A Model for Large $_{13}$ Constructed using the Eigenvectors of the S_4 Rotation Matrices

2013 J. Phys.: Conf. Ser. 447 012043

(http://iopscience.iop.org/1742-6596/447/1/012043)

A Model for Large θ_{13} Constructed using the Eigenvectors of the S_4 Rotation Matrices

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Abstract. A procedure for using the eigenvectors of the elements of the representations of a discrete group in model building is introduced and is used to construct a model that produces a large reactor mixing angle, $\sin^2\theta_{13} = \frac{2}{3}\sin^2\frac{\pi}{16}$, in agreement with recent neutrino oscillation observations. The model fully constrains the neutrino mass ratios and predicts normal hierarchy with the light neutrino mass, $m_1 \approx 25$ meV. Motivated by the model, a new mixing ansatz is postulated which predicts all the mixing angles within 1σ errors.

1. Introduction

We use the group $SU(3)$ and its discrete subgroup S_4 for model building. Both of these groups had been studied extensively as flavour symmetry groups. The S_4 group has the presentation

$$\langle a, b | a^2 = b^3 = (ab)^4 = e\rangle. \tag{1}$$

S_4 is the symmetry group of the cube (Fig. 1) and the elements of the group can be represented as the orientation preserving rotations of the cube. The matrices representing the generators can be written as

$$a = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \tag{2}$$

Here the basis vectors $e_1 = (1,0,0)^T$, $e_2 = (0,1,0)^T$ and $e_3 = (0,0,1)^T$ form the symmetry axes of the cube passing through face centres. If we define the left-handed leptons, $L = (L_e, L_\mu, L_\tau)^T$ where $L_e = (\nu_{eL}, e_L)^T$ etc., as a triplet in this basis, the flavour states L_e, L_μ, L_τ correspond to e_1, e_2, e_3 respectively. Usually in models a set of flavons are introduced whose vacuum expectation values (vevs) produce the desired texture for the fermion mass matrices. In other words, the orientation of fermion flavour states as well as the flavon vevs in the flavour space determines the form of the mass matrices.

Axes of the orientation preserving rotations of the cube are nothing but eigenvectors of the corresponding rotation matrices with eigenvalue equal to $+1$. The basis vectors e_1, e_2 and e_3 are examples. There are also other vectors like the ones passing through the opposite edge centres (e.g. axis$_a$ in Fig. 1) and the ones passing through the opposite vertices (e.g. axis$_b$ in Fig. 1). Compared to vectors pointing in random directions, these vectors are "special" in the context of the S_4 symmetry. The rotation matrices are unitary and so their eigenvalues in general are complex numbers with unit modulus. If non-degenerate, these eigenvalues also correspond to unique eigenvectors and the author argues that these eigenvectors are also "special" like the rotation axes.

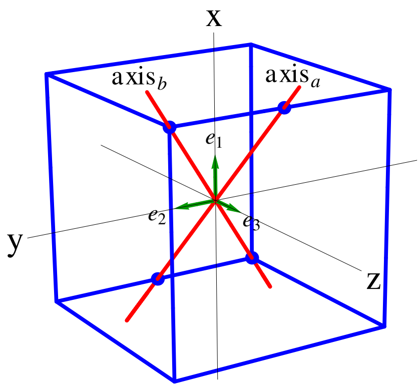

Figure 1. The generators a and b represent π-rotation about axis$_a$ and $\frac{2\pi}{3}$-rotation about axis$_b$ respectively.

As an example consider $v = \frac{1}{\sqrt{3}}(1, \bar{\omega}, \omega)^T$, the normalised ($v^\dagger v = 1$) eigenvector of the matrix b in Eq. (5), corresponding to the eigenvalue ω, where $\omega = e^{i\frac{2\pi}{3}}$ and $\bar{\omega} = e^{-i\frac{2\pi}{3}}$. Since $e^{i\theta}v$, where $e^{i\theta}$ is an arbitrary phase, is also a normalised eigenvector, we impose the following condition to uniquely fix the phase: The component of the eigenvector in the direction of one of the basis vectors should have zero phase ie. $\arg(v^\dagger e_i) = 0$, where $i = 1$, 2 or 3. This is intuitive since the basis vectors are used to define the fermion flavour states. Thus the allowed choices for v are $\frac{1}{\sqrt{3}}(1, \bar{\omega}, \omega)^T$, $\frac{1}{\sqrt{3}}(\omega, 1, \bar{\omega})^T$ and $\frac{1}{\sqrt{3}}(\bar{\omega}, \omega, 1)^T$.

Let g represent an element of the group and l be one of its non-degenerate eigenvalues. The corresponding "special" eigenvector, $\text{eig}(g, l)_i$, is defined using the following normalisation condition and the phase condition:

$$\text{eig}(g,l)_i^\dagger \, \text{eig}(g,l)_i = 1, \quad \arg\left(\text{eig}(g,l)_i^\dagger \, e_i\right) = 0. \tag{3}$$

For example, the basis vector e_3 is $\text{eig}(ab, 1)_3$. Such eigenvectors which will be used later in the model are listed below:

$$\begin{aligned}
\text{eig}(b,1)_1 &= \tfrac{1}{\sqrt{3}}\,(1,1,1)^T, & \text{eig}(a,1)_1 &= \tfrac{1}{\sqrt{2}}\,(1,0,1)^T, \\
\text{eig}(b,\omega)_1 &= \tfrac{1}{\sqrt{3}}\,(1,\bar{\omega},\omega)^T, & \text{eig}(c,i)_1 &= \tfrac{1}{\sqrt{2}}\,(1,0,i)^T, \\
\text{eig}(b,\bar{\omega})_1 &= \tfrac{1}{\sqrt{3}}\,(1,\omega,\bar{\omega})^T, & \text{eig}(c,i)_3 &= \tfrac{1}{\sqrt{2}}\,(\text{-}i,0,1)^T, \\
\text{eig}(d,1)_1 &= \tfrac{1}{\sqrt{2}}\,(1,0,\text{-}1)^T, & \text{eig}(c,\text{-}i)_1 &= \tfrac{1}{\sqrt{2}}\,(1,0,\text{-}i)^T, \\
\text{eig}(d,1)_3 &= \tfrac{1}{\sqrt{2}}\,(\text{-}1,0,1)^T, & \text{eig}(c,\text{-}i)_3 &= \tfrac{1}{\sqrt{2}}\,(i,0,1)^T
\end{aligned} \tag{4}$$

where

$$c = bab = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ \text{-}1 & 0 & 0 \end{pmatrix}, \quad d = a(bab)^2 = \begin{pmatrix} 0 & 0 & \text{-}1 \\ 0 & \text{-}1 & 0 \\ \text{-}1 & 0 & 0 \end{pmatrix}. \tag{5}$$

2. The Model

Daya Bay reactor neutrino experiment [1] showed that the reactor mixing angle, θ_{13}, is non-zero. In this paper we propose a flavour model to accommodate the observed non-zero θ_{13}. We use the Standard Model (SM) framework with the addition of the right-handed neutrino triplet, $\nu_R = (\nu_{1R}, \nu_{2R}, \nu_{3R})^T$ in the context of the type-1 seesaw mechanism. We postulate a global flavour symmetry group,

$$G_f = SU(3)_1 \times SU(3)_2 \times U(1)_f. \tag{6}$$

The fermion fields and a set of postulated flavons belong to specific representations of G_f as shown in Table 1. The $U(1)$ group is introduced to ensure that the flavons couple to only the desired fermions. We write the mass terms at the lowest order, containing the fermions and the minimum number of flavons, invariant under G_f and the SM gauge group. The eigenvectors of the elements of the S_4 subgroup of the $SU(3)$ group are used to construct the vevs of the flavons. The vevs break the flavour symmetry and the required mass matrices are obtained.

	L	e_R	μ_R	τ_R	ν_R	ϕ_e	ϕ_μ	ϕ_τ	ϕ	ξ_1	ξ_2
$SU(3)_1$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\bar{\mathbf{3}}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{6}$	$\mathbf{1}$
$SU(3)_2$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\bar{\mathbf{6}}$
$U(1)_f$	f_l	$f_l + f_e$	$f_l + f_\mu$	$f_l + f_\tau$	0	$-f_e$	$-f_\mu$	$-f_\tau$	0	f_l	0

Table 1. The fields ϕ_e, ϕ_μ, ϕ_τ, ϕ, ξ_1, ξ_2 are the flavons. For the $U(1)$ group the tabulated values are the generators, e.g. $f_l \equiv e^{if_l\theta}$. The SM Higgs is a flavour singlet.

The tensor product expansion of the fundamental representations of $SU(3)$ are

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}, \quad \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = \mathbf{3} \oplus \bar{\mathbf{6}}, \quad \mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}. \tag{7}$$

For the charged leptons, the lowest order mass term is

$$\overline{L}\,(y_e\phi_e e_R + y_\mu\phi_\mu\mu_R + y_\tau\phi_\tau\tau_R)\,H\frac{1}{\Lambda_{\phi_l}} + H.C. \tag{8}$$

where y_α are the coupling constants, H is the SM Higgs, Λ_{ϕ_l} is the cut-off scale for the flavons ϕ_e, ϕ_μ and ϕ_τ. The S_4 group has four irreducible representations: $\mathbf{1}$, $\mathbf{2}$, $\mathbf{3}$ and $\mathbf{3'}$ [2]. The orientation preserving rotations of the cube discussed earlier belong to $\mathbf{3'}$. The flavon triplets ϕ_e, ϕ_μ and ϕ_τ belong to the representation $\mathbf{3}$ of $SU(3)_1$. The restriction of the representation $\mathbf{3}$ of $SU(3)$ to its subgroup S_4 is the representation $\mathbf{3'}$ of S_4. Therefore the "special" eigenvectors of the representation matrices of $\mathbf{3'}$ of S_4 are used to construct the vevs of the flavons. We assign:

$$\langle\phi_e\rangle = \mathrm{eig}(b,1)_1, \quad \langle\phi_\mu\rangle = \mathrm{eig}(b,\omega)_1, \quad \langle\phi_\tau\rangle = \mathrm{eig}(b,\bar{\omega})_1 \tag{9}$$

where the angular brackets are used to denote vevs. We do not discuss the mechanism of flavour symmetry breaking in this paper. To avoid Goldstone bosons, it is necessary to add explicit symmetry breaking terms for the flavon potentials, which break the continuous flavour group G_f, Eq. (6), into an unknown discrete group. The flavon vevs spontaneously break this discrete flavour symmetry. Also the Higgs vev, $(0, h_o)^T$, breaks the weak gauge symmetry. After the symmetry breaking, the charged-lepton mass term, Eq. (8), takes the form

$$\overline{l_L}\mathcal{T}^\dagger M_d l_R + H.C \tag{10}$$

where

$$\mathcal{T}^\dagger = \frac{1}{\sqrt{3}}\begin{pmatrix}1 & 1 & 1\\ 1 & \bar\omega & \omega\\ 1 & \omega & \bar\omega\end{pmatrix}, \tag{11}$$

$l_L = (e_L, \mu_L, \tau_L)^T$, $l_R = (e_R, \mu_R, \tau_R)^T$ and $M_d = \text{diag}(m_e, m_\mu, m_\tau)$ with $m_e = \frac{y_e h_o}{\Lambda_{\phi_e}}$ etc. The charged-lepton mass matrix, $\mathcal{T}^\dagger M_d$, when left-multiplied with $\mathcal{T}$, is diagonalised giving the charged-lepton masses m_e, m_μ, m_τ. $\mathcal{T}$ is the Trimaximal mixing matrix [3]. For later use we define 2×2 maximal matrices, $\mathcal{B}_2$ and $\mathcal{B}_3$:

$$\mathcal{B}_2 = \begin{pmatrix}\frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}}\\ 0 & 1 & 0\\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\end{pmatrix}, \qquad \mathcal{B}_3 = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\\ 0 & 0 & 1\end{pmatrix}. \tag{12}$$

For the neutrinos, the lowest order Dirac mass term is

$$2y_w \overline{L}\xi_1\nu_R\tilde{H}\frac{1}{\Lambda_{\xi_1}} + H.C. \tag{13}$$

where $\tilde{H}$ is the conjugate Higgs and y_w is the coupling. The flavon ξ_1 belongs to the representation **6** of $SU(3)_1$ and can be written as a 3×3 complex-symmetric matrix. The restriction of **6** of $SU(3)$ to the S_4 subgroup is the direct sum of **1**, **2** and **3** of S_4. We assign a very simple choice of vev for ξ_1 where **2** and **3** parts vanish, ie. $\langle\xi_1\rangle$ becomes the identity when written in the matrix form. After the symmetry breaking, the Dirac mass term, Eq. (13), takes the form

$$m_w\left(\overline{\nu_L}\nu_R + \overline{(\nu_R)^c}(\nu_L)^c\right) + H.C. \tag{14}$$

where $\nu_L = (\nu_{eL}, \nu_{\mu L}, \nu_{\tau L})^T$, $m_w = \frac{y_w h_o}{\Lambda_{\xi_1}}$.

The lowest order Majorana mass term for the neutrinos is

$$y_G\overline{(\nu_R)^c}\phi\xi_2\phi^T\nu_R\frac{1}{\Lambda_{\xi_2}\Lambda_\phi^2} + H.C. \tag{15}$$

Note that ϕ transforms as a **3** under both $SU(3)_1$ and $SU(3)_2$. Therefore ϕ can be written as a 3×3 matrix, ϕ_{ij}, the row index i representing $SU(3)_1$ and the column index j representing $SU(3)_2$. The flavon ξ_2 belongs to the representation $\bar{\mathbf{6}}$ of $SU(3)_2$. A $\bar{\mathbf{6}}$ of $SU(3)$, just like a **6**, contains a **1**, a **2** and a **3** of the S_4 subgroup. As was done earlier for the case of $\langle\xi_1\rangle$, here we assign $\langle\xi_2\rangle$ also to be equal to the identity. After the symmetry breaking, the Majorana mass term, Eq. (15), takes the form

$$m_G\overline{(\nu_R)^c}\langle\phi\rangle\langle\phi\rangle^T\nu_R + H.C. \tag{16}$$

where $m_G = \frac{y_G}{\Lambda_{\xi_2}\Lambda_\phi^2}$. The matrix $\langle\phi\rangle\langle\phi\rangle^T$ is complex-symmetric and $\langle\phi\rangle\langle\phi\rangle^T$ contains all the interesting physics in our model. The reason why we use two $SU(3)$s in the flavour group G_f, Eq. (6), is to ensure that the mass matrix, Eq. (16), contains the symmetric product of two $\langle\phi\rangle$s.

To assign vev for the flavon ϕ_{ij}, we use $(S_4)_1\times(S_4)_2$, the subgroup of $SU(3)_1\times SU(3)_2$. The group $(S_4)_1\times(S_4)_2$ has $24\times 24 = 576$ elements. Let g_1 and g_2 be the elements of $(S_4)_1$ and $(S_4)_2$ respectively. If v_1 and v_2 are the eigenvectors of g_1 and g_2 corresponding to the eigenvalues a_1 and a_2, then the direct product $v_1\times v_2$ will be an eigenvector of $g_1\times g_2$ with an eigenvalue a_1a_2. Now we make the following assumption:

$$\langle\phi\rangle = v_1^1\times v_2^1 \;+\; v_1^2\times v_2^2 \;+\; v_1^3\times v_2^3 \;+\; v_1^4\times v_2^4 \tag{17}$$

where the RHS of Eq. (17) is the sum of four eigenvectors. Based on the choices for $v_1^i\times v_2^i$s we get a set of similar cases of solutions described in the following sections. The assumed form of $\langle\phi\rangle$ given in Eq. (17) and the choices for $v_1^i\times v_2^i$s were obtained through educated guesses and also through trial and error to fit the experimental data.

2.1. Case 1

Here we assign

$$\begin{aligned} &v_1^1 = e_2, \quad v_1^2 = \text{eig}(a,1)_1, \quad v_1^3 = \text{eig}(d,1)_1, \quad v_1^4 = \text{eig}(d,1)_3, \\ &v_2^1 = e_2, \quad v_2^2 = e_1, \qquad\qquad\quad v_2^3 = e_1, \qquad\qquad\quad v_2^4 = \text{eig}(c,\text{-}i)_1. \end{aligned} \tag{18}$$

Using Eq. (17), Eqs. (18) and Eqs. (4), we get

$$\langle\phi\rangle = \begin{pmatrix} -\frac{1}{2}+\sqrt{2} & 0 & \frac{i}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & -\frac{i}{2} \end{pmatrix} \tag{19}$$

in matrix form, where the row and the column indices of the matrix correspond to the $(S_4)_1$ and the $(S_4)_2$ indices respectively. If $m_G >> m_w$, the Majorana mass matrix, $m_G\langle\phi\rangle\langle\phi\rangle^T$ from Eq. (16), becomes much larger than the Dirac mass matrix, $m_w \times$ Identity from Eq. (14), resulting in the type-1 see-saw mechanism. We get an effective see-saw mass matrix [4], M_{ss}, proportional to the inverse of $\langle\phi\rangle\langle\phi\rangle^T$:

$$M_{ss} = -k\left(\langle\phi\rangle\langle\phi\rangle^T\right)^{-1} = -k\begin{pmatrix} 2-\sqrt{2} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix}^{-1} \quad \text{where} \quad k = \frac{m_w^2}{m_G}. \tag{20}$$

M_{ss} is diagonalised using the unitary matrix U_ν:

$$U_\nu^\dagger M_{ss} U_\nu^* = -\text{diag}\,(m_1, m_2, m_3) \tag{21}$$

where the neutrino masses m_1, m_2, m_3 are given by

$$m_1 = \frac{k\left(2+\sqrt{2}\right)}{1+\sqrt{2(2+\sqrt{2})}}, \qquad m_2 = k, \qquad m_3 = \frac{k\left(2+\sqrt{2}\right)}{-1+\sqrt{2(2+\sqrt{2})}} \tag{22}$$

and

$$U_\nu = \mathcal{B}_2\mathcal{I}\mathcal{B}_2\mathcal{E}\mathcal{B}_2^T\mathcal{P} \tag{23}$$

with

$$\mathcal{I} = \text{diag}(1,1,i), \quad \mathcal{E} = \text{diag}(e^{i\frac{\pi}{8}},1,1), \quad \mathcal{P} = \text{diag}(e^{-i\frac{\pi}{16}},1,e^{-i\frac{\pi}{16}}). \tag{24}$$

The PMNS matrix becomes

$$U = \mathcal{T}U_\nu = \mathcal{T}\mathcal{B}_2\mathcal{I}\mathcal{B}_2\mathcal{E}\mathcal{B}_2^T\mathcal{P}. \tag{25}$$

The mixing obtained is a constrained form of the Trichimaximal (TχM) mixing [5] with $\chi = \frac{\pi}{16}$:

$$|U| = |\text{T}\chi\text{M}_{(\chi=\frac{\pi}{16})}| \quad \text{where} \quad \text{T}\chi\text{M} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\chi & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin\chi \\ -\frac{\cos\chi}{\sqrt{6}} - i\frac{\sin\chi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & i\frac{\cos\chi}{\sqrt{2}} - \frac{\sin\chi}{\sqrt{6}} \\ -\frac{\cos\chi}{\sqrt{6}} + i\frac{\sin\chi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -i\frac{\cos\chi}{\sqrt{2}} - \frac{\sin\chi}{\sqrt{6}} \end{pmatrix}. \tag{26}$$

The modulus sign is used throughout this paper to indicate that the expression for the mixing matrix is valid only upto right and left multiplication with diagonal phase matrices (which do not affect the phenomenon of neutrino oscillation). The right multiplying diagonal phase matrices, like $\mathcal{P}$ in Eq. (25), do contribute to Majorana phases, the study of which is beyond the scope of this paper. From Eq. (26) and using Eq. (12) in [5] we get

$$|U_{e3}|^2 = \frac{2}{3}\sin^2\frac{\pi}{16} \;\Rightarrow\; \sin^2\theta_{13} = 0.025, \tag{27}$$

$$|U_{e2}|^2 = \frac{1}{3} \;\Rightarrow\; \sin^2\theta_{12} = 0.342, \tag{28}$$

$$\sin^2\theta_{23} = \frac{1}{2}, \tag{29}$$

$$\delta_{CP} = \frac{\pi}{2}. \tag{30}$$

From Eq. (22) we get the ratios of the neutrino masses, $m_1 : m_2 : m_3 = 0.945 : 1 : 2.117$. These ratios are compatible with the mass-squared differences measured experimentally [6, 7] within 1σ errors and thus we can predict the light neutrino mass:

$$24.7 \text{ meV} \lesssim m_1 \lesssim 25.5 \text{ meV}. \tag{31}$$

Even though the set of eigenvectors given in Eqs. (18) results in the matrix $\langle\phi\rangle$ given in Eq. (19), other choices of eigenvectors also exist which produce the same $\langle\phi\rangle$. The matrix $\mathcal{T}^\dagger\langle\phi\rangle\langle\phi\rangle^T\mathcal{T}^*$ is a highly constrained form of the complex-symmetric "Simplest" texture [8, 9].

Let $x = i^n$ where n is an integer and let

$$\Phi_x = \begin{pmatrix} i\frac{x}{2} + \frac{1-ix}{\sqrt{2}} & 0 & -\frac{x}{2} \\ 0 & 1 & 0 \\ -i\frac{x}{2} + \frac{1+ix}{\sqrt{2}} & 0 & \frac{x}{2} \end{pmatrix}. \tag{32}$$

Using Φ_x, Eq. (19) can be rewritten as

$$\langle\phi\rangle = \Phi_i^*. \tag{33}$$

2.2. *Case 2*

Assigning

$$\begin{array}{llll} v_1^1 = e_2, & v_1^2 = \mathrm{eig}(c,i)_1, & v_1^3 = \mathrm{eig}(c,i)_3, & v_1^4 = \mathrm{eig}(d,1)_1, \\ v_2^1 = e_2, & v_2^2 = e_1, & v_2^3 = e_1, & v_2^4 = \mathrm{eig}(c,\text{-i})_3. \end{array} \tag{34}$$

we get

$$\langle\phi\rangle = \Phi_{i^2}^*. \tag{35}$$

In this case, the resulting PMNS matrix is

$$U = \mathcal{T}U_\nu = \mathcal{T}\mathcal{B}_2\mathcal{I}^2\mathcal{B}_2\mathcal{E}\mathcal{B}_2^T\mathcal{P} \tag{36}$$

and it is a constrained form of the Triphimaximal (TϕM) mixing [5] with $\phi = -\frac{\pi}{16}$:

$$|U| = |\mathrm{T}\phi\mathrm{M}_{(\phi=-\frac{\pi}{16})}| \quad \text{where} \quad \mathrm{T}\phi\mathrm{M} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\phi & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin\phi \\ -\frac{\cos\phi}{\sqrt{6}} - \frac{\sin\phi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{\cos\phi}{\sqrt{2}} - \frac{\sin\phi}{\sqrt{6}} \\ -\frac{\cos\phi}{\sqrt{6}} + \frac{\sin\phi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\cos\phi}{\sqrt{2}} - \frac{\sin\phi}{\sqrt{6}} \end{pmatrix}. \tag{37}$$

From Eq. (37) and using Eq. (9) in [5] we get

$$|U_{\mu 3}|^2 = \frac{2}{3}\sin^2\left(\frac{2\pi}{3} - \frac{\pi}{16}\right) \;\Rightarrow\; \sin^2\theta_{23} = 0.613, \tag{38}$$

$$\delta_{CP} = \pi. \tag{39}$$

Here, as well as in the next two cases, Eq. (22) which gives the neutrino masses and Eqs. (27, 28) which give $\sin^2\theta_{13}$ and $\sin^2\theta_{12}$ remain valid.

2.3. Case 3

Assigning $v_1^1 = e_2$, $v_2^1 = e_2$, $v_1^2 = \text{eig}(a, 1)_1$, $v_2^2 = e_1$, $v_1^3 = \text{eig}(d, 1)_3$, $v_2^3 = e_1$, $v_1^4 = \text{eig}(d, 1)_1$ and $v_2^4 = \text{eig}(c, \text{-i})_1$ we get

$$\langle \phi \rangle = \Phi^*_{i3} \tag{40}$$

$$U = \mathcal{T}\mathcal{B}_2\mathcal{I}^3\mathcal{B}_2\mathcal{E}\mathcal{B}_2^T\mathcal{P} \;\Rightarrow\; |U| = |\text{T}\chi\text{M}_{(\chi=-\frac{\pi}{16})}|, \tag{41}$$

$$\sin^2\theta_{23} = \frac{1}{2}, \tag{42}$$

$$\delta_{CP} = \frac{3\pi}{2}. \tag{43}$$

2.4. Case 4

Assigning $v_1^1 = e_2$, $v_2^1 = e_2$, $v_1^2 = \text{eig}(c, \text{-}i)_1$, $v_2^2 = e_1$, $v_1^3 = \text{eig}(c, \text{-}i)_3$, $v_2^3 = e_1$, $v_1^4 = \text{eig}(d, 1)_3$ and $v_2^4 = \text{eig}(c, \text{-i})_3$ we get

$$\langle \phi \rangle = \Phi^*_{i4} \tag{44}$$

$$U = \mathcal{T}\mathcal{B}_2\mathcal{I}^4\mathcal{B}_2\mathcal{E}\mathcal{B}_2^T\mathcal{P} \;\Rightarrow\; |U| = |\text{T}\phi\text{M}_{(\phi=\frac{\pi}{16})}|, \tag{45}$$

$$|U_{\mu 3}|^2 = \frac{2}{3}\sin^2\left(\frac{2\pi}{3} + \frac{\pi}{16}\right) \;\Rightarrow\; \sin^2\theta_{23} = 0.387, \tag{46}$$

$$\delta_{CP} = 2\pi. \tag{47}$$

The values predicted by all the four cases of the model are within 3σ errors of the experimental best fits [6, 7, 10]. In fact the generic prediction $\sin^2\theta_{13} = 0.025$, Eq. (27), is within 1σ errors. However the global analysis [6] shows more than 2σ tension with $\sin^2\theta_{23} = \frac{1}{2}$, the T$\chi$M value (Cases 1 and 3, Eqs. (29, 42)). On the other hand the TϕM values, $\sin^2\theta_{23} = 0.613$ from Eq. (38) in Case 2 and $\sin^2\theta_{23} = 0.387$ from Eq. (46) in Case 4, are well within 1σ errors calculated in [7] and [6] respectively. All the cases predict $\sin^2\theta_{12} = 0.342$, Eq. (28), which is at the edge of the 2σ error range in [6]. A new mixing ansatz called the VS mixing [1] is proposed in the following section which modifies θ_{12} as well as δ_{CP}.

3. The VS Mixing Ansatz

The mixing obtained using the model, Eqs. (25, 36, 41, 45), is of the form

$$|U| = |\mathcal{T}\mathcal{B}_2\mathcal{I}^n\mathcal{B}_2\mathcal{E}\mathcal{B}_2^T|. \tag{48}$$

The matrix $|\mathcal{T}\mathcal{B}_2\mathcal{I}^n|$ gives the Tribimaximal (TBM) mixing [11]. Multiplying $\mathcal{T}\mathcal{B}_2\mathcal{I}^n$ with $\mathcal{B}_2\mathcal{E}\mathcal{B}_2^T$ mixes the first and the third columns of the TBM matrix. Similarly we may also mix the first and the second columns resulting in the new ansatz defined by

$$|\text{VS}_{i^n}(\alpha)| = |\mathcal{T}\mathcal{B}_2\mathcal{I}^n\mathcal{B}_2\mathcal{E}\mathcal{B}_2^T\mathcal{B}_3\mathcal{E}'\mathcal{B}_3^T| \tag{49}$$

where

$$\mathcal{E}' = \text{diag}(e^{i\alpha}, 1, 1). \tag{50}$$

Note that the Cases 1 to 4 are simply $\text{VS}_{i^n}(0)$ with $n = 1$ to 4 respectively. Eq. (49) on simplification gives

$$|\text{VS}_{i^n}(\alpha)| = |\mathcal{T}\mathcal{B}_2\mathcal{I}^n\mathcal{H}_2\mathcal{S}\mathcal{H}'_3| \tag{51}$$

[1] Dedicated to my father K Venugopal and mother J Saraswathi Amma

where

$$\mathcal{S} = \mathrm{diag}(e^{i\frac{\pi}{16}}, 1, 1), \qquad \mathcal{H}_2 = \begin{pmatrix} \boldsymbol{c} & 0 & i\boldsymbol{s} \\ 0 & 1 & 0 \\ i\boldsymbol{s} & 0 & \boldsymbol{c} \end{pmatrix}, \qquad \mathcal{H}_3' = \begin{pmatrix} \boldsymbol{c}' & i\boldsymbol{s}' & 0 \\ i\boldsymbol{s}' & \boldsymbol{c}' & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{52}$$

with

$$\boldsymbol{c} = \cos\frac{\pi}{16}, \qquad \boldsymbol{s} = \sin\frac{\pi}{16}, \qquad \boldsymbol{c}' = \cos\frac{\alpha}{2}, \qquad \boldsymbol{s}' = \sin\frac{\alpha}{2}. \tag{53}$$

We get $\sin^2\theta_{12}$ within 1σ errors for $0.08\pi \lesssim \alpha \lesssim 0.26\pi$. Table 2 lists a few cases of the VS mixing along with the predicted values of the mixing angles. The author finds the choice $\alpha = \frac{\pi}{8}$ to be aesthetically pleasing. When $\alpha = \frac{\pi}{8}$ we get $\mathcal{E}' = \mathcal{E}$ and also $\boldsymbol{c}' = \boldsymbol{c}$, $\boldsymbol{s}' = \boldsymbol{s}$.

	$\mathrm{VS}_{\text{-}1}(\frac{\pi}{4})$	$\mathrm{VS}_1(\frac{\pi}{4})$	$\mathrm{VS}_{\text{-}1}(\frac{\pi}{6})$	$\mathrm{VS}_1(\frac{\pi}{6})$	$\mathrm{VS}_{\text{-}1}(\frac{\pi}{8})$	$\mathrm{VS}_1(\frac{\pi}{8})$
$\sin^2\theta_{23}$	0.613	0.387	0.613	0.387	0.613	0.387
$\sin^2\theta_{12}$	0.323	0.323	0.317	0.317	0.319	0.319
δ_{CP}	1.27π	0.27π	1.18π	0.18π	1.13π	0.13π

Table 2. Note that $\sin^2\theta_{13} = 0.025$ is a generic feature of the VS mixing. Conjugation, $\mathrm{VS}^*_{i^n}(\alpha)$, changes the sign of δ_{CP} without affecting the mixing angles θ_{12}, θ_{23} and θ_{13}.

4. Summary

The symmetries represented by a discrete group are related to the eigenvectors of the group elements. We develop a notation to uniquely identify the eigenvectors and use it to assign vevs for the flavons. An orthonormal set of eigenvectors define the fermions' flavour states. The model thus constructed predicts the reactor mixing angle, $\sin^2\theta_{13} = 0.025$, as well as the ratios of the neutrino masses, $m_1 : m_2 : m_3 = 0.945 : 1 : 2.117$. The T$\phi$M versions of the model provide solutions for θ_{23} in the first octant, $\sin^2\theta_{23} = 0.387$, as well as in the second octant, $\sin^2\theta_{23} = 0.613$. The T$\phi$M as well as the T$\chi$M versions give $\sin^2\theta_{12} = 0.342$. A new mixing ansatz, $\mathrm{VS}_{i^n}(\alpha)$, is introduced which gives reduced values for θ_{12}. The ansatz also predicts various values for δ_{CP}.

Acknowledgments

Acknowledgments I would like to thank Paul Harrison and Bill Scott for helpful discussions. This work was supported by the UK Science and Technology Facilities Council (STFC). I acknowledge support from the University of Warwick and the Centre for Fundamental Physics at the Rutherford Appleton Laboratory.

References

[1] Daya Bay Collaboration 2012 *Phys. Rev. Lett.* **108** 171803 (*Preprint* 1203.1669)
[2] Ishimori H, Kobayashi T, Ohki H, Shimizu Y, Okada H and Tanimoto M 2010 *Prog. Theor. Phys. Suppl.* **183** 1–163 (*Preprint* 1003.3552)
[3] Harrison P F, Perkins D H and Scott W G 1999 *Phys. Lett. B* **458** 79–92 (*Preprint* hep-ph/9904297)
[4] Smirnov A Y 1993 *Phys. Rev. D* **48** 3264–3270 (*Preprint* hep-ph/9304205)
[5] Harrison P F and Scott W G 2002 *Phys. Lett. B* **535** 163–169 (*Preprint* hep-ph/0203209)
[6] Fogli G L, Lisi E, Marrone A, Montanino D, Palazzo A and Rotunno A M 2012 *Phys. Rev. D* **86** 013012 (*Preprint* 1205.5254)
[7] Forero D V, Tortola M and Valle J W F 2012 *Phys. Rev. D* **86** 073012 (*Preprint* 1205.4018)
[8] Harrison P F and Scott W G 2004 *Phys. Lett. B* **594** 324–332 (*Preprint* hep-ph/0403278)
[9] Krishnan R, Harrison P F and Scott W G 2012 (*Preprint* 1211.2000)
[10] Gonzalez-Garcia M C, Maltoni M, Salvado J and Schwetz T 2012 (*Preprint* 1209.3023)
[11] Harrison P F, Perkins D H and Scott W G 2002 *Phys. Lett. B* **530** 167–173 (*Preprint* hep-ph/0202074)