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DESIGN AND ANALYSIS OF  
UNBRACED STEEL FRAMES

BY

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## SUMMARY

The thesis examines the behaviour and design of unbraced steel frames with rigid and semi-rigid connections.

An approximate hand method has been developed for the calculation of second-order elasto-plastic failure loads for single storey frames. Studies were carried out to propose limiting values of frame parameters, so that the first-order plastic theory can be used as a safe design method for single storey frames.

A second-order elasto-plastic computer analysis program has been developed. The program takes into account the main non-linear phenomena that occur in real frame structures. These include geometric non-linearity, material non-linearity and, the most important of all, the non-linear connection behaviour. The program can deal with any non-linear moment-rotation characteristic resulting from test data or analytical curves. The analysis program has then been used to check the adequacy of the wind connection design method.

The program for static load collapse was further developed to investigate the response of the structure to cyclic loading. The program was used to investigate the incremental collapse behaviour, including alternating plasticity and shakedown of multistorey frames with rigid and semi-rigid connections.

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NOTATION

|                  |  |
|------------------|--|
| a                | Stiffness parameter. ( $EA/L$ )                        |
| A                | Cross-sectional area                                   |
|                  | Displacement transformation matrix.                    |
| A'               | Transpose of A   |
| b                | Stiffness parameter. ( $12EI/L^3$ )                    |
| c                | Stability function.                                    |
| C                | Constant   |
| d                | Stiffness parameter ( $-6EI/L^2$ )                     |
| D                | Diameter of a bolt                                     |
|                  | Depth of section                                       |
| D'               | Diameter of a bolt hole                                |
| e                | Stiffness parameter ( $4EI/L$ )                        |
| E                | Young's modulus of elasticity                          |
| f                | Stiffness parameter ( $2EI/L$ )                        |
| $F_C$            | Axial force on column                                  |
| $F_{mb}, F_{mc}$ | Capacity of an unstiffened column flange<br>in tension |
| $F_q$            | Shear capacity of web                                  |
| $F_s$            | Shear capacity of a bolt                               |
| $F_t$            | Tensile capacity of a bolt                             |
| $F_{wo}$         | Capacity of web in compression                         |
| h                | Storey height  |
| H                | Horizontal wind load                                   |
| i                | joint number   |

|                 |                                  |
|-----------------|----------------------------------|
| $I_o$           | Second moment area of column     |
| $I_b, I_s$      | Second moment area of beam       |
| $j$             | Joint number                     |
| $k$             | member stiffness matrix          |
| $K$             | Overall stiffness matrix         |
|                 | Secant stiffness                 |
| $K'$            | Stiffness of column ( $I_o/h$ )  |
| $K''$           | Stiffness of beam ( $I_b/h$ )    |
| $L$             | Length of beam                   |
|                 | Load vector                      |
| $L_E$           | Effective length                 |
| $M$             | General term for bending moment. |
| $M_b$           | Buckling moment of resistance    |
| $M_p$           | Plastic moment of resistance     |
| $M_y$           | Yield moment                     |
| $N$             | Axial load                       |
| $P$             | Total vertical load              |
| $P_o$           | Axial capacity                   |
| $P_L$           | Proof load of a bolt             |
| $P_r$           | Factored vertical load           |
| $P_y$           | Yield strength                   |
| $q, q_o$        | Uniformly distributed load       |
| $s$             | Stability function               |
| $S_1, S_2, S_3$ | Statical wind load coefficients  |
| $t$             | Thickness of web                 |

|                 |  |
|-----------------|--|
| $t_p$           | Thickness of end plate                   |
| $T$             | Thickness of flange                      |
| $V, W$          | Vertical loading                         |
| $x$             | x-direction of overall coordinate system |
| $X$             | Joint displacement matrix                |
| $y$             | y-direction of overall coordinate system |
| $\alpha$        | Load factor                              |
| $\alpha_a$      | Alternate plasticity load factor         |
| $\alpha_f$      | Elastic-plastic failure load             |
| $\alpha_i$      | Incremental collapse load factor         |
| $\alpha_p$      | Rigid-plastic failure load               |
| $\alpha_s$      | Shakedown load factor                    |
| $\alpha_{cr}$   | Elastic critical load                    |
| $\delta$        | Displacement                             |
| $\theta$        | Joint-rotation                           |
| $\theta_h$      | Hinge rotation                           |
| $\theta_{sr}$   | Semi-rigid connection rotation           |
| $\phi$          | Rotation at connection                   |
| $\phi_2-\phi_3$ | Stability functions                      |
| $\phi_r$        | Slope of storey                          |



## CHAPTER ONE

### INTRODUCTION

One of the most commonly used structural systems in modern construction is the framework. These structures are required to support load and to transfer such loads to the foundations. Because of their importance, the study of the behaviour of different types of framework has been the subject of research for decades.

Frame response can be evaluated by any of conventional structural analysis techniques. With the aid of a desk top computer, today there is no doubt that analysis and design have become more sophisticated. This has lead to the development of suitable computational methods to assess more accurately the overall behaviour of the structure from onset of loading to collapse.

In the conventional analysis and design of steel frameworks, the frames are treated under the simplification that connections behave either as ideally pinned or fully rigid. Although the use of these idealized joint behaviours simplifies drastically the analysis and design procedures, the predicted response of the frame may not be realistic as most connections used in actual practice transmit some moment and experience some deformation upon loading. Realizing the importance of the connection flexibility, there is need for analysis

of frameworks with flexible connections. Therefore the major part of this thesis concerns the development of computer analysis programs to permit the effect of joint flexibility to be included in the analysis.

In order to proceed with this research, it is necessary to review some of the methods of structural analysis carried out by other researchers.

### 1.1 Structural analysis.

The accurate analysis of statically indeterminate structures is complicated by the interaction between members. The equilibrium and compatibility conditions must both be used in determining the member forces and moments. Conventional structural analysis of frames is usually carried out by one of the following methods with the usual assumption that the connections joining the beams to the columns are either fully rigid or pinned.

- a) First order elastic analysis.
- b) Second order elastic analysis.
- c) First order, rigid plastic analysis.
- d) First order, elastic-plastic analysis.
- e) Second order elastic-plastic analysis.

Figure (1.1) shows the load-deformation, behaviour of an unbraced frame. Curve(1) is linear because linear elastic response is assumed in the analysis. Frame response can be evaluated by any of the structural analysis techniques such as moment distribution by Cross [1], slope deflection or matrix methods [2,3,4,5].

The second order elastic load deformation curve(2) is generated by including the reduction in the frame stiffness due to compressive axial load, the so-called 'P- $\delta$ ' effect.

Because of the complex nature of the problem, it is realised that the use of a computer and a modern analysis technique such as the stiffness matrix are unavoidable. As the axial forces in the members are themselves unknown and cannot be included initially in the derivation of the equilibrium equations, therefore an iterative solution technique is usually employed [6,7,8,9].

Livesley [10] developed a program with the option of including second order effects due to axial load. These were allowed for by using stability functions. These functions depend on the ratio of axial force to Euler load of a member, and the particular functions used by Livesley have the value unity for zero load.

In contrast to elastic methods, plastic design methods for rigidly jointed steel frames have been advocated by Baker [11]. In that paper, Baker pointed out that the plastic method of design was more rational

than elastic methods and would in consequence lead to more economic use of material. Since then, significant contributions have been made to the plastic theory of structural analysis [12-19].

Referring to figure (1.1), the rigid plastic analysis is shown, once again neglecting axial load, by the vertical axis until suddenly collapse occurs at  $\alpha_p$  (curve 3). The effect of deformation and instability effects limit the use of rigid plastic design to only two storeys to ensure that the frame is reasonably stiff. This fact is emphasised by AISC [20].

The first-order elastic-plastic hinge curve is a series of straight lines as presented by curve 4 of figure (1.1). Each hollow square on the curve represents the formation of a plastic hinge. Between successive plastic hinge formations, a first order elastic analysis is performed on the frame modified for the presence of plastic hinges [21,22]. It can be seen from the figure (1.1) that this curve meets the first order rigid plastic curve at the point of plastic collapse.

Finally second-order elasto-plastic analysis is considered. This is the method adopted for the analysis in this thesis. Generally two types of computerised second order elasto-plastic analysis have been developed, the main difference being the way that the stiffnesses of individual members are evaluated.

The first type uses the plastic hinge idealization and assumes that the member stiffness changes abruptly at the formation of each hinge

[23]. In the second type, yielding is assumed to develop gradually and to spread continuously in the most highly stressed regions of each member. This spread of yielding is taken into account in evaluating the member stiffness. The first approach is simpler and capable of handling large frames. The second approach is more accurate but requires a large amount of computing effort, particularly for tall or complex frames.

The second-order elastic-plastic curve is illustrated by curve (5) in figure (1.1). It is obtained in a similar manner as the first-order elastic-plastic hinge curve except that the P- $\delta$  effect is taken into account. Unless member stability (eg. lateral torsional buckling, local buckling) governs the limit state, the second order elastic plastic curve will give a very good approximation of the true behaviour of the frame.

#### 1.1.1 Computer analysis programs.

With the advent of computers there has been a greater tendency to develop programs on the basis of the systematic stiffness matrix method of structural analysis, rather than other methods.

Jennings and Majid [21] used the matrix displacement method in which unknown joint displacements are obtained by solving the matrix equation:

$$L=K.X$$

where  $L$  is the external load matrix,  $K$  is the overall stiffness matrix derived from the slope deflection equations and  $X$  is the joint displacement matrix. Member forces are then calculated using these joint displacements. In nonlinear analysis, the effect of the reduction of member stiffness due to axial load in the member is obtained by using the stability functions introduced by Livesley [10].

Jennings [24] developed a compact method for the storage and solution of stiffness equations. The method stores all the elements below the leading diagonal in sequence by rows, but with all elements preceding the first non-zero element in each row left out.

The second-order elastic-plastic analysis of large multi-storey frames was carried out by Majid and Anderson [23], by moving from one hinge to another as plastic hinges formed. Once a hinge is detected and inserted in the frame, the load factor at which the next plastic hinge would form is estimated and that load applied to the frame.

The effect of the reduction in plastic moment of resistance  $M_p$  due to presence of axial load is significant in the frames with high axial load. In the approach by Majid and Anderson, the lowest load factor at which the bending moment at the end of a member reaches its reduced plastic moment is found by extrapolation of the axial force and the corresponding reduced plastic moment,  $M_p'$ , by a process of iteration.

Majid and Anderson later [23] incorporated design features into their second order analysis program.

Korn and Galambos [26] made use of previously documented frame analysis, to study eight storey unbraced planar frames to compare the results of first and second order elastic-plastic analysis.

It was concluded that first order analysis is not adequate for the prediction of maximum deformation except when the failure mechanism requires few plastic hinges, i.e. a localized mechanism such as a beam, joint or a single storey sway mechanism, with the maximum sway being relatively small.

All the analytical procedures mentioned above are based on the small deformation theory, implying small member chord deformation. The problem of large deformations in elastic-plastic frames was studied by Kassimail [27]. Numerical solutions were reported for three structures and compared with the technique mentioned earlier [9,26]. It was concluded that the second order elastic-plastic analysis based on small deformation theory will give satisfactory results for rectangular plane frames. Therefore there is no requirement for the consideration of large deformations and flexural bowing effects which would complicate the problem of nonlinear analysis of a frame considerably.

## 1.2 Approximate method of determining the failure load.

Now-a-days, computer facilities and software are available in almost all design offices, but most of this software is written for the elastic method of analysis. Therefore an approximate method of deter-

mining the failure load,  $\alpha_f$ , is an attractive alternative. Such an approach may also be used to check computer methods and satisfy engineers who wish to maintain full control of the design process.

The Merchant- Rankine formula provides the most important approximate method of estimating the load factor at failure. It was suggested by Merchant [28] on a purely empirical basis and has the form:

$$\frac{1}{\alpha_f} = \frac{1}{\alpha_p} + \frac{1}{\alpha_{er}}$$

Or:

1.2

$$\alpha_f = \frac{\alpha_p}{1 + \left(\frac{\alpha_p}{\alpha_{er}}\right)}$$

The formula will give reasonable approximation to the failure load, since the frame collapse is by an interaction of plasticity and elastic instability. Much later [29] Horne showed that the Merchant-Rankine formula had a theoretical basis provided that the plastic collapse mechanism and the lowest buckling mode had similar deflected shapes. Further justification of Horne's conclusion are provided by tests on three, five and seven storey miniature frames by Low [30].

Wood [31] recognised the generally conservative results given by the Merchant-Rankine formula for bare frames and has suggested a modified



version, to account for the beneficial effect of strain hardening and minimal composite action. Wood's modification to the Merchant - Rankine formula is as follows:

$$\text{If } \frac{\alpha_{cr}}{\alpha_p} > 10 \quad \alpha_f = \alpha_p$$

1.3

$$\text{If } 4 \leq \frac{\alpha_{cr}}{\alpha_p} \leq 10 \quad \alpha_f = \frac{\alpha_p}{0.9 + \left(\frac{\alpha_p}{\alpha_{cr}}\right)}$$

More recently, Scholz [32-33] proposed an approximate method which relies on iteration between the rigid plastic collapse load,  $\alpha_p$  and the elastic critical load  $\alpha_{cr}$ . The basis of the method is the equivalent "limiting frame"; each group of "limiting frames" is identified by a common curve which relates the rigid-plastic collapse,  $\alpha_p$ , and the elastic critical load,  $\alpha_{cr}$  to the failure load,  $\alpha_f$ . Consequently, a family of curves for different groups of frames can be related to the two parameters  $\alpha_{cr}$  and  $\alpha_p$ .

All the above methods require the evaluation of the elastic critical load as well as plastic collapse load. While approximate methods are available for the calculations of elastic critical load, generally the calculation of  $\alpha_{cr}$ , whether carried out by hand or by computer, is considerably more difficult than, for example, a linear elastic analysis. This leads to a tendency of establishing some rules by which simple plastic analysis can be carried out safely in the absence of sophisticated second order analysis.

One of the simple rules is the formula given in the British code, BS5950 [34], which allows frames with pinned bases to be designed by first-order rigid-plastic theory if certain criteria are met.

ECCS design recommendations [35] based on work due to Rubin [36] also contain some formulae suitable for the hand calculation to find the limits within which the rigid-plastic analysis is allowed

Preliminary studies by Reynolds [37] suggested that the above recommendation is very conservative and effectively prevents plastic design.

It is clear that new recommendations are necessary to obtain the collapse load factor by manual calculation, particularly for single storey frames.

### 1.3 Semi-rigid connections.

Joint flexibility has long been recognized as an important parameter influencing frame behaviour. By utilizing the inherent strength and stiffness of connections without stiffening, more economy may be achieved in connection cost by reducing the fabrication time. Beyond the possible economical benefit, a design philosophy which recognizes the effect of joint flexibility enables more realistic evaluation of the behaviour and, therefore, of the reliability of structure.

Having recognized the importance of semi-rigid connections, in the past fifty years several hundred tests have been conducted on various types of steelwork connections. In parallel, theoretical studies have sought to model the main structural action present in these connections. The advent of powerful computers, coupled with the development of sophisticated testing and data acquisition equipment have highlighted the subject.

Aware of this problem, Technical Group 8.2 of the European Convention for Constructional Steelwork decided in 1984 to establish a Task Group with the aim of preparing a reference document for the designer.

The review of research carried out on semi-rigid frames with reference to the following topics will give the background necessary to current research work presented in this thesis:

- a) Behaviour and modelling of connections.
- b) Analysis of frames with semi-rigid connections with reference to the following:
  - (i) Linear-elastic analysis.
  - (ii) Nonlinear elastic analysis.
  - (iii) Inelastic analysis.
- c) Design of frames with semi-rigid connections:
  - (i) Braced frames.
  - (ii) Unbraced frames.

### 1.3.1 Joint behaviour and representation.

The behaviour of the connections is certainly a basic need for the analysis of flexibly connected frames. Large amounts of experimental work have been carried out to investigate the connections behaviour. A particularly important factor in the development of a full understanding of the behaviour of the frame is an appreciation of the way in which the connections operate. Recent appraisal [38-39] of the experimental data base covering in-plane behaviour of beam-to-column connections has highlighted the semi-rigid nature of virtually all commonly used types. In addition a bibliography mounted on IBM P.C. disc has been prepared by the Structural Stability Research Council [40].

Tests on beam-to-column connections indicate that when moment is transferred through the connection, deformation occurs in the connection material which results in a relative motion between the beam and the column to which it is connected. Tests were also shown that the predominant movement is the rotation of the beam end relative to the column as shown in figure (1.2). Thus, when moment is applied to the connection, the centre line of the beam does not remain perpendicular to the centre line of the column (as presumed in rigid frame analysis), rather an angular rotation,  $\theta_r$ , occurs due to the flexibility of the connection.

Figure (1.3) illustrates some types of the beam to column connections which are mainly used in practice. It can be seen from figure (1.3),

that single web angle connection represents a very flexible connection and the T-stub represents a rather rigid connection. Several observations can be made from this figure:

- a) All types of connection exhibit a behaviour that falls between the extreme cases of ideally pinned (the horizontal axis) and fully rigid (the vertical axis).
- b) For the same moment, the more flexible the connection is, the larger value of  $\theta$ . Conversely, for a specific value of  $\theta$ , a more flexible connection will transmit less moment. The ultimate moment capacity decreases with more flexible connections.
- c) The  $M-\theta$  relationship for the semi-rigid connections are nonlinear over the entire range of loading.

The non-linear behaviour of connections was first recognised by Pipard and Baker [41] following their full scale studies on the building of the Cumberland Hotel in London, and a building at Imperial College, London, during 1930s. It was difficult to incorporate this effect into the then-used design techniques.

Since the observation of Pipard and Baker [41], several hundred tests have been conducted on beam-to-column connections, but for most of the more popular forms: web-cleat, flush end plate, extended end plates. However, There are very limited numbers of tests available on minor axis beam-column connections [42].

In order to include semi-rigid joint effects in frame analysis it is necessary to represent the connection's  $M-\phi$  test data in a convenient form. One approach is the use of initial connection stiffness; this approach was mainly used at early stages of study on semi-rigid connections (pre-computer). Recently, an analytical procedure was developed by Aziznamini and et al [43] to predict the initial stiffness of a particular type of semi-rigid connection.

To obtain the nonlinear  $M-\phi$  relationship, the simplest approach is to employ curve fitting to experimental data. Various nonlinear connection models had been obtained by this method [44,45,46, 47,48,49].

The most significant development in modelling  $M-\phi$  curves was the contribution of Frye and Morris [44] who first suggested the use of polynomials and also employed curve-fitting techniques to obtain best fit solutions.

Analytical difficulties associated with negative slopes of polynomial curves can occur at some value of  $M$ , which is physically unacceptable. In addition, it may cause numerical difficulties in the analysis of a structural frame using tangent stiffness formulation [49]. But it appears at this time that the use of this technique provides the best tool for prediction of the response of a wide variety of connection types to monotonic loadings.

The Jones - Kirby - Nethercot model [45] divided the experimental data for  $M-\phi$  into a number of subsets, each spanning a small range of  $M$ . A

Cubic B-Spline curve is then used to fit each and every subset of data with continuities of first and second derivatives enforced at their intersections. This model was reported to circumvent the problem of negative stiffness and represents the non-linear behaviour extremely well [45]. However, a large amount of data is required in this curve-fitting process.

In EC3 [97] the behaviour of beam-to-column connections is represented by tri-linear moment-rotation characteristics as shown in fig. (1.4). This can be used in conjunction with joint classification for both unbraced and braced frames. It can be seen from this figure, if the moment rotation characteristic lies above solid line it will be considered to be rigid and if it is below the line, it is semirigid.

In addition to the connection models described above, some progress has been made in devising analytical models to represents joint flexibility in a physical manner (50,51,52). Using analytical models makes it possible to dispense with much costly and inconvenient testing and to use this approach to actually generate curves, providing use is restricted to those areas where it is known to provide a good estimate of actual behaviour.

### 1.3.2 Analysis.

The behaviour and modelling of connections has been discussed in the proceeding section. This behaviour can be incorporated into the following analytical techniques.

#### 1.3.2.1 Linear elastic analysis

In this analysis linear behaviour of material and connections are assumed. Therefore no iteration would be necessary and this makes the approach very convenient. Interest in this method of analysis was first shown sixty years ago [53]. This was followed by more comprehensive and refined methods by other researchers [54,55,56,57].

The linear elastic analysis is only an acceptable tool of analysis for very low value of displacement. In particular in unbraced frames the lateral deflection will be increased considerably by joint flexibility and second order effects and joint nonlinearity may become non-negligible.

#### 1.3.2.2 Nonlinear elastic analysis.

In this analysis, the nonlinear  $M-\phi$  behaviour of the connections as well as geometrical nonlinearities of the framed structure are accounted for. The methods already presented are easily extended to allow for the influence of deformation on the equilibrium of the frame, by using the techniques well established in structural analysis.

Frye and Morris [44] presented an iterative analysis procedure for planar, rectangular steel frames incorporating nonlinearity of beam-to-column connection. The analysis procedure involved repeated



cycles of linear analysis, to determine a set of connection stiffness, that could be used to predict the displacement and internal forces in the real (nonlinear) structure. Later Ang and Morris [47] generalized the Frye and Morris procedure to permit the analysis of three-dimensional rectangular frames with nonlinear flexible connections.

Both the Frye and Morris and Ang and Morris procedures assumed proportional loading. Thus, they did not permit "unloading" of any connection.

When joint flexibility is incorporated in a matrix displacement method of analysis, the size of the stiffness matrix increases in consequence. In a technique proposed by Anderson and Lok [58], the deformations of the joint were allowed for by revising the load vector at each iteration, before solving the simultaneous equations. Although the approach made possible substantial saving in storage and computer time, convergence problems were experienced by the present author for frames which did not have very stiff connections. The program was later modified by Anderson - Benterkia [59] to use successive estimates of the secant stiffness of each connection to ease the convergence problem.

#### 1.3.2.3 Elastic-plastic analysis

In this analysis, the yielding of the beam and column element is considered. The analysis leads to a more realistic behaviour of the

frame, because, consideration for both material and geometric nonlinearities of members and connections are taken into account. This requires highly sophisticated numerical approaches.

Because of the non-linear behaviour of the connection, an iterative analysis procedure is required. Its basis is that the correct structural deflection and internal forces can be obtained from the analysis, provided the correct stiffness is assumed for each connection.

Poggi [60] developed an elastic-plastic finite element beam model, which incorporates joint flexibility. Elements used by Poggi consist of three parts: central elastic-plastic beam, two rigid bars at ends and a set of nonlinear springs (of null length) between each rigid bar and the beam. Joint behaviour is incorporated by the action of these springs, one for each potential deformation, axial, shear and rotation which follow linearized representation of force deformation relationship. The program was used at Sheffield University [61] and good agreement was reported between analysis and experimental results.

Ackroyd and Gerstle [62] described a computer program which accounted for both material and connection nonlinearities. The program uses secant stiffness for all elements and performs repeated cycles of linear analysis to establish the ultimate strength of the frame under proportionately increasing load. Difficulties were reported in the convergence of non-linear connections approaching the collapse load.

Ohta [63] describes the use of one dimensional finite element to represent the behaviour of semi-rigid connections in the analysis of steel frames. Difficulties were associated with his program when analysing a large frame.

A more comprehensive review of existing methods on the structural analysis with the joint flexibility is given in IABSE Surveys [99]

Various approaches are available for the analysis of flexibly connected frames, it is quite difficult to check the accuracy of each different method, and in particular of the different joint model assumed.

#### 1.3.2.4 Iterative analysis procedure.

Generally there are three stiffness values can be used with any moment rotation curve for the analysis described in preceding sections;

i) The initial stiffness.

ii) The tangent stiffness at any point.

iii) The secant stiffness.

#### 1.3.2.4.1 Initial stiffness.

Initial stiffness may only be assumed when a linear moment-rotation relationship exists. Experiments have clearly shown that moment rotation curves were non-linear over the whole range for the most types of connection. This assumption makes the method only strictly applicable for the very low value of rotation. This method was mainly used during early, pre-computer investigations of semi-rigid connection analysis, or when the final moment in the connection falls within the initial portion of the moment-rotation diagram. The method would not require an iterative approach.

Use of the initial stiffness leads to over-estimation of the stability of the frames and also the deflection will be erroneous.

#### 1.3.2.4.2 Tangent stiffness.

Many other investigators (21,57,96) perform stiffness calculations using a tangent stiffness formulation. This method uses the last obtained values of moments to find an appropriate tangent stiffness, and then iterates on the tangent stiffness until acceptable moment tolerance is met on the current load step. Thus, while the local error on any particular step can be controlled, the small acceptable error in one step is propagated through all subsequent steps and control of total error is impossible. Consequently, the tangent stiffness formu-

lation is prone to accumulation of sizeable error, unless the step size is kept very small, in which case the tangent stiffness approach becomes time consuming.

#### 1.3.2.4.3 Secant stiffness.

If the maximum load-carrying capacity of a frame is required, it is desirable to keep the total error to a minimum in the vicinity of the collapse load, i.e., near the end of the loading process. Another reason for using the secant stiffness is, the secant stiffness provides an integrated average of how the connections arrive at the present level of loading.

#### 1.3.3 Design of semi-rigid connection frames.

Present practice in both elastic and plastic design of frames often results in uneconomical structures. This is because both methods require either fully rigid connections or in the case of simple construction some sort of bracing to be provided. Using rigid connections leads in most cases to fully stiffened connections which are expensive. Columns must be designed to resist moment due to gravity load arising at the ends of the beams.

The use of bolted beam-to-column connections without using stiffeners leads to semi-rigidity and partial strength. The use of this type of connection gives the opportunity to optimize cost for beam and column.

The problem which has limited the use of semi-rigid design in practice is the lack of proper guidelines for design procedure in codes of practice eg. [34].

In the United states with the recent publication of the current limit design specification, [64] (Load Resistance Factor Design) use of semi-rigid connections has been recognized. Through the LRFD specification, the designer has the guide lines to produce designs that employ semi-rigid connections [73]. However, frames using the actual moment rotation curves for the connections have not yet been designed in great number in the United States. Thus a limited performance base is available to the profession.

Having recognized the importance of design with semi-rigid connections a brief review on design of braced and unbraced frames will be discussed in the next subsection.

#### 1.3.3.1 Braced frames

Braced frames are defined in accordance with Eurocode 3 [65] and ref [66] as frames that are laterally supported by stiff elements like bracing, shear walls etc. For frames to be classified as braced the shear stiffness of the support should be at least five times the shear stiffness of the frame which has to be supported. If this criterion is fulfilled, all horizontal forces, including those arising from imperfections and second order effects, shall be considered to be transmitted by the bracing element.

In general the design of braced frames with semi-rigid connections, can be based on elastic theory or, alternatively, on plastic theory.

There are two approaches to elastic design of braced frames. Firstly, an approximate design by limited distribution of moment is already outlined in BS5950: Part 1, by applying 10% of the free moment as an end restraint moment. Adopting this procedure, the connections are then designed to transmit the end restraint moment, as well as end reaction from the beam. This requirement increases the complexity of the calculations and may lead to larger sizes of components within the connection. A reduction of only 10% in the bending moment in the beam is not sufficient justification for the use of this method, particularly when there are possible disadvantages involved in the connection design.

A second design approach, based on  $M-\phi$  curves will result in closer representation of the real behaviour and greater economy in beam design. When designing the beam with the end restraint moment resulting from the  $M-\phi$  relation, the end moment is normally more than 10% of free moment. The designer can retain the same joints as those required by "simple" design, at no extra cost.

However design methods using moment-rotation characteristics require reliable information concerning these characteristics and specialised analysis procedures.

In order to check the column stability in the braced frame with semi-rigid connections, the effective length of the column must first be determined. The current state of research into this subject has been summarised in the report on column stability by Nethercot [67], using proposals by Sugimoto and Chen [68] and Galambos [69]. It is possible to assess the effective length factor for the column about the major axis. However, there is lack of information for minor axis buckling, although recent work by Kim [42] suggested a lower effective length factor for the column about the minor axis than those suggested by BS5950, i.e. less than 0.85.

Plastic design can be referred to as strong-column, weak beam design. The beams are designed by rigid plastic theory (beam mechanism). A mechanism is formed with a plastic hinge at the mid-span of the beam and plastic hinges at the end supports of the beam. The plastic hinge will form either in the connection or in the beam alongside the connection, depending on the relative values of moment capacity, whichever is smaller. The columns are designed in such a way that they do not collapse prior to formation of a beam mechanism. In knee connections, though, it is possible that a plastic hinge may form in the column when the reduced moment capacity of column is smaller than moment capacity of beam [70].

When a beam mechanism occurs, the redistribution of moments is necessary. Redistribution of moments can only occur if the components that



yield first have sufficient deformation capacity . In many cases this deformation capacity has be provided by the semi-rigid partial strength connections [71].

Columns must be checked using interaction formula [65] .When there is a plastic hinge in a beam near the column end, the effective length factor should be taken as unity.

#### 1.3.3.2 Unbraced frames.

Frames in which overall stability and resistance to lateral sway is provided by bending stiffness of the frame are classified as unbraced. Designing this type of frame normally results in a more expensive structure than for a braced frame. But in certain instances, bracing in exterior walls cannot be arranged, although normally masonry walls around a stair-well may be considered sufficiently permanent to resist lateral forces.

In Britain and North America it is common to use the "Wind connection" method of design. In this method, connection stiffness is ignored for the gravity load case i.e. beams are designed for the simply supported condition, but its presence is recognised when considering wind loads.

A full description of historical development of the wind connection method has been provided by McGuire [72].

Extensive study has been made by Ackroyd and Gerstle [74], using frames which originated from usual office practice using the AISC version of the method, referred to as "Type 2 Construction" [75]. For comparison the frames were also designed to take account of the behaviour of semi-rigid connections under both gravity and wind loading, referred as "Type 3 Construction" in AISC Specification [76]. Exact analyses of the above frames were carried out at working load level. The method does not include the P- $\delta$  effect in the analysis which becomes very significant in tall buildings, when the axial load is high.

More recently Nethercot [77] and Gerstle [78] summarised some aspects of Type 2 construction with the reference to the previous research carried out on subject, except the most recent studies [79,80]. The general conclusions were that the beams are overdesigned and column and connections are underdesigned, in comparison with the rigid analysis.

#### 1.4 Variable repeated loading.

The behaviour of structures beyond the elastic limit under proportional loading has been the subject of many investigations. While the ability of a structure to withstand constant load will normally ensure satisfactory behaviour at the working level, it may also be necessary to check the performance of the structure with respect to excessive deflections and repeated loading.

In general there are two ways in which failure can occur due to variable repeated loading. The first possibility is known as "alternating plasticity" which may be set up in one or more members of the structure when bent back and forth so that yield occurs in tension and compression. The behaviour may eventually lead of failure by low endurance fatigue.

The second possibility is that the structure may fail by incremental collapse. This occurs when critical combinations of loads follow one another in fairly definite cycles. Then at a load level above the shakedown load, the structure may be rendered useless by the progressive development of excessive deflections.

Clearly any appreciation of the problem of repeated loading of a structure must depend upon a knowledge of the conditions under which a structure may be expected to shakedown, when under subsequent load applications the changes in bending moment are completely elastic. The shakedown theorem was first stated by Bleich [81], but his proof only covered frames with not more than two redundancies. A more general solution was given by Melan [82] for hypothetical pin-jointed trusses, assuming ideal plastic member behaviour in both tension and compression. Melan's proof has been adopted by Neal [83,84] to the cases of frames whose members possess the ideal elastic-plastic bending moment curvature .

Neal extended the shake down theorem to cover the more realistic type of bending moment-curvature relationship which assumes that the elas-

tic range of bending moment remains limited to the range  $\pm M_p$ . Under these conditions, shakedown will take place provided the following inequalities are satisfied at each section.

$$\bar{m}_i + M_i^{\max} \leq M_p$$

$$\bar{m}_i + M_i^{\min} \geq -M_p$$

1.4

$$M_i^{\max} - M_i^{\min} \leq 2M_y$$

In the above,  $M_i^{\max}$  and  $M_i^{\min}$  are the extreme values of the elastic bending moment at the corresponding  $i$  cross section of the structure, for all states of loading under consideration. The third of these three conditions restricts the external load to a range which avoids the onset of alternating plasticity. The standard theoretical procedure to calculate shakedown limiting loads is given in references [13,15,16,17].

In the light of the experimental evidence available, structural failure due to alternating plasticity is unlikely to occur unless a great many cycles of peak loading are applied. In this respect Horne [14] has shown that alternating yield is most unlikely to cause failure when  $\alpha_a > \alpha_p$  where  $\alpha_a$  is the alternating plasticity load limit and  $\alpha_p$  is the static failure load.

While incremental collapse is theoretically possible at a load factor less than the load factor for static collapse, the importance of the effect of variable repeated loading in design depends on the probability of a sufficient number of load variations occurring above the shakedown limit for significant permanent deformation to be induced. A study of the frequencies of varying intensity of load in relation to the design intensity for both floor and wind loading was made with reference to this problem by Horne [85] who drew the following conclusion from his investigations. Repeated floor loading is unlikely to be of importance in a structure when  $\alpha_I > 0.75\alpha_p$  where  $\alpha_I$  is the incremental collapse load. Considering incremental collapse due to wind loads, it is unlikely that variable loading (that is, wind first from one direction, then from the opposite) will be important when  $\alpha_I > 0.64\alpha_p$ .

Only when there are reasons for believing that variable repeated load conditions are particularly severe is it necessary to check the shakedown load factors  $\alpha_a$  (for alternating yield) and  $\alpha_I$  (for incremental collapse). As an example, the design of 275 Kw Switchhouse which was carried out by Heyman and et al (86). The frame was subjected to high wind velocity and the use of cross bracing was prevented because of possible difficulties with electrical clearance. It was concluded that there was a dramatic drop in collapse factor when incremental collapse analysis was considered.

It is evident that [17,86] the amount of calculation becomes excessive as soon as any but the simplest of structures is considered. This situation demonstrates the need for an automatic analysis for shake-

down loads.

Tocher and Popov [87] developed a method based on a modified linear programming procedure. It seems that their method was not efficient for large structures, as all the examples were confined to relatively simple structures.

Davies [88] extended the elastic-plastic method of Jennings and Majid [21] to include hinge reversal 'unloading', shakedown effect and beneficial phenomena of strain hardening. However his method was limited to unloading of one hinge reversal for an increase in the load factor. This ceases to be true as quite often more than one plastic hinge unloads at the given load level.

More recently Guralnick and et al [89-90] have demonstrated an alternative way of characterizing shakedown and defining the incremental collapse load arising from a consideration of the energy imparted to, and recovered from, a structure during an infinite number of loading cycles. Their early studies were mainly concentrated on simple structure and the results obtained agreed with the results by Neal (17). In their recent papers [91-92], analysis of more complex structures, such as multi storey multi bay frames and more complex loading programmes, were examined.

All the methods of analysis regarding variable repeated loading reviewed so far are associated with frames with rigid connections. However there has been very little work on the analysis of variable

repeated loading on frames with semi-rigid connections. The only works known to the author on this subject are described in following paragraphs.

Two frames, two storey single bay and two bay single storey, were tested by Stelmack et al [93] to study the the behaviour of flexible connections under variable repeated loading. These frames were subjected to cyclic lateral loads without gravity load. It was concluded that no evidence of incremental deflection or other instabilities was obtained under a significant number of cycles at high load and the connections will shakedown to their elastic state. However, these conclusion were drawn on the particular connection types and the validity of this conclusion is not necessarily true for other types of connections.

The analysis of a three storey, three bay frame incorporating the joint flexibility was carried out by Cook [94]. In his studies frames were designed in accordance to Type 2 AISC construction "simple framing". The frame was loaded proportionally to the design level of dead, live and wind. The wind load was cycled seven times from extreme positive to negative value. The same conclusion as Stelmack was drawn from his studies stating that the cyclic wind and live loading need not be considered in the design of unbraced steel frames and connections will shakedown to their elastic state under their expected load cycles.

The mathematical model for a semi-rigid connections under alternating loading condition was developed by Mazzolani [95]. The mathematical model can be used to interpret the results of cyclic tests on structural elements.

### 1.5 The scope for the present work

Single storey building frames can often be analysed with sufficient accuracy by first-order plastic hinge theory. However when the frame is subjected to high wind loading and vertical loading, plastic design of unbraced frames is complicated by the need to make adequate allowance for the loss in load carrying capacity induced by in-plane stability effects, of which the  $P-\delta$  moments are the most prominent. To take account of these second order effects, rigorous analysis techniques are necessary, which often require a computer analysis program. However the sophisticated computer program is not easily available in the design office and the engineer requires an alternative approximate technique for the elastic-plastic analysis of unbraced rigid frames.

Chapters 2 and 3 describe simplified methods for the second order elastic-plastic analysis of single storey frames.

The method adopted in Chapter 2 is an extension of a semi-analytical method by Lok [98] which traces the development of plastic hinges. In this method, where and when the first plastic hinges occur is obtained



using the slope deflection equations. An expression is developed which will result in the collapse load. Comparisons are made with an exact computer analysis and previous approximate techniques.

The equations developed in Chapter 2 will result in a quick determination of failure load, but it was not possible to represent the equations obtained in non-dimensional forms to produce a design chart for the designer. Therefore in Chapter 3 a parametric study has been carried out to find the limits within which the first order plastic hinge theory should be allowed. This was done by determining limiting ratios of the elastic critical load,  $\alpha_{er}$ , to the collapse load factor given by first order plastic hinge theory,  $\alpha_p$ , in order that the second-order collapse load,  $\alpha_c$ , does not fall below  $0.9 \alpha_p$ . The studies concluded with sets of limiting values for both pinned and fixed bases, single storey pitched and flat roofed frames. These results can be used as a design document.

The traditional approach of analysis of frames assumes connections are either fully rigid or pinned. However neither is true and all types of connections exhibit a behaviour that falls between the extreme cases of ideally pinned and fully rigid conditions. If joint flexibility is incorporated in the analysis more reliable assessment of both the frame performance (serviceability) and carrying capacity (ultimate limit state) can be achieved.

Several sophisticated approaches are available already for the analysis of flexibly connected frames. It is quite difficult to check the

accuracy of different methods, and yet an analysis technique was needed to assess design methods. Therefore, it was useful to develop a computer analysis program for unbraced semi-rigid frames.

The development of such a computer program is described in Chapter 4 , as an extension of the well established rigidly connected frame analysis program by Majid and Anderson [23]. The program uses the compact storage scheme by Jennings [24] which can deal with the large frames efficiently.

The program developed is an incremental load level approach, which differs to the Majid and Anderson approach which analysed the frame only at the formation of plastic hinges. Using the present author's approach reduces the number of iterations required for convergence on geometric nonlinearities, material nonlinearities and non-linear connection behaviour. The non-linear connections are represented by series of straight lines.

The influence of semi-rigid connections in the design of building frames is usually based on simplifying assumptions on the behaviour of beam to column connections. Present practice in both elastic and plastic design of unbraced frames often results in uneconomical structures. This is because both methods require fully rigid connections. The design procedure described in Chapter 5 will eliminate the need for fully rigid connection in unbraced frames. The design method is known variously as the wind connection method, or as Type 2 Construction [75]. The wind connection method assumes that beam-to-column

connection are flexible enough to undergo relative rotations under gravity load so as to approach a "pinned" condition for gravity loading of beams but are strong enough to transfer wind moment from the column to the beams.

A number of frames have been designed in accordance with above design procedure. The frame's forces and deflections are calculated by the 'exact' analysis program described in Chapter 4 to study the validity of the design method.

The analysis described above is limited by the fact that no treatment is suggested for the irreversible nature of plastic hinges. Davies [88] included this effect in his analysis, but as stated earlier, only one plastic hinge was allowed to 'unload' at a given load level. Therefore there was need for a program to eliminate these shortcomings and also include the effect of cycles of loading on frames with semi-rigid connections, on which very little work has been published up to date.

The computer program described in Chapter 6, can deal with frames subjected to variable repeated loading. The program analyses the frame at a given load level and load case. It solves sets of simultaneous equations (1.1) for joint displacements and stores any hinge rotations. It then searches for any reversal of plastic hinges or semi-rigid rotations. Once these reversals are detected, appropriate treatments are then made to the stiffness matrix.

The program was initially written for frames with the rigid connections. Comparison of some analyses are made with previously documented results. In the same Chapter the program is extended further to include the effect of cycles of loading on semirigid connections.

The analysis program is used to examine the entire spectrum of incremental collapse behaviour, including alternating plasticity and shake-down of single and multistorey structures.

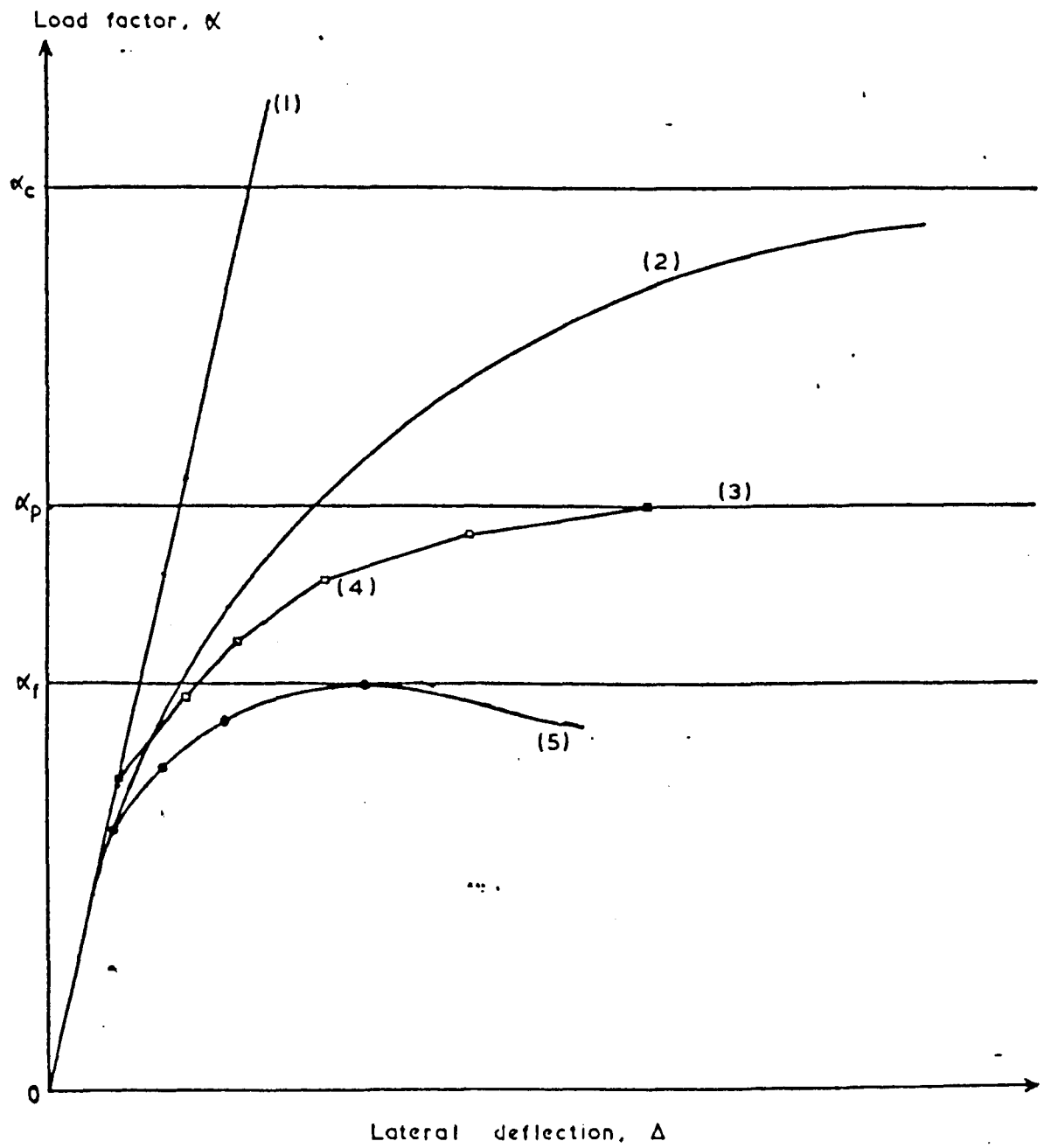


FIG. (1.1) LOAD DISPLACEMENT BEHAVIOUR

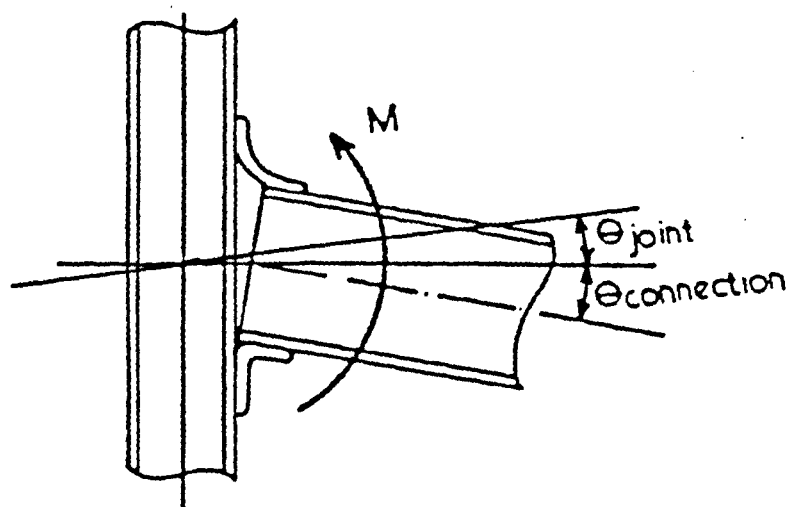


FIG. (1.2) RELATIVE ROTATION OF BEAM TO COLUMN

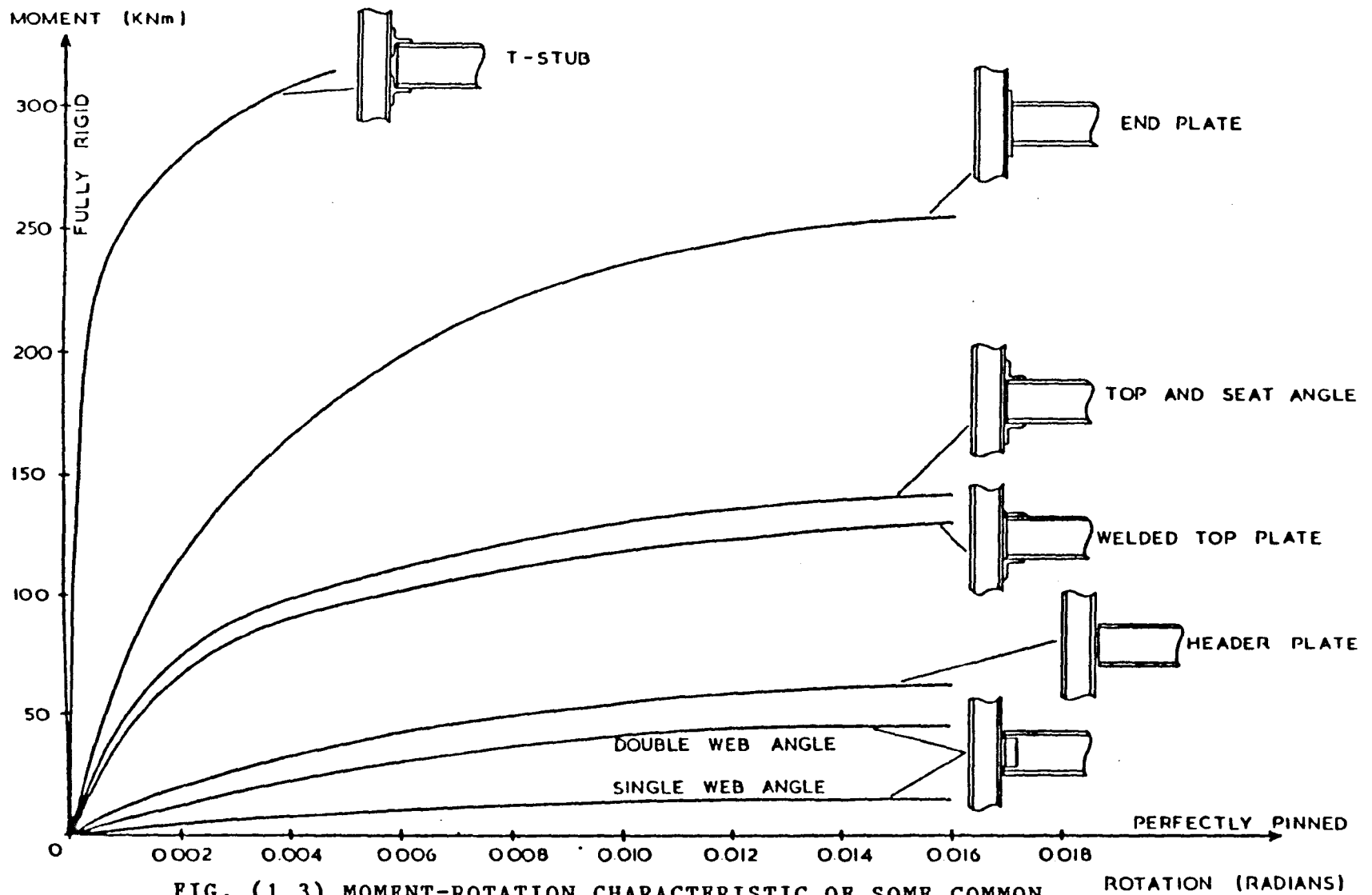


FIG. (1.3) MOMENT-ROTATION CHARACTERISTIC OF SOME COMMON BOLTED CONNECTIONS

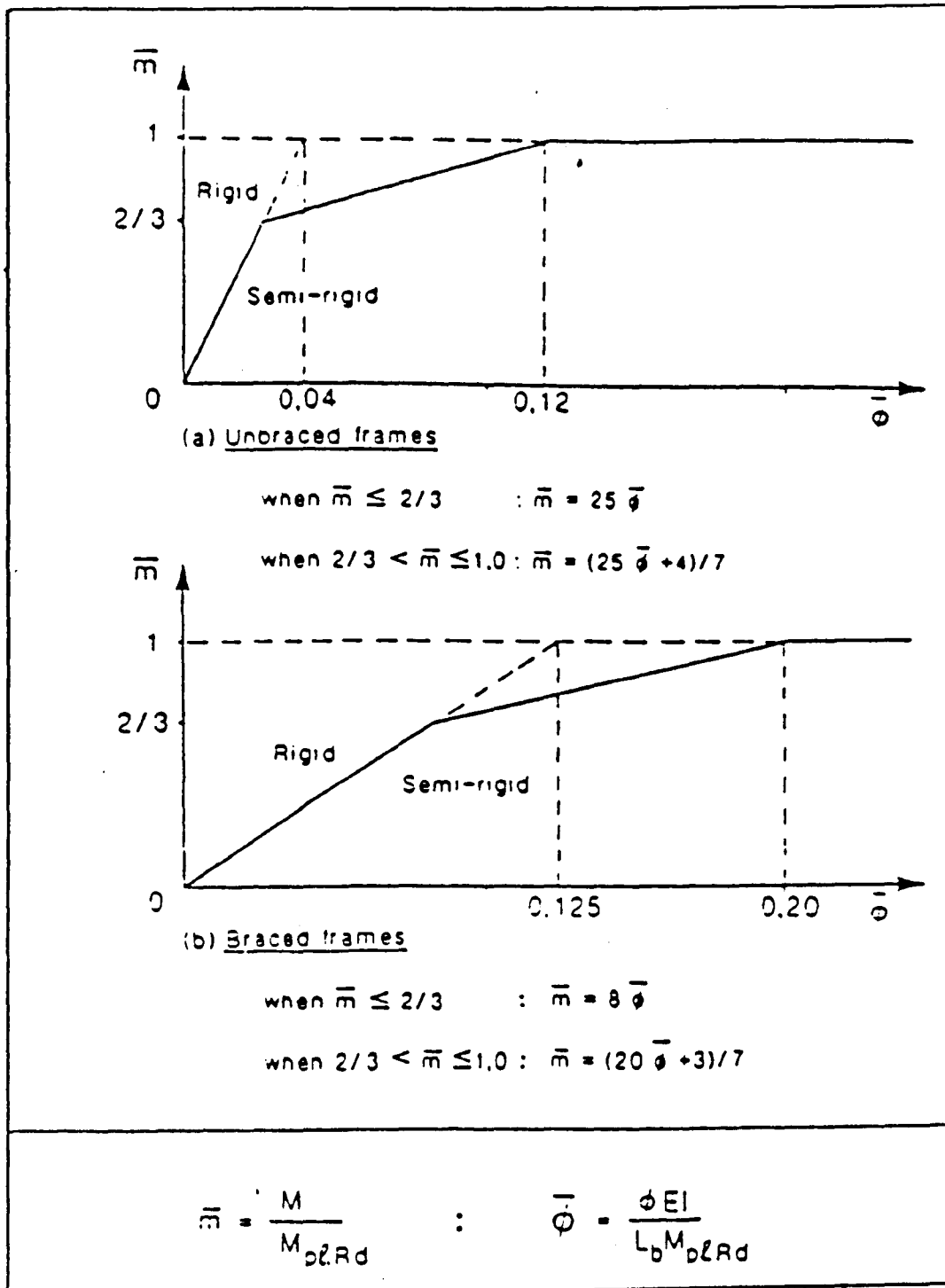


FIG. (1.4) EUROCODE NO.3 CLASSIFICATION FOR CONNECTIONS



## CHAPTER 2

### AN APPROXIMATE DETERMINATION OF THE FAILURE LOAD OF A SINGLE STOREY PIN BASE FRAME.

#### 2.1 Introduction.

The calculation of the failure load by an elastic-plastic analysis is very rigorous and normally the use of computers even for a simple frame is necessary. However the engineer may be interested in only a quick and approximate estimate of the failure load with-out the need to use specialized computer software. This is in particularly true for single storey frames.

There are several approximate methods of finding the failure load  $\alpha_f$ , some of which were described in Chapter 1.

The scope of this chapter is to find simplifying equations for the failure load of a single storey frame using the method adopted by Lok [98]. It also examines the calculation of the failure load recommended in B55950 [34] and the ECCS [35]. The failure load obtained from the exact second-order elasto-plastic computer program were compared with the above methods to demonstrate their accuracy.

## 2.2 Lok's method to determine $\alpha_f$

The method adopted by Lok to find the failure load  $\alpha_f$  of a single storey single bay frame, was based on the slope deflection analysis. It traces the development of the plastic hinges under proportional loading. In this approach the position and load factor at which a plastic hinge forms are located using step-by-step incremental analysis. Second order effects are considered by a combination of stability functions and fictitious horizontal loads, which will be described in section 2.4.

In order to proceed with the proposed simplified equation it is necessary to repeat some of the analysis carried out by Lok [98 ].

## 2.3 Assumptions.

Fig (2.1) shows a pinned base portal frame.  $\alpha V$  is the central point load,  $\alpha(RV)$  is the column end load and  $\alpha H$  a horizontal point load concentrated at eaves level. The frame is proportionally loaded, identified by common load factor  $\alpha$ .

The following assumptions were used by Lok [98] to obtain the approximate failure load:

- a) The reduction in beam stiffness due to compressive axial load is negligible.

- b) The effect of wind loading on the distribution of axial forces in the column can be ignored.
- c) Sway due to axial shortening is neglected.
- d) The members are originally unstressed and lack of fit is neglected.
- e) Out of plane displacements are prevented and failure occurs only in the plane of the frame.
- f) Spread of plasticity and the effects of strain hardening are neglected; the member is assumed to possess its elastic flexural rigidity except at the sections where the plastic moment of resistance ( $M_P$ ) is developed.
- g) Reversal of joint rotation is assumed not to occur under an increase in load.

#### 2.4 Analysis of pinned base single storey frame.

The following elastic bending moments are obtained by superimposing the moments obtained under vertical and horizontal load as shown in fig. (2.2),

$$M_{DC} = M_H + M_V$$

$$M_{BC} = M_H - M_V$$

$$M_{CD} = M_F - M_V$$

where

$$M_H = \text{wind moment} = \frac{\alpha_1 H h}{2} + \alpha_1 (FV) \delta \quad 2.2$$

$$M_V = \text{vertical moment} = \frac{\alpha_1 V L}{8} \left\{ 1 - \frac{2K'}{2K + s(1-c^2)K} \right\} \quad 2.3$$

$$M_F = \text{free moment} = \frac{\alpha_1 V L}{4} \quad 2.4$$

$$\delta = \text{horizontal eaves sway} = \frac{\alpha_1 H h \beta_1}{\left\{ \frac{12EK'}{h} - 2\alpha_1 (FV) \beta_1 \right\}} \quad 2.5$$

$$F(V) = \text{base reaction} = \alpha_1 (RV) + \alpha \frac{V}{2}$$

$$K' = \frac{I_b}{L}$$

$$K'' = \frac{I_g}{h}$$

$$\beta_1 = \frac{1 + 6K''}{s(1-c^2)K''}$$

A plastic hinge forms when the value of bending moment at any section reaches the plastic moment resistance of that section. The plastic hinge in the frame shown in fig (2.1) can either occur directly under the central point load (C) or at the leeward end of the beam (D). The two cases are now examined.

#### 2.4.1 First hinge occurs at midspan of beam.

When the first hinge forms at a load factor  $\alpha_1$ , the frame becomes statically determinate, as shown in fig (2.3). Under the increment  $\alpha V$  the bending moments at B and D are equal but opposite in direction:

$$\Delta M_{BD}(V) = -\Delta M_{DB}(V) = \frac{\Delta \alpha V L}{4} \quad 2.6$$

When the frame is subjected to horizontal load as shown in fig (2.3 b), the bending moments at B and D are of equal magnitude and direction:

$$\Delta M_{BD}(H) = (\Delta \alpha H + H_1) \frac{h}{2} + \alpha_2 (FV) v_1 \quad 2.7$$

where  $H_1$  = Fictitious horizontal load to allow for the increment of vertical loading acting on the sway,  $\delta$ , existing in the frame at  $\alpha_1$ . Sway,  $\delta$ , is shown in fig. (2.3 b)

Thus

$$H_1 = \Delta \alpha \sum V \left( \frac{\delta}{h} \right) \quad 2.8$$

$\Sigma V$  is the total vertical load on the frame at  $\alpha=1$

$v_1$  is the incremental sway due to load factor  $\alpha_2$  as shown in fig. (2.3 b)

$\alpha_2$  is the load factor for formation of second hinge

$$v_1 = \frac{(\Delta\alpha H + H_1)h\beta_1}{\frac{12EK'}{h} - (2\alpha_2(FV)\beta_1)} \quad 2.9$$

$\alpha_2(FV)$  is the base reaction

$\beta_1$  defined earlier but with the stability functions  
calculated base on the total load ,  $\alpha_2$ .

2.4.2 First hinge occurs at leeward end of beam.

Fig. (2.4) shows the portal frame with a leeward hinge. Under  $\alpha$  the incremental bending moments were derived by Lok as follows:

$$\Delta M_{cd}(V) = \frac{\Delta\alpha VL}{4} \quad 2.10$$

$$\Delta M_{bd}(H) = (\Delta\alpha H + H_1 + H_2)h$$

$$\Delta M_{cd}(H) = \frac{\Delta M_{bd}(H)}{2} \quad 2.11$$

$H_2$  is a fictitious horizontal load force which allows for the  
'P- $\delta$ ' moment due to the vertical load shown in fig. (2.5)

$$H_2 = \alpha_2 \sum V \left( \frac{v_2}{h} \right) \quad 2.12$$

$$v_2 = \text{sway deflection} = \frac{H^* h^3 \beta_2}{3EK'} \quad 2.13$$

where  $\beta_2 = 1 + (k'/k'')$

$H^*$  is the horizontal load applied to calculate  $v_2$

and is given by

$$H^* = \Delta\alpha H + (H1 + H2 + H3) \quad 2.14$$

H3 is the horizontal load applied at the eaves to represent the effect on the frame of its lack of symmetry due to the position of the first hinge. The unsymmetrical nature of the frame causes sway. H3 is given by:

$$H3 = \frac{3\Delta\alpha V l}{16h} \left( \frac{1}{1 + \frac{k'}{k}} \right) \quad 2.15$$

## 2.5 Simplified equations

The analysis in which simplified equations were developed by the author using the expressions described in section 2.4, are given in the next two subsections.

### 2.5.1 Simplified equation when the first hinge occurs at mid span.

The load factors at which the hinges form were found by Lok using an iterative procedure. The following analysis is carried out in order to find directly the load factor at which the second plastic hinge forms at the leeward knee of the frame and thus the collapse load. It should be noted that the collapse may occur with only one hinge present in the frame. This is due to severe instability which results from high

axial loads. In this instance the incremental load determined by the simplified equation after formation of the first plastic hinge, tends to zero.

For the plastic hinge to occur at the knee , D (refer to fig. 2.1 ), the moment should be equal to the lesser of  $M_{PB}$  and  $M_{PC}$  , these being the moment resistance of the beam and the column respectively.

$$M_P = M_{DB} + \Delta M_{DB}(V) + \Delta M_{DB}(H) \quad 2.16$$

Where  $M_{DB}$  is the elastic moment at section D at load factor  $\alpha_1$ .  $\Delta M_{DB}(V)$  and  $\Delta M_{DB}(H)$  are the moments due to the incremental load,  $\Delta\alpha$ , and are given in equations (2.6 and 2.7).

$$(M_P - M_{DB}) = \frac{\Delta\alpha VL}{4} + (\Delta\alpha H + H1)\frac{h}{2} + \alpha_2(FV)v_1 \quad 2.17$$

Substituting for  $v_1$  from (2.9) into the above equation and Assuming

$$\frac{12EK'}{h} - 2\alpha_2(FV)\beta_1 = \beta_3$$

gives

$$(M_P - M_{DB}) = \frac{\Delta\alpha VL}{4} + (\Delta\alpha H + H1)h \left\{ 0.5 + \frac{\alpha_2(FV)h\beta_1}{\beta_3} \right\} \quad 2.18$$



The expression  $[\alpha_2(FV)\beta_1/\beta_3]$  is very small and can be ignored. This fact can be demonstrated by referring to example 5 of table (2.1) which will result in the highest value of the above expression, because the base reaction,  $(FV)$ , is high. The value obtained for the above expression was 0.0076, which is very small in comparison to the other factors in equation (2.18). Therefore (2.18) becomes:

$$(M_p - M_{DB}) = \frac{\Delta\alpha VL}{4} + (\Delta\alpha H + H_1) \frac{h}{2} \quad 2.19$$

Rearranging the above equation and substituting for  $H_1$  from equation (2.8) the increment of the load factor over load factor  $\alpha_1$  becomes:

$$\Delta\alpha = \frac{(M_p - M_{DB})4}{(VL + 2(Hh + \sum V\delta))} \quad 2.20$$

Therefore the collapse load obtained is:

$$\alpha_2 = \alpha_1 + \Delta\alpha \quad 2.21$$

2.5.2 Simplified equation when the first hinge occurs at leeward end of the beam

By referring to fig. (2.4), for the second plastic hinge to occur at the midspan of the beam, the bending moment at C should then be equal to the plastic moment resistance of the beam  $M_{PB}$ .

$$M_p = M_{CD} + \Delta M_{CD}(V) + \Delta M_{CD}(H) \quad 2.22$$

Where  $M_{CD}$  is the elastic moment at section C at load factor  $\alpha_1$ .  $\Delta M_{CD}(V)$  and  $\Delta M_{CD}(H)$  are the moments due to the incremental load  $\Delta\alpha$  and are given in equations (2.10 and 2.11). From equation (2.11), it follows that;

$$\Delta M_{CD}(H) = (\Delta\alpha H + H_1 + H_2) \frac{h}{2} \quad 2.23$$

Substitution will now be made for all the terms in the above equation:

$H_2$  from (2.12) is:

$$H_2 = \alpha_2 \sum V \left( \frac{v_2}{h} \right)$$

$v_2$  from equation (2.13) is:

$$v_2 = \frac{H^* h^2 \beta_2}{3EK'}$$

Also

$$H^* = \Delta\alpha H + (H_1 + H_2 + H_3)$$

Substituting for  $H^*$  in the expression  $v_2$  :

$$v_2 = \frac{(\Delta\alpha H + H_1 + H_2 + H_3) h^2 \beta_2}{3EK'} \quad 2.24$$

Substituting for  $H_2$  which contains  $v_2$  and taking all the terms with  $v_2$  into the Left-hand side, equation (2.23) becomes:

$$v_2 * 3EK' - \left( \alpha_2 \sum V \frac{v_2}{h} \right) h^2 \beta_2 = (\Delta \alpha H + H1 + H3) h^2 \beta_2 \quad 2.25$$

Solving for  $v_2$ :

$$v_2 = \frac{(\Delta \alpha H + H1 + H3) h^2 \beta_2}{3EK' - \alpha_2 \sum V h \beta_2} \quad 2.26$$

denote:

$$3EK' - \alpha_2 \sum V h \beta_2 = \beta_3$$

Substituting for term H2 which contains  $v_2$  into equation (2.23),

$$\Delta M_{cd}(H) = (\Delta \alpha H + H1) \frac{h}{2} + \frac{\alpha_2 \sum v (\Delta \alpha H + H1 + H3) h^2 \beta_2}{2\beta_3} \quad 2.27$$

as:

$$\alpha_2 \sum V h \beta_2 = 3EK' - \beta_3$$

the equation (2.27) becomes:

$$\Delta M_{cd}(H) = (\Delta \alpha H + H1) \frac{h}{2} + \left( \frac{3EK'}{\beta_3} - 1 \right) (\Delta \alpha H + H1 + H3) \frac{h}{2} \quad 2.28$$

Rearranging

$$\Delta M_{cd}(H) = (\Delta \alpha H + H1) \left( \frac{3EK'}{\beta_3} - 1 \right) \frac{h}{2} + \left( \frac{3EK'}{\beta_3} \right) \left( \frac{H3h}{2} \right) \quad 2.29$$

Substituting equation (2.29) into the equation (2.22),

$$(M_{ps} - M_{cd}) = \frac{\Delta \alpha VL}{4} + \frac{3EK'}{\beta_3} (\Delta \alpha H + H1) \frac{h}{2} + \frac{H3h}{2} \left( \frac{3EK'}{\beta_3} - 1 \right) \quad 2.30$$

Substituting for H1 and H3 from (2.8 and 2.15 ) and rearranging the above equation:

$$(M_{ps} - M_{cd}) = \frac{\Delta \alpha VL}{4} \left( 1 + \frac{9EK'}{8\beta_2\beta_3} - \frac{3}{8\beta_2} \right) + \Delta \alpha \left( \frac{Hh}{2} + \frac{\sum V\theta}{2} \right) \frac{3EK'}{\beta_3} \quad 2.31$$

Multiplying through by  $\beta_3$ ,

$$(M_{ps} - M_{cd})\beta_3 = \frac{\Delta \alpha VL}{4} \left( \beta_3 + \frac{9EK'}{8\beta_2} + \frac{3\beta_3}{8\beta_2} \right) + \Delta \alpha \left( \frac{Hh}{2} + \frac{\sum V\theta}{2} \right) 3EK' \quad 2.32$$

Therefore the increment of the load factor, over load factor  $\alpha_1$ , becomes

$$\Delta \alpha = \frac{4(M_{ps} - M_{cd})\beta_3}{VL\beta_4 + 6EK'(Hh + \sum V\theta)} \quad 2.33$$

where

$$\beta_3 = 3EK' - 1.1\alpha_1 \sum V h \beta_2$$

$$\beta_2 = 1 + \frac{K'}{K^*}$$

$$\beta_4 = \left( \beta_3 + \frac{9EK'}{8\beta_2} + \frac{3\beta_3}{8\beta_2} \right)$$

Thus the collapse load is equal to;

$$\alpha_2 = \alpha_1 + \Delta\alpha$$

## 2.6 Design code and recommendation.

There are considerable differences between the recent design methods to assess the overall stability of unbraced low-rise frames proportioned in accordance with plastic theory. The relevant provisions of two specifications are now examined.

### 2.6.1 ECCS formula.

ECCS simplified second order plastic hinge theory [35] states that if:

$$i) \quad E = L \sqrt{\frac{N}{EI_c}} \leq 1.6 \quad \text{in all columns}$$

where L is the length of the rafter

N is the axial load

$I_c$  is 2nd moment area of column

ii) There are no plastic hinges between the column end points.

then the following equation can be use:

$$Q_r = H_r + \phi_r P_r + 1.2 \phi_r P_r \quad 2.34$$

where  $\phi_r$  is the column-slope of storey r, calculated iteratively  
by first-order plastic theory.

$\phi_0$  is the column-slope of storey r due geometric  
imperfection

$H_r$  is total sum of factored external horizontal working  
loads above storey r

$P_r$  is total sum of factored vertical working loads above  
storey r

The factor of 1.2 in the above equation is to take account of the P- $\delta$   
effect.

Restrictions (i) and (ii) were specified to avoid local instability of  
highly compressed slender columns.

For the purpose of the studies required here the geometric imperfec-  
tion has been ignored, as is the usual practice in Britain.

Therefore equation (2.34) becomes;

$$Q_r = H_r + 1.2 \phi_r P_r \quad 2.35$$

#### 2.6.2 BS5950: Pt.1 recommendations.

The recommendations for sway stability in Britain are based on the work of Horne [14]. They state that in the absence of rigorous second-order analysis one of the following checks should be carried out.

a) Under 1% of the total factored vertical load applied as a horizontal disturbing force at each eaves joint, the sway deflection of any column should not be allowed to exceed 0.0018h, where h is the height column.

b) The following limitation is imposed on the rafter slenderness in any bay (the formula presented is for a flat-roofed frame):

$$\frac{L}{D} \leq \frac{44}{\Omega} \frac{L}{h} \left( \frac{\rho}{4 + \rho} \right) \frac{275}{P_y} \quad 2.36$$

where L is span of bay (m)

h is the height of column

D is depth of rafter,

$$\rho = \left( \frac{2I_o}{I_b} \right) \left( \frac{L}{h} \right)$$

$I_b$  is the second moment area of rafter section,

$I_o$  is the second moment area of column section,

W is the un-factored vertical load,

$W_o$  is uniformly distributed load which causes plastic

collapse in a fixed ended horizontal roof beam  
of span L with the same cross-section as the actual  
rafter.

is the arching ratio  $W_o/W$

Rearranging the above equation:

$$\Omega = \frac{44DL}{L} \frac{1}{h} \left( \frac{\rho}{4+\rho} \right) \frac{275}{P_y} \quad 2.37$$

where

$$\Omega = \alpha \frac{W}{W_o}$$

This may be further re-arranged to give a load factor for collapse:

$$\alpha_f = \frac{44DW_oL}{L} \frac{1}{W} \frac{1}{h} \left( \frac{\rho}{4+\rho} \right) \frac{275}{P_y} \quad 2.38$$

## 2.7 Comparison of failure loads

The studies were conducted on a frame using a 457x152x52UB for the rafter and a 305x305x137UC for the column section. The reason for choosing the stronger column section is to prevent any hinge forming at the top of the column. This is essential for the ECCS recommendations to be valid (see 2.6.1).

The frame was subjected to the various combinations of vertical and horizontal loading as indicated in tables (2.1) and (2.2). The uniformly distributed loads were replaced by point loads acting at the



midspan of the beam and column head as shown in fig. (2.1). Frame dimensions were also varied. The dimensions together with the factored loading of the frames studied are given in tables (2.1) and (2.2). It can be seen from these tables that some of the frames were analysed with the heavy concentrated loads to act at the head of columns, assumed to come from other supported parts of the structure.

The analyses were conducted on two groups, each containing up to ten frames. The first group are the frames in which the first hinge occurs at mid-span, i.e. the vertical load is dominant; the results of this group are given in table (2.3). The second group are the frames in which the first hinge occurs at the leeward end of beam. These frames were subjected to high wind loading and second-order effects were more significant. The results of this group are given in table (2.4).

The following analyses were carried out to obtain the failure load;

- a) Second-order elasto-plastic analysis using the program  
by Majid and Anderson.
- b) Lok's equation obtained by slope deflection method.
- c) Simplified equation developed by the author.
- d) First order plastic theory .
- e) Simplified second order elasto-plastic analysis recommended

by ECCS and described in sec. 2.5.1.

f) Minimum depth requirement recommended by B55950 and described in sec. 2.5.2.

In order to carry out the analyses mentioned above, a computer program was developed by the author to perform b,c,d, and e.

## 2.8 Discussion of results

The results for the frames with the first hinge occurring at the mid span of the beam are presented in table (2.3). The failure load obtained by the exact second-order computer analysis, Lok's second-order analysis and the simplified equations of the author are very close. It is also evident from these results that there is negligible difference between the second-order analysis and a simple plastic analysis. This is true even for the frames with a very high axial load such as Frame 8. Therefore, for this group of frames, design by plastic theory without reference to frame instability effects is sufficient. The ECCS simplified second-order hinge theory gives close agreement with the exact second-order computer analysis, except for the Frames 8 and 9, where the criteria  $L\sqrt{N/EI}$  exceeds the 1.6 limiting value and no results are given for these frames in table (2.3).

The second group of the frames in which the first hinge occurs at the leeward end of the beam are given in table (2.4). Again very good agreement was reached between the exact second order computer analy-

sis, Lok's method and the author's simplified equations. It can be seen from these results that the simple plastic analysis is unsafe, particularly for the frame with high axial load. This was as expected since the formation of the plastic hinge at the leeward end causes the frames to be more susceptible to the second order effects. The results obtained shows that the ECCS simplified second order elasto-plastic hinge theory over-estimates the failure load in all the cases. in particular, in the frames with the high axial load. For example , in Frame 5 of table (2.4) , ECCS method overestimates the failure load by 22% over the second-order computer analysis.

#### 2.8.1 Results of BS5950 recommendation.

The results of failure load obtained from equation (2.38) are presented in the last column of table (2.3) and table (2.4).

The failure loads obtained by the minimum depth requirement ,in table (2.3), are higher than the failure load obtained by first order plastic hinge theory for all the frames shown in this table . This confirms that the simple plastic analysis can be used for this group of frames.

For the second group of frames shown in table (2.4) , the failure load obtained by the BS5950 recommendations are less than the failure load by simple plastic theory except for Frame 1 . Therefore first order plastic analysis should not be used due to the susceptibility of this

group of the frames to instability. No result is shown for Frame 8, because the sway deflection exceeded  $0.0018h$  as described in sec. 2.6.2.

The validity of the formula given in BS5950 was investigated by referring to the example shown in fig.(2.1) with the following specifications;

$V=30 \text{ kN}$   $R=0.5$  therefore  $W_R=60 \text{ kN}$

$L=15 \text{ m}$   $h=9.0 \text{ m}$

Yield strength=  $250 \text{ N/mm}^2$

BEAM:  $305 \times 127 \times 48 \text{ UB}$   $I_B = 9504 \text{ cm}^4$   $M_{PB} = 176 \text{ kNm}$

COLUMNS:  $152 \times 152 \times 37 \text{ UC}$   $I_C = 2218 \text{ cm}^4$   $M_{PC} = 77.5 \text{ kNm}$

The failure load obtained by simple plastic analysis was 2.250 and failure load obtained by 'exact' 2nd order computer program was 2.215.

The failure load by BS5950, by referring to equation (2.38) is;

By definition:

$$W_o = \frac{16M_{PB}}{L} = \frac{16 \times 176}{15} = 188.3 \text{ kN}$$

$$\rho = \frac{2I_C}{I_B} \cdot \frac{L}{h} = \frac{2 \times 2218}{9504} \times \frac{15}{9} = 0.78$$

$$\Omega = \frac{W_R}{W_0} = 60/188.3 = 0.312$$

Therefore :

$$\alpha_1 = \frac{44 \times 0.310}{15} \times \frac{188.3}{60} \times \frac{15}{9} \times \frac{0.78}{4 + 0.78} \times \frac{275}{250} = 0.85$$

The failure load obtained by this method is much less than the failure load by simple plastic analysis. These results indicate that the frame used for this analysis is susceptible to instability. This is not true since the second-order analysis had shown a failure load factor of 2.215 , compared to a first-order results of 2.250. Therefore the use of this formula can sometimes lead to very uneconomical design.

## 2.9 Conclusion

The following conclusions may be drawn from the studies.

- 1) The simplified equation developed by the Author can be used in all the cases. A good agreement is achieved with the exact second-order computer analysis.
- 2) Simple plastic theory can only be used in the cases where the first plastic hinge occurs at the midspan of the beam ,i.e. when the vertical load is the dominating load.
- 3) Simple plastic theory leads to unsafe results when the first plastic hinge occurs at the leeward end of the beam. Therefore a second-order analysis should be carried out using one of the techniques

described above.

4) The simplified second order elasto-plastic hinge theory given by ECCS overestimates the failure load when the first hinge forms at the leeward end of the beam.

5) BS5950's recommendations can be used as an indication of the frame's susceptibility to instability . However the use of the formula may lead to very conservative results with loss of economy in design.

| FRAME<br>REF. NO | h (m) | L(m) | H (kN) | V (kN) | R    |
|------------------|-------|------|--------|--------|------|
| 1                | 4     | 6    | 24     | 156    | 0.5  |
| 2                | 4     | 6    | 100    | 906    | 0.3  |
| 3                | 4     | 6    | 100    | 1000   | 0.5  |
| 4                | 8     | 6    | 24     | 302    | 0.25 |
| 5                | 8     | 6    | 100    | 1000   | 0.5  |
| 6                | 4     | 12   | 24     | 156    | 0.5  |
| 7                | 4     | 12   | 100    | 1000   | 0.5  |
| 8                | 4     | 12   | 100    | 1000   | 2    |
| 9                | 8     | 12   | 100    | 1500   | 1.33 |
| 10               | 8     | 12   | 50     | 500    | 0.5  |

TABLE (2.1) FRAME DIMENSION AND LOADING FOR FIRST HINGE  
TO OCCUR AT THE CENTRE OF THE BEAM

| FRAME<br>REF. NO | h (m) | L(m) | H (kN) | V (kN) | R   |
|------------------|-------|------|--------|--------|-----|
| 1                | 4     | 6    | 72     | 156    | 0.5 |
| 2                | 4     | 6    | 72     | 156    | 6   |
| 3                | 4     | 6    | 150    | 302    | 5   |
| 4                | 8     | 6    | 90     | 468    | 2   |
| 5                | 8     | 6    | 50     | 302    | 5   |
| 6                | 4     | 12   | 90     | 156    | 6   |
| 7                | 4     | 12   | 75     | 156    | 10  |
| 8                | 4     | 12   | 90     | 156    | 6   |
| 9                | 8     | 12   | 75     | 156    | 10  |
| 10               | 8     | 12   | 75     | 302    | 5   |

TABLE (2.2) FRAME DIMENSION AND LOADING FOR FIRST HINGE  
TO OCCUR AT THE LEEWARD END OF THE BEAM



| FRAME<br>REF.<br>NO. | 2ND ORDER<br>COMPUTER<br>ANALYSIS | 2ND ORDER<br>LOK EQN. | 2ND ORDER<br>SIMPLIFIED<br>EQN. | SIMPLE<br>PLASTIC | 2ND ORDER<br>ECCS SIMPLI-<br>FIED<br>EQN. | BS5950<br>MIN. DEPTH<br>CRITERION |
|----------------------|-----------------------------------|-----------------------|---------------------------------|-------------------|---|-----------------------------------|
| 1                    | 1.842                             | 1.847                 | 1.847                           | 1.86              | 1.843                                     | 6.815                             |
| 2                    | 0.334                             | 0.335                 | 0.335                           | 0.337             | 0.335                                     | 1.408                             |
| 3                    | 0.306                             | 0.307                 | 0.307                           | 0.309             | 0.307                                     | 1.063                             |
| 4                    | 0.933                             | 0.937                 | 0.939                           | 0.957             | 0.935                                     | 1.585                             |
| 5                    | 0.268                             | 0.268                 | 0.268                           | 0.276             | 0.267                                     | 0.363                             |
| 6                    | 1.006                             | 1.014                 | 1.014                           | 1.018             | 1.013                                     | 4.438                             |
| 7                    | 0.162                             | 0.164                 | 0.164                           | 0.164             | 0.164                                     | 0.692                             |
| 8                    | 0.162                             | 0.163                 | 0.163                           | 0.164             | -   | 0.276                             |
| 9                    | 0.104                             | 0.105                 | 0.105                           | 0.107             | -   | 0.102                             |
| 10                   | 0.304                             | 0.304                 | 0.305                           | 0.309             | 0.304                                     | 0.531                             |

Table 2.3 COMPARISON OF THE FAILURE LOAD FOR THE FRAMES OF TABLE 2.1.

| FRAME<br>REF.<br>NO. | 2ND ORDER<br>COMPUTER<br>ANALYSIS | 2ND ORDER<br>LOK EQN. | 2ND ORDER<br>SIMPLIFIED<br>EQN. | SIMPLE<br>PLASTIC | 2ND ORDER<br>ECCS SIMPLI-<br>FIED<br>EQN. | BS5950<br>MIN. DEPTH<br>CRITERION |
|----------------------|-----------------------------------|-----------------------|---------------------------------|-------------------|---|-----------------------------------|
| 1                    | 1.411                             | 1.411                 | 1.378                           | 1.389             | 1.406                                     | 6.815                             |
| 2                    | 1.186                             | 1.186                 | 1.182                           | 1.389             | 1.217                                     | 1.05                              |
| 3                    | 0.611                             | 0.611                 | 0.609                           | 0.697             | 0.620                                     | 0.620                             |
| 4                    | 0.436                             | 0.430                 | 0.427                           | 0.494             | 0.446                                     | 0.313                             |
| 5                    | 0.529                             | 0.529                 | 0.529                           | 0.804             | 0.644                                     | 0.21                              |
| 6                    | 0.724                             | 0.724                 | 0.718                           | 0.810             | 0.744                                     | 0.69                              |
| 7                    | 0.663                             | 0.660                 | 0.660                           | 0.849             | -   | 0.40                              |
| 8                    | 0.401                             | 0.418                 | 0.418                           | 0.634             | 0.468                                     |                                   |
| 9                    | 0.385                             | 0.384                 | 0.384                           | 0.683             | -   | 0.175                             |
| 10                   | 0.326                             | 0.327                 | 0.327                           | 0.435             | -   | 0.156                             |

Table 2.4 COMPARISON OF THE FAILURE LOAD FOR THE FRAMES OF TABLE 2.2.

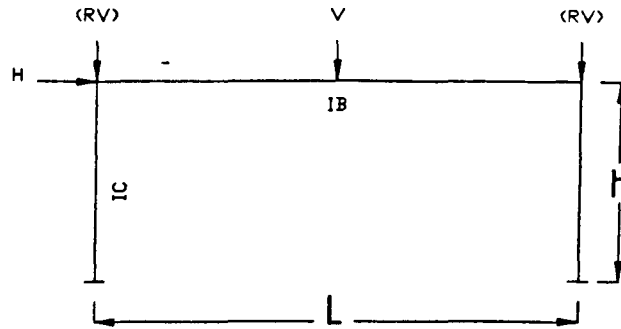


FIG. (2.1) PINNED BASE PORTAL FRAME

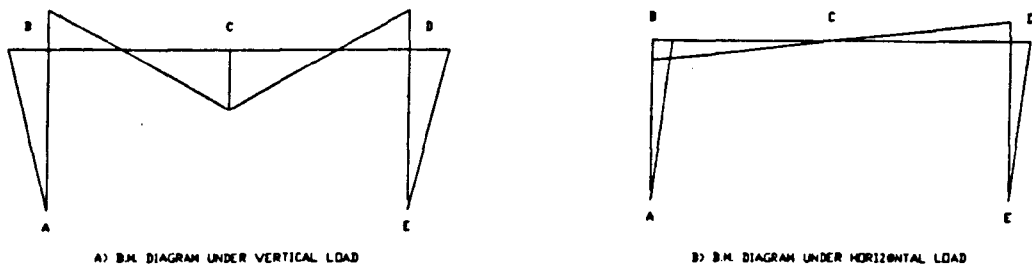


FIG. (2.2) ELASTIC BENDING MOMENT DIAGRAM OF SINGLE STOREY PINNED BASE FRAME

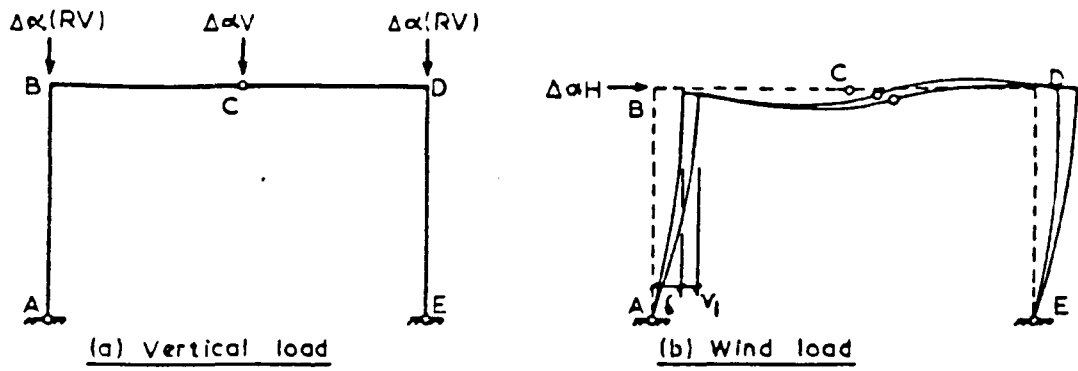


FIG. (2.3) HINGE AT MID-SPAN OF THE BEAM

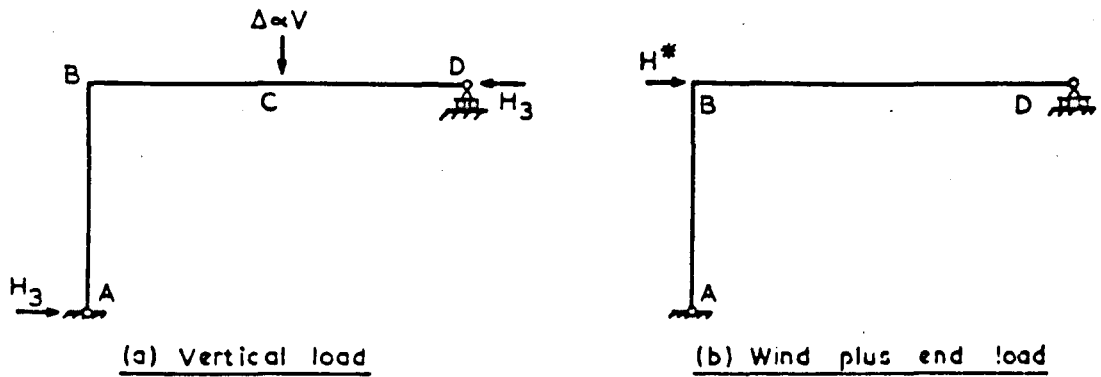
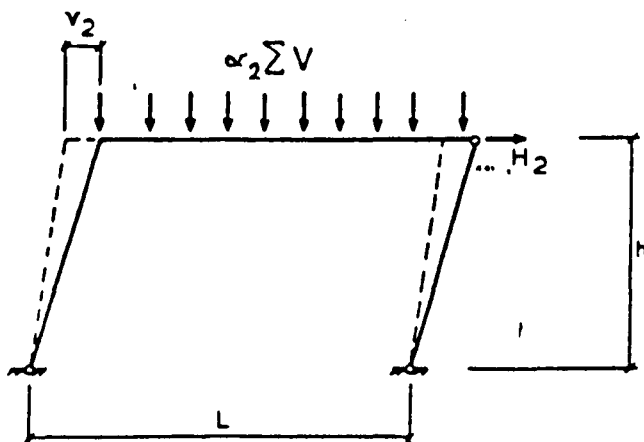


FIG. (2.4) HINGE AT LEEWARD END OF THE BEAM



Equilibrium.

$$H_2 = \alpha_2 \sum V \frac{V_2}{h}$$

FIG. (2.5) FICTITIOUS LOAD  $H_2$

## CHAPTER 3

### FIRST-ORDER PLASTIC DESIGN OF SINGLE STOREY FRAMES.

#### 3.1 Introduction

Chapter two reviewed the determination of the second-order elasto plastic failure load by use of a sophisticated computer program and approximate methods. However first-order plastic analysis has been common in multi-bay portal frame design in Britain for many years. Therefore, there is a need to define limits for the safe use of the method.

It has become common practice to stipulate a limiting ratio of elastic buckling load to plastic collapse load to ensure stability in steel sway frames. If the given limitation is reached, first-order, rigid plastic theory is sufficient to analyse the structure.

In the instance of falling short of the limiting ratio, the frame must either be strengthened or, alternatively, second-order elasto-plastic analysis is required to establish the failure load of the structure.

ECCS publication No. 33 'Ultimate Limit State Calculation of Sway Frames with Rigid Joints' [66] gives minimum stiffness requirements for single storey frames. It states that a frame may be analysed by first-order plastic hinge theory, provided that no hinges form between column end points and if the following equations are satisfied,

$$\frac{3EI_c}{\left(h+L\frac{I_c}{I_b}\right)Ph} \geq 10 \quad \text{for pinned base frames} \quad 3.1$$

$$\frac{6EI_b}{\left(h+L\frac{I_c}{I_b}\right)Ph} \geq 10 \quad \text{for fixed base frames} \quad 3.2$$

where       $h$  = column height  
              $L$  = rafter length  
              $P$  = total vertical load  
              $I_c$  = inertia of column  
              $I_b$  = inertia of beam

This criterion follows from a decision that the critical buckling load be at least ten times the vertical design load just before the development of the last plastic hinge for the most unfavourable mechanism.

Use of the above criterion is restrictive for the frames in which vertical forces act on the frames as distributed loading along the rafter. A frame with the described loading will normally be overdesigned by the ECCS proposals.

The objective of this chapter is to describe a less conservative method than that given by ECCS . The study was concentrated on building frames in which the columns were not subjected to additional concentrated forces resulting from other supported structures. Thus the column axial forces sum to the vertical load applied to the rafter. The aim was to define the limit with inwhich first-order plastic hinge theory is applicable assuming that a ten percent error in the calculated collapse load is acceptable.

A parametric study was used to determine the limiting ratio of elastic critical load  $\alpha_{cr}$  to  $\alpha_p$ , the collapse load factor given by first-order plastic hinge theory. These limits have been found for both pinned-base and fixed-base single bay frames, with flat roofs or roofs of moderate pitch.

### 3.2 Choice of sections.

The studies were conducted on the frames whose general arrangement are shown in figs (3.1-3.2). In order to provide preliminary section sizes for the studies the following values of loading were chosen;

|                         |                        |
|-------------------------|------------------------|
| Unfactored dead load    | 0.43 kN/m <sup>2</sup> |
| Unfactored imposed load | 0.75 kN/m <sup>2</sup> |

These values are reasonably typical for single storey industrial premises or warehouses in the U.K. Using partial safety factor of 1.4

on dead load and 1.6 on imposed load, and assuming frames are positioned at 6m centres longitudinally, the following vertical load was calculated:

$$(1.4 \times 0.43 + 1.6 \times 0.75) \times 6 = 10.8 \text{ kN/m}$$

In order to have a systematic way of referring to the frames studied in this Chapter, the following letters followed by a number were chosen.

PF is Pinned base Flat roof

PP is Pinned base Pitch roof

FF is Fixed base Flat roof

FP is Fixed base Pitch roof

Frames PF1, PF3 and PP5, shown in table (3.1), were designed for the given vertical load by first-order plastic hinge theory assuming mild steel as the structural material. The sections chosen for frame PP5 was based on a height to eaves of 8m. The possible presence of haunches at the eaves (and, in pitched roof frames, at the ridge) was neglected.

Frames PF2, PF4 and PP6 were arbitrary variations of PF1 and PP6. Frame PF2 used a UC section for the legs and the rafter section was chosen to have twice the depth of the column section. A much larger section was chosen for frame PF4. Frame PP6 was similar to frame PP5, but a somewhat smaller section was adopted.



Frames PF7, PP8 and PP9 were devised to study structures in which the plastic moment resistance of the column section was significantly greater than that of the beam or rafter. As the limiting values of  $\alpha_{cr}/\alpha_p$  correspond to frames in which first-order plastic hinge theory may be used, it is more appropriate to take  $M_{Po}/M_{Pg}$  as the measure of the non-uniformity of section, rather  $I_o/I_g$ .

Further frames were devised with fixed bases. The sections chosen for the fixed base frames are presented in table (3.11) and (3.12). In general, frames chosen with fixed bases were identical to the frames in table (3.1). The remaining frames in table (3.11) and (3.12) were variations on frame FF1 or Frame FP1.

### 3.3 Procedure adopted in the studies.

Each frame was subjected to combined vertical and horizontal loading. The failure  $\alpha_f$  was determined from the second order elastic-plastic computer program by Majid-Anderson [23]. The same computer program was used to calculate  $\alpha_p$ . This can be achieved by increasing the stiffness of members using an exaggerated value of Young's modulus  $E$ .  $\alpha_p$  could be calculated quite rapidly by hand but for the large number of structures required to be analysed it was found more convenient to use the computer program. The program automatically calculates the reduction in plastic moment capacity due to axial force.

The elastic critical load  $\alpha_{cr}$  was calculated from the formula given by the ECCS publication [66] for the pinned base and fixed base frames

respectively.

Various combinations of vertical and horizontal loading were examined by varying the ratio of  $H/P$  from 0.1 to 0.4, where  $H$  is the applied horizontal load and  $P$  is the total vertical load on the frame. The vertical load was applied to the frame as a concentrated central point load ( $0.5P$ ), and two column end loads of ( $0.25P$ ) as shown in figures (3.1-3.2).

The ratio of  $H/P$  was limited to 0.4, this being regarded as the maximum side load expected in practice. As the side load increases, it is clear that the sway in the frame will become greater, and for values of  $H/P$  greater than 0.4, the serviceability limit on deflection would be expected to control design.

The ratio of  $H/L$  for each frame was varied from 0.3 to 0.7, where  $h$  is the height of column and  $L$  is the span of the beam.

The aim of the studies was to determine values of  $\alpha_p$  at which  $\alpha_f/\alpha_p=0.9$ . The initial analyses used design strength  $P_y$  of either 250 N/mm<sup>2</sup> or 275 N/mm<sup>2</sup>. The resulting values of  $\alpha_f/\alpha_p$  varied greatly, depending on the susceptibility of each frame to instability. Considering each frame in turn, the design strength was altered and the frame re-analysed. Iteration continued until it was found that  $\alpha_f/\alpha_p$  was approximately 0.9. The corresponding value of  $\alpha_{or}/\alpha_p$  then became

the limiting value for that frame. The device of using artificial  $P_y$  values to find frames for which  $\alpha_f/\alpha_p = 0.9$  was more convenient than adjusting loading or section properties.

Since the limiting values of the frames with the pinned bases are different from those obtained for the fixed base frames, results and conclusions for the former are dealt with first before describing the work on fixed base frames.

### 3.4 Single storey pinned-base frames.

Frames shown in table (3.1) were subjected to the analysis described in the preceding section. The following investigations were made.

#### 3.4.1 Influence of frame dimensions.

A comparison was made between frames of 15m and 20m span to find if the absolute length of members would have a significant influence on the results.

It can be seen from table (3.2) that for the purpose of this study the relationship between  $\alpha_f/\alpha_p$  and  $\alpha_{or}/\alpha_p$  is not unduly dependent on the absolute values of  $h$  and  $L$ .

#### 3.4.2 Influence of section sizes.

Table (3.3) shows a comparison between frames PP5 and PP6. Both have a span of 20m but frame PP6 employs a smaller section. From the results obtained it can be seen that the influence of the size of frame section is small.

Table (3.4) shows a comparison between frames PF3, of uniform section, and frame PF2 in which the column section is smaller than that of the beam. For frame PF2, the ratio of moment resistance of the column,  $M_{Pc}$ , to that of the beam  $M_{Pb}$  is 0.44.

It can be seen from table (3.4) that for  $h/L=0.4$  the agreement between two frames is reasonable. For  $h/L=0.7$  the agreement is not so good. However, in the latter case both frames are very susceptible to instability, as shown by the low value of  $\alpha_r/\alpha_p$ .

It was concluded that within the range of the comparison of table (3.4), namely  $0.44 \leq M_{Pc}/M_{Pb} \leq 1$ , the influence of section sizes can be ignored because;

(i) the limiting values of  $\alpha_{cr}/\alpha_p$  correspond to only a small degree of instability, and

(ii) in making recommendations for design, the limiting values of  $\alpha_{cr}/\alpha_p$  will be rounded up to the nearest whole number.

Frames PF7, PF8 and PF9 are the non uniform frames in which the column section is bigger than that of the beam. Frames PF7 and PF9 employ the

same sections, the moment resistance of the column  $M_{Pc}$  to that one of the beam  $M_{Pg}$  is 1.99 and the corresponding value for frame PP8 is 1.51. It can be seen from tables (3.5-3.7) the limiting values of  $\alpha_{or}/\alpha_p$  are obtained for this groups of the frames are slightly larger than the frames with  $M_{Pc}/M_{Pg} \leq 1$  in tables (3.8-3.9).

Therefore two sets of limits have been proposed, one for  $M_{Pc}/M_{Pg} \leq 1$ , the other for  $1 < M_{Pc}/M_{Pg} < 2$ .

#### 3.4.3 Limiting values of $\alpha_{or}/\alpha_p$

The limiting values of  $\alpha_{or}/\alpha_p$  corresponding to  $M_{Pc}/M_{Pg}$  less than unity are shown in tables (3.8-3.9). These values correspond to  $\alpha_t/\alpha_p=0.9$  approximately. Table (3.8) shows the results for the flat-roofed frames. Tables (3.9) shows the corresponding results for the frames with  $10^\circ$  roof pitch. The recommended limiting values of the latter are of the same order as those for flat-roofed frames.

The limiting value of  $\alpha_{or}/\alpha_p$  corresponding to  $1 < M_{Pc}/M_{Pg} < 2$  are shown in tables (3.5 -3.7). Table (3.5) is for flat roofed frames and tables (3.6-3.7) are for the frames with  $10^\circ$  roof pitch.

It is proposed that the limiting value gives in table (3.10), be used in design of both frames with flat roofs and those whose pitch does not exceed  $10^\circ$ . The limiting results are given for two groups of frames, with  $0.5 \leq M_{Pc}/M_{Pg} \leq 1$  and with  $1 < M_{Pc}/M_{Pg} \leq 2$ . These values have been obtained by taking the larger values of  $\alpha_{or}/\alpha_p$  from tables

(3.5-3.9), whilst retaining the minimum value of  $\alpha_{or}/\alpha_p=5$  recommended by Horne and Morris [14]. The extreme limits of 0.5 and 2.0 on  $M_{Po}/M_{P\pi}$  correspond approximately to the limits of the parametric study.

#### 3.4.4 Conclusion.

It is proposed that first-order plastic hinge theory should be allowed in the design of single bay pinned-base frames provided that the limits in table (3.10) are satisfied.

The limits have been based on parametric studies on frames, whose ranges are given at the head of table. These ranges cover the typical dimensions of present day single storey structures. The limitation of the studies to  $H/P \leq 0.4$  is unlikely to be restrictive in practice because:

- (i) It is difficult to achieve a higher ratio because of the reduction in vertical loading due to wind uplift.
- (ii) For frames with a higher value of  $H/P$ , the need to control deflection at working load will necessitate elastic design.

#### 3.5 Parametric studies on single bay fixed base frame

The studies were conducted on the frames shown in tables (3.11) and (3.12) as described in section 3.2. The procedure adopted for this study is as explained in section 3.3.

### 3.5.1 Flat-roofed frames.

#### 3.5.1.1 Influence of frames dimension.

A comparison was made between frames of span 15m, and frames of span 25m, limiting values of  $\alpha_{or} / \alpha_p$  were obtained for twenty combinations of H/P and h/L. The results are given in tables (3.13) and (3.14).

It is evident from the results obtained that for the purpose of this work the limiting value of  $\alpha_{or} / \alpha_p$  can be assumed to be independent of the absolute value of h and L.

#### 3.5.1.2 Influence of section size.

Frame FF3 retains the same span as FF1, but the larger section size is adopted. The results of this group of frames are shown in table (3.15).

By comparison with FF1 table (3.13), it is concluded that for the purpose of this work the limiting values of  $\alpha_{or} / \alpha_p$  can be assumed to be independent of the size of the section as long as the ratio of  $M_{Po}/M_{Ps}$  is unity, as found with the pinned base frames.

#### 3.5.1.3 Non-uniform frames.

The remaining flat roofed frames FF4-FF10 were chosen to examine the influence of non-uniformity of section. Frame FF4 was subjected to the most extensive study, as shown in table (3.16).

Although for this frame  $I_o/I_g=0.8$ , the use of a UC section for the legs resulted in the ratio of full plastic moments  $M_{F0}/M_{Fg}$  being unity. When the limiting values for this frame were compared with those for the corresponding uniform section frame, FF2, it was found that the two sets of value were of the same order.

Further studies on frames FF5-FF6, given in table (3.17), with  $M_{F0}/M_{Fg}$  being greater than unity but less than or equal to 2. It is evident from these results that the limiting value of  $\alpha_{or}/\alpha_p$  is not affected significantly by section sizes.

### 3.5.2 Frames with $10^\circ$ roof pitch.

#### 3.5.2.1 Influence of frame dimension.

A comparison was made between frame FF2 of span 20m, and frame FF3 of span 10m. From the results obtained in table (3.18) and table (3.19), it is concluded that the limiting value of  $\alpha_{or}/\alpha_p$  can be assumed to be independent of the absolute value of h and L.

#### 3.5.2.2 Influence of section size.



Frame FP3 employs a reduced section by comparison with frame FP1. The ratio of  $M_{Po}/M_{Pg}$  is unity for both the frames. Comparing the results of frame FP3, table (3.20), with frame FP1, table (3.18), it is concluded that for the purpose of this work the limiting values of  $\alpha_{or}/\alpha_p$  can be assumed to be independent of the section as long as a uniform section is used all around the frame.

The remaining pitched roof frames enable the influence of non uniformity in section to be examined.

It can be seen from table (3.21) and table (3.22) that there is a scatter of results. To avoid excessive conservatism, it has been decided to purpose two limiting sets of values. The first is for the frames in which  $M_{Po}/M_{Pg}$  is less than unity. Second group is for frames in which  $M_{Po}/M_{Pg}$  is greater than 1, the maximum value of  $M_{Po}/M_{Pg}$  is equal 2. This should not prove restrictive in practice.

### 3.5.3 Limiting values of $\alpha_{or}/\alpha_p$

For the flat roofed frames ,the results which correspond to  $\alpha_r/\alpha_p$  approximately 0.9 are given in table (3.13), a minimum value of 10 being adopted.

The limiting values of  $\alpha_{or}/\alpha_p$  are strongly influenced by the height span ratio  $h/L$ . It is also evident from this table that as the ratio of  $h/L$  become larger, the influence  $H/P$  become less significant on the

limiting values of  $\alpha_{or} / \alpha_p$ . As an example by referring to table (3.13), for  $h/L=0.7$ , the limiting value of  $\alpha_{or} / \alpha_p$  is being equal to 10 for both  $H/P=0.1$  and  $H/P=0.4$ .

For pitched roofed frames, the limiting values are given in table (3.21) and table (3.22), the choice of table being dependent on whether  $M_{Po}/M_{Pg}$  is less than or greater than unity. The limits are applicable for frames up to  $10^\circ$  pitch.

The limiting value of  $\alpha_{or}/\alpha_p$  is strongly dependent on  $h/L$ . Within the range of  $H/P$  considered, the maximum value of  $\alpha_{or}/\alpha_p$  falls as  $h/L$  increases. As in flat roofed frames the influence of  $H/P$  become less significant on the limiting ratio of  $\alpha_{or}/\alpha_p$  as  $h/L$  increases.

#### 3.5.4 Conclusion

It is proposed that first-order plastic hinge theory should be allowed in the design of single bay fixed base frames, subject to limiting values of  $\alpha_{or} / \alpha_p$  being satisfied.

For the flat roofed frames of uniform section the limiting values are given in table (3.13). For pitched roofed frames, the limiting values are given in table (3.21) and table (3.22).

For both flat roofed and pitched roofed frames the results are applicable provided that  $0.6 < M_{Po}/M_{Pg} < 2.0$ .

| FRAME<br>REF.<br>NO. | PITCH<br>(deg) | L<br>(m) | RAFTER        | COLUMN        | $M_{pc}/M_{pr}$ | P<br>(kN) |
|----------------------|----------------|----------|---------------|---------------|-----------------|-----------|
| PF1                  | 0              | 15       | 254x146x31 UB | 254x146x31 UB | 1.0             | 150       |
| PF2                  | 0              | 15       | 305x127x48 UB | 152x152x37 UC | 0.44            | 150       |
| PF3                  | 0              | 20       | 406x178x54 UB | 406x178x54 UB | 1.0             | 203       |
| PF4                  | 0              | 15       | 457x191x82 UB | 457x191x82 UB | 1.0             | 152       |
| PP5                  | 10             | 20       | 356x171x51 UB | 356x171x51 UB | 1.0             | 216       |
| PP6                  | 10             | 20       | 305x127x42 UB | 305x127x42 UB | 1.0             | 156       |
| PF7                  | 0              | 15       | 254x102x25 UB | 305x127x42 UB | 1.99            | 150       |
| PP8                  | 10             | 20       | 305x102x33 UB | 305x165x46 UB | 1.51            | 216       |
| PP9                  | 10             | 20       | 254x102x25 UB | 305x127x42 UB | 1.99            | 216       |

TABLE (3.1) PINNED BASE FRAMES USED IN PARAMETRIC STUDY

| $h/L$ | $H/P$ | FRAME NO. | DESIGN STRENGTH<br>( $N/mm^2$ ) | $\alpha_f/\alpha_p$ | $\alpha_{cr}/\alpha_p$ |
|-------|-------|-----------|---------------------------------|---------------------|------------------------|
| 0.4   | 0.1   | PF1       | 250                             | 0.97                | 6.4                    |
|       |       | PF3       | 275                             | 0.97                | 6.7                    |
|       | 0.2   | PF1       | 250                             | 0.95                | 7.3                    |
|       |       | PF3       | 275                             | 0.95                | 7.6                    |
|       | 0.3   | PF1       | 250                             | 0.93                | 8.1                    |
|       |       | PF3       | 275                             | 0.93                | 8.5                    |
|       | 0.4   | PF1       | 250                             | 0.92                | 9.4                    |
|       |       | PF3       | 275                             | 0.92                | 9.3                    |
| 0.7   | 0.1   | PF1       | 250                             | 0.91                | 3.0                    |
|       |       | PF3       | 275                             | 0.92                | 3.1                    |
|       | 0.2   | PF1       | 250                             | 0.87                | 3.7                    |
|       |       | PF3       | 275                             | 0.89                | 3.8                    |
|       | 0.3   | PF1       | 250                             | 0.83                | 4.4                    |
|       |       | PF3       | 275                             | 0.83                | 4.5                    |
|       | 0.4   | PF1       | 250                             | 0.81                | 4.8                    |
|       |       | PF3       | 275                             | 0.82                | 5.5                    |

TABLE (3.2) COMPARISON OF FRAMES WITH SPANS OF  
15m AND 20m

| h/L | H/P | FRAME NO. | DESIGN STRENGTH (N/mm <sup>2</sup> ) | $\alpha_f/\alpha_p$ | $\alpha_{or}/\alpha_p$ |
|-----|-----|-----------|--------------------------------------|---------------------|------------------------|
| 0.4 | 0.2 | PP5       | 275                                  | 0.91                | 6.0                    |
|     |     | PP6       | 260                                  | 0.90                | 5.4                    |
|     | 0.4 | PP5       | 210                                  | 0.91                | 9.9                    |
|     |     | PP6       | 180                                  | 0.91                | 9.7                    |
| 0.7 | 0.2 | PP5       | 210                                  | 0.91                | 5.9                    |
|     |     | PP6       | 130                                  | 0.91                | 5.7                    |
|     | 0.4 | PP5       | 135                                  | 0.90                | 10.0                   |
|     |     | PP6       | 115                                  | 0.89                | 9.9                    |

TABLE (3.3) COMPARISON OF PITCHED ROOF FRAMES WITH  
20m SPAN

| $h/L$ | $H/P$ | FRAME<br>NO. | DESIGN<br>STRENGTH<br>( $N/mm^2$ ) | $\alpha_f/\alpha_p$ | $\alpha_{cr}/\alpha_p$ |
|-------|-------|--------------|------------------------------------|---------------------|------------------------|
| 0.4   | 0.2   | PF2          | 250                                | 0.90                | 4.8                    |
|       |       | PF3          | 450                                | 0.91                | 4.6                    |
|       | 0.4   | PF2          | 250                                | 0.86                | 7.6                    |
|       |       | PF3          | 355                                | 0.90                | 7.3                    |
| 0.7   | 0.2   | PF2          | 250                                | 0.69                | 2.4                    |
|       |       | PF3          | 450                                | 0.77                | 2.3                    |
|       | 0.4   | PF2          | 250                                | 0.83                | 4.8                    |
|       |       | PF3          | 275                                | 0.82                | 5.5                    |

TABLE (3.4) COMPARISON OF UNIFORM SECTION AND  
UNEQUAL SECTION FRAMES

| $\frac{H}{P_c}$ | $\frac{h}{L}$ | $\alpha_{cr}$ | $P_y$<br>(N/mm <sup>2</sup> ) | $\frac{\alpha_f}{\alpha_p}$ | $\frac{\alpha_{cr}}{\alpha_p}$ | Proposed limit<br>on $\alpha_{cr}/\alpha_p$ |
|-----------------|---------------|---------------|-------------------------------|-----------------------------|--------------------------------|---|
| 0.3             | 0.3           | 5.5           | 450                           | 0.91                        | 7.7                            | 13  |
|                 | 0.7           | 1.9           | 180                           | 0.91                        | 8.7                            |   |
| 0.4             | 0.3           | 5.5           | 410                           | 0.91                        | 9.1                            | 15  |
|                 | 0.4           | 3.9           | 310                           | 0.91                        | 9.4                            |   |
|                 | 0.5           | 2.9           | 270                           | 0.90                        | 8.9                            |   |
|                 | 0.6           | 2.3           | 140                           | 0.91                        | 14.6                           |   |
|                 | 0.7           | 1.9           | 150                           | 0.90                        | 12.7                           |   |

TABLE (3.5) LIMITING VALUES OF  $\alpha_{cr}/\alpha_p$  FOR FRAME PF7

| $\frac{H}{P}$ | $\frac{h}{L}$ | $\alpha_{cr}$ | $P_y$<br>(N/mm <sup>2</sup> ) | $\frac{\alpha_f}{\alpha_p}$ | $\frac{\alpha_{cr}}{\alpha_p}$ | Proposed limit<br>on $\alpha_{cr}/\alpha_p$ |
|---------------|---------------|---------------|-------------------------------|-----------------------------|--------------------------------|---|
| 0.1           | 0.3           | 3.7           | 440                           | 0.91                        | 4.5                            | 5   |
|               | 0.4           | 2.5           | 390                           | 0.89                        | 3.8                            |   |
|               | 0.5           | 1.9           | 300                           | 0.89                        | 3.8                            |   |
|               | 0.6           | 1.4           | 210                           | 0.91                        | 4.4                            |   |
|               | 0.7           | 1.2           | 180                           | 0.90                        | 4.3                            |   |
| 0.2           | 0.3           | 3.7           | 330                           | 0.91                        | 6.7                            | 8   |
|               | 0.4           | 2.5           | 240                           | 0.91                        | 7.0                            |   |
|               | 0.5           | 1.9           | 210                           | 0.90                        | 6.4                            |   |
|               | 0.6           | 1.4           | 170                           | 0.90                        | 6.5                            |   |
|               | 0.7           | 1.2           | 140                           | 0.90                        | 6.7                            |   |
| 0.3           | 0.3           | 3.7           | 270                           | 0.90                        | 9.0                            | 13  |
|               | 0.4           | 2.5           | 220                           | 0.90                        | 8.6                            |   |
|               | 0.5           | 1.9           | 170                           | 0.90                        | 8.9                            |   |
|               | 0.6           | 1.4           | 140                           | 0.90                        | 9.2                            |   |
|               | 0.7           | 1.2           | 100                           | 0.89                        | 11.1                           |   |
| 0.4           | 0.3           | 3.7           | 220                           | 0.91                        | 12.1                           | 15  |
|               | 0.4           | 2.5           | 170                           | 0.91                        | 12.2                           |   |
|               | 0.5           | 1.9           | 125                           | 0.90                        | 13.7                           |   |
|               | 0.6           | 1.4           | 110                           | 0.90                        | 14.1                           |   |
|               | 0.7           | 1.2           | 115                           | 0.90                        | 12.6                           |   |

TABLE (3.6) LIMITING VALUES OF  $\alpha_{cr}/\alpha_p$  FOR FRAME PP8



| $\frac{H}{P}$ | $\frac{h}{L}$ | $\alpha_{cr}$ | $P_y$<br>(N/mm <sup>2</sup> ) | $\frac{\alpha_f}{\alpha_p}$ | $\frac{\alpha_{cr}}{\alpha_p}$ | PROPOSED<br>LIMIT ON<br>$\alpha_{cr}/\alpha_p$ |
|---------------|---------------|---------------|-------------------------------|-----------------------------|--------------------------------|--|
| 0.1           | 0.3           | 2.2           | 380                           | 0.90                        | 4.8                            | 5  |
|               | 0.4           | 1.5           | 300                           | 0.89                        | 4.6                            |  |
|               | 0.5           | 1.1           | 230                           | 0.91                        | 4.8                            |  |
|               | 0.6           | 0.89          | 190                           | 0.90                        | 4.7                            |  |
|               | 0.7           | 0.73          | 160                           | 0.91                        | 4.8                            |  |
| 0.2           | 0.3           | 2.2           | 290                           | 0.89                        | 7.0                            | 8  |
|               | 0.4           | 1.5           | 200                           | 0.91                        | 7.8                            |  |
|               | 0.5           | 1.1           | 170                           | 0.90                        | 7.5                            |  |
|               | 0.6           | 0.89          | 140                           | 0.91                        | 7.6                            |  |
|               | 0.7           | 0.73          | 110                           | 0.91                        | 8.5                            |  |
| 0.3           | 0.3           | 2.2           | 240                           | 0.89                        | 9.3                            | 13   |
|               | 0.4           | 1.5           | 180                           | 0.90                        | 9.8                            |  |
|               | 0.5           | 1.1           | 130                           | 0.91                        | 11.3                           |  |
|               | 0.6           | 0.89          | 120                           | 0.91                        | 10.3                           |  |
|               | 0.7           | 0.73          | 70                            | 0.91                        | 13.7                           |  |
| 0.4           | 0.3           | 2.2           | 220                           | 0.89                        | 11.0                           | 15   |
|               | 0.4           | 1.5           | 160                           | 0.90                        | 12.1                           |  |
|               | 0.5           | 1.1           | 120                           | 0.90                        | 13.7                           |  |
|               | 0.6           | 0.89          | 100                           | 0.90                        | 15.1                           |  |
|               | 0.7           | 0.73          | 105                           | 0.91                        | 13.8                           |  |

TABLE (3.7) LIMITING VALUES OF  $\alpha_{cr}/\alpha_p$  FOR FRAME PP9

| H/P | h/L | FRAME<br>TYPE | DESIGN<br>STRENGTH<br>(N/mm <sup>2</sup> ) | $\alpha_f / \alpha_p$ | $\alpha_{or} / \alpha_p$ | PROPOSED<br>LIMIT ON<br>$\alpha_{or} / \alpha_p$ |
|-----|-----|---------------|--|-----------------------|--------------------------|--|
| 0.1 | 0.3 | PF2           | 600  | 0.89                  | 2.8                      | 3  |
|     | 0.4 | PF1           | 600  | 0.92                  | 2.7                      |  |
|     | 0.5 | PF2           | 250  | 0.90                  | 2.9                      |  |
|     | 0.6 | PF1           | 355  | 0.90                  | 2.6                      |  |
|     | 0.7 | PF1           | 250  | 0.91                  | 3.0                      |  |
| 0.2 | 0.3 | PF1           | 600  | 0.90                  | 4.3                      | 5  |
|     | 0.4 | PF2           | 250  | 0.90                  | 4.8                      |  |
|     | 0.5 | PF1           | 275  | 0.92                  | 4.9                      |  |
|     | 0.6 | PF1           | 250  | 0.90                  | 4.5                      |  |
|     | 0.7 | PF3           | 275  | 0.89                  | 3.8                      |  |
| 0.3 | 0.3 | PF2           | 285  | 0.91                  | 7.1                      | 8  |
|     | 0.4 | PF3           | 450  | 0.89                  | 5.2                      |  |
|     | 0.5 | PF1           | 250  | 0.91                  | 6.4                      |  |
|     | 0.6 | PF1           | 250  | 0.89                  | 5.2                      |  |
|     | 0.7 | PF1           | 150  | 0.91                  | 7.2                      |  |
| 0.4 | 0.3 | PF2           | 250  | 0.86                  | 9.4                      | 11   |
|     | 0.4 | PF3           | 355  | 0.90                  | 7.3                      |  |
|     | 0.5 | PF1           | 255  | 0.89                  | 7.7                      |  |
|     | 0.6 | PF4           | 250  | 0.90                  | 10.3                     |  |
|     | 0.7 | PF4           | 250  | 0.88                  | 9.2                      |  |

TABLE (3.8) LIMITING VALUES OF  $\alpha_p / \alpha_{or}$  FOR  
FLAT-ROOFED FRAMES

| H/P | h/L | DESIGN<br>STRENGTH<br>(N/mm <sup>2</sup> ) | $\alpha_t/\alpha_p$ | $\alpha_{or}/\alpha_p$ | PROPOSED<br>LIMIT ON<br>$\alpha_{or}/\alpha_p$ |
|-----|-----|--|---------------------|------------------------|--|
| 0.1 | 0.3 | 525  | 0.91                | 3.9                    | 4  |
|     | 0.4 | 400  | 0.91                | 3.7                    |  |
|     | 0.5 | 350  | 0.90                | 3.2                    |  |
|     | 0.6 | 285  | 0.90                | 3.1                    |  |
|     | 0.7 | 210  | 0.91                | 3.4                    |  |
| 0.2 | 0.3 | 375  | 0.90                | 6.1                    | 7  |
|     | 0.4 | 275  | 0.91                | 6.0                    |  |
|     | 0.5 | 240  | 0.91                | 5.4                    |  |
|     | 0.6 | 200  | 0.89                | 5.3                    |  |
|     | 0.7 | 150  | 0.90                | 5.8                    |  |
| 0.3 | 0.3 | 310  | 0.90                | 8.0                    | 9  |
|     | 0.4 | 250  | 0.89                | 7.4                    |  |
|     | 0.5 | 200  | 0.89                | 7.4                    |  |
|     | 0.6 | 152  | 0.90                | 8.1                    |  |
|     | 0.7 | 132  | 0.87                | 8.0                    |  |
| 0.4 | 0.3 | 275  | 0.87                | 9.8                    | 10   |
|     | 0.4 | 210  | 0.91                | 9.9                    |  |
|     | 0.5 | 170  | 0.88                | 9.8                    |  |
|     | 0.6 | 150  | 0.88                | 9.9                    |  |
|     | 0.7 | 135  | 0.90                | 10.0                   |  |

TABLE (3.9) LIMITING VALUES OF  $\alpha_p/\alpha_{or}$  FOR PITCHED ROOF  
FRAMES BASED ON ANALYSIS OF FRAMES PP5

$$0.3 < h/L < 0.7$$

$$\text{PITCH} < 10^\circ$$

| H/P   | $\alpha_{cr}/\alpha_p$ FOR $M_{pc}/M_{ps} < 1$ | $\alpha_{cr}/\alpha_p$ FOR $1 < M_{pc}/M_{ps} < 2$ |
|-------|--|--|
| < 0.1 | 5  | 5  |
| 0.2   | 7  | 8  |
| 0.3   | 9  | 13   |
| 0.4   | 11   | 15   |

TABLE (3.10) LIMITING VALUES OF  $\alpha_{cr}/\alpha_p$  FOR DESIGN BY FIRST-ORDER PLASTIC HINGE THEORY FOR SINGLE STOREY PINNED BASE FRAMES.

| FRAME<br>REF NO. | RAFTER        | COLUMN        | $I_c/I_s$ | $M_{pc}/M_{ps}$ | L<br>(m) | P<br>(kN) |
|------------------|---------------|---------------|-----------|-----------------|----------|-----------|
| FF1              | 254x146x43 UB | 254x146x43 UB | 1.00      | 1.00            | 15       | 150       |
| FF2              | 254x146x43 UB | 254x146x43 UB | 1.00      | 1.00            | 25       | 250       |
| FF3              | 305x127x48 UB | 305x127x48 UB | 1.00      | 1.00            | 15       | 150       |
| FF4              | 254x146x43 UB | 203x203x52 UC | 0.80      | 1.00            | 25       | 250       |
| FF5              | 254x146x31 UB | 203x203x52 UC | 1.18      | 1.43            | 25       | 250       |
| FF6              | 254x146x37 UB | 203x203x86 UC | 1.70      | 2.02            | 25       | 250       |

TABLE (3.11) FLAT ROOFED FIXED BASE FRAMES

| FRAME<br>REF NO. | RAFTER        | COLUMN        | $I_c/I_g$ | $M_{pc}/M_{pg}$ | L<br>(m) | P<br>(kN) |
|------------------|---------------|---------------|-----------|-----------------|----------|-----------|
| FP1              | 356x171x51 UB | 356x171x51 UB | 1.00      | 1.00            | 20       | 216       |
| FP2              | 356x171x51 UB | 356x171x51 UB | 1.00      | 1.00            | 20       | 350       |
| FP3              | 305x127x42 UB | 305x127x42 UB | 1.00      | 1.00            | 20       | 216       |
| FP4              | 254x146x37 UB | 152x152x37 UC | 0.40      | 0.64            | 20       | 216       |
| FP5              | 254x146x31 UB | 152x152x37 UC | 0.50      | 0.78            | 20       | 216       |
| FP6              | 254x146x43 UB | 203x203x52 UC | 0.80      | 1.00            | 20       | 216       |
| FP7              | 254x146x31 UB | 203x203x52 UC | 1.18      | 1.43            | 20       | 216       |
| FP8              | 254x146x31 UB | 254x146x43 UB | 1.47      | 1.43            | 20       | 216       |
| FP9              | 254x102x25 UB | 203x203x52 UC | 1.54      | 1.87            | 20       | 216       |
| FP10             | 254x102x25 UB | 203x203x71 UC | 2.24      | 2.62            | 20       | 216       |

TABLE (3.12) FIXED BASE FRAMES WITH 10° ROOF PITCH

| $\frac{H}{P}$ | $\frac{h}{L}$ | $\alpha_{cr}$ | Design strength<br>(N/mm <sup>2</sup> ) | $\frac{\alpha_f}{\alpha_p}$ | $\frac{\alpha_{cr}}{\alpha_p}$ | Proposed limit<br>on $\frac{\alpha_{cr}}{\alpha_p}$ |
|---------------|---------------|---------------|---|-----------------------------|--------------------------------|---|
| 0.1           | 0.3           | 42.7          | 550                                     | 0.90                        | 19.4                           | 20  |
|               | 0.4           | 26.5          | 500                                     | 0.91                        | 13.1                           | 14  |
|               | 0.5           | 18.2          | 450                                     | 0.90                        | 10.0                           | 10  |
|               | 0.6           | 13.4          | 420                                     | 0.90                        | 7.9                            | 10  |
|               | 0.7           | 10.3          | 350                                     | 0.91                        | 7.3                            | 10  |
| 0.2           | 0.3           | 42.7          | 450                                     | 0.90                        | 23.6                           | 27  |
|               | 0.4           | 26.5          | 400                                     | 0.89                        | 16.4                           | 18  |
|               | 0.5           | 18.2          | 350                                     | 0.89                        | 12.9                           | 16  |
|               | 0.6           | 13.4          | 250                                     | 0.90                        | 13.4                           | 15  |
|               | 0.7           | 10.3          | 200                                     | 0.90                        | 13.2                           | 14  |
| 0.3           | 0.3           | 42.7          | 350                                     | 0.89                        | 30.3                           | 36  |
|               | 0.4           | 26.5          | 300                                     | 0.89                        | 21.8                           | 25  |
|               | 0.5           | 18.2          | 325                                     | 0.89                        | 14.8                           | 18  |
|               | 0.6           | 13.4          | 310                                     | 0.90                        | 12.3                           | 13  |
|               | 0.7           | 10.3          | 300                                     | 0.91                        | 10.4                           | 10  |
| 0.4           | 0.3           | 42.7          | 250                                     | 0.91                        | 42.7                           | 45  |
|               | 0.4           | 26.5          | 300                                     | 0.91                        | 23.9                           | 24  |
|               | 0.5           | 18.2          | 400                                     | 0.91                        | 13.6                           | 14  |
|               | 0.6           | 13.4          | 450                                     | 0.91                        | 9.6                            | 10  |
|               | 0.7           | 10.3          | 500                                     | 0.89                        | 7.2                            | 10  |

TABLE (3.13) LIMITING VALUES OF  $\alpha_{cr}/\alpha_p$  FOR FRAME FP1

| $\frac{H}{P}$ | $\frac{h}{L}$ | $\alpha_{cr}$ | Design strength<br>(N/mm <sup>2</sup> ) | $\frac{\alpha_f}{\alpha_p}$ | $\frac{\alpha_{cr}}{\alpha_p}$ | Proposed limit<br>on $\frac{\alpha_{cr}}{\alpha_p}$ |
|---------------|---------------|---------------|---|-----------------------------|--------------------------------|---|
| 0.1           | 0.3           | 9.2           | 300                                     | 0.91                        | 21.1                           | 20  |
|               | 0.4           | 5.7           | 300                                     | 0.91                        | 13.1                           | 14  |
|               | 0.5           | 3.9           | 280                                     | 0.91                        | 9.7                            | 10  |
|               | 0.6           | 2.9           | 270                                     | 0.90                        | 7.4                            | 10  |
|               | 0.7           | 2.2           | 220                                     | 0.89                        | 7.0                            | 10  |
| 0.2           | 0.3           | 9.2           | 240                                     | 0.91                        | 26.5                           | 27  |
|               | 0.4           | 5.7           | 220                                     | 0.90                        | 17.8                           | 18  |
|               | 0.5           | 3.9           | 200                                     | 0.89                        | 13.5                           | 16  |
|               | 0.6           | 2.9           | 130                                     | 0.90                        | 15.3                           | 15  |
|               | 0.7           | 2.2           | 120                                     | 0.90                        | 13.2                           | 14  |
| 0.3           | 0.3           | 9.2           | 170                                     | 0.91                        | 37.4                           | 36  |
|               | 0.4           | 5.7           | 170                                     | 0.90                        | 23.1                           | 25  |
|               | 0.5           | 3.9           | 165                                     | 0.90                        | 17.5                           | 18  |
|               | 0.6           | 2.9           | 175                                     | 0.91                        | 13.1                           | 13  |
|               | 0.7           | 2.2           | 180                                     | 0.91                        | 10.4                           | 10  |
| 0.4           | 0.3           | 9.2           | 140                                     | 0.91                        | 45.5                           | 45  |
|               | 0.4           | 5.7           | 200                                     | 0.90                        | 21.5                           | 24  |
|               | 0.5           | 3.9           | 240                                     | 0.90                        | 13.5                           | 14  |
|               | 0.6           | 2.9           | 270                                     | 0.91                        | 9.6                            | 10  |
|               | 0.7           | 2.2           | 260                                     | 0.91                        | 8.3                            | 10  |

TABLE (3.14) LIMITING VALUES OF  $\alpha_{cr}/\alpha_p$  FOR FRAME PF2



| $\frac{H}{P}$ | $\frac{h}{L}$ | $\alpha_{cr}$ | Design strength<br>(N/mm <sup>2</sup> ) | $\frac{\alpha_f}{\alpha_p}$ | $\frac{\alpha_{cr}}{\alpha_p}$ | Proposed limit<br>on $\frac{\alpha_{cr}}{\alpha_p}$ |
|---------------|---------------|---------------|---|-----------------------------|--------------------------------|---|
| 0.2           | 0.3           | 61.9          | 500                                     | 0.90                        | 24.8                           | 27  |
|               | 0.4           | 38.3          | 460                                     | 0.89                        | 16.7                           | 18  |
|               | 0.5           | 26.4          | 400                                     | 0.89                        | 13.2                           | 16  |
|               | 0.6           | 19.4          | 280                                     | 0.90                        | 13.8                           | 15  |
|               | 0.7           | 15.0          | 220                                     | 0.91                        | 14.1                           | 14  |

TABLE (3.15) LIMITING VALUES OF  $\alpha_{cr}/\alpha_p$  FOR FRAME FF3,  $H/P=0.2$

| $\frac{H}{P}$ | $\frac{h}{L}$ | $P_y$<br>(N/mm <sup>2</sup> ) | $\frac{\alpha_f}{\alpha_p}$ | $\frac{\alpha_{cr}}{\alpha_p}$ | Proposed<br>Limit |
|---------------|---------------|-------------------------------|-----------------------------|--------------------------------|-------------------|
| 0.1           | 0.3           | 280                           | 0.90                        | 19.6                           | 20                |
|               | 0.4           | 280                           | 0.90                        | 12.1                           | 14                |
|               | 0.5           | 280                           | 0.90                        | 8.3                            | 10                |
|               | 0.6           | 200                           | 0.90                        | 8.4                            | 10                |
|               | 0.7           | 175                           | 0.89                        | 7.4                            | 10                |
| 0.2           | 0.3           | 220                           | 0.90                        | 24.9                           | 27                |
|               | 0.4           | 190                           | 0.90                        | 17.8                           | 18                |
|               | 0.5           | 160                           | 0.90                        | 14.5                           | 16                |
|               | 0.6           | 120                           | 0.90                        | 14.3                           | 15                |
|               | 0.7           | 100                           | 0.89                        | 13.5                           | 14                |
| 0.3           | 0.3           | 160                           | 0.91                        | 34.3                           | 36                |
|               | 0.7           | 160                           | 0.89                        | 10                             | 10                |
| 0.4           | 0.3           | 140                           | 0.90                        | 39.3                           | 45                |
|               | 0.7           | 260                           | 0.90                        | 7.1                            | 10                |

TABLE (3.16) LIMITING VALUES OF  $\alpha_{cr}/\alpha_p$  FOR FRAME FP4

Frame Ref. No  
M<sub>pc</sub>/M<sub>pg</sub>

F5  
1.43

F6  
2.02

Proposed limit on  
 $\alpha_{cr}/\alpha_p$

| H/P | h/L        | Limiting values of $a_{cr}/a_p$ |              |          |
|-----|------------|---------------------------------|--------------|----------|
| 0.1 | 0.3<br>0.7 | 22<br>7                         | 20<br>8.1    | 20<br>10 |
| 0.2 | 0.3<br>0.7 | 28.7<br>14.1                    | 26.5<br>9.1  | 27<br>14 |
| 0.3 | 0.3<br>0.7 | 34.3<br>10.5                    | 30.1<br>10.5 | 36<br>10 |
| 0.4 | 0.3<br>0.7 | 45.7<br>8.3                     | 43.25<br>9.1 | 45<br>10 |

Table (3.17) Limiting values of  $\alpha_{cr}/\alpha_p$  for non-uniform frames F5 and F6.

| $\frac{H}{P}$ | $\frac{h}{L}$ | $\alpha_{cr}$ | Design strength<br>(N/mm <sup>2</sup> ) | $\frac{\alpha_f}{\alpha_p}$ | $\frac{\alpha_{cr}}{\alpha_p}$ |
|---------------|---------------|---------------|---|-----------------------------|--------------------------------|
| 0.1           | 0.3           | 36.0          | 600                                     | 0.91                        | 13.9                           |
|               | 0.4           | 22.3          | 600                                     | 0.89                        | 9.1                            |
|               | 0.5           | 15.4          | 520                                     | 0.89                        | 7.5                            |
|               | 0.6           | 11.3          | 350                                     | 0.89                        | 8.4                            |
|               | 0.7           | 8.7           | 220                                     | 0.90                        | 10.5                           |
| 0.2           | 0.3           | 36.0          | 430                                     | 0.89                        | 18.6                           |
|               | 0.4           | 22.3          | 300                                     | 0.89                        | 18.2                           |
|               | 0.5           | 15.4          | 200                                     | 0.90                        | 19.5                           |
|               | 0.6           | 11.3          | 150                                     | 0.91                        | 20.3                           |
|               | 0.7           | 8.7           | 170                                     | 0.90                        | 14.7                           |
| 0.3           | 0.3           | 36.0          | 230                                     | 0.91                        | 36.2                           |
|               | 0.4           | 22.3          | 270                                     | 0.90                        | 22.3                           |
|               | 0.5           | 15.4          | 280                                     | 0.90                        | 15.7                           |
|               | 0.6           | 11.3          | 320                                     | 0.90                        | 11.1                           |
|               | 0.7           | 8.7           | 370                                     | 0.89                        | 8.0                            |
| 0.4           | 0.3           | 36.0          | 300                                     | 0.91                        | 29.8                           |
|               | 0.4           | 22.3          | 400                                     | 0.91                        | 16.0                           |
|               | 0.5           | 15.4          | 480                                     | 0.91                        | 10.3                           |
|               | 0.6           | 11.3          | 400                                     | 0.89                        | 10.1                           |
|               | 0.7           | 8.7           | 250                                     | 0.91                        | 13.6                           |

TABLE (3.18) LIMITING VALUES OF  $\alpha_{cr}/\alpha_p$  FOR FRAME FP1

| $\frac{H}{P}$ | $\frac{h}{L}$ | $\alpha_{cr}$ | Design strength<br>(N/mm <sup>2</sup> ) | $\frac{\alpha_f}{\alpha_p}$ | $\frac{\alpha_{cr}}{\alpha_p}$ |
|---------------|---------------|---------------|---|-----------------------------|--------------------------------|
| 0.1           | 0.3           | 144           | 1300                                    | 0.91                        | 13.1                           |
|               | 0.7           | 34.8          | 500                                     | 0.89                        | 9.3                            |
| 0.2           | 0.3           | 144           | 900                                     | 0.89                        | 18.5                           |
|               | 0.7           | 34.8          | 350                                     | 0.90                        | 14.5                           |
| 0.3           | 0.3           | 144           | 500                                     | 0.91                        | 34.1                           |
|               | 0.7           | 34.8          | 700                                     | 0.90                        | 8.5                            |
| 0.4           | 0.3           | 144           | 600                                     | 0.91                        | 30.1                           |
|               | 0.7           | 34.8          | 500                                     | 0.91                        | 13.7                           |

TABLE (3.19) LIMITING VALUES OF  $\alpha_{cr}/\alpha_p$  FOR FRAME FP2

| $\frac{H}{P}$ | $\frac{h}{L}$ | $\alpha_{cr}$ | Design strength<br>(N/mm <sup>2</sup> ) | $\frac{\alpha_f}{\alpha_p}$ | $\frac{\alpha_{cr}}{\alpha_p}$ |
|---------------|---------------|---------------|---|-----------------------------|--------------------------------|
| 0.1           | 0.3           | 20.7          | 500                                     | 0.91                        | 14.0                           |
|               | 0.7           | 5.0           | 180                                     | 0.91                        | 10.7                           |
| 0.2           | 0.3           | 20.7          | 370                                     | 0.89                        | 19.0                           |
|               | 0.7           | 5.0           | 160                                     | 0.89                        | 13.2                           |
| 0.3           | 0.3           | 20.7          | 200                                     | 0.91                        | 35.1                           |
|               | 0.7           | 5.0           | 300                                     | 0.90                        | 8.3                            |
| 0.4           | 0.3           | 20.7          | 250                                     | 0.91                        | 30.1                           |
|               | 0.7           | 5.0           | 210                                     | 0.91                        | 13.7                           |

TABLE (3.20) LIMITING VALUES OF  $\alpha_{cr}/\alpha_p$  FOR FRAME FP3

| Frame Ref. No.<br>$M_{pc}/M_{pg}$ |               | P7<br>1.43                                | P8<br>1.43 | P9<br>1.87 | P10<br>2.62 | PROPOSED<br>LIMIT ON<br>$\alpha_{cr}/\alpha_p$ |
|-----------------------------------|---------------|---|------------|------------|-------------|--|
| $\frac{H}{P}$                     | $\frac{h}{L}$ | Limiting values of $\alpha_{cr}/\alpha_p$ |            |            |             |  |
| 0.1                               | 0.3           | 14.0                                      | 16.3       | 21.0       | 21.6        | 21   |
|                                   | 0.4           | 10.4                                      | 10.7       | 15.8       |             | 21   |
|                                   | 0.5           | 8.1                                       | 8.6        | 12.3       | 26.1        | 17   |
|                                   | 0.6           | 7.6                                       | 7.3        | 9.4        | 16.1        | 12   |
|                                   | 0.7           | 7.8                                       | 7.5        | 6.9        | 10.9        | 10   |
| 0.2                               | 0.3           | 19.7                                      | 18.3       | 18.8       | 23.4        | 21   |
|                                   | 0.4           | 14.5                                      | 15.1       | 13.5       | 23.1        | 17   |
|                                   | 0.5           | 15.8                                      | 14.5       | 12.2       | 16.1        | 17   |
|                                   | 0.6           | 16.1                                      | 15.5       | 13.1       | 12.3        | 17   |
|                                   | 0.7           | 16.3                                      | 16.1       | 12.6       | 11.1        | 17   |
| 0.3                               | 0.3           | 27.4                                      | 28.2       | 21.6       | 28.1        | 28   |
|                                   | 0.4           | 26.1                                      | 24.2       | 21.5       | 17.4        | 26   |
|                                   | 0.5           | 21.7                                      | 20.6       | 20.1       | 16.1        | 22   |
|                                   | 0.6           | 19.6                                      | 18.9       | 18.7       | 16.1        | 20   |
|                                   | 0.7           | 15.8                                      | 14.0       | 19.6       | 17.1        | 20   |
| 0.4                               | 0.3           | 37.2                                      | 36.6       | 35.7       | 30.9        | 38   |
|                                   | 0.4           | 29.2                                      | 25.4       | 29.8       | 23.1        | 30   |
|                                   | 0.5           | 18.9                                      | 16.4       | 22.6       | 21.0        | 23   |
|                                   | 0.6           | 13.9                                      | 13.5       | 18.3       | 23.0        | 20   |
|                                   | 0.7           | 11.8                                      | 10.1       | 15.6       | 22.9        | 18   |

TABLE (3.22) LIMITING VALUES OF  $\alpha_{cr}/\alpha_p$  FOR  $1 < M_{pc}/M_{pg} < 2.0$

| Frame Ref. No.<br>$M_{pc}/M_{pg}$ |               | P4<br>0.64                                | P5<br>0.78 | P6<br>1.00 | P1<br>1.00 | Proposed limit<br>on $\alpha_{cr}/\alpha_p$ |
|-----------------------------------|---------------|---|------------|------------|------------|---|
| $\frac{H}{P}$                     | $\frac{h}{L}$ | Limiting values of $\alpha_{cr}/\alpha_p$ |            |            |            |   |
| 0.1                               | 0.3           | 9.6                                       | 9.5        | 13.1       | 13.9       | 14  |
|                                   | 0.4           |   | 7.8        | 8.6        | 9.1        | 11  |
|                                   | 0.5           |   | 7.7        | 8.1        | 7.5        | 11  |
|                                   | 0.6           |   | 9.6        | 10.2       | 8.4        | 11  |
|                                   | 0.7           |   | 11.0       | 9.6        | 10.5       | 11  |
| 0.2                               | 0.3           | 15.4                                      | 18.5       | 19.4       | 18.6       | 20  |
|                                   | 0.4           |   | 22.0       | 22.9       | 18.2       | 23  |
|                                   | 0.5           |   | 19.6       | 18.8       | 19.5       | 20  |
|                                   | 0.6           |   | 17.0       | 17.2       | 20.3       | 20  |
|                                   | 0.7           | 10.9                                      | 13.7       | 14.1       | 14.7       | 15  |
| 0.3                               | 0.3           |   | 35.8       | 32.8       | 36.2       | 36  |
|                                   | 0.4           |   | 20.0       | 24.2       | 22.3       | 25  |
|                                   | 0.5           |   | 12.5       | 18.4       | 15.7       | 19  |
|                                   | 0.6           |   | 8.5        | 12.3       | 11.1       | 13  |
|                                   | 0.7           |   | 7.3        | 9.2        | 8.0        | 10  |
| 0.4                               | 0.3           | 17.9                                      | 20.3       | 28.1       | 29.8       | 30  |
|                                   | 0.4           |   | 13.2       | 18.0       | 16.0       | 18  |
|                                   | 0.5           |   | 10.1       | 10.4       | 10.3       | 11  |
|                                   | 0.6           |   | 10.1       | 10.1       | 10.1       | 11  |
|                                   | 0.7           |   | 13.5       | 14.7       | 13.6       | 15  |

TABLE (3.21) LIMITING VALUES OF  $\alpha_{cr}/\alpha_p$  FOR  $M_{pc}/M_{pg} < 1.0$

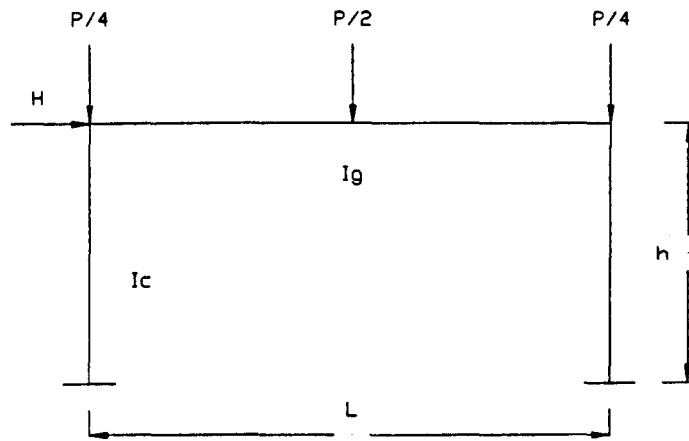


FIG. (3.1) FLAT ROOF FRAMES

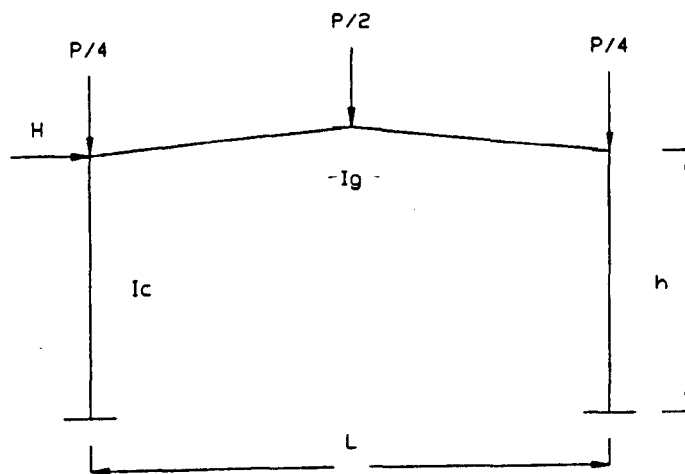


FIG. (3.2) PITCHED ROOF FRAMES



## CHAPTER 4

### ELASTIC-PLASTIC ANALYSIS OF FRAMES WITH SEMI-RIGID CONNECTIONS

#### 4.1 Introduction

Traditional approaches to steel frame design generally neglect the actual behaviour of the connection and refer to two cases, that the end connection of members behave as either fully-rigid or pinned. The use of ideally pinned conditions implies that no moment will be transmitted between the beam and column. As far as the rotation is concerned, the beam and the column that are jointed together by a pin will behave independently. At the other extreme, the use of fully rigid conditions implies that no relative rotation will occur between the adjoining members. Although the use of these idealized joint behaviour simplifies drastically the analysis and design procedures, the predicted response of the frame may not be realistic, as most connections used in actual practice transmit some moment, and experience some deformation upon loading. Thus, the ideally pinned and fully rigid joint assumptions represents only extreme conditions which are rarely encountered in real structures.

The semi-rigid connection or partial strength connection are the terms used to describe the true behaviour of the connection. The objective of development of semi-rigid connections is to produce a better struc-

ture, either are that is more economic or offers improved performance. Economy is achieved by reducing the labour cost of the connections (the cost of the labour has increased far more rapidly than the cost of the material). Improved performance is also achieved in some aspects of serviceability, such as reduced deflection.

Realizing the importance of connection as a structural element, considerable analytical and experimental research has been carried out to measure the moment-relative rotation characteristics of various types of commonly used framing connections. Several types of common building connection are shown in fig (4.1).

Once suitable data for the moment-rotation characteristic of connections are established, it is possible to incorporate the effect of connection flexibility into the analysis by including the complete non-linear moment-rotation characteristics, generally obtained from experimental data or prediction.

The objective of the present Chapter is to develop a computer program which can analyse the frames with semi-rigid connections up to collapse load. The program is for the ultimate load analysis of plane frame with semi-rigid beam-to-column joints. The program permits discrete plastic hinges to form in members. The second order effects are taken into account. At each load level convergence is achieved when the axial force is within a suitable tolerance. The program is capable of analysing any combination of pinned connection, fully rigid joints and connection with any specified moment-rotation relationship.

Analyses of semi-rigid frames are shown for comparison with assumed fully rigid analysis. Comparison is also made with the results of other researchers.

#### 4.2 Joint behaviour and modelling

A review of available methods to determine the  $M-\phi$  curves of connections including mathematical modelling of these joints has already been described in Chapter 1.

From tests results it is found that the behaviour of the joint is non-linear over the whole range of loading. Therefore it is necessary to represent the connection  $M-\phi$  characteristic in a convenient but reasonably accurate form. Early schemes using linear representations only cover initial connection stiffness. Use of the initial stiffness leads to an over estimate of the stability of the frame, but it is very easy to incorporate into the analysis and no iteration would be necessary. Such representation may be suitable for analysis under serviceability loads, where the moments at the connections are within the first portion of non-linear  $M-\phi$  curves.

Certain forms of connections, do actually possess  $M-\phi$  characteristics which correspond quite closely to a linear curve. But it is by no means certain that such  $M-\phi$  curve will always prove sufficient.

In this work the non-linear  $M-\phi$  curves are represented in the form of straight lines. This leads to a more accurate representation of the

M- $\phi$  characteristic. In this manner, the complete loading history of the joint can be realistically followed up to the collapse load.

#### 4.3 The methods of analysis.

A review of the different methods of analysis is given in Chapter 1 . The analysis described here is a non-linear elasto-plastic approach taking into account the effects of joint flexibility. In order to carry out such an analysis, a computer program was developed by the author. The program is an extension of an elasto-plastic program for rigidly connected frames by Majid and Anderson [23].

The program described here uses successive estimates of the secant stiffness of each connection. The secant stiffness method is not only simple to program but is also stable to the point that convergence can always be obtained regardless of the moment-rotation characteristics. Reasons concerning the use of the secant stiffness in comparison with the other methods has been described in Chapter 1.

The loading on the structure in this program is increased proportionally. Therefore the information on displacement and forces in the structure can be obtained at any load level up to collapse.

The effect of axial force on frame stiffness is included by using stability functions calculated from the previous iteration of analy-

sis. At each load step, iteration is continued until the current calculated axial force are within a suitable tolerance with that of the previous iteration.

The reduction of the plastic moment capacity of the cross section as a result of the presence of axial forces is taken into account using the method described by Majid [9]. The assumptions that are used in the analysis of the frames are as follows:

- 1) The member is prismatic and plane sections remain plane after deformation.
- 2) Although large rigid body displacement are allowed, the distortion of each member is small.
- 3) Linear elastic behaviour is assumed in the members except at locations of plastic hinges.
- 4) No plastic hinge is allowed to form at the end of a member with a semi-rigid connection. This is to avoid a joint mechanism.
- 5) The moment-curvature relationship of the connection is idealized as piece-wise linear.

#### 4.4 Overall stiffness matrix

The stability analysis of frames using matrix methods has been described by many authors. The stability of a frame can be measured by the determinant of the overall stiffness matrix of the structure. The point of frame instability occurs when the determinant of the overall stiffness matrix becomes zero.

$$\text{Det } K=0$$

4.1

The analysis leads to the solution of a set of linear equations in the usual form of,

$$L=K.X$$

4.2

in which  $K$  is the stiffness matrix and takes into account the the connection rigidly. The computer program for the analysis of frames with rigid joints can be modified in order to allow for joint flexibility, by means of an appropriate correction to the elements of the stiffness matrix.

##### 4.4.1 Construction of overall stiffness matrix.

Consider an arbitrary prismatic member of a plane frame. Lets  $[L]$  and  $[X]$  denote member end forces and end displacements, respectively, as

shown in fig. (4.2). Let us assume that at end i there is a semi-rigid joint and end j a plastic hinge. The way in which the stiffness matrix for element i-j is constructed is as follows;

The rotation  $\theta^*_1$  at end i will be a combination of joint rotation  $\theta_1$  and rotation due to the presence of the semi-rigid action  $\theta_{sr}$ .

$$\theta^*_1 = \theta_1 + \theta_{sr} \quad 4.3$$

The rotation  $\theta^*_2$  at end j will be a combination of the joint  $\theta_2$  and the plastic hinge  $\theta_h$ .

$$\theta^*_2 = \theta_2 + \theta_h \quad 4.4$$

Therefore the slope deflection equation for this element can be written down as.

$$\begin{aligned} M_{ij} &= \frac{6EI}{L^2} \phi_2 (y_1 - y_2) + \frac{4EI}{L} \phi_3 (\theta_1 + \theta_{sr}) + \frac{2EI}{L} \phi_4 (\theta_2 + \theta_h) \\ M_{ji} &= \frac{6EI}{L^2} \phi_2 (y_1 - y_2) + \frac{2EI}{L} \phi_4 (\theta_1 + \theta_{sr}) + \frac{4EI}{L} \phi_3 (\theta_2 + \theta_h) \\ V_{ij} = -V_{ji} &= \frac{12EI}{L^3} \phi_6 (y_1 - y_2) + \frac{6EI}{L^2} \phi_2 (\theta_1 + \theta_{sr}) + \frac{6EI}{L^2} \phi_2 (\theta_2 + \theta_h) \end{aligned} \quad 4.5$$

The relationship between axial load and axial displacement is,

$$H_1 = -H_2 = \frac{EA}{L}(x_1 - x_2) \quad 4.6$$

where L            length of member.  
A            Area.  
E            Young modulus.  
I            Second moment area.

The functions  $\phi_1, \phi_2, \dots, \phi_5$  are stability functions defined by Livesley [100] in terms of axial load, which is positive when the member is in compression, as follows:

$$\beta = \frac{1}{2} \sqrt{\frac{H}{EI}}$$

$$\phi_1 = \beta \cot 2\beta$$

$$\phi_2 = \frac{1}{3} \beta^2 (1 - \beta \cot \beta)$$

$$\phi_3 = \frac{1}{4} (3\phi_2 + \phi_1) \quad 4.7$$

$$\phi_4 = \frac{1}{4} (3\phi_2 - \phi_1)$$

$$\phi_5 = \phi_1 \phi_2$$

These functions are real for real values of H and equal to unity when  $H=0$ .

The member equations 4.5 and 4.6 can be written in matrix form as:



$$\begin{bmatrix} H'_1 \\ V'_1 \\ M_1 \\ M_{sR} \\ M_{sL} \\ H'_2 \\ V'_2 \\ M_2 \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3}\phi_1 & \frac{6EI}{L^2}\phi_2 & \frac{6EI}{L^2}\phi_3 & \frac{6EI}{L^2}\phi_4 & 0 & \frac{-12EI}{L^3}\phi_1 & \frac{6EI}{L^2}\phi_2 \\ 0 & \frac{6EI}{L^2}\phi_2 & \frac{4EI}{L}\phi_3 & \frac{4EI}{L}\phi_4 & \frac{2EI}{L}\phi_4 & 0 & \frac{-6EI}{L^2}\phi_2 & \frac{2EI}{L}\phi_4 \\ 0 & \frac{6EI}{L^2}\phi_2 & \frac{4EI}{L}\phi_3 & \frac{4EI}{L}\phi_4 & \frac{2EI}{L}\phi_4 & 0 & \frac{6EI}{L^2}\phi_2 & \frac{2EI}{L}\phi_4 \\ 0 & \frac{6EI}{L^2}\phi_2 & \frac{2EI}{L}\phi_4 & \frac{2EI}{L}\phi_4 & \frac{4EI}{L}\phi_3 & 0 & \frac{6EI}{L^2}\phi_2 & \frac{4EI}{L}\phi_3 \\ -\frac{EA}{L} & 0 & 0 & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & \frac{-12EI}{L^3}\phi_1 & \frac{-6EI}{L^2}\phi_2 & \frac{-6EI}{L^2}\phi_3 & \frac{-6EI}{L^2}\phi_4 & 0 & \frac{12EI}{L^3}\phi_1 & \frac{-6EI}{L^2}\phi_2 \\ 0 & \frac{6EI}{L^2}\phi_2 & \frac{2EI}{L}\phi_4 & \frac{2EI}{L}\phi_4 & \frac{4EI}{L}\phi_3 & 0 & \frac{-6EI}{L^2}\phi_2 & \frac{4EI}{L}\phi_3 \end{bmatrix} \begin{bmatrix} x'_1 \\ y'_1 \\ \theta_1 \\ \theta_{sR} \\ \theta_{sL} \\ x'_2 \\ y'_2 \\ \theta_2 \end{bmatrix} \quad 4.8$$

where  $M_{hi}$  is the plastic moment of resistance of member  $i-j$  and  $M_{sR}$  is the bending moment at the semi-rigid connection and is obtained from;

$$M_{sR} = -K\theta_{sR} \quad 4.9$$

$K$  is the secant stiffness obtained from the  $M-\phi$  relationship. The negative sign indicates that the rotation of the connection will be anticlockwise. By referring to fig. (4.2), it can be seen that the clockwise moment will cause deformed shape with reverse curvature. Therefore the sign of the connection is opposite to the sign of bending moment.

Now by substituting for  $M_{sR}$  from equation (4.9) into the equation (4.8) and taking  $M_{sR}$  into the right hand side of the equation (4.8). The row corresponding to  $M_{sR}$  becomes:

$$\begin{bmatrix} 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{6EI}{L^2}\phi_2 & \frac{4EI}{L}\phi_3 & \frac{4EI}{L}\phi_3 + K \frac{2EI}{L}\phi_4 & 0 & \frac{6EI}{L^2}\phi_2 & \frac{2EI}{L}\phi_4 \end{bmatrix} \begin{bmatrix} \theta_{SR} \end{bmatrix} \quad 4.10$$

The equation (4.8) can be partitioned as shown, giving equations in terms of the sub-matrices;

$$\begin{aligned} L'_1 &= k_{11}X'_1 + k_{12}\theta_{SR} + k_{13}\theta_H + k_{14}X'_2 \\ M_{SR} &= k_{21}X'_1 + k_{22}\theta_{SR} + k_{23}\theta_H + k_{24}X'_2 \\ M_H &= k_{31}X'_1 + k_{32}\theta_{SR} + k_{33}\theta_H + k_{34}X'_2 \\ L'_2 &= k_{41}X'_1 + k_{42}\theta_{SR} + k_{43}\theta_H + k_{44}X'_2 \end{aligned} \quad 4.11$$

The sub matrix  $k_{11}, k_{14}, k_{41}, k_{44}$  each contain nine elements, and are functions of the modulus of elasticity. The introduction of a semi-rigid connection at end i and a plastic hinge at end j introduces two further unknown displacements, namely  $\theta_{SR}$  and  $\theta_H$ , and modifies the member stiffness matrix as shown in equation (4.8) by the addition of two extra columns with elements similar to those corresponding to  $\theta_i$  and  $\theta_j$  respectively.

The co-ordinates so far have been considered to be local to the member. It is necessary to have a uniform system of co-ordinates for the structure as a whole. The frame reference for a given member as shown in fig. (4.3) will lie at some angle  $\beta_1$  to the frame of reference for the whole structure, axes X and Y. Therefore the transformation of displacements is necessary. Therefore the member stiffness matrix, k, should be multiplied by the orthogonal transformation matrix, as described by Majid [9]

$K= A'.k.A$

where K overall stiffness matrix  
A Displacement transformation matrix  
A' Transpose of A

Hence the equation (4.8) become

$$\begin{bmatrix} H_1 \\ V_1 \\ M_1 \\ 0 \\ M_h \\ . \\ . \\ H_j \\ V_j \\ M_j \end{bmatrix} = \begin{bmatrix} A & B & -C & -C & -C & . & . & -A & -B & -C \\ B & F & -T & -T & -T & . & . & -B & -F & -T \\ -C & -T & e & e & f & . & . & C & T & f \\ -C & -T & e & e+K & f & . & . & C & T & f \\ -C & -T & f & f & e & . & . & C & T & e \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ -A & -B & C & C & C & . & . & A & B & C \\ -B & -F & T & T & T & . & . & B & F & T \\ -C & -T & f & f & e & . & . & C & T & e \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ \theta_1 \\ \theta_{sr} \\ \theta_h \\ . \\ . \\ x_j \\ y_j \\ \theta_j \end{bmatrix}$$

4.12

where

$A=al_p^2+bl_q^2$   
 $B=al_pm_p+bm_qm_q$   
 $C=dl_q$   
 $F=am_p^2+bm_q^2$   
 $T=dm_q$

4.13

and a,b,d,e and f are:

$$\begin{aligned}
 a &= \frac{EA}{L} \\
 b &= \frac{12EI\phi_5}{L^2} \\
 d &= \frac{-6EI\phi_2}{L^2} \\
 e &= \frac{4EI\phi_3}{L} \\
 f &= \frac{2EI\phi_4}{L}
 \end{aligned}
 \tag{4.14}$$

By referring to fig. (4.3),  $l_p, l_q, m_p$  and  $m_q$  are given as:

$$\begin{aligned}
 l_p &= \cos \beta_1 \\
 m_p &= \sin \beta_1 \\
 l_q &= -\sin \beta_1 \\
 m_q &= \cos \beta_1
 \end{aligned}
 \tag{4.15}$$

Now consider a frame containing  $m$  joints . At some stage of loading let there be ' $n$ ' hinges which may be real, semi-rigid or plastic hinges . There are then  $(3m+n)$  unknown displacements forming displacement vector  $X$ .

$$X = (x_1, y_1, \theta_1, \theta_{N1}, \theta_{N2}, \dots, \dots, \theta_{Nn-1}, \theta_{Nn}, x_m, y_m, \theta_m)
 \tag{4.16}$$

where  $x, y, \theta$  are joint displacements and  $\theta_h$  is either a semi-rigid rotation or a plastic hinge rotation.

The load vector  $L$  of external load will have similar construction having  $(3m+n)$  elements such as

$$L = (H_1, V_1, M_1, M_{h1}, M_{h2}, \dots, \dots, M_{hn-1}, M_{hn}, H_m, V_m, M_m) \quad 4.17$$

where  $H, V$  and  $M$  are respectively the horizontal load, vertical load and applied moment at the specific joints.  $M_{h1}$  to  $M_{hn}$  are the bending moments carried across each hinge. The value of  $M_{h1}$  to  $M_{hn}$  will be equal to the fully plastic moment of the member for a plastic hinge, zero for real hinge and will be equal to equation (4.9) for semi-rigid joints.

The contribution of the other members to stiffness matrix is similar to member  $ij$ , and when more than one member is connected to a particular joint the contributions of these members are accumulative. For most frames, each joint is directly connected to only a small number of the other joints in the frame. This giving the stiffness matrix have two particular features:

- a) a large number of its element are zero, and.
- b) the elements occupy a band of irregular boundaries.

With reference to fig (4.4), the overall stiffness matrix for a frame (A) of fig (4.4a) with semi-rigid joints contains many zero sub-matrices. The non-zero sub-matrices are directly related to the joint connection list. Further,  $K$  is symmetric along the leading diagonal.

The method of Jennings [24] makes use of the symmetrical features of the overall stiffness matrix for the storage and rapid solution of the stiffness equations. This method stores only those elements between the first non-zero sub matrix and the elements on the leading diagonal inclusive as shown in fig (4.4). Null sub-matrices such as  $K_{9,7}$  occurring between these non zero element are also stored. Only the irregular half band width outlined in fig. (4.4) is stored and operated on by the compact elimination technique.

#### 4.5 Program procedure for non-linear elastic-plastic analysis with semi-rigid connections

The procedure adopted in this program follows the response of a structure, by varying the applied load factor by regular increments. At each load step, the calculated bending moments are compared to the plastic moments of resistance. If the difference between bending moment and plastic moment at any given section is within the specified tolerance, then a plastic hinge will be inserted at that section. This process continues to the stage where the frame loses all its stiffness at its elastic-plastic failure load.

Another way of proceeding with the elastic-plastic analysis of the frame is the method adopted earlier by Majid and Anderson [23]. In this method the iteration process finds the load factor at which the first hinge occurs. Once a plastic hinge is detected and inserted in the frame, the load factor at which the next plastic hinge would form is calculated by extrapolation and applied to the frame. The only

advantage of using this method is that the stiffness equations do not need to be solved as many times as the procedure used in this study. With the availability of the fast computers, today this is no longer a problem. Using the present author's procedure will result in a complete load deflection curve with corresponding internal moments and forces. To avoid the hinge development between two consecutive load factors, more refined load increments are chosen. For purely elastic analysis, a high value of yield strength,  $P_y$  can be specified. In this manner a separate program for elastic analysis is no longer required. This is in contrast to Majid and Anderson [23].

#### 4.5.1 Flow chart

To enable the operation of the program to be followed more easily, the flow diagram shown in fig (4.5) has been prepared. The numbers on the flow diagram refer to the following steps.

- 1) a: Read member data, load vector, moment-rotation relationships for various types of connection used and other necessary items of information.
- b: Read the tolerances for convergence on  $M-\phi$  characteristics and for second order analysis.
- c: Read the initial value of load level and the increment of load level.
- d: Read value of section modulus of each member, together with constants for calculation of the reduction of plastic hinge

moments due to axial load.

e: Read the control parameter, in order to carry out first order or second order analysis with rigid or semi-rigid connections.

2) Clear space in stiffness matrix.

3) Take one member at a time.

a: Test if axial load is zero. If so, set the stability functions  $\phi_1$  to  $\phi_5$  to unity or else calculate these functions.

b: Test if end 1 of the member is fixed. If not, calculate the contribution of this end to the stiffness matrix.

c: Test if end 1 has a hinge (real or semi-rigid). If so, add a row and column to the stiffness matrix corresponding to the hinge rotation (the corresponding load vector will be equal to 0,  $-K\theta_{SR}$  and  $M_P$  in the case of real, semi-rigid and plastic joint respectively).

d: Repeat steps (a), (b), and (c) for end 2 of the member.

4) Repeat for next member.

5) a: Solve the set of simultaneous equations  $L=KX$  for the joint displacements.

b: Store the hinge rotations.

6) a: For each member, calculate the stability functions.

b: Calculate the new axial load in the member.

c: Calculate the bending moment at each end of the member, from



the resulting joint displacements, using the slope deflection method.

d: Calculate the reduced plastic hinge moment of each section, using the axial load of step (b).

7) Repeat from (6a) for other members.

8) a: Check if end 1 of a member is semi-rigid. If so obtain the new value of secant stiffness.

b: Check if the present value of secant stiffness is within the specified tolerance of the previous value.

c: If it is not within the tolerance, then predict the new value of secant stiffness as explained in section 4.6.

d: Repeat (a),(b) and (c) for the second end of the member.

9) Repeat from (8) for all other members.

10) If any secant stiffnesses are not within the required tolerance, repeat from step (5).

11) a: Test if a plastic hinge should be added to end 1 of the member, (except if there is semi-rigid connections )

b: If end 1 is semi-rigid, check if the value of bending moment reaches the maximum moment capacity of that joint,  $M_c$ , determined from  $M-\phi$  relationship.

c: If (a) or (b) is satisfied, store the reduced plastic moment

in the case of a rigid joint, the moment capacity of the joint  
in the case of a semi-rigid connection.

d: Repeat (a),(b) and (c) for the second end of the member.

12) Repeat from 11(a) for all other members.

13) If no plastic hinge has formed or if no moment at semi-rigid joint has reached the moment capacity of that joint, increase the load level by the specified amount and repeat from step (16)

14) When a plastic hinge forms, print the position of the hinge, the deflections at the joints, the axial load, the bending moment and value of reduction in  $M_p$  for all the members. When a semi-rigid joint has reached its moment capacity replace the load vector at that joint to the constant value of  $M_o$ .

15) If a plastic hinge has formed.

a: Store the location of the plastic hinge .

b: Increase the size of the stiffness matrix by an extra row and column.

c: Add the value of the reduced  $M_p$  of the member to the load vector at the appropriate location.

16) Calculate the determinant of the stiffness matrix. Stop if this is negative.

17) If the value of stiffness matrix is not negative, increase the

load level by an increment, and repeat from step (6).

#### 4.6 Iterative analysis procedure.

Because of the non-linear behaviour of connections, an iterative analysis procedure is used. Its basis is that the correct structural deflection and internal forces can be obtained from the analysis, provided the correct stiffness is adopted for each connection.

There are three stiffness values which can be used with any moment rotation curve as shown in fig. (6.4);

- i) The initial stiffness.
- ii) The tangent stiffness at any point.
- iii) The secant stiffness.

The advantages and disadvantages of these stiffness values has already been described in Chapter 1.

Many difficulties were experienced in convergence when incorporating the effects of non-linear connection behaviour into the computer analysis program. The method finally developed by the author makes use of the secant stiffness. This is simply calculated as the ratio of the current connection moment to the current angular rotation. The method adopted leads to the rapid convergence of solutions.

The first difficulties were experienced when using the procedure by Anderson - Lok [58]. It was found that, in most of the cases the convergence could not be achieved. The convergence is only attained with the frame with very stiff connections. The latter was only true at the working load level (where the final moment in the connection falls within initial portion of the moment- rotation diagram). Therefore it was decided to use successive estimates of secant stiffness, instead of fixing the rotation of a connection before each iteration as proposed earlier [58].

Consider a structure whose member end connections have a non-linear moment rotation function in the form of:

$$M = K \phi \quad 4.18$$

The nonlinear curve is represented in the form of series of straight lines.

Since the secant stiffness method is used here, the connection stiffness  $K$  was incorporated into the stiffness matrix. This was presented in the form of (M-K) relationship as shown in figure (4.7).  $K$  is simply obtained from the above equation and is given by:

$$K = \frac{M}{\phi} \quad 4.19$$

The moment stiffness (M-K) relationships of all the other connections in the structure are similarly linearized.

In the first iteration the initial connection stiffnesses from the linearized (M-K) relationships are incorporated into the stiffness matrix. Let us assume for a particular connection this is equal to  $K_1$ . Solving the simultaneous equations  $L=K.X$  gives the resulting displacements, and hence member end moments can be calculated. Assume that the moment calculated for  $K_1$  is equal to  $M_1$ . From  $M_1$  a new value of connection stiffness  $K_2$  is obtained using the linearized (M-K) relationship. The connection stiffness  $K_2$  just calculated in the current iteration is compared with previous stiffness  $K_1$ . If the difference between the two stiffnesses is within the acceptable tolerance, then the results obtained from the current analysis are considered correct.

In most of the cases, it was found that  $K_1$  and  $K_2$  were not within the tolerance at the first few iterations. Therefore further iteration was necessary and the above procedure repeated until, the convergence was achieved.

Convergence problems normally occur when very flexible connections are used or when the analysis is close to the collapse load. The next section will investigate some of these problems and the proposed technique to overcome this problem will be discussed.

#### 4.6.1 Convergence problems.

One of the most common convergence problems experienced during the analysis of a structure with semi-rigid end connections was that the connection stiffness oscillated from low value of stiffness in one iteration to a very high value of stiffness in another or vice versa. This leads to an extreme value of connection stiffness being incorporated into the stiffness matrix, and hence a very high value of moment was obtained. This value incorrectly exceeded the maximum moment capacity of the connection (obtained from  $M-\phi$  relationship) considered, which causes the analysis to stop.

Numerical experience shows that to improve the convergence characteristic for the above case, numerical damping of the stiffness predictions is helpful. The damping considered in this study is achieved by using a new predicted stiffness connection which is the average of the current secant stiffness and the previous secant stiffness. It was found that this method only speeds up the convergence in structures with the stiffer connections, but problems still exist in structures with flexible connections.

Finally it was proposed to store two sets of values of connections stiffness obtained from the last two iterations,  $K_0$  and  $K_1$ , where  $K_1$  is the previous connection stiffness and  $K_0$  is that prior to the previous connection stiffness. It will be shown how these two values can be used to predict a better estimation of the secant stiffness.

Let us assume for the first iteration  $K_0$  is incorporated into the stiffness matrix, and the resulting bending moment obtained is equal to  $M_1$ .  $M_1$  is used to obtain a new value of connection stiffness from the (M-K) relationship. Let this value be equal to  $K_1$ . In the second round of iteration, by incorporating the  $K_1$  into the stiffness matrix a new value of moment obtained which is equal to  $M_2$ . The co-ordinates of the moments and connections stiffness obtained are shown in fig (4.7).

Now let  $[X_1]$  have co-ordinates of  $[K_0, M_1]$ , and  $[X_2]$  have co-ordinates of  $[K_1, M_2]$ . The equation of the line determined by connecting  $[X_1]$  and  $[X_2]$  and can be written as;

$$K = m_1 M_1 + C_1 \quad 4.20$$

Now the equation for each segment of the line from the piece-wise linear M-K relationship can be written as;

$$K(I) = m(I)M(I) + C(I) \quad 4.21$$

where  $I$  represents the line number currently used.

The program calculates the co-ordinates of the intersection of the line from equation (4.20) with the line from equation (4.21). From this co-ordinate a new value of the connection stiffness,  $K_2$  is obtained. This value will be used for the next round of iteration. It

was found that by adopting this technique the convergence problem was overcome. Convergence was achieved very rapidly even for very flexible connections and for a frame just prior to the collapse load.

#### 4.8 Numerical examples.

The frames which were analysed by the proposed method are frames A, B and C. These frames were specified by Zandonini [101] for an ECCS task group, in order to check consistency in the prediction of frame response from different research groups. The results of these frames make it possible to check the validity of the program in comparison with the other numerical methods. Additionally, analysis with rigid connections were carried out to compare the results with the semi-rigid connections.

##### 4.8.1 Frames Data.

The configuration and the data of the frames are as follows:

1) Frame A is unbraced with a three storey single bay configuration, as shown in figure [4.8a]. The gravity load of intensity  $q$  is uniformly distributed over the beams. The horizontal force  $H$  applied at each floor, equals to  $0.05qL$ , where  $L$  is the span of the beam.

2) Frame B is unbraced and of two storey three bay configuration as shown in figure (4.8b). This frame has the same type of loading as in frame A.



3) Frame C is braced and of three storey two bay configuration as shown in figure (4.8c). This frame has pattern distributed gravity loading of  $q=40$  KN/m and  $q_0=0.35q$  over the beams. An external axial load of value  $P$  is applied at the outer columns and  $2P$  at centre column, at roof level. The value of  $P$  is equal to  $(q+q_0) L$ .

The loading of the above frames were represented in two types of point load. These types of loading were used to replace the distributed load originally specified. In addition, horizontal forces due to initial imperfections specified by EuroCode3 [65] are shown in figure (4.9), were added to the stated horizontal load.

The sections adopted for these frames, together with area, 2nd moment of area and plastic modulus are presented in table (4.1). All the members are in steel of grade Fe 360 with yield strength of  $P_y=235$  N/mm<sup>2</sup>. Young's modulus of elasticity was taken as  $E=210$  KN/m<sup>2</sup>. The stress strain relationship is idealized as elastic-perfectly plastic. The strain hardening is not taken into the account. The moment-rotation curves of the connection were determined by Tschemmerneegg by means of macro-mechanical model described in reference [93]. Connections A,B and C are extended end plates. The details of these connections are shown in figure (4.10). The  $M-\theta$  curves for these connections are shown in figure (4.11) for each connection.

#### 4.8.2 Results of analysis.

The frames were analysed to determine the following:

- a) To investigate the behaviour at serviceability limit and ultimate strength.
- b) To compare the effect of reduction in plastic moment of resistance  $M_P$ , due to presence of the axial load. The comparisons are made for both rigid and semi-rigid frames.
- c) To compare the accuracy of the results obtained from different patterns of loading .
- d) To investigate the accuracy and the necessity in adopting smaller increments of the load level.
- e) To compare the results obtained from semi-rigid analysis with rigid analysis.
- f) Finally to compare results for semi-rigid analysis made by the other researchers, in order to establish the validity of analysis.

#### 4.8.2.1 Results of frames A-B-C with semi-rigid connections

Frames were analysed using the computer program described. Each frame was loaded up to collapse denoted by  $\alpha_f$  for both types of loading specified. The reduction in  $M_p$  (moment of plastic resistance) due to presence of axial load were taken into account.

The bending moment diagram, deflected shape and load deflection curve for each types of loading of frames are shown in figures (4.12-4.29) These results are used for the following investigation.

#### 4.8.2.2 Effect of reduction in $M_p$

The full plastic moments of the members are reduced appreciably as the axial load level increases . Figure (4.30) shows the reduction in  $M_p$  as the load level increases for frames B and C, and it can be seen that the reduction is appreciable. Normally plastic hinges form at lower values of bending moment when considering the reduction in  $M_p$ . It was evident that when a plastic hinge formed in a member, it causes further loss of stiffness and the frame collapses at a lower load level. Therefore it becomes important to consider the reduction in  $M_p$ . To demonstrate the importance of the latter, the frames were re-analysed by entering the data so no reduction in  $M_p$  takes place. Table (4.2) shows the percentage of reduction in  $M_p$  in comparison with the analysis with the full  $M_p$  due to highest axial load at the collapse load level for three frames A,B and C.

In frame A, it was found that the response of the frame is generally of the same order. This is because the value of  $M_P$  is large and the reduction in  $M_P$  did not cause a plastic hinge in a member.

In frame B with semi-rigid connections the behaviour of the frame with reduction in  $M_P$  was the same as the frame ignoring the reduction, up to a load level 1.49. From this load level up to the collapse load of 1.5, analysis with reduction in  $M_P$  caused two further plastic hinges to be formed at the second end of members 18 and 19, of fig.(4.20). The decrease in  $M_P$  due to the highest axial load at collapse load was found to be 60% .

In Frame B with a rigid connection the decrease in collapse load is 16.6%, when the reduction in  $M_P$  is taken into account. This arises by extra plastic hinges formed at members 18 and 15 of fig. (4.20) .

In frame C, it was found that the plastic hinge formed at member 26 of fig.(4.26), when analysing with reduction in  $M_P$ , in both the rigid and semi-rigid frames. No plastic hinge was formed when ignoring the reduction in  $M_P$ . Analysis of the rigid frame considering the reduction in  $M_P$  decreases the values of the collapse load and plastic moment as much as 12%, and 85% respectively. These values were 7.6% and 73.7% in the frame with the semi-rigid connections.

In general, the value of reduction in  $M_P$  only becomes significant above the load level one and the behaviour of frames are completely elastic prior to this level. It should be borne in mind for frames

with high axial load resulting from supporting the structure the values of  $M_p$  will be reduced appreciably. This may cause the behaviour of the frame to be plastic prior to load level 1.

#### 4.8.2.3 Comparison of different pattern of loading

By referring to the bending moment diagrams and deflected shapes of frames A,B and C of figures (4.12-4.28) it was concluded that, type 1 loading under-estimates the larger beam end moments by 7.5%, 8.2%, 21%, and the sagging moment increases by 7.8%, 3.7%, 7.2% for frames A,B and C respectively. The values of the sway at the top of the structure, were underestimated by 6%, 30% and 15.2%. When analysing under type 1 loading, the difference in the collapse load was negligible for all the frames studied.

From the results obtained it can be concluded that, generally a good representation of behaviour can be obtained with replacing the uniformly distributed loading with the fewer joint loads when strength is considered. When deflection is required for the serviceability limit, it is necessary to increase the number of joint loads in order to achieve a better estimation of sway. Increasing the number of joint loads would cause increases in the size of stiffness matrix and require more data to be input. Therefore a reasonable number of elements must be specified.

#### 4.8.2.4. Comparison of the results adopting different increments of load.

A sensible increment of loading must be specified by the user. If a high value of the increment of load factor is used, it is likely that more than one hinge will develop between two consecutive load factors. The insertion of these hinges all at once brings inaccuracies, and may change the mode of deformation fundamentally. To rectify this defect, a smaller increment of the load factor may be adopted. When a small increment of the load factor is used, it increases the computational time. Therefore in order to keep the balance of time and accuracy, two increments of load factor were incorporated into the computer program. The first increment was used up to the load level at which the frame behaviour is elastic and more refined increment was used from this load level up to collapse load. In all the examples analysed in these studies, values of 0.1 and 0.05 were used as first and a second load increments respectively.

A parametric study was carried out to compare the results of frame B, if different increments were used. It was found that the frame results using the load increments of 0.01 and 0.05 were similar. But when an increment of 0.2 was used, the analysis was largely inaccurate. This is because, as explained earlier, more than one plastic hinge develops between two consecutive load factors.

#### 4.8.2.5 Comparison of semi-rigid and rigid analysis.

In order to make the comparison, all the examples were re-analysed assuming full rigid connections. The comparison of the results obtained for these examples are presented in table (4.3).

Figure (4.31-33) shows the lateral displacement at the top of the structures for the frames studied, for following ;

- a) Second order elastic-plastic analysis with semi-rigid connection.
- b) Second order elastic-plastic analysis with rigid-connection.

It can be seen from figure (4.31-33) that the total sway deflection as a result of incorporating joint flexibility was higher than the deflection calculated assuming fully rigid for all the frames. Further it was noticed from the results that when compressive axial force was taken into account in a non-linear (second order) analysis, the deflections were significantly higher than those assuming fully rigid connection.

The effect of incorporating the connection deformation into the analysis is to reduce the bending moment in the beam-to-column connections, and to increase the mid-span sagging moment at all levels of the structure. From table (4.3) it can be seen that the reduction of the

end moments for the highest value of bending moment of the frames A,B and C was 20%, 33.6%, 28.7% and the corresponding values for the highest mid-span moments were increased by 11.3%, 9% and 13% respectively.

#### 4.8.2.6 Comparison of the results with other researchers.

The results obtained in this studies were compared with the other researchers in order to establish its validity.

The frames specified earlier were analysed by the following researchers;

- 1) Zandonini (Milan)
- 2) Stutzki (Aachen)
- 3) Tschemmenegg (Innsbwck)
- 4) Colson (Cachan)
- 5) Ohta (Warwick)

Figures (4.34-36) shows a set of results for frames A,B and C obtained by the above researchers and the author's analysis. It can be seen from these figures that despite the differences in approach all the results for the frames are quite similar.



#### 4.9 Conclusion.

The computer program has been developed for a second order elastoplastic analysis of flexible jointed steel frames. The present method of analysis can use the real connection data along with any types of analytical curve.

The program makes use of the secant stiffness approach which leads to a more accurate solution of how the connections arrive at the present load in any given moment rotation characteristic. It also provides the necessary numerical stability for all shapes of connection behaviour curves.

Three frames were analysed with the above computer program and the following conclusions were drawn;

- 1) Analysis with flexible connections will decrease end moments and increases mid-span moments of beams. The decrease in end moments is more significant than the increase in mid-span moments.
- 2) The frames with flexible connections deflect more.
- 3) The sway of the frame increases appreciably when P- $\delta$  effects were considered.

4) The effect of reduction in  $M_p$  becomes significant above initial elastic behaviour. This may cause changes in the sequence and load factor at which plastic hinges form.

5) Finally, a good agreement was reached between the author's analysis program and other research groups.

| FRAME<br>ID       | FRAME SECTION | SECOND<br>MOMENT<br>AREA cm <sup>4</sup> | AREA<br>cm | SECTION<br>MODULUS<br>cm <sup>3</sup> |
|-------------------|---------------|--|------------|---------------------------------------|
| FRAME A<br>BEAM   | IPE300        | 8356                                     | 53.8       | 628                                   |
| FRAME A<br>COLUMN | HE200B        | 5696                                     | 78.1       | 642                                   |
| FRAME B<br>BEAM   | IPE300        | 8356                                     | 53.8       | 628                                   |
| FRAME B<br>COLUMN | HE160A        | 1673                                     | 38.8       | 24.6                                  |
| FRAME C<br>BEAM   | IPE300        | 8356                                     | 53.8       | 628                                   |
| FRAME C<br>COLUMN | HE200A        | 3692                                     | 53.8       | 628                                   |

TABLE 4.1 FRAMES SECTIONS

|          | RIGID                                      |   |  | SEMI-RIGID                               |   |   |
|----------|--|---|--|--|---|---|
| FRAME ID | DECREASE IN<br>COLLAPSE LOAD<br>$\alpha_r$ | HIGHEST<br>AXIAL<br>LOAD AT<br>$\alpha_r$ | REDUCTION IN<br>$M_p$<br>DUE TO HIGH-<br>EST<br>AXIAL LOAD | DECREASE IN<br>COLLAPSE LOAD<br>$\alpha$ | HIGHEST<br>AXIAL<br>LOAD AT<br>$\alpha_r$ | REDUCTION IN<br>$M_p$ DUE TO<br>HIGHEST AXIAL<br>LOAD |
| A        | 2.5%                                       | 648.8                                     | 26.3%  | 6.5%                                     | 541                                       | 20%   |
| B        | 16.6%                                      | 437                                       | 40%  | 0.5%                                     | 600                                       | 60%   |
| C        | 7.6%                                       | 979                                       | 73.7%  | 12.25%                                   | 1106                                      | 85.3%   |

TABLE 4.2 EFFECT OF REDUCTION IN  $M_p$ .

|             | RIGID                          |                            |   |  |                                     | SEMI-RIGID                     |                            |   |  |                                     |
|-------------|--------------------------------|----------------------------|---|--|-------------------------------------|--------------------------------|----------------------------|---|--|-------------------------------------|
| FRAME<br>ID | COLLAPSE<br>LOAD<br>$\alpha_p$ | SWAY<br>cm<br>$\alpha = 1$ | MID SPAN<br>BENDING<br>MOMENT<br>KNcm<br>$\alpha = 1$ | END<br>BENDING<br>MOMENT<br>KNcm<br>$\alpha = 1$ | AXIAL<br>LOAD<br>KN<br>$\alpha = 1$ | COLLAPSE<br>LOAD<br>$\alpha_p$ | SWAY<br>cm<br>$\alpha = 1$ | MID SPAN<br>BENDING<br>MOMENT<br>KNcm<br>$\alpha = 1$ | END<br>BENDING<br>MOMENT<br>KNcm<br>$\alpha = 1$ | AXIAL<br>LOAD<br>KN<br>$\alpha = 1$ |
| A           | 1.90                           | 3.26                       | 8341  | 9785   | 338                                 | 1.645                          | 5.31                       | 9290  | 7785   | 336                                 |
| B           | 2                              | 1.05                       | 9745  | 4678   | 203                                 | 1.5                            | 1.39                       | 10631   | 3105   | 208                                 |
| C           | 1.21                           | 1.36                       | 9893  | 5324   | 961                                 | 1.167                          | 1.6                        | 8588  | 3794   | 948                                 |

TABLE 4.3 RESULTS OF ANALYSIS OF RIGID AND SEMIRIGID JOINTS FRAMES.

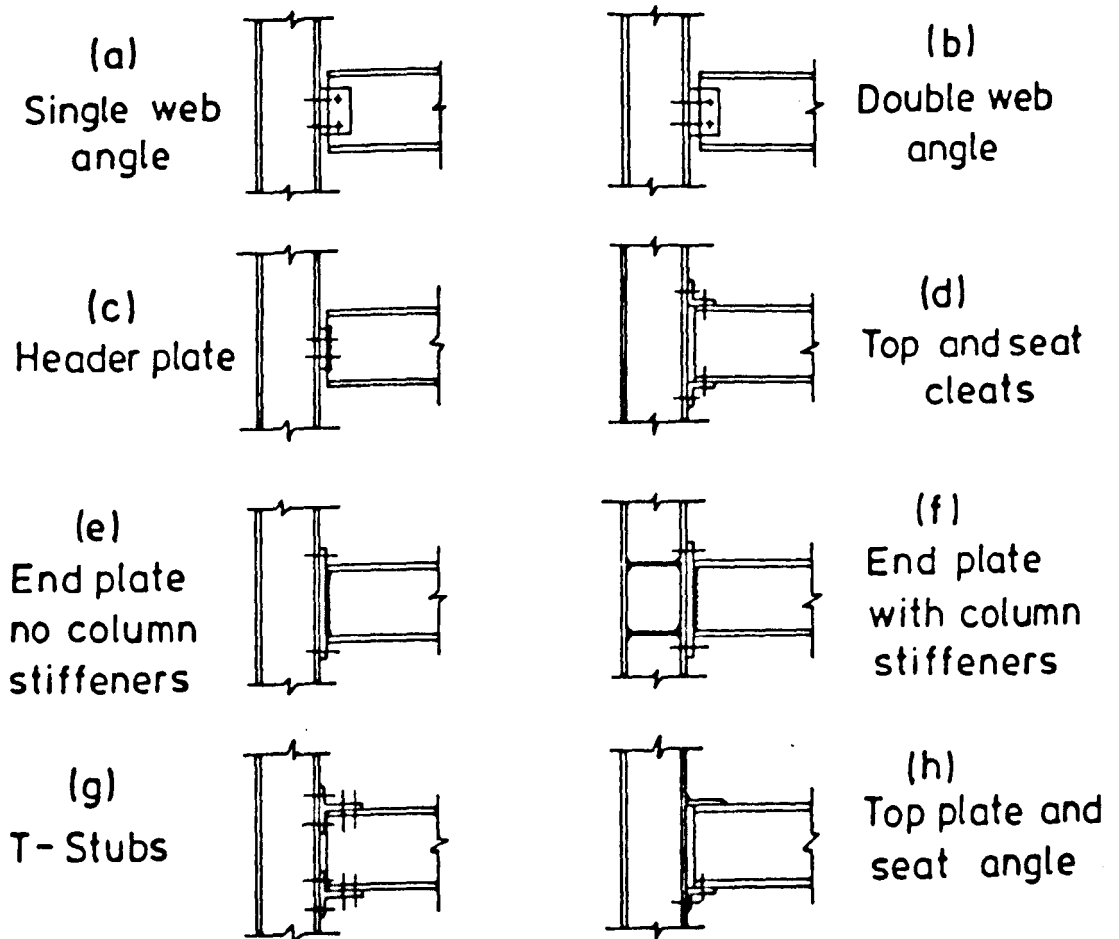


FIG. (4.1) COMMON FORMS OF BEAM-TO-COLUMN CONNECTION

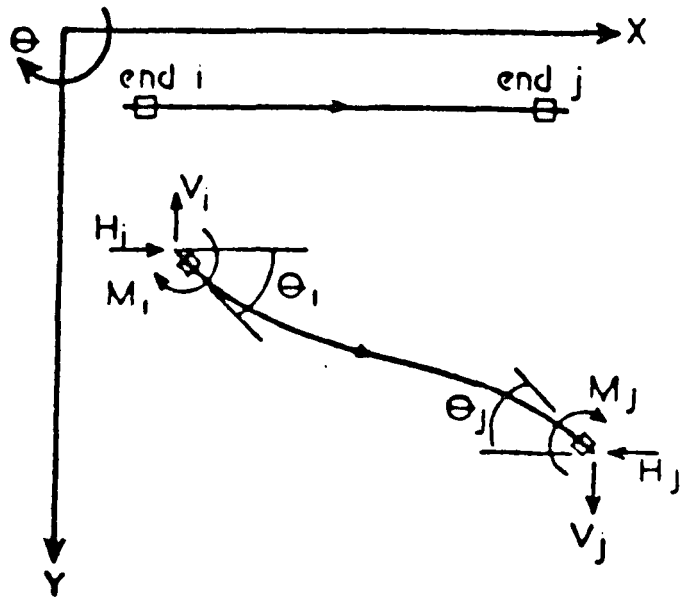


FIG. (4.2) A PRISMATIC MEMBER WITH SEMI-RIGID AT END i AND PLASTIC HINGE AT END j

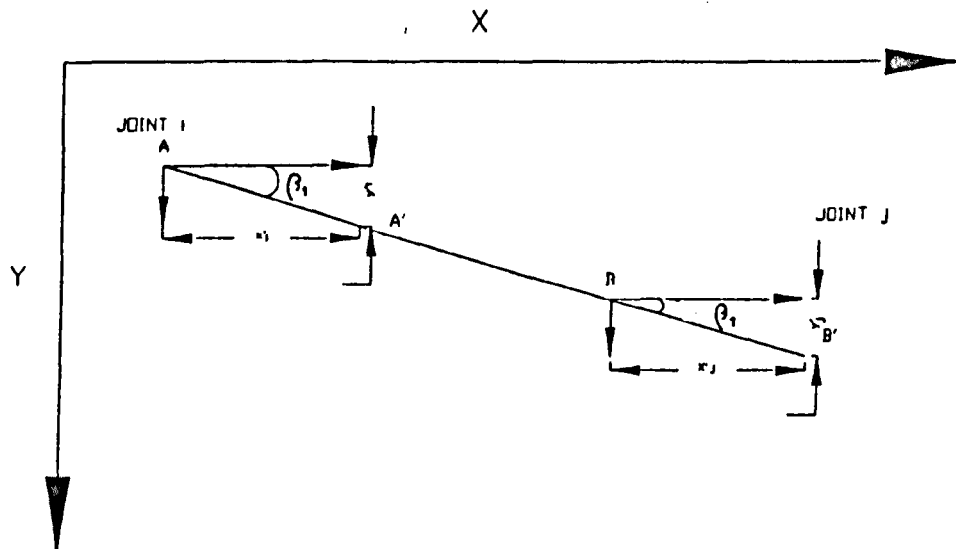


FIG. (4.3) GENERAL DISPLACEMENT OF A MEMBER

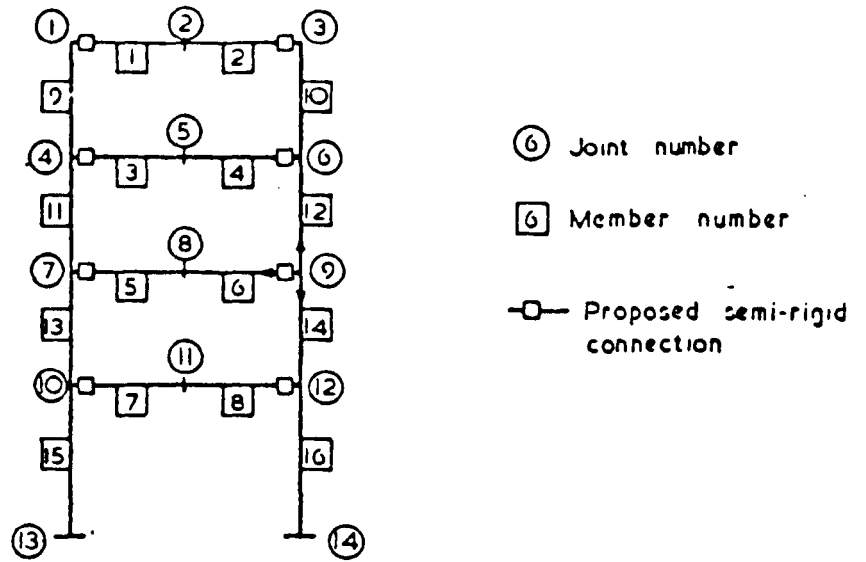


FIG. (4.4 a) FRAME A

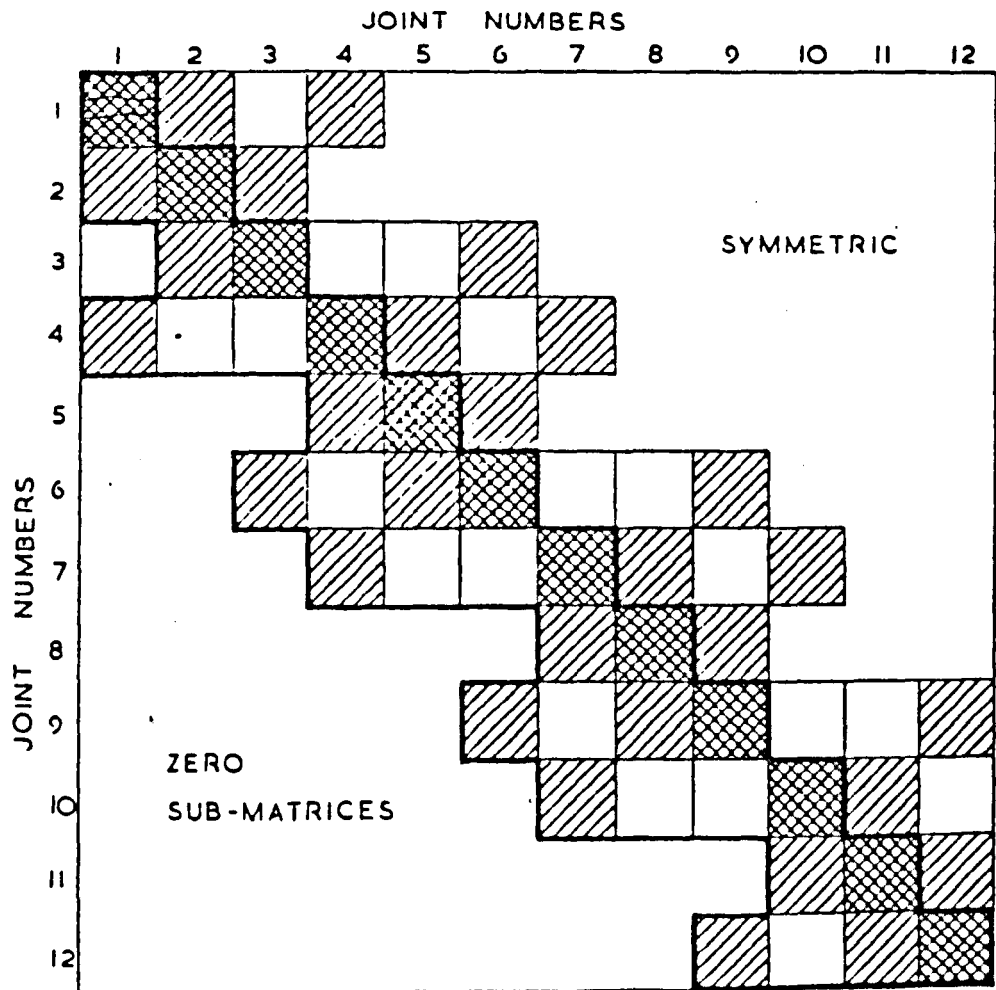


FIG. (4.4) THE OVERALL STIFFNESS MATRIX FOR FRAME 'A'





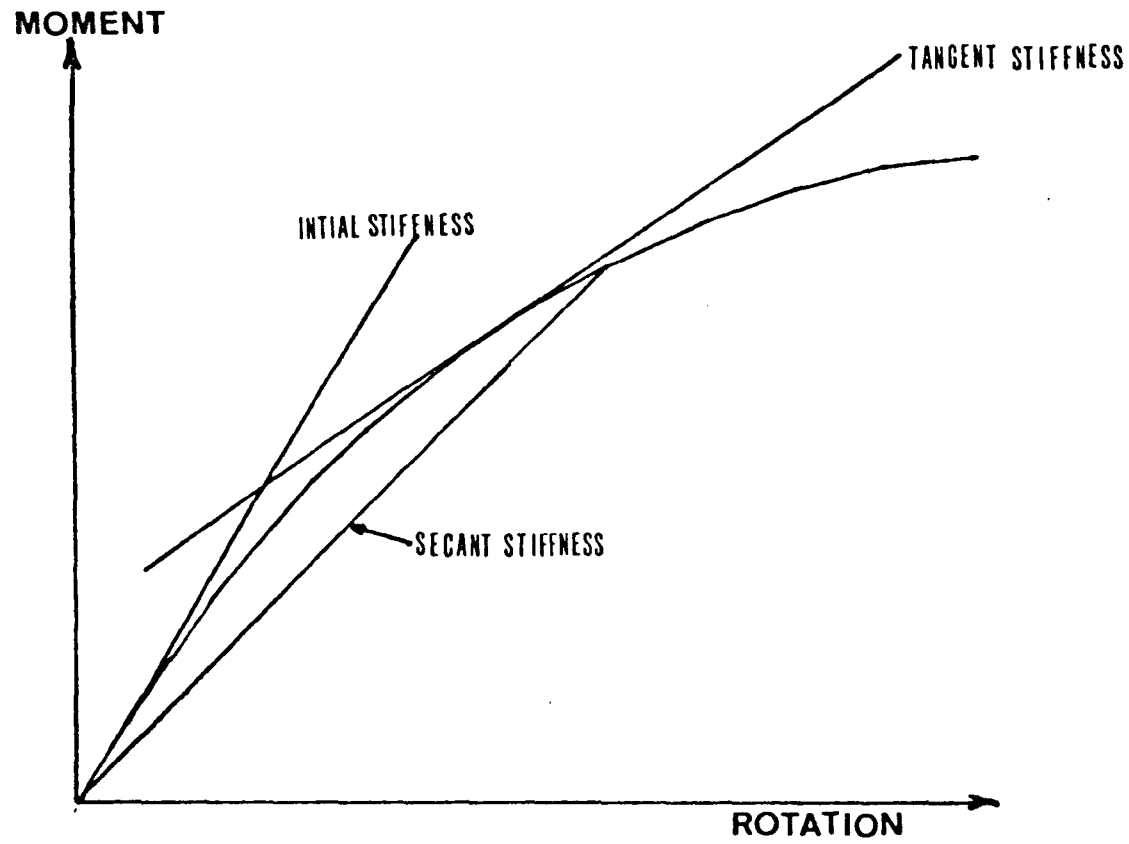


FIG. (4.6) ILLUSTRATION OF INITIAL, SECANT AND TANGENT STIFFNESS

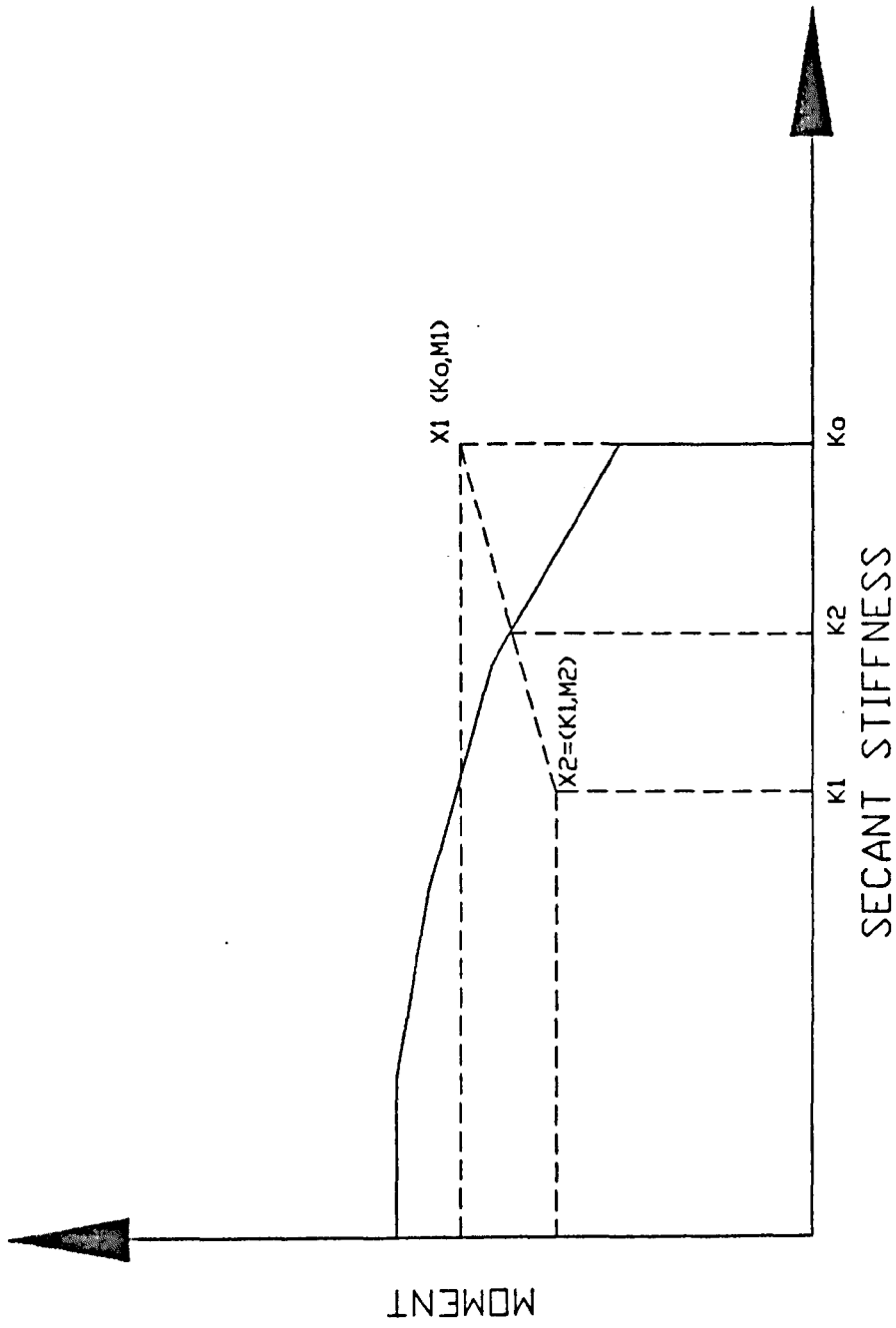


FIG. (4.7) SECANT STIFFNESS ITERATION

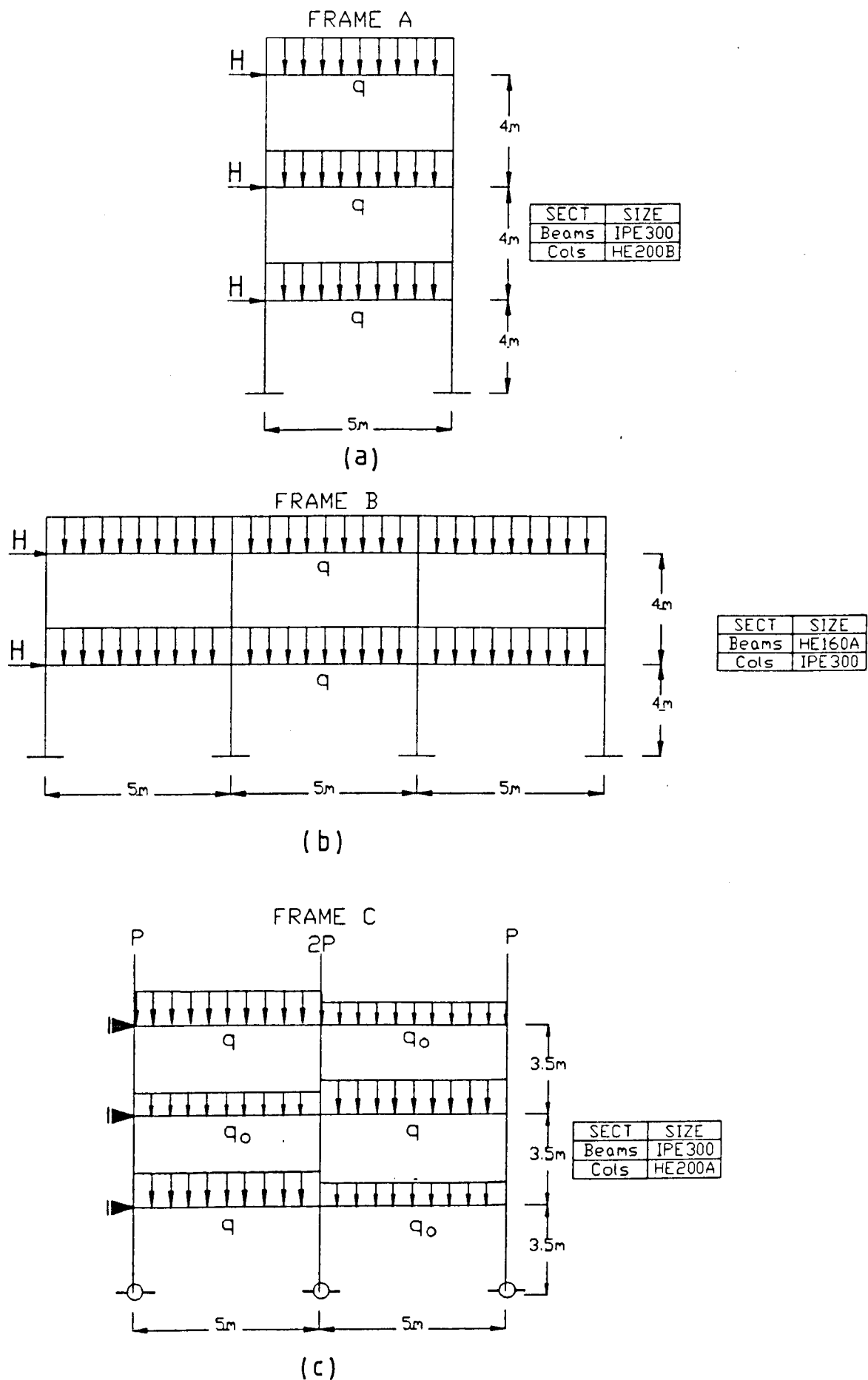
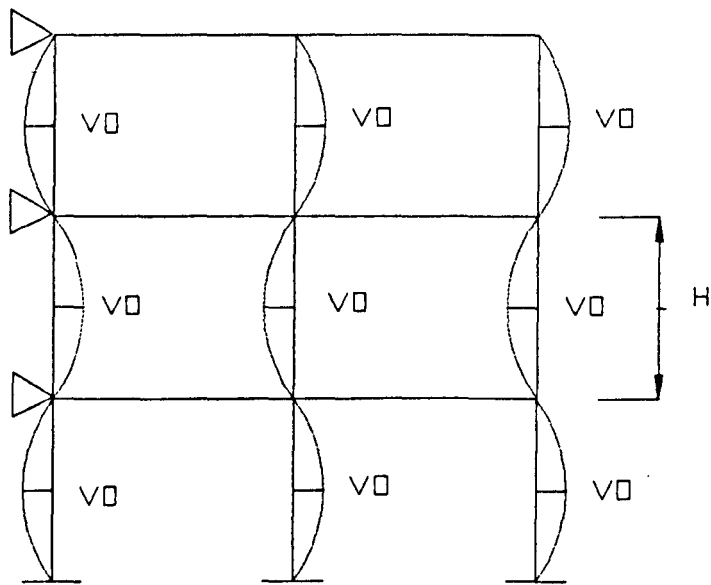
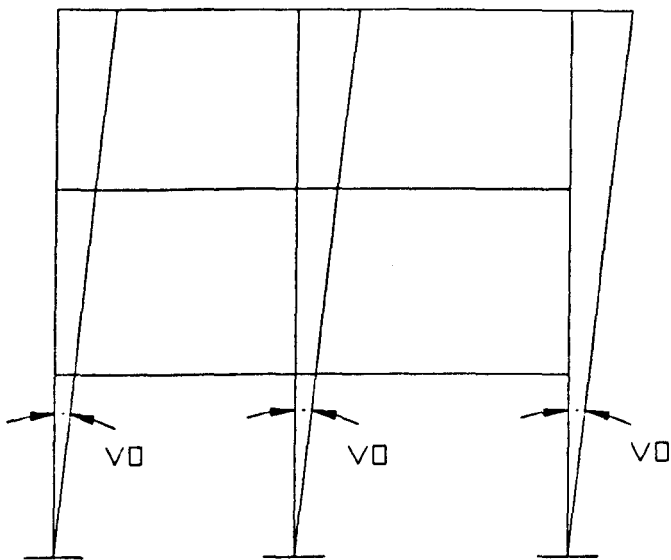


FIG. (4.8) GENERAL CONFIGURATION OF FRAMES A,B AND C



$V_0=1/250H$

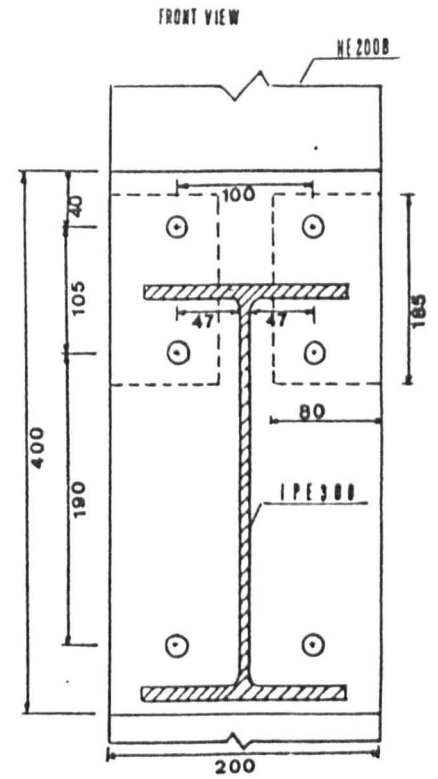
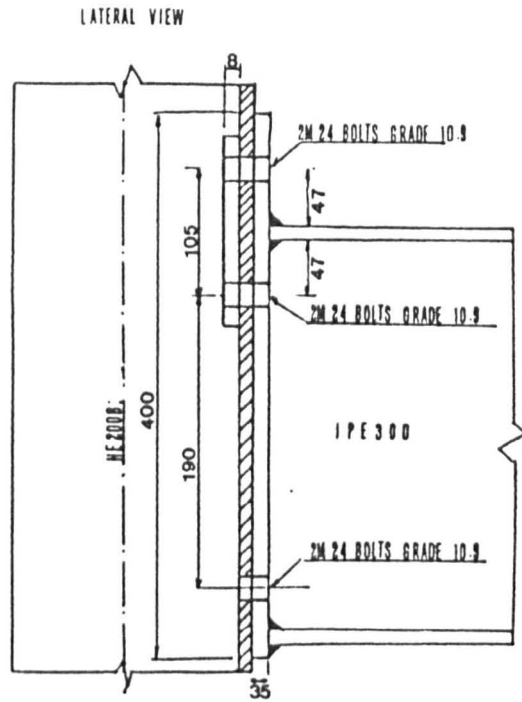
BRACED FRAMES (FRAME C)



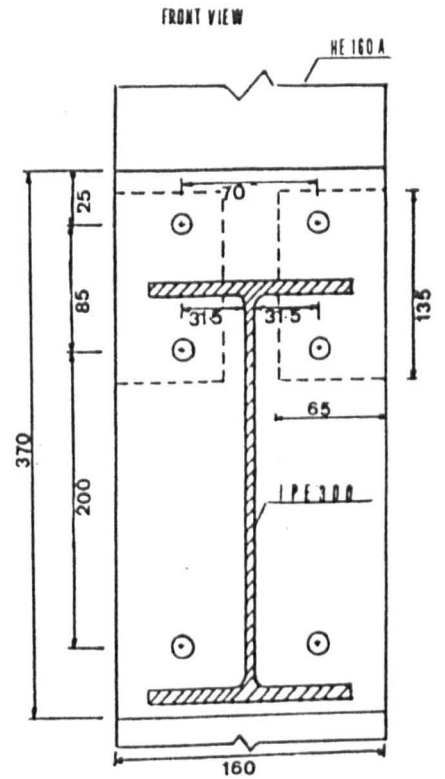
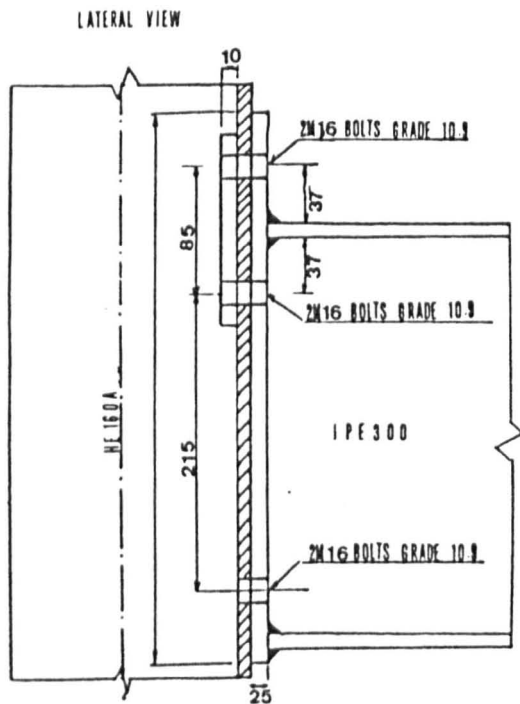
| FRAME | $V_0$ |
|-------|-------|
| A     | 1/287 |
| B     | 1/300 |

UNBRACED FRAMES (FRAME A AND B)

FIG. (4.9) INITIAL IMPERFECTION OF BRACED AND UNBRACED FRAMES  
IN ACCORDANCE WITH EUROCODE NO.3.



JOINT A



JOINT B

FIG. (4.10) THE CONNECTIONS DETAILS OF THE FRAMES A,B AND C

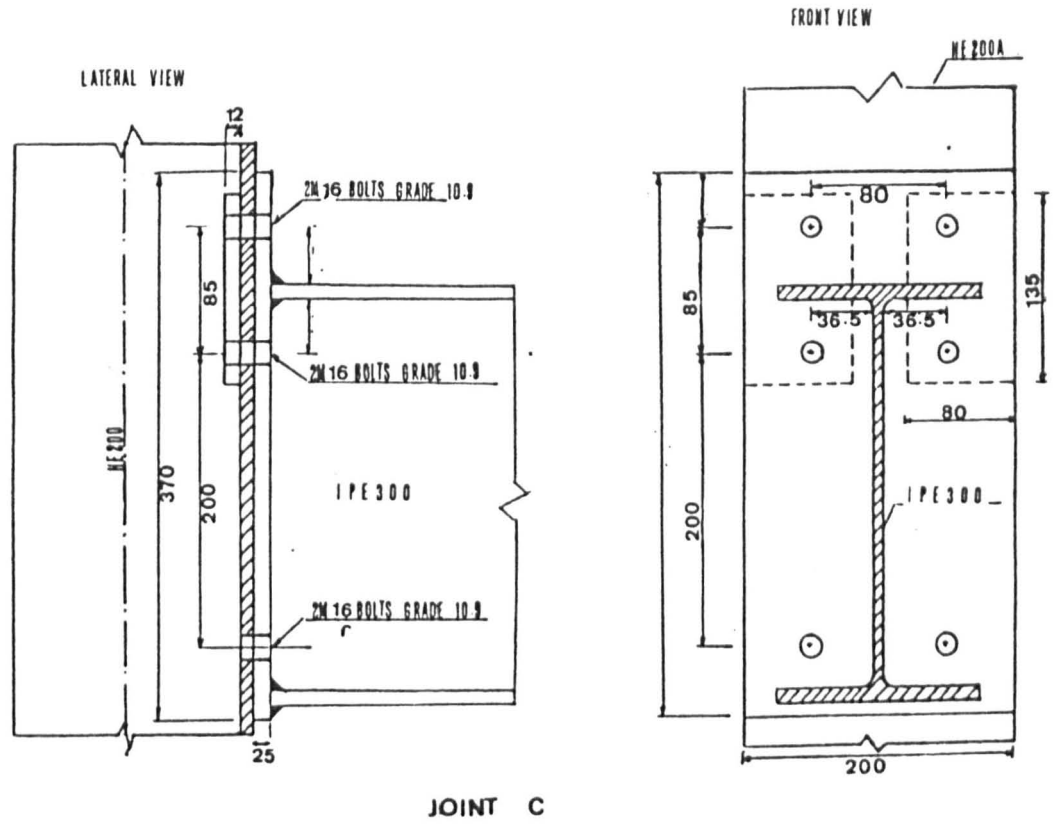


FIG. (4.10) CONTINUED

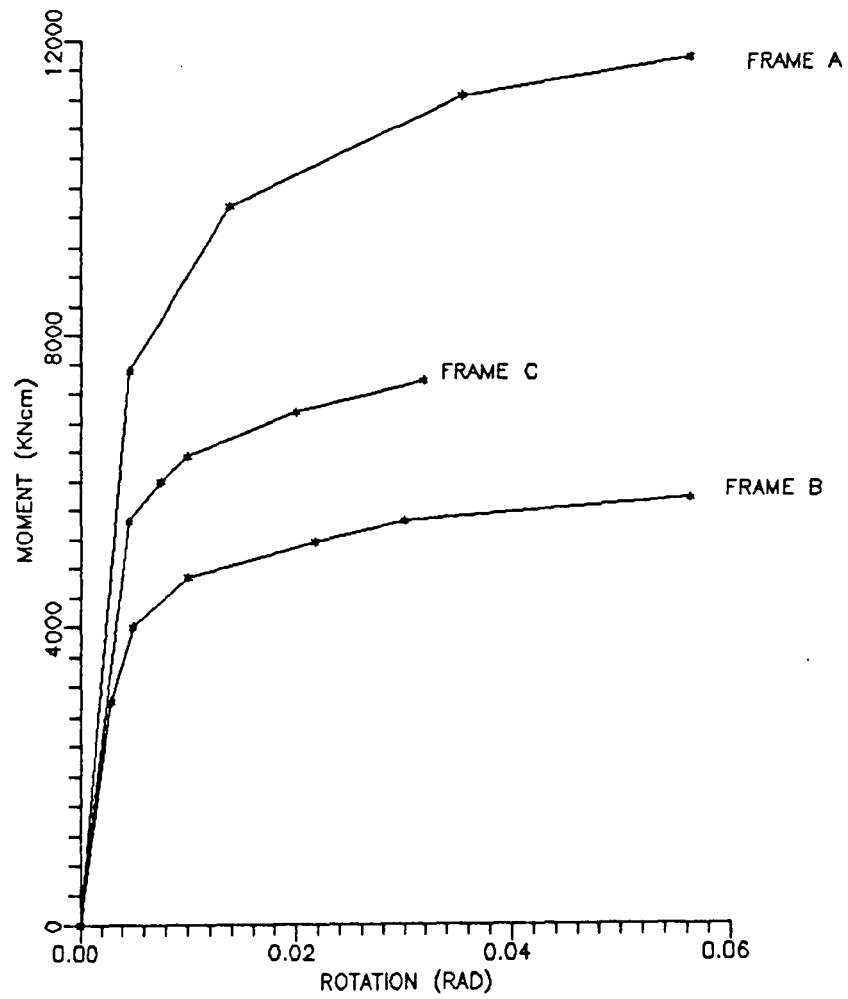


FIG. (4.11) MOMENT-ROTATION RELATIONSHIP OF THE FRAMES A,B AND C



# FRAME A

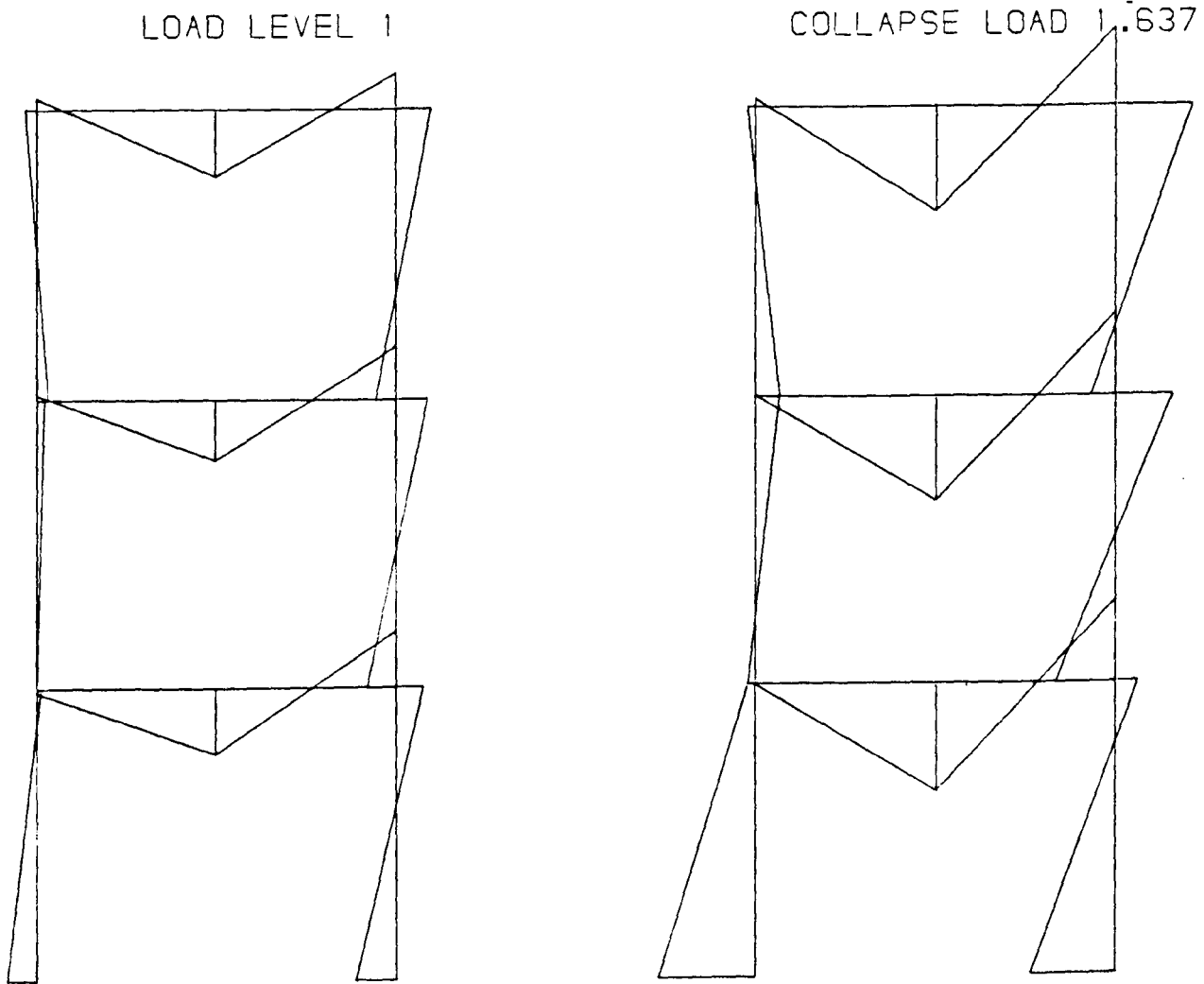


FIG. (4.12) BENDING MOMENT DIAGRAM OF FRAME A  
TYPE 1 LOADING, SCALE 1cm TO 100 kNm

# FRAME A

LOAD LEVEL 1

COLLAPSE LOAD 1.637

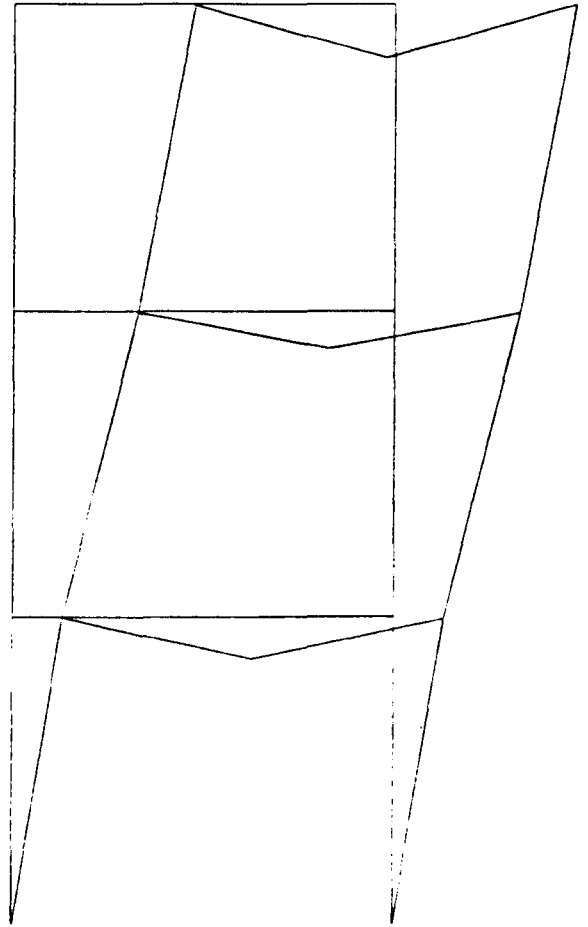
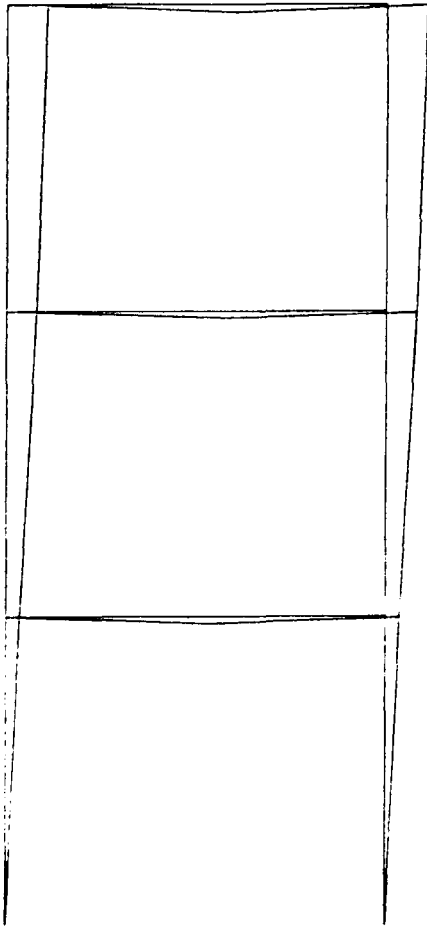


FIG. (4.13) DEFLECTED SHAPE OF FRAME A  
TYPE 1 LOADING, SCALE 1cm TO 10cm

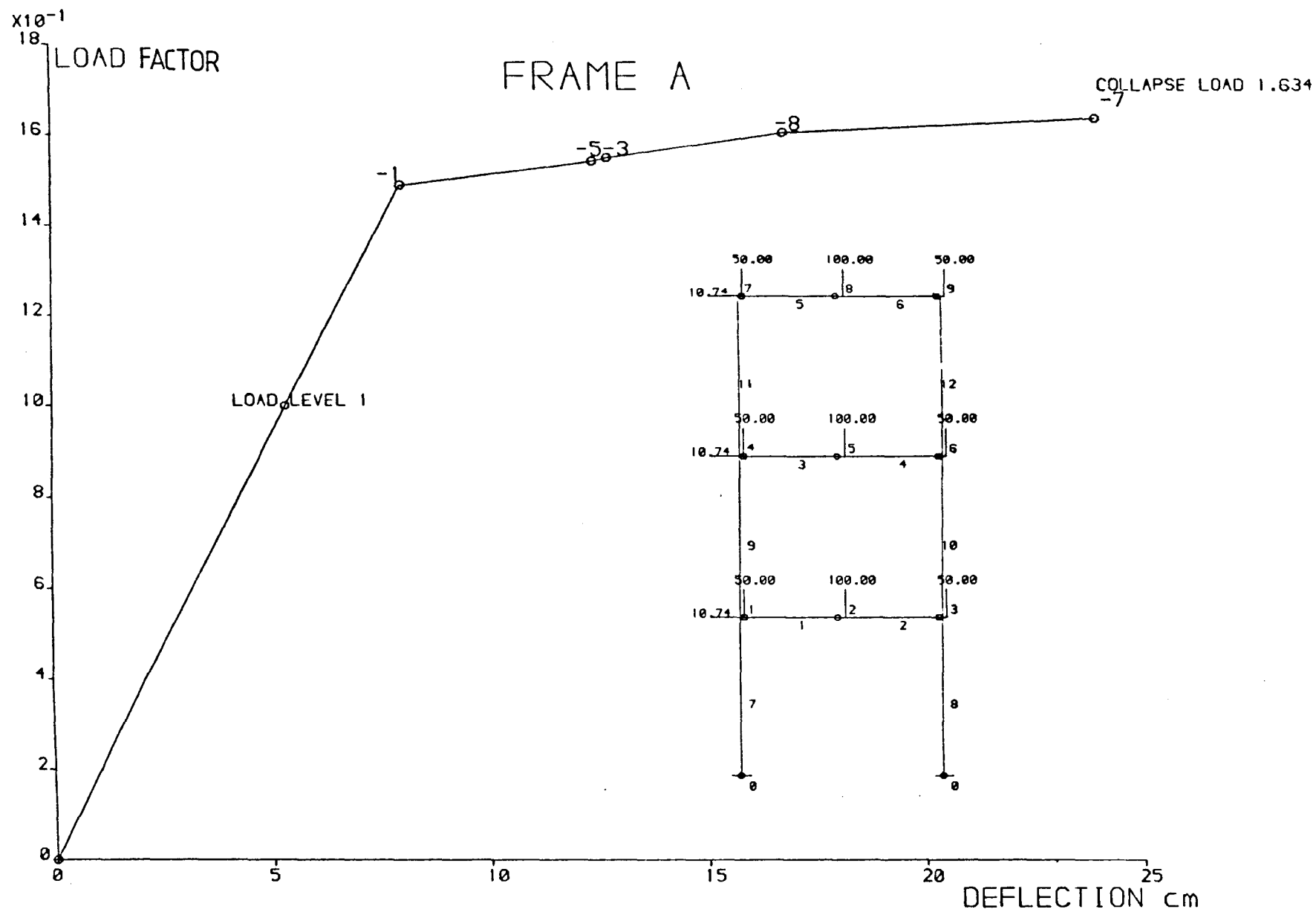


FIG. (4.14) LOAD DEFLECTION CURVE FOR A FRAME A

TYPE 1 LOADING

# FRAME A1

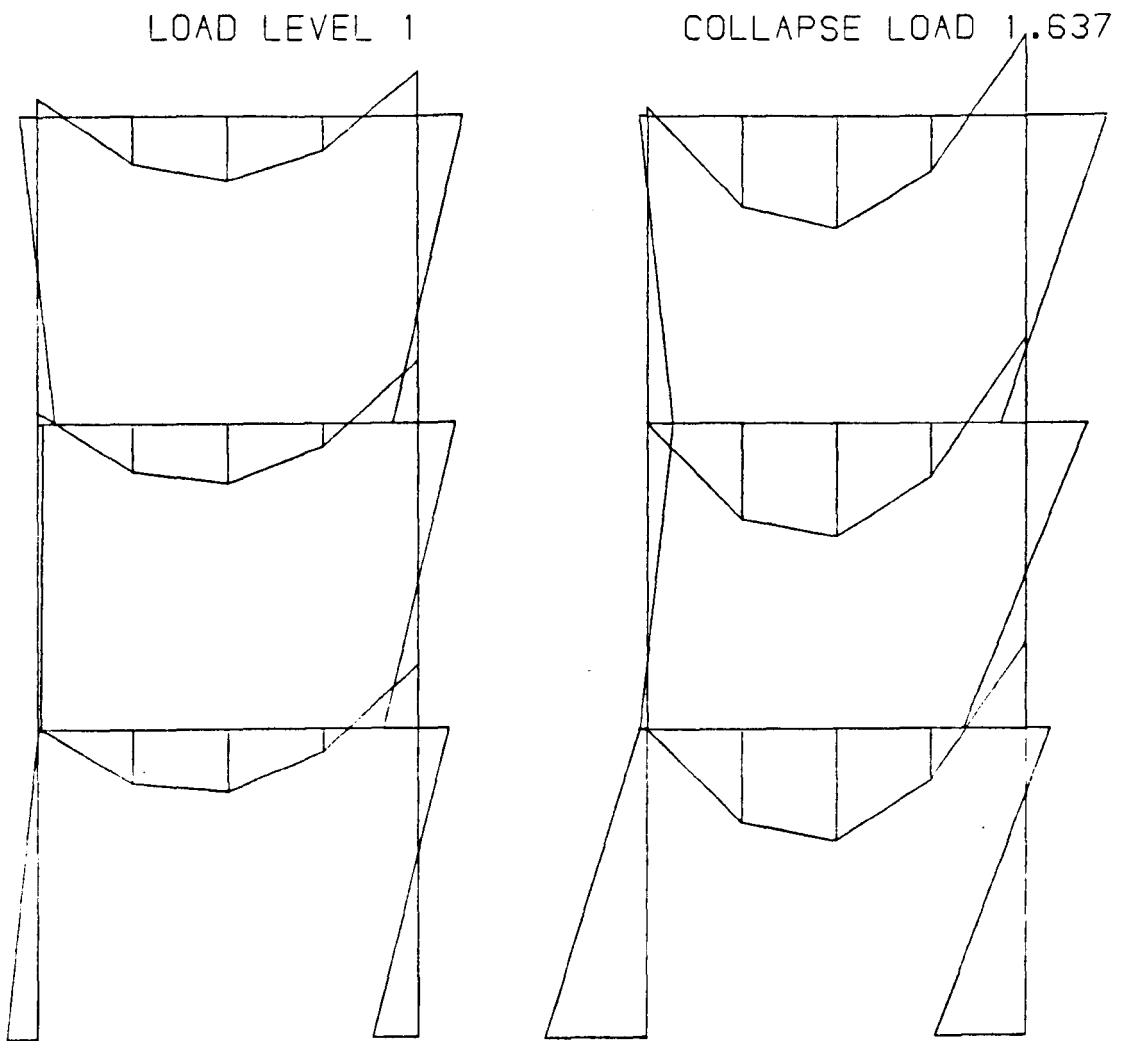
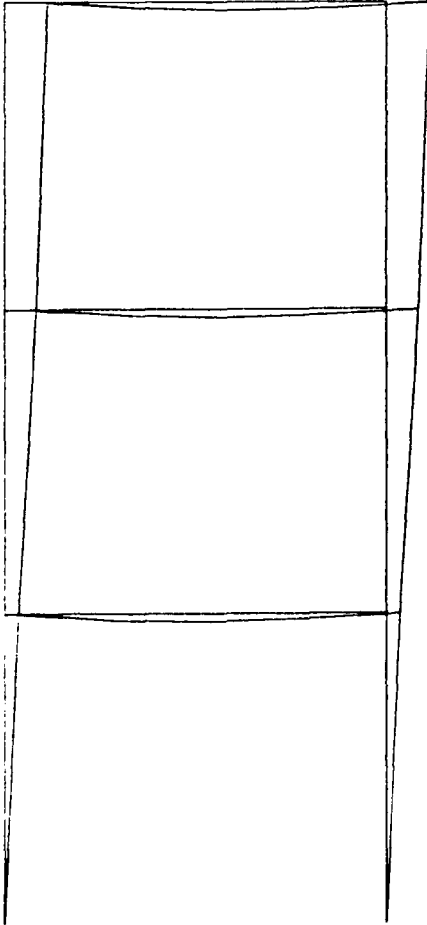


FIG. (4.15) BENDING MOMENT DIAGRAM OF FRAME A1  
TYPE 2 LOADING, SCALE 1cm TO 100 kNm

## FRAME A1

LOAD LEVEL 1



COLLAPSE LOAD 1.637

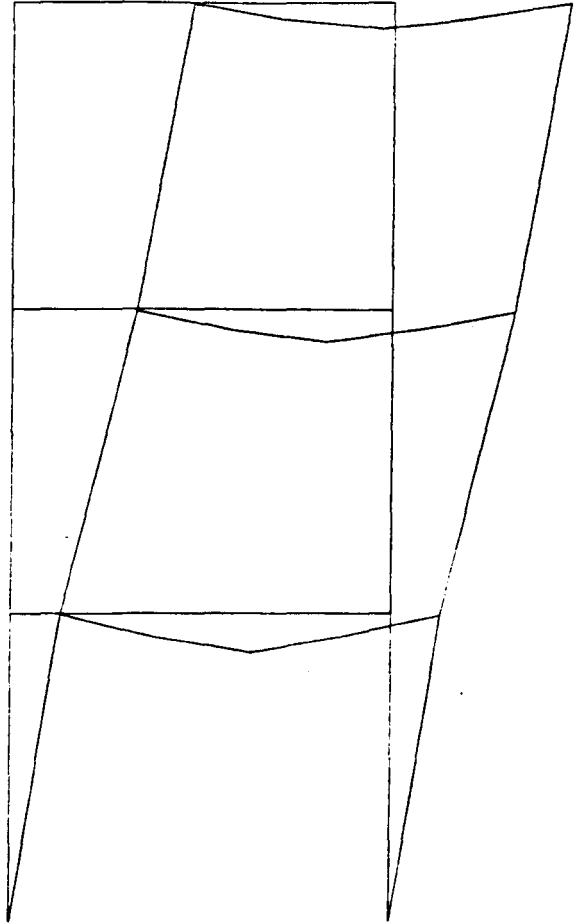
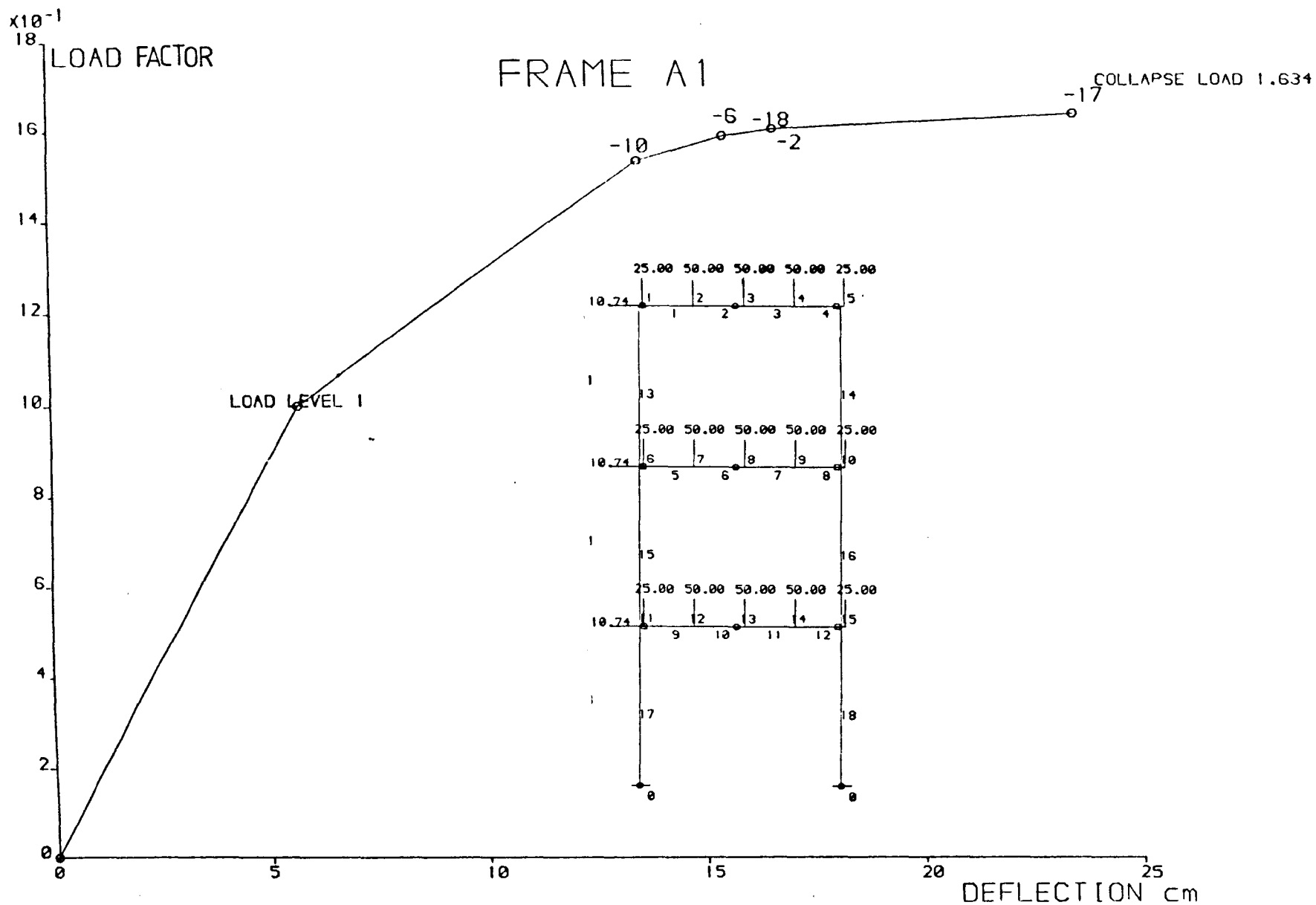


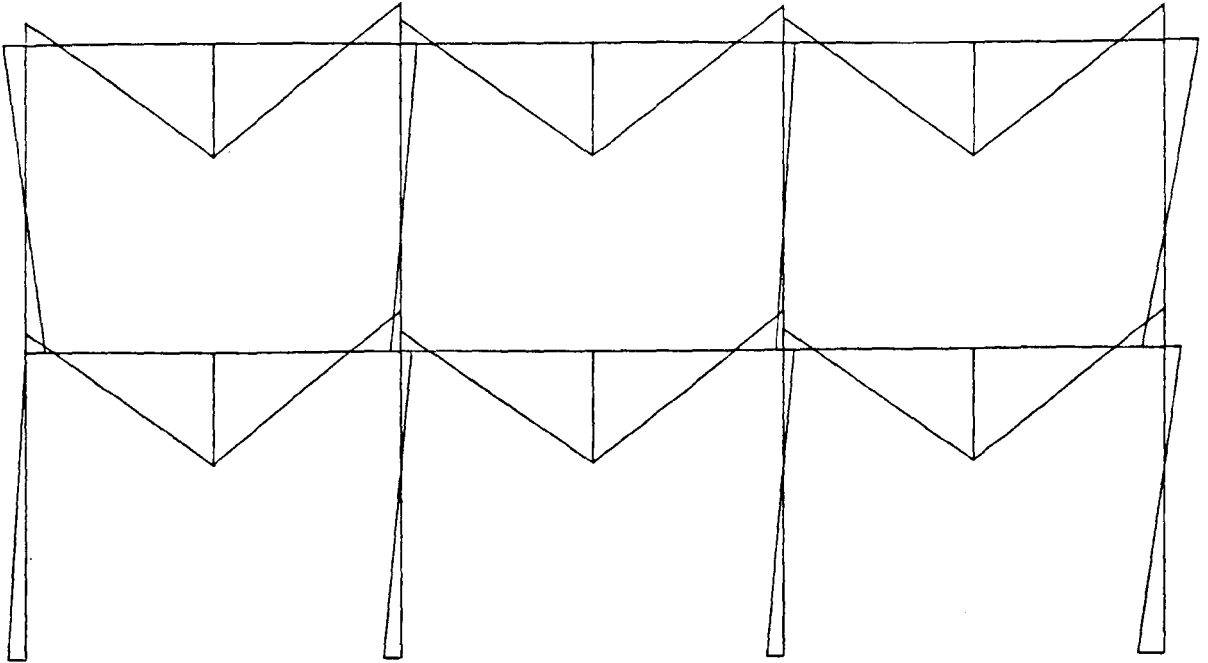
FIG. (4.16) DEFLECTED SHAPE OF FRAME A1  
TYPE 2 LOADING, SCALE 1cm TO 10cm



**FIG. (4.17) LOAD DEFLECTION CURVE FOR A FRAME A1  
TYPE 2 LOADING**

# FRAME B

COLLAPSE LOAD 1.497



LOAD LEVEL 1

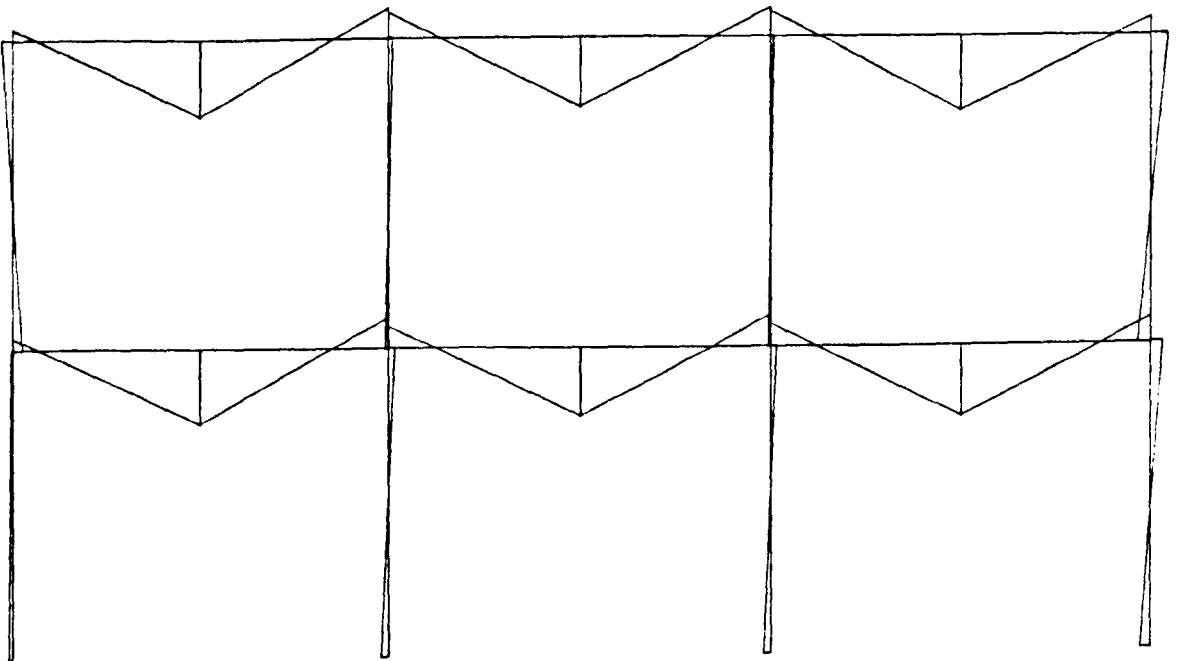
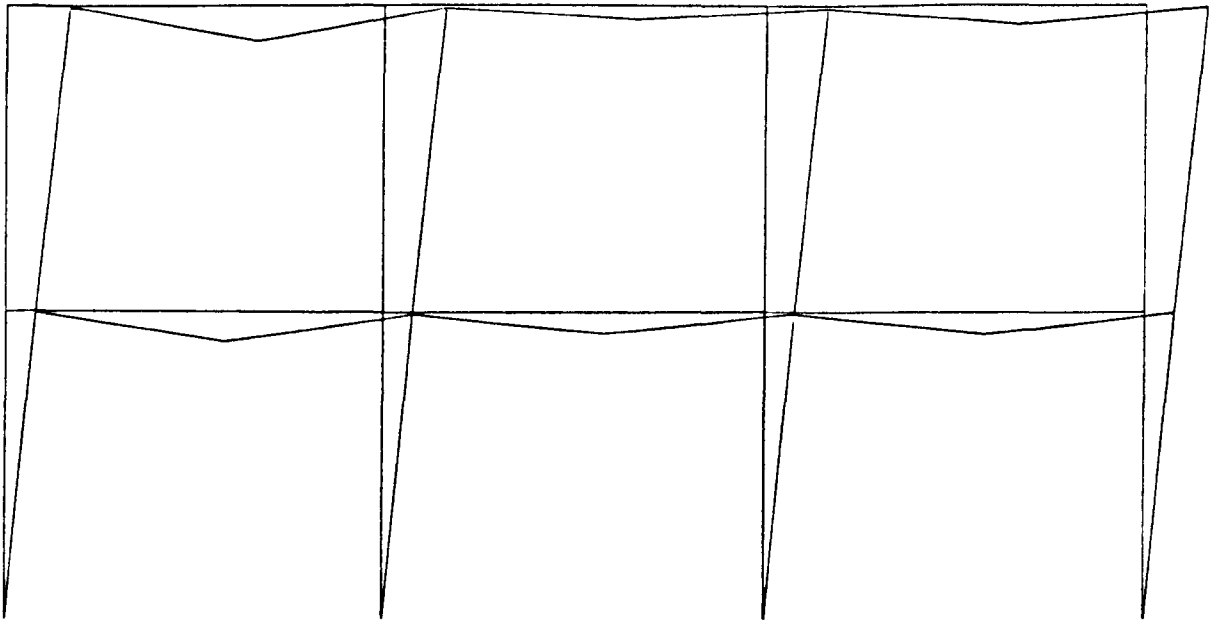


FIG. (4.18) BENDING MOMENT DIAGRAM OF FRAME B  
TYPE 1 LOADING, SCALE 1cm TO 100 kNm

# FRAME B

COLLAPSE LOAD 1.497



LOAD LEVEL 1

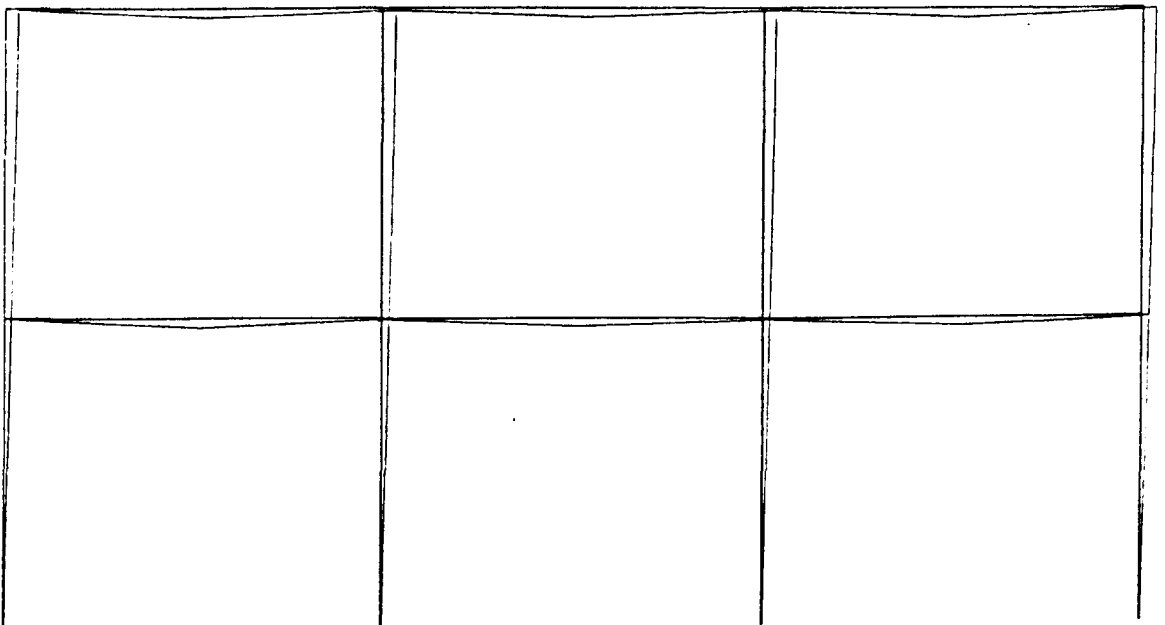


FIG. (4.19) DEFLECTED SHAPE OF FRAME B  
TYPE 1 LOADING, SCALE 1cm TO 10cm



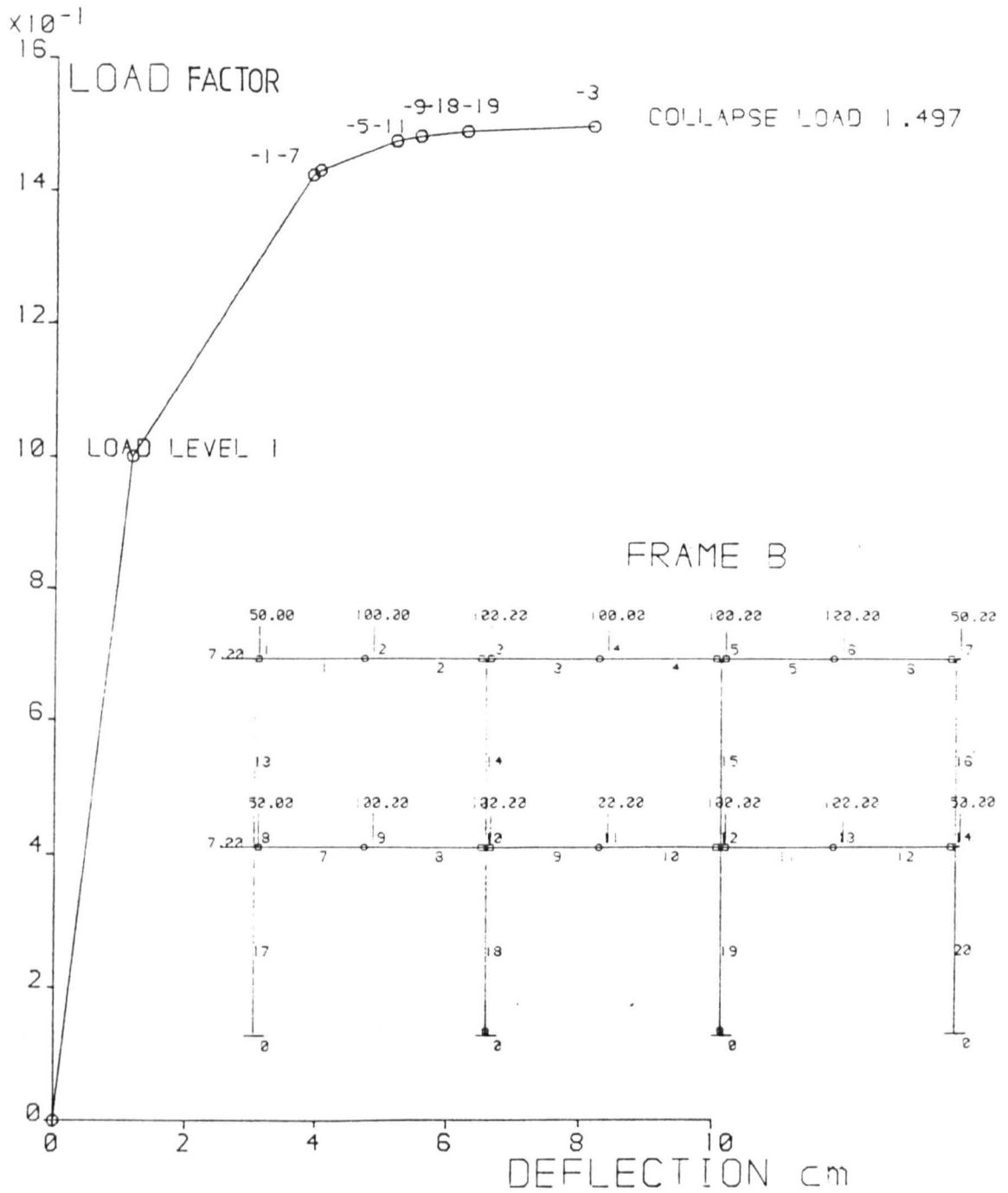
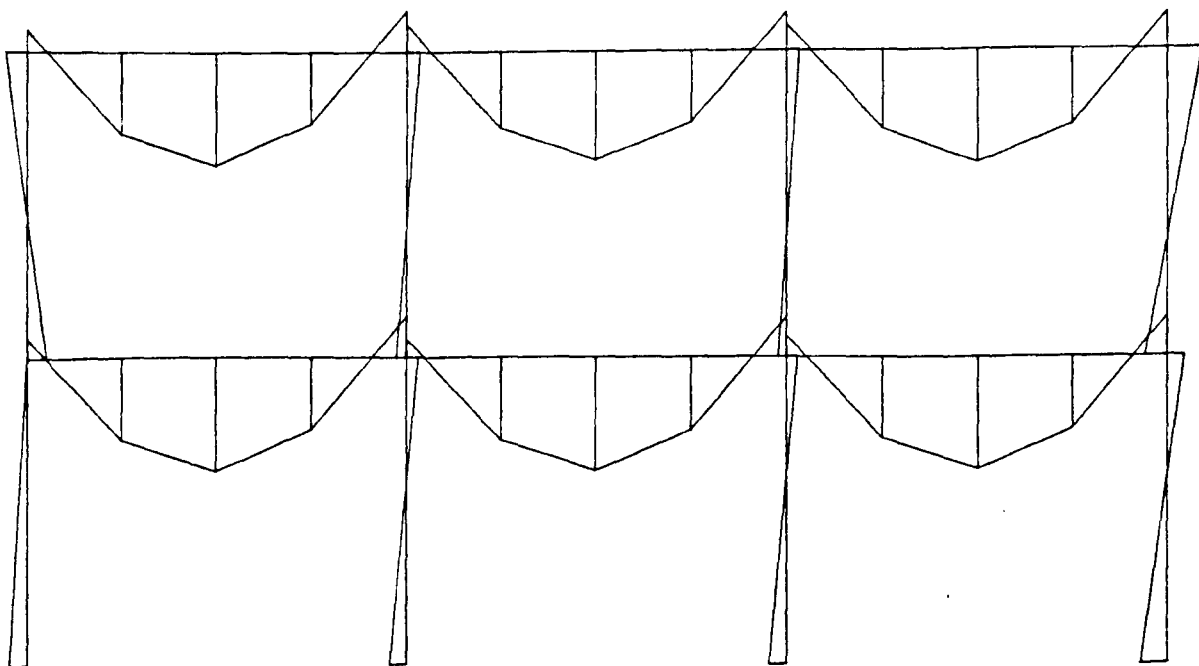


FIG. (4.20) LOAD DEFLECTION CURVE FOR A FRAME B  
TYPE 1 LOADING

# FRAME B1

COLLAPSE LOAD 1.497



LOAD LEVEL 1

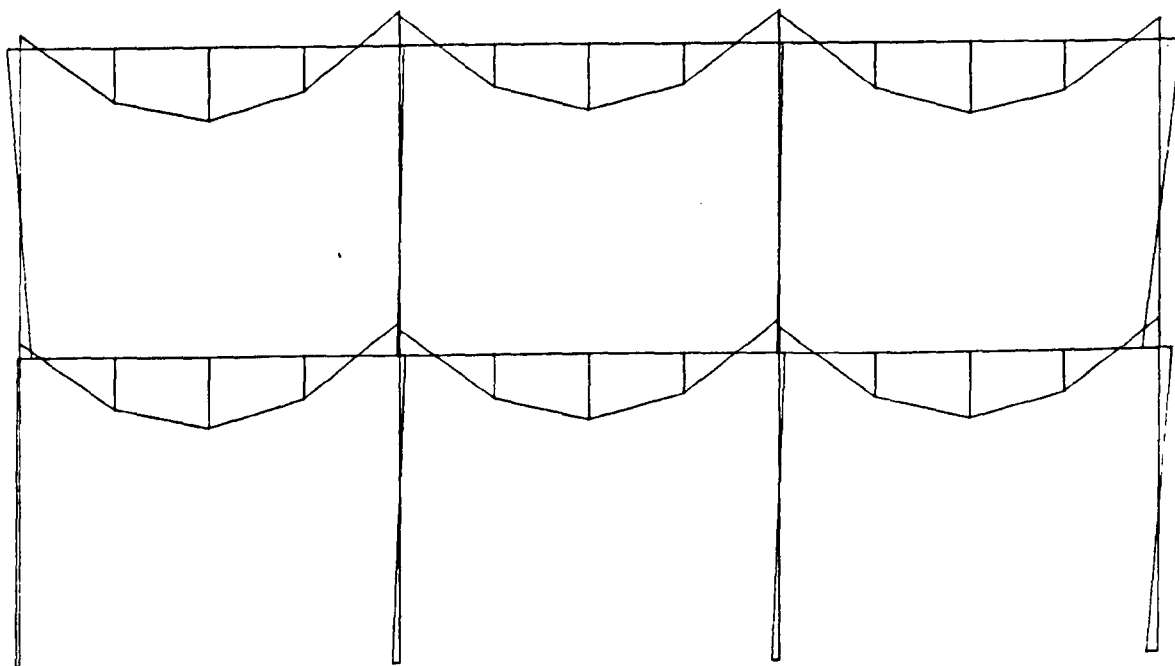
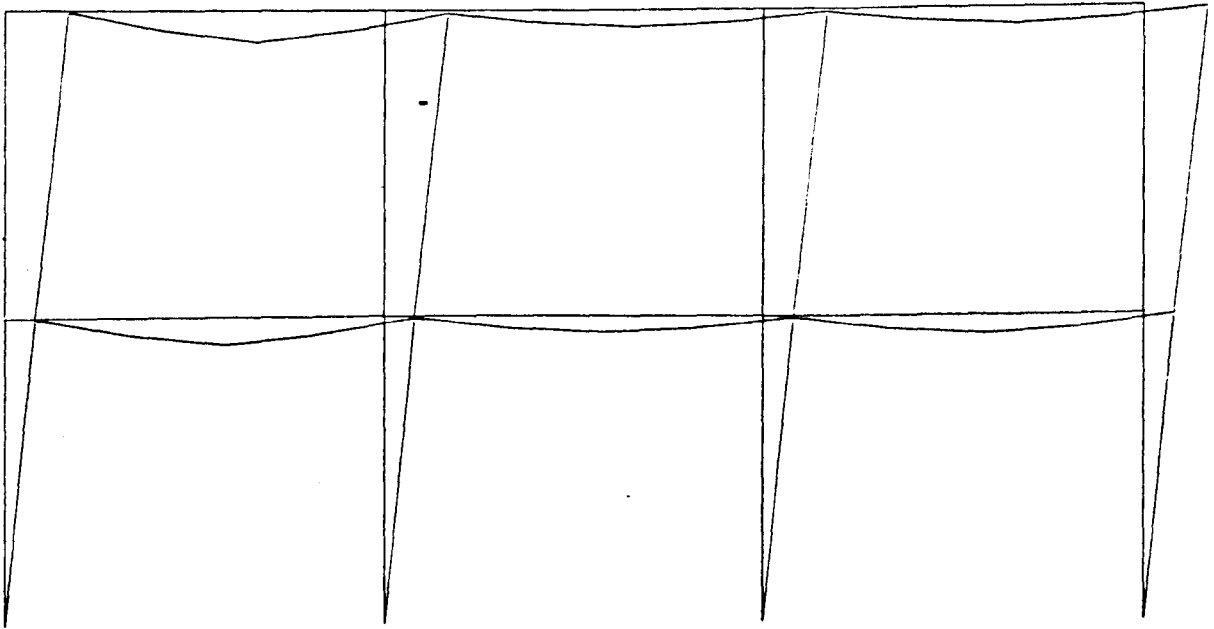


FIG. (4.21) BENDING MOMENT DIAGRAM OF FRAME B1  
TYPE 2 LOADING, SCALE 1cm TO 100 kNm

COLLAPSE LOAD 1.497



LOAD LEVEL 1

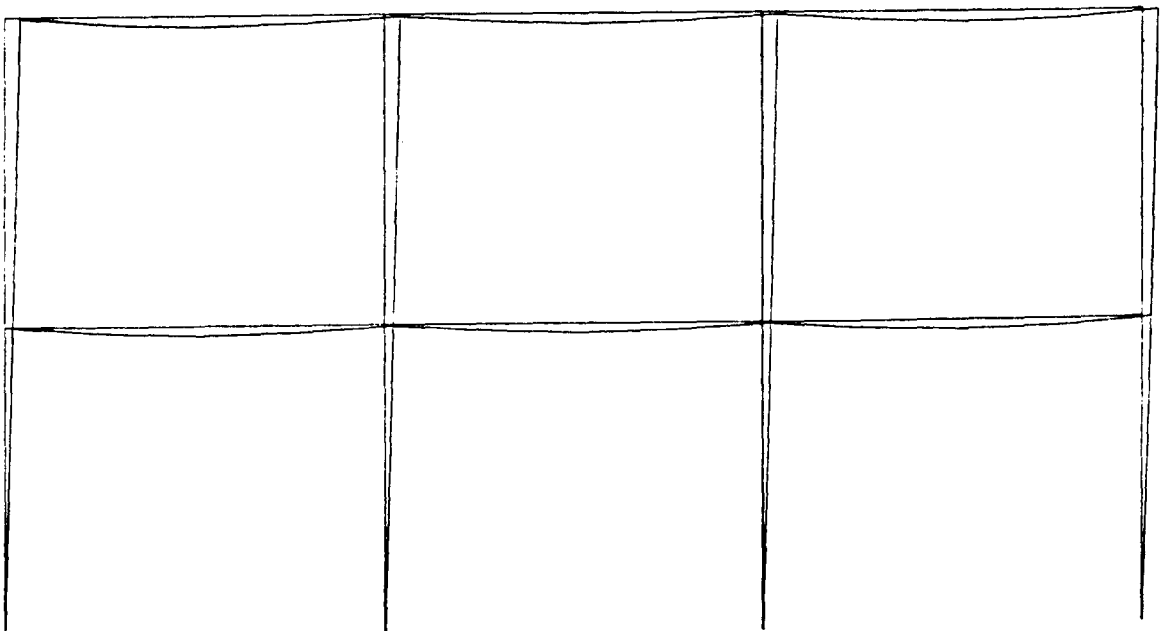


FIG. (4.22) DEFLECTED SHAPE OF FRAME B1  
TYPE 2 LOADING, SCALE 1cm TO 10cm

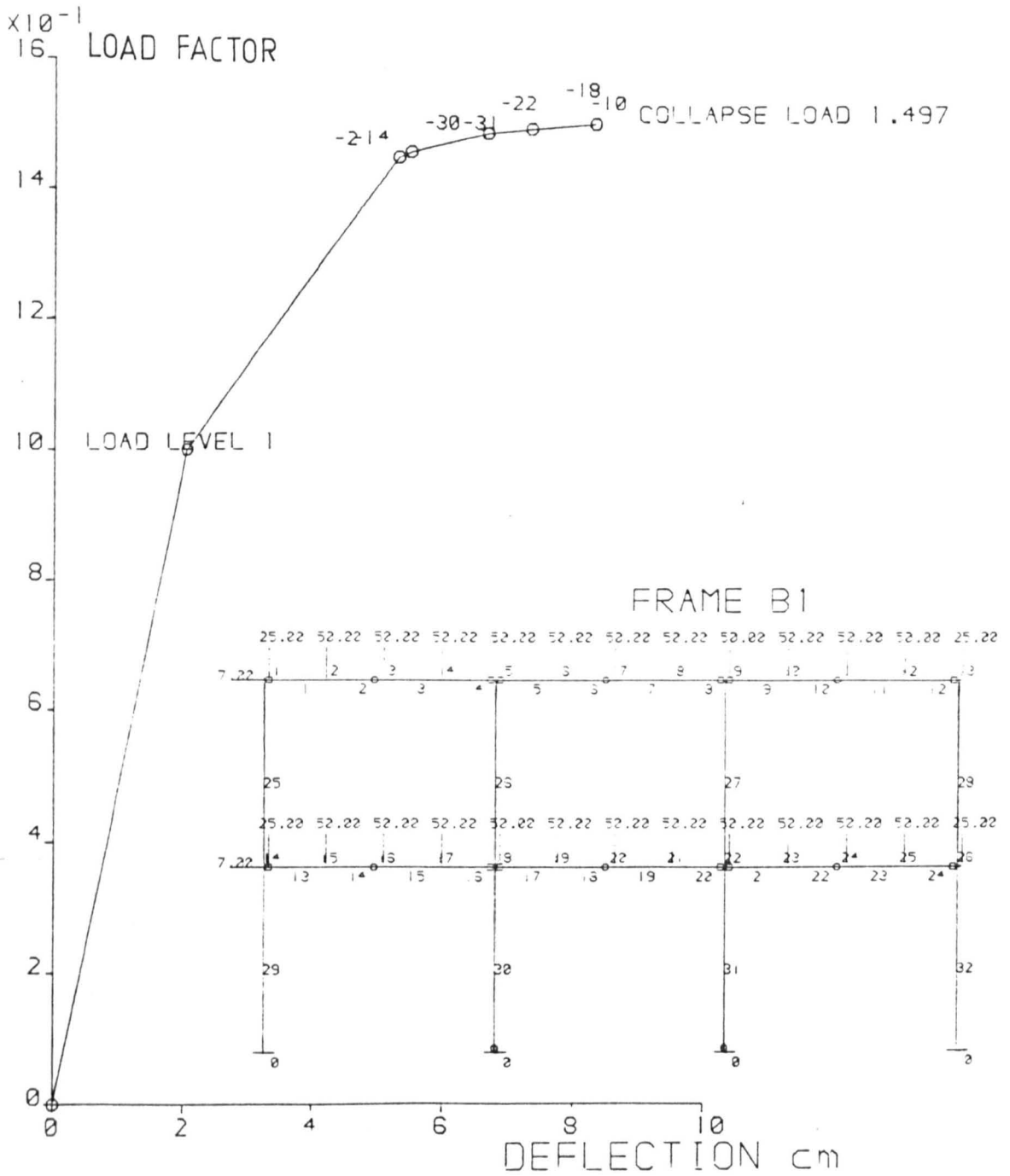


FIG. (4.23) LOAD DEFLECTION CURVE FOR A FRAME B1  
TYPE 2 LOADING

# FRAME C

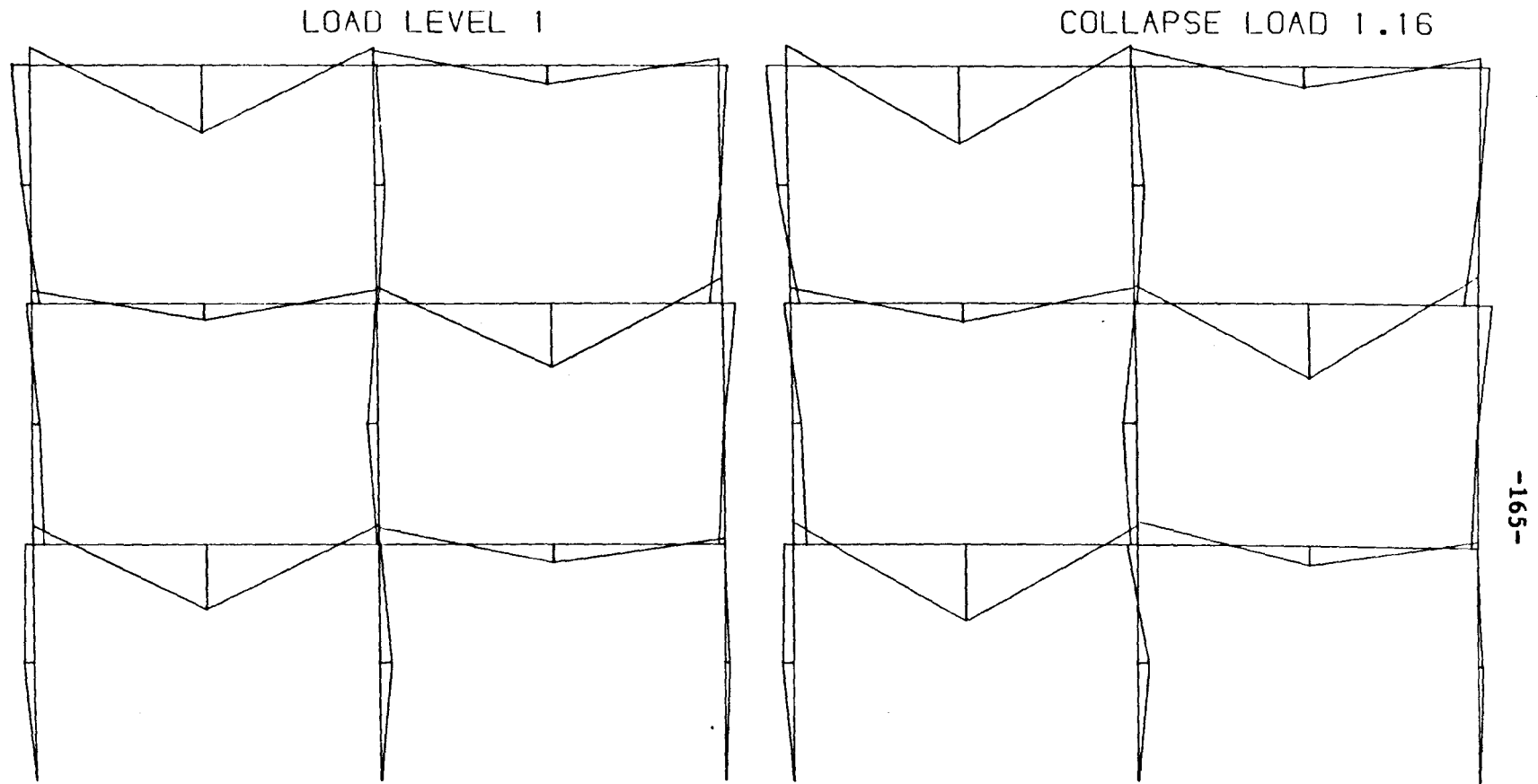
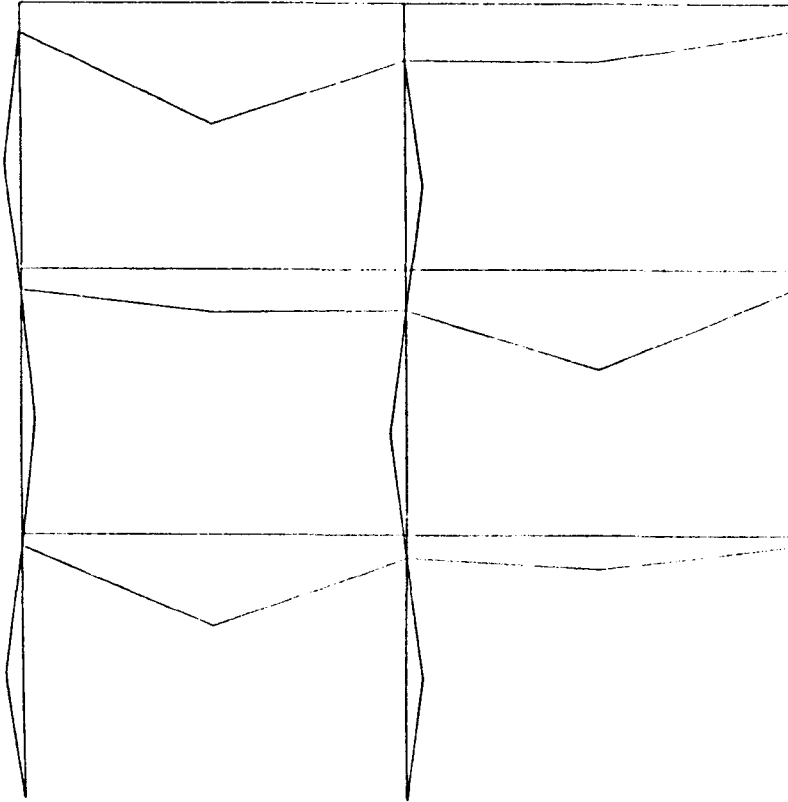


FIG. (4.24) BENDING MOMENT DIAGRAM OF FRAME C  
TYPE 1 LOADING, SCALE 1cm TO 100 kNm

# FRAME C

LOAD LEVEL 1



COLLAPSE LOAD 1.16

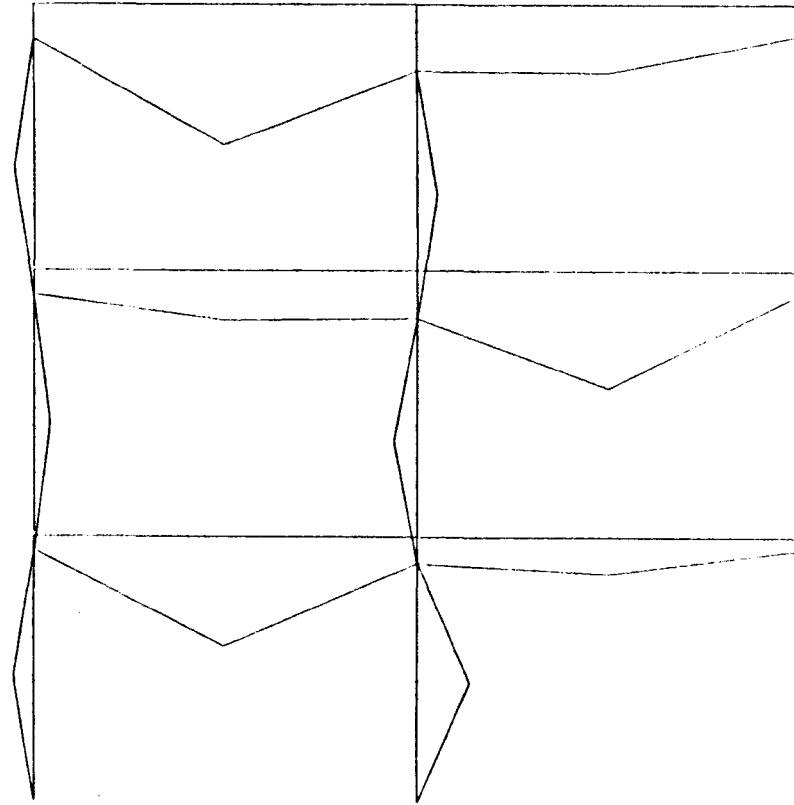


FIG. (4.25) DEFLECTED SHAPE OF FRAME C  
TYPE 1 LOADING, SCALE 1cm TO 10cm

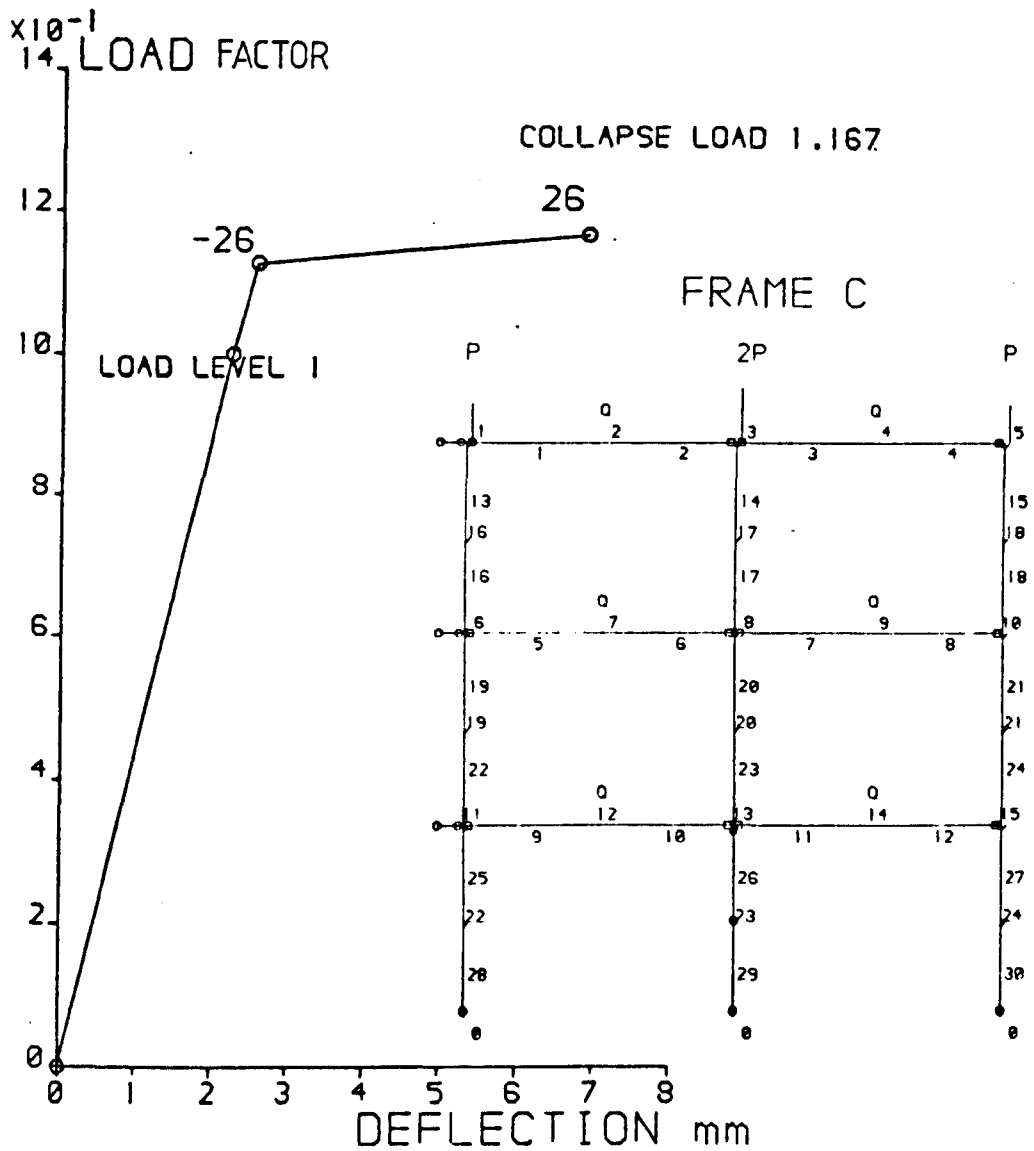


FIG. (4.26) LOAD DEFLECTION CURVE FOR A FRAME C  
TYPE 1 LOADING

## FRAME C1

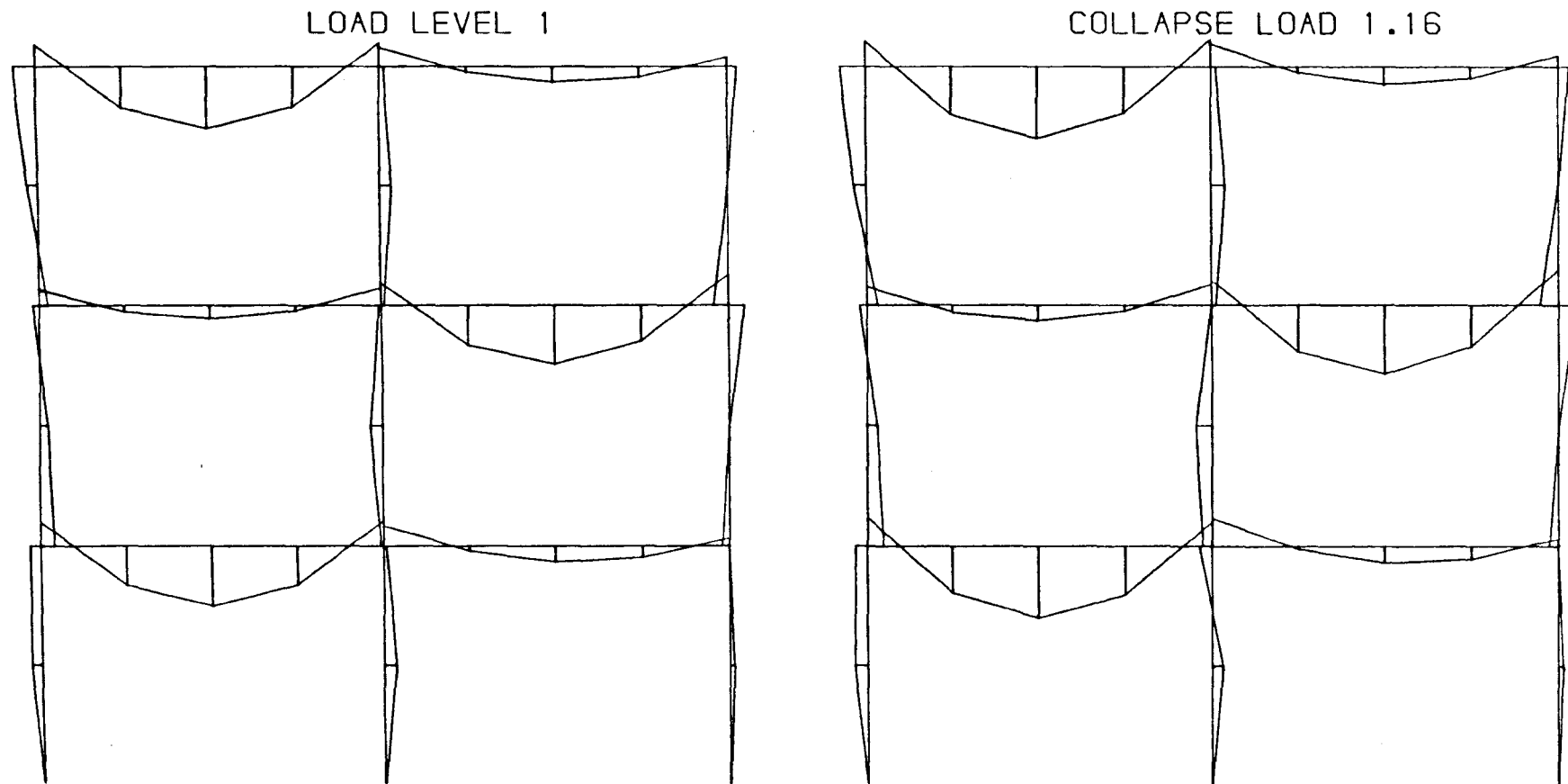
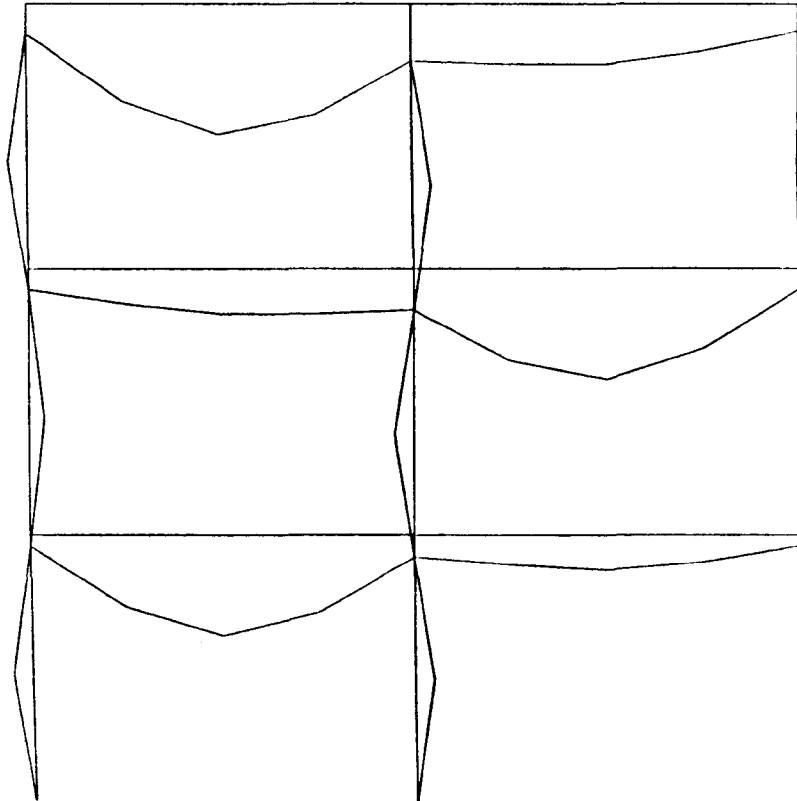


FIG. (4.27) BENDING MOMENT DIAGRAM OF FRAME C1  
TYPE 2 LOADING, SCALE 1cm TO 100 kNm



## FRAME C1

LOAD LEVEL 1



COLLAPSE LOAD 1.16

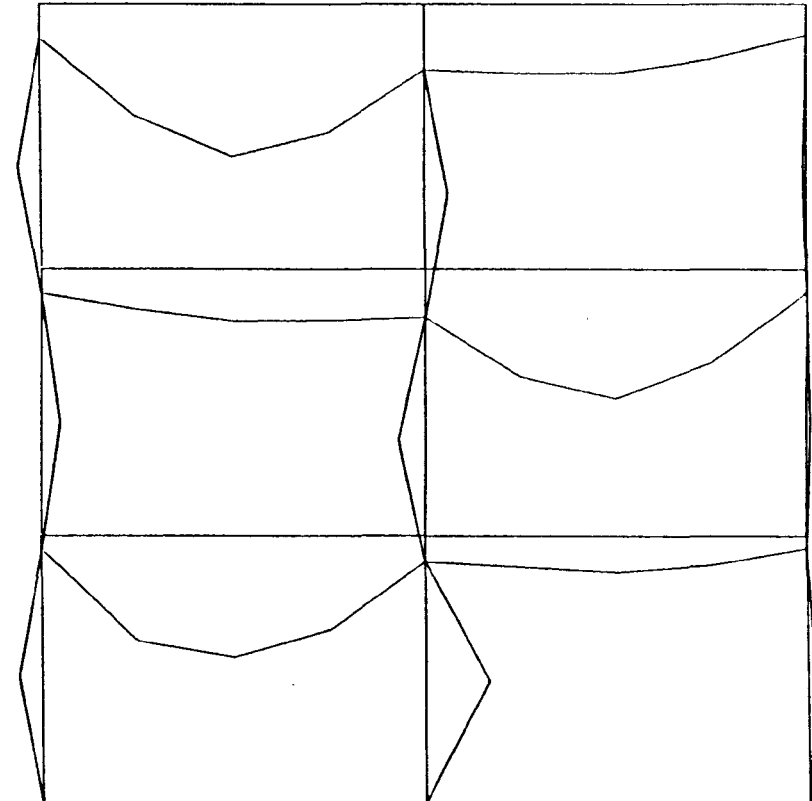


FIG. (4.28) DEFLECTED SHAPE OF FRAME C1  
TYPE 2 LOADING, SCALE 1cm TO 10cm

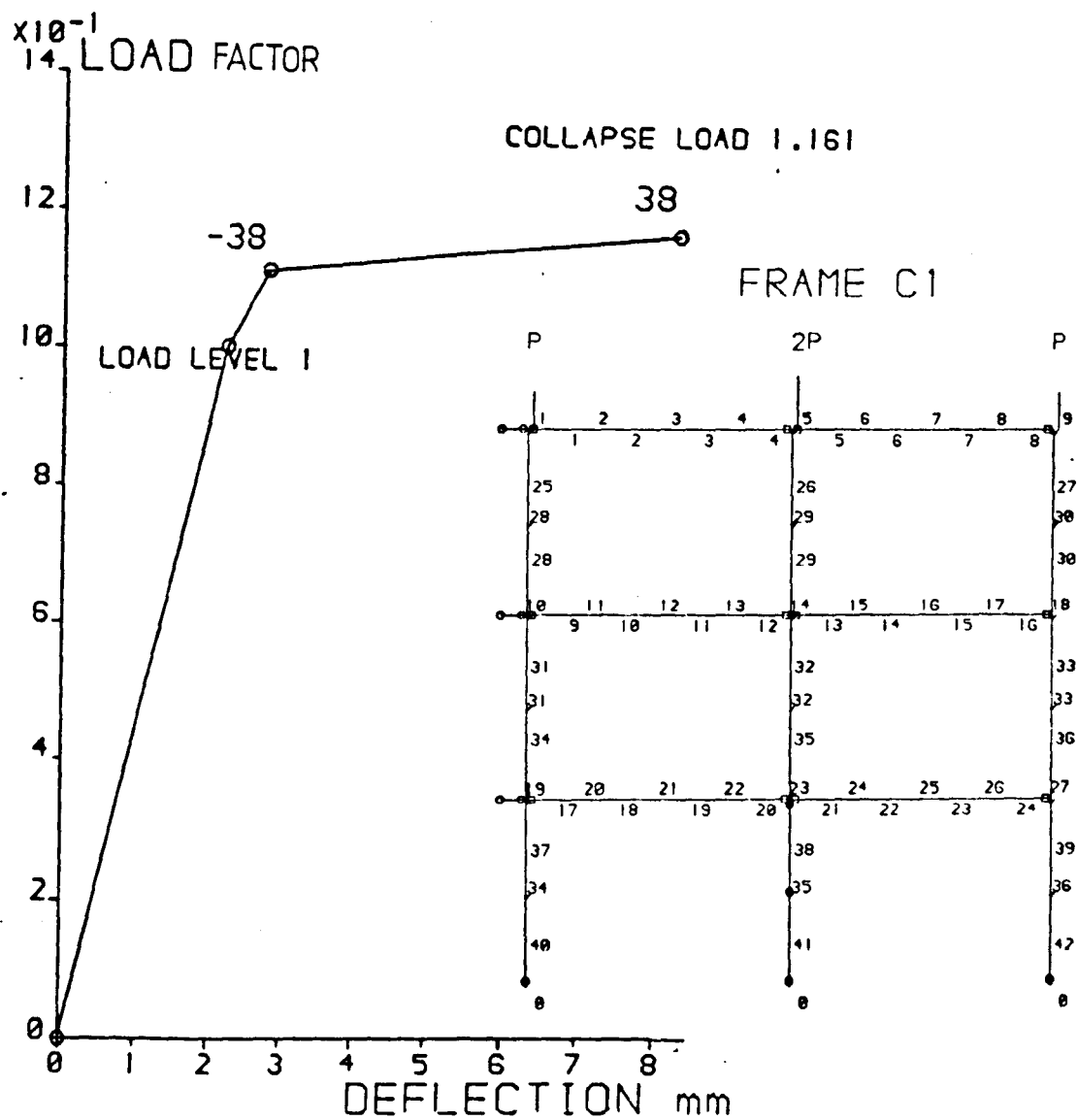


FIG. (4.29) LOAD DEFLECTION CURVE FOR A FRAME C1  
TYPE 2 LOADING

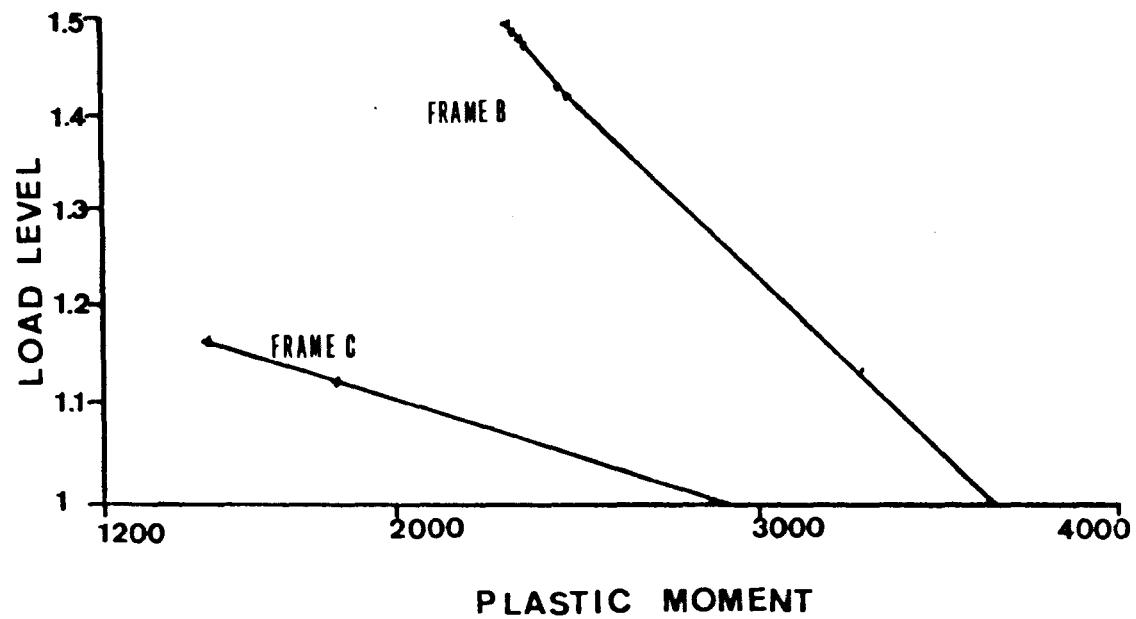


FIG. (4.30) REDUCTION IN PLASTIC MOMENT OF RESISTANCE  
DUE TO INCREASE OF AXIAL LOAD IN FRAMES B AND C.

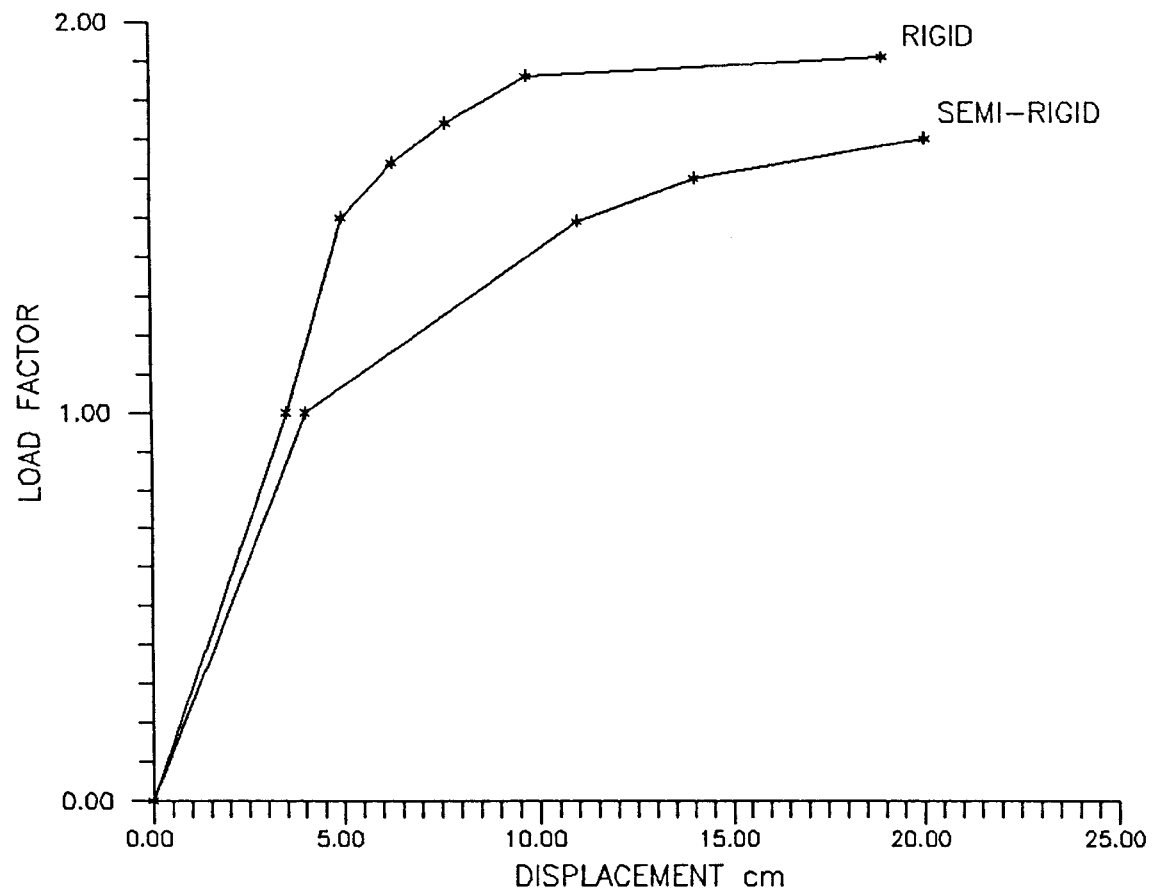


FIG. (4.31) LOAD AGAINST DEFLECTION CURVES OF FRAME A

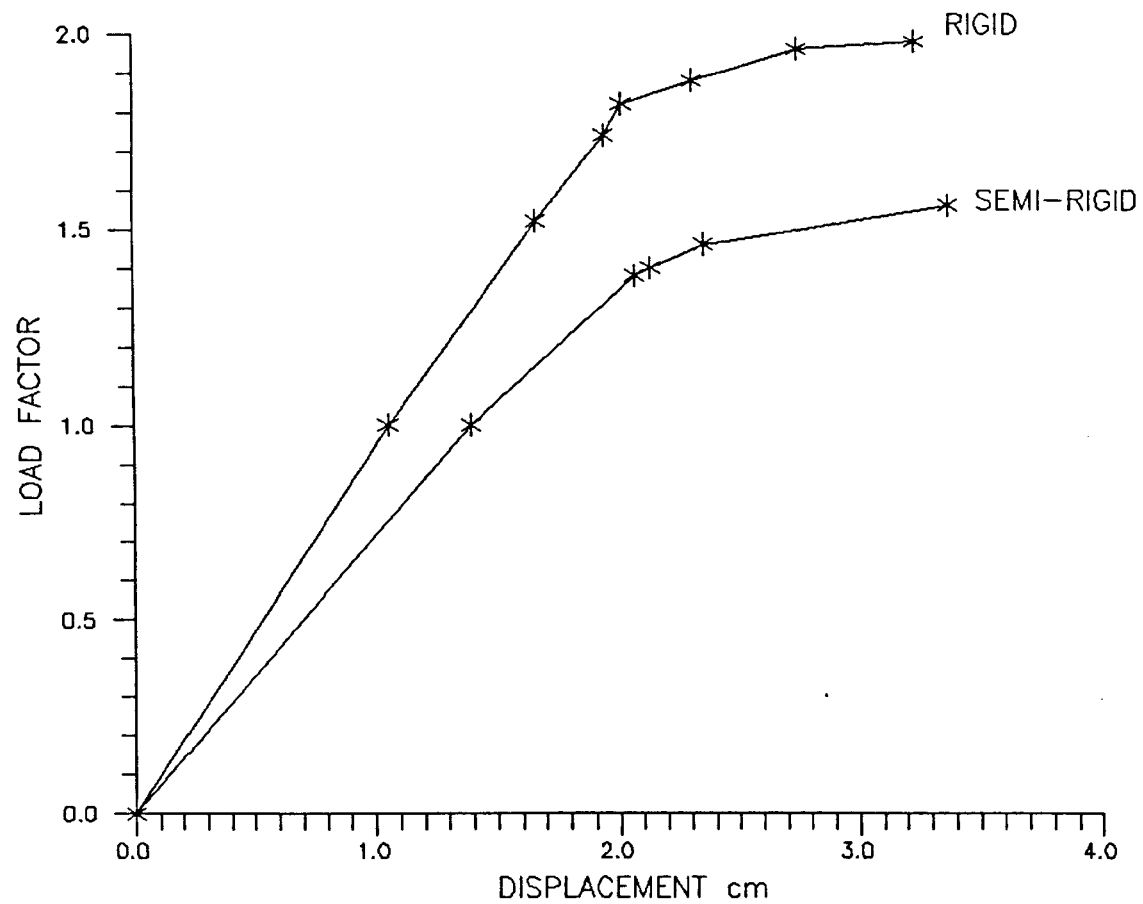


FIG. (4.32) LOAD AGAINST DEFLECTION CURVES OF FRAME B

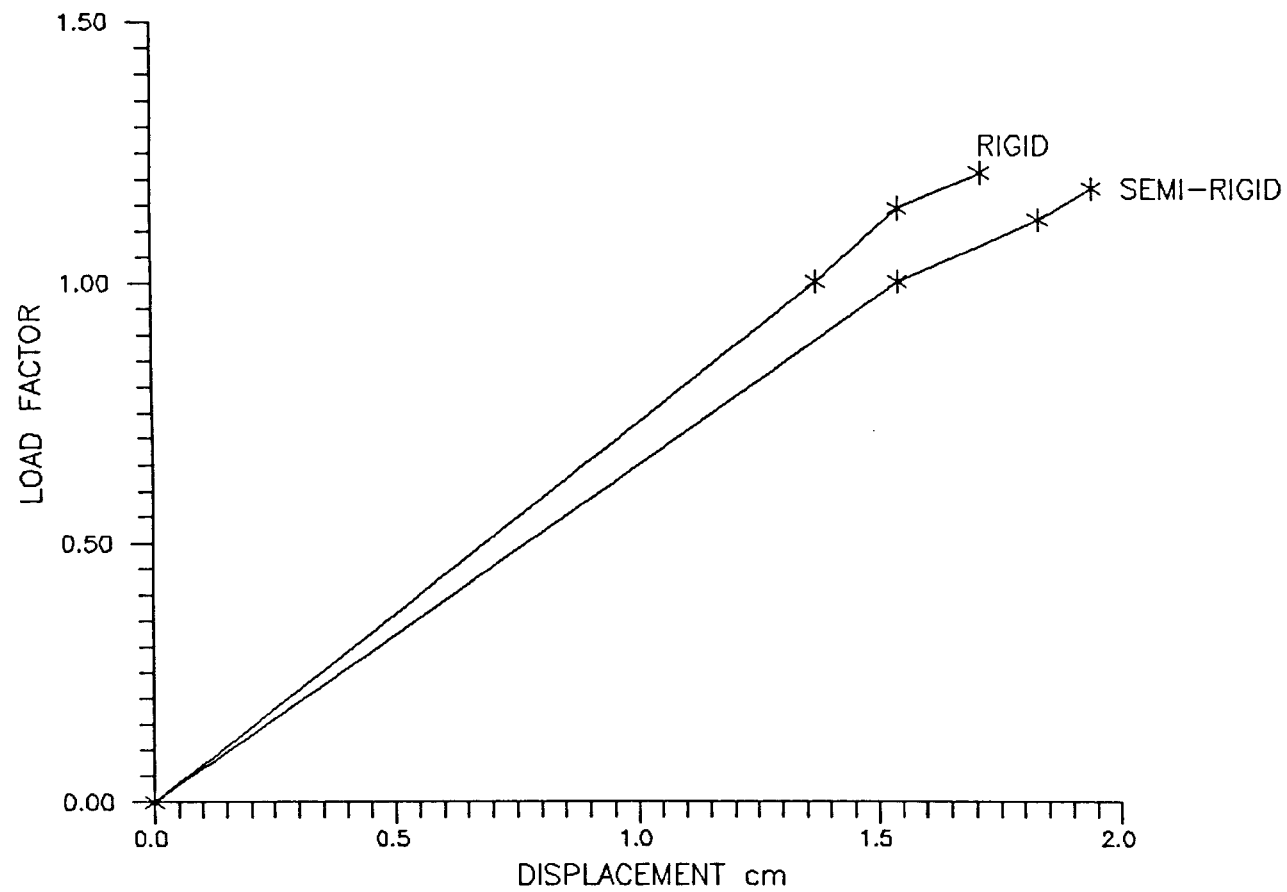


FIG. (4.33) LOAD AGAINST DEFLECTION CURVES OF FRAME C

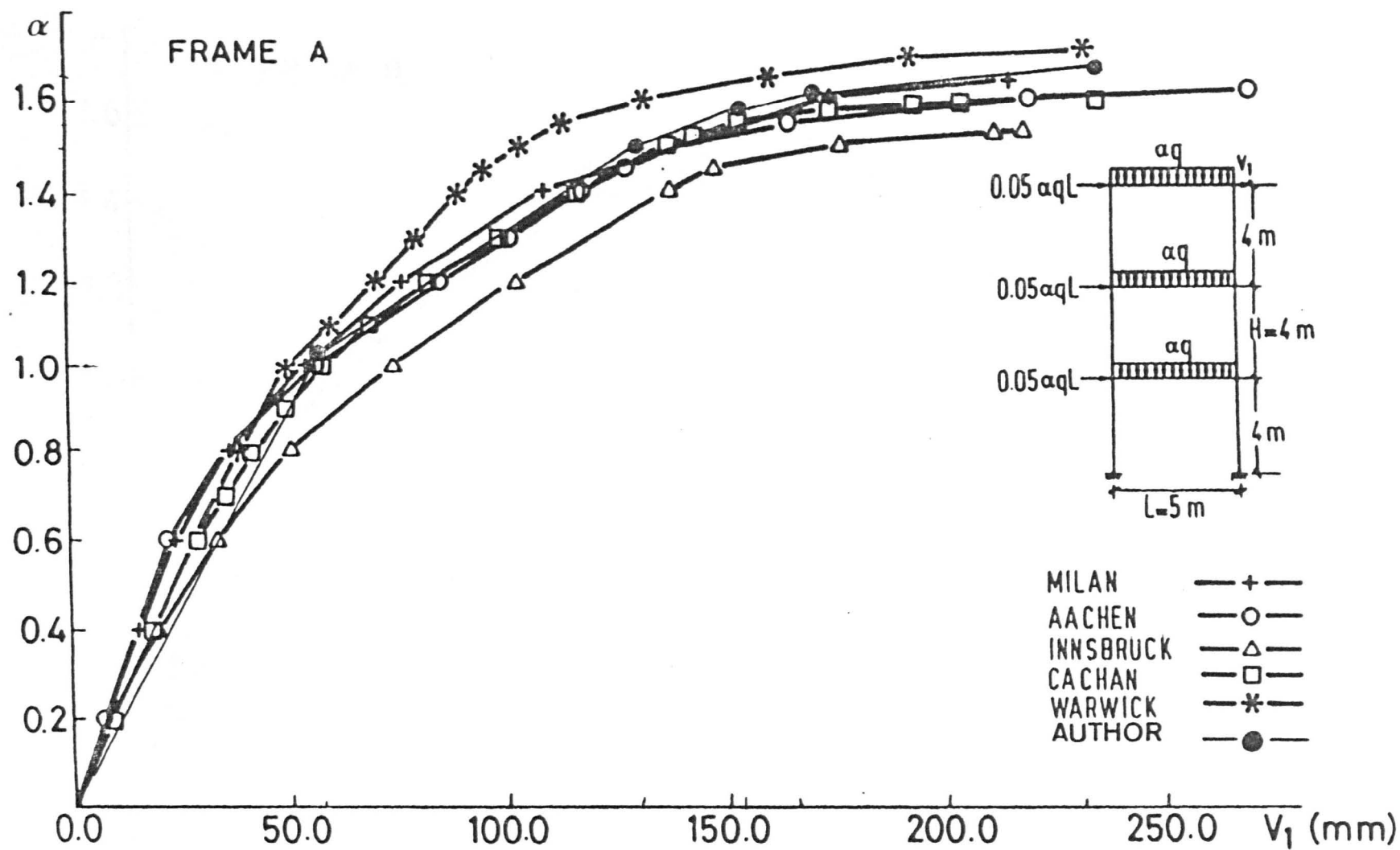


FIG. (4.34) COMPARISON OF THE ANALYSIS OF FRAME 'A' OBTAINED BY DIFFERENT RESEARCHERS.

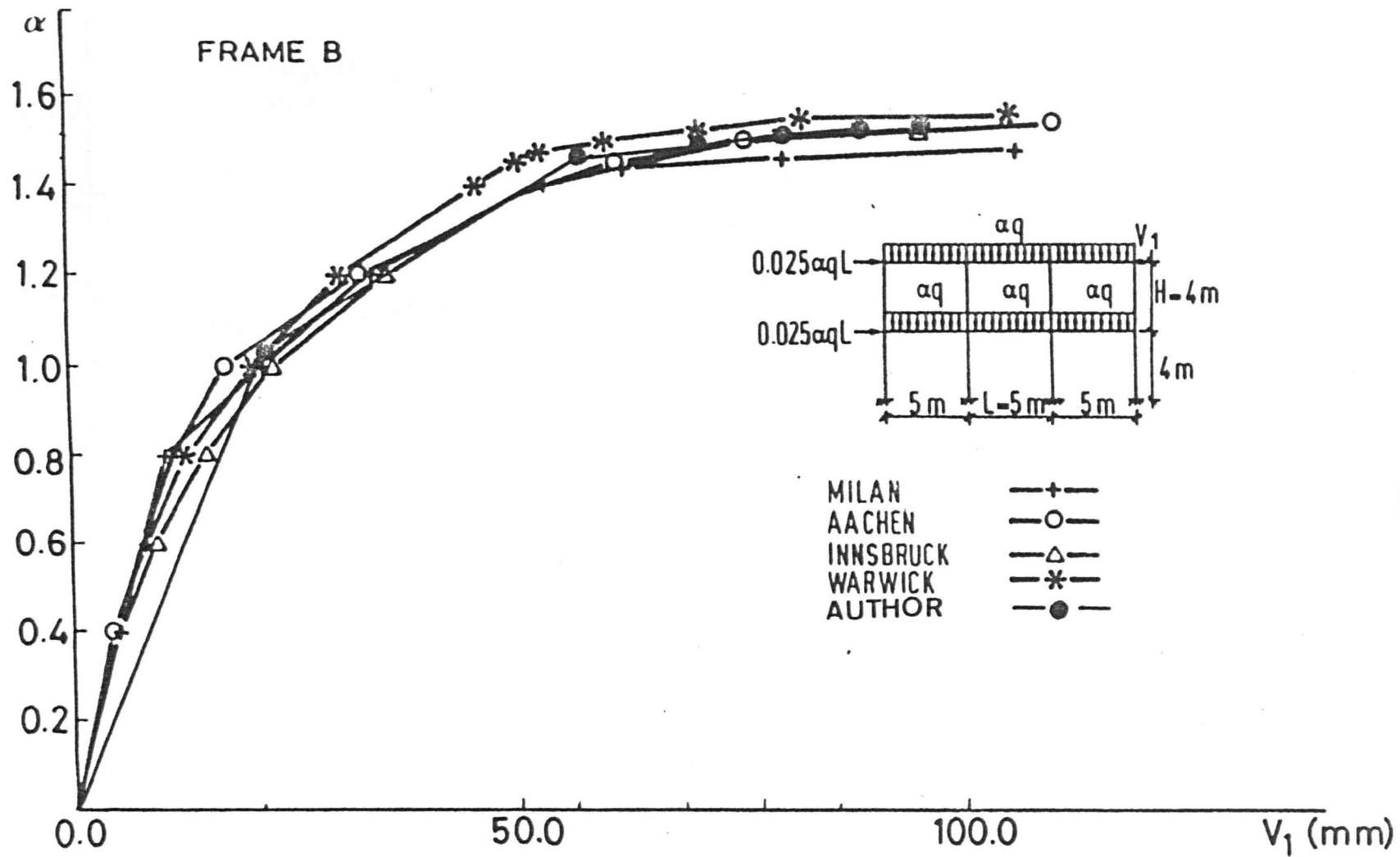


FIG. (4.35) COMPARISON OF THE ANALYSIS OF FRAME 'B' OBTAINED BY DIFFERENT RESEARCHERS.



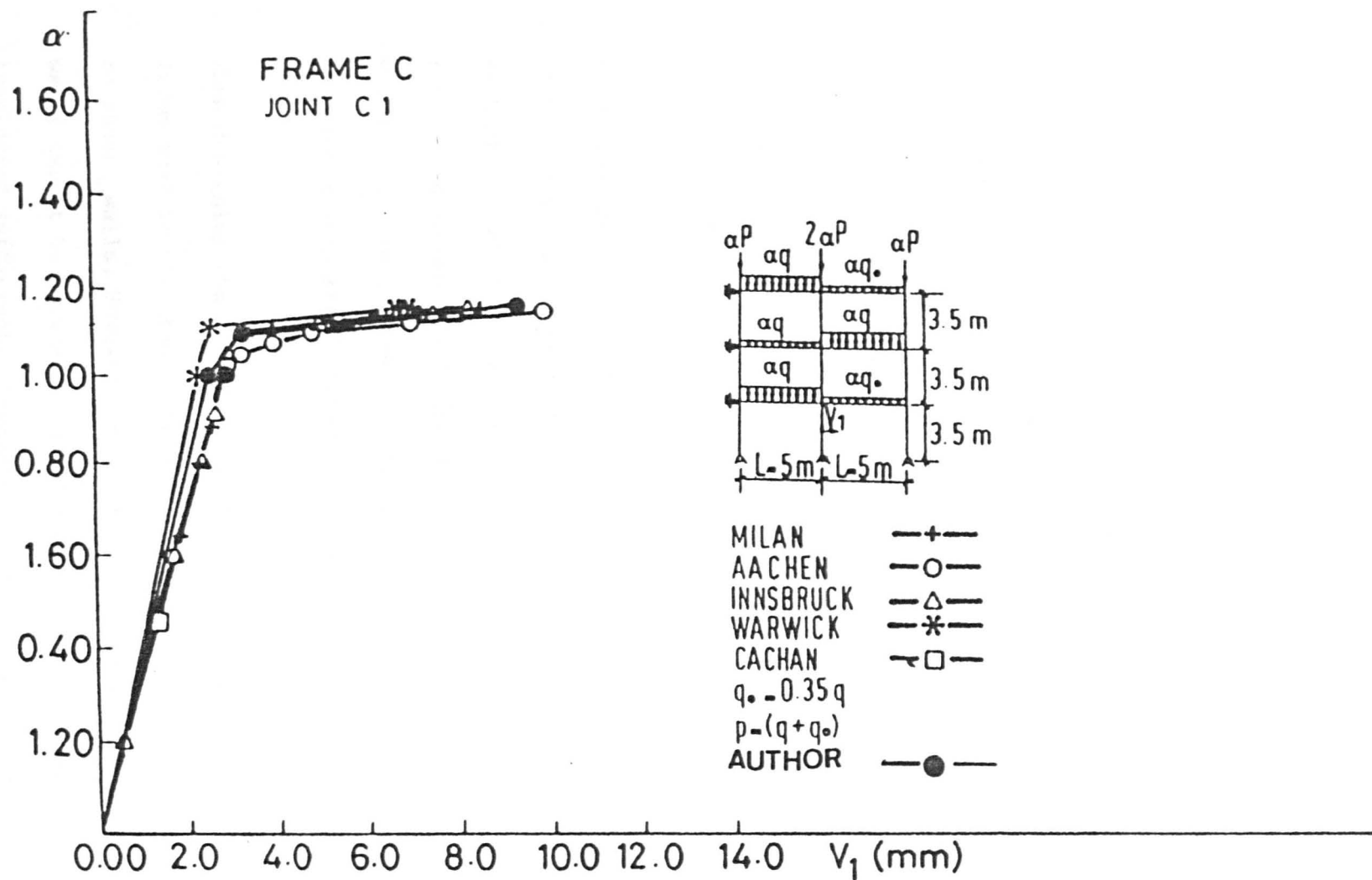


FIG. (4.36) COMPARISON OF THE ANALYSIS OF FRAME 'C' OBTAINED BY DIFFERENT RESEARCHERS.

## CHAPTER 5

### WIND CONNECTION DESIGN METHOD

#### 5.1. Introduction

Design of steel building frames is usually based upon simplifying the assumptions on behaviour of beam-to-column connections. Design codes recognise two main forms types of construction, namely braced and unbraced frames.

In braced frames, design is normally carried out as simple construction. By this it is meant that beam-to-column connections are modelled as hinges and the beams designed accordingly. In British practice [34], each column is designed for axial load and a nominal moment which is due to a 100mm eccentricity taken from the face of column ; the connections are designed for shear only.

When designing the frame in this manner, the wind transfer to the frame must be resisted by an adequately stiff element such as bracing or shear walls. However, in certain instances bracing in exterior walls cannot be arranged , nor may masonry walls around stair-wells be considered sufficiently permanent to resist lateral forces. Therefore, the forces due to wind should be resisted by the plane of the frame in bending, and as a result connections are designed as fully rigid .

Unbraced frames are generally referred to as sway frames in BS5950 [34] or Type 1 construction in AISC specification [76]. These assume that beam-to-column connections are sufficiently stiff such that no distortion occurs between the beam and column centre lines. In unbraced frames the overall frame resistance to lateral sway is provided by the plane of frame in bending.

Considerable economical benefits may arise from the design of unbraced frames by saving material, particularly in floor beams. However, using rigid connections leads in some cases to fully stiffened connections which are expensive and time consuming to fabricate. Finally the assumption of rigid joints may underestimate the sway of the bare frame and might result in heavy columns .

Semi-rigid design or Type 3 construction in AISC [76] is the closest representation of real behaviour . In the case of unbraced frames it will give a better approximation of sway at serviceability limits. Column moments are known with greater certainty if  $M-\phi$  characteristic is known with certainty.

In the case of braced frames, reduction in both span moment and deflection with respect to the pin ended case will result in a more effective design procedure. In particular, use is now made of flexural strength at the ends of a beam, leading to a reduction in beam sections.

A design approach using moment-rotation characteristics has been available for many years. Despite this availability, it appears that little use has been made of it. This is because ,firstly designers lack access to reliable information concerning the moment-rotation characteristics of connections, secondly there is very little guidance on the design procedure in design codes such as BS5950. Finally, the most important obstacle is that the structure is not statically determinate and determination of the internal actions due to the load may necessitate specialized analysis programs which are not readily available.

Therefore ,it appears that there is a need for a design procedure which is simple ,safe and economical. This chapter investigates a simple design procedure which eliminates the need for fully rigid connections in unbraced frames. This design procedure is known as 'wind-connection' or Type 2 construction in AISC [75]. The Type 2 approach has been referred to as simple design because a simple analysis procedure is adopted to determine member design forces.

A number of frames were designed adopting this design procedure in accordance with the BS5950 specification. These frames were then analysed using the computer program described in chapter 4. The member design forces obtained from 'exact' analysis were used to evaluate the following;

- 1) to check the frames have an adequate factor of safety against collapse;

2) to check sway deflection at the serviceability limit.

## 5.2 Design philosophy.

The wind connection design procedure assumes that under vertical loading the connections act as pins, and members and connections are sized using the simple methods included in design recommendations.

The effects of wind loading are examined separately, by assuming that the connections are now acting rigidly. The resulting moments and forces are usually determined by assuming points of contraflexure which renders the structure statically determinate. These internal actions are superimposed on those calculated under vertical loading and design of the members and connections is then completed by amending the proposed sections as necessary to withstand the combined actions.

Wind connections are therefore normally designed to carry only the moments due to wind, without regard to the additional moments caused by gravity loading of the beam. This assumption that a connection is "intelligent" and "knows" which moments to carry and which not to carry may seem paradoxical. However, the validity of such a connection in providing wind bracing for frames will be investigated.

Whilst the design procedure described is quite clear concerning the design of beams, there is vagueness concerning the column design load.

If the connections are to carry the wind moments they must possess significant stiffness, or the frame's sway under wind load may be unacceptable. On the other hand, connections of significant stiffness will transmit gravity load moment to the column. The column sections selected on the basis of the wind-connection design method will not reflect the presence of gravity moments and may be undersized.

The study will also investigate the stability of the column and the adequacy of the connection stiffness under wind to provide justification for continued use of the method.

### 5.3 Design procedure in accordance with BS5950

The British code of practice for the design of structural steel defines two main limit states; the ultimate limit state, where the structure becomes incapable of carrying the applied loads and the serviceability limit state, where the structure becomes unusable from excessive deflection. Therefore in any design the two prime factors which need to be considered are :

- 1) Strength: assuming that the structure is able to support the imposed loadings multiplied by the appropriate factor of safety.
- ii) Stiffness ensuring that the structure does not deflect more than is deemed permissible at the anticipated working load.

The load factors and load combinations adopted for the design of the

frames for the ultimate limit state in this chapter are as follows.

- a)  $1.4 \text{ Dead} + 1.6 \text{ imposed}$ .
- b)  $1.2 (\text{Dead} + \text{imposed} + \text{wind})$ .
- c)  $1.4 (\text{Dead} + \text{wind})$ .

The load factors and load combination for serviceability limit is;

- d)  $\text{DEAD} + 0.8 (\text{Imposed} + \text{Wind})$

These load factor are given in table (2) of the code [34].

Wind loading applied to the frames was calculated using CP3: Chapter 5 [102] .

Each frame was designed using the above load conditions and members were assessed for the critical load condition.

#### 5.3.1 Beam design

The maximum bending moment for each beam was found under combination (a) stated in sec. (5.3) assuming the members to be simply supported. These moments was used to find appropriate sections.

Additional checks were also made for the moments due to horizontal load caused by wind at each floor level, assuming points of contraflexure, and selected sections were increased if necessary.

The beam sections determined were checked to satisfy the following criteria:

- 1) Shear force should not exceed the shear capacity of the section.
- 2) Deflection should be limited so as not to impair the service ability of the structure.
- 3) In the case of an unrestrained beam, the resistance of the beam to lateral-torsional buckling should be checked (this was unnecessary in this study, as all the beams were assumed to be restrained by concrete units)

#### 5.3.2 Column design.

The columns were designed for axial load and the moment calculated assuming that beam reactions act at an eccentricity of 100mm from the face of the column .

In continuous multistorey columns, the moment applied at any one level is divided between the column lengths above and below that level in proportion to their stiffness,  $I/L$  of each length, except that when the ratio of stiffness does not exceed 1.5 the moment may be divided equally.

Reductions in imposed load were taken into account when calculating the axial load at each floor level. This is to allow for the reduced



probability of all floors being fully loaded. The percentage of reduction at various floor levels is tabulated in BS6399, part 1 1984 [103].

The bending moments and axial forces due to wind loading were determined by assuming a point of contraflexure at the mid-height of each column and mid-length of beams. The sections required to sustain combination (a) were then checked under combinations (b) and (c), sections being increased if necessary.

The adequacy of a section was checked using the following equation from BS5950 section 4.8 [34].

$$\frac{F_a}{P_{cy}} + \frac{M_x}{M_b} \leq 1 \quad 5.1$$

Where  $F_a$  is applied load in member.

$P_{cy}$  is the axial capacity for weak axis buckling.

$M_x$  is the applied moment about major axis.

$M_b$  is the lateral torsional buckling resistance moment.

In order to determinate  $P_{cy}$  and  $M_b$ , the effective length factor should be specified. Effective length factors of 1.0 for minor axis failure and 1.5 for major axis buckling were assumed.

### 5.3.3 Connections.

Connection selection and design is a complex problem as many connection types are available. The present study considered only the extended end plate, because of its adequate stiffness and strength. In this connection, the end plate is welded to the beam and bolted to the column as shown in fig. (5.1).

The design procedure followed that recommended by Horne and Morris [14]. All three load combinations were considered. For this type of connection the beam moment was replaced by a couple whose forces act at the beam flange level, that is;

$$F_t = F_c = \frac{M}{d_f}$$

where  $d_f$  is the distance between centroid of the beam flange (see fig. 5.1).

The tension component of the beam moment ( $F_t$ ) is transmitted by tension bolts to the column flange and the compression ( $F_c$ ) by the bearing on the column flange at the bottom of connection.

#### 5.3.3.1 Determination of Bolt Size.

Consider the design procedure for a connection having four tension bolts grouped around the beam's tension flange. This force was divided by four to find the force carried by each bolt. A suitable bolt size was selected to withstand the applied force.

### 5.3.3.2 Design of End Plate.

The design of the end plate, like that of the bolts, is governed by its performance in the zone adjacent to the beam tension flange and by its interaction with the bolts and the column flange.

To calculate the end plate thickness the following equations were applied.

$$t_p = \sqrt{\frac{f_t m^2}{P_{yb} B}} \quad 5.2$$

$$t_p = \sqrt{\frac{f_t}{P_{yb} \left\{ \frac{2B}{c} + \frac{d_t}{A} \right\}}} \quad 5.3$$

where  $P_{yb}$  is the yield strength of the plate

$t_p$  is the end plate thickness.

symbols not defined are shown in fig. (5.1)

The equation (5.2) is applied when the resulting end-plate thickness ( $t_p$ ) is greater than column flange thickness.

The equation (5.3) is applied when the resulting end-plate thickness ( $t_p$ ) is less than column flange thickness.

The two equations (5.2) and (5.3) produce realistic end plate thickness when based on the following end plate geometry.

$$B = 9D \quad A = 5D \quad C = 6D \quad a < 2.5D$$

where D is the diameter of bolt.

#### 5.3.3.3 Weld Size

The weld that connects the end plate to the beam member is crucial. If possible fillet welds should be used rather than butt welding.

The calculation of weld sizes necessary to transfer the forces were as follows;

$$\text{Flange welds} = \frac{T_b}{\sqrt{2}}$$

5.4

$$\text{Web welds} = \frac{t_b}{\sqrt{2}}$$

#### 5.3.3.4 Adequacy of Column Flange.

An investigation by Packer and Morris [104] has shown two modes of failure for the unstiffened column flange. In the first mode it is assumed that failure occurs due to combination of bolt fracture and flange yielding. The maximum force which can be supported by the flange is given by

$$F_{mb} = T_c \left\{ \frac{3.14(m+n) + 0.5C}{m+n} \right\} P_{yc} + 4P_t \left\{ \frac{n}{m+n} \right\} \quad 5.5$$

Where  $m = (A - t_o - 2 \times \text{root fillets})/2$

$$n = (B - A)/2$$

$$n' = (B_c - A)/2$$

$P_L$  is the proof load of bolt

$P_{yo}$  is the yield strength of column.

If double curvature is assumed then a second mode of failure will result. An analysis of this pattern will produce the following maximum flange force:

$$F_{mc} = T_c \left\{ 3.14 + \frac{(2n - C - D)}{m} \right\} P_{ye} \quad 5.6$$

where  $D' = D+2$

The lower value of  $F_{mb}$  or  $F_{mc}$  will indicate the mode of failure of the column flange.

There are three conditions to consider.

- a) If  $f_t$  is less than  $F_{mb}$  and  $F_{mc}$  then the column flange is adequate.
- b) If  $f_t > F_{mb}$  which itself is less than  $F_{mc}$  then the bolt size would be increased to enhance the value of  $P_L$  and hence  $F_{mb}$ . Alternatively the flange could be stiffened.
- c) If  $f_t > F_{mc}$  which itself is less than  $F_{mb}$  then the column needs to be stiffened in order to reduce the amount of cross bending in the flange.

None of the frames reported below required stiffeners. It is assumed that if tension or compression stiffeners were required then it would be more appropriate to design the frame as rigidly-jointed.

#### 5.3.3.5 Column Web in Compression

The maximum compression load that could be sustained by an unstiffened column web given by Horne and Morris [14] is as follows

$$F_{wc} = (T_b + t_s + d + 5K)t_s p_{ys} \quad 5.7$$

where  $K = (T + \text{root fillet})$

#### 5.3.3.6 Column Web in Shear

The shear resistance of column web for an unstiffened column web is

$$F_v = t_w (D_c - 2T_c) p_{ys} \quad 5.8$$

#### 5.4 Analysis of frames designed to wind connection method.

The computer program described in Chapter 4 was used to analyse the frames, with real behaviour of connections represented by semi-rigid moment-rotation relationships. The exact analysis enables the adequacy of the wind connection design method to be checked. For this purpose the program takes into account the main non-linear phenomena that occur in real framing systems.

These non-linearities include geometric non-linearity, material non-linearity and non-linear connection behaviour. The inclusion of these non-linearities into the analysis program was described in Chapter 4. The way in which the connections are modelled for incorporation into the analysis will be described in the next subsection.

For comparison, the frames were analysed with the same computer program assuming fully rigid connections. The results of these different analyses were compared in order to gain an insight into the relative member forces and structure deflection.

#### 5.4.1 Moment-rotation relation $M-\phi$ and modelling

Frye and Morris's mathematical expressions [44] were used to find the moment-rotation relationships for extended end plate connections. Since these relations are non-linear for almost the whole of the range, for computational purposes they were represented as piece wise linear. The expression for the extended end plate is, using metric units:

$$\phi = 1.83(4.873KM) \times 10^{-3} - 1.04(4.873KM)^2 \times 10^{-4} + 6.38(4.873KM)^3 \times 10^{-6} \quad 5.9$$

where  $K = d^{-2.4} t^{-0.4} f^{-1.5}$

$\phi$  is the connection rotation in radians

$M$  is the bending moment in kN cm

$d, t, f$  correspond respectively to depth between extreme bolt, end plate thickness and column flange thickness, all in cm

The maximum moment capacity of the connection is called the connection limiting moment. The limiting moments obtained in this study from Eqn. (5.9) were calculated by limiting the rotation about 0.02 radian.

#### 5.5 Assessment of design procedure.

The bending moments and axial forces at the design load level for each member obtained by 'exact' second order elasto plastic analysis. These were used to check the following criteria:

##### a) Local capacity

The following equation was used from BS5950 to check the local capacity of member.

$$\frac{F_c}{P_z} + \frac{M_x}{M_{cx}} \leq 1 \quad 5.10$$

where  $F_c$  and  $M_x$  are the axial force and maximum moment from the computer analysis.

$P_z$  is the squash load of the section ,and  $M_{cx}$  is the moment capacity about the major axis buckling.

##### b)Lateral torsional buckling.

The following equation was used to check lateral torsional buckling.



$$\frac{F_c}{P_{cy}} + \frac{mM_x}{M_b} \leq 1 \quad 5.11$$

Where  $F_c, M_x$  as defined earlier.

$P_{cy}$  is the compression axial load about minor axis.

$m$  is equivalent moment gradient factor.

The values of  $M_b$  and  $P_{cy}$  are dependent on the effective length, so as the effective length increases these values decrease. Therefore a choice of suitable effective length  $L_E$  for different end restraints of the column is essential.

In the case of the beams, it was assumed that the beams are restrained laterally at the top flange (by supporting roof and floor units). The hogging moment at the end of the beam would cause compression in the bottom flange. This unrestrained portion should be checked for lateral torsional buckling (This check was unnecessary for the examples in this Chapter, as unrestrained portions of the beams were close to metre)

In the case of columns it was assumed that the head of the column is restrained in the lateral direction, so conservatively an effective length of  $L_E = L$  can be taken. This effective length can be used to calculate  $M_b$  and  $P_{cy}$ . As the moments were calculated by a more exact procedure, account can be taken of moment gradient by taking the

equivalent uniform moment factor 'm' to be less than unity when checking the resistance of each column . A full set of values of 'm' is given in BS5950 [34] .

Finally the vertical deflection of the beam and horizontal sway deflection were determined at working load under load combinations specified earlier. These values were checked against the horizontal sway limit and vertical limit allowed in the code of practice.

#### 5.6 Design examples.

In order to demonstrate the application of the design procedure described in the preceding sections four design examples were examined. The results of these ,covering a range of frames, are presented and discussed in the remainder of this chapter. The frames are rectangular in elevation with the following number of storeys and bays:

- 1) four storey one bay;
- 2) four storey four bay;
- 3) seven story four bay;
- 4) six storey two bay;

The storey height was constant at 3.75m in all frames but different bay widths were considered. All the bases were fixed.

Details of roof and floor loading and basic wind speed will be given for each of the frames in forthcoming sections. The uniformly dis-

tributed loads were modelled by point loads acting at the midspan of the beam and at the column head for all the examples in this chapter.

Horizontal forces were calculated from the basic wind speed by using CP3:Chapter V: Part 2. These forces were based on the total height of the frame and were therefore equal of value at each floor level. The force at roof level was taken as half of that at the intermediate floor levels.

S2 factors were obtained from CP3:Chapter V: Part 2, table (3), with appropriate decisions concerning ground roughness and class of building. S1 and S2 factors were taken as 1.

It should be emphasized that the "exact" analysis for the frames takes into account the reduction of full plastic moment due to axial load. This leads to a collapse load lower than analysis when neglecting the reduction in  $M_p$ . The  $P-\delta$  effects are also included in the analysis of the frames using classical stability functions.

In addition to the basic check on frame adequacy using semi-rigid analysis, the following studies were also made:

- 1) Comparison of the results of semi-rigid with rigid analysis. This includes the stability of individual members and the load deflection behaviour up to collapse load.
- 2) The increase of the sway compared to that calculated by the rigid

jointed frame analysis.

3) The increase of sway taking into account P- $\delta$  effects compared to linear analysis for both rigid and semi-rigid joints.

#### 5.6.1 Four storey one bay structure.

This structure was chosen to illustrate the effect of minimum vertical loading combined with the maximum values of wind loading.

The frame shown in fig. (5.2) is subjected to uniformly distributed floor and roof loading. The loadings on the frame were as follows;

|                                   |        |                    |
|-----------------------------------|--------|--------------------|
| Dead load on roof                 | 3.75   | kN/m <sup>2</sup>  |
| Imposed load on roof              | 1.50   | kN/m <sup>2</sup>  |
| Dead load on floor                | 4.75   | kN/m <sup>2</sup>  |
| Imposed load on floor             | 3.75   | kN/m <sup>2</sup>  |
| Characteristic wind load on roof  | 12.375 | kN                 |
| Characteristic wind load on floor | 24.75  | kN                 |
| Design strength of steel          | 275    | N/mm <sup>2</sup>  |
| Young modulus                     | 20500  | kN/cm <sup>2</sup> |

Frames were spaced at 4m longitudinally.

The frame sections were determined in accordance with wind connection design and these are shown, together with the dimension of the frame in fig. (5.2). The columns are taken to be continuous over two storey to reduce fabrication costs.

The design generated by the wind connection method was subjected to the exact analysis program for both rigid and semi-rigid joints.

The moment rotation relationship for different connection sizes are shown in fig. (5.3). These were incorporated into the semi-rigid analysis of the frame.

The failure loads of the frame determined from computer analysis for both rigid and semi-rigid conditions are presented in table (5.1). It can be seen from this table that the lowest margin of safety above the design load level occurred due to load combination of 1.4 (Dead+Wind).

The load deflection curves for all the load combinations concerned are shown in fig. (5.4). It can be seen from this figure that the response of the frame is elastic at the design load level.

The bending moments and axial forces at the design load factor for each member from the computer analysis were used to check the validity of the sections given by the wind connection method.

Table (5.2) presented the stability checks (left hand side of eqn. 5.11) for the sections adopted by the design procedure. It can be observed from this table that the roof and floor bending moments from the computer analysis are less than moment resistance of the proposed sections. The checks for column sections for both local capacity and lateral torsional buckling are less than unity and these values are

presented in the same table. Therefore it can be concluded that the sections adopted by the wind connection design procedure are adequate for strength requirements.

Now consider the second criterion (i.e. serviceability limit). Sway deflection at each story height and beam deflections were determined at working load for both linear and nonlinear analysis (P- $\delta$  effect). The results obtained from the computer analysis are presented in table (5.3). It can be seen from this table that the beams deflection are within the prescribed limit of span/200. But sway deflection, taking into the account the P- $\delta$  effect and nonlinearity of the joints, exceeds the limit of storey height/300. The reason is due to relatively high horizontal load in comparison to the vertical loads.

It is also evident from table (5.3) that sway deflection will increase significantly when the second-order effects are included in the analysis. The maximum increase of sway considering second order effect (P- $\delta$ ) with the flexible joints compared to the first-order analysis, denoted as X2 in the last column of the table, is 10%. But comparing the results of non-linear analysis of semi-rigid with the rigid connections (denoted by X1) the maximum increase in sway was 72%.

It should be emphasised that all the values in table (5.3) presented in parentheses correspond to the analysis with rigid joints. Other notations used in this table are ;

X1 is the increase of sway considering semi-rigid compared to rigid analysis.

X2 is the increase of sway considering the second-order analysis over the first-order analysis.

The above notation was used for other tables in this Chapter.

The assessment of the design suggests that ultimate strength will not be the governing criterion in the choice of the sections of this frame.

#### 5.6.2 Four storey four bay frame.

The frame discussed in this section is for maximum vertical loading to minimum wind, designed by Ohta [105]. The frame geometry and sections adopted for this frame are shown in fig. (5.5). The following values have been adopted for this design.

|                                   |       |                    |
|-----------------------------------|-------|--------------------|
| Dead load on roof                 | 3.75  | kN/m <sup>2</sup>  |
| Imposed load on roof              | 1.50  | kN/m <sup>2</sup>  |
| Dead load on floor                | 4.80  | kN/m <sup>2</sup>  |
| Imposed load on floor             | 5.00  | kN/m <sup>2</sup>  |
| Characteristic wind load on roof  | 2.85  | kN                 |
| Characteristic wind load on floor | 5.68  | kN                 |
| Design strength of steel          | 275   | N/mm <sup>2</sup>  |
| Young modulus                     | 20500 | kN/cm <sup>2</sup> |

Frames were spaced at 4m longitudinally.

The moment-rotation relationships are shown in fig. (5.6). Only load combinations of (a) and (b) of section 5.3 were examined. This is because the low horizontal forces due to the load combination of 1.4(dead+wind) would not be critical.

The load deflection curves obtained from the computer analysis program are shown in fig. (5.7). As expected, the lower failure load occurred due to load combination of dead plus imposed.

Table (5.4) presented the stability factors (left hand side of eqn. 5.11) of the members of this frame. It is evident from this table that none of the stability factors exceeded unity. Therefore the sections adopted for this frame are adequate as far as strength requirement.

Table (5.5) shows the beams deflection and sway drift at each storey height at working load, for both linear and non-linear analysis. Because of the small horizontal forces the sway deflections are very small and are well within the prescribed limits.

This is a classical example of a frame where ultimate strength is the governing criteria.

#### 5.6.3 Seven storey four bay frame.

This frame was designed by the author as an illustration of larger multi-storey, multi-bay frames to the wind connection design method. The full design calculations and design procedures for determining the



sizes of beams, columns and connections, in accordance with BS5950 design recommendations, are presented in appendix A at the end of this thesis.

The design sections for this frame are shown in figure (5.8). Details of loading of this frame are specified in appendix A.

Seven joints sizes were specified for this frame; the moment rotation relationships of these joints are shown in figure (5.9).

The frame was analysed for the three load combination specified in section 5.3. The smallest margin of safety above design load obtained by computer analysis with the semi-rigid connections was 1.35, due to load combination a. The load deflection curves for all the load combinations are shown in figure (5.10). It can be observed from this figure that no plastic hinges were present in the frame at the design load factors. This fact implies that the frame is elastic at load factor unity.

Checks for the stability of the members at the design load levels are presented in table (5.6). Results show that dead plus imposed load is the critical combination. Stability factors are all less than unity and are greater lower down the frame, where axial load is more dominant than the moment.

The results of the sway deflection at working load are presented in table (5.7). It is evident from this table that the overall sway

deflection dose not exceed the limiting value ( $h/300$ ) at any storey level .

Therefore the design was found to posses adequate strength and serviceability under all the load combinations considered.

#### 5.6.4 Six storey two bay frame.

This frame was design by D.Anderson in accordance with the a draft of Eurocode 3 [65]. The wind load on this frame was quite high, as in example 1.

The significant difference between this frame and the proceeding frames lies in the use of European sections rather than Universal Beams and Columns. The frame geometry together with the frame sections adopted for this example are shown in figure (5.11). The frame's specification is as follows;

|                                   |       |                    |
|-----------------------------------|-------|--------------------|
| Dead load on roof                 | 3.75  | kN/m <sup>2</sup>  |
| Imposed load on roof              | 1.50  | kN/m <sup>2</sup>  |
| Dead load on floor                | 4.8   | kN/m <sup>2</sup>  |
| Imposed load on floor             | 3.5   | kN/m <sup>2</sup>  |
| Characteristic wind load on roof  | 17    | kN                 |
| Characteristic wind load on floor | 8.5   | kN                 |
| Design strength of steel          | 235   | N/mm <sup>2</sup>  |
| Young modulus                     | 20500 | kN/cm <sup>2</sup> |

Frames were spaced at 4.5 m longitudinally

The moment rotation relationships for various joints dimensions are shown in figure (5.12). These non-linear curves were incorporated into the computer program and analysis was carried out as before, for both rigid and semi-rigid connections. Figure (5.13) shows the load deflection curves for all of the load combinations specified earlier. As noted from this figure, no plastic hinge forms below the design load level.

Results obtained by computer analysis shows that dead plus imposed plus wind is the critical load combination. The failure load factor is 1.3 due to this load combination.

As experienced from Example one (four storey 1 bay), because of the high wind, the major problem was the sway at serviceability. Therefore this will be examined first. It can be seen from table (5.9) that the beam deflections are within the permissible value, but the sway deflection shows that permissible values are exceeded at floor levels 2,3,4 and 5. It should be emphasised that the sway drifts are within the limits when second-order effects ( $P-\delta$ ) were ignored.

The stability factors for the members of this frame are presented in the table (5.8). It is evident from this table that first storey internal columns exceeded unity and the other columns are very close or equal to unity. It should be noted that, the sections were adopted on this frame were based on preliminary design using interim draft of EuroCode 3.

### 5.7 Comparison of rigid with semi-rigid analysis.

The frames designed by this study were also used to obtain an estimate of the accuracy of rigid frame analysis. To this end ,the moments and sway deflections from the rigid frame analysis were compared with those from the exact analysis for the flexible connections.

Beam and column moments obtained by semi-rigid joint analysis were compared to the corresponding values of rigid joint analysis. This comparison was made on all the frames included in this study ,and the values presented in the parentheses correspond to the analysis with rigid joints. The largest percentage of difference of critical bending moments of beams and columns are presented in table (5.10) in comparison with the rigid analysis. It is evident from this table that when the vertical loading controls the beam design, they are under designed. The latter is true when the design basis is elastic theory, but plastic design would allow redistribution of the moments and ,therefore, the beam may not be under-designed. The decrease of 25% in mid-span sagging moment was noted in comparison with the analysis with the semi-rigid joints.

Column moments obtained by rigid joints were over-estimated by up to 65% compared with an analysis accounting for connection flexibility. However, when the stability checks were made for the columns there was generally negligible difference between the rigid and semirigid joint analysis. This is because the axial load is the dominant part in the stability checks.

Sway deflections obtained from the analysis of rigid and semi-rigid joints, for the frames used in this study, indicate that there is a large increase compared to those with the rigid joints. For example by referring to table (5.3) for the four storey one bay frame, the maximum sway of 4.6 cm may be compared to the corresponding values of 3.05 cm (58% increase) assuming rigid joints.

### 5.8 Sensitivity of the results.

A single storey single bay frame was used to investigate the behaviour of the structure under different degrees of connection flexibility. The member sizes and the applied loading on the frame designed by Reading [80] are shown in figure (5.14)

For the frame designed, six different sets of connection stiffness were used. First the frame was analysed assuming the beam was rigidly connected to the columns. These results correspond to those one would obtain from a conventional rigid frame analysis. Then the frame was analysed assuming the beam was pinned to the columns. Finally four additional analysis were performed in between the above two extremes.

The  $M-\theta$  relationship for the various connection stiffness were used for this frame are shown in figure (5.15). The numbers shown on fig. (5.15) represent the connection flexibility type which were used in the frame. The vertical axis (1) indicated an upper bound on connection stiffness and abscissa (6) a lower bound. Curve (2) indicates the moment rotation characteristic obtained from polynomial expression of

Frye and Morris [44] . Curves 3,4 and ,5 were chosen as 50%,25% and 12.5% of the value of the moments from the curve (2) whilst keeping the same rotations as curve (2).

The preceding information was input into the computer program, a non-linear second order analysis of the frame were performed, accounting for the flexibility of the connections. Member forces and sway deflection were computed for each connection stiffness defined earlier.

Table (5.11) presented the analysis carried out due to different connection stiffness. The following points were concluded.

1) There is small difference in the failure load obtained by Frye and Morris's curve(2) , curve(3) and curve(4) with the failure load by rigid joint analysis. The failure load by rigid analysis is only 8% bigger than the failure load by type a connection with 25% of the Frye and Morris stiffness.

2) The mid span moment and column moments obtained from the analysis with the rigid joints are very close to the analyses with the type 2,3,4 connection stiffness. In fact there is only an increase of 6% in mid span moment and decrease of 18% in the column moment on analysis of type 4 (25% curve) over the rigid analysis.

The reason for the above points is that the connections designed in this frame are very stiff, and reducing its moment capacity to 25%

still results with a high enough moment capacity to have little effect on carrying capacity of the frame.

It is interesting to note that when the analysis is carried out with the type 5 connection stiffness (12.5% curve), considerable changes were observed in the frame. The load carrying capacity and the column moments were decreased by 21% and 34% respectively and the mid span moment was increased by 10%. The changes from type 4 to type 5 connection stiffness caused the formation of a plastic hinge at the mid span of the beam at the lower load factor. Furthermore, due to the flexibility of the connection, the joint can sustain a smaller moment and reaches its moment capacity at the earlier stage of loading.

Finally the analysis with the pinned connection indicates a decrease of 60% in load carrying capacity and increase of 24% in mid span moment over the analysis with the rigid joints. Fig. (5.16) shows the bending moment diagram of this frame for type 1 (rigid), type 5 (semirigid) and type 6 (pinned).

Load deflection curves for all the types of connection stiffness considered in this frame are shown in fig. (5.17). The sway at serviceability was also calculated and these values are presented in the last column of table (5.11). It can be seen from this table that the frame sway gets larger as the connection stiffness reduces. The sway increases more rapidly as the joint become more flexible (i.e. from type 5 onwards).

## 5.9 Conclusions

A broad range of the realistic frames have been studied to determine the validity of the wind connection design method. For comparison, these frames were analysed assuming fully rigid and flexible connections.

On the basis of the examples were presented, the following conclusions may be drawn regarding the validity of various design assumptions for unbraced multistorey steel frames.

- 1) The wind connection method of design may be used with frames resisting low and medium horizontal loads.
- 2) The analysis procedure employed in the wind connection method consistently overestimates the critical values of moment in the beam, whilst underestimating the column moments.
- 3) The axial forces in the column are predicted fairly accurately by the wind connection method, particularly in the lower portion of structure. Because the axial load in the lower storey is large and can be predicted much more accurately than the accompanying moments, these column sizes are less radically under-estimated (see the assessment of design in appendix A).
- 4) None of the frames were plastic at the design load level.
- 5) In general the frames designed in this chapter proved to be ade-



quate on strength criteria.

6) The recommended limit for the horizontal deflection was exceeded for those frames with high horizontal loads . The deflections were computed taking into the account the flexibility of the joints and the second-order P- $\delta$  effect.

7) The deflection limit was satisfied in some frames where previously exceeded, when the connections were modelled as rigid and first-order analysis was performed.

8) The analysis of frames with the fully rigid connections resulted in an under-estimation of frame drift and over-estimation of frame strength.

9) The analysis of frames with the joint flexibility resulted in the reduction in strength and increase in the frame drift.

10) The sequence of plastic hinge formation will be altered if the effect of connection flexibility is considered in the analysis.

| FRAME ID       | 1.4D+1.6I |      | 1.2(D+I+W) |      | 1.4(D+W) |      |
|----------------|-----------|------|------------|------|----------|------|
|                | R         | SR   | R          | SR   | R        | SR   |
| 4 STOREY 1 BAY | 1.7       | 1.5  | 1.45       | 1.4  | 1.45     | 1.25 |
| 4 STOREY 4 BAY | 1.45      | 1.4  | 1.8        | 1.65 | -        | -    |
| 7 STOREY 4 BAY | 1.75      | 1.35 | 1.450      | 1.4  | 2.1      | 1.75 |
| 6 STOREY 2 BAY | 1.7       | 1.7  | 1.350      | 1.3  | 1.4      | 1.5  |

**TABLE (5.1) FAILURE LOAD FACTOR OF FRAME STUDIES FOR RIGID  
AND SEMI-RIGID CONNECTIONS  
R=RIGID SR=SEMI-RIGID**

| MEMBER     | SECTION       | 1.4D+1.6I      |                | 1.2(D+W+I)     |                | 1.4(D+W)       |                |
|------------|---------------|----------------|----------------|----------------|----------------|----------------|----------------|
|            |               | L              | LTB            | L              | LTB            | L              | LTB            |
| ROOF       | 305*165*54 UB | 0.70<br>(0.61) | —              | 0.57<br>(0.50) | —              | 0.48<br>(0.43) | —              |
| FLOOR      | 406*178*67 UB | 0.71<br>(0.62) | —              | 0.58<br>(0.50) | —              | 0.37<br>(0.32) | —              |
| 3rd STOREY | 203*203*46 UC | 0.48<br>(0.59) | 0.44<br>(0.49) | 0.78<br>(0.83) | 0.57<br>(0.59) | 0.69<br>(0.73) | 0.51<br>(0.53) |
| 1st STOREY | 203*203*71 UC | 0.44<br>(0.50) | 0.47<br>(0.49) | 0.95<br>(0.90) | 0.70<br>(0.68) | 0.97<br>(0.91) | 0.67<br>(0.64) |

TABLE (5.2) STABILITY OF FOUR STOREY ONE BAY FRAME

VALUES IN BRACKETS ARE FOR THE RIGID JOINTS

L: LOCAL CAPACITY      LTB: LATERAL TORSIONAL BUCKLING

| MEMBER | DEFLECTION<br>cm | X1  | DEFLECTION TO<br>SPAN RATIO |
|--------|------------------|-----|-----------------------------|
| ROOF   | 2.0<br>(1.6)     | 25% | SPAN/375<br>SPAN/468        |
| FLOOR  | 1.5<br>(1.2)     | 25% | SPAN/500<br>SPAN/625        |

TABLE (5.3 a) MAXIMUM DEFLECTION OF BEAM AT WORKING  
LOAD FOR FOUR STOREY ONE BAY FRAME

| STOREY<br>LEVEL | SWAY<br>cm     |                | SWAY TO<br>HEIGHT RATIO |                  | X1  |     | X2           |
|-----------------|----------------|----------------|-------------------------|------------------|-----|-----|--------------|
|                 | L              | NL             | L                       | NL               | L   | NL  |              |
| 4               | 7.95<br>(4.75) | 8.61<br>(5.0)  | h/188<br>(h/315)        | h/174<br>(h/300) | 67% | 72% | 8%<br>( 5%)  |
| 3               | 6.81<br>(4.21) | 7.40<br>(4.44) | h/165<br>(h/267)        | h/152<br>(h/253) | 62% | 66% | 9%<br>( 5%)  |
| 2               | 4.65<br>(2.97) | 5.10<br>(3.12) | h/161<br>(h/252)        | h/147<br>(h/240) | 56% | 63% | 10%<br>( 5%) |
| 1               | 2.0<br>(1.44)  | 2.18<br>(1.59) | h/187<br>(h/260)        | h/172<br>(h/236) | 38% | 37% | 9%<br>(10%)  |

TABLE (5.3 b) SWAY AT EACH STOREY LEVEL FOR FOUR STOREY  
ONE BAY FRAME

L: LINEAR      NL: NONLINEAR (P-δ EFFECT)

| MEMBER                 | SECTION       | 1.4D+1.6I      |                | 1.2(D+W+I)     |                |
|------------------------|---------------|----------------|----------------|----------------|----------------|
|                        |               | L              | LTB            | L              | LTB            |
| ROOF                   | 406*140*46 UB | 0.70<br>(0.56) | —              | 0.57<br>(0.46) | —              |
| FLOOR                  | 457*152*74 UB | 0.80<br>(0.61) | —              | 0.64<br>(0.50) | —              |
| 3rd STOREY<br>EXTERNAL | 152*152*30 UC | 0.62<br>(0.65) | 0.82<br>(0.80) | 0.58<br>(0.55) | 0.69<br>(0.66) |
| 3rd STOREY<br>INTERNAL | 203*203*46 UC | 0.40<br>(0.43) | 0.63<br>(0.67) | 0.37<br>(0.43) | 0.54<br>(0.59) |
| 1st STOREY<br>EXTERNAL | 203*203*46 UC | 0.56<br>(0.61) | 0.71<br>(0.70) | 0.50<br>(0.55) | 0.60<br>(0.60) |
| 1st STOREY<br>INTERNAL | 203*203*60 UC | 0.64<br>(0.68) | 0.95<br>(0.99) | 0.60<br>(0.60) | 0.85<br>(0.88) |

**TABLE (5.4) STABILITY OF FOUR STOREY FOUR BAY FRAME**

**VALUES IN BRACKETS ARE FOR THE RIGID JOINTS**

**L: LOCAL CAPACITY      LTB: LATERAL TORSIONAL BUCKLING**

| MEMBER | DEFLECTION<br>cm | X1  | DEFLECTION TO<br>SPAN RATIO |
|--------|------------------|-----|-----------------------------|
| ROOF   | 1.9<br>(1.4)     | 35% | SPAN/396<br>SPAN/535        |
| FLOOR  | 1.58<br>(1.1)    | 43% | SPAN/474<br>SPAN/681        |

TABLE (5.5 a) MAXIMUM DEFLECTION OF BEAM AT WORKING  
LOAD FOR FOUR STOREY FOUR BAY FRAME

| STOREY<br>LEVEL | SWAY<br>cm     |                | SWAY TO<br>HEIGHT RATIO |                    | X1  |     | X2            |
|-----------------|----------------|----------------|-------------------------|--------------------|-----|-----|---------------|
|                 | L              | NL             | L                       | NL                 | L   | NL  |               |
| 4               | 0.60<br>(0.38) | 0.69<br>(0.41) | h/2500<br>(h/3947)      | h/2174<br>(h/3658) | 57% | 59% | 15%<br>( 8%)  |
| 3               | 0.51<br>(0.33) | 0.60<br>(0.36) | h/2205<br>(h/3409)      | h/1824<br>(h/3125) | 54% | 57% | 18%<br>( 9%)  |
| 2               | 0.37<br>(0.24) | 0.43<br>(0.27) | h/2027<br>(h/3125)      | h/1744<br>(h/3125) | 54% | 60% | 16%<br>( 13%) |
| 1               | 0.17<br>(0.12) | 0.20<br>(0.14) | h/2205<br>(h/3125)      | h/1875<br>(h/2678) | 42% | 43% | 18%<br>(17%)  |

TABLE (5.5 b) SWAY AT EACH STOREY LEVEL FOR FOUR STOREY  
FOUR BAY FRAME

L: LINEAR                      NL: NONLINEAR (P-δ EFFECT)

| MEMBER                 | SECTION       | 1.4D+1.6I      |                | 1.2(D+W+I)     |                | 1.4(D+W)       |                |
|------------------------|---------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                        |               | L              | LTB            | L              | LTB            | L              | LTB            |
| ROOF                   | 203*102*28 UB | 0.75<br>(0.60) | —              | 0.62<br>(0.50) | —              | 0.52<br>(0.41) | —              |
| FLOOR                  | 305*127*48 UB | 0.73<br>(0.59) | —              | 0.59<br>(0.47) | —              | 0.34<br>(0.27) | —              |
| 6th STOREY<br>EXTERNAL | 152*152*23 UC | 0.58<br>(0.68) | 0.75<br>(0.78) | 0.60<br>(0.69) | 0.69<br>(0.71) | 0.44<br>(0.47) | 0.49<br>(0.49) |
| 6th STOREY<br>INTERNAL | 152*152*30 UC | 0.41<br>(0.43) | 0.88<br>(0.90) | 0.50<br>(0.38) | 0.83<br>(0.76) | 0.38<br>(0.27) | 0.58<br>(0.52) |
| 4th STOREY<br>EXTERNAL | 152*152*37 UC | 0.55<br>(0.60) | 0.81<br>(0.81) | 0.60<br>(0.62) | 0.76<br>(0.75) | 0.45<br>(0.47) | 0.56<br>(0.56) |
| 4th STOREY<br>INTERNAL | 203*203*46 UC | 0.55<br>(0.56) | 0.87<br>(0.89) | 0.62<br>(0.60) | 0.80<br>(0.80) | 0.50<br>(0.50) | 0.60<br>(0.60) |
| 1st STOREY<br>EXTERNAL | 203*203*46 UC | 0.58<br>(0.61) | 0.77<br>(0.77) | 0.66<br>(0.66) | 0.76<br>(0.75) | 0.47<br>(0.53) | 0.57<br>(0.59) |
| 1st STOREY<br>INTERNAL | 203*203*71 UC | 0.53<br>(0.54) | 0.97<br>(0.99) | 0.67<br>(0.63) | 0.91<br>(0.90) | 0.58<br>(0.58) | 0.71<br>(0.71) |

TABLE (5.6) STABILITY OF SEVEN STOREY FOUR BAY FRAME

VALUES IN BRACKETS ARE FOR THE RIGID JOINTS

L: LOCAL CAPACITY      LTB: LATERAL TORSIONAL BUCKLING

| MEMBER | DEFLECTION<br>cm | X1  | DEFLECTION TO<br>SPAN RATIO |
|--------|------------------|-----|-----------------------------|
| ROOF   | 2<br>(1.6)       | 25% | SPAN/250<br>SPAN/312        |
| FLOOR  | 1.75<br>(1.4)    | 25% | SPAN/285<br>SPAN/357        |

TABLE (5.7 a) MAXIMUM DEFLECTION OF BEAM AT WORKING  
LOAD FOR SEVEN STOREY FOUR BAY FRAME

| STOREY<br>LEVEL | OVERALL SWAY<br>cm |                | SWAY TO<br>HEGHT RATIO |                  | X1  |     | X2           |
|-----------------|--------------------|----------------|------------------------|------------------|-----|-----|--------------|
|                 | L                  | NL             | L                      | NL               | L   | NL  |              |
| 7               | 4.47<br>(3.01)     | 5.35<br>(3.47) | h/587<br>(h/872)       | h/490<br>(h/756) | 49% | 54% | 20%<br>(15%) |
| 6               | 4.18<br>(2.84)     | 5.05<br>(3.28) | h/538<br>(h/792)       | h/445<br>(h/658) | 47% | 54% | 21%<br>(16%) |
| 5               | 3.6<br>(2.44)      | 4.34<br>(2.82) | h/520<br>(h/768)       | h/432<br>(h/664) | 48% | 54% | 21%<br>(18%) |
| 4               | 2.98<br>(2.06)     | 3.62<br>(2.40) | h/503<br>(h/728)       | h/414<br>(h/625) | 31% | 51% | 21%<br>(17%) |
| 3               | 2.21<br>(1.55)     | 2.68<br>(1.80) | h/509<br>(h/725)       | h/419<br>(h/625) | 43% | 49% | 21%<br>(16%) |
| 2               | 1.46<br>(1.06)     | 1.77<br>(1.24) | h/513<br>(h/707)       | h/604<br>(h/604) | 38% | 43% | 21%<br>(17%) |
| 1               | 0.62<br>(0.48)     | 0.73<br>(0.56) | h/604<br>(h/781)       | h/513<br>(h/669) | 30% | 30% | 18%<br>(17%) |

TABLE (5.7 b) SWAY AT EACH STOREY LEVEL FOR SEVEN STOREY  
FOUR BAY FRAME

L: LINEAR      NL: NONLINEAR (P-δ EFFECT)



| MEMBER                 | SECTION | 1.4D+1.6I      | 1.2(D+W+I)     | 1.4(D+W)       |
|------------------------|---------|----------------|----------------|----------------|
| ROOF                   | IPE330  | 0.6<br>(0.51)  | 0.5<br>(0.42)  | 0.4<br>(0.34)  |
| FLOOR                  | IPE400  | 0.6<br>(0.50)  | 0.5<br>(0.4)   | 0.34<br>(0.3)  |
| 5th STOREY<br>EXTERNAL | HEB140  | 0.84<br>(0.80) | 0.96<br>(0.94) | 0.82<br>(0.78) |
| 5th STOREY<br>INTERNAL | HEB180  | 0.60<br>(0.63) | 0.85<br>(0.85) | 0.78<br>(0.77) |
| 3rd STOREY<br>EXTERNAL | HEB180  | 0.86<br>(0.87) | 1<br>(1)       | 0.93<br>(0.91) |
| 3rd STOREY<br>INTERNAL | HEB240  | 0.71<br>(0.74) | 0.94<br>(0.94) | 0.82<br>(0.80) |
| 1st STOREY<br>EXTERNAL | HEB220  | 0.74<br>(0.74) | 0.99<br>(0.93) | 0.90<br>(0.82) |
| 1st STOREY<br>INTERNAL | HEB280  | 0.83<br>(0.85) | 1.13<br>(1.07) | 1.00<br>(0.94) |

TABLE (5.8) STABILITY OF SIX STOREY TWO BAY FRAME

VALUES IN BRACKETS ARE FOR THE RIGID JOINTS

| MEMBER | DEFLECTION<br>cm | X1  | DEFLECTION TO<br>SPAN RATIO |
|--------|------------------|-----|-----------------------------|
| ROOF   | 1.32<br>(1.11)   | 19% | SPAN/454<br>SPAN/540        |
| FLOOR  | 1.10<br>(0.91)   | 21% | SPAN/545<br>SPAN/659        |

TABLE (5.9 a) MAXIMUM DEFLECTION OF BEAM AT WORKING  
LOAD FOR SIX STOREY TWO BAY FRAME

| STOREY<br>LEVEL | SWAY<br>cm     |                | SWAY TO<br>HEIGHT RATIO |                  | X1  |     | X2           |
|-----------------|----------------|----------------|-------------------------|------------------|-----|-----|--------------|
|                 | L              | NL             | L                       | NL               | L   | NL  |              |
| 6               | 6.50<br>(4.45) | 7.19<br>(4.77) | h/342<br>(h/505)        | h/309<br>(h/466) | 46% | 51% | 11%<br>( 7%) |
| 5               | 6.01<br>(4.11) | 6.67<br>(4.42) | h/311<br>(h/456)        | h/281<br>(h/424) | 47% | 52% | 11%<br>( 7%) |
| 4               | 4.89<br>(3.25) | 5.43<br>(3.49) | h/306<br>(h/461)        | h/276<br>(h/430) | 50% | 56% | 11%<br>(8%)  |
| 3               | 3.80<br>(2.54) | 4.22<br>(2.73) | h/296<br>(h/443)        | h/267<br>(h/412) | 50% | 55% | 11%<br>( 7%) |
| 2               | 2.34<br>(1.58) | 2.59<br>(1.69) | h/320<br>(h/474)        | h/290<br>(h/443) | 48% | 53% | 11%<br>( 7%) |
| 1               | 0.97<br>(0.72) | 1.06<br>(0.77) | h/386<br>(h/521)        | h/353<br>(h/487) | 35% | 38% | 10%<br>7%)   |

TABLE (5.9 b) SWAY AT EACH STOREY LEVEL FOR SIX STOREY  
TWO BAY FRAME

L: LINEAR                      NL: NONLINEAR (P-δ EFFECT)

| FRAME ID          | 1.4 D+1.6I |      | 1.2(D+I+W) |      | 1.4(D+W) |      |
|-------------------|------------|------|------------|------|----------|------|
|                   | 1          | 2    | 1          | 2    | 1        | 2    |
| 4 STOREY<br>1 BAY | 15%        | -20% | 16%        | -13% | 15%      | -12% |
| 4 STOREY<br>4 BAY | 32%        | -65% | 32%        | -52% | —        | —    |
| 7 STOREY<br>4 BAY | 25%        | -23% | 25%        | -18% | 25%      | -15% |
| 6 STOREY<br>2 BAY | 22%        | -40% | 25%        | -20% | 22%      | -13% |

TABLE (5.10) COMPARISON OF THE ANALYSIS OF RIGID JOINTS WITH SEMI-RIGID JOINTS.

1= INCREASE IN MID SPAN MOMENT      2= DECREASE IN END MOMENT

| DEGREE<br>OF JOINT<br>FLEXIBILITY | COLLAPSE<br>LOAD | MID SPAN<br>MOMENT<br>kNcm | COLUMNS<br>MAX MOMENT<br>kNcm | SWAY<br>DEFLECTION AT<br>SERVICEABILITY<br>cm |
|-----------------------------------|------------------|----------------------------|-------------------------------|---|
| RIGID                             | 2.65             | 195                        | 137                           | 3.03  |
| FRY & MORRIS<br>CURVE             | 2.625            | 198                        | 130                           | 3.23  |
| 50% CURVE                         | 2.575            | 201                        | 125                           | 3.41  |
| 25% CURVE                         | 2.435            | 207                        | 116                           | 3.75  |
| 12.5% CURVE                       | 2.01             | 228                        | 86                            | 4.45  |
| PINNED                            | 1.655            | 255                        | 0                             | 10.18   |

**TABLE (5.11) COMPARISON OF THE RESULTS OF SINGLE STOREY SINGLE BAY  
WITH DIFFERENT END BEAM RESTRAINT**

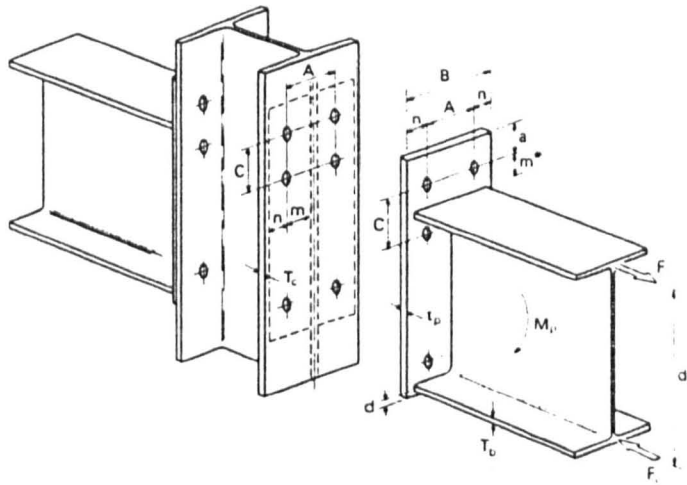
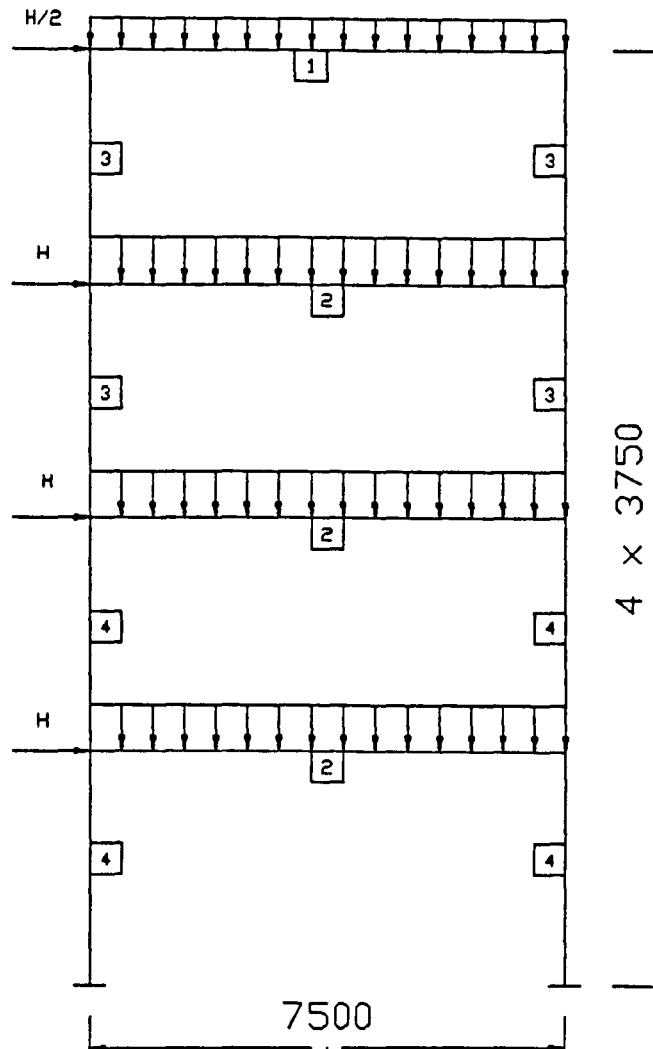


FIG. (5.1) EXTENDED END-PLATE CONNECTION



| GROUP | SECTION           |
|-------|-------------------|
| 1     | 305 x 165 x 54 UB |
| 2     | 406 x 178 x 67 UB |
| 3     | 203 x 203 x 46 UC |
| 4     | 203 x 203 X 71 UC |

FIG. (5.2) GEOMETRY, SECTIONS AND LOADING OF FOUR STOREY ONE BAY FRAME

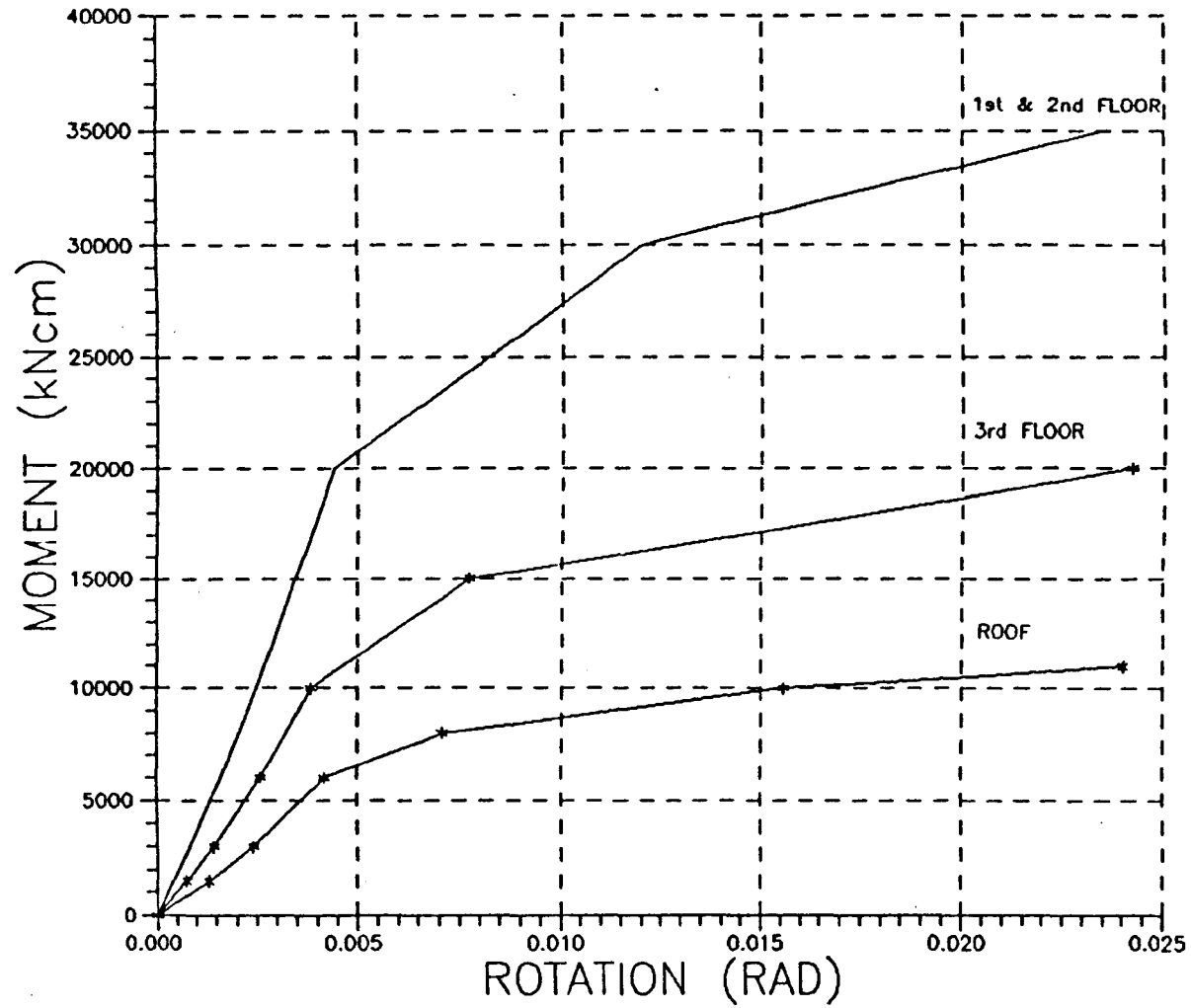


FIG. (5.3) MOMENT ROTATION RELATIONSHIP FOR FOUR STOREY ONE BAY FRAME

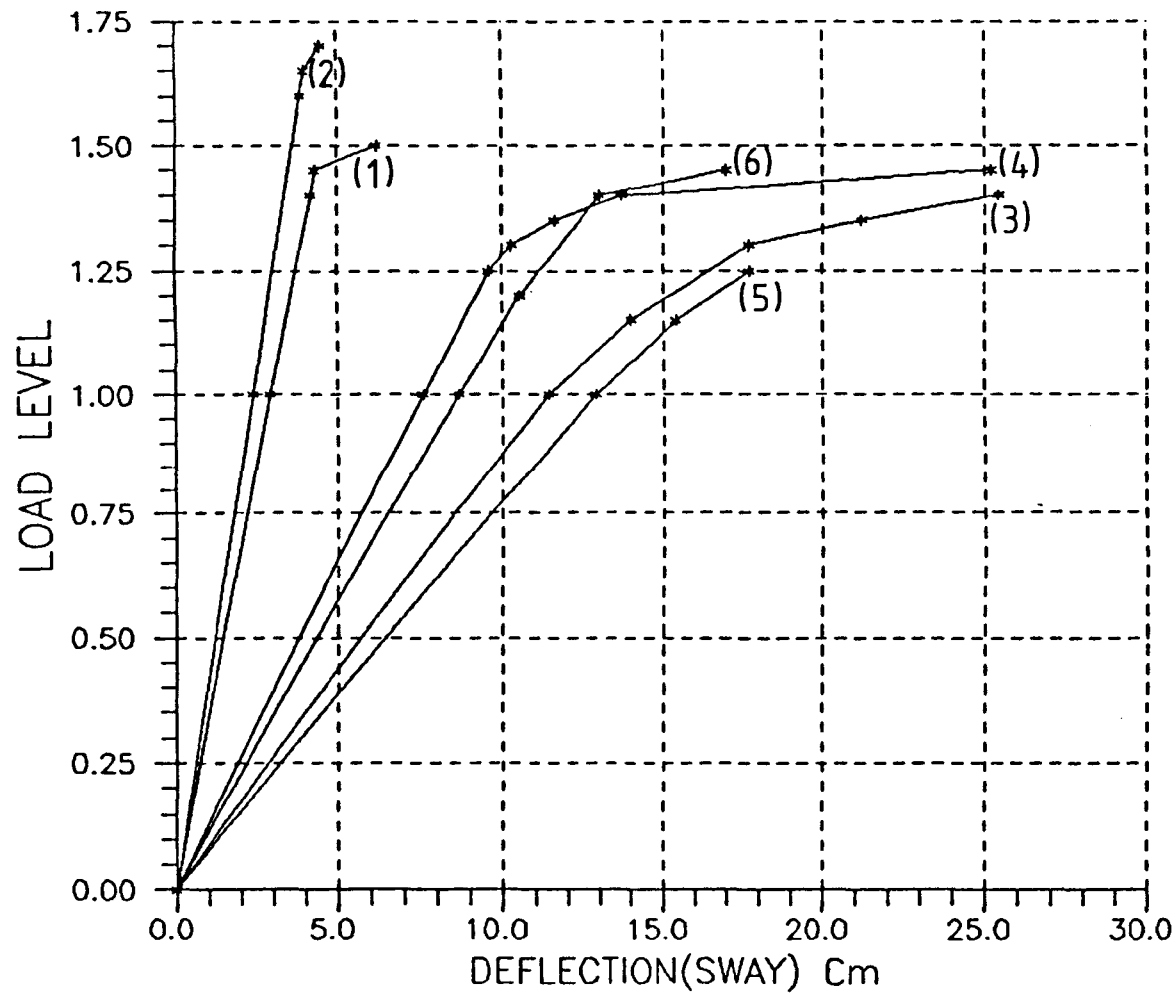
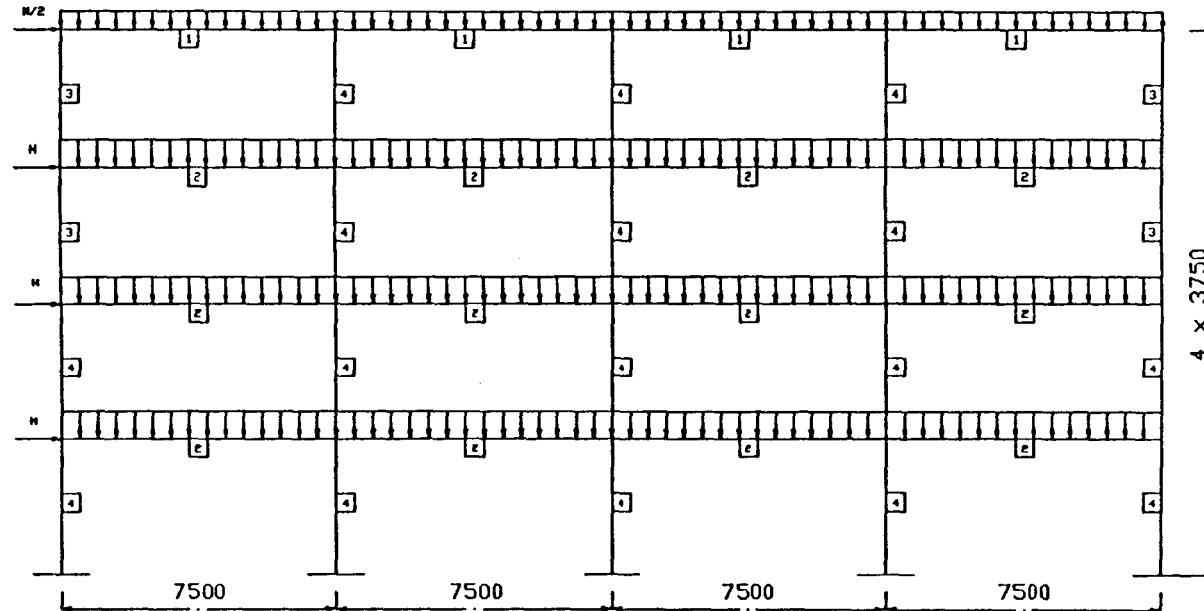


FIG. (5.4) LOAD AGAINST DEFLECTION FOR FOUR STOREY  
ONE BAY FRAME

|                                 |                            |
|---------------------------------|----------------------------|
| CURVE(1) (1.4D+1.6I) SEMI-RIGID | CURVE(2) (1.4D+1.6I) RIGID |
| CURVE(3) 1.2(D+I+W) SEMI-RIGID  | CURVE(4) 1.2(D+I+W)) RIGID |
| CURVE(5) 1.4(D+W) SEMI-RIGID    | CURVE(6) 1.4(D+W) RIGID    |





| GROUP | SECTION           |
|-------|-------------------|
| 1     | 406 x 140 x 46 UB |
| 2     | 457 x 152 x 74 UB |
| 3     | 152 x 152 x 30 UC |
| 4     | 203 x 203 x 46 UC |

FIG. (5.5) GEOMETRY, SECTIONS AND LOADING OF FOUR STOREY FOUR BAY FRAME

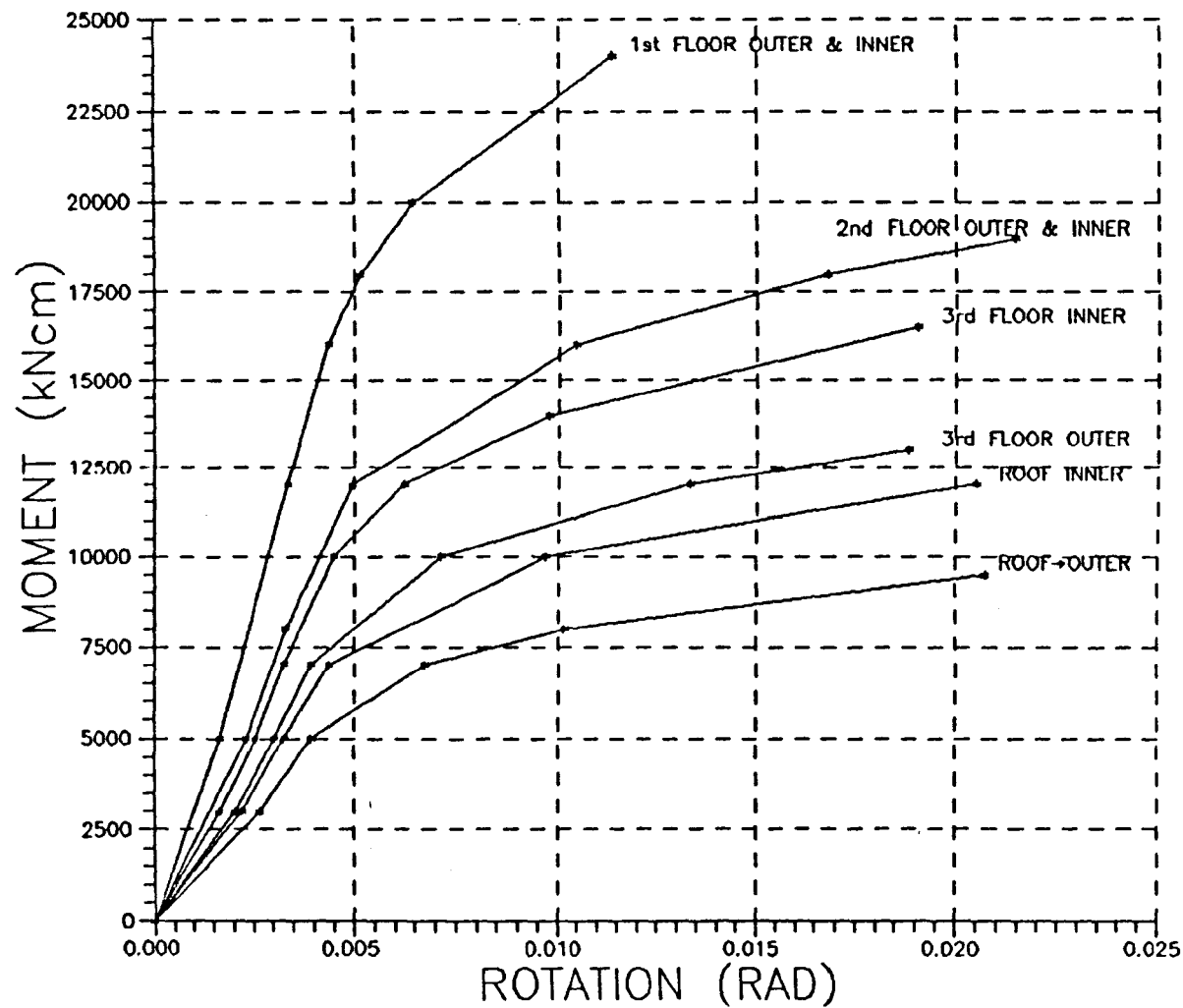


FIG. (5.6) MOMENT ROTATION RELATIONSHIP FOR FOUR  
STOREY FOUR BAY FRAME

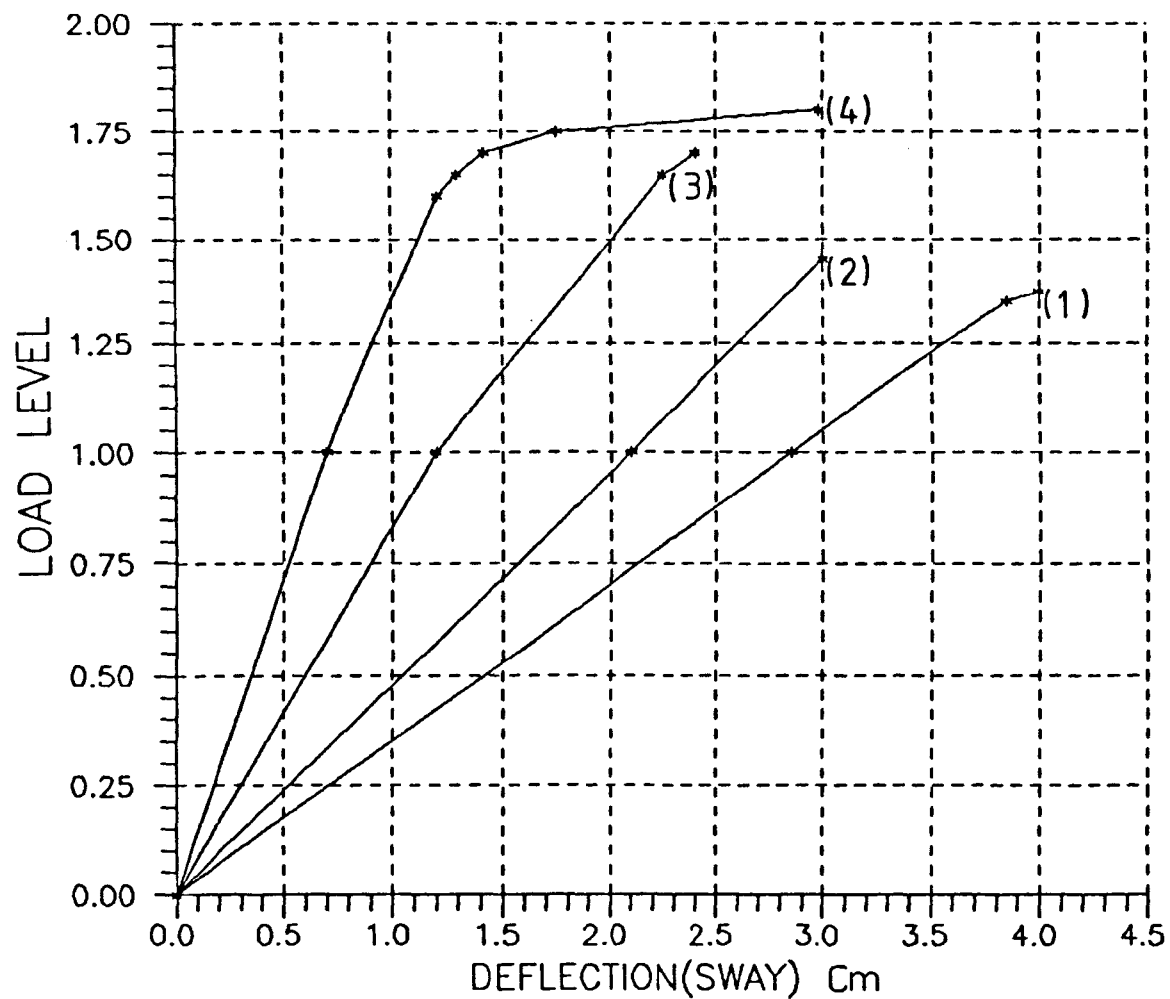


FIG. (5.7) LOAD AGAINST DEFLECTION FOR FOUR STOREY  
FOUR BAY FRAME

|                                 |                            |
|---------------------------------|----------------------------|
| CURVE(1) (1.4D+1.6I) SEMI-RIGID | CURVE(2) (1.4D+1.6I) RIGID |
| CURVE(3) 1.2(D+I+W) SEMI-RIGID  | CURVE(4) 1.2(D+I+W) RIGID  |

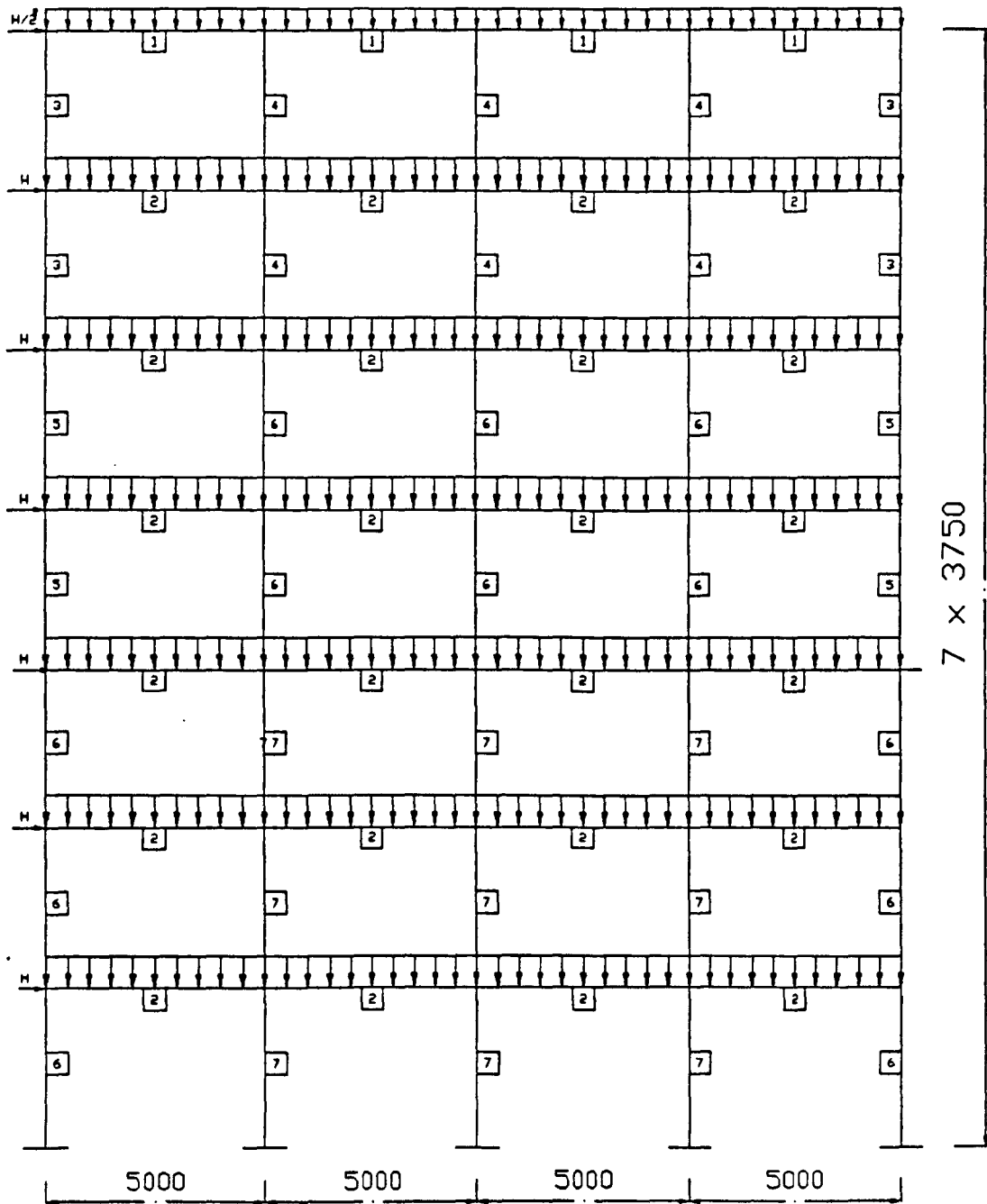


FIG. (5.8) GEOMETRY, SECTIONS AND LOADING OF SEVEN STOREY FOUR BAY FRAME

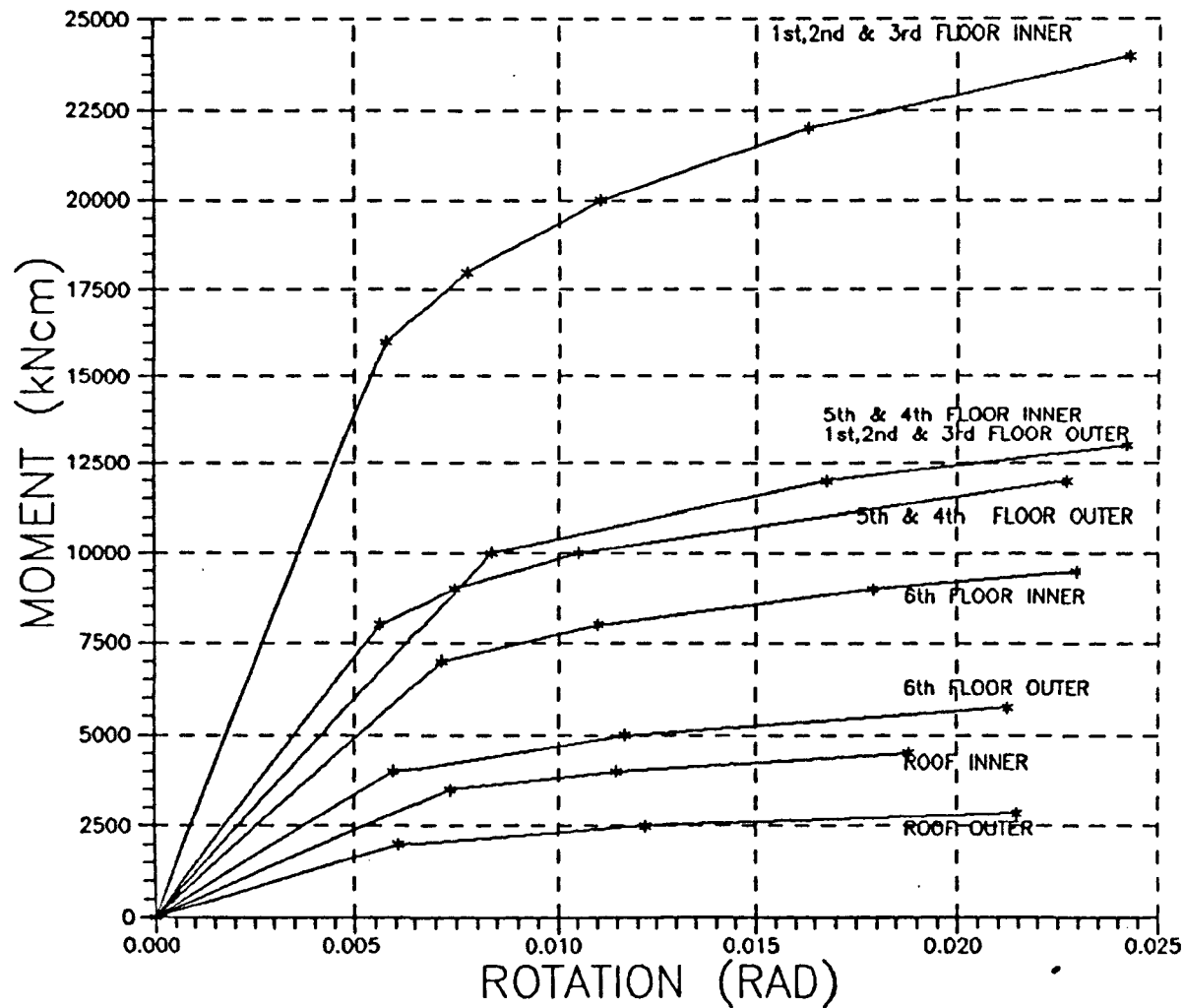


FIG. (5.9) MOMENT ROTATION RELATIONSHIP FOR SEVEN  
STOREY FOUR BAY FRAME

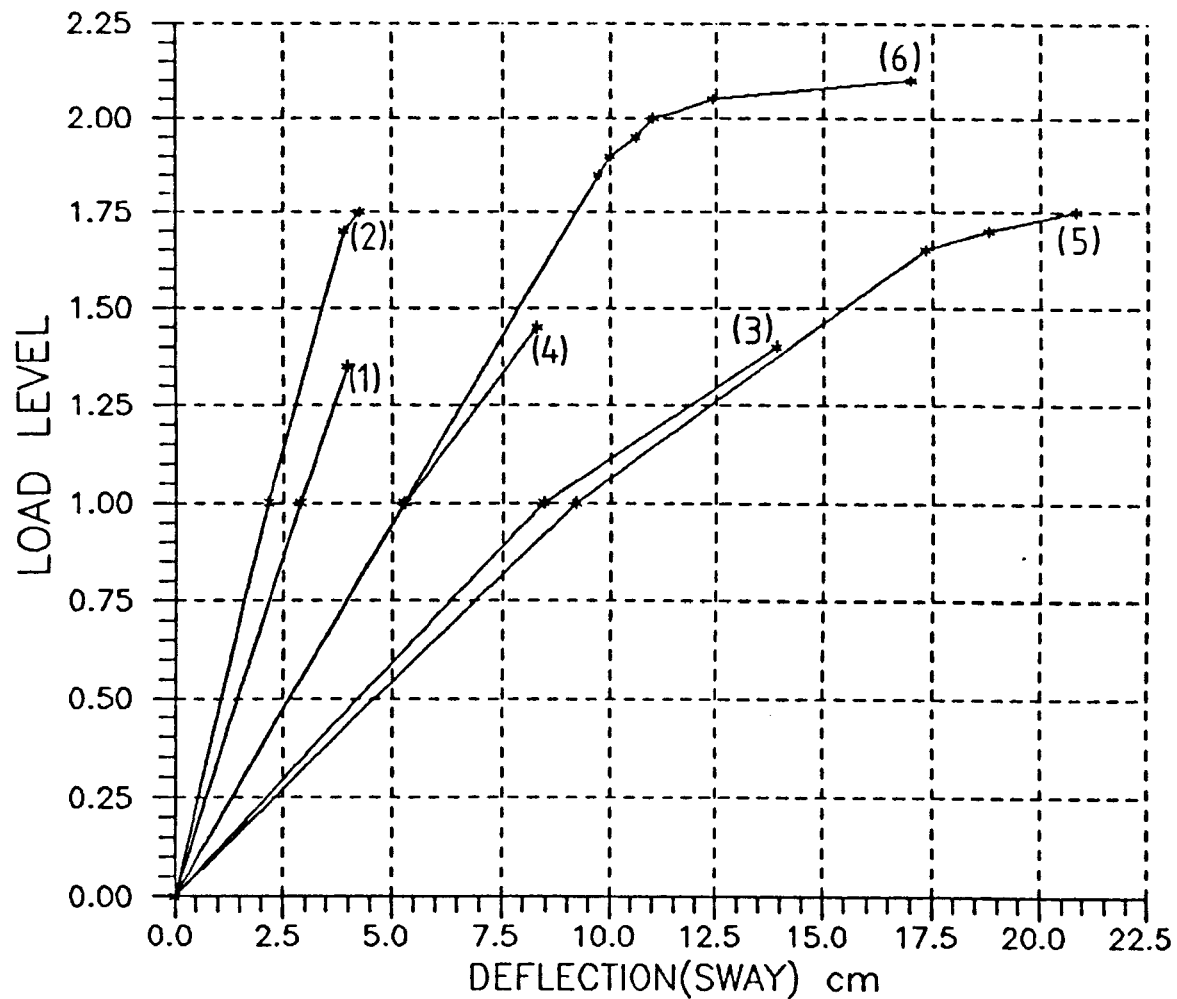
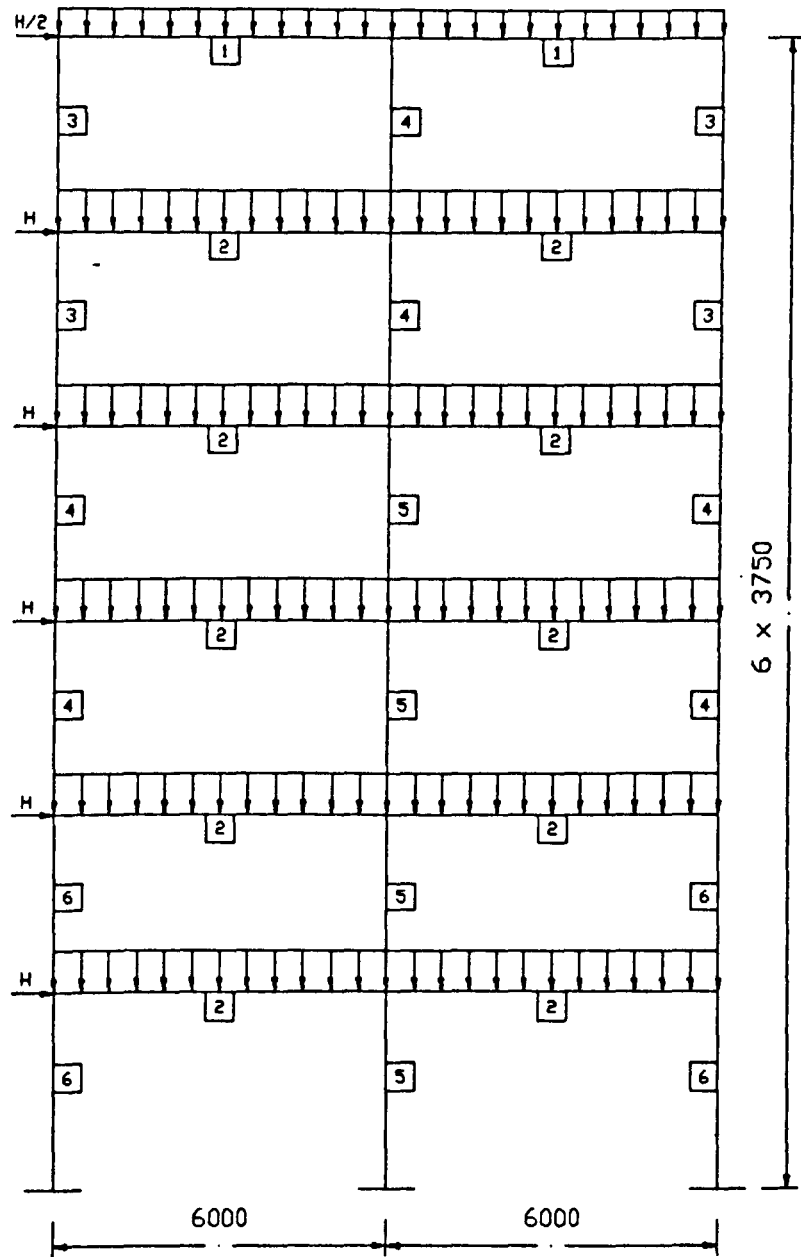


FIG. (5.10) LOAD AGAINST DEFLECTION FOR SEVEN STOREY  
FOUR BAY FRAME

|                                 |                            |
|---------------------------------|----------------------------|
| CURVE(1) (1.4D+1.6I) SEMI-RIGID | CURVE(2) (1.4D+1.6I) RIGID |
| CURVE(3) 1.2(D+I+W) SEMI-RIGID  | CURVE(4) 1.2(D+I+W) RIGID  |
| CURVE(5) 1.4(D+W) SEMI-RIGID    | CURVE(6) 1.4(D+W) RIGID    |



| GROUP | SECTION |
|-------|---------|
| 1     | IPE 330 |
| 2     | IPE 400 |
| 3     | HEB 140 |
| 4     | HEB 180 |
| 5     | HEB 280 |
| 6     | HEB 280 |

FIG. (5.11) GEOMETRY, SECTIONS AND LOADING OF SIX STOREY TWO BAY FRAME

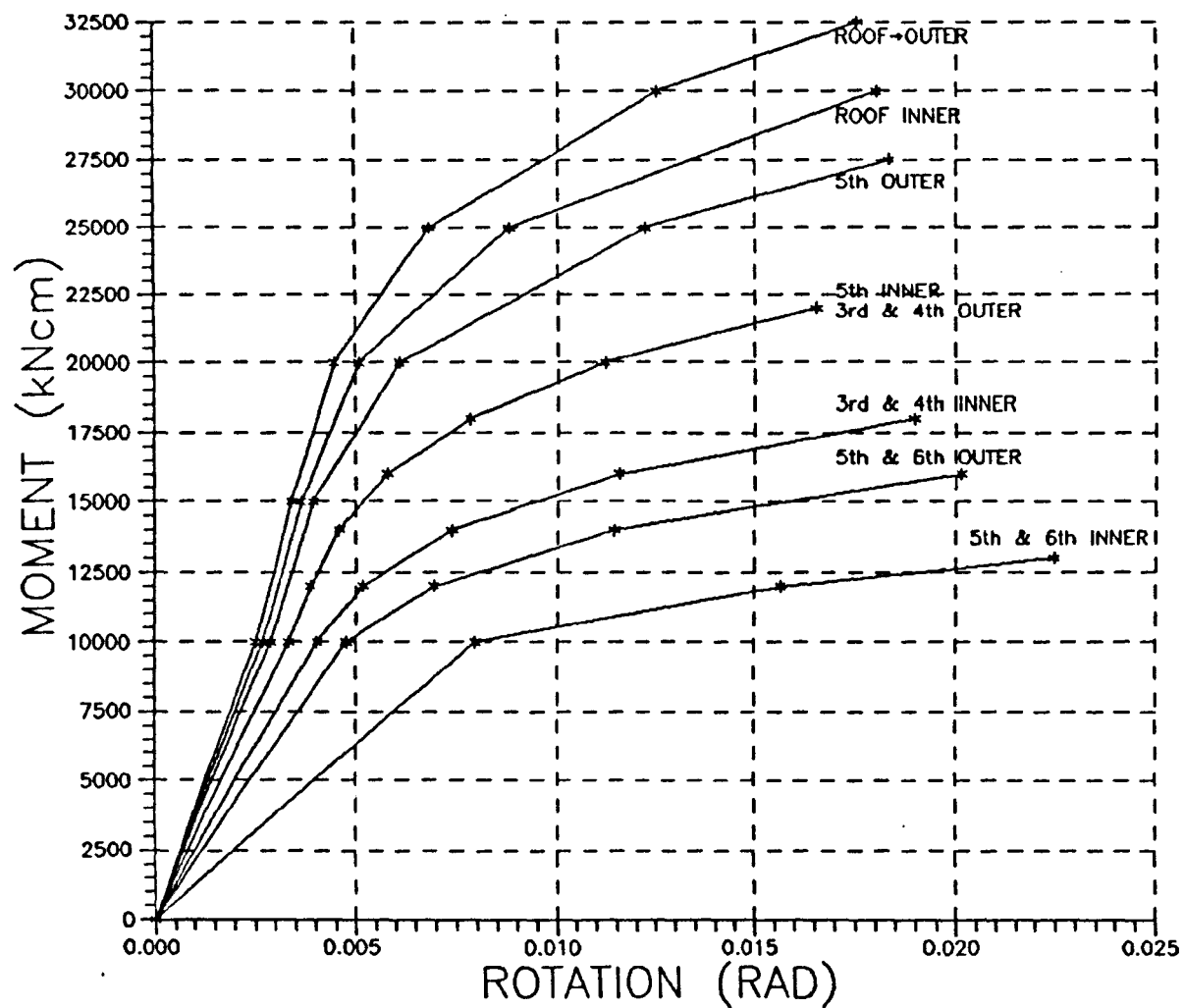


FIG. (5.12) MOMENT ROTATION RELATIONSHIP FOR SIX STOREY TWO BAY FRAME



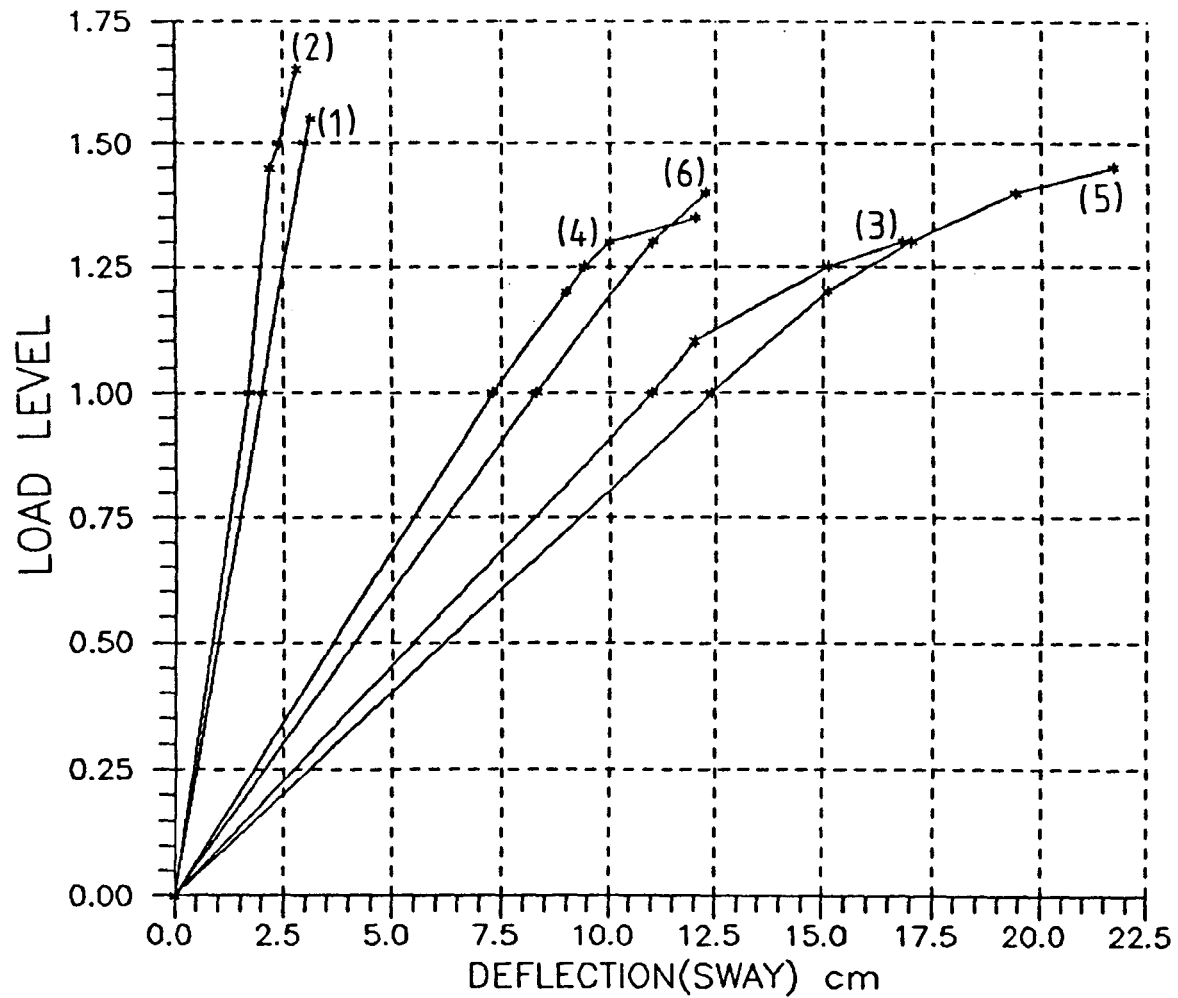


FIG. (5.13) LOAD AGAINST DEFLECTION FOR SIX STOREY  
TWO BAY FRAME

|                                 |                            |
|---------------------------------|----------------------------|
| CURVE(1) (1.4D+1.6I) SEMI-RIGID | CURVE(2) (1.4D+1.6I) RIGID |
| CURVE(3) 1.2(D+I+W) SEMI-RIGID  | CURVE(4) 1.2(D+I+W)) RIGID |
| CURVE(5) 1.4(D+W) SEMI-RIGID    | CURVE(6) 1.4(D+W) RIGID    |

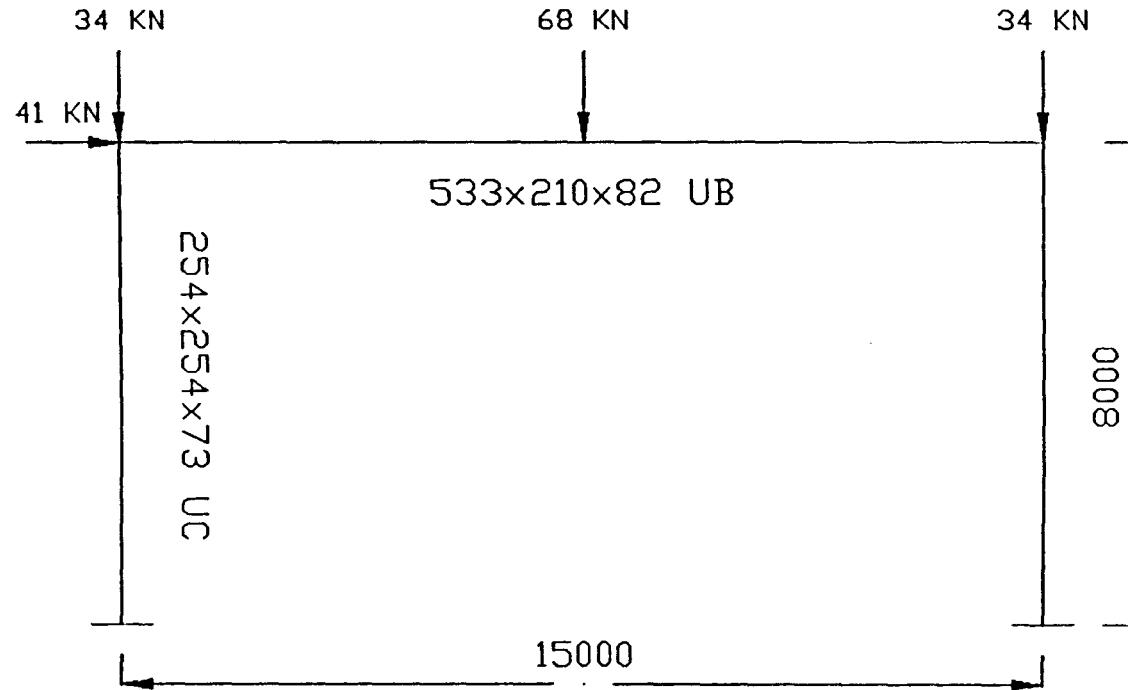


FIG. (5.14) GEOMETRY, SECTIONS AND LOADING OF SINGLE STOREY SINGLE BAY FRAME

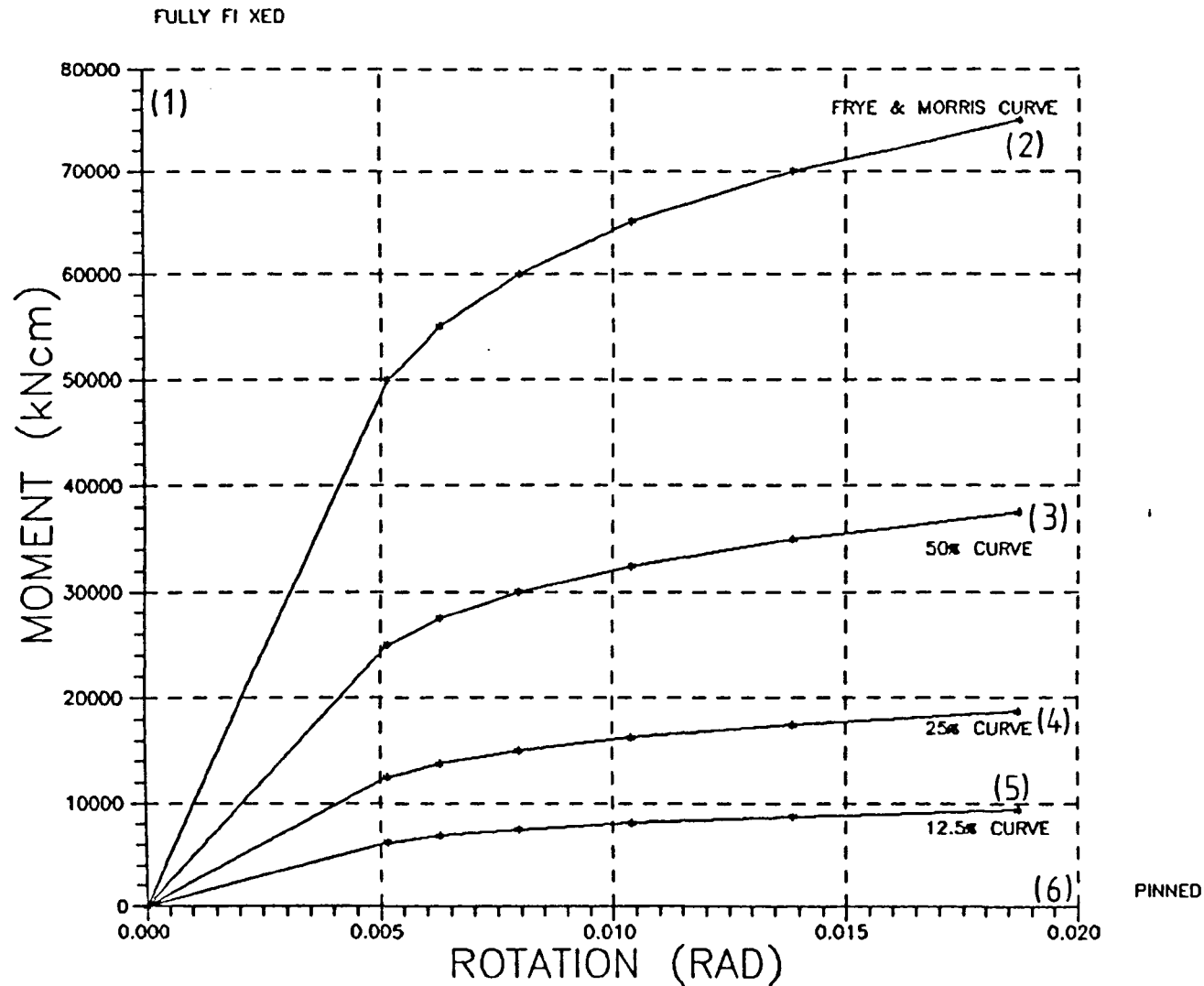


FIG. (5.15) MOMENT ROTATION RELATIONSHIP FOR SINGLE STOREY SINGLE BAY FRAME

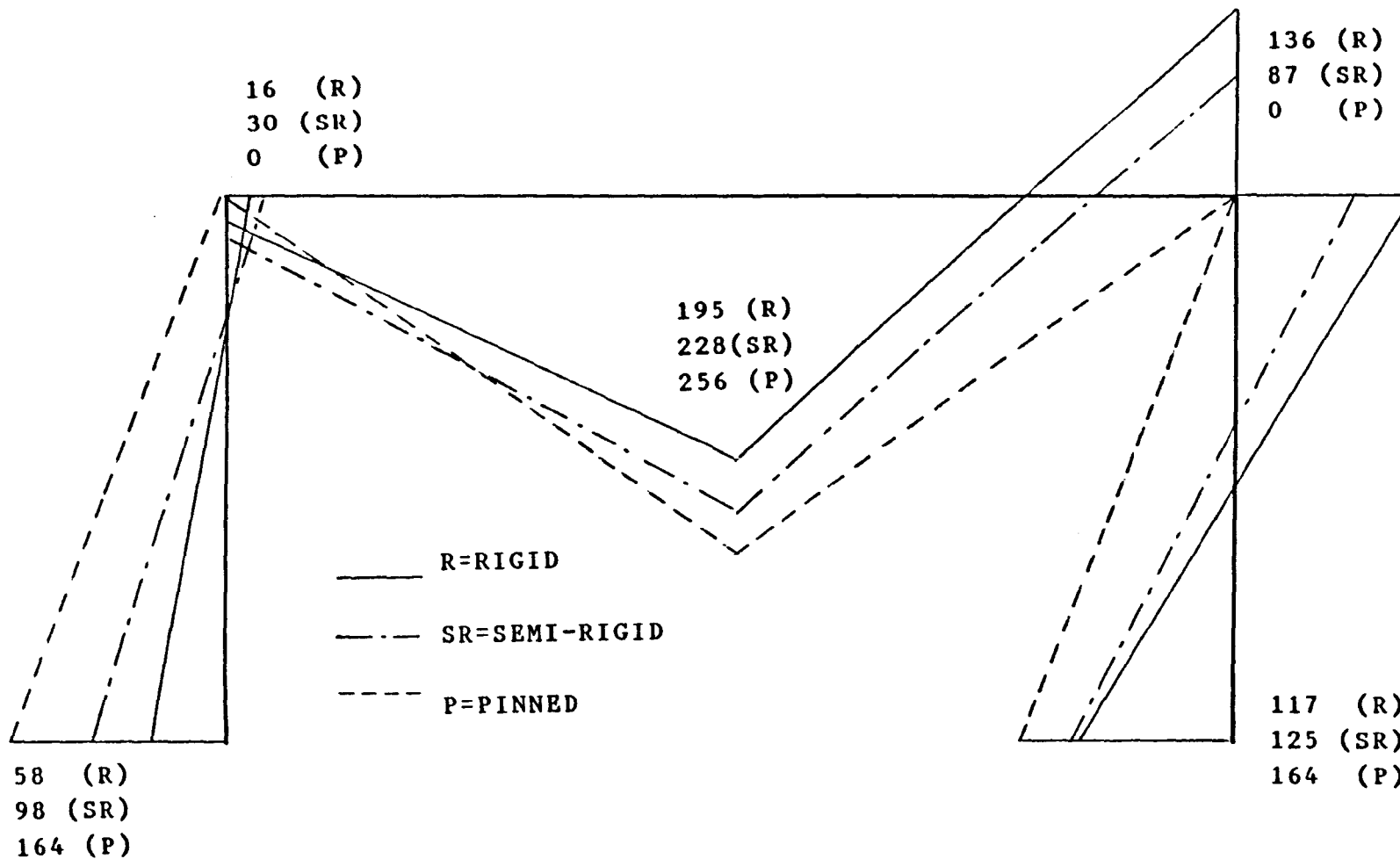


FIG. (5.16) COMPARISON OF BENDING MOMENT DIAGRAM FOR THE RIGID, SEMI-RIGID AND PINNED ANALYSIS

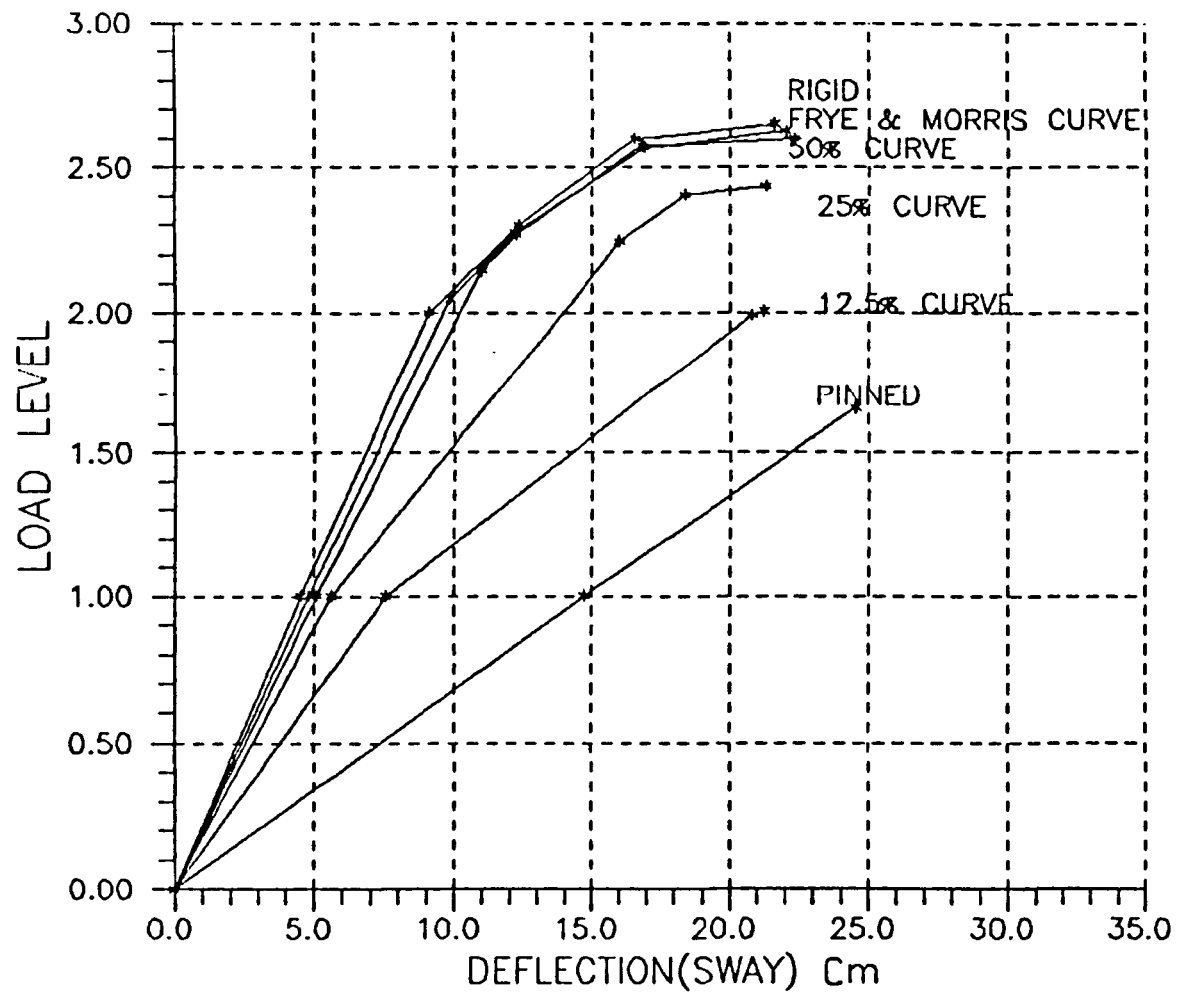


FIG. (5.17) LOAD AGAINST DEFLECTION OF SINGLE STOREY ONE BAY FRAME FOR VARIOUS CONNECTION FLEXIBILITY

## CHAPTER 6

### VARIABLE REPEATED LOADING

#### 6.1 Introduction

The work carried out in the previous chapters assumed that the critical pattern (or patterns) of loading is fixed and the problem was treated as one of a static collapse. The present chapter is concerned with a loading pattern in which the various loads can act randomly and independently within given limits. These limits might simply correspond to the maximum value of a particular load and zero; there may be snow on the roof of a building, or not. The wind may blow in alternate directions and maximum and minimum loads may therefore be positive and negative forms of the same magnitude. It is also clear that, in practice, no building frame ever enjoys proportionally increasing loads but is subject to random fluctuations of loading throughout its working life.

The response of structures to variable repeated loading has been studied extensively by many researchers. The review of their work has already been dealt with in Chapter 1 of this thesis.

The work described in the first part of this Chapter concerns frames with rigid connections. It examines the response of structures to

regular cycles of loads below or above the calculated shakedown load. It is evident that the amount of calculation becomes excessive as soon as any but the simplest of structures is considered. This situation demonstrates the need for an automatic analysis for shakedown load. Some researchers have developed methods based on a modified linear programming procedure or adopted an energy method for calculation of shakedown load  $\alpha_s$ .

The calculation of shakedown load is not the prime concern in this chapter. The main objective is to investigate the behaviour of structures under cycles of loading. For this purpose a computer program was developed. This program is an extension of the elastic-plastic program described in Chapter 4. The program is used to examine the phenomena associated with this particular form of structural behaviour, which are :

- A) Alternate plasticity.
- B) Incremental collapse.
- C) Shakedown theorem.

A short discussion on each of the above phenomena is given in the following sections.

In the second part of this Chapter the program is further developed to deal with the analysis of variable repeated loading on structures with semi-rigid connections. The effect of unloading on a semi-rigid connection is indicated by a reversal in the direction of rotation.

When the rotation of the particular connection is reversed, it will follow a path parallel to the initial stiffness of moment-rotation characteristic.

## 6.2 Alternate plasticity:

One of main affects of the random and repeated loading of a structure is alternate plasticity. It may be that, under a certain combination of the independently varying loads, a plastic hinge develops at a certain cross section of the frame. At a later time, a different combination of loads may produce plasticity at the same cross-section but with the bending moment acting in an opposite sense. Such repeated bending reversal at a cross section may not be very harmful if the number of repetitions is fairly small in the life of the structures. But it is sometimes necessary to compute the permitted range of loading to prevent its occurrence.

In the light of experimental evidence reported by Davies [106], it now seems evident that structural failure due to alternating plasticity is unlikely to occur unless a great many cycles of peak loading are applied. Therefore alternating plasticity does not require serious consideration in the design of structural framework under long duration of imposed loads, snow loads and wind loads. This may not be true in structures with cranes and plant loads.



### 6.3 Incremental collapse.

The second and apparently more important possibility is the structural failure by incremental collapse. It was noted that under a certain combination of the independently varying loads a plastic hinge formed at a cross section and irrecoverable rotation took place at that plastic hinge during each cycle of loading. If the maximum load intensity is unchanged from cycle to cycle, an unvarying regime emerges in which the change in the rotation at any given hinge is a constant from cycle to cycle, so that in each cycle the irrecoverable or residual deflections of the structure increase by a definite amount. After a certain number of cycles of load application has taken place, the residual deflections will have risen to such high value that the structure is rendered useless. For this reason, the structure is then said to have failed by incremental collapse.

This incremental collapse occurs when plastic hinge rotation of a definite amount takes place at a certain section each time one of the critical load combinations is applied. It is important to note that the increments of rotation must always be in the same direction at each section. Under conditions of incremental collapse, there are not, at any one time, sufficient hinges present in the structure to form the complete mechanism, but during each cycle, each hinge undergoes a certain rotation that is limited by the elastic restraint of the remainder of the structure.

The failure of structures in this manner is clearly demonstrated with

the aid of examples in section 6.7 of this Chapter.

#### 6.4 Shakedown theorem.

Understanding of any problem associated with the repeated loading of the structure must depend on a good knowledge of the condition under which a structure may be expected to shakedown.

The phenomena associated with this particular form of structural collapse may be briefly summarized as follows. If the maximum intensity of the load level applied to a particular structure is expressed in terms of  $\alpha$ , then it can be shown that if  $\alpha$  exceeds a certain intensity, "residual moments" due to rotation at "plastic hinge" locations are induced in the structure after each cycle of loading. If the magnitude  $\alpha$  exceeds  $\alpha_p$  but remains smaller than a certain value  $\alpha_s$ , the shakedown load, then the increase in the residual moments which remain in the structure after each cycle of loading becomes progressively smaller as the number of cycles of loading increases. Eventually, a condition is reached where no further change in the plastic hinge rotation occurs and subsequent application of these loads cause the hinge to revert to its elastic state. When this occurs, the structure is said to have shakedown.

The bending moment-curvature relation which is usually assumed in shakedown analysis is shown in fig. (6.1). This relation is appropriate for a beam of ideal plastic material whose cross section has two axes of symmetry, and which is bent about one of these axes. The

magnitude  $M_y$  and  $M_p$  of the yield and plastic moments are then the same for bending in either sense. Furthermore, the yield range of bending moment within which wholly elastic behaviour will occur remains at  $2M_y$  regardless of the previous loading history. The necessary conditions for shakedown can now be written in terms of the conventional elastic solution for a frame and of the residual moments which may exist. It will be appreciated that both contributions are necessary for the formulation of the problem. The elastic solution is required since the value of  $\alpha_e$  is sought for which the response of the frame is entirely elastic. On the other hand, since some plastic deformation can occur, residual moments will be induced which will effect the total value of bending moment at any cross section.

Using the working values of the loads, the value  $M_i$  of the elastic bending moment may be computed for each critical section of the frame. As the individual loads vary between their prescribed maximum values, so the bending moment  $M_i$  will vary, and the greatest and least values  $M_i^{\max}$  and  $M_i^{\min}$  may be calculated. A load factor  $\alpha$  applied to a range of loading will increase these values to  $\alpha M_i^{\max}$  and  $\alpha M_i^{\min}$ . These factored values of the elastic bending moments are those that would occur if the frame remained undistorted, but there will, in general, be a residual moment  $m_i$  at a cross section which must be added to the elastic values to give the total bending moment at that section. Thus the necessary condition for shakedown to occur is;

$$\alpha M_i^{\max} + m_i \leq (M_p)_i$$

$$\alpha M_i^{\min} + m_i \geq -(M_p)_i$$

and , to guard against the possible danger of alternating plasticity

$$\alpha(M_i^{\max} - M_i^{\min}) \leq 2M_y \quad 6.2$$

The conditions of (6.1) and (6.2) will be referred to collectively as the static condition. It is evident that if they are not be satisfied , shakedown cannot occur. These conditions are therefore necessary for shakedown to be possible.

#### 6.5 Computer program

A computer program was developed to analyse the structure subjected to variable repeated loading . Once the shakedown load factor is calculated, by any conventional method or the method which will be described in section 6.6, then the analysis program can be used to investigate the behaviour of the structure above or below this load level.

Since any analysis of variable repeated loading must allow the static collapse as a special case, the computer program in chapter four was used here, but extended to allow for the reversal of rotations. Fig. (6.2) illustrates the flow chart of the program. It can be seen that in addition to the program explained in chapter four, appropriate modifications were made to include the analytical techniques developed. This is for both reversal of plastic hinge rotations and semi-rigid connection rotations.

#### 6.5.1 Modification to overall stiffness matrix.

At a given load level the analysis of the structure is carried out as normal, and sets of simultaneous equations  $L=K.X$  are solved to determine the displacement functions. These displacement are then used to evaluate the moments and axial forces within a given structure. The structure is then checked to see if any moment have reached the reduced value of plastic moment  $M_p$ . If so, plastic hinges are inserted at the appropriate sections. Once the plastic hinges have formed in the next and subsequent load levels, the values of the current set of hinge rotation values are compared with the previous rotations. This is to test whether the analysis implies a reversal in the direction of any plastic hinges.

If rotation of a certain plastic hinges reverses, the section concerned become elastic and there remains a known rotational discontinuity at that section. This state of affairs can occasionally arise in a structure subjected to conventional, single application of load. It will frequently arise when a new phase of cyclic loading is initiated.

Let us assume at some stage of loading a plastic hinge is formed at end i of the member ij as shown in fig. (6.3) therefore the bending moments at this member can be written as follows:

$$\begin{aligned} M_i &= \frac{2EI}{L} \left( 2(\theta_i + \theta_j) + \theta_j - \frac{3\delta}{L} \right) = M_p \\ M_j &= \frac{2EI}{L} \left( (\theta_i + \theta_j) + 2\theta_j - \frac{3\delta}{L} \right) = M_p \end{aligned} \quad 6.3$$

On unloading, the plastic hinge locks at an known rotational discontinuity  $\theta_h'$  and behaves elastically. Therefore equations (6.3) become:

$$\begin{aligned} M_{ij} &= \frac{2EI}{L} \left( 2(\theta_i + \theta_h') + \theta_j - \frac{3\delta}{L} \right) = M_p \\ M_{ji} &= \frac{2EI}{L} \left( (\theta_i + \theta_h') + 2\theta_j - \frac{3\delta}{L} \right) = M_p \end{aligned} \quad 6.4$$

The value of  $\theta_h'$  is constant and equal to the value of the hinge rotation before it began to reverse in direction. The constant hinge rotation  $\theta_h'$  are added to the bending moments as shown in equation (6.4) at all load level or load phase above which the plastic hinge ceases to rotate.

Now assume that the stiffness matrix  $[K]$  has been built up for member  $ij$ , as before, and the contributions have been added for two plastic hinges at ends  $i$  and  $j$  with unknown plastic hinge rotations  $\theta_{hi}$  and  $\theta_{hj}$  at ends  $i$  and  $j$  respectively, as shown in fig. (6.4). When the load is removed both these plastic hinges unload and reverse in their direction and the section becomes elastic. Therefore the rows and columns which were originally added to the stiffness matrix due to the presence of these plastic hinges will now be removed. There remain the two known quantity of  $\theta_{hi}, \theta_{hj}$  which will be taken onto the right hand side of equation  $L=KX$ . A modified load vector, which corresponds to the number of elements of the original load vector, is then used to obtain a new set of displacements and member forces. An example of the modified load vector for member  $ij$  is given by:

$$\begin{aligned}
 H_i & H_i - ds(\theta'_{ni}) - ds(\theta'_{nj}) \\
 V_i & V_i + dc(\theta'_{ni}) + dc(\theta'_{nj}) \\
 M_i & M_i - a(\theta'_{ni}) - f(\theta'_{nj}) \\
 H_j & H_j + ds(\theta'_{ni}) + ds(\theta'_{nj}) \\
 V_j & V_j - dc(\theta'_{ni}) - dc(\theta'_{nj}) \\
 M_j & M_j - f(\theta'_{ni}) - a(\theta'_{nj})
 \end{aligned}$$

6.5

where:

$$\begin{aligned}
 a &= \frac{EA}{L} \\
 b &= \frac{12EI\phi_3}{L^3}
 \end{aligned}$$

$$d = \frac{-6EI\phi_2}{L^2}$$

$$e = \frac{4EI\phi_3}{L}$$

$$f = \frac{2EI\phi_4}{L^3}$$

s,c= direction sine and cosine of the angle of inclination of the member, measured clock wise.

E,I,A,L are Young's modulus of elasticity, the second moment of area, cross sectional area and the length of the member i-j ;  $\phi_1$  to  $\phi_5$  are the usual stability functions.

The load vector is modified as just explained at all load levels after the detection of the inactive hinge. This is stopped as soon as a plastic hinge forms at that section again. New plastic hinges are then added to the structure; consequently, extra rows and columns are added to stiffness matrix and the analysis is carried out as before.

More detailed explanation approach is best achieved by referring to the example of a simple structure.

#### 6.5.2 Verification of computer analysis.

The example of a propped cantilever shown in fig. (6.5) will demonstrate the approach taken for the computer analysis program in this chapter.

##### Load P<sub>1</sub>

Stage 1 ( ref. to Fig. (6.5b)) at load P<sub>1</sub> plastic hinge forms at A . By referring to Steel Designers' Manual [107] the value of bending moment and the joint rotation are given as;

$$\begin{aligned}M_p &= \frac{-3P_1L}{16} \\M_A = M_p &= \frac{-3P_1L}{16} \\ \theta_{A1} &= \frac{-P_1L^2}{32EI} = \frac{-0.03125P_1L^2}{EI}\end{aligned}$$

Therefore



$$\theta_{s1} = \frac{-0.03125P_1 L^2}{EI} \quad 6.6$$

The plastic collapse load for this cantilever can be obtained from virtual work by referring to fig. (6.5c)

$$M_p \theta + M_p (2\theta) = P_2 \times \frac{L}{2} \times \theta$$

Therefore

$$P_2 = \frac{6M_p}{L} = \frac{6}{L} \times \left( \frac{3P_1 L}{16} \right) = 1.125P_1 \quad 6.7$$

Therefore the collapse load  $P_2$  is 12.5% above the formation of first plastic hinge. Let us analyse the cantilever at  $1.1P_1$ .

Load  $1.1P_1$ .

Stage 2 ( Fig. (6.5d)) load increases to  $1.1 P_1$ , therefore the joint rotation and hinge rotation can be derived as:

$$\theta_{h1} = \frac{(0.1 P_1) L^2}{16EI} = \frac{0.00625 P_1 L^2}{EI} \quad 6.8$$

$$\theta_{s2} = \theta_{s1} - \theta_h = (-0.03125 - 0.00625) \frac{P_1 L^2}{EI}$$

Therefore

$$\theta_{s2} = \frac{-0.0375 P_1 L^2}{EI} \quad 6.9$$

Stage 3 (Fig.(6.5e)): decrease the load from  $1.1P_1$  to  $P_1$ . Analysis at load  $P_1$  is as follows;

Load  $P_1$

$$\Delta\theta_2 = \frac{(0.1P_1)L^2}{32EI} = \frac{0.003125P_1L^2}{EI}$$

$$\theta_{23} = (\theta_{22} + \Delta\theta_2) = (-0.0375 + 0.003125) \frac{P_1L^2}{EI}$$

$$\theta_{23} = \frac{-0.034375P_1L}{EI}$$

6.10

$$M_A = \frac{-3P_1L}{16} + \frac{3(0.1P_1)L}{16} = -0.16875P_1L < M_p$$

Since the  $M_A$  is less than  $M_p$  the entire cantilever behaviour is elastic.

Now let us find the load level at which the plastic hinge forms at A under reversal of loading. For a plastic hinge to form at section A in the opposite sense, the load level can be obtained as follows;

$$M_{A2} = +M_p = \frac{-3P_1L}{16} + \frac{3(\Delta P_1)L}{16} = \frac{3P_1L}{16}$$

therefore  $\Delta P_1 = 2P_1$

Thus the plastic hinge forms at A in the opposite sense at a load

$$1.1P_1 - 2P_1 = -0.9P_1$$

The analysis at load level  $0.9P_1$  is as follows:

Load  $-0.9P_1$

Stage 4. Due to  $P_1$  changes in  $\theta_B$  is equal to;

$$\theta_3 = \frac{(4P_1)L^2}{32EI} = \frac{2P_1L^2}{32EI} = \frac{0.0625P_1L^2}{EI}$$

$$\theta_{3,4} = (\theta_{3,2} + \theta_3) = (-0.0375 + 0.0625) \frac{P_1L^2}{EI}$$

$$\theta_{3,4} = \frac{0.025P_1L^2}{EI}$$

6.11

Plastic collapse occurs by referring to equation (6.7) when;

$$P_2 = -1.125P_1$$

Consider an increment of  $-0.2P_1$ . Conduct an analysis at  $-1.1P_1$ .

Load  $-1.1P_1$

Stage 5 (Fig. (6.5f))

$$\theta_{4,2} = \frac{-(0.2P_1)L^2}{16EI} = \frac{-0.0125P_1L^2}{EI}$$

$$\theta_{3,5} = (\theta_{3,4} - \theta_{4,2}) = (0.025 + 0.0125) \frac{P_1L^2}{EI}$$

6.12

$$\theta_{B5} = \frac{0.00375 P_1 L^2}{EI}$$

$$\theta_{A3} = \theta_{A1} + \theta_{A2} = (0.00625 - 0.0125) \frac{P_1 L^2}{EI}$$

$$\theta_{A3} = \frac{-0.00625 P_1 L^2}{EI}$$

6.13

$\theta_{A1}$  and  $\theta_{A3}$  are the hinge rotations at  $1.1P_1$  and  $-1.1P_1$  respectively

It is evident from the analysis that,  $\theta_{A3}$  and  $\theta_{B5}$  obtained at load level  $-1.1P_1$  are of the same magnitude as  $\theta_{A1}$  and  $\theta_{B2}$  obtained at load level  $+1.1P_1$ . Therefore the hinge rotation obtained at load level  $-1.1P_1$  can be obtained directly by increasing the load level from 0 to  $-1.1P_1$  without carrying out the rotations from the analysis of 0 to  $1.1P_1$ .

It can be concluded that in any structure where a plastic hinge forms at a given application of the load and this plastic hinge becomes inactive during the next application of the load. The resulting residual bending moments arising from this inactive hinge will be added to bending moments calculated in the subsequent load application until a plastic hinge reforms at that section. This is the method adopted in the author's program.

## 6.6 Calculation of shakedown load.

The value of  $\alpha_s$  or shakedown load is always less than, or at most equal to,  $\alpha_p$  (static collapse load) where the value of the static load factor is computed from the maximum values of applied loads. In fact, the value of the incremental load factor resulting from the analysis of any assumed mechanism of collapse, not necessarily the correct mechanism, can never exceed the corresponding static value of the load factor for the same assumed mechanism. The formulation given by Ogle [108] stated that the basic incremental collapse equation for any assumed mechanism  $\theta_i$  can be written as:

$$\alpha_s \left\{ \sum_i^0 (M_i^{\max} \theta_i^+ + M_i^{\min} \theta_i^-) \right\} = \sum (M_p)_i |\theta_i| \quad 6.14$$

Where  $M_i^{\max}$  and  $M_i^{\min}$  are the maximum and minimum elastic bending moments at section  $i$  for all different load combinations considered.

Elastic bending moments for different loading combinations can be easily determined by the author's computer program. This is done by adopting very large value of  $P_y$  (yield strength), to prevent the formation of plastic hinges. From these elastic moments  $M_i^{\max}$  and  $M_i^{\min}$  can be calculated.

In the next stage, the frame is analysed using the actual value of  $P_y$ . The frame is then loaded up to its collapse load. It would then be

assumed that incremental collapse would occur by the same mechanism as static collapse. Finally the residual bending moment denoted by  $m_i$  can be determined from one of the following equations.

$$\begin{aligned} m_i + \alpha_i M_i^{\max} &= (M_p)_i & \text{for all } \theta_i^+ \\ m_i + \alpha_i M_i^{\min} &= -(M_p)_i & \text{for all } \theta_i^- \end{aligned} \quad 6.15$$

Since  $m_i$  is statically admissible with zero external load, it follows from the principle of virtual work that,

$$\sum m_i \theta_i = 0 \quad 6.16$$

where the summation covers all the hinge positions in the assumed mechanism. Therefore, the value of  $\alpha_i$  corresponding to any assumed mechanism of incremental collapse can be determined.

### 6.7 Numerical example

The computer program described in section 6.5 is used to follow the response of structures to variable repeated loading. The method described in section 6.6 is used to calculate the shakedown load of such a structure.

The behaviour of three structures under variable repeated loading is described in this section. Comparisons are made between the results obtained from the described computer program and the results of other researchers.

The shakedown loads obtained are compared with the corresponding failure loads under proportional loading and some conclusions are drawn.

#### 6.7.1 Example1: Single storey single bay frame.

The single storey portal frame shown in fig. (6.6), has been extensively treated analytically by Neal using a step by step technique [17] and experimentally by Neal and Symonds [109]. This frame will be analysed to validate the results obtained using of author's computer program.

When analysing this frame two further simplifying assumption are made. Firstly, the reduction in full plastic moment of the stanchions due to axial load is ignored and secondly, the second order effects are ignored. This simplification is not essential but it allows the results obtained by the described program to be compared with the Neal results.

It is not necessary to calculate the values of alternate plasticity  $\alpha_a$  shakedown load,  $\alpha_s$  and the plastic collapse  $\alpha_p$  because these values have already been calculated by Neal [17] to be as follows:

$$\begin{aligned}\alpha_a &= \frac{2.759 M_p}{L} \\ \alpha_s &= \frac{2.857 M_p}{L} \\ \alpha_p &= \frac{3 M_p}{L}\end{aligned}$$

The first loading cycle to be considered is shown in fig. (6.7). The

sequence of loading was as follows;

|       |        |    |          |
|-------|--------|----|----------|
| $V=W$ | $H=W$  | OR | $(W,W)$  |
| $V=0$ | $H=0$  | OR | $(0,0)$  |
| $V=0$ | $H=-W$ | OR | $(0,-W)$ |
| $V=0$ | $H=0$  | OR | $(0,0)$  |

These cycles cause alternating plasticity when  $\alpha=\alpha_w= 2.857$ .

The results of calculations for  $\alpha=2.85 \text{ Mp/EI}$  are summarized in table (6.1). The first row of each result in the table correspond to the author's results ,the second row to Neal's [17] results. This format follows through the table. It can be seen from table (6.1) that there is very close agreement between the two sets of results.

The loading cycle which may cause incremental collapse is shown in fig. (6.8), the sequence of loadings is;

|       |       |    |         |
|-------|-------|----|---------|
| $V=W$ | $H=W$ | OR | $(W,W)$ |
| $V=0$ | $H=0$ | OR | $(0,0)$ |
| $V=0$ | $H=W$ | OR | $(0,W)$ |
| $V=0$ | $H=0$ | OR | $(0,0)$ |

The results of calculations for shakedown load at  $\alpha=2.9\text{Mp/L}$  are summarized in table (6.2). Once more it can be seen that, the results obtained agree closely with the Neal results.



### 6.8.2 Example 2 ,single storey single bay frame.

As a second example of this kind of incremental collapse analysis, the frame of fig. (6.9) has been investigated. The loads shown will be taken as varying between the following limits.

$$V=(90 \alpha, 28 \alpha) \text{ kN}$$

$$H=(56 \alpha, 0) \text{ kN}$$

The frame has a uniform section, with full plastic moment of 141 kNm. A shape factor of 1.15 is taken for an I section. From the three possible mechanisms of fig. (6.10), the combined mechanism is critical for static collapse, and this is taken as possible mechanism of incremental collapse.

The elastic bending moment-distributions in the frame for different combinations of load computed, are given in table (6.3). The maximum positive and negative changes of bending moment are then be determined. The final row of table gives values of elastic bending moments  $\alpha(M_{max}, M_{min})$ . The largest value of the last row of table (6.3) is used to determine alternate plasticity, using the equation (6.2). This is given by;

$$95.4\alpha_s = (2 \times M_p) = \frac{2 \times 141}{1.15}$$

Therefore  $\alpha_s = 2.57$ .

For the combined mechanism of fig. (6.10c) the equation of equilibrium for residual moment is given as:

$$-m_1 + 2m_3 - 2m_4 + m_5 = 0 \quad 6.17$$

The residual bending moments for the possible hinges given by equation (6.15) are as follows:

$$\begin{array}{ll} m_1 + \alpha M_{pl} = -M_p & \theta_1 = -\theta \\ m_3 + \alpha M_{pl} = +M_p & \theta_3 = 2\theta \\ m_4 + \alpha M_{pl} = -M_p & \theta_4 = -2\theta \\ m_5 + \alpha M_{pl} = +M_p & \theta_5 = \theta \end{array} \quad 6.18$$

Using  $M_p = 141$  kNm and substituting for elastic moments from table (6.3) the equation (6.18) becomes :

$$\begin{array}{l} m_1 - 58.8\alpha_s = -141 \\ m_3 + 108\alpha_s = 141 \\ m_4 - 114\alpha_s = -141 \\ m_5 + 106.6\alpha_s = 141 \end{array}$$

By substituting into equation (6.17) for residual moments  $\alpha_s$  can be determined;

$$609.4\alpha_s = 846 \quad \alpha_s = 1.388$$

This analysis determines the critical extreme load limit for shakedown

to be possible. The results of computers analysis at  $\alpha=1.32$  and  $\alpha=1.4$  are tabulated in tables (6.4) and (6.5) for the load combinations shown, in fig. (6.11).

First, let us examine the the results of analysis at load level  $\alpha=1.32$ . This load level is below the shakedown load of  $\alpha_s=1.388$ . In first cycle, during the first application of loading, plastic hinges occur at sections 3,4 and 5. This is followed by a second application of loading where no plastic hinges form. During the second cycle, the plastic hinges only form at section 5 and 3 at the higher load level than the first cycle and the deflection increases. It can be seen from table (6.4) that, after six cycles of loading the structure will behave entirely elastically and there is no further changes in displacement. This demonstrates that the structure has shakedown and the distribution of residual bending moments is such that all further load applications will result in elastic behaviour.

Now examine the results of the analysis at load level  $\alpha=1.42$  (which is above the calculated shakedown load  $\alpha_s$ ). As previously found, in the first application of loading, plastic hinges form at sections 3,4 and 5. But this changes during the second load combination, of  $(28\alpha, 56\alpha)$ , with a further plastic hinge forming at section 1. The same number of plastic hinges occur in all subsequent cycles of loading. The horizontal deflection at the top of the column for each cycle of loading is shown in the last column of table (6.5). It can be seen, from this table that, after four cycles of loading the deflections have built up and failure occurs by incremental collapse.

Fig. (6.12) shows the effect of cycles of loading on horizontal deflection. When  $\alpha=1.42$ , the deflection increased by 52% in comparison with the deflection in the first cycle. For  $\alpha=1.32$  the deflection increases with each cycle, but tends asymptotically to a definite limit.

It is noteworthy that the value of  $\alpha_p$  (plastic collapse load factor) corresponding to a single application of the worst possible load combination was 1,442. This is only 4% above the shakedown load  $\alpha_s$  in this particular example.

#### 6.7.3 Example 3: four storey one bay structure.

As the final example a four storey one bay structure was chosen to illustrate the complex plastic collapse and behaviour to repeated loading of a multistorey frames. This structure was analysed by Davies [106] for both static and repeated loading.

The sections for this frame together with dimensions and applied loading are shown in fig. (6.13). The structure of fig. (6.13) was first analysed for plastic collapse. The designation of possible locations of plastic hinges are shown in the same figure.

As the load parameter increases from a starting value to the final collapse value  $\alpha_p$ , more plastic hinges formed at various locations throughout the structure, until a sufficient number of plastic hinges appear simultaneously to cause a mechanism to form. Table (6.6) shows

the load levels corresponding to the successive formation of plastic hinges. The bottom line entry in table (6.6), indicates that the collapse load  $\alpha_p$  is 3.2. The last column in the table shows the displacement at the formation of each plastic hinge. Fig. (6.14) shows the plastic hinges which caused the structure to fail.

The load deflection curves obtained for this frame are shown in fig. (6.15). The collapse load  $\alpha_p$  obtained by author is in agreement with load factor of 3.18 obtained by Davies. In order to proceed with calculation of the shakedown load, the structure in Fig. (6.13) was subjected to three different loading sequences as shown in Fig. (6.16). The elastic bending moment distributions in the frame for the three loading sequences that are shown in fig. (6.16) were computed by choosing a large  $P_y$  (yield strength) to prevent any formation of plastic hinges. For each loading condition, the  $M_{max}$  and  $M_{min}$  elastic bending moments were determined and are tabulated in table (6.7). The final row of the table gives the value of the elastic bending moment  $\alpha(M_{max}-M_{min})$ . The largest of these values for different sections are used to calculate the alternate plasticity load level as follows;

For section 1 floor level 3 and 4.

$$\alpha_x \times 165.8 = 2 \times M_y = \frac{2 \times 351}{1.15} = 3.70$$

For section 2 floor level 1 and 2;

$$\alpha_x \times 163 = 2 \times M_y = \frac{2 \times 388}{1.15} = 4.14$$

For section 3 all the stanchions;

$$\alpha_s \times 157 = 2 \times M_y = \frac{2 \times 555}{1.15} = 6.148$$

The lowest of the three values calculated for alternate plasticity is still much higher than the collapse load  $\alpha_p = 3.20$ . Therefore, there is no problem concerning alternate plasticity.

To calculate the shakedown load  $\alpha_s$ , the same mechanism for static collapse shown in fig. (6.14) is assumed here. This gives rise to ten equations for values of residual moments;

$$\begin{array}{ll}
 m_2 + \alpha M_{max} = M_p(1) & \theta = 2\theta \\
 m_3 + \alpha M_{min} = -M_p(1) & \theta = -2\theta \\
 m_6 + \alpha M_{max} = M_p(1) & \theta = 2\theta \\
 m_6 + \alpha M_{min} = -M_p(1) & \theta = -2\theta \\
 m_8 + \alpha M_{max} = M_p(2) & \theta = 2\theta \\
 m_9 + \alpha M_{min} = -M_p(2) & \theta = -2\theta \\
 m_{11} + \alpha M_{max} = M_p(2) & \theta = 2\theta \\
 m_{12} + \alpha M_{min} = -M_p(2) & \theta = -2\theta \\
 m_{19} + \alpha M_{min} = M_p(3) & \theta = -\theta \\
 m_{26} + \alpha M_{max} = M_p(3) & \theta = \theta
 \end{array} \tag{6.19}$$

By substituting for  $M_p$  and  $M_{max}$  and  $M_{min}$  from table (6.7) equation (6.19) becomes:

$$m_2 + 104.4 = 353$$

$$m_3 - 115.4 = -353$$

$$m_8 + 92.8 = 353$$

$$m_4 - 136.7 = -353$$

$$m_8 + 89.8 = -388.7$$

$$m_9 - 164.3 = -388.7$$

$$m_{11} + 85.4 = 388.7$$

$$m_{12} - 161.7 = -388.7$$

$$m_{19} - 140 = -555$$

$$m_{26} + 156.7 = 555$$

6.20

For the mechanism of fig. (6.14), the equations of equilibrium for residual moment is given as:

$$2m_2 - 2m_3 + 2m_5 - 2m_6 + 2m_8 - 2m_9 + 2m_{11} - 2m_{12} - m_{19} + m_{26} = 0$$

6.21

Substitute for  $m_2, \dots, m_{26}$  from equations (6.19) into equations (6.21):

$$\begin{aligned} & 2(353 - 104.4\alpha) - 2(-353 + 115.4\alpha) + 2(353 - 92.8\alpha) \\ & - 2(-353 + 136.7\alpha) + 2(388.7 - 89.8\alpha) - 2(-388.7 + 164.3\alpha) \\ & + 2(388.7 - 85.7\alpha) - 2(-388.7 + 161.7\alpha) - (-555 + 140\alpha) + (555 - 156.7\alpha) = 0 \end{aligned}$$

From which

$$\alpha = \alpha_s = \frac{7043.6}{2198.5} = 3.20$$

Thus the shakedown load is the same as static collapse load  $\alpha_p$ . Therefore it indicates that there is no problem with incremental collapse. The shakedown load of 3.20 computed here agrees closely with the

shakedown load found by Davies [106].

The behaviour of the structure was examined below the calculated shakedown load of 3.20, for the loading combinations shown in fig. (6.16). The results of these analyses, obtained from the computer program are summarized in tables (6.8) and (6.9) for  $\alpha=2.96$  and  $\alpha=3$  respectively.

From the analysis of  $\alpha=2.96$  it can be seen that, at the first load application plastic hinges formed at sections 9,12,6,26 and 3, as shown in table (6.8). When the second and third applications of loading were considered all the plastic hinges previously formed were unloaded. In the second cycle of loading, considering the first load application, plastic hinges formed only at sections 26,6 and 9. In the second cycle the deflection was increased slightly in comparison with the first cycle. The loading cycles on the frame were continued until it was found that after four cycles of loading, the structure behaved entirely elastically. Therefore, there was no further change in the displacement, i.e. the structure had shakedown.

For  $\alpha=3$  it can be seen that, generally, the same phenomena occurs as previously shown, except that more cycles of loading are required for the frame to shakedown. The results of this load level are tabulated in table (6.9). In this particular example, seven cycles of loading were required for the frame to behave entirely elastically. It can be concluded that as  $\alpha$  approaches the shakedown load of 3.20 more cycles of loading are required in order for the structure to shakedown.



## 6.8 Conclusion

From the examples studied, the general conclusion appears to be that, for rigidly connected frames, variable repeated loads are unlikely to lead to a revision of designs which already satisfy proportional loading and are of normal proportions for building structures.

For the three examples analysis implies that shakedown loads calculated are very close to the elastic-plastic failure loads under the worst loading condition. In fact in the last example the shakedown load was equal to the static collapse load. Bearing in mind that the initial assumption was that the whole of loading on the structures was live load (except example 2), whereas a large proportion of the total load would in fact be dead load, it is evident that, for each of these structures, the elastic-plastic failure load is an adequate ultimate load for the purpose of design.

Finally, the results using the program developed in this Chapter agree closely with the results of other researchers. This will give the basis needed for the further development of the program to include the effect of cycles of loading on semi-rigid connections.

## 6.9 The effect of cyclic loading on the frames with the semirigid connections.

In the previous sections the effect of cyclic loading on frames with rigid joints was investigated. This section describes an analytical

technique, incorporated into the computer program by the author, to study the effect of cyclic loading on frames with semi-rigid connections. To do this, the program described in section 6.5 was further extended to allow for the reversal of semi-rigid joints, as well as rigid joints.

#### 6.9.1 Modification to the overall stiffness matrix.

As before, the set of simultaneous equations  $L=K.X$  is solved for a given load factor. The resulting displacements which also include the semi-rigid rotations are obtained. The bending moments are calculated from these displacements in the same manner as shown in equation (6.3), except that  $\theta_{hi}$  is replaced by  $\theta_{eri}$ , where semi-rigid joints are present.

During each load step and load cycle the values of the semi-rigid rotations are checked to establish if any are starting to rotate in the opposite direction. If so, the last value of the hinge rotation before it began to reverse in direction is stored and modification is made to the load factor due to the known quantity of semi-rigid rotation.

Consider fig.(6.4) again, but assume that there are two semi-rigid rotations  $\theta_{eri}$  and  $\theta_{erj}$  at ends  $i$  and  $j$  of the member  $ij$  respectively. On unloading the two known quantities of  $\theta_{eri}$  and  $\theta_{erj}$  will be

taken on to the left hand side of equations  $L=K.X$ . The modified load vector obtained is similar to that obtained in equation (6.5) and the value of bending moments in member  $ij$  is given by:

$$M_{ij} = \frac{2EI}{L} \left( 2(\theta_i + \theta_{ori} + \theta'_{ori}) + \left( \theta_j + \theta_{orj} + \theta'_{orj} - \frac{3\delta}{L} \right) \right)$$
$$M_{ji} = \frac{2EI}{L} \left( (\theta_i + \theta_{ori} + \theta'_{ori}) + 2 \left( \theta_j + \theta_{orj} + \theta'_{orj} - \frac{3\delta}{L} \right) \right)$$
6.22

The notation is as defined earlier.

The constant hinge rotations are added to the load vector and bending moments at all load levels and load phases above the detection of reversal of hinge rotations. The constant rotation is deleted as soon as the initial loading path is resumed. This is described best by referring to fig. (6.17) which shows the cyclic moment rotation characteristic.

The loading phase (1) follows the path predicted by Frye and Morris [44]. Unloading from any point on the loading curves follows the path parallel to the initial stiffness  $K$  (path 2). On loading again (3), it will reach the path on point B in the curve where it abruptly turns and again follows the loading response curve (1). At this stage all the modification to the load vector and bending moments are stopped and analysis will be carried out as normal.

#### 6.10 Analytical results.

In order to ensure that the results obtained by the computer program are valid, two examples which were previously investigated for rigid joints were analysed here assuming semi-rigid connections at the ends of the beams. The first example is the single storey single bay frame investigated in section 6.7.1. The second example is the four storey one bay frame described in section 6.7.3. The second example was chosen to demonstrate the effect of repeated loading on the multi-storey frames with semi-rigid connections.

Both these examples were subjected to the same applied loading and load cycle as previously specified. These are shown in fig. (6.6) and fig. (6.13).

In order to place a bound on the behaviour of these frames, three sets of arbitrary linear connection stiffnesses were chosen. These are shown in fig. (6.18). The first connection stiffness  $K_1$  was chosen to be very stiff, so that the results correspond to those one would obtain from the rigid analysis. The latter will also confirm the validity of the computer program. The second and third connection stiffnesses were chosen to be 10 and 20 times more flexible than  $K_1$ , respectively.

The results obtained for a single storey frame are presented in table (6.10). It can be seen from this table that the sway displacements for

the connection stiffness  $K_1$  are very close to those of the rigid analysis. Two further features were observed from the analysis which are not shown in the above table. Firstly the number of the plastic hinges and their locations, obtained from the analysis with  $K_1$ , were in the same order as the analysis described in section 6.7.1. Secondly, the analysis with  $K_1$  requires the same number of the load cycles as the rigid jointed frame for the frame to shakedown.

Further analysis with the connection stiffness  $K_2$  and  $K_3$  were performed. The results of these analysis are shown in table (6.10) for single storey frame and table (6.11) for four storey frame.

Fig. (6.19) shows the load versus deflection hysteresis loops of the single storey frame. These results are for the analysis with the connection stiffness  $K_3$ . It can be seen from this figure that initial elastic behaviour is followed by, firstly, yielding until it reaches point A. Upon unloading (path 2), the behaviour is again elastic and parallel to the initial elastic behaviour. On reverse loading, the curve follows path 3 until it reaches the point C. After unloading (path 4) the loading starts at point D and the curve follows the path 5 until eventually reaches the point A. This type of behaviour under load histories was found by other researchers both experimentally and analytically [93,94,95]

The results obtained for the four storey frame analysis indicate that the values of the sway displacements obtained during the 2nd cycle of loading are very close to those values obtained in the 1st cycle for

both K1 and K2. This implies that the reversal of the semi-rigid connection has very little effect on the displacement of this frame. The analysis also shows that plastic hinges formed during the 1st cycle of loading disappeared during the 2nd cycle of the loading. This means that the response of the frame has become completely elastic; therefore the frame has shakedown.

#### 6.11 Conclusion.

The following conclusions can be deduced from the analysis of the frames mentioned above;

- 1) The frames with semi-rigid connections shakedown to their elastic state in the same forms as rigidly connected frames.
- 2) The steel frame behaviour under load histories (fig. 6.19) found in this study is in agreement with the behaviour shown by others.

These conclusions are based solely on the results of the examples carried out in this study with the linear representation of the connection stiffness. The true behaviour of the moment rotation characteristic are needed before generalization can be drawn.

| VL/M <sub>P</sub> | HL/M <sub>P</sub> | M1/M <sub>P</sub> | M2/M <sub>P</sub> | M3/M <sub>P</sub> | M4/M <sub>P</sub> | M5/M <sub>P</sub> | $\phi_1 EI/M_P L$ | $\phi_3 EI/M_P L$ | $\phi_4 EI/M_P L$ | $\phi_5 EI/M_P L$ |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| 2.85<br>2.85      | 2.85<br>2.85      | -0.823<br>-0.823  | 0.027<br>0.028    | 0.939<br>0.939    | -1<br>-1          | 1<br>1            | 0<br>0            | 0<br>0            | -0.159<br>-0.158  | 0.105<br>0.103    |
| 0<br>0            | 0<br>0            | -0.219<br>-0.217  | 0.064<br>0.063    | 0.084<br>0.084    | 0.104<br>0.104    | -0.179<br>-0.176  | 0<br>0            | 0<br>0            | -0.159<br>-0.158  | 0.105<br>0.103    |
| 0<br>0            | -2.85<br>-2.85    | -0.712<br>-0.715  | -0.490<br>-0.491  | 0.078<br>0.077    | 0.647<br>0.645    | -1<br>-1          | 0<br>0            | 0<br>0            | -0.159<br>-0.158  | 0.105<br>0.069    |
| 0<br>0            | 0<br>0            | -0.172<br>-0.176  | 0.042<br>0.044    | 0.077<br>0.077    | 0.111<br>0.110    | -0.12<br>-0.109   | 0<br>0            | 0<br>0            | -0.159<br>-0.158  | 0.066<br>0.069    |
| 2.85<br>2.85      | 2.85<br>2.85      | -0.823<br>-0.823  | 0.027<br>0.028    | 0.0931<br>0.931   | -1<br>-1          | 1<br>1            | 0<br>0            | 0<br>0            | -0.159<br>-0.158  | 0.105<br>0.103    |

TABLE 6.1 COMPARISON OF THE AUTHOR'S WITH THE NEAL'S RESULTS  
ALTERNATING PLASTICITY :  $W=2.85M_P/L$

| VL/M <sub>P</sub> | HL/M <sub>P</sub> | M1/M <sub>P</sub> | M2/M <sub>P</sub> | M3/M <sub>P</sub> | M4/M <sub>P</sub> | M5/M <sub>P</sub> | ϕ <sub>1</sub> EI/M <sub>P</sub> L | ϕ <sub>3</sub> EI/M <sub>P</sub> L | ϕ <sub>4</sub> EI/M <sub>P</sub> L | ϕ <sub>5</sub> EI/M <sub>P</sub> L |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|
| 2.9<br>2.9        | 2.9<br>2.9        | -0.865<br>-0.865  | 0.035<br>0.035    | 0.0967<br>0.968   | -1<br>-1          | 1<br>1            | 0<br>0                             | 0<br>0                             | -0.188<br>-0.186                   | 0.118<br>0.116                     |
| 0<br>0            | 0<br>0            | -0.251<br>-0.249  | 0.072<br>0.071    | 0.098<br>0.098    | 0.124<br>.124     | -0.199<br>-0.196  | 0<br>0                             | 0<br>0                             | -0.188<br>-0.186                   | 0.118<br>0.116                     |
| 0<br>0            | 2.9<br>2.9        | -1<br>-1          | 0.627<br>0.629    | 0.081<br>0.082    | -0.465<br>-0.465  | 0.808<br>0.806    | -0.078<br>-0.078                   | 0<br>0                             | -0.188<br>-0.186                   | 0.116<br>0.116                     |
| 0<br>0            | 0<br>0            | -0.1<br>-0.094    | 0.085<br>0.085    | 0.083<br>0.082    | 0.08<br>0.079     | -0.105<br>-0.1    | -0.078<br>-0.078                   | 0<br>0                             | -0.188<br>-0.186                   | 0.116<br>0.116                     |
| 2.9<br>2.9        | 2.9<br>2.9        | -0.819<br>-0.818  | 0.081<br>0.082    | 0.990<br>0.991    | -1<br>-1          | 1<br>1            | -0.078<br>-0.078                   | 0<br>0                             | -0.256<br>-0.256                   | 0.172<br>0.171                     |
| 0<br>0            | 0<br>0            | -0.205<br>-0.206  | 0.118<br>0.118    | 0.121<br>0.121    | 0.124<br>0.124    | -0.199<br>-0.196  | -0.078<br>-0.078                   | 0<br>0                             | -0.256<br>-0.256                   | 0.72<br>0.171                      |

TABLE 6.2 COMPARISON OF THE AUTHOR'S WITH THE NEAL'S RESULTS  
INCREMENTAL COLLAPSE W=2.9 M<sub>P</sub>/L



| $V_L/M_P$  | $H_L/M_P$  | $M_1/M_P$        | $M_2/M_P$      | $M_3/M_P$      | $M_4/M_P$        | $M_5/M_P$        | $\phi_1 EI/M_P L$ | $\phi_3 EI/M_P L$ | $\phi_4 EI/M_P L$ | $\phi_5 EI/M_P L$ |
|------------|------------|------------------|----------------|----------------|------------------|------------------|-------------------|-------------------|-------------------|-------------------|
| 0<br>0     | 2.9<br>2.9 | -1<br>-1         | 0.669<br>0.672 | 0108<br>0110   | -0.452<br>-0.451 | 0.779<br>0.777   | -0.131<br>-0.133  | 0<br>0            | -0.256<br>-0.256  | 0.172<br>0.171    |
| 0<br>0     | 0<br>0     | 0.1<br>-0.094    | 0.128<br>0.128 | 0111<br>0110   | 0.094<br>0.092   | -0.134<br>-0.129 | -0.131<br>-0.133  | 0<br>0            | -0.256<br>-0.256  | 0.172<br>0.172    |
| 2.9<br>2.9 | 2.9<br>2.9 | -0.8<br>-0.8     | 0.1<br>0.1     | 1<br>1         | -1<br>-1         | 1<br>1           | -0.131<br>-0.133  | 0.043<br>0.05     | -0.315<br>-0.333  | 0.171<br>0.171    |
| 0<br>0     | 0<br>0     | -0.185<br>-0.184 | 0.138<br>0.136 | 0.131<br>0.130 | 0.124<br>0.124   | -0.199<br>-0.196 | -0.131<br>-0.133  | 0.043<br>0.05     | -0.315<br>-0.333  | 0.216<br>0.216    |
| 0<br>0     | 2.9<br>2.9 | -1<br>-1         | 0.686<br>0.688 | 0.120<br>0.121 | -0.446<br>-0.446 | 0.767<br>0.766   | -0.174<br>-0.179  | 0.043<br>0.05     | -0.315<br>-0.333  | 0.216<br>0.216    |
| 0<br>0     | 0<br>0     | -0.1<br>-0.094   | 0.157<br>0.144 | 0.129<br>0.121 | 0.102<br>0.098   | -0.154<br>-0.140 | -0.174<br>-0.179  | 0.043<br>0.05     | -0.315<br>-0.333  | 0.216<br>0.216    |

TABLE 6.2 CONTINUE§

| $VL/M_p$   | $HL/M_p$   | $M1/M_p$         | $M2/M_p$       | $M3/M_p$       | $M4/M_p$         | $M5/M_p$         | $\phi_1 EI/M_p L$ | $\phi_3 EI/M_p L$ | $\phi_4 EI/M_p L$ | $\phi_5 EI/M_p L$ |
|------------|------------|------------------|----------------|----------------|------------------|------------------|-------------------|-------------------|-------------------|-------------------|
| 2.9<br>2.9 | 2.9<br>2.9 | -0.8<br>-0.8     | 0.1<br>0.1     | 1<br>1         | -1<br>-1         | 1<br>1           | -0.179<br>-0.179  | 0.117<br>0.140    | -0.40<br>-0.423   | 0.253<br>0.261    |
| 0<br>0     | 0<br>0     | -0.185<br>-0.184 | 0.137<br>0.136 | 0.131<br>0.130 | 0.124<br>0.124   | -0.199<br>0.196  | -0.179<br>-0.179  | 0.117<br>0.140    | -0.40<br>-0.423   | 0.253<br>0.261    |
| 0<br>0     | 2.9<br>2.9 | -1<br>-1         | 0.686<br>0.688 | 0.120<br>0.121 | -0.446<br>-0.446 | 0.767<br>0.766   | -0.224<br>-0.224  | 0.140<br>0.140    | -0.40<br>-0.423   | 0.253<br>0.261    |
| 0<br>0     | 0<br>0     | -0.1<br>-0.094   | 0.145<br>0.144 | 0.122<br>0.121 | 0.099<br>0.098   | -0.146<br>-0.140 | -0.224<br>-0.224  | 0.140<br>0.140    | -0.40<br>-0.423   | 0.253<br>0.261    |

TABLE 6.2 CONTINUED]

CROSS SECTIONS

| LOAD COMBINATION                 | 1     | 2      | 3     | 4     | 5     |
|----------------------------------|-------|--------|-------|-------|-------|
| (90,56)                          | -34.5 | -29.6  | 108   | -114  | 10.66 |
| (90,0)                           | 35.9  | -72.17 | 108.6 | -72.2 | 35.9  |
| (28,56)                          | -58.8 | 19.7   | 33.5  | -64.5 | 81.4  |
| (28,0)                           | 11.1  | -22.4  | 33.7  | -22.4 | 11.2  |
| $\alpha_s M_{\max}$              | 35.9  | 19.7   | 108   | -22.4 | 10.66 |
| $\alpha_s M_{\min}$              | -58.8 | -72.17 | 33.5  | -114  | 11.2  |
| $\alpha_s (M_{\max} - M_{\min})$ | 94.7  | 91.87  | 72.5  | 91.6  | 95.4  |

TABLE 6.3 ELASTIC BENDING MOMENT  $V(90\alpha, 28\alpha), H(56\alpha, 0)$

| CYCLE NO | LOAD CASE | LOAD LEVEL | POSITION OF PLASTIC HINGE | DISPLACEMENT AT TOP COLUMN cm |
|----------|-----------|------------|---------------------------|-------------------------------|
| 1        | 1         | 1.241      | 4                         | 6.57                          |
| 1        | 1         | 1.247      | 3                         | 7.236                         |
| 1        | 1         | 1.3        | 5                         | 7.782                         |
| 1        | 1         | 1.32       | -                         | 9.41                          |
| 1        | 2         | 1.32       | -                         | 9.41                          |
| 2        | 1         | 1.307      | 5                         | 9.35                          |
| 2        | 1         | 1.313      | 3                         | 9.41                          |
| 2        | 1         | 1.32       | -                         | 9.51                          |
| 2        | 2         | 1.32       | -                         | 9.5                           |
| 3        | 1         | 1.307      | 4                         | 9.441                         |
| 3        | 1         | 1.32       | -                         | 9.63                          |
| 3        | 2         | 1.32       | -                         | 9.61                          |
| 4        | 1         | 1.313      | 3                         | 9.59                          |
| 4        | 1         | 1.32       | -                         | 9.67                          |
| 4        | 2         | 1.32       | -                         | 9.67                          |
| 5        | 1         | 1.313      | 4                         | 9.65                          |
| 5        | 1         | 1.32       | -                         | 9.75                          |
| 5        | 2         | 1.32       | -                         | 9.73                          |
| 6        | 1         | 1.32       | -                         | 9.75<br>ELASTIC               |

TABLE (6.4) BEHAVIOUR OF FRAME EXAMPLE 2 WHEN  $\alpha=1.32$  BELOW SHAKEDOWN LOAD  
 '- 'INDICATES NO PLASTIC HINGE WAS FORMED AT THAT LOAD LEVEL

| CYCLE NO | LOAD CASE | LOAD LEVEL | POSITION OF PLASTIC HINGE | DISPLACEMENT AT TOP COLUMN<br>cm |
|----------|-----------|------------|---------------------------|----------------------------------|
| 1        | 1         | 1.235      | 4                         | 6.55                             |
| 1        | 1         | 1.285      | 3                         | 7.26                             |
| 1        | 1         | 1.35       | 5                         | 7.85                             |
| 1        | 1         | 1.42       | -                         | 17.81                            |
| 1        | 2         | 1.157      | 1                         | 16.41                            |
| 1        | 2         | 1.42       | -                         | 18.6                             |
| 2        | 1         | 1.314      | 5                         | 18.1                             |
| 2        | 1         | 1.377      | 4                         | 18.73                            |
| 2        | 1         | 1.406      | 3                         | 19.36                            |
| 2        | 1         | 1.42       | -                         | 20.54                            |
| 2        | 2         | 1.13       | 1                         | 18.98                            |
| 2        | 2         | 1.42       | -                         | 21.4                             |
| 3        | 1         | 1.321      | 5                         | 20.89                            |
| 3        | 1         | 1.37       | 4                         | 21.42                            |
| 3        | 1         | 1.406      | 3                         | 22.21                            |
| 3        | 1         | 1.42       | -                         | 23.28                            |
| 3        | 2         | 1.13       | 1                         | 21.83                            |
| 3        | 2         | 1.42       | -                         | 24.25                            |
| 4        | 1         | 1.32       | 5                         | 23.74                            |
| 4        | 1         | 1.37       | 4                         | 24.34                            |
| 4        | 1         | 1.406      | 3                         | 24.97                            |
| 4        | 1         | 1.42       | -                         | 26.14                            |
| 4        | 2         | 1.13       | 1                         | 24.63                            |
| 4        | 2         | 1.42       | -                         | 27.00                            |

TABLE (6.5) BEHAVIOUR OF FRAME EXAMPLE 2 WHEN  $\alpha=1.32$  ABOVE SHAKEDOWN LOAD

| LOAD LEVEL | LOCATION OF FIRST<br>OCCURANCE OF<br>PLASTIC HINGE | LOCATION OF ALL<br>PLASTIC HINGES<br>APPEARING IN<br>STRUCTURE | DISPLACEMENT<br>cm |
|------------|--|--|--------------------|
| 2.40       | 9  | 9  | 5.86               |
| 2.42       | 12   | 9, 12  | 5.94               |
| 2.56       | 6  | 9, 12, 6   | 6.74               |
| 2.76       | 26   | 9, 12, 6, 26   | 8.40               |
| 2.84       | 3  | 9, 12, 6, 26, 3  | 9.16               |
| 3.00       | 19   | 9, 12, 6, 26, 3, 19  | 11.32              |
| 3.10       | 2  | 9, 12, 6, 26, 3, 19, 2   | 13.94              |
| 3.14       | 5  | 9, 12, 6, 26, 3, 19, 2, 5                                      | 15.55              |
| 3.18       | 8  | 9, 12, 6, 26, 3, 19, 2, 5, 8                                   | 18.45              |
| 3.20       | 11   | 9, 12, 6, 26, 3, 19, 2, 5, 8, 11                               | 21.90              |
| FAILURE    |  |  |                    |

TABLE 6.6 PLATIC HINGE FORMATION SEQUENCE OF ONE BAY FOUR STOREY STRUCTURE.

| CROSS<br>SECTIONS<br>LOADING CASE | 1     | 2     | 3      | 4     | 5    | 6      | 7     | 8    | 9      |
|-----------------------------------|-------|-------|--------|-------|------|--------|-------|------|--------|
| 1                                 | -59.4 | 104.4 | -115.4 | -41.7 | 92.8 | -136.7 | -1.2  | 89.8 | -164.3 |
| 2                                 | 27.5  | 0     | -27.5  | 46.75 | 0    | -46.75 | 80.3  | 0    | -80.3  |
| 3                                 | -87.4 | 104.4 | -87.4  | -89   | 92.8 | -89    | -82.7 | 89.7 | -82.7  |
| $M_{max}$                         | 27.5  | 104.4 | -27.5  | 46.75 | 92.8 | -46.75 | 80.3  | 89.8 | -80.3  |
| $M_{min}$                         | -87.4 | 0     | -115.4 | -89   | 0    | -136.7 | -82.7 | 0    | -164.3 |
| $M_{max}-M_{min}$                 | 114.9 | 104.4 | 87.9   | 165.8 | 92.8 | 89.95  | 163   | 89.8 | 84     |

| CROSS<br>SECTIONS<br>LOADING CASE | 10    | 11   | 12     | 13    | 14   | 15   | 16    | 17    | 18    |
|-----------------------------------|-------|------|--------|-------|------|------|-------|-------|-------|
| 1                                 | 7     | 85.4 | -161.7 | 58.6  | -17  | 19.6 | -18.4 | -13.3 | 6.4   |
| 2                                 | 83.2  | 0    | -83.2  | 4.2   | -51  | -20  | -60.2 | -56.7 | -26.5 |
| 3                                 | -77.4 | 85.3 | -77.4  | 54.29 | 34.8 | 39.8 | 42.8  | 44.5  | 32.9  |
| $M_{max}$                         | 83.2  | 85.4 | -77.4  | 58.6  | 34.8 | 39.8 | 42.8  | 44.5  | 32.9  |
| $M_{min}$                         | -77.4 | 0    | -161.7 | 4.2   | -51  | -20  | -60.2 | -56.7 | -26.5 |
| $M_{max}-M_{min}$                 | 160.6 | 85.4 | 85.4   | 54.4  | 85.8 | 59.8 | 103   | 101   | 59.4  |

| CROSS<br>SECTIONS<br>LOADING CASE | 19     | 20    | 21   | 22   | 23    | 24    | 25   | 26    |
|-----------------------------------|--------|-------|------|------|-------|-------|------|-------|
| 1                                 | -124.6 | 140.5 | 86.2 | 59.9 | 104.4 | 103.4 | 58.1 | 156.7 |
| 2                                 | -140   | -3.8  | 50.6 | 19.7 | 60.6  | 57.8  | 25.2 | 138   |
| 3                                 | 17     | 54.3  | 34.7 | 39.8 | 42.84 | 44.5  | 32.9 | 16.7  |
| $M_{max}$                         | 18.7   | 54.3  | 86.2 | 59.9 | 104.4 | 103.4 | 58.1 | 156.7 |
| $M_{min}$                         | -140   | -3.8  | 34.7 | 19.7 | 42.8  | 44.5  | 25.2 | 16.7  |
| $M_{max}-M_{min}$                 | 157    | 58    | 51.5 | 40.2 | 61.6  | 58.7  | 32.9 | 121.3 |

TABLE (6.7) ELASTIC BENDING MOMENT OF FOUR STOREY  
ONE BAY STRUCTURE

| CYCLE<br>NO | LOAD<br>CASE | LOAD<br>LEVEL | POSITION OF<br>PLASTIC HINGE | DISPLACEMENT<br>AT TOP COLUMN<br>cm |
|-------------|--------------|---------------|------------------------------|-------------------------------------|
| 1           | 1            | 2.40          | 9                            | 5.86                                |
| 1           | 1            | 2.42          | 12                           | 5.94                                |
| 1           | 1            | 2.56          | 6                            | 6.74                                |
| 1           | 1            | 2.76          | 26                           | 8.40                                |
| 1           | 1            | 2.84          | 3                            | 9.15                                |
| 1           | 1            | 2.96          | -                            | 10.78                               |
| 1           | 2            | 2.96          | -                            | 10.76                               |
| 1           | 3            | 2.96          | -                            | 3.58                                |
| 2           | 1            | 2.88          | 26                           | 10.56                               |
| 2           | 1            | 2.94          | 6,9                          | 10.76                               |
| 2           | 1            | 2.96          | -                            | 10.90                               |
| 2           | 2            | 2.96          | -                            | 10.87                               |
| 2           | 3            | 2.96          | -                            | 3.70                                |
| 3           | 1            | 2.94          | 3,26                         | 10.85                               |
| 3           | 1            | 2.96          | -                            | 10.93                               |
| 3           | 2            | 2.96          | -                            | 10.91                               |
| 3           | 3            | 2.96          | -                            | 3.73                                |
| 4           | 1            | 2.96          | -                            | 10.93<br>ELASTIC                    |

TABLE (6.8) BEHAVIOUR OF FRAME EXAMPLE 3 WHEN  $\alpha=2.96$   
 '- 'INDICATES NO PLASTIC HINGE WAS FORMED AT THAT LOAD LEVEL



| CYCLE NO | LOAD CASE | LOAD LEVEL | POSITION OF PLASTIC HINGE | DISPLACEMENT AT TOP COLUMN<br>cm |
|----------|-----------|------------|---------------------------|----------------------------------|
| 1        | 1         | 2.40       | 9                         | 5.86                             |
| 1        | 1         | 2.42       | 12                        | 5.94                             |
| 1        | 1         | 2.56       | 6                         | 6.74                             |
| 1        | 1         | 2.76       | 26                        | 8.4                              |
| 1        | 1         | 2.84       | 3                         | 9.15                             |
| 1        | 1         | 3          | -                         | 11.32                            |
| 1        | 2         | 3          | -                         | 11.3                             |
| 1        | 0         |            |                           |                                  |
| 1        | 3         | 3          | -                         | 4                                |
| 2        | 1         | 2.88       | 26                        | 11.13                            |
| 2        | 1         | 2.94       | 19                        | 11.18                            |
| 2        | 1         | 2.98       | 6,9,12                    | 11.38                            |
| 2        | 1         | 3          | -                         | 11.78                            |
| 2        | 2         | 3          | -                         | 11.75                            |
| 2        | 3         | 3          | -                         | 4.77                             |
| 3        | 1         | 2.96       | 26                        | 11.7                             |
| 3        | 1         | 2.98       | 3,19                      | 11.74                            |
| 3        | 1         | 3          | -                         | 11.86                            |
| 3        | 2         | 3          | -                         | 11.84                            |
| 3        | 3         | 3          | -                         | 4.59                             |
| 4        | 1         | 2.98       | 12                        | 11.82                            |
| 4        | 1         | 3          | -                         | 1.88                             |
| 4        | 2         | 3          | -                         | 11.86                            |
| 4        | 3         | 3          | -                         | 4.58                             |

TABLE (6.9) BEHAVIOUR OF FRAME EXAMPLE 3 WHEN  $\alpha=3$

| CYCLE NO | LOAD CASE | LOAD LEVEL | POSITION OF PLASTIC HINGE | DISPLACEMENT AT TOP COLUMN<br>cm |
|----------|-----------|------------|---------------------------|----------------------------------|
| 5        | 1         | 2.98       | 19,26                     | 11.83                            |
| 5        | 1         | 3          | -                         | 11.93                            |
| 5        | 2         | 3          | -                         | 11.91                            |
| 5        | 3         | 3          | -                         | 4.63                             |
| 6        | 1         | 2.98       | 9                         | 11.88                            |
| 6        | 1         | 3          | -                         | 11.96                            |
| 6        | 2         | 3          | -                         | 11.94                            |
| 6        | 3         | 3          | -                         | 4.66                             |
| 7        | 1         | 3          | -                         | 11.96                            |
| 7        | 2         | 3          | -                         | 11.94                            |
| 7        | 3         | 3          | -                         | 4.66<br>ELASTIC                  |

TABLE (6.9) CONTINUE

| VL/MP | HL/MP | LOAD<br>LEVEL | SWAY DISPLACEMENT (cm) |        |        |        |
|-------|-------|---------------|------------------------|--------|--------|--------|
| 2.9   | 2.9   | 0.2           | RIGID                  | K1     | K2     | K3     |
|       |       | 0.4           | 0.22                   | 0.22   | 0.25   | 0.36   |
|       |       | 0.6           | 0.435                  | 0.44   | 0.49   | 0.72   |
|       |       | 0.8           | 0.652                  | 0.66   | 0.74   | 1.08   |
|       |       | 1             | 0.88                   | 0.88   | 0.97   | 1.44   |
| 0     | 0     | 0.2           | 1.418                  | 1.42   | 1.402  | 3.74   |
|       |       | 0.4           | 0.33                   | 0.32   | 0.183  | 1.94   |
|       |       | 0.6           | 0.115                  | 0.1    | -0.059 | 1.581  |
|       |       | 0.8           | -0.1                   | -0.1   | -0.3   | 1.22   |
|       |       | 1             | -0.315                 | -0.316 | -0.54  | 0.86   |
| 0     | -2.9  | 0.2           | -0.53                  | -0.53  | -0.78  | -0.64  |
|       |       | 0.4           | -0.816                 | -0.916 | -1.17  | -3.73  |
|       |       | 0.6           | 0.26                   | 0.49   | 0.0405 | -1.94  |
|       |       | 0.8           | 0.47                   | 0.705  | 0.29   | -1.579 |
|       |       | 1             | 0.71                   | 0.93   | 0.53   | -1.219 |
| 2.9   | 2.9   | 0.2           | 0.91                   | 1.145  | 0.77   | -0.86  |
|       |       | 0.4           | 1.3                    | 1.366  | 1.02   | 0.668  |
|       |       | 0.6           | 1.418                  | 1.586  | 1.402  | 3.74   |
|       |       | 0.8           |                        |        |        |        |
|       |       | 1             |                        |        |        |        |

TABLE (6.10) CYCLIC LOADING ON A SINGLE STOREY ONE BAY FRAME  
WITH THE SEMI-RIGID CONNECTIONS

| LOAD CASE | LOAD LEVEL | SWAY AT TOP OF THE COLUMN cm |       |
|-----------|------------|------------------------------|-------|
|           |            | K1                           | K2    |
| 1         | 2          | 4.91                         | 5.16  |
|           | 2.2        | 5.4                          | 5.67  |
|           | 2.4        | 5.9                          | 6.19  |
|           | 2.6        | 6.4                          | 6.71  |
|           | 2.8        | 6.9                          | 7.22  |
|           | 3          | 7.5                          | 7.9   |
| 2         | 2          | 5                            | 5.3   |
|           | 2.2        | 5.5                          | 5.8   |
|           | 2.4        | 6                            | 6.3   |
|           | 2.6        | 6.5                          | 6.8   |
|           | 2.8        | 7                            | 7.3   |
|           | 3          | 7.5                          | 7.8   |
| 3         | 2          | 0.107                        | 0.13  |
|           | 2.2        | 0.109                        | 0.133 |
|           | 2.4        | 0.11                         | 0.135 |
|           | 2.6        | 0.112                        | 0.136 |
|           | 2.8        | 0.114                        | 0.138 |
|           | 3          | 0.116                        | 0.140 |
| 4         | 2          | 5                            | 5.3   |
|           | 2.2        | 5.5                          | 5.8   |
|           | 2.4        | 6                            | 6.3   |
|           | 2.6        | 6.5                          | 6.8   |
|           | 2.8        | 7                            | 7.3   |
|           | 3          | 7.5                          | 7.9   |

TABLE (6.11) TOP STOREY SWAY OF FOUR STOREY SINGLE BAY FRAME WITH THE DIFFERENT CONNECTIONS STIFFNESS UNDER CYCLIC LOADING

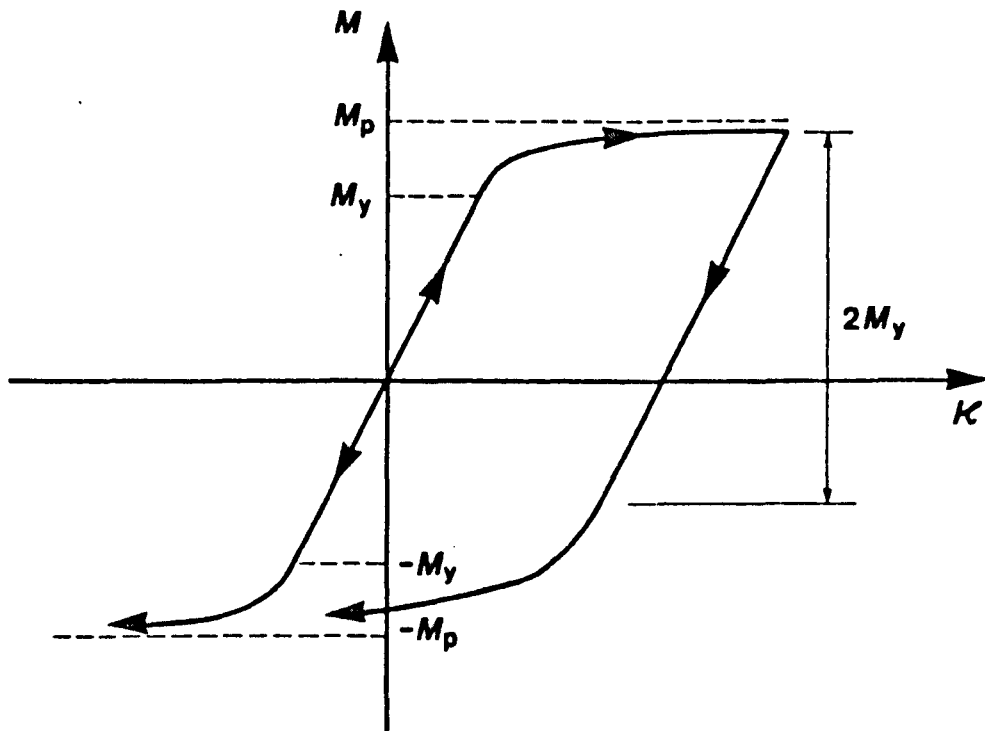


FIG. (6.1) BENDING MOMENT-CURVATURE RELATION ASSUMED FOR SHAKEDOWN THEOREM

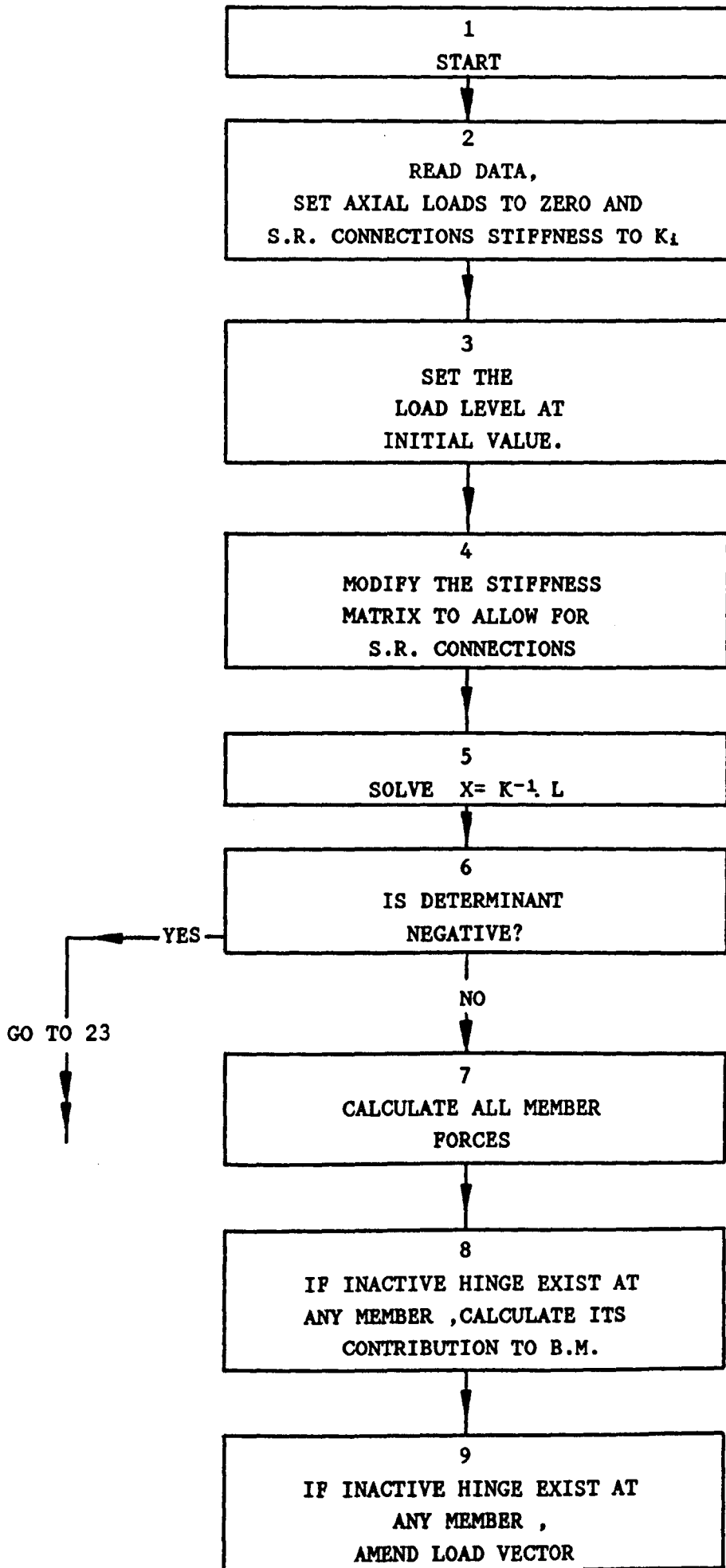


FIG. (6.2) FLOW CHART OF PROGRAM

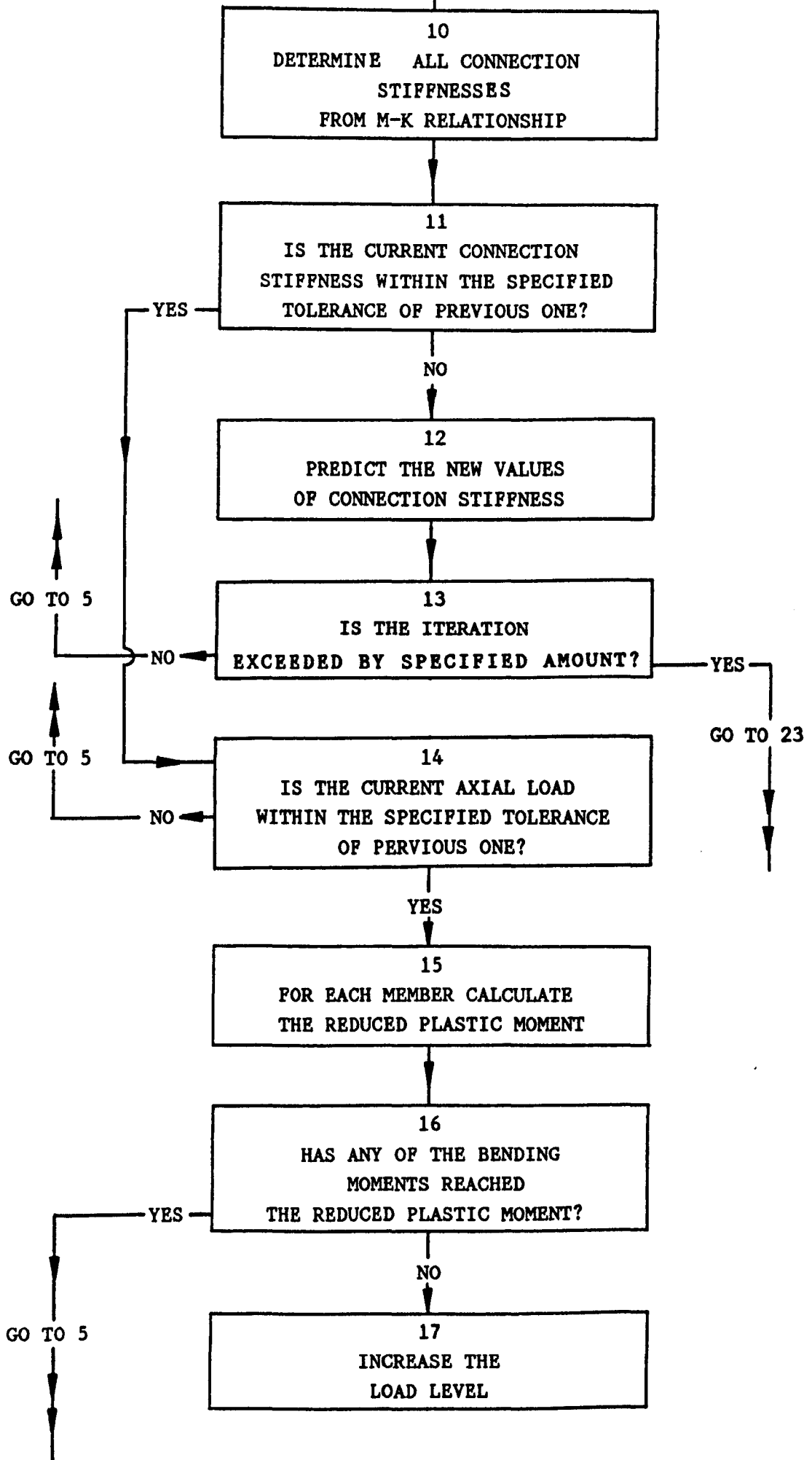


FIG. (6.2) CONTINUED

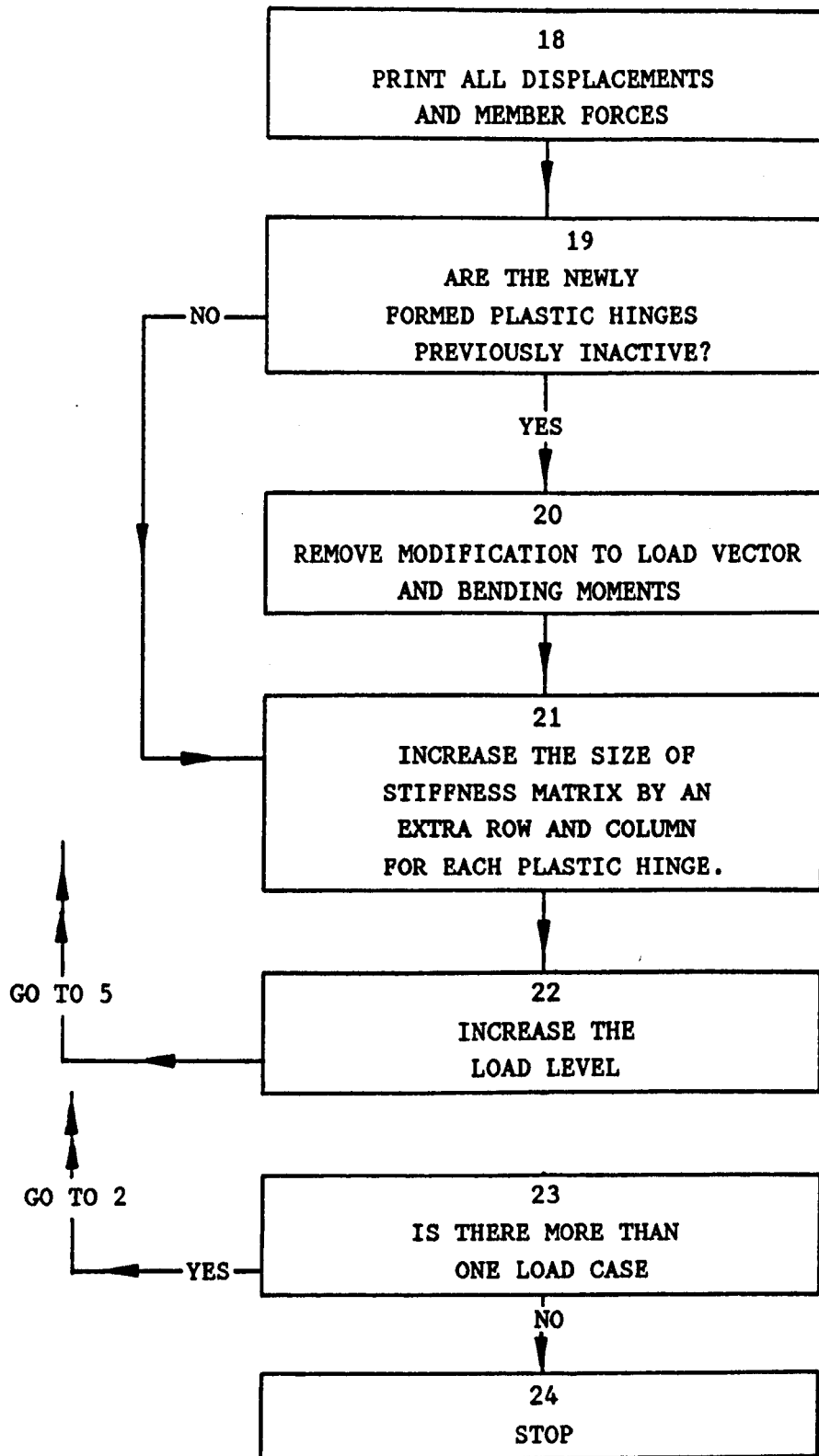


FIG. (6.2) CONTINUED



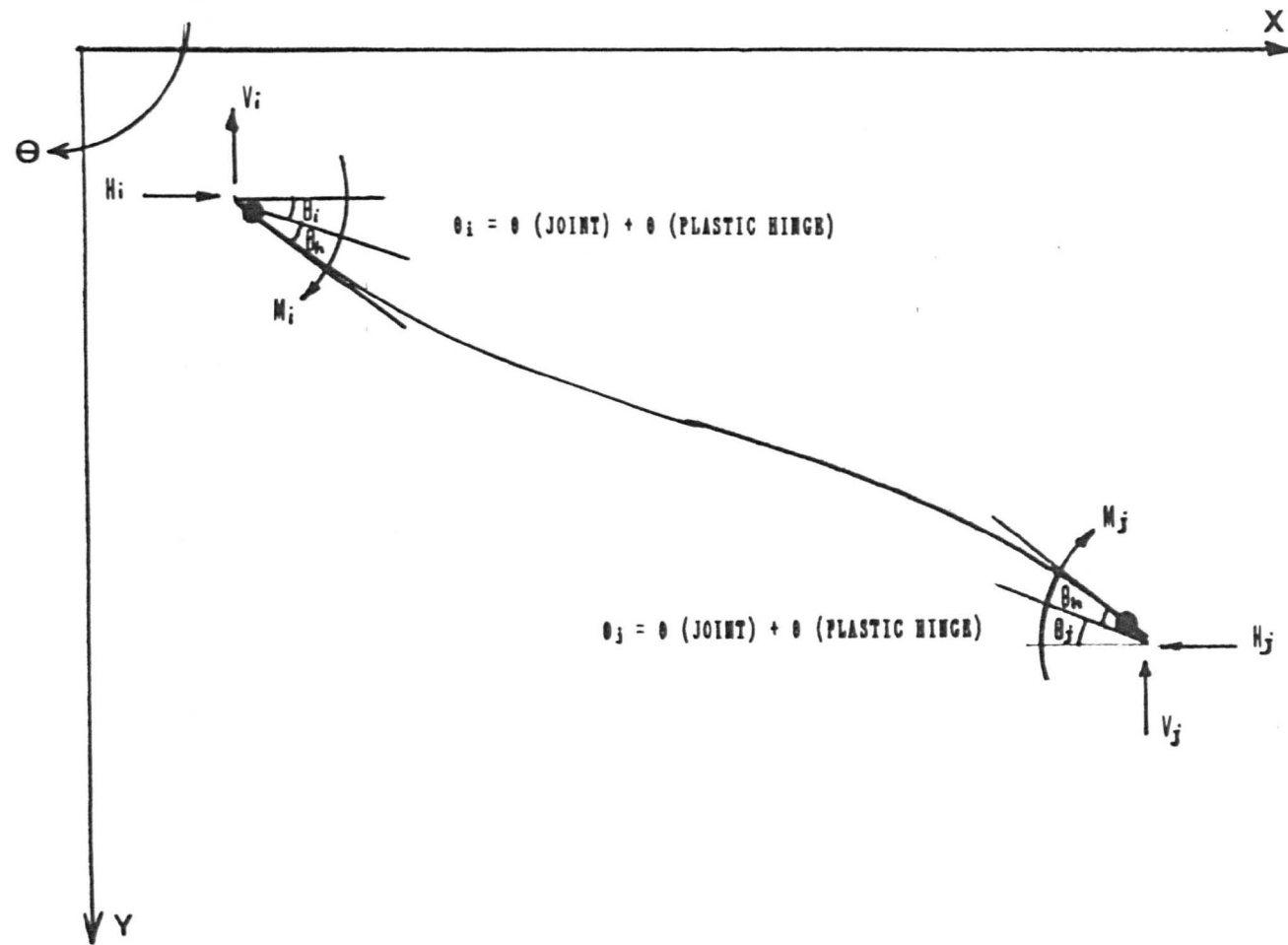
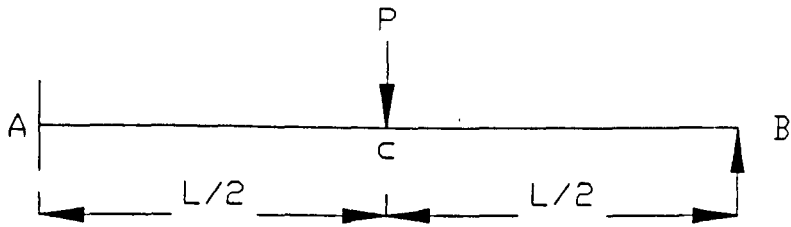


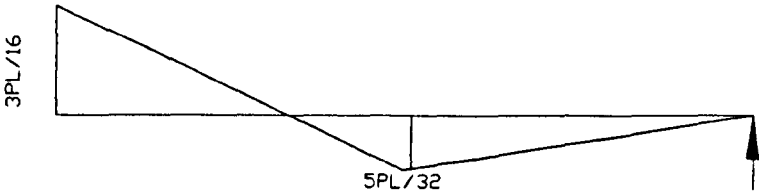
FIG. (6.3) FORMATION OF PLASTIC HINGE AT MEMBER i-j

$$\begin{bmatrix} H_i \\ V_j \\ M_i \\ M_{phi_i} \\ M_{phi_j} \\ H_j \\ V_j \\ M_j \end{bmatrix} = \begin{bmatrix} ac^2+bs^2 & & & & & & & & \\ asc-bac & as^2+bc^2 & & & & & & & \\ ds & -dc & e & & & & & & \\ ds & -dc & e & e & & & & & \\ ds & -dc & f & f & e & & & & \\ -ac^2-bc^2 & -asc+bsc & -ds & -ds & -ds & ac^2+bs^2 & & & \\ -asc+bsc & -as^2-bc^2 & dc & dc & dc & asc-bac & as^2+bc^2 & & \\ ds & dc & f & f & e & -ds & -dc & e & \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ \theta_i \\ \theta_{hi} \\ \theta_{hj} \\ x_j \\ y_j \\ \theta_j \end{bmatrix}$$

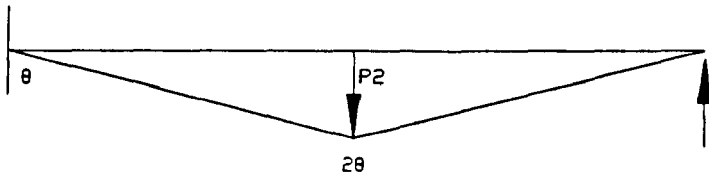
FIG (6.4) CONTRIBUTION OF PLASTIC HINGES TO MEMBER i-j IN THE OVERALL STIFFNESS MATRIX.



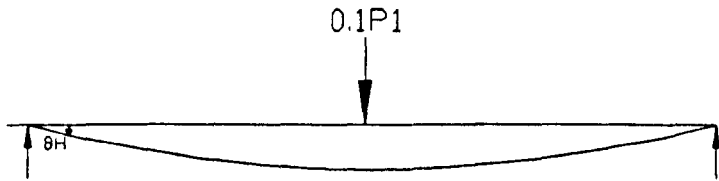
a) PROPPED CANTILEVER



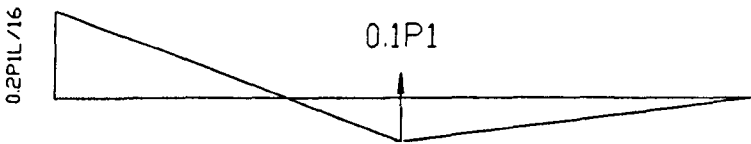
b) B.M. AT LOAD  $P_1$



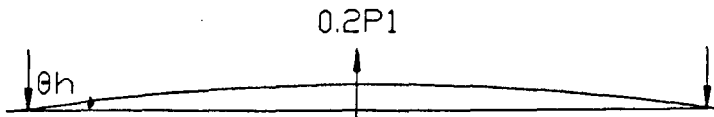
c) COLLAPSE MECHANISM



d) HINGE ROTATION AT LOAD  $1.1P_1$



e) ELASTIC BENDING MOMENT AT  $-0.1P_1$



f) HINGE ROTATION AT LOAD  $-1.1P_1$

FIG. (6.5) ANALYSIS OF PROPPED CANTILEVER

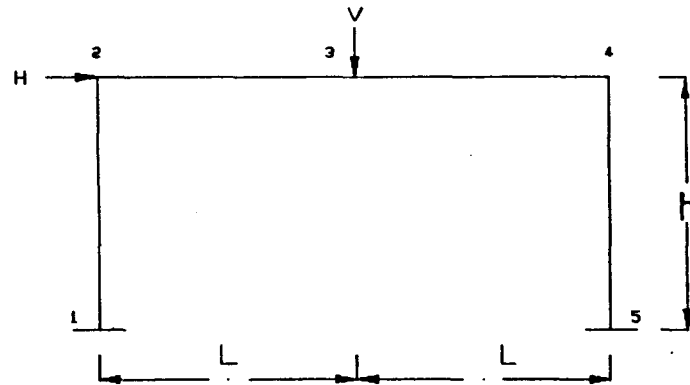


FIG. (6.6) FRAME GEOMETRY AND LOADING OF EXAMPLE 1

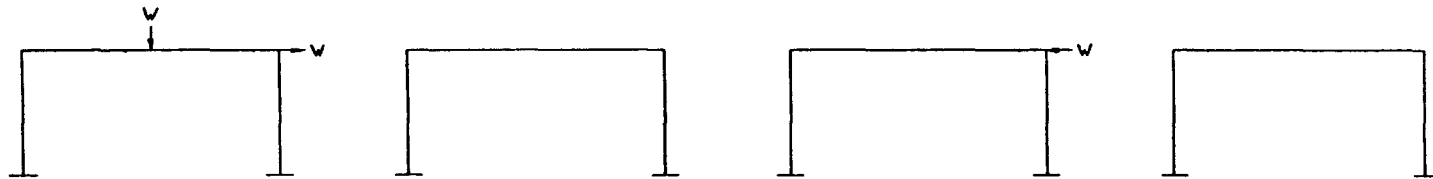


FIG. (6.7) CYCLE OF LOADING WHICH MAY CAUSE ALTERNATING PLASTICITY

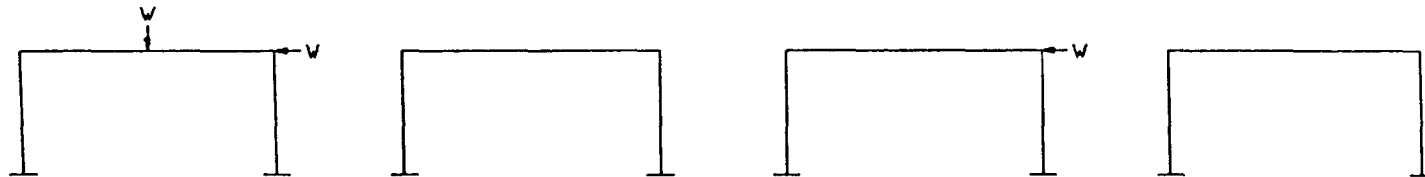


FIG. (6.8) CYCLES OF LOADING WHICH MAY CAUSE INCREMENTAL COLLAPSE

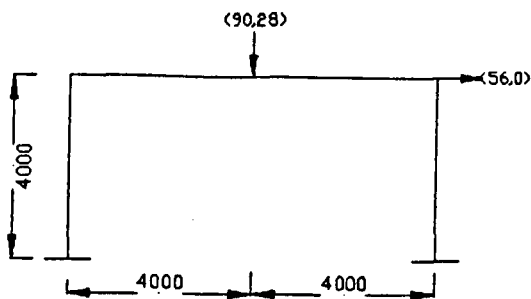


FIG. (6.9) FRAME GEOMETRY AND LOADING OF EXAMPLE 2

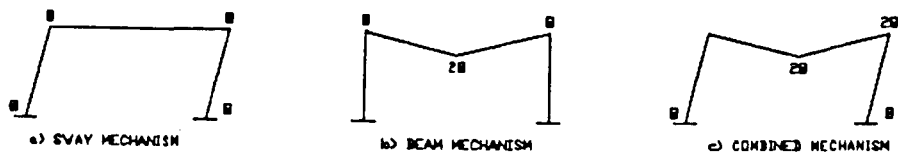


FIG. (6.10) POSSIBLE INCREMENTAL MECHANISM

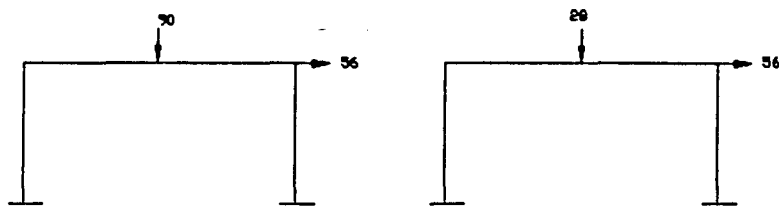


FIG. (6.11) CYCLES OF LOADING WHICH MAY CAUSE INCREMENTAL COLLAPSE

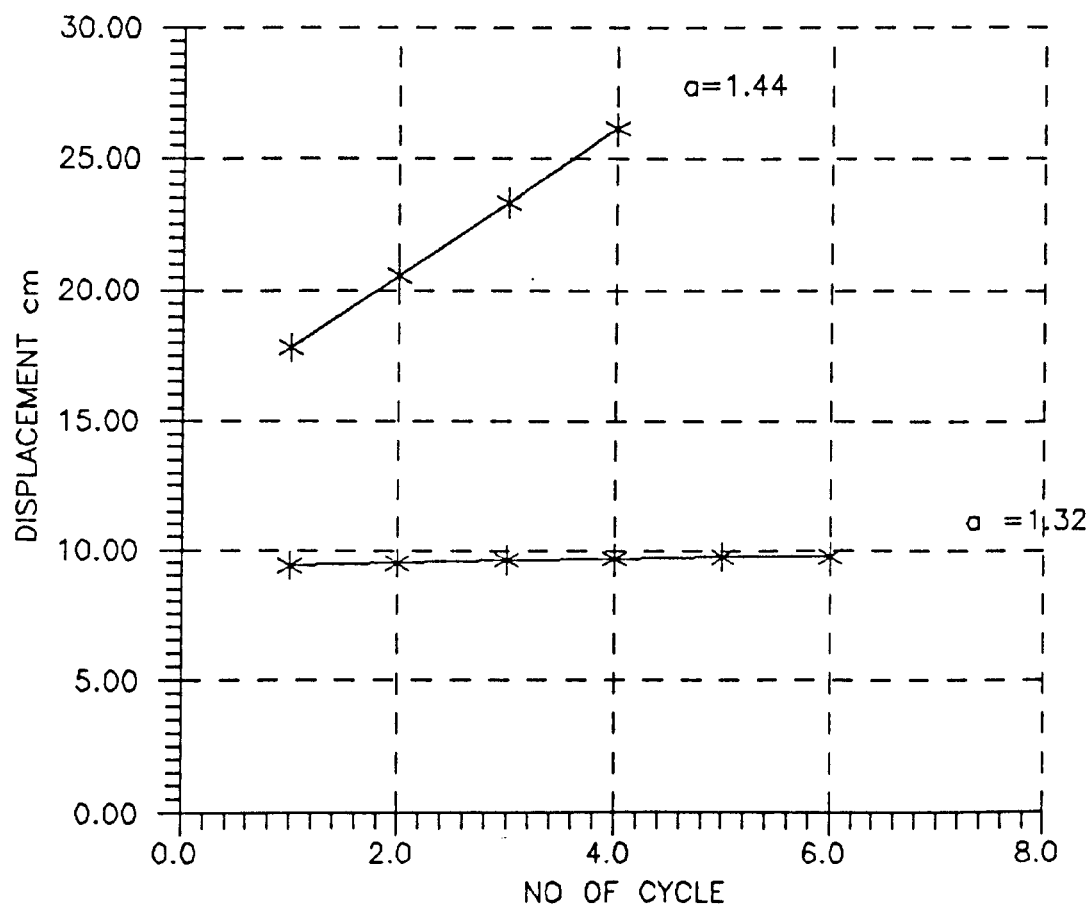
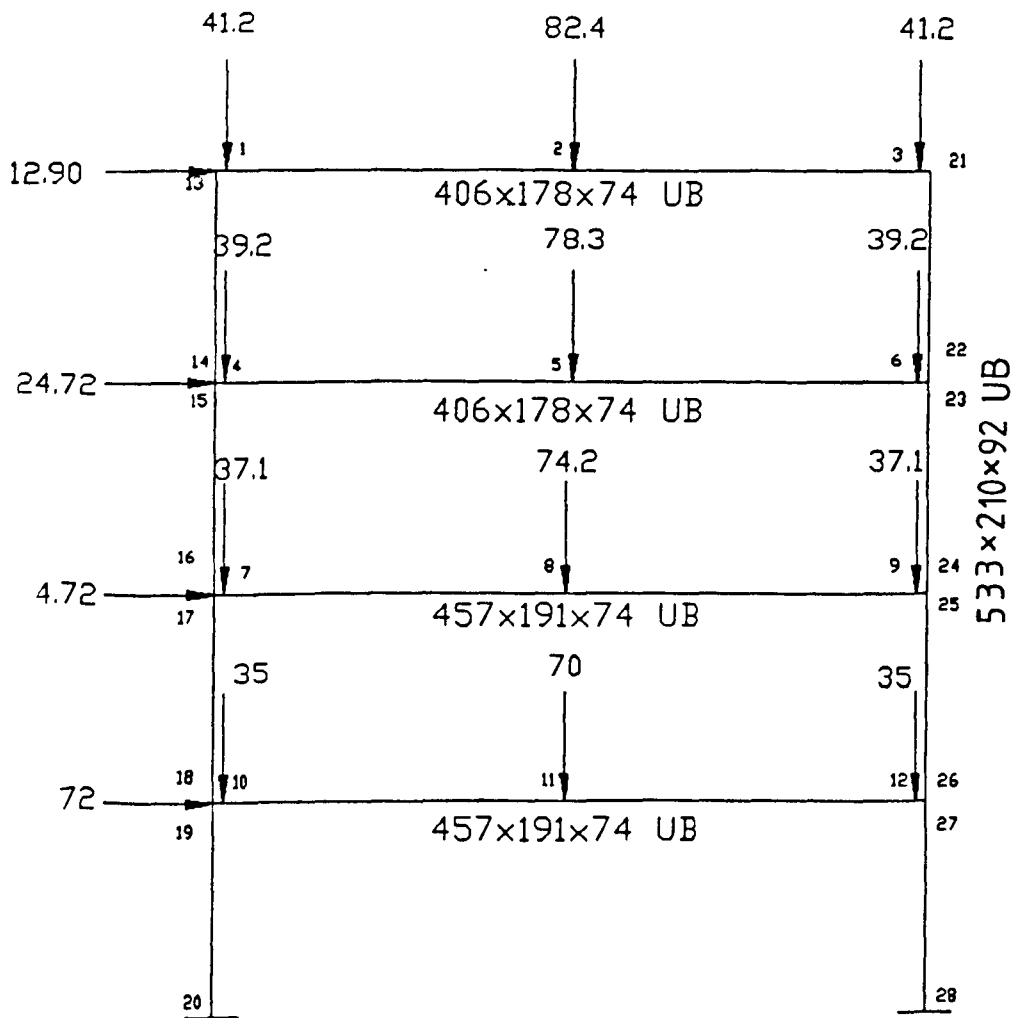


FIG. (6.12) EFFECT OF CYCLIC LOADING ON DEFLECTION



G. (6.13) FOUR STOREY ONE BAY STRUCTURE





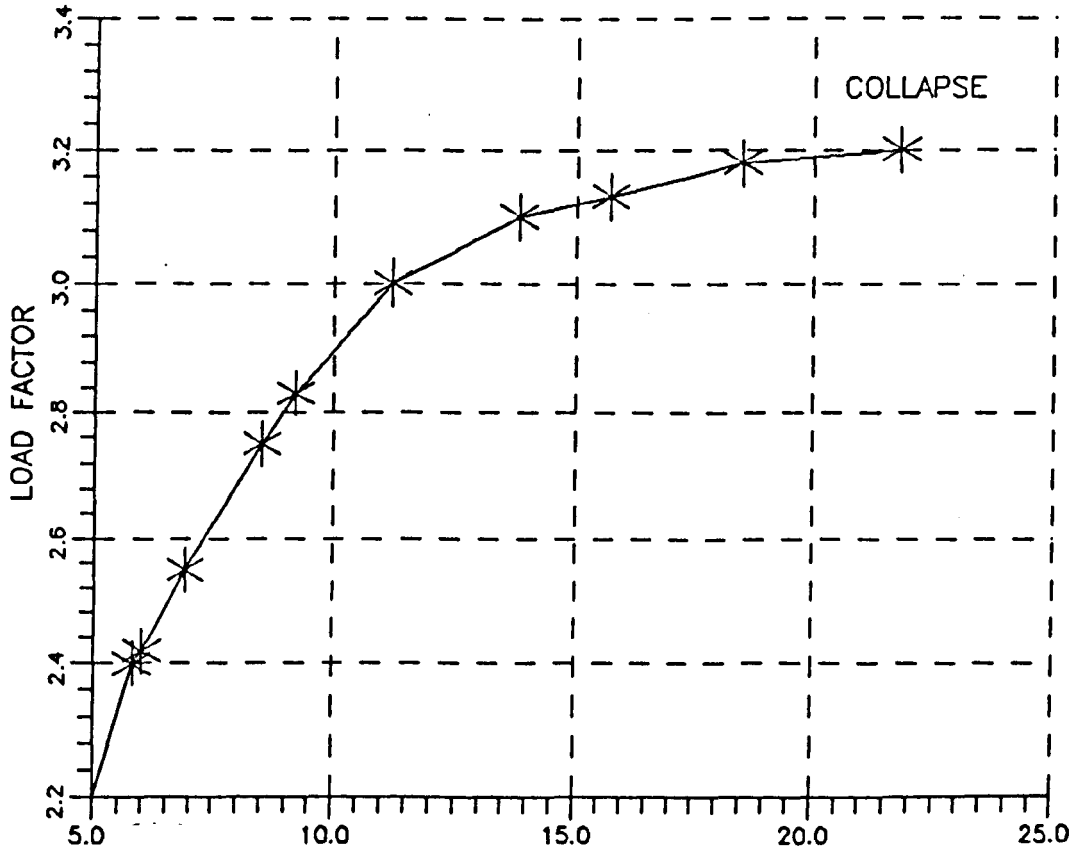
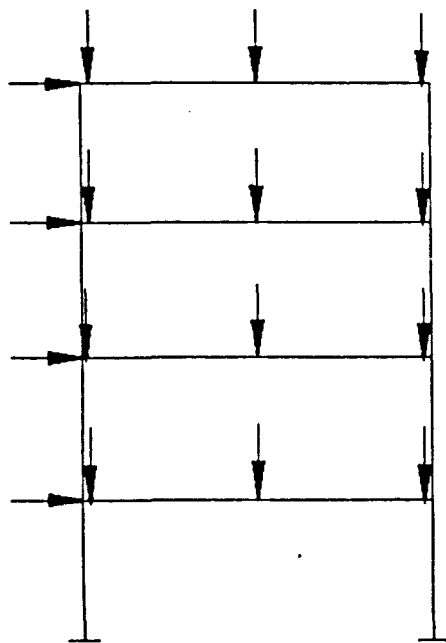
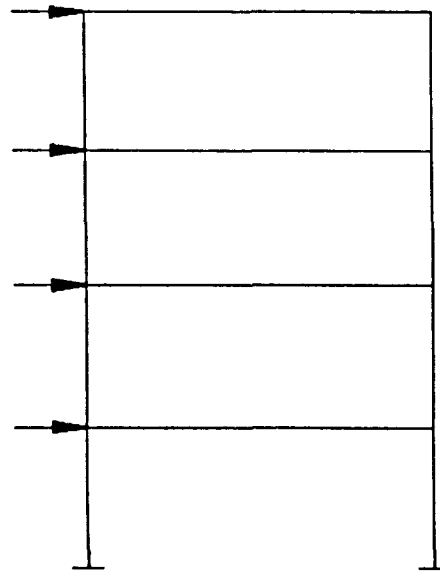


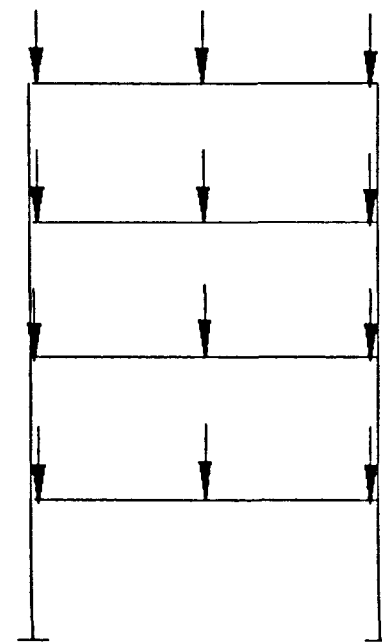
FIG. (6.15) LOAD DEFLECTION CURVE FOR FOUR STOREY SINGLE BAY STRUCTURE



CASE 1



CASE 2



CASE 3

FIG. (6.16) LOAD COMBINATION FOR SHAKEDOWN ANALYSIS  
OF FOUR STOREY SINGLE BAY STRUCTURE

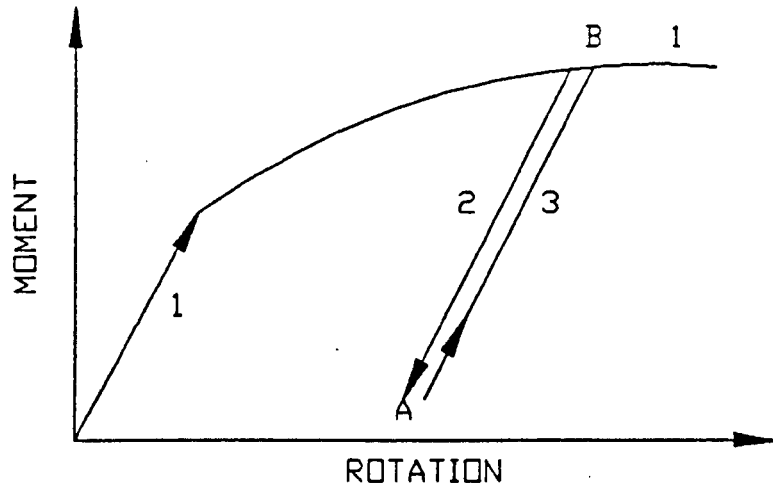


FIG. (6.17) BEHAVIOUR OF CONNECTIONS UNDER CYCLIC LOADING

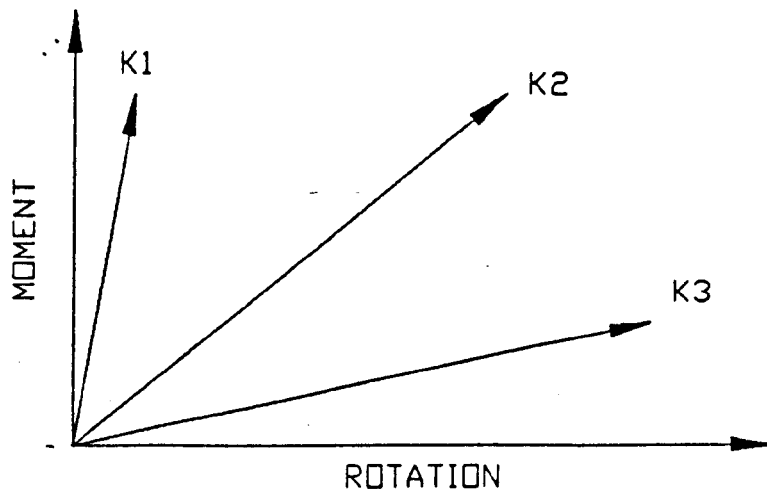


FIG. (6.18) LINEAR CONNECTIONS STIFFNESS ADOPTED FOR THE SINGLE BAY FOUR STOREY FRAME

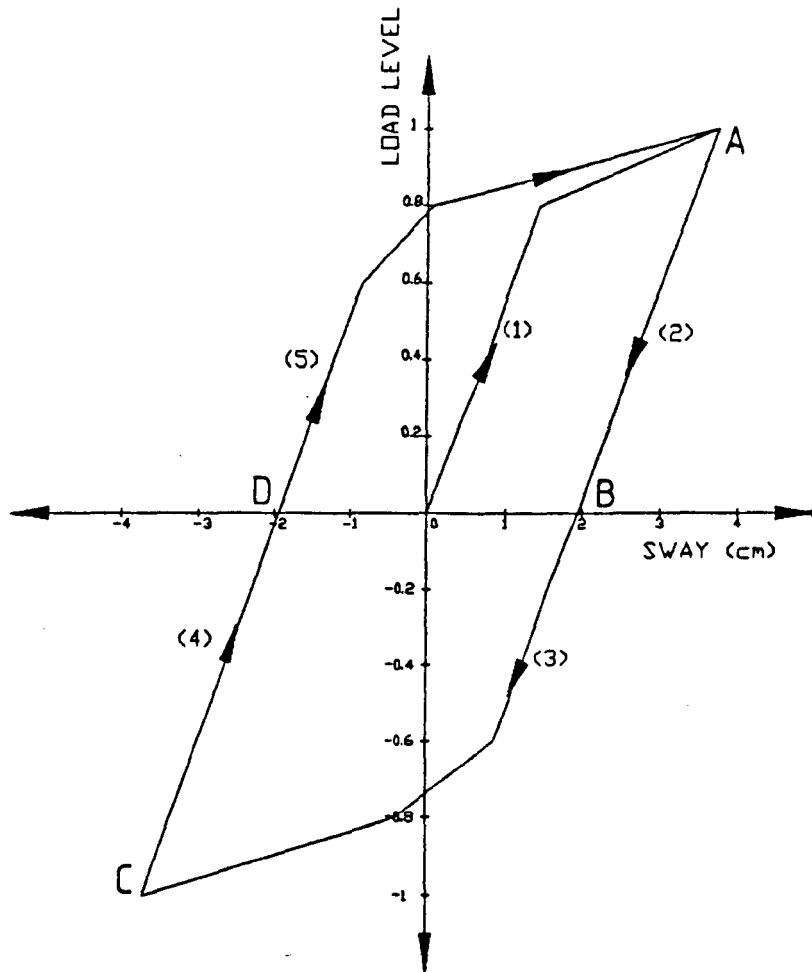


FIG. (6.19) LOAD DEFLECTION CURVES OF SINGLE STOREY SINGLE BAY FRAME

## CHAPTER 7

### CONCLUSION AND SUGGESTIONS FOR FURTHER WORK

The work described in this thesis has examined various aspects in the design and the analysis of unbraced steel frames with rigid or semi-rigid connections.

The approximate method presented in Chapter 2 is able to estimate the second order elasto-plastic failure load of single storey frames. This would enable the failure load to be determined on frames subjected to high concentrated loads at or near the knee joint, which can result in significant problems. The method is a good substitute for a lengthy manual calculation or "exact" second-order elasto-plastic computer analysis. In the same Chapter, two other alternative methods in recent design documents were investigated and conclusions were drawn. However, an investigation to develop an approximate method for the failure load of fixed base frames should be made. Furthermore, an attempt should be made to represent these simplified methods in non-dimensional forms to produce a design chart. This would enable the failure load to be determined swiftly for single storey frames.

In contrast to Chapter 2 which calculates the failure load by second-order analysis, in Chapter 3 parametric studies were carried out to determine the range of parameters for acceptable use of first-order

plastic theory. The purpose of these studies was to devise less conservative rules than those given in an ECCS Publication [66]. Therefore, it was proposed that first-order plastic theory can be used provided the limiting ratios of  $\alpha_{er}$ , the elastic critical load, to  $\alpha_p$ , the first-order plastic theory collapse factor given in Chapter 3, are satisfied. The limits are given for both pinned and fixed base single storey single bay frames up to a  $10^\circ$  pitch roof. A regular pattern for these limits was found for the frames with the pinned bases, but the results associated with the fixed base frames were somewhat scattered. Therefore, there is a need to carry out more analyses with regard to fixed base frames to avoid excessive conservatism.

A second-order elasto-plastic computer analysis program that can simulate the behaviour of unbraced steel frames that have flexible beam-to-column connections has been developed in Chapter 4. The program is capable of analysing any combination of pinned, semi-rigid and rigidly connected frames. A suitable method is developed for the convergence of internal forces due to non-linear moment-rotation characteristics. It was found that by using the proposed method convergence was achieved very rapidly even for very flexible connections.

By including semi-rigid joint behaviour, a better assessment could be made of the real behaviour of such frames, both at the serviceability and ultimate limit state. The true assessment of any design with semi-rigid connections depends on the modelling of its moment-rotation characteristic. This is because small changes in connection characteristics may generate significant changes in strength, stiffness or

stability characteristics in the framework. Therefore, there is a need for simple but reliable  $M-\phi$  curves to predict the generic performance of any sets of connections. More investigation should be made into the use of initial stiffness, bi-linear and tri-linear models. These assumptions simplify connection models and considerably reduces the computational time as well as simplifying the analytical procedure.

Based on the examples analysed with semi-rigid connections, lateral sway limits specified by codes of practice were violated even in very low frames. Therefore, the limits given in codes of practice should be relaxed to encourage the design of structures with the semi-rigid connections. It is possible to allow for stiffening elements such as cladding, heavy partitioning and other incidental infill material.

A particular type of semi-rigid design is known as Type (2) or the wind connection method. In Chapter 5 several frames were designed in accordance with this method. This type of construction has been used often by designers in Britain, but up until recently, there has not been a design document available to provide a degree of reliability of this design method. However, with the development of the "exact" computer program in Chapter 4, more realistic behaviour of this type of construction was examined. In particular, a forthcoming design document [110] will give the necessary rules and guarantee for the future use of the method.

The study in Chapter 5 only considered one type of connection, the extended end plate. It should be possible to design using the above

method for other types of connections, in particular minor axis connections. There is little of information known to the author concerning the accurate evaluation of effective length for minor axis buckling (except the recent work at the University of Warwick [42]. Therefore, the designer has to adopt empirical effective length factors associated with simple design or to provide rigid minor-axis connections. On the other hand, there is sufficient experimental and other information currently available on several major-axis connections, but this has not been classified. This should be classified into subset of low, medium and high moment performance with the corresponding stiffness performance.

Finally, the computer program described in Chapter 4 was further developed in Chapter 6 to examine the effect of cyclic loading on frames with rigid and semi-rigid connections. The general problem of variable repeated loading has been divided into the problem of incremental collapse and that of alternating plasticity. The first problem will be seriously aggravated by frame instability and will lead to complete failure. The second problem will cause localised damage from reversing wind load without significant variation in vertical load.

As a result of the investigation of a small number of structures, it appears that the problems associated with the above phenomena will not take place at the load significantly below the elastic plastic failure load. It is also observed that the connections gradually shakedown, i.e., incremental deformation ceases. Finally as indicated by previous tests results, for the connection to fail under alternating plasticity



a large number of cycles of loading application are required. More analysis should be carried out, though, before any general conclusions can be drawn. Further analysis should include more flexible connections in which loading and unloading characteristics of the connections may have a significant effect on frame behaviour.

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APPENDIX

| REF     | CALCULATION   |
|---------|---|
|         | <u>SEVEN STOREY FOUR BAY UNBRACED FRAME.</u>        |
|         | FRAME SPACED AT 4m CENTRES LONGITUDINALLY           |
|         | LOADING   |
|         | DEAD ON ROOF 3.75 kN/m <sup>2</sup>                 |
|         | IMPOSED ON ROOF 1.50 kN/m <sup>2</sup>              |
|         | DEAD ON FLOOR 4.80 kN/m <sup>2</sup>                |
|         | IMPOSED ON FLOOR 5.0 kN/m <sup>2</sup>              |
|         | WIND LOAD   |
| CP3     |   |
| CHAPTER | BASIC WIND SPEED                                    |
| V       | S1 TOPOGRAPHY FACTOR 1                              |
|         | S2 FOR H=26.25 AND 0.813                            |
|         | GROUND ROUGHNESS 3                                  |
|         | BUILDING CLASS B                                    |
|         | S3 STATICAL FACTOR 1                                |
|         | $V_S=1 \times 0.813 \times 1 \times 38=$ 30.89 m/s  |
|         | $q=0.613 \times (30.89)^2=$ 0.585 kN/m <sup>2</sup> |
|         | WIND FORCE AT EACH LEVEL                            |
|         | $F= 0.585 \times 3.75 \times 4=$ 9.35 kN            |

STRUCTURAL  
STEEL DESIGN

| REF    | CALCULATION                                   |          |
|--------|---|----------|
|        | <u>DEAD+IMPOSED LOADING</u>                   |          |
|        | <u>ROOF</u>                                   |          |
|        | DEAD  |          |
|        | $1.4 \times 3.75 \times 5 \times 4 =$         | 105 kN   |
|        | IMPOSED                                       |          |
|        | $1.6 \times 1.5 \times 5 \times 4 =$          | 48 kN    |
|        | <u>FLOOR</u>                                  |          |
|        | DEAD  |          |
|        | $1.4 \times 4.80 \times 5 \times 4 =$         | 134.4 kN |
|        | IMPOSED                                       |          |
|        | $1.6 \times 5 \times 5 \times 4 =$            | 160 kN   |
|        | <u>COLUMNS AXIAL LOAD DUE TO DEAD+IMPOSED</u> |          |
|        | 6th STOREY OUTER COLUMN                       |          |
|        | DEAD FROM ROOF $105/2 =$                      | 52.5 +   |
|        | DEAD FROM FLOOR $134.4/2$                     | 67.2     |
|        |   | -----    |
|        |   | 119.7 kN |
|        | IMPOSED FROM ROOF $48/2 =$                    | 24 +     |
|        | IMPOSED FROM FLOOR $160/2 =$                  | 80       |
|        |   | -----    |
|        |   | 104 kN   |
| BS6399 | 10% REDUCTION ON IMPOSED LOAD                 |          |
| PART 1 |   |          |
| 1984   | $F_{C06} = 119.7 + 0.9 \times 104 =$          | 213.3 kN |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION  |
|-----|--|
|     | 6th STOREY INNER COLUMN                            |
|     | DEAD FROM ROOF 105 +                               |
|     | DEAD FROM FLOOR 134.4                              |
|     | -----  |
|     | 239.4 kN   |
|     | IMPOSED FROM ROOF 48 +                             |
|     | IMPOSED FROM FLOOR 160                             |
|     | -----  |
|     | 208 kN   |
|     | 10% REDUCTION ON IMPOSED LOAD                      |
|     | $F_{CI6} = 239.4 + 0.9 \times 208 =$ 426.6 kN      |
|     | $F_{CO} =$ APPLIED AXIAL LOAD AT THE OUTER COLUMNS |
|     | $F_{CI} =$ APPLIED AXIAL LOAD AT THE INNER COLUMNS |
|     | 4th STOREY OUTER COLUMN                            |
|     | DEAD FROM ROOF $105/2 =$ 52.5 +                    |
|     | DEAD FROM 3 FLOOR $3 \times 134.4/2$ 201.6         |
|     | -----  |
|     | 254.1 kN   |
|     | IMPOSED FROM ROOF $48/2 =$ 24 +                    |
|     | IMPOSED FROM 3 FLOOR $3 \times 160/2 =$ 240        |
|     | -----  |
|     | 264 kN   |
|     | 30% REDUCTION ON IMPOSED LOAD                      |
|     | $F_{CO4} = 254.1 + 0.7 \times 264 =$ 438.9 kN      |

STRUCTURAL  
STEEL DESIGN

| REF    | CALCULATION  |
|--------|--|
|        | 4th STOREY INNER COLUMN                            |
|        | DEAD FROM ROOF    105=                      105 +  |
|        | DEAD FROM 3 FLOOR 3x134.4=                403.2    |
|        | -----  |
|        | 508.2 kN   |
|        | IMPOSED FROM ROOF 48=                    48 +      |
|        | IMPOSED FROM 3 FLOOR 3x160=               480      |
|        | -----  |
|        | 528 kN   |
|        | 30% REDUCTION ON IMPOSED LOAD                      |
|        | $F_{CI4}=508.2+0.7 \times 528=$ 877.8 kN           |
|        | 1st STOREY OUTER COLUMN                            |
|        | DEAD FROM ROOF    105/2=                    52.5 + |
|        | DEAD FROM 6 FLOOR 6x134.4/2               403.2    |
|        | -----  |
| BS6399 | 455.7 kN   |
| PART 1 |  |
| 1984   | IMPOSED FROM ROOF 48/2=                    24 +    |
|        | IMPOSED FROM 6 FLOOR 6x160/2=            480       |
|        | -----  |
|        | 504 kN   |
|        | 40% REDUCTION ON IMPOSED LOAD                      |
|        | $F_{CO1}=455.7+0.6 \times 504=$ 758.1 kN           |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION   |
|-----|---|
|     | 1st STOREY INNER COLUMN   |
|     | DEAD FROM ROOF    105=                      105 +                       |
|     | DEAD FROM 6 FLOOR 6x134.4=                806.6                         |
|     | -----   |
|     | 911.4    kN   |
|     | <br>IMPOSED FROM ROOF   48=                    48 +                     |
|     | IMPOSED FROM 6 FLOOR 6x160=               960                           |
|     | -----   |
|     | 1008    kN  |
|     | 40% REDUCTION ON IMPOSED LOAD   |
|     | <br>F <sub>CI1</sub> =911.4+0.6x1008=                      1516.2    kN |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION                      |
|-----|----------------------------------|
|     | <u>DEAD+IMPOSED+WIND LOADING</u> |
|     | <u>ROOF</u>                      |
|     | DEAD                             |
|     | 1.2x3.75x5x4= 90 kN              |
|     | IMPOSED                          |
|     | 1.2x1.5x5x4= 36 kN               |
|     | WIND                             |
|     | 1.2x9.35/2= 5.61 kN              |
|     | <u>FLOOR</u>                     |
|     | DEAD                             |
|     | 1.2x4.80x5x4= 115.2 kN           |
|     | IMPOSED                          |
|     | 1.2x5x5x4= 120 kN                |
|     | WIND                             |
|     | 1.2x9.35 9.35 kN                 |



STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION  |
|-----|--|
|     | <u>BEAM BENDING MOMENTS DUE TO HORIZONTAL FORCES</u> |
|     | ROOF   |
|     | WIND MOMENT= $0.7 \times 1.875 =$ 1.31 kN            |
|     | 6th STOREY   |
|     | WIND MOMENT= $1.31 + 3.937 =$ 5.24 kN                |
|     | 5th STOREY   |
|     | WIND MOMENT= $5.24 + 6.562 =$ 11.8 kN                |
|     | 4th STOREY   |
|     | WIND MOMENT= $11.8 + 9.18 =$ 20.1 kN                 |
|     | 3rd STOREY   |
|     | WIND MOMENT= $20.1 + 11.8 =$ 31.91 kN                |
|     | 2nd STOREY   |
|     | WIND MOMENT= $31.91 + 13.68 =$ 45.5 kN               |
|     | 1st STOREY   |
|     | WIND MOMENT= $45.5 + 17.1 =$ 62.56 kN                |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION   |          |          |          |
|-----|---|----------|----------|----------|
|     | <u>COLUMNS BENDING MOMENTS DUE TO HORIZONTAL FORCES</u> |          |          |          |
|     | ROOF  |          |          |          |
|     | EXTERNAL  | 1.31 kN  | INTERNAL | 2.62 kN  |
|     | 6th STOREY  |          |          |          |
|     | EXTERNAL  | 3.937 kN | INTERNAL | 7.87 kN  |
|     | 5th STOREY  |          |          |          |
|     | EXTERNAL  | 6.562 kN | INTERNAL | 13.12 kN |
|     | 4th STOREY  |          |          |          |
|     | EXTERNAL  | 9.18 kN  | INTERNAL | 18.36 kN |
|     | 3rd STOREY  |          |          |          |
|     | EXTERNAL  | 11.8 kN  | INTERNAL | 23.6 kN  |
|     | 2nd STOREY  |          |          |          |
|     | EXTERNAL  | 13.68 kN | INTERNAL | 27.36 kN |
|     | 1st STOREY  |          |          |          |
|     | EXTERNAL  | 17.1 kN  | INTERNAL | 34.2 kN  |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION  |
|-----|--|
|     | <u>AXIAL LOAD DUE TO HORIZONTAL FORCES</u>   |
|     | 6th STOREY   |
|     | $(5.61 \times 5.625) / 5 + (11.22 \times 1.875) / 5 = 10.5 / 4 \text{BAY} = 2.62 \text{ kN}$ |
|     | 4th STOREY   |
|     | $(5.61 \times 13.125) / 5 + (11.22 \times 9.375) / 5 + (11.22 \times 5.62) / 5 +$            |
|     | $(11.22 \times 1.875) / 5 = 52.56 / 4 \text{BAY} 13.14 \text{ kN}$                           |
|     | 1st STOREY   |
|     | $(5.61 \times 24.375) / 5 + (11.22 \times 20.625) / 5 + (11.22 \times 16.875) / 5 +$         |
|     | $(11.22 \times 13.125) / 5 + (11.22 \times 19.375) / 5 + (11.22 \times 5.625) / 5 +$         |
|     | $(11.22 \times 1.875) / 5 = 178.77 / 4 \text{BAY} 44.7 \text{ kN}$                           |
|     | <u>AXIAL LOAD ON COLUMNS DUE TO DEAD+IMPOSED+WIND</u>  |
|     | C  |
|     | 6th STOREY OUTER COLUMN  |
|     | DEAD FROM ROOF 90/2= 45 +  |
|     | DEAD FROM FLOOR 115.2/2 57.6   |
|     | -----  |
|     | 102.6 kN   |
|     | IMPOSED FROM ROOF 36/2= 18 +   |
|     | IMPOSED FROM FLOOR 120/2= 60   |
|     | -----  |
|     | 76 kN  |
|     | 10% REDUCTION ON IMPOSED LOAD  |
|     | $F_{C06} = 102.6 + 0.9 \times 76 + 2.62 = 191.62 \text{ kN}$                                 |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION                                       |
|-----|---|
|     | 6th STOREY INNER COLUMN                           |
|     | DEAD FROM ROOF    90=                      90 +   |
|     | DEAD FROM FLOOR   115.2=                115.2     |
|     | -----   |
|     | 205.2 kN  |
|     | IMPOSED FROM ROOF   36=                36 +       |
|     | IMPOSED FROM FLOOR 120=                120        |
|     | -----   |
|     | 156 kN  |
|     | 10% REDUCTION ON IMPOSED LOAD                     |
|     | $F_{CI6}=205.2+0.9\times156+0=$ 345.6 kN          |
|     | 4th STOREY OUTER COLUMN                           |
|     | DEAD FROM ROOF    90/2=                      45 + |
|     | DEAD FROM 3 FLOOR 3x115.2/2            172.8      |
|     | -----   |
|     | 217.8 kN  |
|     | IMPOSED FROM ROOF   36/2=                18 +     |
|     | IMPOSED FROM 3 FLOOR 3x120/2=        180          |
|     | -----   |
|     | 198 kN  |
|     | 30% REDUCTION ON IMPOSED LOAD                     |
|     | $F_{CO4}=217.8+0.7\times198+13.14=$ 369.54 kN     |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION                                 |
|-----|---|
|     | 4th STOREY INNER COLUMN                     |
|     | DEAD FROM ROOF 90= 90 +                     |
|     | DEAD FROM 3 FLOOR 3x115.2= 345.6            |
|     | -----                                       |
|     | 435.6 kN                                    |
|     | IMPOSED FROM ROOF 36= 36 +                  |
|     | IMPOSED FROM 3 FLOOR 3x120= 360             |
|     | -----                                       |
|     | 396 kN                                      |
|     | 30% REDUCTION ON IMPOSED LOAD               |
|     | $F_{CI4}=435.6+0.7\times396+0=$ 712.8 kN    |
|     | 1st STOREY OUTER COLUMN                     |
|     | DEAD FROM ROOF 90/2= 45 +                   |
|     | DEAD FROM 6 FLOOR 6x115.2/2 345.6           |
|     | -----                                       |
|     | 390.6 kN                                    |
|     | IMPOSED FROM ROOF 36/2= 18+                 |
|     | IMPOSED FROM 6 FLOOR 6x120/2= 360           |
|     | -----                                       |
|     | 378 kN                                      |
|     | 40% REDUCTION ON IMPOSED LOAD               |
|     | $F_{CO1}=390.6+0.6\times378+44.7=$ 622.1 kN |

| REF | CALCULATION                                 |
|-----|---|
|     | 1st STOREY INNER COLUMN                     |
|     | DEAD FROM ROOF 90= 90 +                     |
|     | DEAD FROM 6 FLOOR 6x115.2= 691.2            |
|     | -----                                       |
|     | 781.2 kN                                    |
|     | IMPOSED FROM ROOF 36= 36 +                  |
|     | IMPOSED FROM 6 FLOOR 6x120= 720             |
|     | -----                                       |
|     | 756 kN                                      |
|     | 40% REDUCTION ON IMPOSED LOAD               |
|     | $F_{C01}=781.2+0.6 \times 756+0=$ 1234.8 kN |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION                                    |
|-----|--|
|     | <u>DEAD+WIND LOADING</u>                       |
|     | <u>ROOF</u>                                    |
|     | DEAD   |
|     | $1.4 \times 3.75 \times 5 \times 4 =$ 105 kN   |
|     | WIND   |
|     | $1.4 \times 9.35 / 2 =$ 6.55 kN                |
|     | <u>FLOOR</u>                                   |
|     | DEAD   |
|     | $1.4 \times 4.80 \times 5 \times 4 =$ 134.4 kN |
|     | WIND   |
|     | $1.4 \times 9.35$ 13.1 kN                      |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION  |
|-----|--|
|     | <u>BEAM BENDING MOMENTS DUE TO HORIZONTAL FORCES</u> |
|     | ROOF   |
|     | WIND MOMENT= $0.82 \times 1.875 =$ 1.54 kN           |
|     | 6th STOREY   |
|     | WIND MOMENT= $1.54 + 4.61 =$ 6.15 kN                 |
|     | 5th STOREY   |
|     | WIND MOMENT= $6.15 + 7.68 =$ 13.8 kN                 |
|     | 4th STOREY   |
|     | WIND MOMENT= $13.8 + 10.76 =$ 24.56 kN               |
|     | 3rd STOREY   |
|     | WIND MOMENT= $24.56 + 13.83 =$ 38.4 kN               |
|     | 2nd STOREY   |
|     | WIND MOMENT= $38.4 + 16.89 =$ 55.29 kN               |
|     | 1st STOREY   |
|     | WIND MOMENT= $55.29 + 20 =$ 75.29 kN                 |



STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION   |          |          |          |
|-----|---|----------|----------|----------|
|     | <u>COLUMNS BENDING MOMENTS DUE TO HORIZONTAL FORCES</u> |          |          |          |
|     | ROOF  |          |          |          |
|     | EXTERNAL  | 1.54 kN  | INTERNAL | 3.08 kN  |
|     | 6th STOREY  |          |          |          |
|     | EXTERNAL  | 4.6 kN   | INTERNAL | 9.20 kN  |
|     | 5th STOREY  |          |          |          |
|     | EXTERNAL  | 7.69 kN  | INTERNAL | 15.38 kN |
|     | 4th STOREY  |          |          |          |
|     | EXTERNAL  | 10.76 kN | INTERNAL | 21.52 kN |
|     | 3rd STOREY  |          |          |          |
|     | EXTERNAL  | 13.83 kN | INTERNAL | 27.66 kN |
|     | 2nd STOREY  |          |          |          |
|     | EXTERNAL  | 16.89 kN | INTERNAL | 33.78 kN |
|     | 1st STOREY  |          |          |          |
|     | EXTERNAL  | 20 kN    | INTERNAL | 40.00 kN |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION   |
|-----|---|
|     | <u>AXIAL LOAD DUE TO HORIZONTAL FORCES</u>  |
|     | 6th STOREY  |
|     | $(6.55 \times 5.625)/5 + (13.1 \times 1.875)/5 = 12.28/4\text{BAY} = 3.1 \text{ kN}$  |
|     | 4th STOREY  |
|     | $(6.55 \times 13.125)/5 + (13.1 \times 9.375)/5 + (13.1 \times 5.62)/5 +$<br>$(13.1 \times 1.875)/5 = 61.40/4\text{BAY} = 15.35 \text{ kN}$   |
|     | 1st STOREY  |
|     | $(6.55 \times 24.375)/5 + (13.1 \times 20.625)/5 + (13.1 \times 16.875)/5 +$<br>$(13.1 \times 13.125)/5 + (13.1 \times 19.375)/5 + (13.1 \times 5.625)/5 +$<br>$(13.1 \times 1.875)/5 = 208.78/4\text{BAY} = 52.2 \text{ kN}$ |
|     | <u>AXIAL LOAD ON COLUMNS DUE TO DEAD+WIND</u>   |
|     | 6th STOREY OUTER COLUMN   |
|     | DEAD FROM ROOF $105/2 = 52.5 +$   |
|     | DEAD FROM FLOOR $134.4/2 = 67.2$  |
|     | -----   |
|     | 119.7 kN  |
|     | $F_{C06} = 119.7 + 3.1 = 122.8 \text{ kN}$  |
|     | 6th STOREY INNER COLUMN   |
|     | DEAD FROM ROOF    105 +   |
|     | DEAD FROM FLOOR    134.4  |
|     | -----   |
|     | 239.4 kN  |
|     | $F_{CI6} = 239.4 = 239.4 \text{ kN}$  |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION  |
|-----|--|
|     | 4th STOREY OUTER COLUMN  |
|     | DEAD FROM ROOF    105/2=                    52.5 +               |
|     | DEAD FROM 3 FLOOR 3x134.4/2                201.6                 |
|     | -----  |
|     | 254.1 kN   |
|     | F <sub>CO4</sub> =254.1+15.35=                    269.45 kN      |
|     | 4th STOREY INNER COLUMN  |
|     | DEAD FROM ROOF    105=                    105 +                  |
|     | DEAD FROM 3 FLOOR 3x134.4=                403.2                  |
|     | -----  |
|     | 508.2 kN   |
|     | F <sub>CI4</sub> =508.2                                508.2 kN  |
|     | 1st STOREY OUTER COLUMN  |
|     | DEAD FROM ROOF    105/2=                    52.5 +               |
|     | DEAD FROM 6 FLOOR 6x134.4/2                403.2                 |
|     | -----  |
|     | 455.7 kN   |
|     | F <sub>CO1</sub> =455.7+52.2=                    507.9 kN        |
|     | 1st STOREY INNER COLUMN  |
|     | DEAD FROM ROOF    105=                    105 +                  |
|     | DEAD FROM 6 FLOOR 6x134.4=                806.6                  |
|     | -----  |
|     | 911.4 kN   |
|     | F <sub>CI1</sub> =911.4+                                911.4 kN |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION  |
|-----|--|
|     | <u>BEAMS DESIGN</u>  |
|     | <u>ROOF BEAMS</u>  |
|     | DEAD+IMPOSED   |
|     | $W=104+48=$ 152 kN   |
|     | MOMENT AT CENTRE OF BEAM   |
|     | $M=152 \times 5/8=$ 96 kNm   |
|     | MAXIMUM MOMENT DUE TO WIND 1.54 kNm  |
|     | <u>SELECT 254x102x28</u>   |
|     | TREAT BEAM AS FULLY RESTRAINED BY SLABS .                                    |
|     | FROM CONSTRADO GUIDE TO BS5950 VOLUME 1                                      |
|     | $M_{CX}=97 > 96$   |
|     | SHEAR FORCE= $153/2=$ 76.5 kN  |
|     | SHEAR CAPACITY= $0.6 \times 275$   |
|     | DEFLECTION AT THE CENTRE OF THE BEAM DUE TO UNFACTORED<br>IMPOSED LOAD.      |
|     | $(5 \times 30 \times 500^3)/(384 \times 21000 \times 4004) \times 10$ 5.8 mm |
|     | ALLOWABLE DEFLECTION= $5000/360=$ 13.9 mm                                    |
|     | <u>USE 254x102x28</u>  |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION   |
|-----|---|
|     | <u>FLOOR BEAMS</u>  |
|     | DEAD+IMPOSED  |
|     | $W=134.4+160=$ 294.4 kN   |
|     | MOMENT AT CENTRE OF BEAM  |
|     | $M=294.4 \times 5/8=$ 184 kNm   |
|     | MAXIMUM MOMENT DUE TO WIND 75.26 kNm  |
|     | <u>SELECT 305x127x48</u>  |
|     | TREAT BEAM AS FULLY RESTRAINED BY SLABS .                                     |
|     | FROM CONSTRADO GUIDE TO BS5950 VOLUME 1                                       |
|     | $M_{CX}=194 > 96$   |
|     | SHEAR FORCE= $294.4/2=$ 147.2 kN  |
|     | SHEAR CAPACITY= $0.6 \times 275$  |
|     | DEFLECTION AT THE CENTRE OF THE BEAM DUE TO UNFACTORED<br>IMPOSED LOAD.       |
|     | $(5 \times 100 \times 500^3)/(384 \times 21000 \times 9485) \times 10$ 8.1 mm |
|     | ALLOWABLE DEFLECTION= $5000/360=$ 13.9 mm                                     |
|     | <u>USE 305x127x48</u>   |

STRUCTURAL  
STEEL DESIGN

| REF         | CALCULATION                                       |
|-------------|---|
|             | <u>COLUMNS DESIGN</u>                             |
|             | <u>EXTERNAL COLUMN STOREY 6</u>                   |
|             | EFFECTIVE LENGTH= $L_E=L=$ 3.75 m                 |
| CL<br>4.7.7 | <u>SELECT 152x152x23 UC</u>                       |
|             | FROM CONSTRADO GUIDE BOOK TO BS5950               |
|             | FOR $L_E=3.75$ m $P_{cy}=365.5$ kN $M_b=34.5$ kNm |
|             | ECCENTRICITY= $152.4/2+100=$ 0.176 m              |
|             | DEAD+IMPOSED                                      |
|             | $M_x=0.176(67.2+80)/2=$ 12.9 kNm                  |
|             | $P_{CO1}=$ 213.3 kN                               |
| CL<br>4.8.3 | $213.3/365.5+12.9/34.5=0.957<1$                   |
|             | DEAD+IMPOSED+WIND                                 |
|             | $M_x=0.176(57.6+60)/2+3.93=$ 14.29 kNm            |
|             | $P_{CO1}=$ 191.62 kN                              |
|             | $191.62/365.5+14.29/34.5=0.938<1$                 |
|             | DEAD+WIND   |
|             | $M_x=0.176(67.2)/2+4.6=$ 10.5 kNm                 |
|             | $P_{CO1}=$ 122.8 kN                               |
|             | $122.8/365.5+10.5/34.5=0.0.61<1$                  |
|             | <u>USE 152x152x23 UC</u>                          |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION                                     |
|-----|---|
|     | <u>INTERNAL COLUMN STOREY 6</u>                 |
|     | EFFECTIVE LENGTH= $L_E=L=$ 3.75 m               |
|     | SELECT <u>152x152x30 UC</u>                     |
|     | FROM CONSTRADO GUIDE BOOK TO BS5950             |
|     | FOR $L_E=3.75$ m $P_{cy}=491.5$ kN $M_b=49$ kNm |
|     | ECCENTRICITY=0                                  |
|     | DEAD+IMPOSED                                    |
|     | $P_{C01}=$ 426.6 kN                             |
|     | $426.6/491.5=0.87<1$                            |
|     | DEAD+IMPOSED+WIND                               |
|     | $M_x=$ 7.88 kNm                                 |
|     | $P_{C01}=$ 345.6 kN                             |
|     | $345.6/491.5+7.88/49=0.86<1$                    |
|     | DEAD+WIND                                       |
|     | $M_x=$ 9.2 kNm                                  |
|     | $P_{C01}=$ 239.4 kN                             |
|     | $239.4/491.5+9.2/49=0.675<1$                    |
|     | USE <u>152x152x30 UC</u>                        |

| REF | CALCULATION  |
|-----|--|
|     | <p><u>EXTERNAL COLUMN STOREY 4</u></p> <p>EFFECTIVE LENGTH=<math>L_E=L=</math> 3.75 m</p> <p><u>SELECT 152x152x37 UC</u></p> <p>FROM CONSTRADO GUIDE BOOK TO BS5950</p> <p>FOR <math>L_E=3.75</math> m    <math>P_{cy}=619.5</math> kN    <math>M_b=66</math> kNm</p> <p>ECCENTRICITY=<math>157.5/2+100=</math> 0.179 m</p> <p>DEAD+IMPOSED</p> <p><math>M_x=0.179(67.2+80)/2=</math> 13.3 kNm</p> <p><math>P_{C01}=</math> 438.9 kN</p> <p><math>489.9/619.5+13.3/66=0.91&lt;1</math></p> <p>DEAD+IMPOSED+WIND</p> <p><math>M_x=0.179(57.6+60)/2+9.18=</math> 19.8 kNm</p> <p><math>P_{C01}=</math> 369.54 kN</p> <p><math>369.54/619.5+19.8/66=0.896&lt;1</math></p> <p>DEAD+WIND</p> <p><math>M_x=0.179(67.2)/2+10.76=</math> 16.81 kNm</p> <p><math>P_{C01}=</math> 269.45 kN</p> <p><math>269.45/619.5+16.81/66=0.7&lt;1</math></p> <p><u>USE 152x152x37 UC</u></p> |



STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION                                       |
|-----|---|
|     | <u>INTERNAL COLUMN STOREY 4</u>                   |
|     | EFFECTIVE LENGTH= $L_E=L=$ 3.75 m                 |
|     | <u>SELECT 203x203x46 UC</u>                       |
|     | FROM CONSTRADO GUIDE BOOK TO BS5950               |
|     | FOR $L_E=3.75$ m $P_{cy}=1024$ kN $M_b=114.5$ kNm |
|     | ECCENTRICITY=0                                    |
|     | DEAD+IMPOSED                                      |
|     | $P_{C01}=$ 877.8 kN                               |
|     | $877.8/1024=0.857<1$                              |
|     | DEAD+IMPOSED+WIND                                 |
|     | $M_x=$ 18.36 kNm                                  |
|     | $P_{C01}=$ 712.8 kN                               |
|     | $712.8/1024+18.36/114.5=0.856<1$                  |
|     | DEAD+WIND   |
|     | $M_x=$ 21.52 kNm                                  |
|     | $P_{C01}=$ 508.2 kN                               |
|     | $508.2/1024+21.52/114.5=0.68<1$                   |
|     | <u>USE 203x203x46 UC</u>                          |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION                                       |
|-----|---|
|     | <u>EXTERNAL COLUMN STOREY 1</u>                   |
|     | EFFECTIVE LENGTH= $L_E=L=$ 3.75 m                 |
|     | <u>SELECT 203x203x46 UC</u>                       |
|     | FROM CONSTRADO GUIDE BOOK TO BS5950               |
|     | FOR $L_E=3.75$ m $P_{cy}=1024$ kN $M_b=114.5$ kNm |
|     | ECCENTRICITY= $203.2/2+100=$ 0.2 m                |
|     | DEAD+IMPOSED                                      |
|     | $M_x=0.2(67.2+80)/2=$ 14.83 kNm                   |
|     | $P_{C01}=$ 758.1 kN                               |
|     | $758.1/1024+14.83/114.5=0.87<1$                   |
|     | DEAD+IMPOSED+WIND                                 |
|     | $M_x=0.2(57.6+60)/2+17.1=$ 28.9 kNm               |
|     | $P_{C01}=$ 622.1 kN                               |
|     | $622.1/1024+28.9/114.5=0.86<1$                    |
|     | DEAD+WIND   |
|     | $M_x=0.2(67.2)/2+20=$ 26.75 kNm                   |
|     | $P_{C01}=$ 507.9 kN                               |
|     | $507.9/1024+26.75/114.5=0.729<1$                  |
|     | <u>USE 203x203x46 UC</u>                          |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION                                     |
|-----|---|
|     | <u>INTERNAL COLUMN STOREY 1</u>                 |
|     | EFFECTIVE LENGTH= $L_E=L=$ 3.75 m               |
|     | <u>SELECT 203x203x71 UC</u>                     |
|     | FROM CONSTRADO GUIDE BOOK TO BS5950             |
|     | FOR $L_E=3.75$ m $P_{cy}=1750$ kN $M_b=149$ kNm |
|     | ECCENTRICITY=0                                  |
|     | DEAD+IMPOSED                                    |
|     | $P_{C01}=$ 1516.2 kN                            |
|     | $1516.2/1750=0.87<1$                            |
|     | DEAD+IMPOSED+WIND                               |
|     | $M_x=$ 34.2 kNm                                 |
|     | $P_{C01}=$ 1234.8 kN                            |
|     | $1234.8/1750+34.2/149=0.88<1$                   |
|     | DEAD+WIND                                       |
|     | $M_x=$ 40 kNm                                   |
|     | $P_{C01}=$ 911.4 kN                             |
|     | $911.4/1750+40/149=0.79<1$                      |
|     | <u>USE 203x203x71 UC</u>                        |

| REF | CALCULATION  |
|-----|--|
|     | <u>CONNECTIONS</u>                                       |
|     | <u>ROOF</u>  |
|     | ROOF BEAM      254x102x28                                |
|     | D=260.4   B=102.1   t=6.4   T=10   r=7.6   ALL UNITS mm  |
|     | INTERNAL COLUMN      152x152x30                          |
|     | D=157.5   B=152.9   t=6.6   T=9.4   r=7.6   ALL UNITS mm |
|     | <u>DEAD+IMPOSED</u>                                      |
|     | SHEAR=      76.5 kN                                      |
|     | <u>DEAD+IMPOSED+WIND</u>                                 |
|     | SHEAR=      64 kN  |
|     | MOMENT=      1.31 kNm                                    |
|     | <u>DEAD+WIND</u>   |
|     | SHEAR=      52.5 kN                                      |
|     | MOMENT      1.54 kNm                                     |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION                                     |
|-----|---|
|     | <u>BOLT GROUPS</u>                              |
|     | <u>SELECT 16mm DIA. GRADE 4.6</u>               |
|     | $P_S=160 \times 157 / 1000 =$ 25.1 kN           |
|     | $P_T=195 \times 157 / 1000 =$ 30.6 kN           |
|     | DEAD+IMPOSED                                    |
|     | SHEAR LOAD PER BOLT= $74 / 6 =$ 12.5 kN         |
|     | $12.5 / 25.1 = 0.5 < 1.4$                       |
|     | DEAD+IMPOSED+WIND                               |
|     | SHEAR LOAD PER BOLT= $64 / 6 =$ 11 kN           |
|     | $F_T=1.31 \times 1000 / (260.4 - 10) =$ 5.23 kN |
|     | TENSILE LOAD PER BOLT= $5.23 / 4 =$ 1.3 kN      |
|     | $11 / 25.1 + 1.3 / 30.6 = 0.49 < 1.4$           |
|     | DEAD+WIND                                       |
|     | SHEAR LOAD PER BOLT= $52.5 / 6 =$ 8.75 kN       |
|     | $F_T=1.54 \times 1000 / (260.4 - 10) =$ 6.2 kN  |
|     | TENSILE LOAD PER BOLT= $6.2 / 4 =$ 1.55 kN      |
|     | $8.75 / 25.1 + 1.55 / 30.6 = 0.4 < 1.4$         |
|     | <u>USE 16mm DIA. GRADE 4.6</u>                  |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION   |
|-----|---|
|     | <p><u>JOINT DIMENSION</u></p> <p>B=9x D=9x16=144      SAY      150 mm</p> <p>A=5x D=5x16=80      80 mm</p> <p>C=6x D=6x16=96      SAY      100 mm</p> <p>a=2.5D=2.5x16=40      40 mm</p> <p><u>WELD SIZE</u></p> <p>FLANGE WELD= <math>10 / 2 = 7.07</math>      USE 8mm F.W.</p> <p>WEB WELD= <math>6.4 / 2 = 4.6</math>      USE 6mm FW</p> <p><u>END PLATE</u></p> <p>END PLATE THICKNESS USING EQUATION</p> <p>5230</p> <p><math>t_p = \frac{5230}{275(300/100+250.4/80)} = 1.83</math></p> <p>USE THE END PLATE THICKNESS OF 8 mm      <math>t_p = 8</math> mm</p> <p>MOMENT CAPACITY =</p> <p><math>8^2 \times 275 [300/100+250.4/80] \times 250.4 \times 10^{-6} =</math>      25 kNm</p> <p><u>ADEQUACY OF COLUMN FLANGE</u></p> <p>ADEQUACY OF COLUMN FLANGE USING THE EQUS. (5.5) AND (5.6)</p> <p><math>m = (80 - 6.6 - 2 \times 7.6) / 2 =</math>      29.1 mm</p> <p><math>n = (150 - 80) / 2 =</math>      35 mm</p> <p><math>n' = (152.9 - 80) / 2 =</math>      36.2 mm</p> |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION   |
|-----|---|
|     | $F_{mb} = 9.4^2 \frac{3.14 \times 65.3 + 0.5 \times 100}{275 + 4 \times 34826 \times \frac{35}{64.1}} = 64.1$ |
|     | $F_{mb} = 172744/1000 = 173 \text{ kN}$   |
|     | $F_{mc} = 9.4^2 [3.14 + (2 \times 36.2 + 100 - 18)/29.1] \times 275/1000 = 199 \text{ kN}$                    |
|     | i.e. $F_t < F_{mb} < F_{mc}$ COLUMN FLANGE ADEQUATE   |
|     | <u>COLUMN WEB IN COMPRESSION ZONE</u>   |
|     | BY REFERRING TO EQN. (5.7)  |
|     | $k = 9.4 + 7.6 = 17 \text{ mm}$   |
|     | $F_{wc} = (10 + 8 + 15 + 5 \times 17) \times 6.6 \times 275/1000 = 214 \text{ kN}$                            |
|     | i.e. $F_t < F_{wc}$ COLUMN WEB IN COMPRESSION ZONE O.K.   |
|     | <u>COLUMN WEB IN SHEAR</u>  |
|     | BY REFERRING TO EQN. (5.8)  |
|     | $F_q = 6.6 (152.9 - 2 \times 9.4) \times 275/1000 = 242 \text{ kN}$   |
|     | i.e. $F_t < F_q$ COLUMN WEB IN SHEAR OK.  |

# STRUCTURAL STEEL DESIGN

| REF | CALCULATION  |
|-----|--|
|     | FLOOR  |
|     | FLOOR BEAM      305x127x48 UB                                  |
|     | D=310.4   B=125.2   t=8.9   T=14   r=8.9   ALL UNITS<br>mm     |
|     | INTERNAL COLUMN    203x203x71 UC                               |
|     | D=215.9   B=206.2   t=10.3   T=17.3   r=10.2   ALL UNITS<br>mm |
|     | <u>DEAD+IMPOSED</u>  |
|     | SHEAR=                          148 kN                         |
|     | <u>DEAD+IMPOSED+WIND</u>                                       |
|     | SHEAR=                          118 kN                         |
|     | MOMENT=                        63 kNm                          |
|     | <u>DEAD+WIND</u>   |
|     | SHEAR=                          67.2 kN                        |
|     | MOMENT                         73 kNm                          |



STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION  |
|-----|--|
|     | <p><u>BOLT GROUPS</u></p> <p><u>SELECT 20mm DIA. GRADE 8.8</u></p> <p><math>P_S = 375 \times 245 / 1000 =</math> 92 kN</p> <p><math>P_T = 450 \times 245 / 1000 =</math> 110 kN</p> <p>DEAD+IMPOSED</p> <p>SHEAR LOAD PER BOLT= <math>148 / 6 =</math> 25 kN</p> <p><math>25 / 92 = 0.27 &lt; 1.4</math></p> <p>DEAD+IMPOSED+WIND</p> <p>SHEAR LOAD PER BOLT= <math>118 / 6 =</math> 20 kN</p> <p><math>F_T = 63 \times 1000 / (310.4 - 14)</math> 212 kN</p> <p>TENSILE LOAD PER BOLT= <math>212 / 4 =</math> 53 kN</p> <p><math>20 / 92 + 53 / 110 = 0.7 &lt; 1.4</math></p> <p>DEAD+WIND</p> <p>SHEAR LOAD PER BOLT= <math>67.2 / 6 =</math> 11.2 kN</p> <p><math>F_T = 73 \times 1000 / (310.4 - 14)</math> 246 kN</p> <p>TENSILE LOAD PER BOLT= <math>246 / 4 =</math> 61.5 kN</p> <p><math>11.2 / 92 + 61.5 / 110 = 0.68 &lt; 1.4</math></p> <p><u>USE 20mm DIA. GRADE 8.8</u></p> |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION   |
|-----|---|
|     | <u>JOINT DIMENSION</u>  |
|     | B=9x D=9x20=180 180 mm  |
|     | A=5x D=5x20=100 100 mm  |
|     | C=6x D=6x20=120 120 mm  |
|     | a=2.5D=2.5x20=50 50 mm  |
|     | <u>WELD SIZE</u>  |
|     | FLANGE WELD= $14 / 2 = 9.9$ USE 10mm FW   |
|     | WEB WELD= $8.9 / 2 = 6.3$ USE 8mm FW  |
|     | <u>END PLATE</u>  |
|     | END PLATE THICKNESS USING EQUATION  |
|     | $m^X = (120 - 14 - 2 \times 10) / 2 = 43 \text{ mm}$                                |
|     | $t_p = \frac{246 \times 43 \times 1000}{275 \times 180} = 14.6 \text{ mm}$          |
|     | USE THE END PLATE THICKNESS OF 18 mm $t_p = 18 \text{ mm}$                          |
|     | MOMENT CAPACITY =   |
|     | $18^2 (180 \times 275) / 43 \times (310.4 - 14) \times 10^{-6} = 110.5 \text{ kNm}$ |
|     | <u>ADEQUACY OF COLUMN FLANGE</u>  |
|     | ADEQUACY OF COLUMN FLANGE USING THE EQUS.(5.5) AND (5.6)                            |
|     | $m = (100 - 10.3 - 2 \times 10.2) / 2 = 34.65 \text{ mm}$                           |
|     | $n = (180 - 100) / 2 = 50 \text{ mm}$   |
|     | $n' = (206.2 - 100) / 2 = 53.1 \text{ mm}$  |

| REF | CALCULATION   |
|-----|---|
|     | $F_{mb} = 17.3^2 \frac{3.14 \times 87.8 + 0.5 \times 120}{84.65} \frac{275 + 4 \times 140283 \times 50}{84.65}$ $F_{mb} = 657833 / 1000 = 658 \text{ kN}$ $F_{mc} = 17.3^2 [3.14 + (2 \times 53.1 + 120 - 22) / 34.65] \times 275 / 1000 =$ $F_{mc} = 485 \text{ kN}$ <p>i.e. <math>F_t &lt; F_{mc} &lt; F_{mb}</math> COLUMN FLANGE ADEQUATE</p> <p><u>COLUMN WEB IN COMPRESSION ZONE</u></p> <p>BY REFERRING TO EQN. (5.7)</p> $k = 17.3 + 10.2 = 27.5 \text{ mm}$ $F_{wc} = (14 + 18 + 20 + 5 \times 27.5) \times 10.3 \times 275 / 1000 = 536 \text{ kN}$ <p>i.e. <math>F_t &lt; F_{wc}</math> COLUMN WEB IN COMPRESSION ZONE OK.</p> <p><u>COLUMN WEB IN SHEAR</u></p> <p>BY REFERRING TO EQN. (5.8)</p> $F_q = 10.3 (215.9 - 2 \times 17.3) \times 275 / 1000 = 513 \text{ kN}$ <p>i.e. <math>F_t &lt; F_q</math> COLUMN WEB IN SHEAR OK.</p> |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION   |
|-----|---|
|     | <p>CALCULATION OF REDUCTION IN IMPOSED LOAD. INPUT</p> <p>IMPOSED LOADS FOR COMPUTER ANALYSIS.</p> <p>IMPOSED(UNFACTORED)</p> <p>ROOF 30 kN</p> <p>FLOOR 100 kN</p> <p>6th STOREY 10% REDUCTION</p> <p>DEAD+IMPOSED</p> <p><math>1.6(30+100)/2=104 \times 0.1=10.4</math></p> <p><math>73.6-10.4=</math> 63.2 kN</p> <p>DEAD+IMPOSED+WIND</p> <p><math>1.2(30+100)/2=78 \times 0.1=7.8</math></p> <p><math>58.8-7.8=</math> 51 kN</p> <p>5th STOREY 20% REDUCTION</p> <p>DEAD+IMPOSED</p> <p><math>1.6(30+2 \times 100)/2=184 \times 0.2=36.8</math></p> <p><math>36.8-10.4=26.4</math></p> <p><math>73.6-26.4=</math> 47.2 63.2 kN</p> <p>DEAD+IMPOSED+WIND</p> <p><math>1.2(30+2 \times 100)/2=138 \times 0.2=27.6</math></p> <p><math>27.6-7.8=19.8</math></p> <p><math>58.8-19.8=</math> 39 kN</p> <p>4th STOREY 30% REDUCTION</p> <p>DEAD+IMPOSED</p> <p><math>1.6(30+3 \times 100)/2=264 \times 0.3=79.2</math></p> <p><math>79.2-10.4-26.4=42.2</math></p> |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION   |
|-----|---|
|     | $73.6 - 42.2 = 31.2 \text{ kN}$<br><br>DEAD+IMPOSED+WIND<br>$1.2(30+3 \times 100)/2 = 198 \times 0.3 = 59.4$<br>$59.4 - 7.8 - 19.8 = 31.8$<br>$58.8 - 31.8 = 27 \text{ kN}$<br><br>3rd STOREY 40% REDUCTION<br><br>DEAD+IMPOSED<br>$1.6(30+4 \times 100)/2 = 344 \times 0.4 = 137.6$<br>$137.6 - 10.4 - 26.4 - 42.2 = 58.4$<br>$73.6 - 58.4 = 15.2$ $15.2 \text{ kN}$<br><br>DEAD+IMPOSED+WIND<br>$1.2(30+4 \times 100)/2 = 258 \times 0.4 = 103.2$<br>$103.2 - 7.8 - 19.8 - 31.8 = 43.8$<br>$58.8 - 43.8 = 15 \text{ kN}$<br><br>2nd STOREY 40% REDUCTION<br><br>DEAD+IMPOSED<br>$1.6(30+5 \times 100)/2 = 424 \times 0.4 = 169.6$<br>$169.6 - 10.4 - 26.4 - 42.4 - 58.4 = 32$<br>$73.6 - 32 = 41.6$ $41.6 \text{ kN}$<br><br>DEAD+IMPOSED+WIND<br>$1.2(30+5 \times 100)/2 = 318 \times 0.2 = 127.2$<br>$127.2 - 7.8 - 19.8 - 31.8 - 43.8 = 24$<br>$58.8 - 24 = 34.8 \text{ kN}$ |

STRUCTURAL  
STEEL DESIGN

| REF                   | CALCULATION   |                      |                       |                              |                       |                              |               |     |   |       |  |               |     |    |       |  |                     |     |    |       |  |                     |     |    |       |  |                     |      |    |       |  |                     |    |    |       |  |                       |    |    |       |  |                       |      |    |       |  |
|-----------------------|---|----------------------|-----------------------|------------------------------|-----------------------|------------------------------|---------------|-----|---|-------|--|---------------|-----|----|-------|--|---------------------|-----|----|-------|--|---------------------|-----|----|-------|--|---------------------|------|----|-------|--|---------------------|----|----|-------|--|-----------------------|----|----|-------|--|-----------------------|------|----|-------|--|
|                       | <p>SUMMARY OF CONNECTIONS DETAILS FOR FRYE AND MORRIS</p> <p>MATHEMATICAL MODELLING.</p> <table><tr><th>POSITION</th><th>T<sub>cf</sub><br/>mm</th><th>t<sub>p</sub><br/>mm</th><th>EFFECTIVE DEPTH<br/>mm</th><th>PLATE MOMENT<br/>CAPACITY kNm</th></tr><tr><td>ROOF EXTERNAL</td><td>6.8</td><td>8</td><td>260.4</td><td></td></tr><tr><td>ROOF INTERNAL</td><td>9.4</td><td>10</td><td>260.4</td><td></td></tr><tr><td>6-7 STOREY EXTERNAL</td><td>6.8</td><td>18</td><td>310.4</td><td></td></tr><tr><td>6-7 STOREY INTERNAL</td><td>9.4</td><td>18</td><td>310.4</td><td></td></tr><tr><td>5-4 STOREY EXTERNAL</td><td>11.5</td><td>18</td><td>310.4</td><td></td></tr><tr><td>5-4 STOREY INTERNAL</td><td>11</td><td>18</td><td>310.4</td><td></td></tr><tr><td>1,2,3 STOREY EXTERNAL</td><td>11</td><td>18</td><td>310.4</td><td></td></tr><tr><td>1,2,3 STOREY INTERNAL</td><td>17.3</td><td>18</td><td>310.4</td><td></td></tr></table> | POSITION             | T <sub>cf</sub><br>mm | t <sub>p</sub><br>mm         | EFFECTIVE DEPTH<br>mm | PLATE MOMENT<br>CAPACITY kNm | ROOF EXTERNAL | 6.8 | 8 | 260.4 |  | ROOF INTERNAL | 9.4 | 10 | 260.4 |  | 6-7 STOREY EXTERNAL | 6.8 | 18 | 310.4 |  | 6-7 STOREY INTERNAL | 9.4 | 18 | 310.4 |  | 5-4 STOREY EXTERNAL | 11.5 | 18 | 310.4 |  | 5-4 STOREY INTERNAL | 11 | 18 | 310.4 |  | 1,2,3 STOREY EXTERNAL | 11 | 18 | 310.4 |  | 1,2,3 STOREY INTERNAL | 17.3 | 18 | 310.4 |  |
| POSITION              | T <sub>cf</sub><br>mm   | t <sub>p</sub><br>mm | EFFECTIVE DEPTH<br>mm | PLATE MOMENT<br>CAPACITY kNm |                       |                              |               |     |   |       |  |               |     |    |       |  |                     |     |    |       |  |                     |     |    |       |  |                     |      |    |       |  |                     |    |    |       |  |                       |    |    |       |  |                       |      |    |       |  |
| ROOF EXTERNAL         | 6.8   | 8                    | 260.4                 |                              |                       |                              |               |     |   |       |  |               |     |    |       |  |                     |     |    |       |  |                     |     |    |       |  |                     |      |    |       |  |                     |    |    |       |  |                       |    |    |       |  |                       |      |    |       |  |
| ROOF INTERNAL         | 9.4   | 10                   | 260.4                 |                              |                       |                              |               |     |   |       |  |               |     |    |       |  |                     |     |    |       |  |                     |     |    |       |  |                     |      |    |       |  |                     |    |    |       |  |                       |    |    |       |  |                       |      |    |       |  |
| 6-7 STOREY EXTERNAL   | 6.8   | 18                   | 310.4                 |                              |                       |                              |               |     |   |       |  |               |     |    |       |  |                     |     |    |       |  |                     |     |    |       |  |                     |      |    |       |  |                     |    |    |       |  |                       |    |    |       |  |                       |      |    |       |  |
| 6-7 STOREY INTERNAL   | 9.4   | 18                   | 310.4                 |                              |                       |                              |               |     |   |       |  |               |     |    |       |  |                     |     |    |       |  |                     |     |    |       |  |                     |      |    |       |  |                     |    |    |       |  |                       |    |    |       |  |                       |      |    |       |  |
| 5-4 STOREY EXTERNAL   | 11.5  | 18                   | 310.4                 |                              |                       |                              |               |     |   |       |  |               |     |    |       |  |                     |     |    |       |  |                     |     |    |       |  |                     |      |    |       |  |                     |    |    |       |  |                       |    |    |       |  |                       |      |    |       |  |
| 5-4 STOREY INTERNAL   | 11  | 18                   | 310.4                 |                              |                       |                              |               |     |   |       |  |               |     |    |       |  |                     |     |    |       |  |                     |     |    |       |  |                     |      |    |       |  |                     |    |    |       |  |                       |    |    |       |  |                       |      |    |       |  |
| 1,2,3 STOREY EXTERNAL | 11  | 18                   | 310.4                 |                              |                       |                              |               |     |   |       |  |               |     |    |       |  |                     |     |    |       |  |                     |     |    |       |  |                     |      |    |       |  |                     |    |    |       |  |                       |    |    |       |  |                       |      |    |       |  |
| 1,2,3 STOREY INTERNAL | 17.3  | 18                   | 310.4                 |                              |                       |                              |               |     |   |       |  |               |     |    |       |  |                     |     |    |       |  |                     |     |    |       |  |                     |      |    |       |  |                     |    |    |       |  |                       |    |    |       |  |                       |      |    |       |  |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION  |
|-----|--|
|     | <u>ASSESSMENT OF DESIGN</u>                                      |
|     | MEMBER MOMENTS AND FORCES OBTAINED FROM EXACT COMPUTER ANALYSIS. |
|     | 1.4D+1.6I=LC1  |
|     | 1.2(D+I+W)=LC2   |
|     | 1.4(D+W)=LC3   |
|     | <u>BEAMS</u>   |
|     | <u>ROOF</u>  |
|     | SECTION 254x102x28 UB  |
|     | MCX=97 kNm   |
|     | LC1 SEMIRIGID= 73< 97  |
|     | RIGID= 58< 97  |
|     | LC2 SEMIRIGID= 60< 97  |
|     | RIGID 48< 97   |
|     | LC3 SEMIRIGID= 50< 97  |
|     | RIGID= 40< 97  |
|     | <u>FLOOR</u>   |
|     | SECTION 305x127x48 UB  |
|     | MCX=194 kNm  |
|     | LC1 SEMIRIGID= 143< 194  |
|     | RIGID= 114< 194  |
|     | LC2 SEMIRIGID= 114< 194  |
|     | RIGID 91< 194  |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION  |
|-----|--|
|     | <p>LC3 SEMIRIGID= 65 &lt; 194</p> <p>RIGID= 52 &lt; 194</p> <p><u>COLUMNS</u></p> <p><u>6th STOREY EXTERNAL</u></p> <p>SECTION 152x152x23 UC</p> <p><math>M_{CX}=45 \text{ kNm}</math> <math>P_z=820 \text{ kN}</math></p> <p><math>L_E=3.75 \text{ m}</math> <math>M_b=35 \text{ kNm}</math> <math>P_{cy}=366 \text{ kN}</math></p> <p><u>LOCAL CAPACITY</u></p> <p>LC1 SEMIRIGID= <math>206/820+15/45 = 0.58 &lt; 1</math></p> <p>RIGID = <math>196/820+20/45 = 0.68</math></p> <p>LC2 SEMIRIGID= <math>170/820+18/45 = 0.60</math></p> <p>RIGID = <math>161/820+22/45 = 0.69</math></p> <p>LC3 SEMIRIGID= <math>120/820+13/45 = 0.44</math></p> <p>RIGID = <math>113/820+15/45 = 0.47</math></p> <p>BS5950 <u>LATERAL TORSIONAL BUCKLING</u></p> <p>FROM TABLE 18 <math>m=0.43</math></p> <p>LC1 SEMIRIGID= <math>206/366+0.43 \times 15/35 = 0.75 &lt; 1</math></p> <p>RIGID = <math>196/366+0.43 \times 20/35 = 0.78</math></p> <p>LC2 SEMIRIGID= <math>170/366+0.43 \times 18/35 = 0.69</math></p> <p>RIGID = <math>161/366+0.43 \times 22/35 = 0.71</math></p> <p>LC3 SEMIRIGID= <math>120/366+0.43 \times 13/35 = 0.49</math></p> <p>RIGID = <math>113/366+0.43 \times 15/35 = 0.49</math></p> |



STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION   |
|-----|---|
|     | <p><u>6th STOREY INTERNAL</u></p> <p>SECTION      152x152x30 UC</p> <p><math>M_{CX}=67 \text{ kNm}</math>      <math>P_z=1050 \text{ kN}</math></p> <p><math>L_E=3.75 \text{ m}</math>      <math>M_b=50 \text{ kNm}</math>      <math>P_{cy}=492 \text{ kN}</math></p> <p><u>LOCAL CAPACITY</u></p> <p>LC1      SEMIRIGID= <math>432/1050</math>      = 0.41 &lt;1</p> <p>         RIGID      = <math>444/1050</math>      = 0.43</p> <p>LC2      SEMIRIGID= <math>365/1050+10/67</math> = 0.50</p> <p>         RIGID      = <math>360/1050+ 7/67</math> = 0.38</p> <p>LC3      SEMIRIGID= <math>242/1050+10/67</math> = 0.38</p> <p>         RIGID      = <math>248/1050+ 2/67</math> = 0.27</p> <p><u>LATERAL TORSIONAL BUCKLING</u></p> <p>FROM TABLE 18      <math>m=0.43</math></p> <p>LC1      SEMIRIGID= <math>432/492</math>      = 0.88 &lt;1</p> <p>         RIGID      = <math>444/492</math>      = 0.90</p> <p>LC2      SEMIRIGID= <math>365/492+0.43 \times 10/50</math> = 0.83</p> <p>         RIGID      = <math>360/492+0.43 \times 7/50</math> = 0.76</p> <p>LC3      SEMIRIGID= <math>242/492+0.43 \times 10/50</math> = 0.58</p> <p>         RIGID      = <math>248/492+0.43 \times 2/50</math> = 0.52</p> |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION  |
|-----|--|
|     | <u>4th STOREY EXTERNAL</u>   |
|     | SECTION      152x152x37 UC   |
|     | $M_{CX}=85 \text{ kNm}$ $P_z=1300 \text{ kN}$  |
|     | $L_E=3.75 \text{ m}$ $M_b=66 \text{ kNm}$ $P_{cy}=620 \text{ kN}$                                    |
|     | <u>LOCAL CAPACITY</u>  |
| LC1 | SEMIRIGID= $425/1300+19/85 = 0.55 < 1$<br>RIGID      = $402/1300+25/85 = 0.60$                       |
| LC2 | SEMIRIGID= $361/1300+27/85 = 0.60$<br>RIGID      = $341/1300+31/85 = 0.62$                           |
| LC3 | SEMIRIGID= $265/1300+21/85 = 0.45$<br>RIGID      = $253/1300+23/85 = 0.47$                           |
|     | <u>LATERAL TORSIONAL BUCKLING</u>  |
|     | FROM TABLE 18 $m=0.43$   |
| LC1 | SEMIRIGID= $425/620+0.43 \times 19/66 = 0.81 < 1$<br>RIGID      = $402/620+0.43 \times 25/66 = 0.81$ |
| LC2 | SEMIRIGID= $361/620+0.43 \times 27/66 = 0.76$<br>RIGID      = $341/620+0.43 \times 31/66 = 0.75$     |
| LC3 | SEMIRIGID= $265/620+0.43 \times 21/66 = 0.56$<br>RIGID      = $253/620+0.43 \times 23/66 = 0.56$     |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION  |
|-----|--|
|     | <p><u>4th STOREY INTERNAL</u></p> <p>SECTION      203x203x46 UC</p> <p><math>M_{CX}=137 \text{ kNm}</math>      <math>P_Z=1620 \text{ kN}</math></p> <p><math>L_E=3.75 \text{ m}</math>      <math>M_B=115 \text{ kNm}</math>      <math>P_{CY}=1024 \text{ kN}</math></p> <p><u>LOCAL CAPACITY</u></p> <p>LC1      SEMIRIGID= <math>892/1620</math>      = 0.55 &lt;1</p> <p>         RIGID      = <math>915/1620</math>      = 0.56</p> <p>LC2      SEMIRIGID= <math>725/1620+24/137</math> = 0.62</p> <p>         RIGID      = <math>743/1620+20/137</math> = 0.60</p> <p>LC3      SEMIRIGID= <math>514/1620+25/137</math> = 0.50</p> <p>         RIGID      = <math>525/1620+23/137</math> = 0.50</p> <p><u>LATERAL TORSIONAL BUCKLING</u></p> <p>FROM TABLE 18      <math>m=0.43</math></p> <p>LC1      SEMIRIGID= <math>892/1024</math>      = 0.87 &lt;1</p> <p>         RIGID      = <math>915/1024</math>      = 0.89</p> <p>LC2      SEMIRIGID= <math>725/1024+0.43 \times 24/115</math> = 0.80</p> <p>         RIGID      = <math>743/1024+0.43 \times 20/115</math> = 0.80</p> <p>LC3      SEMIRIGID= <math>514/1024+0.43 \times 25/115</math> = 0.60</p> <p>         RIGID      = <math>525/1024+0.43 \times 23/115</math> = 0.60</p> |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION  |
|-----|--|
|     | <p><u>1st STOREY EXTERNAL</u></p> <p>SECTION      203x203x46 UC</p> <p><math>M_{CX}=137 \text{ kNm}</math>      <math>P_Z=1620 \text{ kN}</math></p> <p><math>L_E=3.75 \text{ m}</math>      <math>M_b=115 \text{ kNm}</math>      <math>P_{CY}=1024 \text{ kN}</math></p> <p><u>LOCAL CAPACITY</u></p> <p>LC1      SEMIRIGID= <math>723/1620+19/137 = 0.58 &lt; 1</math><br/> RIGID      = <math>698/1620+24/137 = 0.61</math></p> <p>LC2      SEMIRIGID= <math>642/1620+36/137 = 0.66</math><br/> RIGID      = <math>620/1620+38/137 = 0.66</math></p> <p>LC3      SEMIRIGID= <math>495/1620+23/137 = 0.47</math><br/> RIGID      = <math>484/1620+32/137 = 0.53</math></p> <p><u>LATERAL TORSIONAL BUCKLING</u></p> <p>FROM TABLE 18      <math>m=0.43</math></p> <p>LC1      SEMIRIGID= <math>723/1024+0.43 \times 19/115 = 0.77 &lt; 1</math><br/> RIGID      = <math>698/1024+0.43 \times 24/115 = 0.77</math></p> <p>LC2      SEMIRIGID= <math>642/1024+0.43 \times 36/115 = 0.76</math><br/> RIGID      = <math>620/1024+0.43 \times 38/115 = 0.75</math></p> <p>LC3      SEMIRIGID= <math>495/1024+0.43 \times 23/115 = 0.57</math><br/> RIGID      = <math>484/1024+0.43 \times 32/115 = 0.59</math></p> |

STRUCTURAL  
STEEL DESIGN

| REF | CALCULATION  |
|-----|--|
|     | <p><u>1st STOREY INTERNAL</u></p> <p>SECTION      203x203x71 UC</p> <p><math>M_{CX}=213 \text{ kNm}</math>      <math>P_Z=2920 \text{ kN}</math></p> <p><math>L_E=3.75 \text{ m}</math>      <math>M_b=191 \text{ kNm}</math>      <math>P_{CY}=1590 \text{ kN}</math></p> <p><u>LOCAL CAPACITY</u></p> <p>LC1      SEMIRIGID= <math>1550/2920</math>      = 0.53 &lt;1</p> <p>         RIGID      = <math>1577/2920</math>      = 0.54</p> <p>LC2      SEMIRIGID= <math>1267/2920+50/213</math> = 0.67</p> <p>         RIGID      = <math>1283/2920+42/213</math> = 0.63</p> <p>LC3      SEMIRIGID= <math>930/2920+56/213</math> = 0.58</p> <p>         RIGID      = <math>936/2920+55/213</math> = 0.58</p> <p><u>LATERAL TORSIONAL BUCKLING</u></p> <p>FROM TABLE 18      <math>m=0.43</math></p> <p>LC1      SEMIRIGID= <math>1550/1590</math>      = 0.97 &lt;1</p> <p>         RIGID      = <math>1577/1590</math>      = 0.99</p> <p>LC2      SEMIRIGID= <math>1267/1590+0.43 \times 50/191</math> = 0.91</p> <p>         RIGID      = <math>1283/1590+0.43 \times 42/191</math> = 0.90</p> <p>LC3      SEMIRIGID= <math>930/1590+0.43 \times 56/191</math> = 0.71</p> <p>         RIGID      = <math>936/1590+0.43 \times 55/191</math> = 0.71</p> |

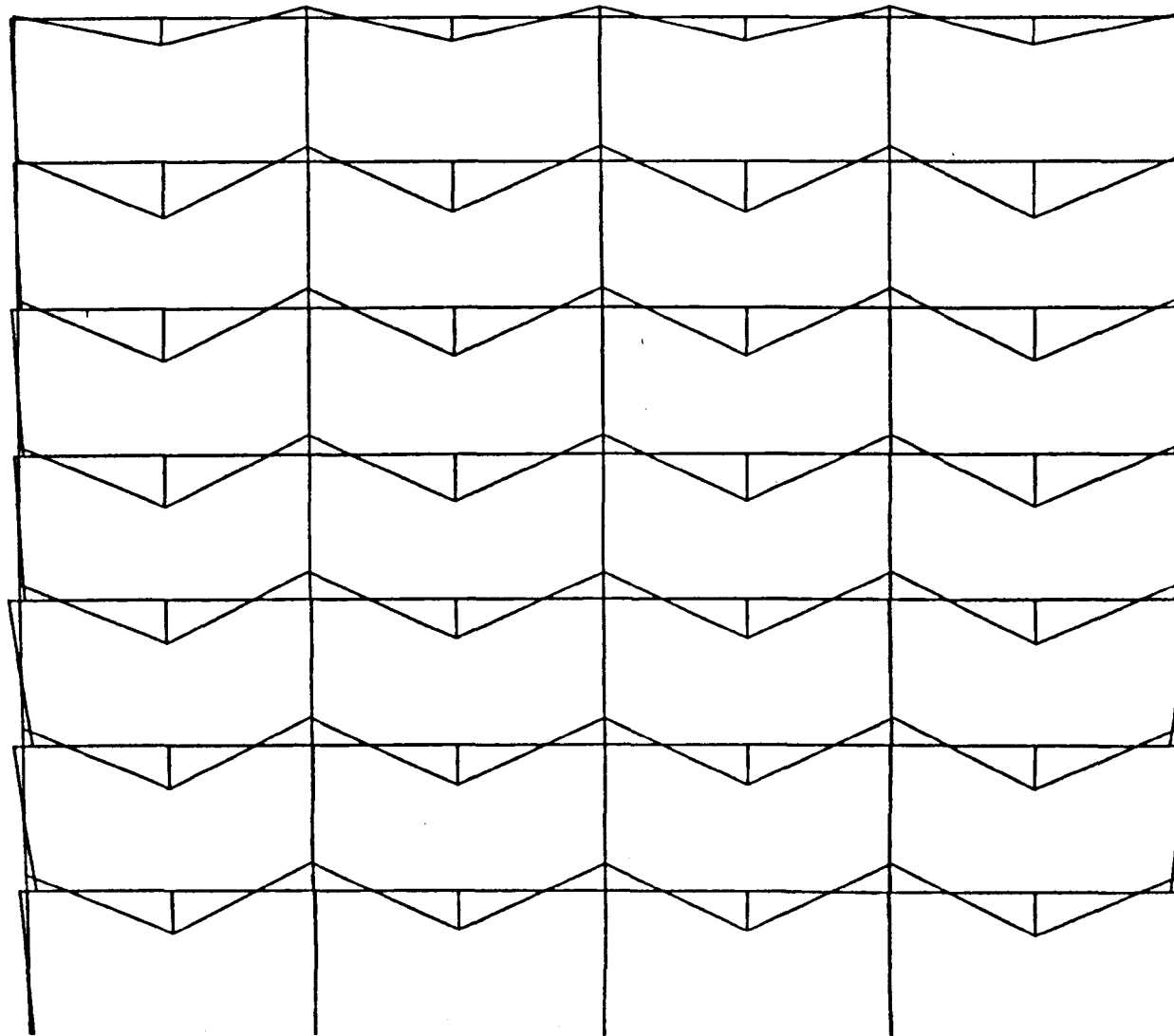
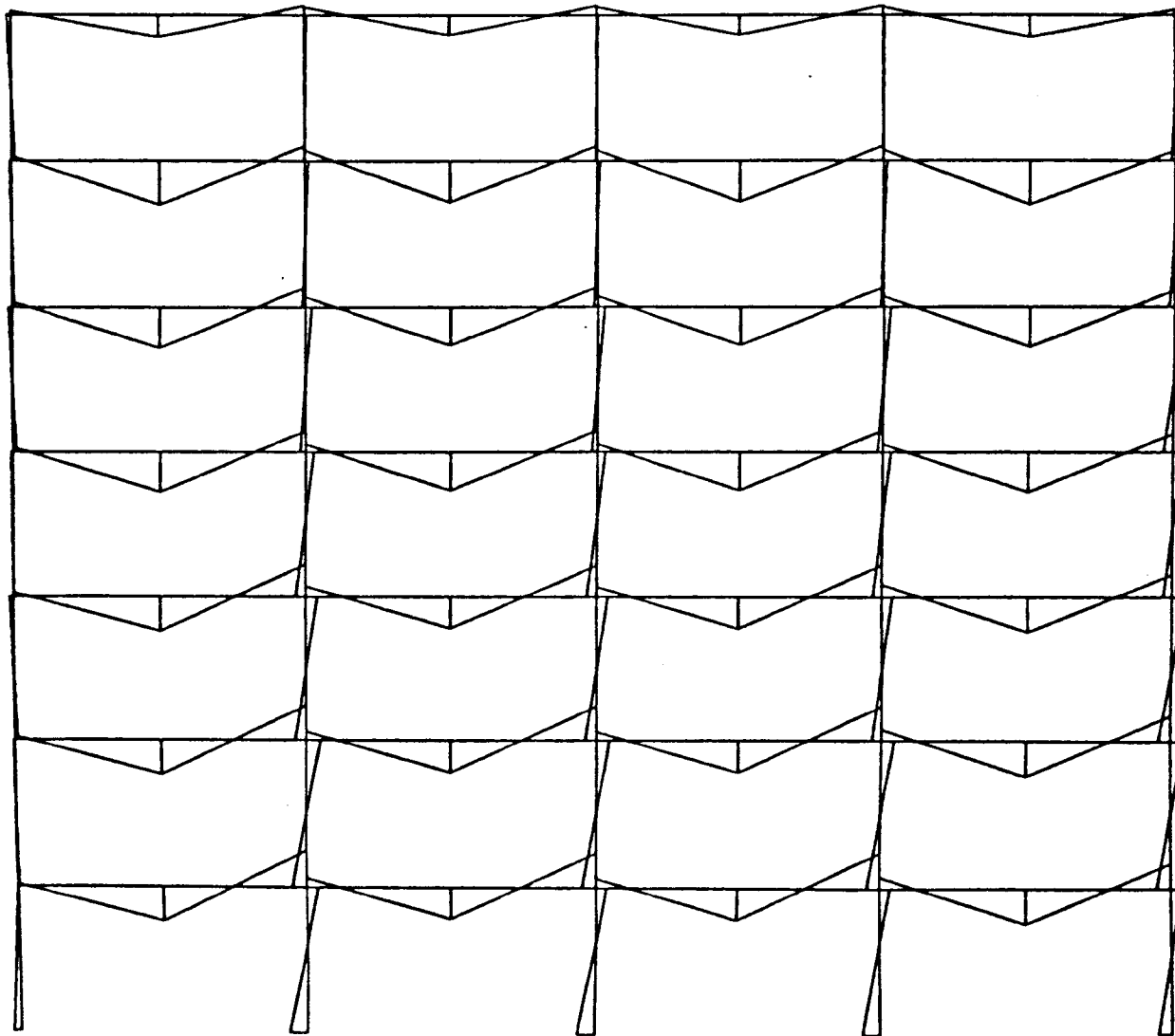


FIG. (A1) BENDING MOMENT DIAGRAM OF SEVEN STOREY FOUR BAY FRAME  
AT DESIGN LOAD LEVEL, COMBINATION (1.4D+1.6I)

SCALE: 0.5Cm:1 kNm



SCALE : 0.5cm:1kNm

FIG. (A2) BENDING MOMENT DIAGRAM OF SEVEN STOREY FOUR BAY FRAME  
AT DESIGN LOAD LEVEL, COMBINATION  $1.2(D+I+W)$

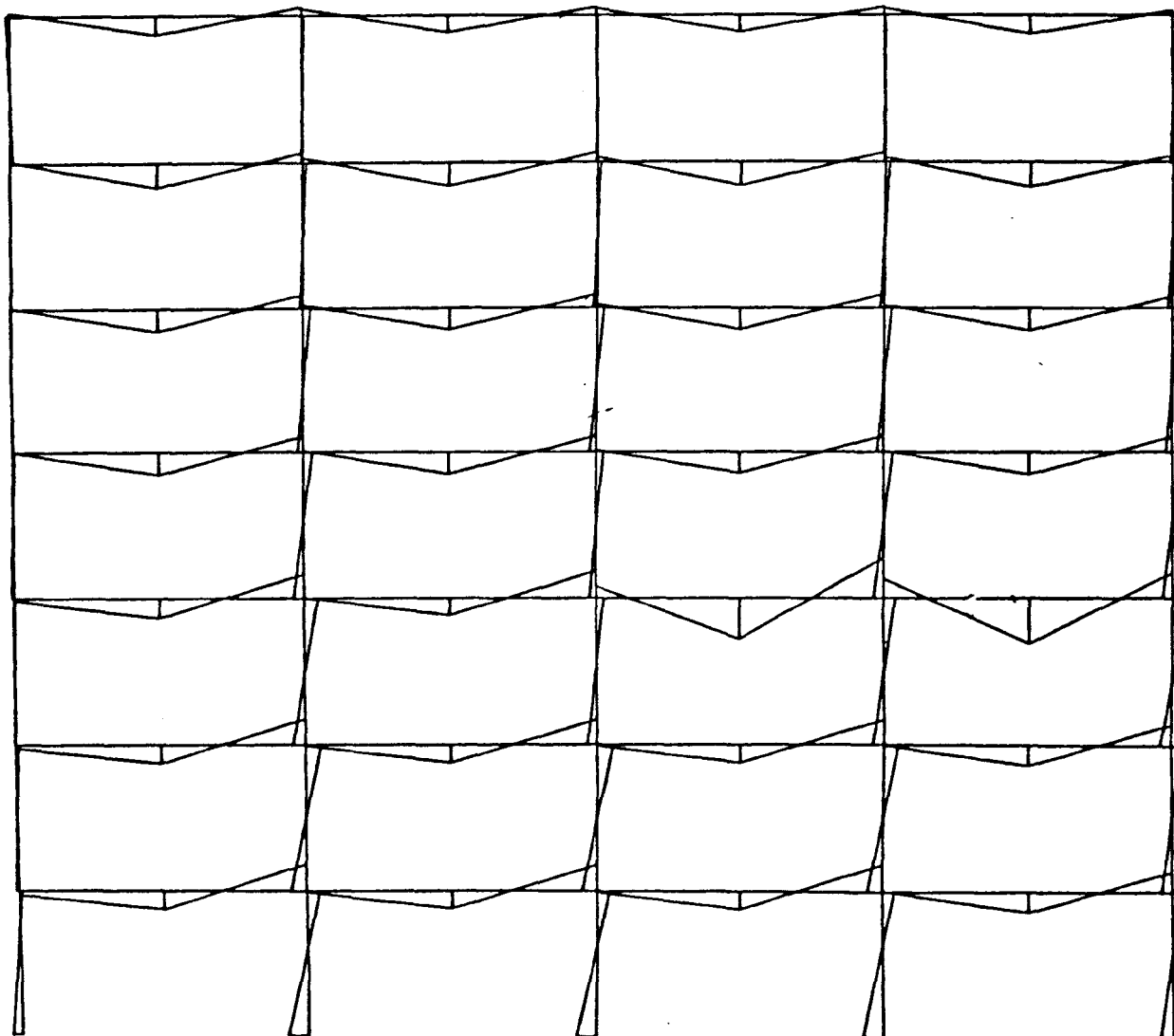


FIG. (A3) BENDING MOMENT DIAGRAM OF SEVEN STOREY FOUR BAY FRAME  
AT DESIGN LOAD LEVEL, COMBINATION 1.4(D+W)