs. Such models cannot provide full information for the actual reaction of the surface acting at the stylus tip and the collision and impact forces on the surface at the end of tip flight. A new dynamic model is needed to accurately predict the dynamic behavior of the stylus instrument.

The novel dynamic model of stylus contact and fidelity introduced here takes account of Hertzian contact stiffness and damping forces. It is used to investigate the effects of tip flight on signal fidelity for severe cases of random and periodic surfaces. The method and procedure for generating a random surface profile are briefly described, with cubic spline interpolation used to smooth the generated profiles. The influences of the inherent parameters of a stylus instrument and the parameters of the measurement process on the signal fidelity are explored. The slope and cycloidal signals are investigated in order to reduce residual vibration at the beginning of each scanning process.

## 2. Dynamic model

Fig. 1 illustrates the dynamic model of the stylus instrument and surface, and the coordinate system. The equivalent mass $m$ of moving parts is supported by a spring with stiffness $k$ and damper with damping coefficient $c$. The Hertzian contact between the tip and the measure surface is described by a Hertzian stiffness $k_c$ and a damping coefficient $c_c$. The measured surface moves from right to left with a velocity $v$. The effective surface roughness that is swept through the contact region during scanning forms the dynamic input to the stylus system. The dynamics of such a stylus instrument can be modelled as a second order damped mass-spring system. Thus, the displacement of the stylus tip during contact conditions is given by

$$m\ddot{y} + c\dot{y} + k(y + \delta) = k_c(y_i - y) + c_c(\dot{y}_i - \dot{y}) \quad (1)$$

where $y$ and $y_i$ are the vertical displacement of the stylus tip and random surface, respectively. $\delta$ is the initial static displacement. The adhesive forces (e.g., from adsorbed water films) at the contact between the tip and the surface are normally negligible for conventional stylus instruments. Thus, tip flight occurs during scanning when the Hertzian contact stiffness and damping forces reduce to zero. At loss of contact, the governing equation of the stylus system simplifies to

$$m\ddot{y} + c\dot{y} + k(y + \delta) = 0 \quad (2)$$

The simulation programme automatically switches to Eq. (2) whenever loss of contact is detected and switches back to Eq. (1) when contact resumes. The tip trajectory is calculated using the Runge-Kutta method. In order to analyse the stability of the stylus instrument tracking, the tip is initially located at the first point of a generated profile with zero velocity. Scans are then simulated for various surface parameters, damping ratio, and scanning speed.

## 3. Generation of random profile and stylus tip

Random surfaces can be obtained using both experimental and numerical methods. For consistency, numerical generation is the best choice to simulate natural and engineering surfaces with different bandwidths. There are a number of approaches for generating random surfaces [9, 10]. In this work, a digital filter and Johnson transfer system are used to generate random profiles with given skewness and kurtosis [11]. In order to implement the dynamic simulations, these profiles must be smooth enough to have finite second order derivatives. Thus, the generated discrete points must be interpolated. The vertical displacement and velocity during scanning serve as the inputs to the established system models. The displacement, velocity and acceleration time histories (i.e., the actual tip trajectories) of the stylus tip are found using a Runge-Kutta subroutine. Rather than use a piecewise finite difference approximation to the second derivative in evaluating the dynamic response of the stylus, it is preferable, for reasons of numerical stability, to use a smoothed analytical approximation to the discrete profile and then calculate the derivatives analytically. The cubic spline function approximation is chosen as it can go through the discrete points exactly without the ripple of a comparable polynomial approximation.

Especially with bridging effect, the tip size and shape affect not only the kinematics but also the dynamics of the stylus system. Thus, an ideal tip shape with a triangular section and vertex angle of 90° is generated and used in the dynamic performance investigation. It has a base of 20 μm and a height of 10 μm. To simulate a practical stylus (roughly 5 μm tip radius) and a worn version, the perfect tip is truncated with a rounded shape as shown in Fig. 2. The parameters used in this study are based on a practical stylus system with $m$=1.5 g, $k$=11 N/m, $c$=0.00516 N·s/m ($\xi$=0.02), $\omega_n$=86 rads$^{-1}$ [7]. Based on the Hertzian contact theory and the typical conditions for a Talysurf instrument, a rigid sphere of $r$=5 μm against a steel flat with a force of 1 mN is utilized to estimate the parameters of Hertzian contact. The stiffness and damping coefficient of Hertzian contact is set as: $k_c$=150 N/mm, and $c_c$=0.01 N·s/m.

## 4. Numerical Simulation

This study concentrates on profile fidelity under relatively severe cases where the instrument should not ideally be used but might well be in practice. Particularly, it considers the effect of finite size tip in short wavelength surface metrology, where the radius of the stylus tip can be of the order of the wavelength and the measured signal may be seriously distorted by the bridging effect of the stylus. Random test surface profiles for the computer simulations were therefore generated as discussed above with autocorrelation lengths chosen as 1 μm and 10 μm. The random profiles were generated from 140 data points with interval of 1 μm, and then interpolated with extra 20 points between the adjacent original points. The cubic spline interpolation method can guarantee the existence of the second derivative, and thus the achieved curves can be used to conduct dynamic simulation. The difference at the original points is zero and the maximum value is located at the midpoint between the adjacent original points. Fig. 3 shows the generated profiles before and after interpolation: clearly the interpolation preserves the statistics well.

Finite size tip will affect not only the kinematic analysis but also vibration characteristics of stylus tip. The tip profiles for different sizes and the subsequent loci are first generated by the established kinematic methodology. The number of contacts with the random profile at each point around the stylus tip is also provided for insight into the characteristics of the stylus measurement. Fig.4 shows example loci and contact distributions for different tip shapes. The tip loses contact with the profile where there are narrow, deep valleys. The distortion of the loci increases with increasing truncation length. The contact number is at a maximum at the centre of the tip and symmetrically reduces towards the edges of the tip. With the increase of truncation length, the distribution of contact number becomes wide and flat. This indicates the contact point may not be limited within the convex section of the tip. However, in all cases the measurement lateral uncertainty will be enlarged, adversely affecting signal fidelity: this kinematic error is often overlooked. The kinematic profiles provide the forcing input to the dynamic simulations, which modify them through contact phenomena and, most notably, stylus flight. In principle, these changes could feed back into the driving function.

Comparisons of kinematic and dynamic measurements on random profiles are shown in Fig.5. The dynamic loci separate from kinematic loci when scanning speed increases, as expected of acceleration-related behaviour. Similarly, tip flight occurs most at the steep peaks of the measured profiles, and the flight height becomes significantly large with increased scanning speed. With a damping ratio of 0.02, as is common in commercial stylus instruments, tip flight will significantly distort the signal even at low scan speeds, as shown in Fig.5 (a). If the damping ratio increases to 0.5, this tip flight can be suppressed and signal fidelity is improved. However, increased damping must not significantly affect the response speed of the system, i.e. the stylus must be able to follow sharp downwards profile features during the scanning process. The simulations tend to confirm previous work [8]: a damping ratio between 0.4-0.5 is suitable for high dynamic performance of stylus measurement. The new results emphasize that there is relatively large tip flight at the beginning of each scanning process even at very low scanning speed. It arises from the transient response and consequent residual vibration commonly occurring when the stylus starts to move suddenly from a static state to a motion state with constant scanning speed. This residual vibration also reduces the signal fidelity of stylus measurement and must be constrained within an acceptable level.

As shown in Fig.6, further confidence in the model arises when as it predicts the obvious condition that that even at very low scanning speed the stylus tip cannot follow the profile at a valley when the preload displacement is less than the maximum depth of that valley. Thus, preload displacement must be large enough to keep the stylus tip in contact with the surface throughout measurement. However, large preload displacement leads to large static forces being applied near the peaks of the profile, which might be sufficient to cause scratching and tip wear, especially when combined with dynamic forces.

To further investigate the effect of transient response on the signal fidelity, a sinusoidal profile of 1 μm amplitude and 5 μm wavelength was used as the input signal for the dynamic system. Signal distortions from an ideal tip under different scanning speeds are shown in Fig.7. At very low scanning speed (0.01 mm/s), the tip follows the profile and residual vibration at the beginning is negligible small. There are two peak values in the frequency response. The first peak (2 Hz) denotes the frequency of the sinusoidal profile and the second, much smaller, peak (159 Hz) shows the first natural frequency of the stylus system in Hertzian contact condition. When the scanning speed increases to 0.03 mm/s, the contact vibration energy causes these two frequencies to shift into the region of 100-150 Hz, indicating the onset of nonlinearity in the stylus scanning system. The time history plot shows slight tip flight at the beginning of the scan. As the scanning speed is further increased, the measured signals are significantly distorted by the residual vibration at the beginning of each scan. A distinct peak at just below 100 Hz appears in the system frequency response. This frequency relates to the natural frequency of the stylus system, but varies somewhat with scan speed. It indicates that tip has separated from the profile during scanning. Thus, examining the frequency spectrums of the profiles for the appearance of natural frequency of the stylus instrument is an effective method for tip flight detection.

Transient-induced residual vibration of the stylus will have a major effect on the signal fidelity in high scanning speed conditions. Thus, this phenomenon should be suppressed during surface metrology operations. Trajectory planning is very widely used in robotics and machine tools and offers one of the best choices for improving the performance of stylus measurement. The residual vibration is mainly due to the sudden velocity change at the beginning of the scan. Thus, a jerk-free velocity input command for the scan should be introduced to improve signal fidelity. Slope and cycloidal command signals are commonly used in trajectory planning. Their effectiveness for improving profile signal fidelity is shown in Fig. 8. In all cases, the steady state scanning speed is 0.07 mm/s. The height of tip flight is significantly reduced by using a slope velocity command signal with rising time 0.06 s (Fig. 8a). Thus, signal fidelity is greatly improved. Using a cycloidal velocity signal (Fig. 8b) gives slightly better signal fidelity than the slope velocity input. This is because that cycloidal signal is a jerk free signal and cannot excite high frequency vibration of the stylus system. The results show that velocity control strategy should be considered in closed-loop controlled stylus instruments and other contact scanning probe microscopes.



## Conclusions

A novel vibration model for stylus measurement method, which accounts for Hertzian contact stiffness and damping effects, has been proposed. For brevity and without loss generality, the bilinear lumped mass model was used to investigate the vibration characteristics of the stylus instrument during the scanning process. The resulting non-linear equations were solved by the Runge-Kutta method to predict the tip traverse trajectory under some metrologically severe conditions.

Models of random surfaces and stylus tips were generated using previously published approaches. The tip size effect was examined using a kinematic model. With the increase of tip radius (or tip wear), the contact point distribution will extend more to the edges of the tip profile, and the uncertainty of the measured signal will increase. However, the smoothing function of the tip as a mechanical filter will tend to reduce tip flight during stylus measurement.

The new dynamic model demonstrated effects of damping ratio and preload displacement on the signal fidelity similar to those in other recent studies. The damping ratio is best chosen as a higher value than on most commercial stylus instruments.

Residual vibration of stylus system arising from transients at the start of each scan can have a serious impact on signal fidelity. Using a sinusoidal profile, the effects of residual vibration on the signal fidelity were explored in both time and frequency domains. The frequency spectrum of the profile changed in distinctive ways, including the appearance of the natural frequency of the stylus system, when tip flight occurred. Spectral analysis can be used to detect the presence of tip flight from the measured profile data. In order to eliminate and/or avoid the effects of residual vibration, the use of slope and cycloidal velocity command signals for the stylus scan were shown to reduce residual vibration compared to the usual step control, even at quite low steady-state scan speeds. Rising times that made little difference to overall measurement time could significantly reduce the transient effects and improve signal fidelity for surface metrology.

**Acknowledgements**

This research is supported by National Natural Science Foundation of China (grant no. 50705064), Natural Science Foundation of Tianjin (grant no. 08JCYBJC01400), Australian Research Council (ARC) Discovery (grant nos. DP0450944, DP0666366), and ARC Linkage Infrastructure, Equipment and Facilities (grant nos. LE0347024, LE 0775692).

**References**

[1] M.N.H. Damir, Error in measurement due to stylus kinematics, Wear, 26 (1973) 219–227.

[2] J.I. McCool, Assessing the effect of stylus tip radius and flight on surface topography measurements, Trans. ASME, J. Tribol., 106(1984) 202–210.

[3] J.F. Song, T.V. Vorburger, Stylus flight in surface profiling, Trans. ASME, J. Manuf. Sci. Eng., 118(1996) 188–198.

[4] P. Pawlus, M. Smieszek, The influence of stylus flight on change of surface topography parameters, Precis. Eng., 29(2005) 272–280.

[5] D.J. Whitehouse, Dynamic aspects of scanning surface instruments and microscopes, Nanotechnology, 1(1990) 93–102.

[6] D.J. Whitehouse, Enhancement of instrument and machine capabilities, Nanotechnology, 7(1996) 47–51.

[7] X. Liu, D.G. Chetwynd and S.T. Smith et al., Improvement of the fidelity of surface measurement by active damping control, Meas. Sci. Tech., 4(1993) 1330–1340.

[8] Y. Tian, X. Liu, D. Zhang, D. Chetwynd, Dynamic modelling of the fidelity of random surface measurement by the stylus method, Wear, 266(2009) 555–559.

[9] J.J. Wu, Simulation of non-Gaussian surfaces with FFT, Tribol. Int., 37(2004) 339–346.

[10] V. Bakolas, Numerical generation of arbitrarily oriented non-Gaussian three-dimensional rough surface, Wear, 254(2003) 546–554.

[11] S. Chilamakuri, B. Bhushan, Contact analysis of non-Gaussian random surfaces. Proc. Inst. Mech. Eng. Part J.-J. Eng. Tribol., 212(1998) 19-32.

## List of figure captions

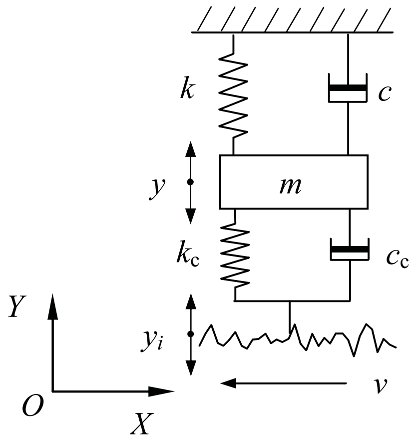



Fig.1. Dynamic model of the stylus pick-up system

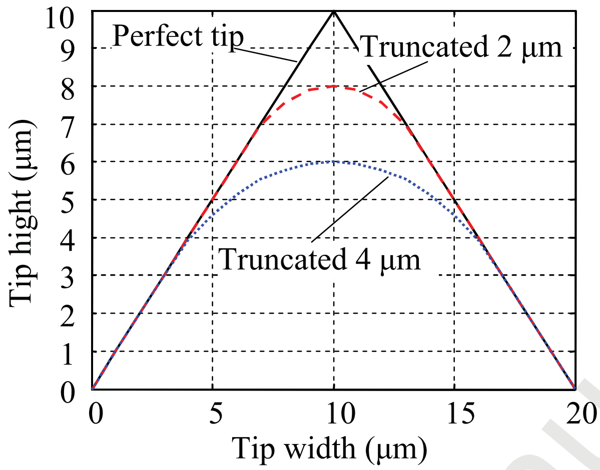
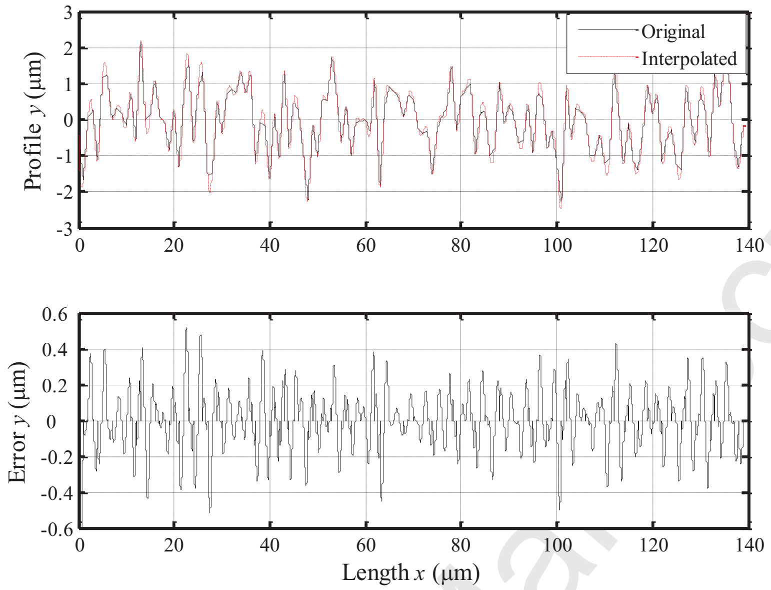
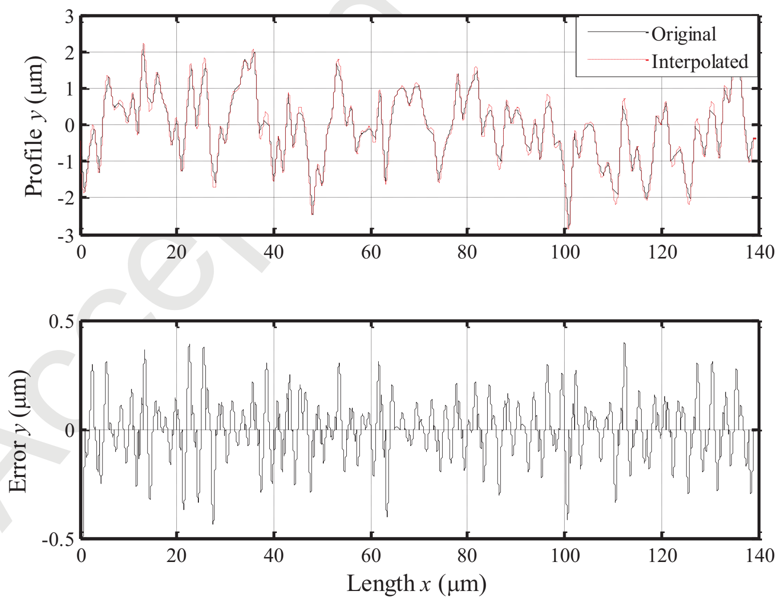



Fig.2. Tip shapes

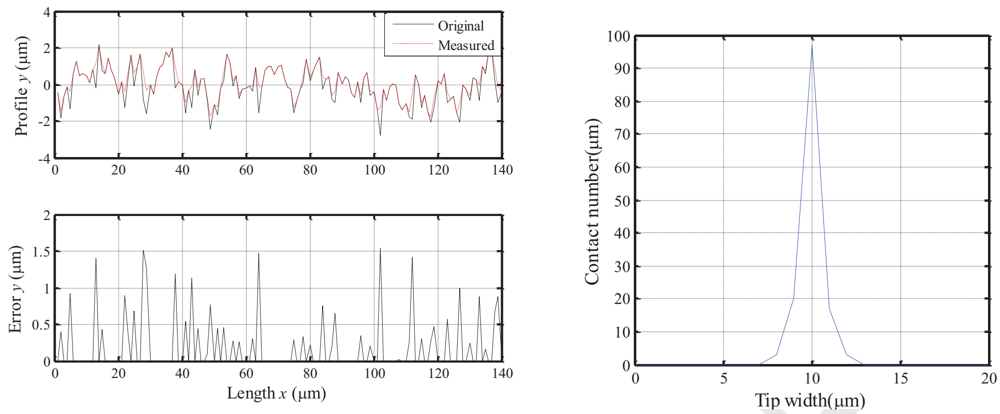



(a) Autocorrelation length 1 μm

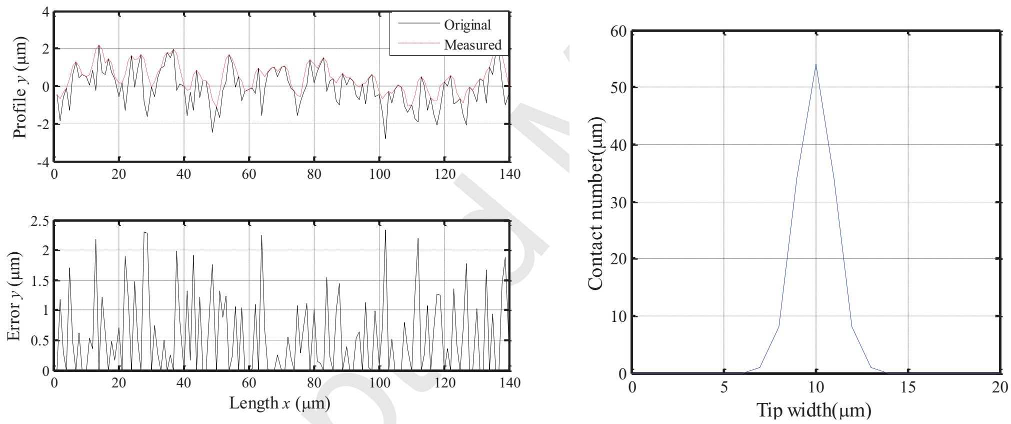



(b) Autocorrelation length 10 μm

Fig. 3. Generated profiles with different autocorrelation

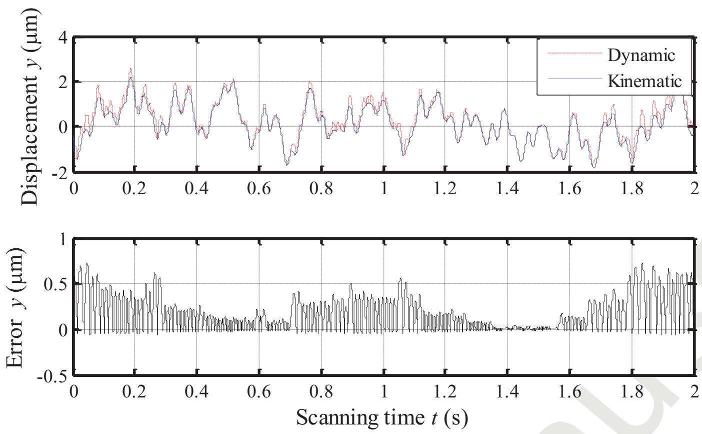



(a) Measured profile (perfect tip) (b) Contact distribution (perfect tip)

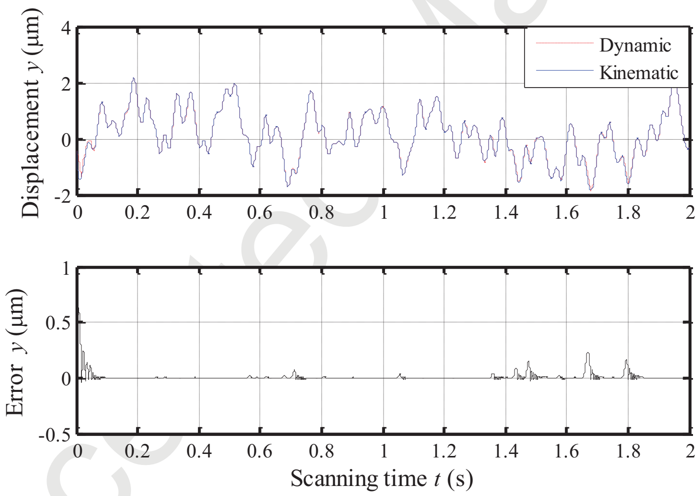



(c) Measured profile (truncated 1μm) (d) Contact distribution (truncated 1μm)

Fig. 4 Finite size tip effect of the stylus instrument

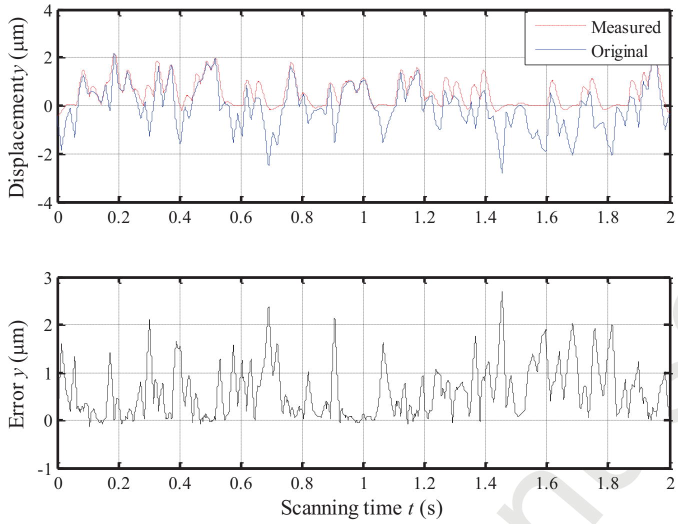



(a) $\xi$=0.02

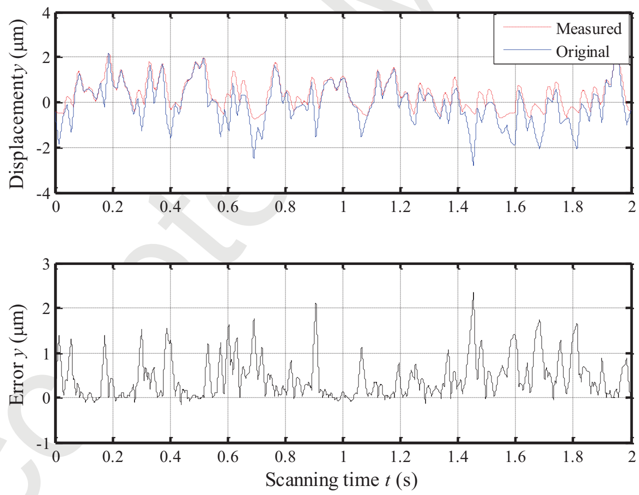



(b) $\xi$=0.5

Fig.5. Comparisons of kinematic and dynamic measurements ($v$=0.07 mm/s)

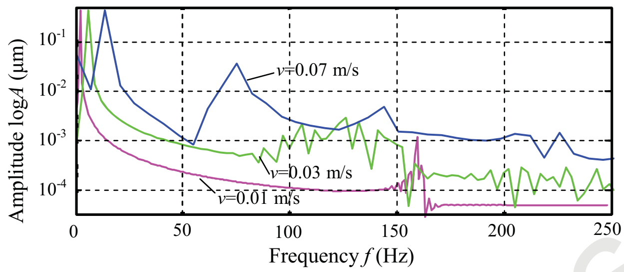



(a) Preload displacement 0 μm

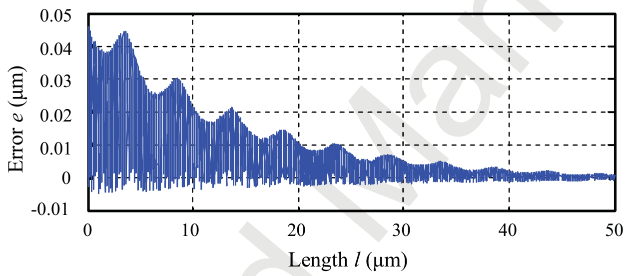



(b) Preload displacement 0.5 μm

Fig.6. Effect of preload displacement on signal fidelity




(a) Frequency response of stylus instrument




(b) Measurement error ($v$=0.03 mm/s)

Fig. 7. Signal fidelity for sinusoidal profiles

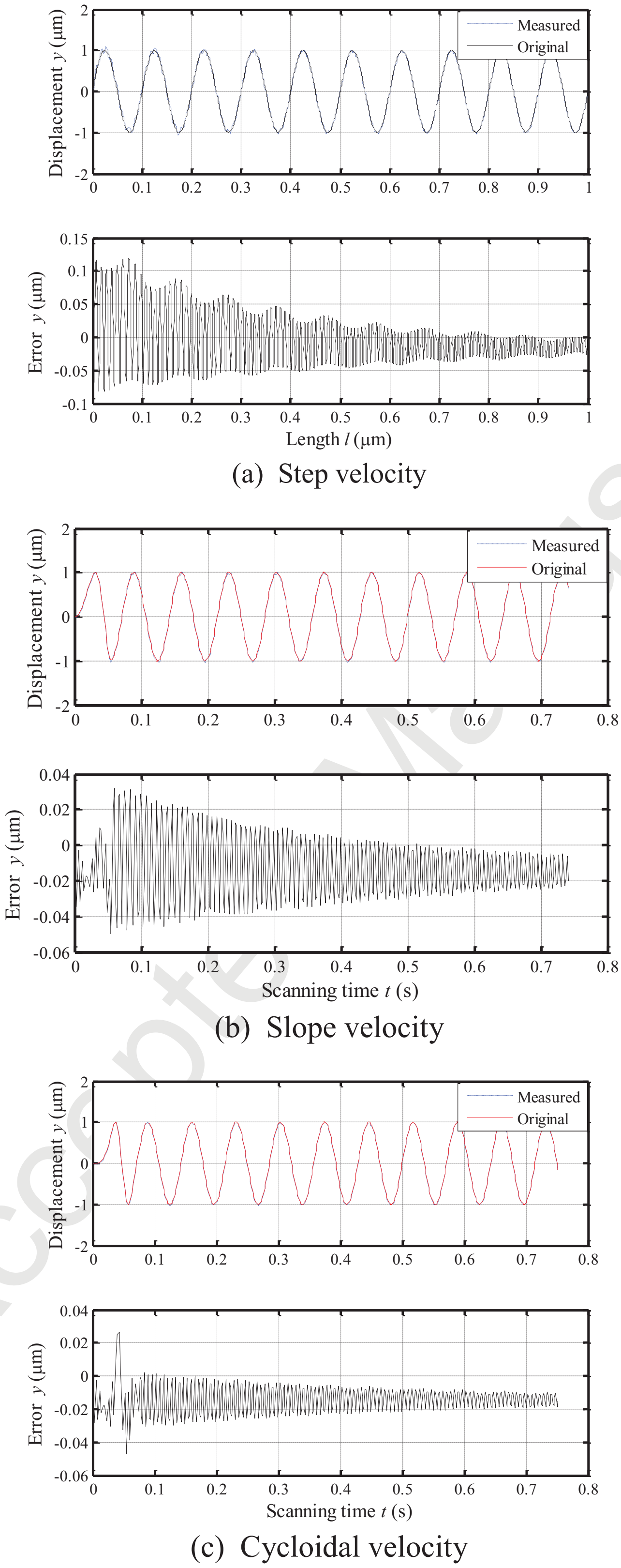



Fig. 8. Comparison signal fidelity of different velocity command signals