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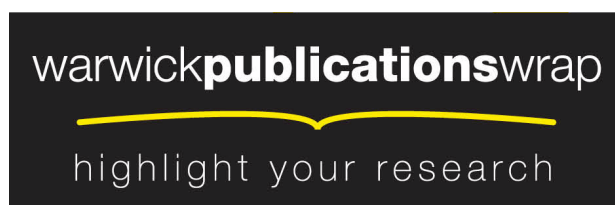
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# The Optimal Use of Return Predictability: An Empirical Study

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## Abstract

In this paper we study the economic value and statistical significance of asset return predictability, based on a wide range of commonly used predictive variables. We assess the performance of dynamic, unconditionally efficient strategies, first studied by Hansen and Richard (1987) and Ferson and Siegel (2001), using a test that has both an intuitive economic interpretation and known statistical properties. We find that using the lagged term spread, credit spread, and inflation significantly improves the risk-return trade-off. Our strategies consistently outperform efficient buy-and-hold strategies, both in and out of sample, and they also incur lower transactions costs than traditional conditionally efficient strategies.

## I. Introduction

Asset return predictability has profound implications for asset pricing and asset allocation. An important issue in this regard is how to use predictability *optimally* in forming actively managed portfolios that outperform uninformed buy-and-hold strategies, when evaluated by an investor without access to the predictive information. From a theoretical viewpoint, this issue was first addressed by Hansen and Richard (1987) and later by Ferson and Siegel (2001). Both studies show how to optimally utilize conditioning information to construct dynamically managed, unconditionally efficient portfolio strategies.

In this paper we study the empirical performance of unconditionally mean-variance efficient strategies for a large number of predictive instruments, both in as well as out of sample. We use a variety of ex post performance measures, the first being the difference in maximum achievable Sharpe ratios, with and without the optimal use of predictability. We show that this difference in maximum

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Sharpe ratio can be expressed in terms of the coefficient of determination ( $R^2$ ) in a predictive regression. Under the null hypothesis, this difference is equal to the Wald test statistic for the slope coefficient in the regression. Our test thus measures the economic gains from predictability and at the same time has a known distribution. This allows us to assess the statistical significance of the extent to which the optimal use of predictive information expands the efficient frontier and thus improves the risk-return trade-off for the investor.

We assess the economic gains from return predictability both *ex ante* (implied by the estimated moments of asset returns) and *ex post* (by assessing the step-ahead performance of the portfolio). The portfolio-based approach facilitates an out-of-sample evaluation of the model predictions. Our study focuses on the optimal use of conditioning information on portfolio choice using common predictive variables. Bansal, Harvey, and Dahlquist (2005) study the performance of mean-variance trading strategies with conditioning information, while Ferson, Siegel, and Xu (2006) develop unconditional multifactor minimum variance strategies, and Chiang (2005) develops unconditionally efficient strategies relative to a benchmark.

We compare the performance of unconditionally efficient strategies with more traditional conditionally efficient ones from an investment-based perspective. These two types of strategies reflect different ways of exploiting predictive information. While unconditionally efficient strategies are necessarily also conditionally efficient, the converse is not normally true (Hansen and Richard (1987)). As Dybvig and Ross (1985) point out, a conditionally optimal strategy can appear inefficient when evaluated *ex post* without knowledge of the *ex ante* conditional moments. Using portfolios based on industry, size, and book-to-market (BM) ratio, Ferson and Siegel (2009) show that the use of unconditionally efficient strategies can significantly increase unconditional Sharpe ratios.

In our empirical analysis, we use 3 sets of base assets: a single-market index, 5-industry portfolios, and 6 portfolios sorted on size and BM ratios. Using monthly return data from 1960 to 2004, we estimate a predictive model using a variety of predictive instruments. Based on the estimated conditional moments of the base asset returns, we then construct several dynamically efficient active portfolio strategies: strategies that minimize volatility for given expected return, maximize expected return for given variance, or maximize an investor's expected utility for given risk aversion. We assess the performance of these strategies, both in and out of sample. As criteria, we use a variety of industry-standard performance measures, including Sharpe ratios, alphas, and utility premia (Fleming, Kirby, and Ostdiek (2001)). We also compare the transaction costs incurred by the various types of strategies.

We first study the relative performance of the different predictive variables, capturing information about the state of the economy, the financial markets, and the term structure of interest rates.<sup>1</sup> Similar variables have been the focus of

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<sup>1</sup>Specifically, the predictive variables we use include the lagged return on the market index, dividend yield (Campbell (1987)), the short rate of interest (Fama and Schwert (1977)), the term spread (Fama and French (1988)), the convexity of the yield curve, the credit yield spread, and changes in inflation.

much recent research. For example, Fama and Schwert (1977) and Avramov and Chordia (2006a) investigate the economic value of lagged business cycle variables in trading portfolios sorted on size and BM ratios, while Ang and Bekaert (2007) find that the T-bill rate is a prime instrument for predicting returns.<sup>2</sup>

Our in-sample results, using data from 1960 through 2004 (we use data up to 2009 for our out-of-sample analysis), show that (lagged) term spread and convexity of the Treasury yield curve, the credit yield spread, and inflation consistently perform well as predictors for all sets of assets. The optimal use of (any one of) these variables significantly expands the unconditionally efficient frontier. However, the dividend yield, which has been studied extensively in the past (Campbell (1987)), does not produce any significant results in any of our samples, confirming the findings of Goyal and Welch (2003). Interestingly, while lagged market returns do not seem to help in forming successful market-timing strategies, they produce significant gains when used in an asset allocation context. Although still significant, the *relative* gains from predictability are less pronounced in the case of the size and BM portfolios because the well-documented size and value premia (Fama and French (1992)) allow even a static portfolio to achieve Sharpe ratios in excess of 1.0.

To investigate whether the potential gains from predictability can be realized in practice, we construct several dynamically efficient portfolio strategies and assess their ex post performance, both in and out of sample. While the in-sample performance closely matches that predicted by the model, out of sample the difference between “good” and “bad” predictors widens. Our results indicate that successful strategies use the predictive information to “decouple” themselves from the market index (with betas of 0.5 or less, allowing them to achieve annual alphas between 5% and 10%), while less powerful predictors effectively yield “index tracking” strategies (with betas closer to 1.0 and low alphas). We also evaluate the performance of these portfolios by estimating the “management fee” that a risk-averse investor would be willing to pay for the superior strategy (Fleming et al. (2001)). The results are consistent with our earlier findings, with “good” predictors supporting management premia of from 2.50% to more than 6% per annum. These figures far exceed those found by Fleming et al., who, in contrast to our study, use predictive information to model the time variation of return volatility. To facilitate a direct comparison, we also estimate a model in which conditional volatility is a time-varying function of the conditioning instruments, following the approach of Ferson and Siegel (2003). We find that the performance of this model does not even come close to that of a simple linear predictive model. More specifically, our results indicate that time-varying volatilities are indeed important in the construction of conditionally efficient portfolios, while adding little to the performance of the unconditionally efficient strategies.

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<sup>2</sup>Although (lagged) consumption-wealth ratio (CAY) has been shown (Lettau and Ludvigson (2001)) to possess considerable predictive power, we chose not to include CAY in this study, mainly because of the look-ahead bias that is inherent in its construction (Brennan and Xia (2005)). Constructing our own bias-free version of CAY was unfortunately not an option, as the required data are only available at quarterly frequency.

To assess the out-of-sample performance of our strategies, we conduct two types of experiments: First, we estimate the predictive model using a subsample of data, and then we evaluate the performance of the resulting strategies over the entire remaining sample period. As cutoff points for the out-of-sample periods we chose Jan. 1995 (the beginning of the “dot.com” boom) and 2000 (the beginning of the collapse of the bubble). Of course, the out-of-sample performance, in particular in the latter case, does not match that predicted by the estimation (because the model was estimated over a bull run, but the strategy was run through a bear market). However, the performance *relative* to the market index (or the efficient fixed-weight strategy) is largely consistent with our in-sample results. Specifically, strategies using term and credit spread continue to show strong performance, outperforming the benchmark by as much as 6%–12% per annum, while lagged market returns and inflation produce mixed results.

Second, we conduct a “true” out-of-sample analysis by assessing the performance of our strategies using data from 2005 to 2009 (which includes the “credit crunch”), which was not used in any of our in-sample tests. We find that up to the end of 2007, our strategies continue to perform well in accordance with our in-sample results. However, with the advent of the “credit crunch” and the ensuing global economic crisis in 2008/2009, it appears that even predictability cannot avoid losses. However, our strategies still fare better than the market index or a mean-variance optimal buy-and-hold strategy, incurring significantly lower losses during this period.

To assess the practicability of our strategies, we estimate the total transaction costs incurred each period as a fraction of portfolio value, for both the in- and out-of-sample applications. Overall, transaction costs destroy only a very small fraction of the portfolios’ performance, ranging from 0.10% to 0.30% per annum for market-timing strategies, and 0.50% to 1.00% per annum in the multiasset cases.

The remainder of this paper is organized as follows: In Section II, we briefly review the theoretical background, establish our notation, develop our measures of the economic value of predictability, and construct our statistical test. Our empirical results, for different choices of base assets and predictor variables, are reported in Section III. Section IV concludes.

## II. Measuring the Gains from Predictability

There is a wide range of methods to test for statistical evidence of predictability, the most basic being derived from a simple linear regression of asset returns on lagged predictive instruments. It is not a priori obvious how these statistical measures translate into real economic gains that could be realized by an investor. Our analysis addresses several main questions:

- i) Is there statistical evidence for return predictability?
- ii) Does statistical predictability (if any) translate into economic gains?

While i) can be addressed by standard statistical techniques, we answer ii) by measuring the extent to which the optimal use of predictability can expand the

mean-variance efficient frontier. We propose a simple test to assess whether the improvement in risk-return trade-off is statistically significant. This is important, as even a weak statistical relationship between predictive instruments and asset returns can generate large and significant economic gains.<sup>3</sup> While questions i) and ii) concern the *potential* gains that are theoretically possible, we must also ask,

iii) Can the potential gains (if any) be realized in practice?

We construct dynamically managed trading strategies that use predictive information optimally, and assess their performance both in and out of sample, using a variety of industry-standard performance measures. Finally, we study how the performance of our strategies is affected by market frictions such as transaction costs and short-sale constraints.

## A. Dynamically Managed Portfolio Strategies

There are  $N$  risky assets, indexed  $k = 1, \dots, N$ . Denote by  $r_t^k$  the gross return in period  $t$  on asset  $k$  (i.e., the future value at time  $t$  of \$1 invested in asset  $k$  at time  $t - 1$ ) and by  $R_t = (r_t^1 \dots r_t^N)'$  the  $N$ -vector of returns. In addition to the risky assets, a risk-free asset is traded whose gross return we denote  $r_{t-1}^f$ . The difference in time indexing indicates that, while the return  $r_{t-1}^f$  on the risk-free asset is known at the beginning of the period (time  $t - 1$ ), the returns  $r_t^k$  on the risky assets are uncertain ex ante and only realized at the end of the period (time  $t$ ). Note, however, that we do *not* assume  $r_{t-1}^f$  to be constant through time. In other words,  $r_{t-1}^f$  is conditionally but not unconditionally risk free.

To define a portfolio strategy, denote by  $\theta_{t-1}^k$  the fraction of portfolio value invested in asset  $k$  at time  $t - 1$ . The return at time  $t$  on such a strategy is hence

$$(1) \quad r_t(\theta) = r_{t-1}^f + (R_t - r_{t-1}^f \mathbf{1})' \theta_{t-1},$$

where  $\theta_{t-1} = (\theta_{t-1}^1 \dots \theta_{t-1}^N)'$  is the  $N$ -vector of portfolio weights. We wish to allow for *dynamically managed* strategies (i.e., those for which the portfolio weights are time varying). To this end, we assume that the  $\theta_{t-1}^k$  are stochastic processes, measurable with respect to the information set  $\mathcal{F}_{t-1}$  available to the investor at the beginning of the period.<sup>4</sup> For notational convenience, we write  $E_{t-1}(\cdot)$  for the conditional expectation with respect to  $\mathcal{F}_{t-1}$ .

## B. Measuring the Gains from Predictability

We wish to assess the economic gains that an investor can derive from return predictability. One such possible measure is the extent to which the optimal use of

<sup>3</sup>As shown, for example, in Kandel and Stambaugh (1996), Campbell and Vicera (2001), and Avramov and Chordia (2006b).

<sup>4</sup>In our empirical applications, we assume that  $\mathcal{F}_{t-1}$  is generated by a vector  $Z_{t-1}$  of (lagged) instruments, variables observable at time  $t - 1$  that contain information about the distribution of asset returns at time  $t$ .

the predictive information contained in  $\mathcal{F}_{t-1}$  expands the mean-variance efficient frontier. In contrast to most of the existing literature that has focused on conditionally efficient strategies (i.e., strategies that maximize conditional return given conditional variance), we consider instead strategies that are unconditionally efficient given the conditioning information. As these two types of strategies display considerable differences in both behavior and performance, we provide a more detailed discussion in Sections II.E and II.F.

Because there is a risk-free asset, the conditionally efficient frontier has the familiar “wedge” shape, touching the zero-risk axis at  $r_{t-1}^f$ . However, as  $r_{t-1}^f$  is only conditionally risk free, the same is not true for the unconditionally efficient frontier (see also Figure 1). The shape of the latter is hence described by 3 parameters: the *location* (mean and volatility) of the global minimum-variance (GMV) return (achieved using the conditioning information), and the *asymptotic slope* of the frontier. The latter is captured by the maximum “hypothetical” Sharpe ratio relative to the zero-beta rate that corresponds to the mean of the GMV. We thus define

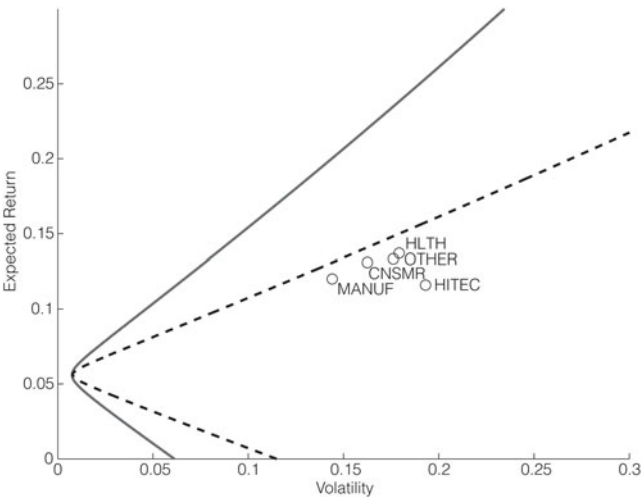
*Definition 1.* The maximum hypothetical Sharpe ratio is defined as

(2) 
$$\lambda_* := \sup_{\theta} \frac{E(r_t(\theta)) - \nu}{\sigma(r_t(\theta))},$$

where  $\nu$  is the expected return of the GMV, and the supremum is taken over all returns of the form in equation (1) that are attainable by dynamically managed strategies  $\theta$ .

FIGURE 1  
Efficient Frontiers

Figure 1 shows the fixed-weight (dashed line) and dynamically optimal (solid line) efficient frontiers. The base assets are the 5-industry portfolios (shown as circles), and the instruments are TSPR, CSPR, and INFL.



Because the risk-free asset has very low volatility, the GMV is very close to  $(0, E(r_{t-1}^f))$  in mean-standard deviation space (see also Figure 1). Hence, the asymptotic slope  $\lambda_*$  is very close to the traditional Sharpe ratio relative to  $E(r_{t-1}^f)$ . In a slight abuse of notation, we will therefore often refer to  $\lambda_*$  simply as the Sharpe ratio. Moreover, because we find that predictability does not significantly alter the location of the GMV, we focus on  $\lambda_*$  as our main measure of interest. To make it usable in empirical applications, we need to derive an explicit expression for  $\lambda_*$ :

*Proposition 2.* Up to a 1st-order approximation,  $\lambda_*$  can be written as

$$(3) \quad \lambda_*^2 \approx E(H_{t-1}^2), \quad \text{where} \quad H_{t-1}^2 = (\mu_{t-1} - r_{t-1}^f)' \Sigma_{t-1}^{-1} (\mu_{t-1} - r_{t-1}^f),$$

where  $\mu_{t-1} = E_{t-1}(R_t)$  and  $\Sigma_{t-1} = E_{t-1}(R_t - \mu_{t-1})(R_t - \mu_{t-1})'$  denote the conditional mean vector and variance-covariance (VCV) matrix, respectively, of the base asset returns.

*Remark.* We use the Taylor approximation in Proposition 2 in order to obtain an expression that has a known statistical distribution (see below). The exact expression is  $\lambda_* = E(H_{t-1}^2 / (1 + H_{t-1}^2)) / E(1 / (1 + H_{t-1}^2))$ , and the error term is proportional to  $\text{cov}(H_{t-1}^2, 1 / (1 + H_{t-1}^2))$ . As this term is negative, the approximate value  $E(H_{t-1}^2)$  tends to overstate the maximum Sharpe ratio attainable by the optimal strategy. In our empirical applications, we found the error term to be on average (across all predictive instruments) about 0.004 (or 0.8%) in the case of a single risky asset, and about 0.019 (or 2.5%) in the case of multiple risky assets. This is less than 2.5% on average (and in no case more than 5%) of increase in the Sharpe ratio due to predictability. For example, in the case of a single risky asset, using the term spread (TSPR) as a predictive instrument increases the Sharpe ratio by more than 66%, from 0.328 to 0.544, if the approximation is used. With the correct value, the Sharpe ratio is 0.539 (an increase of 64% relative to the fixed-weight strategy).

*Proof of Proposition 2.* This is an extension of Theorem 3 in Ferson and Siegel (2001): Equations (20) and (21) in their paper imply  $\lambda_*^2 = \alpha_3 / (1 - \alpha_3)$ . Reformulating  $\alpha_3$  to include the conditionally risk-free asset (extending  $A_{t-1}$  accordingly and using the Sherman-Morrison formula) and then applying a Taylor expansion yields the desired result.  $\square$

From traditional mean-variance theory, we know that  $H_{t-1}$  is the maximum conditional Sharpe ratio, given the conditional moments of returns  $\mu_{t-1}$  and  $\Sigma_{t-1}$ . Hence, the above result states that the maximum gain achievable by optimally exploiting the predictive information contained in  $\mathcal{F}_{t-1}$  is given by the 2nd moment of the conditional Sharpe ratio.<sup>5</sup> As a consequence, time variation in the conditional Sharpe ratio  $H_{t-1}$  improves the ex post risk-return trade-off for the mean-variance investor who has access to predictive information.

<sup>5</sup>For the case of only one risky asset, this result is also shown in Cochrane (1999).



### C. Statistical Significance

To measure the *incremental* effect of predictability, we denote by  $\lambda_0$  the quantity corresponding to expression (3) with the conditional moments  $\mu_{t-1}$  and  $\Sigma_{t-1}$  replaced by their unconditional counterparts. This corresponds to the maximum Sharpe ratio that is attainable by *static* portfolio strategies (i.e., those whose weights do not depend on the predictive information  $\mathcal{F}_{t-1}$ ). The test statistic  $\Omega := \lambda_*^2 - \lambda_0^2$  thus measures the value added by the optimal use of predictability. Of course, as static portfolios are a subset of managed strategies, we always have  $\Omega \geq 0$ .

Although  $\Omega$  is well defined for any (parametric or nonparametric) specification of the conditional return moments, in the special case of a linear predictive model,  $\Omega$  has a known distribution that allows us to assess its statistical significance. Therefore, we assume that the risky asset returns  $R_t$  follow a linear predictive model of the form

$$(4) \quad R_t = \mu_0 + B \cdot Z_{t-1} + \varepsilon_t,$$

where  $Z_{t-1}$  is an  $M$ -vector of lagged predictive variables. The vector of conditional expected returns in this case becomes,  $\mu_{t-1} = \mu_0 + B \cdot Z_{t-1}$ . We assume that the residuals  $\varepsilon_t$  are serially independent and independent of  $Z_{t-1}$ . Hence, the conditional VCV matrix does not depend on  $Z_{t-1}$ , and we write  $\Sigma$  instead of  $\Sigma_{t-1}$ . However, because we estimate equation (4) jointly across all assets, we do *not* assume the  $\varepsilon_t$  to be cross-sectionally uncorrelated, that is, we do not assume  $\Sigma$  to be diagonal. In the context of equation (4), our null hypothesis ( $H_0$ ) is that  $B \equiv 0$ , that is, that the predictive instruments do not affect the distribution of asset returns. Obviously, under the null we have  $\Omega = 0$ .

*Proposition 3.* In the case of a single predictive instrument ( $M = 1$ ), under the null hypothesis ( $H_0$ ) in the linear predictive model (4), we have

$$(5) \quad \frac{T - N - 1}{N} \cdot \Omega \sim F_{N, T-N-1}$$

in finite samples, and  $T \cdot \Omega \sim \chi_N^2$  asymptotically. Here,  $N$  is the number of base assets, and  $T$  is the number of time-series observations.

*Proof.* This result is standard; see Jobson and Korkie (1982), Shanken (1987). A formal proof is given in the Appendix.  $\square$

This result allows us to assess the statistical significance of the economic gains generated by the optimal use of the predictive information contained in the information set  $\mathcal{F}_{t-1}$ . Moreover, Lo and MacKinlay (1997) show that  $\lambda_*$  is directly related to the maximum<sup>6</sup>  $R^2$  of the predictive regression (4). As a consequence, even a (statistically) small amount of predictability can lead to substantial economic gains. For example, even a moderate 2% monthly  $R^2$  would increase an annualized Sharpe ratio of 0.3 to almost 0.6.

<sup>6</sup>To compute the maximum  $R^2$  in a multivariate regression, one needs to find the linear combination of the right-hand side variables for which the corresponding univariate  $R^2$  is maximized.

## D. Exploiting Return Predictability

So far, we have measured the ex ante potential gains from predictability. In this section, we give an explicit characterization of the dynamically managed strategies that attain these maximal gains ex post.

*Proposition 4.* The weights in equation (1) of any dynamically efficient strategy are of the form

$$(6) \quad \theta_{t-1} = \frac{w - r_{t-1}^f}{1 + H_{t-1}^2} \cdot \Sigma_{t-1}^{-1} \left( \mu_{t-1} - r_{t-1}^f \mathbf{1} \right),$$

where  $w \in \mathbb{R}$  is a constant, related to the unconditional mean of the strategy.

*Proof.* This follows from Theorem 3 in Ferson and Siegel (2001), after extending  $A_{t-1}$  in their equation (19) to account for the conditionally risk-free asset, and applying the Sherman-Morrison formula.  $\square$

In the case of an unconditionally risk-free asset,<sup>7</sup> this result reduces to Theorem 2 in Ferson and Siegel (2001). By choosing  $w$  in equation (6) appropriately,<sup>8</sup> we can construct efficient strategies that track a given target expected return or target volatility, or strategies that maximize a quadratic utility function (see also Section II.F). In particular, the Sharpe ratio (relative to the zero-beta rate corresponding to the mean of the GMV) of any such strategy will converge to  $\lambda_*$  as  $w$  becomes sufficiently large. In our empirical analysis, we compare the ex ante efficiency gains as measured by  $\lambda_*$  with the ex post performance of efficiently managed strategies. Note also that Proposition 4 holds for any specification of the conditional return moments  $\mu_{t-1}$  and  $\Sigma_{t-1}$ , not only for the linear specification considered in Section II.C.

## E. Comparison with Conditionally Efficient Strategies

In contrast to most of the existing literature that focuses on *conditionally* efficient portfolios, the strategies defined in equation (6) are designed to be *dynamically optimal* (unconditionally efficient). It can be shown that any unconditionally efficient strategy is necessarily also conditionally efficient, while the converse is generally not true.<sup>9</sup> As Dybvig and Ross (1985) show, when portfolio managers possess information not known to outside investors, their conditionally efficient strategies may appear inefficient to outside observers. Moreover, conditional efficiency is difficult to verify empirically, as conditional moments are not observed ex post. In fact, almost all commonly used measures of portfolio performance are based on unconditional estimates of the portfolio's ex post risk and return characteristics.

<sup>7</sup>See also Abhyankar, Basu, and Stremme (2007) for a discussion of the case with a conditionally risk-free asset.

<sup>8</sup>Note that  $w$  is in fact the coefficient on  $z^*$  in the Hansen and Richard (1987) decomposition  $r^* + wz^*$  of the unconditionally efficient frontier.

<sup>9</sup>In fact, conditionally efficient portfolios can be obtained from equation (6) by replacing the constant  $w$  by an  $\mathcal{F}_{t-1}$ -measurable, time-varying coefficient (Hansen and Richard (1987)).

Note first that for small values of  $\mu_{t-1} - r_{t-1}^f$ , the efficient weights in equation (6) respond almost linearly to changes in  $\mu_{t-1}$ , shifting more money into the risky assets the higher their expected returns are relative to the risk-free asset. However, for extreme values of  $\mu_{t-1}$ , the behavior of the weights is dominated by the denominator  $1 + H_{t-1}^2$ , creating the “conservative response” to extreme signals first observed by Ferson and Siegel (2001).

To shed additional light on the difference in behavior between the two types of strategies, consider for a moment an investor who chooses an optimal asset allocation, such as to maximize conditional quadratic utility with conditional risk-aversion coefficient  $\Gamma_{t-1}$ . The weights of the resulting strategy will be of the form

$$(7) \quad \theta_{t-1} = \frac{\text{const}}{\Gamma_{t-1}} \cdot \Sigma_{t-1}^{-1} \left( \mu_{t-1} - r_{t-1}^f \mathbf{1} \right).$$

In other words, the dynamically efficient weights in equation (6) correspond to a conditionally optimal strategy with time-varying risk aversion proportional to  $1 + H_{t-1}^2$ . In particular, the implied conditional risk-aversion coefficient  $\Gamma_{t-1}$  increases when the conditional expected return  $\mu_{t-1}$  takes on extreme values, thus causing the strategy to respond more conservatively to extreme information. In contrast, a conditionally optimal strategy for constant risk aversion tends to “overreact” to extreme signals. In other words, the portfolio weights of a conditionally efficient strategy tend to be more volatile than those of the corresponding dynamically efficient strategy, an important consideration, in particular in view of transaction costs. We study the difference in behavior, performance, and cost between conditional and unconditional strategies in our empirical analysis (see Section III.F).

## F. Economic Value of Predictability

In addition to the difference in Sharpe ratios, we also employ a utility-based framework to assess the economic value of return predictability. Ferson and Siegel (2001) show that dynamically efficient portfolios maximize a quadratic utility function. Following Fleming et al. (2001), we consider a risk-averse investor whose preferences over future wealth are given by a quadratic von Neumann-Morgenstern utility function. They show that, if relative risk aversion  $\gamma$  is assumed to remain constant, the investor’s expected utility can be written as

$$\bar{U} = W_0 \left( E(r_t) - \frac{\gamma}{2(1+\gamma)} E(r_t^2) \right),$$

where  $W_0$  is the investor’s initial wealth, and  $r_t$  is the (gross) return on the portfolio they hold. Consider now an investor who faces the decision whether or not to acquire the skill and/or information necessary to implement the active portfolio strategy that optimally exploits predictability. The question is, how much of their expected return would the investor be willing to give up (e.g., pay as a management fee) in return for having access to the superior strategy? To solve this problem, we need to find the solution  $\delta$  to the equation

$$(8) \quad E(r_t^* - \delta) - \frac{\gamma}{2(1+\gamma)} E((r_t^* - \delta)^2) = E(r_t) - \frac{\gamma}{2(1+\gamma)} E(r_t^2),$$

where  $r_t^*$  is the optimal strategy, and  $r_t$  is a fixed-weight strategy that does not take predictability into account. The solution  $\delta$  is the management fee that the investor would be willing to pay in order to gain access to the superior strategy. Graphically, the premium can be found in the mean-standard deviation diagram by plotting a vertical line downward, starting from the point that represents the optimal strategy  $r_t^*$ , and locating the point where this line intersects the indifference curve through the point that represents the inferior strategy  $r_t$ .

### III. Empirical Analysis

There is considerable evidence that stock returns are predictable using, for example, dividend yield (Fama and French (1988)), the nominal short interest rate (Fama and Schwert (1977)), earnings-price ratio (Lamont (1998)), or consumption-wealth ratio (Lettau and Ludvigson (2001)). In this paper, we employ a set of 7 variables as predictive instruments, capturing different types information available to the investor. We study both simple market-timing strategies (allowing the investor only to allocate funds between a single risky asset and the risk-free asset) as well as dynamically managed asset allocation strategies (allowing the investor to allocate funds across several risky assets).

#### A. Data and Methodology

Using monthly data covering the period from Jan. 1960 to Dec. 2004, we estimate a linear predictive model of the form

$$(9) \quad R_t = \mu_0 + B \cdot Z_{t-1} + \varepsilon_t,$$

where  $R_t$  is the vector of risky asset returns, and  $Z_{t-1}$  is the vector of (lagged) predictive instruments. The vector of conditional expected returns is, hence,  $\mu_{t-1} = \mu_0 + B \cdot Z_{t-1}$ , and the conditional VCV matrix is given by  $\Sigma_{t-1} = E_{t-1}(\varepsilon_t \varepsilon_t')$ . For most of our analysis, we assume the residuals  $\varepsilon_t$  to be independent and identically distributed (i.i.d.), so that  $\Sigma_{t-1} \equiv \Sigma$  is constant over time. However, we do *not* assume the  $\varepsilon_t$  to be cross-sectionally independent (i.e., we do not assume  $\Sigma$  to be diagonal).

##### 1. Predictive Instruments

Campbell (1987) finds that dividend yields predict stock returns. Fama and Schwert (1977) show that the 1-month U.S. T-bill rate is a proxy for future economic activities, a fact confirmed by Ang and Bekaert (2007), who find that it outperforms the dividend yield in terms of predicting asset returns. Fama and French (1988) find that the term spread is closely related to the short-term business cycle. Fama and French (1989) also show that the credit spread tracks long-term business cycle conditions, documenting that this variable is higher during recessions and lower during expansions, and that the credit spread predicts the long-term business cycle.

Motivated by these results, we use as predictive instruments the lagged return on the market index (MKT), the dividend yield (DY) on the index, the

1-month T-bill rate (TB1M), the term spread (TSPR, defined as the difference in yield between the 30-year and the 1-year T-bond), the convexity of the term structure (CONV, defined as a weighted average, 20/29 and 9/29, respectively, of the 1-year and 30-year yields, minus the 10-year yield), the credit yield spread (CSPR, defined as the difference in yield between 10-year BAA-rated corporate bonds and the corresponding T-bond), and inflation (INFL, derived from changes in Consumer Price Index (CPI)). While MKT and DY were obtained from the Center for Research in Security Prices (CRSP), all other variables were constructed using data from the economic database at the Federal Reserve Bank of St. Louis (FRED). To reduce the problem of spurious regressions, we follow Ferson, Sarkissian, and Simin (2003) and detrend our predictive variables.<sup>10</sup> Also, because many of these regressors are highly persistent (although we reject the unit-root hypothesis for all of them), we work with 1st differences in most cases (except for lagged MKT).

## 2. Base Assets

We use 3 sets of base assets. For the case of a single risky asset, we use the total return (including reinvested distributions) on the CRSP value-weighted market index. For multiple risky assets, we use the 5-industry portfolios of Fama and French (available from Kenneth French's Web site at [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)), as well as their  $2 \times 3$  portfolios sorted on size and BM ratio. Using inflation data, we also convert the returns of each of the base asset sets into real returns, but as the results for real returns are qualitatively very similar to those for nominal returns, we choose to report only the latter.

Our empirical analysis is organized as follows: First, we conduct an in-sample analysis of the model, using each of the 3 base asset sets and each of the predictor variables individually (Section III.B). Using Proposition 2, we estimate the increase in the ex ante Sharpe ratio due to predictability, and we assess its statistical significance using Proposition 3. Using Proposition 4, we construct several efficient strategies, based on the in-sample model estimates, and we assess their ex post performance, both in and out of sample (Sections III.C and III.G). We analyze portfolio performance on the basis of the Sharpe ratio, capital asset pricing model (CAPM) alpha (relative to the market index portfolio), management premium (relative to an uninformed buy-and-hold strategy), transaction costs, and sensitivity to short-sale constraints. We compare both performance and cost of our dynamically optimal strategies to their corresponding conditionally efficient counterparts (Section III.F). Finally, we investigate the robustness of our estimates using bootstrap simulation (Section III.H).

## B. In-Sample Estimation Results

Table 1 presents the in-sample (using data from 1960 to 2004) summary statistics for the single risky asset case. We report the correlation coefficients

<sup>10</sup>This is done by simply fitting a linear regression of the variable in question on a calendar-time index and deducting the resulting trend. We did not consider nonlinear trends. We do, however, find that detrending does not significantly change our results.

between the market index and the lagged (demeaned and detrended) predictive instruments. Clearly, candidates for “good” predictors are those for which the correlation with the index is high in absolute value (e.g., DY, TSPR, CSPR, and possibly INFL). There is relatively little collinearity between the equity market instruments (MKT and DY) and the term structure indicators (TB1M, TSPR, and CONV), with most coefficients falling in the range of about  $\pm 0.10$ . However, the term structure variables themselves have higher correlations with one another, probably because most of the variability in these variables is due to changes in the short rate, while the long end of the term structure is less volatile.

TABLE 1  
Summary Statistics

Table 1 presents the summary statistics (averages, standard deviations, and correlations) for the market index and the 7 predictive instruments (lagged MKT, DY, TB1M, TSPR, CONV, CSPR, and INFL).

|                         | Market Index | Predictive Instrument |         |         |         |         |         |        |
|-------------------------|--------------|-----------------------|---------|---------|---------|---------|---------|--------|
|                         |              | MKT                   | DY      | TB1M    | TSPR    | CONV    | CSPR    | INFL   |
| Average                 | 12.83%       |                       |         |         |         |         |         |        |
| Standard deviation      | 15.13%       | 0.1516                | 0.0348  | 0.0076  | 0.0347  | 1.9574  | 0.0403  | 0.1353 |
| Correlation Coefficient |              |                       |         |         |         |         |         |        |
| MKT                     | 0.0454       | 1.0000                |         |         |         |         |         |        |
| DY                      | 0.1022       | 0.1009                | 1.0000  |         |         |         |         |        |
| TB1M                    | -0.0063      | -0.0622               | 0.6995  | 1.0000  |         |         |         |        |
| TSPR                    | 0.0711       | 0.1142                | -0.0324 | -0.5500 | 1.0000  |         |         |        |
| CONV                    | 0.0366       | 0.0632                | -0.0522 | -0.4255 | 0.8917  | 1.0000  |         |        |
| CSPR                    | 0.0703       | 0.1268                | -0.1130 | -0.6007 | 0.9657  | 0.8546  | 1.0000  |        |
| INFL                    | -0.0579      | -0.1501               | 0.4750  | 0.5656  | -0.4363 | -0.4529 | -0.4437 | 1.0000 |

There is relatively little persistence in market returns (with a 1st-order autocorrelation of only 0.05), which could be interpreted as evidence in support of weak-form efficiency. Consistent with standard business cycle theory, market returns are negatively correlated with interest rates and inflation. However, lagged CSPR is positively correlated with MKT, which seems counterintuitive at first but is consistent with the intuition developed in Fama and French (1989).

To begin with, we estimate the predictive regression (9) for each of the 3 sets of base assets, using each predictive instrument on its own in turn. We use monthly data from Jan. 1960 through 2004. The in-sample results are summarized in Table 2.

### 1. Single-Index Case

Panel A of Table 2 reports the in-sample estimation results in the single risky asset (using only the market index portfolio) case. Unsurprisingly, the fixed-weight Sharpe ratio (0.38) in this case only marginally exceeds that of the market index itself (0.375). The regression coefficients (not reported in Table 2 to save space) are consistent in size and sign with the correlations reported in Table 1. Although all  $R^2$ s are less than 1%, the efficient use of TSPR, CSPR, and INFL leads to significant increases in the Sharpe ratio (with  $p$ -values of 3.7%, 4.2%, and 4.8%, respectively). These results indicate that the optimal use of these variables in dynamic asset allocation can potentially produce significant economic gains

TABLE 2  
In-Sample Results

Table 2 reports the in-sample estimation results of the predictive model, using different sets of base assets and each of the instruments (lagged MKT, DY, TB1M, TSPR, CONV, CSPR, and INFL) as predictive variables at a time. Panel A reports the results for a single-market index, while Panels B and C report the results using Fama and French's 5-industry portfolios (available from Kenneth French's Web site at <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data.library.html>), and the 2 × 3 portfolios sorted by size and BM ratio, respectively, as base assets. The (annualized) Sharpe ratios are computed as in Proposition 2, and the *p*-values are obtained from the standard Wald test for the slope coefficient in the predictive regression (4).

|  |              | Optimally Managed |        |        |         |        |         |        |
|--|--------------|-------------------|--------|--------|---------|--------|---------|--------|
|  | Fixed Weight | MKT               | DY     | TB1M   | TSPR    | CONV   | CSPR    | INFL   |
| <i>Panel A. Single-Market Index</i>  |              |                   |        |        |         |        |         |        |
| Maximal $R^2$  |              | 0.38%             | 0.46%  | 0.21%  | 0.80%   | 0.17%  | 0.75%   | 0.71%  |
| Sharpe ratio   | 0.379        | 0.437             | 0.447  | 0.413  | 0.492*  | 0.406  | 0.486*  | 0.481* |
| $p$ -value   |              | 14.74%            | 12.98% | 27.93% | 3.66%   | 33.59% | 4.24%   | 4.79%  |
| <i>Panel B. Fama-French 5-Industry Portfolios</i>  |              |                   |        |        |         |        |         |        |
| Maximal $R^2$  |              | 3.11%             | 1.61%  | 1.18%  | 2.93%   | 1.42%  | 3.15%   | 1.70%  |
| Sharpe ratio   | 0.479        | 0.788**           | 0.656  | 0.612  | 0.773** | 0.636  | 0.792** | 0.663  |
| $p$ -value   |              | 0.35%             | 14.50% | 25.68% | 0.55%   | 16.27% | 0.32%   | 9.21%  |
| <i>Panel C. Fama-French 2 <math>\times</math> 3 Portfolios Sorted by Size and BM Ratio</i> |              |                   |        |        |         |        |         |        |
| Maximal $R^2$  |              | 6.97%             | 1.50%  | 0.91%  | 1.88%   | 2.67%  | 1.81%   | 1.87%  |
| Sharpe ratio   | 1.198        | 1.563**           | 1.284  | 1.248  | 1.301   | 1.343* | 1.297   | 1.300  |
| $p$ -value   |              | 0.01%             | 21.73% | 47.46% | 7.29%   | 1.12%  | 8.57%   | 7.52%  |

for the mean-variance investor. Interestingly, although lagged DY has the highest correlation with market returns, it does not produce significant gains when used in a market-timing strategy (with a Sharpe ratio of 0.44, an insignificant increase with a *p*-value of 14.7%). This is consistent with the findings of, among others, Goyal and Welch (2003), who document that the predictive power of DY has diminished since the 1990s.

## 2. Multiple Risky Assets

Panel B of Table 2 reports the analogous results using the 5-industry portfolios as base assets. The fixed-weight Sharpe ratio (0.48) is only marginally higher than in the single-index case, while the optimally managed Sharpe ratios now reach levels of almost 0.8. The regression coefficients (not reported in Table 2 to save space) are broadly consistent with those in the single-index case (with all instruments except TB1M and INFL having positive coefficients for all assets). The single exception is the health sector (HLTH), for which the sign of most coefficients is reversed, indicating the countercyclical nature of this sector. Again, even though the maximal  $R^2$  range from only 1% to just over 3%, the efficient use of predictive information (in particular, lagged MKT, TSPR, and CSPR) considerably improves the investor's risk-return trade-off (with *p*-values for the Wald test for predictability of 0.4%, 0.6% and 0.3%, respectively). The ranking of predictive instruments is broadly consistent with the single-index case, the only exceptions being lagged MKT (which is now highly significant) and INFL (which has lost some of its predictive power).

Figure 1 shows the efficient frontiers, with (solid line) and without (dashed line) the optimal use of predictive information (here, TSPR, CSPR, and INFL are used as instruments). While the 5-industry portfolios are quite close to being

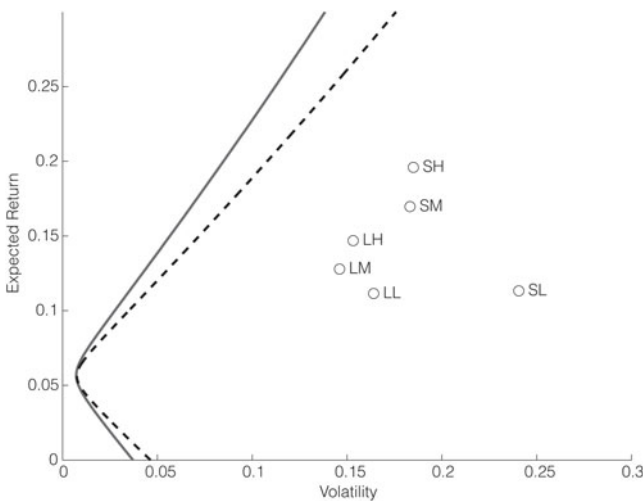
efficient in the fixed-weight sense, their average returns are about 10% lower than the return of a dynamically managed portfolio with similar risk!

Panel C of Table 2 finally reports the analogous results using the  $2 \times 3$  size and BM portfolios as base assets. Again, the coefficients of the predictive regression (not reported in Table 2 to save space) are broadly consistent with the single-index case (with all instruments except TB1M and INFL having positive coefficients for all assets). However, unlike in the case of the industry portfolios, there is a clear pattern in the magnitude of the coefficients: For almost all predictors, the coefficients on the small-cap portfolios are considerably larger in absolute value than those for the large-cap portfolios. This indicates that returns on small stocks are more predictable than those of large stocks. For the size and BM portfolios, even the fixed-weight Sharpe ratio now more than triples (1.20). This is because when the investor is allowed to select stocks on the basis of size and BM ratio, even a static portfolio can take advantage of the well-documented size and value premia (Fama and French (1992)). However, even though the absolute levels are very different for the 2 asset sets, the *relative* increase in Sharpe ratio due to the optimal use of predictive information is remarkably similar. The best-performing predictors are still lagged MKT, TSPR, CONV, CSPR, and INFL, although the ranking has changed slightly (with CONV now playing a significant role, while TSPR and CSPR have lost some of their power).

Figure 2 shows the efficient frontiers, with (solid line) and without (dashed line) the optimal use of predictive information (here, TSPR, CONV, and INFL are used as instruments). In contrast to the industry portfolios, the size and BM portfolios themselves are far from being even fixed-weight efficient. This is because even a static portfolio can exploit the size and value premium by trading the

FIGURE 2  
Efficient Frontiers

Figure 2 shows the fixed-weight (dashed line) and dynamically optimal (solid line) efficient frontiers. The base assets are the  $2 \times 3$  size and BM portfolios (shown as circles), and the instruments are TSPR, CONV, and INFL.





spread between small and large stocks, and value and growth stocks. However, the optimal use of predictability still yields substantial economic gains, achieving almost 5% more return at 10% volatility than the efficient fixed-weight portfolio.

C. Portfolio Performance

While the preceding sections documented the *potential* gains from return predictability, in this section we focus on analyzing the ex post performance of the dynamically managed strategies that exploit this predictability. To do this, we estimate the predictive model (9) and use the estimated coefficients to construct dynamically efficient portfolio strategies as in Proposition 4. We then compute the returns on these strategies and evaluate their performance using a variety of industry-standard performance measures. The results are reported in Table 3 (although we tested 3 types of strategies—maximum-return, minimum-variance, and maximum-utility—we only report the performance figures for the minimum-variance portfolios to save space, but the results for the other strategies are very similar).

TABLE 3  
Portfolio Performance

Table 3 reports the ex post performance of fixed-weight and dynamically managed portfolios. In Panel A the single-market index is used as base asset, while Panel B reports the results for the 5-industry portfolios. The predictive model is estimated for each set of assets and each of the predictive instruments. Based on the parameter estimates, efficient minimum-variance portfolios (with a target mean of 15%) are constructed as in Proposition 4, and their performance is evaluated over the entire sample period. Return, volatility, Sharpe ratio, and alpha are annualized.

|   |              | Optimally Managed |        |        |         |        |         |        |
|---|--------------|-------------------|--------|--------|---------|--------|---------|--------|
|   | Fixed Weight | MKT               | DY     | TB1M   | TSPR    | CONV   | CSPR    | INFL   |
| <i>Panel A. Single Index</i>                      |              |                   |        |        |         |        |         |        |
| Model estimates (see Table 2)                     |              |                   |        |        |         |        |         |        |
| Sharpe ratio                                      | 0.379        | 0.437             | 0.447  | 0.413  | 0.492   | 0.406  | 0.486*  | 0.481* |
| Average return                                    | 14.93%       | 14.92%            | 15.05% | 14.87% | 14.99%  | 14.95% | 14.95%  | 14.90% |
| Volatility  | 22.53%       | 15.28%            | 18.51% | 19.15% | 15.81%  | 19.80% | 16.49%  | 17.17% |
| Sharpe ratio                                      | 0.378        | 0.497             | 0.447  | 0.442  | 0.542   | 0.431  | 0.517   | 0.494  |
| CAPM beta   | 1.473        | 0.887             | 1.063  | 1.133  | 0.708   | 1.195  | 0.785   | 0.732  |
| CAPM alpha  | 0.03%        | 3.44%             | 1.61%  | 1.96%  | 4.59%   | 1.67%  | 4.08%   | 4.36%  |
| Management fee                                    |              | 5.76%             | 1.32%  | 3.72%  | 7.00%   | 3.10%  | 6.36%   | 5.68%  |
| Transaction costs                                 | 0.06%        | 1.56%             | 0.14%  | 0.28%  | 0.32%   | 0.30%  | 0.32%   | 0.98%  |
| <i>Panel B. Fama-French 5-Industry Portfolios</i> |              |                   |        |        |         |        |         |        |
| Model estimates (see Table 2)                     |              |                   |        |        |         |        |         |        |
| Sharpe ratio                                      | 0.479        | 0.788**           | 0.656  | 0.612  | 0.773** | 0.636  | 0.792** | 0.663  |
| Average return                                    | 14.95%       | 14.99%            | 15.10% | 15.07% | 15.15%  | 15.05% | 15.16%  | 14.82% |
| Volatility  | 17.88%       | 10.90%            | 13.29% | 14.04% | 11.38%  | 13.58% | 10.87%  | 12.61% |
| Sharpe ratio                                      | 0.477        | 0.786             | 0.626  | 0.614  | 0.765   | 0.635  | 0.802   | 0.667  |
| CAPM beta   | 1.018        | 0.065             | 0.581  | 0.560  | 0.331   | 0.528  | 0.352   | 0.536  |
| CAPM alpha  | 2.70%        | 6.54%             | 4.77%  | 5.54%  | 7.02%   | 5.72%  | 6.90%   | 5.45%  |
| Management fee                                    |              | 5.53%             | 2.49%  | 3.38%  | 5.29%   | 3.71%  | 5.60%   | 4.18%  |
| Transaction costs                                 | 0.10%        | 6.00%             | 0.44%  | 0.92%  | 0.66%   | 1.00%  | 0.73%   | 2.52%  |

1. Single Risky Asset Case

We begin with an in-sample exercise, using data from 1960 to 2004 to estimate the model and then evaluate the strategies over the same period. Panel A of Table 3 reports the ex post performance of the dynamically efficient minimum-variance strategies in the single-index case. The portfolio Sharpe ratios closely

match the theoretically predicted ones and exceed the latter for “good” predictors (in the single-index case, TSPR, CSPR, and INFL). The minimum-variance strategies match their target mean (15%) almost perfectly, while the volatility of the maximum-return strategies (not reported in Table 3) often falls below the target (15%) (in particular for “good” predictors). The dynamically managed strategies clearly outperform their fixed-weight efficient counterparts (e.g., the maximum-return strategy using TSPR achieves almost 2% more return at almost 2.5% less volatility, while the corresponding minimum-variance strategy reduces volatility from 23% to 16% without sacrificing return).

The quality of the predictive instrument is reflected in the strategy’s beta relative to the market index: While strategies based on “bad” predictors effectively track the index (with betas close to 1.0), “good” predictors allow the strategy to successfully “decouple” itself from the index (with betas of 0.60, 0.66, and 0.61 for TSPR, CSPR, and INFL, respectively). In other words, the predictive information allows the strategy to “time the market,” dynamically shifting funds between the index portfolio and the risk-free asset.

While the efficient fixed-weight strategies fail to “beat the market” (with alphas of about 0.03%), the dynamically managed portfolios outperform the market benchmark by a wide margin (with alphas of up to 4.5% per annum). Interestingly, the minimum-variance strategies achieve higher alphas than their maximum-return counterparts. This is probably due to the fact that the former have lower tracking errors.

## 2. Multiple Risky Assets

Panel B of Table 3 reports the analogous results using the 5-industry portfolio as base assets. The results are qualitatively very similar to the single-index case: The ex post performance matches the theoretically predicted one closely. Being able to dynamically allocate funds across different assets allows portfolio managers to “decouple” their strategies even more from the index (with betas as low as 0.3). Although the fixed-weight efficient portfolios now outperform the benchmark (with alphas of 2.3% and 2.7%), the dynamically managed strategies achieve in-sample alphas in excess of 9% per annum! In contrast to the market-timing strategies considered in the preceding section, the alphas of maximum-return strategies are now consistently higher than those of the corresponding minimum-variance strategies. One possible explanation is that in the market-timing case, maximum-return strategies are constrained to track a given target volatility. Overall, the increase in performance compared to Panel A of Table 3 indicates that return predictability is not just about “timing the business cycle,” but that dynamic asset allocation plays just as important a role.

Figure 3 shows the ex post performance (average return and volatility) of the fixed-weight and dynamically optimal minimum-variance and maximum-return strategies, respectively, relative to the ex ante efficient frontiers. For this figure, we used TSPR, CSPR, and INFL as predictive instruments. Figure 4 shows the performance of the corresponding utility-maximizing portfolios for different levels of risk aversion. Obviously, the lower the risk aversion, the “further up” the frontier the investor will locate the portfolio. As the spread between the fixed-weight and dynamically optimal frontier widens with increasing volatility,

FIGURE 3  
Minimum-Variance and Maximum-Return Portfolios

Figure 3 shows the ex post performance (average return and volatility) of the fixed-weight and optimally managed minimum-variance and maximum-return portfolios, respectively, relative to the fixed-weight (dashed line) and dynamically optimal (solid line) ex ante efficient frontiers. The base assets are the 5-industry portfolios (shown as circles), and the instruments are TSPR, CSPR, and INFL.

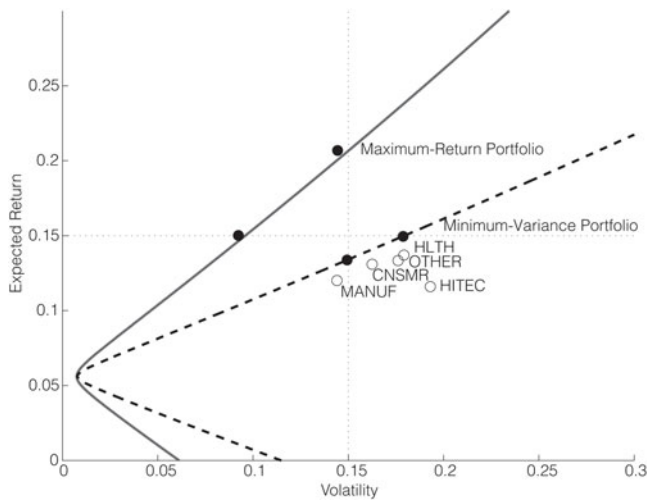
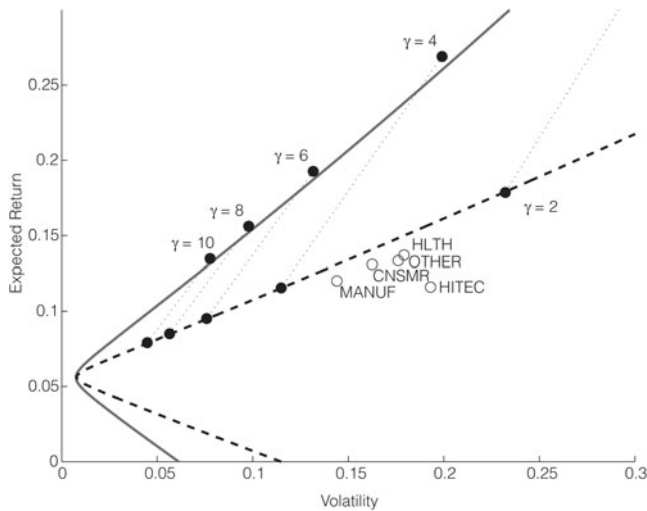


FIGURE 4  
Utility-Maximizing Portfolios

Figure 4 shows the ex post performance (average return and volatility) of the fixed-weight and optimally managed utility-maximizing portfolios for different levels of risk aversion, relative to the fixed-weight (dashed line) and dynamically optimal (solid line) ex ante efficient frontiers. The base assets are the 5-industry portfolios (shown as circles), and the instruments are TSPR, CSPR, and INFL.



less risk-averse investors stand to gain more from predictability (see also next section).

## D. Economic Value

To assess the economic value of predictability, we estimate the “management fee” that a risk-averse investor would be willing to pay for the superior strategy (Fleming et al. (2001)). The results (selected figures are reported in Tables 3 and 4) are consistent with our earlier findings, with “good” predictors supporting management premia from 2.50% to more than 6% per annum (for these figures, we used a moderate risk-aversion coefficient of 5). While for maximum-return strategies, the management premium is only very slightly decreasing in risk aversion (ranging from 4.5% to 4.2%, using TSPR as predictor), the premium for minimum-variance strategies strongly increases with the investor’s risk aversion (ranging from 1.2%, for a very low risk-aversion coefficient of 1, to 10.9%, for a coefficient of 10). This makes sense, as the maximum-return strategy is constrained to track a fixed volatility target, so that its utility is largely unaffected by the risk-aversion coefficient. For the utility-maximizing strategies (see Table 4), the management premium strongly decreases with increasing risk aversion. This is due to the fact that risk-tolerant investors will choose a portfolio “further up” the efficient frontier, where the “spread” between fixed-weight and optimally managed portfolios is wider (see also Figure 4).

TABLE 4  
Comparison of Unconditionally and Conditionally Efficient Strategies

Table 4 compares the performance of fixed-weight, unconditionally and conditionally efficient maximum-return, minimum-variance, and maximum-utility portfolios, respectively. The rows labeled “(net)” refer to the management premium that an investor would pay after transaction costs have been deducted. The base assets are the 5-industry portfolios, and the instruments are TSPR, CSPR, and INFL. All figures are annualized.

|  | Fixed Weight               | Unconditionally Efficient | Conditionally Efficient | Fixed Weight                | Unconditionally Efficient | Conditionally Efficient |
|--|----------------------------|---------------------------|-------------------------|-----------------------------|---------------------------|-------------------------|
| <i>Panel A. Maximum-Return Portfolio</i>     |                            |                           |                         |                             |                           |                         |
| Sharpe ratio                                 | 0.477                      | 0.923                     | 0.917                   |                             |                           |                         |
| Transaction cost                             | 0.08%                      | 2.60%                     | 2.72%                   |                             |                           |                         |
| Management premium (net)                     |                            | 6.84%<br>(4.17%)          | 6.71%<br>(3.88%)        |                             |                           |                         |
| <i>Panel B. Minimum-Variance Portfolio</i>   |                            |                           |                         |                             |                           |                         |
| Sharpe ratio                                 | 0.477                      | 0.922                     | 0.814                   |                             |                           |                         |
| Transaction cost                             | 0.10%                      | 1.64%                     | 2.40%                   |                             |                           |                         |
| Management premium (net)                     |                            | 6.32%<br>(4.69%)          | 5.82%<br>(3.39%)        |                             |                           |                         |
| <i>Panel C. Utility-Maximizing Portfolio</i> |                            |                           |                         |                             |                           |                         |
|  | Risk Aversion $\gamma = 5$ |                           |                         | Risk Aversion $\gamma = 10$ |                           |                         |
| Sharpe ratio                                 | 0.478                      | 0.923                     | 0.923                   | 0.479                       | 0.921                     | 0.921                   |
| Transaction cost                             | 0.05%                      | 2.88%                     | 2.88%                   | 0.03%                       | 1.41%                     | 1.41%                   |
| Management premium (net)                     |                            | 6.05%<br>(3.08%)          | 6.05%<br>(3.08%)        |                             | 2.90%<br>(1.49%)          | 2.90%<br>(1.49%)        |

The figures we obtain far exceed those found by Fleming et al. (2001), who, in contrast to our study, use predictive information to model the time variation of return volatility. To facilitate a direct comparison, we also estimate a model in which conditional volatility is a time-varying function of the conditioning

instruments. However, we find that the performance of this model does not even come close to that of a simple linear predictive model.

E. Transaction Costs

To assess the practicability of our strategies, we estimate the total transaction costs<sup>11</sup> incurred each period as a fraction of portfolio value, for both the in- and out-of-sample (see later) applications. Overall, transaction costs destroy only a very small fraction of the portfolios’ performance (ranging from 0.10% to 0.30% per annum for market-timing strategies, and 0.50% to 1.00% per annum in the multiasset cases; see Table 5). This still leaves net management premia of up to 6%. Interestingly, the relation between the predictive ability of an instrument and the cost of the corresponding strategy is not always monotonic: While strategies based on “bad” predictors (e.g., DY or TB1M) are generally cheaper (because the strategy has no meaningful signal to respond to), the cost of “good” strategies varies substantially across instruments. Strategies based on INFL or lagged MKT are 3–5 times more expensive than those based on term structure instruments (i.e., TSPR, CONV, and CSPR). In fact, in the multiasset cases, the transaction costs of strategies based on lagged MKT and INFL wipe out the entire management fee, making the strategy unattractive to a moderately risk-averse investor. The transaction cost effects of the different instruments likely reflect their relative

TABLE 5  
Out-of-Sample Portfolio Performance (5-industry portfolios)

Table 5 reports the *out-of-sample* performance of fixed-weight and dynamically managed maximum-return and minimum-variance portfolios, respectively. The model is estimated for the in-sample period, using the 5-industry portfolios as base asset and each of the instruments as predictor variables. Based on the parameter estimates, efficient portfolios are constructed as in Proposition 4, and their performance evaluated over the out-of-sample period. Return, volatility, Sharpe ratio, and alpha are annualized.

|  | Out-of-Sample Period 2000:01–2004:12 |                   |        |        |        | Out-of-Sample Period 1995:01–2004:12 |                   |        |        |        |
|--|--------------------------------------|-------------------|--------|--------|--------|--------------------------------------|-------------------|--------|--------|--------|
|  |                                      | Optimally Managed |        |        |        |                                      | Optimally Managed |        |        |        |
|  | Fixed Weight                         | MKT               | TSPR   | CSPR   | INFL   | Fixed Weight                         | MKT               | TSPR   | CSPR   | INFL   |
| <i>In-Sample Estimates</i>                 |                                      |                   |        |        |        |                                      |                   |        |        |        |
| Sharpe ratio                               | 0.511                                | 0.853             | 0.805  | 0.834  | 0.771  | 0.429                                | 0.776             | 0.866  | 0.882  | 0.726  |
| (p-value)                                  |                                      | 0.23%             | 0.89%  | 0.39%  | 2.10%  |                                      | 1.26%             | 0.14%  | 0.09%  | 3.57%  |
| <i>Panel A. Maximum-Return Portfolio</i>   |                                      |                   |        |        |        |                                      |                   |        |        |        |
| Average return                             | −3.52%                               | 4.28%             | 8.74%  | 9.62%  | −1.69% | 11.55%                               | 16.47%            | 10.08% | 10.45% | 12.76% |
| Volatility                                 | 17.63%                               | 17.89%            | 24.21% | 23.94% | 26.59% | 16.49%                               | 16.57%            | 34.16% | 27.88% | 17.94% |
| Sharpe ratio                               | −0.356                               | 0.084             | 0.236  | 0.273  | −0.166 | 0.432                                | 0.722             | 0.169  | 0.220  | 0.458  |
| CAPM beta                                  | 0.971                                | 0.681             | 0.720  | 0.845  | 1.064  | −0.074                               | −0.014            | −0.676 | −0.237 | 0.403  |
| CAPM alpha                                 | −3.17%                               | 3.71%             | 8.27%  | 9.58%  | −1.05% | 8.03%                                | 12.24%            | 12.20% | 8.46%  | 4.87%  |
| <i>Panel B. Minimum-Variance Portfolio</i> |                                      |                   |        |        |        |                                      |                   |        |        |        |
| Average return                             | −3.97%                               | 3.77%             | 6.94%  | 7.39%  | −0.48% | 13.60%                               | 12.90%            | 7.89%  | 8.07%  | 10.62% |
| Volatility                                 | 18.94%                               | 12.09%            | 16.88% | 16.20% | 19.32% | 20.75%                               | 12.06%            | 22.20% | 17.87% | 13.72% |
| Sharpe ratio                               | −0.356                               | 0.083             | 0.239  | 0.275  | −0.165 | 0.432                                | 0.691             | 0.169  | 0.220  | 0.458  |
| CAPM beta                                  | 1.043                                | 0.457             | 0.504  | 0.573  | 0.774  | −0.093                               | −0.010            | −0.438 | −0.149 | 0.037  |
| CAPM alpha                                 | −3.40%                               | 2.48%             | 5.77%  | 6.44%  | −0.74% | 10.20%                               | 8.75%             | 7.76%  | 5.33%  | 3.70%  |

<sup>11</sup>We ignore fixed costs and assume transaction costs of 20 basis points (bp) (as a percentage of the value of the transaction). While this is a conservative estimate, we ignore any costs implicit in the construction of the base asset portfolios.

persistence. Lagged MKT and INFL deliver more volatile fitted expected returns, and thus more trading, than the persistent interest rate variables.

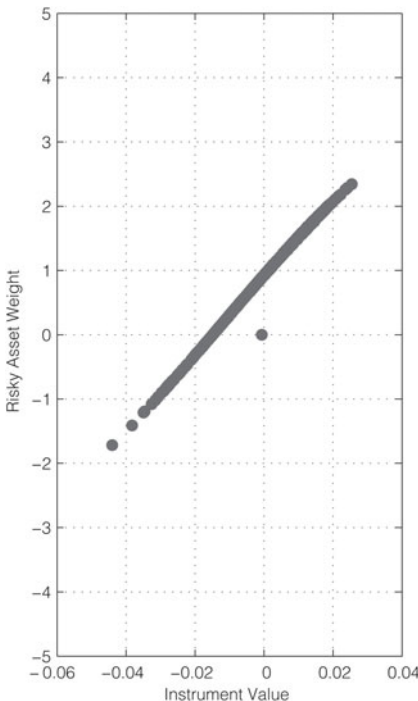
## F. Comparison with Conditionally Efficient Strategies

When we compare the performance of our dynamically efficient strategies with that of their corresponding conditionally efficient counterparts (selected results reported in Table 4), three consistent patterns emerge: First, conditionally efficient strategies never outperform their dynamically efficient counterparts, and in many cases they significantly underperform. Second, dynamically efficient strategies are consistently cheaper to run, the difference in costs being most pronounced (see Table 4) for minimum-variance strategies. Third, conditionally optimal strategies are much more sensitive to short-sale constraints, as they often require extreme shifts between long and short positions in excess of 200% (see Figure 5). In fact, we find that in contrast to our dynamically efficient strategies, conditionally efficient strategies often improve in performance when short-sale constraints are imposed. This is in line with the findings of Frost and Savarino (1988) and Jagannathan and Ma (2003).

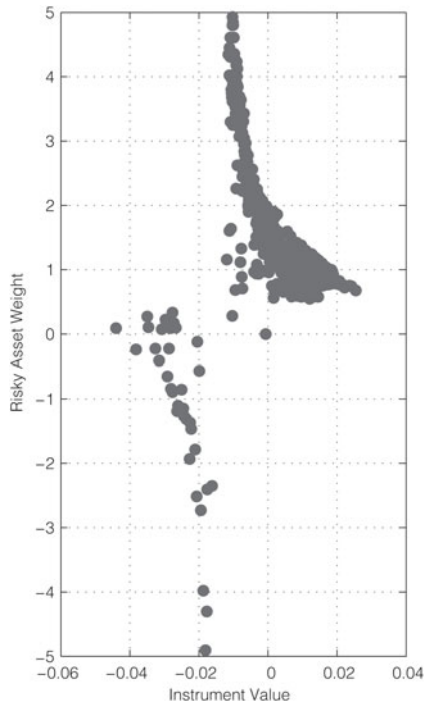
FIGURE 5  
Efficient Portfolio Weights

Figure 5 shows the efficient weights on the risky asset as a function of (the optimal linear combination of) the predictive instruments, for the case of a single-market index as risky asset. Graph A shows the weights of the unconditionally efficient (dynamically efficient) strategy, while Graph B shows the weights for the conditionally efficient (myopically optimal) strategy. The predictive instruments are TSPR, CSPR, and INFL.

Graph A. Dynamically Efficient



Graph B. Myopically Optimal



These effects are due to the fact that dynamically efficient strategies display a much more “conservative” response to extreme changes in the predictive instrument (see also Section II.E), a fact also observed by Ferson and Siegel (2001). For example, Figure 5 shows the weights of the 2 types of strategies on the single risky index, as a function of (the optimal linear combination of) the predictive instruments. The figure shows that even for very small changes in the predictive instruments, the weights of the conditionally optimal strategy (Graph B) often require sudden changes between extreme long and short positions.

## G. Out-of-Sample Performance

To assess the out-of-sample performance of our strategies, we conduct 3 experiments: First, we estimate the predictive model using a subsample of the original 1960–2004 data, and then we evaluate the performance of the resulting strategies over the entire remaining sample period. As cutoff points for the out-of-sample periods, we choose Jan. 1995 (the beginning of the “dot.com” boom), and 2000 (the beginning of the collapse of the bubble). Second, we run “rolling window” experiments, in which the model is estimated over a 5- or 10-year period, the resulting strategy is run over the 12 months following the estimation period, and then the estimation window is rolled forward by 1 year. Finally, we estimate the performance of our strategies over the 2005–2009 period (which includes the “credit crunch” and subsequent economic downturn), using data that has not been utilized in any of our in-sample estimations.

Table 5 presents the out-of-sample performance of several dynamically efficient strategies in the 2 chosen out-of-sample periods, using the 5-industry portfolios as base assets. Of course, the out-of-sample performance, in particular, in the 2000–2004 period case, does not match that predicted by the estimation (because the model was estimated over a bull run, but the strategy was run through a bear market). However, the performance *relative* to MKT (or the efficient fixed-weight strategy) is largely consistent with our in-sample results. Specifically, we find that strategies based on TSPR and CSPR outperform the market index by a margin of 6%–12% in both cases. While the term structure variables generate good performance in both out-of-sample periods, the results for other predictors are mixed: While lagged MKT perform well in the “dot.com” boom (1995–2004), they do not add much value during the collapse of the bubble (2000–2004). Conversely, strategies based on INFL in fact underperform during the bear market, while showing moderate performance during the rise of the bubble. In other words, our findings suggest that the level and shape of the term structure is the only information that is consistently useful in predicting stock returns, while other variables do well in bull markets but fail in bear markets or vice versa.

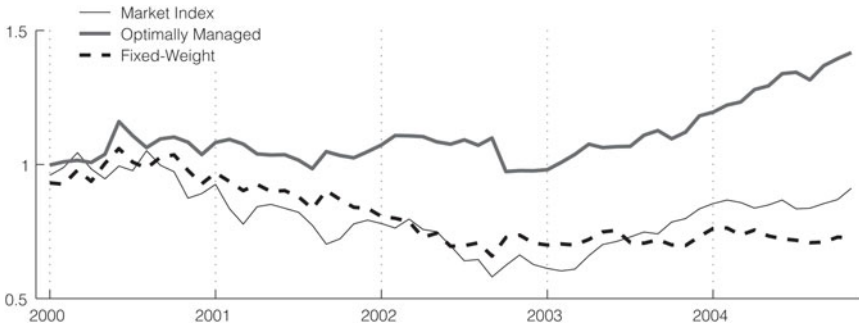
Figure 6 shows the out-of-sample performance of the efficient maximum-return strategies throughout the collapse of the “dot.com” bubble. While the fixed-weight strategy (dashed line) closely follows the market index (light-weight line), thus incurring significant losses, the dynamically optimal strategy (bold-faced line), using TSPR and CONV as predictive instruments, avoids all losses and achieves a gain of about 40% during the 5 years until the end of 2004.

FIGURE 6

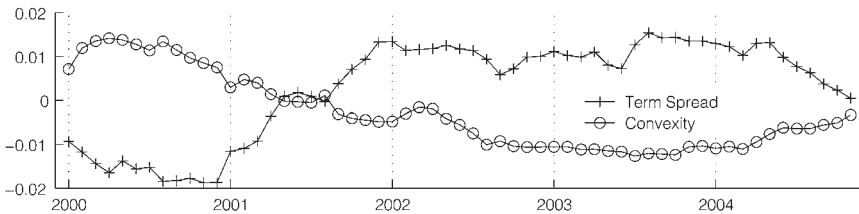
### Out-of-Sample Portfolio Performance (5-industry portfolios)

Graph A of Figure 6 shows the *out-of-sample* performance (cumulative return) of the fixed-weight (dashed line) and optimally managed (bold-faced line) maximum-return portfolio, relative to the performance of the market index (light-weight line). The model is estimated using data until Dec. 1999, and the strategy is evaluated in the out-of-sample period from 2000 through 2004. Graph B shows the time series of the instruments used: TSPR (“+”) and CONV (“o”).

Graph A. Cumulative Return



Graph B. Predictive Instruments



## 1. Rolling Window

Second, we run “rolling window” experiments, in which the model is estimated over a 5- or 10-year period, the resulting strategy is run over the 12 months following the estimation period, and then the estimation window is rolled forward by 1 year. We find that the strategies obtained this way perform much worse than in the simple out-of-sample experiments. We attribute this to the fact that dynamically efficient strategies are designed to be optimal with respect to long-run unconditional moments. Reestimating the model each year and allowing the strategy to run only for 1 year at a time is contrary to this philosophy, as the long-run mean cannot be attained over such short periods, while the risk is amplified due to the lack of time aggregation (because the strategy “thinks” that there is only 1 period to respond to the information, it tends to “overreact”).

## 2. Performance after 2004

We estimate the predictive model using the original 1960–2004 data, then formed optimal dynamic portfolio strategies on the basis of the estimated model coefficients and assessed the performance of these strategies in the period after 2004.

We first restrict the out-of-sample period to the end of 2007 (before the “credit crunch” really took hold). In this period, the performance of our strategies



remains strong. For example, using the 4 predictive variables that had shown the strongest performance in the in-sample estimation (TSPR, CONV, CSPR, and INFL), we found that the single-index market-timing strategy achieved a Sharpe ratio of 1.3 from 2005 through 2007, compared with only 0.74 for the fixed-weight strategy. Interestingly however, the out-of-sample performance of both the fixed-weight and the dynamically optimal strategy exceeded the in-sample predictions (which were 0.48 and 0.85, respectively).

While the fixed-weight strategy underperformed relative to the market index (with a negative alpha), the dynamically managed strategies achieved alphas of between 5% (for the maximum-return strategy) and 7.4% (for the minimum-variance strategy). Even after the deduction of transaction costs, risk-averse investors would still be willing to pay an annual fee of about 150–400 bp for this strategy. Moreover, the unconditionally efficient strategies beat their conditionally efficient counterparts by a wide margin in this period; the latter achieved lower Sharpe ratios while experiencing much higher volatility in portfolio weights, resulting in higher transaction costs and a negative management fee (i.e., investors would rather “burn money” than invest in these strategies).

On the other hand, strategies based on the 5-industry portfolios do not sustain their superior performance in the 2005–2007 period. In fact, while the fixed-weight strategies in this case still achieve a respectable Sharpe ratio and a marginally positive alpha, all managed strategies (both unconditionally and conditionally efficient, with or without short-sale constraints) underperform. This suggests that, in the period after 2004, market timing still worked (i.e., the predictive variables told investors when to “get out” of the market), but the predictive relationship between the instruments and optimal asset allocation seems to be broken.

An interesting observation is that when we remove CSPR from the set of predictive variables, the performance of our strategies increased dramatically (from underperforming the market to achieving a Sharpe ratio of almost 1.0 and earning a 250-bp management fee). This suggests that the predictive ability of the *average* credit spread was not sufficient to select among different industries in the post-2004 period. This observation, together with the fact that market-timing strategies still worked well using CSPR, seems to indicate that there were a lot of idiosyncratic (sector-specific) movements during this period that the average spread was not able to discern.

In our final experiment, we keep the in-sample period (1960–2004) but extend the out-of-sample period all the way to June 2009. In this case, we have to admit that none of our strategies managed to avoid the losses incurred by the market. In fact, most of the dynamically managed strategies even underperformed during the final 18 months (i.e., during the “credit crunch” and the subsequent economic meltdown). This (quite plausibly, given recent events) suggests a regime shift in the relation between predictive instruments and market returns.

## H. Robustness

To assess the robustness of our estimation results, we perform a bootstrap analysis. For each set of base assets and predictive instruments, we run two

simulation experiments under the null hypothesis and the alternative, respectively. In both cases, we first fit a (vector) AR(1) model to the original time series of instrument(s), and then simulate 100,000 new time series by resampling from the residuals of this estimation and then reconstructing the instrument values using the AR(1) coefficients.

### 1. Simulation under the Null Hypothesis

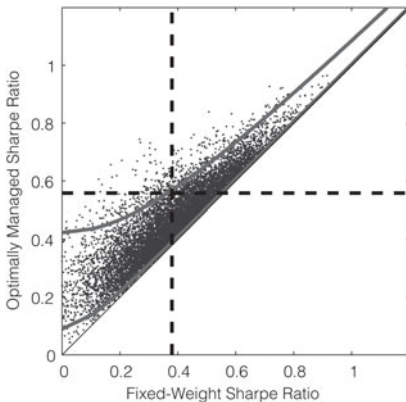
To simulate the corresponding time series of asset returns under the null hypothesis (of no predictability), we draw independent samples from the unconditional empirical distribution of asset returns. In other words, this procedure assumes that the time series of asset returns is i.i.d. (with the marginals given by the empirical distribution of the original data) and independent of the instruments, thus modeling the null hypothesis of no predictability. For each of the 100,000 simulated time series of returns and instruments, we then reestimate the predictive model (9) and compute the implied Sharpe ratios from equation (3).

Figure 7 shows the result of the bootstrap simulation for the single-index model, using TSPR, CONV, and CSPR as predictors. Graph A plots the optimally managed (vertical axis) against the fixed-weight (horizontal axis) annualized Sharpe ratios, estimated from the simulated time series.<sup>12</sup> The dashed lines show the Sharpe ratios derived from the original model estimation (0.379 fixed-weight and 0.557 optimal). The empirical  $p$ -value of 6.13% obtained from

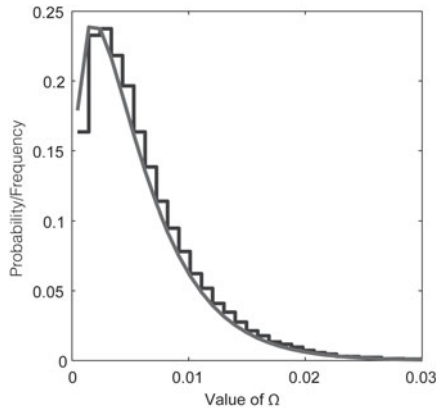
FIGURE 7  
Bootstrap (null hypothesis)

Figure 7 shows the result of the bootstrap experiment (see Section III.H) under the null hypothesis. Graph A plots the optimally managed (vertical axis) against the fixed-weight (horizontal axis) annualized Sharpe ratios, estimated from the simulated time series of asset returns and instruments. The solid lines show the 95% confidence interval for the test statistic  $\Omega$ . Also shown are the Sharpe ratios estimated from the original data (dashed lines). Graph B plots the theoretical (asymptotic  $\chi^2$ ) distributions of the Wald test for the slope coefficient in the predictive regression (4), and the empirical distribution obtained from the bootstrap simulation.

Graph A. Bootstrap



Graph B. Empirical and Theoretical ( $\chi^2$ ) Distribution



<sup>12</sup>For clarity of the graphical representation, we only plot the first 10,000 samples, although 100,000 samples were simulated.

the simulation closely matches the theoretical one of 6.21% (derived from the asymptotic  $\chi^2$  distribution of the test statistic, see Proposition 3).

Graph B of Figure 7 shows the empirical distribution of the test statistic  $\Omega$ , obtained from the simulation, compared with the  $\chi^2$  distribution of the Wald test statistic of the slope coefficient in the predictive regression. Although the empirical distribution displays very slight excess skewness (1.69 compared with the theoretical 1.63) and kurtosis (7.52 compared with 7.50), the match is very close. The bootstrap thus confirms our theoretical results, and moreover shows that neither nonnormality nor finite-sample problems significantly affect our empirical findings. We conduct the same experiment for all other combinations of assets and/or instruments and find in all cases a similarly close match between theoretical and empirical distribution.

## 2. Simulation under the Alternative

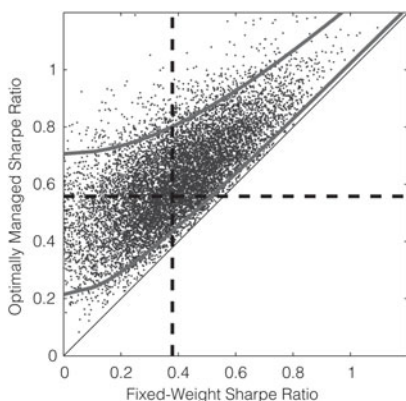
To construct the corresponding experiment under the alternative, we first estimate the predictive model (9) using the original data and then construct 100,000 simulated time series by simultaneously resampling the residuals of the AR(1) model for the instruments and the predictive model for asset returns. In other words, the originally estimated predictive relation between lagged instruments and asset returns is assumed to be the true data-generating process for this simulation, thus modeling the alternative to the null hypothesis of no predictability.

The results for the single-index model, using TSPR, CONV, and CSPR as predictors, are shown in Figure 8. To match the empirical distribution of the test statistic  $\Omega$  under the alternative, we fit a noncentral  $\chi^2$  distribution, with the

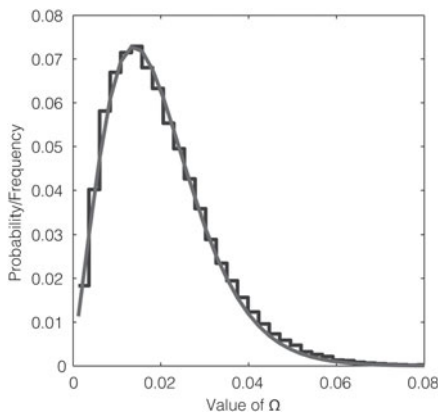
FIGURE 8  
Bootstrap (alternative)

Figure 8 shows the result of the bootstrap experiment (see Section III.H) under the alternative. Graph A plots the optimally managed (vertical axis) against the fixed-weight (horizontal axis) annualized Sharpe ratios, estimated from the simulated time series of asset returns and instruments. The solid lines show the 95% confidence interval for the test statistic  $\Omega$ . Also shown are the Sharpe ratios estimated from the original data (dashed lines). Graph B plots the theoretical (asymptotic  $\chi^2$ ) distributions of the Wald test for the slope coefficient in the predictive regression (4), and the empirical distribution obtained from the bootstrap simulation.

Graph A. Bootstrap



Graph B. Empirical and Theoretical ( $\chi^2$ ) Distribution



noncentrality parameter given by the original estimate  $T\Omega$ . As before, the empirical distribution displays only marginal excess skewness (1.11 instead of 0.96) and kurtosis (4.81 instead of 4.23).

## IV. Conclusions

This paper explores the economic value of optimally utilizing asset return predictability in portfolio formation. We construct dynamically efficient portfolio strategies that optimally exploit predictability and assess their performance both in and out of sample. Overall we find that predictability can significantly improve the risk-return trade-off. We find that dynamic unconditionally efficient strategies always outperform their conditionally efficient counterparts, while incurring lower transaction costs and being less sensitive to the short-sale constraints. Specifically, our unconditionally efficient strategies outperform the market by a margin of 5%–12%, justifying a management premium of up to 10% per annum.

We test several commonly used predictive variables, including lagged market returns, dividend yield, the level and shape of the term structure, credit spreads, and inflation. We find that the ranking of predictors is consistent across different sets of assets, trading strategies, and sampling periods. Our results are remarkably robust across sampling periods. Overall, we find that (lagged) term and credit spread consistently produce significant economic gains, while the results for inflation and lagged market returns are mixed, depending on asset universe and sample period. In contrast to earlier research but in line with recent evidence, we find that dividend yield possesses no significant predictive ability.

Our results show not only the significance of return predictability in asset allocation, but they also highlight the importance of using predictive information *optimally*. Moreover, our results suggest that, despite a vast range of literature studying and advocating various predictive instruments, the term structure of interest rates remains the most powerful and consistent source of information in forecasting stock returns. However, recent events (the “credit crunch” and the subsequent economic downturn) seem to have caused a structural break in the relationship between some of the predictive variables and market returns, and it remains to be seen if these relationships will reestablish themselves or not.

## Mathematical Appendix

### 1. Proof of Proposition 3

We begin with the predictive model, as specified in equation (4):

$$(A-1) \quad R_t = \mu_0 + B \cdot Z_{t-1} + \varepsilon_t.$$

This implies  $\mu_{t-1} = \mu_0 + B \cdot Z_{t-1}$ . From Proposition 2 we know that, up to a 1st-order approximation, the maximum hypothetical Sharpe ratio can be written as  $\lambda_*^2 = E(H_{t-1}^2)$ , where  $H_{t-1}$  is the conditional Sharpe ratio as defined in expression (3). From this, we get

$$(A-2) \quad \lambda_*^2 = E \left( (\mu_0 - r_{t-1}^f)' \Sigma_{t-1}^{-1} (\mu_0 - r_{t-1}^f) \right) + E \left( Z_{t-1}' B' \Sigma_{t-1}^{-1} B Z_{t-1} \right).$$

Under the null hypothesis,  $B = 0$ , and hence  $\Sigma_{t-1} = \Sigma_0$  (where  $\Sigma_0$  is the unconditional VCV matrix of asset returns), which implies  $\Omega = 0$ . Conversely, when  $M = 1$  (i.e.,  $Z_{t-1}$  is scalar), then  $\Omega = 0$  implies  $B = 0$ , which proves the 2nd part of Proposition 3.  $\square$

## 2. Specification with Time-Varying Volatility

Following the specification outlined in Ferson and Siegel (2001), we chose a (vector of) systematic risk factor(s)  $F_t$  and postulate that asset returns and the factor(s) follow a predictive regression of the form

$$(A-3) \quad R_t = \mu_0 + BZ_{t-1} + \varepsilon_t \quad \text{and}$$

$$(A-4) \quad F_t = \nu_0 + DZ_{t-1} + \eta_t,$$

where  $R_t$  is the vector of base asset returns,  $F_t$  is the (vector of) systematic factor(s), and  $Z_{t-1}$  is the (vector of) predictive instrument(s). At this stage, we do not make any assumptions on the residuals  $\varepsilon_t$  and  $\eta_t$ , except that they have zero conditional mean.

To model the factor structure, we then regress the asset residuals on the factor residual(s),  $\varepsilon_t = H\eta_t + \xi_t$ . Here, we assume that  $\xi_t \sim \text{iid}(0, \Sigma_\xi)$  with constant VCV matrix  $\Sigma_\xi$ .

To model the time variation in conditional variance, we regress  $\log \eta_t^2 = \alpha + \gamma Z_{t-1} + \zeta_t$ . If we assume that  $\zeta_t \sim N(0, \sigma_\zeta^2)$ , then

$$(A-5) \quad E(\eta_t^2 | Z_{t-1}) = \exp \left( \alpha + \gamma Z_{t-1} + \frac{1}{2} \sigma_\zeta^2 \right) =: \underbrace{C \cdot \exp(\gamma Z_{t-1})}_{=: v_{t-1}}.$$

Note, to match the correct in-sample unconditional expectation, the constant  $C$  above should be chosen so that  $E(\eta_t^2) = C \cdot E(\exp(\gamma Z_{t-1}))$  in sample.

From this, we can compute the conditional VCV matrix of  $R_t$  as  $\Sigma_{t-1} = v_{t-1} H' H + \Sigma_\xi$ . Hence, using the Sherman-Morrison formula, we get

$$(A-6) \quad \Sigma_{t-1}^{-1} = \Sigma_\xi^{-1} - \frac{v_{t-1} \Sigma_\xi^{-1} H' H \Sigma_\xi^{-1}}{1 + v_{t-1} H \Sigma_\xi^{-1} H'}.$$

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