EXCHANGE RATES, EXPECTATIONS AND INTERNATIONAL TRADE: THEORY AND EVIDENCE

by

Christopher John Williams

December 1990.

Thesis submitted for the Degree of Doctor of Philosophy.

University of Warwick,
Department of Economics.

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A SUMMARY.

Unprecedented movements in real exchange rates during the 1980s led to suspicions of instability in the exchange rate - trade relationships in the UK and elsewhere. The research in this thesis investigates the sensitivity of UK trade volumes to movements in the real exchange rate, and considers various interpretations of the alleged parameter instability: econometric misspecification; theoretical inadequacy due to the neglect of possible hysteresis effects and/or the neglect of supply side factors; and the Lucas critique effects of a changed policy regime on expectations formation. Against the background of UK experience we examine specific questions of theory and evidence within partial equilibrium frameworks. These share a common concern: considering the (macroeconomically important) case of mean reversion in real exchange rate expectations.

Chapters two and three introduce mean reversion into Dixit's (1989a) theory model of sunk cost hysteresis in trade. This research uses both analytic and numerical methods to characterise solutions with mean reversion in greater detail than elsewhere and uncovers some striking and unexpected results. Most important is the possible reversal of the stochastic and perfect foresight triggers under asymmetric sunk costs which reflects the essential difference between costly reversibility and strict irreversibility in investment. Uncertainty does not always delay action, because the possibility of reversal must be allowed for. Chapter four explores the wider significance of the analysis for similar stochastic saddlepoint models such as the analysis of exchange rate target zones.

Chapters five and six consider the significance of the short run dynamic specification of quarterly UK manufactured export volumes equations to the reported instability in estimates of the long run competitiveness elasticity in the light of evidence that UK competitiveness measures follow stationary processes within an institutionally identified policy regime. Hausman specification tests, show that the long run competitiveness elasticity is misspecified and underestimated in recent (error correction mechanism) specifications of UK manufactured export volume equations. This inadequacy reflects the omission of long 'smoothing' lags on the competitiveness variable

Subsequently, chapter seven considers simulation evidence from the Dixit model as to the potential relevance of such effects to the UK experience under the large shock to competitiveness of 1980-1 but emphasises that the aggregate implications are not clear cut. Chapter eight considers whether the expectational effects of the 1979 Thatcher government's change in policy regime can be separated out from the other influences at work behind reduced form models but finds that the data do not support the particular approach adopted.

Concluding, we emphasise that the potential importance and complexity of expectational factors and theory combines with the our empirical findings to suggest that exchange rate uncertainty may be crucial to trade behaviour, and that macroeconomic adjustment may be inhibited by excess exchange rate uncertainty. Overall export performance may also reflect supply factors which are not captured in existing models, such as hysteretic exit, or expected cost changes. But we doubt whether future research will achieve a data consistent aggregate econometric model of UK trade which is fully grounded in appropriate optimising economic theory with realistic adjustment costs. We may have to settle for approximations to the data generation process which do not employ recent theoretical insights. In that event, the use of such models in policy design should be circumscribed due to the possibility of Lucas critique effects, hysteresis mechanisms and supply side factors.

This thesis is dedicated to the memory of my grandparents, Edward and Nellie Williams and Adam and Muriel Kessen. They suffered the vicissitudes of the great depression and the collapse in international trade, but they did not lose their values.

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ACKNOWLEDGEMENTS.

I acknowledge the financial support of the Economic and Social Research Council under linked award number B00428525063 entitled "Macroeconomic and Econometric Problems in Modelling the UK Economy".

However this research could not have been completed without the extraordinary financial and material support provided by Joan McFarland, Millicent English, Frank and Norah Williams and Laurel McFarland. For this kind assistance, I express my deep gratitude. Invaluable logistical help has also been provided by Betty, Roy and Paul Davies, by the Department of Economics and by the Institute of Employment Research at the University of Warwick.

I owe a substantial intellectual debt of gratitude to my supervisors, Professors Marcus H. Miller and Kenneth F. Wallis for their guidance, advice and encouragement. Not the smallest part of this debt is for their good natured tolerance of the particular set of agendas and priorities - both geographical and intellectual - by which I constrained the process of research. Their incisive questions often pointed the way through difficulties. If something of my supervisors' contrasting but complementary approaches to macroeconomics is discernible in the finished product, it should be credited to their role as mentors. But insofar as the research fails to carry off the combination of theoretical and empirical approaches that is my responsibility.

A special debt is due to Avinash Dixit whose generous attention to my work was of itself a great encouragement to persist at a difficult early stage. I also thank Professor Dixit for his prompt and serious consideration of my theoretical findings in relation to his continuing research on "Investment and Hysteresis".

I have greatly appreciated the friendly support of many of the staff of the Department of Economics at Warwick. I have benefitted from discussions at Warwick with Martin Cripps, Mark Salmon, Alan Sutherland, and Dave Turner who have helped me with specific problems arising in my research. I also benefitted from the advice of Andrew Carverhill and Nick Webber of the ESRC Financial Options Research Centre.

I have been grateful for the questions and comments of seminar participants at Warwick, the Johns Hopkins S.A.I.S., and the Congressional Budget Office, and for the opportunity to attend conferences by CEPR and the Brookings Institution.

Amongst my fellow graduate students I have gained from specific discussions with Ioannis Ganoulis, Richard Hammersley, Ming-Sheng Hwang, Reza Moghadam, Nehemiah Ngeno, Barry Reilly, Mary Zephirin, Lei Zhang, and most of all, Paul Warren.

I am grateful for the assistance or advice on logistical matters of both staff and students in the Department of Economics, the ESRC Macroeconomic Modelling Bureau, the Development Economics Research Centre, and the Institute for Employment Research at the University of Warwick, especially Graham Wright, Lisa Hayes, and Liz Thompson. In addition I must record my particular thanks to Paul Fisher for his unfailing helpfulness on all matters relating to macroeconomic models and computing, and to Rob Wilson in IER for his tolerance of my thesis priorities.

The completion of doctoral research is a major rite of passage, and as such it is fitting to acknowledge the wide range of teachers and colleagues who have shaped my professional outlook as an economist over a decade of study and research in different places. I am conscious of very many personal contributions, and, at the risk of unintended omissions, I wish to recognise them. In addition to those already mentioned I must thank: David Begg, Christopher Smallwood, Peter Kenen, Bill Branson, Alan Blinder, John Taylor, Carl Walsh, Charlie Bean, Jim Trussell, 'Bill' Brown, Angus Deaton, Jan Magnus, Andrew Harvey, Jim Davidson, Wiji Narendranathan, Bobby Willig, Carl Shapiro, David Bradford, Kevin Roberts, Paul Weller, Norman Ireland, Keith Cowling, Richard Layard, Richard Jackman, Jaime Garcia, Olympia Bover, Elias Dinenis, Ze Pedro Barosa, Jim Symons, Andy Newell, Kevin Lee, Andy Sentance, Jon Haskell, Andrew Oswald, Martyn Andrews, Sailesh Tanna, John Whitley, Chris Harris, Anne Case, Calum Carmichael, Julie Nelson, Linda Goldberg, Duncan Thomas, Becky Blank, Claudio Lucifora, Henrik Vetter and Ann Gibbons.

I record the encouragement I received from fellow graduate students at Warwick and elsewhere as they completed their doctoral studies. From a long list I must single out Hazel Williams, Mike Taylor, Massimo, Ngeno, Mary, Ioannis, Hwang, Mabel and Elmer.

My family and friends have given me unfailing (even when uncomprehending) support during my doctoral research. My special appreciation goes to Geoff and Coral, Tim, Andy and Mary, Kirsten English and David Kessen, and to Stephen, Martin and Barbara, who all kept asking the same question in the gentlest possible way; to the Kenpas crew for episodes of light relief; and to Karen and Paul for their friendship. Adam, Benjamin, Thomas and Heather merit a special mention just for being themselves.

I am especially grateful to my parents, Frank and Norah Williams, for their understanding and concern, and to Joan McFarland, who kept "my eyes on the prize."

My final thoughts are reserved for the incomparable Laurel McFarland, with my thanks for her belief, encouragement, cajoling, inspiration, example and unconditional but not uncritical support; for her invaluable advice on the forging of a coherent thesis; and most of all for making everything worth the effort.

LIST OF ABBREVIATIONS.

BE	Bank of England.
COMC	LBS Cost Competitiveness (See appendix 5.1).
COMT	LBS Tax Adjusted Cost Competitiveness (see appendix 5.1).
DGP	Data Generation Process.
ECM	Error Correction Mechanism.
HMT	Her Majesty's Treasury.
I(0)	Integrated of order zero.
I(1)	Integrated of order one.
iid	identically and independently distributed.
IV	Instrumental Variables.
LBS	London Business School.
MUKE	Models of the UK Economy.
NIESR	National Institute of Economic and Social Research.
OLS	Ordinary Least Squares.
PDL	Polynomial Distributed Lag.
XMAN	Volume of UK Manufactured Exports (see appendix 5.1)
XWM	Volume of World Manufactured Exports (see appendix 5.1)

Chapter One.

AN INTRODUCTION TO EXCHANGE RATES, EXPECTATIONS AND INTERNATIONAL TRADE: THEORY AND EVIDENCE.

At the start of the 1980s the British economy experienced an unprecedented appreciation of some 30 or 40 per cent, (depending on definitions), in the Sterling real exchange rate.[1] Yet, despite the magnitude of this deleterious shock to competitiveness, UK export volumes turned out notably higher than predicted in both 1980 and 1981. One explanation offered for this striking predictive failure was that some kind of structural change had occurred in underlying UK export behaviour: Walters (1986), for example, argued that the quality of UK exports had improved, and also that UK export suppliers may have been forced to improve productivity by the severity of the macroeconomic squeeze. Alternative explanations were put forward by Buiter and Miller (1981), who argued that exports might actually respond to "'permanent' or long run rather than current competitiveness", particularly if "large set up costs" were important; they argued further that, even if the shock to competitiveness were expected to be permanent, the response of export volumes would normally involve a considerable lag. Thus UK export behaviour of the early 1980s exposed limitations in the existing empirical models of export behaviour, and also, as the conjectured explanations indicate, in their theoretical underpinnings. The modelling of the trade linkages in international adjustment has also been challenged by the experience of the global imbalances of the 1980s, with the associated large swings in the dollar and yen. This thesis aims to contribute to the continuing re-assessment of the trade component in international macroeconomic analysis.

The research presented in this thesis addresses specific theoretical and empirical issues in the modelling of export volumes, with reference to the UK experience. Our concern with export volumes is complementary to the emphasis on export pricing in much of the recent literature stemming from US experience. Our particular focus is on the sensitivity of trade volumes to movements in real exchange

1. The real exchange rate can be thought of as the relative price of exportables to importables, or of traded to non-traded goods, depending on the simplifying assumptions underlying the conceptual model employed. Under constant mark-up pricing assumptions it can be thought of as relative unit labour costs.

rates, and on the hypothesis that predictive failure reflected a fall in the estimated competitiveness elasticity of export behaviour. We consider whether predictive failure could be explained by inappropriate assumptions in the dynamic specification of empirical models of UK manufactured export volumes. Turning to the theoretical underpinnings of export behaviour, we examine the neglect of expectational influences, and explore the possibility of sunk cost hysteresis effects in export supply. We seek to link our theoretical and empirical considerations: to implement data-based expectations models of export volumes; and to assess misspecification, the Lucas Critique and structural change as explanations of instability in the estimated long run competitiveness elasticity. The specific elements of this research relate more or less directly to the partial equilibrium analyses which themselves serve as the building blocks for the macroeconomic analysis of international adjustment.

A unifying theme in our search for theoretical and empirical explanations of apparent parameter instability is the importance of the stationarity of the real exchange rate. In our theoretical work we investigate the Dixit model of sunk cost hysteresis in trade when the real exchange rate follows a mean reverting process. In our empirical work we re-examine the dynamic specification and parameter stability of export volume equations under the assumption that the real exchange rate is stationary. At the heart of these considerations is the notion that expectations concerning future real exchange rates are important determinants of export volumes: the sensitivity of exports to a given exchange rate movement will depend on the expected permanence of the shock.

Our theoretical research confirms some predictions, but also reveals some unlooked-for features of the Dixit model with mean reversion. The impact of mean reversion on the solution depends on the degree of uncertainty concerning the future. Most notably, we show how uncertainty can hasten state transitions rather than delay them. Our empirical research confirms that the estimated long run response of exports to the real exchange rate is sensitive to assumptions concerning the variable's stationarity; but our attempts to connect this result with a formal treatment of expectations prove unsuccessful. Nevertheless, taken as a whole, our theoretical and empirical studies support the hypothesis that expectations concerning the future complicate the modelling of the relationship between exports and exchange rates.

In the remainder of this introductory chapter we will highlight the background to this research in the literature on international macroeconomic adjustment and in the experience of the UK and elsewhere. We will then set out the structure of the research presented in subsequent chapters.

The research in this thesis can be placed within a broad literature on the international transmission mechanism which has developed in response to the evolving experience of macroeconomic management with floating, managed floating and pegged but adjustable exchange rates and increased capital mobility. This literature has included attempts at the integration of asset and goods market approaches to the theory of exchange rate determination, and the often discouraging task of the empirical implementation and evaluation of theoretical frameworks.[2] Nevertheless, uncertainty persists concerning the sources and transmission of shocks between markets and between countries.[3] From a policymaker's perspective the search continues for a consensus macroeconomic framework within which the choice of targets and the appropriate balance between stabilisation and adjustment in response to shocks can be assessed.[4] Active debate concerns the choice of exchange rate regime, in which context familiar macroeconomic policy arguments concerning rules versus discretion and policy autonomy versus co-ordination have been developed and deployed.[5]

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2. For theoretical frameworks see eg. Dornbusch and Fischer (1980), Hooper and Morton (1982), Frenkel and Mussa (1985), and chapter four of Wallis et al (1985). Bruce and Purvis (1985) consider the implications of various goods market specifications while Branson and Henderson (1985) consider asset markets. Barr (1984), Blackburn (1987) and Smith (1987) are among continuing developments of the sticky-price monetary and portfolio balance approaches developed from the work of, *inter alia*, Dornbusch (1976), Branson (1977), Buiter and Miller (1982) and Mussa (1984).

The implications of intertemporal optimisation are pursued by Persson and Svensson (1985) and Frenkel and Razin (1987), while the equilibrium determinants of real exchange rates have been considered by Balassa (1964), Bruno (1976), Hseih (1982), Neary (1988) and Edwards (1989). (Williamson (1983) has attempted to apply such notions).

Isard (1988) and MacDonald and Taylor (1989) offer reviews of the state of exchange rate modelling. For UK empirical work see Barr (1989), Smith and Wickens (1987), and Fisher et al (1989).

3. See eg. Marquez and Ericsson (1990) and other papers from the Brookings conference on Empirical Evaluation of Alternative Policy Regimes, March 1990.

4. See eg. Kenen (1988)'s view that basic disagreements about macroeconomics dominate any subsequent analysis of specific issues of international macroeconomics.

5. References to the exchange rate policy debate include: Krugman (1987), Miller, Weller and Williamson (1988), Miller, Eichengreen and Portes (1989), Krugman and Rotemburg (1990).

Within this literature on international macroeconomics, our research addresses the sensitivity of trade flows to exchange rate movements. In doing so it draws on two quite different approaches, which we argue are complementary rather than contradictory. Our theoretical analysis follows the literature seeking optimising microeconomic foundations for aggregate behaviour, while our empirical work seeks to re-evaluate existing relationships within conventional macroeconometric models.[6] What is common to both approaches is the attempt to explain alleged or apparent instability in the relationship between trade flows and exchange rates, and thereby improve on our analysis of the international transmission mechanism.

After the switch to flexible exchange rates during the 1970s, the first half of the 1980s demonstrated that a regime of flexible exchange rates did not provide complete insulation for the domestic economy from the effects of international shocks.[7] A central feature of the monetary transmission mechanism during the early 1980s was that swings of unprecedented magnitude occurred in the real exchange rates of the UK, and shortly thereafter, the US. The Japanese Yen underwent a major real appreciation after the Plaza accord of 1985. Much of this volatility in real exchange rates reflected volatility in nominal exchange rates. A substantial literature has considered whether this measured volatility has been accounted for by economic fundamentals, or whether it is better understood as having been generated within exchange markets themselves, through phenomena such as 'bubbles', 'fads', and 'noise trading'.[8] It is not clear however that the *source* of the exchange rate volatility as opposed to the fact of volatility itself should matter to the behaviour of trade in goods.

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6. The optimising approach to macroeconomics is presented in the textbooks by Sargent (1979), (1987), while the standard reference for the 'conventional' macroeconometric approach to trade flows is Goldstein and Khan (1985).

Attempts to combine these approaches and model import demand as the demand for a factor of production are made in Kollintzas and Husted (1981, 1983) and Husted and Kollintzas (1987). Diewert and Morrison (1983) tackle export supply and import demand from production theory.

7. The international transmission and spillover of output and price shocks proved sufficient that hitherto sceptical governments adopted measures of international policy coordination during the mid 1980s. Funabashi (1988) analyses the political economy of the Plaza and Louvre accords.

8. Krugman (1989a) discusses some of these possibilities. See Miller and Weller (1990a) for further references.

Krugman (1989a) has argued that the large amount of work which the international transmission mechanism was called upon to perform in the 1980s exposed inadequacies in previous modelling approaches to these areas. In particular, the responsiveness of international trade imbalances to the large actual movements in real exchange rates was claimed to have differed from the predictions of existing models. As has already been noted, such a phenomenon was reported in UK export volumes equations in 1980/1. The Krugman thesis of 'exchange rate instability' developed in (1989a) suggests that the optimal response of exporters to exchange rate changes would depend on their perceived volatility, so that exchange rate uncertainty would therefore inhibit adjustment; ironically, this effect would itself magnify exchange rate volatility.

We now turn to a review of an important episode in UK experience. In 1979 and 1980 the nominal effective exchange rate for Sterling appreciated around 20 per cent.[9] Given sticky prices and wages, the UK tradeables sector suffered a severe squeeze on its profitability: indeed, given the contemporary surge in domestic cost inflation, the appreciation in the real exchange rate over these years was notably larger than that in the nominal exchange rate. Cost competitiveness worsened by almost 50 per cent, while, even allowing for UK exporters' reluctance to pass cost increases into prices, price competitiveness worsened by more than 20 per cent.[10] Walters (1986) argues that:

"Even if one ignores the sharp peak of January 1981, the real appreciation of sterling clearly exceeded 30 per cent - and perhaps the best figure to be used for this period is a real appreciation of 35 to 40 per cent." (p 161)

A principal cause of the Sterling real appreciation was the tight monetary policy adopted by Mrs. Thatcher's government in 1979. Burns (1988, p 435) concludes that the "dominant mechanism" of monetary policy at this time in the UK worked through the exchange rate, while, as Walters (1986, p 142) puts it: "The rise in Sterling was part and parcel of the squeeze". Minford (1988) argues that the

9. Buiter and Miller (1981b, p 330) report that the Sterling effective index with IMF MERM weights increased 21/6% from 1979:1 to 1980:4.

10. Buiter and Miller (1981b, p 330) report that IMF Relative Normalised Unit Labour Costs show a 46.2% loss of competitiveness from 1979:1 to 1980:4, while relative export prices increased by 21%, and relative wholesale prices by 30%.

great severity of this monetary squeeze was unintended, and was a consequence of the misleading behaviour of the chosen monetary indicator, Sterling M3.

Buiter and Miller (1981a, 1982) have analysed the mechanisms by which a real appreciation can be explained as an overshooting phenomenon in a sticky price monetary model. While acknowledging that:

"North Sea oil, together with the rise in world oil prices, may have contributed to the appreciation of Sterling and the loss of competitiveness", (1981b, p 336)

Buiter and Miller (1981b) regard the monetary policy effect as "probably even more important". However, Buiter and Miller (1983) give a more circumspect account of these events, suggesting that, *ex post*, "the loss of competitiveness is not to be attributed to economic fundamentals", (p 321). In his comment on Buiter and Miller (1983), Branson offers an *ex post* rationale to buttress the monetary policy explanation of the appreciation by assuming that the inflation differential was unanticipated, and that monetary policy was expected to remain reasonably tight.[11]

We have already reported that, in the face of the sharp worsening in competitiveness, UK export volumes in 1980 and 1981 were surprisingly high. Buiter and Miller (1981b, p 330) report that "exports were maintained relatively well while inventories and production were cut back". Walters (1986) notes that:

"while the production of manufactures fell approximately 15 per cent, exports of manufactures actually *increased* slightly by about 1 per cent", (p 166).

before drawing the conclusion that:
"clearly, the manufacturing sector was not decimated by the decline in exports due to the appreciation of Sterling", (p 166).

This surprising strength of UK export volumes in 1980/1 revealed an inadequacy of the pre-existing empirical models. Attempts to explain the phenomenon reveal further theoretical concerns, which are not addressed in the pre-existing analyses of export behaviour.

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11. Bean (1987a) attributes a large part of the appreciation to the short run excess demand effects of the resource discovery of North Sea oil and a somewhat smaller proportion to the monetary policy channel than do the other studies considered above.

Buiter and Miller (1981a, p 363) offered three types of explanation for the behaviour of export volumes in 1980/1. The first argument is that the smaller than predicted effects of the appreciation were a consequence of the importance of expectations: agents viewed the 1980 shock to competitiveness as a temporary one. The predictive failure may thus have reflected the Lucas Critique effects of the change in exchange rate policy in 1979. The second argument is presented with the first. Given "large set up costs of re-entering foreign markets", exporters could be expected to incur losses in the short run. This is the same insight subsequently developed in the theoretical literature on sunk cost hysteresis, in Baldwin and Krugman (1989) and Dixit (1989a) etc. Although it is a distinct effect, this second point also suggests that behaviour will be sensitive to shifts in expectations of competitiveness. The third argument points to the plausibility of a long lag structure in the response of export volumes to changes in competitiveness due to delivery lags, etc. This point suggests that predictive failure may simply reflect errors in the dynamic specification of export volume equations. In this research we consider aspects of all three explanations proposed by Buiter and Miller: the dynamic specification of export volume equations, sunk cost hysteresis effects, and the effects of shifts in the expected behaviour of competitiveness.

A significantly different approach to the performance of UK exports in the early 1980s is adopted by, *inter alia*, Walters (1986) and Maynard (1988). This approach argues that export behaviour in 1980/1 and thereafter reflects fundamental improvements in the supply side of the UK economy. A variety of distinct mechanisms have been sketched out through which government policies could have influenced trade behaviour through the supply side, but, as yet, few formalisations of these arguments have been put forward. These accounts suggest that supply side improvements might be revealed in export volume equations, and therefore they emphasise the need for caution in interpreting export volume equations in the conventional way as demand equations. Walters (1986, p 141) conjectures that an improvement in the quality of UK exports had occurred by 1980/1. Walters (1986, p 168) also acknowledges the 'Cold Shower' argument that "export industries were forced to improve productivity more than anyone thought possible", before noting that this argument "is difficult to develop in terms of economic theory".

A number of different types of mechanism might produce supply side improvements which could, in turn, be detected in export performance. Firstly, labour market reforms could provide for an increase in productivity by weakening union power and altering bargaining outcomes, as considered in Layard and Nickell (1988) and Bean and Symons (1990).[12] Secondly, as Maynard (1988) argues, the removal of restrictions on capital mobility may have increased the pressures on managers to deploy capital to its maximum efficiency. A third mechanism is through the change in the overall macroeconomic framework which established, (eventually - see Minford (1988) on this point), a credible commitment not to validate inflationary pay settlements, and to be prepared to tolerate the consequent unemployment. Buiter and Miller (1983) characterise the government's strategy as one of "changing the rules", with consequences for aggregate labour market outcomes. The labour market consequences would in turn have an impact, (as yet still to be formalised), on export supply.

Maynard (1988), offers a particular variant on the 'Cold Shower' story, which identifies "the central role of the exchange rate" (p 158), arguing that the refusal:

"to 'accommodate' rising costs and poor productivity with exchange rate depreciation ... imposed pressure on industry to raise productivity, lower costs, and generally up-market its products." (Maynard 1988, p 158).

On this view, the 'shock treatment' of sharp appreciation may have been a "blessing in disguise", (p 158), "encouraging a change in industrial structure and a rise in labour and capital productivity", (p 159). The appreciation of Sterling may not only have made it necessary, but also possible, for management to overcome the costs of bargaining and of re-organisation in order to achieve gains in cost control and productivity.

We have not attempted to develop a formal treatment of the effects of improvements in supply side performance on trade behaviour in this research. Instead we confine our attention to these arguments to

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12 Layard and Nickell (1988) have offered an assessment of 'The Thatcher Miracle' which stresses the inflationary labour market implications of real exchange rate depreciation. Bean and Symons (1990) focus on explanations of permanent productivity catch up which stems from labour market developments, but acknowledged that a real appreciation would temporarily strengthen management resistance to wage pressure.

two aspects: the overlap with the supply side element in sunk cost hysteresis; and the possibility that there may be structural changes in the competitiveness elasticity if UK export volumes which could be viewed as manifestations of such supply side developments.

A particular starting-off point for the empirical research in this thesis was provided by the 1985 Autumn Statement to Parliament by the then UK Chancellor of the Exchequer, Nigel Lawson. In 1985, (not unlike the circumstances of mid-1990), Sterling had appreciated markedly over the period prior to the Chancellor's statement: the Chancellor therefore sought to downplay fears as to the possible consequences of the appreciation. In doing so, an important part of his argument was that the sensitivity of the UK tradeables goods sector to a rise in the real exchange rate had fallen during the 1980s, and in particular, that the long run competitiveness elasticity of UK manufactured export volumes had fallen. The UK government's view was evidently that some kind of structural change had occurred in the trading performance of the UK economy, with the implication for macroeconomic policy that the anti-inflationary benefits of a strong currency could be enjoyed at a lower cost in terms of demand for UK exports than hitherto. However, although parameter instability might reflect structural change due to supply side improvements in the trading performance of the British economy, there are other possible explanations to be considered: dynamic misspecification; the expectational effects of a regime change as in the Lucas critique; and hysteresis effects reflecting the loss of steady state exporting capacity.[13]

Krugman and Baldwin (1987), Krugman (1989a), the Bryant, Holtham and Hooper (1988) volume and Hooper and Mann (1987, 1989) are amongst studies of the US experience in the 1980s during which the dollar real exchange rate took a 'round trip' beginning in 1980, peaking in 1985 after an appreciation of some 40 per cent, before returning by the end of 1987 to near its start of decade value. As in the UK, the huge swing in competitiveness proved a stern test for existing models of US trade, but given the full US macroeconomic context, the question was posed as one as to why, after a large real depreciation, the external deficit appeared to persist. Was the traditional adjustment mechanism

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13. See Williams (1990) for a simplified review of these issues in the policy context of the 1985 Autumn Statement.

working, or had the traditional models been tested to destruction by the 1980s? The comparison with the UK case is instructive.

Baldwin and Lyons (1988) summed up the results of earlier empirical studies as having shown that "import and export demand elasticities have been stable in the UK since the 1960s". However, researchers suspected instability in US import pricing equations, and sought to explain the behaviour of exchange rate pass-through into prices. The contributions of Dornbusch (1987), Baldwin (1988), Froot and Klemperer (1989) and Ohno (1989) were followed by Hooper and Mann (1989)'s judgement that "changes in the pricing behaviour of foreign manufacturers" were "weakening the link between exchange rates and prices". When the dollar fell, importers allowed their margins to be squeezed and apparently sought to maintain market share.

The US literature has therefore focussed on possible instabilities in trade pricing rather than on trade volumes. Nevertheless, Hooper and Mann (1989) do report overprediction of US export volumes as the real exchange rate fell in 1986/7, while Helkie and Hooper (1988, Figure 2.7) report the underprediction of export volumes during the real appreciation of 1984/5. It turns out that these results are qualitatively similar to the underprediction of UK exports reported in 1980/1, which we have already indicated has been judged in the literature to be worthy of further investigation.

Bryant and Holtham (1988) take as "strong evidence against the assertion that there has been a structural break in historical relationships" the fact that prediction errors in simulations of the US trade balance are not sustained or cumulating. But it appears that the US experience may be consistent with some kind of *temporary* instability in the export volume relationships during the extreme swings in competitiveness, despite the understandable emphasis in the literature on stronger effects in pricing. Even if statistically significant parameter instability is not detected, the evidence is at least suggestive of the importance of expectational effects in export volumes.

Our reading of the literature on the US experience suggests that the behaviour of export volumes, though not so striking a feature of the data, may nevertheless be consistent with that reported from the

UK appreciation. The literature has offered alternative expectations and structural change explanations of observed UK export behaviour, in addition to careful attention to the dynamic specification of export volumes.

Corker (1989) considered the stability of econometric models of Japanese trade in the face of the sharp appreciation of the Yen after the Plaza agreement of 1985. He was unable to reject parameter stability and he found that Japanese trade had indeed responded to the changes in competitiveness as the traditional elasticities approach suggests. It is worth noting a contrast with the alleged instability in UK and US trade equations: whereas both the pound Sterling and the dollar experienced short-lived 'overshooting' round trips, the Plaza Yen revaluation appears to have been a one-off, durable realignment.

A brief outline of the contents of the thesis now follows.

The first part of the thesis is theoretical, and its main aim is to introduce mean reversion into the stochastic process for the exchange rate in the Dixit model of entry and exit under uncertainty. Sunk cost hysteresis effects in trade are a potential cause of instability in estimated trade equations, but consideration of the UK experience of the early 1980s suggests that the properties of the model with a mean reverting exchange rate may be of particular interest.

In chapter two we introduce the Dixit model, explaining its solution as one to a stochastic control problem and as an exercise in option pricing. We show that the trigger price solutions are independent of the endpoint boundary conditions applied to the value functions, and we offer an economic explanation of this property.

In chapter three we introduce mean reversion into the model, and we use both analytic and numerical approaches to explore the effects on the model's solution of adding increasing degrees of mean reversion. We confirm the 'band widening' effect of mean reversion on the (non-certainty equivalent) solution with uncertainty, and offer an economic explanation as why the deterministic effect of mean

reversion dominates the non-certainty equivalent effect. We document further aspects of the model's non-certainty equivalent properties, linking these to economic insights. Our most important result arises when sufficient mean reversion is added to models with asymmetric sunk costs of entry and exit. When entry costs dominate, optimal entry under uncertainty can occur before it would be optimal under certainty. This result is in sharp contrast with Dixit's model where the presence of uncertainty serves to delay actions that are costly to reverse. It vividly demonstrates the essential interrelatedness of reversible entry and exit decisions. In an appendix to chapter three a simple two period model with binomial uncertainty is developed which provides analytic intuitions into the results derived for the Dixit model with mean reversion.

Chapter four relates the results of chapter three to the literature on stochastic saddlepoint systems. This suggests how the theoretical properties explored in chapter three may be extended beyond hysteresis in trade and models of entry and exit with sunk costs, for example to models used in the literature on exchange rate target zones.

Having explored the effects of introducing mean reversion into the Dixit single firm model of hysteresis in trade, we move on to the second, empirical, part of the thesis, in which we consider the effect of allowing for the stationarity of the real exchange rate in the empirical analysis of UK export volumes. The immediate starting point for this empirical analysis is in the suggestion of HM Treasury in 1985 that a fall had occurred in the long run competitiveness elasticity of manufactured export volumes in models of the UK economy.

In chapter five we review the background literature on UK export equations, and link this to a review of approaches to dynamic specification, both when regressors are, and when they are not, co-integrated. Following on from these discussions we consider whether measures of UK competitiveness should be considered to be stationary, and caution against the risk of misspecification if competitiveness is wrongly judged to be nonstationary and co-integrated with export volumes and a world activity variable.

In chapter six we test the proposition that the fall in the long run competitiveness elasticity reflects dynamic misspecification in 'error correction' type models, using a number of test procedures, most notably Hausman's specification test. We find that the omission of short run dynamics on competitiveness has a statistically significant effect on estimates of the long run competitiveness elasticity. However, it appears that part of the alleged fall in the estimated elasticity cannot be explained by changes in specification. We employ recursive estimation and parametric tests of parameter stability to explore this possibility: the data are suggestive of parameter instability but cannot reject stability in conventional tests.

We next consider whether the parameter change indicated by our recursive estimation reflects either some kind of structural change, due perhaps to supply side shifts, or else whether it reflects the Lucas critique consequences of a change in the expected behaviour of competitiveness. In chapter seven we calibrate the Dixit model of hysteresis in trade and suggest that it may be relevant to the UK experience in the early 1980s. It also indicates how sensitive behaviour might be to assumptions about expectations concerning the future path of the exchange rate. In chapter eight we attempt to estimate various models of export volumes incorporating an explicit role for expectations. Although these efforts are not successful, we discuss whether it is nevertheless reasonable to assume forward-looking behaviour, and we consider the possibility of testing for structural stability conditional on some form of that assumption.

Overall we find ourselves unable satisfactorily to discriminate between expectational and structural explanations of the possible parameter change after 1979. However, while it is plausible that hysteretic exit may have occurred from some UK export markets, and while conjectures of 'structural improvements' may merit formal examination, neither yields an unambiguous prediction of parameter instability. In contrast, both changes in the certainty equivalent dynamics and in the level of uncertainty associated with the real exchange rate appear to point to a fall in the estimated competitiveness elasticity of UK export volumes after 1979. We are therefore inclined to retaining the null hypothesis that the estimated elasticity is not invariant to regime changes in competitiveness,

while the underlying price elasticity has not undergone structural change as a result of government policies.

In summary, this thesis contributes to the re-examination of the empirical modelling of UK export behaviour and its theoretical underpinnings which began in 1980/1, when the reaction of export volumes to the sharp appreciation of the Sterling real exchange rate confounded modellers' predictions. We consider both theory and evidence concerning the effects of exchange rates and the influence of exchange rate expectations on export volumes. We examine the possibility that supply side factors were important to observed behaviour, whether through the hysteresis effects of sunk costs, or through some other structural change in fundamentals. We show how the findings of existing theory and empirical models are sensitive to the use of the assumption that the real exchange rate may follow a stationary process within a given policy regime: this leads us to attempt an explicit empirical treatment of the role of expectations. Having identified specific issues, we make original contributions to both the theoretical and empirical literature on exports: moreover, both theoretical results and empirical methods have wider relevance. Taken overall, our research makes a small but significant contribution towards the re-assessment of the partial equilibrium analyses of exports which form a component of the international trade linkages in UK and global macroeconomic models.

Chapter Two.

SUNK COSTS, HYSTERESIS IN TRADE AND THE DIXIT-KRUGMAN MODEL OF ENTRY AND EXIT UNDER UNCERTAINTY.

2.1 INTRODUCTION

Our aim is to investigate the properties of the Dixit (1989a) landmark model of entry and exit under uncertainty when the stochastic price process is extended to the case of exogenous mean reversion.[1] In this chapter we therefore introduce the basic Dixit model and describe its solution and properties. We discuss the solution of the model in terms of the difference between the value functions for the firm in the active and idle states, showing that the model's solution can be derived using only a subset of the boundary conditions employed by Dixit. We comment on an economic implication of this property.

2.2 THE SUNK COST APPROACH TO HYSTERESIS IN TRADE.

2.2.1 The Theoretical Literature on Hysteresis in Trade.

The severity of the squeeze on the production of tradeables in the UK and the USA which was associated with high real exchange rates in the early 1980s led to speculation that there were permanent consequences for the structure of the respective national economies. UK and US exporters had been forced to give up long established market positions, while import penetration of the domestic markets had increased.[2]

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1. Dixit (1989a, p 634) suggests this extension himself.

2. In the US case, Baldwin and Krugman (1989) characterised this viewpoint as that of businessmen who did not believe in "the conventional economic wisdom and conventional econometric estimates" (p 637) which suggested that losses could be reversed.
In the UK context, Bean (1987a) quoted evidence given by the Vickers engineering business to a House of Lords Select Committee that "the withdrawal by the UK from a number of important sectors of manufacturing industry represents a permanent loss to the UK economy".

A literature on hysteresis effects in trade has emerged to formalise the intuition that large shocks to the real exchange rate might have permanent (or at least difficult to reverse) effects on economic structure, and to explain why the effects of 'large' shocks might be qualitatively different from those of 'small' shocks. The key feature of the literature is its recognition of the possible role of sunk costs.

The explanatory power of sunk market entry costs as a cause of hysteresis effects in trade was first explored by Baldwin (1986). These ideas were developed in Baldwin and Krugman (1989), where consideration was given to exchange rate expectations and to the robustness of the intuition to aggregation.

In a sequence of important papers Dixit (1989a, 1989b, 1989c) significantly extended our understanding of the nature and potential scope of the phenomena of sunk cost hysteresis by the use of continuous time methods and concepts from financial theory to analyse market entry and exit as investment decisions under uncertainty. Dixit (1989a) examined the case of a single firm, while (1989b) aggregated to the industry level and (1989c) applied a similar approach to resource allocation. Krugman adopted the Dixit approach, with some modifications to the model, for his Robbins lectures (1989a), while he employed similar ideas in a macroeconomic context in (1988).

An alternative analytic approach, using discrete time techniques was employed by Baldwin (1988) in a deterministic treatment extending to pricing decisions and market equilibrium. Baldwin (1989) has applied a stochastic extension of the discrete time approach to the consideration of sunk cost hysteresis under general specifications of the stochastic process for prices, and in Baldwin and Lyons (1989), this is applied to the consequences of hysteresis in trade for exchange rate determination.[3]

In this chapter we focus on the single firm analysis of sunk cost hysteresis, with particular reference to Dixit's model of entry and exit under uncertainty which can be expected to serve as the starting point for many future applications of sunk cost hysteresis.

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3. See also Williams (1987) and Sutherland (1989) for different approaches to the integration of hysteresis in trade with exchange rate determination.

2.2.2 Hysteresis, Sunk Costs, and Sunk Cost Hysteresis.

Before proceeding to an exposition of the basic sunk cost hysteresis result, we should clarify the more general concept of hysteresis itself. This concept has its roots in the physical sciences where it has been applied, for example, to the irreversibility of induced magnetic fields and also to mechanical examples.[4] Dixit (1989a) refers to hysteresis as "the failure of an effect to reverse itself as its underlying cause is reversed", (p 622), and Baldwin (1989) quotes a similar definition. A more formal definition is of use for our purposes: hysteresis can be defined as *"path dependence of the steady state"*.

Hysteresis is therefore a property of a system with multiple steady state equilibria, in which the actual equilibrium achieved is not independent of the path taken en route. The equilibrium properties of the system depend on history, ie. on the sequence of shocks which have impinged on the system.

Much use of the concept of hysteresis in economics has been in labour market contexts, as for example in the work of Blanchard and Summers (1986).[5] The basic idea has been that temporary shocks may have a permanent effect on an equilibrium quantity, such as the non-accelerating inflation rate of unemployment (NAIRU). One simple way in which this hysteresis property can be modelled empirically is through the existence of a unit root in the dynamics of the equilibrium quantity: in a random walk process, temporary shocks are expected to have permanent effects.

Since Baldwin (1986), the literature on hysteresis effects in trade has taken the existence of *sunk costs* in international trade as its starting point. Sunk costs are those which cannot be recouped through, for

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4. See Purcell (1965), p 399. Purcell points out that the hysteresis property in ferro-magnetism is the basis of magnetic computer memory. Thompson (1982), p 195 notes that hysteresis phenomena have also been suggested as a physiological basis for short term human memory, while giving the behaviour of a tied arch under loads as a simple mechanical example of hysteresis (p 12).

5. An interesting history of the use of the concept of hysteresis in economics is given in Cross (1988). This recognises the influence of Paul Samuelson but fails to mention a noteworthy paper by Kemp and Wan (1974) cited by Dixit, in which hysteresis is considered in the allocation of a mobile factor between sectors.

example, the re-sale of assets, and their potential influence on behaviour has been explored in the industrial organisation literature.[6]

It is suggested that the establishment of distribution networks, product identity and consumer goodwill etc. require significant market specific expenditure which cannot be recovered when a firm quits an export market. The sunk component of market presence is lost altogether on exit (due to what Dixit (1989a) describes as 'rusting'), and must therefore be rebuilt from scratch if the firm is to re-enter the market.

The assumption that sunk costs may be particularly relevant to export markets is a powerful simplification. Sunk costs in production and distribution are surely a feature of domestic as well as export markets. However the focus in the hysteresis in trade literature is on the sensitivity of trade to movements in the real exchange rate, and the key idea is that there may be specific sunk costs associated with particular export markets.

Sunk costs in entry and exit can give a system a hysteresis property.[7] But there is more to *sunk cost hysteresis* than the simple unit root ideas in, say, Blanchard and Summers (1986). In fact, much of the theoretical, empirical and policy interest of the sunk costs hysteresis result is due to its fundamental *nonlinearity*, which corresponds to the intuition that 'large' shocks may have qualitatively different effects from 'small' shocks. The path dependence of the steady state is thus a selective, nonlinear, dependence. Only shocks which are 'large' enough to cross particular thresholds will give rise to entry or exit and hence affect market structure in the steady state equilibrium.

The basic intuitions of the sunk cost hysteresis result are nevertheless very simple and are captured in the following example. Consider a firm facing lump sum sunk costs of K on entry into a particular industry and assume that the firm has perfect foresight concerning the path of its product price. Further

6. See, for example, Dixit (1980).

7. Baldwin (1989), p 14, emphasises 'that we demonstrate that hysteresis is a possibility - not an inevitability' in the presence of sunk costs..

assume that, once active in the industry, the discounted sum of current and future profits are increasing in the current product price, and are given by $\pi(P)$. For simplicity, exit is assumed to be costless.

An inactive firm will be indifferent between remaining idle and entry into the industry when the current price is such that the expected present discounted value of profits is equal to the sunk entry costs, ie. when $\pi(P) - K = 0$. At any higher price the initially inactive firm will enter the industry.

In contrast, assuming no sunkenness of exit costs, an active firm will be indifferent between continued activity and exit at a current price with a zero expected present discounted value of profits, ie. when $\pi(P) = 0$. At any lower price the initially active firm will exit the industry.

Given the assumption that the expected present discounted value of profits is increasing in the current price, the price at which an inactive firm is induced to enter the industry is greater than that at which an active firm is induced to quit the industry. Hence, in the presence of sunk entry costs, the minimum level of profitability required to induce entry must be greater than that which will prevent an active firm from quitting: the difference offsets the sunk cost incurred at entry. If, moreover, the value of current and expected future profits is an increasing function of current prices, then the price level required to trigger entry of idle firms is greater than that which will trigger the exit of incumbents.

We have given a simple motivation for the existence of a gap between distinct entry and exit trigger prices in the presence of a sunk entry or exit cost. This result is the primary feature of sunk cost hysteresis. This range between the trigger prices has been variously termed a 'zone of inaction' or ' range of inaction', (Dixit (1989a)), a 'range of no change', (Krugman (1989)), an 'interval of hysteresis' (Kemp and Wan (1974), and 'hysteresis band' or 'hysteresis zone', (Baldwin (1986)). Given the existence of such a band, the hysteresis result follows: when the price lies in this band, whether the firm will choose to be in or out of the industry will depend on history as captured by its status at the start of the current decision period. If previously active, the firm will remain so; and if previously idle, it will remain idle. Thus for prices within the band the current optimal state is dependent on the state inherited from the past, and the model therefore displays a hysteresis property.

The three way partition of possible values for the current price resulting from sunk entry costs is shown in figure 2.1. For high values of P it will be optimal for the firm to be active in the industry regardless of its initial status, while for low values of P it will be optimal for the firm to be out of the industry regardless of its initial status. The effect of sunk entry and/or exit costs is seen in the existence of the intermediate *hysteresis zone*, for which values of P it is optimal for firms to continue their initial state.

The presence of sunk entry (or exit) costs makes it possible that the equilibrium configuration of an export or import competing industry depends on previous history in those cases where the current price falls within the hysteresis zone. Moreover, given this hysteresis zone, only movements of the current price of a sufficient magnitude will reach the trigger thresholds and cause further entry or exit. Hence, sunk costs give rise not only to a hysteresis property, but also to the nonlinearity of the response to price movements, whereby large movements have a qualitatively different effect from small ones. Large cumulative price movements cause entry or exit, while small ones do not.[8]

Before moving on to consider the complications of a stochastic approach to sunk cost hysteresis we refer to Williams (1987) which presents a simple analytic formulation of a two period deterministic dynamic programming solution for a single firm facing sunk entry costs.

2.2.3 Hysteresis, Expectations and Uncertainty.

So far we have said little concerning the expected future path of the price variable. But of course the firm's entry and exit decision is taken in the context of a dynamic system, and so a forward looking decision framework is appropriate for a formal analysis. It is important to consider the sensitivity of the model's solution to alternative paths for future prices.[9]

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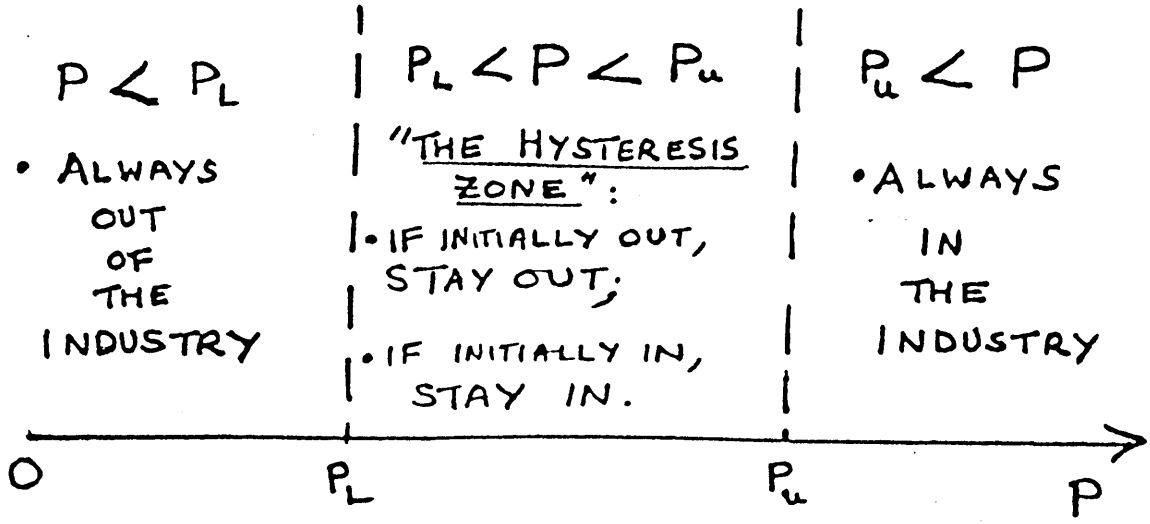
8. Note that a sequence of small movements in the same direction can cumulatively constitute a large price movement, as pointed out by Baldwin (1988).

9. Dixit (1989a) briefly considers cases amounting to static expectations and deterministic mean reversion, before concentrating on Brownian motion price expectations.

Figure 2.1

THREE WAY PARTITION OF CURRENT PRICES SHOWING THE 'HYSTERESIS' ZONE.



Horizontal axis represents prices, P, on the positive real line.

We must take account of both the displacement and the duration of the shock to the initial price path, because the relevant quantity in the objective function is the discounted integral of expected future profits. Thus, as Baldwin (1988) has emphasised, a departure from the initial price path 'large' enough to trigger entry or exit could involve either a large displacement of temporary duration, or a smaller displacement of permanent duration. This concern translates to an analysis of the sensitivity of model solutions to the degree of persistence, or, conversely, the degree of mean reversion in the price process. The basic intuition which will operate here is that the more persistent a shock is expected to be, the greater its impact on behaviour.

A major feature of the future is its uncertainty. Thus, though it is possible to consider solution properties under alternative perfect foresight price paths, it is important to consider the entry and exit decision under uncertainty. Baldwin and Krugman (1989) suggested a stochastic framework for the analysis, but reported (p 640) that: "we could not come up with any specific examples that prove particularly enlightening". Specific consequences of uncertainty were not identified.

A successful treatment of the stochastic case was a principal contribution of Dixit (1989a), and of the related papers (1989b), and (1989c). Dixit established the failure of certainty equivalence in a continuous time stochastic treatment of the single firm case. Moreover, he indicated that the practical effect of uncertainty was to increase the magnitude and economic significance of the hysteresis property of the solution, even for very small sunk costs.[10]

10. While sunk cost hysteresis is not dependent on uncertainty, Dixit has thus emphasised the potential importance of uncertainty to the phenomena.

2.3 THE DIXIT MODEL OF ENTRY AND EXIT WITH UNCERTAINTY

2.3.1 The Model

Dixit (1989a) provides a framework for the analysis of entry and exit decisions under uncertainty which builds on the key feature of sunk costs of entry and/or exit. The model is of quite general applicability in economic theory, but was originally conceived for the analysis of firms engaged in trade and facing fluctuating real exchange rates.[11]

The model is stochastic and in continuous time, and in common with much of the finance literature, analytic results are facilitated by the assumption of an infinite horizon and also by martingale properties.[12] Dixit employs geometric Brownian motion to describe the evolution of the relative price or real exchange rate. The problem is one of optimal entry into and exit from a market in the face of a stochastic process for relative prices and subject to lump sum sunk costs of entry and/or of exit.

The firm faces a lump sum sunk entry cost of K and a lump sum sunk exit cost of L, one of which may be zero without compromising the hysteresis property.[13] When active, the firm produces a unit flow of output at constant variable cost w. The constant discount rate is r. The firm is risk neutral and maximises its expected net present value, facing the relative price P_t in the product market which evolves as geometric Brownian motion with drift, described by:

$$dP/P = \mu\, dt + \sigma\, dz \qquad (2.1)$$

where $E(dz) = 0$ and $E(dz^2) = dt$. To ensure convergence it is assumed that $\mu < r$.[14]

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11. See Dixit's original version of the paper, (1987a). In (1989a) Dixit suggests applications ranging from hysteresis in trade to university hiring decisions.

12. The infinite horizon plays an important role in the stationarity property of the model and its solution. The Brownian motion assumption on the price process helps deliver closed form solutions to the value functions.

13. In what follows we adopt a notation largely consistent with that in Dixit (1989a), with a few minor exceptions. We use P_U instead of Dixit's P_H, and r instead of Dixit's ρ.

14. In the hysteresis in trade application we think of the firm as supplying an export market and the product price P is given by the real exchange rate. Following arguments stated by Krugman (1989) we

The state of the dynamic system is fully described by the current value of P_t along with the value of a second, discrete state variable which indicates whether the firm is currently in or out of the market. Given the state vector and the dynamics of the state variables the firm chooses either to continue or else to terminate its current market status so as to maximise its expected present discounted value: entry and exit thus reflect the optimal choice of control.[15]

Given that at least one of K and L is nonzero, the model's solution is assumed to take the form that distinct trigger or threshold prices for entry and exit exist. Above the entry trigger the firm will never be out of the market, while below the exit trigger it will never be in: as in the deterministic case the pattern is of trigger prices separated by a hysteresis zone.

The model's solution can be explained in terms of two distinct literatures. In what follows we first characterise the model as a stochastic control problem. We subsequently consider the alternative financial options interpretation.[16]

2.3.2 A Control Theoretic Interpretation of the Solution.

Following Dixit (1987) we characterise the firm's problem as involving two interrelated optimal stopping problems in stochastic dynamic programming, which must be solved jointly, the value function of each serving as a terminal value for the other.[17] In order to extract the most economic

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may then conclude that a predominant source of randomness in the typical exporter's environment is due to the variability in nominal exchange rates.

15. The structure of the problem is essentially the same as Brennan and Schwartz's (1985) characterisation of the optimal opening and closing of a mine with infinite resources.

16. In his original version of the paper, Dixit presented a control theory derivation of the solution to the model, before adding an alternative option value interpretation: subsequent revision of the paper has focussed attention on the financial options analogy, with the stochastic control treatment relegated to an appendix. Both approaches offer helpful and complementary intuitions concerning the model.

17. Dixit (1987) put it thus: "The value function in each condition, net of the cost of the switch, is the terminal payoff function for the other state. Interrelated solution of the two stopping problems completes the solution of the full problem". (p 23).

intuition from the control approach, we will set out the components of what may otherwise be an unfamiliar argument in greater detail.

Optimal stopping problems involve the application of the general principles of control theory (dynamic programming) to a particular class of problems, namely those where a termination point must be chosen at which to end the continuation of an activity.[18] An optimal control rule is characterised as one which satisfies Bellman's principle of optimality, which states that:

"An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision." Bellman, (1957), p 83.

An optimal stopping policy will satisfy this principle.

The mathematical formulation of the principle of optimality is the fundamental recurrence relation of dynamic programming. This focusses attention on a single period decision embedded within the entire multi-stage problem.[19] We must now introduce the concept of an (optimal) value function, which measures the expected value of the objective function for the dynamic optimisation calculated at time t under the optimal policy for time t and all subsequent periods, conditional on the initial state vector (X_t). We denote this quantity $J^*_t(X_t)$. We denote the state vectors at time t by X_t, the control variable at time t by u_t, and the single period contribution to the expected value of the objective function by $h(X_t,u_t)$, while the evolution of X_t is given by $X_{t+1}=f(X_t,u_t)$. The optimal control u^*_t is given by the function $u^*_t = u(X_t)$, and it satisfies Bellman's recurrence relation. This can finally be stated:

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18. Dynamic programming seeks a control rule which will specify the optimal setting of the control variables for each point in time as a function of the state vector describing the dynamic system. This approach can be applied to intertemporal optimisation problems with a recursive structure. (See Sargent, 1987), p 13. Useful insight to the required recursive structure can be gained from the finite horizon discrete time case, were we solve backwards one period at a time from a terminal condition.

19. The Dixit problem is an infinite horizon continuous time stochastic control problem. Nevertheless, in giving the basic intuitions behind the principle of optimality here we employ a discrete time formulation. Moreover we do not attend to any of the technical issues raised by the stochastic nature of the problem. Technical references for continuous time stochastic control include: Krylov (1980), Whittle (1983), Kamien and Schwartz (1981).

$$J^*_t(X_t) = \max_{u_t(.)} \; h(X_t,u_t) + \delta J^*_{t+1}(f(X_t,u_t)) \tag{2.2}$$

where δ is the relevant discount factor.

The continuous time analogue of the discrete time recurrence equation is the Hamilton-Jacobi-Bellman equation. As the discrete time interval considered tends to an infinitessimal, the principle of optimality is captured by a partial differential equation.[20]

$$\partial J(X,u)/\partial t = \max_{u_t(.)} \{ h(X,u) + (\partial J(X,u)/\partial X) \,.\, f(X,u)\} \tag{2.3}$$

The decision facing the firm in the Dixit model is whether to be in or out of the industry at any given time. However it is more useful to emphasise entry and exit decisions as opposed to decisions to continue. As Dixit (1987a), p 23, puts it, the problem is "to switch optimally from one (state) to another". Thus the immediate problem facing a firm in each state is one of optimal stopping: when to cease continuation of the current state.

The key to a satisfactory characterisation of the entire solution to the Dixit model as a stochastic control problem is to treat the optimal strategy as consisting of a succession of alternating periods of activity and inactivity, within each of which an appropriate continuation rule is assumed to be followed. The switches between discrete states (ie. entry or exit) occur at the boundaries to the continuation periods. Hence the principal task for control theory is to characterise the transitions of entry and exit.[21]

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20. Jacobs (1974), pp 169-170.

21. The discontinuity of the state variable (market participation) appears to violate a requirement of the 'traditional' statements of control problems and their solutions as in eg. Intriligator (1971). By treating the problem as a succession of distinct optimal stopping problems this continuity requirement is replaced by conditions for the boundaries of a solution, (cf. transversality conditions in the 'traditional' solution).

What Dixit described as the "interrelated" nature of the two stopping problems follows from the fact that the stopping point for the period spent in the active state is the starting point for the idle state, and vice versa. Formally this means that the terminal condition on the value function for the firm in the active state is the initial condition for the value function for the idle state. Thus the optimal stopping of the active state and the optimal stopping of the idle state cannot be solved independently.

The characterisation of the optimal strategy is further simplified by the stationarity of the problem. This property follows from the assumption of an infinite horizon, along with the absence of explicit time dependence in the objective function and state vector.[22] Stationarity implies that the firm always faces the same decision problem conditional on the state vector (current price plus market status) and hence that the solution is time invariant. In this model the practical implication is that once we have found the optimal boundary for entry we have solved it for all idle - to - active state transitions. Similarly, there is only one exit boundary for the active - to - idle state transition. The model's solution will be characterised by one value for an entry trigger price and one value for an exit trigger price. The existence of sunk costs ensure that the entry and exit triggers will be distinct.

We assume the existence of two value functions: one which holds in the continuation zone for the active state (V(P)), and one in the continuation zone for the idle state (W(P)). These value functions satisfy the following second order stochastic differential equations. The Bellman equation for the idle state, when it is optimal to stay out of the market, is given by:

$$\tfrac{1}{2}\,\sigma^2 P^2 W''(P) + \mu\, P\, W'(P) - r\, W(P) = 0 \qquad (2.4)$$

Similarly, the Bellman equation for the active state when it is optimal to remain in the market is:

$$\tfrac{1}{2}\,\sigma^2 P^2 V''(P) + \mu\, P\, V'(P) - r\, V(P) + P - w = 0 \qquad (2.5)$$

---

22. Sargent (1987), p 20, argues that for dynamic programming problems, "In general a different policy function $u=u(x_t)$ mapping the state at t into the control at t is to be used at each date. This is a consequence of two features of the problem: the fact that the horizon T is finite, and the fact that the functions... (the objective and the state dynamics) ... have been permitted to depend on time in an arbitrary way. For many practical applications it is inconvenient that the policy function varies over time." Dixit's infinite horizon problem achieves time invariance of the policy function, and Baldwin (1989)'s extension to the infinite horizon case also yields a stationary entry-exit strategy.

Note that these two equations share the same homogeneous part and differ only by the flow value of profits in the active state.[23]

The formal theory of optimal stopping provides us with conditions to ensure that Bellman's principle is upheld at the boundary points, where a state transition is optimal. Value Matching and Smooth Pasting conditions will characterise entry and exit under the optimal policy.[24]

The simple intuition behind the Value Matching conditions is that in an optimal trajectory there can be no jumps in the expected value of the firm net of the sunk costs of entry and exit as incurred at state transitions. Optimality requires continuity of the value function along the trajectory. When entry is optimal (at trigger price P_U) value matching implies:

$$V(P_U) - K = W(P_U) \tag{2.6}$$

and, when exit is optimal, at (P_L):

$$V(P_L) = W(P_L) - L \tag{2.7}$$

The Smooth Pasting condition amounts to a requirement of continuity of the first derivatives of the value functions across the state transition. Smooth Pasting is in effect an optimality condition reflecting the fact that the boundary is freely chosen. If Smooth Pasting did not hold at the boundaries then the firm could gain from adjusting the trigger prices.[25]

Smooth Pasting requires, at the entry threshold:

$$V'(P_U) = W'(P_U) \tag{2.8}$$

---

23. A familiar economic intuition can be obtained by rearranging these Bellman equations as the arbitrage requirement that the return on the optimally managed firm, given by the instantaneous flow of profits plus the instantaneous capital gain, is equal to the instantaneous discount rate.

24. A formal statement of these conditions is given in Krylov (1980). See also Whittle (1983) and Malliaris and Brock (1982).

25. Dixit gives a detailed, (but from a mathematical point of view still heuristic), discussion of smooth pasting in (1988). Dumas (1989) has expanded on Dixit's discussion to emphasise smooth pasting's status as an optimality condition. He also clarifies the significance of the distinction between lump sum and incremental costs for the nature of the optimality condition.

and at the exit threshold:

$$V'(P_L) = W'(P_L) \tag{2.9}$$

The solution to the firm's problem is therefore characterised by the trigger prices P_L and P_U. These are obtained when the Bellman equations satisfy the boundary conditions given by the Value Matching and Smooth Pasting. These boundary conditions effectively determine the relative positions of the value functions V(P) and W(P).

We can illustrate the solution graphically, as in figure 2.2. The *certainty equivalent* or *perfect foresight* triggers for entry and exit are given by P_{UC} and P_{LC} repectively, while the *fully optimal* triggers are P_U for entry and P_L for exit. The difference between the fully optimal and certainty equivalent triggers defines the *non-certainty equivalent* or *caution* component of the solution.

An important alternative presentation of the model can be given in terms of the difference between the value functions V(P) and W(P). Dixit (1989a) defines the function G(P) as V(P)-W(P) in our notation, and uses it to discuss some analytic results. The linearity of the Bellman equations in W(P) and V(P) allows us to form a similar Bellman equation in G(P):

$$\tfrac{1}{2}\,\sigma^2P^2G''(P) + \mu\, P\, G'(P) - r\, G(P) - (w - P) = 0 \tag{2.10}$$

Both Value Matching and Smooth Pasting boundary conditions can be reformulated as conditions on G(P) and G'(P), so that the trigger prices P_L and P_U are determined by the Value Matching conditions:

$G(P_L) = -L$ and $G(P_U) = K$

and Smooth Pasting conditions:

$G'(P_L) = 0$ and $G'(P_U) = 0$

The solution is illustrated in terms of G(P) in figure 2.3.

Figure 2.2

THE SOLUTION TO THE DIXIT (1989) MODEL OF ENTRY AND EXIT WITH UNCERTAINTY IN TERMS OF THE VALUE FUNCTIONS V(P) AND W(P).

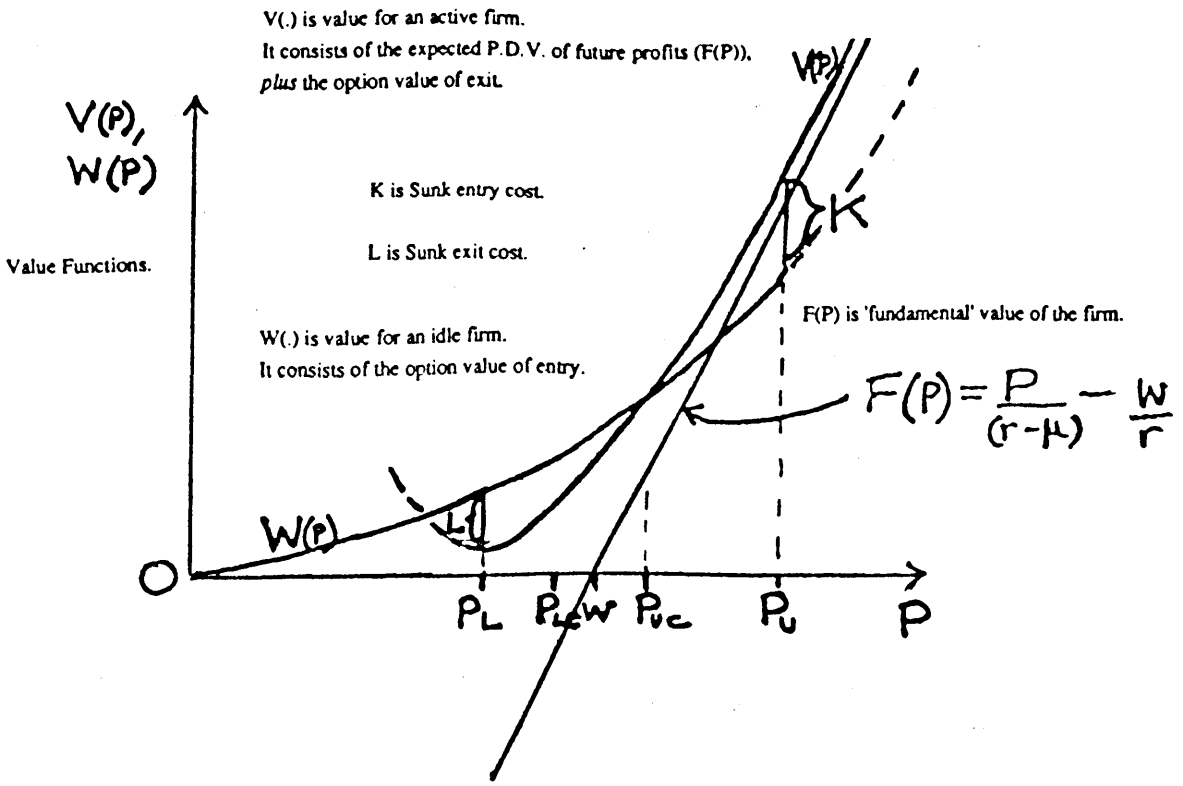
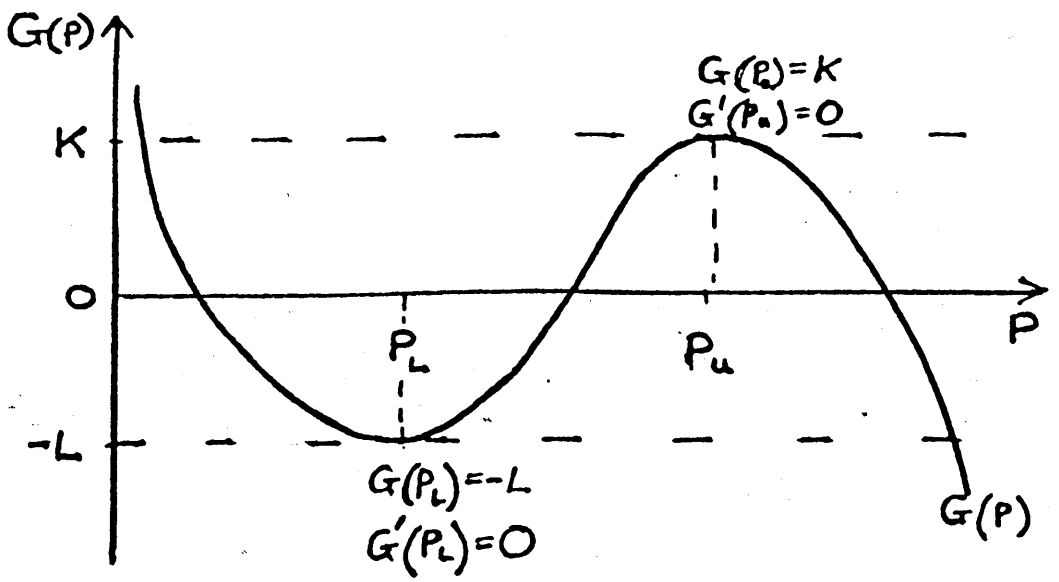


Figure 2.3

THE SOLUTION TO THE DIXIT MODEL IN TERMS OF THE VALUE DIFFERENCE FUNCTION G(P).



2.3.3 Financial Options, Entry and Exit.

We now look at the solution of Dixit's model from the financial options perspective. In doing so we take advantage of the fact that under the Brownian motion assumption for the price process we can derive closed form solutions for the value functions. We set out the algebra and interpret it in terms of financial options.

Dixit exploits the analogy between real and financial investments to characterise entry and exit decisions in his model, drawing on the financial options literature. In what follows we aim to introduce the concept of financial options, present the algebra of the closed form solutions for the value functions, and then relate the financial options interpretation to the solution.

A financial option is an asset which consists of the right to buy (a 'call' option) or to sell (a 'put' option) another asset, usually on payment of an exercise fee or 'strike' price. Merton (1977) argued that financial option pricing techniques would be applicable: "whenever a security's return structure is such that it can be described as a contingent claim", (p242),
(where a contingent claim appears to be defined as: "one whose value on a specified date is uniquely determined by the price of another security").

In recent years the variety of contingent claims so analysed has been extended, and Dixit's paper refers to some of the work which has extended the analysis to real investment decisions.[26]

Financial options theory has been concerned with the question of how to value options or contingent claims of this kind. Nature provides a probability distribution of returns for the primary asset underlying the contingent claim, be it a stock price, manufacturing plant or a position of market incumbency. The holder of the contingent claim, (we need only consider options to buy at any time, ie.

26. Dixit refers to McDonald and Siegel (1986), Brennan and Schwartz (1985), Constantinides (1986), Pindyck (1988) and Bertola (1987) among recent applications of options pricing beyond strictly financial instruments.

'American call' options), is free, on payment of the specified exercise fee, (such as a sunk cost of entry), to take possession of the underlying asset whenever it is advantageous to do so.[27] The existence of this alternative effectively modifies the probability distribution of returns faced by the holder of the option: given the key assumption of voluntary exercise, the expected value can only be increased by the existence of the option.

In general terms the value to the holder of the right to exercise the contingent claim stems from the change in the expected value, (net of exercise costs) of the distribution of returns which the optimal exercise of the contingency makes possible. For example the possibility that a firm could exit a particular market would have a value related to the losses that could be avoided by judicious exercise of that freedom.

The analogy with financial options therefore gives us a strong intuition as to why, in the presence of uncertainty, the possibility of contingent future action has a value. Under uncertainty, the existence of the option values of entry and of exit must be taken into account in order to maximise the expected value of the firm, and their existence may cause the solution to differ from the certainty case.

In what follows we apply the financial options approach to interpret the closed form solutions for the value functions in the Dixit model. We will draw on the options interpretation in subsequent analysis of the trigger prices etc.

As already stated, entry occurs at a trigger price P_U and exit at a trigger price P_L, where, if either or both of K and L, the sunk costs of entry and exit respectively, are strictly positive, $P_U > P_L$. The two Bellman equations (given above as equations 2.4 and 2.5) share the same homogeneous part,

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27. It is worth noting that when option pricing methods are applied to some general contingent decisions in economics there may be a zero exercise price. For example, in the absence of any sunk costs there is a zero exercise price of entry and exit from an industry. Nevertheless, given a fixed scale of output, the options of entry and exit will still be valuable: active firms would not be indifferent to legislation prohibiting them from ceasing operations in the event of their incurring losses.

characteristic equation and roots, γ_1 and γ_2, which can be shown to satisfy $\gamma_1 > 1$, $\gamma_2 < 0$. The complementary functions for the solutions are assumed to take the form:

$$\beta_i \quad P^{\gamma 1} + \beta_{i+1} \; P^{\gamma 2}$$

where the β_i are arbitrary constants.

The value of the firm when idle, W(P),is given as:

$$W(P) = \beta_1 \quad P^{\gamma 1} + \beta_2 \; P^{\gamma 2} \tag{2.11}$$

The value of the firm when active, V(P), is non-homogeneous, with a particular integral given by the expected present discounted value of the stream of future profits, $P/(r-\mu) - w/r$. Hence:

$$V(P) = \beta_3 \quad P^{\gamma 1} + \beta_4 \; P^{\gamma 2} + P/(r-\mu) - w/r \tag{2.12}$$

We have given expressions for the value functions V(P) and W(P), but there are still six unknowns to determine: β_1, β_2, β_3, β_4, P_L, P_U.

Value Matching and Smooth Pasting at both the upper and lower trigger prices provide us with four boundary conditions which determine the relative position of the two value functions. To obtain a full solution for the six unknowns, two further independent boundary conditions are required. Dixit (1989a), Krugman (1989a), and Brennan and Schwartz (1985) invoke two conditions corresponding to extreme values of the process P, which serve to pin down the lowest point of W(P) and the highest point of V(P).

Brennan and Schwartz give these 'endpoint' conditions as:

i) W(P) must remain finite as $P \rightarrow 0$.

Given $\gamma_2 < 0$, we must force the term $\beta_2 \; P^{\gamma 2}$ in the general solution for W(P) to be bounded by requiring that $\beta_2 = 0$.

ii) that $V(P)/P$ must remain finite as $P \rightarrow \infty$

Given $\gamma_1 > 1$, we must require $\beta_3 = 0$ in order that the term $\beta_3 \quad P^{\gamma 1}$ remains bounded in the limit.

Imposing the endpoint conditions onto the proposed solutions for the value functions we conclude that:

$$W(P) = \beta_1 \quad P^{\gamma 1} \tag{2.13}$$

$$V(P) = \beta_4 \quad P^{\gamma 2} \quad + P/(r-\mu) - w/r \tag{2.14}$$

We can now apply the financial options interpretation to the value functions. We draw on Dixit's exposition in an earlier version of the paper, (Dixit 1987). The asset represented by an idle firm is an option, (the option to enter), to buy an active firm with an exercise or strike price equal to the sunk cost of entry. Simultaneously the asset represented by the active firm has two components: a stream of expected profits from the operation of the firm, and the option to buy an idle firm by the exercise of the option of exit on payment of the sunk cost of exit.

The firm's value in each state depends on the value of the option to convert it to the other state. The two options are essentially interwoven, and so their values must be simultaneously determined.[28] Considering the option of exit, the intuition here is that, given the possibility of entry and exit, the value of exit to an active firm is the probability weighted value of all contingent optimal strategies which will involve exit at some point (including all those in which subsequent entry and exit may take place) net of the sunk costs incurred. It incorporates both the value of immediate exit and that of waiting for possible future exit.[29]

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28. Dixit's (1989a) statement of the option pricing problem is as follows:
"The asset that is acquired by exercising the option to invest includes another option, namely to abandon the investment and revert to the original situation. We have two interlinked option pricing problems, which must be solved simultaneously, and the prices of both must be obtained in terms of the underlying uncertainty in exchange rates, demand etc.", (p 621).

29. In a recent private correspondence Professor Dixit and the author have considered whether, and how, these two elements of the option value should be distinguished, ie. can we isolate the pure 'Value of Waiting' from the intrinsic value of immediate exercise.

Consider first the particular integrals in the solution for W(P) and V(P): these give the expected present discounted value of the firm under the continuation policy. In the active state, ie. for V(P), the particular integral is $P/(r-\mu) - w/r$. In the idle state the continuation policy has zero expected present discounted value, which corresponds to the absence of a particular integral from the general solution for W(P).

However, the full value of the firm, whether active or idle initially, incorporates a further component reflecting the value of the contingency of entry or exit. Hence the complementary functions of the solutions for the value functions V(P) and W(P) can be interpreted respectively as the value of the option to quit when initially active, $(\beta_4 \quad P^{\gamma 2})$, and the value of the option to enter when initially idle, $(\beta_1 \quad P^{\gamma 1})$.

As the level of the exchange rate approaches zero, the probability of the Brownian process ever returning to a level at which entry would be optimal also tends to zero. Hence the option value of possible entry or re-entry must also go to zero, ie $W(P) \rightarrow 0$ as $P \rightarrow 0$. As the exchange rate approaches infinity, the probability under the Brownian assumption of a return to a level precipitating exit falls (towards zero), and we can imagine that the value of the option of exit similarly falls towards zero. In the limit the value function V(P) measures only the value of the continuation policy, given by the discounted value of future net operating profits.

We illustrate the roles of the options of entry and exit in figures 2.4 and 2.5.

Note how the value of the option of entry is zero at $P = 0$, and how it increases with P. In contrast the value of the option of exit approaches zero for high values of P. The value of the option to quit is inversely related to the price level, while the value of the option of entry increases with the price level.

Figure 2.4

THE OPTION VALUE OF ENTRY.

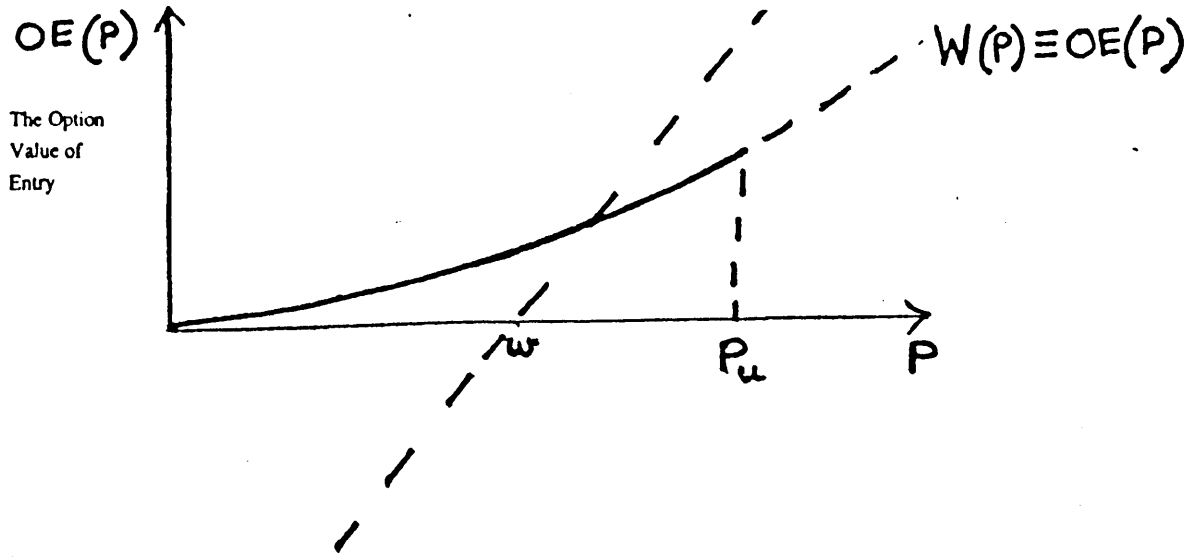
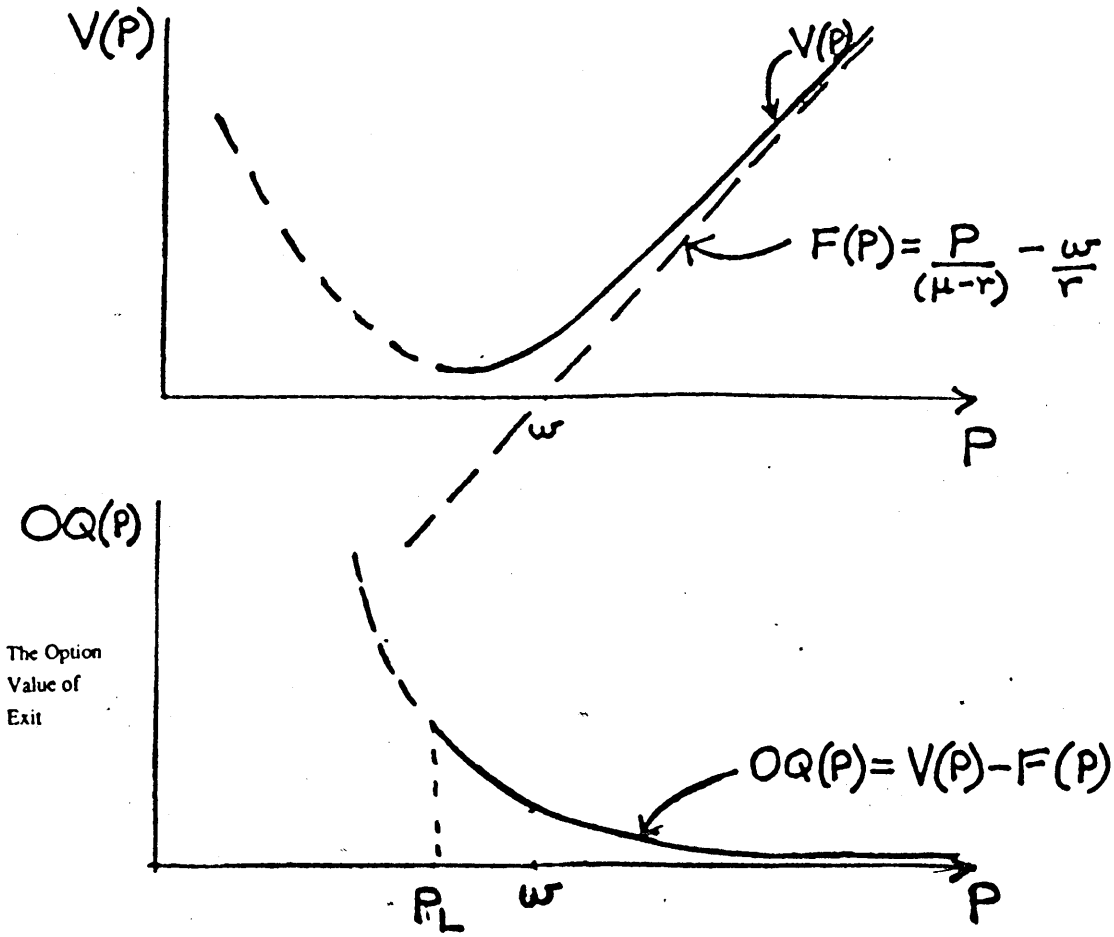


Figure 2.5

THE OPTION VALUE OF EXIT.



After imposing the endpoint conditions $\beta_2 = \beta_3 = 0$, the solution for the price thresholds now requires us to solve for β_1, β_4, P_L, and P_U from the value matching and smooth pasting conditions which can be written out using the functional form as:

$$\beta_1 \; P_L^{\gamma 1} - L = \beta_4 \; P_L^{\gamma 2} + P_L/(r-\mu) - w/r \tag{2.15}$$

$$\beta_1 \; P_U^{\gamma 1} + K = \beta_4 \; P_U^{\gamma 2} + P_L/(r-\mu) - w/r \tag{2.16}$$

$$\gamma_1 \beta_1 \; P_L^{(\gamma 1)-1} - L = \gamma_2 \; \beta_4 \; P_L^{(\gamma 2)-1} + 1/(r-\mu) \tag{2.17}$$

$$\gamma_1 \beta_1 \; P_U^{(\gamma 1)-1} - L = \gamma_2 \; \beta_4 \; P_U^{(\gamma 2)-1} + 1/(r-\mu) \tag{2.18}$$

These four equations in β_1, β_4, P_L, P_U are nonlinear in P_L and P_U, but can in principle be solved for P_L and P_U. Solving for β_1 and β_4 and then substituting back will leave nonlinear equations in P_L and P_U which must in general be solved numerically.[30]

A brief discussion of various approaches to the solution properties now follows before we move on to a consideration of the main conclusions concerning the solution.

2.4 SOLUTION METHODS.

The Dixit model, as we have seen, gives closed form solutions to the value functions V(P) and W(P), which are amenable to an options interpretation. Analysis of the trigger prices is more difficult, as the solutions must be obtained from nonlinear equations and closed forms do not exist. Dixit has employed a variety of techniques to investigate the properties of the solution.

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30. Note therefore that the 'closed form' nature of the solution does not extend to the trigger prices, although Dixit shows how analytic comparative statics involving these triggers may be possible.

Firstly, it is possible to get some analytical results concerning solution properties. The general shape of the value difference function G(P), and in particular the sign of the derivatives at the entry and exit thresholds allow Dixit (1989a) to establish the non certainty equivalence of the solution. Further results are obtained by the use of limiting arguments as K and L both go to zero, and as sigma goes to zero. Total differentiation of the value matching and smooth pasting conditions permits Dixit to obtain comparative static results for changes in the flow cost, w, and in the sunk costs K and L.

Secondly, the value matching and smooth pasting conditions can be solved numerically for the trigger prices. Dixit (1989a) uses this approach and hence is able to confirm the importance of the non certainty equivalence. He is also able to investigate a wider range of comparative static properties of the solution.

In a subsequent paper, Dixit (1989d) has employed a third approach: he derives analytic approximations to the solution of the nonlinear equations for the trigger prices. After manipulating the value matching and smooth pasting conditions, and changing variables, Dixit derives expressions which he can expand on the assumption that certain parameters are small or negligible. First he takes the case of a zero discount rate, then he considers small values for the sunk costs K and L: the procedure yields some further insight into the solution. This approach gives us locally valid approximations after fairly complex algebraic manipulations, but it is not automatically applicable in practice: Dixit is unable to find an approximation for the case when σ is small. [31]

Dixit's numerical methods involved the solution of the nonlinear equations for the value matching and smooth pasting equations using the closed form expressions for the value functions in the process. An alternative numerical approach to the solution for the trigger prices is employed in our research because it permits us to solve the case with mean reversion for which closed form expressions for the value functions are not available. This numerical method uses a simple shooting technique as employed

31. Note that the use of these approximations depends of the existence of closed forms for the value matching and smooth pasting conditions, and hence these examples cannot be applied to the case with mean reversion considered in the next chapter.

in Miller and Weller (1988) to find the value difference (G(P)) function which satisfies the necessary value matching and smooth pasting boundary conditions.[32]

2.5 PRINCIPAL PROPERTIES OF THE DIXIT MODEL.

We now identify the principal qualitative features of the solution to the model.

i) Hysteresis Property

We have already noted that the solution for nonzero sunk costs is given as two trigger or threshold prices which define between them a band or range of inaction. At prices above this zone, entry is always optimal: below the zone exit is always optimal. But within the zone the optimal policy is one of no change: ie. idle firms stay out of the market, and active firms stay in it.

This feature of the solution amounts to the path dependence of the steady state: this is the model's hysteresis property. This result does not depend on uncertainty but does depend on the existence of sunk costs of entry and/or exit. Dixit (1989a, p629) reports the limiting result that:

"when both K and L tend to zero, both P_U and P_L tend to the common limit w; thus sunk costs are essential for hysteresis." (Our notation).

When both K and L are zero, value matching and smooth pasting can only be satisfied at the single threshold $P_U = P_L = w$, and the hysteresis property does not apply to the system.

The hysteresis property of the full stochastic solution is similar to the solution to simple deterministic models with sunk costs of entry and exit, such as those in Baldwin (1986), and Williams (1987).

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32. We employ this shooting techniques to investigate the effects of adding mean reversion to the model in chapter three. Details of the shooting procedure are given in an appendix to chapter three.

ii) Non-Certainty Equivalence

Central to Dixit's (1989a) contribution is the argument that the full stochastic solution is non-certainty equivalent. Moreover, Dixit shows (pp 623-4) that the non-certainty equivalent properties of the solution may be of major quantitative significance, so that the zone of inaction may be substantially larger in the stochastic case than would be implied by certainty equivalent analysis.[33]

The noncertainty equivalence of the solution is demonstrated (Dixit, p 628) by substituting value matching and smooth pasting conditions into the Bellman equation in G(P). For example, at P_U, when $G(P_U) = K$ and $G'(P_U) = 0$, the Bellman yields $P_U = w + rK - \{ {}^1/_2\sigma^2 P^2 G''(P_U)\}$. Dixit's numerical results (pp 630-1) indicate the importance of this result:

"hysteresis is very significant, and a major part of the full gap between P_U and P_L arises from the uncertainty". (Our notation).

We can use both the control theory and the financial options approach to understand the non-certainty equivalence of the problem. From the control perspective the difference between the certainty equivalent solution and the full stochastic solution is termed the 'caution' component of the solution.[34] So called 'cautious control' strategies are widely found in the control literature, as certainty equivalence is a characteristic of the solutions of only a limited class of problems, of which the most common in economics are linear quadratic Gaussian problems. The Dixit model does not fit this class.

The uncertainty in the model concerning the future path of prices tends to make firms wary of changing state: they must allow for the possibility that any price movement may be reversed, thereby causing them to regret an overhasty entry or exit decision. Hence we expect to find that the zone of inaction will be wider under the presence of uncertainty than in its absence.[35]

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33. Our simulation examples in chapter seven based on UK experience reinforce this insight.

34. See 2.3.2 and figure 2.2 above where this terminology is introduced.

35. Note however that with mean reversion we find that for some cases characterised by sufficient asymmetry of entry and exit costs, it is not true *both* that the exit threshold is lower *and* that the entry threshold is higher under uncertainty than in its absence. See below.

The financial options approach provides us with an alternative interpretation of the non-certainty equivalence of the solution. The introduction of uncertainty brings into being the option values attached to the possibilities of entry and exit - where the option value reflects the gains to being able to depart from the strategy associated with the actual realisation of the point expectations for the future path of prices. The difference between the two interrelated option values which are part of the value of the firm in the active and idle states influences the firm's decision to change states. In general the difference will be nonzero and therefore the optimal decision under uncertainty will differ from that under certainty.

iii) Large Consequences of Small Sunk Costs and Uncertainty.

An important pair of results reported by Dixit (1989a) are that "even a little uncertainty matters a lot" (p 632) in generating a non-certainty equivalent hysteresis zone, and that "when there is some uncertainty, hysteresis emerges very rapidly even for very small sunk costs" (p 630). These results imply that the combination of small sunk costs with a small measure of uncertainty gives rise to a large, non-certainty equivalent hysteresis zone in the model's solution. Even a small amount of uncertainty or sunkenness is not negligible, and so we might expect sunk cost hysteresis to be a pervasive economic phenomenon.

These results can be seen clearly using one of the analytic approximations developed in Dixit (1989d). If the sunk costs of entry and exit and the discount rate are each sufficiently small, then twice the logarithm of the ratio of the trigger prices (x), which is a convenient measure of the hysteresis band width, can be approximated by:

$$x = \{ {}^{3}/_{4} (\sigma^2 (K+L)/w \}^{1/3} \tag{2.19}$$

The partial derivatives of x with respect to uncertainty (σ^2) and to the sunk costs (K+L) are:

$$\partial x/\partial\sigma^2 = {}^{1}/_{3} (x/\sigma^2) \tag{2.20}$$

$$\partial x/\partial(K+L) = {}^{1}/_{3} (x/(K+L)) \tag{2.21}$$

It is clear from these expressions that as σ^2 goes to zero, the partial derivative $\partial x/\partial\sigma^2$ goes to infinity. A small amount of uncertainty has a large effect on the band width. Similarly, as (K+L) goes to zero, the partial derivative $\partial x/\partial(K+L)$ goes to infinity.

Dixit derives the same limiting comparative static results for the model as a whole in (1989a, appendix B). The large quantitative significance of the noncertainty equivalent hysteresis zone is also illustrated by Dixit's numerical results, (1989a, pp 630-634).

iv) Other Results.

First we group together several comparative static results reported in Dixit (1989a) on the basis that they share an unambiguous effect on the expected value of the permanent continuation policy for the active state relative to the idle state.

An increase in the flow cost (w), an increase in the discount rate (r), and a fall in the trend growth of prices, (μ), will all reduce the expected value of continued activity relative to the idle state. The effect of this shift in relative values will be to make idle firms less likely to enter and active firms more likely to exit the industry. In terms of the trigger prices, both P_L and P_U will be shifted upwards: the price has to rise higher than before to trigger entry for an idle firm, and it does not have to fall as far as before to trigger exit for an active firm.[36]

Two other cases noted by Dixit are the solutions when entry or exit costs become infinite: in the former case entry will never occur but a closed form exists for the exit trigger; in the latter case exit will never occur but the entry trigger can be specified in closed form.

The final property of the solution which we wish to record here is one identified as 'surprising' by Dixit in his consideration of his analytic approximations, but which is borne out by his earlier numerical results. When the sunk costs K and L are themselves small, a change in one of them has an

36. Dixit (1989a) establishes the effect of the increased flow cost analytically, but the effects of changes in r and μ are reported as numerical results.

approximately symmetric effect on both the entry and exit price triggers, regardless of the discount rate, (Dixit 1989d, p 9).

2.6 CONCERNING THE VALUE DIFFERENCE ISSUE.

In order to solve his model, Dixit (1989a) first imposes endpoint conditions on the value functions V(P) and W(P). He then imposes the necessary conditions for the optimality of entry and exit by value matching and smooth pasting of the value functions at both entry and exit thresholds. In order to derive some analytical results, Dixit then introduces the value difference function G(P) and shows how value matching and smooth pasting conditions can conveniently be expressed in terms of G(P) and its derivative G'(P).

In this section we wish to emphasise some of the implications of this feature of the model that the solution for the trigger prices can be obtained from conditions on the value difference function G(P).

The value difference treatment of the problem is helpful in three ways.

Firstly, it indicates the property of the model that the solution for the trigger prices is independent of the absolute levels of the value functions. This is not a feature emphasised by Dixit, but it is a result of some importance, and for example, supports Krugman's assumption, in his re-industrialisation paper (Krugman 1988), that the objective should be framed in terms of a value difference.

Secondly, the value difference is the form of the model we employ to shoot for the trigger price solutions using the methods of Miller and Weller (1988). This property is particularly convenient when we extend the model to include a mean reverting price process in chapter three, when suitable end-points for the value functions in absolute terms are hard to specify.

Thirdly, the value difference formulation links neatly to the analysis of stochastic saddlepoint systems in Miller and Weller (1988). In chapter four we draw on this point to suggest that some of our results concerning the effects of mean reversion may have wider applicability to other stochastic saddlepoint systems.

Formally, the first of these points is an obvious one, and, perhaps for that reason, is not made by Dixit. Given the function G(P) we can solve for the price triggers through value matching and smooth pasting. But the value difference G(P) is of course independent of the absolute levels of the value functions V(P) and W(P). This in turn implies that the solution for the trigger prices, P_L and P_U is independent of the absolute values of the firm in its idle and active states. Thus the trigger prices depend only on the *relative* slopes and levels of the value functions for the firm in the active and idle states.

Another way of expressing the same point is that the trigger price solutions are invariant to the presence in *both* value functions of the *same* arbitrary function or constant. This suggests an economic interpretation which we now pursue.

The financial options interpretation enables us to characterise the complementary functions of the value functions as the option valuations of the contingencies of entry and exit (which, as we have seen, include all possible strategies beginning from the present state, other than those implying permanent continuation of the present state). In keeping with this we offer the interpretation of any arbitrary term as a 'misvaluation' or a 'bubble term' in the valuation of the option of exit for the active firm and of the option of entry for the idle firm.

We conclude that the optimal switching strategies between idle and active states are robust to errors in the valuation of the associated options, provided that the same 'bubble' term appears in both the options to enter and to exit.[37] Errors in the absolute valuation of such claims may occur in practice due to the

37. To see this, compare the position of an idle firm with the active position an instant later upon the payment of the entry fee. Value matching implies that the option values of entry and exit differ only by

complexity of the claims' nature.[38] However the model indicates that consistency in *relative* valuation is sufficient for the invariance of behaviour.

We should note however that if this analysis were incorporated into a larger model with other investment possibilities, misvaluations such as these, would have real consequences. An over-estimation of that part of the value of a trading firm which reflects the option of closure would cause excessive investment in trading firms relative to other investments, just as much as would an over-estimation of the fundamental value of the trading firm's output.[39]

Examination of the model's solution supports a further observation. This is that the trigger prices can be derived without the use of the endpoint conditions.[40] The model thus has a recursive property in that the trigger price solutions can be derived using only a subset of the full set of boundary conditions. This observation does not appear to have been recorded elsewhere: Brennan and Schwartz (1985), Dixit (1989a), and Krugman (1989) all make use of the endpoint conditions in their solutions.

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the constant present values of sunk costs and future returns. Hence an erroneous change in the valuation of the option to quit which has no effect on sunk costs and the returns to the active state must be incorporated into the value of the option to enter.

38. That even skilled financial practitioners can fail to value complicated options correctly is a conjecture encouraged by anecdotal evidence raised by various participants from both the academic and financial communities at the Warwick Options Seminar.

39. I am grateful to Professor Dixit for noting the probable implications of such misvaluations in a larger model.

40. In the context of the full solution the endpoints serve to pin down the absolute levels and slopes of the value functions. We have already seen that the trigger prices can be solved in terms of the value difference function, so this result should not surprise us. Note however that Dixit used the endpoint conditions before proceeding to calculate the difference between the value functions.

2.7 CONCLUSIONS

This chapter begins from the conjecture that hysteresis effects may be important in international trade. A brief review of the literature identifies sunk cost hysteresis as the pivotal theoretical concept in this area. After introducing the basic ideas of sunk cost hysteresis, we have focussed on Dixit's (1989a) important microfoundations model of entry and exit under uncertainty.

We have set out the elements of the Dixit model and reviewed its solution in some detail, judging that this model will prove to be an important starting point for future research in a range of fields. From the perspective of stochastic control we have explained the model as an example of interrelated optimal stopping, and indicated how the firm's proposed solution strategy satisfies the principle of optimality. We have also noted the simplifying role of the property of stationarity in this particular model. We have also expounded Dixit's financial options interpretation of the model and its solution, without presuming prior knowledge of the options literature. We have sought to develop further intuitions concerning the Dixit model through a graphical presentation of the solution.

Closed form expressions do not exist for the trigger prices which are central to the model's solution, and we have reported that Dixit therefore employs a variety of techniques to explore the model's properties, including numerical methods and analytic approximations. We have reviewed the principal findings of Dixit's analyses. In particular we have emphasised the model's hysteresis property, which is dependent on the existence of sunk costs in either entry or exit; the non-certainty equivalence of the stochastic model's solution (and the potential quantitative importance of this feature); and the property that small amounts of sunkenness and uncertainty combine to produce relatively large hysteresis effects, suggesting that sunk cost hysteresis may be a widespread phenomenon.

We have drawn attention to the fact that the Dixit model's trigger price solutions can be derived using only that subset of the boundary conditions for the full control problem which relate to the relative

position of the value functions in the two discrete states, and we have suggested an economic interpretation of this fact.

The main task of this chapter has been to identify and present the Dixit model as a framework for the analysis of sunk cost hysteresis, in preparation for the subsequent analysis of the model under the assumption of exogenous mean reversion in the price process. The properties of the model with mean reversion are investigated in the next chapter.

Chapter Three.

MEAN REVERSION IN THE DIXIT MODEL OF HYSTERESIS IN TRADE WITH BOTH SYMMETRIC AND ASYMMETRIC PATTERNS OF SUNK ENTRY AND EXIT COSTS

3.1 INTRODUCTION.

We investigate Dixit's model of entry and exit under uncertainty when it is extended, (following the suggestion in 1989a, pp 634-5), to include exogenous mean reversion in the price process. We explore how mean reversion affects the properties of this important model.

Although we use other analytical evidence to develop and support our argument where we can, we must solve for the entry and exit trigger price solutions to the Dixit model with mean reversion by numerical means. We have used a simple shooting method to explore solutions for a wide range of the parameter space.[1] We find that, while mean reversion has the predicted hysteresis band-widening effect, it also reduces the non-certainty equivalence of the solutions. However, our major finding is a result of great qualitative importance to the study of entry and exit under uncertainty. We now describe this quite unexpected result and go on to explain its significance.

In all previous analyses, (including Dixit's), the effect of allowing for uncertainty has been that a firm facing sunk entry costs should delay its entry relative to a perfect foresight (certainty equivalent) decision. This means that the optimal entry trigger under uncertainty is higher than the certainty equivalent trigger. The most important result of this chapter is that when uncertainty is added to the model it can actually be optimal for a firm to enter sooner than it otherwise would under perfect foresight. Compared with previous results, the qualitative impact of uncertainty on entry is therefore reversed in this case.

1. In appendix 3.2 we describe a simplified two period discrete time analogue of the Dixit model which does have closed form solutions for the trigger prices and which supports the numerical results.

The conditions under which this outstanding result occurs cannot be described as extreme or uninteresting, and indeed they can be satisfied by adding a fairly small degree of mean reversion to the "central case" of Dixit's (1989a) numerical analysis. When analogous conditions are satisfied, a similar result can be found to apply to exit decisions, so that exit is then hastened rather than delayed by the addition of uncertainty to the analysis.

We refer to this result as a 'crossover' result, because the optimal trigger solution has crossed over the associated certainty equivalent solution. This reversal of the ordering of the triggers is quite unexpected on the strength of the existing literature, and it is a strong counterexample to the pattern of previous results. Such a surprising or counterintuitive finding must challenge our existing understanding of the model: what then is its significance to the study of entry and exit?

The crossover result spotlights the essential difference between reversible and strictly irreversible entry decisions under uncertainty, and forces us to recognise the practical impact of the simultaneous determination of entry and exit decisions in the reversible case. Although his analysis captures this simultaneity, Dixit's results fail to give a strong intuition of its importance; qualitatively, Dixit's results are similar to the case with irreversible entry. In sharp contrast, the intuitions from the irreversible case are quite incompatible with our crossover results. Hence these results vividly demonstrate the interdependence ofentry and exit: future possibilities of both entry and exit influence current decisions about entry; equally, both entry and exit possibilities affect current exit decisions.

Our main conclusions concerning the Dixit model with mean reversion can be illustrated by reference to figures 3.1, 3.2 and 3.3.[2]

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2. Figures 3.1, 3.2, and 3.3 plot the value functions in the active state (V_i) and in the idle state (W_i) as a function of the current price p, for the case of zero exit costs (L=0), and nonzero sunk entry costs (K), and over varying degrees of mean reversion.

We can take 3.1 to correspond to Dixit's solution with Brownian motion, while 3.2 shows the effect of the introduction of a small amount of mean reversion. The certainty equivalent or perfect foresight solution band is given by the entry and exit triggers P_{CU} and P_{CL} respectively. The fully optimal solution but is given by entry and exit triggers P_U and P_L.

Figure 3.3 has sufficient mean reversion in the price process to deliver the particularly interesting 'crossover' solution property at the entry threshold.

Figure 3.1

SOLUTION TO DIXIT MODEL WITH BROWNIAN MOTION.

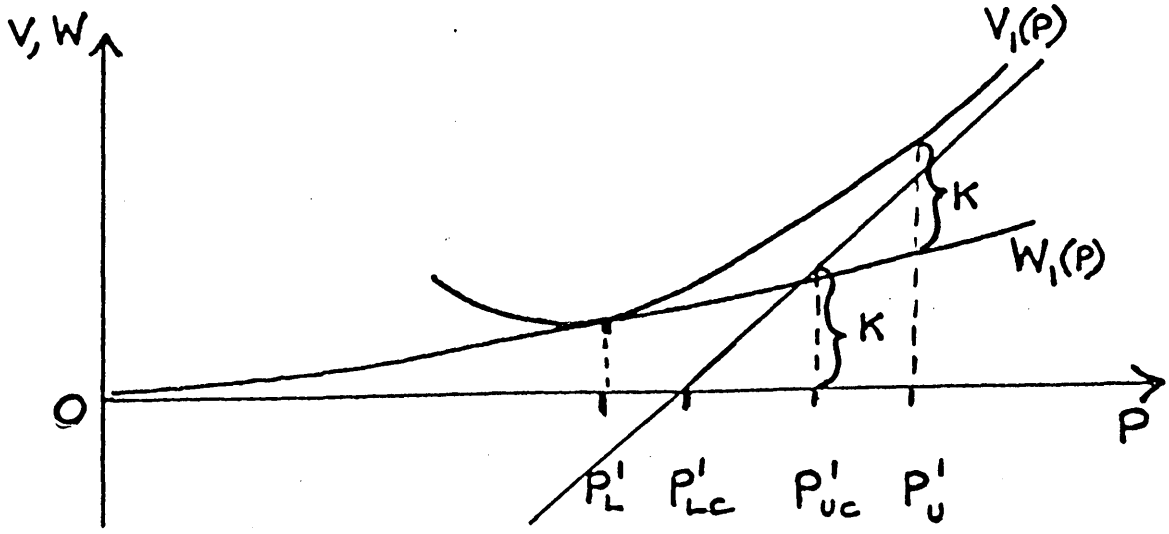


Figure 3.2

SOLUTION TO DIXIT MODEL WITH MEAN REVERSION.

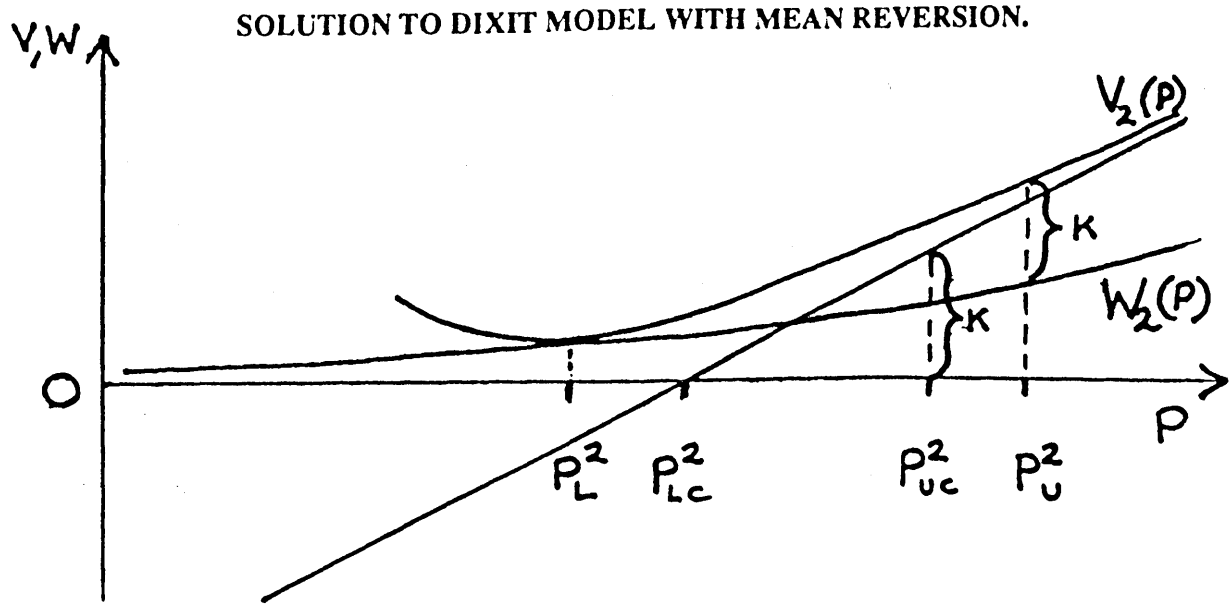
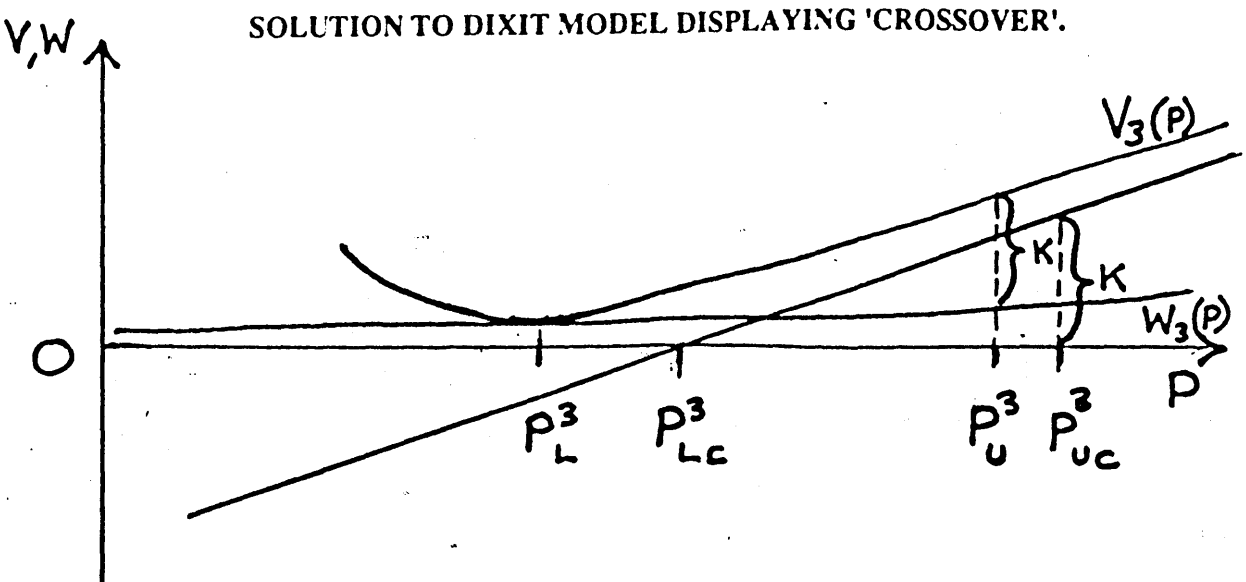


Figure 3.3

SOLUTION TO DIXIT MODEL DISPLAYING 'CROSSOVER'.



Firstly, an increase in mean reversion widens the overall hysteresis band between the optimal entry trigger P_U and the optimal exit trigger P^L. The degree of mean reversion increases as we move from figure 3.1 to 3.2 to 3.3. As the diagrams illustrate, the horizontal distance $P_U - P_L$ increases with mean reversion. This confirms the band-widening predictions in the literature.

Secondly, increased mean reversion reduces the non-certainty equivalent part of the solution. In terms of the illustrations this part of the solution can be seen in the difference between the optimal solution band $P_U - P_L$ and the certainty equivalent solution band $P_{CU} - P_{CL}$. The difference between these quantities is reduced as mean reversion increases from figure 3.1 to 3.2 to 3.3.

Mean reversion therefore has opposing effects on the certainty equivalent part of the solution ($P_{CU} - P_{CL}$) and on the remaining non-certainty equivalent part. Our intuition as to why the certainty equivalent effect of mean reversion dominates is that increased mean reversion affects whether or not it pays to exercise the options whose value is reflected in the non-certainty equivalent part of the solution.

Nevertheless, when the initial non-certainty equivalence is relatively large, changes in mean reversion have relatively little net effect on the overall band width compared with when the certainty equivalent part predominates. Hence we must conclude that the deterministic comparative statics with respect to mean reversion are not a reliable guide to those of the stochastic solution.

So far we have referred to figures 3.1 and 3.2 to illustrate some general effects on solutions to the Dixit model of the addition of mean reversion to the price process. In addition to these general properties, figure 3.3 illustrates our most interesting result, which only applies for a particular part of the parameter space. When sufficient mean reversion has been added to a case in which the sunk costs of entry dominate the (in this example, zero) sunk exit costs, we find that <u>the optimal entry trigger P_U can be lower than the certainty equivalent entry trigger P_{CU}</u>. This result is illustrated in figure 3.3. This

very important counterexample establishes the conclusion that allowing for uncertainty will not always delay entry in sunk cost models with both entry and exit.

In order to make sense of these results with asymmetric sunk costs we consider the non-certainty equivalent part of the sunk cost hysteresis model. As a starting point for explanations we must recognise that the non-certainty equivalent or caution component of the solution reflects the difference in the value of two options. Entry and exit costs affect this option difference in opposite ways: hence when entry and exit costs are substantially different, the 'usual' sign of the difference between the options of entry and exit may be reversed. Moreover, mean reversion reduces the value of distant possibilities relative to the immediate future in which the different impact effect of entry and exit costs really counts. (The explanatory power of these two effects within the model's solution is clarified by the simplified model developed in appendix 3.2.)

The chapter begins by explaining why it is of interest to investigate the effects of exogenous mean reversion in the stochastic price process in the Dixit model: both theoretical and empirical arguments are given. We discuss the way in which price expectations have been modelled in the sunk cost hysteresis literature, and note the predictions that have been made concerning the probable effects of adding mean reversion.

We then review Dixit's proposal for the addition of mean reversion to his model and discuss the various ways in which the model might then be solved. We explain our choice of a relatively simple numerical shooting method.

We endeavour to deploy a variety of analytic arguments to shed light on the effects of adding mean reversion before we present the results of our numerical experiments. To begin with we discuss the implications of various choices of the level to which the price is expected to revert, and we argue that our primary focus should be on cases in which the model has a long run hysteresis property. We then review the comparative statics of the deterministic trigger prices with respect to changes in mean reversion, before extending the approach to the case with uncertainty, using the intuitions from the

options interpretation of the model to offset the absence of closed forms for these derivatives. We draw on our simplified analogue of the Dixit model to help disentangle the role of different influences, and note the possibility it raises that mean reversion with uncertainty may not always widen the hysteresis band.

We move on to a presentation of our numerical results. As indicated above, we report three findings: the failure to uncover a band narrowing result; the damping effect of the stochastic component on the deterministic response; and what we have termed an 'asymmetric crossover' property. In order to focus on the qualitative properties in which we are interested, detailed accounts of the numerical work are relegated to an appendix.[3] In addition to reporting and characterising the results from the numerical experiments we seek to understand them. We identify their role in the extension of the Dixit model and related literature, and we also comment on their relation to the results obtained by Baldwin (1989) within a more general but discrete time context.

3.2 WHY CONSIDER THE STOCHASTIC CASE WITH MEAN REVERSION?

There are three kinds of reasons why we wish to consider the properties of the Dixit model under uncertainty with exogenous mean reversion in the price process.[4]

The first kind may be described as reasons of theoretical completeness. The case with mean reversion is an intermediate case between the iid stochastic specification in Baldwin and Krugman (1989) and the Brownian motion specification in the Dixit model itself. It is a natural question to ask whether these limiting cases are useful guides to the description of the intermediate case.

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3. See appendix 3.2 below.

4. Baldwin (1989) makes the case thus:
"sunk cost hysteresis has been formally characterised for only three highly special assumptions on firms beliefs about the process governing future uncertainty. This lack of theoretical generality is a significant omission since it hinders efforts to empirically test for hysteresis as well as the development of econometric techniques to estimate models that allow for sunk costs. Moreover, it hinders the integration of the partial equilibrium sunk cost model into a general equilibrium macro model". (p 3)

The second class of reasons is based on the theoretical plausibility of stationarity in the price process. The finance literature predicts a martingale property for asset prices such as exchange rates, assuming rational expectations, efficient markets and martingale fundamentals. In contrast, a variety of arguments familiar to debates in macroeconomics, (such as the sluggishness of goods price and wage adjustment, and the possibility of temporary phenomena such as fads in asset markets), suggest that price processes may tend to revert over time to an equilibrium level.[5]

A third kind of argument for a consideration of mean reversion in the price process is empirically based and relates to the hysteresis in trade application of the model in which the relevant price process is the real exchange rate. In the empirical analysis of subsequent chapters we argue that UK competitiveness behaves as a stationary variable in the policy periods prior and subsequent to the regime change in UK economic policy in 1979.[6] If we wish to apply the sunk costs hysteresis arguments to UK trade, we should wish to consider the case with mean reversion.[7]

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5. The Dornbusch (1976) model gives one account of why real exchange rates might be expected to display mean reversion.

A survey of the theoretical literature on bubbles is given in Blanchard and Fischer (1989), while West (1988) assesses the empirical literature. Miller and Weller (1990a) give a qualitative analysis of currency bubbbles in the stochastic saddlepoint framework described in chapter four below.

Poterba and Summers (1987) put the case for modelling autoregressive 'fads' in financial markets, an approach developed in the context of 'noise traders' by Campbell and Kyle (1988).

6. See chapter five for the development of these arguments.

7. We consider the application of the Dixit model to the UK case in chapter seven, where we argue that the difference between results with and without mean reversion in the real exchange rate appears to be of some practical relevance.

3.3 THE EXISTING LITERATURE.

We have surveyed the existing literature on hysteresis in trade in the previous chapter, where we have acknowledged the relevance of a general theoretical literature on sunk cost hysteresis.[8] This literature has recognised that the case with a mean reverting stochastic process would be of interest, but, with the recent exception of Baldwin (1989), has not attempted to explore it in any detail. We can however record the view of the literature as to the likely impact of the introduction of mean reversion. As we have seen, Dixit allowed for this extension, but did not pursue the numerical implementation, commenting that:

"It is intuitively obvious that the result (of adding mean reversion to the model) can only be a widening of the range of inaction, ...when the current price is high, mean reversion makes the future outlook less favourable and therefore the firm is more reluctant to enter".(Dixit, 1989a, p 634)

In a related model, Krugman reported the numerical solution in the presence of both mean reversion and uncertainty for a single set of parameter values.[9] However, he failed to explore the detail, including the consequences for the solution of varying the degree of mean reversion, reporting only that:

"the consequence of the combination of expected mean reversion and uncertainty is a drastic widening of the range" (of the zone of no change or inaction). (p 20)

We note, moreover, that Krugman's method is, by construction, restricted to analysis of the symmetric case, in which entry and exit costs are equal.

Baldwin (1989) adopts a very general stochastic dynamic programming formulation of sunk costs hysteresis using discrete time methods. Greater generality can thus be claimed for the results than in the particular parametric framework of Dixit's model. Baldwin seeks to characterise solutions

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8. We will refer to the aggregate literature on hysteresis in trade in chapter seven.

9. Krugman (1988)

"for a broad class of assumptions concerning firm's beliefs about the process generating the forcing variable", (1989, p1),

noting that this class will include model consistent expectations. His conclusion concerning the effects of changes in mean reversion is that:

"Greater persistence in shocks has the effect of making well-entrenched firms more likely to exit, of narrowing the band for marginal firms and of making unlikely entrants more likely to enter." (p 25)

Greater persistence is equivalent to decreased mean reversion in the price process, and so the result for 'marginal firms' is effectively that increased mean reversion widens the band.[10]

We will indicate that these conclusions fail to add anything to the characterisation of the solution with mean reversion that is not available from a purely deterministic analysis. Baldwin does not address the non-certainty equivalent aspects of sunk cost hysteresis with a mean reverting price process which we investigate here.[11]

On what basis does the literature predict that the introduction of mean reversion will widen the hysteresis band? Mean reversion, in effect, makes price departures from the long-run level temporary relative to the Brownian motion case, in which price shocks appear permanent. For a temporary price shock to be equivalent to a permanent one, it must initially be a larger one. But this is a deterministic or certainty equivalent intuition which fails to take account of the non-certainty equivalent aspect of the full stochastic solution. In fact the caution component of the solution, (reflecting the option values associated with the possibility of entry and exit), responds in a quite different way to the introduction of mean reversion than does the deterministic component. Thus a full description of the effects of introducing mean reversion into the model requires a richer account than is predicted in the literature.

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10. We will return later to the distinctions between marginal firm, well entrenched firms and unlikely entrants.

11. We will contrast Baldwin's analysis with our approach.

3.4 INTRODUCING MEAN REVERSION INTO THE DIXIT MODEL AND SELECTING A SOLUTION METHOD.

3.4.1 Modelling Mean Reversion.

Dixit considers a number of directions in which this model can be extended. In order to allow for the possibility that prices or real exchange rates tend towards long-run equilibrium levels despite their short-run fluctuations, Dixit proposes the use of a mean-reverting process for the price or competitiveness variable of the linear form:

$$dP = \lambda (P^{*}-P)\, dt + \sigma P\, dz \qquad (3.1)$$

where P^{*} is the fixed long-run equilibrium.[12] In contrast to the Brownian motion case we thus assume the price process to be (covariance) stationary. Dixit also allows that a Poisson process with discrete price jumps might be appropriate. However, we will investigate the effects of the mean reverting stochastic specification given above.

3.4.2 Solutions.

Dixit (1989a) argues that, when the stochastic specification is modified in this fashion, the model's Bellman equations "do not have closed form solutions". (p 634) Hence this extension of the Dixit model is taken to require the use of numerical procedures. In this study we employ the shooting techniques made use of by Miller and Weller (1988a) and Krugman (1988) in order to investigate the effects on the solution to this model of the introduction of varying degrees of mean reversion into the exogenous price process. As given by Dixit (1989a, p 634), the Bellman equations become:

$$\tfrac{1}{2}\sigma 2\, P^{2}W''(P) + \lambda (P^{*}-P)W'(P) - rW(P) = 0 \qquad (3.2)$$

$$\tfrac{1}{2}\sigma^{2}\, P^{2}V''(P) + \lambda (P^{*}-P)V'(P) - rV(P) + P - w = 0 \qquad (3.3)$$

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12. See Dixit (1989a), p 634.

These equations are to be solved subject to Value Matching and Smooth Pasting conditions which hold at the trigger prices for entry and exit, P_L and P_U.

Dumas (1989a) indicated that general solutions for the value functions exist in similar problems with an exogenous mean reverting stochastic process and that these expressions involve confluent hypergeometric functions.[13] Alan Sutherland (1989) has also noted that these methods may be applied to the stochastic saddlepoint systems of Miller and Weller for the restricted case corresponding to exogenous mean reversion.[14] In fact, as Sutherland reported, the closed form of the solution does not facilitate analytic solution of the model in the sense that Kummer's function must itself be evaluated numerically. Hence there is no gain in analytic tractability from this approach, leaving a methodological decision to be taken as to whether the additional numerical accuracy which the use of Kummer's functions may facilitate is worth the additional complexity. In the light of the continuing research of Miller and Weller and Sutherland, the simple shooting method of solution employed by Krugman (1987) and Miller and Weller (1988a) was adopted.[15]

3.4.3 Using a Different Model with Closed Form Solutions for the Trigger Prices as a 'Maquette' for the Dixit Model.

A disadvantage with numerical solutions is that they can suffer from 'black box' syndrome: the explanation of particular properties of a solution is not necessarily clear. In order to supplement our approach to the Dixit model with mean reversion we consider an entirely distinct (albeit related) model of sunk cost hysteresis, which stands in relation to the Dixit model as a maquette to a finished sculpture.[16]

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13. Dumas (1989a), p 11. See Abramowitz and Stegun (1965), Erdelyi et al (1953) and Slater (1960) for mathematical presentations of confluent hypergeometric functions. These are also known as 'Kummer's function'.

14. Sutherland (1989), p 3.

15. As part of continuing research Alan Sutherland has solved a more general stochastic saddlepoint model by using power series expansions. He reports that these solutions can behave strangely at singularities.

The simplified model is a two period discrete time set up, based on the deterministic model described in the appendix to the previous chapter, but with uncertainty added in a simple binomial form. In this simplified analogue of the Dixit framework we can obtain closed form solutions for the first period entry and exit trigger prices. The simplicity which allows the closed form solutions for the price triggers is bought with some strong restrictions relative to the Dixit model, including the restriction to two periods, and the restriction of the future to one of two possible outcomes.[17] We must further restrict the possible future outcomes to ensure that the options of entry and exit are nonzero and hence that the *ex post* realisation of uncertainty makes a material difference to the optimality of different strategies.

Despite its stark simplicity the model captures some important properties of the Dixit model, including its non-certainty equivalence. Moreover, we find that the model is able to explain many of our numerical results when we add mean reversion to the Dixit model. Hence it appears that the model serves as a simplified but valid representation of key theoretical features of the Dixit model. The model is developed in Williams (1990b), but some details of its solutions and properties are presented in appendix 3.2.

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16. I thank Marcus Miller for arranging my introduction to the smaller works of Henry Moore as a means of suggesting this analogy. A similar usage of "maquette" can be found in Balasko (1984).

17. If pressed to justify this approach we could always refer to the following quote from Dixit (1989a, p 623):
"Suppose that it (the firm) waits one period. If at the end of this period the price has gone up, it can invest and get positive expected net present value. If the price has gone down, it need not invest, so the expected present value is zero. If we weight these by the probabilities of each and add, the expected present value of waiting one period is positive".
The two period binomial approach is implicit in Dixit's thought experiment.

3.5 ANALYTIC EVIDENCE ON THE SOLUTION WITH MEAN REVERSION.

3.5.1 Introduction.

Before beginning our numerical studies we are able to use some analytic methods to gain insight into the effects of mean reversion on the solution. First we consider the issue of the long run level to which the price process reverts, classifying models into three according to whether this long run price falls above, below or within the hysteresis zone defined by the trigger prices. We argue that the third of these cases is of primary interest, and restrict our subsequent attention to this case.

We proceed to the model with mean reversion but without uncertainty. In this deterministic case the addition of exogenous mean reversion will widen the band between the trigger prices. It is relative to the deterministic analysis that we measure the further complications introduced by the simultaneous presence of mean reversion and uncertainty.

We emphasise two factors affecting the non-certainty equivalent part of the solution. One concerns the different impact entry and exit costs have on the difference between the option values of entry and exit. Another concerns the role of mean reversion in damping future uncertainty. These factors are important mechanisms behind the results of our subsequent numerical analysis.

3.5.2 Steady State Profits and the Long Run Price Level

A basic modelling decision which has to be made in order to implement the numerical solution of the Dixit model with a mean-reverting price process is as to the relative levels of the full flow cost (w) and the long-run equilibrium price (P^*). There are three logical possibilities to consider, classified by the position of the long-run equilibrium. The long-run price level might be so high relative to flow costs, that, no matter what the intervening events and possible future shocks, when that equilibrium price is

achieved, the firm will be active. Such a case would be characterised by $P^* > P_U$. Alternatively, the long-run price level might be so low relative to flow costs that no matter what, when the equilibrium price is achieved the firm will be inactive. This would correspond to $P_L > P^*$.

Much the most interesting possibility is the third, in which the long-run equilibrium to which the price process tends to revert lies within the hysteresis zone. This corresponds to the case where $P_L < P^* < P_U$: in this case the model's steady state equilibrium (at $P=P^*$) is path dependent. Whether the firm is active or idle at $P=P^*$ will depend on the initial value of P and on any subsequent shocks which modify its path to the long-run equilibrium. We choose to restrict our numerical studies to this hysteresis case.[18]

Having chosen to locate the long-run equilibrium price P^* within the zone of inaction there is still a decision to be made as to the relative position of P^* and w. It seems appropriate to avoid the implications of either steady state profits or losses, and so we concentrate on the case where $P^*=w$.[19]

3.5.3 Mean Reversion: the Deterministic Solution.

We examine the properties of the deterministic solution with a mean-reverting price process, but with no uncertainty. We first set out solutions for the entry and exit thresholds and then consider the comparative static effects of an increase in mean reversion on those solutions.

In order to simplify the analysis of the mean-reverting case we wish to maintain the convenient separation such that if the conditionally expected path for the price were realised, an active firm would

18. Baldwin (1989) considers all three possibilities with respect to the location of the long run or mean price, referring to them as the case of the 'marginal firm', (our hysteresis case), the case of the 'unlikely entrant', and the case of the 'well-entrenched' firm. Baldwin's characterisation of these solutions corresponds to a the deterministic predictions.

19. On the one hand firms that projected long term losses might be expected to adjust their costs accordingly, or else to exit. On the other hand, while in the presence of sunk costs we might reasonably expect prices to be able to exceed avoidable costs, this may not be true in the long run: for example, under Chamberlinian monopolistically competitive assumptions we would expect long-run profits to be eliminated. The appropriate answer to the relation of long-run price to costs would depend on the wider equilibrium model within which the firm were located, and also on whether, within that context, the firm perceived its environment as atomistically competitive or else as inherently strategic.

remain active and an idle firm would choose to remain idle.[20] By restricting the certain long-run real exchange rate *a priori* to lie within the hysteresis band (or "zone of no change") we are still able to analyse the case with mean reversion by using infinite horizon approximations of the future profits stream.

Given this simplifying restriction, the value of an idle firm is zero and the value of an active firm is given by the present discounted value of future profits under the deterministic price path. For the deterministic mean-reverting process:

$$dP/dt = \lambda (P^{*}-P) \qquad (\lambda > 0) \tag{3.4}$$

The present discounted value of profits for the active firm, V(P), is thus given by the two components:

$$V(P) = (P^{*}-w)/r + (P-P^{*})/(r+\lambda) \tag{3.5}$$

The solution to the entry and exit thresholds is determined by Value Matching alone for our certainty case.

The Value Matching condition at the exit threshold P_L is thus given as:

$$(P_L-P^{*})/(r+\lambda) + (P^{*}-w)/r + L = 0 \tag{3.6}$$

Similarly the value matching condition at the entry threshold P_U is given as:

$$(P_U-P^{*})/(r+\lambda) + (P^{*}-w)/r - K = 0 \tag{3.7}$$

Solving for the desired trigger prices or thresholds and rearranging we obtain:

$$P_L = w + (\lambda/r)(w-P^{*}) - (r+\lambda) L \tag{3.8}$$

and:

$$P_U = w + (\lambda/r)(w-P^{*}) + (r+\lambda) K \tag{3.9}$$

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20. We would prefer not to have to calculate the present value of an initially active firm by integrating over profits to a particular date of exit, and, for an initially idle firm, over profits subsequent to some entry date.

The price triggers differ from the unit cost w due to the term reflecting the direct effect of sunk costs (K and L), and/or a term reflecting the long run profitability of the active firm (the term in (P^*-w)).[21] Note that in the absence of mean reversion the thresholds are $P_L = w - rL$ and $P_U = w + rK$, which correspond to Dixit's limiting results as σ goes to zero.[22]

In the absence of sunk costs of either kind there is of course no hysteresis and $P_L = P_U = w$.

We can now differentiate these solutions for the price triggers to obtain the effect of changing the speed of adjustment and of changing the long-run equilibrium price. The derivatives are as follows:

$$\partial P_L/\partial\lambda = (1/r)(w-P^*) - L \tag{3.10}$$

$$\partial P_U/\partial\lambda = (1/r)(w-P^*) + K \tag{3.11}$$

and:

$$\partial P_L/\partial P^* = \partial P_U/\partial P^* = -(\lambda/r) \quad 0 \quad (\text{NB. } r>0, \lambda>0) \tag{3.12}$$

Raising the long-run equilibrium price lowers the certainty-equivalent component of both thresholds by an identical amount. The effect of changing the speed of adjustment on the overall band width is given by the difference between the derivatives.

$$\partial(P_U - P_L)/\partial\lambda = (\partial P_U/\partial\lambda - \partial P_L/\partial\lambda) = K + L \tag{3.14}$$

Regardless of the value of the long run equilibrium price this derivative is positive for nonzero sunk costs. So we may conclude that <u>deterministic mean reversion always widens the hysteresis band.</u>

In general the effect of mean reversion on the individual price triggers is ambiguous. We can, however, establish conditions under which raising the speed of adjustment will both raise P_U the entry threshold and lower P_L the exit threshold.

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21. The long-run profitability effect can be likened to the effect of adding a drift term to the Brownian motion case.

22. Dixit sets λ=0.

For $\partial P_L/\partial\lambda < 0$ we require $w - P^* < rL$

Increasing the speed of adjustment will make exit less likely as long as there is no long run gain to exit, ie. losses are less than the coupon equivalent exit cost.

For $\partial P_U/\partial\lambda > 0$ we require $w + rK > P^*$.

Increasing the speed of adjustment will make entry less likely as long as there are no long run profits to entry, ie. the long-run equilibrium price is greater than the full cost including the coupon-equivalent of entry costs. Combining these conditions, both entry and exit thresholds move outwards if the long-run price lies within the hysteresis zone as defined for the model *without* mean reversion.

$W + rK > P^* > W - rL$

This condition ensures that the deterministic effect if mean reversion does not unambiguously increase the value of one state relative to the other. Our working assumption that $P^* = w$ satisfies this condition.

The intuition for the basic result that the band should widen as the speed of reversion increases is that mean reversion reduces the present value of both profits and losses associated with a given level of the current price or real exchange rate. Hence, under certainty, entry or exit require more extreme values of the current rate to make their exercise rational.

3.5.4 On Mean Reversion in the Presence of Uncertainty.

When we add uncertainty to the model, closed form expressions for the trigger prices no longer exist. Nevertheless, we can still separate out the certainty equivalent from the non-certainty equivalent ('options difference' or 'caution') components of the trigger prices in expressions for P_L and P_U.

We define OQ(P) as the value of the option of exit (option to quit) to an active firm, evaluated at price P. This is a complex function of the parameters of the model: no convenient closed form representation of this function exists. Similarly, we define the function OE(P) as the value of the option of entry to an idle firm, evaluated at price P.

Employing these option value expressions we can write the Value Matching conditions which must hold at the entry and exit trigger prices respectively as follows.

Value Matching at the entry threshold, P_U:

$$(P_U-P^*)/(r+\lambda) + (P^*-w)/r + OQ(P_U) - K = OE(P_U) \tag{3.15}$$

Value Matching at the exit threshold, P_L:

$$(P_L-P^*)/(r+\lambda) + (P^*-w)/r + OQ(P_L) + L = OE(P_L) \tag{3.16}$$

Without solving fully for P_U and P_L we can rewrite these conditions as expressions for P_U and P_L as follows:

$$P_U = w + (\lambda/r)(w-P^*) + (r+\lambda)\,K + (r+\lambda)\,\{\,OE(P_U) - OQ(P_U)\,\} \tag{3.18}$$

$$P_L = w + (\lambda/r)(w-P^*) - (r+\lambda)\,L - (r+\lambda)\,\{\,OQ(P_L) - OE(P_L)\,\} \tag{3.18}$$

In addition to the flow cost w, each of these trigger price expressions consists of three components.

The first term, $(\lambda/r)(w-P^*)$, gives the effect on the trigger of the long run profitability of the active state.[23] In the analyses that follow, including our numerical experiments, we choose to set $P^* = w$, and so leave aside issues concerning the long-run equilibrium price.

The second term gives the remainder of the deterministic solution, $(r+\lambda)$ K for P_U, and - $(r+\lambda)$ L for P_L. Note that the sunk cost of entry is discounted by $(r+\lambda)$ to allow for the reversion inherent in the price process as well as for the discount rate.

The non-certainty equivalent part of the solution is given by the third, *option difference*, term $\{OE(P_U) - OQ(P_U)\}$: this is the similarly discounted (by both λ and r) coupon or flow equivalent of the

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23. The effect of an increase in long run equilibrium profits is to make the active state more valuable relative to the idle state at all levels of the current price. Hence the deterministic effect is to lower both entry and exit triggers. When we consider the stochastic context, an improvement in the valuation of the active state relative to the idle state will increase the option to enter relative to the option of exit. We acknowledge the possibility that this will tend to damp the deterministic effect.

difference between the options to enter and exit, both evaluated at the entry trigger. To understand how the full stochastic solution differs from the deterministic case, we must focus on this option difference term.

A slightly more general treatment would write Value Matching at the entry trigger P_U as:

$$EV(P_U) = K + OD(P_U) \tag{3.19}$$

As before, K represents the lump sum cost of entry. EV(P) represents the expected present discounted value of the continuation strategy, which is assumed to be increasing in the current price. $OD(P_U)$ is the option difference given as $OE(P_U) - OQ(P_U)$, and is also increasing in the current price.

The trigger price solution is located where the expected value of the continuation strategy matches the sunk cost of entry plus the option difference term.[24] The trigger price solution thus has components corresponding to both the sunk cost and the option difference.

3.5.5 The Option Difference Term.

Firstly we attempt a benchmark characterisation of the option difference term. We know that for higher values of P, the option of entry (OE(P)) will become more valuable and the option of exit (OQ(P)) will become less valuable (ie. that $OE'(P) > 0$, $OQ'(P) < 0$). Hence the difference between the options of entry and exit, expressed as OE(P) - OQ(P), should be increasing in P, (although without a closed form). Finally, we know that, by definition, option values are non-negative, as they would not be exercised at a negative value: however, the difference between two option values can obviously take on a negative value.

In figure 3.4 we illustrate the properties of the option difference as a function of P as in the Brownian motion solutions examined in Dixit (1989a). We would expect that at the entry threshold P_U, the value

24. Smooth Pasting must also hold of course to ensure optimality of the switching boundary..

Figure 3.4

THE OPTION DIFFERENCE AS A FUNCTION OF P: TYPICAL VALUE WITH SYMMETRIC SUNK COSTS.

$$OD(P) = OE(P) - OQ(P)$$

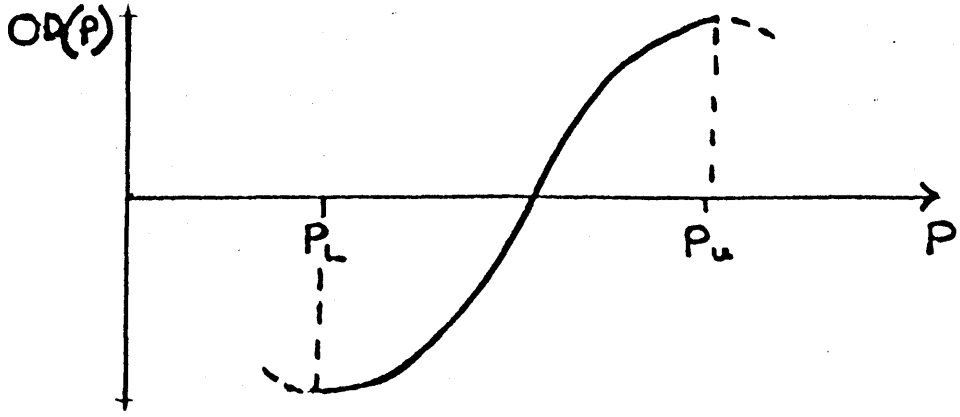
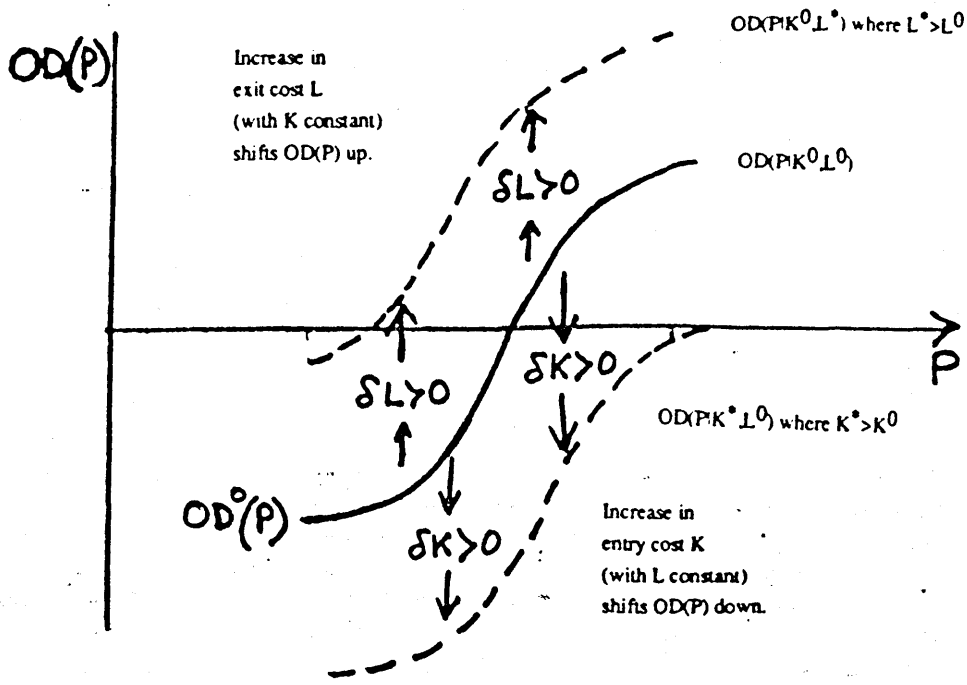


Figure 3.5

NET EFFECTS ON THE OPTION DIFFERENCES OF CHANGES IN THE SUNK COSTS.



of the option to enter would exceed the value of the option to exit so that $OE(P_U)-OQ(P_U) > 0$. Similarly, we might expect that at the exit threshold P_L, we would find $OQ(P_L)-OE(P_L) > 0$.[25]

The results of our numerical experiments, reported below, draw attention to the fact that the characterisation of the option difference function given in the preceeding paragraph is not universally valid. In particular, we conclude that the option difference given as OE(P)-OQ(P) may on occasion take negative values at the entry trigger P_U or positive values at the exit trigger P_L. These results arise when the entry and exit costs differ substantially.

3.5.6 The Impact of Asymmetric Sunk Costs of Entry and Exit.

When sunk costs of entry and exit diverge they can have an important impact on the qualitative nature of the model's solution. The key insight here is that sunk costs of entry and exit each affect the option values of entry and of exit differently, and therefore the assumptions on the sunk costs have implications for the options difference.

We begin from the observation that, *ceteris paribus*, an increase in the exercise fee will reduce the value of an option. It is clear that an increase in the entry fee should reduce the value of the option of entry while an increase in the exit fee will reduce the option value of exit. However, because optimal strategies may involve successive entry and exit transitions, the option values of entry and exit must be viewed as interrelated, in that the value of the option to exit for an active firm must take account of those future paths which will involve re-entry, and *vice versa* for the value of the option to enter.

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25. These intuitions concerning the option difference would certainly be valid if we could assume that the model was strictly symmetric in the following senses: identical entry and exit costs; a long run equilibrium corresponding to zero profits; and a symmetric distribution of prices about the zero profit value. Under such conditions the value of the option of entry OE(P) would be an exact reflection of the value of the option of exit OQ(P) in the line given by P=w. At P=w the option difference would be zero. The option difference would be a symmetric S-shaped curve as illustrated in figure 3.4. We know that in the Dixit model shocks are proportional to the price level so the distribution is not strictly symmetric in levels even when the other conditions hold.

Nevertheless, while the options of entry and exit are intertwined, there is a significant difference between them. *Every* possible future path incorporated into the value of the option of entry involves the payment of the sunk cost of entry. This is *not* true for the option of exit, although as we have noted, *some* of the paths incorporated into its value will involve entry. Hence we can see that at any given initial level of the price, while a rise in the sunk cost of entry will lower the value of both the option to enter (OE(P)) and value of the option to exit (OQ(P)), *the reduction in the option of entry will necessarily be greater.* So the following inequality will hold:

$\partial OE(P)/\partial K < \partial OQ(P)/\partial K < 0$

Similarly, for changes in the sunk cost of exit, L:

$\partial OQ(P)/\partial L < \partial OE(P)/\partial L < 0$

An increase in K will, with L unchanged, shift the option difference function down, while an increase in L with K fixed shifts the option difference upwards. We illustrate these points in figure 3.5.

In principle it is possible that the net option difference term (OE(P)-OQ(P)) could be negative for all P if the entry cost were sufficiently large relative to the exit cost and other parameters. Similarly, the term might be negative for all P if the exit costs were sufficiently large relative to the entry cost and other parameters. The consequences of a sufficient shift in the option difference function due to the effects of asymmetric sunk costs may be solutions which differ qualitatively from the 'typical' solution based on symmetric sunk costs as described below.[26]

3.5.7 The Direct Effect of Mean Reversion on Option Values.

A second mechanism begins from the recognition that a change in the degree of mean reversion in the exogenous process will directly affect the value of the options of entry and exit. We now develop the arguments behind this claim.

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26. In principle it seems possible that this effect could operate in the Brownian motion, and that it is not dependent on mean reversion. Our simplified analytic model reveals such a possibility. However, Dixit's analysis of such cases failed to uncover such results.

In terms of continuous time processes the effect of mean reversion on the variance of future realisations may be captured in the covariance of realisations over an interval h. Cox and Miller (1965), p 289, give the correlation ρ between realisations, separated by interval h, of the process:

$$dx = -\lambda x\, dt + \sigma\, dz$$

as $\rho(h) = e^{-\lambda h}$.

Noting that the instantaneous variance of x is σ, we can infer that:

$$g(h) = \mathrm{cov}(x(t),x(t+h)) = \sigma\, e^{-\lambda h} \tag{3.20}$$

A rise in the coefficient of mean reversion λ reduces the covariance.[27] In effect, increasing mean reversion reduces the variation in future realisations of the process.

We now identify the effects reduced future variability of prices due to increased mean reversion will have on option valuations. One of the fundamental results of option pricing is that a reduction in the variability of the underlying asset will reduce the value of the associated option.[28] The effect of increasing mean reversion in the exogenous price process is to reduce the value of both the option to enter to an idle firm and of the option to exit for an active firm. Intuitively, the option value reflects the likelihood of exercising the right to enter (or exit): increased mean reversion reduces that likelihood.

Our concern is the consequences of mean reversion for the option difference term which enters the full stochastic solution. We thus wish to sign the derivative of the difference between the options of entry and exit, ie.

$\partial\{ OE(P_U) - OQ(P_U) \}/\partial\lambda$ and $\partial\{ OE(P_L) - OQ(P_L) \}/\partial\lambda$.

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27. This result is similar to the familiar result for the infinite step ahead forecast variance in a discrete time first order autoregression. ($\sigma^2/(1 - \phi^2)$

28. Familiar in texts such as Brealey and Myers (1984).

3.5.8 The Overall Effects of Mean Reversion in the Dixit Model of Entry and Exit with Uncertainty

We use the expressions for the trigger prices to consider the comparative statics with respect to exogenous changes in the coefficient of mean reversion.

With $P^*=w$, we obtain by differentiating:

$$\partial P_U/\partial\lambda = [K + \{ OE(P_U) - OQ(P_U)\} + (r+\lambda)\, \partial\{ OE(P_U) - OQ(P_U) \}/\partial\lambda].A(P_U) \qquad (3.21)$$

$$\partial P_L/\partial\lambda = [-L + \{ OQ(P_L) - OE(P_L)\} + (r+\lambda)\, \partial\{ OQ(P_L) - OE(P_L) \}/\partial\lambda].A(P_L) \qquad (3.22)$$

where $A(P) = (1 - (r+\lambda)\{\partial(OE(P) - OQ(P))/\partial P)^{-1}$.

Both of these derivatives can be thought of in terms of three components. The first of these is identical with the certainty equivalent derivative, and corresponds to the effect of the increased speed of mean reversion in further discounting the present value of profits (losses) implied by a given initial price. This term implies that an increased value of the entry threshold (decreased value of the exit threshold) is required to offset the sunk cost of entry (exit). The second term can be explained in a similar way: it reflects the increased discounting of the current price due to an increased speed of mean reversion, but in this case, it is required to offset the difference of the options to enter and exit given the initial state.

The third term in each of our expressions reflects the fact that a change in the speed of mean reversion in the exogenous stochastic price process will also have an effect on the option values involved in the problem. This effect will reduce the option difference, and therefore operates on the solution in the opposite direction from the certainty equivalent effect. Thus the presence of uncertainty could damp the comparative static effect of mean reversion predicted under the certainty case

3.5.9 Expected Value versus Option Difference.

The trigger prices are determined using Value Matching and Smooth Pasting boundary conditions, and these can be expressed in terms of the expected value of continuation (EV) and option difference (OD) components. At the entry trigger we know that EV = K + OD, while at the exit trigger EV = OD - L. Smooth pasting requires that $\partial EV/\partial P = \partial OD/\partial P$ at the price triggers.[29] The relationship between, and behaviour of, EV and OD is crucial to the properties of the solution.

In the absence of sunk costs, EV and OD are identically equal, as the options of entry and exit between them include all possible realisations of the stochastic process. Nonzero sunk costs imply that it is not always optimal to exercise either the option of entry or the option of exit: it is no longer the case that the option difference term reflects the entire distribution of outcomes of the stochastic process. In general we can assume that the option difference term will only be a proportion of the expected value.

We conjecture that a band widening effect of mean reversion requires that the expected value of the continuation policy for the active firm is more sensitive to mean reversion than is the option difference term.

We would expect this band widening condition to be satisfied because there is a choice involved in the optimal exercise of an option. While increased mean reversion reduces the value of the active firm, the possibility of a change of strategy means that the value of the option difference will fall by less than the value of the continuously active firm.

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29. Note that the smooth pasting condition would take a different form with *proportional* as opposed to lump sum costs.

3.5.10 Conclusions.

We end this section with a brief review of our various analytic investigations. We have examined the deterministic consequences of adding mean reversion to the Dixit model. We have examined the issue of the level of the long run price and argued that we are most interested in reversion to the hysteresis zone. Although closed forms for the trigger prices are not available we have written down expressions which provide a framework for the analysis of the model's non-certainty equivalence with mean reversion.

We have identified two important factors influencing the option difference term and hence the non-certainty equivalent component of the solution. One of these is the differential impact of entry and exit costs on the option difference. The other is the effect of mean reversion in damping future forecast variance.

Our analysis has shed light on the net effect of mean reversion on the hysteresis band. A band widening result requires the expected value of continuation to be more sensitive to mean reversion than is the option difference. Such a result is likely because, with sunk costs, it is not always optimal to exercise options of entry and exit.

3.6 PRINCIPAL FINDINGS OF THE NUMERICAL STUDIES.

3.6.1 Introduction.

We now present the findings from our numerical studies of the Dixit model as the coefficient of mean reversion is varied. The solutions divide naturally into two kinds of cases. Firstly we deal with cases which have broadly the same qualitative properties as the solution to the model under symmetric boundaries: these include many cases which are strictly asymmetric in the sense that the boundaries are not identical. Secondly we consider the important phenomena we term 'crossover' effects which arise with asymmetric sunk costs and mean reversion.

In order to maintain clarity we present the results in general terms, relegating details of the numerical experiments to an appendix. We concentrate on explaining the way in which the non-certainty equivalence of the solution to the Dixit model is further complicated by the introduction of mean reversion.

In all cases considered we have found that the Dixit model displays the property that the hysteresis zone widens with mean reversion. For trigger prices corresponding to those state transitions with nonzero sunk costs, we find that the threshold price appears consistently to move in the direction predicted by the certainty case as mean reversion is increased: entry prices appear to be strictly increasing in λ for nonzero entry costs and exit prices strictly decreasing in λ for nonzero exit costs.[30]

We have found no cases in which the change in the caution component actually dominated the certain component. Nor have we found any cases where the increased discounting effect was greater than the reduced options value effect, and hence where the caution component of the solution widened (in the direction predicted by the certainty effect) with increased mean reversion.

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30. We could check results for cases with ε sunk costs instead of zero sunk costs at one threshold.

3.6.2 Symmetric and Nearly Symmetric Cases.

We must rely on numerical results to disclose the effects on the caution component of the solution in the full Dixit model with mean reversion. The principal findings for cases with symmetric sunk costs can be summarised as follows.

Firstly, increases in mean reversion reduce the caution component.

Secondly, in all the cases we have examined, the net effect of increased mean reversion is to widen the zone of inaction.

Thirdly the caution components are found to be monotonically decreasing in the coefficient of mean reversion.

Fourthly, the caution component appears to be convex in the coefficient of mean reversion at the entry trigger, and concave at the exit trigger.

Overall these factors combine so that, as the degree of mean reversion increases, the stochastic and deterministic solutions approach each other: the caution components of the solution decrease.[31] We find that the damping effects of the 'options difference' component on the deterministic component of the solution may be such as to render changes in mean reversion of little practical significance in cases where uncertainty is a relatively important factor.

We thus conclude that the *response* of the Dixit model's solution to changes in the degree of mean reversion is not certainty equivalent.

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31. This is not necessarily to imply that the limit of the stochastic solution as mean reversion increases is the deterministic solution, or that the caution component of the solution should, in general, go to zero.

The typical response we have found of the trigger prices to the introduction of mean reversion is summarised in figure 3.6 for cases which are symmetric or nearly symmetric. The full and certainty equivalent trigger prices are plotted against λ. In figure 3.7 we plot the value difference function for these solutions.

3.6.3 Asymmetric Boundaries and 'Crossover' Solutions.

We now turn to our most important result. Where the sunk costs of entry and exit differ to a sufficient extent, the solution can display properties which are qualitatively different from those described above.[32] These further complicate the non-certainty equivalence of the full stochastic solution with mean reversion.[33] We are convinced that these features of asymmetric cases are not mere artefacts of the numerical techniqes employed: moreover, they can be explained in terms of the behaviour of the difference between the options to enter and exit.

To illustrate our points we will consider the extreme asymmetric cases in which the sunk component of either the entry or exit cost is zero. Dixit's 'central case' is one such example, where values of $K = 4$ and $L = 0$ are assumed.

The most striking effect we have uncovered is the possibility that the threshold corresponding to the significantly larger sunk cost may in fact cross the certainty equivalent solution, so that for higher values of the coefficient of mean reversion the stochastic boundary lies *inside* the certainty equivalent

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32. We offer no *a priori* basis for identifying those cases in which the difference between the entry and exit costs is sufficient to generate this effect, though it will certainly happen when one or the other is strictly zero. Some of our numerical experiments suggest that crossover may occur when the smaller sunk cost is as big as 75% of the larger. (See appendix 3.3 for a report of cases investigated).

33. The quantitative importance of these effects in practical cases remains an entirely different matter: our evidence is that they will be common, but will be relatively small in magnitude.

Figure 3.6

EFFECT OF VARYING MEAN REVERSION ON THE TRIGGER PRICES (SYMMETRIC SUNK COSTS).

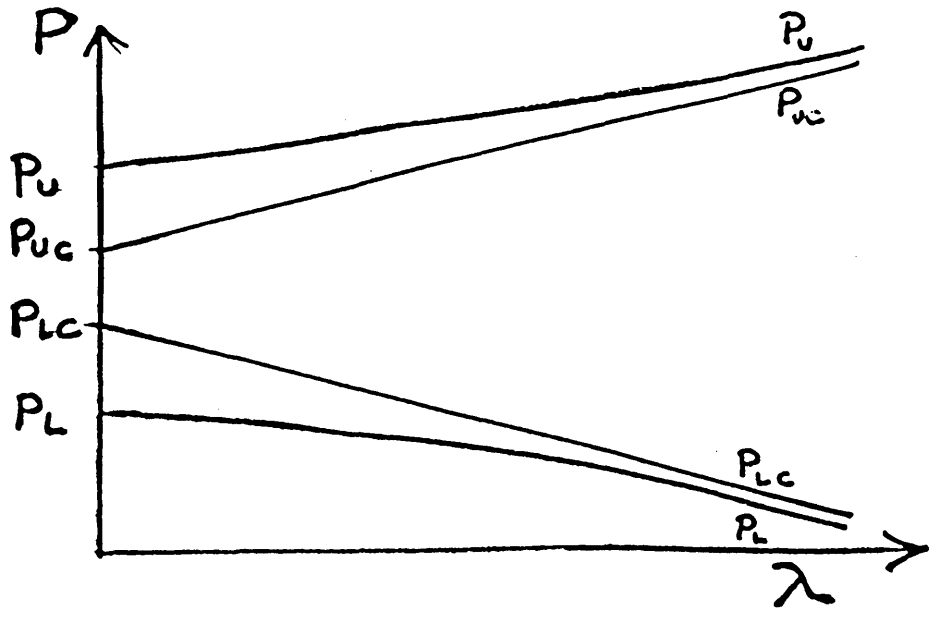
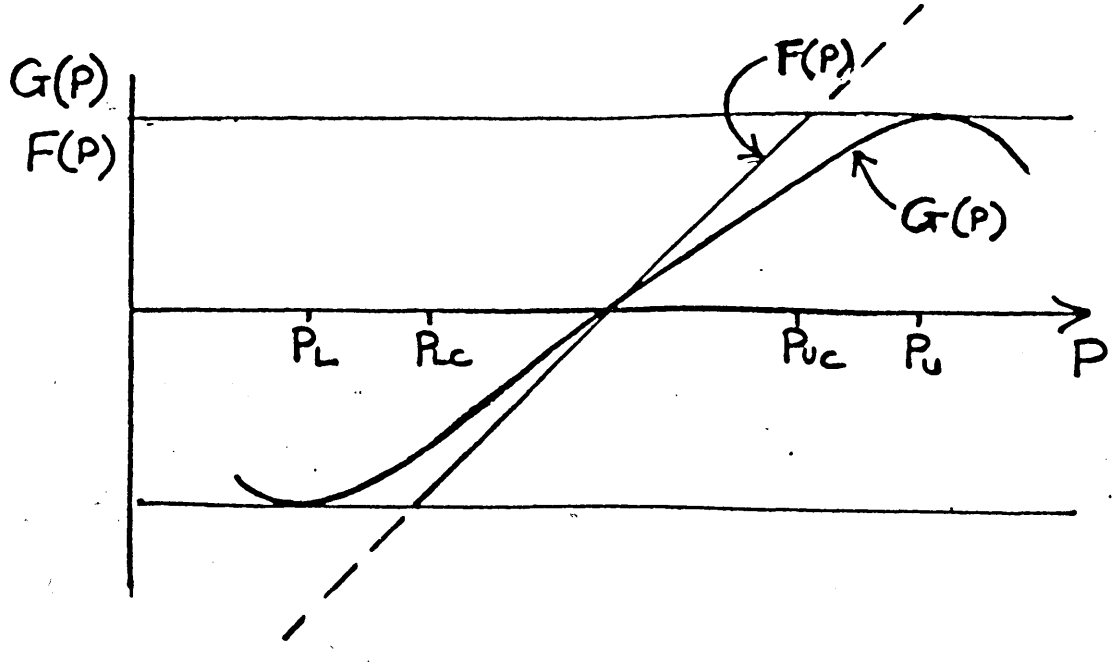


Figure 3.7

VALUE DIFFERENCE FUNCTION (G(P)) FOR SYMMETRIC SOLUTIONS.



one.[34] The sign of the caution component is reversed and the full stochastic solution *crosses* the certainty equivalent solution.

The crossover finding is illustrated in figures 3.8 and 3.9, in which the price triggers are plotted against λ. The crossover effect occurs in the upper threshold in the first case and in the lower threshold in the second.

We are also satisfied that the basic crossover phenomena are not merely numerical artefacts, but reflect the nature of the solution to the underlying problem.[35]

The crossover effect to be robust: it does not depend on strictly zero sunkenness at one state transition. We have found crossover effects where the smaller sunk cost is as much as 75% of the larger.

A further observation concerning the crossover effect is that we have found some cases where a further increase in mean reversion appears to cause the crossover effect to be reversed again. Note that this would require the option difference to be non monotonic in P.

A second feature of the crossover solution with asymmetric sunk costs applies to the other threshold, ie. to that at which there is little or no sunk cost. In many (but by no means all) cases of crossover we find that the opposite threshold to that displaying crossover will display a *non-monotonic* response to increased mean reversion, (see figure 3.10). Moreover the *curvature* of the option difference with respect to the degree of mean reversion appears to change sign in such a case. This result appears to require that the smaller sunk cost is very close to zero.

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34. Note that there is no suggestion that the stochastic zone of inaction as a whole is narrower than the deterministic one, which would be a much stronger claim.

35. Our initial response to the finding of the crossover effect on the entry trigger was to suspect our numerical method. However the finding of a crossover effect at both triggers indicated that the crossover effect itself is robust to the numerical.

Figure 3.8

'CROSSOVER' RESULT AT UPPER THRESHOLD.

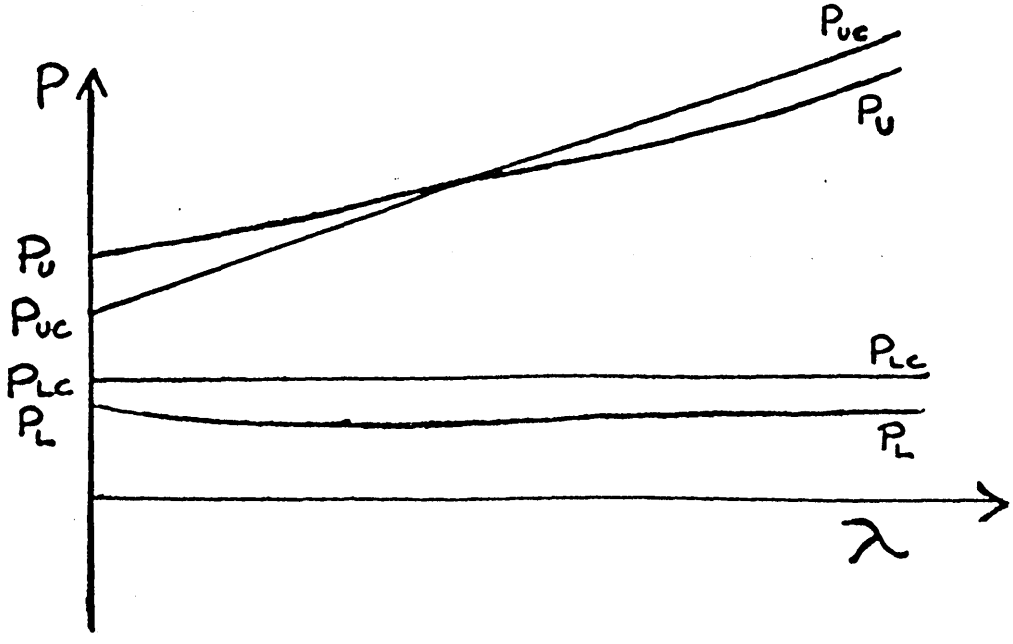


Figure 3.9

'CROSSOVER' RESULT AT LOWER THRESHOLD.

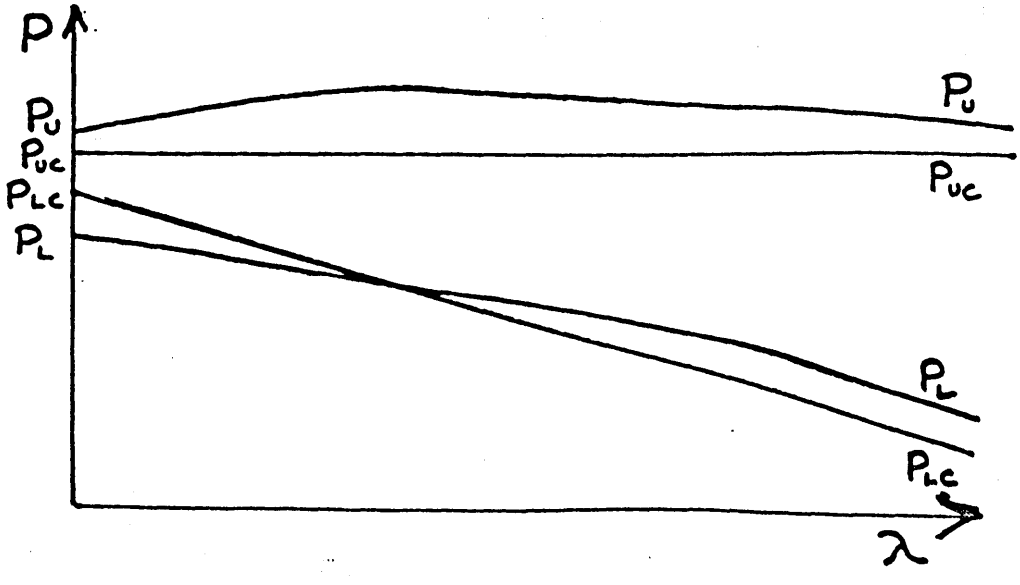
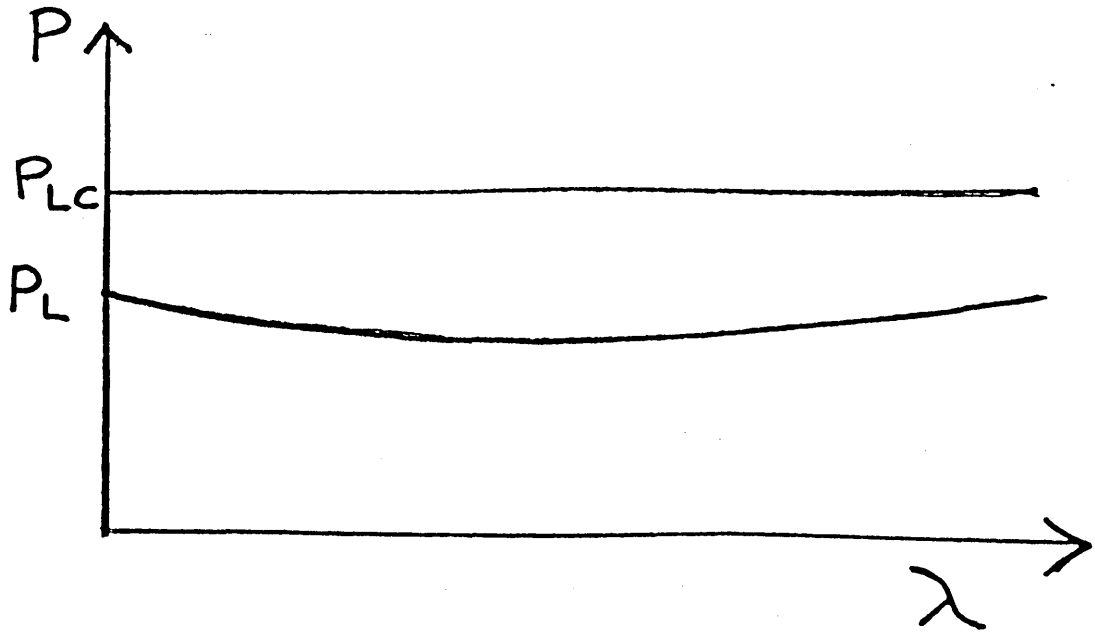


Figure 3.10

NON MONOTONICITY OF CAUTION COMPONENT AT OPPOSITE THRESHOLD TO CROSSOVER.

(FOR A ZERO EXIT COST)



3.7 EXPLANATIONS AND IMPLICATIONS OF THE NUMERICAL RESULTS CONCERNING MEAN REVERSION IN THE DIXIT MODEL.

3.7.1 Introduction.

There are two principal features of the results. The first is that the non-certainty component of the solution damps, though apparently does not reverse, the deterministic effects of mean reversion. Thus the sensitivity of the solution to the introduction of mean reversion is itself non-certainty equivalent. The second and most interesting feature is that differences between the sunk costs of entry and exit can cause the costlier state transition to occur more readily with uncertainty than without it, in contrast to the cases with more equal sunk costs, where uncertainty always delays state transitions.

3.7.2 Non-Certainty Equivalence and Damping the Deterministic Effect.

Increased mean reversion reduces the 'option difference' component of the solution. We can explain this as the effect of mean reversion in reducing the forecast variance associated with the stochastic process, which reduces the values of the option of entry and the option of exit.

In some cases, the response of the caution component can almost entirely nullify the certainty equivalent effect, so that, although the response of the full solution to increased mean reversion retains the sign predicted by the certainty equivalent case, the magnitude of the stochastic response may be drastically different. Thus, in the presence of uncertainty, the sensitivity of the entry and exit thresholds to changes in mean reversion may be quite different to what the deterministic analysis would suggest.

We can put these findings a different way. When the initial full solution is dominated by the effects of uncertainty, so that the caution component is much larger than the certain component, then the response to increased mean reversion largely reflects the caution component's damping effect on the certainty equivalent response. When uncertainty is relatively *less* important then the solution will be

more sensitive to the assumed degree of mean reversion. We must specify agents' level of uncertainty as well as their point expectations before we can characterise their response to mean reverting process.

Imagine that a particular firm reveals a zone of inaction in its entry and exit decisions from an overseas market, and that we wish to predict whether its behaviour will be materially affected by a change in its (subjective or objective) expectations concerning real exchange rate dynamics. The firm's response will differ considerably depending on the relative importance of the caution component in the firm's original plan. The more important uncertainty in shaping the original plans, the less they will be affected by changes in the real exchange rate dynamics. For example, in a case where the hysteresis zone is founded on epsilon sunk costs, while the zone will, strictly speaking, widen with increased mean reversion, that effect will be negligible relative to the overall band width.

3.7.3 Cases With Asymmetric Sunk Costs of Entry and Exit.

Our most important finding is that asymmetric sunk costs can give rise to qualitively different solution patterns where the costlier state transition is hastened rather than delayed relative to the certain case by the addition of uncertainty.

The reversal of the optimal and perfect foresight triggers confirms the intuitions derived from our two period model of the importance of the differential effects of entry and exit costs on the difference between the value of the options of entry and exit. The presence of mean reversion in the price process lowers the variance of future paths and hence the initial difference between the two options, so that an increase in one sunk cost or exercise fee is able to reverse the sign of the option difference.

When 'crossover' occurs at the entry trigger we must find $OE(P_U) - OQ(P_U) < 0$. When this result obtains, with higher values of mean reversion, uncertainty makes entry occur more, rather than less, easily: the qualitative effect of adding uncertainty is reversed.

The two period model of appendix 3.2 confirms what we have found in the Dixit model, that this effect can only arise when there is a sufficient degree of mean reversion. The finding that this effect does not require strictly zero sunk costs at one threshold is consistent with the difference between the sunk costs being the crucial factor.

The theoretical interest of such a crossover finding is clear: we cannot maintain as a general rule that the certainty equivalent or perfect foresight zone of inaction provides a minimum dimension for the stochastic zone of inaction in the sense of lying everywhere within it. The economic implications of this property are that when entry costs are large relative to exit costs, entry can take place at an initial price which would be unprofitable in a deterministic world; while when exit costs are large, exit can occur at a price which would still be profitable in the deterministic case.

Dixit's analysis of the case without mean reversion showed that uncertainty made it optimal to change state *later* than would be the case under certainty. We have now shown how our notion of the cautious control behaviour must allow for the interrelated nature of the entry and exit decisions, finding that with sufficient asymmetry of sunk costs and mean reversion, the addition of uncertainty can sometimes make changing state optimal *sooner* than in the certain case. When entry occurs sooner in the stochastic than the deterministic case it is essentially because the large sunk cost of entry serves to make us even more cautious about subsequent exit than it does about initial entry.

This 'crossover' result is of great significance to the literature on entry and exit under uncertainty. For despite the simultaneous analysis of reversible entry and exit, Dixit's results correspond to the intuition of Pindyck's (1989) *irreversible* investment decisions, where uncertainty causes delay. This result, in which uncretainty can hasten the investment or entry decision does *not* conform to the irreversible intuition, and threfore, provides a concrete example of the essentially interrelated nature of reversible entry and exit decisions.

3.7.4 Why the Damping Effect of Uncertainty Cannot Reverse the Deterministic Effect of Mean Reversion.

The net effect of mean reversion is to widen the hysteresis band. What does this result mean? We have already shown that the band widening result requires that the expected value is more sensitive to mean reversion than is the option difference. Our consideration of the simplified two period model (See Appendix 3.2) has suggested why this is so. Mean reversion should generally be expected to affect the optimality of exercise, and when this effect is allowed for it widens the band in the two period model. This intuition should carry over to explain band widening in the Dixit model.

3.8 RELATION OF RESULTS TO THE BALDWIN PAPER

We now contrast our approach to mean reversion in models of sunk cost hysteresis to Baldwin's (1989) paper.

Considering cases in which the long run equilibrium price falls within the model's hysteresis zone, Baldwin offers (pp 17-20) a *proof* that reduced persistence (increased mean reversion) widens the band within a general discrete time stochastic dynamic programming formulation of sunk cost hysteresis.

Baldwin also challenges the claim that increased volatility will always widen the solution band, by pointing to the certainty equivalence of the discrete time sunk cost hysteresis model when the stochastic process is iid.[36] This result can be understood if we note that with an iid process the conditional expectation of future prices is entirely independent of the current realisation of the price. Given additional assumptions the option values of entry and exit will be equal thereby ensuring certainty equivalence.

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36. Baldwin (1989), p 20.

But it is immediately clear that Baldwin's certainty equivalent result is also dependent on entry and exit costs. Using different exercise prices the option values of entry and exit will now differ and hence the solution will no longer be certainty equivalent under an iid process. This argument illustrates the greater understanding of the model which has been gained through our explicit consideration of the effects of mean reversion on the non-certainty equivalent part of the solution. This research therefore complements Baldwin's more general approach.

3.9 CONCLUSIONS.

In this chapter we have examined the extension of the Dixit (1989) model of entry and exit under uncertainty to include exogenous mean reversion. We have explained why this extension of the model is of interest, and we have examined how these issues have been handled in the literature. We have explored the model's properties through numerical solutions and in doing so we have allowed for the general case where costs of entry and exit can differ.[37] We have also used analytic arguments as far as possible to indicate the various ways in which mean reversion can be expected to influence the solution. In explaining our numerical results we have drawn valuable insight from a simplified two period discrete time model (developed in appendix 3.2) which yields analytic expressions capable of explaining many of our numerical results.

Our analysis has suggested ways in which the properties of deterministic and stochastic solutions with mean reversion can differ. In this regard we have emphasised the role of mean reversion in damping future forecast variance for the price process; also the differential effect of sunk costs of entry and exit on the option values of entry and exit. The simple two period model makes these effects stand out.

The key to band widening in the case with uncertainty is in the relative magnitude of the changes in the net option difference term and in the value of the continuation strategy when mean reversion shifts the

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37. The numerical methods employed to investigate the solution properties in the presence of mean reversion are explained in appendix 3.1.

underlying distribution of returns to the active firm. For band widening, the effect of mean reversion on the option difference should be the smaller. Because it excludes unprofitable contingencies, the option difference term should be less sensitive to changes in mean reversion than is the conditional expected value of continuation, thereby ensuring that the net effect of mean reversion is indeed to widen the band.

Although the effect of mean reversion in the Dixit model is to widen the hysteresis bands, the deterministic effect of mean reversion dominates. The deterministic effect can however be almost entirely damped by the offsetting non-certainty equivalent effect when sunk costs are relatively small.

Our numerical experiments do show that the effects of mean reversion are more complicated in the stochastic than the deterministic case. The effects on the caution component are found to dampen, the deterministic effects of changes in mean reversion on these solutions. The sensitivity of the model's solution to changes in the speed of mean reversion is found to vary, increasing as increased mean reversion diminishes the caution component relative to the certain component.

The most important effect of adding mean reversion occurs when one sunk cost, (say, the exit cost), is either zero or else is relatively small compared to the size of the other sunk cost, (say, the entry cost). For some parts of the parameter space the threshold associated with the larger entry cost is such that it is optimal for a change of state to occur sooner under uncertainty than would be the case under certainty. This 'crossover' phenomenon can be explained in terms of the differential effects of the asymmetric exercise fees (sunk costs of entry and exit) on the options to enter (OE(P)) and to quit (OQ(P)). This effect combines with the effects of mean reversion on option values to make crossover phenomena possible. These results establish that the presence of uncertainty in a model of entry and exit does not necessarily delay state transitions, and thereby distinguish the case with both entry and exit from the case of strictly irreversible entry.

In closing we should set our conclusions in the context of the literature. As we have seen, Dixit himself suggested that his model should be extended to include exogenous mean reversion, but he did not

pursue the necessary numerical solution method. Krugman (1988) did solve a related model which combined both mean reversion and uncertainty numerically, but he did not explore its properties and merely reported a solution for one set of parameter values. Baldwin (1989) characterised sunk cost hysteresis under general schemes for the forcing variable, which would include mean reverting processes. The common feature in all three authors' work is the prediction or reported result that increasing mean reversion will widen the hysteresis zone in the solution to a sunk cost hysteresis model with uncertainty.

Our research is complementary to the work referred to above. For these authors have not studied the specific effects of mean reversion on the non-certainty equivalent part of the model's solution. That is precisely what we have sought to illuminate.

In contrast to Baldwin's 'incumbency premium' method, Dixit's specific parametric approach allows us to isolate the non-certainty equivalence of the model, and through its option value interpretation to give this a clear economic meaning. Thus by following Dixit's suggested extension we have been able to explore the effects of introducing mean reversion on the non-certainty equivalent part of the model's solution. In this way we have added to the broad prediction of band widening made in the literature: a prediction which adds no qualitative detail over and above the properties of the deterministic analysis.

In seeking to explain the detail of our numerical solutions we have increased our understanding of the complex option difference which generates the model's non-certainty equivalence. We also developed a simplified two period model with a binomial form of uncertainty which serves as a simplifying representation (a 'maquette') for the full Dixit model with mean reversion. This simple model ennabled us to identify the damping role of mean reversion on option values and the differential effects of entry and exit costs on the option difference; it also provides us with an explanation for the net band widening effect of mean reversion we encounter in our numerical solutions.

Finally, our 'crossover' result demonstrates in concrete terms how the nature of the solutions with reversible entry and exit differs from the strictly irreversible case.

In conclusion, this research has supplemented and extended our previous understanding of the role of mean reversion in the Dixit model, giving a richer account than hitherto of the mechanisms involved, and exploring further aspects of the model's non-certainty equivalence, and clarifying the essentially joint nature of the entry and exit decisions.[38]

38. This research has subsequently led to a private correspondence with Professor Dixit concerning what further generalisations can be made in extending his results to cover 'Investment and Hysteresis' for general stochastic processes. In this connection there was a particular concern with isolating the 'value of waiting'.

Chapter Four.

A STOCHASTIC SADDLEPOINT INTERPRETATION OF THE DIXIT MODEL OF HYSTERESIS WITH SYMMETRIC AND ASYMMETRIC PATTERNS OF SUNK ENTRY AND EXIT COSTS; WITH SOME SUGGESTIONS FOR FUTURE RESEARCH.

4.1 INTRODUCTION.

In this chapter we recognise that the addition of mean reversion to the stochastic process for the price gives the Dixit model a formal structure akin to the stochastic saddlepoint systems analysed in Miller and Weller (1988a). We therefore consider how the Dixit model of sunk cost hysteresis might be analysed within such a framework. This provides a convenient vehicle for us to restate and underpin the major findings of the previous chapter. Moreover, it suggests how those results may have wider relevance beyond the Dixit model, thereby raising directions for future research.

We give a stochastic saddlepoint interpretation of the numerical results reported in the previous chapter. As in the previous chapter, we report two patterns of results: those which follow the patterns in the existing literature; and those which display what we have termed an 'asymmetric crossover' property. The saddlepoint interpretation adds to our understanding of these solutions and their properties.

Before concluding the chapter, we discuss the likelihood that our findings in the Dixit model with mean reversion might extend to the class of logarithmic stochastic saddlepoint models such as Krugman's (1988) macroeconomic model of hysteresis effects in trade, his (1987, 1990) monetary model of exchange rates, and the versions of the closed economy ISLM model and the Dornbusch model considered by Miller and Weller (1988a).

4.2 STOCHASTIC LINEAR SADDLEPOINT SYSTEMS.

The introduction of mean reversion has a significant qualitative impact on the Dixit model of sunk cost hysteresis. The system becomes stationary, in the sense that it possesses a well defined steady state towards which the systematic dynamics operate.[1] With mean reversion, the model assumes the character of a saddlepoint system.

In order to indicate how the conclusions possible from our analysis of the Dixit model with mean reversion might be more widely applicable, we place the results within the framework put forward by Miller and Weller (1988a) for the qualitative characterisation of solutions to stochastic linear saddlepoint systems. A brief introduction to the work of Miller and Weller now follows.

Miller and Weller consider the framework of a linear rational expectations model consisting of the backward looking dynamics on a sticky 'fundamentals' variable X, and a forward looking arbitrage equation giving the expected dynamics of the asset price Y. This structure, familiar from examples such as Dornbusch (1976)'s open economy ISLM model, is converted into a stochastic system by the addition of white noise to the dynamics of X.

The stochastic system is given by:

$$dX = \alpha X\,dt + \beta Y\,dt + \sigma\,dz \tag{4.1a}$$

$$EdY = \gamma X\,dt + \delta Y\,dt \tag{4.1b}$$

The solution to the deterministic version of the system is well known, and is considered in, for example, Buiter (1984). The solution of the model requires the use of boundary conditions, and

1. This well defined steady state is given in terms of the price level (P) and the difference in expected value between the firm in the idle and active states (V). The issue of the actual market status of the firm is a distinct one: in respect of this the model displays a hysteresis property. However there is a unique steady state equilibrium in V and P.

transversality conditions may be invoked to complete the system.[2] However our interest is in solutions under conditions which place finite bounds on the possible movements of the asset price. In the deterministic case these finite bounds have a trivial consequence as, between the boundaries, the solution is identical to the case in which the bounds are only at infinity. The system is saddlepoint stable, and a unique solution path is described by the eigenvector associated with the stable eigenvalue or root.

When we turn to consider the stochastic case, the choice of boundary conditions affects the solution path. Taking expectations through the system, we obtain EdX and EdY, the expected dynamics, which are identical with the deterministic case: hence the same roots and eigenvectors apply. The actual solution path is given by a deterministic relationship between the asset price and the fundamental. The effect of the finite boundary conditions is to produce nonlinear solution paths (as in Krugman's (1987) 'bias in the band'), through their impact on the probability distribution of outcomes.

Miller and Weller consider the qualitative aspects of solution paths to the stochastic system in those cases in which finite boundaries on the asset price are symmetric about the equilibrium value: given the symmetric distribution these paths will pass through the deterministic equilibrium. The properties of an infinity of solutions are then established, each solution corresponding to a pair of symmetric boundaries. Miller and Weller's map of potential solutions to the symmetric case is given at figure 4.1.

Miller and Weller also raise the question of the solutions to cases with asymmetric boundaries on the asset price.[3] Our examination of Dixit's model has led us to consider many cases of asymmetric boundaries, and our most interesting results arise from some of these cases.[4] These indicate that, in the Dixit model with its exogenous mean reversion, we can go further than Miller and Weller in characterising the possible form of a solution with asymmetric boundaries.

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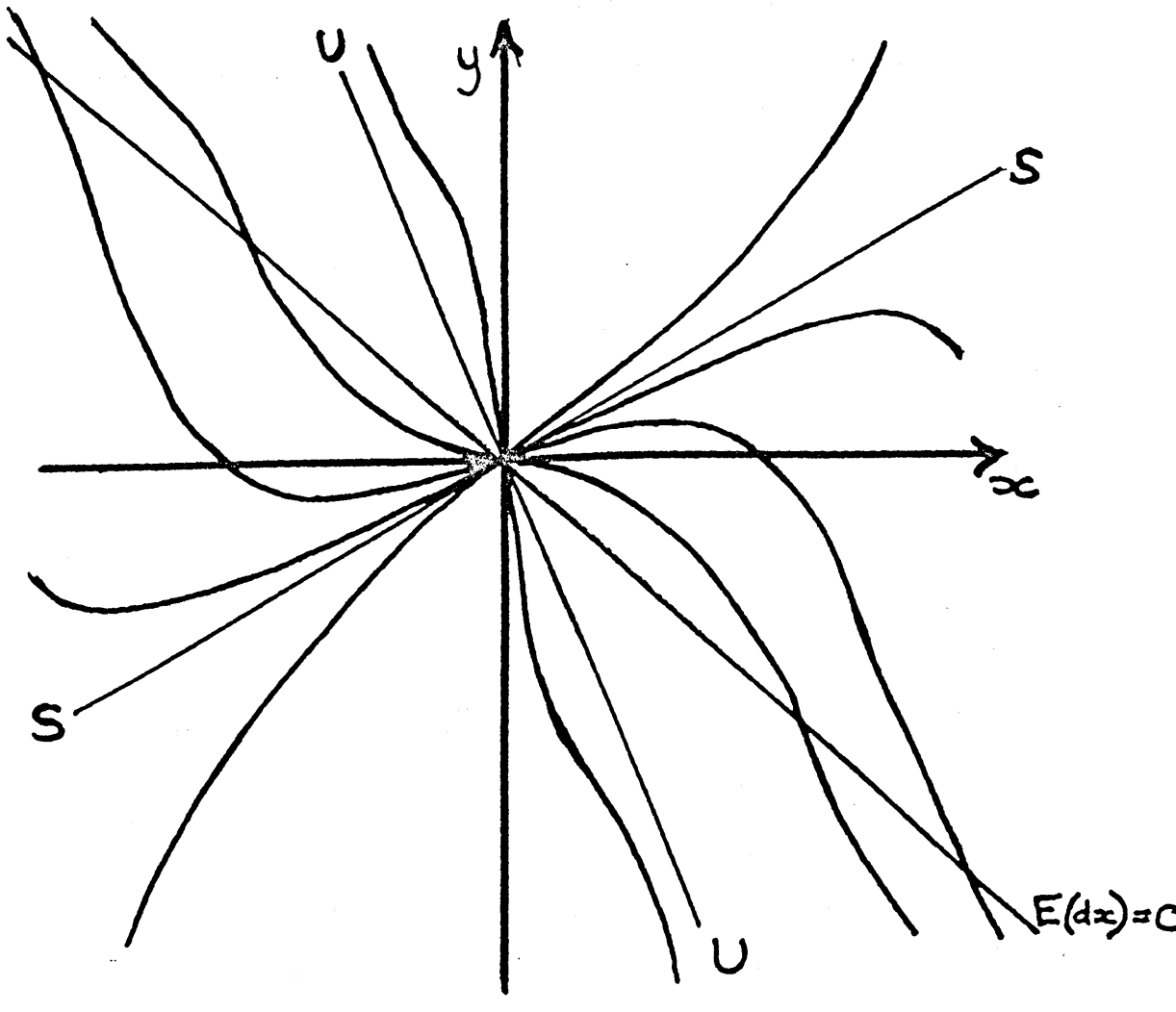
2. See Buiter (1984), p 669.

3. Miller and Weller (1988a), pp 22-23.

4. Note that Dixit's 'central case', with its nonzero sunk entry costs and zero sunk exit costs, is a prime example of asymmetric boundaries.

Figure 4.1

MAP OF POTENTIAL SOLUTIONS TO THE GENERAL STOCHASTIC SADDLEPOINT SYSTEM WITH SYMMETRIC BOUNDARIES.



SOURCE: MILLER AND WELLER
T.W.E.R.P. no. 309

A typical example of the solution to the system under symmetric bounds on Y is illustrated in figure 4.2.

The symmetric bounds on the asset price are given by BC_1 and BC_2, and the stable eigenvector or saddlepath by SS. The solution to the deterministic model is given by ABOCD, while the solution to the stochastic model is given by the S-shaped curve AEOFD.

The same solution is shown in figure 4.3 in which we identify some further points of interest. Given some exogenously determined bounds on the asset price Y, we will often wish to determine at what values of the fundamental X those bounds on Y are attained. The certainty equivalent solution in figure 4.3 (ABOCD) gives the answers X_{LC} for BC_2 and X_{UC} for BC_1. The horizontal difference between the stochastic and certainty equivalent solutions is the caution component of the full stochastic solution for the implied bounds on X.

4.3 THE DIXIT MODEL IN THE MILLER AND WELLER FRAMEWORK OF STOCHASTIC SADDLEPOINT SYSTEMS.

The introduction of mean reversion into the Dixit model has the effect of giving the model a saddlepoint property. In what follows we show how the Dixit model with mean reversion can be given a linear saddlepoint analysis. This provides an alternative interpretation of the model's solutions, in which we can relate the value difference functions and the trigger price solutions to the certainty equivalent dynamics of a saddlepoint system.

The Dixit model of entry and exit with uncertainty is basically similar to the class of models covered by the Miller and Weller framework. Strictly speaking its specification differs from that of the Miller and Weller examples in a number of respects. Firstly the certainty equivalent dynamics are non homogeneous. Secondly the Dixit model is specified in terms of the price level and not the logarithm of the price level, so that the fundamental is bounded below by zero with no corresponding upper

Figure 4.2

STOCHASTIC SADDLEPOINT SOLUTION UNDER SYMMETRIC BANDS.

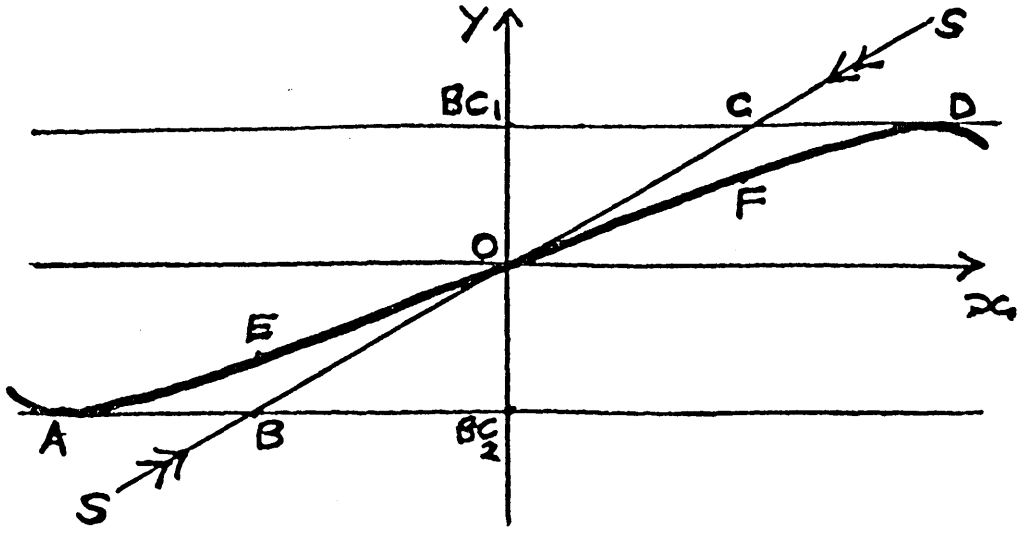
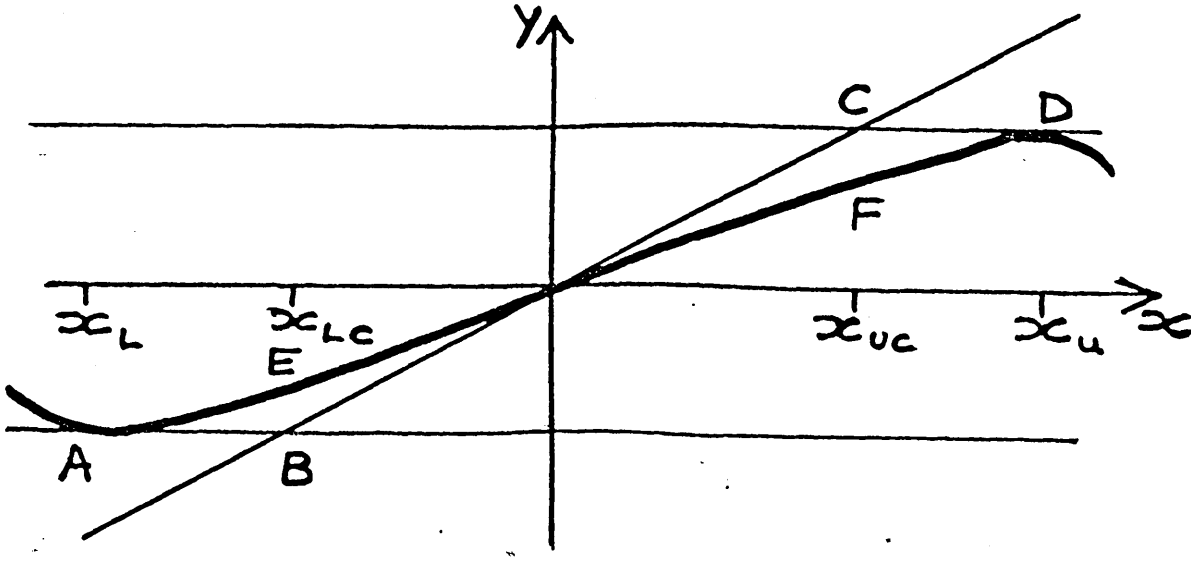


Figure 4.3

STOCHASTIC AND CERTAINTY EQUIVALENT TRIGGER SOLUTIONS.



bound. Thirdly the stochastic price process is defined so that innovations are scaled by the price level. Nevertheless a stochastic saddlepoint analysis is possible. We will conjecture that it is close enough to the Miller and Weller specification that results in the one should be a good guide to the results in the other.

In order to express the Dixit model in this framework we must choose to define the forward looking asset price as the difference between the value of the active firm and the value of the idle firm. This choice is consistent with the argument made in the previous chapter that the solutions for the trigger prices depend only on the relative positions of the value functions.

We denote the value difference as V, and define it as the value of the active firm less the value of the idle firm. An arbitrage equation in V is implicit in Dixit's model, and is given as:

$$r\,V = E(dV/dt) + (P - w) \tag{4.2}$$

The fundamentals variable is the relative price (the real exchange rate or competitiveness), which is denoted P, and follows the (possibly mean-reverting) stochastic process:

$$dP = -\lambda\,(P - P^*)\,dt + \sigma\,P\,dz \tag{4.3}$$

We can now write the Dixit model as a saddlepoint system thus:

$$\begin{pmatrix} dP \\ EdV \end{pmatrix} = \begin{pmatrix} -\lambda\,, 0 \\ -1, r \end{pmatrix} \begin{pmatrix} P\,dt \\ V\,dt \end{pmatrix} + \begin{pmatrix} \lambda\,P^*\,dt \\ w\,dt \end{pmatrix} + \begin{pmatrix} \sigma\,P\,dz \\ 0 \end{pmatrix} \tag{4.4}$$

This is a non-homogeneous first order stochastic differential equation system in P and V. To analyse it we will consider the homogeneous deterministic system which is defined around the deterministic steady state values of P and V.

We first analyse the deterministic dynamics before considering stochastic solutions. The equilibrium point of the deterministic system is characterised by $dP = 0$ and $dV = 0$, which define lines of stationarity for P and V. The system solves recursively, first for the equilibrium price $P^{O} = P^{*}$, and then for the equilibrium asset price, $V^{O} = (P^{*} - w)/r$. Given these steady state values we rewrite the system in the following homogeneous form:

$$\begin{pmatrix} dP \\ EdV \end{pmatrix} = \begin{pmatrix} -\lambda , 0 \\ -1 , r \end{pmatrix} \begin{pmatrix} (P - P^{O})\, dt \\ (V - V^{O})\, dt \end{pmatrix} \tag{4.5}$$

The recursive nature of the solution also allows us to identify the two roots of the system, $-\lambda$ and r by inspection. These roots ensure the system's saddlepoint property, the stable root, $-\lambda$, being associated with the predetermined variable P, and the unstable root, r, being associated with the forward looking variable, V. The stable eigenvector, θ_s, takes the positive value given by: $\theta_s = 1/(r+\lambda)$. This gives us the slope of the certainty equivalent saddlepath. The unstable eigenvector, θ_u, has an infinite slope, and lies along the vertical V axis. These deterministic dynamics are illustrated in figure 4.4.

This system yields the following solutions:[5]

$$V_t - V^{O} = P_t/(r+\lambda) \tag{4.6}$$

and:

$$P_t - P^{O} = (P_0 - P^{O})\, e^{-\lambda (t-0)} \tag{4.7}$$

---

5. Buiter (1984) gives details of a solution method for general non-homogeneous continous time saddlepoint systems under certainty equivalence. We could use Buiter's method, employing the diagonalising matrix (V in Buiter's notation) given by:

$$\begin{matrix} (r+\lambda)^{-1}, & 0 \\ -1 & , (r+\lambda) \end{matrix}$$

From Buiter's equations 17 and 18 (p 671) we obtain, after completing the definite integrals, the same expressions as those implied above for P_t and V_t.
The Buiter method is however more general than the simple transformation to the homogeneous form employed above, and thus would be capable of giving the solution to cases where P^{*} and w are time varying forcing variables.

Figure 4.4

DETERMINISTIC DYNAMICS FOR THE DIXIT MODEL.

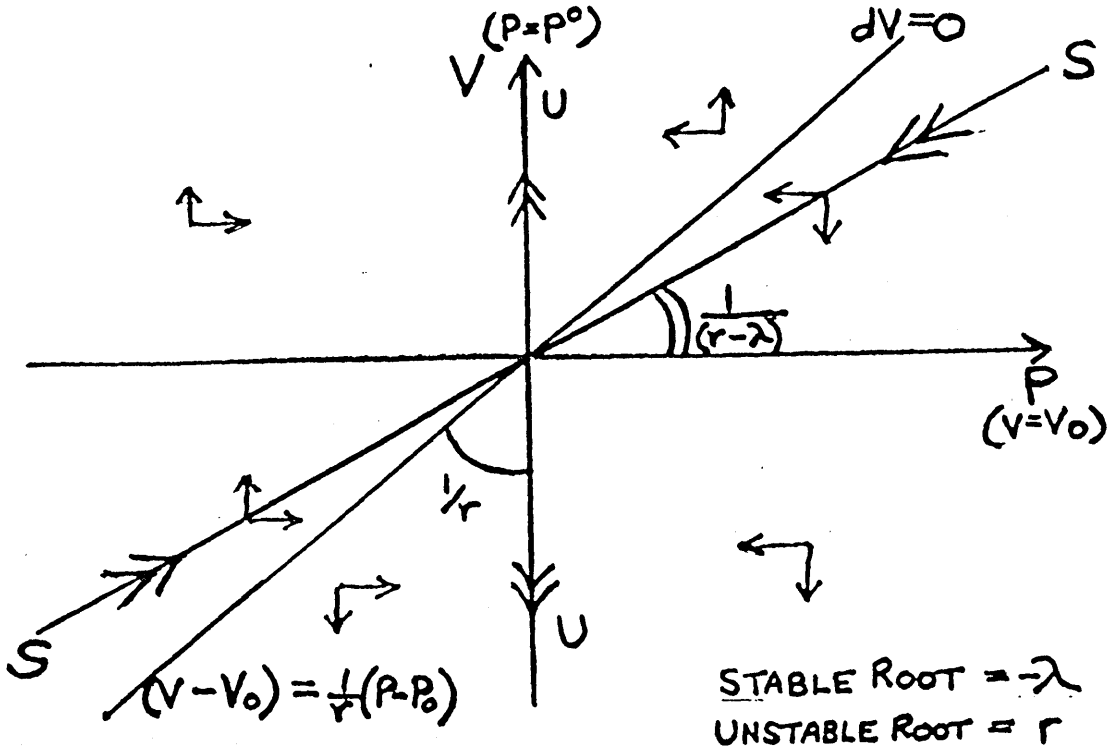
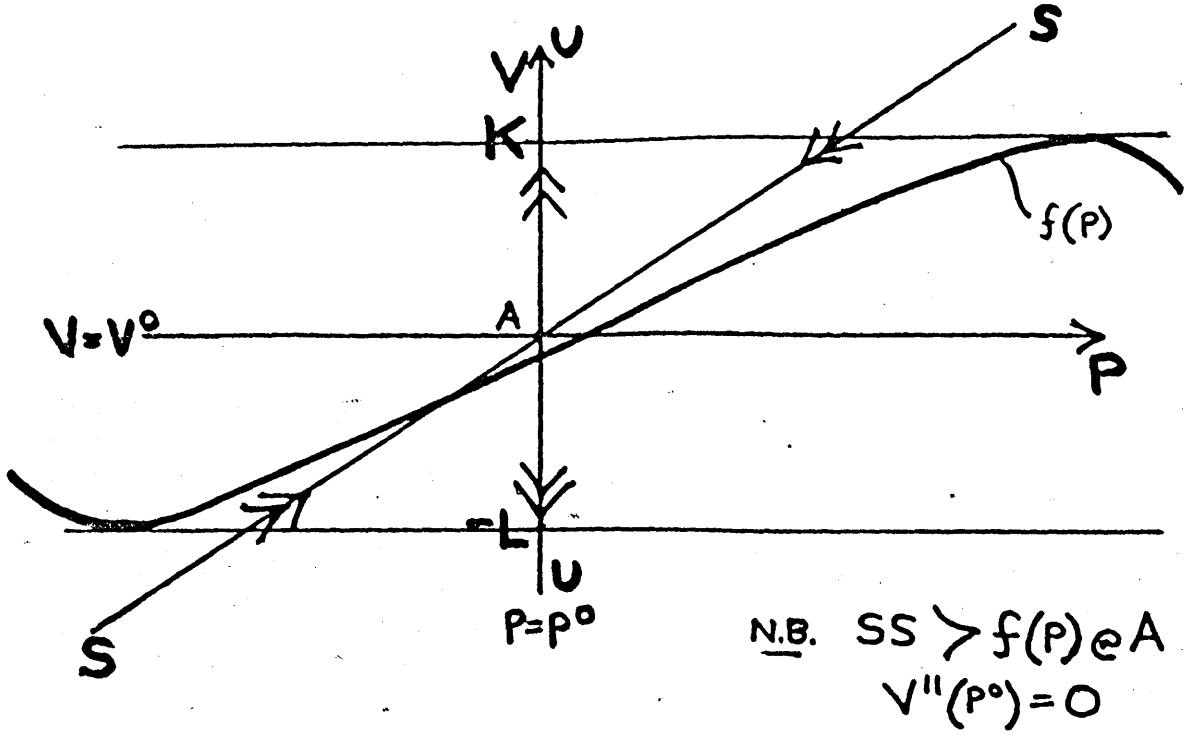


Figure 4.5

STOCHASTIC SADDLEPOINT SYTEM IN V AND P.



where P^{O} and V^{O} are as defined above.

We now turn to a consideration of the full stochastic solution. The solution to the bounded stochastic system is characterised by a nonlinear relationship between V and P which satisfies the relevant Bellman equation and which value matches and smooth pastes to the boundary surfaces at the trigger prices for entry (ie. at V = K) and exit (ie. at V = -L).[6] As Miller and Weller have explained (p 4), this bounded stochastic solution will diverge from the certainty equivalent one: nevertheless it is still subject to the influence of the certainty equivalent dynamics, with its eigenvalues, eigenvectors and loci of expected stationarity.

Between them the loci of expected stationarity define a point in V,P space for which the expected rate of change of these variables is zero. For a general specification of entry and exit costs we find that the stochastic solution path for V as a function of actual P will *not* pass through this point of expected stationarity. In general, the stochastic solution will not attain expected stationarity due to the probability of entry and exit.

In the Dixit model the symmetric bounds solution does *not* pass through the certainty equivalent steady state, and hence is similar to what Miller and Weller (pp 22-23) identify as a typical solution with asymmetric boundaries in their model.[7]

In figure 4.5 we show the saddlepoint system in V and P under the simplifying assumptions $P^{\star} = w$ and K = L. Under these assumptions the model should resemble the symmetric cases considered by Miller and Weller. The certainty equivalent solution is given by the SS locus corresponding to the stable eigenvector. The stochastic solution is given by the S-shaped curve which satisfies smooth

6. The use of Bellman equations in solutions to the Dixit model is described in chapter two (section 2.3.2).

7. Note however that we have previously shown how sufficiently asymmetric boundaries can give rise to the qualitatively different 'crossover' property. The failure of the stochastic solution to pass through the certainty equivalent steady state is therefore not an exhaustive characterisation of the consequences of asymmetric boundaries for the solution.

pasting conditions by its tangency to the boundaries V = -L and V = K.[8] Note how the full solution in this symmetric boundaries case passes below and to the right of the certainty equivalent steady state A. We also find from our numerical results (with $P^{\star}$=w) that the point of inflection appears to be located at $P=P^{\star}$, so that $V''(P^{\star})=0$.

4.4 A SADDLEPOINT INTERPRETATION OF SOLUTION PROPERTIES WITH MEAN REVERSION.

In the previous chapter we investigated the comparative static effects of introducing mean reversion into the Dixit model. We will now interpret our main qualitative findings in the context of the saddlepoint framework.[9]

In our numerical results we found in many cases that the full stochastic solution gives rise to entry and exit trigger prices which lie outside the certainty equivalent trigger prices due to the non-certainty equivalent component of the solution (the options effect). However the combination of a sufficient degree of mean reversion with sunk costs of entry and exit which are sufficiently different from one another in magnitude give rise the to the important and qualitatively distinct 'crossover' solutions.

We now illustrate the saddlepoint interpretation of the effects of mean reversion on the solution. This is given in figure 4.6, which gives only the positive quadrant for an example where the deterministic and stochastic steady states coincide.

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8. The representation in figure 4.5 can easily be seen to be a transformation of the representation considered in chapter 3 at figure 3.1 : that illustration in its turn followed Dixit's representation of the system without mean reversion, ie. $\lambda = 0$.

9. We have already noted that in the restricted case, with no feedback from the asset price to fundamentals, the coefficient of mean reversion in the relative price process is the stable root of the system. In Miller and Weller's general case, the stable root itself would be the parameter of economic interest, as it would measure the mean reverting tendency of the system as a whole.
In Miller and Weller's notation, the stable root is given by the negative solution to the characteristic equation:
$\theta^2 - (\alpha+\delta)\theta - (\alpha\delta-\gamma\beta) = 0.$

Figure 4.6

EFFECTS OF CHANGING MEAN REVERSION IN DIXIT MODEL.

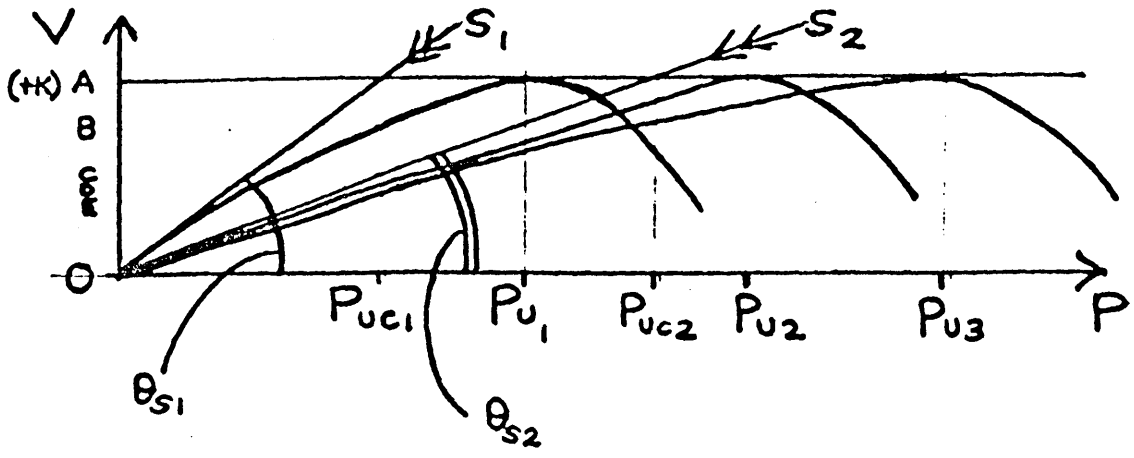
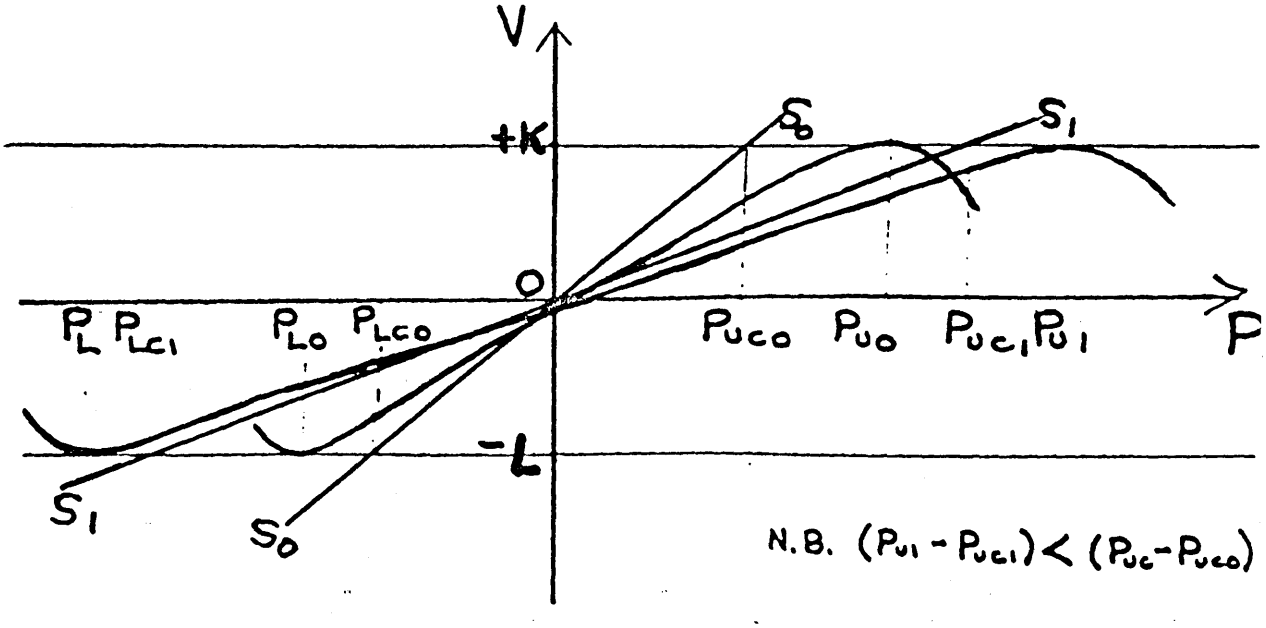


Figure 4.7

MEAN REVERSION AND THE STOCHASTIC SADDLEPATH WITH SYMMETRIC SUNK COSTS IN THE DIXIT MODEL.



The most obvious effect of an increase in the coefficient of mean reversion, λ, is to reduce the slope of the stable eigenvector θ_σ ($\theta_\sigma=1/(r+\lambda)$) from $\theta_{\sigma 1}$ to $\theta_{\sigma 2}$. This shifts the certainty equivalent solution path from the locus S_1 to S_2, and the certainty equivalent entry trigger from P_{UC1} to P_{UC2}. The increase in λ implies a faster speed of movement along the solution path towards the equilibrium point O.[10]

In the full stochastic solution the change in mean reversion affects the non-certainty equivalent part in two ways. On the one hand the stochastic solution path tends to shift in the same way as the stable eigenvector in order to relect the 'discounting' effect of mean reversion. On the other hand the option difference is directly reduced by mean reversion.

The overall effect of the increase in λ on the entry threshold in the full stochastic solution is to shift P_{U1} to P_{U2}. We can disentangle the two effects of the change in λ on the caution component in the following way. If the innovation variance were to be increased in order to offset the effect of the increase in λ on the relevant measure of forecast variance, we would obtain the solution P_{U3}.[11] At P_{U3} the caution component is increased by the same proportion as the certainty equivalent component.

The value of the option difference is given by the vertical distance between a nonlinear solution and its associated eigenvector. The compensating effect of adjusting the innovations variance to keep the

---

10. Note that if we attempt to change the stable root without changing the slope of the eigenvector (by a compensatory re-scaling of all other parameters incoporated into the expression for the stable eigenvector) we find that we have simply re-scaled the entire model. (We can show this easily in the Bellman equation.) Hence we conclude in the Dixit model that a meaningful change in the stable root must be associated with a change in the eigenvector.

11. In a private communication Professor Dixit has suggested that this solution, in which the interaction between uncertainty and mean reversion has been removed by maintaining the variation in future price realisations at a constant level, may be a more natural parameterisation of the model.

We nevertheless prefer not to employ this alternative parameterisation in our analysis, for while such an exercise may be of analytic interest, it does not seem appropriate as a way of thinking abour the example of the trading firm. When a firm perceives a change in real exchange rate dynamics it may reasonably expect this to have implications for the future variability of the real exchange rate: it does not necessarily expect a concomitant and offsetting change in the variance of the shocks. It may be of course that a large change in the variance of shocks might lead to a change in real exchange rate dynamics, but even in such a case we should not assume that the effects on future price variation are offsetting.

option difference constant can therefore also be seen in the equality of the vertical differences AB and CE. In contrast the direct effect of increased mean reversion on the option difference is captured in the comparison of CD, the option difference in the uncompensated case at price level P_{UC1}, with CE, the compensated case at P_{UC1}.

The results of our numerical experiments into the net effects of mean reversion on the solution to the Dixit model with uncertainty are reflected in figure 4.7 for a case with symmetric boundaries. Note that the stochastic steady state must differ from the deterministic steady state in this case. The full solution lies outside the certainty equivalent solution, in that $P_U > P_{UC}$. In addition, $P_{U2} > P_{U1}$, reflecting the result that increased mean reversion widens the band. Finally increased mean reversion reduces the caution component so that $P_{U1}-P_{UC1} > P_{U2}-P_{UC2}$.

The caution component of the solution is reduced by increasing mean reversion, and partly offsets the certainty effect of an increase in mean reversion. The net effect of increased mean reversion is to widen the band but also, simultaneously, to reduce the non-certainty equivalence of the solution.

4.5 ASYMMETRIC BOUNDARIES AND 'CROSSOVER' SOLUTIONS.

4.5.1 Solutions with Asymmetric Boundaries.

After expounding their qualitative analysis for the general saddlepoint problem with symmetric boundary conditions, Miller and Weller (1988a, pp 22-23) briefly considered cases where the boundary conditions are not symmetric around equilibrium. By perturbing one of the symmetric boundries they showed that the solution for V(P) would no longer pass through the certain solution at the deterministic equilibrium, but lies between it and the larger of the two boundaries. The Miller and Weller characterisation of 'asymmetric' boundaries is presented in figure 4.8, and is otherwise similar to the symmetric case.

Figure 4.8

STOCHASTIC SADDLEPOINT WITH ASYMMETRIC (BUT 'NEARLY' SYMMETRIC) BOUNDARIES.

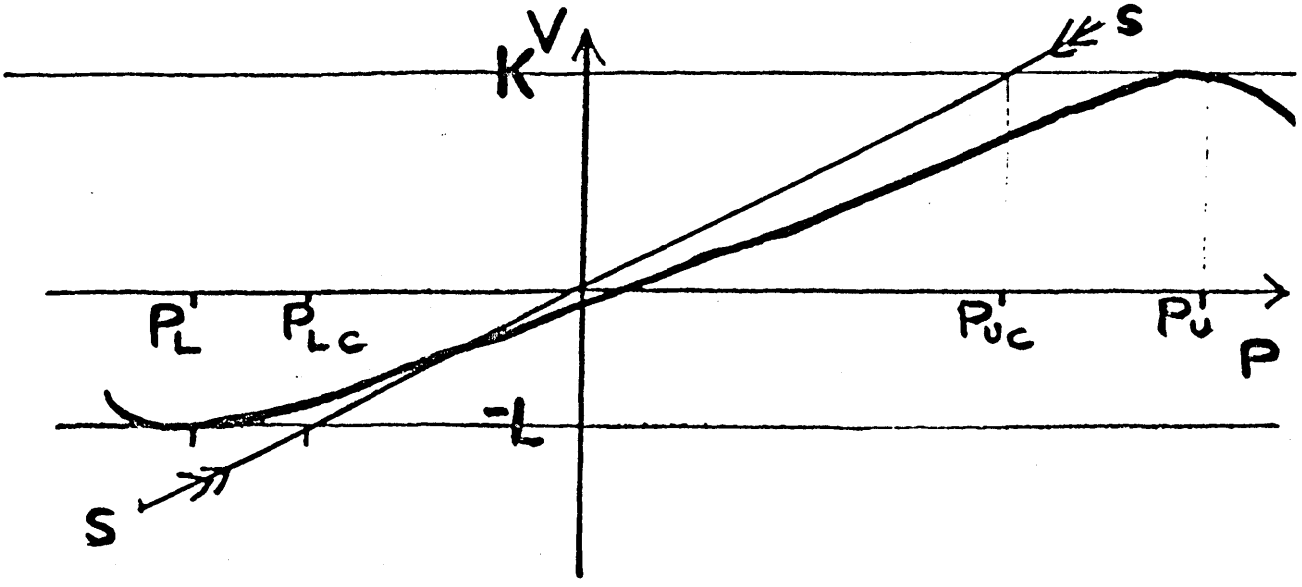
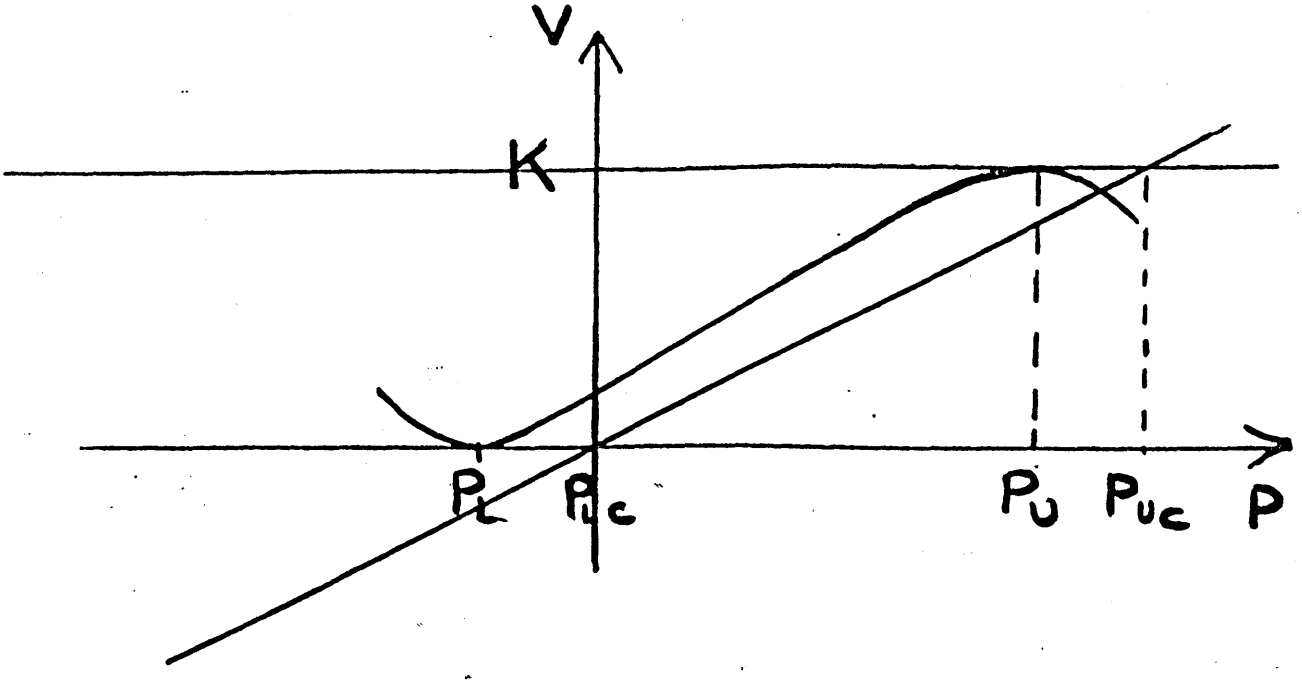


Figure 4.9

SADDLEPOINT INTERPRETATION OF THE 'CROSSOVER' PHENOMENON (WITH A ZERO EXIT COST).



4.5.2 'Crossover' with Asymmetric Boundaries.

Our investigations indicate that when the boundaries are sufficiently different, the solution with asymmetric boundaries can display features which are qualitatively distinct. We now set these 'crossover' results in the saddlepoint context.

To illustrate the crossover result we will consider the extreme asymmetric cases in which the sunk component of either the entry or exit cost is zero. (Dixit's 'central case' is one such example, with $K = 4$ and $L = 0$.) In figure 4.9 we present the crossover phenomenon in the saddlepoint framework for a case where the exit cost is zero. We can see that the stochastic solution lies everywhere to one side of the stable eigenvector which corresponds to the deterministic solution. The option difference takes the same sign at both entry and exit triggers.

The crossover result establishes that boundaries may be reached sooner with uncertainty than without it. This is in sharp contrast to Miller and Weller's analysis of stochastic saddlepoint systems in which smooth pasting is always reported at values of the fundamental which lie outside those for the deterministic saddlepoint analysis. We have shown that, with sufficient asymmetry of the bands and a sufficiently high stable root, the addition of uncertainty can give rise to qualitatively different solutions, taking the stochastic saddlepoint form shown above. We have therefore extended the Miller and Weller characterisation of stochastic saddlepoint solutions under asymmetric boundaries.

4.6 SOME EXAMPLES WITH SIMILAR STRUCTURES.

4.6.1 Introduction.

The saddlepoint analysis of the Dixit model emphasises the formal structure of models in which our characterisation of solutions with varying degrees of exogenous mean reversion has been established: a wide range of economic models might take this form. However, many macroeconomic saddlepoint models employ the slightly different specification in the Miller and Weller analysis of stochastic saddlepoint systems, in which the fundamental is specified logarithmically and the arbitrage relationship is log-linearised.[12]

We conjecture that the qualitative characterisation of solutions we have obtained using the Dixit model, including crossover solutions arising with asymmetric boundaries, should apply in the recursive Miller and Weller saddlepoint systems as well as in the Dixit model. The ability of the strictly symmetric simplified two period model to explain these qualitative results also suggests they are not unique to the Dixit specification. Limitations built into existing computer programmes for the solution of the Miller and Weller examples have hindered intended attempts to confirm this conjecture by numerical means, and so we leave this to subsequent research.

In this section we indicate a number of macroeconomic models to which we believe we may generalise our results concerning the qualitative solution properties of the Dixit model with symmetric and asymmetric boundaries on the asset price. These include Krugman's (1988) macroeconomic model of hysteresis in trade, Krugman's (1990) monetary model of exchange rate target zones, and versions of the closed economy ISLM model in which there is no feedback from the asset price to the dynamics of the predetermined fundamental. We indicate what relevance our findings might have in terms of the economics of these alternative models.

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12. See Miller and Weller (1990a, p173) for a discussion of the advantages of this approximation in relation to the Siegel paradox.

Miller and Weller (1988a, p 7) argued that the feedback effects which are ruled out in the recursive saddlepoint models constitute "the feature of the stochastic saddlepoint systems which is peculiarly macroeconomic". It is therefore of interest whether the features of the solutions we have explored, particularly the 'crossover' property with sufficiently asymmetric boundaries, might carry over to the non-recursive cases. We do not address that question further here, although we identify the Buiter and Miller modified Dornbusch model as an important case in point.

4.6.2 The Krugman 'De-Industrialisation' Model.

The first model we consider within the Miller and Weller framework is Krugman's (1988) macroeconomic model of de-industrialisation and re-industrialisation. This model is closely related to the microeconomic model of entry and exit under uncertainty. The model describes an economy in which it is costly to move productive resources between the tradeable and non-tradeable sectors, and illustrates the responsiveness of resource reallocation to real exchange rate movements.

The exogenous fundamental is the trade balance (or, by identity, capital flow), B, which is assumed to follow the stochastic process:

$$dB = -\rho B\, dt + \sigma\, dz \tag{4.8}$$

where dz is the increment of a Wiener process.

The relationship between the real exchange rate (e) and the trade balance is given by:

$$e = \alpha - \beta (R_T + B) \tag{4.9}$$

where R_T is the state variable recording the inherited level of resources in the traded goods sector.

The value of a unit of resources in the traded as opposed to the non-traded sector is given by the value function V(R,B). This is like a forward looking jump variable or asset price, and it is assumed to satisfy the arbitrage condition:

$$rV = E(dV/dt) + (e-c) \tag{4.10}$$

where the flow return to resources allocated to the traded sector is given by (e-c), assuming a constant flow cost of production given by c. Krugman restricts the demand side of the model to exclude income effects from the demand for the traded good, and so facilitates the 'quasi-partial equilibrium' analysis of resource allocation in production.

The key to the model's analysis of resource allocation is the specification of the costly nature of adjustment. Shifting one unit of the resource either into or out of the tradeables sector costs γ units of resource in the nontradeable sector: crucially, these adjustment costs are nondifferentiable at zero.[13] As Krugman argues, "as long as $-\gamma < V < \gamma$, there is no incentive to move resources between sectors". On the other hand, "resources will be reallocated when V reaches its upper or lower limit". Reallocation of resources alters the value of the state variable R_T.

In the general case in which capital flows follow a mean reverting stochastic process this model constitutes a stochastic saddlepoint system in V and B, and can be analysed as such. The stochastic saddlepoint representation is given by:

$$\begin{pmatrix} EdV \\ dB \end{pmatrix} = \begin{pmatrix} r, & B \\ 0, & -\rho \end{pmatrix} \begin{pmatrix} V\,dt \\ B\,dt \end{pmatrix} + \begin{pmatrix} 0 \\ \sigma\,dz \end{pmatrix} + \begin{pmatrix} \beta R_T - \alpha - c \\ 0 \end{pmatrix} \tag{4.11}$$

It is convenient to analyse the case in which $R_T = (\alpha + c)/\beta$.

Conditional on an initial value for R_T, the solution to the model can be represented by the stochastic saddlepath solution subject to the boundary conditions $(\gamma, -\gamma)$ on the value function V(R,B). This takes the familiar s-shaped form, and it yields the range of values of the fundamental, B, for which no recursive reallocation occurs. This solution is shown in figure 4.10.

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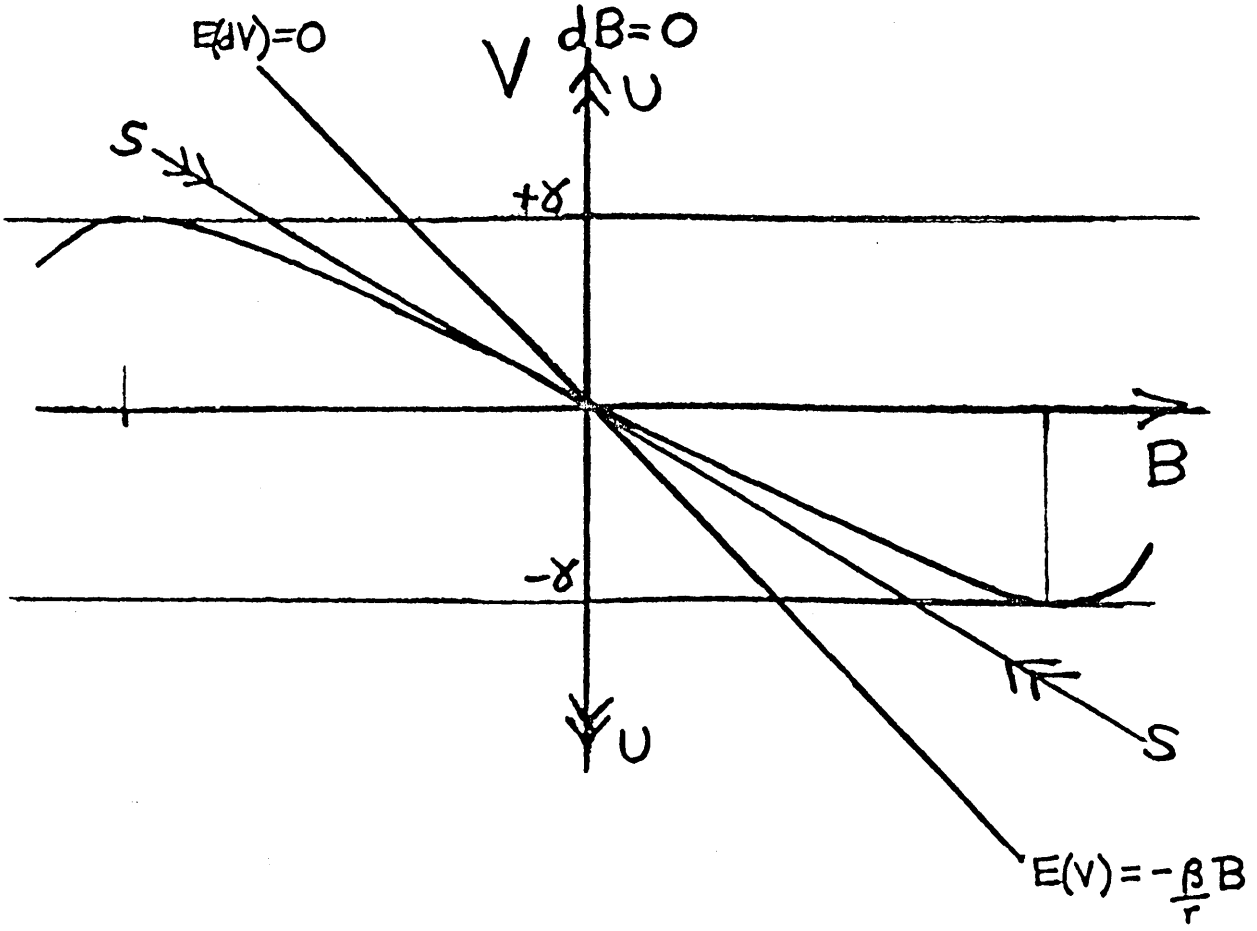
13. Krugman's solution employs Value Matching and Smooth Pasting as conditions on the level and derivative of the value function V. This suggests that infinitessimal resource reallocation at proportional cost is ruled out - otherwise Dumas's 'supercontact' form of Smooth Pasting as a second derivative condition would be appropriate. (See Dumas (1989)).

Figure 4.10

STOCHASTIC SADDLEPATH SOLUTION TO KRUGMAN'S MODEL OF "DE-INDUSTRIALISATION".



The model features a hysteresis property in that the steady state resource allocation is path dependent, and is shaped by the historical experience of 'large' shocks. The solution is also non-certainty equivalent, due to the option values associated with the possibility that future shocks might require costly resource movements to be reversed. Both these properties correspond to those of the Dixit (1989a) model.

The model has a recursive form, as no feedback is suggested from the relative valuation of resources in the traded sector to the exogenous process driving capital flows. Krugman considered the case with mean reversion and uncertainty, but did not explore the effects of changes in the degree of mean reversion in capital flows. Note, moreover, that Krugman's numerical method restricted his analysis to the case of symmetric sunk costs. Hence the analysis of the effects of varying mean reversion and asymmetric boundaries in the Dixit model suggests extensions to Krugman's examination of this model.

Assuming our qualitative results carry over to the logarithmic specifications, they would confirm Krugman's claim that increased mean reversion will widen the model's hysteresis band. Our analysis of solutions adds the further insight that the non-certainty equivalence of the solution is reduced as the mean reversion of capital flows increases. Hence the impact of changes in mean reversion on the overall solution band will depend on the initial level of uncertainty: the greater initial uncertainty, the smaller the effect of changes in mean reversion on the overall band.

Finally we consider how the crossover property would apply to the Krugman model. If, for example, the sunk cost of transferring resources out of the tradeables sector were zero, and mean reversion made shocks sufficiently temporary in nature, a crossover solution would imply that the magnitude of the capital outflow required to depreciate the real exchange rate and trigger a shift of resources into the tradeables sector would be larger under certainty than under uncertainty. Uncertainty would hasten the costly shift into tradeables.

4.6.3 A Simple Monetary Model of the Exchange Rate with Target Zones.

Our second example is based on the basic log-linear monetary model of the exchange rate employed by Krugman (1987, 1990) for the stochastic analysis of target zones, serving as a basis for much of the subseqent literature on speculative attacks and target zones.[14] Flexible pricing in goods markets, perfect capital mobility and purchasing power parity allow the exchange rate to be determined by the realtive supply and demand of domestic and foreign currencies.

In Krugman's notation, the exchange rate is given by:

$$s = m + v + \gamma E(ds/dt) \tag{4.12}$$

where s is the log of the price of foreign exchange, m is the log of the ratio of the domestic to foreign money supplies, and v is a term expressing income and velocity shocks to relative money demand. The final term captures the effect of expected depreciation on relative money demand. The random component of the model is given by v.

Krugman and Rotemberg (1990,p 5), have acknowledged that a simple Brownian motion specification for v could be substituted by "more complex processes, notably autoregressive ones". We give the model a recursive stochastic saddlepoint structure by specifying exogenous mean reversion in the process for v, thus:

$$dv = -\rho\, v\, dt + \sigma\, dz \tag{4.13}$$

The stochastic saddlepoint structure for the model is thus given by:

$$\begin{pmatrix} Eds \\ dv \end{pmatrix} = \begin{pmatrix} 1/\gamma, -1/\gamma \\ 0, \ -\rho \end{pmatrix} \begin{pmatrix} s\, dt \\ v\, dt \end{pmatrix} + \begin{pmatrix} 0 \\ \sigma\, dt \end{pmatrix} + \begin{pmatrix} -m/\gamma \\ 0 \end{pmatrix} \tag{4.14}$$

The model has a saddlepoint property about the steady state given by $s=m/\gamma$, $v=0$.

---

14. See the use of this simple monetary model in Flood and Garber (1989), Buiter and Grilli (1990), Delgado and Dumas (1990) and Miller and Williamson (1990).

Krugman (1987, 1990) used this stochastic saddlepoint for the analysis of exchange rate target zones in a world in which exchange rate movements reflect only relative monetary considerations. The target zone is given in terms of upper and lower boundaries on the movements of the exchange rate s. Relative monetary policy interventions occur to enforce these boundaries, in a similar manner to the resource reallocation in the previous model.[15]

The qualitative properties of the model correspond to the stochastic saddlepoints analysed above: there is a range of values of the stochastic fundamental which correspond to policy inaction, and the solution is non-certainty equivalent - what Krugman (1987) termed the "bias in the band".

Once more the formal structure of the model is essentially similar to that employed in chapter three, and our results concerning changes in the degree of mean reversion and our results under asymmetric boundaries can be expected to carry over. Increasing the degree of mean reversion in the income and velocity shocks to relative money demand will of course widen the range of realisations of relative money demand - the 'band width' - for which no monetary intervention in defence of the exchange rate target is required. Our results in chapter three indicate the further result that the sensitivity of the band width on v to changes in the degree of mean reversion in the process on v will vary with the degree of uncertainty.

We can easily imagine that the bands on the exchange rate might be specified asymmetrically about the expected steady state value of the exchange rate. For example, policy may seek to prevent depreciation from an initial equilibrium, but might allow a significantly wider degree of latitude for appreciation. With such bands, when relative money demand shocks are sufficiently lacking in autocorrelation, a 'crossover' phenomenon could be observed. In the case where depreciation from the initial equilibrium

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15. See Miller and Weller (1990b) for a treatment of the different intervention policies required to support real and nominal exchange rate bands. The formal structure of the smooth pasting solution is supported in the case of nominal exchange rate bands, as in the simple monetary model of Krugman (1987, 1990), by discontinuous but reversible switches in regime from a floating to a fixed exchange rate. These regime switches bound the movements of the *asset price* (the nominal exchange rate).

Miller and Weller analyse real exchange rate bands in the non-recursive extended Dornbusch framework, where they find that the formal structure of the smooth pasting solution is maintained by infinitessimal interventions to regulate the *economic fundamental* (real balances) within bands.

is ruled out by policy, but appreciation would be tolerated up to some upper limit, this means that intervention to limit the appreciation would be required for a smaller departure of relative money demand from its mean than would be the case under certainty. Uncertainty works to hasten rather than delay action in the crossover case.

4.6.4 A Closed Economy ISLM Model with Intervention Bands for the Stabilisation of Long Term Interest Rates.

In this example we use a model similar to the closed economy example given by Miller and Weller (1988a, p 16), but we use it to analyse the impact of a monetary policy which seeks to stabilise random output shocks, but only when they cause long term interest rates to reach specified upper and lower bounds. We take the general saddlepoint structure but restrict it to obtain the recursive case in which output dynamics are exogenous.

In order to achieve the recursive saddlepoint structure we require that aggregate demand for output is completely interest inelastic. The evolution of output is independent of the bond and money markets, and so long term interest rates must therefore only be of interest as an indicator through the term structure of expectations of future demands for real balances.[16]

Our restricted form of the model consists of the following elements. Firstly the LM curve describing the money market is conventional, given fixed prices:

$$m = -\lambda r + \kappa y \qquad (4.15)$$

in which y denotes aggregate demand and r the money market interest rate.

An interest inelastic IS curve ensures the recursive property of the saddlepoint system with respect to demand shocks, and is represented by:

$$x = \alpha y + g \qquad (\alpha < 1) \qquad (4.16)$$

16. This restriction may be judged so strong as to deprive the model of real world relevance. Nevertheless it retains analytic interest.

in which x is aggregate output, and g represents government spending.

A Phillips curve type slow adjustment of demand is specified, and these dynamics for demand incorporate a random component in the form of the Wiener process with increments dz. The resultant stochastic differential equation is:

$$dy = \phi\,(x-y)\,dt + \sigma\,dz \tag{4.17}$$

Finally a linearised arbitrage relationship between the bond and money markets is given by the deterministic differential equation:

$$E(dR) = \pi\,(R-r)\,dt \tag{4.18}$$

The recursive stochastic saddlepoint system for this model is thus given by:

$$\begin{pmatrix} EdR \\ dy \end{pmatrix} = \begin{pmatrix} (\pi\,\kappa)/\lambda,\ \pi \\ \phi\,(\alpha-1)\,,\ 0 \end{pmatrix} \begin{pmatrix} R\,dt \\ y\,dt \end{pmatrix} + \begin{pmatrix} 0 \\ \sigma\,dz \end{pmatrix} \tag{4.19}$$

The familiar non-certainty equivalent hysteresis zone property indicates in this model the levels of output (relative to its mean) at which monetary intervention to stabilise interest rates will occur. The presence of uncertainty is expected to widen the band relative to the deterministic case.

An increase in mean reversion in output can arise from two sources. Firstly there may be an increase in the responsiveness of output to excess aggregate supply or demand. Secondly there could be an increase in the savings rate, thereby reducing the multiplier effect from output to aggregate demand.

The predicted band widening effect of mean reversion indicates that an decline in the persistence of output will have the effect that larger fluctuations in output will occur before the interest rate stabilisation policy is triggered.

On the basis of our Dixit model results we emphasise that the sensitivity of the overall band to the degree of mean reversion in output will depend on the level of uncertainty concerning future output. Changes in the system's deterministic dynamics will have little effect on the levels of output at which interest rate stabilisation is triggered if future levels of output are highly variable.

We must allow for possible 'crossover' solutions, if, for example, the authorities were prepared to see little rise in interest rates above their mean level, while being much more tolerant of temporary falls in interest rates. In the case suggested above, where the lower bound on long term interest rates is the larger, a crossover solution would mean that the level of output at which the interest rate floor was activated would be achieved sooner in the stochastic solution than under certainty equivalence. Intervention at the larger boundary would be hastened by the effect of uncertainty.

4.6.5 Exchange Rate Target Zones in a Modified Dornbusch Model.

Miller and Weller (1988a, 1989b, 1989) have analysed target zones in a more developed macroeconomic framework than the simple monetary model of Krugman given above: the Buiter and Miller extension of the Dornbusch model with sticky goods prices and sluggish output adjustment. However, we find that there is no restriction of this model which both yields the convenient recursive structure of our previous analyses, and also has a sensible interpretation in terms of exchange rate target zones.[17] There is thus no direct application of our findings concerning the noncertainty equivalence of the response of the recursive stochatic saddlepoint system to mean reversion, nor of those concerning the possibility of crossover solutions with asymmetric boundaries. This tends to reinforce Miller and Weller's contention (1988a, p 7) that interesting macroeconomic applications of stochastic saddlepoint models should incorporate endogenous mean reversion in the fundamentals.

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17. Buiter and Miller (1982) suggested restrictions on the wage-price process which gave rise to recursive versions of the model, but these do not suit the target zone application.

It therefore remains for future research to extend our investigations to the general saddlepoint case. It will be of particular interest to confirm whether the qualitative features we have described as 'crossover' can occur with asymmetric boundaries in the case of endogenous mean reversion.

4.7 CONCLUSIONS.

In this chapter we have shown how our results concerning mean reversion in the Dixit (1989a) model of entry and exit under uncertainty can be linked to the literature on stochastic saddlepoint solutions. We have indicated the similarity between the Dixit model and a restricted form of Miller and Weller's (1988a) stochastic saddlepoint framework. The stochastic linear saddlepoint approach offers an alternative insight concerning the properties of the solution to the Dixit model with mean reversion. It also indicates that our results concerning the Dixit model with mean reversion might be generalised to linear saddlepoint systems with symmetric and asymmetric boundaries. We have therefore considered a number of models to see how the findings of qualitative properties of stochastic saddlepoint solutions implied by our analysis of the Dixit model with mean reversion might be applied to other contexts.

We argue that future research should seek to confirm whether our results derived in the Dixit model can be found to apply to the models suggested above. In particular we suggest that future research could seek to confirm the possibility of 'crossover' results. We should consider first the crossover possibility in the logarithmic Miller and Weller type specification with exogenous mean reversion where we strongly expect that the result will hold. We have encountered some numerical complications with the particular programmes we have attempted to use to confirm this, and so we have chosen to leave this exercise for future work.[18]

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18. Following the suggestions in this research, Alan Sutherland has recently confirmed that 'crossover' type solutions can indeed occur within the Miller and Weller saddlepoint cases with sufficiently asymmetric bands. He also conjectures that the nature of these solutions could also be established analytically by use of the properties of confluent hypergeometric functions.

It is quite a different matter whether or not we could find parts of the parameter space for which the solution displays crossover results in the general model without feedback from the asset price to the fundamentals. This is something that remains to be established by future research.

Chapter Five.

INSTABILITY IN UK EXPORT ELASTICITIES AND THE IMPORTANCE OF THE STATIONARITY OF COMPETITIVENESS.

5.1 INTRODUCTION

This chapter proceeds as follows. Firstly we document the reported changes in elasticities and explain the background of the development of model specifications. We then put some representative models on a common basis by estimating them on the LBS choice of variables. We recognise the regime change in competitiveness in 1979, and so we split our analyses into sub-periods before and after 1979. We employ a variety of methods to study the time series properties of the variables and establish that co-integration analysis cannot be applied in respect of the competitiveness variable. We show that estimates of the world trade elasticity appear to be superconsistent but that, as expected, this is not true of the competitiveness elasticity. The recognition of the stationarity of competitiveness can thus prevent misleading inference concerning the long run elasticity.

5.2 INSTABILITY IN UK EXPORT COMPETITIVENESS ELASTICITIES: THE ISSUES.

The UK is a country where debates over trade imbalances, the external adjustment mechanism and exchange rate policy have in recent years returned to centre stage in public affairs. Most obviously, the balance of payments constraint on growth, which was central to the 'stop-go' policy cycle of the 1960s returned to centre stage as a concern of policy makers and financial markets after 1987.[1] However, prior to this recent focus on relative growth rates and the trade balance, the debate had concerned the price or competitiveness elasticity of trade flows. This debate followed on from the experience of the early 1980s, in which the UK manufacturing sector was hit hard by a sharp appreciation of the real

1. See Turner (1988) for an explanation of the developments since 1987 in terms of high relative rates of growth in the UK. See Landesmann and Snell (1989) for evidence that the income elasticity of UK exports may have risen during the 1980s.

exchange rate. The sensitivity of trade to movements in competitiveness was, (and remains), a relevant consideration to continuing debates concerning exchange rate policy, the merits of Sterling's recent entry into the exchange rate mechanism of the EMS or participation in a European monetary union. In the context of such debates, Chancellor Lawson's 1986 Autumn Statement included the claim that the elasticity of trade volumes to movements in competitiveness had fallen. This was taken to support the view that the government need not be too concerned about the consequences of higher competitiveness for the demand for real output, and hence could take a relatively relaxed view of volatility in the nominal exchange rate.

The primary source of econometric evidence for the Chancellor's 1986 claim was the Treasury quarterly forecasting model. In table 5.1 we present the estimates for the long run competitiveness elasticity derived from versions of this model and related research papers over the last decade.

From 1979 to 1987 Treasury estimates of the competitiveness elasticity fell from around -1.3 to -0.39. However, the evolution of the equation specification suggests the reported fall in elasticities may be the consequence of model development itself. Nevertheless, there are also a number of plausible explanations for changes in the behavioural relationships: these may have different policy implications.[2]

One class of explanation might account for parameter change as a consequence of changes in the fundamental structure of and behaviour in the market for UK traded goods. Such structural change could have occurred in the composition, quality and product differentiation of UK manufactured goods. Maynard (1988) has given an account of supply side improvements under the Conservative economic policy since 1979: he refers to the pressure on UK firms to " up-market its products" to high quality product areas "where price competition is less important than non-price competition" (pp 157-8). Landesmann and Snell (1989) conclude that the consequences of Mrs Thatcher on UK manufacturing exports appear in the form of increased income elasticities. A reduction in price elasticities such as is

2. While primarily designed for forecasting, the Treasury's quarterly macroeconomic model is widely used for policy analysis.

Table 5.1.

EVOLUTION OF THE LONG RUN ELASTICITY ON COMPETITIVENESS IN H.M.TREASURY MANUFACTURED EXPORTS EQUATIONS SINCE 1979.

From Treasury Quarterly Models and Related Research.

Sample Period.	Long Run Competitiveness Elasticity.	Treasury Model Version and/or Reference.
1966:1 -1977:4	-1.3	Treasury Model Manual 1979
1966:1 -1978:4	-0.825	1980 Treasury Model Brooks (1981)
1965:2 -1979:4	-0.89	5th order Almon, 17 lags. Ritchie and Hicklin (1981)
1965:2 -1979:4	-0.39	3rd order Almon, 10 lags. Ritchie and Hicklin (1981)
1972:1 -1983:4	-0.14 (+ margins term)	Treasury Model Manual 1985
1966:1 -1985:4	-0.56	Treasury re-estimation of NI Mark 9 in Milne (1986).
1973:3 -1985:4	-0.39	Treasury Model Manual 1987

Notes:
These equations are estimated on versions of the Treasury measure of normalised unit labour costs.

alleged to have occurred would be entirely consistent with an increase in income elasticities. However, as Landesmann and Snell acknowledge (p 23) such parameter change in export volume equations could either be caused by the kind of supply side improvements detected by Maynard across all firms, or else as the consequence of the elimination of 'weaker' UK producers from their export markets in the (possibly hysteretic) consequences of the large appreciation in 1980/1. While these alternative explanations for 'structural change' may have differing policy importance, they share the implication that the sensitivity of UK exports to permanent shocks in UK competitiveness would be reduced as a consequence of the supposed structural change.[3]

An alternative class of explanations takes account of possible forward looking behaviour by market participants and of the so-called 'Lucas critique'.[4] To the extent that behavioural relationships depend on expectations of future competitiveness, observed trade relationships might be expected to reflect perceived changes in the exchange rate policies of UK governments.[5]

We maintain that a significant regime change occurred in the time series properties of UK competitiveness during 1979.[6] Institutional changes support this contention. Exchange controls were lifted in July 1979, and Chancellor Howe's tight monetary policy was implemented without regard for the exchange rate: competitiveness movements were sharply changed relative to those of the previous two decades.

Under the expectations story an observed fall in the competitiveness elasticity in the reduced form could simply reflect a change in the way market participants filter movements in competitiveness.

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3. Whether policymakers could expect to achieve a credible permanent shock to competitiveness is a different matter - this would probably require a change in the supply side.

4. Lucas (1976). But these arguments were first raised in the early work of the Cowles Commission.

5. While it is the regime for the real exchange rate or competitiveness that is expected to determine trade flows, given price stickiness, nominal exchange rate policy will also matter. One of the stylised facts of modern international finance is that real and nominal exchange rates are substantially correlated. See Krugman (1989) pp 14-19.

6. We examine the time series properties of competitiveness in more detail below, allowing for the regime change.

The underlying relationship is represented by export volume equations of the general form:

$$A(L)\ X_t = B(L)\ W_t + G(L)\ C_t + \text{dummies} + u_t \qquad (5.1)$$

where A(L), B(L) G(L) are lag polynomials (whose order can vary considerably between specifications), X_t is UK manufactured export volumes, W_t is a world activity variable such as the volume of world trade manufactures, C_t is a price or cost measure of relative competitiveness. Dummy variables are used to eliminate the effects of dock strikes etc. Apart from the possible inclusion of a time trend, this general form characterises most of the export volume equations estimated by UK macroeconomic modellers, including the Treasury.

It is usual to assume that we can characterise the market for manufactured export goods by imperfectly competitive behaviour.[7] We imagine a monopolistically competitive UK exporter making profit-maximising decisions to supply the foreign demand for its differentiated good. It is a common assumption that the export volume equation traces the demand curve, while the supply decision is captured in a reduced form export pricing equation in which the demand curve and production costs are reflected.[8] However we note that the price variable used by the Treasury (and also by LBS) in its export volume equations is actually a cost measure: this is more in keeping with the notion that the export volume equation is also a reduced form mixture of supply and demand.[9] The effects of supply influences on the estimated equation may for example include decisions by UK exporters to exit from markets, thereby changing the composition of observed aggregate export volumes. We are therefore

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7. Goldstein and Khan (1985).

8. See Goldstein and Khan (1978), (1985), and Haynes and Stone (1983). A recent application is in Corker (1989).

9. To a large extent, price and cost measures are collinear and serve as proxies for each other. However we should not overlook the substantial recent literature on the extent of pass through to prices of exchange rate changes, as in Dornbusch (1987), Baldwin (1988), Froot and Klemperer (1989) and Ohno (1989).

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complications arise for the interpretation of aggregate relationships when the microfoundations of supply or demand may be characterised by discontinuous adjustment, due, for example, to sunk cost hysteresis effects: we return to these matters in chapter seven below. We take the view that, given long delivery lags, expectations formation and 'smoothing', possible nonlinearities due to kinky adjustment costs, and frequent changes in policy regimes, it will prove difficult for researchers to develop a given reduced form equation into a convincing, theoretically based, structural econometric model of export behaviour.[12]

5.3 A BRIEF HISTORY OF MANUFACTURED EXPORT VOLUME EQUATIONS IN UK QUARTERLY FORECASTING MODELS.

5.3.1 Introduction.

We now will report a range of other estimates of the elasticity and seek to present a stylised history of the research conducted by the proprietors of the principal UK quarterly forecasting models.[13]

We draw on papers recording the successive re-specification exercises carried out by the various modelling teams. These have been prompted by the breakdown of existing estimated relationships, but they have also reflected the steady integration into the model proprietors' methodology of dynamic model specification and testing procedures within the 'London School of Economics' tradition.[14] Particularly useful from the point of view of this section are the published papers by Brooks (1981) and Anderton and Dunnett (1987) in which explicit comparisons are made across the model specifications

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12. Pessimism concerning the potential complexity of the task has not prevented eg. Holly and Wade (1989) from giving stuctural supply and demand interpretations to relatively straightforward equations estimated on UK aggregate data. In a related paper, Dinenis and Holly (1989) have modelled the supply decision and pricing to alternative domestic and export markets.

13. The forecasting models referred to are those of H.M.Treasury, (HMT); the London Business School, (LBS), the National Institute of Economic and Social Research (NI), and (to a lesser extent) the Bank of England (BE). These models are all widely used for policy analysis purposes as well as forecasting.

14. See Hendry, Pagan and Sargan (1984) (in the Handbook of Econometrics, volume II) for an exposition, also the collected papers in Hendry and Wallis (1984).

employed by the various modelling teams, in addition to the expected critical evaluation of the particular model subject to revision.[15]

Specification exercises have thus been carried out according to increasingly similar (and arguably more thorough) model selection procedures. Not surprisingly perhaps, there has been a marked convergence in the form of the estimated relationships for UK manufactured export volumes in the quarterly models over the last decade.[16] It might be hoped that this process has constituted a 'progressive research programme' resulting in successively closer approximations to the underlying data generation process. However, in important respects this optimistic interpretation of the research endeavour may not be appropriate; some of what was learned from earlier studies appears to have been lost. In particular the specification of short run dynamics on competitiveness requires re-examination if satisfactory estimates of the long run competitiveness elasticity are required.

In table 5.2 we set out estimated values for the long run competitiveness elasticity for range of UK macroeconomic models since the late 1970s. As we have already seen in the Treasury case, the estimates reflect different specifications as well as different sample periods. With the exception of the LBS model the estimated long run competitiveness elasticity has indeed fallen over time, though not as much as have the Treasury estimates. The LBS model differs: its general specification has changed little during the period. Lower elasticities may reflect equation specification rather than changing sample data.

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15. Ritchie and Hicklin (1981) made an explicit study of the then current LBS model to inform their re-specification of the Treasury equation.

16. The nature, status and legitimacy of the statistical procedures implicit in cumulative specification search as reflected in a body of literature of this type remains a matter of debate, with particular doubts concerning the significance levels involved in successive test-based model revision. If it is fair to judge that "intelligent model building undoubtedly reflects an in informal Bayesian viewpoint", (Hendry and Wallis (1984), p11), we must all come to terms with the Bayesian dilemma of needing to bring "the state of knowledge prior to data analysis" into our priors, while at the same time tolerating the consequent influence of a single sample of non-experimental data. (See Trivedi (1984), p193, on this point).

Table 5.2

ESTIMATES OF LONG RUN COMPETITIVENESS ELASTICITIES IN UK MANUFACTURED EXPORT VOLUME EQUATIONS OF VARIOUS RESEARCH TEAMS SINCE 1979.

Sample Period.	Long Run Competitiveness Elasticity.	Competitiveness Variable.	Model and/or Reference.
Bank of England			
	-0.604	NULC	BE in Brooks, (1981).
1967:1-1981:4	-0.37	Relative Prices	Hotson & Gardiner,(1983).
National Institute (NI) equations.			
1964:1-1977:3	-0.687	RPR	NI in Brooks (1981).
1966:1-1978:1	-0.678	RPR	Brooks (1981) 17 lag. specification.
1967:2-1983:2	-0.579	RPR	NI Mark 8.
1966:1-1985:4	-0.58	RPR	NI Mark 9.
1966:2-1987:1	-0.58	RPR	NI Mark 10.
London Business School (LBS) equations.			
1964:1-1978:2	-0.27	Relative Costs	LBS in Brooks (1981).
1963:4-1981:4	-0.35	COMT	LBS, 1985
1963:4-1983:4	-0.34	COMT	LBS, 1987
H.M.Treasury (HMT) equations.			
1966:1-1977:4	-1.3	Relative Costs	HMT, 1979
1966:1-1978:4	-0.825	RNULC	HMT in Brooks (1981).
1965:2-1979:4	-0.89	RNULC	Ritchie and Hicklin (1981)
1972:1-1983:4	-0.14 (+0.51	Total Costs. on Margins)	HMT, 1985
1966:1-1985:4	-0.56	RNULC	NI Mk 9 on HMT data Milne (1986).
1973:3-1985:4	-0.39	RNULC	HMT, 1987
Other research.			
1972:1-1986:2	0.69	Relative Export Prices.	Landesmann Snell (1989)

Notes:
Variable names in capitals are as coded by research teams.

Details of the specification are given in table 5.3. Note that the stability of unrestricted estimates of the elasticity of world trade contrast with the range of estimates obtained for the elasticity on competitiveness.

5.3.2 Differing Measures of Competitiveness.

Different modelling teams have employed different measures of competitiveness in export volume equations. The Treasury equations have employed normalised unit labour costs, the LBS has employed tax adjusted relative cost measures and the National Institute relative export prices, while the Bank of England has used both cost and price measures at various times.

Enoch (1978) has discussed the relative merits of different price and cost variables. He concludes that the matter should ultimately be resolved by purely statistical criteria. The theoretical arguments can only discriminate between the various variables on the basis of excessively strong priors concerning the underlying behavioural and structural characteristics of the relevant market(s). More recently, Anderton and Dunnett (1987) conclude that:

"empirical results did not identify any competitiveness measure as clearly superior to the other measures. It seems that the overall specification of the exports equation... is as important as the specification of the proxy variable representing UK competitiveness". (p 51).

In what follows we therefore take the view that the various price and cost competitiveness measures are to a substantial degree interchangeable.

In table 5.4 we report the variances of some different competitiveness measures. The variance of relative costs is larger than the variance of relative prices, suggesting, as might be expected, that producers are unable (or unwilling) to pass on all cost shocks.[17] The implication of these relative variances is that the coefficient on relative prices will be greater than that on relative costs in explaining the same data on export volumes.

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17. See the literature on pricing to market eg. Dornbusch (1987), Froot and Klemperer (1989).

Table 5.3.

SPECIFICATION DETAILS, INCLUDING SHORT RUN DYNAMICS ON COMPETITIVENESS FOR VARIOUS EXPORT VOLUME EQUATIONS FEATURED IN TABLE 2.

Model and/or Reference.	Specification on Competitiveness.	Lagged Dependent Variable.	Other Details.
BE EQUATIONS.			
BE in Brooks	2nd order Almon on 16 lags.	No	WT=1.0 Quadratic trend.
Hotson & Gardiner.	Linear Almon from 3rd to 7th lags.	Yes	WT=1.0 Trend variable. (specialisation)
NI EQUATIONS.			
NI in Brooks	3rd order Almon on 6 lags.	No	WT=0.57
Brooks' 17 lag	7th order on 17 lags	Yes	WT=0.60
NI Mark 8.	5th order Almon 5th to 12th lags.	No	WT=0.60
NI Mark 9.	Restricted 4 lag autoregression.	Yes	WT=0.59
NI Mark 10.	Restricted 4 lag autoregression.	Yes	WT=0.59
LBS EQUATIONS.			
LBS in Brooks.	Restricted 4 lag autoregression.	Yes	WT=1.0 Trend variable. (World Ind Production)
LBS, 1985	Restricted 4 lag autoregression.	Yes	WT=0.60
LBS, 1987	Restricted 4 lag autoregression.	Yes	WT=0.57
HMT EQUATIONS.			
HMT, 1979	Almon, 17 lags.	No Time trend.	WT=0.9
HMT in Brooks	3rd order Almon, 17 lags.	No Time Trend.	WT=0.86
Ritchie & Hicklin.	5th order Almon, 17 lags.	No	WT=0.93 Time trend. Capacity utilisation.
HMT, 1985	Rectangular lag, lags 1-24.	Yes	WT=1.0 Time trend. Margins.
NI9, HMT data Milne	Restricted 4 lag autoregression.	Yes	
HMT, 1987	Restricted 4 lag autoregression.	Yes	WT=1.0 Time trend.
OTHER.			
Landesmann & Snell.	7 lags.	No	WT=0.69 or spline on WT after 1979

Table 5.4.

SAMPLE MEANS AND VARIANCES FOR DIFFERENT MEASURES OF UK COMPETITIVENESS.
The measures are all taken from the LBS databank.

Sample Means.

	COMT	COMC	COMP	COMW
1968:3 - 1987:2	90.5	78.76	85.22	85.48
1968:3 - 1979:2	84.12	74.58	80.63	81.16
1979:3 - 1987:2	99.28	84.51	91.52	91.42

Sample Variances.

	COMT	COMC	COMP	COMW
1968:3 - 1987:2	176.64	104.92	56.41	63.06
1968:3 - 1979:2	42.65	40.37	24.15	30.31
1979:3 - 1987:2	230.73	138.91	32.15	47.64

Notes:

COMT is LBS cost competitiveness adjusted for payroll taxes.
COMC is LBS cost competitiveness, unadjusted.
COMP is LBS export price competitiveness.
COMW is LBS wholesale price competitiveness.

All series are indices scaled with 1980=100.

In table 5.5 we report the long run properties of two export volume specifications, both estimated using each of four measures of competitiveness. The estimated competitiveness elasticities range from -0.346 to -0.522 for the LBS equation, and from -0.329 to -0.463 for the '(4,4,4)' specification. As expected the price elasticity is the greatest, while the tax adjusted cost elasticity is the smallest. Note that the spread of the estimates with the change in competitiveness variable is considerably less than the range reported in the literature: variable definitions alone cannot account for the variation in reported estimates.

In order to facilitate study of the effects of different equation specifications we re-estimate equations similar to some of those we have reported but employing a common data base. We (somewhat arbitrarily) choose to use the LBS database with its tax adjusted cost competitiveness measure.

5.3.3 Contrasting Approaches to Equation Specification.

The development of the manufactured export volume equations over the last decade can be represented in the following stylised form. At the beginning of the decade (as described by Brooks) there were, broadly speaking, two kinds of equation in use: the first, (as employed by HMT, BE and also NI), featuring long polynomial distributed lags on the competitiveness variable; the second, (the LBS equation), employing a restricted autoregressive distributed lag form with an implicit rational distributed lag on competitiveness and a possible error-correction interpretation. By the end of the decade (as described by Anderton and Dunnett) the modelling approach reflected in the LBS equation had come to dominate, and the Treasury and National Institute equations were similar in specification to what had, after some minor modifications, proved to be a robust specification of the LBS equation.

Before we consider the treatment of the short run dynamics more closely we deal briefly with some other specification issues which arise out of Treasury research.

Table 5.5.

ESTIMATES OF LONG RUN PROPERTIES OF MANUFACTURED EXPORT VOLUME EQUATIONS ESTIMATED WITH DIFFERENT MEASURES OF COMPETITIVENESS.

We estimate two models using each of four different measures of competitiveness. The equations are the LBS 1985 equation and the 4th order autoregressive model of which the LBS equation is a restricted form. The latter is denoted (4,4,4). All estimates are over the sample period 1969:3-1985:2.

1) COMT

(4,4,4)

XMAN = 0.589 XWM - 0.329 COMT + s.run dynamics + cons + dum
(0.048) (0.080)

LBS

XMAN = 0.574 XWM - 0.346 COMT + s.run dynamics + cons + dum
(0.041) (0.064)

2) COMC

(4,4,4)

XMAN = 0.555 XWM - 0.341 COMT + s.run dynamics + cons + dum
(0.041) (0.075)

LBS

XMAN = 0.539 XWM - 0.362 COMT + s.run dynamics + cons + dum
(0.036) (0.062)

3) COMP

(4,4,4)

XMAN = 0.621 XWM - 0.463 COMT + s.run dynamics + cons + dum
(0.053) (0.132)

LBS

XMAN = 0.601 XWM - 0.522 COMT + s.run dynamics + cons + dum
(0.053) (0.134)

4) COMW

(4,4,4)

XMAN = 0.581 XWM - 0.418 COMT + s.run dynamics + cons + dum
(0.034) (0.079)

LBS

XMAN = 0.561 XWM - 0.459 COMT + s.run dynamics + cons + dum
(0.037) (0.081)

Notes: COMT is LBS cost competitiveness adjusted for payroll taxes.
COMT is LBS cost competitiveness adjusted for payroll taxes.
COMT is LBS cost competitiveness adjusted for payroll taxes.
COMT is LBS cost competitiveness adjusted for payroll taxes.
All series are indices scaled with 1980=100.
Heteroskedastic standard errors in parentheses.

The first concerns the inclusion of trend variables and the imposition of a unitary long run coefficient on world trade. Treasury modelling of export volumes has been marked by a peculiar concern for time trends, which is apparently not shared by other modelling teams. Another Treasury concern, still apparent in recent research, has been the desire to impose a unitary coefficient on the long-run world trade effect. On this point Anderton and Dunnett (p 50) conclude that "the results as a whole do not readily support the imposition of a unitary coefficient for world trade combined with an export-share equation".[18] Overall it appears that the practical justification for the inclusion of a trend is that it allows the Treasury to constrain the income elasticity to unity without doing too much violence to the data. Nevertheless it seems preferable to recognise that low income elasticities for UK exports may be a reality (as argued in Landesmann and Snell) than to attribute this effect to a 'trend'.[19]

The second point is that the initial Treasury responses to the breakdown of existing equations in the early 1980s involved the search for additional explanatory variables to augment the familiar specification of export volumes. These attempts sought to capture supply side factors which were thought to be important: Ritchie and Hicklin (1981) sought to incorporate capacity measures, and Milne (1986) included profit margins. These attempts proved unsatisfactory and were subsequently abandoned in favour of the adoption of the LBS type approach and approximate error correction specifications.

In the older form of equation, export volumes were modelled as a relatively short distributed lag on a measure of world activity and a long (Almon) polynomial distributed lag of up to four years (16 or 17 quarters) on a relative price or relative cost variable. The HMT and BE specifications both conformed with this characterisation, as described by Brooks (1981). While the NI model, though of similar form, featured only some six quarters of lags, Brooks found himself unable in his subsequent specification

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18. It is true that the LBS equation considered by Brooks also imposed a unitary coefficient, but this restriction has been relaxed in subsequent LBS work.

19. This point is strengthened if we acknowledge Granger's (1966) observation that the existence of a trend is often a matter of definition and sample selection.

testing procedures to reject the presence of long lags of around four years on a relative export price measure of competitiveness, unless they were ruled out on *a priori* grounds.[20]

A second, quite different, approach was employed for the LBS equation.[21] This equation has a possible error-correction interpretation, such as has been emphasised recently in the context of the Co-integration literature by Engle and Granger (1987).[22] It was derived by testing, and then imposing where appropriate, successive restrictions on an initial autoregressive distributed lag model with maximum lags of four quarters in all variables, including a relative cost measure of competitiveness. The rational distributed lag formulation is capable of satisfactorily approximating more complex dynamics and lag structures; but lagged dependent variables may also be thought of as reflecting cost-of-adjustment dynamics and/or adaptive response mechanisms in the underlying behavioural relationships. The LBS modelling procedure produced a parsimonious equation, avoided the complications of the joint restrictions implied by the Almon form on lag length and order, and was consistent with the emerging systematic approach to dynamic model specification documented by Hendry et al (1984).[23] In this revised form the LBS equation has stood up well in the face of diagnostic testing and has shown remarkable parameter stability through a period in which there has been substantial variability in the data.[24]

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20. Brooks found good grounds for rejecting the particular BE and HMT models he had to examine as misspecified, but was able to suggest that some of this may have reflected the over-restrictive use of the Almon procedures in these specific cases.

21. LBS model manual 1979.

22. Error correction models have been familiar in the UK context since the work of Phillips (1954, 1957), further popularised through the work of Sargan (1964) and, under his influence, in work such as DHSY(1978). See also Hendry et al (1984), 4.2 p1069.

23. See Hendry et al (1984) 3.2 to 3.6 pp 1050-1066

24. See Dave Turner (1988) for an examination of stability in relation to the activity coefficient.

The performance of the LBS approach as an approximation to the DGP has evidently satisfied the model proprietors, in sharp contrast to the polynomial distributed lag formulations of HMT and NI which were found wanting at the start of the decade.[25]

Both the Treasury and the National Institute ultimately adopted similar starting points to the LBS for the application of specification testing procedures, with the result, reported in Anderton and Dunnett (1987), that the preferred specifications of the different modelling teams are now broadly similar in form, differing principally in the choice of relative price or competitiveness variable.[26]

5.3.4 Dynamic Misspecification and Long Lags on Competitiveness.

We have suggested that the changes in model specification described above could account for much of the reported fall in elasticities.[27] Given the LBS approach we might assume that its lower estimates of the competitiveness elasticity are to be preferred. We argue that this assumption is a risky one, and seek to argue that the inclusion of long lags on competitiveness may after all be justified.[28]

The earlier studies threw up two points of concern. Firstly Brooks (1981) indicated that long distributed lags on (relative price) competitiveness could not be rejected unless they were discounted by prior assumption.[29] Secondly we find that subsequent studies have rejected the possibility of long

25. The basic problem was predictive failure. Both Brooks and Ritchie and Hicklin writing in 1981 warned that the huge upswing in the real value of Sterling would constitute a powerful test of the estimated equations. It turned out that the equations underpredicted export performance for 1981 and 1982 as Sterling peaked and then began to fall.

26. See our previous discussion on the choice of competitiveness variable, which refers to Enoch (1978) and Anderton and Dunnett (1987). Note also that the Treasury equation still incorporates a time trend and a unitary coefficient on world trade.

27. Brooks (1981) reported that the coefficient on (price) competitiveness was sensitive to the specification of the lag structure. Conversations with Alan Sutherland of the Warwick Parliamentary Unit and Dave Turner of the ESRC Macroeconomic Modelling Bureau during 1987 provided independent confirmation of the similar findings of this research.

28. The notion of a 'progressive research programme' is expounded in eg. Lakotos (1971).

29. The null tested was of the form $\beta = 0$. Such a restriction was rejected. Given that rejection of null hypotheses in cumulative testing procedures can reflect their high implicit size, it must be appropriate

lags on competitiveness by assumption. Hence, despite the evidence offered by Brooks, the potential role of long lags specifically on competitiveness (cost or price) has subsequently been ignored.[30] In this important respect the literature has apparently not even attempted to encompass the findings of previous studies.

There are of course legitimate reasons why an investigator might choose to discount a possible explanatory role for anything like as many as 17 lags of competitiveness. Firstly, conflict with the requirement of parsimony, particularly in the context of the relatively small sample sizes typical to time-series economics. Secondly, doubts about inference due to the small number of degrees of freedom and also the complications of significance levels in a successive testing procedure. Thirdly, a belief that the rational distributed lag structures allowed for by the general model specification should in any case be sufficiently rich to capture any long lag effects. Fourthly, the assumed absence of strong prior justification from economic theory for such lags.[31]

We do not believe that the above reasons are sufficient to justify *a priori* rejection of Brook's findings, without any further recourse to the data. Furthermore, we believe that economic justifications for the presence of long lags on competitiveness can be given. One of the most convincing of these is the suggestion that the relevant behavioural variable is a smoothed measure of competitiveness, reflecting a notion of 'permanent' competitiveness. For the period up until the late 1970s this might be seen as smoothing competitiveness through the infamous UK 'Stop-go' business cycle.[32] An alternative kind of rationale for the role of lags of 4 years or more would be in terms of investment and capacity

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to treat Brooks' results with some scepticism. But his evidence is treated as establishing *prima facie* that the long lags case should be re-examined.

30. There is an important point to be clarified here. A rational distributed lag can approximate arbitrarily closely any distributed lag, providing the order of the two rational polynomials employed are sufficiently large. Whether or not the LBS version of the rational distributed lag actually can encompass the long distributed lags model must be resolved empirically.

31. Brooks found himself with considerable problems of interpretation, especially given the 'two humped' shape of the lag distribution and his priors that the equation reflected demand behaviour: he was wary of the shape of the lags rather more than their length *per se*. (pp 74-77).

32. We find some evidence, detailed below, of a role for a moving average component in competitiveness, which would in principle require the semi-infinite data record for the formation of expectations of future competitiveness as a function of the infinite past.

adjustment. Does current competitiveness affect future investment plans, and does that future capacity, when installed, then affect export volumes?[33] We thus subject the LBS equation to the misspecification analysis in respect of long lags on the competitiveness variable which it has hitherto avoided.[34] We repeat the dynamic misspecification analysis in the context of an unrestricted 'general' autoregressive model with an arbitrary maximum lag length of four quarters, such as appears to have been the starting point for the LBS modelling exercise.[35]

5.4 CO-INTEGRATION, AND TESTING FOR DYNAMIC MISSPECIFICATION, IN RELATION TO THE LONG-RUN ELASTICITY ON COMPETITIVENESS.

5.4.1 The Parameter of Interest: the Long-run Competitiveness Elasticity.

Our motivation for the study is not from the perspective of short-term forecasting but rather follows from debate over alleged structural change in this long-run elasticity. We are concerned to understand what may have happened to this long-run relationship, and also, insofar as possible, to interpret its significance, if any, for policy-makers. Of course this interest reflects the experience of previous researchers, (discussed elsewhere in this work), which has found the dynamic relationship between competitiveness and export volumes to be the 'weak link' in equations estimated on the available non-experimental sample. This experience can be given a particular interpretation in the context of the recent and developing literature on co-integration, to which we now turn.

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33. See Milne (1986) for attempts to use such a rationale. Such explanations could of course also fit in with and augment the pure expectational stories. They might also link with the hysteresis literature where capacity investment involves sunk costs.

34. Our dynamic misspecification analysis is largely carried out using the LBS cost competitiveness variable. However similar kinds of results were obtained using export prices and wholesale price based measures.

35. The unrestricted specification reported in most detail by Brooks (1981) excluded lagged dependent variables from consideration (p 74), but Brooks did report some testing of specifications with lagged dependent variables (Table 5, p 76). We also allow both for long lags on competitiveness and for lags on the dependent variable.

5.4.2 Co-integration, Superconsistency and the Competitiveness Elasticity.

The co-integration literature has formalised the notion that economic forces (typically associated with long-run equilibrium concepts) will influence the movements of certain economic variables so that they "cannot drift too far apart" (Engle and Granger (1987)) or are "stochastically trended together" (Stock (1987)). Engle and Granger (EG) have put forward a definition which describes a particular relationship between stochastically trended series such that there exists a (linear in the case examined in the paper) combination of these series which is integrated of a lower order than the component series.[36] The definition characterises "a special kind of relationship with interpretable and testable consequences" (EG, p 254), and the suggested interpretation is that the co-integrating relationship is the empirical counterpart to the long-run equilibrium constraints.[37]

The superconsistency results in Stock (p 1036) would appear to promise a powerful tool for extracting information concerning long-run equilibrium relationships from systems of co-integrated variables. In an example such as the UK manufactured export equations we might see how to 'settle' disputes concerning the long-run elasticities, given only that the underlying assumption of co-integration were appropriate, and moreover, that the sample size were sufficient to invoke superconsistency results.[38] However, the theoretical literature on co-integration is far from complete, as the extent to which the kinds of results described above can be applied to a wide range of cases remains to be established. For instance, Wickens and Breusch (1988) assert (p 202) superconsistency in cases with deterministic trend but with *stationary* stochastic components.

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36. Where the component series are all difference-stationary, there is a relationship between the levels of variables which is itself levels-stationary. This stationary vector is readily picked out from the non-stationary alternatives by least squares estimates: this effect is 'superconsistency'.

37. We do not employ a multi-equation approach and so the co-integrating vector is determined by our arbitrary normalisation on the volume of manufactured exports.

38. The findings of Banerjee et al (1986) encourage scepticism with respect to the practical relevance of the superconsistency result suggesting that "often when co-integration obtains we do not find it and often when we find it our estimate of.... (the co-integrating parameter) ...is very inaccurate". (p 260)

In the context of the results of estimated equations for UK manufactured export volumes the remarkable precision and robustness to specification changes of the coefficient on world trade we might seems to reflect a superconsistency result. In contrast the long-run elasticity on competitiveness is less well defined and varies considerably with the equation's specification.[39]

But we would be wise to note Hendry's (1986) caution that the theory of co-integration offers modellers no 'free lunch':

"it is essential to check that *all* variables in any static regression are I(1) (and that no subset is co-integrated) if the super-consistency result of Stock (1987) is to apply". (p 208).

Wickens and Breusch (1988) make essentially the same point in the following fashion:

"it is possible to misspecify the dynamic structure of models with non-stationary variables by omitting higher order lags.... ...without affecting the consistency of the estimates of the long-run multipliers associated with the non-stationary variables", (p 203),

whereas "for stationary series such misspecification results in inconsistent estimates" (p 204).

We continue to assume that competitiveness does not share a common stochastic trend with the volume of UK exports. UK competitiveness is under this hypothesis a trendless, stationary series, and so the competitiveness elasticity must be expected to be sensitive to dynamic specification, in complete contrast to the world trade elasticity.

The warnings in Hendry (1986) and in Wickens and Breusch (1988) concerning the importance of checking the non-stationarity of regressors in putative co-integrating relationships may need to be strengthened. Given that the usual tests for unit roots have weak power to reject the null of non-stationarity in the face of local alternatives, it is entirely possible that *despite* attempting to check that all variables are non-stationary before formulating a co-integrating equations, researchers may, being unable to reject it, erroneously proceed on the assumption that all variables are non-stationary.[40] The

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39. Under a maintained hypothesis that there exists a co-integrating relationship involving export volumes, world trade and competitiveness, we might put the erratic behaviour of the estimated competitiveness coefficient down to finite sample bias.

40. See the section on testing for co-integration below for comments on the risks of invalid inferences that series are co-integrated.

export volumes equations may provide an important example of the dangers of the too-easy application of the co-integration approach to the specification of dynamic relationships. The outstanding issue appears therefore to be whether competitiveness is a stationary or a non-stationary variable.

5.5 THE WICKENS AND BREUSCH APPROACH TO DYNAMIC SPECIFICATION.

In order to investigate our hypothesis we find it convenient to draw on the recommendations made by Wickens and Breusch (1988).[41] The authors seek to address those cases "when interest focuses mainly on the long-run properties of a model" (p 189).In what follows we note those aspects of the Wickens and Breusch paper which have proved particularly helpful in the development of the overall arguments presented in this work, or else have influenced the particular methods employed in the supporting research.

Renormalisation and Direct Estimation of the Long-Run Effects.

To begin with, Wickens and Breusch take up Bewley's (1979) approach to the direct estimation of the equilibrium response. Given our interest in the long-run elasticities there is a practical convenience in being able to estimate them (and their standard errors) directly. A number of alternative transformations or renormalisations are put forward which achieve Bewley's convenient result, but in which further convenience is gained. For our purposes, the most convenient of the renormalisations proves to be the form in which short-run dynamics are specified as a distributed lag in the differenced variables (lags of first differences only). From a practical point of view it makes easier the use of polynomial distributed lags in order to achieve a parsimonious representation of the short-run dynamics - which are not themselves the primary focus of our interest. Such restrictions may win valuable degrees of freedom for the examination of the long-run elasticity on competitiveness.

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41. Wickens and Breusch (1988) on 'Dynamic Specification, the Long Run and the Estimation of Transformed Regression Models'.

For a general export volume equation:

$$X_t = \Sigma_{i=1}^{m} \alpha_i \; X_{t-i} + \Sigma_{i=0}^{n} \beta_i \; W_{t-i} + \Sigma_{i=0}^{p} \delta_i \; C_{t-i} + u_t \qquad (5.2)$$

taking X_t to represent export volumes, W_t, the level of world trade, and C_t, competitiveness.

The Wickens-Breusch renormalisation which we employ is:

$$\begin{aligned} X_t = {} & (\Sigma_{i=0}^{n} \beta_i)/\phi \; W_t + (\Sigma_{i=0}^{p}) \delta_i)/\phi \; C_t \; - \\ & \{\Sigma_{i=0}^{m-1} (\Sigma_{j=i+1}^{m} \alpha_j)/\phi \} \; \Delta X_{t-i} \; - \\ & \{\Sigma_{i=0}^{n-1} (\Sigma_{j=i+1}^{n} \beta_j)/\phi \} \; \Delta W_{t-i} \; - \\ & \{\Sigma_{i=0}^{p-1} (\Sigma_{j=i+1}^{p} \delta_j)/\phi \} \; \Delta C_{t-i} \; + u_t/\phi \qquad (5.3) \end{aligned}$$

where $\phi = 1 - \Sigma_{i=1}^{m} \alpha_i$.

The renormalisations entail the presence of first differences of the dependent variable among the regressors. Wickens and Breusch propose the use of the ordinary instrumental variables estimator in which the instruments are the regressors from the original equation (before renormalisation). (p 197). These estimates are shown to be identical to the least squares estimates on the original equation. It is also shown that the estimated variance-covariance matrix is the same, and moreover, where θ is the normalising constant defining the particular transformation,

"that the usual residual based tests which are invariant to θ will give the same results".(p 199).

Wickens and Breusch comment on the implications for their analysis of non-stationary variables:

"Engle and Granger's co-integrating regression is identical to estimating the long-run multipliers from a model misspecified through the omission of short-run dynamics". (p 203).

But, referring to work of Gourieroux et al (1987), Phillips (1985, a,b, 1986,a,b) and Perron and Phillips (1986), Wickens and Breusch conjecture "that it is probable that the finite sample biases of the long-

run estimates will be reduced by not omitting the short-run dynamics" (p 203) in the case of co-integrating relationships.

Wickens and Breusch draw attention to the possible contrast between the properties of estimates of the long-run coefficients on non-stationary and stationary variables - in particular their asymptotic sensitivity to the misspecification of the short-run dynamics. They conclude with a proposal for a dynamic modelling strategy in the light of these matters. This proposal has strongly influenced the strategy employed in this research. We therefore quote it in full below:

"First estimate the long-run multipliers using one of the alternative formulations. For non-stationary series it may not be important to specify the length of the lag correctly. *For stationary series it may be best to overparameterise by including too many lags.* The conventional tests of misspecification can be carried out on this equation. If the equation passes these tests we may be reasonably confident that the long-run estimates have good statistical properties. If interest centres on the long-run multipliers we can stop at this point. *Any subsequent re-estimation for example to achieve parsimony in the short-run dynamics should yield very similar long-run estimates. If they are different then this is evidence that invalid restrictions have been imposed.*" (p 204, italics added)

We should be concerned at the evidence (described in our survey of the UK manufactured export volume literature) that the long-run competitiveness elasticity appears to be sensitive to the specification of the short-run dynamics.

5.6 RE-ESTIMATION OF VARIOUS SPECIFICATIONS ON COMMON DATA AND RENORMALISED FOR DIRECT ESTIMATION OF THE LONG-RUN PROPERTIES.

5.6.1 Introduction.

To provide a starting point for our empirical work we re-estimate a number of specifications using common variable definitions drawn from the LBS dataset.[42] The specifications are intended to enable us to represent the development of UK export volume equations while using a common competitiveness variable. We estimate the equations over the approximate sample period for which similar specifications were developed. Thus the resultant estimates of the long run competitiveness

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42. See data appendix for details.

elasticity are corrected for the choice of competitiveness variable, but still reflect variations in both dynamic specification and sample period.

5.6.2 The Equations.

The equations estimated were selected to represent the development of manufactured export volume equations among UK modellers, but on common data. These equations reflect the consequences of key differences in modelling procedures. All these results are obtained with an instrumental variables procedure which permits the direct estimation of the long run properties, as suggested in Wickens and Breusch (1988).[43]

Four broad approaches to the dynamic specification of export volumes are reflected in our re-estimated equations.

Firstly we estimate the kind of long distributed lag model suggested in Brooks (1981) featuring around four years (17 quarters) of lags on competitiveness, using fifth order Almon lag restrictions on the short run dynamics. Such an equation is also similar to the then prevailing Treasury model considered by Brooks.

Secondly, two models of similar form but employing shorter lags on competitiveness (two years) are estimated. These correspond to the equation reported by Brooks in 1981 as the then prevailing NIESR model and to the NIESR model 8. In the former case the distributed lag begins at the current value,

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43. In principle this procedure will produce identical estimates to those which would be obtained without renormalisation. Some complications arise in our use of the procedure. Firstly we must decide how to deal with the use of Almon lag restrictions on the levels of a variable in the original specification. We choose to reflect this by the specification of an Almon lag of similar order on the first difference of the variable in the renormalised model. We then include as instruments a lag structure in levels which omits any Almon restrictions. A second issue arises when an original specification omits intermediate lags. In order to ensure that unintended further restrictions are not built into the long run estimates we may choose to over-specify the renormalised form. A related point is that, regardless of the original model, the levels term capturing the long run effect in the renormalised equation is entered as the first included lag - usually the current value.

whereas in the latter it begins at the fifth quarter: in both cases a fifth order Almon lag of seven quarters is fitted.

Thirdly we estimate the autoregressive distributed lag model favoured by the LBS, including lags of up to four quarters on all variables.

The final approach used is a more restricted autoregressive specification corresponding to a first order error correction model, and very similar to the Treasury's preferred 1987 model.

Note that in the case of the first and fourth cases, each of which correspond to sometime Treasury approaches, the equations are estimated both with and without a time trend. An additional variation is added by the imposition of a unitary coefficient on world trade along with the time trend.

Further details of the actual specifications are set out in table 5.6.

In each case the sample period over which the equation is estimated corresponds to the period over which similar models were developed and estimated.

The estimated long run properties for these various specifications are given in table 5.7.

5.6.3 Results.

The results show a large range of values for the long run competitiveness elasticity, from high values of over -0.9 for the long lag models estimated up to 1979 to a low value of below -0.2 for the error correction models estimated from 1973 to 1985. The variation in estimated elasticities can thus be seen *not* to be merely a product of differing choices of competitiveness variables.

Table 5.6.

SPECIFICATION DETAILS FOR VARIOUS UK MANUFACTURED EXPORT VOLUME EQUATIONS AS RE-ESTIMATED USING A COMMON MEASURE OF COMPETITIVENESS.

All equations use COMT as a competitiveness measure.

Sample Period.	Specification.
1) 'LONG LAGS' APPROACH.	
cf. Brooks (1981) and HMT 1979.*	
1967:3 -1979:2	X = W, C, Almon(5,16,far) on DC, T, DD, cons
1967:3 -1979:2	X-W = C, Almon(5,16,far) on DC, T, DD, cons
1967:3 -1979:2	X = W, C, Almon(5,16,far) on DC, DD, cons
2) 'MEDIUM LAGS' APPROACH.	
cf. NI in Brooks (1981).	
1965:2 -1979:2	X = W, C, Almon(5,7,far) on DC, DD, cons
cf. NI Mark 8.	
1967:3 -1979:2	X = W, C(-5), Almon(5,16,far) on DC(-5), DD, cons
3) 'LBS TYPE' ERROR CORRECTION APPROACH.	
...LBS 1985 equation.	
1963:4 -1983:4	X = W, C, DX, DX(-1), DC, DC(-1), DC(-2), DD, cons
cf HMT 1987.	
1973:3 -1985:4	X = W, C, DX, DW, DC, T, DD, cons
1973:3 -1985:4	X-W = C, DX, DW, DC, T, DD, cons
1973:3 -1985:4	X = W, C, DX, DW, DC, DD, cons

Notes: In the above we represent export volumes by X, world trade by W, and competitiveness (COMT) by C. The prefix D denotes first differences and the suffix (-i) denotes the ith lag. T represents a time trend, DD a composite strike dummy, and 'cons' the constant term.
In each case the sample period is chosen as typical of research using similar specifications.

Table 5.7.

RESULTS OF RE-ESTIMATION OF VARIOUS UK MANUFACTURED EXPORT VOLUME EQUATIONS USING COMMON VARIABLES.

Sample Period. of Equation.	Standard Error	Long Run Elasticities: World Trade.	Competitiveness
1) 'LONG LAGS' APPROACH.			
1967:3 -1979:2	0.0288	0.709 (0.064)	-1.099 (0.247)
1967:3 -1979:2	0.0321	1.0 (Imposed)	-1.083 (0.293)
1967:3 -1979:2	0.0296	0.565 (0.037)	-0.924 (0.216)
2) 'MEDIUM LAGS' APPROACH.			
1965:2 -1979:2	0.036	0.617 (0.023)	-0.231 (0.129)
1967:3 -1979:2	0.0346	0.607 (0.015)	-0.398 (0.052)
3) 'LBS TYPE' ERROR CORRECTION APPROACH.			
1963:4 -1983:4	0.0688	0.580 (0.016)	-0.360 (0.061)
1973:3 -1985:4	0.0393	0.959 (0.110)	-0.179 (0.026)
1973:3 -1985:4	0.0394	1.0 (Imposed)	-0.180 (0.027)
1973:3 -1985:4	0.0347	0.477 (0.025)	-0.179 (0.026)

Notes:

All equations use COMT as a competitiveness measure.
Heteroskedastic consistent standard errors in parentheses.

In contrast to the widely fluctuating results for the competitiveness elasticity, the results for the long run world trade elasticity are much more stable. When time trends are excluded estimates of this quantity are all close to 0.6, except for the estimates for the most recent sample considered (1973-1985) for which the estimated elasticity falls to 0.5.

The inclusion of a time trend appears to have little effect of the estimated competitiveness elasticity though it does affect the estimation of the long run elasticity on world trade. The evidence illustrates the view, expressed above, that the presence of a time trend permits the long run elasticity on world trade to be restricted to unity without too much violence to the data. Nevertheless, for reasons already referred to, we still prefer to omit time trends from the export volume equation.

The use of lagged dependent variables does not appear to have a clear cut effect on the estimated competitiveness elasticity. The contrasting approaches of the National Institute Mark 8 model and its LBS contemporary (both from the 1985 vintage of models) are illuminating on this point. The first omits lagged dependent variables while the second includes them, but they produce similar elasticities on common data. By way of contrast our results show that there is a notable difference between the estimates produced by the LBS and Treasury/ECM models, both of which include lagged dependent variables but to different lag lengths.

The Treasury have argued that the estimated competitiveness elasticity falls as the sample period moves forward in time. With the highest values obtained for the period 1973:3 to 1985:4, the results of our re-estimations might be taken to support this conclusion. However, the 'medium lag' specification estimated for 1965:2 to 1979:2, with an estimated long run competitiveness elasticity of -0.23, provides an important counterexample to the hypothesis that the sample period is the prime cause of differing elasticity estimates: something else must be going on.

We conclude that our results indicate that the variety of the estimates of the long run competitiveness elasticity in large measure reflects differences in the short run dynamic specification of the different equations with respect to competitiveness. The sensitivity of this estimate to short run dynamics is in

marked contrast to the behaviour of the estimates of the long run elasticity on world trade. Thus these results justify the direction of our subsequent research in which we explore the extent to which the lag structure on competitiveness, rather than any other aspects of equation specification, is what matters.

5.7 A LOOK AT THE DATA: TIME SERIES PROPERTIES OF THE VARIABLES.

5.7.1 Introduction.

We observe that export volumes and world trade are strongly trended variables, and it is clear that competitiveness does not share in this dominant characteristic. However if "competitiveness follows a random walk" as is often asserted, it could share in a common stochastic trend..

Our principal aim is thus to establish whether the competitiveness series is best characterised by a stochastic trend. We consider various qualitative features which "allow the data to speak for themselves" in order to distinguish I(1) and I(0) behaviour.[44] We find that the qualitative evidence indicates that COMT is an I(0) variable before 1979. However, for the period after 1979 we cannot discriminate so clearly between the alternative I(1) and I(0) interpretations.

We also consider formal tests of the hypotheses that the univariate time series feature a unit root. These may fail to reject the hypothesis of a unit root in competitiveness when other evidence points clearly to the absence of a unit root. There is an important practical point here: if we relied on such formal tests alone we might employ a co-integration approach for the whole equation, even though other evidence show this to be inappropriate. We estimate cointegrating regressions in this way and show how this gives rise to misleading estimates of the long run coefficient on competitiveness.

We have taken XMAN and XWM to be obviously trended variables, but we have not investigated their trends exhaustively. We certainly would suspect that they share a common trend, and so, regardless of

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44. As detailed in Engle and Granger (1987).

the properties of competitiveness, we look for (and, at least prior to 1979, find) a co-integrating relationship between XMAN and XWM. We have chosen not to investigate the nature of the trends in XMAN and XWM any further. We note that these variables could feature deterministic trends, stochastic trends, or possibly both deterministic and stochastic trends. It is not however evident that we could discriminate between these alternatives.[45]

5.7.2 Overview of the Series.

To begin with we plot the time series for UK manufactured export volumes (XMAN), world trade in manufactures (XWM), and UK tax adjusted cost competitiveness (COMT). These time series are shown in figures 5.1.

On inspection we see that XMAN and XWM are both trended upwards. XMAN appears to show more short run variations relative to trend than does XWM, which is consistent with XWM's higher level of aggregation. It seems possible to adopt the working hypothesis that XMAN and XWM share a common trend, with UK exports assumed to reflect the general growth in world trade.[46]

In contrast, COMT displays no strong trend throughout the sample period.[47] A further striking feature of the series is its cyclic component. However, on the basis of this inspection alone we cannot dismiss a possible unit root from consideration.

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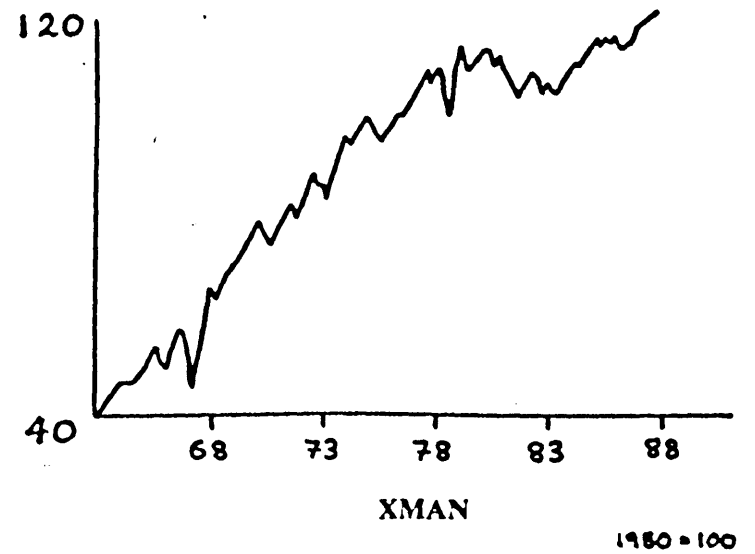
45. Some attempts, (not reported here), to apply Augmented Dickey Fuller (ADF) tests for a unit root after deterministic detrending suggested that we might be able to reject the combination of stochastic and deterministic trends. However, we would still be unable to choose between deterministic and stochastic trends.

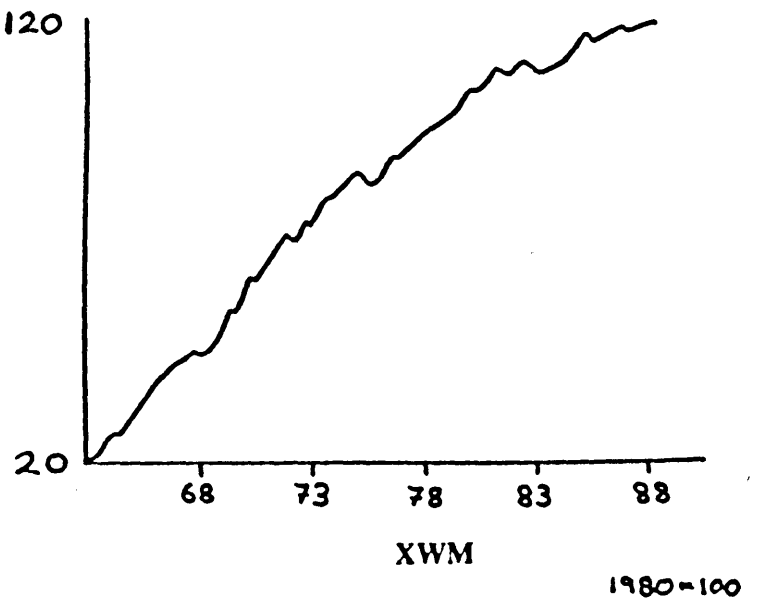
46. Note that under this working hypothesis we would not need to include a separate trend in a regression of XMAN on XWM. This is something that the Treasury have persistently done, and as we have suggested, permits the restriction of the income elasticity to unity. We do not favour this approach.

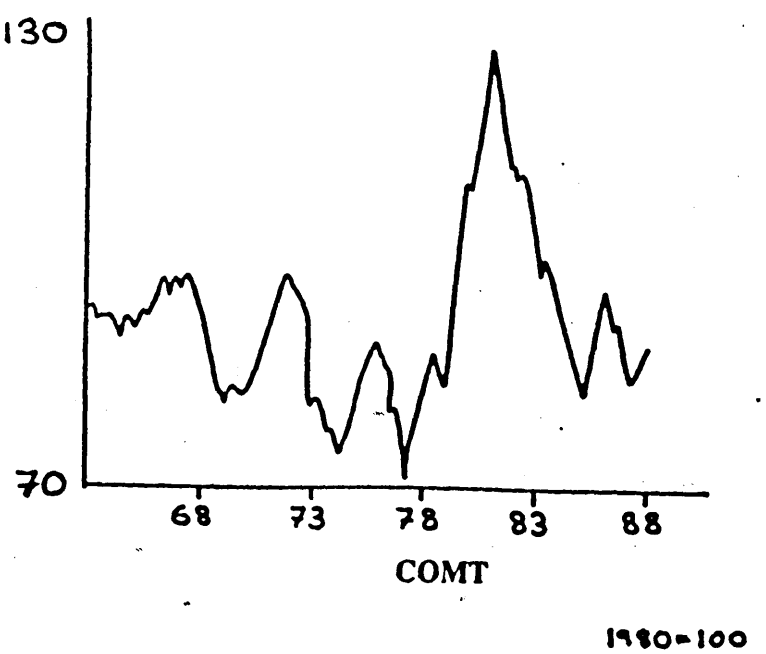
47. It is possible to argue for the existence of a relatively weak downward trend in the 1960s, but this is not something that has been maintained since, reflecting Granger's (1966) comments to the effect that the existence of trends can depend on the choice of sample.

Figure 5.1

PLOTS OF TIME SERIES.







We have already referred to the other outstanding feature of the time series properties of COMT, which is the existence of a sharp change in its behaviour after 1979. This corresponds to the regime change associated with the change of government and its approach to monetary and exchange rate policy. After this point the amplitude and frequency of the series fluctuations appear to be increased.

A question of particular interest is whether the effect on UK exports of the large shock to competitiveness in the early 1980s was a temporary or a permanent one. To the extent that the effect on export volumes is permanent we might infer the presence of hysteresis effects in trade.[48]

However, the key question we must address before proceeding is whether competitiveness contains a unit root, or else follows a stationary process (once externally identified regime shifts have been allowed for).

5.7.3 Qualitative Features.

Engle and Granger (1987), pp 252-3, set out five qualitative aspects in which the theoretical properties of I(0) and I(1) series differ. The differences concern the duration of the effect of innovations, the frequency of crossings of the mean or starting value, the limiting variance of the series, the pattern of sample autocorrelations and the shape of the spectrum of the series. In order to employ these qualitative features we must of course consider the sample analogues to the theoretical properties. We consider the five points in turn.

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48. Note that if we have difficulty in identifying the nature of the upwards trend in export volumes we will find it difficult to identify whether particular episodes represent permanent or transitory departures from that trend, and hence to identify whether or not a large shock to competitiveness has a permanent hysteresis type effect on export volumes.

a) Innovations.

In theory innovations have only a temporary effect on the value of a time series which is I(0), in contrast to a permanent effect on a series which is I(1). In practice we cannot examine this aspect without the use of a formal model to yield us estimates of the innovations.[49]

b) Crossings.

In theory, for an I(0) series, the expected time between crossings of the series mean is finite, whereas for an I(1) series, the expected time between crossings of the initial value is infinite. In practice, we employ the sample time between crossings as an estimator of the expected time. Details of the evidence concerning crossings for the three time series are given in Table 5.8.

For the entire sample 1963:3 to 1987:2, and for the sub-period 1963:3 to 1979:2, the patterns of crossings identify XMAN and XWM as I(1) and COMT as I(0).

The sub-period 1979:3 to 1987:2 is less clear cut. Once again XWM displays no crossings and is clearly I(1). However, the behaviour of XMAN is influenced by short run fluctuations and is consistent with an I(0) process rather than the I(1) process.[50] As for COMT, the crossings of the initial value are the best evidence in favour of the I(0) property of the series, with an expected time between crossings of 2 years.

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49. We can view formal tests for a unit root in the time series as addressing this point, conditional on the assumed form for the process.

50. If, after 1979, XMAN is close to I(0) we should be cautious about the precision of estimates of the income elasticities, and also expect these estimates to be sensitive to the specification of short-run dynamics. This suggests a sceptical approach to Landesmann and Snell's results concerning the income elasticity after 1979.

Table 5.8.

EXPECTED TIME BETWEEN CROSSINGS OF INITIAL VALUE OR SAMPLE MEAN.

Results for three periods on export volumes (XMAN), world trade (XWM), and competitiveness (COMT).

1967:3 - 1987:2	98 quarterly observations.
XMAN	No crossings of initial value.
XWM	No crossings of initial value.
COMT	8 crossings of sample mean value of COMT = 90.58 with a sample period of 12.25 quarters between crossings. The expected time to crossing is 3 years.
1967:3 - 1979:2	66 quarterly observations.
XMAN	No crossings of initial value.
XWM	No crossings of initial value.
COMT	6 crossings of sample mean value of COMT = 86.36 with a sample period of 11 quarters between crossings. The expected time to crossing is 2 years 9 months.
1979:3 - 1987:2	32 quarterly observations.
XMAN	2 crossings of initial value to yield an expected time to crossing of 16 quarters or 4 years. 3 crossings of the sample mean of 103.0 yield an expected time to crossing of 2.66 years.
XWM	No crossings of initial value.
COMT	1 crossing of sample mean value of COMT = 99.28 with a sample period of 16 quarters between crossings yields an expected time to crossing of 4 years. 2 crossings of the initial value give an expected time to crossing of 2 years.

C) Limiting Variance.

In theory the variance of an I(1) process goes to infinity as the sample size increases, whereas the variance of an I(0) process will approach a finite limit. In practice we must use sample variances and aim to draw conclusions concerning the limiting variances from these. To do this we use the device of contrasting the variance of each series with the variances of its own first and second differences.

In table 5.9 we present results for the three variables, XMAN, XWM and COMT. The variance of the original series, its first difference and second difference are given in each case for the periods 1963:3 to 1987:2, 1963:3 to 1979:2 and 1979:3 to 1987:2. We also give the approximate ratio of the variance of each series in levels to that in its first differences. The results also show very clearly that XWM is I(1). In contrast the behaviour of COMT corresponds to a series with a finite limiting variance ie. to an I(0) series. In the case of XMAN, the crucial evidence is that the ratio does increase with the sample size. This is indicative that the variance of the series does not approach a finite limit, and hence that XMAN is I(1).

d) Sample Autocorrelations.

In theory the autocorrelations for an I(1) series will approach unity at all lags as the sample size increases relative to the lag length of the autocorrelation. In the case of an I(0) series, autocorrelations decrease steadily with their lag length so that the sum of all autocorrelations remains finite. These autocorrelation properties correspond to the result that the effect of a shock on an I(1) series is permanent, whereas the effect of a shock on an I(0) series will decay and so be temporary.

In practice we employ sample autocorrelations.[51] We calculate sample autocorrelations for the three series XMAN, XWM and COMT over the three periods, and then plot them against lag length to form the correlogram. These results are plotted at figures 5.2, 5.3 and 5.4.

Table 5.9.

SAMPLE VARIANCES OF TIME SERIES.

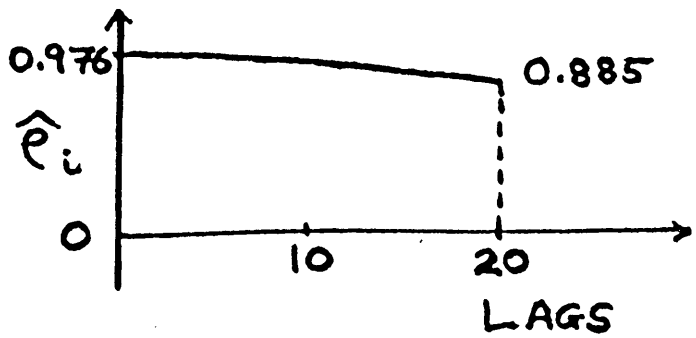
	a) Variance of series	b) Variance Differenced 1st Diff	c) of Series: 2nd Diff	d) =(a)/b)) Ratio of Variances in Levels to 1st Differences. (Approx)
1963:3 - 1987:2				
	a)	b)	c)	d)
XMAN	455.62	23.85	72.63	19
XWM	953.44	4.02	8.72	237
COMT	140.54	15.87	23.1	9
1963:3 - 1979:2				
	a)	b)	c)	d)
XMAN	339.14	27.72	74.83	12
XWM	481.15	2.93	6.78	164
COMT	44.39	9.02	13.39	5
1979:3 - 1987:2				
	a)	b)	c)	d)
XMAN	74.07	14.63	39.8	5
XWM	90.4	6.55	13.63	14
COMT	230.73	29.21	43.28	8

Notes:
XMAN is volume of UK manufactured exports.
XWM is volume of world trade.
COMT is LBS cost competitiveness adjusted for payroll taxes.

Figure 5.2

CORRELOGRAMS FOR UK MANUFACTURED EXPORT VOLUMES (XMAN).

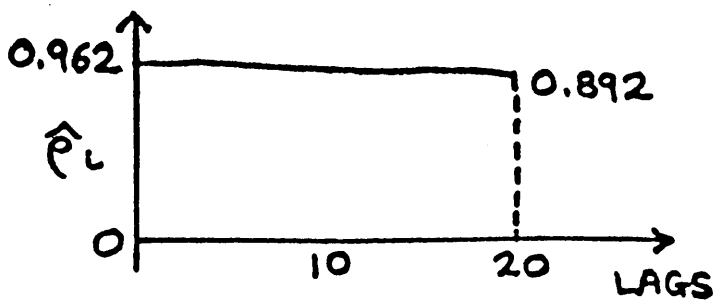
a) 1963:1 TO 1987:2



ρ_i^ is the ith sample autocorrelation.

(PC-GIVE: GIVA)

b) 1963:1 TO 1979:2



c) 1979:2 TO 1987:2

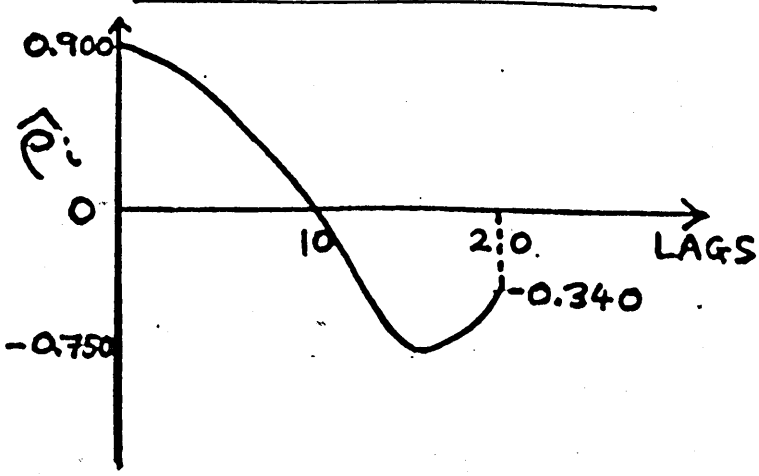
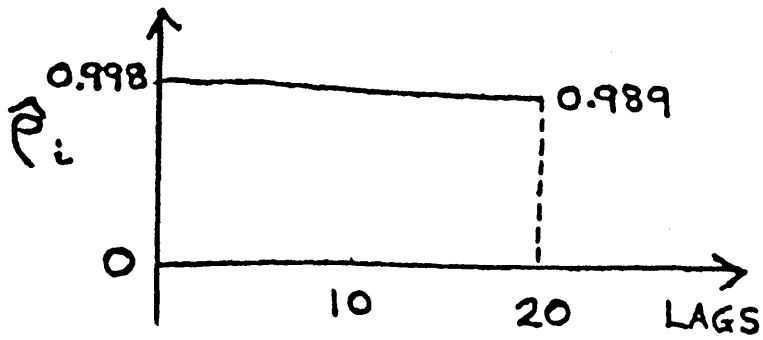


Figure 5.3

CORRELOGRAMS FOR VOLUME OF WORLD EXPORTS OF MANUFACTURES (XWM).

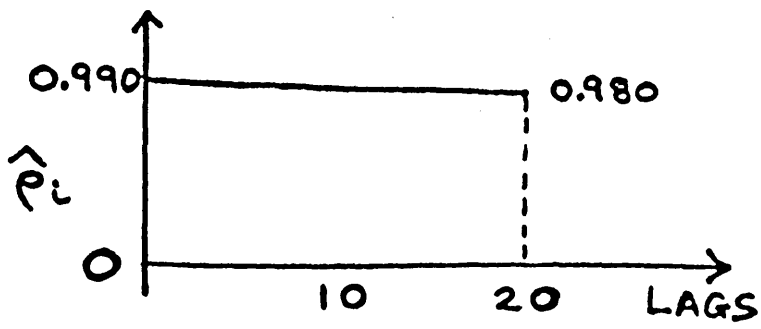
a) 1963:1 TO 1987:2



ρ_i^ is the ith sample autocorrelation.

(PC-GIVE: GIWA)

b) 1963:1 TO 1979:2



c) 1979:2 TO 1987:2

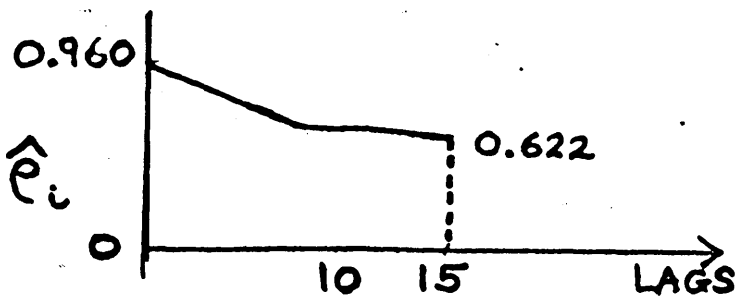
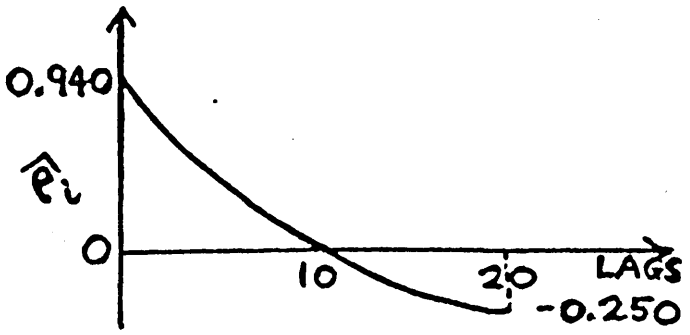


Figure 5.4

CORRELOGRAMS FOR UK COST COMPETITIVENESS (COMT).

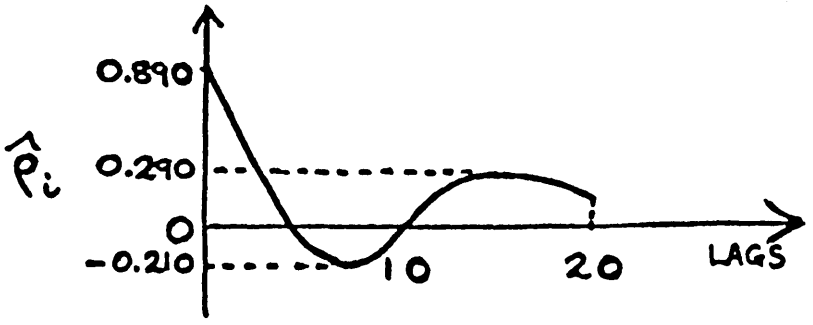
a) 1963:1 TO 1987:2



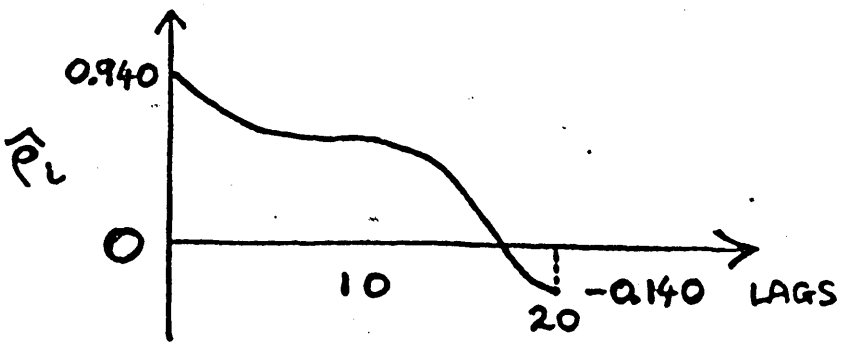
ρ_i^ is the ith sample autocorrelation.

(PC-GIVE: GIVA)

b) 1963:1 TO 1979:2



c) 1979:2 TO 1987:2



For the whole sample, and also for the pre-1979 sub-sample, the autocorrelations for COMT display the properties associated with an I(0) series, in that they decline and approach zero after around ten lags. The correlogram for the post-1979 period declines less rapidly, although not to the extent that we *must* rule out stationarity.

In contrast the autocorrelations for the trended variables do not approach zero. This is most clearly the case for XWM, but it is also true of XMAN. Note that the sample evidence for XMAN is once again less clear cut for the post-1979 period, when the correlogram is similar in shape to that for COMT over the same period.[52]

e) Spectral Density.

In theory the spectral density is infinite at the zero frequency for an I(1) variable, whereas it is finite at the zero frequency for an I(0) variable.[53]

In practice we employ a smoothed version of the periodogram as our estimator of the spectral density.[54] The key theoretical distinction on which we seek inferences from the sample evidence is between infinite and finite values at the zero frequency.

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51. We have calculated these using both the TSP BJIDENT routines and the PC-GIVE GIVA procedure. The results obtained differ slightly, probably reflecting the use of different degree of freedom corrections.

52. A different presentation of the autocorrelation evidence is the treatment of the estimated spectra. Will the alternative presentation qualify our conclusions from this section at all?

53. In the I(1) case, we know, further, that the spectral density can be approximated for low frequencies as f(w) = A w(-2d). Engle and Granger, p 251.

54. We experimented with five different patterns for the weighting function - the spectral window or kernel. The weighting patterns were 1,2,1, 1,2,3,2,1, 1,2,2,2,1, 1,2,3,4,3,2,1, and 1,2,2,2,2,2,1. The first of these appeared sufficient to perform the smoothing task. See Fuller (1976), Granger and Newbold (1986).

In figures 5.5, 5.6, and 5.7 we present the sample spectra derived employing the preferred spectral window for the three variables XMAN, XWM, and COMT over the three sample periods.

The sample spectra for XWM all display Granger's typical shape in which the lowest frequency components predominate. The sample spectra for XMAN are similar. The largest contribution to the spectra for COMT, is from the lower frequencies. But prior to 1979 a distinct strong peak stands out corresponding to a period of about three years. In the post-1979 sample period the spectrum for COMT appears to have relatively more weight in its middle 'business cycle' frequencies than is the case for the trended variables XMAN and XWM.

The spectral evidence suggests that COMT is a stationary variable when taken over the whole sample, and especially so over the period before 1979. If the post-1979 period alone were considered we would be less confident that we could rule out I(1) behaviour. The estimated spectra do reflect a noticeable change in regime for COMT at 1979, with this regime change being manifest in a more irregular cyclic component in the later period.

5.7.4 Formal Tests of the Unit Root Hypothesis.

We present values of the DW statistic for XMAN, XWM and COMT for a number of sample periods at table 5.10.[55]

An examination of the results reveals that in no case can we reject the null even at a 5% level. We may however note a pattern familiar from our qualitative explorations: it is clearly going to be harder to reject the unit root hypothesis for COMT for the post-1979 period than the pre-1979 period, given these test results.

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55. See Sargan and Bhargava (1983) in which critical values are given for the Durbin Watson statistic based on the distribution under the unit root null hypothesis.

Figure 5.5

SAMPLE SPECTRAL DENSITIES FOR XWM.

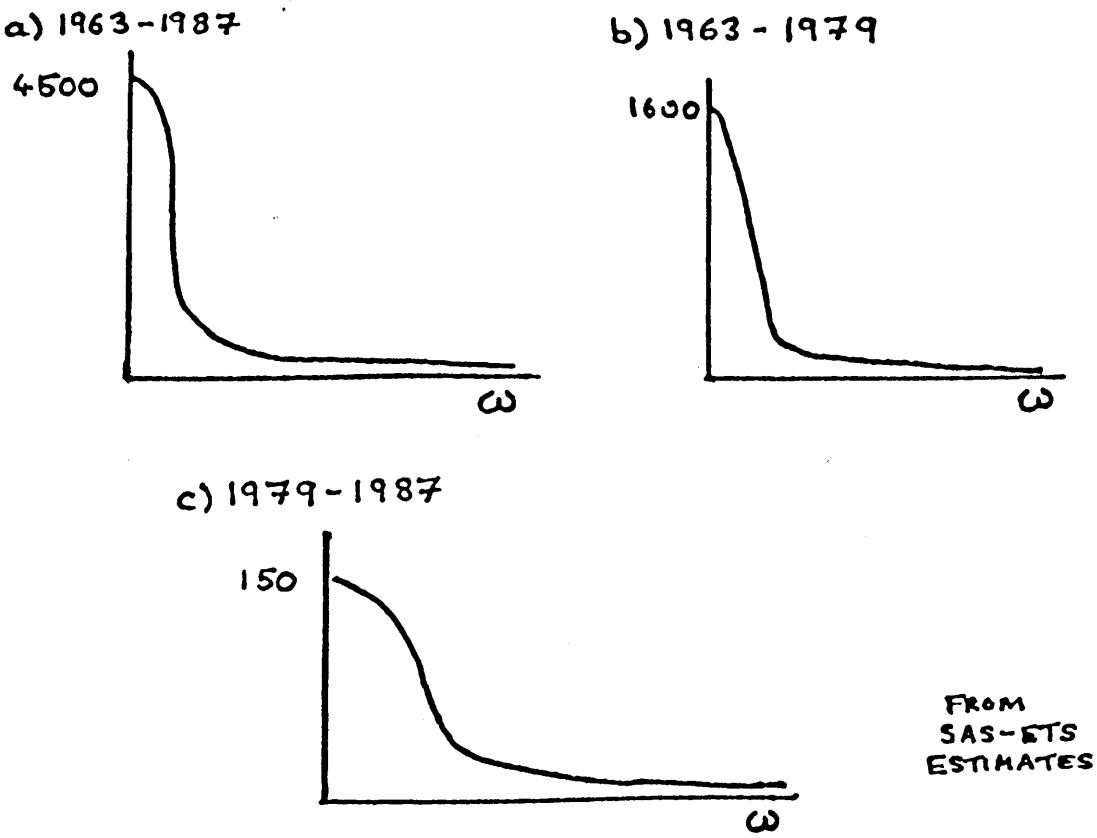

Figure 5.6

SAMPLE SPECTRAL DENSITIES FOR XMAN.

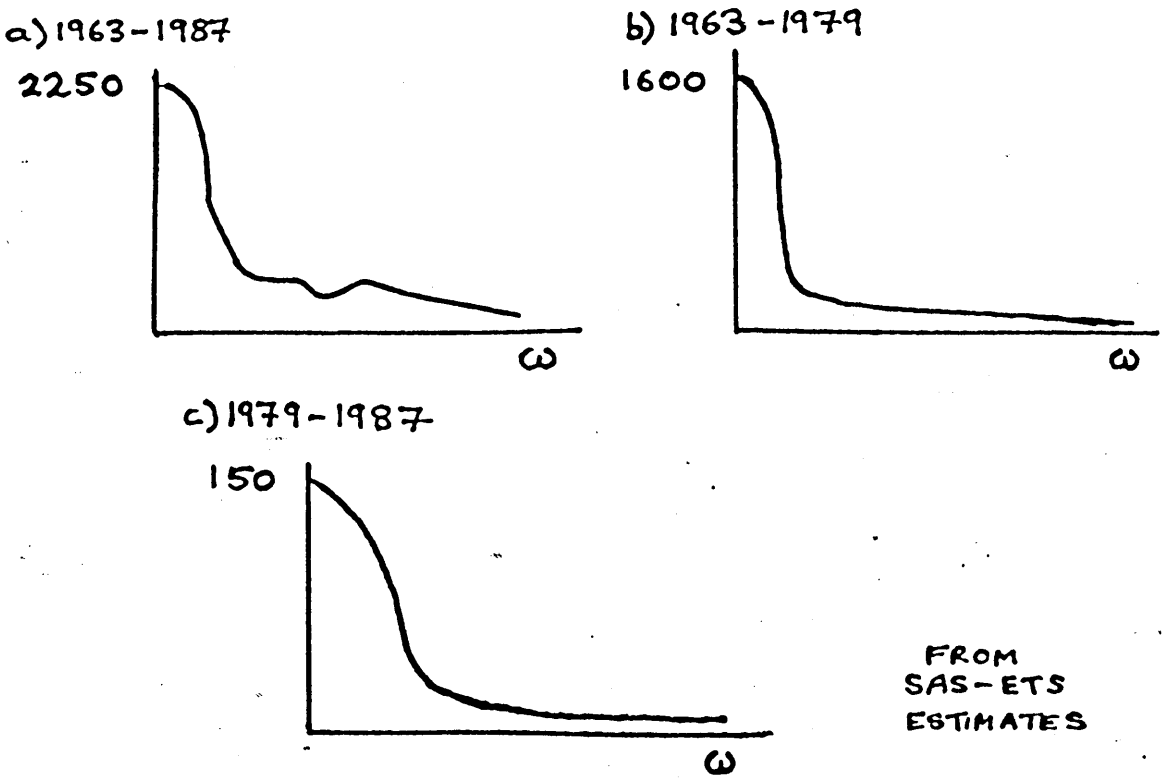

Figure 5.7

SAMPLE SPECTRAL DENSITIES FOR COMT.

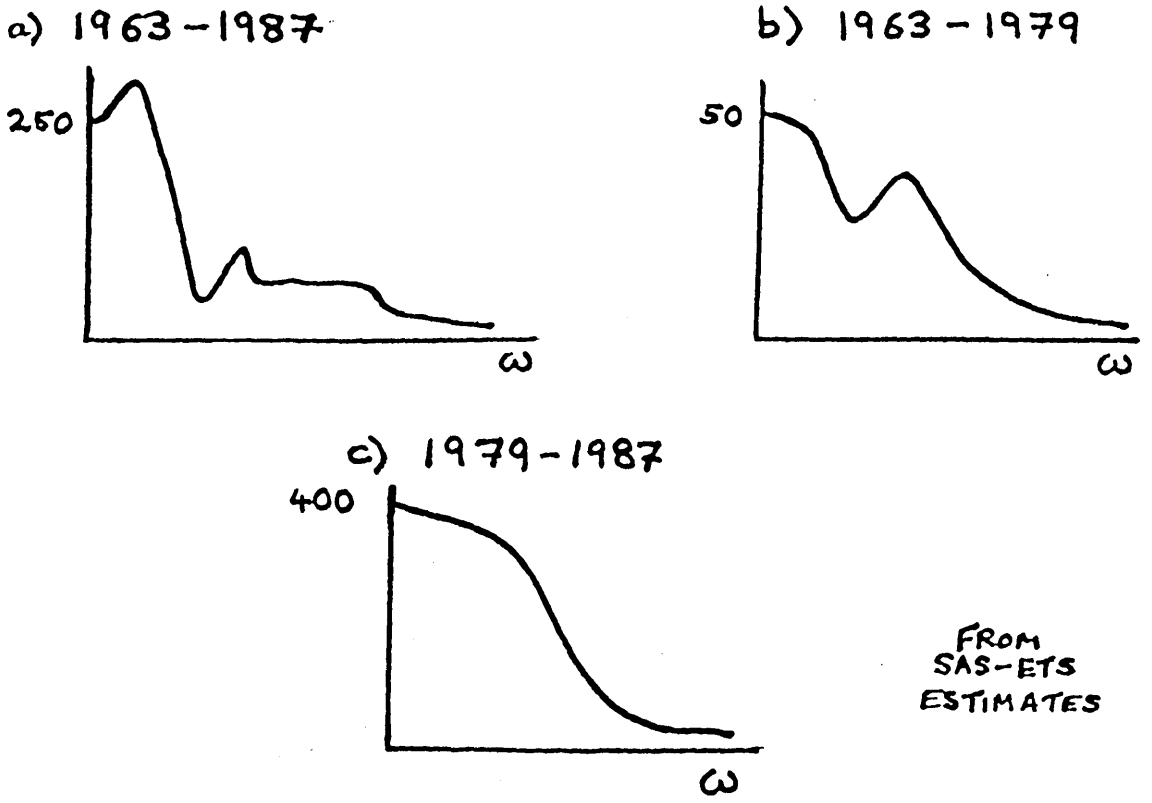

Table 5.10.

DURBIN WATSON (DW) STATISTICS AS TESTS OF INTEGRATION.

Sample Period.	Time Series		
	XMAN	XWM	COMT
1964:1 - 1987:2	0.0487	0.0046	0.1185
1967:2 - 1987:2	0.1016	0.0094	0.1176
1964:1 - 1979:2	0.0872	0.0087	0.2098
1979:2 - 1987:2	0.2195	0.0778	0.1319

Notes:
XMAN is volume of UK manufactured exports.
XWM is volume of world trade.
COMT is LBS cost competitiveness adjusted for payroll taxes.

Critical values for the DW statistic as a test of the null of a unit root for a univariate time series are given in Sargan and Bhargava (1983).

Taken on their own, the formal test results cannot reject the hypotheses that all three variables are I(1) in all periods.[56] We might typically proceed to test for the existence of a co-integrating regression on this basis.

An alternative approach to testing for the presence of unit roots based on the correct distribution of the t statistic under the null has been developed by Dickey (1976) and Dickey and Fuller (1979, 1981). Of particular interest for our purposes are the 'augmented Dickey-Fuller' tests for a pth order autoregressive representation.

The results of Dickey-Fuller tests for the presence of unit roots are presented (at table 5.11) for COMT, and also for XMAN and XWM, for the entire sample and for pre and post-1979 sub-periods. The null of a unit root is tested in maintained models of 1st, 4th, 8th, 12th and 16th order autoregressions.

Some of the augmented Dickey-Fuller tests appear to offer a gain in power relative to the first order treatment. For the period prior to 1979 the results confirm the view that COMT is best represented as a stationary process (of at least 4th order). However, for the period from 1979 we are still unable to reject the null of a unit root.

When we consider XMAN we see that we cannot reject the null that it contains a unit root, for full sample and sub sample periods. Similarly we cannot reject this null for XWM over the sub-periods.

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56. There are reasons to suspect that the test may lack power against alternatives with roots close to unity. For example, in their application of similar unit root tests to co-integrating regression residuals, Engle and Granger studied power against different autoregressive alternatives (pp 268-276) and reported a sharp drop in power from a first order alternative with a coefficient of 0.8 to one with a coefficient of 0.9: this drop in power occurring for both Sargan-Bhargava and Dickey-Fuller tests.

Table 5.11.

DICKEY FULLER AND AUGMENTED DICKEY FULLER TESTS OF NULL HYPOTHESIS OF UNIT ROOT IN TIME SERIES.

Time Series.	Sample Period.	Dickey Fuller. DF	Augmented Dickey Fuller Statistics. ADF(4)	ADF(8)	ADF(12)	ADF(16)
XMAN						
	1963:3 - 1987:2	-1.82	-1.49	-1.62	-1.6	-1.43
	1963:3 - 1979:2	-1.42	-1.16	-1.07	-1.01	-0.97
	1979:3 - 1987:2	-0.85	-0.52	0.3	1.14	-1.55
With Trend						
	1963:3 - 1987:2	-3.6 (10%)	-2.04	-1.63	-1.56	-1.87
	1963:3 - 1979:2	-4.91 (1%)	-2.52	-2.08	-1.14	-1.17
	1979:3 - 1987:2	-2.69	-2.12	-1.06	-1.42	-4.23 (10%)
XWM						
	1963:3 - 1987:2	-2.4	-2.6 (10%)	-3.0 (5%)	-3.62 (1%)	-2.91 (10%)
	1963:3 - 1979:2	-1.58	-1.67	-2.02	-2.59	-2.13
	1979:3 - 1987:2	-0.93	-0.72	-0.64	-0.32	0.29
With Trend						
	1963:3 - 1987:2	-1.96	-1.72	-1.47	-1.53	-1.48
	1963:3 - 1979:2	-1.92	-1.56	-0.77	-0.003	0.31
	1979:3 - 1987:2	-2.16	-2.09	-2.5	-2.36	-2.97
COMT						
	1963:3 - 1987:2	-1.5	-2.44	-2.44	-2.04	-1.96
	1963:3 - 1979:2	-1.77	-3.37 (5%)	-2.68 (10%)	-2.06	-1.64
	1979:3 - 1987:2	-0.67	-1.24	-1.69	-1.49	-1.66

Notes:
Figures in parentheses are approximate significance levels at which null of unit root can be rejected. All statistics allow for a constant. For XMAN and XWM we also test for a unit root in the detrended series.

5.7.5 Overall Conclusions.

The qualitative evidence and the results of formal tests concerning the stationarity of the time series, leads us to some clear conclusions.

Firstly COMT is a stationary variable for the period 1963-1979. While the formal tests cannot all reject the null of a unit root, some can, and the qualitative aspects undoubtedly tip the balance of evidence.

Secondly, while the strongly trended behaviour of XWM may reflect a deterministic trend it is also consistent with I(1) behaviour (ie. a stochastic trend). Given the I(0) nature of competitiveness there is no urgent need to distinguish more finely the exact nature of the trend component.

Thirdly, the trended behaviour of XMAN is clearly consistent with I(1) behaviour prior to 1979. After 1979, while less clear cut, the evidence remains consistent with the I(1) assumption. The behaviour of XMAN after 1979 may reflect the changed behaviour of COMT in the post-1979 period.

The evidence is much less clear cut with regard to COMT in the post 1979 period. None of our formal tests are able to reject the null hypothesis of a unit root (ie. that COMT is I(1)). Turning to the qualitative evidence, the sample spectral density, the sample time to crossings and the autocorrelations all fail to give a decisive indication.

We have taken the failure of the variance of the series to increase with the sample size as suggestive of an I(0) process. However this argument is vulnerable due to apparently increased innovation variance after 1979.[57] Our choice between representations for COMT in the post-1979 period will therefore have to depend on our priors.

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57. This last point suggests two further important factors. Firstly, the sample size of the post-1979 period is relatively small, and this hinders statistical discrimination. Secondly we must acknowledge that the I(1) aspect of the behaviour of COMT after 1979 may in fact reflect the presence within the post-1979 sub-sample of another regime change, roughly corresponding to the departure of Sir Geoffrey Howe from the Treasury, marking the end of a policy of indifference to the exchange rate. The small sample size makes it quite unrealistic to attempt such further division of our analysis, but we must acknowledge that such a non-homogeneity in the data may have undesirable consequences.

One view is that following 1979 the British government had a policy of neglect with respect to the level of the real exchange rate. In the absence of an explicit stabilising policy, the random walk notion that today's value is the best guess of tomorrow's has appeal as a model. An alternative view would reject the unit root implication of unbounded forecast variance, arguing that regardless of relatively short run developments, inherent dynamics in the economy (or else in the whole political economy process), would eventually tend to return the real exchange rate to a mean value.

We are aware of the risks associated with an incorrect assumption that COMT is I(1) in terms of the possible misspecification of long run dynamics that can result. Mindful of this risk, we thus choose to adopt the working assumption that COMT follows an I(0) process after 1979 as well as before 1979. This choice will appear to be justified if we find that the estimate of the long-run coefficient on competitiveness is sensitive to the specification of short-run dynamics after 1979.

In any case we will explore the consequences of the I(1) assumption by proceeding as if with tests for co-integrating relationships. But our main argument will proceed on the basis that competitiveness is a stationary (I(0) variable).

5.8 ON TESTING FOR COMMON TRENDS.

5.8.1 Introduction.

We have concluded that both XMAN and XWM feature strong upward trends which may be deterministic, but are also consistent with the I(1) type of behaviour of a stochastic trend. It seems entirely plausible that UK exports should be influenced by a trend in world trade, and we thus test for the existence of a co-integrating relationship between XMAN and XWM. The tests indicate that we can treat XMAN and XWM as sharing a common trend during the pre-1979 period. However, we cannot reject the null of no co-integration for the period after 1979.[58]

In contrast to XMAN and XWM, the competitiveness variable COMT is undoubtedly I(0) for the period prior to 1979. However, for the period subsequent to 1979 the evidence cannot reject the alternative I(1) interpretation of COMT. Although we prefer, for reasons explained above, to maintain the I(0) hypothesis for COMT, we acknowledge the *possibility* that COMT could include a stochastic trend after 1979, and thus (we can and do) consider whether such a trend might be shared with XMAN. Our results indicate that no co-integrating relationship exists between XMAN, XWM and COMT for the period after 1979 and so we reject the idea of a common stochastic trend in both COMT and XMAN. A further implication of this result is that the rejection of co-integration between XMAN and XWM for the post-1979 period cannot be explained by reference to a possible stochastic trend in COMT.

Note that if evidence were restricted to Sargan Bhargava and first order Dickey-Fuller tests alone, a logical development would then be to proceed to a co-integration analysis. Yet if COMT is I(0) then the results of such a co-integration approach would be meaningless. This serves further to emphasise the importance of our detailed analysis of the properties of the time series in advance of further work on equation specification.

5.8.2 Results

In table 5.12 we present the results for two alternative specifications of putative co-integrating regressions estimated over both the pre and post-1979 periods, along with various test statistics based on the residuals from these equations.[59]

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58. We will consider possible explanations for this possible lack of co-integration, including those that assert that XMAN reflects some other source of non-stationarity over and above that of XWM in the period after 1979.

59. Note that in each case we assume that the equation should be estimated with XMAN as the dependent variable. This assumption is consistent with a demand side or reduced form interpretation of the export volume equations, and it enables us to by-pass the issue of the appropriate normalisation of the co-integrating vector (E&G, p 261). Note that we also avoid consideration of systems of equations, confining our ourselves to a single equation analysis. Hence we assume that any co-integrating regression will correspond to the long-run properties of the underlying export volume equation.

Table 5.12.

ESTIMATES OF CANDIDATE CO-INTEGRATING REGRESSIONS.

1) Regressions of XMAN on XWM, constant and dummy (DD).

1967:3 - 1979:3 49 observations.

XMAN = 1.52 + 0.68 XWM - 0.09 DD
(t statistics) (19.9) (37.3) (7.3)

DW = 1.1249
DF = 4.2467 48 obs.
ADF(4) = 3.1718 44 obs.

1979:2 - 1987:2 33 observations.

XMAN = 0.83 + 0.81 XWM - 0.17 DD
(t statistics) (1.96) (8.96) (3.76)

DW = 0.7894
DF = 2.7125 32 obs.
ADF(4) = 2.3537 28 obs.

2) Regressions of XMAN on XWM, COMT, constant and dummy.

1967:3 - 1979:3 49 observations.

XMAN = 2.03 + 0.679 XWM - 0.110 COMT - 0.09 DD
(t statistics) (7.03) (37.6) (1.82) (7.44)

DW = 1.1979
DF = 4.4842 48 obs.
ADF(4) = 3.3103 44 obs.

1979:2 - 1987:2 33 observations.

XMAN = 2.38 + 0.613 XWM - 0.135 COMT - 0.139 DD
(t statistics) (2.07) (3.75) (1.44) (2.71)

DW = 0.7572
DF = 2.630 32 obs.
ADF(4) = 1.9737 28 obs.

Notes:
The choice of significance levels for the tests of the null hypothesis of no co-integration is treated in the text. For the co-integrating Durbin Watson 5% points we use 0.7 at 51 observations and 1.1 at 31 observations. The 5% critical value for the Dickey Fuller in the Engle and Granger example is 3.37.

The first pair of estimates are based on the assumption of a common trend between XMAN and XWM. The second pair are based on the assumption that COMT may contain a stochastic trend and that there is a common trend between COMT and XMAN as well as XWM and XMAN. For the period up to 1979 the second alternative is invalid due to the fact that COMT is I(0); this alternative may however be valid for the period from 1979, when an I(1) description of COMT is sustainable.

Following Engle and Granger (1987) we know that, under the assumption of co-integration, the point estimates on the variables in the co-integrating regressions will be consistent estimates of the long-run relationships.[60] In each case we thus present statistics which, conditional on the assumed I(1) behaviour of the component series, test the null of no co-integrating relationship by considering whether the residuals from the relevant 'co-integrating regression' are stationary or not.[61]

The results we present are for the Co-integrating Durbin Watson and Dickey Fuller tests, and also for a fourth order Augmented Dickey Fuller test.[62] Based on interpolation from calculated tables, we choose 0.7 at 51 observations and 1.1 at 31 observations as working 5% critical values for the Co-integrating Durbin Watson, and we approximate the critical values for the Co-integrating Dickey Fuller in smaller samples by the usual value for 100 observations.

We consider first the results for the regressions featuring XMAN and XWM only. The Durbin Watson statistics take values of 1.1249 for the pre-1979 period (49 observations) and 0.7894 for the post-1979 period (33 observations). We can reject the hypothesis of no co-integration convincingly at the 5% level for the pre-1979 period, whereas in the post-1979 period we do not come near to rejecting the hypothesis of no co-integration at the 5% level. The Dickey Fuller statistics of 4.2467 and 2.7125

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60. Note that this is a *large* sample result.

61. See EG (1987).

62. In the case of the Durbin Watson and Dickey Fuller type tests we propose working values for the purposes of interpretation which are, strictly speaking, applicable for the 2 variable case. More detailed tables of critical values will no doubt be developed by other authors. (We do not attempt to guess appropriate working values for the fourth order Augmented Dickey Fuller, but we still present the results for the record.) Nevertheless we hope to make some use of the examples given in Engle and Granger for a sample size of 100 and a 2 variable, first order case.

imply a similar pattern: non co-integration is rejected at 1% prior to 1979; after 1979, however, non co-integration can only be rejected at about a 15% level. The results of the Augmented Dickey Fuller tests are similar. The underlying pattern thus remains that the residuals from the co-integrating regression are clearly stationary prior to 1979, but may not be so thereafter.

The estimates for the pre-1979 period are very well determined, with a coefficient of 0.68 on world trade. In contrast the coefficient on world trade is less sharply defined after 1979, with a t statistic of 8.96 compared with 37.3 . The point estimate is higher after 1979 than before, at 0.81.

We can conclude that there is strong evidence in favour of a common stochastic trend in UK exports and world trade prior to 1979.

After 1979 the evidence is less clear, and we may be inclined to reject the hypothesis of a common trend: we should certainly be cautious of assuming superconsistency for these long run estimates. Further doubts are raised by the observation that our re-estimation of an error correction specification for the post 1973 period suggested that the long run trade elasticity may have fallen rather than increased as the post-1979 'co-integrating regression' appears to indicate.

Nevertheless, our priors in favour of a common trend between XMAN and XWM are relatively strong ones, and rather than interpret the evidence as a straightforward rejection of this hypothesis we might wish to look for alternative explanations.

A possible explanation may be provided by the work of Landesmann and Snell (1989), who argue for the occurrence of structural change in the income elasticity of UK manufactured exports during the early 1980s. Such a structural change would cause the post-1979 co-integrating regression, based on a parameter constancy assumption, to be misspecified, and might produce apparent non-stationarity in the residuals.

no evidence of anything resembling 'superconsistency' here. We reject these estimates as indicators of long run properties.

In any case, as we have seen, COMT is indeed I(0) for the pre-1979 period, so the attempt to employ a co-integrating regression is strictly invalid. Note however that if we fail to recognise this point, we would then accept the estimated co-integrating relationship along with its estimated long-run coefficient of 0.11 on COMT. (We expect stationary residuals from a regression of XMAN and XWM, and the extra regressor COMT is also stationary). The recognition that COMT is I(0) is thus very important, as it prevents such invalid inference.

For the post-1979 period as we have seen, the data cannot reject an I(1) interpretation of COMT, and so we must allow that a valid co-integrating regression may in principle exist. However, we can only reject the non-stationarity of residuals at relatively high significance levels, and so we should be inclined to dismiss the hypothesis that a valid co-integrating regression includes XMAN, XWM and COMT. The instability of the estimates for the post-1979 period certainly discredits the hypothesis that the difficulties in finding a well defined common trend between XMAN and XWM after 1979 might be due to the omission of COMT and the influence of a further common trend.

5.8.3 Conclusions.

Overall our search for common trends yields the following conclusions. Firstly there is strong support for a common stochastic trend between XMAN and XWM prior to 1979: the long-run coefficient on XWM should be robust to short-run specification. Secondly, support for a common stochastic trend between XMAN and XWM after 1979 is relatively weak: we should be cautious about the neglect of short-run specification, but also beware of the possibility of structural change in the long-run coefficient on XWM. Thirdly there is no evidence to support a co-integration analysis involving COMT in the period after 1979, even though COMT itself may be thought to be I(1). We thus cannot neglect short run dynamics on competitiveness, and so we should proceed as if COMT were I(0)

anyway. The pre-1979 period also suggests the dangers of failing to recognise that a regressor is stationary.

5.9 CONCLUSIONS.

Our study of UK manufactured export volume equations has reported the large range of estimates of the long run competitiveness elasticity offered since 1979 (from -1.3 to -0.14). Further investigation has indicated that the evolution of equation specifications is an important explanation for the reported fall in estimated elasticities over time. We have focussed in particular on the short run dynamics of competitiveness.

We have reviewed the argument, put forward for example by Wickens and Breusch, that the correct specification of the short run dynamics is important for correct inference concerning the long run response of an equation to a stationary explanatory variable. On the basis of both qualitative features of the time series and formal tests we have shown that UK competitiveness followed a stationary process in the period prior to 1979. Thus the specification of the short run dynamics must be expected to affect the estimation of the long run competitiveness elasticity.

For the post-1979 period we cannot draw as clear cut a conclusion regarding the stationarity of competitiveness, but stationarity cannot be ruled out, and moreover, a cautious approach to dynamic specification suggests that we should treat competitiveness as a stationary variable after 1979 if we wish to insure against misspecification of the long run competitiveness elasticity.

Confirmatory evidence is provided by the application of methods for the estimation of and testing for co-integrating relationships to the data. This reveals a sharp contrast between the apparent superconsistency properties of estimates of the world trade elasticity before 1979 and the apparent lack of such properties in the estimates of the competitiveness elasticity. These studies also emphasise the importance of the recognition of the stationarity of competitiveness: without this recognition

researchers could proceed to assume co-integration of export volumes and competitiveness and hence make invalid inferences concerning the long run elasticity.

The clear implication of the analysis of this chapter is that more recent specifications of the export volumes equation, which limit the short run dynamics on competitiveness *a priori* to the extent of for example the LBS model, may in consequence produce inconsistent estimates of the long run competitiveness elasticity. We pursue this in the next chapter.

Chapter Six.

THE LONG RUN COMPETITIVENESS ELASTICITY OF UK MANUFACTURED EXPORT VOLUMES: MISSPECIFICATION TESTING, RE-SPECIFICATION AND A RETURN TO THE INSTABILITY QUESTION.

6.1 INTRODUCTION.

In this chapter we test for the misspecification of recent error correction type models of UK manufactured export volumes due to the omission of lags on competitiveness: particular attention is focussed, through the use of Hausman tests, on the misspecification of the long run competitiveness elasticity. We find that at least 3 years of lags on competitiveness should be included if misspecification of the long run elasticity is to be avoided. Subsequently, recursive estimation and parametric tests are used in order to study parameter stability in proposed alternative specifications which incorporate the longer lags required to avoid the misspecification of the long run competitiveness elasticity. Formal tests cannot reject parameter stability, but recursive estimation is strongly suggestive of a fall in the long run elasticity after 1979.

6.2 TESTING FOR MISSPECIFICATION OF THE LONG RUN COMPETITIVENESS ELASTICITY AND FOR THE OMISSION OF SHORT RUN DYNAMICS ON COMPETITIVENESS.

6.2.1 Introduction.

We have established that we should treat the competitiveness variable COMT as an I(0) time series. Therefore we cannot proceed on the basis that the short run dynamics on COMT do not matter for the investigation of the long-run elasticity of XMAN with respect to COMT. The estimated long run competitiveness elasticity of manufactured export volumes derived by popular 'error correction' type specification methods (eg. the LBS specification) may be biased due to the misspecification of the short run dynamics. This may have misled policymakers as to the consequences of permanent shocks to competitiveness.

In what follows we employ three distinct approaches to this issue. Firstly we examine graphical evidence of the effect on the long run competitiveness elasticity of adding further lags on competitiveness to the LBS model and to a less restricted version of the same basic approach. This presentation focuses attention on the expected trade off between inconsistency due to too few short run dynamics and inefficiency due to too many short run dynamics. Our second approach seeks to formalise the consistency versus efficiency trade-off by the use of Hausman type tests of misspecification with the long run elasticity as the parameter of interest. Our third approach is to employ conventional tests of dynamic specification to test for the elimination of short run dynamics from more general alternatives. Breusch and Mizon (1984), pp 246-7, have urged the use of classical F tests in such cases on grounds of practical effectiveness, and our findings tend to confirm the value of employing more than one approach.

Overall the evidence supports the hypothesis that there is an explanatory role for up to four years of lags in the short run dynamics on competitiveness, and that neglect of these long lags causes the estimate of the long run competitiveness to be significantly biased downwards. There is however a difference between the two periods, before and after 1979. While the inclusion of extra short run dynamics produces a larger change in the point estimate of the long run elasticity prior to 1979, it is easier to show that the change in that coefficient is significant for the post-1979 period. This may be a feature of 'experimental design', reflecting the greater variance of competitiveness under the post-1979 regime.

6.2.2 The Maintained 'Short Lag' Equations.

Our aim is to test the kind of specification which has emerged in recent years as a 'consensus' specification amongst UK macroeconomic modellers against alternatives with more lags on competitiveness. Our null hypothesis is that the recent UK models of export volumes are not misspecified due to insufficient lags on competitiveness.

We choose two equations to serve as the maintained models in our testing procedures. One of these is the general specification typically taken as a starting point, with four quarterly lags on all variables, including competitiveness: we denote this the (4,4,4) specification. We also use the specific restricted form of the LBS equation (as introduced in 1985).

The equations are estimated over the sample period 1967:3 to 1987:2. Recognising the regime change in competitiveness in 1979 they are also estimated over the sub-periods 1967:3 to 1979:2 and 1979:1 to 1987:2. The alternative specifications considered in the testing procedures are derived by adding lags of competitiveness, two at a time, to each of the maintained models. This provides us with a range of alternative models with from six to eighteen lags on competitiveness.

Details of the specification of the 'short lag' maintained models are given in table 6.1. The equations are estimated in renormalised form using the instrumental variable procedure detailed in Wickens and Breusch (1988), and the resultant estimates of the long run properties of the 'short lag' equations are presented in table 6.2.

The estimates obtained from the two models for the pre-1979 period are rather more precise than for the post-1979 period. It is also notable that, in contrast to the pre-1979 period, the LBS restricted form fits less well than the general model in the post-1979 period.[1]

The long run world trade elasticity falls from around 0.65 in the pre-1979 period to around 0.55 in the post-1979 period. This is consistent with the findings of our earlier re-estimation work, but not with the results of the attempted co-integrating regression between XMAN and XWM.

There is no evidence of a change in the long run competitiveness elasticity between the two sub periods, particularly if we compare the estimates of the (4,4,4) model over the two. Hence if, (like HMT, NI and LBS), we were to accept these 'short lag' specifications, we would have to conclude that

1. Some further diagnostic results are reported in Appendix 6.1.

Table 6.1.

SPECIFICATION DETAILS FOR 'SHORT LAG' UK MANUFACTURED EXPORT VOLUME EQUATIONS EMPLOYED AS THE BASIS FOR TESTS OF MISSPECIFICATION.

We employ two 'short lag specifications' which we test for misspecification of the long run competitiveness elasticity and for omission of short run dynamics on competitiveness. These equations are augmented by additional lags on competitiveness up to a maximum lag length of eighteen in the course of these tests.
In order to facilitate direct estimation of the long run coefficients and to test for misspecification we employ a Wickens-Breusch re-normalisation of these equations.
Details of the actual equations estimated as the null hypotheses are given below.

1) The LBS equation.

This equation is closely based on the specification of the LBS 1985 model.

Estimating Equation:

XMAN = DXMAN DXMAN(-1) XWM COMT DCOMT DCOMT(-1) DCOMT(-2) DD Constant.

Instruments:
XMAN(-1) XMAN(-2) XWM COMT(-3) DCOMT DCOMT(-1) DCOMT(-2) DD Constant.

2) The (4,4,4) equation.

This equation is equivalent to the general autoregressive specification of which the LBS 1985 model is a restricted version.

Estimating Equation:

XMAN = DXMAN DXMAN(-1) DXMAN(-2) DXMAN(-3) XWM DXWM DXWM(-1) DXWM(-2) DXWM(-3) COMT DCOMT DCOMT(-1) DCOMT(-2) DCOMT(-3) DD Constant.

Instruments:
XMAN(-1) XMAN(-2) XMAN(-3) XMAN(-4) XWM XWM(-1) XWM(-2) XWM(-3) XWM(-4) COMT COMT(-1) COMT(-2) COMT(-3) COMT(-4) DD Constant.

Notes: XMAN is UK manufactured export volumes, XWM is world trade volume and COMT is tax adjusted cost competitiveness. The prefix D denotes first differences and the suffix (-i) the ith lag. DD is a composite strike dummy.

Table 6.2.

ESTIMATES OF 'SHORT LAG' MODELS FOR VARIOUS PERIODS.

We estimate two models over the whole sample and for the pre and post 1979 periods. The equations are the LBS 1985 equation and the 4th order autoregressive model of which the LBS equation is a restricted form. The latter is denoted (4,4,4).

1) 1967:3 - 1987:2

LBS

XMAN = 0.603 XWM - 0.377 COMT + s.run dynamics + cons + dum
(0.025) (0.058)
80 obs. 9 regressors SE=0.0671 SSR=0.3193

(4,4,4)

XMAN = 0.615 XWM - 0.357 COMT + s.run dynamics + cons + dum
(0.022) (0.066)
80 obs. 15 regressors SE=0.0657 SSR=0.2802

2) 1967:3 - 1979:2

LBS

XMAN = 0.655 XWM - 0.266 COMT + s.run dynamics + cons + dum
(0.024) (0.117)
48 obs. 9 regressors SE=0.0525 SSR=0.1074

(4,4,4)

XMAN = 0.656 XWM - 0.283 COMT + s.run dynamics + cons + dum
(0.023) (0.128)
34 obs. 15 regressors SE=0.0555 SSR=0.0586

3) 1979:1 - 1987:2

LBS

XMAN = 0.568 XWM - 0.361 COMT + s.run dynamics + cons + dum
(0.138) (0.112)
34 obs. 9 regressors SE=0.0722 SSR=0.1303

(4,4,4)

XMAN = 0.523 XWM - 0.288 COMT + s.run dynamics + cons + dum
(0.109) (0.104)
80 obs. 15 regressors SE=0.0530 SSR=0.0928

Notes:
Heteroskedastic consistent standard errors are given in parentheses.

there was no evidence that the actual long run competitiveness elasticity had fallen during the entire sample period.

6.2.3 Graphical Evidence.

We consider first the graphical evidence concerning the estimate of the long-run competitiveness coefficient and its estimated variance. Under the null hypothesis that long-lags on competitiveness do matter, the estimate of the long-run elasticity on competitiveness in the LBS type model will be biased and inconsistent. When the relevant lags on competitiveness have been added the estimate will be consistent, and the addition of further lags will merely increase the estimated variance, ie. causing inefficiency. Should the LBS-type model be correctly specified, the addition of extra lags on competitiveness should cause inefficiency in what was already a consistent estimate.

The evidence is presented in graphical form (see the graphs in figures 6.1 and 6.2) in which the estimated long-run competitiveness elasticity and its two-standard-error bands are plotted against the number of additional included dynamic lags on competitiveness for the LBS (1985) equation over the periods 1967(3) to 1987(2), 1967(3) to 1979(2), and 1979(1) to 1987(2); also for the more general model, (for which we use the notation "(4,4,4)"), for the same periods.

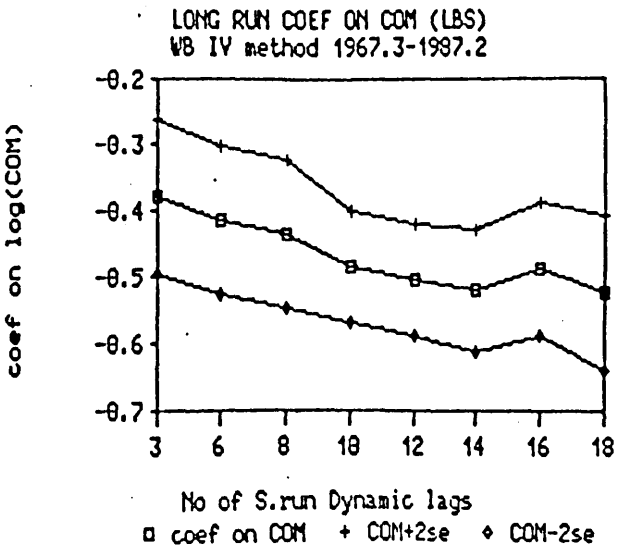
In all cases the long-run competitiveness elasticity is raised by the addition of short-run dynamics. There is also some suggestion in all cases that the variance of the estimate falls somewhat as lags are added up to a point beyond which it then begins to increase - at which point the estimated coefficient appears to have stabilised (allowing for its increasing variance).

We can note that the results for the entire period are less sharp than for the two sub-periods. In particular the evidence is very clear for the pre-1979 period. In both equations the evidence is strongly suggestive that the lags on competitiveness up to the 14th quarter are required, with the point estimate rising from -0.27 to between -0.6 and -0.9 in the LBS case,and from -0.29 to between -0.6 and -0.8 for the (4,4,4) case.

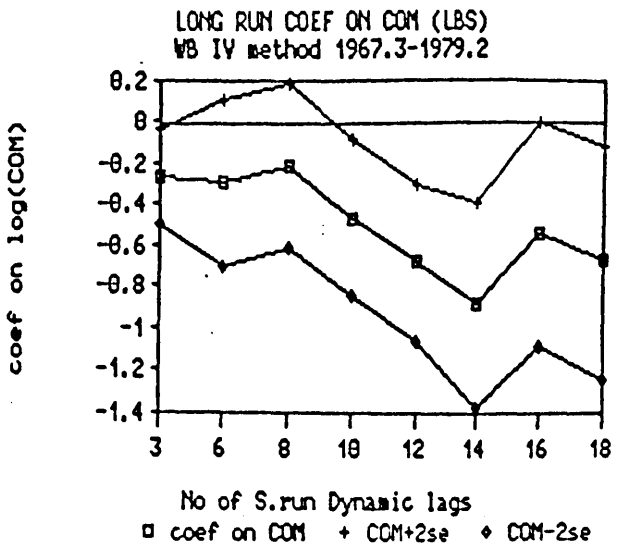
Figure 6.1

LONG RUN COMPETITIVENESS COEFFICIENT IN AUGMENTED LBS MODEL.

a)



b)



c)

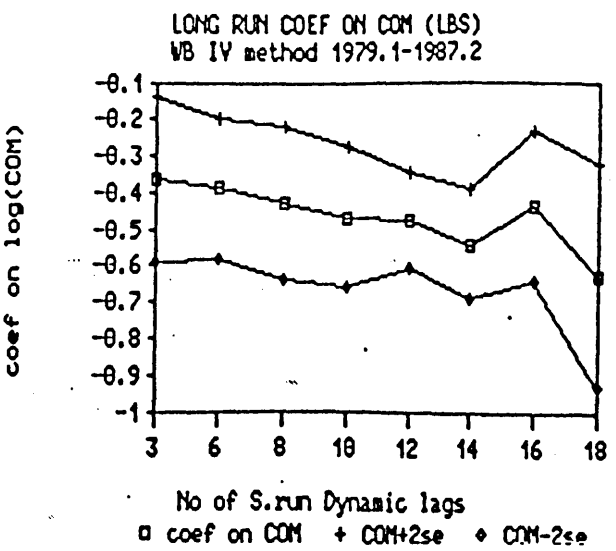
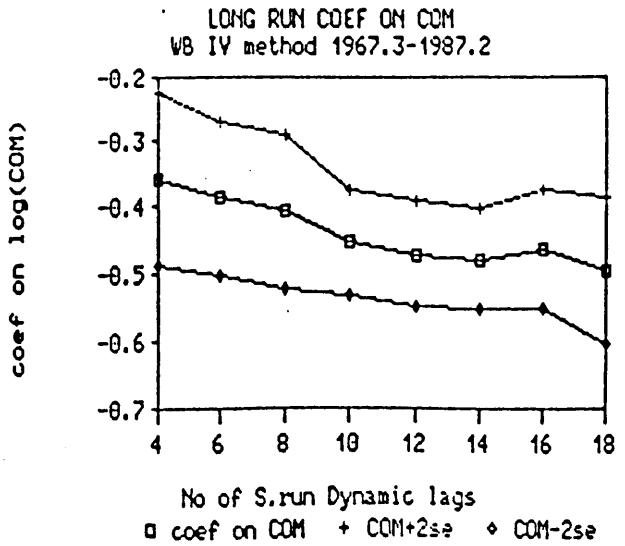


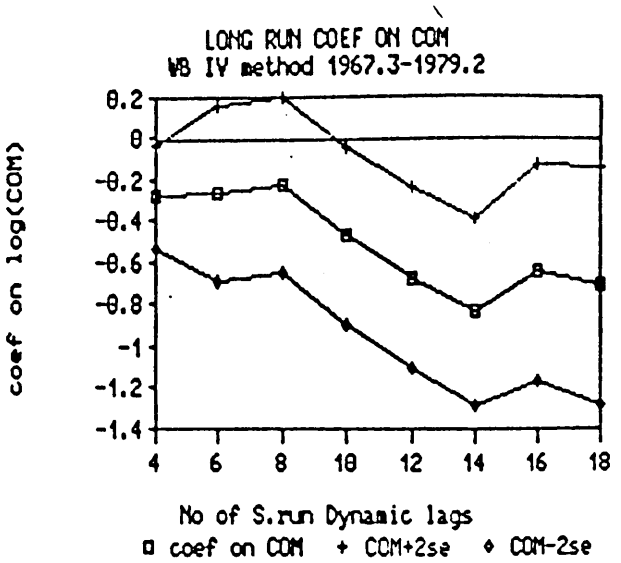
Figure 6.2

LONG RUN COMPETITIVENESS COEFFICIENT IN AUGMENTED (4,4,4) MODEL.

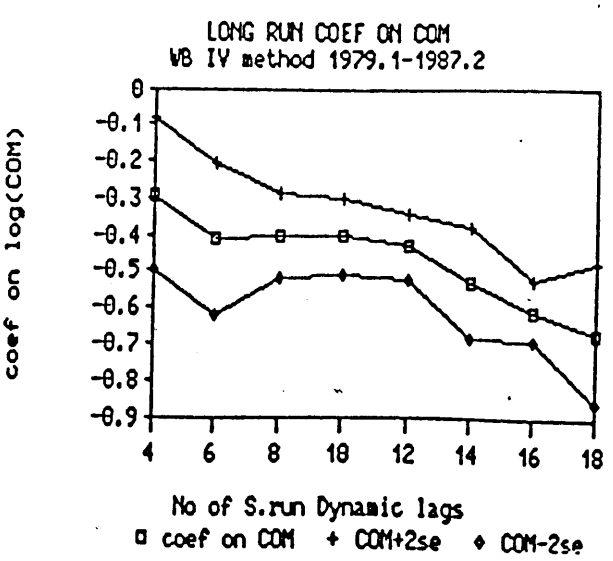
a)



b)



c)



The evidence for the post-1979 period is mixed. In the LBS case the elasticity rises, but by a smaller amount, from -0.36 to -0.48 in the 12th or -0.54 in the 14th quarter, at which point the variance of the estimate appears to be minimised. In contrast, in the (4,4,4) case, the estimated coefficient first rises, from -0.29 to -0.41, but then flattens out somewhat between 6 and 12 lags, before increasing again, and is at a maximum of -0.66 with 18 lags.[2]

On the basis of the above we would wish to include 12 to 14 lags of competitiveness in equations before 1979, and probably 16 in equations after 1979. We would expect the long-run elasticity to be increased in both cases, though perhaps by rather more in the first period.

Note that many of the diagrams show relatively little change in the long-run elasticity for additional lags from four up to around eight quarters. This helps to explain why either Hausman tests or classical tests of dynamic specification fail to detect problems against alternatives including further short run dynamics up to a maximum of two years, and yet do indicate misspecification against alternatives with longer short run dynamics of three or four years.

6.2.4 Hausman Tests of the Misspecification of the Long Run Competitiveness Elasticity.

In accordance with the Wickens and Breusch proposal referred to above, our primary interest, however, is in the consistent estimation of the long run properties of the equation rather than just in goodness of fit. Inconsistency of the estimated long run competitiveness elasticity due to too few short run dynamics can be avoided albeit at a risk of inefficiency due to too many short run dynamics. This contrast of possibly inconsistent versus possibly inefficient estimators of a parameter was the starting point for Hausman's (1978) approach to testing, and so it seems natural for us to consider whether to adopt it for our purposes.

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2. It is not clear that 18 lags are enough to prevent the long-run elasticity from being misspecified, except using the evidence that the variance of the estimate has increased here.

The work by Holly (1982) and Hausman and Taylor (1982) did much to elucidate the relationship between Hausman tests and the Classical procedures, placing the Hausman test in a Classical framework and identifying what was termed its 'implicit null hypothesis'. Ruud (1984), summarises the conclusions of Holly and Hausman and Taylor thus:

"If one is interested only in the verity of H_0, then... (the Classical tests) ....are the obvious candidates, being locally uniformly most powerful for alternatives to H_0. If one is interested only in consistency and efficiency of the restricted estimator, then a form of the Hausman specification test is optimal for alternatives to that hypothesis".(Ruud, 1984, p 221).

Moreover, in our analysis the number of restricted coefficients is less than the number of exclusion restrictions in any of the classical tests. This has the implication that the classical tests are not equivalent to the Hausman tests. In our examples, the Hausman test is always a single parameter test. It is then tempting to assume that the Hausman will be a more powerful test. (Formally, it is locally Uniformly Most Powerful for its particular implicit null, as defined in Holly (1982)). However, Breusch and Mizon (1984, pp 246-247) express a strong caution in respect of cases in which the simplification of a complete parametric alternative is considered. They argue that the case for preferring the Hausman approach cannot be made on grounds of power. Instead we must argue for the suitability of the Hausman approach to our particular purposes on different criteria.

Ruud (p 222) prefers to view the Hausman test primarily as a "pure significance test" (after Cox and Hinkley (1974)), for which we do not need to specify the alternative hypothesis, and which thus can make sense for many alternative models. Hence his practical recommendation is that researchers should choose between Hausman and Classical approaches based on convenience and on intuitions concerning the nature of relevant alternative models. Breusch and Mizon adopt a similar position. They see the practical value of the Hausman test in:

"its facility to be used in limited information situations in which the investigator is unable or unwilling to detail fully an alternative model that incorporates some additional feature in parametric form." (Breusch and Mizon, p246).

The nature of our research into UK export volume equations is one of misspecification as opposed to specification analysis.[3] Firstly we begin from a conventional wisdom which provides the LBS type of model as a null hypothesis: our intent is to suggest that there are problems with such models. Secondly, while we have an idea of the general direction of the misspecification - that it concerns the relative lack of short run dynamics on competitiveness - the inherent nature of short run dynamics is such that we cannot reasonably offer a fully specified particular alternative (or at least not without further specific assumptions concerning adjustment and expectational methods.) Hausman tests thus appear to be indicated.

Not only do we approach the LBS model in the spirit of misspecification analysis but our practical interest is in whether it provides us with consistent estimates of the long-run competitiveness elasticity. What the Hausman approach does, which the classical tests of dynamic specification cannot do, is to address directly whether the truncation of the short run dynamics in the LBS model prevents it from serving as a good approximation in respect of the long run competitiveness elasticity. Once again, the Hausman approach seems well suited to our particular purposes.

Actual Test Procedures Employed.

The tests employed the full statistic of the general form:

$$H = (b-B)' [\sigma^2_i S'\{ Q_{IV} + (X'M_W X)^{-1} - (X'M_V X)-1\} S]^{-1} (b-B) \qquad (6.1)$$

where:

X denotes the matrix of regressors in the re-normalised model under H_0, and M_V is the projection matrix on the corresponding set of instruments.

Z denotes the matrix of additional regressors in the re-normalised model under the richer dynamic specification of H_A, and M_W is the projection matrix on the corresponding set of instruments.

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3. Mizon (1977) characterises tests of specification as optimal sequential tests with a specific maintained hypothesis. In contrast, with tests of misspecification we begin by estimating the parameters corresponding to a particular hypothesis, and then form test statistics as a function of the restricted parameter estimates. The aim of misspecification tests is thus "to check whether more general hypotheses are required".

S is a selection vector which picks out only the element of the covariance matrix corresponding to the long run competitiveness coefficient.

B is the IV estimator of the long run elasticity on competitiveness, obtained directly in the re-normalised model under H_0.

b is the IV estimator of the long run elasticity on competitiveness, obtained directly in the re-normalised model under H_A.

σ^2_i is a scalar quantity which in practice we model by a consistent estimator of the innovation variance. For the reasons given in Appendix 6.2, we employ σ^2_A the estimate based on the model under H_A to form the variant of the statistic denoted H_2.

and $Q_{IV} = [(X'M_WX)^{-1} X'M_WZ \{Z'M_WZ - (Z'M_WX (X'M_WX)^{-1} X'M_WZ)\}^{-1} Z'M_WX (X'M_WX)^{-1}]$

For comparison we also calculate a variant of the statistic in which different estimates of the innovation variance σ^2 in each component of the variance difference. As we have argued in Appendix 6.2, this variant can give rise to negative values for the statistic. We denote this variant as H_1.

$$H_1 = (b-B)' [S'\{\sigma^2_A [Q_{IV} + (X'MWX)^{-1}] - \sigma^2_O (X'M_VX)^{-1}\} S]^{-1} (b-B) \qquad (6.2)$$

Results.

Hausman type tests of both forms described above as H_1 and H_2 were applied as follows to the two periods 1967:3 to 1979:2 and 1979:2 to 1987:2. The null hypothesis was taken to be the (re-normalised) LBS specification for one set of results, and a more general (4,4,4) specification of the error correction variety for another set. Tests were constructed using inefficient estimators of the null associated with a variety of alternatives in which the short run dynamics on competitiveness had been augmented to lengths 6, 8, 10, 12, 14, 16 and 18. In addition polynomial distributed lags of 5th and 4th orders were also employed at lag lengths of 14, 16 and 18, (actually in the form of Almon lags on 13, 15 and 17 lags on the first difference). All the tests yield statistics which are distributed in large samples as a chi-squared variate with one degree of freedom.

The hypothesis being tested is as to whether the long run competitiveness elasticity in the maintained model is robust to the possible misspecification of the short run dynamics on competitiveness. A rejection of the hypothesis of no misspecification requires both that there should be inconsistency of sufficient magnitude in the estimates obtained using the maintained model as compared with the augmented model and also that *both* sets of estimates are sufficiently precise to enable us to recognise the inconsistency as statistically significant. A failure to reject the hypothesis of no misspecification could reflect the absence of inconsistency in the original estimates, but it could also result from either the inconsistency of an inadequate alternative or else the imprecision of both sets of estimates. In practice we use a variety of alternative models. An appropriate length for the alternative is then suggested by choosing that length beyond which the corresponding test statistic falls as if due to increased inefficiency.[4]

a) LBS Model.

The results relating to the null hypothesis of the LBS model are presented in table 6.3. For the period 1967:3 to 1979:2 we find that H_1 and H_2 give similar results.

Excluding the PDL examples, we find that the hypothesis of no misspecification cannot be rejected at conventional significance levels against successive alternatives of 6, 8, 10 and 12 lags on COMT. For the 6 and 8 lag alternatives the statistic takes values close to zero, but by the 12 lag alternative the associated P value is over 0.75. At about a 5% significance level the null of no misspecification can be rejected against an alternative of 14 lags using the H_1 statistic. The H_2 statistic takes a slightly lower value, but its P value still comfortably exceeds 0.9.

The use of PDLs for the short run dynamics does not appear to improve the precision of the estimates of the long run elasticity over this sub sample.

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4. We then presumably test the null of no misspecification as if against this particular alternative. It is not clear how appropriate significance levels should be calculated if the lag length in the alternative model has been chosen in this way.

Table 6.3.

HAUSMAN TESTS OF MISSPECIFICATION OF THE LONG RUN ELASTICITY OF EXPORT VOLUMES WITH RESPECT TO COMPETITIVENESS IN THE LBS MODEL.

Alternative Model.	Elasticity Estimate	Hausman H_1	Statistics H_2
1967:3 - 1979:2		48 observations.	
Parameter estimate under the null is		-0.2656	
(lags on COMT)			
6 lags	-0.2948	0.0273	0.0293
8 lags	-0.2146	0.0394	0.0416
10 lags	-0.4648	0.42402	0.42781
12 lags	-0.6809	1.81315	1.74715
14 lags	-0.8883	3.8298	3.5655
16 lags	-0.5478	0.46918	0.45052
18 lags	-0.6823	0.71913	0.69371
(Almon lags on DCOMT)			
PDL(5,13)	-0.9025	4.13107	3.73853
PDL(4,13)	-0.756	1.64961	1.58904
PDL(5,15)	-0.3068	0.00409	0.0412
PDL(4,15)	-0.9564	3.24884	3.07331
PDL(5,17)	-0.0122	0.11742	0.11523
PDL(4,17)	-1.2759	3.92506	3.82413
1979:2 - 1987:2		33 observations.	
Parameter estimate under the null is		-0.3777	
(lags on COMT)			
6 lags	-0.4658	*	1.01299
8 lags	-0.6037	*	7.0957
10 lags	-0.6542	*	17.21162
12 lags	-0.6215	*	15.56209
14 lags	-0.667	*	15.88656
16 lags	-0.5982	*	4.61454
18 lags	-0.6881	*	3.10851
(Almon lags on DCOMT)			
PDL(5,13)	-0.6881	*	34.40079
PDL(4,13)	-0.6381	*	26.30716
PDL(5,15)	-0.7095	*	16.16835
PDL(4,15)	-0.7381	*	21.35098
PDL(5,17)	-0.644	*	4.89954
PDL(4,17)	-0.7096	*	1.70343

Notes:
These are tests of the null hypothesis of no misspecification of the long run competitiveness elasticity in the LBS equation for UK manufactured export volumes. The tests are conducted against a range of alternative specifications in which the short run dynamics on competitiveness are augmented.
Two forms of the test statistic are presented. H_2 is modified so as to be a function only of one estimate of the innovations variance (from the alternative), while H_1 is the usual form. Conventional OLS covariance estimates are used in the tests.
Under the null, H_1 & H_2 are distributed as $X^2(1)$ variates. See the text for explanation of negative quadratic forms, denoted by above by *.

For the period 1979:2 to 1987:2 the values taken by the H_2 statistic are uniformly higher than for the earlier period. This appears to reflect more precise estimates rather than a larger difference in point estimates. Our preferred test statistic H_2 enables us to reject the null of no misspecification at the 5% significance level for alternatives with 8, 10, 12, 14, and 16 lags on COMT. The employment of PDLs appears to increase the efficiency of estimates somewhat, thereby working to increase the probability of rejection of the no misspecification hypothesis.

b) The (4,4,4) Model.

The results relating to the (4,4,4) null hypothesis are presented in table 6.4.

For the period 1967:3 to 1979:2 the pattern of results is similar to those for the LBS model. The statistics are close to zero for the 6 and 8 lag alternatives, but the P value rises above 0.75 for 12 lags. The statistics are still higher at 14 lags, for which H_1 rejects the null of no misspecification at a 5% level, and although the corresponding H_2 statistic takes a somewhat lower value it still has a P value of more than 0.9.

However, for the period 1979:2 to 1987:2 we find that *both* H_1 and H_2 take negative values. (H_2 is negative for all but a few cases). Given that H_2 is constructed so as to be positive we must assume that this result reflects the relatively few degrees of freedom available in estimating the (4,4,4) model and its augmented alternatives of this sample. These negative results for H_2 must be due to the failure of the sums of squared instrumented regressors to converge sufficiently towards the usual fixed quantities of asymptotic theory. Note that the larger pre-1979 sample gives no suggestion of this problem.

Table 6.4.

HAUSMAN TESTS OF MISSPECIFICATION OF THE LONG RUN ELASTICITY OF EXPORT VOLUMES WITH RESPECT TO COMPETITIVENESS IN THE (4,4,4) MODEL.

Alternative Model.	Elasticity Estimate	Hausman statistics H_1	H_2
1967:3 - 1979:2		48 observations.	
Parameter estimate under the null is		-0.269	
(lags on COMT)			
6 lags	-0.2494	0.020502	0.0216527
8 lags	-0.2047	0.084857	0.0867259
10 lags	-0.4347	0.29816	0.29055
12 lags	-0.5822	1.37885	1.1827
14 lags	-0.6947	4.08781	2.89017
16 lags	-0.6075	1.06205	0.94143
18 lags	-0.6785	0.61322	0.59043
(Almon lags on DCOMT)			
PDL(5,13)	-0.7006	9.32435	4.59589
PDL(4,13)	-0.743	1.49408	1.46477
PDL(5,15)	-0.6297	4.13455	2.53924
PDL(4,15)	-0.8235	3.45029	3.0113
PDL(5,17)	-0.4699	0.67381	0.5064
PDL(4,17)	-0.8933	3.90453	3.43527
1979:2 - 1987:2		33 observations.	
Parameter estimate under the null is		-0.269	
(lags on COMT)	*(Beware few degrees of freedom)*		
6 lags	-0.4322	*	*
8 lags	-0.5838	*	*
10 lags	-0.6321	*	*
12 lags	-0.6035	*	*
14 lags	-0.6415	*	345.9
16 lags	-0.649	*	95.17
18 lags	-0.7128	*	3.066
(Almon lags on DCOMT)			
PDL(5,13)	-0.5714	*	*
PDL(4,13)	-0.5548	*	*
PDL(5,15)	-0.6018	*	*
PDL(4,15)	-0.6079	*	*
PDL(5,17)	-0.5596	*	*
PDL(4,17)	-0.6087	*	*

Notes:
These are tests of the null hypothesis of no misspecification of the long run competitiveness elasticity in the (4,4,4) model of UK manufactured export volumes. The tests are conducted against a range of alternative specifications in which the short run dynamics on competitiveness are augmented. Two forms of the test statistic are presented. H_2 is modified so as to be a function only of one estimate of the innovations variance (from the alternative), while H_1 is the usual form. Conventional OLS covariance estimates are used in the tests.
Under the null H_1 & H_2 are distributed as $X^2(1)$ variates. Negative values are denoted by * .

6.2.5 Classical Tests of Dynamic Specification.

The Classical tests of dynamic specification test only for the omission of short run dynamics on competitiveness, without regard to the effect on the long run competitiveness elasticity. Nevertheless, we supplement the Hausman tests with classical tests.

Various forms of the classical tests are available, implementing the Wald, Likelihood Ratio and Lagrange Multiplier principles in different ways. All these tests are asymptotically equivalent for a given parametric null and alternative, although small sample properties would be of interest for many of our examples.

The Classical tests are conducted on the original rather than on the renormalised forms of the equations in order to avoid the complications of testing under instrumental variables.

We apply tests of dynamic misspecification to two different maintained models, the LBS and (4,4,4) formulations of the short lag approach. Once again we apply the tests to the two sub periods, before and after 1979, as well as to the entire sample.

Firstly, we employ F tests of the restrictions inherent in the omission of longer lags on competitiveness from successively 'longer' alternative models. These are presented for the LBS null in table 6.5, and for the (4,4,4) null in table 6.6.

Consider the LBS formulation. For the entire sample, the null is rejected at a 5% level against 13 or more lags. Turning to the pre-1979 estimates the F-tests are unable to reject the validity of the restrictions in the basic (short lags) model relative to longer lag alternatives at conventional significance levels, although the P value associated with the null is lowest at 13 lags. After 1979, we can reject the null against alternatives with 7 or more lags at a 5% level.

Table 6.5.

F TESTS OF DYNAMIC SPECIFICATION IN LBS MODEL.

Sample Period	Alternative Lag Length on Competitiveness)	F test of LBS Null	R,n-k	Reject Null At (per cent)
1967:3 - 1987:2		80 observations.		
	5	1.119	2,69	-
	7	1.766	4,67	-
	9	1.430	6,65	-
	11	1.846	8,63	-
	13	2.488	10,61	5%
	15	2.433	12,59	5%
	17	2.160	14,57	5%
1967:3 - 1979:2		48 observations.		
	5	0.0167	2,37	-
	7	0.0706	4,35	-
	9	0.0599	6,33	-
	11	0.5148	8,31	-
	13	1.431	10,29	-
	15	1.696	12,27	**
	17	1.636	14,25	-
1979:2 - 1987:2		33 observations.		
	5	1.044	2,22	
	7	3.859	4,20	5%
	9	3.263	6,18	5%
	11	3.173	8,16	5%
	13	3.108	10,14	5%
	15	2.446	12,12	10%
	17	2.853	14,10	5%

Note:

In each case the null hypothesis tested is that the exclusion restrictions implied by the LBS equation for the short run dynamics on competitiveness in the successive alternative models can be validly imposed.

** Most likely to reject null against 15 lag alternative, with a 5% critical value of 2.08.

Table 6.6

F TESTS OF DYNAMIC SPECIFICATION IN (4,4,4) MODEL.

Sample Period	Alternative (Lag Length on Competitiveness)	F test of LBS Null	R,n-k	Reject Null At (per cent)
1967:3 - 1987:2		80 observations.		
	5	1.037	1,63	-
	7	1.433	3,61	-
	9	1.161	5,59	-
	11	1.566	7,57	-
	13	3.779	9,55	1%
	15	3.092	11,53	1%
	17	2.682	13,51	1%
1967:3 - 1979:2		48 observations.		
	5	0.0053	1,31	-
	7	0.0956	3,29	-
	9	0.1210	5,27	-
	11	0.5190	7,25	-
	13	1.564	9,23	**
	15	1.423	11,21	-
	17	1.317	13,19	-
1979:2 - 1987:2		33 observations.		
	5	0.8374	1,16	-
	7	1.733	3,14	-
	9	1.550	5,12	-
	11	1.397	7,10	-
	13	6.153	9,8	1%
	15	3.786	11,6	10%
	17	3.293	13,4	-

Note:

In each case the null hypothesis tested is that the exclusion restrictions implied by the (4,4,4) equation for the short run dynamics on competitiveness in the successive alternative models can be validly imposed.

** Most likely to reject null against 13 lag alternative, with a 5% critical value of 2.28.

The null of the (4,4,4) basic model is rejected for the entire sample against alternatives with 13 or more lags. For the pre-1979 period the null cannot be rejected against augmented alternatives, but the P value associated with the null is lowest for 13 lags. On the post 1979 data, the null is rejected at a 1% level against the 13 lag alternative.

We also conducted tests of successive restrictions (two lags at a time) on the length of the short-run dynamics on competitiveness from the maximum of seventeen down to the LBS and (4,4,4) models. These successive restrictions cannot be rejected at conventional significance levels. We have not reported these restrictions in detail.

By way of comparison with the F tests considered above, and in order to avoid problems which may arise from the use of two different estimators for σ^2 in the same statistic, we consider alternative classical approaches. First we present Wald statistics for the exclusion of short-run dynamics on competitiveness in tables 6.7 and 6.8 for the augmented LBS and (4,4,4) models respectively. The tests compare the various augmented specifications with the original LBS and (4,4,4) cases.[5]

Consider the Wald tests relative to the LBS null. On the entire sample we can reject the null at the 5% level relative to the 7 and 9 lag alternatives, and at the 1% level relative to higher order alternatives. On the pre-1979 period the Wald statistics reject the LBS null at the 1% level against 13 or more lags in the alternative. Post-1979, 1% rejections of the null occur relative to the null against 7 or more lags.

The (4,4,4) null is rejected at a 1% level over the entire sample for alternatives of 11 or more lags. On the pre-1979 sample the null is rejected relative to 13 or more lags at 1%. After 1979 the rejections occur for 7 or more lags in the alternative. We should be cautious of the few degrees of freedom in these alternatives on the post 1979 sample.

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5. The Wald test is given by:
$W=S'B\{(S'Var(B)S)^{-1}\}B'S$
where S is a selection vector which picks out the coefficients on the additional short run dynamics on competitiveness in the unrestricted model.

Table 6.7.

WALD TESTS OF DYNAMIC SPECIFICATION IN LBS EQUATION.

Sample Period	Lag Length of Alternative	Wald Statistic	No. of Restric -tions	Reject Null At (%)
1967:3 - 1987:2		80 observations.		
	5	3.191	2	-
	7	10.007	4	5%
	9	12.974	6	5%
	11	25.355	8	1%
	13	46.109	10	1%
	15	57.363	12	1%
	17	65.523	14	1%
1967:3 - 1979:2		48 observations.		
	5	0.0424	2	-
	7	0.5576	4	-
	9	0.5671	6	-
	11	11.383	8	-
	13	37.213	10	1%
	15	94.374	12	1%
	17	95.984	14	1%
1979:2 - 1987:2		33 observations		
	5	3.345	2	-
	7	25.899	4	1%
	9	40.674	6	1%
	11	85.035	8	1%
	13	141.83	10	1%
	15	183.70	12	1%
	17	491.459	14	1%

Note:
In each case the null hypothesis tested is that the exclusion restrictions implied by the LBS equation for the short run dynamics on competitiveness in the successive alternative models can be validly imposed.

Table 6.8.

WALD TESTS OF DYNAMIC SPECIFICATION IN (4,4,4) EQUATION.

Sample Period	Lag Length of Alternative	Wald Statistic	No. of Restric -tions	Reject Null At (%)
1967:3 - 1987:2		80 observations.		
	5	1.206	1	-
	7	7.071	3	10%
	9	8.888	5	-
	11	18.723	7	1%
	13	89.020	9	1%
	15	96.429	11	1%
	17	96.064	13	1%
1967:3 - 1979:2		48 observations.		
	5	0.0098	1	-
	7	0.609	3	-
	9	1.368	5	-
	11	8.930	7	-
	13	42.703	9	1%
	15	79.967	11	1%
	17	85.751	13	1%
1979:2 - 1987:2		33 observations		
	5	0.9987	1	-
	7	11.837	3	1%
	9	31.419	5	1%
	11	48.870	7	1%
	13	285.31	9	1%
	15	300.04	11	1%
	17	525.47	13	1%

Note:
In each case the null hypothesis tested is that the exclusion restrictions implied by the (4,4,4) equation for the short run dynamics on competitiveness in the successive alternative models can be validly imposed.

A different comparison with the F tests is provided by the calculation of LM statistics presented in table 6.9 for the LBS null and 6.10 for the (4,4,4) case. These employ the statistic TR^2 derived from the regression of the residuals from the null regression against the additional lags of COMT. This simple derivation of the LM test is as suggested by Harvey (1981a).

On the full sample the LBS null is rejected at a 1% level against 15 lags, and is significantly rejected against 13 and 17 lags also. On the pre-1979 sample the null is rejected at about 10% against 13, 15 or 17 lags. On the post-1979 sample the null is strongly rejected against all augmented alternatives.

The (4,4,4) null is rejected on the full sample against 11 or more lags. It is rejected at the 1% level against 13 or more lags on the pre-1979 sample, and at the 1% level against alternatives with 7 or more lags.

Note however that we should be cautious of the results for the post-1979 sample, due to the few degrees of freedom, particularly relative to the (4,4,4) case.

6.2.6 Conclusions on Testing for Misspecification.

We have sought to establish whether the current class of specification, as represented by the LBS equation, are misspecified in respect of short run dynamics on competitiveness, and in particular, whether this has resulted in the significant misspecification of the long run competitiveness elasticity.

On the basis of graphical evidence, Hausman tests and classical tests of specification, we can conclude that there is a good case that the response of export volumes to competitiveness should be smoothed over a lag of perhaps 3 or 4 years, and that failure to do this results in estimates of the long run elasticity which are only half of what they would otherwise be. An important feature of the results is that the estimates after 1979 are more precise, perhaps reflecting the greater variation in the competitiveness series in the 1980s. This greater precision explains why, although the consequence of

Table 6.9.

LM TESTS OF DYNAMIC SPECIFICATION IN LBS EQUATION.

Using the T x R squared form of the LM statistic.

Sample Period	Lag Length of Alternative	LM Statistic	No. of Restric -tions	Reject Null At (%)
1967:3 - 1987:2		80 observations		
	5	2.513	2	-
	7	7.630	4	-
	9	9.329	6	-
	11	15.192	8	10%
	13	23.177	10	5%
	15	26.483	12	1%
	17	27.731	14	5%
1967:3 - 1979:2		48 observations		
	5	0.0433	2	-
	7	0.3844	4	-
	9	0.5167	6	-
	11	5.6287	8	-
	13	15.858	10	10%
	15	20.633	12	10%
	17	22.946	14	10%
1979:2 - 1987:2		33 observations		
	5	2.862	2	-
	7	14.374	4	1%
	9	17.192	6	1%
	11	20.241	8	1%
	13	22.751	10	5%
	15	23.425	12	5%
	17	26.393	14	5%

Note:
In each case the null hypothesis tested is that the exclusion restrictions implied by the LBS equation for the short run dynamics on competitiveness in the successive alternative models can be validly imposed.

Table 6.10.

LM TESTS OF DYNAMIC SPECIFICATION IN (4,4,4) EQUATION.

Using the T x R squared form of the LM statistic.

Sample Period	Lag Length of Alternative	LM Statistic	No. of Restric -tions	Reject Null At (%)
1967:3 - 1987:2		80 observations		
	5	1.295	1	-
	7	5.268	3	-
	9	7.168	5	-
	11	12.905	7	10%
	13	30.566	9	1%
	15	31.273	11	1%
	17	32.483	13	1%
1967:3 - 1979:2		48 observations		
	5	0.0082	1	-
	7	0.4702	3	-
	9	1.052	5	-
	11	6.090	7	-
	13	18.226	9	5%
	15	20.499	11	5%
	17	22.754	13	5%
1979:2 - 1987:2		33 observations		
	5	1.641	1	-
	7	8.938	3	5%
	9	12.947	5	5%
	11	16.316	7	5%
	13	28.835	9	1%
	15	28.844	11	1%
	17	30.180	13	1%

Note:
In each case the null hypothesis tested is that the exclusion restrictions implied by the (4,4,4) equation for the short run dynamics on competitiveness in the successive alternative models can be validly imposed.

the misspecification of short run dynamics in terms of point estimates appears to be greater prior to 1979, the statistical significance of the misspecification is greater thereafter.

There is also evidence in the conventional tests of dynamic specification that the LBS type specifications are misspecified in the period before 1979 against alternatives which include lags of upwards of 3 years. However, the interpretation of such tests will depend heavily on our priors concerning an appropriate general model. Given the small sample size, doubts are also raised by the reported F statistics.[6]

Our use of the Hausman tests provides important additional support to the argument. The hypothesis of no misspecification of the long run competitiveness elasticity is rejected against alternative estimates employing 3 years of lags on COMT. Moreover, the properties of the estimates and of the test statistics are entirely consistent with the alternative hypothesis insofar as estimator efficiency falls off as lags of COMT are added beyond this number, with little change in the point estimate.

We must allow that the inclusion of long lags on competitiveness is at odds with the principle of parsimony in modelling.[7] We concede that others may set relatively little store by the prior beliefs we bring to the analysis that adjustment and delivery lags, and also, perhaps, expectational lags, may require the smoothing of the response to competitiveness movements over a period of several years. Nevertheless, we claim that the results presented here must at least cast serious doubts over reliance on the point estimates of competitiveness elasticities derived from the LBS type of specification.

The evidence we have presented shows that the LBS type of specification tends to underestimate the long run competitiveness elasticity. We conclude that the LBS model provides a poor approximation of the underlying data generation process with respect to the long run elasticity of UK manufactured

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6. See Pagan's (1984) survey of the use of variable addition methods in diagnostics which recommends F tests, at least in part due to their small sample characteristics relative to the chi squared forms. See also the PC-GIVE manual, (Hendry, 1989), which notes that the chi-squared forms may reject acceptable nulls too often in small samples.

7. See the introduction to Hendry and Wallis (1984).

export volumes to movements in competitiveness. A better approximation might imply a long run elasticity on competitiveness twice the magnitude of that implicit in the LBS analysis.[8]

6.3 ESTIMATES OF AUGMENTED MODELS.

The omission of long lags on COMT from models such as the LBS model has the consequence of causing the estimate of the long run elasticity to be underestimated. We now report estimates and diagnostic test results for the two short lag specifications (LBS and (4,4,4)) which we have tested, as well as for versions of each which are augmented respectively to include 13 and 17 lags of competitiveness. These augmented equations are put forward as candidate specifications for a model with correctly specified long run properties.

We employ two augmented specifications. The first is a 'most parsimonious possible specification', in which 14 lags of competitiveness are added to the restricted LBS equation from the 1986 model.[9] This reflects the minimum acceptable modification to the maintained model on the basis of our Hausman procedures.

The second is a 'most general possible' specification, adding 18 lags of competitiveness to the general autoregressive model behind the LBS model specification procedures. This equation may be a suitable starting point for a general to specific testing procedure.

We present estimates for two versions of each of the augmented models. Firstly we add the extra lags in unrestricted form. However, due to the small samples relative to the many parameters of the

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8. Doubts which remain concerning the alternative models implicit in the augmented specifications will carry more force in respect of the appropriate progressive modelling strategy for formulating a satisfactory alternative to the LBS type model than in respect of our critique of the LBS model itself.

9. Note that in the diagnostic testing carried out on these equations and reported elsewhere it appears that three common factor restrictions are not rejected on the more general model. If these restrictions were imposed they would approximately reduce the more general model to the LBS equation augmented with 14 lags.

augmented models, we also consider the application of 5th order Almon polynomial distributed lags to the short run dynamics on competitiveness.

Estimates are presented for the entire sample and also for the two sub-periods corresponding to the regime change in 1979. Table 6.11 gives the results on the augmented LBS model and table 6.12 on the augmented (4,4,4) model.

Firstly we consider the results on the world trade elasticities. These are similar across different augmented variants for each sample period. For the period prior to 1979, the augmented models' estimates are around 0.6. The elasticity is lower for the post-1979 period, as was the case in the estimates of the 'short lag' models: in these cases it takes values just over 0.4.

We now turn to the long run competitiveness elasticity, which, as we have seen, is increased in absolute value when the 'short lag' models are augmented.

Prior to 1979 the unrestricted point estimates of the long run competitiveness elasticity are -0.888 for the augmented LBS equation and -0.678 for the augmented (4,4,4) model. The use of 5th order Almon restrictions on the short run dynamics has the effect of reversing these results, to -0.683 for the augmented LBS model and -0.887 for the augmented (4,4,4) model. Overall these results suggest that the long run elasticity lies between -0.6 and -1.0 prior to 1979.

After 1979 the unrestricted point estimates are -0.667 for the augmented LBS equation and -0.713 for the augmented (4,4,4) model. The use of 5th order Almon restrictions on the short run dynamics yields estimates of -0.639 for the augmented LBS equation and -0.562 for the augmented (4,4,4) model. The Almon restrictions appear to affect the augmented (4,4,4) estimates more than those for the LBS equation. Overall we might conclude from these results that the long run elasticity lies between -0.5 and -0.8.

Table 6.11.

ESTIMATES OF AUGMENTED LBS MODELS FOR VARIOUS PERIODS.

We consider the LBS equation augmented by 14 unrestricted lags on COMT and by a 5th order Almon on 13 lags of DCOMT. We estimate both equations over the whole sample, and also for the pre 79 and post 79 periods.

1) 1967:3 - 1987:2

LBS + 14 unrestricted lags on COMT

XMAN = 0.626 XWM - 0.519 COMT + s.run dynamics + cons + dum
(0.014) (0.047)
80 obs. 20 regressors SE=0.0408 SSR=0.0999

LBS + Almon(5,13,far) on DCOMT

XMAN = 0.629 XWM - 0.519 COMT + s.run dynamics + cons + dum
(0.014) (0.045)
80 obs. 10 regressors SE=0.0367 SSR=0.0944

2) 1967:3 - 1979:2

LBS + 14 unrestricted lags on COMT

XMAN = 0.567 XWM - 0.888 COMT + s.run dynamics + cons + dum
(0.039) (0.246)
48 obs. 20 regressors SE=0.0406 SSR=0.0460

LBS + Almon(5,13,far) on DCOMT

XMAN = 0.600 XWM - 0.683 COMT + s.run dynamics + cons + dum
(0.034) (0.206)
48 obs. 10 regressors SE=0.0338 SSR=0.0574

3) 1979:2 - 1987:2

LBS + 14 unrestricted lags on COMT

XMAN = 0.409 XWM - 0.667 COMT + s.run dynamics + cons + dum
(0.077) (0.045)
33 obs. 20 regressors SE=0.0201 SSR=0.0053

LBS + Almon(5,13,far) on DCOMT

XMAN = 0.436 XWM - 0.639 COMT + s.run dynamics + cons + dum
(0.062) (0.027)
33 obs. 10 regressors SE=0.0169 SSR=0.0065

Notes:
Heteroskedastic consistent standard errors in parentheses. All equations are estimated in re-normalised form. (Full details of the specification of short run dynamics have been omitted).

Table 6.12.

ESTIMATES OF AUGMENTED (4,4,4) MODELS FOR VARIOUS PERIODS.

We consider the (4,4,4) model augmented by 18 unrestricted lags on COMT and by a 5th order Almon on 17 lags of DCOMT. We estimate both equations over the whole sample, and also for the pre-79 and post-79 periods.

1) 1967:3 - 1987:2

(4,4,4) + 18 unrestricted lags on COMT

XMAN = 0.624 XWM - 0.491 COMT + s.run dynamics + cons + dum
(0.010) (0.041)
80 obs. 30 regressors SE=0.0293 SSR=0.0429

(4,4,4) + Almon(5,17,far) on DCOMT

XMAN = 0.623 XWM - 0.476 COMT + s.run dynamics + cons + dum
(0.011) (0.045)
80 obs. 16 regressors SE=0.0291 SSR=0.0541

2) 1967:3 - 1979:2

(4,4,4) + 18 unrestricted lags on COMT

XMAN = 0.606 XWM - 0.678 COMT + s.run dynamics + cons + dum
(0.038) (0.274)
48 obs. 30 regressors SE=0.0389 SSR=0.0273

(4,4,4) + Almon(5,17,far) on DCOMT

XMAN = 0.569 XWM - 0.887 COMT + s.run dynamics + cons + dum
(0.041) (0.300)
48 obs. 16 regressors SE=0.0363 SSR=0.0421

3) 1979:2 - 1987:2

(4,4,4) + 18 unrestricted lags on COMT

XMAN = 0.409 XWM - 0.713 COMT + s.run dynamics + cons + dum
(0.062) (0.027)
33 obs. 30 regressors! SE=0.0085 SSR=0.0022

(4,4,4) + Almon(5,17,far) on DCOMT

XMAN = 0.432 XWM - 0.562 COMT + s.run dynamics + cons + dum
(0.052) (0.046)
33 obs. 16 regressors SE=0.0116 SSR=0.0023

Notes:
Heteroskedastic consistent standard errors in pararentheses. All equations are estimated in re-normalised form. There are only 3 degrees of freedom in the unrestricted equation after 1979. The Almon lag raises this to 17.

The evidence suggests that there may after all have been a fall in the long run elasticity of competitiveness from the pre-1979 to the post-1979 period. The evidence is not decisive, as our informal confidence ranges for the true value of the parameter before and after 1979 overlap in the range -0.6 to -0.8. But it would certainly appear that we can confirm the proposition that the estimates of the long run competitiveness elasticity are higher in the augmented models than in their unaugmented equivalents. (See table 6.2).

In Appendix 6.1 we present some further diagnostic results for the two augmented specifications. The equations appear to satisfy these tests.

6.4 STABILITY OF THE LONG RUN COMPETITIVENESS ELASTICITY

6.4.1 Introduction.

We have strong priors that there may have been a structural break corresponding to a regime change in the time series behaviour of competitiveness in 1979. Institutional knowledge informs us that capital controls were lifted, and also that monetary policy was changed so that it took no account of the exchange rate (at least until Geoffrey Howe gave way to Nigel Lawson as Chancellor of the Exchequer in 1983).[10] Economic theory tells us that a regime change can produce apparent structural change in reduced form models of behaviour which have forward-looking or expectational components. Given a change in the time series properties of competitiveness, the filtering process employed by participants to extract a signal relevant for their economic behaviour will also change.[11]

Despite the background of changing regimes, analysis of the properties of the prevailing 'LBS-type' equations shows the long-run elasticities with respect to both world trade and competitiveness to be

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10. Details of the changes in policy regime in 1979 are set out in Buiter and Miller (1981b) and (1983). The change of properties in the time series for competitiveness have been examined in chapter five above.

11. See Lucas (1976).

remarkably constant.[12] But these 'LBS-type' short-lag equations appear to be misspecified with respect to short-run dynamics. An analysis of parameter constancy should be conducted on the augmented reduced forms.

In the following analysis of parameter constancy we consider the augmented LBS equation, which we denote as LBS(+13) in its unrestricted form and as LBS(+PDL13) in its Almon restricted form. We also consider an augmented (4,4,4) model, in this case augmented with 17 (as opposed to the previous 18) lags on competitiveness, and which we denote (4,4,17) in its unrestricted form and (4,4,(PDL(17)) in its Almon restricted form.

In keeping with recent work of eg. Hendry (1988), Patterson (1986), the issue of parameter stability is investigated by the method of recursive least squares.[13] Our parameter of interest has been explicitly identified as the long-run competitiveness elasticity, and so we concentrate our analysis on that coefficient.[14] The necessary instrumental variable recursive estimates for even the more parsimonious of these models had to be derived from successive non-recursive estimation.

6.4.2 Graphical Presentation of Recursive Estimates.

To begin with we plot the long-run competitiveness elasticity for the LBS model, estimated recursively from 1967. These results are presented at figure 6.3. The point estimate appears to be quite stable over time. However there is a qualitative change in the results around 1980 (from 1979(3)), when the

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12. See eg. Turner (1988) on the coefficient on world trade, and the diagram at figure 6.3 below. The existence of an apparently stable 'feedback' model in the presence of an alleged change in regime for an important regressor suggests a link to the 'feedback versus feed-forward' debate in Hendry (1988), and Cuthbertson (1988).
We have already referred to the work of Landesmann and Snell who have employed a different specification to the LBS model and have found evidence for parameter change in the world trade coefficient.

13. For theoretical background see Dufour (1982).

14. We must be wary of the issues raised eg. by Patterson (1986) concerning significance levels.

estimate becomes much more sharply defined: the putative regime change coincides with the notably increased precision of the estimate.[15]

We next (at figure 6.4a) present recursive estimates of the elasticity (from 1967) in a model in which 14 free lags have been added to the LBS model. Once again the coefficient is well-defined and stable for the 1980s at a value of around -0.5. We have established that the addition of lags consistently raises the elasticity over this period - it does not obviously lead to unstable parameters.

The recursive estimates from 1967 are similar to those for the original LBS equation in that the variance of the estimates appears to be notably higher before the structural change in 1979 (especially if we allow for the extra 2 years of lags it may take the regime change to work through the lag structure). However, in contrast to the original LBS equation, the point estimate prior to 1979 appears - insofar as can be ascertained given the variance of the estimate - to be higher in the pre-1979 period.

To determine whether the parameter estimate is stable for the pre-1979 period it is necessary to increase the degrees of freedom. We therefore consider the use of polynomial distributed lags. However, in contrast to the previous literature, the Almon lags are applied here in the context of the Wickens-Breusch re-normalisation, and hence only to the short-run dynamics.

After some initial experimentation on the full sample estimates we chose to use a fifth order polynomial with no end-point constraints. Although the estimate is still marked by higher variance in the pre-1979 period, the use of the Almon restrictions appears to deliver a fairly stable point estimate of around -0.75 over the period up to 1980, subsequently falling during 1981 to the value of -0.5 obtained before.

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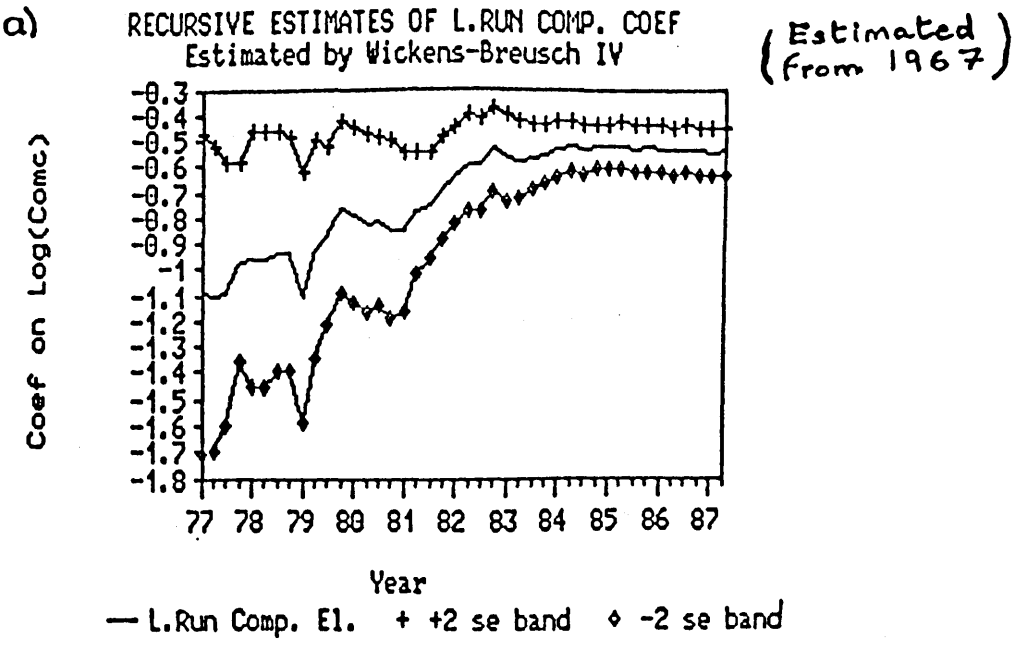
15. Of course the variance in competitiveness increases sharply here so it is not surprising that the precision of the estimate increases too, as can readily be seen from the formula for the variance of a least squares regression slope coefficient.

An alternative and more complicated explanation for the increase in the precision of the estimate from around 1979 might be that after 1979 the behaviour of competitiveness becomes non-stationary, that co-integrating superconsistency results begin to apply etc. However we would then not expect to find, as we have, that the estimates were variant to the short-run dynamics.

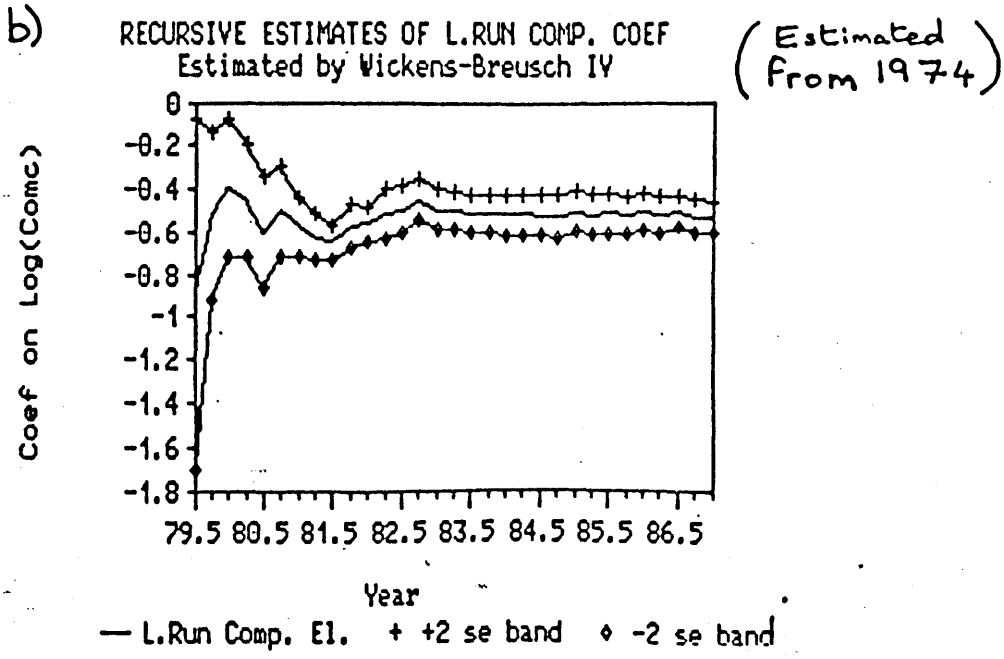
Figure 6.4

RECURSIVE ESTIMATION OF LONG RUN COMPETITIVENESS COEFFICIENT IN LBS MODEL(+13).

a)



b)



Our analysis is thus suggestive that the long-run coefficient in the estimated equation augmented with 13 additional lags on competitiveness is stable before 1979 at a value of around -0.75 and stable from 1981 onwards at -0.5.[16] This evidence is consistent with the notion that the regime change in competitiveness gives rise (via the Lucas critique) to parameter instability.

While it is possible to test successive restrictions within an Almon framework, a problem arises that the inclusion of too many lags in the Almon framework can introduce inconsistency and not just inefficiency into the estimates.[17] We must therefore consider carefully the effect on our results of introducing the Almon lag. To do this we plot the point estimates for the long-run elasticity with and without the Almon restriction alongside each other in figure 6.5. A plausible interpretation of the result would be that the introduction of the Almon restrictions does not introduce inconsistency, but merely appears to stabilise the estimate in earlier years: the Almon restrictions eliminate some of the small sample variability from the estimates of the long-run elasticity.

A version of the general (4,4,4) model with additional lags on competitiveness added up to as many as 18 lags is a possible starting point for a reformulation of the export volume equation with 'long lags' on competitiveness. Such an over-parameterised model will be even more prone to sample size and degrees of freedom problems. Nevertheless the steps considered above are repeated for what we may term the (4,4,17) formulation, and plotted at figure 6.6..

The evidence is once again that the long-run elasticity is sharply defined and stable for the period after 1982 and that the variance of the estimates is sharply increased prior to 1979. As would be expected, the small sample variability problem is at least as bad as it was for the augmented LBS equation, so once more an Almon lag is fitted to the short-run dynamics. Again this appears to remove much of the small sample size variability from the estimate, and suggests that the long-run elasticity is fairly stable

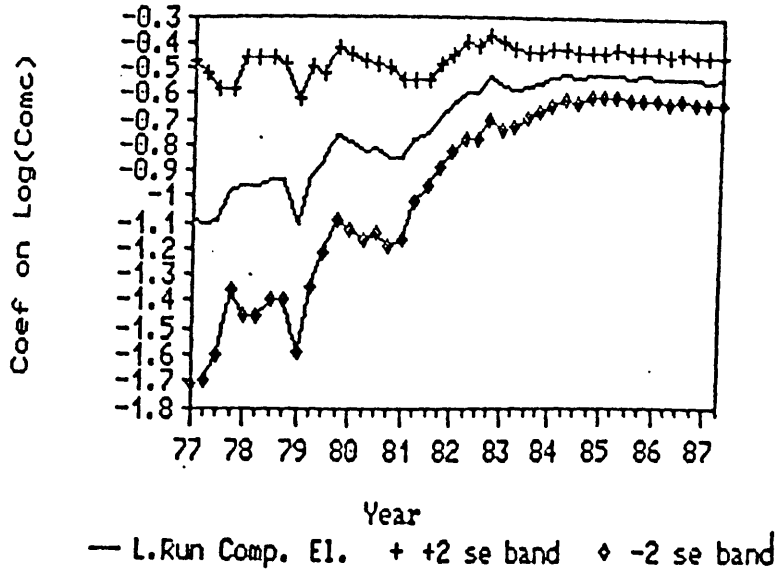
16. An alternative interpretation is that the coefficient is not stable prior to 1979, but is falling steadily until it stabilises after 1979. This reading of the estimates does not encourage an interpretation in terms of a Lucas regime shift.

17. See Sargan (1980), Pagan and Trivedi (1979), Trivedi (1984) and Hendry et al (1984).

Figure 6.5

CONTRAST OF LBS MODEL(+13) WITH AND WITHOUT ALMON PDL ON COMPETITIVENESS.

a) RECURSIVE ESTIMATES OF L.RUN COMP. COEF (13 LAG UNRESTRICTED)
Estimated by Wickens-Breusch IV



b) RECURSIVE ESTIMATES OF L.RUN COMP. COEF (5 TH ORDER ALMON)
Estimated by Wickens-Breusch IV

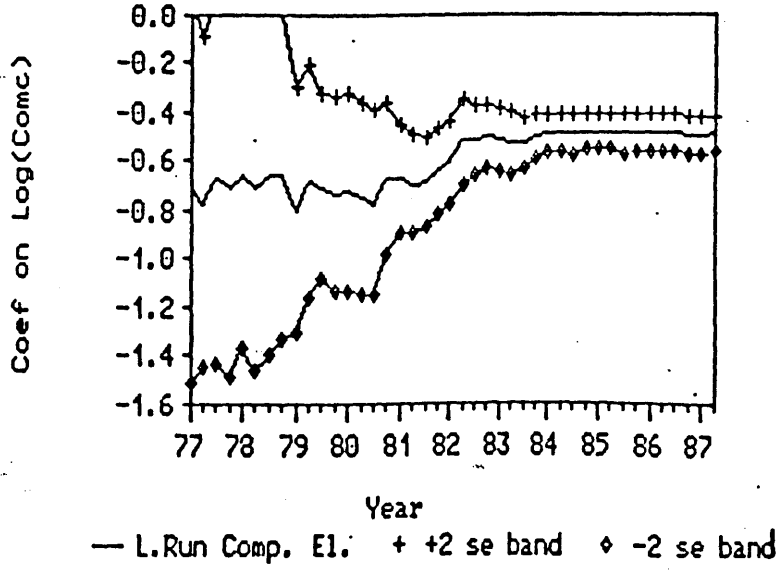
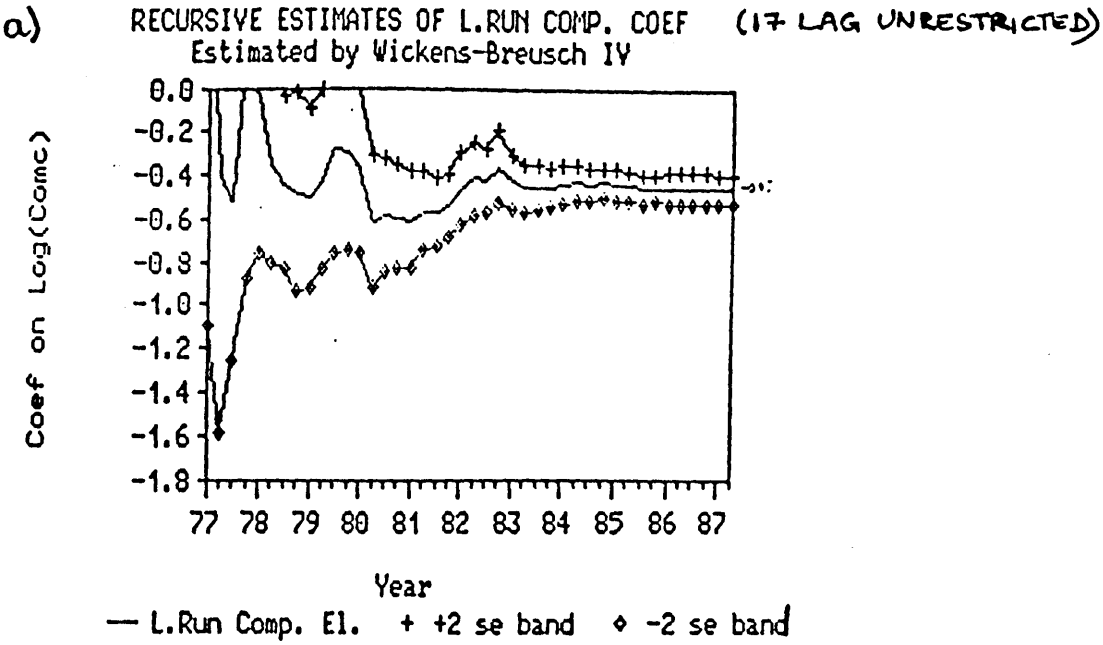


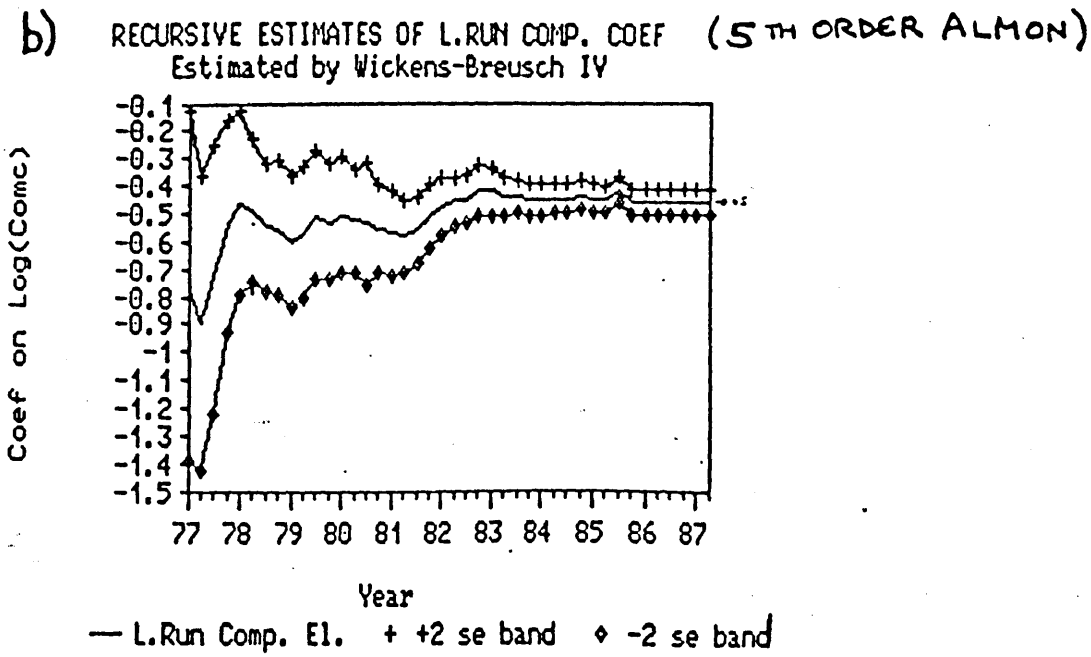
Figure 6.6

RECURSIVE ESTIMATION OF LONG RUN COMPETITIVENESS COEFFICIENT IN (4,4,17) MODEL: UNRESTRICTED AND ALMON LAG SPECIFICATIONS.

a)



b)



before 1980, at around -0.55, shifting during 1981 to a stable value of -0.45 from that point onwards. By comparing the recursive estimates of the elasticities with and without the Almon restrictions we can satisfy ourselves that the restrictions serve principally to eliminate the small sample size variability.

6.4.3 Parametric tests of stability.

The graphical evidence presented above suggests that a shift in the long run coefficient on competitiveness in the reduced form equation for manufactured export volumes took place following the 1979 regime change. We now attempt to find support for this conclusion by the use of Chow type tests of parameter stability.

Direct Tests of the Constancy of the Long Run Coefficient on Competitiveness.

Tests of the stability of this coefficient can be calculated by the insertion of a dummy for all observations in the second half of the sample to estimate any shift in the coefficient. Allowance was also made for a shift in the short run dynamics.

Such tests have been performed with both the (4,4,PDL(17)) and the LBS(+PDL(13)) formulations, variously splitting the sample after 1979(2), 1980(2), and 1981(2). The results are presented in table 6.13.

In the case of the LBS(+PDL(13)) model allowing the sample coefficients to vary appears to have no explanatory power. In no case is the dummy on either the levels term or the short run dynamics significant. In the (4,4,PDL(17)) case it is possible to reject the hypothesis of parameter constancy at a 5% level in some of the cases considered. But the point estimate of the parameter change is minute - only about 1% of the overall parameter value, although the sign is consistent with the recursive results.

Hence we find it difficult to confirm the evidence of the recursive estimation by formal statistical procedures.

Table 6.13.

TESTS OF PARAMETER CONSTANCY AROUND 1979 REGIME CHANGE.
Goldfeld Quandt (variance ratio) and Wald tests.

Our primary interest is in the stability of the long run competitiveness elasticity. But the innovation variance may also be expected to have changed in 1979. We apply parametric tests to examine the stability of two models around the regime change of 1979. The models are the LBS equation augmented by a 5th order Almon on 13 lags of DCOMT, and the (4,4,4) model augmented by a 5th order Almon on 17 lags of DCOMT.

We test for the equality of the innovation variances by use of the Goldfeld Quandt test. This is independent of the slope coefficient and is distributed as an F under the null. We use a Wald test for the constancy of the slope coefficient giving a chi-squared variate under the null. This test is independent of the constancy of the innovations variance.

1) LBS + Almon(5,13,far) on DCOMT

Pre-1979 estimates over 1967:3 - 1979:2.

Coefficient on COMT = -0.807 SSE=0.0353

Post-1979 estimates over 1979:2 - 1987:2.

Coefficient on COMT = -0.592 SSE=0.0055

Goldfeld Quandt, F(38,23) 3.894

Wald test, $X^2(1)$ 1.686

2) (4,4,4) + Almon(5,17,far) on DCOMT

Pre-1979 estimates over 1968:2 - 1979:2.

Coefficient on COMT = -0.646 SSE=0.0055

Post-1979 estimates over 1979:2 - 1987:2.

Coefficient on COMT = -0.588 SSE=0.0010

Goldfeld Quandt, F(28,16) 3.254

Wald test, $X^2(1)$ 0.302

Tests of Variance constancy or of Slope constancy.

We believe that the 1979 regime change to have been associated with an increased variance of shocks to the traded sector. A variance ratio F-test (Goldfeld-Quandt test) allows us to test for the equality of innovation variances independently of the slope coefficient and so it is natural to apply the test at this point. Hence we calculate the variance ratio statistic for the LBS(+13) and (4,4,PDL(17)) models split at 1979(2). The results are given in table 6.13.

It was possible to reject the null of constant variance at a 1% level in both cases.

Given that the variance ratio tests reject the constancy of the variance parameter we must use a Wald test of the constancy of the slope coefficients: this is set out in Pesaran, Smith and Yeo (1985). This is an asymptotic test, and our sample size is relatively small, but we can calculate it. The results are given in table 6.13.

In neither case was it possible to reject the null of a constant slope coefficient at even a 10% significance level. The Wald statistic for the more restricted case was 1.686, with a P value of about 0.8, and for the more general case it was 0.302, with a P value of about 0.4. The data do not allow us to discriminate formally between the point estimates of the competitiveness elasticity obtained for the two sub periods.

6.4.4 Conclusions on Parameter Stability.

Our attempts at parametric testing cannot reject the null of parameter constancy in a convincing manner. This is hardly surprising: given the general lack of precision of the estimates of the competitiveness elasticity we probably should not expect to be able to discriminate on this point by formal statistical means.

Our look at the stability of the long run competitiveness elasticity in the augmented or 'long lag' models has suggested that a fall in the parameter may have occured after 1979, from late 1980 onwards. This

suggestion arises from our recursive estimates of the parameter, but as we have seen, we are unable to confirm it with a formal test result. This combination of outcomes leaves us with two choices: either scepticism with respect to the findings of the recursive estimation; or else a belief that the recursive procedure has enabled us to find evidence of a phenomenon which we could not otherwise have discovered.

6.5 CONCLUSION.

In this chapter we have shown that tests reject the hypothesis that extra lags on competitiveness are irrelevant to the short run dynamic specification of the UK manufactured export volume equation. Moreover, the evidence suggests that 3 or 4 years of lags on competitiveness should be included in order to produce an equation with a satisfactory long run competitiveness elasticity.

In the absence of the required 'long lags' the current type of specification underestimates the long run competitiveness elasticity. Our results suggest that the true value may be twice the absolute value given by the current type of specification. An estimate of -0.75 for this important elasticity would appear to be preferable as a working value to one of -0.35. Such a conclusion is different from the view implicit in the Chancellor's 1986 Autumn Statement.

Once a sufficient number of lags have been allowed for, we can return to the question of alleged parameter instability. Were the Chancellor's claims in 1985/6 of a fall in elasticities based on nothing more than a failure to recognise the implications of changing equation specifications - or rather misspecifications?

Using data-acceptable Almon restrictions on the short-run dynamics in order to increase the available degrees of freedom, we find that recursive estimation of the long-run coefficient reveals a structural break in the parameter estimate at around 1980: the estimated parameter falls by some 20% to 30%

after this point.[18] This instability is in contrast to the parameter constancy in the models estimated using only one year of direct lags on competitiveness.

The apparent failure to reject stability by the use of conventional parametric tests leaves us with a judgement to make: are we inclined to believe that the recursive procedures indicate the presence of instability which the parametric tests fail to reveal? Prior information and beliefs suggest the possibility of both expectational and structural change, and this reinforces the evidence from the recursive estimation.

Overall we have shown that the existing 'consensus' models of UK manufactured export volumes are misspecified in respect of short-run dynamics on competitiveness, and that as a result they understate the long run competitiveness elasticity. Moreover, the existing models fail to detect the structural change in the long-run competitiveness elasticity which appears to occur around 1980 in the augmented reduced forms. We conclude that a structurally stable, backward looking model of UK trade volumes is neither econometrically nor economically convincing over the period. Subsequent research should seek to determine whether this instability in the augmented reduced form can be explained by a shift in expectations in response to the 1979 change in UK monetary and exchange rate policy regime. It must also consider whether the parameter change reflects a shift in the underlying structure, ie. in the fundamental behavioural parameters characterising the response of trade in manufactured exports to a change in competitiveness.

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18. The recursive estimation procedures served in this case to reveal a parameter shift. Establishing the shift as a statistically significant one is a further question. It may be optimistic to assume that the data will enable us to discriminate in this way.

Chapter Seven.

CAN HYSTERESIS IN TRADE EXPLAIN INSTABILITY IN UK TRADE EQUATIONS?

7.1 INTRODUCTION.

In this chapter we consider hysteresis effects in trade in the context of UK macroeconomic experience. We seek to apply the Dixit microfoundtions model of sunk cost hysteresis to the experience of the UK in the early 1980s, when large swings in the Sterling exchange rate produced similar movements in UK competitiveness. It is commonly believed that these shocks caused the exit of UK firms from export markets and the further import penetration of UK domestic markets, with the added suggestion that such changes might prove difficult to reverse.[1] Such beliefs can be expressed formally in the hypothesis that hysteresis effects exist in UK trade.

The UK government claimed in 1986 that the elasticity of manufactured exports with respect to competitiveness had fallen, and we have reported (in chapter six) that the long run elasticity may indeed have fallen in the early 1980s.[2] We consider whether possible hysteresis effects in UK trade could explain what we have observed in terms of parameter instability in UK export volume equations.

In order to determine whether supply side sunk cost hysteresis effects could explain instability in estimated trade equations we consider their aggregate implications. Moreover, we should also consider the possibility that demand side inertia could give rise to hysteresis effects in trade with observable implications for aggregate trade equations. The literature indicates that hysteresis effects of various kinds can yield predictions of instability in trade equations but also acknowledges the difficulty of testing for the presence of these effects.

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1. See for example the evidence given by the Vickers engineering group to the House of Lords Select Committee (quoted in Bean (1987a)) for an expression of such beliefs.

2. Our evidence of instability is based on the interpretation of recursive least squares estimates. However we were unable to reject the hypothesis of stability in formal tests.

In the absence of satisfactory and powerful *tests* of the hypothesis that instability in trade equations is due to some form of hysteresis in trade, we apply the Dixit model of supply side hysteresis in trade to UK experience. Suitable calibration indicates that the sterling appreciation of 1980 might have triggered entry and exit which would not have occurred during the less volatile experience of the 1960s and 1970s. We consider the extent to which this conclusion is robust to the specification of expectations, including the choice of stationary or nonstationary representations for competitiveness.

Using an expectations process for UK competitiveness based on first order univariate time series estimates, our simulations show that it is possible, although not certain, that firms which did not enter or exit markets prior to the 1979 regime change would have done so in response to the 1980/1 shock. Firms able to control their long run variable costs in line with expected long run revenues and with sunk costs of the magnitude suggested by Dixit would *not* have exited under consistent expectations of the real exchange rate. However, UK exporters would have exited from overseas markets in 1980/1 if expected long run losses outweighed the cautious behaviour associated with volatility.

Alternatively we could explain the exit of UK firms from their export markets as due to myopia or inconsistent expectations concerning future real exchange rate volatility. Differing expectations concerning exchange rate dynamics can also make a difference. For example, newly efficient UK producers would have contemplated re-entry into overseas markets in 1983 if they thought competitiveness were nonstationary; whereas if they thought competitiveness followed a mean reverting process they would not have re-entered.

Our simulation exercise indicates that the Dixit model of sunk costs hysteresis may have relevance to UK experience and that expectations of future exchange rates would be an important determinant of behaviour in that case. However, our attention is drawn to two factors which are not developed by Dixit. These are the determinants of long run costs and the possibility of credit constraints.

Given that hysteresis effects could reasonably be expected in UK trade in 1980/1 according to the Dixit model, we ask whether the observed reduction in the estimated long run competitiveness elasticity could be explained as an aggregate consequence of hysteresis phenomena. Hysteresis episodes of entry and exit might be marked by temporarily higher competitiveness elasticities of aggregate trade volumes: however, after the episode of entry or exit was over, the consequences for aggregate elasticities would depend on compositional effects. In fact the literature does not give clear cut predictions as to the form of instability in export equations which *may* be associated with entry and exit following from large exchange rate movements. Moreover, we also acknowledge that there are a range of other possible alternative explanations for observed parameter instability which should be considered.

Thus the hysteresis in trade model cannot provide a clear cut explanation of the apparent fall in the long run competitiveness elasticity. Yet the evidence from the calibrated Dixit model tends to confirm that the large movements in real exchange rates may have had permanent effects on industrial structure - what Baldwin and Krugman (1989) refer to as "the businessman's view". And there is a degree of confirmation of such claims in the finding of instability in export volume equations. On this evidence it would be foolish to discount the hypothesis that hysteresis in trade may account for some of the observed fall in UK export elasticities. The difficult-to-reverse exit of UK firms from export markets may indeed constitute part of "the Thatcher effect on British Manufacturing Exports".[3]

3. See Landesmann and Snell (1989) for an alternative investigation of structural change in UK export performance. They claim to find evidence for a "Thatcher effect" in an increase in the income elasticity of demand for aggregate UK exports.

7.2 AGGREGATE HYSTERESIS EFFECTS IN TRADE: THE LITERATURE ON THEORY, TESTING AND EVIDENCE.

7.2.1 Theoretical Aspects of Hysteresis in Aggregate Trade.

The literature shows that the hysteresis effects of sunk costs of entry and exit which we have examined in the single firm context survive aggregation. Baldwin and Krugman (1989) establish this point for the special case of aggregation when all firms have identical sunk costs and differ only in their variable costs. The number of firms alters when exchange rate movements are large enough to trigger the entry or exit thresholds of the marginal firm. Baldwin and Krugman go on to conclude that, with a more general distribution of industry characteristics:

"...the peculiar behaviour we have derived for a single industry will not in general get averaged away in the aggregate". (p 647).

Dixit (1989b) offers an industry model (p 213) as "a very natural generalisation of the firm model" of treatment of sunk cost hysteresis in his (1989a). The single firm is embedded within explicit assumptions concerning the structure of the industry. There is a single undifferentiated product in the industry; firms have rational expectations and are competitive. The nonlinear hysteresis property of the model is upheld, and two features are emphasised:

"Very large deviations of the exchange rate from the normal are needed to alter the numbers of foreign firms and the extent of import penetration", and "Once such a change in import share has occurred, even larger deviations of the exchange rate in the other direction are necessary to restore the former condition." (p 219).

Dixit's numerical studies suggest moreover that "the phenomenon is likely to have considerable quantitative significance" (p 221) in the aggregate model. However the industry analysis uncovers a "difference of degree" (p 215) relative to the single firm analysis, due to the fact that rational firms allow for the effect on equilibrium prices of any entry or exit precipitated by large exchange rate movements. This analysis also indicates that the hysteresis effects will have implications for pricing and the degree of pass-through to prices of exchange rate changes. The model, with competitive pricing, predicts that, in the absence of entry or exit, 'small' exchange rate shocks will only be passed

through to the market price to a very small degree. In contrast, when shocks are large enough to precipitate entry or exit, they will be substantially passed through to market prices.

Baldwin (1988) considers the sunk costs model under Cournot-Nash Chamberlinian monopolistic competition with heterogeneous varieties. There is still a hysteretic gap between entry and exit. Large exchange rate movements which provoke entry into a market will increase competition and tend to reduce mark ups (over costs which will have reduced import components). Baldwin's prediction for import volumes is that, in response to the expectation of a large appreciation of the domestic currency, the price elasticity of demand should rise "synchronous with the break in the pass through equation". There are no associated restrictions on the income elasticity or constant.

The explanation for the shift in the price elasticity is that, with heterogeneity, additional entry increases the aggregate price elasticity, while exit decreases the aggregate price elasticity. However, this prediction "is not robust to small changes in the market structure assumptions", (mimeo, p 13), and particularly if the heterogeneous product assumption is dropped in favour of homogeneous products.[4]

So far we have discussed the aggregate implications of supply side hysteresis effects in trade. However we must recognise the strong possibility that hysteresis effects may occur in trade due to demand side mechanisms. Hysteresis effects might occur in consumption patterns for a variety of reasons.

One of the first applications of the idea of hysteresis effects in trade to the experience of the 1980s was in Bean (1987a). This put a greater emphasis on *demand* side relative to supply side hysteresis mechanisms.[5] Bean referred to arguments due to Schmalensee concerning product variety (1978), and uncertainty over product quality (1982), and to arguments by Klemperer (1987) concerning switching

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4. The prediction concerning export volumes is also tied to predictions concerning price mark up, which are beyond the scope of the work in this thesis, We thus have not checked pricing evidence for corroboration.

5. Bean acknowledged the work of Baldwin and Krugman on supply side hysteresis but in practice Bean adopted non-hysteretic quasi-convex adjustment cost assumptions for his treatment of the supply side.

costs, before settling in practice for the direct assumption that, for whatever reason, consumer tastes are related to the past history of consumption. If current consumption depends on past consumption with a unit root this model has a formal hysteresis property.

Froot and Klemperer (1989) responded to the fall in the pass-through of exchange rate changes into the price of imported manufactures into the USA in the early 1980s, and argued that the implications of Baldwin and Krugman type supply side hysteresis for pricing would operate in the opposite direction than the observed fall in pass-through. Hence they offered demand side hysteresis effects as an explanation for US import prices. An interesting finding of their research was the sensitivity of the model's properties to *expectations* of future exchange rates, with the possibility that the observed fall in exchange rate pass-through in US import prices in the early 1980s reflected expectations of subsequent depreciation.

In this research we have however chosen to focus on supply side sunk cost hysteresis and on the aggregate behaviour of export volumes.

We should not leave this survey of literature on the aggregate implications of the theory of hysteresis in trade without a consideration of work which attempts a general equilibrium approach or at least seeks to incorporate wider feedback effects. We have already referred to Baldwin and Krugman (1989) in which an attempt was made to build in feedback effects from trade flows to the exchange rate; and to Krugman (1988), in which exchange rates were endogenised in a similar way. The authors nevertheless conceded that these approaches were 'quasi-partial equilibrium' due to simplifying assumptions elsewhere in the models.[6]

Sutherland (1989) has incorporated sunk cost hysteresis effects into a Dornbusch type macroeconomic model in continuous time, and Baldwin and Lyons (1989) combined them in discrete time. In both

6. Williams (1987) had previously extended the Baldwin and Krugman approach in a skeletal model which combined a forward looking exchange rate with an optimising treatment of supply side hysteresis. An analytic solution showed that supply side hysteresis would damp large exchange rate shocks through its effect on the expected long run exchange rate.

cases the entry and exit thresholds were assumed rather than explicitly solved for in the macroeconomic treatment.[7]

General equilibrium models incorporating discontinuous adjustment costs (albeit in the form of proportional rather than lump sum costs) have been developed by Dumas in a model of trade (1988b) and in a model of capital formation (1988a). Dumas's research indicates that the pattern of a discontinuous response with trigger thresholds may carry over into the general equilibrium context, although the pass-through results in pricing may be altered (Dumas, 1988b, p 3).

A different approach to general equilibrium considerations is suggested in Baldwin's (1988) analysis of hysteresis in import prices and also in his (1989) treatment of sunk cost hysteresis for a broad class of assumptions on the process followed by the forcing variable. Baldwin (1988), p 3, omits the macro model which would underlie the formation of exchange rate expectations, "simply assuming that firms perfectly anticipate the exchange rate path". and goes on to argue (p 10) that his treatment of the forcing process is sufficiently general to allow for hysteresis effects which alter the parameters of the macroeconomic model. By introducing mean reversion into the exogenous process for the price variable we obtain solutions which correspond to the general equilibrium case, as long as the assumed process is consistent with the actual mean reversion arising from the general equilibrium. This argument can be used to support our use (later in this chapter) of the partial equilibrium Dixit model to analyse the behaviour of a single firm within the aggregate context of general equilibrium.

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7. Baldwin and Lyons (1988) used a similar, highly simplified framework to analyse the effects of pricing rather than entry and exit. They demonstrated the basic economic logic of Krugman's (1989a) thesis of exchange rate instability in the "two way amplification effect" by which unresponsive trade prices amplify the volatility of exchange rates and exchange rate volatility amplifies the responsiveness of trade prices.

7.2.2 Empirical Aspects of Hysteresis in Aggregate Trade.

We now review some attempts to allow for hysteresis effects in trade in empirical work, and we comment on some of the methodological difficulties.

Krugman and Baldwin (1987) looked for hysteresis effects in trade by estimating dummy variables on the *constant* in trade volume equations. These dummies proved insignificant and Krugman and Baldwin argued that the "evidence does not support the view of hysteresis in the trade balance". However Baldwin (1988) indicates that dummies on the constant term in the export volume equation would not register the presence of hysteresis effects. Baldwin's theoretical analysis predicted that episodes of entry and exit would appear as a structural break in the constant term for estimated import pricing equations, and that the price elasticity of import volumes could also display structural change due to entry and exit (depending on market structure). Baldwin's empirical work on aggregate US non-oil import prices and volumes did not aim to test for the presence of sunk costs hysteresis directly, but sought to establish evidence of structural instability in aggregate trade equations that was not inconsistent with these predictions. Tests for the significance of slope dummies supported a structural break in the pass-through (pricing) equation, but not in the import volume equation.

The inability of tests to reject the stability of the price elasticity of US import volumes is similar to the result of the parametric tests of stability conducted in chapter six above. We should note however that this may reflect the lack of power of the test, and that recursive least squares methods might still give a strong indication of the presence of parameter instability.

Bean's (1987a) empirical study of UK exports deserves some consideration. He estimated both demand and supply side effects in different ways which were intended to allow for hysteresis mechanisms. On the demand side Bean searched for a unit root in the relative demand for UK exports. He found that the neglect of a levels term could not be rejected: either the hysteresis property should be accepted or else we should conclude that the reversion to equilibrium was very slow. Quadratic costs of adjustment

were assumed in export supply, but with higher costs to increase exports than to reduce them. Diagnostics indicated that this equation may have been misspecified.[8] Bean carefully used a long time series (from 1900) to avoid obtaining results dominated by one historical episode (the 'single event' problem), and concluded that there was evidence favourable to the idea of hysteresis in UK exports, particularly on the demand side. Bean failed to allow for the nonlinear response of trade to exchange rate movements, which would reflect discontinuities in optimal adjustment with sunk costs. There would be periods between 'large' shocks when the data displayed no sign of a unit root, in the sense that there would be no change in the equilibrium relationship underlying export volumes.[9]

What is clear from the literature is that the econometric methodology of estimation and testing remains problematic given the possibility of hysteresis effects. Baldwin and Krugman (1989) express scepticism concerning the usual dynamic techniques of continuous response, and stable lag structures to circumstances featuring hysteresis properties. They, like Bean, also emphasise the single event problem. In order to counteract these difficulties, they advocate the use of case studies. Baldwin (1988) concludes that a "direct test on macroeconomic data is impossible", despite the possibility of confirmatory type work, as there is little power to reject alternative hypotheses.

Dixit (1989b) is very critical of the 'dummy variable' approach and suggests that the development of better theories will uncover testable predictions. While it is inappropriate to treat dummy variable applications as tests of the hysteresis hypothesis, the examination of the stability of suitably chosen parameters may nevertheless be of value when taken together with other evidence. Ohno (1989) looks at disaggregated data as well as aggregate data on the pass through implications, and finds a

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8. It might nevertheless be the case that Bean's specification could serve as an approximation of hysteresis effects in the data.

9. While we may expect hysteresis shifts in the equilibrium relationship to be manifested as a nonstationarity in the time series for the relative demand for UK exports versus the rest of the world, Bean (1987a) has shown that such nonstationarity is not an automatic corollary of demand side hysteresis.

remarkable agreement in his findings across levels of aggregation.[10] However, he acknowledges that findings of structural breaks could be due, for example, to changes in the stochastic environment.

To complete this section we turn to Bertola and Caballero (1990) which considers the consequences of aggregation for models of discontinuous adjustment, and reports progress made towards the empirical application of this microeconomic insight in aggregate models. From the premise that real life microeconomic decisions may best be characterised by non-convex adjustment costs which, moreover, are often non-differentiable ('kinky') at the no adjustment point, they argue that:

"real life individuals are not solving the representative agent's convex adjustment cost problem, and its parameters have no clear 'deep structural' interpretation". (p 1).

The authors seek to characterise the patterns of aggregate behaviour which arise from such underlying microeconomics of kinky adjustment, including for example, the lump sum costs considered in chapters two, three and four of this research. They recognise (p 5) that the work of Baldwin and Krugman (1989), Dixit (1989a,b,c) and Dumas (1988b) has shown that:

"more generally, the dynamic relationship between exchange rates, activity levels and trade balances is highly nonlinear". (p 2).

For our purposes the chief interest of the paper is in two general conclusions, which we discuss below.

Firstly, whether or not microeconomic adjustment will give rise to observed aggregate time series behaviour of a discontinuous and nonlinear kind will, according to the authors, depend:

"in an intuitive way on the relative importance of on-going aggregate and idiosyncratic uncertainty". (p 2).

When aggregate uncertainty - for example, from exchange rates - predominates:

"the aggregate behaves much as any one of the individuals would, displaying strong history dependence and sluggishness". (p 2).

On the other hand, as idiosyncratic shocks - for example, shocks specific to the demand for a single firm's product - become more important:

"the aggregate behaves more and more as an individual would in the absence of any obstacle to adjustment and therefore quite unlike any one of the actual individuals in the aggregate". (p 2).

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10. See Bertola and Caballero (1990) for the conditions under which aggregate and disaggregated behaviour should be similar in the presence of lumpy adjustment costs.

What is the significance of these findings for our research? Idiosyncratic unctainty qualifies confidence in the robustness of the properties of discontinuity and nonlinearity which arise from kinky adjustment costs at the microeconomic level. Thus, unless we are able to discount idiosyncratic uncertainty, we cannot use the absence of discontinuity in aggregate trade equations as evidence against sunk cost hysteresis effects in trade. These arguments also warn against attempts to go beyond reduced form descriptions of the data towards explicit parametric models incorporating sunk costs *unless and until* aggregate and idiosyncratic uncertainty can be satisfactorily modelled.[11] Such steps are beyond the scope of this research. Nevertheless, evidence of discontinuity in aggregate relations should still prompt us to consider the hypothesis of kinky adjustment costs.

The second point of interest is that, even where:

"the presence and kinkiness of adjustment costs results in optimality of inaction, implying a wide dispersion of outcomes at a point in time and rich, history dependent dynamics",

the authors claim that:

"in the long run, endogenous variables should be well predicted on average by models of costless adjustment". (p 4).

On the face of it, this suggests that discontinuous adjustment might be irrelevant to measurement of the long run effects. However, costless adjustment models could only hope to work 'on average' if they were applied over a long enough period that all hysteresis shifts in the steady state relationship cancelled each other out. In the trade equation case this would require a long enough sample that 'large' exchange rate shocks of appreciation and depreciation averaged each other out. In the context of UK manufactured export volumes, where competitiveness is not co-integrated with export volumes, we know that estimation of the long run properties is sensitive to the short run dynamics.[12] In such a context discontinuous adjustment and nonlinear, history dependent dynamics may have an effect.

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11. We note in passing the implication that discontinuous behaviour due to kinky adjustment costs may in the meantime be more easily detected with disaggregated data.

12. Even in a model where a cointegrating relationship could be estimated, we conjecture that its superconsistent estimation in the presence of discontinuous adjustment costs would require the distribution of shocks to the hysteretic steady state to be stationary.

It would thus appear that the full implications of discontinuous microeconomic adjustment for estimated aggregate long run elasticities remain to be established.

7.2.3 Conclusions Concerning Aggregate Applications of Hysteresis in Trade.

Despite the progress reported by Bertola and Caballero, it seems unlikely that a satisfactory model of this type will be developed for UK aggregate export volumes. Such a model would have to capture expectations of future exchange rates as well as the nonlinear dynamics of possible hysteresis effects in both supply and demand: on which we have few priors. The exercise would also be frustrated by the changes of policy regime which break up the sample. The difficulty of estimating a theoretically based model of the determination of trade volumes does not prevent the search for reduced form characterisations But we judge it unlikely that an empirical implementation of the theory will enable us to test for hysteresis effects in trade. However, other kinds of evidence may point to the relevance of such effects, and this suggests a role for case studies and also for simulations and calibration.

An important dimension has so far been neglected in our discussion of aggregate aspects of hysteresis in trade. Perhaps the central point of the theoretical treatment offered by Dixit (1989a) concerns the non-certainty equivalence of optimal behaviour with sunk adjustment costs. Yet the implications of non-certainty equivalence in aggregate have not emerged as an issue from the preceeding discussion. We note, in particular, that Bertola and Caballero (1990) do not address how exogenous changes in uncertainty can be expected to affect non-certainty equivalence in aggregate patterns of behaviour. Our ignorance on these issues is regrettable if non-certainty equivalence is as important in practice as Dixit (1989a) suggests.

The complex nonlinearities and distinction between temporary and permanent shocks which arise when sunk costs are important nevertheless suggest the case for fresh consideration of the aggregate time series data. There is a case for focussing on the long run reduced form elasticities, estimated on data for

periods between large shocks. (Such measures would not claim to model behaviour during periods of entry or exit).

7.3 THE APPLICATION OF THE DIXIT MODEL TO THE UK CASE.

7.3.1 Introduction.

It is possible that there may have been threshold levels of competitiveness during the 1960s and 1970s which were not breached by real exchange rate movements until the dramatic changes subsequent to 1979. The large upswing in the real exchange rate would then have precipitated entry to the UK market by overseas firms, and exit from overseas markets by UK firms. Once such entry and exit had occurred, the subsequent path of the real exchange rate may not have fallen far enough to reverse the phenomena.

Before we calculate results for our UK estimates we can assess the plausibility of this approach by examining the orders of magnitude of the swings in UK competitiveness before and after 1979 and comparing them with the existing results on the widths of zones of inaction in Dixit's model and in Krugman's macroeconomic model. We now consider the behaviour of UK competitiveness. In our pre-1979 period (1964-1979) we find that the UK cost competitiveness index (COMT, as used in chapters five and six above) has a mean of 87 and that its oscillations are bounded below by about 75 and above by about 100. In percentage terms, the upper and lower bounds are some 14% of the mean.

From 1979 the UK competitiveness index moved rapidly to 135 in 1981 before falling back to a subsequent minimum of 75 in 1983, from which it rebounded again. The mean value for the 1979-183 period is 112, and the upper and lower bounds to the series are respectively some 20% above and 33% below the mean. However, in terms of the behaviour of the series prior to 1979, the peak of 135 represents a value some 55% above the mean of 87, while the lower bound is unchanged from the 1960s and 1970s.

In order for it to be plausible that firms which had maintained their market position throughout the 1960s and 1970s should have changed their state during the 1980s we must be able to argue that the zone of inaction is more than 14% above and below the mean for the 1960s and 1970s, but that the appropriate band width for the 1980s is less than 55% above the same value. We now consider whether existing results for the Dixit (1989a) and Krugman (1988) models are of similar orders of magnitude.

We first consider Dixit's 'central case'. Except for the absence of mean reversion, these assumptions are similar to those we will make for the UK. The solution to this case yields an entry threshold 47.3% above the variable cost and an exit threshold 23% below the variable cost.[13]

These results are consistent with the idea that the 1960s and 1970s, with maximum swings of only 14%, would not have induced entry or exit. However, whether or not the UK experience of the early 1980s could be expected to have induced entry appears to be an open question. While a 55% swing from variable costs does breach a 47.3% threshold, the result is close enough to suggest that detailed calculations with UK estimates is warranted.

Turning to Krugman's 'general case' of a mean reverting real exchange rate with uncertainty, we find that the solution to the model produces a 'range of no change' or band width of plus or minus 24.8%.[14] These numbers again suggest that no entry or exit would have taken place during the 1960s and 1970s. But this Krugman example suggests that the large real exchange rate movement of the early 1980s was large enough to have produced a resource re-allocation.

The numerical 'order of magnitude' examples we have reported from the work of Dixit and Krugman suggest that a more careful consideration of the UK experience is indeed justified.

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13. The parameterisation of the Dixit model in these calculations is $w=1$, $\sigma=0.1$, $r=0.025$, $K=4$, $L=0$.

14. The full parameterisation of the Krugman model in these calculations is $\alpha=2$, $\beta=1$, $r=0.2$, $R_T=1$, $\sigma=0.1$, $\gamma=0.5$, $r=0.1$.

We will use Dixit's single firm model, which was originally developed (Dixit (1987)) in terms of entry to and exit from an overseas market, with reference to the position of firms exporting to the USA during 1983-5. Given the way the cost competitiveness variable is defined, it is convenient for us to consider the optimal entry and exit thresholds for an overseas from exporting to the UK (ie. an analysis of UK imports). However the formal structure of the model enables us consider the behaviour of UK firms exporting to overseas markets by reversing the interpretation of the entry and exit costs, and also of the entry and exit thresholds.

Initially we assume that exit costs are zero, while sunk entry costs are significant. We solve the model for both exporters and importers for a range of different entry costs. Subsequent analysis will allow for the possibility of nonzero exit costs. We solve the model for both the pre and post-1979 UK competitiveness regimes, assuming (to begin with) that variable costs are equal to the mean value of competitiveness in each case. We choose to employ both the stationary mean reverting and nonstationary Brownian motion (random walk) representations of UK competitiveness on each occasion, which will indicate how robust historical interpretations based on the Dixit model are to the assumption of mean reversion.

7.3.2 A First Order Representation of the Time Series on UK Competitiveness.

We have already considered the time series properties of UK competitiveness in chapter five, where we used the LBS measure of tax adjusted cost competitiveness. We noted there that a clear regime change occurs in 1979 corresponding to the institutional and policy changes of that year. We also considered whether UK competitiveness is best represented as a stationary or a nonstationary variable: we concluded that the series was clearly stationary prior to 1979, and though the evidence was mixed, we also favoured the stationarity assumption after 1979.

In what follows we seek estimates to parameterise the geometric Brownian motion process for competitiveness in the Dixit model. We approximate the continuous process with estimates from a first order autoregressive process in levels on competitiveness.

We fit univariate first order autoregressive (AR1) processes for two separate time periods to reflect the regime change in 1979. The first covers the period 1964-1979. The second is chosen to cover the years of Sir Geoffrey Howe's Chancellorship from 1979-1983.[15]

We approximate the continuous time process employed in Dixit's model:

$$dP = \lambda (P^{*} - P)\, dt + \sigma P\, dz \tag{7.1}$$

by the following discrete time first order autoregression:

$$P_t = \lambda P^{*} + (1-\lambda) P_{t-1} + u_t = \alpha + \phi P_{t-1} + u_t \tag{7.2}$$

where u_t has standard deviation σ_u.

The estimate of the autoregressive coefficient, ϕ^, yields our estimate for the mean reverting parameter σ. This enables us to derive P^{*} from the estimate of the constant term α^, and σ from the estimate σ_u^, for which we assume $\sigma P^{*} = \sigma_u$^.

The restriction to first order processes prevents us from using the best fitting approximation of the time series. Hence the estimates of σ may be exaggerated. This restriction also tends to cause us to overlook the lower frequency aspects of mean reversion corresponding to the several year 'cycle' observed in chapter five.[16] On the basis of the evidence in chapter five we employ the stationary or

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15. In previous econometric work our second sample was form 1979-1987. We would have preferred to have been able to identify a further regime change in 1983, after which, under Nigel Lawson, policy once again sought to prevent large fluctuations in competitiveness. But this would have made the sample too small for the purposes of econometric estimation of multivariate dynamic models. However, in this case, as we are restricted to first order models there is no problem with the sample size and so we select the referred period 1979-1983. This ensures that the contrast in regimes before and after 1979 is made clear.

16. Using only first order representations we are unable to reject a unit root in Dickey Fuller tests.

mean reverting assumption, employing the point estimates of ϕ^ which are less than unity. However we are interested in the robustness of results to this assumption and so we will also consider the unit root case by imposing ϕ^ = 1.

The results of our estimated AR1 process (approximate point estimates only) are given in table 7.1.

From these results we approximate parameters for the stationary competitiveness process in the Dixit model (see table 7.2). The mean reverting coefficient of 0.1 prior to 1979 corresponds to a mean persistence of competitiveness innovations (given by 1/ϕ^) of 10 quarters or 2.5 years. The mean persistence after 1979 falls to 5 quarters or 1.25 years. Competitiveness innovations are thus reduced at 10% and 20% per quarter in the pre- and post-1979 regimes. Comparing the pre- and post-1979 regimes we see that the estimated standard deviation of the innovations after 1979 is doubled as compared with before 1979. At the same time the point estimate of the mean reverting coefficient has also doubled, which will partly offset the increased magnitude of the innovations. The net effect of the two changes is captured by the expression for the asymptotic variance.[17] We calculate the asymptotic variance (based on the quarterly estimates) of 6.5% for the pre-1979 stationary process and of 9.5% for the post-1979 stationary process. Hence the effect of the regime change is to increase the asymptotic variance by some 50%.

Our estimates of the processes are based on quarterly data. The standard deviation is proportional to the square root of time, and so to convert from quarterly estimates to an annual figure we simply multiply by two. The annual equivalent standard deviation of the real exchange rate is thus found to be 6% prior to 1979 and 12% thereafter. Dixit (1989a) employs 10% as his estimate of the annual standard deviation of real exchange rates. Thus our UK data appear to fall on either side of Dixit's working assumptions concerning the variability of real exchange rates.

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17. Once again we employ the expression given in the Miller and Weller mimeo for the asymptotic variance of a stationary process as $\sigma^2/(2\lambda)$.

Table 7.1

UK COMPETITIVENESS;POINT ESTIMATES OF PARAMETERS OF AR1 PROCESS

Sample Period.	Constant. (α)	Autoregressive Coefficient. (ϕ)	Standard Deviation. (σ)
1964:1 -1979:3	10	0.9	3.1
1979:3 -1983:4	27	0.8	6.7

Notes:
The autoregressive process is estimated in levels.
Approximate point estimates are given.
Standard errors of estimates are not reported.

Table 7.2

PARAMETER VALUES FOR USE IN CONTINUOUS TIME BROWNIAN MOTION REPRESENTATION OF COMPETITIVENESS. CALCULATED USING POINT ESTIMATES OF PARAMETERS OF AR1 PROCESS.

Sample Period.	Long Run Mean. P^*	Mean Reverting Coefficient. (λ)	Standard Deviation. (σ)
1964:1 -1979:3	87	0.1	0.03
1979:3 -1983:4	112	0.2	0.06

The other salient feature of the pre- and post-1979 regimes is that the mean value of the process for the second period is increased by more than a fifth. In what follows we find that the treatment of this shift will have a strong influence on our analysis.

7.3.3 Calibrating the Model Parameters.

In addition to the estimates for the competitiveness process, the calculation of plausible band widths for the UK case using the Dixit model requires two further parameter choices. Firstly the discount rate must be chosen so as to be commensurate with the time unit implicit in the competitiveness process. Secondly, we calculate the magnitude of sunk costs whose flow equivalent, given the discount rate, takes a plausible value in relation to the level of variable costs w.

The time unit in our competitiveness estimates is one quarter. We choose a discount rate of 1% which corresponds to an annual rate of over 4%. The discount rate is important because of its influence on the choice of the sunk cost parameters, (L and K). We follow Dixit's (1989a) reading of the industrial organisation literature which suggests that the flow equivalent of sunk costs is around 10% of variable costs.[18] Based on the 1960s mean value of 87 and using the discount rate 0.01 we would obtain a 10% sunkenness result by using a sunk cost of 870.

The solution to the model is sensitive to the choice of sunk cost figures, and so we must take care in this aspect of the model calibration. Firstly, Dixit's assumption that the flow equivalent of sunk costs are of the order of magnitude of 10% of variable costs may not be appropriate for the UK. More importantly, the choice of discount rate directly affects the flow equivalent of any given choice of sunk cost.[19]

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18. See Dixit (1989a), pp 630-1 for further details. Note that he first considers treating all capital costs as sunken so that sunk costs would be 50% of capital costs. However, allowing that many capital costs are either recurrent due to depreciation, or recoverable on exit, although conceding that some labour costs are sunk, Dixit settles on the assumption that sunk costs are 10% of variable flow costs.

19. A clear implication of the sensitivity of the solution to changes in the flow equivalent of given sunk costs brought about by a change in the discount rate is that a permanent change in real interest rates would change the balance between flow profits or losses and sunk costs and hence the trigger prices. In our analysis of the regime change of 1979 we have kept the discount rate constant: but real interest rates have risen.

In view of the above we let the sunk costs take a variety of values in our numerical studies on the UK case. We employ the following values: 100, 300, 600, 900 and 1800. With a 1% quarterly discount rate and constant variable costs of 87 these values imply approximate flow equivalents of sunk costs expressed as a percentage of variable costs of 1.1%, 3.3%, 6.6%, 10% and 20%. Our chosen range of values thus correspond to a spread on either side of the Dixit 'central case' of 10%.

7.3.4 Results Under the Pre-1979 Competitiveness Regime.

To begin with we report the solutions to the model of an importer to the UK facing various magnitudes of sunk entry costs and zero exit costs.[20] In table 7.3a we set out the entry and exit thresholds under the *nonstationary* competitiveness process for the period before 1979. In table 7.3b we set out the same, but this time under the *mean reverting* process for the pre-1979 period.

Turning to the case of a UK exporter facing zero exit costs but sunk entry costs, in table 7.4a we give entry and exit thresholds under the nonstationary competitiveness process for the pre-1979 period. In table 7.4b we give results under the mean reverting process. We plot all these solutions in figures 7.2.

For importers to UK markets we expect neither entry nor exit of foreign products into UK markets with sunk entry costs during the 1960s unless sunk costs were 1% or less of variable costs. (See table 7.3a). The practical implication of the mean reversion in table 7.3b is limited. The UK competitiveness regime prior to 1979 would not have been associated with the entry and exit from the UK market of overseas suppliers facing significant sunkenness in entry costs.

Similar results are obtained in tables 7.4a and 7.4b for the case of UK exporters facing entry costs to overseas markets. The pre-1979 competitiveness regime appears to have been such that UK firms

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20. Alternatively these solutions could be taken to represent the UK exporter facing zero entry costs and various nonzero exit costs, but the conventional wisdom appears to suggest that this is an unlikely combination. In this case the entry and exit thresholds would be reversed.

Table 7.3

BAND WIDTHS FOR IMPORTER INTO THE UK FACING VARIOUS SUNK ENTRY COSTS AND NO EXIT COSTS.

1960s and 1970s

a) Brownian Motion ('Random Walk') Representation.

	ENTRY COSTS (K)				
	100	300	600	900	1800
Entry P_U	96.9	102.4	108.6	113.6	126.9
P_{UC}	88	90	93	96	105
P_{LC}	87	87	87	87	87
P_L Exit	79.9	77.4	75.6	74.6	72.9

b) Stationary (Mean Reverting) Representation.

	ENTRY COSTS (K)				
	100	300	600	900	1800
		CROSSOVER RESULTS			
Entry P_U	99	118.4	150.9	183.9	na
P_{UC}	98	120	153	186	na
P_{LC}	87	87	87	87	na
P_L Exit	79.5	79.3	?	?	na

Notes:

Incomplete results due to failure of programme to report lower threshold in some cases.

$P^*=w=87$
$\sigma=0.03$
$L=0$
$r=0.01$

Table 7.4

BAND WIDTHS FOR EXPORTER FROM THE UK FACING VARIOUS SUNK ENTRY COSTS AND NO EXIT COSTS.

1960s and 1970s

a) Brownian Motion ('Random Walk') Representation.

	ENTRY COSTS (L)				
	100	300	600	900	1800
Exit P_U	95.55	99.18	101.76	103.14	106.1
P_{UC}	87	87	87	87	87
P_{LC}	86	84	81	78	69
P_L Entry	79.05	77.34	74.68	70.25	66.64

b) Stationary (Mean Reverting) Representation.

	ENTRY COSTS				
	100	300	600	900	1800
Entry P_U	96.84	97.97	?	?	na
P_{UC}	87	87	87	87	na
P_{LC}	76	54	21	-12	na
P_L Exit	77.34	58.97	?	?	na
	CROSSOVER RESULTS				

Notes:

Incomplete results due to failure of programme to cope with a negative lower threshold.

$P^*=w=87$
$\sigma=0.03$
$K=0$
$r=0.01$

facing significant entry costs would not have entered overseas markets, and were unlikely to have exited overseas markets in which they were incumbent. The assumption of mean reversion does not affect optimal exit but entry might just become optimal under mean reversion.

On this analysis, during the pre-1979 period we would only expect to find entry and exit occurring from UK trade for firms facing sunk entry costs of 1% or less of variable costs. Any greater degree of sunkenness would have made it profitable for incumbents and outsiders to remain as they were.

Table 7.5 shows that a small amount of exit costs widens the solution sufficiently to ensure that entry and exit would not have taken place during the 1960s.

7.3.5 Results Under the 1979-1983 Regime.

We now consider what would be optimal strategies for a firm facing the 1979-1983 competitiveness regime. In tables 7.6 we consider the importer to UK markets. We conclude that entry would not occur unless sunk costs of entry were less than 1%. In contrast exit would have occurred. Once again, mean reversion does not materially affect these conclusions.

As they stand, these conclusions do not square with what we have hypothesised. But it is important to note that these numbers have taken no account of the fact of a regime change.

In tables 7.7a and 7.7b we consider the UK exporter to overseas markets. We conclude that exit would not have occurred for firms facing sunk entry costs (flow equivalent) of more than 1% of variable costs. On the other hand we find that the conclusion with respect to possible entry is dependent on the mean reversion assumption.

Under unit root competitiveness, (table 7.7a), UK exporters would have entered overseas markets in about 1983 despite facing sunk entry costs up to and including 10% of variable costs. Under mean reversion (in table 7.7b) entry could not have occurred in this period. Once again it is true that when

Table 7.5

BAND WIDTHS FOR UK EXPORTER WITH SUNK EXIT COSTS IN ADDITION TO SUNK ENTRY COSTS.

1960s and 1970s

Brownian Motion ('Random Walk') Representation.

	VARIOUS EXIT COSTS (K)		
	0	100	300
Exit P_U	103.14	104.9	108.01
P_{UC}	87	88	90
P_{LC}	78	78	78
P_L Entry	70.25	66.4	66.01

Table 7.6

BAND WIDTHS FOR IMPORTER INTO THE UK FACING VARIOUS SUNK ENTRY COSTS AND NO EXIT COSTS.

1979-1983

a) Brownian Motion ('Random Walk') Representation.

	ENTRY COSTS (K)				
	100	300	600	900	1800
Entry P_U	129.7	138.8	149.2	157.6	177.71
P_{UC}	113	115	118	121	130
P_{LC}	112	112	112	112	112
P_L Exit	98.8	93.8	90.3	88.1	84.21

b) Stationary (Mean Reverting) Representation.

	ENTRY COSTS (K)				
	100	300	600	900	1800
		CROSSOVER RESULTS			
Entry P_U	133.86	168.7	?	?	na
P_{UC}	133	174	238	301	na
P_{LC}	112	112	112	112	na
P_L Exit	97.4	94.2	?	?	na

Notes:

Incomplete results due to failure of programme to report lower threshold in some cases.

$P^*=w=112$
$\sigma=0.06$
$L=0$
$r=0.01$

Table 7.7

BAND WIDTHS FOR EXPORTER FROM THE UK FACING VARIOUS SUNK ENTRY COSTS AND NO EXIT COSTS.

1979-1983

a) Brownian Motion ('Random Walk') Representation.

	ENTRY COSTS (L)				
	100	300	600	900	1800
Exit P_U	128.83	136.26	142.57	146.57	154.92
P_{UC}	112	112	112	112	112
P_{LC}	111	109	106	103	94
P_L Entry	97.83	91.23	85.17	80.57	69.92

b) Stationary (Mean Reverting) Representation.

	ENTRY COSTS				
	100	300	600	900	1800
Entry P_U	130.98	135.18	?	?	na
P_{UC}	112	112	112	112	na
P_{LC}	91	49	-14	-77	na
P_L Exit	94.98	62.18	?	?	na
	CROSSOVER RESULTS..				

Notes:

Incomplete results due to failure of programme to cope with a negative lower threshold.

$P^*=w=112$
$\sigma=0.06$
$K=0$
$r=0.01$

sunk entry costs (flow equivalent) are less than 1% of variable costs, both entry and exit would have occurred.

In this section we have found that sunk costs of entry or exit greater than 1% would probably be enough to ensure that no entry or exit took place from UK trade under the competitiveness regime prior to 1979. Under the regime after 1979 a similar magnitude of sunk costs would have prevented entry to UK markets by overseas firms and exit from overseas markets by UK firms. If competitiveness displayed a tendency to return to the mean, entry by UK firms into overseas markets would also have been inhibited.[21] As it stands, therefore, we have not yet accounted for episodes of entry and exit in our UK application of the Dixit model. To do so we must allow for unexpected regime change or for expectations of long run losses. Though we have not pursued them here, we could also consider the possible role of borrowing limits (credit constraints) and bankruptcy.

7.3.6 Expected Long Run Profitability and Myopic Expectations.

So far we have considered the solutions to the model when one competitiveness regime was taken as given, and we assumed that a steady state had evolved. In contrast the real world experience we wish to consider involves a change in the competitiveness regime at 1979. Ideally we would wish to study what would have happened to the pattern of trading activities when the pre-1979 regime was abruptly discontinued and UK competitiveness took on drastically different behaviour. Restricting ourselves to the single firm Dixit model raises problems for such an analysis: but we can still draw some conclusions.

Expected Long Run Profitability.

The 1979 regime change saw a sharp increase in the variance of competitiveness process and a partly offsetting increase in the speed of return to the mean.

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21. These conclusions are conditioned on, amongst other assumptions, the assumption that $P^* = w$. We have not checked for robustness, as it is not obvious what a reasonable alternative might be here.

However, there is a third feature of the regime change: the estimated mean of the process during the Howe chancellorship is some 29% higher than under the pre-1979 regime. As consider the effects of a transition from one regime to another, this forces us to recognise the limitation assumption that flow costs are constant.

If variable costs remain fixed after 1979 at the previous level, the consequences of the regime change are dominated by the term in P^*-w, to the extent that, for plausible entry and/or exit costs, temporary shocks to competitiveness will not influence the optimal strategy.

Although mean reverting short run shocks to competitiveness can be large enough to cause entry or exit decisions to be taken by firms facing sunk costs of entry and exit, the impact of shocks to the long run level of competitiveness will be of much greater importance.[22] Therefore, faced with the regime change in 1979, the most important decision facing UK exporters may have been to form their expectations of the long run real exchange rate relative to their cost structure. To the extent that UK exporters were unable to cut their costs and yet believed that the deterioration in their underlying competitiveness was not to be quickly reversed, we should expect the exit from overseas markets of many firms who would not have withdrawn from those markets in the smaller fluctuations prior to 1979. Hence the expectation of a relatively long lasting cost disadvantage may have led to a widespread hysteresis reaction to the regime change after 1979.

The Dixit model of the single firm treats both the long run level of competitiveness and the level of variable costs as exogenous, and does not attempt to explain how these crucial factors are determined. Having identified the relevance of these two parameters we can say little more about them in this framework.[23]

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22. See Campbell and Clarida (1987) for evidence as to the relative importance of shocks to the long run equilibrium real exchange rate amongst total exchange rate variability.

23. The focus on long run price - cost margins is moreover in accord with Maynard's (1988) interpretation of the supply side (cost cutting) consequences of the real exchange rate shock. The story is that the shock somehow enabled firms to cut unit costs in a way that would not previously have been profitable. The shock somehow affected 'X-inefficiency', or more plausibly, expectations and

There are nevertheless a number of ways in which we can use the single firm Dixit model to think about the possible consequences of the 'Sterling round trip' of the early 1980s.

Myopic Expectations Based on the Pre-1979 Regime.

Firstly we consider a firm whose variable costs are fixed at the pre-1979 level, and assume that the response to the appreciation of 1980 is given by the solutions under the pre-1979 regime rather than those under the post-1980 regime. We offer two justifications for the use of this approach. The first is that the nature of the regime change in competitiveness was quite unexpected.[24] Hence it may be reasonable to assume that the initial response would be on the basis of the past behaviour of competitiveness, as expectations had not yet changed.

The second justification takes a longer view. The arrival of Nigel Lawson at the Treasury, as we have already observed, marked a resurrection of policy concern for the exchange rate. It is possible to interpret the period since 1983 as involving a return to broadly the same competitiveness regime as might be identified for the pre-1979 period, with the Howe years as a short lived aberration. Under this interpretation the appropriate response to the experience of 1980/1 might be to treat it as an anomalous outlier to the pre-existing regime.

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bargaining power in industrial relations. In their study of the 'Thatcher Miracle?', Layard and Nickell (1988) similarly explain the improvement in British productivity growth as reflecting the fact that "managers have regained control over their work arrangements". The huge reduction in competitiveness "gave both workers and managers little alternative but to increase productivity or go under". (Layard and Nickell p 13).

24. Geoffrey Maynard (1988) suggests that the real exchange rate shock was quite unforeseen by the government itself. Amongst other commentators sympathetic to the government's broad approach, Minford (1987, 1988, 1989) has argued that problems with the Sterling M3 aggregate first caused a "failure of credibility" for an intended policy of monetary gradualism, and then, fortuitously, delivered shock treatment. Walters (1986) accepts that real exchange rate appreciation was "part and parcel" of the tighter than expected squeeze due to the "misleading" monetary indicator. Burns (1989), who does not attribute so much responsibility to the initial use of Sterling M3, nevertheless regards the exchange rate as having been "the dominant mechanism" of the monetary squeeze: but he does not suggest that this outcome was planned in the design of the Medium Term Financial Strategy.

In order to employ this approach to interpret the 1980/1 experience we return to tables 7.3 and 7.4, based on expectations of the pre-1979 regime. With the solitary exception of importers under mean reversion, the thresholds are such that the shock of 1980/1 would produce entry into UK markets and exit of UK traders from overseas markets for firms with 10% sunk costs, under the myopic expectation of a continuation of the pre-1979 regime.

Myopic Expectations Based on the Post-1979 Regime.

Suppose that the first of our two justifications is correct. Entry of importers into the UK market and exit of UK exporters from overseas markets is assumed to take place before firms had adjusted to a new regime. However, in contrast to the second explanation advanced above, we do not assume that the expectation is of a return to the pre-1979 regime. Instead we assume that the 1979 to 1983 regime is expected to be relevant for the foreseeable future. Can we now make any use of the results from tables 7.5 and 7.6.

It seems unlikely, in the case of an overseas firm which has just entered the UK market at a variable cost of 87, that as soon as it realises that a regime change has taken place it would allow costs to rise to 112 to eliminate excess profits.[25] We would rather expect monopolistic competition to work to eliminate the profits by lower prices: this effect is not captured in a single firm treatment.

However, we may find it more reasonable in the case of the UK exporter that a large real exchange rate shock could force significant cost cutting: in particular a firm that had closed an operation might be well placed to restructure it and consider whether to re-open with lower unit costs. Hence we can use tables 7.7a and 7.7b to consider whether UK exporters forced to exit in 1980/1 due to the change in exchange rate regime would have considered re-entry in 1983/4 if they could adjust their costs in line with the mean price. We conclude that re-entry would require nonstationary expectations of competitiveness.

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25. Yet see the Sunday Times, 25th September, 1989, "Courting of the Big Cat", for the argument that such a rise in costs was part of the response of Jaguar to the rise of the US dollar in 1984/5.

7.3.7 Conclusions Based on the Dixit Model in the UK Context.

At this point we draw together our conclusions based on applying the Dixit model to the UK experience of the 1980s.

Firstly, where sunk costs (flow equivalent) amounted to less than 1% of variable costs, they would not have prevented entry and exit from industries within the fluctuations of the 1960s or the 1980s.

Secondly, where sunk costs (flow equivalent) amounted to the 10% of variable costs suggested by Dixit, (on the basis of the industrial economics literature), they would be enough to prevent entry and exit during the pre-1979 regime and to prevent some entry and exit in 1979 to 1983 regime, provided that the regimes were rationally expected. We have relaxed this assumption below.

Thirdly, the assumption of mean reversion in competitiveness would have significant effects on the qualitative nature of the solutions in contrast to a nonstationary process, particularly with regard to the relative importance of certainty equivalent and caution components: with asymmetric sunk costs crossover effects may occur. However, the practical significance of these differences may be relatively unimportant. Questions as to whether importers to the UK should exit the market in 1981, and whether UK exporters should re-enter markets given up in 1981 to 1983, are the principal ones to which the answers are sensitive to the role of mean reversion in the competitiveness regime.

Fourthly, a consideration of the regime change in 1979 within the Dixit model focuses our attention of the importance of the long run level of competitiveness and also of the response of costs to external shocks: points on which the Dixit model itself is silent. Moreover the shift in the expected long run level of competitiveness implicit in the mean reverting process for the 1979 to 1983 regime would itself justify exit for UK exporters facing significant entry costs.

Fifthly, under the assumption that the response to the 1980/1 appreciation might have been based on myopic expectations of a continuation of the previous competitiveness regime, it is entirely plausible that firms with significant sunk costs which did not change status prior to 1979 may have entered or exited markets following the 1980/1 shock, thereby justifying a hysteresis in trade effect.

Sixthly, if UK exporters were able to adjust their costs into line with the long run mean level of competitiveness, they might have re-entered overseas markets in 1983. Whether or not they did would depend on whether or not they expected competitiveness to be nonstationary (leading to re-entry) or stationary (leading them to remain idle).

A final point is one we suggest for future research. We have indicated that myopic expectations and the expectation of long run losses can explain episodes of exit within our application of the Dixit model to UK experience. A further possibility, which would require development of the Dixit model would be the existence of credit constraints. Even firms with a rational expectation of mean reversion in the exchange rate might be unable to persuade financial markets or bankers to sustain large short term losses.

7.4 ON THE POSSIBLE AGGREGATE IMPLICATIONS OF THE MICROFOUNDATIONS STORY FOR THE UK CASE.

7.4.1 Introduction.

We now wish to consider whether the hysteresis effects suggested by the Dixit microfoundations could account for the observed change during 1980 and 1981 in the aggregate export volume relationship reported in chapter six.

The theoretical and empirical literature on the aggregate implications of hysteresis effects in trade suggests we are unable to test formally the hypothesis that instability in the competitiveness elasticity

is due to sunk costs hysteresis effects in supply.[26] However we can consider whether there is consistency in qualitative properties between the observed instability and the existence of sunk costs of entry and/or exit at the microeconomic level.

Once again we acknowledge that we do not know whether, or how, microeconomic non-certainty equivalence will be reflected in aggregate relationships such as export volume equations. This is a potentially serious difficulty for the interpretation of reduced form equations, particularly if they do not include an explicit role for some measure of uncertainty about competitiveness.

7.4.2 Aggregation and the Exports - Competitiveness Relationship in the Presence of Hysteresis Effects.

How would UK aggregate export volumes have behaved given hysteresis effects in trade and given the UK real exchange rate of the early 1980s. There are two parts to this exercise. The first is to contrast the properties of the exports - competitiveness relationship in export volume equations before and after the alleged episode of hysteretic exit in 1980 and 1981. The second is to give a description of the consequences for the exports competitiveness relationship of the episode of exit itself.

Baldwin (1988) has dealt with the first of these points, arguing that, under Chamberlinian assumptions "additional entry increases the aggregate price elasticity" of the observed demand curve. Transposing Baldwin's argument to the UK case in the early 1980s, and assuming product heterogeneity, it implies that the large appreciation in the real value of Sterling should have resulted in a fall in the price elasticity of UK manufactured exports at the point that UK firms exited the market. This corresponds to the observed instability in our recursive estimates in chapter six.[27]

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26. This represents little methodological progress beyond the simple dummy variable test of Krugman and Baldwin (1987). However we do refer to Bertola and Caballero's (1990) report on continuing research which seeks to relate aggregate behaviour to the microeconomics of discontinuous adjustment.

27. Note that Baldwin is unable to reject the hypothesis of parameter stability in formal tests, even where the price elasticity in his import volume equations appears to fall after the sample split. This is similar to our findings that Chow tests cannot reject stability, but we have taken our recursive estimates as indicating a likely fall in the competitiveness elasticity.

Baldwin (1988) uses the Spence-Dixit-Stiglitz assumption that the demand curves for different varieties are identical. If we relax this assumption non-identical demand curves with different elasticity properties then there would be *compositional* effects on the aggregate elasticity.[28] A change in the composition of the aggregate demand curve due to entry or exit would have an effect on the steady state competitiveness elasticity, but we cannot predict the sign.[29]

Thus the overall effect in the presence of sunk cost hysteresis of the competitiveness shock of 1980/1 on UK exports could be to lower the competitiveness elasticity of the reduced form: which is what we find in the aggregate data. The combination of hysteresis and compositional effects suggest that the shifts in the aggregate elasticity would occur in discrete episodes, corresponding to shocks sufficiently large to produce hysteretic entry or exit.

Nevertheless we must emphasise once again that the observed consistency between the empirical findings and the possible properties of a theoretical model cannot be treated as a test of hypotheses expressed in the theory model. It is, at best, corroborative evidence.

We now turn to the probable observed effects of the alleged episode of hysteretic exit in 1980/81. How might the process of exit itself affect observed relationships? Our treatment is informal and qualitative: we draw on Dixit's (1989b) treatment of industry equilibrium for price taking firms facing sunk entry costs and also on Baldwin and Krugman (1989).[30] The latter authors are able to show that:

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28. Chamberlin's (1933) development of his 'group theory' proceeded under the "heroic assumption that both demand and cost curves for all the 'products' are uniform throughout the group." (8th ed., p 82). This assumption was later relaxed, on the recognition that "Actually, of course, they differ widely" (p 110).

29. Theory gives us no guidance to the relationship between the competitiveness elasticity of demand for particular exporters' outputs and the order in which those exporters enter or exit from overseas markets. Perhaps suppliers of products whose demand is more elastic will be more likely to exit due to adverse competitiveness conditions than those whose demand is less elastic. Were this true, it would imply that the effect of exit would reduce the observed competitiveness elasticity, while entry would increase it.

30. As we have seen, Bertola and Caballero (1990) argue that the discontinuous dynamics of individual adjustment may be smoothed away in the aggregate in a stochastic model in the event that idiosyncratic uncertainty dominates aggregate uncertainty: we *assume* that this is not the case here.

Figure 7.1

TRIGGER THRESHOLDS IN F,C SPACE.

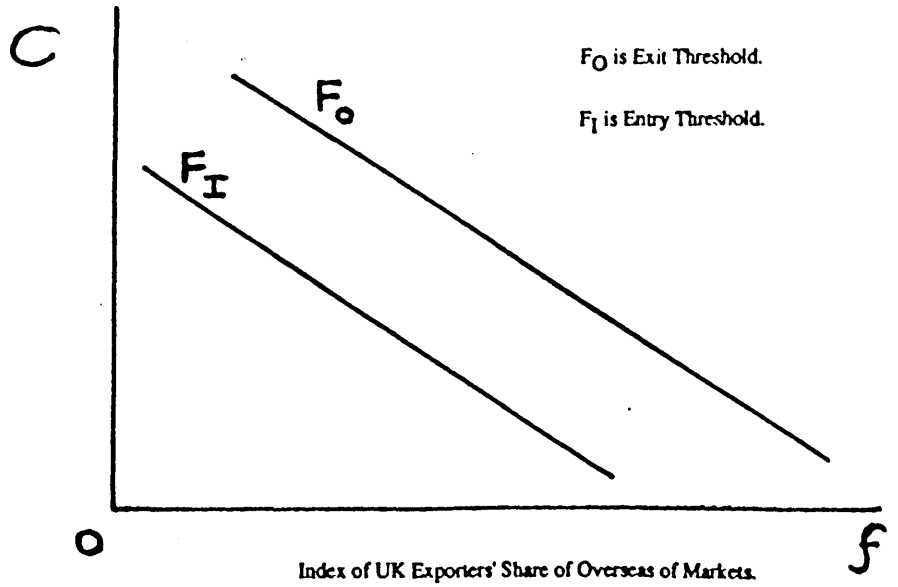
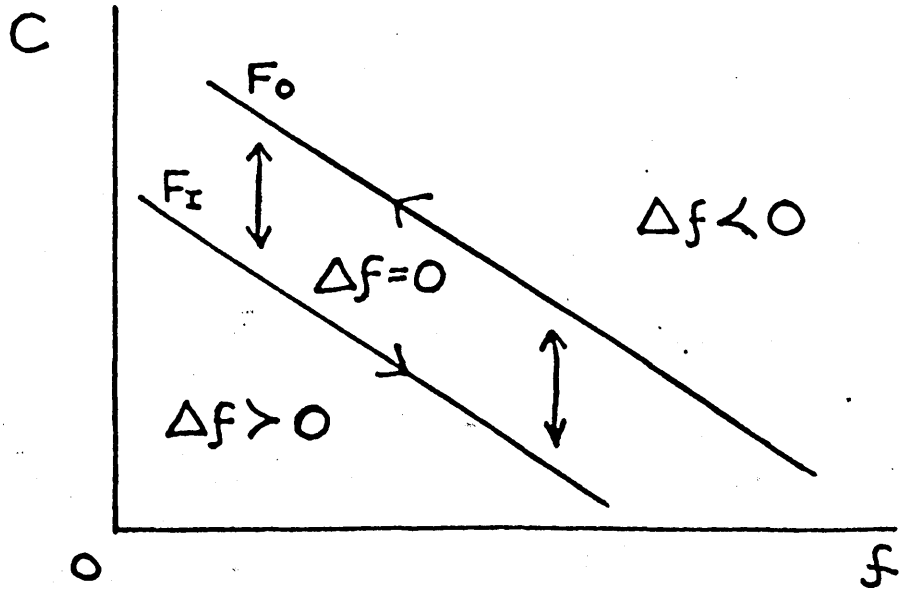


Figure 7.2

DYNAMICS OF THE SYSTEM.



"for an interesting special case the aggregate behaviour will be similar to the behaviour we have analysed for a single industry". (1989, p 642)

The special case is one in which *industries* vary little or not at all in their degree of sunk entry costs, but differ in the relative position of their variable costs.[31]

The hysteresis property of the Dixit model can be represented through the use of a diagram in f,C space, where f is taken to indicate the degree of market penetration by exporters into overseas markets (or importers into domestic markets), and C signifies the real exchange rate or competitiveness. On this we plot the trigger levels of C as a function of f.[32] These are given by the loci F^O and F^I in figure 7.1, with the 'range of no change' or 'zone of inaction' between them, for the marginal export firm corresponding to a given level of market penetration.

The dynamics of the system are also indicated in figure 7.2. Within the central band (the 'range of no change'), motion is vertical , as fluctuations in competitiveness do not cause entry or exit. However, when the entry (F^I) or exit (F^O) thresholds are reached (conditional on the initial value of f), the system moves downwards and to the right along F^I as entry occurs, and aggregate export market share (f) increases; and upwards and to the left along F^O as exit occurs.

We can translate these considerations into the exports - competitiveness space which is relevant to the reduced form exports volume relationship. In the absence of hysteresis effects in trade, the familiar reduced form relationship is that exports increase as competitiveness falls. This reduced form is usually taken to represent the aggregated demand curves for individual UK exports. We now add the hysteretic behaviour described above, in which suppliers facing sunk costs may decide to exit or enter the overseas markets, thereby shifting f, the index of UK export market share: *compositional* changes take place in the familiar aggregate trade relationship.

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31. We note that Dixit's (1989b) basic assumption is essentially similar: firms (as opposed to industries in the hysteretic sector) have fixed sunk costs and differ only in terms of variable costs. He does consider the effects of relaxing this assumption and finds the basic analysis is robust.

32. This diagram can be compared with figure III in Dixit (1989b) and figure III in Baldwin and Krugman (1989).

Assume that all UK exporters face identical overseas demand curves. When competitiveness reaches entry or exit thresholds, the aggregate demand curve is shifted parallel to itself, outwards as entry occurs, or inwards as exit occurs. At the same time the *observed* aggregate reduced form relationship displays a nonlinearity as entry and exit starts or finishes. By grafting the diagram in f,C space onto the diagram in X,C space we obtain figure 7.3. Between the entry and exit thresholds, export volumes respond to competitiveness in accordance with the slope of the demand relationship. However, as entry and exit occur, export volumes respond to competitiveness not only due to the slope of the demand curve but also due to the changing market share of UK exports as firms enter or exit: this response is greater than in the absence of entry or exit and is given by the slope of F^O and F^I.

We now apply this analysis to the circumstances of 1980/1 in which we believe exit may have taken place as competitiveness reached its peak. Figure 7.4 plots the path of competitiveness and the possible value of the competitiveness elasticity of export volumes against time under such an account. During the episode of exit the elasticity is higher than otherwise. After the episode of exit it will fall again, and may indeed be expected to return to a lower value than before the hysteretic shock for the reasons given above concerning the Chamberlinian effect of fewer varieties or due to compositional effects.

On the basis of the story sketched above the competitiveness elasticity should be higher during 1980 and 1981 than at other times as a result of the 'impact effect' of an episode of hysteretic exit. We have found no suggestion in our empirical work of the kind of 'impact effect' on the elasticity suggested by the stylised and simplified analysis presented above. However, this may not count against the possibility of hysteresis, as it is not even clear we should expect such an effect to register in our estimates of the *long run* competitiveness elasticity. As indicated above, the implications of hysteresis properties due to kinky adjustment costs for the estimates of long run properties is an area for future research.[33]

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33. Hysteresis shifts in the steady state equilibrium relationship between two variables would seem to prevent our finding a stationary relationship between them and thereby hinder the consistent estimation of long run properties: unless, perhaps, those hysteresis shifts themselves could be considered as draws from a stationary distribution. We have already expressed doubts on related points in our previous

Figure 7.3

REDUCED FORM FOR EXPORTS IN THE PRESENCE OF ENTRY AND EXIT.

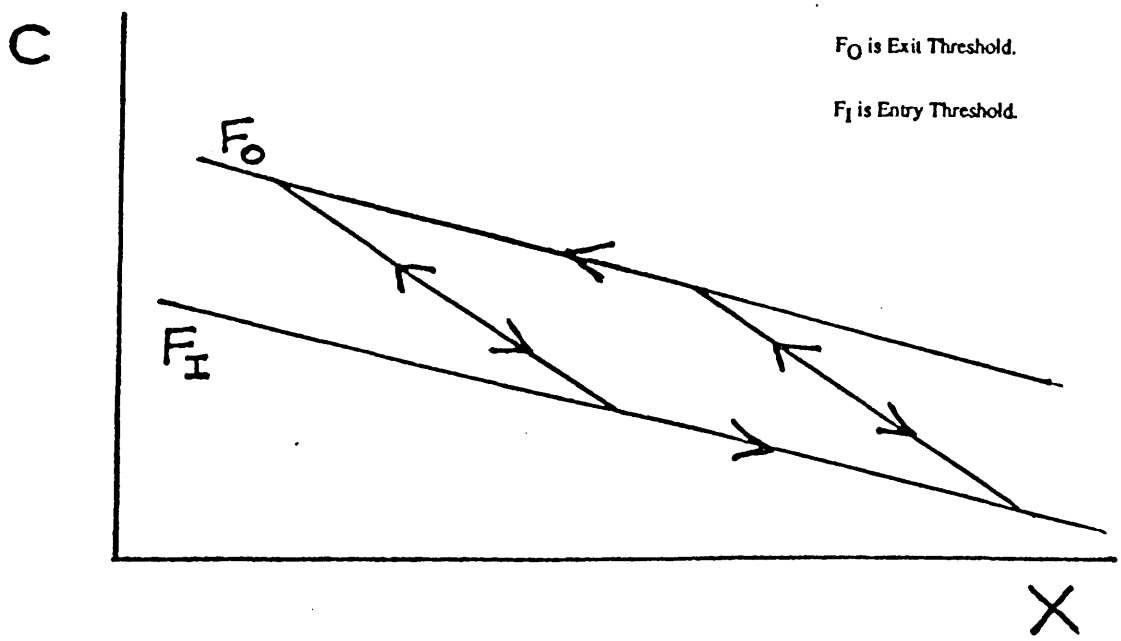
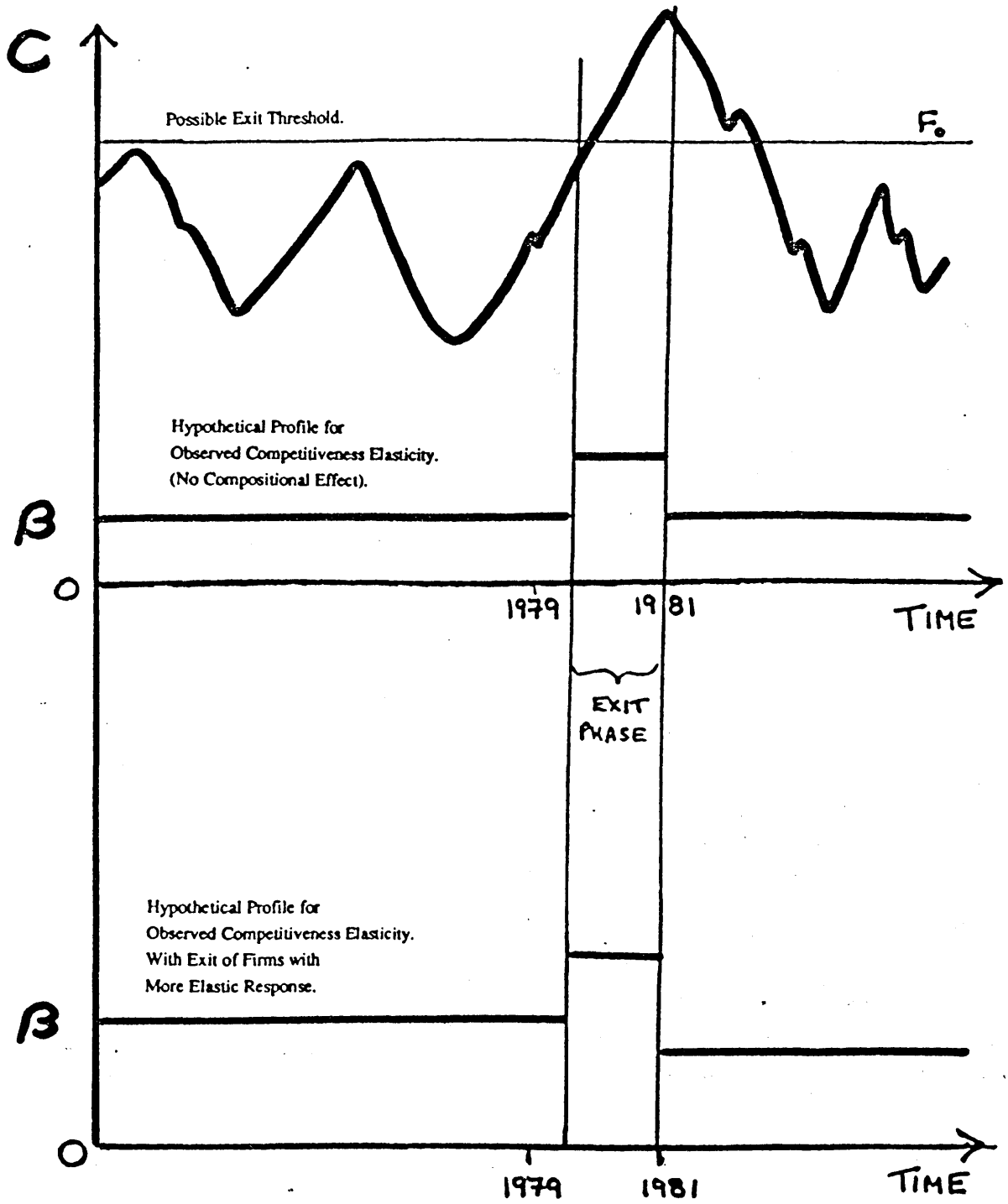


Figure 7.4

TIME PROFILE OF COMPETITIVENESS AND OBSERVED COMPETITIVENESS ELASTICITY UNDER DIFFERENT EXIT ASSUMPTIONS.



7.5 CONCLUSIONS.

In this chapter we have considered whether the Dixit model of hysteresis in trade could be employed to explain the fall in the long run competitiveness elasticity of UK manufactured exports which our recursive estimation procedures suggest takes place after 1979.

The previous work of Krugman and Dixit suggested that the magnitude of the 1980/1 shock to UK competitiveness might have been sufficient to precipitate entry and exit which would have previously been inhibited by plausible sunk costs of entry or exit. However, our work suggested that firms who correctly anticipated the future variability of competitiveness might find it optimal *not* to change state even in response to the shock of 1980/1.

On the other hand, alternative assumptions concerning the expectations of firms suggested that the 1980/1 shock could indeed have been responsible for hysteresis changes. An important possible cause of entry and exit is clearly revealed as expectations of medium term disparities between costs and international competitiveness.

The most important feature of a regime change may concern this longer term view as to the consequences of the new regime for profitability. The extraordinary productivity gains which Layard and Nickell (1988) view as UK manufacturing's life or death response to the shock of 1988 may in practice have averted what would have been much worse hysteresis effects. If costs had not been able to respond to the extraordinary extent they did through the elimination of 'overmanning' in the early 1980s, the deterioration in expected medium or long term profitability would have dominated the consequences of the new real exchange rate regime.[34]

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discussion in section 7.2.3 of comments by Bertola and Caballero on the long run with kinky adjustment costs.

34. We should be careful here, as we believe that a general equilibrium response should operate to offset severe cost disparities. Hence it might be a justifiable restriction on our partial equilibrium that costs and prices should be expected to be brought back into line.

Nevertheless, a *temporary* shock to competitiveness, such as that of 1980/1, could be expected to lead to entry and exit if it was viewed as an outlier, that is, as a shock that is large relative to expectations concerning the underlying variation in the behaviour of competitiveness.

If we view the events of 1979 to 1983 as constituting a quite different policy regime for competitiveness, with sharply increased volatility, then the most likely cause of possible entry and exit in 1980/1 may have been myopia and the unexpected nature of the new regime. Increased volatility of the dimensions of the Howe years, rationally expected, might simply have caused firms to be prepared to put up with more severe short run losses rather than give up their initial state. If the volatility was underestimated, then the likely speed of return from the shock may also have been underestimated.[35]

We also found that in *some* circumstances a decision to enter or exit could depend on whether the competitiveness process was expected to revert to a mean value or not: entry and exit thresholds would tend to be further apart with mean reversion, though not necessarily enough to make a practical difference to behaviour.

We have noted that the Dixit model does not allow for credit constraints which might precipitate exit when it would otherwise have been optimal to stay in the market. This is a possible direction for the extension of research into hysteresis effects in UK trade.

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35. Many observers of the UK industrial scene in 1980/1 might be surprised at the conclusion that firms with rational expectations should have accepted losses for the duration of the competitiveness shock. Exit may, as we have suggested, reflected the unanticipated nature of the regime shift; or it may have occurred because of beliefs about a permanent deterioration in average price cost margins, as suggested by the sensitivity of model results to the term P^*-w.

However, even where firms had rational expectations concerning the regime change and believed that they could successfully address the longer term competitiveness issue, they may still have had to close down because their bankers would not accept huge short run losses. The assumption in the Dixit model that firms are unconstrained in capital markets for rational borrowing to finance the 'short run' losses associated with maintaining the asset of incumbency may not match UK industrialists' experience.

We have considered the likely implications of Dixit type hysteresis phenomena for aggregate trade relationships. We conclude that such phenomena do not offer a fully convincing explanation for the observed behaviour of the estimated long run competitiveness elasticity of UK manufactured export volumes after 1979. While they could lead to a fall in the competitiveness elasticity from that prevailing before the episode of exit to that prevailing afterwards we could also expect to find a temporary increase in the estimated elasticity during the period of exit itself. Although we have observed some fall in the elasticity from before 1980/81 to afterwards, we have not observed any temporary increase coinciding with the alleged episode of exit: this latter observation is mitigated by our doubts that such a property would be captured in estimates of long run elasticities.

A further difficulty with the aggregate modelling of hysteresis effects is that we have no account of how microeconomic non-certainty equivalence will be translated into aggregate relationships, and hence of what the effect on those aggregate relationships may be of changes in the second moment of the competitiveness regime.

Overall then, while we judge it plausible on the basis of our numerical work that Dixit type hysteresis effects could have been in operation in UK trade in 1980/1, the aggregate data on export volumes do not appear to provide clear evidence to support the contention.[36] At the same time, while hysteretic entry and exit nevertheless may have occurred in 1980/1, we cannot with any confidence assert that such entry and exit can account for the fall in the long run elasticity of competitiveness which we have detected in the aggregate data.

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36. This tends to confirm the conclusions of other authors that this issue might be best addressed on a firm or industry level.

Chapter Eight.

CAN THE LUCAS CRITIQUE EXPLAIN INSTABILITY IN UK TRADE EQUATIONS?

8.1 INTRODUCTION.

This chapter investigates the role of *expectations* in explaining the observed behaviour of UK manufactured exports. Firstly, we ask whether the evidence supports the interpretation of export volume equations in terms of forward-looking behaviour. Secondly, we consider whether forward-looking behaviour can explain observed parameter change and hence makes it unnecessary to seek explanations in terms of structural change, of possibly unspecified form. Because the 'structural change' and 'expectational change' interpretations of the observed instability in the estimated long run competitiveness elasticity have differing economic and policy interpretations, we should assess their relative importance.

The chapter begins by setting out a general signal extraction framework within which a regime change in competitiveness will imply a different optimal filtering rule and could thus explain the decrease in the long run elasticity reported earlier in the thesis. We discuss the plausibility of expectations and errors in variables rationales for signal extraction in the context of export volume determination.

The chapter proceeds to a univariate time series analysis of the competitiveness process such as we would employ in a weakly rational expectations model of export behaviour. This sheds light on two questions. Firstly, are long lags on competitiveness helpful in order to forecast current and future realisations? Secondly, can we identify empirical evidence of regime changes, and in particular, of any regime change with a significant effect on the long run filter?

The chapter then employs the 1979 regime change in the estimation of various expectational models for the pre- and post-1979 periods. Some doubts remain as to the homogeneity of the post-1979 data and these suggest that difficulties may arise with the estimation of explicit expectations models.

Aggregate UK export volume equations may be vulnerable to the Lucas critique insofar as they fail to identify invariant *deep parameters* underlying behaviour due to the absence of an explicit treatment of expectations. In the context of a particular expectations model, the observed parameter change in the long run elasticity estimates might be explained by a shift in the time series behaviour of competitiveness, and hence a shift in expectations formation.

We can investigate the relative importance of the alternative explanations of observed parameter change - 'structural' or 'expectational' - as long as we are prepared to make assumptions concerning one of the two. We attempt to test for structural stability conditional on various assumptions as to the role of forward-looking expectations formation in export behaviour.

The chapter attempts to draw together the fragmentary pattern of evidence from these investigations. We are unable to find a satisfactory explicit treatment of the expectations interpretation of the export volumes equation; and yet the time series properties of univariate competitiveness offer some encouragement to an expectational interpretation of the role of long lags on competitiveness in the estimated reduced form. Moreover the theory of intertemporal optimisation indicates forward looking behaviour. We therefore choose to make assumptions as to the relevance and implications of forward-looking behaviour for the estimated elasticities and attempt a test of the hypothesis of structural stability conditional on the expectations assumption. Assuming an expectational interpretation, we find that the undoubted instability in the expectations process does not translate into a clear cut shift in the univariate *long run* signal extraction filter. But the variance of the estimated long run univariate filter is such that we cannot reject the hypothesis that the observed parameter instability in the reduced form reflects the Lucas critique consequences of the assumed forward-looking behaviour and that the fundamental parameters of the underlying behavioural model display structural stability.

However, the results as a whole give only lukewarm support to the Lucas critique interpretation of the observed fall in the competitiveness elasticity that no fundamental structural change had occurred. Non-homogeneities in the data, (suggesting that the true DGP is characterised by many short episodes), and also insufficient variability in the data, (the instability in the long run parameter and the change in the univariate long run filter are both too small relative to their precision), combine to prevent either a convincing rejection of structural stability, or a clear endorsement of the Lucas critique explanation of regime change.

The research reported in this chapter neglects one potentially important issue. Our estimation treats the aggregate export volumes relationship as *certainty equivalent*, despite the possibility that, for example, sunk cost hysteresis effects will give rise to significant non-certainty equivalence. If non-certainty equivalence is important, then its neglect is a misspecification, giving rise to a Lucas Critique in the second moment of competitiveness. A large change in exchange rate uncertainty, for example, could give rise to apparent parameter instability in the misspecified equations.

We bring the chapter to a close with a discussion of why it is nevertheless important for policymakers to consider both expectations and structural change. The failure of particular expectations model does not prove the irrelevance of the Lucas critique. So policymakers should not draw conclusions concerning structural change in UK manufactured exports behaviour without independent theoretical and empirical evidence for some specific type or types of structural change.

8.2 AN ANALYTIC FRAMEWORK FOR THE TREATMENT OF EXPECTATIONS IN EXPORT EQUATIONS.

8.2.1 Introduction.

In chapters five and six we have estimated autoregressive distributed lag reduced form relationships for UK manufactured export volumes. The general form of these equations is given by:

$$A(L)\,X_t = B(L)\,W_t + C(L)\,C_t + v_t \qquad (8.1)$$

where, as before, X_t represents UK manufactured export volumes, W_t the volume of world trade, and C_t tax adjusted UK cost competitiveness. A(L), B(L), and C(L) are polynomial lags of varying lengths, and v_t is an iid random disturbance term.

The estimated long run competitiveness elasticity is biased if the short run dynamics are restricted. That UK competitiveness follows a stationary process confirms the potential relevance of the expectations interpretation of the long lags on competitiveness. We consider whether a model of export volumes incorporating an explicit treatment of expectations can be developed within the reduced form equation of chapter six.[1]

Instability in the reduced form could be due to the hysteretic exit explored in chapter seven, or due to the expectational effects of regime change considered in this chapter. Alternatively, it could reflect some less explicitly specified 'structural change', such as the supply side improvements in product quality espoused by Walters (1986, p 141, footnote). We outline a framework within which to consider these possibilities.

8.2.2 The Analytic Framework.

We now set out our analytic framework for a signal extraction approach to export volume equations which does not depend on a particular structural model of behaviour, and which allows for both

1. As explained in chapter five, in declining to determine the structural supply and demand components we have followed a familiar approach to the specification of UK export volumes: a variety of alternatives theories of market structure would suggest different behavioural specifications. (But see Holly and Wise (1989) for a recent formal attempt to separate supply and demand). The task of specifying an aggregate structural model would be all the more difficult in the presence of lumpy adjustment costs (See Bertola and Caballero (1990))

explicit forward expectations behaviour and errors in variables. Within this framework the applicability of the Lucas critique to the long run properties is made clear.[2]

In order to implement such a "signal extraction" interpretation of our reduced form equation we do not seek to frame the Lucas critique in terms of a specific optimising model.[3] Instead we posit that the reduced form relationship exists as a general feedback relationship of unknown form on a set of variables which include the unobserved signal which we proxy by suitably filtered competitiveness. One interpretation of this approach is that we conduct a specification search with few priors apart from the assumption of 'weakly rational' or 'partly rational' expectations of competitiveness. We have so far estimated the unrestricted reduced form equation, (see 8.1 above), which we can rewrite in Wickens - Breusch form as:

$$A(1)X_t = B(1)W_t + C(1)C_t + a(L)\hat{}X_t + b(L)\hat{}W_t + c(L)\hat{}C_t + v_t \qquad (8.2)$$

where long run elasticities can be derived using only the coefficients A(1), B(1), C(1).

We seek to explain the instability of the estimates of the long run elasticity on competitiveness, C(1), through the consideration of signal extraction models.

Under our general signal extraction approach let us characterise the data generation process (DGP) by the following equations:[4]

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2. Lucas (1976) drew attention to the fact that the parameters of models which failed explicitly to identify forward-looking behaviour when it was relevant could not be expected to prove invariant to interventions (eg. policy regime changes) which altered the way in which expectations were formed. Lucas summarised his argument thus: "given that the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models." (p 41).

3. Optimal behavioural rules in intertemporal decision problems justify including forward expectations in econometric models: given preferences and technologies, we can solve for optimal decision rules. But, where we have not settled on an appropriate specification of the decision problem, cost structures, etc, (or, as argued by Bertola and Caballero (1990) adjustment costs are lumpy), the representative agent's solution to a particular version of the decision may not characterise aggregate behaviour. Hence we choose not to rely on this approach.

$$\alpha(L)\, X_t = \beta(L)\, W_t + \gamma(L)\, E(C_{t+1}{}^{P} | C_t) + v_t \quad v_t = (0, \sigma^2{}_v) \tag{8.3}$$

$$(1 - H(L))\, C_t{}^{P} = u_t \qquad u_t = (0, \sigma^2{}_u) \tag{8.4}$$

$$C_t = C_t{}^{P} + e_t \qquad e_t = (0, \sigma^2{}_e) \tag{8.5}$$

As before, X_t, W_t and C_t denote the volume of UK manufactured exports, the volume of world trade and UK competitiveness respectively. $C_t{}^{P}$ denotes the unobserved signal corresponding to economically relevant or 'permanent' competitiveness, and $E(C_{t+1}{}^{P} | C_t)$ denotes the one step ahead expectation of permanent competitiveness conditioned on the current value of observed competitiveness.

Equation 8.4 is the state equation describing the evolution of permanent competitiveness which we assume to take a purely autoregressive form, while 8.5 is the measurement equation showing that we observe permanent competitiveness with error.[5] Equation 8.3 gives the usual reduced form relationship, but allowing a role for expectations of future competitiveness.[6] Note, however, that equation 8.3 does incorporate the restriction that competitiveness enters only in the form of its expected value. (Relaxing this restriction would be straightforward).

In order to arrive at a version of the reduced form equation which incorporates the restrictions implied by the signal extraction framework of expectations and/or errors in variables we must express $E(C_{t+1}{}^{P} | C_t)$ as an explicit function of observable competitiveness.

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4. We acknowledge Hendry's stated methodological preference (Hendry, 1988) for beginning with a VAR system from which we could proceed to test the validity of various conditioning assumptions. As in Hendry (1988) however, our starting point here is a widely accepted single equation specification.

5. The observed process corresponding to an autoregressive signal subject to uncorrelated independent measurement error is an ARMA process. (See Granger and Newbold (1986, chapter 1.8)

6. We do not commit ourselves to a particular theoretical interpretation of this equation, but note tht one convenient explanation for its reduced form status is as representing, in aggregate, monopolistically competitive supply to the demand for differentiated UK manufactures.

Combining these formulae for conditional prediction and updating, and invoking the steady state Kalman gain, ($\phi = \phi_t$ for all t), we obtain the expression:

$$E(C_{t+1}{}^{P}|C_t) = [\ \{\phi\ (H(L)/L)\ \}/\{1-(1-\phi)H(L)\}\ ]\ C_t = F(L)\ C_t \qquad (8.6)$$

Substituting back into 8.3 we obtain the following expression for the restricted reduced form for this signal extraction problem.

$$\alpha(L)\ X_t = \beta(L)W_t + \gamma(L).F(L)\ C_t + v_t \qquad (8.7)$$

We can now see directly the relevance of Kelly's (1985) cautionary note. The 'structural' parameters $\gamma(L)$ are not estimated by the unrestricted estimates of C(L) in equation 8.1, which are equivalent to the convolution $\gamma(L).F(L)$.[7] The long run competitiveness elasticity estimated in the unrestricted reduced form is given by C(1)/A(1), (where C(1) denotes the sum of the coefficients of the lag polynomial C(L)). This estimate reflects the convolution of 'structural' and 'expectational' (or 'filter') dynamics, and we can thus write:

$$C(1)/A(1) \ = \ [\ \gamma(1).F(1)\ ]/\alpha(1) \qquad (8.8)$$

The observed parameter instability estimates of the long run competitiveness elasticity could be due to any of three different causes: a change in the 'structural' parameters given by $\gamma(1)$; a change in the persistence of the signal, as measured by H(1); or a change in the signal to noise ratio as captured by the Kalman gain, ϕ.

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7. A more formal treatment of the problem is given by Hendry and Neale (1988, p 810), whose argument, applied to this example, is that, since the parameters $\gamma(L)$ cannot be recovered from the conditional model (what we have referred to as the unrestricted reduced form) without knowledge of the parameters $(H(L),\phi)$ in the marginal model for C_t, then it follows that Ct is not *weakly exogenous* for $\alpha(L)$, $\beta(L)$, $\gamma(L)$. Weak exogeneity is a sufficient (and usually a necessary) condition for valid conditional inference. Moreover, weak exogeneity is necessary for super exogeneity, which is in turn a requirement for a conditional model to be structurally invariant to interventions such as changes in policy regimes. See Engle, Hendry and Richard (1983) for the definitions and implications of the concepts of weak and super exogeneity, and of structurally invariant conditional models.

We can apply this analysis to the fall in the estimates of the long run competitiveness elasticity (C(1)) after 1979 as claimed in chapter six. This could reflect a fall in the long run response of export volumes to a change in permanent competitiveness, $\gamma(1)$. Alternatively it could reflect a fall the signal extraction filter, F(1), with a given movement in observed competitiveness implying a smaller change in permanent competitiveness than before.

8.2.3 Errors in Variables and Pure Expectations Interpretations of the Signal Extraction Approach.

An explicit role for expectational terms will arise when the behavioural basis for the export volumes equation is one of intertemporal decision making. Note however that a very simple justification will suffice to ensure an explanatory role for one step ahead expectations of competitiveness, even in an otherwise entirely static model. We need only assume that either the importers or the exporting firms bear some of the currency risk (ie. it is not all hedged in financial markets) and that settlement takes place with a single period lag. In such circumstances the estimating equation should incorporate the one period ahead expected level of competitiveness.[8]

An errors in variables interpretation is always available for a rational expectations model, in that the observed outcome of the expected variable consists of the expected value *plus* the forecast error. However, regardless of this expectations effect it is quite plausible that the economically relevant measure of competitiveness should differ from the observed variable. There is considerable ambiguity as to the appropriate price or cost competitiveness variable for the reduced form export volume equation (see Enoch (1978) etc). Forward cover could also cause the price relevant to export behaviour to differ from the observed spot rate. Concerning forward cover, some evidence has been offered to support the view that although there is often a desire to hedge currency risk, actual practice for exports

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8. Note also that such a model would require as many 'expectational' lags (ie. observations on which predictions are conditioned) as would a more complex model depending on expectations of several periods ahead.
Modest further complications would be introduced to the estimating equation if currently observed export volumes depend on lagged expectations, as they might in the presence of delivery lags.

from the UK varies, with some exporters bearing the currency risk themselves.[9] We can probably safely assume that institutional developments during the 1980s have increased the extent of forward cover, and hence decreased the signal to noise ratio. However, although errors in variables will invalidate weak exogeneity for 8.2, if the Lucas critique is to apply to the *long run* properties of the unrestricted reduced form, then there must be some expectations component involved.[10]

8.2.4 The Particular Vulnerability of the Long Run Competitiveness Elasticity to Expectations Arguments.

Our focus is on the long run properties of the export volume equation. Hendry and Neale (1988, pp 814-5), establish that Kelly's warning concerning the inconsistency of long run parameter estimates is not relevant for regressors which share a common trend.[11] If a regressor is I(1) and co-integrated with the endogenous variable, then estimates of the long run relationship will be consistent.

We have already (in chapter five) made the case that export volumes and world trade share a common trend or are co-integrated while, in contrast, competitiveness is a stationary variable. Therefore, given our concern for long run properties, we need only consider signal extraction and Lucas critique issues with regard to the coefficient on competitiveness.

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9. See Carse, Williamson and Wood (1980).

10. When we evaluate the long run we set the lag operator to unity. So the pure errors in variables filter $\{\phi/(1-(1-\phi)L)\}$ becomes $\{\phi/(1-(1-\phi))\}$, which is unity.

11. Hendry and Neale argue that "if Y_t and X_t are co-integrated, then, despite invalidly taking X_t as weakly exogenous, no bias results in the long run solution." Hence, "the desired levels solution is ... correct, and an 'expectations' critique is irrelevant." On this basis sufficient conditions for the validity of the long run properties of the unrestricted reduced form are thus that the variables are I(1) and co-integrate. The 'expectations critique' will still apply to the estimates of the short run dynamics.

8.3 UNIVARIATE TIME SERIES PROPERTIES OF COMPETITIVENESS.

8.3.1 Introduction.

What kind of expectational lags might be justified within an expectations version of the export volumes relationship? How would they have been affected by the 1979 regime change? In order to answer these questions we consider two questions concerning the univariate time series representation of competitiveness. Firstly we consider whether the time series is one for which an appropriate filter involves long lags. Is competitiveness usefully represented by a process involving a long autoregressive component or else by a process with a significant moving average component? Secondly, given our focus on the long run competitiveness elasticity of export volumes, we must consider whether the long run filter for univariate competitiveness changes with the regime shift at 1979.

Recall the main results of our estimation of first order univariate representations for competitiveness in chapter seven. We concluded that the persistence of shocks to competitiveness may have fallen after 1979 although, allowing for larger shocks, the net effect of the 1979 regime change was to increase the variability of future competitiveness.

The fall in the autoregressive coefficient suggests that the long run competitiveness elasticity can be expected to fall after 1979 if expectations are important determinants of export behaviour.

8.3.2 Preferred Low Order Univariate Representations.

We can approximate any stationary univariate time series by a low order ARMA model.[12] Hence our first efforts to improve on the first order representations of chapter seven explore a range of low order

12. See eg. Box and Jenkins (1976), Fuller (1976) etc.

autoregressive moving average and mixed processes. For each of the sample periods considered a preferred representation was chosen on the basis of smallest estimated standard error. Further sample partitions were also considered in order to indicate whether further regime changes demand recognition: a break in mid 1973 could reflect the breakdown of Smithsonian parities and one in mid 1985 could reflect the end of the Howe chancellorship during which external considerations were neglected in UK monetary policy. We set out the point estimates of the parameters of the preferred processes in table 8.1 along with the estimated equation standard errors.[13]

On the basis of equation standard error, the preferred representation for the entire sample is a second order autoregression (AR2). However, when the sample period is broken at 1979, an ARMA(2,2) process is preferred for the post-1979 sample, with an AR2 process prior to 1979.[14] We can also consider the estimates of the equation standard error for the various sample periods. If we split the sample only at 1979, the estimated standard error approximately doubles from 0.034 to 0.037 prior to 1979 to 0.078 afterwards.

What can we learn from these estimated univariate processes?

Firstly, we have found that first order autoregressive representations can be improved on.

Secondly, we have found that for some sub-periods the data supports an ARMA(2,2) representation. It encourages the notion that the relatively distant past provides information for forecasting future values.

Thirdly, we find that the variance of innovations increases after 1979 as compared with before, which is consistent with the view that the removal of capital controls and continued internationalisation of capital markets would be associated with an increase in the volatility of the nominal exchange rate.

Fourthly, while there is strong confirmation of a regime change in 1979, there is evidence of further regime changes in the time series. It seems unlikely that the data could discriminate between many of

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13. Note that we do not present standard errors for the individual coefficients: this reflects the exploratory nature of the original estimations.

14. In the case of second order autoregressions fitted prior to 1979 the evidence of the 12th order Box-Pierce diagnostic tests suggests that there may be residual autocorrelation present. This would indicate that we might improve on the AR2 representation.

Table 8.1

UK COST COMPETITIVENESS (TAX ADJUSTED)
POINT ESTIMATES OF UNIVARIATE TIME SERIES PROCESSES.
(S.E.s omitted to emphasise exploratory purpose.)

i) Full Sample.

1964:1 - 1987:1

AR2 $(1 - 1.21L + 0.29L^2)\,C + 0.36 = e$

Equation S.E. = 0.041 (cf. 0.043 for AR1)

1964:1 - 1983:4

AR2 $(1 - 1.28L + 0.35L^2)\,C + 0.30 = e$

Equation S.E. = 0.039

1968:3 - 1987:2

AR2 $(1 - 1.19 + 0.27L^2)\,C + 0.35 = e$

Equation S.E. = 0.039

ii) Sample split only at 1979.

1964:1 - 1979:2 * (See notes)

AR2 $(1 - 1.06L + 0.21L^2)\,C + 0.66 = e$

Equation S.E. = 0.034 (cf. 0.036 for AR1)

1968:3 - 1979:2 ** (See notes)

AR2 $(1 - 1.04L + 0.23L^2)\,C + 0.85 = e$

Equation S.E. = 0.037 (cf 0.039 for AR1)

1979:2 - 1987:1

ARMA(2,2) $(1 - 0.67L + 0.24L^2)\,C + 0.35$
$= (1 + 0.59L + 0.03L^2)e$

Equation S.E. = 0.078 (cf 0.336 for AR1)

iii) Shorter sub-periods.

1964:1 - 1973:2

AR2 $(1 - 1.13L + 0.27L^2)\,C + 0.65 = e$

Equation S.E. = 0.026

1973:3 - 1979:2

ARMA(2,2) $(1 - 1.30L + 0.75L^2)\,C$ + constant
$= (1 - 0.67L + 0.87L^2)e$

Equation S.E. = 0.075

1979:2 - 1983:4

ARMA(2,2) $(1 - 1.89L + 0.99L^2)\,C + 0.48$
$= (1 - 0.47L - 0.41L^2)e$

Equation S.E. = 0.047 (cf 0.50 for AR1)

1983:3 - 1987:1

AR2 $(1 - 0.88L + 0.40L^2)\,C + 2.31 = e$

Equation S.E. = 0.045 (cf 0.049 for AR1)

Notes:

Using the 12th order Box-Pierce statistic, the null of no autocorrelation among residuals is barely accepted in the marked cases at the levels

* 5% level

** 1% level

these hypotheses in practice, so for practical reasons we will continue to employ a sample separation based only on the single regime change in 1979.[15]

8.3.3 Higher Order Autoregressive Representations.

So far we have found evidence of moving average components in the post-1979 regime for competitiveness. In the pre-1979 regime there was some suggestion of residual autocorrelation in the AR2 processes. We now try and extend the autoregressive specification to higher order processes to capture these effects. Higher order autoregressions could justify an expectations interpretation of the long lags in the reduced forms of chapter six. In table 8.2 we present the estimated equation standard errors for AR2, AR13 and AR17 representations of competitiveness for the entire sample, for the pre-1979 period and the post-1979 period. These higher order processes were chosen to correspond to the lag lengths of the reduced forms in chapter six.

An examination of the estimated equation standard errors shows strong evidence of improved fit relative to the AR2 representation in the post-1979 period for the AR13 and AR17 representations. But there is at best only a small reduction in the standard error for the pre-1979 period, and also for the entire sample. The explanatory power of higher order lags is examined by F tests of the zero restrictions on higher order lags implicit in the AR2 representation relative to the AR13 and AR17 alternatives. These test statistics are reported in table 8.3.

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15. Following Hendry (1988), we would hope for a change in time series properties so clear cut that the implied change in the parameters of the reduced form under the expectations hypothesis must be statistically significant. A particular worry must now be that the practical need to treat the entire period after 1979 as homogeneous will understate the extent of regime change in 1979, and thereby blur any difference between alternative contingent plans and expectations models. From the econometrician's point of view it seems it would have been preferable if Nigel Lawson had continued with Geoffrey Howe's relative neglect of external considerations.

Table 8.2

HIGHER ORDER AUTOREGRESSIVE REPRESENTATIONS OF COMPETITIVENESS: ESTIMATED EQUATION STANDARD ERRORS.

Sample Period	Specification	Equation Standard Error
1964:1-1987:2	AR2	0.04136
	AR13	0.04149
	AR17	0.04144
1968:2-1987:2	AR2	0.04398
	AR13	0.04368
	AR17	0.04381
1964:1-1979:2	AR2	0.0347
	AR13	0.03518
	AR17	0.03424
1968:3-1979:2	AR2	0.03726
1968:2-1979:2	AR13	0.03586
	AR17	0.03494
1979:2-1987:1	AR2	0.257
	AR13	0.06066
	AR17	0.06599
	(ARMA(2,2) gives	0.078)

Table 8.3

F TESTS OF AR2 SIMPLIFICATION OF AR13 AND AR17 REPRESENTATIONS OF COMPETITIVENESS.

Sample Period	Alternative Hypothesis	Degrees of Freedom	F-Statistic
1964:1-1987:2	AR13	11,80	0.966
	AR17	15,76	0.977
1968:3-1987:2	AR13	11,63	1.000
	AR17	15,59	0.969
1964:1-1979:2	AR13	11,48	0.854
	AR17	15,44	1.107
1968:2-1979:2	AR13	11,31	1.205
	AR17	15,27	1.307
1979:2-1987:1	AR13	11,18	45.666 *
	AR17	15,14	28.393 *

Notes:

Null hypothesis is that AR2 is an acceptable restriction of alternative hypothesis.

* signifies rejection of null at 1% level.

These results show that the restrictions inherent in the simplification of the AR13 and AR17 alternatives to an AR2 representation cannot be rejected at conventional significance levels for the period prior to 1979 (or for the sample as a whole). In contrast, the restrictions associated with the AR2 process are strongly rejected for the post-1979 period.

These results reinforce the idea of a sharp regime change at 1979. The distant past becomes much more important to prediction after 1979 than before, whether through higher order autoregressions or a moving average error process. However we must acknowledge that there is little or no support here for the hypothesis of long expectational lags prior to 1979.

The inclusion of long lags in the reduced form may have to be explained by delivery lags etc. ie by 'structural lags'. But the equation may still depend on expectations and hence be vulnerable to the Lucas critique.

8.3.4 Long Run Filters and Regime Change.

We first review the measure of regime change that is most relevant to our concerns: change in the long run properties of the one step ahead expectations filter. We then consider the values which this measure takes under a variety of representations and how it changes with the sample period.

The case of a pth order autoregressive signal plus white 'measurement error' noise gives rise to an ARMA(p,p) process for the observed variable.[16]

For a general ARMA(p,q) process given by:

$$(1 - \phi(L))\, Ct = (1 - \theta(L))\, e_t \qquad (8.9)$$

the long run expectational dynamics will be given by:

16. Granger and Newbold (1986), chapter 1.8.

$$F(1) = \{ (\phi(1) - \theta(1)) / (1 - \theta(1)) \} \tag{8.10}$$

$$F(1) = \{(\Sigma_{i=0}^{p} \phi_i - \Sigma_{j=0}^{q} \theta_j)/(1 - \Sigma_{j=0}^{q} \theta_j)\} \tag{8.11}$$

We can use this formula to calculate the *point estimates* for the long run filter based on any given ARMA representation. We can thus determine directly from the point estimates whether a particular regime change should have Lucas critique implications for the long run reduced form competitiveness coefficient.[17]

We will later consider the standard errors of the estimated quantity F(1) in order to determine whether a particular regime change would yield a statistically significant change in the long run filter.

In table 8.4 we set out point estimates of the long run filters implicit in a range of univariate representations of competitiveness. We give the long run filters corresponding to the preferred representations considered above in table 8.1; and also those corresponding to first order autoregressive processes (the autoregressive coefficient). In general the results in table 8.4 show that the longer the sample period considered the higher the estimated long run filter. This may reflect the ngeglect of structural breaks in the series.[18] The consequences of a regime change at 1979 may not be understood if the post-1979 estimate of the long run filter is biased by neglect of a further break at, say, 1983.

On the basis of preferred representations for the short sub-periods in table 8.4 it would appear from the point estimates that the long run filter falls somewhat after 1973 from 0.86 to 0.63 before falling much further after 1979 to 0.17. After 1983 it rises again to 0.48. Numbers for the longer periods are in sharp

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17. When the time series is non-stationary, and hence has a value of $\phi(1)$ equal to one, then the long run filter F(1) must also take the value of unity. This is consistent with Hendry and Neale's (1988) observation that with non-stationary regressors, (and *only* in this case), a failure to allow for expectational effects will *not* affect the estimated long run properties.

18. The order of integration of variables subject to structural breaks or non-homogeneities tends to be overestimated. See Fisher et al (1989), p 15, Perron (1989).

contrast with those obtained from the four shorter sub-periods, though the major divergence between the estimates occurs after 1979.

We favour the interpretation that the consequence of regime change at 1979 was to increase the mean reverting tendency of the series, ie. to reduce the persistence of shocks. This is based on a view of agents attempting to filter out short run movements. (We implicitly assume that agents are able to identify regime changes immediately and adjust their expectations accordingly.)

The results of table 8.4 complicate the search for an expectations model of UK manufactured exports volumes. We have identifired a single parameter (the long run filter) which could focus on the Lucas critique effect of regime change. If the persistence of competitiveness falls after 1979 then a fall in the long run competitiveness elasticity is consistent with a Lucas critique explanation. But if we use the single sample break at 1979 (as we have to in chapter six) this interpretation would appear to be lost. It thus appears that attempts to fit an expectations model will be defeated by the non-homogeneity in the data.

8.3.5 Direct Estimation of the Long Run Filter.

If we wish to test hypotheses concerning the constancy or otherwise of the long run filter we need to obtain standard errors. In the case of purely autoregressive processes direct estimates of the long run filter and its standard error can be obtained by a rearrangement of the model similar to those considered by Wickens and Breusch for multivariate dynamic models. For example, the second order autoregression is given by:

$$C_t = h_1 C_{t-1} + h_2 C_{t-2} + e_t \qquad (8.12)$$

should be rewritten thus:

Table 8.4

LONG RUN FILTERS IMPLICIT IN VARIOUS REPRESENTATIONS OF COMPETITIVENESS. (POINT ESTIMATES ONLY)

Sample Period	Preferred Process	Preferred Long Run Filter	cf. AR(1) Filter
1964:1-1987:2	AR(2)	0.92	0.94
1968:3-1987:2	AR(2)	0.90	0.94
1964:1-1979:2	AR(2)	0.85	0.88
1968:3-1979:2	AR(2)	0.81	0.84
1979:2-1987:1	ARMA(2,2)	0.94	0.96
1964:1-1973:2	AR(2)	0.86	n.a.
1973:3-1979:2	ARMA(2,2)	0.63	n.a.
1979:3-1983:4	ARMA(2,2) (cf. AR(2)	0.17 0.59)	0.85
1983:3-1987:1	AR(2) (ARMA(2,1)	0.48 0.70)	0.63

$$C_t = (h_1 + h_2) C_{t-1} - h_2 (C_{t-1} - C_{t-2}) + e_t \qquad (8.13)$$

In this form the coefficient on the lagged levels term is the desired long run filter and can be estimated directly.

We estimate four different autoregressive representations over the whole sample and over pre- and post-1979 samples. The standard errors give us a guide to the precision of these estimates. The four autoregressive specifications we estimate are a first order autoregression, a second order autoregression, a thirteenth order autoregression and a restricted fourteenth order autoregression. The restriction on the fourteenth order regression is that the lagged differences of competitiveness are included in the form of a third order Almon lag.[19]

The results of these estimations are given in table 8.5.

The effect of increasing the order of the autoregression appears to be to lower the point estimate of the long run filter. In the cases of the AR13 and restricted AR14 process there is also a loss of efficiency. For the thirteenth order autoregression the estimated long run filter falls from 0.738 before 1979 to 0.725 after 1979. This is a negligible fall and a hypothesis of parameter stability would not be rejected. For the fourteenth order autoregression the effect of the regime change at 1979 is to lower the estimated long run filter from 0.754 before 1979 to 0.362 after 1979.

Unlike the pre-1979 period we find that the restricted (Almon lag on difference terms) AR14 estimate differs sharply from the unrestricted AR13 estimate. We must prefer the unrestricted estimate as a possible model of expectations.[20]

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19. This is done to save degrees of freedom, and mirrors the use of Almon lags in the Wickens and Breusch specifications of the export volumes equations in chapter six.

20. Note that when recursive estimates of the univariate long run filter were obtained, the observed tendency was for the estimates to rise after 1979. This is consistent with the results of Table 8.5 in which the estimates are uniformly higher for the entire sample than when truncated at 1979.

Table 8.5

LONG RUN FILTERS FOR 1 STEP AHEAD FORECASTS FOR VARIOUS UNIVARIATE REPRESENTATIONS OF COMPETITIVENESS.

Sample Period	Process	Long Run Filter (H.C.S.E.)	Equation S.E.
1963:2-1987:2	AR(1)	0.943 (0.0414)	0.0423
1963:3-1987:2	AR(2)	0.925 (0.0385)	0.0410
1967:3-1987:2	AR(13)	0.875 (0.0414)	0.0446
1967:1-1987:2	AR(14) (PDL)	0.867 (0.0639)	0.0439
1963:2-1979:2	AR(1)	0.902 (0.0505)	0.0353
1963:3-1979:2	AR(2)	0.871 (0.0547)	0.0341
1967:3-1979:2	AR(13)	0.738 (0.136)	0.0379
1967:1-1979:2	AR(14) (PDL)	0.754 (0.1432)	0.0355
1979:3-1987:2	AR(1)	0.955 (0.0679)	0.0549
1979:3-1987:2	AR(2)	0.927 (0.0631)	0.0536
1978:2-1987:2	AR(13)	0.725 (0.123)	0.0502
1979:3-1987:2	AR(14) (PDL)	0.362 (0.1535)	0.0487

Notes: H.C.S.E. are White's Heteroskedasticity Consistent Standard Error estimates.

AR(14) estimates employ Almon polynomial lags

8.3.6 Conclusions on Univariate Time Series Representations.

We have considered two questions relating to the competitiveness time series. Does univariate prediction of the time series require higher order autoregressive terms or a moving average component, so that expectational lags might involve long distributed lags? Did the policy regime change of 1979 require a significant change in the filter through which agents might forecast future competitiveness?

We have found mixed evidence on the possibility of long expectational lags. After 1979 there appears to be a role for either a moving average component in an ARMA representation or else for a high order autoregressive process. However there is little support for long expectational lags prior to 1979. (This of course does not discount the possibility that expectational factors may still be important.)

There is evidence of regime change at 1979, if only in the change in the necessary short run dynamics. However this regime change is not associated with clear-cut evidence concerning a change in the long run properties of the one step ahead forecast filter. Hence even if expectational lags were important we cannot be sure that in practice the theoretically relevant Lucas critique would apply to the 1979 regime change.

Although the evidence concerning the effects of the 1979 regime change on the long run filter is mixed, we suspect that the recognition of a further structural break at 1983 would lead to support for the notion that the persistence of competitiveness fell after 1979 in addition to an increase in the innovation variance. Reduced persistence would be consistent with a possible Lucas critique explanation of the observed fall in the long run competitiveness elasticity. But the successful recovery of a stable expectations model would become more difficult from the smaller sub samples associated with multiple structural breaks.

Overall, this explanation of the univariate properties of competitiveness casts doubts as to the likely success of our attempts to establish whether export volumes are sensitive to expectations of competitiveness and whether observed parameter change can be explained by the Lucas critique. Sceptical of the chances of success, we will nevertheless continue with the analysis.

8.4 CONCERNING THE ESTIMATION OF EXPECTATIONS MODELS.

8.4.1 Introduction.

We now consider attempts to estimate models of UK manufactured export volumes which include an explicit role for expectations of competitiveness. These attempts have not uncovered a satisfactory expectations interpretation of the reduced form equations of chapter six.[21] We nevertheless report the approach adopted and the results of various attempts to implement an empirical model corresponding to the expectations hypothesis. We note that these attempts may have been unduly restricted by their implicit certainty equivalence, given the theoretical grounds for suspecting that, in the presence of sunk costs, uncertainty concerning future competitiveness could inhibit action.

8.4.2 Specification and Estimation Procedures.

In order to implement the possible estimation methods we must formulate a particular form of our (weakly rational) expectations hypothesis. We search for an explicit weakly rational expectations (univariate filtering) model corresponding to the unrestricted reduced forms of chapter six. Seven different particular forms of the expectations hypothesis were estimated and these are detailed in tables 8.6 and 8.7, where the estimation methods employed for each are also recorded.

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21. The failure to find a convincing particular specification does not of course require us to reject the general notion that expectations of competitiveness may play an important role in determining export volumes.

Table 8.6

CANDIDATE SPECIFICATIONS AND ESTIMATION METHODS FOR EXPECTATIONS MODELS WITH CURRENT AND LAGGED EXPECTATIONS ONLY.

'Error Correction' Specifications

PagGR	X = DX, W, DW, C^, constant, DD
I.V.s:	X(-1), W, W(-1), C^, constant, DD
McCsub	X = DX, W, DW, C, constant, DD
I.V.s:	X(-1), W, W(-1), C^, constant, DD

'4,4,4e' Specifications

PagGR	X = DX,...DX(-3), W,DW,...DW(-3), C^, DC^,...DC^(-3),cons, DD
I.V.s:	X(-1),...X(-4), W,W(-1),...W(-4), C^, C^(-1)...C^(-4), cons, DD
McCsub	X = DX,...DX(-3), W,DW,...DW(-3), C, DC,...DC(-3),cons, DD
I.V.s:	X(-1),...X(-4), W,W(-1),...W(-4), C^, C^(-1)...C^(-4), cons, DD

'LBS type' Specifications

McCsub	X = DX, DX(-1), W, C, DC, DC(-1), DC(-2), cons, DD
I.V.s:	X(-1), X(-2), W, C^, C^(-1), C^(-3), cons, DD

In all the above specifications we generate C^ from the following augmented process for C:

C = C(-1), DC(-1)...DC(-12), X(-1), X(-2), W, W(-1)

Notes:

PagGR denotes estimation by generated regressors method, following Pagan (1984).
McCsub denotes estimation using the McCallum substitution technique.
I.V.s denotes the instrumental variables employed.
X is export volumes, W is world trade, and C is competitiveness. DX is the 1st difference of X and C^ is the conditional expectation of C.

Table 8.7

CANDIDATE SPECIFICATIONS AND ESTIMATION METHODS FOR EXPECTATIONS MODELS WITH FUTURE EXPECTATIONS AND CURRENT/LAGGED ACTUAL VALUES OF COMPETITIVENESS.

'Error Correction + Futexp' Specifications

PagGR X = DX, W, DW, C, C^(+1), cons, DD

I.V.s: X(-1), W, W(-1), C, C^(+1), cons, DD

McCsub X = DX, W, DW, C, C(+1), cons, DD

I.V.s: X(-1), W, W(-1), C, C^(+1), cons, DD

'4,4,4 + Futexp' Specifications

PagGR X = DX,...DX(-3), W,DW,...DW(-3), C, DC,...DC(-3), C^(+1), cons, DD

I.V.s: X(-1),...X(-4), W,W(-1),...W(-4), C, C(-1)...C(-4), C^(+1), cons, DD

McCsub X = DX,...DX(-3), W,DW,...DW(-3), C, DC,...DC(-3), C(+1), cons, DD

I.V.s: X(-1),...X(-4), W,W(-1),...W(-4), C, C(-1)...C(-4), C^(+1), cons, DD

'LBS type + Futexp' Specifications

McCsub X = DX, DX(-1), W, C, DC, DC(-1), DC(-2), C(+1), cons, DD

I.V.s: X(-1), X(-2), W, C, C(-1), C(-3), C^(+1), cons, DD

In all the above specifications we generate C^ with the augmented process for C given in Table 8.6.

The following modified LBS equation was estimated jointly with an AR(14) process for C:

X = X(-1),X(-2),W,C(-1),C(-2),C(-3),C(-4),C^,cons,DD

where C^ is the conditional expectation from:

C = C(-1), ... C(-14), cons.

I.V.s: X(-1),X(-2),W,C(-1),...C(-13),cons,DD

Notes:

PagGR denotes estimation by generated regressors method, following Pagan (1984) McCsub denotes estimation using the McCallum substitution technique. I.V.s denotes the instrumental variables employed.

X is export volumes, W is world trade, and C is competitiveness. DX is the 1st difference of and C^ is the conditional expectation of C.

Three specifications of the expectations hypothesis for the export volumes equation are estimated in which competitiveness enters only in the form of current and lagged expectations, (pure expectations models). These three differ from each other in terms of dynamic specification, and are detailed in table 8.6.

One is similar to an error correction specification, featuring current values and first lags of the dependent variable and world trade, but with the expected level of current competitiveness. A second specification corresponds to the 4,4,17 reduced form of chapter six. This is achieved by substituting current and lagged expectations based on a 13th order representation for actual values of competitiveness into a 4,4,4 specification. A third case considered corresponds to the LBS specification detailed in chapter five, except that current and three lagged values of competitiveness are replaced by the corresponding expectations.

Three 'mixed' specifications are considered in which competitiveness enters the export volume equation both in the form of one step ahead forecasts of competitiveness (equivalent to forecasts of any number of steps ahead) as well as in the form of current and lagged values of actual competitiveness. One case adds the one step forecast of competitiveness to a backward looking error correction type model; another adds forecast competitiveness to a 4,4,4 model in actual competitiveness; a third model consists of the LBS model augmented by a term in expected future competitiveness.

A fourth variant on the null hypothesis of a 'mixed' forward and backward looking model is used in experiments at joint estimation of an export volumes equation with a univariate 14th order process for competitiveness. The export volumes equation takes the form of an LBS type model, but with four lags on actual competitiveness along with expectations of current competitiveness included. Details of all these 'mixed' specifications are given in table 8.7.

Initially estimates were obtained by the generated regressors technique in which the predicted values of competitiveness from the assumed exogenous process for competitiveness were used as regressors in

the export volume equation. In order to take account of issues raised by Wickens (1982) and Pagan (1984) the chosen procedure was close to the 2SLS procedure which Pagan (1984), p229, indicates is required for correct specification of the standard errors. The risk of inconsistency in the estimated standard errors is not crucial at this initial stage of our exploration of possible specifications. Moreover, as Pagan (1984) also points out, the OLS (as opposed to 2SLS) standard errors are appropriate under the null of a zero coefficient on the expectations terms, implying that the t-statistics on those terms are satisfactory.

Alternative estimates were obtained using an errors in variables estimation procedure in which the expected value of competitiveness is replaced by the actual realisation. This procedure, (the McCallum (1976) substitution), requires instrumental variables estimation. Following Pagan (1984) we use the predicted values from the process for the competitiveness variable as instruments; we employ as our process for competitiveness 13th order autoregressions augmented by current and lagged world trade and two lags of export volumes. We note that Pagan (1984), p 229, suggests this errors in variables procedure as an alternative to 2SLS for the consistent estimation of standard errors.

The third procedure is the joint estimation of the univariate process for competitiveness (in this case we use a fourteenth order univariate autoregression) along with the exports equation. The cross equation restrictions between the univariate process and the expectations filter are made explicit in this approach.

8.4.3 Estimation Results

We now present the results of our attempts to estimate expectations versions of the UK manufactured exports equation. The various specifications described above were each estimated over sample periods prior to 1979 and subsequent to 1979.

In table 8.8, by way of preliminaries, we set out salient statistics for the 13th order autoregressive processes for competitiveness and also for the augmented version employed in the generated regressor approach and as the instruments for the errors in variables estimator.

The point estimate of the long run value implicit in the forecast filter for the purely autoregressive case is 0.738 for the period 1967:3 - 1979:2 and is 0.725 for the period 1978:2 - 1987:2. Recalling the estimates of chapter six (see table 6.11) the equation standard errors for the LBS(+13) model are 0.0406 before 1979 and 0.0201 after 1979. The estimated long run competitiveness elasticity in this reduced form falls (in absolute value) from -0.888 before 1979 to - 0.667 after 1979.

We set out statistics for the pure expectations models for the period prior to 1979 in table 8.9, and for the period subsequent to 1979 in table 8.10. We set out details for the mixed expectations and feedback models in tables 8.11 and 8.12 for the pre- and post-1979 periods respectively. Where models contain both forward looking expectations and feedback dynamics the long run response to expected competitiveness must be re-scaled to derive the implied response to a given shift in observed competitiveness. The resultant elasticity corresponds to the reduced form elasticity between export volumes and observed competitiveness. The comparison with the unrestricted reduced form estimates of chapter six indicates the impact of the restrictions imposed by specific assumptions concerning the role of expectations and structural dynamics.

A simpler method of calculating the net responsiveness is simply to sum the long run responses to observed and expected competitivneess. The result has validity as a measure of the responsiveness of export volumes to an anticipated, permanent shift in competitiveness, in which case there will be a one to one correspondence between shifts in expected and observed competitiveness.

In table 8.13 we give both these contrasting measures of the net long run competitiveness elasticity for the mixed cases. In order to calculate the reduced form equivalents the long run filter coefficients are taken from the univariate AR13 processes given above, with 0.738 as the filter prior to 1979 and 0.725 as the filter after 1979. We can also calculate the implied reduced form elasticities corresponding to the

Table 8.8

AR(13) PROCESSES FOR COMPETITIVENESS ESTIMATED
FOR USE AS GENERATED REGRESSORS IN EXPECTATIONS MODELS.

1967:3 - 1979:2

AR(13)

$$C = 0.738\ C(-1) + DC(-1)....DC(-12) + 1.159$$
(HCSE) (0.136) (0.604)

Equation SE = 0.03793 SSR = 0.04891
Adjusted R^2 = 0.7589 n = 48, k = 14

Augmented AR(13)

$$C = 0.704\ C(-1) + DC(-1)....DC(-12) + 1.107 + X(-1) + X(-2) + W + W(-1)$$
(HCSE) (0.298) (1.627)

Equation SE = 0.03827 SSR = 0.04394
Adjusted R^2 = 0.7545 n = 48, k = 18

1978:2 - 1987:2

AR(13)

$$C = 0.725\ C(-1) + DC(-1)....DC(-12) + 1.256$$
(HCSE) (0.123) (0.563)

Equation SE = 0.05022 SSR = 0.05802
Adjusted R^2 = 0.8821 n = 37, k = 14

Augmented AR(13)

$$C = 0.792\ C(-1) + DC(-1)....DC(-12) + 1.017 + X(-1) + X(-2) + W + W(-1)$$
(HCSE) (0.269) (2.030)

Equation SE = 0.05377 SSR = 0.05494
Adjusted R^2 = 0.8648 n = 37, k = 18

Notes: C is competitiveness, X is export volumes and W is world trade. DC is the first difference of C, and C(-i) is the ith lag on C.
HCSE are Heteroskedasticity Consistent SEs.

Table 8.9

PURE EXPECTATIONS MODELS. PRE-1979 ESTIMATES.
(Wickens-Breusch renormalised form)

i) Generated Regressors, Error Correction Mechanism.

1967:4 - 1979:2 — Equation SE = 0.044534
n = 47, k = 6 — Adjusted R^2 = 0.94329

X = 0.661 W - 0.153 E(C) + diffs, cons, dums
(HCSE) (0.026) (0.084)

ii) Generated Regressors, (4,4,4e).

1968:4 - 1979:2 — Equation SE = 0.0536412
n = 43, k = 16 — Adjusted R^2 = 0.881605

X = 0.625 W - 0.245 E(C) + diffs, cons, dums
(HCSE) (0.048) (0.148)

iii) McCallum Substitution, LBS type.

1968:3 - 1979:2 — Equation SE = 0.259193
n = 44, k = 9 — Adjusted R^2 = 0.71333

X = 0.603 W - 0.479 E(C) + diffs, cons, dums
(HCSE) (0.067) (0.841)

N.B. Estimation of this model in levels form
yields SE = 0.03353 and AdjR^2 = 0.951641

iv) McCallum Substitution, Error Correction.

1967:4 - 1979:2 — Equation SE = 0.0444814
n = 47, k = 6 — Adjusted R^2 = 0.943420

X = 0.661 W - 0.154 E(C) + diffs, cons, dums
(HCSE) (0.025) (0.084)

v) McCallum Substitution, (4,4,4).

1968:3 - 1979:2 — Equation SE = 0.05686
n = 44, k = 16 — Adjusted R^2 = 0.874848

X = 0.633 W - 0.201 E(C) + diffs, cons, dums
(HCSE) (0.062) (0.296)

Notes: Equations estimated in Breusch and Wickens renormalised form by IV methods.
E(C) is the expected value of C.

Table 8.10

MIXED EXPECTATIONS AND FEEDBACK MODELS.
PRE-1979 ESTIMATES.

i) Generated Regressors, ECM with Future Expectations.

1967:4 - 1979:1 — Equation SE = 0.0440248
n = 46, k = 7 — Adjusted R^2 = 0.942528

X = 0.653 W + 0.310 E(C) - 0.437 C +... . + diffs, cons, dums.
(HCSE) (0.023) (0.235) (0.226)
Net C effect = -0.127

ii) Generated Regressors, (4,4,4)+Future Expectations.

1968:4 - 1979:1 — Equation SE = 0.0619752
n = 42, k = 17 — Adjusted R^2 = 0.833964
(For levels est: AdjR^2 = 0.9590)

X = 0.604 W + 0.328 E(C) - 0.549 C +... . + diffs, cons, dums.
(HCSE) (0.061) (0.555) (0.470)
Net C effect = -0.221

iii) McCallum Sub, LBS + Future Expectations.

1968:3 - 1979:1 — Equation SE = 0.0644186
n = 43, k = 10 — Adjusted R^2 = 0.831568

X = 0.605 W + 0.407 E(C) - 0.616 C +... . + diffs, cons, dums.
(HCSE) (0.046) (0.691) (0.535)
Net C effect = -0.209

iv) McCallum Sub., ECM + Future Expectations.

1968:3 - 1979:1 — Equation SE = 0.0436979
n = 43, k = 7 — Adjusted R^2 = 0.922496

X = 0.630 W + 0.174 E(C) - 0.353 C +... . + diffs, cons, dums.
(HCSE) (0.028) (0.257) (0.240)
Net C effect = -0.179

v) McCallum Sub., (4,4,4). + Future Expectations.

1968:3 - 1979:1 — Equation SE = 0.0592028
n = 43, k = 17 — Adjusted R^2 = 0.857738

X = 0.621 W + 0.254 E(C) - 0.502 C +... . + diffs, cons, dums.
(HCSE) (0.052) (0.449) (0.379)
Net C effect = -0.248

Notes: Where IV estimation of the renormalised equation yields a -ve AdjR^2 we also report levels AdjR^2.

Table 8.11

PURE EXPECTATIONS MODELS. POST-1979 ESTIMATES.
(Wickens-Breusch renormalised form)

i) Generated Regressors, Error Correction Mechanism.

1979:3 - 1987:2 Equation SE = 0.0607673
n = 31, k = 6 Adjusted R^2 = 0.414669
(For levels est: AdjR^2 = 0.85034)

X = 0.621 W + 0.254 E(C) - 0.502 C +... . + diffs, dums, cons.
(HCSE) (0.052) (0.449) (0.379)
Net C effect = -0.248

ii) Generated Regressors, (4,4,4e).

1979:3 - 1987:2 Equation SE = 0.159107
n = 32, k = 5 Adjusted R^2 = -2.75029
(For levels est: AdjR^2 = 0.89166)

X = 0.991 W - 0.118 E(C) + diffs, cons. (No dums)
(HCSE) (0.586) (0.377)

iii) McCallum Substitution, LBS type.

1979:2 - 1987:2 Equation SE = 0.067729
n = 33, k = 6 Adjusted R^2 = 0.306115
(For levels est: AdjR^2 = 0.889141)

X = 0.668 W - 0.244 E(C) + diffs, cons, dums
(HCSE) (0.318) (0.240)

iv) McCallum Substitution, Error Correction.

1979:2 - 1987:2 Equation SE = 0.0713413
n = 33, k = 5 Adjusted R^2 = 0.230132

X = 0.749 W - 0.174 E(C) + diffs, cons, dums
(HCSE) (0.260) (0.139)

v) McCallum Substitution, (4,4,4).

1979:2 - 1987:2 Equation SE = 0.186744
n = 33, k = 15 Adjusted R^2 = -4.27508
(For levels est: AdjR^2 = 0.892463)

X = 0.737 W - 0.309 E(C) + diffs, cons, dums
(HCSE) (0.816) (0.835)

Notes: Equations estimated in Breusch and Wickens renormalised form by IV methods. E(C) is the expected value of C.

Table 8.12

MIXED EXPECTATIONS AND FEEDBACK MODELS.
POST-1979 ESTIMATES.

i) Generated Regressors, ECM with Future Expectations.

1979:3 - 1987:1 Equation SE = 0.0607693
n = 31, k = 6 Adjusted R^2 = 0.414669

X = 0.701 W + 0.464 E(C) - 0.620 C +... + diffs, cons. (No dums).
(HCSE) (0.0218) (0.270) (0.312)
Net C effect = -0.156

ii) Generated Regressors, (4,4,4)+Future Expectations.

1979:3 - 1987:1 Equation SE = 0.126069
n = 31, k = 16 Adjusted R^2 = - 1.519
(For levels est: AdjR^2 = 0.901692)

X = 0.408 W + 1.447 E(C) - 1.935 C +... + diffs, cons, dums.
(HCSE) (0.395) (1.104) (1.426)
Net C effect = -0.488

iii) McCallum Sub, LBS + Future Expectations.

1979:2 - 1987:1 Equation SE = 0.0570799
n = 32, k = 10 Adjusted R^2 = 0.473946
(For levels est: AdjR^2 = 0.826316)

X = 0.667 W + 0.500 E(C) - 0.722 C +... . + diffs, cons, dums.
(HCSE) (0.205) (0.258) (0.204)
Net C effect = -0.222

iv) McCallum Sub., ECM + Future Expectations.

1979:2 - 1987:1 Equation SE = 0.0611744
n = 32, k = 7 Adjusted R^2 = 0.395768

X = 0.825 W + 0.502 E(C) - 0.576 C +... . + diffs, cons, dum.
(HCSE) (0.237) (0.305) (0.287)
Net C effect = -0.074

v) McCallum Sub., (4,4,4). + Future Expectations.

1979:2 - 1987:1 Equation SE = 0.0907001
n = 32, k = 17 Adjusted R^2 = -.328252
(For levels est: AdjR^2 = 0.800380)

X = 0.841 W + 1.187 E(C) - 1.341 C +... + diffs, cons, dum.
(HCSE) (0.285) (0.508) (0.601)
Net C effect = -0.154

pure expectations models, and in table 8.14 these are presented alongside estimates for the pure expectations model.

Overall the estimates do not offer much encouragement towards the expectations hypothesis, and do not merit detailed individual examination. However we will attempt to emphasise some of the main features of the results.

The first feature we observe is that there is a contrast between estimates obtained for the pre- and post-1979 period. The various models appear to fit relatively poorly in the second period. Further, the loss of explanatory power which seems to be associated with the Wickens and Breusch renormalisation is greater in the second period.[22]

If we examine the pre-1979 estimates we find that the long run world trade elasticity appears to be well determined, close to 0.6, and varies little from specification to specification, with a minimum value of 0.603 and a maximum of 0.661. (These values are similar to the estimate of 0.567 for the unrestricted reduced form (LBS+14) in table 6.11)

However, we observe that the point estimates of the competitiveness elasticity are more varied, ranging for the pure expectations model between -0.153 for the ECM case under generated regressors and -0.479 for the 'LBS' variant as estimated by the McCallum substitution. For the mixed examples the net long run competitiveness elasticity varies between -0.127 for the ECM case with generated regressors and -0.248 for the 4,4,4 case under the McCallum substitution. These estimates are all low relative to the estimates of the long run competitiveness elasticity obtained for the unrestricted reduced form in chapter six.

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22. In our estimation of expectations models of export volumes we find that the use of the Wickens and Breusch type renormalisation for the direct estimation of long run properties has the consequence of worsening the fit of the equation. Adjusted $\bar{R}^2$ measures fall and estimated equation standard errors rise, although the point estimates of long run prpoerties appear unaffected.
We do not have a satisfactory explanation for this phenomenon, although it may reflect small sample problems with the use of instruments. We note also that, in contrast to the Wickens and Breusch analysis, the equation must be estimated by instrumental variables even in the absence of renormalisation.

Table 8.13

NET COMPETITIVENESS ELASTICITIES FOR MIXED EXPECTATIONS AND FEEDBACK MODELS.

Pre 1979. AR13 Filter = 0.738 (point estimate)

Model	Net Elasticity	Filtered Elasticity (Corresponds to RF)
i) ECM + GR	-0.127	-0.208
ii) 4,4,4 + GR	-0.221	-0.307
iii) McCsub LBS	-0.209	-0.316
iv) McCsub ECM	-0.179	-0.225
v) McCsub 4,4,4	-0.248	-0.316

Post 1979. AR13 Filter = 0.725 (point estimate)

Model	Net Elasticity	Filtered Elasticity (Corresponds to RF)
i) ECM + GR	-0.156	-0.284
ii) 4,4,4 + GR	-0.488	-0.886
iii) McCsub LBS	-0.222	-0.36
iv) McCsub ECM	-0.074	-0.212
v) McCsub 4,4,4	-0.154	-0.48

Notes: All elasticities are long run competitiveness elasticities.
The net elasticity is the sum of the long run terms for the feedback and expectations components. It gives the long run response to a fully anticipated permanent shock.

The filtered elasticity converts the expectations term to be a function of observed competitiveness before combining terms to give the equivalent of the reduced form elasticity.

Table 8.14

ACTUAL VERSUS REDUCED FORM EQUIVALENT COMPETITIVENESS ELASTICITY IN PURE EXPECTATIONS MODELS.

Pre 1979. AR13 Filter = 0.738 (point estimate)

Model	Actual Elasticity	R.F. Equivalent Elasticity
i) ECM + GR	-0.153	-0.113
ii) 4,4,4 + GR	-0.245	-0.181
iii) McCsub LBS	-0.479	-0.354
iv) McCsub ECM	-0.154	-0.114
v) McCsub 4,4,4	-0.201	-0.148

Post 1979. AR13 Filter = 0.725 (point estimate)

Model	Actual Elasticity	R.F. Equivalent Elasticity
i) ECM + GR	-0.133	-0.096
ii) 4,4,4 + GR	-0.118	-0.086
iii) McCsub LBS	-0.244	-0.177
iv) McCsub ECM	-0.174	-0.126
v) McCsub 4,4,4	-0.309	-0.224

Notes: All elasticities are long run competitiveness elasticities. The actual elasticity is estimated within the pure expectations model itself. It gives the long run response to a fully anticipated permanent shock.

The RF equivalent elasticity modifies the actual elasticity by allowing for the filtering of observed competitiveness, using the point estimates for the long run filter from the AR13 process.

Comparing these ranges of values for the long run competitiveness elasticity we see that the restrictions inherent in these formulations of the weakly rational expectations assumption lower the estimated long run elasticity relative to the unrestricted reduced forms. The misspecification of the long run competitiveness elasticity identified in chapter six is not avoided in the expectations specifications we have considered.

In summary, the key features of the pre-1979 expectations results are that the equations by and large fit reasonably well, but that they cannot reproduce the higher values for the competitiveness elasticity which reflect long lags on competitiveness.

Turning to the post-1979 estimates, we find that the fit of the equations is notably poorer than the fit of the pre-1979 estimates. We note the relative inefficiency of the estimates of the world trade elasticity and the wide range of point estimates of this parameter from 0.408 in the mixed 4,4,4 plus future expectations case estimated by using generated regressors to the 0.841 for the same model estimated by McCallum substitution. However, relative instability of the world trade coefficient was a feature of our earlier examination of post-1979 data in chapter six, and it appears to reflect the extent to which trend movements in world trade over this period are masked as an influence on export volumes in the short run by the huge swings in competitiveness.

The point estimates of the long run elasticity of competitiveness in these models for the post-1979 period are very low compared with the unrestricted reduced form values of chapter six. For the pure expectations models, they range from -0.118 for the 4,4,4e model with generated regressors to -0.309 for the same model estimated by McCallum substitution. For the mixed expectations and feedback models the net elasticities range from -0.074 for the ECM model estimated by the McCallum substitution to -0.448 for the 4,4,4 case using generated regressors.

The post-1979 estimates can thus be characterised by bad fits, with unstable estimates of the trade elasticity. The estimated competitiveness elasticities do not appear to correspond to the higher values of the long lag reduced form models of chapter six.

A further contrast that can be made between the pre-1979 and post-1979 estimates concerns their sensitivity to the choice of estimation procedure. The estimates obtained by the generated regressors and McCallum substitution methods prior to 1979 are similar, but in the post-1979 period they appear to diverge. This is consistent with the general observation that the post-1979 attempts to estimate specific expectations models are less satisfactory than the attempts prior to 1979.

We have not yet commented on the attempts at joint estimation of an expectations version of the export volume equation along with the process for competitiveness. Although it appeared that the reported export volume estimates had converged, the cross equation restrictions for the (AR14) univariate process on competitivness entailed huge prediction errors (which we have not reported). It thus appears that the structural restrictions inherent in our estimated equations are not compatible with the partially rational schema for expectations formulation. Further, while the estimated structural parameters for the export volume equation in the pre-1979 period were not wildly at variance with reduced form estimates, those for the post-1979 period were drastically different. (See for example the estimated elasticity on world trade, which is over 4.0).

Another striking feature of the estimated results in those cases with mixed expectations and feedback is that, (leaving the jointly estimated equation aside), the sign of the estimated coefficient on the expectation of future competitiveness is positive.[23] This is counterintuitive, in that we would expect behaviour to respond less to a noisy signal than to movements in the fundamental signal alone, as we do find for the pure expectations cases. Given our scepticism concerning the success of the estimates reported here in capturing underlying behaviour we do not pursue this point.

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23. This effect is not necessarily implausible, as it could reflect intertemporal substitution in demand; we note that the long run response to an anticipated, permanent shift in competitiveness still has the familiar negative sign.

8.4.4 Conclusions.

We have briefly presented the results of our attempts to estimate export volume equations with an explicit role for expectations and corresponding to the reduced form equations of chapter six. None of the examples considered amounts to a satisfactory approximation of the data generation process congruent with the unrestriced reduced form estimations. A number of different interpretations of our attempted estimations are possible.

If we accept the 'long lags' reduced forms of chapter six with their higher competitiveness elasticities as good approximations to the underlying data generation process, we must conclude that the specific expectational models we have considered involve unsatisfactory restrictions relative to the underlying DGP.

It remains possible that expectations of future competitiveness are an important factor but that we failed to capture the expectations hypothesis within the correct particular null specification. But if partially or weakly rational expectations do play an important role, they do not appear to be sufficient to explain the relatively high long run elasticity of the reduced forms: long 'structural' lags may be necessary to explain these. In the reduced form estimates, export volumes respond to smoothed competitiveness: this may still reflect the influence of some kind of expectations mechanism.

Another explanation for the failure of our attempts to estimate equations with an explicit role for expectations of competitiveness is the implicit certainty equivalence of our approach. We did not allow for an explanatory role for, say, the forecast variance of competitiveness. Yet the presence of sunk costs, for example, would make non-certainty equivalence potentially relevant. We note that Kenen

and Rodrik (1986) found some support for an explicit role for exchange rate uncertainty in trade equations. But we have not pursued that approach further here.[24]

If we had settled on a satisfactory specific form for the expectations model we could now subject it to formal test procedures and use it to assess the relative importance of regime change and structural change. As we have not found such a satisfactory expectations model we leave these questions open, although somewhat more sceptical of the empirical value of the rational expectations approach in this context than hitherto. We cannot, however, go so far as to reject the relevance of expectations of competitiveness to UK export behaviour.

8.5 CONCLUSIONS, EXPLANATIONS AND INTERPRETATIONS.

8.5.1 Review of Findings.

In this chapter we have explained why expectations and errors in variables might both help to account for observed parameter instability in export volume equations. Within a general signal extraction framework we have shown that the Lucas critique could be relevant to the long run elasticity on competitiveness.

Statistically significant moving average components and/or high order autoregressive elements in the univariate representation of competitiveness would encourage the hypothesis that the explanatory role of long lags on competitiveness in estimated UK manufactured export volume equations could be explained in terms of weakly rational forecasts of current and future competitiveness. Univariate forecasts of competitiveness can indeed be improved by the inclusion of long lags for the post-1979 period but not for the pre-1979 data.

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24. See also Cushman (1983) and De Grauwe (1988).

Univariate representations of competitiveness confirms that a major regime change in the underlying time series properties occurred in 1979. However there is also evidence that a further regime change occurred around 1983: allowing for this would make the sub-sample too small for satisfactory estimation, while ignoring it might impose invalid restrictions on the estimation.

The most important aspect of regime change for our purposes is that which implies a change in the long run properties of the optimal univariate filter. Our evidence of the impact of the regime change at 1979 on the long run signal extraction filter is inconclusive. The failure of the data to yield a precise estimate of the long run filter indicates how inference may suffer from non-homogeneities in the data.

When we attempt the estimation of a variety of specifications of export volumes incorporating the assumption of weakly rational expectations on competitiveness we meet with little success: these particular specifications are unsatisfactory.

8.5.2 Explanation and Interpretation of the Findings.

We have considered whether there is a role for expectations of competitiveness in the determination of export volumes. We have been unable to find satisfactory specifications in which long lags on competitiveness in the export volume equation were due to weakly rational expectations derived from high order autoregressive representations. Nevertheless we must recognise:

firstly, that expectations could play an important role in the export volumes equation without their being able to explain the explanatory power of long lags on competitiveness. The long lags may, wholly or in part, reflect other factors such as time-to-build and delivery lags etc;

secondly, that expectations may not be adequately represented by the weakly rational expectations hypothesis and that the assumption of no systematic forecasting or filtering errors may not be appropriate. Long lags on competitiveness may reflect the role of expectations after all, albeit under some other expectations hypothesis.

Therefore we cannot interpret the results of our attempted estimations as a general rejection of the relevance of the Lucas critique. It appears likely that observed parameter change may reflect either or both the Lucas critique effect or structural change. We may be unable to discriminate between these rival explanations.

We could test the hypothesis of structural stability conditional on the assumption of forward-looking behaviour. This reflects our priors as to structural stability and forward-looking behaviour. The theoretical justification of the Lucas critique is at a high level of generality, and yet it also provides a framework which yields relatively specific predictions concerning change in the estimated parameter. On the other hand, structural change (what Landesmann and Snell (1989) have termed a 'Thatcher effect') provides no clear prediction for the observed parameter: we have no well articulated model as to how some specific policy changes should affect the reduced form parameter in question. Hysteresis effects in trade gives no clear prediction for the observed long run price elasticity.

Therefore, we accord primacy to the expectational account. We make the joint assumptions that forward-looking expectations of competitiveness are relevant to the export volumes equation, and also that the underlying behavioural relationships between suitably filtered variables are structurally invariant. We should consider structural change only if we can show that the long run filter on competitiveness cannot explain observed parameter instability. Structural change would be deduced as a residual.

We could test whether the change in the long run univariate filter was significantly different from the change in the long run competitiveness elasticity in the reduced form. The lack of precision in the relevant estimates, combined with the relatively small magnitudes of the reported changes in the two relevant parameters, indicates that we would be unable to reject the hypothesis of structural stability, conditional on forward-looking behaviour within the framework of section 8.2.

Two problems arise from the nature of the data, which are difficulties of "experimental design". The first problem is the non-homogeneity of the post-1979 sample, where it seems that the post-1979

sample combines at least two distinct regimes for the behaviour of competitiveness. The result is that we do not have a sufficiently large sub-sample to estimate models which reflect the additional regime change. The second problem is that there may be insufficient variation in the data. Even though the regime change at 1979 is sharply defined in the time series properties for competitiveness, it is not well defined as a change in the long run properties of the optimal filter. Hence there may only be small and difficult to observe implications of the Lucas critique for the long run parameter in practice.[25]

Overall we conclude first that the evidence in support of long lags on competitiveness, and our priors from economic theory support the working assumption of forward-looking behaviour. Secondly, given this hypothesis, we would be unlikely to reject the conditional hypothesis of no structural change in the underlying parameters. Essentially, the data are unable to overturn the priors reflected in the maintained hypothesis.

Nevertheless, caution is in order, as there are good reasons for expecting that some parameter change may have occurred, even though we may not be able to predict its effects. Furthermore, we have been unable to implement a weakly rational expectations model. So while we have established the potential relevance of both expectational and structural change around 1979 to the long run competitiveness elasticity, we are unable to discriminate between them other than by the assertion of our priors. This is a significant practical limitation to the reduced form export volumes equations presented in chapter six.

8.5.3 Concerning Policy Significance.

Whether the observed fall in the estimated long run competitiveness elasticity is a reflection of structural change or of the Lucas critique is an important matter for policymakers. We have argued that the difficulties experienced with the various explicit expectational models described above do not

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25. Note that these two problems may interact. We suspect that the failure to find a clear change in the long run filter at 1979 in higher order representations reflects our inability to allow for the possible break at 1983.

imply that the reduced form equation of chapter six will be invulnerable to the Lucas critique. It is appropriate for policymakers to be wary of the possible role of expectations in this regard.

The Lucas critique suggests that policymakers should not expect simple and familiar multivariate correlations between macroeconomic variables to be robust to changes in the policy regime. In our case the argument must be turned around, as we describe below.

The reference by Chancellor Lawson in the 1985 Autumn Statement to the apparent fall in the competitiveness elasticity of UK manufactured export volumes carried the implication that there had been a change in the underlying structure of UK trade in manufactures. We have interpreted the implicit policy argument as being that the government could accept a permanently higher real exchange rate (worsened competitiveness) and reap the associated anti-inflationary gains at a lower cost in terms of the squeeze on factors employed in the export sector (or in unemployment in manufacturing) than before. The assumed change in the underlying structural parameters would thus justify a change in policy towards the level of the real exchange rate.[26]

We judge that policymakers would have been well advised to exercise caution before inferring that some kind of structural change had taken place and that the apparent change in the elasticity provided a basis for an adjustment in policy towards the real exchange rate. Although we have been unable to settle the expectations versus structural change debate in the empirical parts of this chapter, we have made the argument that the appropriate maintained hypothesis is one of the relevance of the Lucas critique and of structural invariance. The riskier inference would be that the observed parameter change after 1979 (once corrected for misspecification as per chapter six) reflected structural change that could be exploited by policy rather than merely the Lucas critique. The prudent conclusion for policymakers would be that the observed parameter change reflected the Lucas critique effect and that the sensitivity of exports to a permanent shift in the real exchange rate had not been reduced.

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26. We have elaborated on these arguments in Williams (1990).

Chapter Nine

CONCLUSIONS

This thesis has addressed the interaction of exchange rates, expectations and international trade, against the background of the UK experience of the early 1980s. The appreciation in the UK real exchange rate after the 1979 regime change in UK economic policies was of unprecedented magnitude, and, perhaps not surprisingly, such an event tested the adequacy of existing analyses of trade and exchange rates. In fact UK export volumes outstripped predictions in the face of this appreciation. One possible explanation of this outcome was the suggestion of a 'fundamental improvement' in the export performance of British industry; while an alternative pointed to the inadequate treatment of expectations in the pre-existing analyses. At the same time, despite the higher-than-expected export volumes many observers nevertheless feared that the appreciation was producing a permanent loss of export markets from UK producers. Such macroeconomic questions have not been resolved in the literature, and remain open to debate: they have set the context for this research.

This research has pursued both theoretical and empirical questions concerning the exact nature and stability of the relationship between trade and exchange rates, and the relevance of various aspects of exchange rate expectations to the determination of trade flows. Of necessity, the specific issues we have examined are much narrower than the 'big' questions prompted by the events of the early 1980s, (although we finally return to comment on the 'big' questions below). And yet we have found that both our theoretical results and empirical methods have their own broader economic significance beyond the specific context of trade equations and UK export performance.

In this concluding chapter we will first review the principal theoretical and empirical findings of our research. Having done that, we then judge it appropriate to return to the starting point and ponder some of the 'big' issues provoked by the UK trade experience of the 1980s in the light of our own particular research.

9.1 REVIEW OF THE RESEARCH.

Our theoretical analysis (in chapters two, three and four) studied the effects of adding mean reversion to the Dixit (1989a) model of entry and exit with uncertainty, which can serve as a prototype theory model of hysteresis effects in trade. We have used both numerical and analytic techniques to investigate this issue, and we have obtained three principal findings, as detailed below.

Firstly, we have confirmed that the addition of mean reversion to the Dixit model widens the hysteresis band between the trigger price solutions. Our analysis suggests that this result reflects the interaction of mean reversion with sunk costs in determining whether the exercise of options of entry and exit is optimal for a given realisation of the price. Only profitable possibilities affect option values and this fact makes the option difference term less sensitive to changes in mean reversion than is the expected value of the continuation strategy. Thus the certainty equivalent response of the model to mean reversion dominates the non-certainty equivalent effect on the option difference, and this, in turn, delivers the band widening result. We conclude that, in respect of band widening, our results are consistent with Dixit's predictions and also with the findings of other authors (eg. Baldwin (1989)); we go beyond the literature in offering an economic intuition as to why the band widening result is found to hold in the case with uncertainty.

Secondly we have shown how the non-certainty equivalence of the Dixit model is complicated by the introduction of mean reversion. The sensitivity of the band width to mean reversion varies with uncertainty concerning future prices. The model's response to mean reversion is complicated because the non-certainty equivalent part of the solution responds to mean reversion in the opposite direction from the deterministic part. With a high level of uncertainty, the damping effect of mean reversion on the option difference term (the non-certainty equivalent or caution component of the solution) can almost entirely offset the deterministic band widening effect of increased mean reversion. Our findings in respect of the complicated non-certainty equivalent response to mean reversion are not duplicated

elsewhere. We have therefore added to the detailed understanding of the model, and in doing so have supplemented and complemented other research on sunk costs hysteresis.

Our third, but most significant, result is as follows. In the presence of sufficiently asymmetric sunk costs and mean reversion, the state transition associated with the larger sunk cost can occur sooner than it would in the deterministic case. This result reflects the differential effects of the sunk costs of entry and exit on the value of the options of entry and exit. This striking qualitative finding is not anticipated in the literature, and it carries the implication that the addition of uncertainty to the entry and exit model does not always introduce inertia into both the entry and exit decisions. For example, when entry costs dominate exit costs, and the stochastic price process is mean reverting, the possibility of regrets about future exit can overwhelm possible regrets about entry and cause an idle firm to hasten its entry. This result demonstrates the interrelated nature of the options of entry and exit in a way the previous literature cannot, and hence is of considerable importance to the study of entry and exit.

Our theoretical study benefitted from our development of a two period 'maquette' which served to simplify the mechanisms operating in the Dixit model and hence clarified the results revealed in our numerical analyses. Overall our theoretical research has developed and documented a detailed characterisation of the Dixit model with mean reversion, and has added to our understanding of the model's workings. We have also emphasised (in chapter four) that our findings concerning the Dixit model should generalise to a class of stochastic saddlepoint models.

One implication that has been drawn in the literature from the kind of hysteresis theory models considered above is that the search for stable time series econometric relationships of the conventional form used to describe trade relationships may be greatly complicated, (or even confounded), by the widespread occurrence of sunk cost hysteresis effects in trade. Nevertheless our empirical work (in chapters five, six and eight) has taken the existing time series literature on UK trade equations as its starting point. We now turn to assess our empirical findings.

Our empirical analysis focussed on the long run competitiveness elasticity of UK manufactured export volumes using quarterly data. A particular motivation for this part of the thesis was provided by claims made by Chancellor Nigel Lawson in 1985 to the effect that the long run competitiveness elasticity of UK manufactured export volumes had fallen. The government suggested that structural change had occurred in the economy, with consequences for the price sensitivity of trade, and hence for the appropriate stance towards the real exchange rate. We have considered the following points in the context of estimated export volume equations, taking the 1985 LBS manufactured export volumes equation as a particular example to relate to Chancellor Lawson's claims.

Firstly, following Wickens and Breusch, we noted that the misspecification of the short run adjustment lags on a stationary regressor in a dynamic model will lead to inconsistent estimation of the long run properties, in contrast to the consequences of a similar omission in the event that the regressor were co-integrated with the other regressors. Based on an examination of the autocorrelation properties of the series as proposed by Engle and Granger (1987), we have argued that competitiveness merits treatment as a stationary regressor in UK export volume equations. Our study of the UK export volume literature, in addition to our own estimation, indicated that estimates of the important long run competitiveness elasticity are indeed sensitive to the model's short run dynamic specification, in sharp contrast to the estimates of the elasticity with respect to the activity variable, world trade. Furthermore, we have argued that the fall in elasticities reported by the government in 1985 appears in the first instance to reflect changes in equation specification with respect to the short run dynamics on competitiveness.

Secondly, following from the recognition of the stationarity of competitiveness we have employed a variety of procedures, including Hausman tests, to make the case that the long run competitiveness elasticity is misspecified in equations such as the 1985 LBS model. A side effect of the use of error correction forms for the dynamic specification of more recent UK export volume equations appears to have been the inconsistent estimation of the long run competitiveness elasticity. The inconsistency reflects the omission of lags from the short run dynamics on competitiveness which has caused the long run competitiveness elasticity to be under-estimated by at least a third. The need for smoothing

over up to four years of lags on competitiveness may reflect a role for expectations of competitiveness: trade may depend on a filtered variable akin to 'permanent' competitiveness.

Thirdly, we have found that, when sufficient lags on competitiveness are included in the reduced form export volume equation so as to avoid the misspecification of the long run properties, there is evidence from recursive estimation of a fall in the estimated long run competitiveness elasticity following the 1979 change in the UK economic policy regime. However, this recursive least squares evidence of instability is contestable, and the hypothesis of parameter stability cannot be rejected in classical tests.

Parameter instability may in general be taken to reflect the inadequacy of a conditional model. However, in the context of UK export volume equations we have identified three alternative interpretations which offer potential economic rationales for parameter instability. Firstly, parameter instability could reflect structural change in the fundamental parameters of the economy, such as supply side changes following from government policies. Alternatively, parameter instability might reflect the shifts in the relationship between export volumes and competitiveness which follow from the threshold effects of sunk cost hysteresis in trade. Neither of these two hypotheses has been formulated so as to permit estimation and formal testing on aggregate data: we have however attempted to consider their plausibility and relevance.

A third possible cause of parameter instability in UK export volume equations is more amenable to formulation as a parametric hypothesis. Parameter instability in the early 1980s could reflect the Lucas critique effects of the evident change in policy regime which followed the election of the Conservative government in 1979. The time series behaviour of competitiveness has been shown to have changed substantially after 1979, and so, if forward looking behaviour were important to UK export determination, we would expect the regime change to have produced parameter instability in equations estimated without an explicit treatment of expectations. We have therefore attempted to estimate export volume equations including an explicit role for partially rational expectations of univariate competitiveness. But, as we reported in chapter eight, we have been unable to uncover a satisfactory specification of this type.

Our inability to find a satisfactory expectations formulation is open to a number of interpretations. Forward looking behaviour and the Lucas critique may not in fact be relevant to UK exports. Alternatively, forward looking expectations might indeed be relevant, but not in the rational expectations forms we have considered. Finally, in the light of the Dixit theory considered in chapters two, three and four above, the failure of our efforts may reflect the restriction of our search to the class of certainty equivalent models with no explicit role for the variance of the innovations in competitiveness.

Given the theoretical appeal of forward-looking agents, plus the apparent empirical role for smoothing over long lags of competitiveness shown in chapter six, we judge that it would be foolish to reject the Lucas critique explanation; and wise to treat the assumption of forward looking behaviour as a maintained hypothesis.

We therefore proceed to compare the long run reduced form elasticity with the long run univariate prediction filter. Rather than examine the expectations issue conditional on the assumption of parameter stability (as for example in Hendry (1988)), we endeavour to consider the hypothesis of parameter stability conditional on the assumption that competitiveness expectations are weakly rational in the long run. While, in the absence of a successful explicit expectations specification, this approach may have some promise, we are disappointed in practice, finding that the variation across regimes is too small relative to the poor precision of our estimates for formal statistical tests to be worth pursuing.

We must acknowledge that doubts remain concerning the evidence of parameter instability we have presented, and that our interpretation of the search for an expectations model is also debateable. Nevertheless our empirical work has succeeded in establishing that the long run competitiveness elasticity obtained from error correction specifications of the export volumes equation should be treated with some scepticism. We have also shown that the neglect of the stationarity of competitiveness in the dynamic specification procedure may have led to the significant underestimation of this important long run property.

9.2 REFLECTIONS ON THE UK EXPERIENCE OF THE EARLY 1980s.

So far we have reviewed the main theoretical and empirical findings of this research. Understandably these make fairly narrow contributions, which have their significance in relation to specific parts of the literature. However, the motivation for this research was provided by broad UK macroeconomic questions of trade and exchange rates, and it is appropriate that we should return to these issues before closing. We therefore comment on a number of questions which arise in particular from the UK experience of export performance in the early 1980s. In order to form an overall judgement on the matters in hand we draw on a variety of factors, ranging from prior theoretical beliefs and assumptions, to our empirical results, both conclusive and inconclusive. Once again, we emphasise that, our research having directly addressed more specific questions, what follows might be more properly considered as "reflections upon" rather than "answers to" the questions.

With the foregoing proviso, we consider the following questions. Why did UK exports hold up so well in 1980/1 in the face of unprecedentedly unfavourable competitiveness with the huge appreciation of Sterling? Did the early 1980s witness a fundamental and lasting reduction in the sensitivity of UK exports to fluctuations in the real exchange rate? Are UK trade flows subject to sunk cost or other hysteresis effects? What light, if any, does this research throw on the Krugman thesis of 'Exchange Rate Instability'? What light does this research throw on the nature and use of econometric export equations?

Why Were UK Export Volumes Underpredicted in 1980/1?

Our first comments concern the underprediction of UK export volumes in 1980/1 by contemporary econometric models of UK trade. Why did UK exports hold up so well in the face of a sharp real appreciation of unprecedented magnitude? Despite our singular failure to formulate a satisfactory

econometric model of export volumes with an explicit treatment of expectations, our research as a whole points to the major significance of expectational factors in the answer to this question.

A variety of theoretical consideration in both supply and demand emphasise the intertemporal nature of export determination: the very existence of trade in a particular product can sensibly be treated as if it were an asset. Behavioural decisions then depend on expectations as to the future trading environment, and the careful specification of expectations of competitiveness is especially important. In the medium run, UK competitiveness appears to be stationary, and therefore predictable in the sense that it will be expected to return towards some mean value: moreover, the evidence that export volumes may respond to long 'smoothing' lags on competitiveness is consistent with this view. In this context, the fact that the appreciation of the real exchange rate had less than the predicted effect on export volumes suggests that the real appreciation of 1980/1 was discounted relative to previous movements; that the shock was considered more likely to be reversed - and more quickly - than were previous movements. This belief would correspond to a faster speed of mean reversion in the exchange rate dynamics under the post-1979 regime.

The Dixit model of entry and exit offers a potent further intuition. In a stochastic world, the response of trade to movements in competitiveness may be non-certainty equivalent. In many cases, (although see the 'crossover' effect of chapter three for a partial counterexample), the presence of uncertainty combines with sunk costs to delay the optimal response to a given price shock. When actions are costly to reverse, and agents believe that they may subsequently wish to reverse them, they will be less inclined to act. Increased uncertainty will, *ceteris paribus*, tend to reduce the (possibly discontinuous) response to a given stimulus. Notice that such 'cautious control' results may arise under a range of other theoretical assumptions.

Although our empirical work has not found a way successfully to operationalise the effect on export volumes of mean reversion and of forecast uncertainty in exchange rate expectations, both factors seem relevant to the UK of 1980/1. Firstly, agents faced with the unprecedented exchange rate behaviour of this period could reasonably have viewed the appreciation as a temporary phenomenon: a case of "what

goes up, must come down", due to the mean reverting tendencies of adjustment when short run macroeconomic properties are Keynesian. Secondly, given increased volatility in exchange markets, and subjective (if not objective) uncertainty concerning the nature of the government's policy towards the exchange rate over these years, agents facing lumpy transactions costs should have found it optimal to "wait and see" before taking such drastic steps as ending trade in UK products.

We argue that estimated export volume equations are reduced forms reflecting elements of both supply and demand behaviour; and we contend that the interaction of sunk transactions costs and uncertainty could apply on both sides of the market. We are thus strongly persuaded of the potential relevance of a non-certainty equivalent Lucas Critique, which could, in principle, explain the surprisingly strong behaviour of UK export volumes in 1980/1. Demanders will have been less likely to switch suppliers and suppliers more likely to sustain losses and not quit the market, if they expected a reversal in the appreciation, and perhaps more importantly if they recognised notably increased exchange rate volatility and uncertainty over future exchange rate developments.

Those who were most likely to respond to the appreciation by ceasing trade in UK products would have included: those whose switching costs were lowest; those who underestimated the possibility that the appreciation would reverse itself within a medium term time horizon; those who underestimated the increased uncertainty associated with exchange rates, or were able somehow to significantly hedge the risk. With specific reference to the supply side, the exit of UK exporters would also have been more likely in response to the appreciation when firms lacked confidence in their ability to control costs in line with an underlying permanent shock to the real exchange rate.

It is instructive to note that a naive, deterministic model of discontinuous adjustment costs would predict that large exchange rate changes would have proportionately large effects. This is precisely *not* what was observed in the UK in 1980/1. This realisation serves to re-emphasise the crucial role of *both* mean reversion and uncertainty in exchange rate expectations: it is these features which offer an explanation of the events of 1980/1. In the light of this assessment, it would appear that a shortcoming

of our empirical investigation in chapter eight was its implicit use of certainty equivalence: the chief significance of the Lucas Critique may have operated through the second moment of competitiveness.

Despite our stress on exchange rate expectations, our research does suggest that another factor merits attention if we are to explain UK export volumes in 1980/1. Our simulation analysis using the Dixit model of hysteresis suggested that optimal behaviour of UK exporters was very sensitive to expected long run price-cost margins: these could dominate the effects of sunk costs, uncertainty and mean reversion. We must therefore conclude that the Dixit model's assumption that flow costs are exogenous is too strong a simplification for the analysis of UK export performance in the 1980s.

It has been suggested that the Government's supply side policies may have led to improved trade performance. If this conjecture were to be formalised it would require an explanation of the mechanism which compensated for the loss of competitiveness associated with the real appreciation. Walters (1986) conjectured that an "improvement in the quality of British exports" took place at this time, but did not spell out a mechanism, (p 141, footnote). Alternatively, labour market analyses might seek to explain how government policies have strengthened management's hand in raising current and future productivity. Maynard (1988) suggests that it was in part the very severity of the exchange rate shock itself which precipitated costs adjustments in 1980/1: Walters (1986, p 168) has described this view as "the 'Cold Shower' argument". On this view, the magnitude and prospective costs of the appreciation may have outweighed the large sunk costs associated with the re-organisation of work patterns.

Our consideration of the issues indicates the importance of extending research to explain trade performance in conjunction with a formal analysis of labour market developments (such as in Layard and Nickell (1988) and Bean and Symons (1990)). It seems plausible that some part of the explanation for the surprisingly strong performance of UK trade in 1980/1 may have been cost developments traceable back to the labour market. This is entirely consistent with our sceptical approach to export volume equations, whereby we treated them as reduced forms, allowing that they reflect aspects of supply as well as demand behaviour.

In summary, in answer to the question as to why UK trade volumes were underpredicted in 1980/1, we would argue strongly that the existing econometric models failed to allow for expectations that the huge appreciation was liable to unwind and, most interestingly, that they failed to incorporate effects by which the increased level of uncertainty concerning exchange rates might inhibit actions. But we would also emphasise that, with export volumes reflecting supply as well as demand influences, cost cutting and labour market effects must also warrant investigation.

Has there been a Reduction in the Exchange Rate Sensitivity of UK Exports?

Our second question concerns whether there has been a fundamental and lasting reduction in the sensitivity of UK exports to real exchange rate movements. Our focus on the price elasticity follows the 1985 Autumn Statement, but is in contrast with Landesmann and Snell's (1989) examination of the income elasticity of export volumes for a 'Thatcher effect' in British trade.

We must be sceptical as to what can be learned from a consideration of this reduced form equation given that we are unable to distinguish supply and demand elements within it. However, such equations are employed in macroeconometric models for purposes of analysis, forecasting and policy. Therefore possible changes in the parameter are of practical importance and we must attempt to interpret them. We concentrate on the long run competitiveness elasticity, taking our underlying concern to be whether the long run response of the UK economy to a permanent exogenous shift in competitiveness has changed. Such a shock might reflect a shift in the relative price of importables and exportables, due, say, to a permanent rise in commodity prices.

Our judgement is that relatively little (if any) change has occurred in UK trade performance as measured by long run competitiveness elasticities. Suggestions to the contrary appear to have reflected differences in specification, including probable misspecification of the short run dynamics on competitiveness, a stationary regressor. Furthermore, to the extent that some fall in the estimates may have occurred in the early 1980s, it is probable that this could have reflected the 'Lucas Critique'

consequences of the regime change in the exchange rate dynamics and volatility. There is little evidence to persuade us that the fundamental sensitivity of the economy to a fully expected, permanent shift in competitiveness had occurred.

Nevertheless, we concede that hysteresis effects in trade, for example, could have led to some *compositional* effects on UK trade and its sensitivity to exchange rate movements. But while we argue that such hysteresis effects are entirely plausible, there is no real evidence that the aggregate price elasticity has actually shifted in this way.

On the basis of this research, we are strongly inclined to reject the wishful thinking implicit in the 1985 Autumn Statement that UK exports had somehow become fundamentally less price sensitive as a result of government policies. The kind of supply side improvements the government may have had in mind may be indicated by Alan Walters' (1986, p 141) suggestion that improvements in quality had occurred. Even supposing that misspecification and possible Lucas Critique effects were proven incapable of explaining all the reported change in the long run competitiveness elasticity, the attribution of parameter change to the effects of government policies must be treated as speculative conjecture until it is supported by independent theoretical and empirical evidence of some kind. In this research, we have reported theory and simulation evidence to give credence to the possibility of sunk costs hysteresis effects in trade. Proponents of a fundamental transformation in UK export performance should offer some evidence for their particular interpretation of parameter change if they wish to be taken seriously. Having said that, we believe that such research might make some progress if it were tied to the analysis of labour market developments, and perhaps sought to trace the influence of government policies on expected long run cost control in tradeable goods production. Improvements in product quality might also seem plausible if explained in terms of labour market developments and productivity gains.

Are There Hysteresis Effects in UK Trade?

Our third question is whether or not UK trade flows are subject to sunk cost or other hysteresis effects. We have taken as a starting point widespread anecdotal evidence of the "businessman's view" that many UK manufacturers were driven to closure through the effect of the real exchange rate appreciation on their exported and import-competing products, and that, once closed, the firms' had permanently lost their market position. This pointed to the plausibility of the assumptions of sunk costs in production and switching costs in demand. To these assumptions we now add the evidence of the performance of UK exports in 1980/1, interpreted through the Dixit model of entry and exit, as above. The combination of increased uncertainty, increased mean reversion, and sunk costs offers a stylised explanation of the observation that export volumes did better than predicted in 1980/1. On these counts hysteresis effects appear to fit the picture.

Yet we must emphasise that we cannot claim to have tested for the presence of hysteresis effects. Our simulations indicate how such effects might have been relevant in explaining events in 1980/1 and reported parameter change would not be inconsistent with a Chamberlinian model with hysteresis effects. However the need to set limits on this research has meant that we have not considered UK export pricing, which studies of US experience in the mid 1980s have looked to for confirmation of hysteresis effects; nor have we considered the behaviour of UK imports during this period. Furthermore, it is widely held in the literature that convincing evidence of hysteresis effects is most likely to come from microeconometric studies of industry or firm level data: to which might be added the support of case studies of the company level response to the events of 1980/1.

The hysteresis question remains important for a number of reasons. One example is the assessment of the welfare costs of a tight monetary policy acting primarily through exchange rate appreciation in circumstances when there are hysteresis consequences for tradeable goods industries: this might be expected to have an inhibiting effect on investment in the tradeable goods sector. However, our attempts to apply such analysis to the UK experience have only been explanatory.

Krugman and Exchange Rate Instability.

Our fourth question concerns the Krugman (1989a) thesis of 'Exchange Rate Instability'. Krugman's argument is that exchange rate volatility inhibits the response of trade flows and prices necessary for external adjustment, and that this damped responsiveness of trade feeds back to accentuate exchange rate volatility. More predictable, less frequent, and less easily reversed exchange rate movements could therefore do the necessary job of external adjustment more effectively than volatile free-floating exchange rates.

Our research, insofar as it goes, is supportive of Krugman's case. In trying to explain the behaviour of UK export volumes in 1980/1 we have argued that exchange rate expectations, and particularly, exchange rate uncertainty, offer a powerful intuition as to why UK export volumes may have been underpredicted in 1980/1. Krugman's argument encourages us to turn this around: the very uncertainty in the exchange rate and about exchange rate policy over this period may have necessitated a much greater appreciation than otherwise in order to achieve the squeeze on activity in the tradeables sector required for disinflationary purposes. Moreover, our consideration of the long run competitiveness elasticity suggests that, if the stationary nature of medium term movements in competitiveness is allowed for, there is little reason to believe that the response of the UK economy to a permanent and credible shift in the real exchange rate has fallen over time. On this count, at least, external adjustment through exchange rate policy could still be effective.

We note that our failure to do more than explore the relevance of hysteresis effects in trade does not hinder the Krugman argument: Baldwin and Lyons (1988), for example, have shown that the 'mutual amplification' effect does not depend on the assumption of hysteresis effects.

On Econometric Trade Equations.

We turn now to our final question. In the light of our research, what views have we formed concerning the nature and use of econometric trade equations. Despite having confined our empirical work to aggregate UK manufactured export volume equations, we take the liberty here to generalise our resultant impressions quite freely.

Firstly, for better or for worse, it is clear that the typical trade equations in use in macroeconometric models in the UK and elsewhere are far removed from the ambitions of the modern optimising approach to macroeconomics. This would be all the more true if the optimising approach were to be combined with what may be realistic assumptions about market structure and behaviour, adjustment and transactions costs. In contrast to the essentially reduced form specification of existing equations (notwithstanding the traditional identification by assumption of demand with volumes and supply with pricing), the theorist would look for consistent expectations; the separation of supply and demand influences, possibly with allowance for strategic behaviour; and a treatment of the discontinuous adjustment, hysteresis, nonlinearity, and non-certainty equivalence which might all be present if adjustment costs are indeed lumpy. Compared with the potential richness of such theory-driven models, the traditional approaches offer an apparently pedestrian - if by now well tried - interpretation of aggregate phenomena. However, theoretical analyses suggest that we could surely do better.

The wild swings in real exchange rates of the 1980s at times appeared to have tested the old equations to destruction: the predictive failure of UK export volume equations in 1980/1 was one example, to be followed in the US by evidence of structural shifts in exchange rate pass-through coefficients in pricing equations. Nevertheless the well developed recent literature on modelling the US external deficit has been conducted using such equations, albeit with some qualifications, as in Hooper and Mann (1989). For all this, the theoretical simplification of the traditional equations may be seen as a drawback for analysis, to be mitigated only if they prove themselves as robust approximations to the system driving the observed data (the DGP). On this score, an important piece of work by Marquez and Ericsson

(1990) considers a variety of existing models of US trade behaviour and is able to show that all the models are rejected by the data.

By Marquez and Ericsson's relatively strict criteria, existing models are not sufficiently good approximations of the data, (and are rejected by their nonlinear forecast encompassing test procedures). Moreover, better approximations of the data are shown, in principle to be available. Assuming that the best possible approximations to the data generation process were eventually obtained in the UK case, we might hope that they would support a more powerful theoretical interpretation than do the current models.

Does it seem possible that satisfactory empirical models will be developed which also satisfy the kinds of theoretical concerns raised in the literature? Perhaps: but the experience of this research suggests caution. The development of convincing replacement equations may prove all the more difficult in a world of frequent exchange rate policy shifts. We can nevertheless take Marquez and Ericsson's result as an optimistic one and continue the search for improved statistical models of trade behaviour. It remains to be seen whether theoretical insights as to the nature of the underlying DGP may contribute to improved empirical analysis in this case.

By way of example, consider a particular theme of this research: the vulnerability of export volume equations to shifts in exchange rate expectations. We suspect that this vulnerability is not simply confined to the short run dynamic response, although, as long as expectations are consistent over the long run, it may prove easier to improve the modelling of long run properties than to do the same for the short run response. Yet our efforts to incorporate this theoretical insight explicitly into empirical models have met with little success. We doubt that an empirically successful aggregate model of UK trade will be developed which is also firmly grounded in the theory of optimisation with realistic adjustment costs.

In the meantime we are left with the 'trusty' (evidently not completely trustworthy!) old style econometric models of trade behaviour. On reflection, the theory and evidence of this research should

teach us that, while the old equations may be relatively robust approximations of the data, (and while, though flawed, they may be as good as any existing alternatives), they must be used with care. In particular, these equations should not be used uncritically as part of the evaluation of alternative policies for external adjustment or of alternative exchange rate regimes.

9.3 SOME IMPLICATIONS FOR WELFARE AND POLICY

It is appropriate that we should make some comments concerning welfare implications raised by this research. We should acknowledge the point made by Dixit that under the assumptions of his model, all behaviour is optimal: there is no suggestion of any externalities. However, there are real resource costs associated with the loss or the establishment of a market position. Thus if the exchange rate movements do not themselves reflect social costs and benefits, (which is a possible interpretation of excess volatility, fads and bubbles in financial markets), the privately optimal response of industry to the exchange rate signal may not be socially optimal. In a non-certainty equivalent world, such as one with lumpy adjustment costs, if financial market failures cause prices to reflect something other than the social level of risk, then that will lead to socially sub-optimal decisions. However, we should also acknowledge that there may be beneficial as well as deleterious social externalities associated with the private sunk costs of resource re-allocation. For example, there might be aggregate consequences for labour market behaviour from a large exchange rate shock as in the 'Cold Shower' stories, or through reputational and information transmission channels. The welfare implications of such a large macroeconomic shock will be not easily be calculated.

We should also make a number of policy related observations in the light of our research.

Firstly, following on from our theoretical work we know that the behaviour of export volumes may be sensitive to expectations of future competitiveness in a relative complex, non certainty equivalent fashion. Trade behaviour will be sensitive both to whether policy implies permanent or temporary

shocks to competitiveness, and also to the extent to which policy reduces uncertainty by a commitment to stabilise future shocks.

Secondly, our research as a whole emphasises the need for policymakers to take a sceptical view of particular estimates of important empirical magnitudes. In the context of exports behaviour, the competitiveness elasticity is evidently not robust to differing specifications, and this sensitivity should be borne in mind. Moreover, policymakers should consider the purposes for which they intend to use empirical estimates.

Thirdly, policymakers should consider the possibility that the response of the economy to large changes in policy may be different from the response to small changes in policy when behavioural relationships are marked by sunk cost hysteresis phenomena.

Finally we return to the UK government's claim in 1985 that competitiveness elasticities had fallen. This claim was made in support of a policy judgement that a higher real exchange rate than hitherto could be tolerated, in view of the belief that the costs in terms of demand for UK tradeables of such a policy had fallen. We have suggested firstly that the bulk of the perceived structural change reflected a misspecification of the long run competitiveness elasticity; we have also raised the likelihood that any residual parameter change may have reflected the Lucas critique consequences of the government's change in exchange rate policy.

9.4 OVERALL SIGNIFICANCE OF THIS RESEARCH.

In closing, what can we conclude concerning the overall significance of the research presented in this thesis?

Firstly, while this research has provided a useful background against which to consider to such 'big questions' as whether the UK economy in the 1980s was characterised by hysteresis in trade, and

whether trade relationships underwent any structural change as a consequence of a 'Thatcher effect', we must recognise that our reflections on these questions should not be treated as definitive answers.

Secondly, our research makes original contributions to both theoretical and empirical analysis and these can be related through the partial equilibrium analysis of trade flows to the overall debate on open economy macroeconomic policy. Hooper and Mann (1989) have given a robust defence of the utility of partial equilibrium approaches to trade behaviour as part of their policy research into US macroeconomic imbalances: our detailed results concerning partial equilibrium analyses of sunk cost hysteresis and concerning UK manufactured export volumes can similarly be viewed as a contribution to the wider debates on macroeconomic analysis and policy in the UK.

Thirdly, although we have motivated this research in terms of exchange rates, expectations and trade, the contributions of this research may be of some value in other areas of economics. Having begun from the theoretical literature on hysteresis in trade, we have indicated that our theoretical results may have much more general relevance to stochastic saddlepoint systems with a recursive restriction on the evolution of the predetermined variable. Equally, our empirical analysis may be of as much significance as an example of the application of a Wickens and Breusch approach to dynamic specification and long run properties in the presence of a stationary regressor, as it is in terms of findings concerning UK export volume behaviour. Some possible directions for the extension of this research follow these lines rather than being confined to trade and exchange rates.

Our theoretical work has added to the understanding of entry and exit with uncertainty through the identification of the effects by which increased mean reversion affects the non-certainty equivalence of the Dixit model. Our major finding is a 'crossover' result, which vividly demonstrates the difference between reversible and irreversible investment decisions.

Our empirical work has cast doubt on the validity of a popular error correction specification for export volume equations in terms of the misspecification of the long run competitiveness elasticity. Although we have been unable to suggest clear directions for the improvement of the model, there nevertheless

appears to be a role for long lags of competitiveness which may reflect smoothing to form a measure of 'permanent' competitiveness.

Nevertheless, taking both theoretical and empirical work together, it is plausible that both expectational and sunk cost hysteresis effects may have an important role to play in export determination. These factors suggest that a satisfactory empirical model of export determination may be quite difficult to achieve, particularly if it is intended for policy analysis purposes: nevertheless, research should continue to explore the microfoundations of aggregate trade behaviour. However, we do not wish to go so far as Dixit, Baldwin and Krugman appear to do in rejecting the value of the traditional aggregate literature on trade equations. We would rather conclude that due care should be taken in the use of traditional estimates, particularly when changes in exchange rate expectations or exchange rate uncertainty appear to be important, or when other evidence points to the relevance of discontinuities in adjustment.

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APPENDICES.

Appendix 3.1

THE NUMERICAL APPROACH.

Our numerical method uses the elementary shooting technique employed in Miller and Weller (1988a) and Krugman (1988). We solve the differential equations describing the value functions in the active and idle states, using the boundary conditions provided by the optimal stopping conditions at the entry and exit triggers. In practice, the numerical method is based on the simultaneous shooting of value-difference (corresponding to Value Matching, and defined as W(P) - V(P)) and slope difference (corresponding to Smooth Pasting and defined as W'(P) - V'(P)) functions.

The procedure begins with an initial guess for the exit trigger price P_L. From this value of P, at which both Value Matching and Smooth Pasting are made to hold by construction, both the value and slope difference functions are projected by the shooting technique. The initial value of the value difference at the assumed exit threshold is given by L, the exit cost, while the initial value of the slope difference at the exit threshold is zero. The shoot proceeds until the slope difference equals zero, and so the value difference is at a minimum. If at this point the Value Matching condition is satisfied, (which requires the value difference plus the entry cost K to be within a prescribed tolerance of zero), then a solution pair for the trigger prices has been found.

If the upper Value Matching and Smooth Pasting conditions do not hold simultaneously, then the initial guess for the exit trigger is incorrect and must be revised by a line search. Simple rules for the systematic revision of the initial condition in response to the failure of the upper Value Matching condition are usually stable. However, numerical instability problems can arise: in these cases the numerical procedure is only stable if started in the neighbourhood of the solution. Valid solutions can nevertheless be obtained with care.

A few points should be made with respect to the accuracy of this numerical procedure for the solutions of the differential equations.

Firstly, the differential equations are approximated by discrete steps in the shooting method: a smaller stepsize will presumably improve the approximation. A larger stepsize has the effect of raising the values of both exit and entry thresholds somewhat in the proposed solution. Hence we conclude that the procedure involves an upwards bias to both exit and entry thresholds which can be made arbitrarily small by the choice of sufficiently small stepsize.

Secondly we can choose the tolerance of our procedure in terms of the accuracy with which the upper Value Matching criterion is required to hold. Typically this has been set sufficiently small that it does no act as a binding constraint on the accuracy of the proposed solution.

A third degree of freedom in the accuracy of a solution is given by the precision with which the initial condition can be given. These three parameters, (stepsize, tolerance of the upper Value Matching requirement, and the precision of initial condition) give us a considerable degree of control over the accuracy of our procedures, and this control can exercised to ensure that proposed solutions are acceptably robust.

The potential for controlling the accuracy of the solution method is most valuable for those parameter values for which the method displays strongest signs of numerical instability.[1]

This appears to arise when relatively high values of the mean reverting coefficient make the cautious component of the solution small relative to the total band-width. The numerical instability arises in conjunction with extreme sensitivity of the proposed solution for the upper or entry trigger to the initial condition or exit trigger. In general we find that the exit threshold can be solved to an arbitrary accuracy (given by the initial conditions) but that the entry threshold may be much less accurate. The

1. The numerical instability which arises operates so that for values only slightly below the initial condition associated with the solution value, the usual revision rules for the initial conditions are unstable.

differing accuracy of the two trigger price solutions is an important consideration when we wish to check the accuracy of unexpected findings.

The Fortran 77 code of the simple shooting programme which we employed for this research follows.

```
      PROGRAM DOUBLE
*********************************************************************
*
**Christopher John Williams, University of Warwick, January 1989.
**(This program is a Fortran77 modification of a February 1988 123
setup)
*********************************************************************
***
**  DOUBLE PRECISION VERSION OF DIXIT PROGRAM.
**
**  (INITIALISATION OF WVDD CORRECTED MAY 5TH 1989)
*********************************************************************
**

*THIS PROGRAM SOLVES THE DIXIT MODEL OF HYSTERESIS IN TRADE IN THE
PRESENCE
*OF MEAN-REVERSION IN THE EXCHANGE RATE OR PRICE PROCESS.

*A SHOOTING METHOD, AFTER THE USAGE OF KRUGMAN OR MILLER & WELLER IS
EMPLOYED.

*FROM AN INITIAL GUESS FOR THE LOWER THRESHOLD (EXIT) PRICE, THE
PROGRAM
*ITERATES UNTIL VALUE MATCHING AND SMOOTH PASTING CONDITIONS ARE
SATISFIED
*AT BOTH THE LOWER AND UPPER THRESHOLDS. THESE ARE SOLUTION VALUES.

*********************************************************************
***

C**DECLARE VARIABLES

C**Variables in Shoot

      DOUBLE PRECISION P,WV,WVK,WVD,WVDD,LP,LWVK,LWVD,HH,
     & LL,MD,PLC,PUC,TOL

C** Parameters

      DOUBLE PRECISION STP,PL,PBAR,W,THETA,RHO,SIGMA,L,K .

C** Counts I for shoot steps, J for interim write, II for attempts

      INTEGER I,II,J,MAXIT,MAXSTP

      OPEN(3,FILE='DOUBLE.IN')
```

```
      OPEN(7,FILE='DOUBLE.OUT')
C***LOAD MAXIT AND MAXSTP*************************************
      READ(3,*) MAXIT, MAXSTP, TOL
      WRITE(6,) "MAXIT =", MAXIT, "MAXSTP = ", MAXSTP,"TOL = ",TOL
      WRITE(7,) "MAXIT =", MAXIT, "MAXSTP = ", MAXSTP,"TOL = ",TOL
C***STEP SIZE*************************************************
      READ(3,*) STP, HH, LL
      WRITE(6,*) "STEP =", STP
      WRITE(7,*) "STEP =", STP
      WRITE(6,*) 'HH = ', HH, ' LL = ', LL
      WRITE(7,*) 'HH = ', HH, ' LL = ', LL

C***MODEL PARAMETERS******************************************
      READ(3,*) PL,PBAR,W,THETA,RHO,SIGMA,L,K
      WRITE(6,*) "PL = " , PL
      WRITE(7,*) "PL = " , PL
      WRITE(6,*) "PBAR = " , PBAR
      WRITE(7,*) "PBAR = " , PBAR
      WRITE(6,*) "W = " , W
      WRITE(7,*) "W = " , W
      WRITE(6,*) "THETA = " , THETA
      WRITE(7,*) "THETA = " , THETA
      WRITE(6,*) "RHO = " , RHO
      WRITE(7,*) "RHO = " , RHO
      WRITE(6,*) "SIGMA = " , SIGMA
      WRITE(7,*) "SIGMA = " , SIGMA

      WRITE(6,*) "L = " , L
      WRITE(7,*) "L = " , L
      WRITE(6,*) "K = " , K
      WRITE(7,*) "K = " , K

C****CERTAINTY EQUIVALENT SOLUTION

      PLC=W+(THETA/RHO)*(W-PBAR)-(RHO+THETA)*L
      PUC=W+(THETA/RHO)*(W-PBAR)+(RHO+THETA)*K

      WRITE(6,*) "CERTAINTY EQUIVALENT LOWER AND UPPER TRIGGER
PRICES"
      WRITE(7,*) "CERTAINTY EQUIVALENT LOWER AND UPPER TRIGGER
PRICES"
      WRITE(6,*) "PLC = " , PLC
      WRITE(7,*) "PLC = " , PLC
      WRITE(6,*) "PUC = " , PUC
      WRITE(7,*) "PUC = " , PUC



C**STARTING VALUES FOR SHOOT*****************

C** First Load a value for P sub L OR ELSE derive it from cert.
equiv.

     IF ((HH-PLC).GE.0) GOTO 60
C**                    ( At 60 Write 'Invalid value for PL' and
stop.)
```

```
C**INITIALISE HIGH AND LOW GUESSES AND ITERATION COUNT

      II=0


C**Start of Iteration Loop

10      MD=(HH+LL)/2

**Initial values for level and gradient differences are L and 0

      WV=L
      WVD=0

C**Initial values for P from P sub L (PL)

       P=MD
       PL=MD

C** Initialise counts for within shoot variables

       I=0
       J=0
C** Write Titles for output.

      WRITE(6,200)
'PL','P','WV(P)','WVD(P)','WVK(P)','LP','LWVD','LWVK'
      WRITE(7,200)
'PL','P','WV(P)','WVD(P)','WVK(P)','LP','LWVD','LWVK'

C**BEGIN SHOOT BY UPDATING SECOND DERIVATIVE

       WVDD=(2/((SIGMA*P)**2))*(P-W+RHO*L)

C**UPDATE P, GRADIENT AND LEVEL DIFFERENCES
       WRITE(6,*) 'P = ', P,'WVDD = ', WVDD
       P=P+STP

       WVD=WVD+(WVDD*STP)
       WRITE(6,*) 'P = ' , P , 'WVD = ', WVD
       WV=WV+(WVD*STP)
       WRITE(6,*) 'WV = ', WV
       WVK=WV+K
       WRITE(6,*) 'WVK = ', WVK
       WRITE(6,*) 'I =', I , 'J = ', J
       I=I+1
       J=J+1

C**DEFINE VARIABLES FOR PREVIOUS STEP COMPARISONS

       LP=P
       LWVD=WVD
       LWVK=WVK


C**Start Loop for Shoot steps.
```

```
C**CONTINUE SHOOT FOR SUBSEQENT STEPS BY ITERATION

20        WVDD=(2/((SIGMA*P)**2))*(P-W+(RHO*WV)-THETA*(PBAR-P)*(WVD))

C**UPDATE P, GRADIENT AND LEVEL DIFFERENCES

        P=P+STP

        WVD=WVD+(WVDD*STP)

        WV=WV+(WVD*STP)

        WVK=WV+K


        I=I+1
        J=J+1

        IF (J.NE.99)  GOTO 35

C**REPORT INTERMEDIATE VALUES

        WRITE(6,*) 'Shoot is continuing. Interim values at step', I
        WRITE(7,*) 'Shoot is continuing. Interim values at step', I
C** Write Titles for output.

      WRITE(6,200)
'PL','P','WV(P)','WVD(P)','WVK(P)','LP','LWVD','LWVK'
      WRITE(7,200)
'PL','P','WV(P)','WVD(P)','WVK(P)','LP','LWVD','LWVK'

        WRITE(6,201) PL, P, WV, WVD, WVK,LP,LWVD,LWVK
        WRITE(7,201) PL,P, WV, WVD, WVK, LP, LWVD, LWVK
        J=0


C**When WVD=0, ie min Value diff, end shoot.

35        IF (WVD.GE.0.AND.LWVD.LE.0)  GOTO  40

C** When large -ve or +ve numbers (make parameters of critical
values)

        IF (WVK.GE.20*(K+L)) THEN
               WRITE(6,*) 'SHOOT DUMP, TOO HIGH, RESTARTING'
               WRITE(7,*) 'SHOOT DUMP, TOO HIGH, RESTARTING'
               GOTO  45
        ELSE
               CONTINUE
        ENDIF

        IF (WVK.LE.-20*(K+L)) THEN
               WRITE(6,*) 'SHOOT DUMP, TOO LOW, RESTARTING'
               WRITE(7,*) 'SHOOT DUMP, TOO LOW, RESTARTING'
            GOTO  45
        ELSE
               CONTINUE
        ENDIF
```

```
C       IF (P.GT.(PUC+(10*(PLC-LL))))   GOTO 90

C***STOPPING RULES FOR SHOOT IF TOO MANY STEPS

      IF (I.GT.MAXSTP) STP=2*STP
      IF (I.GT.MAXSTP) WRITE(6,*) 'MAXSTP REACHED, RESET STP = ',STP
      IF (I.GT.MAXSTP) WRITE(7,*) 'MAXSTP REACHED, RESET STP = ',STP


C**UPDATE VARIABLES FOR PREVIOUS STEP COMPARISONS
       LP=P
       LWVD=WVD
       LWVK=WVK


C**Loop Return

      GOTO  20

C**REPORT END OF SHOOT
C** Write Titles for output.

40    WRITE(6,200)
'PL','P','WV(P)','WVD(P)','WVK(P)','LP','LWVD','LWVK'
      WRITE(7,200)
'PL','P','WV(P)','WVD(P)','WVK(P)','LP','LWVD','LWVK'

      WRITE(6,201) PL, P, WV, WVD, WVK, LP, LWVD, LWVK
      WRITE(7,201) PL, P, WV, WVD, WVK, LP, LWVD, LWVK
      WRITE(6,*) 'NO OF ITERATIONS (II) = ',II, 'HH =', HH, 'LL =',LL
      WRITE(7,*) 'NO OF ITERATIONS (II) = ',II, 'HH =', HH, 'LL =',LL

      WRITE(6,*) 'ABOVE RESULTS FROM STARTING MD = ' ,MD
      WRITE(7,*) 'ABOVE RESULTS FROM STARTING MD = ' ,MD

C**STOPPING RULES FOR WHOLE SEARCH

      IF (WVK.LE.TOL.AND.WVK.GE.-TOL) GOTO 50
C**  Goto write out results and stop.



C**REVISION RULES FOR INITIAL GUESS OF P sub L

C**When to lower
45    IF (WVK.GT.0) THEN
                  IF (ABS(HH-LL).LE.0.001) WRITE(7,*) 'HH-LL LE .001'

                  HH=MD
      ENDIF

C**When to raise

      IF (WVK.LE.0) LL=MD

C**  Iteration Loop Return
```

```
      WRITE(6,200)
'PL','P','WV(P)','WVD(P)','WVK(P)','LP','LWVD','LWVK'
      WRITE(7,200)
'PL','P','WV(P)','WVD(P)','WVK(P)','LP','LWVD','LWVK'

      WRITE(6,201) PL, P, WV, WVD, WVK, LP, LWVD, LWVK
      WRITE(7,201) PL, P, WV, WVD, WVK, LP, LWVD, LWVK
      WRITE(6,*)  'HH =', HH, 'LL =',LL
      WRITE(7,*)  'HH =', HH, 'LL =',LL

      IF (II.GT.MAXIT) GOTO 70

      II=II+1

      WRITE(6,*) 'BEGINNING ITERATION NO =', II
      WRITE(7,*) 'BEGINNING ITERATION NO =', II
      GOTO 10

C**REPORT END OF SEARCH
C** Write out results of shoot and then stop.

50     WRITE(6,*) 'CONVERGENCE ACHIEVED: PROPOSED SOLUTION'
       WRITE(7,*) 'CONVERGENCE ACHIEVED: PROPOSED SOLUTION'

       WRITE(6,300) 'PL', 'PU', 'LP','LWVK', 'WVK'
       WRITE(6,301)  PL, P, LP, LWVK, WVK

       WRITE(7,300) 'PL', 'PU', 'LP','LWVK', 'WVK'
       WRITE(7,301)  PL, P, LP, LWVK, WVK

       WRITE(6,*) 'STOP'
       CLOSE(7)
       STOP

C**Warn Invalid value of PL and Stop
60     WRITE(6,*) 'WARNING: HH GT PLC. TERMINATION ADVISED.'
       WRITE(7,*) 'WARNING: HH GT PLC. TERMINATION ADVISED.'

       WRITE(6,*) 'PROGRAM CONTINUES IN ABSENCE OF INTERVENTION'
       II=0
       GOTO 10

70     WRITE(6,*) 'WARNING: STOPPING AS NUMBER OF ITERATIONS = ', II
       WRITE(7,*) 'WARNING: STOPPING AS NUMBER OF ITERATIONS = ', II

       WRITE(6,*) 'P = ', P, 'WV = ' , WV, 'WVD = ', WVD, 'WVK = ',
WVK
       WRITE(6,*) 'LP = ', LP, 'LWVD = ', LWVD, 'LWVK = ', LWVK
       WRITE(6,*) 'HH = ',HH,'LL = ',LL,'I = ', I, 'J = ', J ,'II =
', II

       WRITE(7,*) 'P = ', P, 'WV = ' , WV, 'WVD = ', WVD, 'WVK = ',
WVK
       WRITE(7,*) 'LP = ', LP, 'LWVD = ', LWVD, 'LWVK = ', LWVK

       WRITE(6,*) 'STOP'
       CLOSE(7)
```

```
      STOP

C******ABNORMAL DUMP FOR PROGRAM
TESTING*********************************

90    WRITE(6,*) 'ABNORMAL DUMP FOR TESTING CHECK'
      WRITE(7,*) 'ABNORMAL DUMP FOR TESTING CHECK'

      WRITE(6,*) 'P = ', P, 'WV = ' , WV, 'WVD = ', WVD, 'WVK = ',
WVK
      WRITE(6,*) 'LP = ', LP, 'LWVD = ', LWVD, 'LWVK = ', LWVK
      WRITE(6,*) 'HH = ',HH,'LL = ',LL,'I = ', I, 'J = ', J ,'II =
', II

      WRITE(7,*) 'P = ', P, 'WV = ' , WV, 'WVD = ', WVD, 'WVK = ',
WVK
      WRITE(7,*) 'LP = ', LP, 'LWVD = ', LWVD, 'LWVK = ', LWVK
      WRITE(7,*) 'HH = ',HH,'LL = ',LL,'I = ', I, 'J = ', J ,'II =
', II

      CLOSE(7)
      STOP
*******************************************************************
*****
C**FORMATS*******************************************************
***

 200  FORMAT(1X,A2,8X,A1,9X,A5,5X,A6,4X,A6,4X,A2,8X,A4,6X,A4)

 201  FORMAT(1X,8G10.4)

 300  FORMAT(1X,A2,10X,A2,10X,A2,10X,A4,8X,A3)

 301  FORMAT(1X,5G12.6)



      END
```

Appendix 3.2

A TWO PERIOD STOCHASTIC MODEL OF SUNK COST HYSTERESIS WITH MEAN REVERSION IN THE PRICE PROCESS.

In this appendix we present the principal analytic results from our two period discrete time treatment of entry and exit under uncertainty. The model and its properties are developed in detail in Williams (1990b). A single firm faces sunk costs of entry and exit into an industry, with an exogenous net revenue (price less fixed flow cost). There are two periods, and we wish to consider optimal decisions made in the current period (period one) but allowing for the future (period two). The stochastic element in the problem is introduced by binomial uncertainty concerning the second period (future) price.

The notation is chosen to be comparable with the Dixit model where possible. Hence, K and L denote sunk costs of entry and exit. The model is normalised as if the flow cost w=0 so that the variable P_i represents revenues (price less zero costs) in period i.

Conditional on the period one value of the price (P_1), mean reversion operates so that the expected period two price is given by the expression ϕP_1. Due to the discrete time context, ϕ measures increasing persistence (as opposed to λ which measures increasing mean reversion in the continuous time case). The process for P reverts monotonically towards P=0 for $0 < \phi < 1$; is a martingale process when $\phi = 1$; and has an iid distibution for $\phi = 0$. Binomial uncertainty concerning period two outcomes is increasing in the parameter σ, so that the second period outcomes differ from the expectation by the realisation of the shock, which takes the values +/- σ. A discount factor of δ is applied to second period values.[1]

The firm is assumed to choose its optimal entry and exit threshold or trigger prices in order to maximise the discounted value of expected profits. The particular advantage of this two period model is that it gives rise to analytic solutions for these trigger prices. These analytics provide important insights into the solution to the more complex Dixit model investigated in chapters two and three.

1. Other details and restrictions are presented in Williams (1990b).

To begin with we make a further simplifying assumption, and confine our consideration to those parts of the parameter space for which both the options of entry and exit take nonzero values. Positive option values are ensured by the restriction that:

$$(K - \sigma)/\phi) < P_1 < (\sigma - L)/\phi$$

It turns out to be essential to relax this restriction in order that changes in uncertainty or changes in the degree of mean reversion will affect the overall optimal trigger solutions. But this simplifying assumption helps clarify the analytic structure at the outset.

i) Trigger Prices.

Given these preliminaries, we can present the closed form solutions for the trigger prices in period one under the restriction of nonzero option values:

$$P_U = [\, K + \delta \ \{ (L-K)/2 \} \,]$$

$$= [\, K/(1+ \delta \ \phi) + \delta \ \{ \ \phi K/(1+ \delta \ \phi) + (L-K)/2 \} \,] \qquad (1)$$

$$P_L = -[\, L + \delta \ \{ (K-L)/2 \}]$$

$$= -[\, L/(1+ \delta \ \phi) - \delta \ \{ \ \phi L/(1+ \delta \ \phi) + (K-L)/2 \} \,] \qquad (2)$$

These price triggers imply the following band width:

$$P_U - P_L = (K+L) = (K+L)/(1+\delta \ \phi) + \delta \ \phi/(1+\delta \ \phi)$$

for which: $\partial(P_U - P_L)/\partial K = 1$, $\partial(P_U - P_L)/\partial L = 1$

For comparison, the deterministic solutions for the triggers are given by:

$$P_{UC} = K/(1+\delta \ \phi) \qquad (3)$$

$$P_{LC} = -L/(1+\delta \ \phi). \qquad (4)$$

ii) Option Difference Terms and Non-Certainty Equivalence.

Given equations (1) to (4), the non-certainty equivalent or option difference components of the trigger prices are given by:

$$P_U - P_{UC} = [\ \delta\{\ \phi K/(1 + \delta\ \phi) + (L-K)/2\ \}\ ] \tag{5}$$

for the entry trigger P_U, and by:

$$P_L - P_{LC} = -[\ \delta\{\ \phi L/(1 + \delta\ \phi) + (K-L)/2\ \}\ ] \tag{6}$$

for the exit trigger P_L.

The option difference terms have two components: one of these reflects the different effect of the entry and exit costs on the option values of entry and exit; the other reflects the possibility of future gains associated with both options.

This simple analytic model mimics manyh properties of the Dixit model of entry and exit with uncertainty.

iii) Derivatives with respect to Sunk Costs

The effects of changes in the sunk costs are shown below:

$$\partial P_U/\partial K = [\ 1/(1+\delta\ \phi) + \delta\phi/(1+\delta\ \phi) - \delta/2\ ]$$

$$= (1 - \delta/2) > 0 \tag{7}$$

$$\partial P_L/\partial L = -[\ 1/(1+\delta\ \phi) - \delta\phi/(1+\delta\ \phi) + \delta/2\ ]$$

$$= -(1 - \delta/2) < 0 \tag{8}$$

$$\partial P_L/\partial K = -(\delta/2) < 0 \tag{9}$$

$$\partial P_U/\partial L = (\delta/2) > 0 \tag{10}$$

An increase in the sunk cost of entry (K) increases P_U and decreases P_L, thereby widening the hysteresis band. An increase in the sunk cost of exit also widens the hysteresis band. These results are consistent with Dixit (1989a).

iv) Derivatives with respect to Persistence.

These derivatives are as follows:

$$\partial P_U/\partial\phi = -\ \delta K/(1+\delta\ \phi)^2 - \delta\phi K/(1+\delta\ \phi)^2 + \delta K/(1+\delta\ \phi) = 0 \tag{11}$$

$$\partial P_L/\partial\phi = \delta L/(1+\delta\ \phi)^2 + \delta\phi K/(1+\delta\ \phi)^2 - \delta L/(1+\delta\ \phi) = 0 \tag{12}$$

These results show that there is no band narrowing effect of persistence (band widening effect of mean reversion) in the model as it stands. The effects of mean reversion on the option difference completely offset the deterministic prediction of band widening, so that there is no net effect. Mean reversion simply increases the proportion of the solution due to the deterministic band at the expense of the non-certainty equivalent component. In order to obtain band widening effects it is necessary to relax the restriction that option values are nonzero, as we consider below.

Nevertheless, these results from the restricted case allow us to distinguish various components of the solution. The derivatives each comprise three terms: a deterministic effect; the effect of discounting the option difference; the direct effect of mean reversion on the option value.[2]

We can avoid any further complications due to asymmetric sunk costs by setting K=L initially. Within this model the direct effect of mean reversion on option values always offsets the 'discounting' effect because $\delta\,\sigma\,(1+\delta\phi)$ exceeds $-\,\delta^2\,\sigma\,\phi$ in absolute value. Hence we see that the effect of mean reversion on the non-certainty equivalent component of the solution is the opposite of its effect on the deterministic component.

v) On Band Widening and the Optimal Exercise of Options.

In the two period model as presented so far, the overall hysteresis band is independent of mean reversion. We can show that this result stems in this particular specification from the restriction that the options of entry and exit are always nonzero.[3] We lift this restriction and consider, from an initial case in which it is not optimal to exercise the option of exit for the negative realisation of revenues, the effects of an increase of mean reversion sufficient to make the exercise of the option of exit profitable. We show that such an increase in mean reversion has the effect of reducing the expected value of

2. The third term reflects our *ad hoc* specification to capture the effects of mean reversion on forecast variance.

3. We also show in Williams (1990b) that this restriction to cases in which both option values are positive appears to explain why an increase in uncertainty does not widen the solution in the two period model, in contrast to the Dixit case.

continuation by more than it does the option difference, and that it will therefore widen the solution band. An outline of the essential elements of the argument follows.

Firstly, a change in mean reversion alters the conditional distribution of future returns. Moreover, with nonzero sunk costs, this does not simply change the value of existing strategies: it can give rise to a change in the optimal choice of strategies. We must consider how mean reversion affects whether it pays to exercise an option, knowing that if the prospective return does not exceed the transctions cost non-exercise will be optimal and the option will have no value.

For an initially high price relative to the unconditional mean of the process, an increase in mean reversion makes it more likely that the option of entry is of no value and also more likely that the option of exit is of value. The non-exercise due to increased mean reversion of the option of entry will exclude a negatively valued contingency from the option, thereby offsetting part of the fall in the option value which would otherwise have occurred.

At the same time, if increased mean reversion causes an initially worthless option of exit to become valuable, then the change in the option value of exit due to mean reversion is less than it would have been if the option of exit had previously reflected negatively valued contingencies. Therefore the effects of mean reversion on changing whether the exercise of options of entry and exit is optimal both work to damp the direct effect of increased mean reversion (which reduces the mean return conditional on an initially high price relative to the unconditional mean).

For an initially low price relative to the unconditional mean these effects are reversed. Increased mean reversion makes it more likely that the option of exit is of no value and that the option of entry is valuable. Once again, when we allow that non-exercise discounts negatively valued contingencies from option valuation, the effect of mean reversion on the optimal exercise of options serves to damp the direct effect of increased mean reversion (which increases the mean return conditional on a low initial price relative to the unconditional mean).

The deterministic effect of mean reversion is to reduce the present value of a given initial departure from the unconditional mean. In the presence of uncertainty there is a value associated with the possibilities of changing state which can be expressed as the difference between the option values of entry and exit: mean reversion also reduces this term directly, and thus the non-certainty equivalent component of the solution responds to increased mean reversion in a way which offsets the band widening deterministic effect. It is the essential fact that option values ignore negatively valued contingencies which provides us with our economic intuition as to why the non-certainty equivalent response of the trigger price solutions to mean reversion is smaller than the certainty equivalent response; and hence to why the net effect of mean reversion on the solution in the Dixit model with uncertainty is one of band widening.

We conclude that changes in parameters such as persistence (mean reversion) and uncertainty affect the band width because they affect the optimal choice of strategies.

vi) Asymmetric Sunk Costs.

If we relax the condition that $K = L$ then we can investigate asymmetric cases. A first observation here is that this will not affect the band width $P_U - P_L$, as the effects of asymmetric sunk costs on the individual triggers cancel out in the expression for the band width.

The terms in (K-L) affect the option difference so that the more asymmetric the sunk costs the more the options difference component of the solution is displaced to the price trigger with the smaller sunk cost.

If the sunk costs are sufficiently asymmetric then the bands may be shifted so far that:

- if $K > L + 2\phi\sigma$, the stochastic entry trigger will lie inside the deterministic solution, ie. $P_{CL} < P_U < P_{CU}$.
- if $L > K + 2\phi\sigma$, the stochastic exit trigger will lie inside the deterministic solution, ie. $P_{CL} < P_L < P_{CU}$.

The two period model clarifies the effects at work in the Dixit model with mean reversion in producing the results reported in chapter three, including the important and unexpected 'crossover' result. The two period model also replicates the Dixit model in that persistence (mean reversion) is necessary for the 'crossover' result.

vii) Conclusions.

The major gain from the drastic simpliying restrictions is that the model yields closed form analytic solutions, which develop insights for the Dixit model, and corroborate the findings of the numerical findings in chapter three. These include the fact that increased mean reversion reduces the non-certainty equivalence of the solution and that uncertainty acts to damp the deterministic effects of mean reversion.

The two period model shows clearly that sunk costs of entry and exit have different impacts on the difference between the option values of entry and exit. It is this which makes possible the important 'crossover' result of chapter three.

A particular difference between this and the Dixit model is that the two period model cannot allow for the possibility of repeated future entry and exit among its solution strategies. Hence it omits the interrelated nature of the valuation problems for the options of entry and exit emphasised in Dixit (1989a): it is precisely the resultant independence of the option valuations which gives the model its analytic tractability.

The two period model simplifies the Dixit model and emphasises that the reversible entry decision depends on both the value of the option of future entry and the value of the option of future exit.

Appendix 3.3

DETAILS OF NUMERICAL EXPERIMENTS CONDUCTED.

1) Introduction.

In chapter three we have investigated the effects of adding mean reversion to the Dixit (1989a) model of entry and exit with uncertainty. This extension of the Dixit model requires the use of numerical methods. In chapter three we presented the outcome of our numerical experiments in broad, stylised terms. In this appendix we give details of some of the particular experiments conducted (using the numerical techniques described in appendix 3.1), and on which the acount of the solution properties given in chapter three is based.

2) Experimental Details.

i) The first example employed the parameters adopted by Dixit as his 'central' case in his original (1987) paper. The parameter values are given by: $P^* = w = 1$. $\sigma = 0.1$, K=20, L=0, r=0.025. (With a stepsize set at 0.001). The mean reverting coefficient (λ) was varied between 0 and 0.2. This experiment revealed the previously predicted band-widening effect of increased mean reversion, but also that the option components of the entry and exit triggers were reduced by increased mean reversion. The results also showed that for $0.15 < \lambda < 0.20$, the entry trigger (P_U) in the full solution displayed a 'crossover' property relative to the certainty equivalent trigger (P_{CU}). However, this solution displayed some signs of numerical instability.

ii) This example employed the Dixit (1989a) 'central case' parameter values, in which the sunk entry costs are reduced relative to i) above, in an effort to establish the 'crossover' result before any numerical instability problems set in. The parameter values were given by $P^* = w = 1$. $\sigma = 0.1$, K=4, L=0, r=0.025. (with stepsize once again set at 0.001). The mean reverting coefficient λ was varied between 0 and 0.45. The experiments confirmed the overall band-widening effect of mean reversion. A

crossover property of the entry trigger (P_U) was confirmed for values of λ equal to 0.2 and above. The accuracy of this crossover solution was shown to be robust for $\lambda = 0.35$ when the initial value of P was refined to 15 decimal places. The solution also shared non-monotonicity in the response of the exit trigger (P_L) to mean reversion.

iii) This example lowered the sunk entry cost further partly in the hope of being able to push the solution further into or beyond the 'crossover' result before numerical instability affects it. The parameter values were given by: $P^* = w = 1$. $\sigma = 0.1$, K=2, L=0, r=0.025, and λ is allowed to vary from 0 to 0.9. The experiments confirmed the existence of a crossover property in the solution for P_U at a value of $\lambda = 025$. The solution was shown to be very accurate, and the crossover property to be robust to concerns about possible numerical instability. The solution appeared to be vulnerable to numerical instability for values of λ greater then 0.55, and so we were unable to establish whether there were limiting properties of P_U-P_{CU} as λ increased: however, there was some suggestion that P_U-P_{CU} might approach a negative constant.

iv) This example considered the addition of mean reversion to the model with symmetric costs of entry and exit. The parameter values were given by: $P^* = w = 1$, σ=0.1, K = L = 4 r = 0.025. The mean reverting coefficient, λ, varied between 0 and 0.19. This experiment revealed that the band widening result applies, but that the trigger price's caution components (P_U-P_{CU}) and (P_{CL}-P_L) are reduced by increased mean reversion. These caution components were reduced at a diminishing rate by increased mean reversion.

v) This example considered a case with asymmetric sunk costs of entry and exit but in which (unlike the previous example) the smaller sunk cost was nonzero. The parameter values were given by: $P^* = w = 1$, $\sigma = 0.1$, K = 4, L = 2, r=0.025. The coefficient of mean reversion, λ, is allowed to vary between 0 and 0.35 (after which numerical instability appears to set in). The experiment found overall band widening and a reduced caution component of the trigger solutions in response to increased mean reversion. However, numerical instability affected the solutions without any 'crossover' result having

occurred in the entry trigger. See example ix) below for a successful search for crossover results with asymmetric but nonzero sunk costs.

vi) This example considered a case in which the zero sunk cost is the entry cost, (rather than the exit cost of previous examples). It enabled us to counter the suggestion that the crossover property was a numerical artefact due to a downward bias in the numerical solutions for the triggers relative to the certainty equivalent triggers. The parameter values were given by: $P^* = w = 1$. $\sigma = 0.1$, K=0, L=2, r=0.025. The mean reverting coefficient, λ, was allowed to vary from 0 to 0.4. The experiment showed that the crossover solution is reversible, in the sense that it can was also found as a property of the exit trigger (P_L) when the exit cost (L) was relatively large: in this case the entry trigger also displays non-monotonicity.

vii) This example repeated the previous example but with a reduced sunk cost of exit, given by L = 1, while the assumption of K = 0 was maintained. This permitted λ to vary from 0 to 0.75 before numerical instability appeared to affect the solutions.

viii) This example increased the variance of the stochastic process in the context of the previous parameterisation. The parameter values were given by $P^* = w = 1$. $\sigma = 0.2$, K=0, L=1, r=0.025, and λ is again allowed to vary from 0 to 0.75.

This experiment confirmed the qualitative solution properties of the two previous examples. One effect of the increased variance was, as in the Brownian motion case, to widen the overall solution. Increased uncertainty also served to increase the values of λ at which crossover in P_L and a turning point in the response of P_U occur.

ix) In this example we explored further whether zero sunk costs at one threshold were necessary to obtain a crossover solution. The parameters were given by $P^* = w = 1$. $\sigma = 0.1$, L=0, and r=0.025. Solutions were calculated for the following values of the sunk entry cost K: 0.01, 0.1, and 0.5. The experiments established that the crossover result did not require the smaller sunk cost to be zero.

Note that further experiments were conducted with the various combinations of sunk costs of entry and exit given by the following (K,L) pairs: 2,0; 2,0.01; 2,0.1; 2,0.5; 2,1; 2,1.25; 2,1.5; 2,1.5; 2,1.9. A crossover result was found to occur when the smaller sunk cost was as much as two thirds of the larger.

x) In this example we considered a different set of symmetric costs of entry and exit. The parameter values were given by $P^* = w = 1$. $\sigma = 0.1$, K=1, L=1, r=0.025, and λ is allowed to vary up to 0.7. The results duplicated the qualitative features of example iv).

xi) In this example we attempted to consider the effects of changes in the variance on the response of the solution to mean reversion. The parameter values were given by $P^* = w = 1$, K=0.2, L=0, r=0.025. with λ allowed to vary from 0 to 2. The following values for σ were considered: 0, 0.1, 0.5, 1.0, 2.0.

xii) This example made another attempt to consider the effects of varying the level of uncertainty, to study this effect on the response to mean reversion. The sunk cost of entry was reduced by a factor of ten, making its coupon equivalent very small relative to the flow cost and mean price. The parameters were given by $P^* = w = 1$, K=0.02, L=0, r=0.025, with λ varying from 0 to 2 as in the previous example. One noteworthy feature of the results was that for the range of mean reversion considered, crossover solutions were not found as the variance increases.

xiii) This example involved a rather different parameterisation of the model, exploring whether solutions had similar qualitative properties in different parts of the parameter space. The parameters values were given by $P^* = w = 10$. $\sigma = 0.5$, K=5, L=5, and r=0.025 with λ allowed to vary from 0 to 1. The entry and exit costs were symmetric, and the qualitative results were as in example iv).

xiv) This example repeated the previous example but with a smaller variance parameter, reduced by a factor of ten. The parameter values were therefore given by $P^* = w = 10$. $\sigma = 0.05$, K=5, L=5, r=0.025. Until numerical instability set in (for λ =0.65) the results again confirmed the qualitative properties of example iv).

Appendix 5.1

DATA APPENDIX.

The quarterly time series data employed in this research were drawn from the LBS Databank lodged with the ESRC Macroeconomic Modelling Bureau at the University of Warwick. Details of the particular series used are given below.

XMAN: UK Manufactured Export Volumes (excluding erratics). A seasonally adjusted index variable with 1980=0. Source: CSO. Available from 1958:2.

XWM: World Manufactured Export Volumes. A seasonally adjusted index variable with 1980=100. Source: OECD Monthly Economic Indicators Available from 1958:2.

COMC: Cost Competitiveness. A seasonally adjusted index variable with 1980=100. Source: LBS Available from 1963:1.

COMP: Export Price Competitiveness. A seasonally adjusted index variable with 1980=100. Source: LBS Available from 1968:3.

COMW: Wholesale Price Competitiveness. A seasonally adjusted index variable with 1980=100. Source: LBS Available from 1968:3.

COMT: Cost Competitiveness Adjusted for Payroll Taxes. A variable constructed by the LBS modelling team as COMT= COMC*{(TNIS+YEP)/(YFP+YWS)}

The Payroll Tax factor employed by the LBS derivation of COMT is constructed using the following variables:

TNIS: National Insurance Surcharge.

YEC: Employers' Contributions.

YWS: Total Wage and Salary Bill.

YFP: Forces Pay.

We now list the data for XMAN, XWM and COMT for the sample period 1963:1 to 1987:2.

Date	XMAN	XWM	COMT
1963:1	46.000000	24.301500	92.335110
1963:2	47.000000	25.721800	91.212000
1963:3	49.000000	26.090000	90.171660
1963:4	49.000000	27.405000	90.143630
1964:1	49.000000	26.668600	88.477650
1964:2	50.000000	27.510200	90.205200
1964:3	49.000000	27.615400	90.132500
1964:4	50.000000	28.562200	89.401020
1965:1	50.000000	29.246000	90.233470
1965:2	51.000000	30.613600	92.392680
1965:3	53.000000	30.876600	92.862400
1965:4	53.000000	32.139000	94.572880
1966:1	54.000000	33.769700	93.110350
1966:2	50.000000	34.295700	95.424090
1966:3	54.000000	34.821700	94.072740
1966:4	55.999990	36.189300	95.898660
1967:1	55.999990	36.715300	94.098270
1967:2	55.000000	37.030900	93.666850
1967:3	53.000000	36.084100	94.610140
1967:4	47.000000	37.030900	85.907030
1968:1	60.000000	40.555190	80.110170
1968:2	58.000000	40.923400	80.112500
1968:3	60.999990	45.026200	80.888190
1968:4	62.000000	43.921600	81.385570
1969:1	62.999990	44.921000	81.805390
1969:2	66.000010	48.866100	81.440430
1969:3	71.000020	49.918100	82.608570
1969:4	69.999990	49.812900	83.761760
1970:1	69.050000	51.127900	85.902080
1970:2	67.309990	52.811100	86.603480
1970:3	63.800000	52.390300	89.174690
1970:4	72.449990	54.021010	90.561410
1971:1	68.760010	55.125590	93.173320
1971:2	74.369990	55.967210	92.859090
1971:3	74.469990	57.597810	94.277830
1971:4	76.810000	54.967810	94.903920
1972:1	73.249980	58.965410	95.393920
1972:2	74.379990	60.596000	92.660710
1972:3	64.809980	60.070000	88.002780
1972:4	84.260030	65.856090	83.730020
1973:1	81.699980	67.802400	79.058020
1973:2	83.089990	69.485590	80.344560
1973:3	84.620030	69.696010	74.866800
1973:4	85.760020	71.484410	74.770830

Date.	XMAN	XWM	COMT
1974:1	85.860020	74.430020	73.676540
1974:2	88.240020	77.428300	73.378250
1974:3	90.849980	76.954920	77.399100
1974:4	88.669980	76.376300	83.085160
1975:1	86.730020	71.852590	84.685130
1975:2	84.810000	72.851980	88.251880
1975:3	83.519990	72.273390	88.686730
1975:4	88.640010	76.008120	87.432590
1976:1	90.200000	79.374490	86.759180
1976:2	94.729990	83.003980	79.186560
1976:3	92.700000	81.846710	79.061300
1977:4	95.680010	84.318980	72.010060
1977:1	97.279990	84.981680	74.495060
1977:2	100.660000	84.981680	76.784070
1977:3	104.410000	86.446900	78.345410
1977:4	98.990010	87.912100	82.701710
1978:1	98.580010	87.179470	86.963370
1978:2	102.380000	90.109890	81.459300
1978:3	98.380010	90.109890	83.791050
1978:4	99.870010	93.772900	87.566440
1979:1	85.460010	93.040310	91.889470
1979:2	107.790000	93.040310	95.345350
1979:3	99.979980	95.238120	102.880800
1979:4	102.910000	98.168480	101.600100
1980:1	105.570000	102.564000	107.381500
1980:2	101.040000	100.366000	115.024200
1980:3	97.140010	97.435880	122.596400
1980:4	96.249990	99.633710	129.865800
1981:1	91.220000	100.366000	131.339200
1981:2	92.160000	102.564000	122.390300
1981:3	95.160000	106.227000	112.008500
1981:4	97.870000	104.762000	110.926800
1982:1	96.910020	103.000000	112.700600
1982:2	97.739980	99.000010	109.595900
1982:3	92.700000	100.000000	109.820700
1982:4	95.760030	100.000000	107.577400
1983:1	93.860000	101.000000	93.298300
1983:2	93.270010	102.000000	98.673590
1983:3	95.989970	105.000000	97.194730
1983:4	98.690020	108.000000	94.657160
1984:1	101.280000	114.000000	91.714990
1984:2	101.640000	110.000000	87.772240
1984:3	101.490000	115.000000	86.210290
1984:4	112.000000	118.000000	83.673940
1985:1	109.667000	119.000000	79.402500
1985:2	112.333000	118.000000	86.972120
1985:3	109.000000	120.000000	93.256980
1985:4	112.333000	120.000000	91.622800
1986:1	107.000000	118.000000	87.074870
1986:2	115.000000	119.000000	87.969290
1986:3	113.000000	123.000000	82.235050
1986:4	120.000000	124.000000	77.675480
1987:1	119.333000	123.000000	79.758710
1987:2	118.000000	124.000000	82.168140

Appendix 5.2

LBS (1985) MODEL MANUFACTURED EXPORT VOLUMES EQUATION AND DIAGNOSTIC STATISTICS.

The following equation is taken from the 1985 LBS model manual, as are the accompanying diagnostic statistics.

XMAN = exp { [0.49609]*lnXMAN(-1) + [0.11242]*lnXMAN(-2) + [0.22615]*lnXWM - [0.1759]*(lnCOMT - lnCOMT(-1)) - [0.13595]*lnCOMT(-3) + [1.38222] } - [0.1457]*D1 + [0.0753]*D2 - [0.1546]*D3 - [0.1546]*D4

$R^2 = 0.990$	FORE(8) = 9.2
SE = 2.61%	CHOW(8.63) = 1.13
DH = 1.23	SPEC(14,49) = 0.69
LM(5) = 0.65	ARCH(1) = 0.0474
Un = 0.65	

Estimation period: 1963:4 - 1983:4

Estimation method: OLS.

XMAN, XWM and COMT are as given in appendix 5.1. D1, D2, D3 and D4 are dummy variables (1,-1) for the dock strikes at 67:4-68:1, 70:3-70:4, 72:3-72:4, and 79:1-79:2.

Implicit Point Estimate for:

Long Run World Trade Elasticity = 0.578

Lomg Run Competitiveness Elasticity = -0.347

Appendix 6.1

DIAGNOSTIC TESTS OF 'LONG LAG' SPECIFICATIONS.

The primary emphasis of our empirical work is on the examination of pre-existing reduced form export volume equations for evidence of misspecification in respect of the long run competitiveness elasticity. Nevertheless, an important secondary motivation is that 'long lag' specifications augmented by additional short run dynamics (in the form of distributed lags on the competitiveness variable) may provide reduced form starting points for the development of empirical models incorporating expectational dynamics, additional theory-based restrictions, etc.

We therefore present a range of diagnostic tests performed upon the augemented forms of both the "LBS" and "(4,4,4)" specifications, as derived in chapter six. We view these tests as design criteria (see Wallis (1987)). As Harvey (1981) puts it, "any model which is seriously entertained must satisfy various diagnostic testing procedures."

The diagnostic testing procedures were implemented using the PC-GIVE package because of its practical convenience for these purposes. However, this choice appears to restrict both the length and order of Almon lags which can be successfully implemented. As a result we only present diagnostics for specifications which feature unrestricted distributed lags on competitiveness. These diagnostic procedures are therefore denied the benefit of the extra degrees of freedom which are gained in chapter six by the use of data-acceptable Almon lags.

For each of the (4,4,17) and LBS(+13) 'long lag' specifications described in chapter six we would prefer to calculate diagnostic statistics for the entire sample and for both the pre- and post- 1979 sub-samples. This does not prove literally possible due to degrees of freedom considerations: we explain our response to this difficulty below.

Figures in parentheses following the reported values of test statistics are the 95% critical values. For further details of procedures, see the PC-GIVE manual.

1) The (4,4,17) Specification.

We first present diagnostics for the more general augmented specification considered, which includes four distributed lags on both XMAN and XWM, and seventeen quarterly lags on COMT.

a) 1968:2 - 1987:2.

77 observations. $R^2 = 0.989$. $\sigma = 0.0236$. F(28,48) = 155.48 (9.71).

Sargan's Information Criterion = -6.334.

We cannot reject the null hypothesis of no error autocorrelation, given the following:

	7th order residual autoregression:	F(7,34) = 0.45 (2.3)	
	12th order Box-Pierce statistic:	chi-squared(12) = 13.166 (19.7)	
and	LM tests of no autocorrelation:		
	versus 1st order alternative.	chi-squared(1) = 0.27 (3.84)	F(1,47) = 0.17 (4.05)
	versus 4th order alternative.	chi-squared(4) = 7.02 (9.48)	F(4,44) = 1.11 (2.59)
	versus 10th order alternative	chi-squared(10) = 12.99 (18.3)	F(10,38) = .77 (2.09)

We cannot reject the null hypothesis of homoskedastic errors given the following test results:

versus ARCH(1) alternative.	F(1,46) = 0.29 (4.05)
versus ARCH(4) alternative.	F(4,40) = 0.42 (2.61)
versus ARCH(12) alternative.	F(12,24) = 0.25 (2.18)

We cannot reject the null of normally distributed errors given the Bowman-Shenton test statistic: chi-squared(2) = 1.191 (5.99).

b) 1968:2 - 1979:2.

Note that the pre-1979 sub-sample offers few degrees of freedom for diagnostic testing with this (4,4,17) specification.

45 observations. $R^2 = 0.990$ $\sigma = 0.0276$. $F(28,16) = 57.09$ (2.21).

Sargan's Information Criterion = -5.762.

We cannot reject the null hypothesis of no error autocorrelation, given the following:

4th order residual autoregression: $F(4,8) = 0.28$ (3.84)

8th order Box-Pierce statistic: chi-squared(8) = 6.145 (15.5)

and LM tests of no autocorrelation:

versus 1st order alternative. chi-squared(1) = 0.051 (3.84) $F(1,15) = 0.02$ (4.54)

We cannot reject the null hypothesis of homoskedastic errors given the following test results:

versus ARCH(1) alternative. $F(1,14) = 0.42$ (4.60)

versus ARCH(4) alternative. $F(4,8) = 0.32$ (3.84)

We cannot reject the null of normally distributed errors given the Bowman-Shenton test statistic: chi-squared(2) = 0.385 (5.99).

c) 1975:2 - 1987:2.

The post-1979 sample period (1979:2 to 1987:2) is too short to support a specification with as many parameters as the unrestricted 4,4,17 case: hence our advocacy of data-acceptable Almon lags on the short-run dynamics in chapter six. Because Almon lags of this length proved unobtainable within the PC-GIVE environment, we chose to lengthen the sample for our diagnostic testing, by initialising it from 1975:2. Although these diagnostics do not strictly correspond to the post-1979 estimates of chapter six, they may nevertheless warn of any severe misspecification.

49 observations. $R^2 = 0.974$. $\sigma = 0.0219$. $F(28,20) = 26/27$ (2.05).

Sargan's Information Criterion = -6.239.

We cannot reject the null hypothesis of no error autocorrelation, given the following:

4th order residual autoregression:	F(4,12) = 0.55 (3.26)	
8th order Box-Pierce statistic:	chi-squared(8) = 11.12 (15.5)	
and LM tests of no autocorrelation:		
versus 1st order alternative.	chi-squared(1) = 6.792 (3.84)	F(1,19) = 3.06 (4.38)
versus 4th order alternative.	chi-squared(4) = 14.361 (9.48)	F(4,16) = 1.66 (3.01)

Note how the chi-squared forms appear able to reject the null of no autocorrelation against the alternatives of first and fourth order error autocorrelation at a 5% significance level. However, the F - form of the statistic is considered more reliable (especially in small samples), and we note that these statistics cannot reject the null at the 5% level.

We cannot reject the null hypothesis of homoskedastic errors given the following test results:

versus ARCH(1) alternative.	F(1,18) = 074 (4.41)
versus ARCH(4) alternative.	F(4,12) = 0.21 (3.26)

We cannot reject the null of normally distributed errors given the Bowman-Shenton test statistic: chi-squared(2) = 0.384 (5.99).

We also conduct forecast tests by estimating the model from 1975:2 to 1985:2 and then forecasting from 1985:3 to 1987:2. There were no individually significant forecasting errors and the out-of-sample forecast test statistics were: chi-squared(8) = 3.08 (15.5) and CHOW(8.12) = 0.70 (2.85). We were thus unable to reject the null of our of sample structural stability on the basis of the above.

Overall these diagnostic statistics do not draw attention to any serious deficiencies in the validity of the (4,4,17) specification. However, the power of these tests to detect deficiencies maybe low, particularly given the few degrees of freedom (too few for asymptotic results) in the split sample cases.

2) The LBS(+13) Specification.

We now turn to the model in which the LBS export volume equation is augmented to incorporate some 13 lags on COMT. The relative parsimony of the model compared with the 4,4,17 specification facilitates our diagnostic procedures.

a) 1967:2 - 1987:2.

81 observations. $R^2 = 0.989$ $\sigma = 0.0249$ F(18,62) = 307.63 (1.77).

Sargan's Information Criterion = -6.619.

We cannot reject the null hypothesis of no error autocorrelation, given the following:

	8th order residual autoregression:	F(8,46) = 0.92 (2.15)	
	12th order Box-Pierce statistic:	chi-squared(12) = 15.956	
and	LM tests of no autocorrelation:		
	versus 1st order alternative.	chi-squared(1) = 1.128 (3.84)	F(1,61) = 0.86 (4.0)
	versus 4th order alternative.	chi-squared(4) = 8.04 (9.48)	F(4,58) = 1.60 (2.53)
	versus 12th order alternative	chi-squared(12) = 14.887 (18.3)	F(12,50) = .94 (1.95)

We cannot reject the null hypothesis of homoskedastic errors given the following test results:

versus ARCH(1) alternative.	F(1,60) = 0.01 (4.00)
versus ARCH(4) alternative.	F(4,54) = 0.49 (2.54)
versus ARCH(12) alternative.	F(12,38) = 1.21 (2.02)

We cannot reject the null of normally distributed errors given the Bowman-Shenton test statistic: chi-squared(2) = 2.558 (5.99).

b) 1967:2 - 1979:2.

49 observations. $R^2 = 0.989$. $\sigma = 0.0263$. F(18,30) = 151.67 (1.96).

Sargan's Information Criterion = -6.260.

We cannot reject the null hypothesis of no error autocorrelation, given the following:

4th order residual autoregression: F(4,22) = 0.36 (2.82)

8th order Box-Pierce statistic: chi-squared(8) = 10.387 (15.5)

and LM tests of no autocorrelation:

versus 1st order alternative. chi-squared(1) = 0.405 (3.84) F(1,29) = 0.24 (4.18)

versus 4th order alternative. chi-squared(4) = 3.095 (9.48) F(4,26) = 0.44 (2.74)

We cannot reject the null hypothesis of homoskedastic errors given the following test results:

versus ARCH(1) alternative. F(1,28) = 0.02 (4.2)

versus ARCH(4) alternative. F(4,22) = 0.10 (2.82)

We cannot reject the null of normally distributed errors given the Bowman-Shenton test statistic: chi-squared(2) = 0.399 (5.99).

Forecast tests do not reject out-of-sample predictive stability. We estimate 1967:2 to 1977:2 and forecast 8 quarters from 1977:3 to 1979:2. There are no individually significant forecast errors, and the model gives rise to the forecast chi-squared(8) = 3.36 (15.5) and the CHOW(8,22) = 1.38 (2.4).

c) 1979:2 - 1987:2.

Note that in the absence of Almon lags there is still an uncomfortably small number of degrees of freedom for the LBS(+13) model in the post-1979 period.

33 observations. $R^2 = 0.989$. $\sigma = 0.0236$. F(28,48) = 155.48 (9.71).

Sargan's Information Criterion = -6.334.

We cannot reject the null hypothesis of no error autocorrelation, given the following:

3rd order residual autoregression: F(3,8) = 0.40 (4.07)

6th order Box-Pierce statistic: chi-squared(6) = 6.022

and LM tests of no autocorrelation:

versus 1st order alternative. chi-squared(1) = 5.063 (3.84) F(1,13) = 2.36 (4.67)

versus 2nd order alternative. chi-squared(2) = 5.123 (5.99) F(2,12) = 1.10 (3.89)

Note that, (as in the later sub-sample for the (4,4,17) model), the chi-squared form appears able to reject the null of no error autocorrelation against an alternative first order autoregressive scheme at a 5% level. Once again we should not the very few degrees of freedom, and once again recall the preference for the F form of the statistic over the chi-squared version, particularly due to the superior small sample properties of the F-test.

We cannot reject the null hypothesis of homoskedastic errors given the following test result:

versus ARCH(1) alternative. F(1,12) = 0.006 (4.75)

We cannot reject the null of normally distributed errors given the Bowman-Shenton test statistic: chi-squared(2) = 0.443 (5.99).

Forecast tests are suggestive of inadequacy in the application of the LBS(+13) model to the post-1979 period, although the evidence is not overwhelming. We estimate from 1979:2 to 1985:2, and then make 8 forecasts up to 1987:2. We find significant individual forecast errors for 1985:3 (postively signed) and 1985:4 (negatively signed) and 1987:2. (Note however that the significant errors in 1985:3/4 appear to be offsetting). The forecast chi-squared(8) = 17.23 (15.5), and this rejects out-of-sample stability at the 5% level, while the CHOW(8,6) fails to reject out-of-sample stability at the 5% level with its value of 3.57 (4.15).

Overall, these diagnostic statistics do not draw attention to any serious deficiencies in the within-sample performance of the LBS(+13) specification. However, we must be concerned by the failures of forecast tests for the 8 quarters after 1985:2. Our of smple performance may, furthermore, be regarded as a more powerful test of model adequacy than are within-sample statistics, and so these failures should, at the very least, cause us to treat the post-1979 augmented specification with some suspicion.

We note that our discussion of possible expectations models in chapter eight acknowledges that there appears to have been another change in exchange rate policy when Nigel Lawson succeeded Sir Geoffrey Howe as Chancellor of the Exchequer, with consequences for the behaviour of competitiveness. Such a policy change could plausibly be expected to give rise to non-homogeneity in the post-1979 sample, and hence to poor forecast performance after 1985.

Appendix 6.2

ISSUES ARISING IN THE IMPLEMENTATION OF HAUSMAN TESTS.

In practice we found that a number of complications arose in the implementation of Hausman tests. The most notable was that in many cases negative values were obtained in the first instance for the relevant quadratic form. Careful attention to the construction of the statistic enabled such results to be explained and avoided.

We are concerned here with asymptotic properties and consistency, for Hausman's approach exploits the fact that the variance of the difference between two estimators is given *asymptotically* by the difference of their variances.

However, for simplicity we give the algebra below as if in a simple regression case with fixed regressors and normal disturbances. This reflects the least squares calculations employed, but omits the convergence conditions etc. of asymptotic theory. Our aim is to illustrate the mechanics of the computations as clearly as possible.

For analytic convenience we simplify the analysis by assuming initially that all parameters in the null hypothesis are to be tested for consistent estimation.
We also specify the form of the alternative hypothesis which corresponds to the inefficient estimator employed for the null.

We write the null hypothesis as:

$$H_0 \quad Y = X\beta + u \qquad u \sim n(0,\sigma^2_u) \tag{1}$$

We estimate β using B the usual OLS estimator. We write the alternative hypothesis (the null could be a simplification in respect of short run dynamics) as:

$$H_A \quad Y = X\beta^* + Z\gamma + e, \quad e \sim n(0,\sigma^2_e) \tag{2}$$

We estimate β^* using b, the usual OLS estimator. The variance of the estimators B and b are given respectively by:

$$\mathrm{var}(B) = \sigma^2_i (X'X)^{-1} \tag{3}$$

$$\mathrm{var}(b) = \sigma^2_i [(X'X)^{-1} + Q] \tag{4}$$

where the scalar σ^2_i denotes the true innovation variance, and we employ the formula for a partitioned inverse in the second case, with:

$$Q = [(X'X)^{-1} X'Z \{ Z'Z - (Z'X (X'X)^{-1} X'Z) \}^{-1} Z'X (X'X)^{-1}]$$

Hence the theoretical Hausman (1978) test statistic employs the quadratic form:

$$(b-B)' [M]^{-1} (b-B) \tag{5}$$

where M denotes the matrix difference var(b)-var(B).[1]

Note that the theoretical test statistic uses a variance quantity (M) which is P.S.D. by definition, so that the resultant quadratic form must always be non-negative.

In practice, however, the true innovation variance, (σ^2_i), must be replaced by an estimated quantity. The usual output from regression packages will calculate the estimated variances for B and b by using estimators for the innovations process which are based on different sums of squared residuals - in each case those associated with the model under estimation. This can lead the estimated variance difference to fail to be P.S.D so that negative values may arise in the quadratic form.

---

1. Consider writing M as:
$\sigma^2_i [(Z'Z)^{-1} Z'X(X'X)^{-1} X'Z(Z'Z)^{-1}]$

Under H_0, the true value σ^2_i is given by σ^2_u and the usual OLS estimators for both σ^2_u and σ^2_e are both consistent for σ^2_u.

However, under H_A, the true value σ^2_i is given by σ^2_e and we now find that, while the usual OLS estimator σ^2_e^ will be consistent for σ^2_e, the OLS estimator σ^2_u^ will in general be *inconsistent* for σ^2_e.

The estimated variance difference calculated using the usual regression output can be written as:

$$\text{var(b)-var(B)} = \sigma^2_e\hat{}\{ Q \} + \{(\sigma^2_e\hat{}-\sigma^2_u\hat{}) (X'X)^{-1} \} \qquad (6)$$

where Q is as defined above.

The first term is a consistent estimate of the variance difference: the explanation of any large sample problems with the estimated variance difference must therefore lie in the second term.

Under the null hypothesis, due to the consistency of its components, the term ($\sigma^2_e\hat{} - \sigma^2_u\hat{}$) will tend to zero. However, under the alternative hypothesis, given that σ^2_u^ is inconsistent for σ^2_e, the limiting value of ($\sigma^2_e\hat{}-\sigma^2_u\hat{}$) is nonzero, and so the overall estimate of the variance difference is inconsistent. In the least squares context, we know that $\sigma^2_e\hat{} \leq \sigma^2_u\hat{}$ *even in large samples* because the restricted sum of squared residuals cannot be smaller than the unrestricted sum of squared residuals. If, *when the null fails to hold*, the inconsistency term dominates, then a negative quadratic form will result.

Negative quadratic forms may arise due to the finite sample problems referred to by Ruud (1984) p 214, but the inconsistency identified above would not be eliminated in large samples. Ruud (1984) reports methods to avoid the finite sample problems, referring to Newey (1983) and Ruud (1982) and to White (1982). Ruud's own "more straightforward" proposal to ensure a PSD variance difference is: "to evaluate both estimates of the estimators' covariances at one of the estimators", (Ruud, p 214).

The equivalent of the information matrices should be evaluated "at the same point in the parameter space". In this context this means using the same estimate for σ^2_i in both components of the variance difference.

Ruud's small sample recommendation is effective in our case. Using the same estimate for σ^2_i instead of *both* σ^2_u^ and σ^2_e^ will eliminate the large sample possibility of negative quadratic forms. Note moreover, that in order to guarantee consistent estimation of the variance difference we should choose σ^2_e^ rather than σ^2_u^ as the standard estimator for σ^2_i.

We must also mention two additional features of our case which serve to distinguish it from the example used in our exposition above, and which serve to complicate the implementation of the test.

The first feature is the fact that our test focuses on only a subset (of one) of the slope coefficients in the null hypothesis, (in Holly's terminology, on only one of the nuisance parameters).

The second point concerns instrumental variables. The actual structure of the Hausman tests employed in our analysis is further complicated by the fact that the estimation procedure for our re-normalised equations is instrumental variables. Part of the added complexity is due to the straightforward replacement of the matrix of regressors with the projection of the regressors on their instruments. However, the effect of the IV procedures on the test statistic is also complicated because the set of instruments employed for the estimators corresponding to the two hypotheses are different. To the extent that the two sets of instruments differ, an additional term is added to the difference between the two estimator variances.

Denote the set of instruments for X under H_0 as V. Let the projection matrix M_V be defined by $M_V=\{V(V'V)^{-1}V'\}$.

Denote the set of instruments for (X,Z) under H_A as (V,T) and let W=(V,T). Let the projection matrix M_W be defined by $M_W=\{W(W'W)^{-1}W'\}$ so that X_A^=M_WX and Z_A^=M_WZ.

Under the null hypothesis the variance of $(b_{IV}-B_{IV})$ is given by $var(b_{IV})-var(B_{IV})$.

$$Var(b_{IV})-Var(B_{IV}) = \sigma^2_i \, [\, Q_{IV} - (X'M_VX)^{-1} \,] \qquad (7)$$

The first term contains Q_{IV}, which is the IV analogue of Q in the OLS case considered above. The second term reflects the difference between the two instrument sets: this will vanish if the two instrument sets are identical. However under the Wickens and Breusch renormalisation the instrument sets differ.