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TAX COMPETITION RECONSIDERED

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DEPARTMENT OF ECONOMICS

Tax Competition Reconsidered[1]

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ABSTRACT: In a classic model of tax competition, we show that the level of public good provision and taxation in a Nash equilibrium can be efficient or inefficient with either too much, or too little public good provision. The key is whether there exists a unilateral incentive to deviate from the efficient state and, if so, whether this entails raising or lowering taxes. A priori, there is no reason to suppose the incentive is in either one direction or the other. In addition, we demonstrate conditions ensuring existence of an asymmetric Nash equilibrium with efficient public good provision. As in prior literature, local amenities enhance capital's productivity. Prior literature, however, focuses on under-provision of public goods.

KEYWORDS. Amenities, Asymmetric Nash equilibrium, Efficiency, Tax competition.

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1 Introduction

Academics and policy makers alike are becoming increasingly concerned about the constraints on policy imposed by tax competition. According to received wisdom, competition between governments for mobile capital will result in a 'race to the bottom' with tax rates too low and with public goods underprovided. It is argued that, by taxing at a lower rate, in order to prevent capital from fleeing elsewhere, each government has an incentive to engage in wasteful competition with the consequence of underprovision of public goods. George R. Zodrow and Peter Mieszkowski (1986) and John D. Wilson (1986) were the first to formalize the intuition of this argument, expounded by Wallace E. Oates (1972).

The purpose of this paper is to present a new challenge to the standard conclusions of the tax competition literature, that when capital is mobile and a major source of taxation then a race to the bottom will be the result. Specifically, we show that when public goods have a positive impact on productivity, then the presence of tax competition does not necessarily result in a race to the bottom. Alternatively, the outcome can be efficient, or there can be a 'race to the top' where there is over-provision of public goods and tax rates are too high. There is no reason, *a priori*, to suppose that one sort of outcome will prevail.

Our model is sufficiently general to incorporate and extend the model that has come to be thought of as standard, where jurisdictions are ex-ante identical. This 'standard' model is the one on which we carry out most of our analysis. Three types of Nash equilibrium are demonstrated; the economic outcome is efficient, or inefficient with too little public good provision, or inefficient with too much public good provision.

In a departure from analysis of the standard model, we show that when jurisdictions are ex ante asymmetric in that the technology of public good provision varies, the Nash equilibrium outcome cannot be efficient. Under-provision or over-provision must result. A policy dilemma is raised by this finding that has, to our knowledge, not been documented previously. On the one hand, governments are unable to achieve efficiency by acting unilaterally. On the other, harmonization in the sense of equating taxes does not achieve efficiency either, because when technology varies efficient tax rates must vary as well.[3]

[3] Asymmetries between jurisdictions have been examined formally by Amrita Dhillon et al (1999),

The standard model is also used in the present paper to prove existence of an asymmetric Nash equilibrium. This equilibrium differs from the standard symmetric equilibrium in two key respects. First, one jurisdiction undertakes all production, while citizens of the other jurisdiction engage in no production at all, being content to lend out all their capital, and live only on the rental payments. Second, the jurisdiction that undertakes production offers an optimal 'policy', in which amenities and taxes are both set at levels that yield a efficient outcome, whilst the other jurisdiction offers no public goods at all. We show that the difference in outcome, that is whether equilibrium is symmetric or asymmetric, depends on the nature of technology.

The all-nothing character of this equilibrium may appear at first sight to be an esoteric curiosum for economic theorists. However, we think of this as characterizing the OECD as one jurisdiction versus the rest of the world as the other. More than 90% of the world's capital is employed in production in the OECD countries. Our analysis highlights the role of public good provision in preventing capital from fleeing to poorer countries even though they might offer lower taxes.

An alternative interpretation of the asymmetric outcome suggests that if there are multiple produced commodities then some jurisdictions may optimally specialize whilst others do not produce the good at all, and this may be an equilibrium. This contrasts with previous multi-product analyses which have imposed symmetry on the outcome (see for example Wilson 1987).

We are not the first to show that the outcome of provision of local public amenities can be efficient. For example, Oates and Robert M. Schwab (1988) present a model where the level of public service provision is chosen by the median voter. They demonstrate a Tiebout type mechanism in which voters vote with the vote rather than with their feet, and the same efficient outcome conjectured by Charles Tiebout results. Dan A. Black and William H. Hoyt (1989) examine the process where jurisdictions bid for firms. They consider a situation where the marginal cost of providing a firm and its workers with public services is less than the tax revenue. In that case, a government may offer the firm

Ravi Kanbur and Michael Keen (1993) and Wilson (1991), and discussed informally by Roger Gordon (1990). However, Dhillon et al focus on a situation where jurisdictions differ in their taste for public goods, and the other papers consider situations where one jurisdiction is larger than the other. The effect of variations in technology has not, to our knowledge, been considered.

subsidies that actually reduce the distortions the average cost pricing of the public service creates, thus increasing efficiency. David E. Wildasin (1989) shows that the inefficiency created by competition for mobile capital can be corrected by a subsidy[4]. Myrna Wooders (1985) demonstrates that when local public goods are financed by lump sum taxation and consumers can 'opt out' to provide the public goods for themselves, then the outcome is near-optimal, where the closeness of the outcome to efficiency depends on the costs of opting out.

Our results are in keeping with those of other previous work in showing that there may be over-provision of public goods. Excessive levels of taxation can arise when there is tax exporting as in Shelby D. Gerking and John H. Mutti (1981) or when policy-makers have Leviathan tendencies as in Jack Mintz and Henry Tulkens (1996). A recent paper by Michael Keen and Maurice Marchand (1997) emphasizes not the level but the composition of public expenditure. They show that when there are two types of public good, one that enhances the marginal productivity of capital (of the kind that we examine) and one that enhances consumer welfare, in any given noncooperative equilibrium the first will be over-provided and second will be under-provided. The point we make is that while other research has introduced a number of different additional factors to the standard basic framework to derive a variety of outcomes, we show that all of these outcomes are possible in the standard basic model.

Other authors have shown that a 'race to the bottom' may not occur due to the presence of asymmetries in the model. For example, in Richard Baldwin and Paul Krugman (2000) there is no symmetric race to the bottom because regions are asymmetric ex ante. In addition, the sequence of play in their paper is asymmetric, with policy makers moving sequentially in Stackelberg fashion, yielding a familiar Stackelberg type outcome. A surprising result that we present is that our outcome can be efficient and asymmetric, even though the framework of our model is of classic Zodrow-Mieszkowski-Wilson form, where all aspects of the model, including the timing of play, are symmetric[5].

The empirical literature questions the extent to which a race to the bottom in tax

[4] Contrastingly, Wildasin (1988) shows that when governments compete using expenditures rather than taxes, the result is an even greater divergence from efficiency.

[5] Models in this tradition are set up as a static game where policy makers move simultaneously in setting policy; see Wilson (1999) for a comprehensive survey.

rates actually occurs. The most comprehensive empirical investigation of this question has been undertaken by Michael Devereux et al (2001). They bring together a number of different measures for ten or more OECD countries over the period 1970-1998. The universally quoted Statutory Tax Rate (STAT) is compared with others such as the Implicit Tax Rate on Corporate Profits (ITR-COR). The nature of their findings is summarized in the following quote: "The differences in the development of STAT and ITR-COR over time is striking. The former clearly fell over time while the latter did not, and if anything rose." Mintz and Michael Smart (2001) present and examine evidence that corporate income tax rates have remained the same or increased slightly since 1986 across provinces in Canada. Richard Baldwin and Paul Krugman (2000) also present empirical evidence (as well as a theoretical model) which counters the idea that historically high taxation countries have had to lower their capital tax rates across the board as capital markets have become more integrated. Richard Higgott (1999) draws attention to a number of other papers which cast doubt over the pervasiveness of the 'race to the bottom' hypothesis.

In this paper, we define an 'amenity' as a public good that has a positive impact on productivity. As we model them, amenities enter the production function. Most of the literature has studied public goods that enter the consumption function instead, although an exception in common with our approach is considered by Zodrow and Mieszkowski (1986). This feature of the model, that the amenity enhances the marginal productivity of capital, is the key motivating force that leads to the possible variety of outcomes.

To help explain the results of our analysis more clearly, we define the notion of *marginal public good valuation* (*mpgv*) which measures the extent to which output is increased - through productivity enhancement - by the marginal unit of the public good. Zodrow and Mieskowski (1986) simply assume that the *mpgv* is never as great as the marginal cost of the public good. Under their assumption, efficient public good provision can never occur through taxation of mobile capital. In our model, the full range of possibilities are covered. To do this, we introduce an assumption related to one that is used in the growth literature; see Robert Barro (1990) and Barro and Xavier Sala-i-Martin (1995 Chapter 4, Section 4). Our assumption implies that the *mpgv* is high at low levels of public good provision, but then declines as public good provision is increased, eventually falling below the marginal cost. The situation examined by Zodrow and Mieskowski -

a race to the bottom - is a special case, where the *mpgv* is *everywhere* lower than the marginal cost. In addition, we consider the possibility that the *mpgv* matches exactly the marginal cost in an efficient state of the economy. Consequently, there is no incentive to deviate from the efficient state, and the outcome of a Nash equilibrium is efficient. And we analyze the situation where the *mpgv* is greater than the marginal cost in the efficient state, creating a unilateral incentive to deviate by *raising* taxes. Complaints are frequently heard that government is 'too big'. This paper demonstrates the existence of a Nash equilibrium in a standard tax competition model that characterizes just such a situation.[6]

Our modelling of government behavior is in the tradition of the so called 'benevolent dictator' who maximizes the welfare of the representative citizen in his jurisdiction. This is in keeping with the approach adopted by Zodrow and Mieskowski (1986) and the papers that followed in the tax competition literature.[7] The assumption that the government behaves as a benevolent dictator contrasts markedly with the approach taken in much of the more recent literature looking at taxation and public good provision, where the government is assumed to be employed as an agent by the electorate (the principle in this setting). See Timothy Besley and Anne Case (1995) and Kenneth Rogoff (1990) for prominent examples that highlight asymmetries of information that give rise to political agency problems. Since the novelty of our approach is to examine the incentive to deviate from the efficient state of the economy, and the conventional tax competition effect is sufficient to motivate this, we leave aside the more complex 'agency problem' affects. The agency problems can be seen in relation to our work as creating additional incentives to deviate from an efficient state of the economy.

The paper proceeds as follows. In the next section, the primitives of the model are set up, and the conditions both necessary and sufficient for efficiency are stated. Two cases are considered. One is where production occurs in both jurisdictions; the 'standard' case. The other is where production occurs in just one jurisdiction. In Section 3 the

[6] In our model, government is too big when too many public goods are provided. A common illustration would be where there are too many roads. This is to be distinguished from the complaint that government is too big in the sense that public goods could be more efficiently provided by the private sector. Here in our model, the government is no less efficient a provider than the private sector would be.

[7] The same approach has also been adopted in the closely related literature on commodity taxation in jurisdictions that are members of a federation; see for example Jack Mintz and Henry Tulkens (1986).

strategic game played by jurisdictional governments is set up. A Nash equilibrium in policies - tax rates and public good levels - is defined in the policy variables. Section 4 then restricts the general framework to consider the Zodrow-Mieskowski model. It is in this section that we show conditions under which the state of the economy is efficient in Nash equilibrium, inefficient with too little public good provision and inefficient with too much. Section 5 then considers the situation where jurisdictions are ex ante asymmetric, in that the technology of public good provision is not identical across jurisdictions. It is shown that under these circumstances the efficient plan cannot be decentralized in a Nash equilibrium. Section 6 returns to the standard model where jurisdictions are ex ante symmetrical and examines the asymmetric Nash equilibrium, where all public good provision and production takes place in one jurisdiction at the efficient level.

2 Primitives and Production Efficiency

The model is of just two jurisdictions[8], 1 and 2. Each jurisdiction consists of a representative citizen, a government and a firm. The firms produce a homogeneous consumer good, the sole consumption good in the economy.

Citizens of jurisdiction i own a quantity of capital $\overline{k}_i$; total capital supply is denoted by $\overline{k} = \overline{k}_i + \overline{k}_j$. Capital is perfectly mobile between jurisdictions: $\overline{k}_i = k_{ii} + k_{ij}$ is capital owned by citizens in jurisdiction i and rented to producers in i and j respectively; $k_i = k_{ii} + k_{ji}$ is domestic and foreign owned capital employed in production by firms in jurisdiction i ($0 \leq k_{ij} \leq k_i \leq \overline{k}$). A *capital allocation* is a vector of capital demands across the two jurisdictions: $\mathbf{k} = (k_i, k_j) \in \mathcal{R}^2_+$.

2.1 Public amenities and the means of production

Production in either jurisdiction results in output of a homogeneous consumer good, which will also serve as the numeraire. The production function is denoted by

$$f_i\left(k_i, y_i\right) \tag{1}$$

[8] The model and results could be generalised to n jurisdictions, but without adding insight (except perhaps for convergence results).

The overall functional form is designed to represent a production technology that depends on the level of a publicly provided amenity y_i as well as capital k_i. We make relatively mild assumptions about the functional form of (1):

A1. Let the function $f_i : \mathcal{R}_+^2 \to \mathcal{R}_+$ be quasi-concave on the domain $k_i \in \left[0, \overline{k}\right]$. Moreover, for $y_i > 0$ assume that f_i has a convex segment on the domain $k_i \in [0, \widetilde{k})$ and a strictly concave segment on the domain $k_i \in [\widetilde{k}, \infty]$, where $\widetilde{k}$ is a unique point in the interval $\widetilde{k} \in \left[0, \overline{k}\right]$ Let $f_i(0, y_i) = 0$. Finally, let the concave segment be twice continuously differentiable.

A2. Let $f_i : \mathcal{R}_+^2 \to \mathcal{R}_+$ be twice continuously differentiable with respect to y_i. Let $f_i(k_i, 0) = 0$. In addition, assume that $\partial f_i(k_i, y_i) / \partial y_i \to \infty$ as $y_i \to 0$, $\partial f_i(k_i, y_i) / \partial y_i \to 0$ as $y_i \to \infty$, and $\partial^2 f_i(k_i, y_i) / \partial y_i^2 < 0$ for $k_i \in \left(0, \overline{k}\right)$ and $y_i > 0$.

These assumptions allow for a wide variety of production functions to be encompassed by our model. Various possibilities familiar from the literature are illustrated in Figures 1a-c. Figure 1a shows the standard case, where $\widetilde{k} = 0$ and the production function is everywhere strictly concave. Figures 1b and 1c show possibilities where $\widetilde{k} > 0$. Figure 1b illustrates the production function pictured in most undergraduate textbooks, where the segment in the domain $[0, \widetilde{k})$ is strictly convex. Figure 1c shows an alternative functional form, where the segment $[0, \widetilde{k})$ is linear. Both of these latter forms have been used in the growth literature; see for examples Paul Romer (1990a, b) and Philippe Askenazy and Cuong Le Van (1999). An example of a function satisfying our conditions, for the case of strict concavity everywhere, is a Cobb-Douglas production function

$$f_i(k_i, y_i) = k_i^{\alpha} y_i^{1-\alpha},$$

as illustrated in Figure 1a.

An example of a function satisfying our conditions for the case of first increasing then decreasing returns to capital is of the form

$$f_i(k_i, y_i) = \left(\frac{\alpha}{2} k_i^2 - \frac{\delta}{3} k_i^3\right) \sqrt{y_i} \tag{2}$$

where $\alpha, \delta > 0$. This functional form proves to be very versatile and can be used to construct an example, under appropriate parameter restrictions, of all the cases that we discuss at a general level in the present paper. For example, it is used as the basis for the illustration of a key theorem in the paper (Theorem 1), in Section 4.1.1.[9]

Finally, an example of the production function in Figure 1c is:

$$f_i\left(k_i, y_i\right) = \beta k_i + \left(\left(1-\beta\right) k_i\right)^{\alpha} y_i^{1-\alpha}$$

where $0 \leq \beta \leq 1$.

Each of the illustrations in Figure 1 is drawn under the assumption that y_i is constant. The impact on the production function (drawn in k_i space) of a change in y_i is illustrated in Figure 2 (drawn of the production function pictured in Figure 1b). Here we see that for each given level of k_i the level of output is increasing in y_i. Because A2 implies that $f_i\left(k_i, y_i\right)$ is strictly concave in y_i, the amount by which output increases is decreasing in y_i.

The opportunity cost to producing the public amenity y_i is expressed as a function $g_i\left(y_i\right)$ of the quantity of the public amenity, denominated in terms of the numeraire.

A3. Let $g_i : \mathcal{R}_+ \rightarrow \mathcal{R}_+$ be C^2 such that:

(i) $\partial g_i\left(y_i\right)/\partial y_i > 0$;

(ii) $\partial^2 g_i\left(y_i\right)/\partial y_i^2 \geq 0$.

The function $g_i\left(y_i\right)$ is in fact the inverse of the production function for the public good. This may seem unfamiliar, but turns out to be a very convenient way to represent the function when considering the planner's problem. Implicit in the previous literature is the assumption that the numeraire is transformed into the public good via a linear function; the non-linearity of $g_i\left(y_i\right)$ allows a higher level of generality to be considered in our analysis. Under A3, we allow for the possibility that there is increasing opportunity cost of the private good to produce more of the public good.[10] The subscript i on g_i allows for the possibility that the opportunity cost of providing the amenity can vary across jurisdictions, the implications of which receive close examination in Section 5.

[9] This functional form appears for the first time to our knowledge in this present paper.

[10] At a technical level, this assumption is consistent with concavity of the objective function in the range where production takes place, both of the planner and of individual agents.

The expression for *net output x_i in jurisdiction i* is therefore given by

$$x_i(k_i, y_i) = f_i(k_i, y_i) - g_i(y_i).$$

The concept of net output is needed in the definition of efficiency of production, which we turn to next.

2.2 The efficiency of production

The definition of production efficiency adapts a standard definition to the context of the present model.

Definition 1. A *plan*, consisting of a capital allocation $\mathbf{k}^E = \left(k_i^E, k_j^E\right) \in \mathcal{R}_+^2$ and vector of amenities $\mathbf{y}^E = (y_i^E, y_j^E)$, is *efficient* if:

1.1 $k_i^E + k_j^E = \overline{k}$ and

1.2. For all other capital allocations $\mathbf{k} = (k_i, k_j) \in \mathcal{R}_+^2$ satisfying $k_i + k_j = \overline{k}$ and amenities $\mathbf{y} = (y_i, y_j)$, it holds that

$$\begin{aligned} & x_i\left(k_i^E, y_i^E\right) + x_j\left(k_j^E, y_j^E\right) \\ \geq \; & x_i(k_i, y_i) + x_j(k_j, y_j). \end{aligned}$$

Under Definition 1, a capital allocation and vector of amenities is efficient if it entails the largest possible surplus for division between citizens in the two jurisdictions. It will be convenient to represent efficient capital allocations and amenities in terms of the allocations of capital and public goods to the respective jurisdictions; $\left(k_i^E, y_i^E\right)$ and $\left(k_j^E, y_j^E\right)$. Thus, if $\mathbf{k}^E = \left(k_i^E, k_j^E\right)$ and $\mathbf{y}^E = (y_i^E, y_j^E)$ constitute an efficient plan, we also call the induced outcome $\mathcal{E} = \left\{\left(k_1^E, y_1^E\right), (k_2^E, y_2^E)\right\}$ efficient.

In general terms the planner's problem can be expressed as the maximization of the following objective function:

$$\begin{aligned} \max_{k_i \; y_i \; k_j \; y_j} \; \Omega(k_i, y_i, k_j, y_j) &= x_i(k_i, y_i) + x_j(k_j, y_j) \\ &= f_i(k_i, y_i) + f_j(k_j, y_j) - g_i(y_i) - g_j(y_j) \\ &\quad \text{subject to} \\ \overline{k} &= k_i + k_j \text{ where } (k_i, k_j) \in \mathcal{R}_+^2. \end{aligned}$$

Using the feasibility of total capital usage in the maximand, the problem simplifies to

$$\max_{k_i\ y_i\ y_j} \ \Omega\left(k_i, y_i, y_j\right) = f_i\left(k_i, y_i\right) + f_j\left(\overline{k} - k_i, y_j\right) - g_i\left(y_i\right) - g_j\left(y_j\right)$$

The following set of first order conditions are necessary when efficiency implies that output is positive in both jurisdictions:

$$\frac{\partial f_i\left(k_i^E, y_i^E\right)}{\partial k_i} = \frac{\partial f_j\left(\overline{k} - k_i^E, y_j^E\right)}{\partial\left(\overline{k} - k_i\right)}; \tag{3}$$

$$\frac{\partial f_i\left(k_i^E, y_i^E\right)}{\partial y_i} = \frac{\partial g_i\left(y_i^E\right)}{\partial y_i}; \tag{4}$$

$$\frac{\partial f_j\left(\overline{k} - k_i^E, y_j^E\right)}{\partial y_j} = \frac{\partial g_j\left(y_j^E\right)}{\partial y_j}. \tag{5}$$

Because the capital feasibility condition $k_i^E + k_j^E = \overline{k}$ is used to substitute for k_j^E, solving the planner's problem for an efficient plan involves solving three first order conditions for the three unknowns, k_i, y_i and y_j.

The first condition states the familiar requirement that at an efficient plan the marginal unit of capital in each jurisdiction is equally productive. The second condition states, for jurisdiction i, that the marginal cost of foregoing a unit of the consumption good to produce the marginal unit of the public good must be equal to the marginal product of the public good in production. The third condition states the same thing for jurisdiction j.

We use standard short hand notation for the second derivative of the planner's problem. So, for example, $\partial^2\Omega\left(k_i^E, y_i^E, y_j^E\right)/\partial k_i \partial y_i$ will be written as $\Omega_{k_i y_i}$. The following lemma characterizes necessary and sufficient conditions under which efficiency implies that production takes place in *both* jurisdictions:

Lemma 1. *(Sufficiency) Assume that A1-A3 hold and that there exists a plan* $\mathcal{E} = \left\{\left(k_i^E, y_i^E\right), \left(k_j^E, y_j^E\right)\right\}$ *satisfying* (3), (4) *and* (5). *Then if the following conditions are satisfied at* $\mathcal{E}$*, it is efficient.*

(i) $\Omega_{k_i k_i}\Omega_{y_i y_i} > \left(\Omega_{k_i y_i}\right)^2$;

(ii)

$$\Omega_{y_j k_i} \begin{vmatrix} \Omega_{k_i y_i} & \Omega_{k_i y_j} \\ \Omega_{y_i y_i} & \Omega_{y_i y_j} \end{vmatrix} < \Omega_{y_j y_j} \begin{vmatrix} \Omega_{k_i k_i} & \Omega_{k_i y_i} \\ \Omega_{y_i k_i} & \Omega_{y_i y_i} \end{vmatrix}.$$

(iii) The function $\Omega(k_i, y_i, y_j)$ is strictly quasi-concave.

(Necessity) Assume that $\mathcal{E} = \left\{\left(k_i^E, y_i^E\right), \left(k_j^E, y_j^E\right)\right\}$ is an efficient plan. Then (3), (4) *and* (5) *are satisfied.*

Proof. See appendix.

Conditions (3)-(5) are the standard first order conditions for $\mathcal{E}$ to be a unique interior maximum. Conditions (i) and (ii) are second order conditions, but are assumed to hold only at the plan $\mathcal{E}$, not globally. Therefore we need the additional assumption that $\Omega(k_i, y_i, y_j)$ is strictly quasi-concave - a weakening of the usual concavity assumption - to ensure that the point is indeed a maximum. The reason we need to assume quasi-concavity is because the functions $f_i(k_i, y_i)$ and $f_j(k_j, y_j)$ are themselves quasi-concave and not necessarily concave (A1). If they had been concave, then we would have known, by a standard result, that the same must be true of $\Omega(k_i, y_i, y_j)$. The same result does not hold for quasi-concavity, hence assumption (iii). The other possibility that we need to consider, where $\Omega(k_i, y_i, y_j)$ is quasi-convex, is covered by Lemma 2.

We now state the first order conditions that must hold when efficiency implies that all production occurs in just one jurisdiction. Suppose, without loss of generality that we label by j the jurisdiction where no production takes place; then by the notation introduced above we allow for the possibility that $\left(k_j^E, y_j^E\right) = (0,0)$. In this case, the first order conditions are as follows:

$$\frac{\partial f_i\left(k_i^E, y_i^E\right)}{\partial k_i} \geq \frac{\partial f_j\left(\overline{k} - k_i^E, y_j^E\right)}{\partial\left(\overline{k} - k_i\right)} \tag{6}$$

$$\frac{\partial f_i\left(k_i^E, y_i^E\right)}{\partial y_i} = \frac{\partial g_i\left(k_i^E, y_i^E\right)}{\partial y_i} \tag{7}$$

$$\frac{\partial f_j\left(\overline{k} - k_i^E, y_j^E\right)}{\partial y_j} \geq \frac{\partial g_j\left(\overline{k} - k_i^E, y_j^E\right)}{\partial y_j} \tag{8}$$

Note that under conditions (6)-(8) we may or may not have a critical point. The first order conditions have been adjusted to allow for the possibility that a corner solution exists. We may have $\partial f_i\left(k_i^E, y_i^E\right)/\partial k_i > \partial f_j\left(\overline{k} - k_i^E, y_j^E\right)/\partial\left(\overline{k} - k_i\right)$ where $k_i^E = \overline{k}$. Then the output per unit of capital could be increased further were it not for the fact that the firm in one jurisdiction already uses all the capital that is available.

It remains to characterize the necessary and sufficient conditions under which efficiency implies that production takes place in a single jurisdiction:

Lemma 2. *(Sufficiency) Assume A1-A3 and that the plan* $\mathcal{E} = \left\{\left(\overline{k}, y_i^E\right), (0,0)\right\}$ *satisfies* (6), (7) *and* (8). *If the following conditions are satisfied, then* $\mathcal{E}$ *is efficient.*

(i) $\Omega_{k_ik_i}\Omega_{y_iy_i} > \left(\Omega_{k_iy_i}\right)^2$;

(ii)

$$\Omega_{y_jk_i}\begin{vmatrix} \Omega_{k_iy_i} & \Omega_{k_iy_j} \\ \Omega_{y_iy_i} & \Omega_{y_iy_j} \end{vmatrix} < \Omega_{y_jy_j}\begin{vmatrix} \Omega_{k_ik_i} & \Omega_{k_iy_i} \\ \Omega_{y_ik_i} & \Omega_{y_iy_i} \end{vmatrix}.$$

(iii) $\Omega\left(k_i, y_i, y_j\right)$ *is strictly quasi-convex and* $\Omega\left(\overline{k}, y_i^E, y_j^E\right) \geq \Omega\left(0, y_i^E, y_j^E\right)$[11].

(Necessity) Assume that $\mathcal{E} = \left\{\left(\overline{k}, y_i^E\right), (0,0)\right\}$ *is efficient. Then* (6), (7) *and* (8) *are satisfied.*

Proof: See appendix.

The quasi-convexity assumption, coupled with the fact that $\Omega\left(0, y_i, y_j\right) \leq \Omega\left(\overline{k}, y_i, y_j\right)$, ensures that the maximum occurs at the corner point $\overline{k}$. Quasi-convexity implies that all interior points of the function must lie below the end points. And $\Omega\left(0, y_i, y_j\right) \leq \Omega\left(\overline{k}, y_i, y_j\right)$ ensures that the maximal end-point lies at $k_i = \overline{k}$. There is no loss of generality here. Suppose we find that $\Omega\left(0, y_i, y_j\right) > \Omega\left(\overline{k}, y_i, y_j\right)$. Then switching labels i and j will ensure that the condition holds.

[11] If we assume common technology, $f_i(\cdot,\cdot) = f_j(\cdot,\cdot)$, then the property that $\Omega\left(\overline{k}, y_i, y_j\right) > \Omega\left(0, y_i, y_j\right)$ follows from the very mild condition that $y_i > y_j$. By A2, if $y_i > y_j$ then $f_i\left(\overline{k}, y_i\right) > f_j\left(\overline{k}, y_j\right)$. Set $k_i = \overline{k}$, by which capital market feasibility implies $k_j = k_i - \overline{k} = 0$. Recalling that $f_j\left(0, y_j\right) = 0$, the planners problem at $\overline{k}$ can be written $\Omega\left(\overline{k}, y_i, y_j\right) = f_i\left(\overline{k}, y_i\right) - g_i\left(y_i\right) - g_j\left(y_j\right)$. Alternatively, setting $k_j = \overline{k}$ and $k_i = 0$, we have $\Omega\left(0, y_i, y_j\right) = f_j\left(\overline{k}, y_j\right) - g_i\left(y_i\right) - g_j\left(y_j\right)$. Because $f_i\left(\overline{k}, y_i\right) > f_j\left(\overline{k}, y_j\right)$ when $y_i > y_j$, we have $\Omega\left(\overline{k}, y_i, y_j\right) > \Omega\left(0, y_i, y_j\right)$. By exactly the same sequence of argument, when $y_i = y_j$ we have $\Omega\left(\overline{k}, y_i, y_j\right) = \Omega\left(0, y_i, y_j\right)$.

Figures 3a and 3b illustrate the possibilities covered by Lemmas 1 and 2 respectively. (Again, they are drawn for production functions of the form shown in Figure 1b; it is easy to imagine what they would look like for other functional forms.) Implicit in the illustrations is that production technologies used by firms are identical across jurisdictions $(f_i(\cdot,\cdot) = f_j(\cdot,\cdot))$ which we assume to be the case throughout the analysis. Then fixing $y_i = y_i^E$ and $y_j = y_j^E$ the function $\Omega(k_i, y_i, y_j)$ can be plotted for $k_i \in [0, \overline{k}]$. If the function is quasi-concave, as in Figure 3a, then k_i^E occurs at a unique interior maximum. If it is quasi-convex, then k_i^E occurs at a corner $(k_i^E = \overline{k})$ as shown in Figure 3b[12]. In this sense, technology is a key determinant of the nature of the outcome.

3 The Strategic Game

The main purpose of this section is to set up the game played by governments when they set policy. The game is general enough to allow a wide range of outcomes to be derived in the subsequent sections. Amongst these is the familiar 'race to the bottom' of taxes and public service provision. Section 4 also shows conditions under which the efficient state of the economy derived in Section 3 coincides with the Nash equilibrium . And in addition to showing that there can be underprovision of public goods in Nash equilibrium - a race to the bottom - we show that there may alternatively be a 'race to the top', where taxes and public good provision are too high in Nash equilibrium. Special care is taken to compare the assumptions on which equilibrium in our model is based with those of Zodrow and Mieszkowski. This facilitates a direct comparison of our efficient and 'race to the top' results with the 'race to the bottom' result that has been the focus of earlier research.

The first task is to make explicit the role of the representative firm and the government in each jurisdiction. In the previous section the model of production was set out, and this was sufficient to describe the problem of the planner in solving for efficiency. The objective now is to show when the efficient plan does and does not arise in a decentralized

[12] Of course, it may be the case that $\Omega(k_i, y_i, y_j)$ is neither quasi-concave nor quasi-convex, which is formally outside the scope of our analysis. However, given that under symmetry of $f_i(\cdot,\cdot)$ either the maximum occurs at an interior point or a corner, our framework covers all the economically interesting possiblities.

(Nash) equilibrium, with each government optimizing individually.

Because the previous literature has focused on a standard model in which production occurs in all jurisdictions and the transformation of the consumer good into the public good is essentially linear, we first focus on this well known case. One of the properties of equilibrium derived from this conventional model, where the primitives are ex ante symmetric across jurisdictions and production occurs in both jurisdictions, is that the equilibrium is symmetric. Taxes and public good provision are the same across jurisdictions, whether efficient or inefficient in equilibrium. However, our framework allows a number of additional cases to be considered.

In Section 5 we show that when efficiency implies that production occurs in both jurisdictions but that the technology of public good provision is asymmetric across jurisdictions, the result is that efficient levels of public good provision vary across jurisdictions. Consequently, taxes to support efficient public good provision would have to vary across jurisdictions as well. But we show that precisely because of this variation of efficient tax rates, the efficient state of the economy cannot be decentralized in Nash equilibrium; a result that contrasts starkly with the case where technology is symmetric across jurisdictions. This result simultaneously calls into question the wisdom of tax harmonization policies and the ability to set taxes at efficient levels in a decentralized equilibrium when jurisdictions are not symmetric. In doing so, it brings to light for the first time what must in practice be a long-standing dilemma for policy makers.

Then in Section 6 an asymmetric Nash equilibrium is derived which has similar properties to the symmetric equilibrium of Section 4. It is important to emphasize that the model of Section 6 is ex ante symmetric. And the state of the economy is efficient in the Nash equilibrium that we demonstrate. But production and public good provision occur in just one jurisdiction, with no public good provision or private production being undertaken in the other.

3.1 The firm's view of production

In our model firms are not strategic players. The profit maximizing conditions characterizing a competitive firm play a role in determining the payoffs to jurisdictions. The representative firm in jurisdiction i is assumed to behave competitively in capital and goods markets and to have no influence over policy making. It takes as given the level of public good provision, y_i, and the user cost of capital, p_i. Its objective is simply to maximize profits by choosing the appropriate quantity of capital;

$$\max_{k_i \geq 0} \ \pi_i = f_i (k_i, y_i) - p_i k_i, \tag{9}$$

where p_i is the per unit user cost of capital. The first order condition of the firm in jurisdiction i is

$$\frac{\partial f_i (k_i, y_i)}{\partial k_i} - p_i \begin{cases} = 0 \text{ for } k_i \in [0, k_i) \\ \geq 0 \text{ for } k_i = \overline{k}. \end{cases} . \tag{10}$$

The first order condition holds with equality at an interior solution. But at a corner solution $k_i = \overline{k}$, it may be the case that $\partial f_i (k_i, y_i) / \partial k_i \geq p_i$. It may be that the firm could increase profits were it able to increase its capital use.

The user cost of capital is given by the identity

$$p_i = p_i (r, t_i) = r + t_i.$$

Because capital is perfectly mobile between jurisdictions there is a single world price of capital r. Tax revenue is deducted on the destination basis, in the sense that the local tax rate must be paid on all the capital used in production within the jurisdiction, whether of local or foreign origin.[13]

Figure 4a illustrates the firm's problem of profit maximization. It is based on the production function of Figure 1b, given the level of amenity provision y_i, which the firm takes as fixed. Figure 4b illustrates the same thing in terms of the profit function. Obviously, the firm's problem is to reach the highest point of this function by choosing k_i; achieved at k_i^* in the figure. Figure 4c shows the profit function when $y_i = 0$. From A1, $f(k_i, 0) = 0$ for all k_i. We can see straight away that $k_i = 0$ solves $\max_{k_i} \pi_i$ in that case. The demand function for capital can be written in reduced form as $k_i = k_i (r + t_i)$.

[13] Here the tax is modelled as specific. From the work of Ben Lockwood (2001) and Lockwood and Kar-yiu Wong (2000), we anticipate that our results would change quantitatively but not qualitatively if an ad valorem and/or origin based regime were modelled instead.

3.2 Capital market clearing

Capital market clearing is the result of competitive behavior by the firms in both jurisdictions defined in a standard way as follows:

A *capital market equilibrium* is a pair $(r^*, \mathbf{k}^*)$, where r^* is the equilibrium rate of interest and $\mathbf{k}^*$ is the capital allocation $k^* = \left(k_i^*, k_j^*\right) \in \mathcal{R}_+^2$, such that:

(i) For each $i = 1, 2$, $k_i^* \in \arg\max_{k_i} \pi_i(r^*, t_i^*)$;

(ii) $\sum_{i=1,2} k_i^* = \overline{k}$

It is sufficient for our analysis to restrict attention to the case where $r^* > 0$ and therefore (ii) holds with equality. Section 3.1 stated the firm's problem for a single jurisdiction. The first condition says that one such problem must be solved by each firm in each jurisdiction. The second is a feasibility condition, saying that (for a positive price of capital) total demand for capital must be equal to supply.

3.3 Welfare and the feasibility of consumption

The output available for consumption by citizens of jurisdiction i is

$$c_i \leq \pi_i + r\overline{k}_i. \tag{11}$$

That is, citizens of jurisdiction i receive the profits from the firm in that jurisdiction and also the revenue from rental of their endowment of capital. We assume that consumers have monotonically strictly increasing preferences for output.

3.4 The government's view of production

The availability of the public good within a jurisdiction is given by the government budget condition

$$t_i k_i = g_i\left(y_i^f\right) \geq g_i\left(y_i\right). \tag{12}$$

The feasible level of public good provision y_i^f is given by the same convex function $g_i(\cdot)$ as for the planner (see Section 2.2). However, in general the government budget constraint need not necessarily hold with equality; $y_i \leq y_i^f$.

The decentralized problem differs from that of the planner in that the opportunity cost of the public good is expressed not directly in terms of the quantity of the consumer good forfeited but in terms of the capital tax base and the tax rate, as in a standard tax competition model. This reflects the fact that, whilst the planner can simply pick the optimal level of public good provision, the government has to raise the revenue through taxation in order to produce the public good.

Equilibrium requires that the government budget condition holds identically;

$$t_i k_i = g_i(y_i) \tag{13}$$

A standard assumption in the literature is that public good provision is directly proportional to the tax rate and tax base; $y_i = t_i k_i$. We now extend A3 to take account of the public good cost function given by (12), which allows for a richer set of possibilities[14]:

A3′. Let $g_i : \mathcal{R}_+ \to \mathcal{R}_+$ be twice continuously differentiable with respect to its arguments such that:

(i) $\partial(t_i k_i)/\partial y_i = \partial g_i(y_i)/\partial y_i > 0$;

(ii) $\partial^2(t_i k_i)/\partial y_i^2 = \partial^2 g_i(y_i)/\partial y_i^2 \geq 0$.

The government seeks to maximize (11) by setting taxes on behalf of the representative citizen, taking as given the actions of the government in the other jurisdiction. To facilitate comparison with the analysis of Zodrow and Mieskowski (1986) and Wilson (1986), we assume as they do that each government takes the interest rate r as given. Thus we effectively assume that each jurisdiction regards themselves as small.[15] The problem of the government in jurisdiction i can then be expressed as choosing t_i and y_i

[14] In Section 5 we consider the case where $g_i(\cdot) \neq g_j(\cdot)$.

[15] Wildasin (1987) analyses the case where jurisdictions regard themselves as large, and take into account the impact of their tax setting on r. In that case the choice of government policy variable t_i or y_i makes a difference. We adopt the 'small jurisdiction' assumption to keep the analysis tractable. But the basic thinking behind our approach of analysing the incentive to deviate from a Pareto efficient solution is applicable to the situation where governments take into account the impact of their actions on r.

to maximize the consumption of the residents of the jurisdiction. Defining $h_i(t_i, y_i, r)$ as the payoff to the government of the strategy (t_i, y_i), we have:

$$\begin{aligned} h_i(t_i, y_i; r) \quad &: \quad = \max_{t_i} c_i = \pi_i + r\overline{k}_i \\ &= \quad f\left(k_i, y_i\right) - \left(r + t_i\right) k_i + r\overline{k}_i \end{aligned}$$

subject to the constraints that:

1. The production of public good is feasible:

$$t_i k_i = g_i(y_i)$$

2. The firm in jurisdiction i maximizes profit at k_i. Thus, assuming differentiability,

$$\partial f_i\left(k_i, y_i\right) / \partial k_i = r + t_i.$$

The fact that the government views production in a different way to firms should also be highlighted. Whilst firms take the level of the public good as given, governments account fully for the impact of providing the public good in making decisions on the level of provision, and the requisite level of taxation. The problem of the government in jurisdiction i is solved by the following first order condition;

$$\begin{aligned} \frac{dc_i}{dt_i} \quad &= \quad \left(\frac{\partial f_i}{\partial k_i} - (t_i + r)\right) \frac{\partial k_i}{\partial t_i} \\ &\quad + \frac{\partial f_i}{\partial y_i}\left(\frac{\partial y_i}{\partial t_i} + \frac{\partial y_i}{\partial k_i}\frac{\partial k_i}{\partial t_i}\right) - k_i = 0. \end{aligned} \tag{14}$$

Note that since the first order condition of the firm is required to hold then the term on the first line disappears.

Under appropriate conditions the function $h_i(t_i, y_i; r)$ is a (single-valued) function of r, $y_i$and t_i. For example, if $f_i\left(k_i, y_i\right) = k_i^{\alpha} y_i^{1-\alpha}$, then given r, the optimizing choice of t_i and the value of the government's payoff, $h_i(t_i, y_i; r)$, are uniquely determined. This implies that optimizing values of k_i and y_i exist and are unique. So the government payoff function can also be written $h_i\left(t_i, y_i; r\right)$; that is, not just as a function of t_i given r but of y_i as well. This will be useful in the definition of equilibrium itself.

Implicit in the formulation of the government's problem is the assumption that lump-sum transfers are not available as a policy instrument. This is a standard assumption in the tax competition literature. It is well recognized that if lump-sum transfers are possible then efficiency can be achieved directly, and the whole efficiency question of tax competition vanishes.

The two following definitions introduce concepts that will be useful in the definition of equilibrium itself (Definition 4).

Definition 2. A *policy* (t_i, y_i) consists of a tax rate and a level of amenity provision.

Definition 3. A pair of policies $\mathcal{Q} = ((t_i, y_i), (t_j, y_j))$ is *feasible* if, when they are adopted simultaneously by governments,

(i) There exists a rate of interest r and an capital allocation $k = (k_1, k_2)$ such that the capital market is in equilibrium

(ii) Budgets balance. That is, $t_i k_i = g(y_i)$, $i = 1, 2$.

A policy solved for using (14) must be feasible because it takes account of the balanced budget constraint (1.) above.

3.5 Definition of equilibrium

We now introduce a formal definition of equilibrium:

Definition 4. A *Nash equilibrium in policies* is a pair of feasible policies $\mathcal{Q} = \left(\left(t_i^N, y_i^N\right), \left(t_j^N, y_j^N\right)\right)$ such that, for each jurisdiction i,

$h_i(t_i^N, y_i^N; r) \geq h_i(t_i, y_i; r)$ for all other feasible choices by jurisdiction i of (t_i, y_i)

where both $h_i(t_i^N, y_i^N; r)$ and $h_i(t_i, y_i; r)$, $i \neq j$, are evaluated at a rate of interest corresponding to the feasibility of the policies. Let $\mathbf{k}^N = (k_1^N, k_2^N)$ denote the capital allocation uniquely determined by the Nash equilibrium.

Observe that because the government maximizes c_i we can write

$$h_i(t_i^N, y_i^N; r) \geq h_i(t_i, y_i; r) \Longleftrightarrow c_i(t_i^N, y_i^N; r) \geq c_i(t_i, y_i; r).$$

The *outcome induced by a Nash equilibrium* $\mathcal{Q} = \left(\left(t_i^N, y_i^N\right), \left(t_j^N, y_j^N\right)\right)$ is the corresponding set $\left\{\left(k_1^N, y_1^N\right), \left(k_2^N, y_2^N\right)\right\}$.

We now have a complete statement of equilibrium: governments in each jurisdiction behave strategically in setting tax rates; firms behave competitively in their production decisions, taking interest rates as given and choosing capital to maximize profits.

4 The Zodrow-Mieskowski (Z-M) Model

The model of Zodrow and Mieskowski (1986) Section 3 (henceforth Z-M), which allows the public good to affect the productivity of capital, can be considered as a special case of the general framework set out above. It is generally understood from Z-M that when capital is mobile the incentive to attract capital through a reduction of taxes will bring about an under-provision of the public good, even when its role is to enhance the marginal productivity of capital. In this section we show that by using an alternative assumption to describe the impact of public good provision on the marginal productivity of capital, the efficient plan can arise in Nash equilibrium or a race to the top, not just a race to the bottom.

We now restrict our general framework to yield the Z-M model, by imposing what we call the Z-M assumptions:

A4. (Z-M assumptions) In addition to the conditions on technology imposed by A1, A2 and A3$'$, assume there is a common endowment of capital, $\overline{k}_i = \overline{k}_j$ and common technology across jurisdictions; $f_i(\cdot,\cdot) = f_j(\cdot,\cdot)$ and $g_i(\cdot) = g_j(\cdot)$. Moreover, $g_i(y_i) \equiv y_i$ so (13) takes the form $t_i k_i = y_i$. Also, efficiency implies that production occurs in both jurisdictions. In addition,[16]

$$t_i(\partial^2 f_i / \partial k_i \partial y_i) < -\partial^2 f_i(k_i, y_i) / \partial k_i^2. \tag{15}$$

where $t_i = y_i / k_i$.

[16] The following condition appears in Zodrow and Mieskowski (1986) as equation (17). Zodrow and Mieskowski suggest that it holds globally. We use a weaker version, which is required only to hold for the efficient plan and supporting policy.

The equation (15) reflects the responsiveness of the (diminishing) marginal returns to capital to a change in taxes with an accompanying change in public good provision. Note that (15) is the second derivative, with respect to capital, of the derivative of the production function with respect to capital,

$$\frac{df_i(k_i,y_i)}{dk_i}=\frac{\partial f_i(k_i,y_i)}{\partial k_i}+\frac{\partial f_i(k_i,y_i)}{\partial y_i}\frac{\partial y_i}{\partial k_i}.$$

Thus, the negativitiy of $t_i(\partial^2 f_i/\partial k_i \partial y_i)+\partial^2 f_i(k_i,y_i)/\partial k_i^2$ implies that the derivative of the production function with respect to capital is downward sloping – simply that, when capital is used for production of output and output can be used to produce public goods, which further enhance marginal productivity of capital, the production function exhibits *overall diminishing marginal product of capital* (ODMP$_K$), a natural assumption.[17] If an outcome for a jurisdiction were not at some point satisfying ODMP$_k$ and both jurisdictions were undertaking production, then that outcome could not be efficient and, in addition, it could not be an equilibrium.

Now we introduce an alternative assumption:

A5. Assume that for any positive value of y_i, the function

$$1-k_i\frac{\partial^2 f_i(k_i,y_i)}{\partial k_i \partial y_i}$$

is monotonically increasing and the following two conditions hold:

$$\lim_{k_i\to 0}\left(1-k_i\frac{\partial^2 f_i(k_i,y_i)}{\partial k_i \partial y_i}<0\right);$$
$$\lim_{k_i\to \overline{k}}\left(1-k_i\frac{\partial^2 f_i(k_i,y_i)}{\partial k_i \partial y_i}>0\right).$$

Note that A5 implies that

$$k_i\frac{\partial^2 f_i(k_i,y_i)}{\partial k_i \partial y_i}$$

declines monotonically with k_i. A5 replaces the assumption made by Z-M that[18]

$$1-k_i\frac{\partial^2 f_i(k_i,y_i)}{\partial k_i \partial y_i}>0 \text{ for all } k_i\in\left[0,\overline{k}\right].$$

[17] This is not the same as the standard assumption of diminishing marginal productivity of capital, which would be that $\frac{\partial f_i(k_i,y_i)}{\partial k_i}$ is negatively sloped.

[18] This assumption appears as equation (16) in Zodrow and Mieskowski (1986).

Our alternative assumption is certainly no stronger than the one adopted by Z-M, and could be argued to be more reasonable. Given the importance of the term, we define the *marginal public good valuation* (*mpgv*) as

$$mpgv\text{: } k_i \partial^2 f_i (k_i, y_i) / \partial k_i \partial y_i.$$

This valuation is made from the viewpoint of the jurisdiction by its representative citizen or government, and not for the economy as a whole (from the viewpoint of the planner). The mpgv measures the extent to which output is increased - through productivity enhancement - by the marginal unit of the public good. Our assumption says that the *mpg value* is higher than the marginal cost when capital use is relatively low, but then declines as more capital is used - and therefore more of the public good is provided, given tax rates - eventually falling below marginal cost when capital use is relatively high. This contrasts with the assumption originally made by Zodrow and Mieskowski (1986), which stipulates that the *mpg value* is *never* as high as the marginal cost.

Our assumption essentially says that the public good becomes less important for enhancing the marginal productivity of capital as capital use - and therefore public good provision - increases. This seems reasonable. Arguably, as a firm gets larger public goods provided by the government become less important to it. For example, the larger the firm the more likely it is to have its own intranet, its own transportation networks for goods and people, its own security arrangements.

This is similar to the Barro-Sala-i-Martin assumption, although their point of emphasis is slightly different. Although they too focus on the impact of a change in the marginal impact of the public good on the marginal productivity of capital $(\partial^2 f_i (k_i, y_i) / \partial k_i \partial y_i)$, they look at its variation with respect to the tax rate, holding output constant. Here we look at its variation with respect to capital, holding the tax rate constant.

In effect, Zodrow and Mieskowski introduce the property, by assumption, that starting from any balanced budget position a jurisdiction can make itself unilaterally better off by lowering taxes and public good provision. There is always an incentive for a government to deviate from the efficient plan by cutting taxes. Consequently, the state of the economy *must* be inefficient in Nash equilibrium; the familiar 'race to the bottom'. Our alternative assumption, embodied in A5, introduces a wider range of possibilities.

As we shall see, three situations can arise. One is the same as Z-M's; there is a unilateral incentive to deviate from the efficient plan by reducing taxes. Another equally valid situation is one where an incentive exists to deviate by *raising* taxes. This introduces the possibility that Nash equilibrium supports inefficiently high levels of taxation and public good provision - 'government is too big'. Finally, it is possible that the state of the economy is efficient in Nash equilibrium.

Having introduced A5, the following lemma establishes that there must exist a level of capital usage at which the associated increase in output due to the increased marginal productivity of capital is exactly equal to the marginal cost:

Lemma 3. *Assume A1, A2, and A5 so $f_i(k_i, y_i)$ is C^2 on the compact set $k_i \in [0, \overline{k}]$ and C^2 on the set $y_i \in \mathcal{R}$. Then by A5, there exists, for any given y_i, a value $k_i^I \in [0, \overline{k}]$ such that*

$$1 - k_i^I \frac{\partial^2 f_i(k_i^I, y_i)}{\partial k_i \partial y_i} = 0.$$

Proof: Follows from a straightforward application of the intermediate value theorem. □

This lemma just says that if at low levels of k_i the *mpg value* is greater than the marginal cost, and at high levels of capital use it is less than the marginal cost, then there must exist a level of capital use, which we call k_i^I, at which they are equal. While Lemma 3 is obvious, the result is important for what follows.

By following exactly the same steps as in Z-M, we can derive the change in capital demand within a jurisdiction due to a change in t. Differentiate (13) and (10) and combine the results to yield

$$\frac{\partial k_i}{\partial t_i} = \frac{1 - k_i \partial^2 f_i(k_i, y_i) / \partial k_i \partial y_i}{\partial^2 f_i(k_i, y_i) / \partial k_i^2 + t_i \partial^2 f_i(k_i, y_i) / \partial k_i \partial y_i}. \tag{16}$$

The sign of this expression depends on the level of capital usage and public good provision. The denominator is negative at the efficient plan by A4.[19] If the public good

[19] The assumption that $t_i^E < \left(-\partial^2 f(k_i^E, y_i^E) / \partial k_i^2\right) / \left(\partial^2 f(k_i^E, y_i^E) / \partial k_i \partial y_i\right)$ ensures that the denominator of (16) is negative under the conditions of Lemma 1 at the Pareto efficient plan and induced policy. To see this, recall that $\partial^2 f(k_i^E, y_i^E) / \partial k_i^2 < 0$ and $\partial^2 f(k_i^E, y_i^E) / \partial k_i \partial y_i > 0$ at (k_i^E, y_i^E) under the conditions of Lemma 1.

had no impact on the marginal productivity of capital (as is usually assumed) then $k_i \partial^2 f_i (k_i, y_i) / \partial k_i \partial y_i = 0$. The numerator compares the *mpg value* to the marginal cost. By Lemma 3 we know that for any given value of y_i there exists a value $k_i^I \in [0, \overline{k}]$ for which $\partial k_i / \partial t_i = 0$. And $\partial k_i / \partial t_i \gtreqless 0$ for $k_i \lesseqgtr k_i^I$.

As we shall see, the sign of $\partial k_i / \partial t_i$ at any given level of k_i and y_i determines the *incentive to deviate*. In the next subsection we shall see that if $\partial k_i / \partial t_i = 0$ at the efficient capital allocation k_i^E then there is no incentive for the government of jurisdiction i to deviate by changing taxes, and the efficient plan induces a pair of policies that are efficient and solve the Nash equilibrium conditions. If $\partial k_i / \partial t_i > 0$ at k_i^E then there is a unilateral incentive to deviate from the efficient plan by raising taxes, and a race to the top occurs in Nash equilibrium, with over-provision of the public good. The standard race to the bottom occurs when $\partial k_i / \partial t_i < 0$ at k_i^E.

4.1 Efficient Nash Equilibrium in the Z-M model

Let (k_i^E, y_i^E) and (k_j^E, y_j^E) describe an efficient plan. Let (t_i^N, y_i^N) and (t_j^N, y_j^N) denote policies set by governments i and j respectively in Nash equilibrium. The following theorem shows the conditions under which the plan will induce an efficient Nash equilibrium.

Theorem 1. *Assume A1-A3′-A5 and that there exists a symmetric efficient plan* $\mathcal{E} = \{(k_i^E, y_i^E), (k_j^E, y_j^E)\}$, *characterized by first order conditions* (3)-(5). *Let* $\mathcal{Q} = \{(t_i^N, y_i^N), (t_j^N, y_j^N)\}$ *be a symmetric Nash equilibrium. Assume that* $k_i^I = k_i^E$ *when* $y_i = y_i^N$. *Then the pair of policies induced by the efficient plan,* $((t_i^E, y_i^E), (t_j^E, y_j^E))$ *is a Nash equilibrium.*

Proof: (i) We will show that $y_i^N = y_i^E$, $y_j^N = y_j^E$, $t_i^N = t_i^E$ and $t_j^N = t_j^E$, with $k_i = k_i^E = k_j = k_j^E = \overline{k}/2$.

First, it will be useful to recall the conditions on efficiency under the present assumptions (3)-(5). By A3′, $\partial g_i (y_i) / \partial y_i = \partial y_i / \partial y_i = 1$, so the first order conditions for efficiency (3)-(5) become:

$$\begin{aligned} \frac{\partial f_i (k_i^E, y_i^E)}{\partial k_i} &= \frac{\partial f_j (\overline{k} - k_i^E, y_j^E)}{\partial (\overline{k} - k_i)} \text{ and} \\ \frac{\partial f_i (k_i^E, y_i^E)}{\partial y_i} &= \frac{\partial f_j (k_j^E, y_j^E)}{\partial y_j} = 1. \end{aligned}$$

Now, a requirement of Nash equilibrium is that, for the induced policy $\left(t_i^N, y_i^N\right)$ and capital demand k_i^N it holds for some r, $i = 1, 2$, that

$$\frac{dc_i}{dt_i} = \left(\frac{\partial f_i\left(k_i^N, y_i^N\right)}{\partial k_i} - (t_i + r)\right)\frac{\partial k_i}{\partial t_i} +$$

$$\frac{\partial f_i\left(k_i^N, y_i^N\right)}{\partial y_i}\left(\frac{\partial y_i}{\partial t_i} + \frac{\partial y_i}{\partial k_i}\frac{\partial k_i}{\partial t_i}\right) - k_i^N = 0.$$

This is just a restatement of the government's first order condition (14). (If not, then the equilibrium requirement $c_i(t_i^N, y_i^N; r) \geq c_i(t_i, y_i; r)$ could not hold.)

Note that because the Nash equilibrium is symmetric, and the efficient plan is symmetric, it must be the case that $k_i^N = k_i^E = \overline{k}/2$ $i = 1, 2$. Because the demand for capital is symmetric across jurisdictions in equilibrium, $k_i^N = k_j^N = \overline{k}/2$. Also, because the efficient plan is symmetric we must have $k_i^E = k_j^E = \overline{k}/2$. Therefore, in what follows, we can write k_i^E and k_i^N interchangeably.

We will now show that if $k_i^I = k_i^E$ when $y_i = y_i^N$ then $dc_i/dt_i = 0$ implies $\partial f_i\left(k_i^E, y_i^N\right)/\partial y_i = 1$, which is exactly the same as the condition for efficient level of public good provision (given $k_i = k_i^E$) state above. Then by the equivalence of the Nash equilibrium condition to the first order condition on efficiency, the level of public good provision that solves the Nash equilibrium condition must be efficient; $y_i^N = y_i^E$.

To determine that $\partial f_i\left(k_i^E, y_i^N\right)/\partial y_i = 1$ when $k_i^I = k_i^E$, notice that if $\partial k_i\left(k_i^E, y_i^N\right)/\partial t_i = 0$ then the first order condition of government i's objective function, $dc_i/dt_i = 0$, becomes

$$\frac{\partial f_i\left(k_i^E, y_i^N\right)}{\partial y_i} = 1.$$

To see this, first note that when $\partial k_i\left(k_i^E, y_i^N\right)/\partial t_i = 0$, the first line of the governments first order condition dc_i/dt_i disappears, and that the final term in brackets on the second line also disappears. Then note that $\partial y_i/\partial t_i = t_i \partial k_i\left(k_i^E, y_i^N\right)/\partial t_i + k_i$, but that this becomes $\partial y_i/\partial t_i = k_i$ when $\partial k_i\left(k_i^E, y_i^N\right)/\partial t_i = 0$. So we are left with

$$\frac{\partial f_i\left(k_i, y_i\right)}{\partial y_i} k_i^E = k_i^E,$$

and cancelling k_i from both sides yields the result.

It remains to establish that $\partial k_i\left(k_i^E, y_i^N\right)/\partial t_i = 0$. The sign of $\partial k_i\left(k_i^E, y_i^N\right)/\partial t_i$ is given by the expression (16). Because we assume ODMP$_k$, (A4) the denominator of

(16) is negative and so defined. Therefore, the sign of (16) depends on the numerator, $1 - k_i^E \partial^2 f_i \left(k_i^E, y_i^N\right) / \partial k_i \partial y_i$. From $k_i^I = k_i^E$ it holds by Lemma 3 and A5 that

$$1 - k_i^E \frac{\partial^2 f_i \left(k_i^E, y_i^N\right)}{\partial k_i \partial y_i} = 0.$$

Consequently, $\partial k_i \left(k_i^E, y_i^N\right) / \partial t_i = 0$ when $k_i^I = k_i^E$. Thus $\partial f_i \left(k_i^E, y_i^N\right) / \partial y_i = 1$ when $k_i^I = k_i^E$. So the Nash equilibrium condition does indeed coincide with the first order condition on efficiency, from which it follows that $y_i^N = y_i^E$.

We can obtain t_i^N and t_j^N by rearranging (13) to get $t_i = y_i / k_i$. Because $y_i^N = y_i^E$, it must be that $t_i^N = 2y_i^N / \overline{k} = 2y_i^E / \overline{k} = t_i^E$ $i = 1, 2$.

We have so far shown that the necessary conditions for efficiency are satisfied. That is, we know we have a critical point. It remains to confirm that this critical point is indeed a maximum for each individual jurisdiction $i = 1$, 2. Suppose not. Then for at least one jurisdiction, t_i must be set such that c_i is at a minimum or a point of inflection. But by the symmetry of the two jurisdictions, if this applies to one jurisdiction then it must apply to both. In that case we could not be at an efficient point $\mathcal{E}$; a contradiction.

Thus we have shown that the pair of policies induced by the efficient plan, $\left((t_i^E, y_i^E), \left(t_j^E, y_j^E\right)\right)$ is a Nash equilibrium. □

Theorem 1 relates to a special case in which $k_i^I = k_i^E$. There is no reason to expect, a priori, that k_i^I and k_i^E will coincide, as each can occur anywhere in the domain of $k_i \in \left[0, \overline{k}\right]$. The remaining possibilities are considered in the next section. But first we present an example in which it is the case that $k_i^E = k_i^I$ and all other conditions of Theorem 1 are satisfied as well.

4.1.1 An example

Substitute values $\alpha = 1$, $\delta = \frac{3}{4}$ into (2) in order to obtain a production function of the form

$$f_i = \left(\frac{k_i^2}{2} - \frac{k_i^3}{4}\right) \sqrt{y_i}.$$

First looking at how the production function varies with k_i, we differentiate to obtain $\partial f_i / \partial k_i = \left(k_i - \frac{3k_i^2}{4}\right) \sqrt{y_i}$. It is evident straight away that $\partial f_i / \partial k_i = 0$ for $k_i = \frac{4}{3}$, with

$\partial f_i/\partial k_i > 0$ for all $k_i \in (0, \frac{4}{3})$ and $\partial f_i/\partial k_i < 0$ for all $k_i > \frac{4}{3}$. It is this property that gives the production function its 'S' shape (see Figure 1b). Differentiating twice with respect to k_i we have $\partial^2 f_i/\partial k_i^2 = \left(1 - \frac{3k_i}{2}\right)\sqrt{y_i}$. We can solve for $\widetilde{k}$ by setting $\partial^2 f_i/\partial k_i^2 = 0$ and solving using k_i, from which we find that $\widetilde{k} = \frac{2}{3}$. From this, $\partial^2 f_i/\partial k_i^2 > 0$ for $k_i \in [0, \frac{2}{3})$ and $\partial^2 f_i/\partial k_i^2 < 0$ for $k_i \in (\frac{2}{3}, \infty)$, which satisfies all the requirements of A1.

Turning attention now to the way that the level of y_i affects production, first note that if $y_i = 0$ then $f_i(k_i, 0) = 0$. Differentiating with respect to y_i, we have $\partial f_i/\partial y_i = \left(\frac{k_i^2}{2} - \frac{k_i^3}{4}\right) / \left(2\sqrt{y_i}\right)$. Observe from this that $\partial f_i/\partial y_i \to 0$ as $y_i \to \infty$ and $\partial f_i/\partial y_i \to \infty$ as $y_i \to 0$. Finally, $\partial^2 f_i/\partial y_i^2 = -\left(\frac{k_i^2}{2} - \frac{k_i^3}{4}\right) / \left(2\sqrt{y_i}\right)$. So as long as $\overline{k} \leq 2$ we have $\partial^2 f_i/\partial y_i^2 < 0$ for all $k_i \in \left(0, \overline{k}\right)$, $y_i > 0$, as required by A2.

Looking at the cross partial derivative, $\partial f_i^2/\partial k_i \partial y_i = \left(k_i - \frac{3k_i^2}{4}\right) / \left(2\sqrt{y_i}\right)$. We have that $\partial f_i^2/\partial k_i \partial y_i > 0$ for $k_i \in (0, \frac{4}{3})$, but $\partial f_i^2/\partial k_i \partial y_i = 0$ for $k_i = \frac{4}{3}$ and $\partial f_i^2/\partial k_i \partial y_i < 0$ for $k_i > \frac{4}{3}$.

In this example we will assume that the government budget identity (12) takes the form $t_i k_i \geq y_i$. Given that in equilibrium this holds with equality, $\partial (t_i k_i) / \partial y_i = 1$, and $\partial^2 (t_i k_i) / \partial y_i^2 = 0$. So the requirements of A3$'$ are satisfied.

The common technology requirements of A4 - $f_i(\cdot, \cdot) = f_j(\cdot, \cdot)$ and $g_i(\cdot, \cdot) = g_j(\cdot, \cdot)$ are straightforward to impose.

To fulfill the requirements of A4, we also need to show that the requirement $t_i^E < \left(-\partial^2 f_i\left(k_i^E, y_i^E\right) / \partial k_i^2\right) / \left(\partial^2 f_i / \partial k_i \partial y_i\right)$ holds at the efficient plan. This can only be done once the efficient plan $\mathcal{E} = \left\{\left(k_i^E, y_i^E\right), \left(k_j^E, y_j^E\right)\right\}$ has been solved for.

We now have all the components in place to be able to solve the efficient conditions for $\mathcal{E}$. For our example, the first condition (3) is

$$\left(k_i - \frac{3k_i^2}{4}\right)\sqrt{y_i} = \left(k_j - \frac{3k_j^2}{4}\right)\sqrt{y_j}.$$

Condition (4) takes the form

$$\left(\frac{k_i^2}{2} - \frac{k_i^3}{4}\right) / 2\sqrt{y_i} = 1$$

and (5) is the same but with j subscripts instead of i. Using the capital feasibility condition $k_i + k_j = \overline{k}$ to eliminate k_j we then have three equations in three unknowns.

Solving simultaneously these expressions for (3)-(5), the following solutions are obtained:

$$\begin{aligned} k_i^E &= \frac{\overline{k}}{2}; \\ y_i^E &= y_j^E = \frac{\overline{k}^4 \left(\overline{k}-4\right)^2}{4096}. \end{aligned}$$

If we let $\overline{k} = 2$ then $k_i^E = 1$ and $y_i^E = y_j^E = \frac{1}{64}$.

We have just solved the necessary conditions for a unique interior solution. Further, we need to show that the sufficient conditions of Lemma 1 hold. Rather than work through these for the example, we simply plot $\Omega\left(k_i, y_i, y_j\right)$ and show its unique interior maximum graphically. This is displayed in Figure 5. To construct this plot, it has been assumed that $\overline{k} = 2$. However, we can from the above solutions for the example that $y_i^E = y_j^E$. Given symmetric technology, it is each to check that $\Omega\left(k_i, y_i, y_j\right)$ is symmetrical in $\left[0, \overline{k}\right]$ about the point $\frac{\overline{k}}{2}$ when $y_i = y_j = y$; that is, $\Omega\left(\frac{\overline{k}}{2} - \epsilon, y, y\right) = \Omega\left(\frac{\overline{k}}{2} + \epsilon, y, y\right)$, where $\epsilon \leq \frac{\overline{k}}{2}$. It is clear from Figure 5 that the unique interior maximum occurs at $k_i = \frac{\overline{k}}{2} = 1$.

We can now return to the requirements of A4 and A5. First consider the condition that $t_i < -\left(\partial^2 f_i / \partial k_i^2\right) / \left(\partial^2 f_i / \partial k_i \partial y_i\right)$. The level of taxation consistent with the efficient plan is given by $t_i^E = y_i^E / k_i^E = 2 y_i^E / \overline{k}$. From the primitives of the model we also have that

$$\begin{aligned} -\left(\partial^2 f_i / \partial k_i^2\right) / \left(\partial^2 f_i / \partial k_i \partial y_i\right) &= \left(1 - \frac{3k_i}{2}\right) \sqrt{y_i} / \left(k_i - \frac{3k_i^2}{4}\right) / \left(2\sqrt{y_i}\right) \\ &= \frac{6}{4 - 3k_i} - \frac{2}{k_i}. \end{aligned}$$

Using $k_i = \overline{k}/2$ and setting $\overline{k} = 2$, we have $t_i^E = \frac{1}{64}$ and $-\left(\partial^2 f_i / \partial k_i^2\right) / \left(\partial^2 f_i / \partial k_i \partial y_i\right) = 4$, so the condition imposed by A4, that $t_i^E < -\left(\partial^2 f_i / \partial k_i^2\right) / \left(\partial^2 f_i / \partial k_i \partial y_i\right)$, holds.

The conditions imposed by A5 guarantee under plausible assumptions that a point k_i^I exists such that

$$1 - k_i^I \frac{\partial^2 f_i\left(k_i^I, y_i\right)}{\partial k_i \partial y_i} = 0.$$

Because in this example all aspects of the model are specified we can solve directly for the point k_i^I and show that $k_i^I = k_i^E$ when $y_i = y_i^E$ and $\overline{k} = 2$. Using $y_i^E = \frac{\overline{k}^4 \left(\overline{k}-4\right)^2}{4096}$, $\overline{k} = 2$ and $k_i\left(\partial^2 f_i / \partial k_i \partial y_i\right) = k_i\left(k_i - \frac{3k_i^2}{4}\right) / \left(2\sqrt{y_i}\right)$, it is easy to see that the above equation is solved by $k_i^I = 1$. Because $k_i^E = \overline{k}/2 = 1$ we have $k_i^I = k_i^E$ as required.

Theorem 1 then tells us that as a result, $\partial k_i/\partial t_i = 0$, from which it follows that neither government has an incentive to deviate from the efficient plan by changing taxes.

4.2 Inefficient Nash Equilibrium in the Z-M model

To exhaust the full range of possibilities, we also need to consider the situation where $k_i^E > k_i^I$ and where $k_i^E < k_i^I$ as well. These two possibilities are dealt with simultaneously in the following theorem. We continue to assume that A1-A3′-A5 hold.

Theorem 2. *Assume A1-A3′-A5 and that there exists a symmetric efficient plan* $\mathcal{E} = \{(k_i^E, y_i^E), (k_j^E, y_j^E)\}$ *characterized by the first order conditions* (3)*-*(5)*. Let* $\mathcal{Q} = \{(t_i^N, y_i^N), (t_j^N, y_j^N)\}$ *be a symmetric Nash equilibrium. Assume that* $k_i^I < (>)\, k_i^E$ *when* $y_i = y_i^N$, $i = 1, 2$*. Then the equilibrium outcome is inefficient, with under- (over-) provision of the public good, depending on* $k_i^I < (>)\, k_i^E$ *when* $y_i = y_i^N$.

Proof: (i) We will show that $y_i^N < (>)\, y_i^E$, $y_j^N < (>)\, y_j^E$, $t_i^N < (>)\, t_i^E$ and $t_j^N < (>)\, t_j^E$, with $k_i = k_i^E = k_j = k_j^E = \overline{k}/2$.

First, it will be useful to recall the conditions on efficiency under the present assumptions (3)-(5). By A3′, $\partial g_i(y_i)/\partial y_i = \partial y_i/\partial y_i = 1$, so the first order conditions for efficiency (3)-(5) become:

$$\begin{aligned}
\frac{\partial f_i\left(k_i^E, y_i^E\right)}{\partial k_i} &= \frac{\partial f_j\left(\overline{k} - k_i^E, y_j^E\right)}{\partial\left(\overline{k} - k_i\right)} \text{ and} \\
\frac{\partial f_i\left(k_i^E, y_i^E\right)}{\partial y_i} &= \frac{\partial f_j\left(k_j^E, y_j^E\right)}{\partial y_j} = 1.
\end{aligned}$$

Now, a requirement of Nash equilibrium is that, for the induced policy $\left(t_i^N, y_i^N\right)$ and capital demand k_i^N it holds for some r, $i = 1, 2$, that

$$\begin{aligned}
\frac{dc_i}{dt_i} &= \left(\frac{\partial f_i\left(k_i^N, y_i^N\right)}{\partial k_i} - (t_i + r)\right)\frac{\partial k_i}{\partial t_i} + \\
&\frac{\partial f_i\left(k_i^N, y_i^N\right)}{\partial y_i}\left(\frac{\partial y_i}{\partial t_i} + \frac{\partial y_i}{\partial k_i}\frac{\partial k_i}{\partial t_i}\right) - k_i^N = 0.
\end{aligned}$$

This is just a restatement of the government's first order condition (14). (If not, then the equilibrium requirement $c_i(t_i^N, y_i^N; r) \geq c_i(t_i, y_i; r)$ could not hold.)

Note that because the Nash equilibrium is symmetric, and the efficient plan is symmetric, it must be the case that $k_i^N = k_i^E = \overline{k}/2$, $i = 1, 2$. Because the demand for capital is symmetric across jurisdictions in equilibrium, $k_i^N = k_j^N = \overline{k}/2$. Also, because the efficient plan is symmetric we must have $k_i^E = k_j^E = \overline{k}/2$. Therefore, in what follows, we can write k_i^E and k_i^N interchangeably.

We will now show that if $k_i^I < (>) \, k_i^E$ when $y_i = y_i^N$ then $dc_i/dt_i = 0$ implies $\partial f_i \left(k_i^E, y_i^N\right) / \partial y_i \neq 1$, but because $\partial f_i \left(k_i^E, y_i^N\right) / \partial y_i = 1$ is required for efficiency, the Nash equilibrium cannot therefore be efficient.

To determine the value of $\partial f_i \left(k_i^E, y_i^N\right) / \partial y_i$ in equilibrium, notice that $\frac{\partial f_i\left(k_i^N, y_i^N\right)}{\partial k_i} - (t_i + r) = 0$ and $\partial y_i / \partial t_i = t_i \partial k_i \partial t_i + k_i$; using both of these facts, the first order condition of government i's objective function $dc_i/dt_i = 0$ becomes

$$\frac{\partial f_i \left(k_i^E, y_i^N\right)}{\partial y_i} = \frac{1}{1 + \left(t_i^N / k_i^E\right) \left(\partial k_i / \partial t_i\right)}.$$

Given k_i^E, $t_i^N > 0$, it is the case that $\partial f_i \left(k_i^E, y_i^N\right) / \partial y_i > 1$ if $\partial k_i \left(k_i^E, y_i^N\right) / \partial t_i < 0$ and $\partial f_i \left(k_i^E, y_i^N\right) / \partial y_i > 1$ if $\partial k_i \left(k_i^E, y_i^N\right) / \partial t_i > 0$.

The sign of $\partial k_i \left(k_i^E, y_i^N\right) / \partial t_i$ is given by the expression (16). Because we assume ODMP$_k$, (A4) the denominator of (16) is negative. Therefore, the sign of (16) depends on the numerator, $1 - k_i^E \partial^2 f_i \left(k_i^E, y_i^N\right) / \partial k_i \partial y_i$. From $k_i^I < (>) \, k_i^E$ it holds by Lemma 3 and A5 that

$$1 - k_i^E \frac{\partial^2 f_i \left(k_i^E, y_i^N\right)}{\partial k_i \partial y_i} > (<) \, 0.$$

Consequently, $\partial k_i \left(k_i^E, y_i^N\right) / \partial t_i < (>) \, 0$ when $k_i^I < (>) \, k_i^E$. Thus $\partial f_i \left(k_i^E, y_i^N\right) / \partial y_i > (<) \, 1$ for $t_i^N > 0$ and $k_i^E > 0$ when $k_i^I < (>) \, k_i^E$.

Finally, because $\partial^2 f_i \left(k_i^E, y_i^N\right) / \partial y_i^2 < 0$, $\partial f_i \left(k_i^E, y_i^N\right) / \partial y_i > 1$ implies underprovision of the public good in equilibrium, and $\partial f_i \left(k_i^E, y_i^N\right) / \partial y_i < 1$ implies overprovision.

We can obtain t_i^N and t_j^N by rearranging (13) to get $t_i = y_i / k_i$. Because $y_i^N < (>) \, y_i^E$, it must be that $t_i^N = 2y_i^N / \overline{k} < 2y_i^E / \overline{k} = t_i^E$ $i = 1, 2$.

Thus we have shown that if $k_i^I < (>) \, k_i^E$ then the economic outcome of the Nash equilibrium is inefficient because there is under- (over-) provision of the public good. $\square$

The insight gained from the analysis of this section is that the state of the economy in Nash equilibrium depends on whether there is an incentive to deviate from the Nash equilibrium, and if so in which direction. If the marginal increase in output facilitated by increasing public good provision is exactly equal to the marginal cost in the efficient state then there is no incentive to deviate unilaterally. Then the economy will be in an efficient state in Nash equilibrium. It is the unilateral incentive to deviate from the efficient state in either direction that leads Nash equilibrium to be inefficient.

5 Asymmetry in the Technology of Public Good Provision

The main results established in the previous section are based on the standard highly stylized framework in which jurisdictions are symmetric in all respects. In this section we examine the robustness of these results to a small step away from symmetry. Specifically, we solve for asymmetry in the technology of public good provision across jurisdictions. To make this precise, A4 is replaced by the following set of revised assumptions:

A4′. In addition to the conditions on technology imposed by A1 and A2, assume there is common firm production technology across jurisdictions - $f_i(\cdot,\cdot) = f_j(\cdot,\cdot)$ - but differing technology for the production of public goods $g_i(\cdot) \neq g_j(\cdot)$. Also, efficiency implies that production occurs in both jurisdictions. In addition[20],

$$t_i^E < -\frac{\partial g_i^{-1}\left(t_i^E k_i^E\right)}{\partial\left(t_i^E k_i^E\right)} \frac{\partial^2 f_i\left(k_i^E, y_i^E\right)/\partial k_i^2}{\partial^2 f_i\left(k_i^E, y_i^E\right)/\partial k_i \partial y_i}.$$

Because we have assumed that $\partial g_i(y_i)/\partial y_i > 0$ (A3′(i)) by the inverse function theorem the function $g(y_i)$ has a well defined inverse, written as $y_i = g_i^{-1}(t_i k_i)$.

One jurisdiction could still have the technology $g_i(y_i) \equiv y_i$ assumed by Z-M, but now both cannot. Because we allow for more general public good production technology, A5 must be adjusted to encompass this.

[20] This is an appropriate modification of the corresponding Z-M condition (see A5)

A5′. Assume the following three conditions hold for any positive level of y_i:

$$\lim_{k_i \to 0} \left(1 - k_i \frac{\partial g_i^{-1}(t_i k_i)}{\partial (t_i k_i)} \frac{\partial^2 f_i(k_i, y_i)}{\partial k_i \partial y_i} < 0 \right);$$
$$\lim_{k_i \to \overline{k}} \left(1 - k_i \frac{\partial g_i^{-1}(t_i k_i)}{\partial (t_i k_i)} \frac{\partial^2 f_i(k_i, y_i)}{\partial k_i \partial y_i} > 0 \right);$$
$$k_i \frac{\partial g_i^{-1}(t_i k_i)}{\partial (t_i k_i)} \frac{\partial^2 f_i(k_i, y_i)}{\partial k_i \partial y_i} \text{ declines monotonically with } k_i.$$

In line with this adjustment, Lemma 3 must also be generalized:

Lemma 4. *Assume A1, A2, and A5′. Then $f_i(k_i, y_i)$ is a function that is C^2 on the compact set $k_i \in [0, \overline{k}]$ and C^2 on the set $y_i \in \mathcal{R}$. Then by A5′ there exists, for any given y_i, a value $k_i^I \in [0, \overline{k}]$ such that*

$$1 - k_i^I \frac{\partial g_i^{-1}(t_i k_i)}{\partial (t_i k_i)} \frac{\partial^2 f_i\left(k_i^I, y_i\right)}{\partial k_i \partial y_i} = 0.$$

Proof: Follows from a straightforward application of the intermediate value theorem.
□

With these adjustments, it is possible to analyze a variation in the technology of public good provision across jurisdictions.

5.1 A counterexample to Theorem 1 when public good technology is not symmetric

We now use an example to show why, under this more general framework, Theorem 1 can no longer hold. When $g_i(y_i) \neq g_j(y_j)$ there cannot exist a Nash equilibrium in policies for the example such that the economy is in an efficient state.

Let the production function take the following specific form:

$$f_i = \left(\frac{(k_i)^2}{2} - \frac{(k_i)^3}{4} \right) \sqrt{y_i}.$$

It is straightforward to verify that this function satisfies assumptions A1 and A2, where $\widetilde{k}_i = \frac{2}{3}$. It is strictly convex over the interval $k_i \in [0, \frac{2}{3})$ and strictly concave over the interval $k_i \in [\frac{2}{3}, \infty)$. And it is strictly concave in $y_i \in \mathcal{R}_+$.

In addition, assume that

$$\begin{aligned} g_1(y_1) &= y_1, \\ g_2(y_2) &= 2y_2. \end{aligned}$$

Again, it is straightforward to verify that these linear functions satisfy A3.

Using the capital feasibility condition and solving for (3), (4) and (5) we obtain, respectively,

$$\begin{aligned} \left(k_1 - \frac{3(k_1)^2}{4}\right)\sqrt{y_1} &= \left(\left(\overline{k} - k_1\right) - \frac{3\left(\overline{k} - k_1\right)^2}{4}\right)\sqrt{y_2} \\ \frac{\left(\frac{(k_1)^2}{2} - \frac{(k_1)^3}{4}\right)}{2\sqrt{y_1}} &= 2 \\ \frac{\left(\frac{\left(\overline{k}-k_1\right)^2}{2} - \frac{\left(\overline{k}-k_1\right)^3}{4}\right)}{2\sqrt{y_2}} &= 1 \end{aligned}$$

These latter two conditions can be solved simultaneously to yield

$$\begin{aligned} y_1^E &= \frac{1}{64}(k_1 - 2)^2 k_1^4, \\ y_2^E &= \frac{1}{144}\left(k_1 - \overline{k}\right)^4\left(2 + k_1 - \overline{k}\right)^2, \end{aligned}$$

and specifying the value $\overline{k} = 2$, these can be used in the first to obtain a solution for k_1;

$$k_1^E = \frac{2}{45}\left(20 + 5 \cdot \sqrt[3]{10} - \sqrt[3]{10^2}\right) \approx 1.16136$$

From the capital and public good allocations derived, we can use the balanced budget condition (13) to show what taxes associated with an efficient plan would be, were these to be supportable as decentralized choices by governments in Nash equilibrium;

$$\begin{aligned} t_1^E &= \frac{y_1^E}{k_1^E} = \frac{245 + \sqrt[3]{13110} - \sqrt[3]{4610^2}}{18225} \approx 0.01721 \\ t_2^E &= \frac{y_2^E}{k_2^E} = \frac{2\left(1070 - \sqrt[3]{14210} + \sqrt[3]{6710^2}\right)}{164025} \approx 0.00552 \end{aligned}$$

However, an apparently insurmountable difficulty arises with the decentralization of this efficient plan in equilibrium. Suppose it were the case that policies $\left(t_1^E, y_1^E\right)$ and $\left(t_2^E, y_2^E\right)$ had been adopted by the governments of jurisdictions 1 and 2. Now look at the firms' first order conditions. Recall that from (10),

$$\frac{\partial f_i\left(k_i, y_i^E\right)}{\partial k_i} - \left(r + t_i^N\right) = \frac{\partial f_j\left(k_j, y_j^E\right)}{\partial k_j} - \left(r + t_j^N\right) = 0.$$

In an efficient plan, by (3) we must have that $\partial f_i\left(k_i^E, y_i^E\right)/\partial k_i = \partial f_j\left(k_j^E, y_j^E\right)/\partial k_j$. It can readily be checked that this condition holds for the values of k_1^E, y_1^E and y_2^E that we have just derived for this example. The term r cancels from both sides. But due to the fact that taxes differ across jurisdictions - $t_1^E \neq t_2^E$ - we cannot derive (3) from the firm's decentralized equilibrium condition because t_i^N and t_j^N cannot be cancelled from either side. Consequently, it is not possible to conclude that the allocations that are efficient, $\left(k_i^E, y_i^E\right)$ and $\left(k_j^E, y_j^E\right)$, will also satisfy the firms' profit maximizing conditions. So it is not possible to say that the efficient allocations can be decentralized. With symmetric technology, by contrast, it was possible to do this because tax rates across jurisdictions were the same (see Theorem 1 and its proof).

A further difficulty is introduced by the asymmetry of technology to produce public goods. It appears to be practically impossible (although not logically impossible) to construct a situation in which the condition of Lemma 4 holds for both jurisdictions simultaneously; that is, a situation in which $k_i^E = k_i^I$ for $i = 1$ *and* 2. This is necessary in order to have $\partial k_i/\partial t_i = \partial k_j/\partial t_j = 0$, which is required for the choices of public good provision made by individual governments to be consistent with the efficient plan; in technical terms, for (4) and (5) to coincide with (14).

Recall that all conditions of Theorem 1 *are* satisfied and the solution for efficient plan also solves the Nash equilibrium conditions for the example production function $f_i = \left((k_i)^2/2 - (k_i)^3/4\right)\sqrt{y_i}$, setting $\overline{k} = 2$, when common technology is adopted for public service provision; $g_i(\cdot) = g_j(\cdot)$; see Section 4.1.1.

6 All Production in One Jurisdiction

In this section we turn to the standard (Z-M) model, and use it to prove existence of an asymmetric Nash equilibrium. We shall see that in a Nash equilibrium outcome, all production occurs in one jurisdiction, where the public good is provided at the efficient level. Citizens of the other jurisdiction engage in no production at all, lending out all their capital, and living only on the rental payments; the government provides no public good.

As mentioned in the introduction, we think of this equilibrium as characterizing the OECD as one jurisdiction - where the vast majority of the world's capital is employed in production - versus the rest of the world as the other. Our analysis suggests that even if poorer countries, or poorer states within a federation, charge lower taxes capital will not flee there if public good provision is sub-optimal.

Another way to interpret the analysis of this section is to think of if it as characterizing situations where there are multiple jurisdictions, producing a number of commodities, with specialization in each jurisdiction. The analysis suggests that production levels do not have to be symmetric across jurisdictions as long as a higher overall production level and the net export of goods from one jurisdiction is exactly balanced by a flow of payments for capital lending to the other.

Formally, we return to the set of assumptions A1-A3′-A5 of Section 4. Recall that under these assumptions, both jurisdictions where ex ante symmetrical in all respects, and the technology of public good provision is simplified to the standard linear case, so that (13) becomes $t_i k_i = y_i$. We show that under these assumptions and the conditions of Lemma 2 it is possible for the economy to be in an efficient state in Nash equilibrium when all production of goods and provision of public goods is undertaken by a single jurisdiction.

As before, let $\left(k_i^E, y_i^E\right)$ and $\left(k_j^E, y_j^E\right)$ describe an efficient state of the economy. Let $\left(t_i^N, y_i^N\right)$ and $\left(t_j^N, y_j^N\right)$ denote policies set by governments i and j respectively in Nash equilibrium.

Theorem 3. *Assume A1-A4, and that conditions (i)-(iii) of Lemma 2 hold, so that* *the point $\mathcal{E} = \left\{\left(\overline{k}, y_i^E\right), (0,0)\right\}$ is efficient. In addition, assume that $r^* = f_i\left(\overline{k}, y_i\right)/\overline{k} - y_i/\overline{k}$, and that*

$$\frac{\partial f_i\left(\overline{k}, y_i\right)}{\partial k_i} \geq r + \frac{y_i}{\overline{k}}.$$

Then there exists an asymmetric Nash equilibrium in policies such that the economic outcome is efficient:

(i) $y_i^N = y_i^E > 0$, $y_j^N = 0$ (the amenity is provided at an optimal level in one jurisdiction, and not at all in the other)

$t_i^N = t_j^N = y_i^E/\overline{k}$ (taxes are set at a rate consistent with optimal public good provision in both jurisdictions).

(ii) $k_i^E = \overline{k}$, $k_j^E = 0$ (all capital locates in one jurisdiction);

(iii) $t_i^N = t_j^N$ - taxes are equal across jurisdictions;

(iv) the policies $\left(t_i^N, y_i^N\right)$ and $\left(t_j^N, y_j^N\right)$ are feasible; when they are adopted simultaneously by governments, capital markets clear and budgets balance.

We now introduce the following lemma, which will be helpful in proving the theorem itself.

Lemma 5. *Assume A1-A4, $r^* = f_i\left(\overline{k}, y_i\right)/\overline{k} - y_i/\overline{k}$, $t_i = t_j = y_i/\overline{k}$ and that*

$$\frac{\partial f_i\left(\overline{k}, y_i\right)}{\partial k_i} \geq r + \frac{y_i}{\overline{k}}.$$

Then a capital market equilibrium exists if and only if $y_i > 0$ and $y_j = 0$.

Proof. See the appendix.

The implication of this lemma is quite straight forward. It says that the profit function is not downward sloping at the point where $k_i = \overline{k}$, then only one jurisdiction can offer a positive level of public good provision in equilibrium. Because all capital can be productively employed in one jurisdiction, the other cannot finance any public good provision through taxation.

Proof of Theorem 3. To prove the theorem, we will once again show that $\mathcal{E} = \left\{\left(\overline{k}, y_i^E\right), (0,0)\right\}$ solves the conditions of a Nash equilibrium.

The proof is in three stages.

(i) Suppose that the government of jurisdiction j announces the policy $\left(t^N, 0\right)$. Then by A2, $f_i\left(k_i, 0\right) = 0$. Therefore, the firm in jurisdiction j can make non-negative profits only at $k_j = 0$, given $t_j^N > 0$ and $r^* = f_i\left(\overline{k}, y_i^E\right)/\overline{k} - y_i^E/\overline{k} > 0$.

(ii) Analyze jurisdiction i's best response $(i \neq j)$. As in the proof of Theorem 1, we must first show that at $\left(\overline{k}, y_i^E\right)$, it holds that for some $r, t_i, i = 1, 2$,

$$\frac{\partial c_i}{\partial t_i} = \left(\frac{\partial f_i\left(\overline{k}, y_i^E\right)}{\partial k_i} - (t_i + r)\right)\frac{\partial k_i}{\partial t_i} + \frac{\partial f_i\left(\overline{k}, y_i\right)}{\partial y_i}\left(\frac{\partial y_i}{\partial t_i} + \frac{\partial y_i}{\partial k_i}\frac{\partial k_i}{\partial t_i}\right) - k_i = 0.$$

Adopting exactly the same method as used in the proof of Theorem 1, if we can show that $\partial k_i/\partial t_i = 0$ at $k_i^E = \overline{k}$ then (14) collapses to $\partial f_i\left(k_i^E, y_i^E\right)/\partial y_i = 1$, which is the condition on efficiency (4) under A4. Then $\left(\overline{k}, y_i^E\right)$ also solves the Nash equilibrium condition.

As in the proof of Theorem 1, we assume firms demand $k_i^E = \overline{k}$, and then show that this is consistent with profit maximization by the firm, given the government's optimizing behavior.

We know by Lemma 5 that given $y_j = 0$, we must have $y_i > 0$ in equilibrium. Moreover, Lemma 5 shows that $k_i^* = \overline{k}$ for all $y_i > 0$. By , it follows that $k_i^* = \overline{k}$ for all $t_i > 0$.[21] An equivalent way to state this is to say that $\partial k_i/\partial t_i|_{t_i \gg 0} = 0$. Then (14) collapses to the efficient condition (4), as in Theorem 1, and as we set out to show. Given that k_i^E, y_i^E solves (4), then setting $y_i^N = y_i^E$ must solve this Nash equilibrium condition as well.

To see why $y_i^E > 0$, suppose not. Then $y_i^E = 0$ and $\partial f_i\left(\overline{k}, y_i^E\right)/\partial y_i \to \infty$ by A2. But by (7) we must have $\partial f_i\left(\overline{k}, y_i^E\right)/\partial y_i = 1$ and by A2 this cannot occur when $y_i^E = 0$; a contradiction.

Given $k_i^E = \overline{k}$ and y_i^E, we can work out the tax by rearranging (13) to obtain $t_i^N = y_i^E/\overline{k}$. Thus we have a policy $\left(t_i^N, y_i^N\right)$ for which $c_i(t_i^N, y_i^N; r) \geq c_i(t_i, y_i; r)$, all $(t_i, y_i) \neq \left(t_i^N, y_i^N\right)$, as required by the definition of Nash equilibrium. It follows from $y_i^E > 0$ and (13) that $t_i^N = t_j^N = y_i^E/\overline{k} > 0$.

[21] We cannot have $t_i = 0$ in equilibrium for the same reason that we cannot have $y_i = 0$.

Finally, we can affirm that $k_i = \overline{k}$ given $y_i^E > 0$ from Lemma 5, which shows that $k_i^E = \overline{k}$ for all $y_i > 0$, given $y_j^E = 0$. So we have validated the starting assumption that $k_i = \overline{k}$ is profit maximizing for the firm given optimizing behavior by the government.

(iii) Check whether jurisdiction j has an incentive to deviate from the policy $\left(t^N, 0\right)$, given the best response by i. If not, then we have a Nash equilibrium.

In order to establish their choice $\left(t^N, 0\right)$ as a best response, we need to consider and reject each of the two possible deviations available to jurisdiction j:

(a) That given $t_i^N = t_j^N = t^N$, $y_j = 0$ is a best response. (b) that $t_j = t_i^N = t^N$ is itself a best response.

(a) Given that both jurisdictions set t^N, and $y_i = t^N \overline{k}$, check that $y_j = 0$ is a best response, given that $t_j = t^N$. As we have already seen, the assumption that $r^* = f_i\left(\overline{k}, y_i^E\right) - t^N$ ensures that $\pi_i = 0$. Recall that (11) holds with equality in equilibrium: $c_i = \pi_i + r^* \overline{k}_i$. With $\pi_i = 0$ and all capital productively employed (in the home jurisdiction), we have $c_i = r^* \overline{k}_i$.

Now write the equivalent for j; $c_j = \pi_j + r^* \overline{k}_j$. With $y_j = 0$, no production is undertaken in j because we have ; $f_j(0,0) = 0$ and $0 \cdot t^N = 0$. So we have $\pi_j = 0$. Therefore $c_j = r^* \overline{k}_j$ (the representative citizen in jurisdiction j is earning rental income $r^* \overline{k}_j$ for capital lent to the producer in jurisdiction i). Using $\overline{k}_i = \overline{k}_j$, it follows immediately that $c_i\left(t^N, t^N \overline{k}; r^*\right) = c_j\left(t^N, 0; r^*\right)$.

With $t_j = t^N$ the only feasible deviation is to set $0 < y_j$, but subject to the constraint that $y_j < y_i$. Note that $y_j = y_i = t^N \overline{k}$ is not feasible because both jurisdictions cannot simultaneously have tax base $\overline{k}$. But by Lemma 5, if $y_j < y_i$ then there can be no capital market equilibrium.

(b) Check that $t_j = t^N$ is a best response. Suppose that the government of jurisdiction j announces $t_j \neq t^N$. But in order for jurisdiction j to attract capital from jurisdiction i, it must be possible to show that $\hat{r} > r^*$ when $t_j \neq t^N$. Otherwise, capital will stay in jurisdiction i, where it is paid r^*.

We will show that $\hat{r} > r^*$ is not feasible by demonstrating that the efficient plan maximizes r. To see this, observe that at the efficient point $\mathcal{E} = \left\{\left(\overline{k}, y_i^E\right), (0,0)\right\}$, the planner's problem can be written $\Omega\left(\overline{k}, y_i^E, y_j^E\right) = f_i\left(\overline{k}, y_i^E\right) - y_i^E$. This follows because $f_j(0,0) = 0$ (A1) and because $y_j^E = 0$. And by the definition of efficiency, the values $k_i = \overline{k}$, $y_i = y_i^E$, $y_j = 0$ maximize the planner's problem $\Omega(k_i, y_i, y_j)$. Writing $r^* = \left(f_i\left(\overline{k}, y_i^E\right) - y_i^E\right)/\overline{k} = \Omega\left(\overline{k}, y_i^E, y_j^E\right)/\overline{k}$, we see that r must also be maximized under efficiency. Therefore, $\hat{r} > r^*$ is not feasible. □

When it is efficient for all production to take place in a single jurisdiction, then the efficient allocations and outcome can be decentralized in a Nash equilibrium. The government of one jurisdiction - say jurisdiction i - announces amenity provision at the efficient level, while the government of jurisdiction j announces that no amenity will be provided at all. The efficient capital allocations are the profit maximizing choices of firms, with the firm in jurisdiction i demanding all capital - $k_i^* = k_i^E = \overline{k}$ - and the firm in jurisdiction j demanding none - $k_j^* = k_j^E = 0$. The tax rate set by jurisdiction i is just sufficient to ensure that the government budget balances, by (12); $t_i^N = y_i^E/k_j^E$. The government of jurisdiction j announces the same tax rate $t_j^N = y_i^E/k_j^E$, 'standing ready' to adopt optimal amenity provision if jurisdiction i fails to do so. The interest rate is determined by the net return on capital in jurisdiction i, where all capital is located in production. The marginal productivity of capital in jurisdiction j is zero.

7 Conclusions

The purpose of this paper has been to show that a very much wider set of outcomes is possible within the context of a standard tax competition model than has previously been suggested. The past literature tends to focus on a 'race to the bottom' of tax rates and public good provision. Where other outcomes such as efficient taxation or a 'race to the top' are shown to arise, this is due to the presence of other mechanisms, for example a type of Tiebout mechanism where the representative citizen is able to vote for their preferred policies. We show that all of these outcomes are possible within the same standard tax competition framework. We also show, for the first time to our knowledge, that an efficient state occurring in Nash equilibrium is not necessarily symmetric, and that all production and public good provision may occur in just one jurisdiction, even when the model is ex ante symmetric in all respects.

We use the version of the standard model where the public good enters the production function of firms. This is distinguished from the more familiar approach of simply assuming that the good produced by the government enters the utility function. The way that we obtain this more general set of results is to weaken a standard assumption. In the past literature it has been assumed that the additional output obtained from provision of the public good through taxation is never as great as the opportunity cost in terms of tax revenue. Therefore, in the conventional set-up there is always a unilateral incentive to deviate from the efficient level of public good provision. Under our assumptions there may be a unilateral incentive to deviate upwards, downwards or not at all from the efficient level of public good provision. Thus, all three possibilities can arise in Nash equilibrium.

One possibility revealed by our analysis that has been overlooked completely by the past literature is that there may be an efficient state of the economy in Nash equilibrium that is asymmetric, even when the economic structure of the model and the sequence of play is completely symmetric. We think of this result as characterizing the position in which many developing economies currently find themselves. If the public goods' positive impact on productivity is powerful enough then it may be efficient for all capital to locate in a single jurisdiction. More loosely, our analysis suggests that it may not be possible for developing economies to lure away capital from developed economies simply by offering lower taxes, if the necessary public goods are not in place. Developing economies may have to be in a position to provide good infrastructure, an efficient legal system and a competent labor force before firms can offer a sufficiently high return for capital owners to consider investing their capital there.

Our results potentially pose a difficult question in the debate on whether taxation policy should be harmonized across jurisdictions. We show that even a small step away from the usual symmetry assumptions bring about a situation where an efficient state cannot be achieved in equilibrium. Yet at the same time, it appears that tax harmonization is not the answer because tax rates necessarily differ when technology is asymmetric, as seems likely in practice.

The analysis of this paper focuses on a model where taxation provides firms with a good that they value, because it increases the productivity of their capital. The model could be cast in a consumer setting by looking at taxation associated with consumption.

There is already a literature on this area, which looks at how the 'earmmarking' of taxes for specific purposes valued by consumers can reduce the free rider problem. See, for example, Amrita Dhillon and Carlo Perroni (2001). The analysis of this present paper suggests that in a situation where consumers value that public good being provided along parallel lines to the valuation placed on public goods by firms, the conventional free rider problem may under certain circumstances disappear completely.

One interesting question posed for future research in this area is whether the problem of tax setting can be treated as a mechanism design problem where the tax system is set up in such a way that each jurisdictional government sees the efficient plan as coinciding with their own best response in the policy setting game.

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A Appendix

Proof of Lemma 1: (Sufficiency). Because $\mathcal{E} = \left\{ \left(k_i^E, y_i^E\right), \left(k_j^E, y_j^E\right) \right\}$ satisfies (3), (4) and (5), we know we have the necessary conditions in place for a critical point to exist.

It is well known that $D^2\Omega\left(k_i^E, y_i^E, y_j^E\right)$ is negative semi-definite, and therefore concave at $\mathcal{E} = \left\{ \left(k_i^E, y_i^E\right), \left(k_j^E, y_j^E\right) \right\}$, if the leading principal minors alternate in sign, with $\Omega_{k_ik_i} < 0$, that is, if

$$(-1)^r \begin{vmatrix} \Omega_{k_ik_i} & \Omega_{k_iy_i} & \Omega_{k_iy_j} \\ \Omega_{y_ik_i} & \Omega_{y_iy_i} & \Omega_{y_iy_j} \\ \Omega_{y_jk_i} & \Omega_{y_jy_i} & \Omega_{y_jy_j} \end{vmatrix} \geq 0, \text{ for } r = 1, 2, ..., n.$$

The conditions for the signs of the principal minors to alternative in an appropriate way are established in three steps.

(a) We can easily establish that the first principal minor is negative; $\Omega_{k_ik_i} < 0$. By a well known result, efficiency must occur for a choice of k_i^E within the concave segment; in the non-concave segment it is always possible to increase (or at least not decrease) net output x_i by increasing k_i. The same is true for k_j^E. Therefore $\partial^2 f_i\left(k_i^E, y_i^E\right)/\partial k_i^2 < 0$ and $\partial^2 f_j\left(k_j^E, y_j^E\right)/\partial k_j^2 < 0$. Since $\Omega_{k_ik_i} = \partial^2 f_i\left(k_i^E, y_i^E\right)/\partial k_i^2 + \partial^2 f_j\left(k_j^E, y_j^E\right)/\partial k_j^2$ it follows immediately that $\Omega_{k_ik_i} < 0$.

(b) For the second principal minor to take positive value requires

$$\begin{vmatrix} \Omega_{k_ik_i} & \Omega_{k_iy_i} \\ \Omega_{y_ik_i} & \Omega_{y_iy_i} \end{vmatrix} > 0.$$

By Young's theorem, $\Omega_{k_iy_i} = \Omega_{y_ik_i}$ so this determinant is positive if and only if condition (i) of Lemma 1 - $\Omega_{k_ik_i}\Omega_{y_iy_i} > \left(\Omega_{k_iy_i}\right)^2$ - is satisfied. Given $\Omega_{k_ik_i}$, $\Omega_{y_iy_i} > 0$, (A1, A2) the only requirement is that the square of this determinant is small relative to the product of the diagonal elements.

(c) There are a number of ways to calculate the third principle minor. In general we know that for any matrix A,

$$\det A = \sum_{i=1}^{n} a_{ij} A_{ij} = \sum_{j=1}^{n} a_{ij} A_{ij} \text{ for all } i, j,$$

where a_{ij} is the $(i,j)\,th$ element of A and A_{ij} is the $(i,j)\,th$ cofactor. Now note that by A1 and A2, $\Omega_{y_i y_j} = \Omega_{y_j y_i} = 0$. Therefore, the third principal minor, which is the determinant of the full Hessian matrix, can be calculated as

$$\Omega_{y_j k_i} \begin{vmatrix} \Omega_{k_i y_i} & \Omega_{k_i y_j} \\ \Omega_{y_i y_i} & \Omega_{y_i y_j} \end{vmatrix} + \Omega_{y_j y_j} \begin{vmatrix} \Omega_{k_i k_i} & \Omega_{k_i y_i} \\ \Omega_{y_i k_i} & \Omega_{y_i y_i} \end{vmatrix}.$$

For concavity at the point $\mathcal{E} = \left\{ \left(k_i^E, y_i^E\right), \left(k_j^E, y_j^E\right) \right\}$ we require that this is negative. Note that the determinant in the second term is just the second principle minor, which we have assumed to be positive. And by A2, $\Omega_{y_j y_j} < 0$ so the second term is negative. Turning now to the first term, observe that $\Omega_{y_i y_j} = 0$ so the determinant in the first term simplifies to $-\Omega_{y_i y_i} \Omega_{k_i y_j}$;. $\Omega_{y_i y_i} < 0$ (by A2). To ensure that the third principle minor is negative we must impose condition (ii) of Lemma 1;

$$\Omega_{y_j k_i} \begin{vmatrix} \Omega_{k_i y_i} & \Omega_{k_i y_j} \\ \Omega_{y_i y_i} & \Omega_{y_i y_j} \end{vmatrix} < \Omega_{y_j y_j} \begin{vmatrix} \Omega_{k_i k_i} & \Omega_{k_i y_i} \\ \Omega_{y_i k_i} & \Omega_{y_i y_i} \end{vmatrix}.$$

We have established conditions under which there exists a point $\mathcal{E} = \left\{ \left(k_i^E, y_i^E\right), \left(k_j^E, y_j^E\right) \right\}$ which is a critical point and that it is a local maximum.

(iii) It remains to show that the point $\mathcal{E}$ is a global maximum; that is $\Omega\left(k_i^E, y_i^E, y_j^E\right) > \Omega\left(\hat{k}_i, y_i^E, y_j^E\right)$ for any $\hat{k}_i \neq k_i^E$. This is ensured by condition (iii) of Lemma 1 - quasi-concavity of $\Omega\left(k_i, y_i, y_j\right)$ - as we now explain.

Suppose that there exists a point $\hat{k}_i \neq k_i^E$ such that $\Omega\left(\hat{k}_i, y_i^E, y_j^E\right) > \Omega\left(k_i^E, y_i^E, y_j^E\right)$. By definition, strict quasi-concavity implies

$$\Omega\left(\left((1-\lambda)\, k_i' + \lambda k_i''\right), y_i^E, y_j^E\right) > \min\left(\Omega\left(k_i', y_i^E, y_j^E\right), \Omega\left(k_i'', y_i^E, y_j^E\right)\right)$$

for all k_i', $k_i'' \in \left[0, \overline{k}\right]$, and all $\lambda \in (0,1)$.

Now because $\Omega_{k_i k_i} < 0$, k_i^E must be a local maximum. Because the domain of $\Omega\left(k_i, y_i, y_j\right)$ is compact, there must exist a point $k_i^\lambda = k_i^E + \varepsilon$ in the neighborhood of k_i^E which lies strictly between k_i^E and $\hat{k}_i$ such that $\Omega\left(k_i^E, y_i^E, y_j^E\right) > \Omega\left(k_i^\lambda, y_i^E, y_j^E\right)$. We can express k_i^λ as a linear combination of k_i^E and $\hat{k}_i$ thus; $k_i^\lambda = (1-\lambda)\,k_i^E + \lambda \hat{k}_i$ for some $\lambda \in (0,1)$. Then we have

$$\begin{aligned}\Omega\left(\left((1-\lambda)\,k_i^E + \lambda\hat{k}_i\right), y_i^E, y_j^E\right) &< \Omega\left(k_i^E, y_i^E, y_j^E\right)\\ &= \min\left(\Omega\left(k_i^E, y_i^E, y_j^E\right), \Omega\left(\hat{k}_i, y_i^E, y_j^E\right)\right),\end{aligned}$$

But comparing this expression to the above definition, we see that strict quasi-concavity is violated; replace k_i^E by k_i' and $\hat{k}_i$ by k_i'' and note that the definition holds for all k_i', $k_i'' \in \left[0, \overline{k}\right]$.

(Necessity) Suppose not. (3), (4) and (5) are necessary conditions for a efficient point to exist. If (3), (4) and (5) are not satisfied then $\mathcal{E} = \left\{\left(k_i^E, y_i^E\right), \left(k_j^E, y_j^E\right)\right\}$ cannot be an efficient point. $\square$

Proof of Lemma 2. (Sufficiency). Because $\mathcal{E} = \left\{\left(\overline{k}, y_i^E\right), (0,0)\right\}$ satisfies (6), (7) and (8), we know either that we have a critical point at $k_i = \overline{k}$, $k_j = 0$ or that $\Omega\left(k_i, y_i^E, y_j^E\right)$ is increasing in k_i at $k_i = \overline{k}$.

(Sufficiency) The same as the proof of Lemma 1 up to and including Lemma 1(ii), replacing (3), (4) and (5) by (6), (7) and (8) respectively.

(iii) It remains to show that the point $\mathcal{E}$ is a global maximum; that is $\Omega\left(\overline{k}, y_i^E, y_j^E\right) > \Omega\left(\hat{k}_i, y_i^E, y_j^E\right)$ for any $\hat{k}_i \neq \overline{k}$. This is ensured by the assumed quasi-convexity of $\Omega\left(k_i, y_i, y_j\right)$, as we now explain.

Suppose that there exists a point $\hat{k}_i \neq 0, \overline{k}$ such that $\Omega\left(\hat{k}_i, y_i^E, y_j^E\right) > \Omega\left(\overline{k}, y_i^E, y_j^E\right)$. By definition, quasi-convexity implies

$$\Omega\left(\left((1-\lambda)\,k_i' + \lambda k_i''\right), y_i^E, y_j^E\right) < \max\left(\Omega\left(k_i', y_i^E, y_j^E\right)\right), \Omega\left(k_i'', y_i^E, y_j^E\right)$$

for all k_i', $k_i'' \in \left[0, \overline{k}\right]$, and all $\lambda \in (0,1)$.

Because the domain of $\Omega\left(k_i, y_i, y_j\right)$ is compact, it must be possible to express $\hat{k}_i$ as a linear combination of 0 and $\overline{k}$ thus; $\hat{k}_i = \lambda\overline{k}$ for some $\lambda \in (0,1)$. Then we have

$$\begin{aligned}\Omega\left(\left((1-\lambda)\,0 + \lambda\overline{k}\right), y_i^E, y_j^E\right) &> \Omega\left(\overline{k}, y_i^E, y_j^E\right)\\ &= \max\left(\Omega\left(0, y_i^E, y_j^E\right), \Omega\left(\overline{k}, y_i^E, y_j^E\right)\right),\end{aligned}$$

which violates strict quasi-convexity.

(Necessity) The same as Lemma 1, replacing (3), (4) and (5) by (6), (7) and (8) respectively. $\square$

Proof of Lemma 5. (Sufficiency) We show that if $y_i > 0$ and $y_j = 0$ then $\max_{k_i} \pi_i$ is solved by $k_i^* = \overline{k}$ or $k_i^* = 0$ and $\max_{k_j} \pi_j$ is solved by $k_j^* = 0$. The plan $k_i^* = \overline{k}$, $k_j^* = 0$ satisfies the conditions for capital market clearing.

For the purpose of this proof, define the following function:

$$g(k_i) = \pi(k_i)|_{r=r^*} = f_i(k_i, y_i) - \frac{f_i(\overline{k}, y_i)}{\overline{k}} k_i. \tag{17}$$

This function defines the profit function of the firm in country i when it faces $r^* = f_i(\overline{k}, y_i)/\overline{k} - y_i/\overline{k}$.

Set $y_i > 0$. Suppose it is not the case that $\max_{k_i} \pi_i$ is solved either by $k_i^* = \overline{k}$ or $k_i^* = 0$. Then given $r = r^*$ there must exist some $k_i^* = \hat{k}_i \in (0, \overline{k})$ (ie $\widetilde{k}_i \neq 0, \overline{k}$), such that $\hat{k}_i$ maximizes π_i and $\pi_i \geq 0$. By A1, A2, the function $g : [0, \overline{k}] \to \mathcal{R}$ is continuous and differentiable on $[0, \overline{k}]$, and $g(0) = g(\overline{k}) = 0$. Differentiating once, we have

$$g'(k_i) \geq \frac{\partial f_i(k_i, y_i)}{\partial k_i} - \frac{f_i(\overline{k}, y_i)}{\overline{k}}.$$

Substituting $r^* = f_i(\overline{k}, y_i)/\overline{k} - y_i/\overline{k}$ into $\partial f_i(\overline{k}, y_i)/\partial k_i \geq r + y_i/\overline{k}$, we get

$$\frac{\partial f_i(\overline{k}, y_i)}{\partial k_i} - \frac{f_i(\overline{k}, y_i)}{\overline{k}} \geq 0. \tag{18}$$

At $k_i = \overline{k}$, notice that $g'(\overline{k}) \geq 0$ by (18). This will be used to establish a contradiction.

Differentiating again,

$$g''(k_i) = \frac{\partial^2 f_i(\overline{k}, y_i)}{\partial k_i^2}.$$

Now a necessary condition for $\hat{k}_i$ to be an interior maximum is $g'(\hat{k}_i) = 0$. Because $g(k_i)$ is continuous and $g_i(0) = g(\overline{k}) = 0$, we know by Rolle's Theorem that such a point exists.

Recall by A1 that the production function $f_i(k_i, y_i)$ has a convex segment in the domain $k_i \in [0, \widetilde{k})$ and a concave segment in the domain $k_i \in [\widetilde{k}, \infty]$. By a well known result, the profit maximum must occur for a choice of capital in the concave segment; $\hat{k}_i \in [\widetilde{k}, \infty]$. In the convex segment, it would always be possible to increase revenue per unit of cost by increasing k_i.

Concavity of $f_i(k_i, y_i)$ in the segment $[\widetilde{k}, \infty]$ implies that for $k_i \in [\widetilde{k}, \infty]$, it must be the case that $g''(k_i) = \partial^2 f_i\left(\overline{k}, y_i\right)/\partial k_i^2 < 0$. Given that $\hat{k}_i$ is a maximum, so that $g'\left(\hat{k}_i\right) = 0$, and $g''(k_i) < 0$ on the concave segment, it must be the case that $g'(k_i) < 0$ at $\overline{k}$ because it is to the right of $\hat{k}_i$. This follows by Taylor's formula;

$$g\left(\hat{k}_i\right) - g\left(\overline{k}\right) - \frac{g''\left(\overline{k}\right)\left(\hat{k}_i - \overline{k}\right)^2}{2} = g'\left(\overline{k}\right)\left(\hat{k}_i - \overline{k}\right) + E_n.$$

The left hand side is unambiguously positive. To make the right hand side positive, given that $\left(\hat{k}_i - \overline{k}\right) < 0$, we must have $g'\left(\overline{k}\right) < 0$. But this contradicts the fact that $g'\left(\overline{k}\right) > 0$ by (18). Thus any interior point $\hat{k}_i \in [\widetilde{k}, \overline{k})$ producing a contradiction.

No such contradiction is produced at the point $k_i^* = \overline{k}$ because such a point is consistent with $g'\left(\overline{k}\right) \geq 0$ and $g''(k_i) < 0$. Therefore we must have $k_i^* = \overline{k}$.[22]

Note that $k_i^* = 0$ is also a profit maximizing solution. We have just established that $\pi_i = 0$ at $k_i^* = \overline{k}$ given $r^* = f\left(\overline{k}, y_i\right)/\overline{k} - y_i/\overline{k}$. But it is immediately obvious that $\pi_i = 0$ at $k_i^* = 0$ as well; observe that $f_i(0, y_i) = 0$ and $(r + t_i) \cdot 0 = 0$.

The result that $k_j^* = 0$ when $y_j = 0$ simply follows by assumption; by A2, $f_j(k_j, 0) = 0$ so $\pi_j = -(r + t_j) k_j$, which is maximized at $k_j^* = 0$.

Only the solutions $k_i^* = \overline{k}$, $k_j^* = 0$ satisfy the equilibrium condition $\sum_{i=1,2} k_i^* = \overline{k}$.

(Necessity) We show that if $y_i \ngtr 0$ and $y_j \neq 0$, then no equilibrium exists in the capital market. There are two possibilities that must be considered and ruled out: (i) $y_i = 0$ and $y_j = 0$ and (ii) $y_i > y_j > 0$.

(i) If $y_i = 0$ and $y_j = 0$ then $\max_{k_i} \pi_i$ is solved by $k_i^* = 0$, $\max_{k_j} \pi_j$ is solved by $k_j^* = 0$, and $\sum_{i=1,2} k_i^* = 0$ so we cannot have equilibrium in the capital market.

[22] Note that $k_i^* = 0$ is also a profit maximising solution; at $k_i^* = 0$ or $k_i^* = \overline{k}$, profits are maximised at $\pi_i = 0$. But as we shall see in (ii), with $r^* = f\left(\overline{k}, y_i\right)/\overline{k} - y_i/\overline{k}$ and $y_i > y_j$, the only profit maximising solution for the firm in jurisdiction j is $k_j^* = 0$. And $k_i^* = 0$, $k_j^* = 0$ is not a market clearing solution. Therefore, $k_i^* = \overline{k}$ is the only solution consistent with market clearing under the assumptions of the lemma.

(ii) If $y_i > y_j > 0$ then by A2 and A4, we have that $f_i(k_i, y_i) > f_j(k_j, y_j)$ for $k_i = k_j$. As in the proof of Sufficiency above, given $y_i > 0$ and $r^* = f_i\left(\overline{k}, y_i^E\right)/\overline{k} - y_i^E/\overline{k}$, $\max_{ki} \pi_i$ is solved by $k_i^* = \overline{k}$ and $\pi_i = f_i\left(\overline{k}_i, y_i\right) - (r + t_i)\,\overline{k} = 0$. Then given $f_i(k_i, y_i) > f_j(k_j, y_j)$, we must have $\pi_j < 0$ at $k_j = \overline{k}$. For the same sequence of arguments as in (Sufficiency), there cannot exist an interior maximum. Therefore, the only point at which $\pi_j \geq 0$ is at $k_j^* = 0$ where, by A1, $\pi_j = 0$.

But with $k_j^* = 0$ we cannot have $y_j > 0$ because this violates the government budget condition; $t_j k_j = y_j$. $\square$

(ii) If $y_i > y_j > 0$ then by A2 and A4, we have that $f_i(k_i, y_i) > f_j(k_j, y_j)$ for $k_i = k_j$. As in the proof of Sufficiency above, given $y_i > 0$ and $r^* = f_i\left(\overline{k}, y_i^E\right)/\overline{k} - y_i^E/\overline{k}$, $\max_{ki} \pi_i$ is solved by $k_i^* = \overline{k}$ and $\pi_i = f_i\left(\overline{k}_i, y_i\right) - (r + t_i)\overline{k} = 0$. Then given $f_i(k_i, y_i) > f_j(k_j, y_j)$, we must have $\pi_j < 0$ at $k_j = \overline{k}$. For the same sequence of arguments as in (Sufficiency), there cannot exist an interior maximum. Therefore, the only point at which $\pi_j \geq 0$ is at $k_j^* = 0$ where, by A1, $\pi_j = 0$.

But with $k_j^* = 0$ we cannot have $y_j > 0$ because this violates the government budget condition; $t_j k_j = y_j$. $\square$

Figure 1a

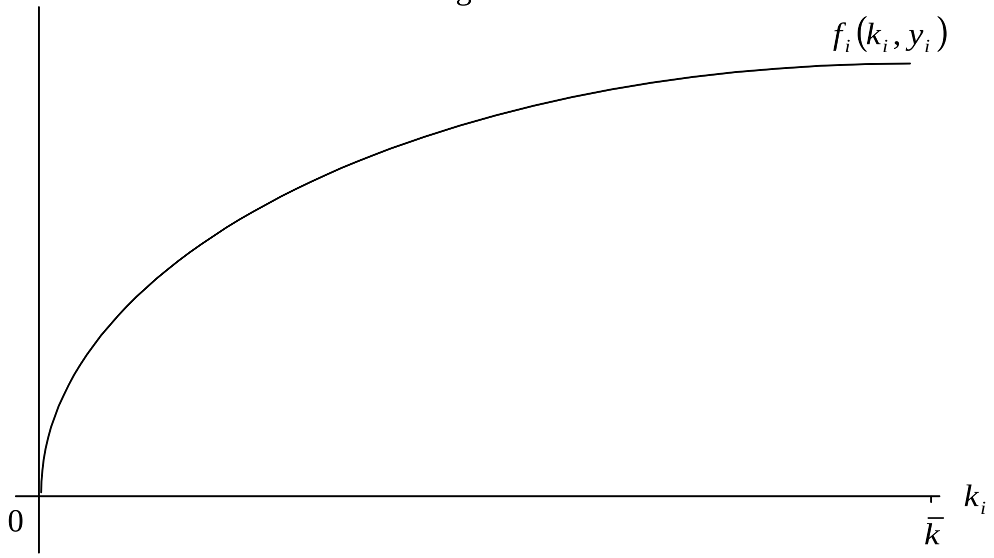


Figure 1b

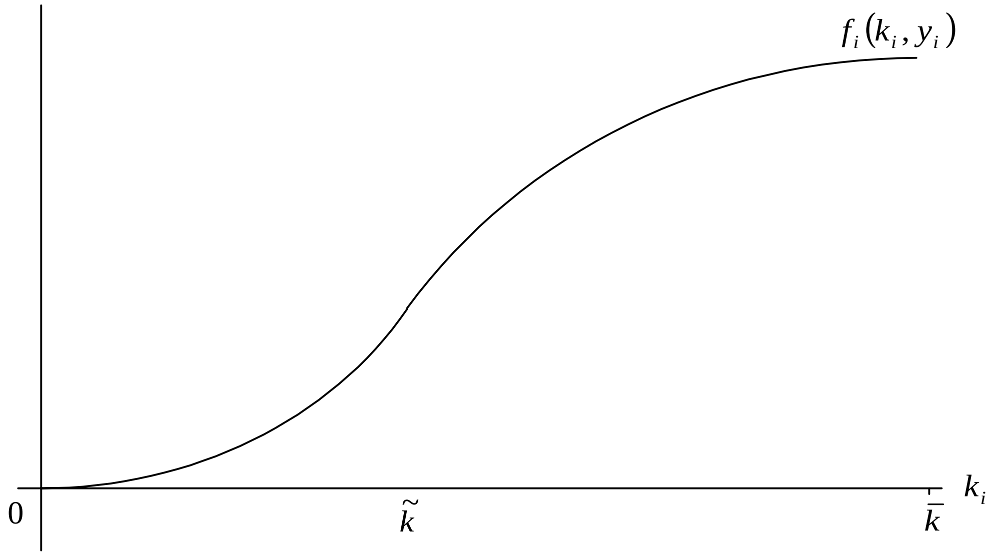


Figure 1c

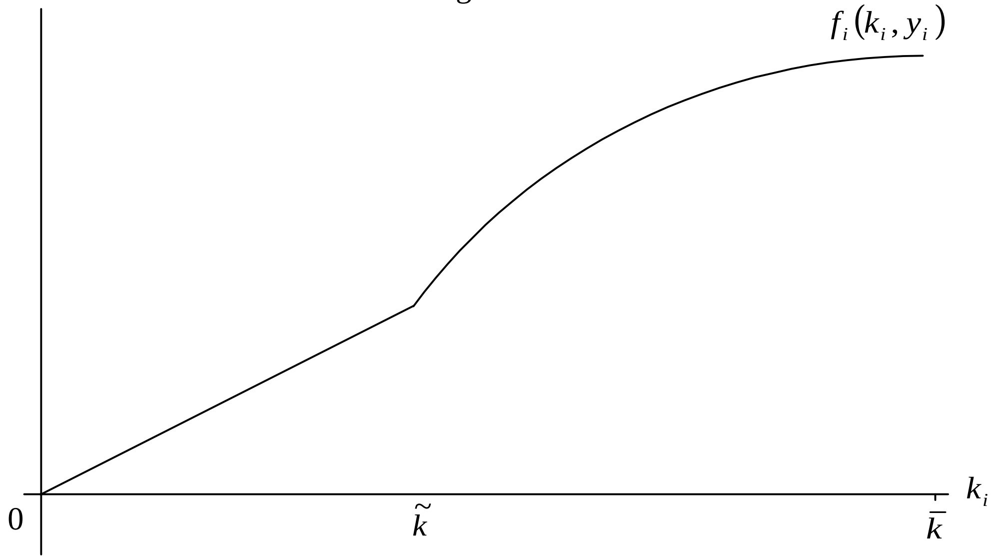


Figure 2

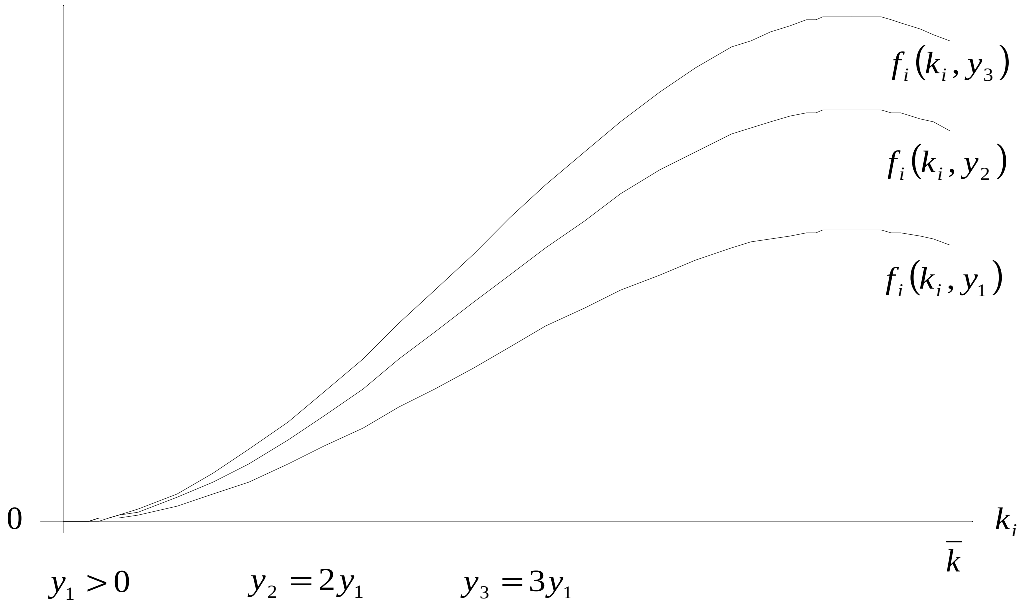


Figure 3a

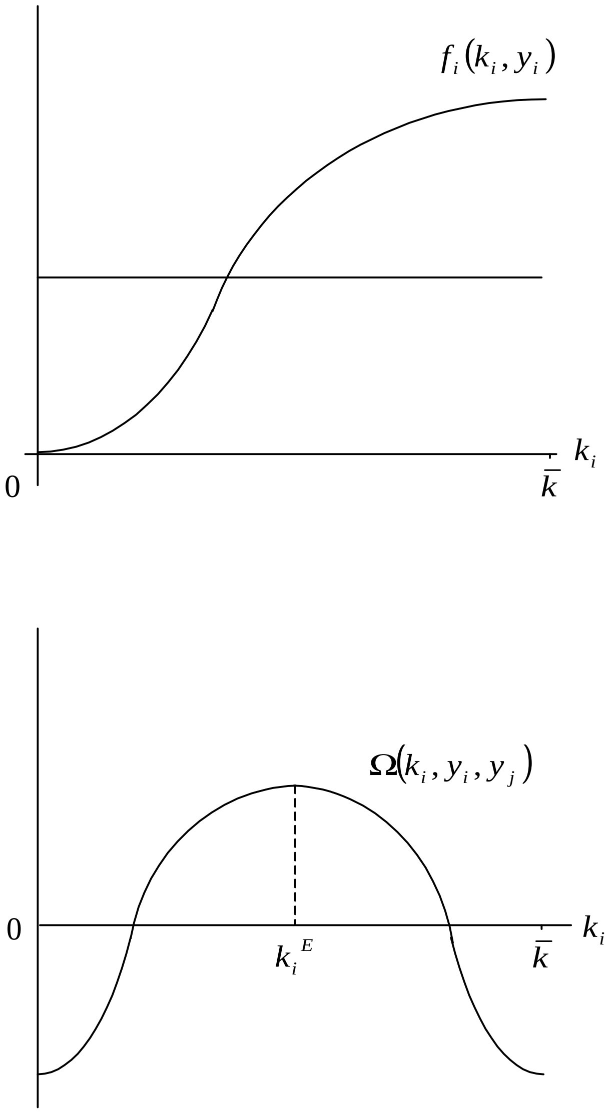


Figure 3b

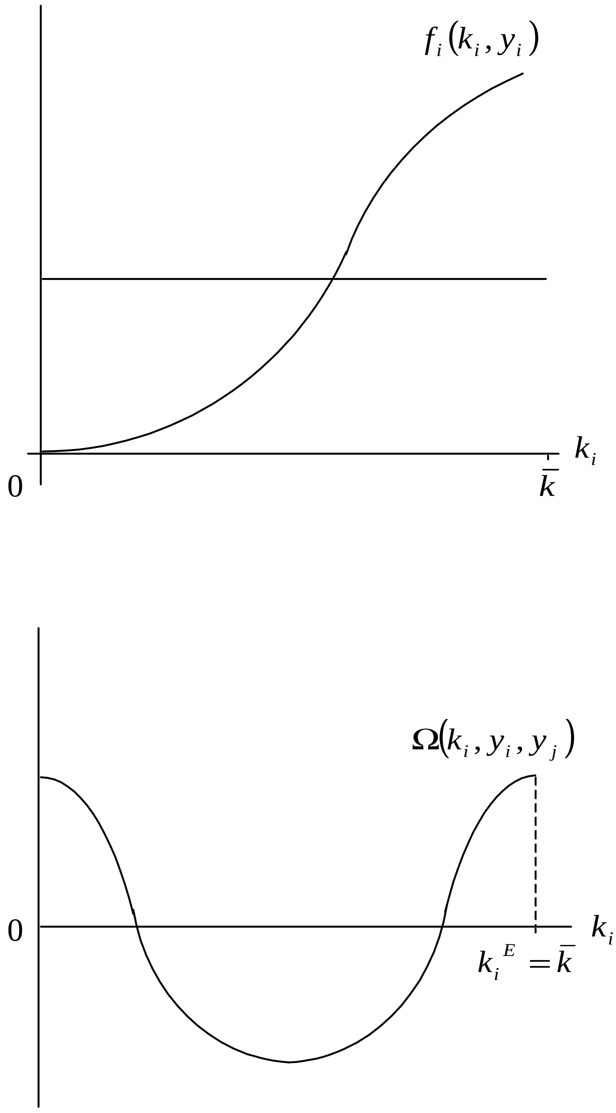


Figure 4a

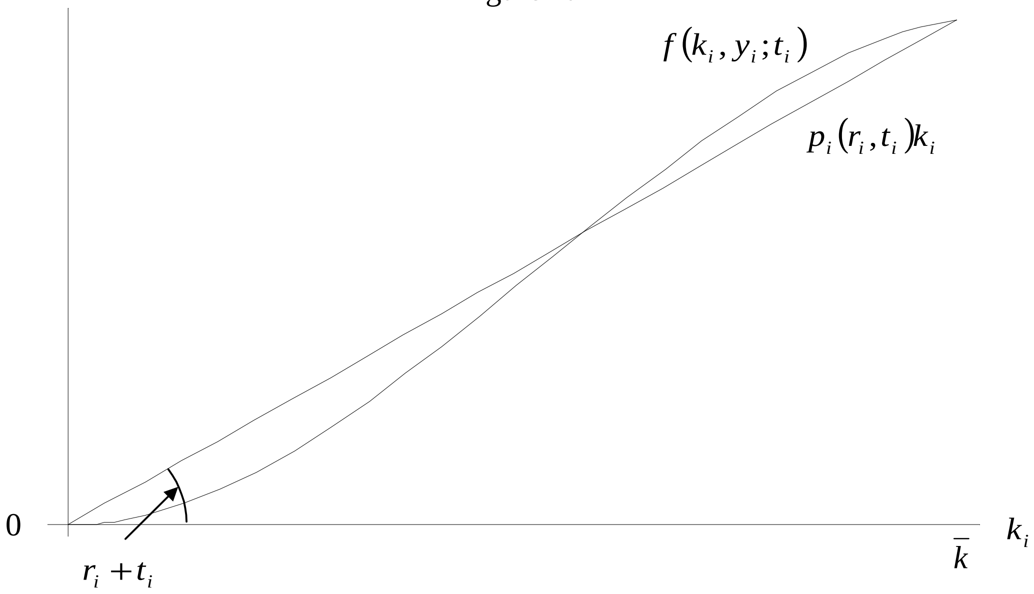


Figure 4b

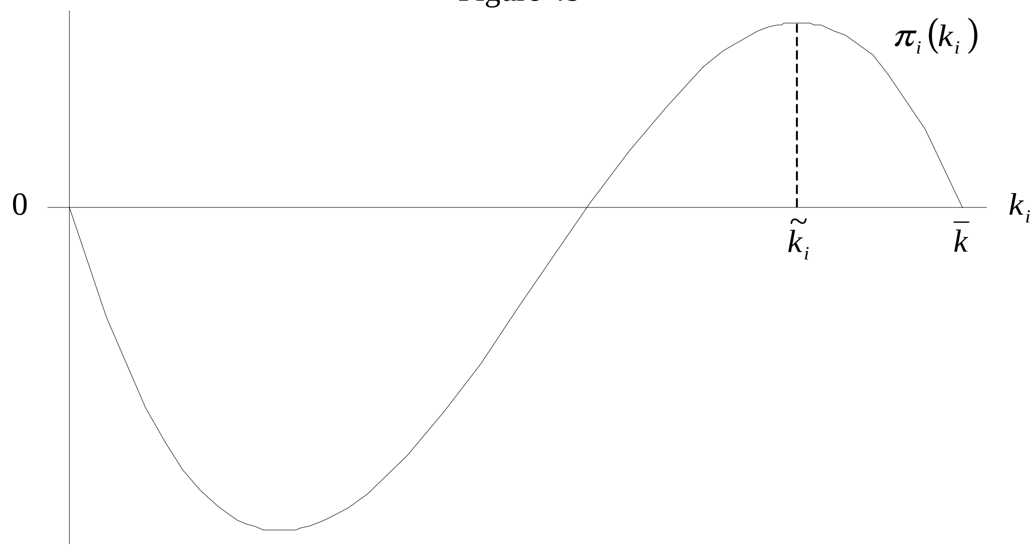


Figure 4c

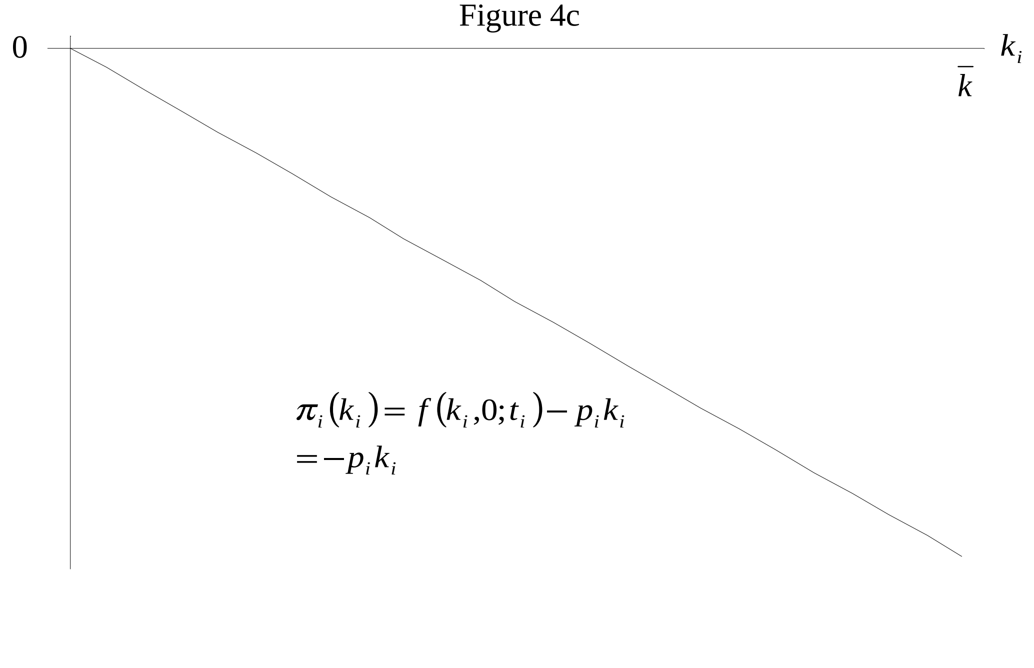
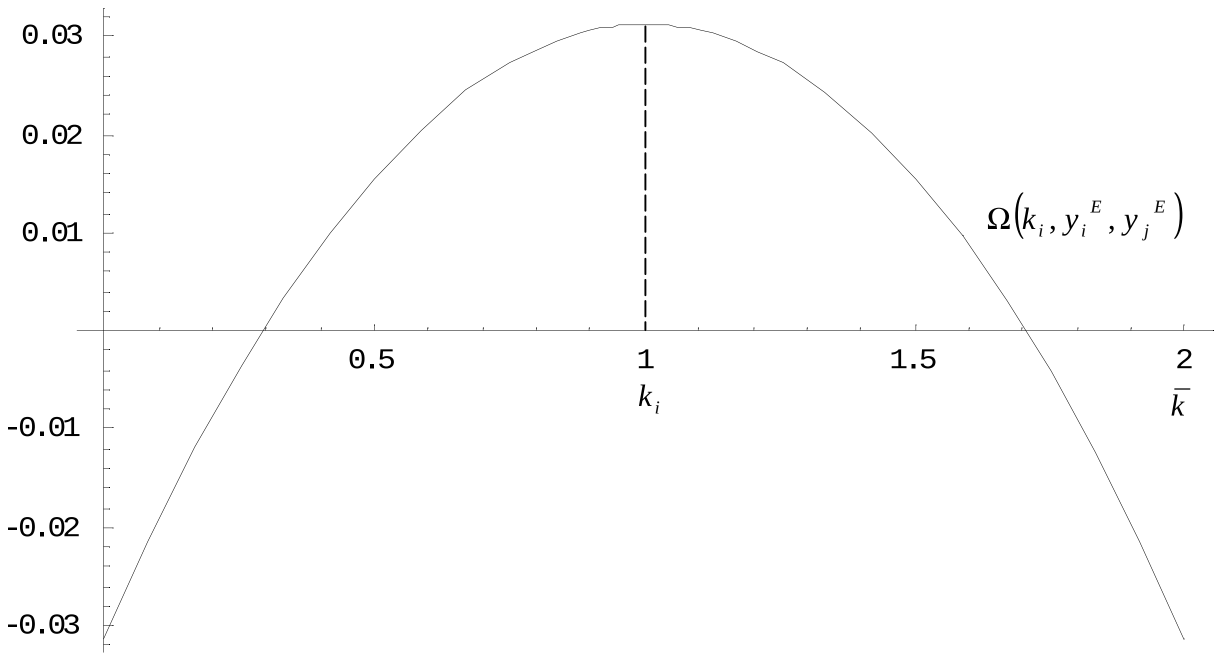


Figure 5



0.03
0.02
0.01
-0.01
-0.02
-0.03
0.5
1
1.5
2
k_i
$\bar{k}$
$\Omega\left(k_i, y_i^E, y_j^E\right)$