On Zerotree Quantization for Embedded Wavelet Packet Image Coding

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Abstract

Wavelet packets are an effective representation tool for adaptive waveform analysis of a given signal. We first combine the wavelet packet representation with zerotree quantization for image coding. A general zerotree structure is defined which can adapt itself to any arbitrary wavelet packet basis. We then describe an efficient coding algorithm based on this structure. Finally, the hypothesis for prediction of coefficients from coarser scale to finer scale is tested and its effectiveness is compared with that of zerotree hypothesis for wavelet coefficients.

1 Introduction

There has been a surge of interest in wavelet transforms for image and video coding applications, in recent years. This is mainly due to the nice localization properties of wavelets in both time (or space) and frequency. The zerotree quantization, proposed by Shapiro [8], is an effective way of exploiting the self-similarities among high-frequency subbands at various resolutions. The main thrust of this quantization strategy is in the prediction of corresponding wavelet coefficients in higher frequency subbands at the finer scales, by exploiting the parent-offspring dependencies. This prediction works well, in terms of efficiently coding the wavelet coefficients, due to the statistical characteristics of subbands at various resolutions, and due to the scale-invariance of edges in high frequency subbands of similar orientation. Various extensions of zerotree quantization, such as [3, 9], have been proposed ever since its introduction.

Wavelet packets [1] were developed in order to adapt the underlying wavelet bases to the contents of a signal. The basic idea is to allow non-octave subband decomposition to adaptively select the *best basis* for a particular signal. Results from various image coding schemes based on wavelet packets show that they are particularly good in coding images with oscillatory patterns, a special form of *texture*. While wavelet packet bases are well adapted to the corresponding signal (image), one loses the parent-offspring dependencies, or the *spatial orientation trees*, as defined in [8, 7].

In this work, we address the following question: can the zerotree quantization be applied to wavelet packet transformed images? The question is valid since one does not generally observe as much self-similarities among the wavelet packet subbands as among the wavelet subbands. Moreover, even if there does exist self-similarity among wavelet packet subbands, the question arises: *how would the spatial orientation trees, or zerotrees, be defined in order to predict the insignificance of corresponding wavelet packets at a finer scale, given a wavelet packet at a coarser scale*? The issue was addressed partially by Xiong et al. in [10]. In their work, however, the tree decomposition was restricted not to have a basis causing the *parenting conflict*, a problem described in the next section. We present a solution to this problem and define a set of rules to construct the zerotree structure for a given wavelet packet geometry, thus offering a general structure for an arbitrary wavelet packet decomposition. This generalized zerotree structure is termed as the *compatible zerotree* for reasons mentioned in the next section. The new tree structure provides an efficient way of encoding the wavelet packet coefficients. We combine the idea of zerotree quantization with wavelet packet transforms for image coding purposes.

2 Compatible Zerotree Quantization

The wavelet packet basis [1] is adaptively selected in order to tailor the representation to the contents of a signal (an image, in our case). The nodes in a full subband tree (or short-term Fourier transform, namely STFT, tree) are pruned following a series of split/merge decisions using certain criterion (see [2] for entropy-based best basis selection and [6] for optimizing the best basis from a rate-distortion viewpoint). Suppose the best basis has been selected using one of these methods. The issue is how to organize the spatial orientation trees so as to exploit the self-similarities, if any, among the subbands. The basis selected by any of the above methods does not, in general, yield the parent-offspring relationships like those in wavelet subbands. Moreover, there can be an instance in the wavelet packet tree where one or more of the child nodes are at a coarser scale than the parent node. This results in the association of each coefficient of such a child node to multiple parent coefficients, in the parent node, giving rise to a *parenting conflict*. To make the point clear, let us consider the segmentation shown in Figure 1, for a 3-level wavelet packet transform. The nodes C_1, C_2, C_3, and C_4 at a coarser scale are children of the node P at a finer scale. The problem with such a situation is that there are four candidates in P claiming the parenthood of the set of four coefficients belonging to the four children nodes. In [10], the best basis was selected in such a way that whenever a conflict like this arises, the four children are merged so as to resolve the conflict. This suboptimal approach constrains selection of the best basis at the loss of the freedom of adapting the wavelet packet basis to the contents of a particular signal.

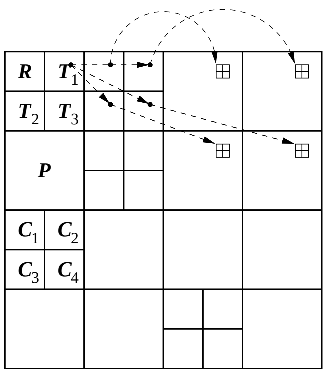

Figure 1: Sample segmentation in a 3-level wavelet packet decomposition and the parent-offspring dependencies for compatible zerotree originating from T_1.

We organize the zerotree structure, called *compatible zerotrees*, dynamically so that there is no restriction on selection of the wavelet packet basis (we call these the compatible zerotrees, since they are generated taking into account both the scale and orientation compatibility). It is to be noted that the compatible zerotrees differ from the spatial orientation trees of [7] in the sense that nodes of a compatible zerotree represent a full subband, rather than one or more coefficients. We utilize the compatible zerotrees to find the coefficients which are zerotree roots, for a given threshold. A set of rules is defined to construct the compatible zerotrees so that the overall zerotree structure can be constructed for any arbitrary wavelet packet basis. The only *assumption* made is that the basis has its lowest frequency bands at the coarsest scale. This is based upon the fact that a significant amount of the signal energy is concentrated in the lowest frequency subbands, which cannot be merged by any of the tree pruning algorithms.

Let R denotes the node representing the lowest frequency subband, situated in the top-left corner of a conventional subband decomposed image. Then R represents the root node of overall compatible zerotree. It has three children (T_1, T_2, and T_3 as shown in Figure 1) which represent the coarsest scale high-frequency subbands. These children are themselves root nodes of three compatible zerotrees with different global orientations; horizontal, vertical, and diagonal respectively. These zerotrees are generated in a recursive manner (only once for a given wavelet packet basis) using the following set of rules, each of them applied to nodes with similar global orientation.

a) If a node P is followed by four nodes C_1, C_2, C_3, and C_4 (at the same scale), then P is declared to be the parent of all these four nodes.

b) If four subbands P_1, P_2, P_3, and P_4 at a coarser scale are followed by four subbands C_1, C_2, C_3, and C_4 at immediately next finer scale, then node P_i is declared to be the parent of node C_i (for $i = 1, 2, 3, 4$).

c) If a node P at a coarser scale is followed only by a node C at the immediately next finer scale (like in a wavelet transform), the node P is declared as a parent of C.

d) If a node P is at a finer scale than four of its children, say C_1, C_2, C_3, and C_4, then P is disregarded as being the parent of all these nodes and all this bunch is moved up in the tree.

Let us consider the segmentation shown in Figure 1 to explain these rules. The construction of final compatible zerotrees proceeds in two steps. In the first step, the primary compatible zerotrees T_1, T_2, and T_3 are constructed using rules a, b, and c, as shown in Figure 2. In the second step, the overall tree is re-organized (as shown in Figure 3) using rule d, in order to resolve the parenting conflict under node P. This rule first identifies nodes with the parenting conflicts and when such nodes are found, the whole bunch (consisting of C_1, C_2, C_3, and C_4) is plucked from its current position in the tree. The algorithm then climbs up the tree to look for an appropriate node, a compatible ancestor at the nearest scale and having the nearest orientation, and glues the bunch under this newly found compatible parent. The parent-offspring dependencies for the primary compatible zerotree T_1 are shown in Figure 1. For more details of the algorithm to construct the compatible zerotrees, please refer to [5].

Given an arbitrarily segmented wavelet packet basis, the compatible zerotrees are constructed using the set of rules described as above. In order to be able to make predictions for wavelet packet coefficients at finer scale subbands, the *compatible zerotree hypothesis* is defined as follows: If a wavelet packet coefficient of a node from the compatible zerotree is insignificant, it is more likely that the wavelet packet coefficients at similar spatial locations in all the descendent nodes of the same compatible zerotree will be insignificant as well.

We tested the success of compatible zerotree hypothesis, as defined above, for wavelet packets. The plots of wavelet packet coefficients' amplitude against the coefficient indices for the wavelet packet decomposition, using the entropy-based basis selection criterion as in [4], of a 512×512 Barbara image are shown in Figures 4 and 5. The amplitude axis in both plots was restricted to ± 1000 to facilitate the display of smaller coefficients. While the subbands were arranged in an increasing frequency order for the plot in Figure 4, the plot in Figure 5 refers to the subbands as organized in the compatible zerotrees. The three families of primary compatible zerotrees $T1$, $T2$, and $T3$ are clearly visible in the latter of these plots showing that the new arrangement was successful in isolating the coefficients of similar orientation in a hierarchy to be used for prediction in a top-down order.

3 The Coder Algorithm

Once the compatible zerotrees have been generated, based upon knowledge of the best basis, the encoding takes place by repeatedly running the *detection stage* and the *fine-tuning stage* until the bit budget is expired. Our algorithm requires only one list of detected coefficients (LDC) to be maintained, as opposed to two (three) lists kept by the EZW (SPIHT). The decoder first reads geometry of the best basis and generates the compatible zerotrees, and then proceeds by entropy decoding and interpreting the codewords (see [5] for more details of the algorithm).

4 Experiments and Conclusions

The idea of compatible zerotree quantization was coupled with the wavelet packet transform for progressive image coding and its performance was tested against the fast adaptive wavelet packet (FAWP) [4] image codec algorithm. A comparison of the peak signal to noise ratio (PSNR) achieved by CZQ and FAWP for three standard 512×512 images encoded at target bit rates of 0.25 and 0.10 bits per pixel (bpp) is presented in Table 1. While being relatively faster, by a factor ranging from two to four, than the bitplane coding of FAWP, the CZQ performs comparably well. We note that the effectiveness of zerotree quantization depends strongly upon the scale-invariance of edges among subbands of similar global orientation. The success of prediction of finer scale coefficients (set of all descendents $\mathcal{D}_i$ at similar spatial location) based upon a given coefficient i at a coarser scale is a function of the conditional probability of $\mathcal{D}_i$ being insignificant, given i is insignificant. We compared the number of zerotree root nodes detected, for various values of threshold, both for wavelet and wavelet packet bases. It was observed that the probability of detection of the zerotree root nodes in a wavelet decomposition is far greater than that in a wavelet packet decomposition.

It is important to note here that the wavelet packet basis was selected by simply using first-order entropy as a criterion for the split/merge decisions. The coefficients of basis were, therefore, imposed to be zerotree quantized. As opposed to wavelets, the wavelet packet basis might not necessarily produce coefficients that help towards success of the zerotree hypothesis (that is, produce more zerotrees). Figure 6 shows the geometry of wavelet packet basis selected for the 512×512 Barbara image. In fact, we found that using the entropy as a criterion for the best basis selection might result into many coarse scale high frequency subbands (whose coefficients represent oscillatory patterns in some parts of the image), as is clear from basis geometry in Figure 6. It is the encoding of coefficients in these types of subbands which takes considerable amount of the bit budget for the following reason. If a coefficient somewhere at the bottom of a compatible zerotree is found to be significant, the significance of all the corresponding coefficients in sibling and the parent nodes needs to be encoded as well, even if some or most of them are insignificant. Our future work aims to look at the methods of optimizing the selection of best basis using a criterion which results into optimal number of zerotrees and optimal distortion value, given the bit budget. Moreover, the ordering of wavelet packet coefficients with respect to increasing frequency (that is, decaying magnitude) within the compatible zerotrees will provide a gain in the coding performance, by increasing the efficiency of arithmetic coder to encode the zerotree codewords.

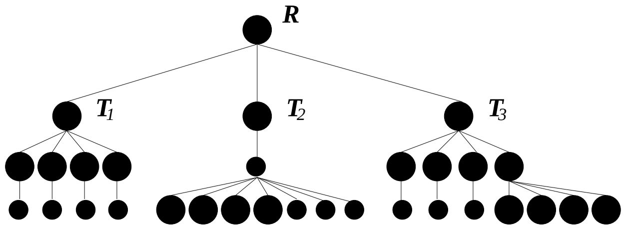

Figure 2: The overall compatible zerotree structure comprising of the three primary compatible zerotrees T_1, T_2, and T_3; each node represents a whole wavelet packet subband, and the node radius is directly proportional to the support of wavelet packets at that transform level.

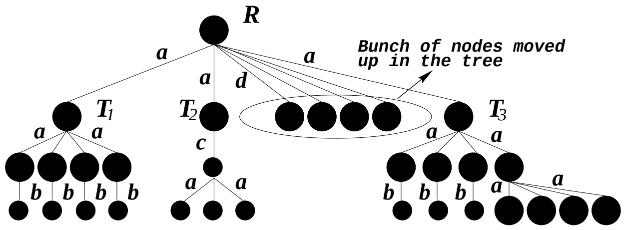

Figure 3: The overall compatible zerotree structure, after re-organization to resolve the parenting conflicts; the edge labels depict the rules used to generate the links between parent and the children nodes.

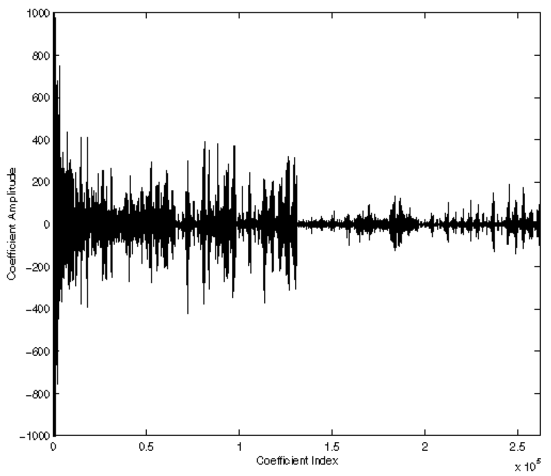

Figure 4: Plot of wavelet packet coefficients' amplitude versus coefficient indices for the wavelet packet decomposition of 512×512 Barbara image; the subbands were arranged in an increasing frequency order.

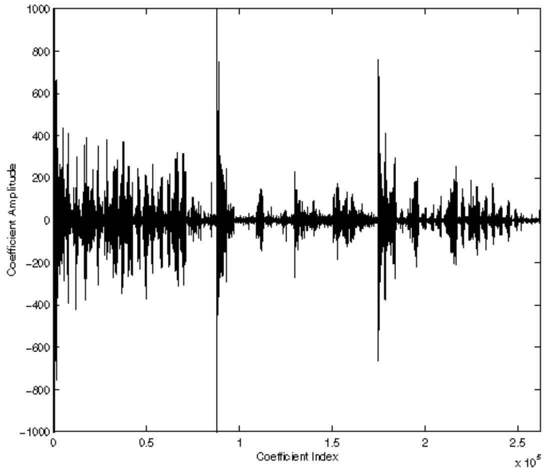

Figure 5: Plot of wavelet packet coefficients' amplitude versus coefficient indices for the wavelet packet decomposition of 512×512 Barbara image; the subbands were organized in compatible zerotrees with the help of rules mentioned in Section 2.

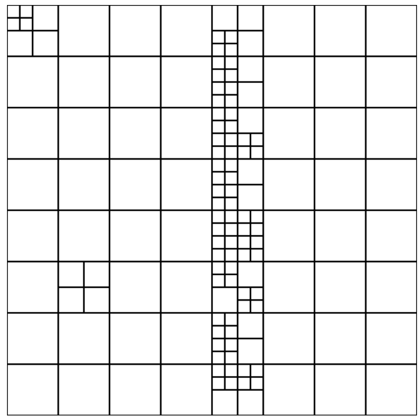

Figure 6: Wavelet packet basis selected for the 512×512 Barbara image using first-order entropy as selection criterion.

Image	Rate (bpp)	CZQ	FAWP
Fingerprint	0.25	26.68	27.21
	0.10	23.25	23.81
Goldhill	0.25	29.19	30.32
	0.10	26.66	27.54
Lena	0.25	31.88	33.26
	0.10	28.07	29.33

Table 1: Coding results for 512×512 standard images at 0.25 bpp and 0.10 bpp.

Acknowledgement

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