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Minimum energy channel codes for molecular communications

Chenyao Bai, Mark S. Leeson and Matthew D. Higgins

Due to the limitations of molecular nanomachines, it is essential to develop reliable, yet energy efficient communication techniques. In this Letter, two error correction coding techniques are compared under a diffusive molecular communication mechanism, namely, Hamming codes and Minimum Energy Codes (MECs). MECs, which previously have not been investigated in a diffusive channel, maintain the desired code distance to keep reliability whilst minimising energy. Results show that MECs outperforms the Hamming codes, both in aspects of BER and energy consumption.

Introduction: Nanomachines are biologically or artificially created tiny devices or components which are capable of implementing only very simple tasks, such as computation, sensing or actuation [1]. Molecular communications, which operates in aqueous environments and uses molecules to encode and transmit information among nanomachines, represents a new communication paradigm. It is beginning to become established that employing channel coding at the nanoscale is necessary for reliable communication [2]. In addition, with their extremely small size, nanomachines can only utilise limited energy, which makes it essential to develop energy efficient communication techniques. Thus, any coding schemes for nano communications should consider energy dissipation as an essential metric. In this Letter, a novel MEC is applied for the first time and compared with the more traditional Hamming codes.

Communication channel model: In diffusive molecular communications, the information molecules, typically protein complexes, peptides or DNA sequences [3], propagate through a fluidic transmission medium between the transmitter and receiver via diffusion. Here, a 3-D diffusion based communication system is considered where, to simplify the analysis, the medium is assumed to be of extremely large dimensions compared to the size of the information molecules. Furthermore, collisions between the information molecules are neglected and the diffusion coefficient, $D = 79.4\mu\mathrm{m}^2\mathrm{s}^{-1}$ is used, given that it is a known value for insulin in water at human body temperature [3].

At a certain time *t*, the hit time probability is given by:

$$h(t) = \frac{R}{R+d}\frac{d}{2\sqrt{\pi D}}\frac{1}{t^{\frac{3}{2}}}\exp\left(-\frac{d^2}{4Dt}\right) \quad (1)$$

where *d* is the distance away of the information molecule from the receiver with radius *R* = 5μm [3]. To effectively represent the transmitted symbols, the propagation time is divided into time slots which have equal length. Only one symbol propagates in single time slot which is denoted by t_s . The information is encoded by concentration with binary representation. Specifically, if the number of information molecules arriving at the receiver at a certain time slot exceeds a threshold τ, the symbol is represented as "1". Otherwise, it will be interpreted as "0". However, errors may be caused by inter symbol interference (ISI), which is an unavoidable consequence of both wired and wireless communication systems and is known to have adverse effects in communication systems, particularly when the system is stochastic [4].

In the diffusive communication system here, some information molecules may arrive at the receiver after the current time slot according to the diffusion dynamics, which will lead to the incorrect decoding of the received symbol of the next time slot. The channel model which is proposed in [3] is applied as the basis for the following work and channel noise is introduced by the ISI effect. To maintain brevity, we refer the reader to the work in [3] to obtain the BER calculations used here.

Error correction coding: For any communication system, the energy budget is a fundamental design requirement, and in the nano-domain, this limit will tend towards the pW order of magnitude given current achievements in energy harvesting devices. This therefore limits the use of state-of-the-art recursive coding schemes [5]. As such, although relatively simple by today's standards (in performance terms), both Hamming Codes and MECs, are very efficient in terms of energy. Subsequently they are thought to be suitable for enhancing the performance of a nanoscale system.

A. Hamming Coding

Hamming codes, which are described as a $(2^m-1, 2^m-m-1)$ code, are used to form coded output blocks of length $n = 2^m - 1$, where *m* is the number of parity check bits. The minimum distance, d_m, of this type of block code is 3, which means that only one error can be corrected in each block. The BER for the Hamming coded operation can be approximated by [2]:

$$P_{ec} = \frac{1}{n}\sum_{j=t_m+1}^{n} j\binom{n}{j}p^j(1-p)^{n-j} \quad (2)$$

where *n* is the length of codeword, $t_m = \lfloor (d_m-1)/2 \rfloor$ is the maximum number of errors that the code can correct, and *p* is the probability of one bit error. In this case, *p* is set as the value of the optimised probability of error appropriate to the code rate [2]. Encoding the transmission information with (7, 4) and (15, 11) Hamming codes, produces coding rate of 4/7 and 11/15 respectively.

B. Minimum Energy Coding

A novel, minimum energy coding scheme, which takes energy into consideration, is provided in [6] for a THz wireless nanosensor network. In theory, by using on-off keying (OOK) modulation, minimum energy codes with Hamming distance constraints can reduce energy consumption by minimising the average weight of codewords [6]. In this Letter, the MEC proposed in [6], which is considered reliable and suitable for nano communications, is used as the channel code to improve the system performance in a diffusive system. Codewords with a lower weight result in reduced energy consumption, because transmission of a "0" symbol requires less energy than the transmission of a "1" symbol. The average codeword energy is minimised by minimising the average code weight. The source message, which is of length *k*, can be encoded into a codeword which is of length *n* in the following way. For a given set of source symbols, which have a specific source distribution, and a given set of codewords, sorting codewords in increasing code weight order and assigning source symbols in decreasing probability order yields the optimum average code weight [6]. For example, the least probable source symbol is mapped to the largest weight codeword. For OOK modulation, transmitting a ''0'' symbol requires no energy. Thus, minimising energy consumption means the minimisation of the average codeword weight. The weight enumerator of a code is the polynomial $W_{\mathbb{C}}(z) = \sum_i c_i z^i$, where c_i is the number of codewords with weight *i* and *z* is a symbol which is called an indeterminate that does not represent any value. Assuming that *M* is the number of codewords, d_m is the minimum Hamming distance and p_{max} is the maximum probability in the source probability distribution, the weight enumerators of MEC codewords are given by [6]:

$$\mathrm{W}_{\mathbb{C}}(z) = \begin{cases} z^0 + (M-1)z^{d_m} & p_{max} > 0.5 \\ z^{\left\lfloor \frac{d_m}{2} \right\rfloor} + (M-1)z^{\left\lceil \frac{d_m}{2} \right\rceil} & p_{max} \leq 0.5 \end{cases} \quad (3)$$

The MEC only provides the limitation of the length of codeword and the maximum weight, rather than the actual codeword. Thus, different codebooks can exist for a single Hamming distance. For MECs, the decoding method is minimum distance decoding which means that the received *n*-tuple is mapped to the closest codeword in terms of Hamming distance. More errors can be corrected when the minimum Hamming distance increases with the codeword length but this leads to a larger number of error patterns, which will decrease the reliability of the MEC [6]. It can be derived that the minimum codeword length is given by:

$$n_{min} = d_m + (M-2)\left\lceil \frac{d_m}{2} \right\rceil \quad (4)$$

After minimum distance decoding of a MEC, the probability that transmitted codeword is correctly decoded is given by:

$$\varepsilon_d = \sum_{i=0}^{\left\lfloor \frac{d_m-1}{2} \right\rfloor} \binom{n_{min}}{i} p^i (1-p)^{n_{min}-i} \quad (5)$$

where p is the crossover probability.

Power consumption for codeword i is $P_i = w_i P_s$, where w_i is the codeword weight and P_s is the symbol power, which is here normalised to be unity. For simplicity, we assume that each codeword carries $\log(M)$ bits of information so M transmitted codewords contain $M\log(M)\varepsilon_d$ bits of information. The average energy per information bit is given by:

$$\eta = \frac{E(P) \times t}{\log(M)\varepsilon_d} \quad \text{Joules/bit} \qquad (6)$$

where *E(P)* is the expected value of power consumption per codeword and *t* is the transmission time.

Analytical Results: MECs are compared with Hamming (7,4) and (15,11) codes in terms of BER and energy consumption. MECs satisfy the minimum Hamming distance required by Hamming codes so here this is set to three. The corresponding MECs are thus $M = 2^4$ and $M = 2^{11}$. The error correction performances of MECs and Hamming codes over a 4μm transmission distance are illustrated in Fig. 1.

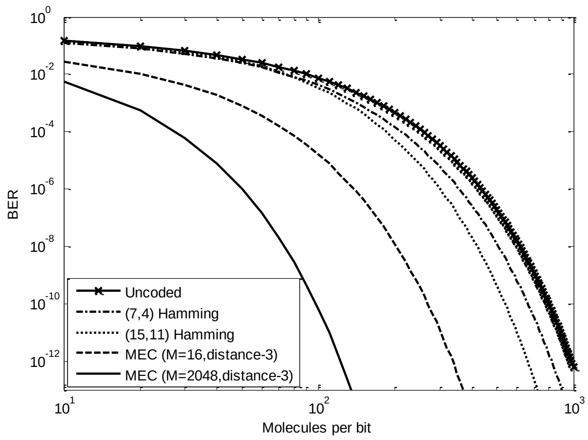

Fig. 1 BER comparison between MECs and Hamming codes.

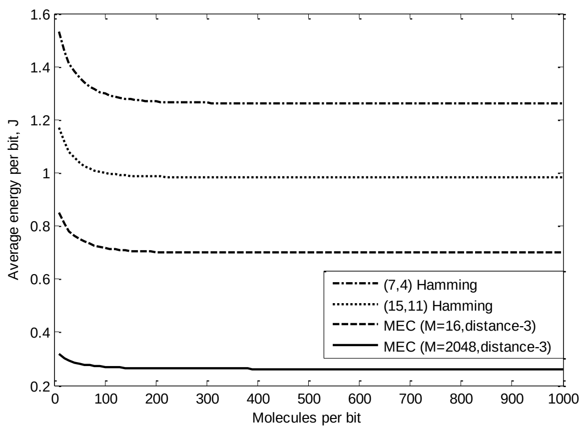

Fig. 2 Average energy per bit comparison between MECs and Hamming codes.

Fig. 1 shows that the system performance is improved by using both Hamming Codes and MECs. The coding gain can be determined by taking the ratio of the number of molecules for a given BER in the uncoded and coded cases since there is an approximately linear relationship between the transmission energy and the number of molecules per bit [2]. Thus the coding gains for the Hamming codes are 0.89 dB and 1.71 dB for the (7, 4) and (15, 11) codes respectively, and for the MECs, the figures are 4.97 dB and 9.44 dB for $M = 2^4$ and $M = 2^{11}$ respectively. In general, MECs have a better performance than Hamming codes with a larger coding gain. Also, the system performance is better with a lengthy codeword. However, for MECs, since increasing the number of codewords means increasing the amount of information to be transmitted, it requires more reliable channels to transmit the codewords, which is intuitively expected. Fig. 2 shows that MECs exhibit superior average energy per bit values. For small numbers of molecules per bit extra energy is needed to deal with unreliable decoding but this effect levels out as the number of molecules per bit increases.

Conclusion: Hamming codes and MECs, with OOK modulation, have been developed and applied to a diffusion based molecular communication system. Analytical results show that both codes offer coding gains which can be several dBs. MECs offer better BER performance and lower energy consumption than Hamming codes but MECs require large codeword lengths.

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