# INVESTMENT AND FINANCING DECISION MODELS UNDER THE IMPUTATION TAX SYSTEM

John Pointon

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University of Warwick

School of Industrial and Business Studies

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## Declaration

An extract from chapter 5 appeared in Accountancy in July 1978 under the title "Towards a corporate cash flow tax system".

Chapter 2 is closely based on a paper entitled "Investment and risk: the effect of capital allowances", which was accepted for publication in Accounting and Business Research in May 1979 and printed in Autumn 1980.

Ashortened version of chapters 3 and 6 was accepted for publication in March 1980 in Accounting and Business Research and will be printed under the heading "Optimal Capital Structure under the Imputation System".

Summary

The aim of this thesis it to explore the effects of the imputation tax system on the relationships between corporate investment and financing decision variables. Under the Capital Asset Pricing Model it is shown that except with instantaneous relief for capital expenditure at 100 per cent, the present system of capital allowances may reduce the expected return to an amount below that required by the post-tax level of risk. A complex equation is derived to show the relationship in partial equilibrium between the after-tax valuation of the levered firm and that of an equity financed firm of equivalent operating risk. This includes the effects of income tax, capital gains tax, corporation tax and risky debt. Sufficient conditions for a neutral tax system are found although these are shown to be violated in practice, with a general preference for debt finance rather than new issues of shares. In general equilibrium capital structure is found to be irrelevant under the UK tax system. For the partial equilibrium model however even in a world of certainty it is shown that the borrowing versus retention decision is complex. It is observed that financial policies may vary over time and are sensitive to the effects of capital investment decisions on (i) Advance Corporation Tax setoff restrictions, (ii) debenture interest carried forward and (iii) the marginal tax rate at which debenture interest is relieved. In turn capital investment decisions are shown to be sensitive to financial decisions in a market which is perfect apart from tax complexities. To accommodate both the peculiarities of the tax rules and the simultaneous solution of investment and financing decisions, a mathematical programming model is presented although it isnoted that in practice it could be difficult to solve.

# CHAPTER 1

## Introduction

The thesis of this work is that in the current state of the theory of business finance there are significant unexplored relationships between taxation and corporate investment and financing decisions. These stem primarily from the peculiarities of the UK imputation tax system.

## 1.1. The case for tax neutrality

In our society the presence of a tax system is inevitable. Musgrave and Musgrave (1976) have described three functions of taxation concerned with the allocation of goods, the redistribution of wealth and the stabilisation of the economy. Very briefly they may be summarised in turn as follows. In the case of a private good the consumer derives benefits to the exclusion of others in return for the price. By contrast that there are some public goods, e.g. national defence, public parks and motorways, which once bought by one individual or group could be enjoyed by others at no marginal cost. For an efficient use of resources the price should equal marginal cost, but at a maximum acceptable price of zero there is a problem in obtaining the goods through a free market. However, from a tax system, designed and enforced by a political voting process, the public goods may be acquired. Second, in order to move towards a socially desirable distribution of income and wealth, tax revenues are required to be raised through central or local government. Third, together with monetary and incomes policies, taxation has been used as a vehicle to influence aggregate demand in the economy.

Although taxation is inevitable the design of the system is of crucial importance. It has been recognised (Musgrave and Musgrave (1976)) that the system should be designed so as not to disturb economic efficiency, assuming the market to be otherwise efficient.

Although the taxpayer suffers a reduction in spending power through the payment of tax, he/she obtains benefits to the extent that public goods are acquired as a consequence. This loss in spending power represents a burden on the economy but the acquisition of public goods is a gain. Provided the tax system does not incur heavy administrative costs and provided there are low costs of compliance, e.g. loss of leisure time through filling in tax forms, then the net loss may be approximately zero. The loss of income or 'income effect' is not an economic inefficiency but an inevitable result of raising revenue to finance expenditure. However a second effect of taxation is the 'substitution effect' which may result in a loss of welfare. If the loss of utility resulting from paying the tax exceeds the minimum loss of utility necessary to acquire the public goods, financed by tax revenues, then there is an excess burden of taxation. The problem arises as a result of distortions in economic choices.

We may define a neutral tax system as one which does not interfere with efficient choices and which results in a minimum excess burden. A requirement of economic efficiency is that "the marginal rate of substitution of X for Z,i.e. the amount of Z which the consumer is willing to surrender for an additional amount of X, should be equal to the marginal rate of transformation of X for Z, being the amount by which the output of Z must be cut to produce an additional unit of X ... In a competitive market both rates are equal to the price ratio of two products ... ( Furthermore) the marginal rate of substitution of future for present consumption, as valued by consumers or savers, should be equal to the marginal rate of transformation of present into future goods in production with both equal to 1 / (1+i), where i is the rate of interest" (Musgrave and Musgrave p.463 (1976)).

For a neutral tax system these marginal rates of substitution before tax should be the same as the respective marginal rates of substitution after tax. Otherwise economic choices are altered, utilities are changed and an excess burden results. The actual measurement of any excess burden is however outside the scope of this piece of research. Indeed it would probably require an explicit utility function for society as a whole (Musgrave and Musgrave (1976) ]. Nevertheless in this thesis we shall be concerned with the less philosophical task of identifying cases of excess burden and formulating investment and financing decision models which aim to be realistic in the treatment of taxation.

## 1.2. Tax imperfections and financial theory

The theory of finance is concerned with the allocation of resources over different points in time both by firms and individuals. It has been argued by Fisher (1930) that an individual's impatience to consume depends on the following characteristics of his income stream:

"1. The size of his expected real income stream.

2. Its expected distribution in time, or its time shape - that is whether it is constant, or increasing, or decreasing, or sometimes one and sometimes the other.

3. Its composition - to what extent it consists of nourishment, or shelter, of amusement, of education, and so on.

4. Its probability, or degree of risk or uncertainty."

Any initial wealth may also affect the analysis. As to firms, "by their production - investment decisions, (they) provide a means for individuals to transform current resources physically into resources to be available in the future" (Fama and Miller p.1. (1972)). Through capital markets individuals can rearrange their patterns of spending and trasnfer resources from different time periods by borrowing or investing in stocks, shares or other securities.

Without a neutral corporate tax system the marginal rate of transformation of present into future goods in production before tax may differ from that after tax. Specifically there will be an excess burden if the net present value of a project before tax is positive, indicating a recommendation of acceptance, yet the net present value after tax is negative. It will be demonstrated that with a neutral tax system the net present value after tax may be a positive fraction of the net present value before tax. This implies that the internal rate of return before tax may be the same as the internal rate of return after tax. In this instance the accept or reject decision of the project is not affected by taxation. Consequently with a neutral tax system the marginal rate of transformation of present into future goods in production after tax equals the rate before tax and also equals 1 / (1+i), where i is the rate of interest.

Consider the rates of substitution of financial instruments. In a competitive market the price ratio of debt to equity should be equal to the marginal rate of substitution of debt for equity and the marginal rate of transformation of debt for equity. With a nil excess burden, taxation does not interfere with the efficient choice between the two. However, if the tax system favours debt, for instance, this may result in an increase in bankruptcy costs which will be an extra burden on the economy. Furthermore if the tax system differentiates between retentions and dividends then the marginal rate of substitution of future for present consumption after tax may differ from that before tax. Again there will be a loss of social utility through the interference with efficient choice.

The differential treatment of dividends and capital gains for tax purposes is of course recognised in the literature of the theory of finance but is not studied in depth. For instance in Fama and Miller (1972) it is mentioned in a footnote on page 84. Emphasis has been placed on the tax deductibility of corporate interest payments following the famous papers by Modigliani and Miller (1958 and 1963). Fama and Miller (1972) p.175, go on to state " We could extend a little further this analysis of the effects of the market imperfections that arise from tax laws. Rapidly however, the conclusions that we could obtain would become more and more ambiguous, and the discussion would become more philosophical than analytical ... Thus rather than speculate about the none too clear-cut effects of these and other market imperfections on the relationships between the financing decisions of firms and their market values, we leave the study of these effects to future research, both theoretical and empirical". It is to this problem of tax imperfections that the thesis is addressed.

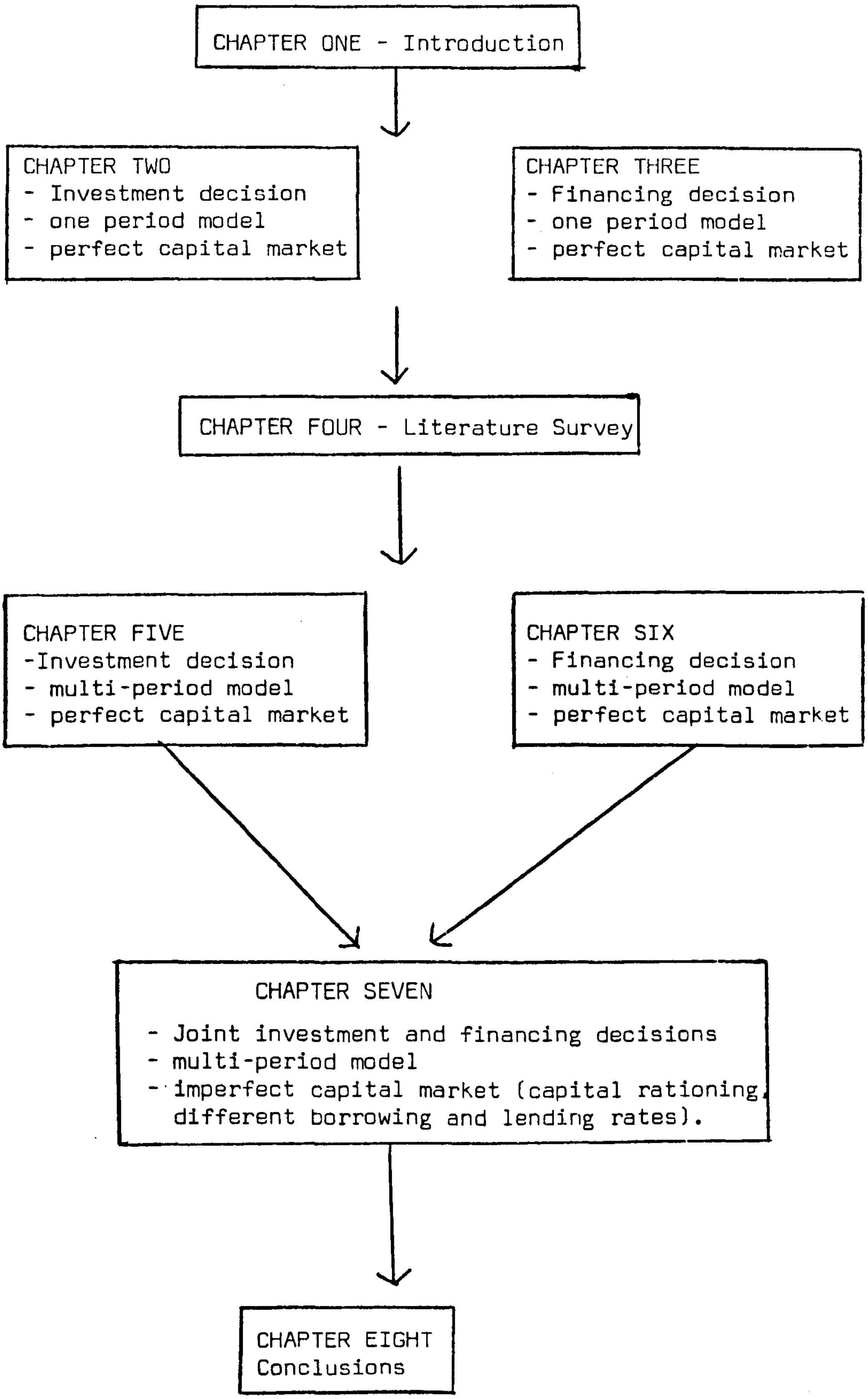
A fundamental theorem in the modern theory of finance is the principle of separation of the firm's production-investment decision from its financing decision in a perfect capital market. In perfect capital markets "markets for consumption goods and investment assets are assumed to be perfect in the sense that all goods and assets are infinitely divisible; any information is costless and available to everybody, there are no transactions costs or taxes; all individuals pay the same price for any given commodity or asset; no individual is wealthy enough to affect the market price of any asset; and no firm is large enough to affect the opportunity set facing consumers" (Fama and Miller p.277 (1972)). Under the UK imputation tax system this separation principle no longer holds. Investment and financing decisions become interrelated through the effect of dividend policy on Advance Corporation Tax and hence on the marginal rate of corporation tax applicable to both capital investment outlays and taxable profits arising from projects; and also through the effect of capital allowance and stock appreciation relief on the carry forward of tax charges on debenture interest. This applies even in capital markets which are perfect apart from tax imperfections.

In a world of uncertainty the theory of finance assumes that investor tastes are based on a model of expected utility. For operational rules however "we must somehow determine either additional restrictions on investor tastes - for example, some assumption about the form of utility functions - or assumptions about common properties of probability distributions of returns - for example, all are normal - that allow us to describe the different alternatives available to an investor in terms of a finite number of parameters." (p.146 Fama and Miller (1972)). The two parameter mean-variance model and the resultant capital asset pricing model (CAPM) have therefore played an important role in the development of the modern theory of finance.

It is simpler to begin the thesis by developing separately the investment and financing decision models. The analysis will start at the point where the existing modern theory of finance under perfect capital markets has presently reached. In analysing the investment decision (chapter 2) it will be shown how there is an insufficient reduction in risk to compensate for the expected slice of taxation. The result is a system of corporate taxation which provides an excess burden on the private sector through financial disincentives to undertake risky investments. As to the financing decision model under CAPM (chapter 3) an important extension is developed to Modigliani and Miller's work under Corporation Tax by determining a solution where the interest on debt capital is not risk-free. Furthermore, a personal tax framework is added in order to determine the effects of dividend policy on optimal capital structure.

After the brief literature survey in chapter 4 the finer details of the UK tax system are discussed within a discounted cash flow framework, initially for the investment decision (chapter 5) and then for the financing decision (chapter 6). Analytically, these chapters represent on their own retrograde steps compared with the theoretical niceties of the CAPM analysis in chapters 2 and 3. Nevertheless the tedious complexities of the UK tax system need to be spelt out within a fairly straightforward model otherwise we would be in danger of losing sight of the wood for the trees. It has already been argued that the UK imputation tax system invalidates the principle of separation of investment and financing decisions and so they are brought together in chapter seven. The form of the model is based on a mathematical programming formulation normally used in business finance in situations of capital rationing. However, it will be shown that the complexities of the UK tax system can be incorporated in the form of a programming model under perfect capital markets as well. Furthermore the carry forward provisions of the legislation require a multiperiod model, and since the CAPM is essentially a one period model, for operational reasons the final model is based on conditions of certainty. Uncertainty could to some extent be introduced by sensitivity analysis, or formally included by the addition of constraints to represent permissible levels of variability of returns although in both cases the principle of parsimony (Bhaskar (1978)) would long since have been violated and the model would lose all usefulness. The grand design is shown in figure 1.

Figure 1 The design



Since the tax legislation is constantly changing either by statute law, case law or extra- statutory concessions it is worth emphasising that this piece of work is based on the UK legislation as described by the Finance Acts 1972 to 1979 and other legislation still valid between those dates.

It also needs to be stated that in the conclusion to each chapter I shall at times express some cursory thoughts on the implications of the material. By contrast the rigour of the analysis will be contained within the inner contents of the chapters.

Finally a word on notation. Because of the complexities of the tax system and hence the numerous variables needed for modelling, the notation used is peculiar to each chapter. My original workings had attempted to achieve absolute consistency but inevitably resulted in the heavy use of subscripts and superscripts. (The lists of notation are to be found at the beginning of each chapter). However, since each item of notation is described in the text when it is introduced, since the variables to represent tax rates are consistent throughout the text, and since there is consistency in each chapter, I sincerely think that the compromise reached leads to a net increase in utility.

# CHAPTER 2

## Investment and risk:  the effect of corporate taxation

*2.1. Abstract*

This chapter presents an analysis of the effect of taxation on the investment decision of the corporate enterprise. Since the Revenue participates in profit-sharing and to some extent loss subsidies, the possibilities of high profits, and perhaps heavy losses, are limited. However, the reduced variability of returns is a reduction in risk, and in itself constitutes an investment incentive. Unfortunately, under the present system of capital allowances there is an insufficient reduction in risk to compensate for the expected slice of Government Revenue. The result is a system of corporate taxation which provides an excess burden on the private sector through financial disincentives to undertake risky investments.

## 2.2. Notation (for chapter 2 only)

$\alpha$ = present value of capital allowances, where $\alpha = 1$ represents 100 per cent capital allowances.

$\beta$ = beta coefficient in a tax-free situation.

$\beta_{T\alpha}$ = beta coefficient with taxation.

$\beta_j$ = $\mathrm{cov}(k_j, k_M)/\mathrm{var}(k_M)$.

$\mathrm{cov}(k_j, k_M)$ = covariance of the rates of return of project j with the rates of return on the "efficient market".

J = investment outlay.

k = discount rate.

$\bar{k}$ = mean rate of return in a tax-free situation.

$\bar{k}_{T\alpha}$ = mean rate of return with taxation.

$\bar{k}_{To}$ = mean rate of return after taxes on inflows but with no relief for capital expenditure.

$\bar{k}_M$ = mean rate of return on the efficient market portfolio.

$\phi$ = an angle such that $\tan \phi = (\bar{k}_M - R_F)$.

$R_F$ = the risk-free rate of interest.

$\bar{r}$ = the minimum required mean rate of return in a tax-free situation.

$\bar{r}_j$ = the minimum required mean rate of return on an individual project or security j.

$\bar{r}_{To}$ = the minimum required mean rate of return after taxes on inflows but with no relief for capital allowances.

$T^*$ = the marginal rate of corporation tax discounted by the time value of money to allow for the time lag between the end of each accounting period and the tax payment date.

$var(k_M)$ = variance of the rates of return on the "efficient market" portfolio.

## 2.3. Introduction

We have stated the importance of examining whether there is a loss of economic efficiency under the imposition of taxation. Indeed, one of the desirable requirements of a fiscal structure is that excess burden is minimised, the existence of excess burden being shown whenever economic choices under a tax system differ from those that would have been made had the tax not been introduced.

We shall now conduct an analysis of the effects of corporate taxation on the risk-return relationship for a particular firm within the framework of the Capital Asset Pricing Model (Sharpe (1964), Lintner (1965), Mossin (1966)).

## 2.4. A neutral tax system

The assumptions of the model may be summarised as follows (quoting Weston and Brigham (1978) and Jensen (1972)):

(1) all investors are single-period expected utility of terminal wealth maximizers who choose among alternative portfolios on the basis of mean and variance (or standard deviation) of returns;

(2) all investors can borrow or lend an unlimited amount at an exogenously given risk-free rate of interest, $R_F$, and there are no restrictions on short sales of any asset;

(3) all investors have identical subjective estimates of the means, variances and covariance of return among all assets, that is, investors have homogeneous expectations;

(4) all assets are perfectly divisible, perfectly liquid (that is, marketable at the going price), and there are no transactions costs;

(5) there are no taxes;

(6) all investors are price-takers;

(7) the quantities of all assets are given.

However, some of these restrictions may be relaxed without very serious consequences to the nature of the analysis:

(a) where the portfolios are not explicitly based on means and variances the same results hold provided the returns on assets are normally distributed or at least symmetric (Fama (1965))

(b) Lintner (1969) has shown that the basic CAPM remains very similar even if investors do not have homogeneous expectations.

(c) The CAPM has been extended to deal with no risk-free asset (Black (1972)) in which case the model is still linear and beta is still the appropriate measure of risk, although the form of the equation is of course slightly different.

(d) Mayers (1972) has considered the case where investors hold some nonmarketable assets. The appropriate measure of risk is now the covariance between the rates of return on the security in question and the rates of return on both the marketable and nonmarketable assets together.

Instead of assumption (5) we shall treat any tax imperfections on the risk-return relationship with respect to the "efficient market" to have been determined prior to the study of the effects of alternative treatments of tax for the particular firm in question. Although this may at first seem a little strange, it is certainly true that with the peculiarities of the present tax system different firms do have different tax profiles. This is reflected, for instance, in different balances of capital allowances brought forward and different reliefs for capital expenditure according to the type of asset.

The question which now presents itself is whether it is feasible to consider disequilibrium states in an equilibrium model. For instance in a perfect frictionless market where any information is costless and available to everybody, "The market value of a firm's securities always fully reflects the market value of any extraordinary production-investment opportunities, with the result that returns on these securities are always in conformance with the equal rate of return principle" (Fama and Miller (1972). "If markets are simply in equilibrium, the exercise (of computing the NPV of a capital budgeting proposal) has a maximum expected incremental value of zero because all expected rents have already been capitalised" (Findlay and Williams (1980)). The problem is however endemic and we can only assume that we are considering a disequilibrium situation whereby a particular firm is considering a project unknown to and unanticipated by the market.

Under the model, the minimum required mean rate of return for a given level of risk, where risk is priced according to the co-variability of a project with the efficient market is

$$\bar{r}_j = R_F + (\bar{k}_M - R_F)\ \beta_j \qquad (1)$$

,

where

$R_F$ = the risk-free interest rate ,

$\bar{k}_M$ = the mean rate of return on the "efficient market" ,

$\beta_j$ = $\mathrm{cov}(k_j, k_M) \,/\, \mathrm{var}(k_M)$ ,

$\mathrm{cov}(k_j, k_M)$ = covariance of the rates of return of project j with the rates of return on the "efficient market",

$\mathrm{var}(k_M)$ = variance of the rates of return on the "efficient market" portfolio ,

$\bar{r}_j$ = the minimum required mean rate of return on an individual project j.

A simplified illustration of the model is shown in Table 1. Although the initial example is partly numerical, the same results obtain if we adopt a general algebraic analysis.

The mean rate of return in a tax-free situation is

$$\bar{k} = \frac{90{,}000}{J} \qquad (2) ,$$

and the beta coefficient in a tax-free situation is

$$\beta = \frac{1}{\mathrm{var}(k_M)} \times \frac{140}{J} \qquad (3) .$$

Let us now consider the effects of tax relief on the capital expenditure at the rate of $\alpha$ where $\alpha$ = 1 for a 100 per cent capital allowance. With proportional tax rates under a cash flow tax system, we can calculate the mean rate of return after tax and a new beta coefficient (see Table 2).

We now observe that the mean rate of return under our special tax system will be

$$\bar{k}_{T\alpha} = \frac{90{,}000\ (1-T^*)}{J\ \ (1-\alpha T^*)} \qquad (4) ,$$

and the beta coefficient will be

$$\beta_{T\alpha} = \frac{1}{\mathrm{var}(k_M)} \times \frac{140\ (1-T^*)}{J\ (1-\alpha T^*)} \qquad (5) ,$$

giving

$$\bar{K}_{T\alpha} = \frac{\bar{K}\,(1-T^*)}{(1-\alpha T^*)} \qquad (6),$$

and

$$\beta_{T\alpha} = \frac{\beta(1-T^*)}{(1-\alpha T^*)} \qquad (7).$$

Substituting for $\alpha = 1$ in equations (6) and (7) we derive

$$\bar{K}_{T1} = \bar{K} \qquad (8),$$

Table 1 : Covariance before tax

| | Possible outcomes | | | |
|---|---|---|---|---|
| | A | B | C | All outcomes |
| Probability of outcome | 0.20 | 0.60 | 0.20 | 1.00 |
| Yield on the 'efficient" Market | 0.05 | 0.06 | 0.12 | - |
| Expected, or average, market yield | 0.010 | 0.036 | 0.024 | 0.070 |
| Deviation of yield on "efficient" market from average market yield for all outcomes | -0.02 | -0.01 | +0.05 | - |
| Project returns, £ per annum | 80,000 | 90,000 | 100,000 | - |
| Rate of return on investment outlay of £s J | $\frac{80{,}000}{J}$ | $\frac{90{,}000}{J}$ | $\frac{100{,}000}{J}$ | - |
| Expected, or average, rate of return (using probabilities given) | $\frac{16{,}000}{J}$ | $\frac{54{,}000}{J}$ | $\frac{20{,}000}{J}$ | $\frac{90{,}000}{J}$ |
| Deviation of project rate of return from average for all outcomes | $\frac{-10{,}000}{J}$ | 0 | $\frac{+10{,}000}{J}$ | |
| Market deviation | -0.02 | -0.01 | +0.05 | - |
| Covariance of rate of return on the project with that of the market | $\frac{40}{J}$ | 0 | $\frac{100}{J}$ | $\frac{140}{J}$ |

Table 2 : Covariance after tax

| | Possible outcomes | | | |
|---|---|---|---|---|
| | A | B | C | All |
| Project returns after tax, £ per annum | 80,000 (1-T*) | 90,000 (1-T*) | 100,000 (1-T*) | - |
| Project rate of return on net investment of $J(1-\alpha T^*)$ | $\frac{80{,}000\ (1-T^*)}{J\ (1-\alpha T^*)}$ | $\frac{90{,}000\ (1-T^*)}{J\ (1-\alpha T^*)}$ | $\frac{100{,}000\ (1-T^*)}{J\ (1-\alpha T^*)}$ | |
| Deviation of project rate of return, for a given outcome, from the average for all outcomes | $\frac{-10{,}000\ (1-T^*)}{J\ (1-\alpha T^*)}$ | 0 | $\frac{+10{,}000\ (1-T^*)}{J\ (1-\alpha T^*)}$ | - |
| Covariance of rate of return on project with that of the market | $\frac{40\ (1-T^*)}{J\ (1-\alpha T^*)}$ | 0 | $\frac{100\ (1-T^*)}{J\ (1-\alpha T^*)}$ | $\frac{140\ (1-T^*)}{J\ (1-\alpha T^*)}$ |

Note: $T^*$ = the marginal corporate tax rate discounted by the time value of money to allow for the time lag between the end of each accounting period and the tax payment date.

and

$$\beta_{T1} \quad = \quad \beta \qquad (9),$$

Hence although the tax system under consideration reduces returns by the amount of the tax bill, it does not alter the *rates* of return if 100 per cent capital allowances are given. Moreover, since rates of return are unaltered, **deviations** of those rates of return and covariance are also unaltered. Even if the returns had been negative in each "state of nature", the returns after tax would have been reduced to (1-T*) times the loss, assuming perfect loss relief, with the same general results. Therefore, given the assumptions behind the Capital Asset Pricing Model, a cash flow tax system, with the characteristics of constant tax rates, constant tax time lags, perfect loss relief and free depreciation, offers neither incentives nor disincentives to risk-taking.

2.5. *The risk disincentive with "imperfect" tax relief*

Let us now examine possible imperfections in the tax system. Consider an extreme case where there is no relief for capital allowances. With $\alpha$ = 0, we have

$$\bar{R}_{T0} \quad = \quad \bar{R}(1-T^*) \qquad (10),$$

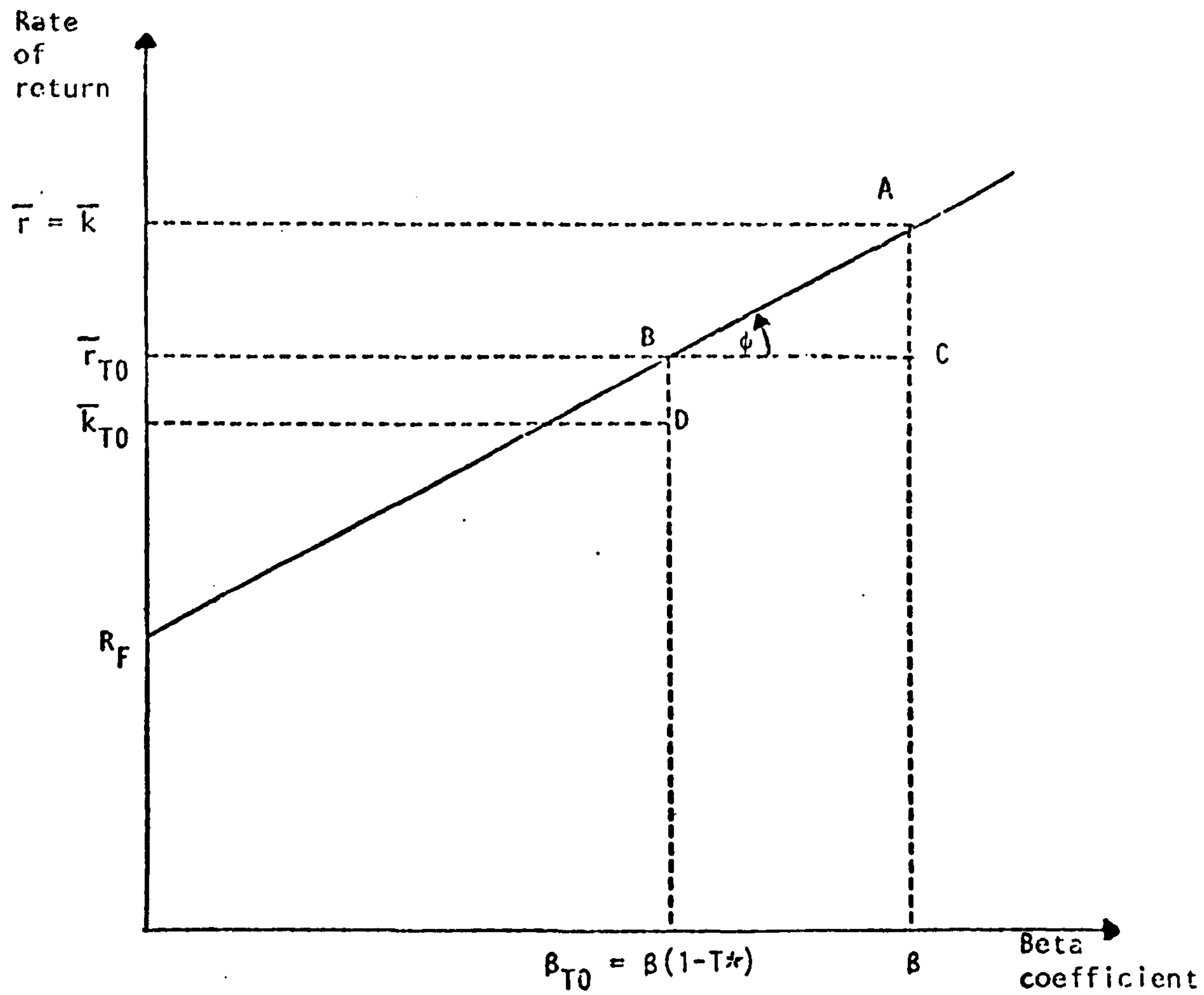
and

$$\beta_{T0} \quad = \quad \beta(1-T^*) \qquad (11),$$

from equations (6) and (7).

The minimum required mean rate of return for the new level of risk, denoted $\beta_{T0}$, is shown in the diagram of Figure 2. Point A represents a project whose mean rate of return in a tax free situation, denoted $\bar{R}$ , is equated with the minimum required mean rate of return, denoted $\bar{r}$, for a given level of risk, $\beta$. With taxes on inflows but no relief on capital expenditure we have a new level of risk, denoted $\beta(1-T^*)$, which determines a new minimum required

Figure 2 *The risk disincentive with taxes on inflows but no capital allowance*



mean rate of return of $\bar{r}_{TO}$. The reduction in the required rate of return is given by

$$AC \quad = \quad \tan\phi\left[\beta-\beta(1-T^*)\right] \qquad (12)\ .$$

From equation (1)

$$\tan\phi \quad = \quad (\bar{K}_M - R_F) \qquad (13)\ ,$$

therefore

$$AC \quad = \quad T^*\beta(\bar{K}_M - R_F) \qquad (14)\ .$$

Now, although full tax relief for capital expenditure would leave the *rate* of return unchanged, since we are considering the absence of capital allowances, the introduction of taxes on inflows decreases the mean rate of return by an expected tax bill at the rate of $(T^*\bar{K})$. Since we are considering a marginal case in the tax-free situation then

$$T^*\bar{K} \quad = \quad T^*\bar{r} \qquad (15)\ .$$

By substitution in equation (1)

$$T^*\bar{K} \quad = \quad T^*R_F + T^*(\bar{K}_M - R_F)\beta \qquad (16)\ .$$

By comparing equations (14) with (16) we note that $T^*\bar{K}$ exceeds AC by the term $T^*R_F$. Hence the expected rate of return, after taxes on inflows but without relief for capital expenditure, is represented by point D in Figure 1, which lies below the line of the minimum required mean rate of return for a given level of risk. Since the expected rate of return is reduced through taxation by more than the required mean rate of return for the new level of risk, there is an economic inefficiency or "excess burden", indicating that the marginal project before tax becomes unattractive after tax.

Effectively, we have considered the following situation. Two firms are about to invest in a similar risky project. The pre-tax returns and their interactions with existing projects are assumed to be identical regardless of which firm proceeds with the investment. Firm X is in a tax-free situation, although Firm Y pays tax it does not receive any tax reliefs on the capital expenditure. Hence if the project is marginal for Firm X, it will be unacceptable for Firm Y and if it is marginal for Firm Y it will be acceptable to Firm X. Let us now measure the extent by which Firm X's project is financially attrative in Firm Y's marginal case.

The minimum required mean rate of return after tax is

$$\bar{r}_{TO} = R_F + (\bar{k}_M - R_F)\ \beta(1-T^*) \qquad (17),$$

being derived directly from Figure 2 and equation (1).

Note that we are now considering an investment which is marginal after tax, and not before tax. Hence

$$\bar{k}_{TO} = \bar{r}_{TO} \qquad (18).$$

where

$\bar{k}_{TO}$ = the expected rate of return after tax on inflows but with no relief for capital expenditure.

By substitution in equation (17) we have

$$\bar{k}_{TO} = R_F + (\bar{k}_M - R_F)\ \beta(1-T^*) \qquad (19).$$

From equation (10) we can substitute $\bar{k}(1-T^*)$ for $\bar{k}_{TO}$, where $\bar{k}$ represents the expected rate of return in a tax-free situation, giving

$$\bar{k} = \frac{R_F}{1-T^*} + (\bar{k}_M - R_F)\beta \qquad (20)$$

However, from equation (1) and from Figure 2 we have by definition

$$\bar{r} = R_F + (\bar{k}_M - R_F)\beta \qquad (21),$$

where

$\bar{r}$ = the minimum required mean rate of return in a non-taxpaying situation.

Hence from equations (20) and (21), the extent by which the expected before tax rate of return, in a tax-paying situation but with no relief for capital expenditure, must exceed the minimum required rate of return in a non-taxpaying situation is

$$\bar{k} - \bar{r} = R_F\left(\frac{1}{1-T^*} - 1\right) \qquad (22),$$

giving

$$\bar{k} - \bar{r} = R_F\left(\frac{T^*}{1-T^*}\right) \qquad (23).$$

When

$T^*$ = 50 per cent,

$\bar{k} - \bar{r} = R_F$ .

Therefore if the risk-free rate is, say 6 per cent, then a firm paying taxes on inflows, but receiving no relief for capital expenditure, needs to find projects with a before tax rate of return of at least 6 per cent more than the required mean rate of return in a non-taxpaying situation. If the effective marginal tax rate is more than 50 per cent, a "tax premium" of more than 6 per cent is required. If the effective marginal tax rate is less than 50 per cent, a "tax premium" of less than 6 per cent is required. The "tax premium" is particularly significant where stock relief is claimed and Advance Corporation Tax Setoff is not restricted. For instance, if net taxable income is more than £85,000 and a claim for stock relief is made throughout the life of the project, then it can be shown that the marginal corporate tax rate is 59.8 per cent (see chapter 5). Ignoring the tax time lag, which will create a slightly reduced effective rate, the tax premium is given by

$$R_F \quad \left(\frac{T^*}{1-T^*}\right) \quad = \quad 0.06 \quad \left(\frac{0.598}{1 - 0.598}\right) \quad = \quad 0.089.$$

Consider a risky project with a required rate of return of 12 per cent if no tax were payable and an expected return of 18 per cent before tax. The effect of a tax-paying situation, but with no capital allowances, is to raise the before-tax required mean rate of return to 12 per cent plus 8.9 per cent, giving 20.9 per cent. Firms paying taxes on inflows but receiving no relief for capital allowances will not be able to undertake such a risky project since the 18 per cent expected rate of return is inadequate; yet a firm not paying tax will find the project attractive since the expected yield will be 6 per cent above the required threshold.

Let us now consider the more general case of $\alpha \neq 0$. From equation (1).

$$\beta_j \quad = \quad \frac{\bar{r}_j - R_F}{\bar{k}_M - R_F} \tag{24}$$

Let

$$\bar{r}_j \quad = \quad \left(\frac{1 - T^*}{1 - \alpha T^*}\right)\bar{r} \tag{25}$$

Therefore

$$\beta_j \quad = \quad \frac{1}{\bar{k}_M - R_F} \quad \left[\frac{(1 - T^*)\bar{r}}{1 - \alpha T^*} \quad - \quad R_F\right] \tag{26}$$

Multiplying by

$(\frac{1 - T^*}{1 - \alpha T^*})$ throughout in equation (21) we obtain

$$\frac{(1 - T^*)\bar{r}}{1 - \alpha T^*} = \frac{(1 - T^*)R_F}{1 - \alpha T^*} + \frac{(1 - T^*)}{(1 - \alpha T^*)} (\bar{R}_M - R_F)\beta \qquad (27).$$

From equations (26) and (27)

$$\beta_j = \frac{1}{\bar{R}_M - R_F} \left[ \frac{(1 - T^*)R_F}{1 - \alpha T^*} + \frac{(1-T^*)(\bar{R}_M - R_F)\beta}{1 - \alpha T^*} - R_F \right]$$

giving

$$\beta_j = \frac{(1 - T^*)}{(1 - \alpha T^*)}\beta - \frac{R_F}{\bar{R}_M - R_F} \left[ 1 - \frac{(1 - T^*)}{(1 - \alpha T^*)} \right] \qquad (29).$$

Since

$$\bar{R}_M > R_F > 0,$$

and

$$0 < \frac{1 - T^*}{1 - \alpha T^*} < 1,$$

then the second term on the right-hand side of equation (29) is negative. Therefore

$$\beta_j < \frac{(1 - T^*)}{(1 - \alpha T^*)} \beta \qquad (30).$$

However, given a marginal investment before tax, the mean rate of return before tax is equated with the minimum required mean rate of return for a given level of risk, denoted $\beta$. Therefore

$$\bar{R} = \bar{r} \qquad (31).$$

and from equation (6)

$$\bar{k}_{T\alpha} = \frac{(1 - T^*)}{(1 - \alpha T^*)} \bar{r} \qquad (32),$$

and the new level of risk is given by equation (7) reproduced below

$$\beta_{T\alpha} = \frac{(1 - T^*)}{(1 - \alpha T^*)} \beta \qquad (33).$$

If we compare equations (25) with (32), and (30) with (33), we find that when

$$\bar{r}_j = \bar{k}_{T\alpha} \quad ,$$

then

$$\beta_j < \beta_{T\alpha} \quad .$$

The relationship is shown in Figure 3 where

$$\beta^* = \beta_j \quad ,$$

for

$$\bar{r}_j = \bar{k}_{T\alpha} \quad .$$

Point A of Figure 3 corresponds with point A of Figure 2. When $\alpha = 0$, symbolising no capital allowance, point D of Figure 2 is represented by point F in Figure 3. The greater the rate of capital allowance the closer $\bar{k}_{T\alpha}$ moves up the vertical axis towards $\bar{k}$, and the smaller the discrepancy between $\beta^*$ and $\beta_{T\alpha}$. The effect of taxation on a project which is marginal before tax is to reduce the expected mean rate of return from $\bar{k}$ to $\bar{k}_{T\alpha}$. At this lower rate of return the maximum degree of risk acceptable is reduced from $\beta$ to $\beta^*$. However, the actual level of risk after tax, denoted $\beta_{T\alpha}$, exceeds the permitted level of $\beta^*$ with the consequence that taxation imperfections make financially unattractive a project which was marginal before tax. Referring back to the introductory comments, we experience an "excess burden".

**2.6.** *A Classification of "imperfect" relief by type of expenditure*

In Table 3 we derive the values of $\alpha$ for different types of expenditure. Note that time lags between (i) the end of each accounting period and the annual tax payment date and (ii) the time of the expenditure and the date of the accounting year-end have been ignored,

Figure 3 *The risk disincentive with taxes on inflows but partial capital allowances*

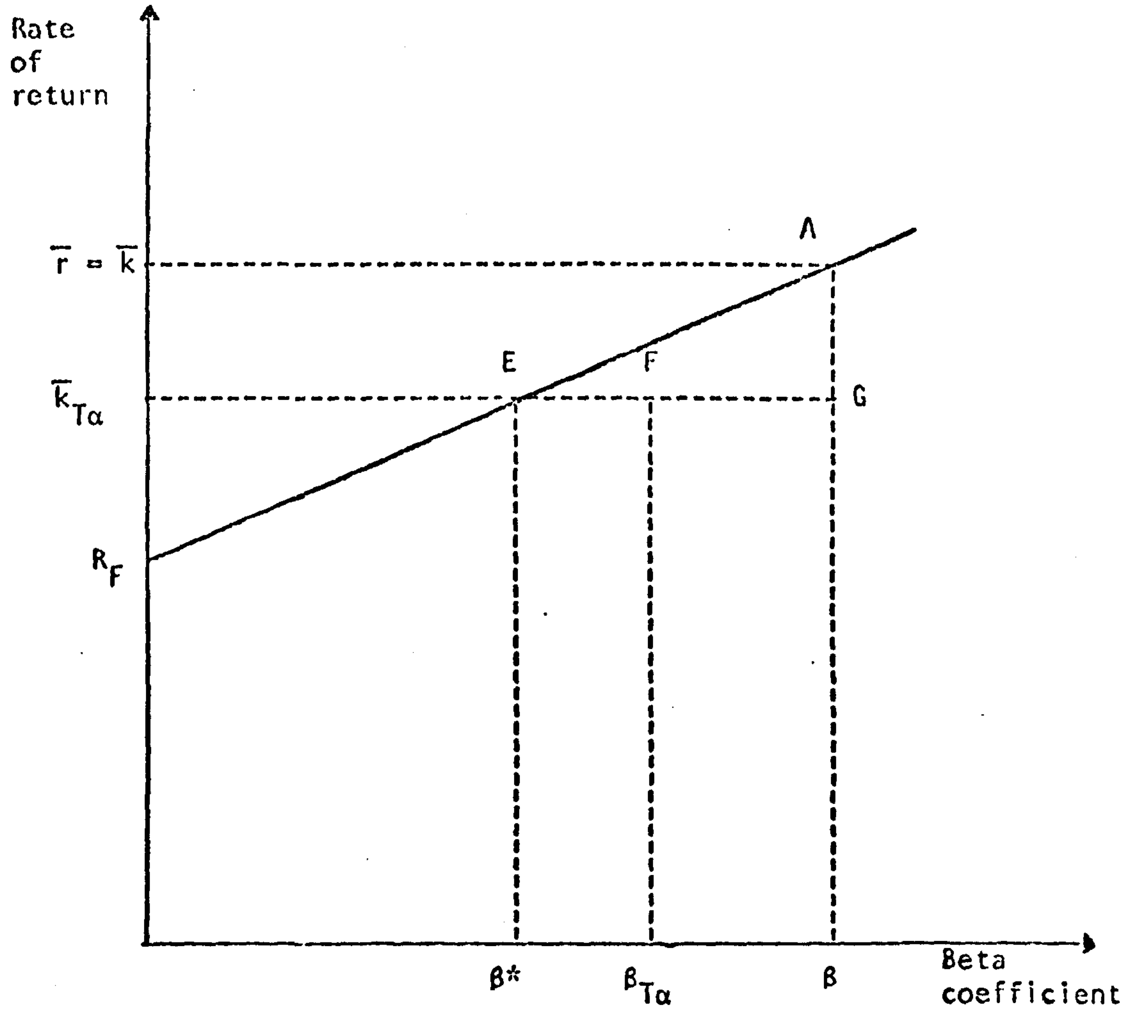


Table 3 *The present value of capital allowances as a proportion of cost*

| Type of expenditure | Values of $\alpha$ k=10% | Values of $\alpha$ k=20% |
|---|---|---|
| Plant and machinery[1] | 1.00 | 1.00 |
| Scientific research | 1.00 | 1.00 |
| New industrial building[2] | 0.81 | 0.72 |
| Second-hand industrial building first acquired in October 1962: | | |
| (a) where the second-hand purchase price in October 1978 does not exceed the cost to the initial owner[3] | 0.32 | 0.26 |
| (b) where the second-hand purchase price in October 1978 is twice the cost to the initial owner[4] | 0.16 | 0.14 |
| (c) where the second-hand purchase price in October 1978 is four times the cost to the initial owner[5] | 0.08 | 0.07 |
| Second-hand industrial building first acquired in December 1962: | | |
| (a) where the second-hand purchase price in December 1978 does not exceed the cost to the initial owner[6] | 0.70 | 0.54 |
| (b) where the second-hand purchase price in December 1978 is twice the cost to the initial owner[7] | 0.35 | 0.27 |
| (c) where the second-hand price in December 1978 is four times the cost to the initial owner[8] | 0.17 | 0.13 |
| Agricultural buildings and works[9] | 0.68 | 0.50 |

| | | |
|---|---|---|
| Patents | | |
| (a) new patents[10] | 0.53 | 0.34 |
| (b) patent with ten years to run[11] | 0.68 | 0.50 |
| Know-how [12] | 0.80 | 0.67 |

*Notes*

1. The calculation of the present value of capital allowances as a proportion of cost ignores (i) time lags between the end of each accounting period and the annual tax payment date, and (ii) the timing of the expenditure in relation to the date of the accounting year end.

2. $$\alpha = 0.50 + 0.04\left[1 + \frac{1}{1+k} + \frac{1}{(1+k)^2} + \ldots\ldots + \frac{1}{(1+k)^{11}}\right] + 0.02\left[\frac{1}{(1+k)^{12}}\right]$$

3. $$\alpha = \frac{1}{34}\left[1 + \frac{1}{1+k} + \frac{1}{(1+k)^2} + \ldots\ldots + \frac{1}{(1+k)^{33}}\right]$$

4. $$\alpha = \frac{1}{2} \times \frac{1}{34}\left[1 + \frac{1}{1+k} + \frac{1}{(1+k)^2} + \ldots + \frac{1}{(1+k)^{33}}\right]$$

5. $$\alpha = \frac{1}{4} \times \frac{1}{34}\left[1 + \frac{1}{1+k} + \frac{1}{(1+k)^2} + \ldots\ldots + \frac{1}{(1+k)^{33}}\right]$$

6. $$\alpha = \frac{1}{9}\left[1 + \frac{1}{1+k} + \frac{1}{(1+k)^2} + \ldots\ldots + \frac{1}{(1+k)^{8}}\right]$$

7. $$\alpha = \frac{1}{2} \times \frac{1}{9}\left[1 + \frac{1}{1+k} + \frac{1}{(1+k)^2} + \ldots\ldots + \frac{1}{(1+k)^{8}}\right]$$

8. $$\alpha = \frac{1}{4} \times \frac{1}{9}\left[1 + \frac{1}{1+k} + \frac{1}{(1+k)^2} + \ldots\ldots + \frac{1}{(1+k)^{8}}\right]$$

9. $$\alpha = \frac{1}{10}\left[1 + \frac{1}{1+k} + \frac{1}{(1+k)^2} + \ldots\ldots + \frac{1}{(1+k)^{9}}\right]$$

10. $$\alpha = \frac{1}{17}\left[1 + \frac{1}{1+k} + \frac{1}{(1+k)^2} + \ldots\ldots + \frac{1}{(1+k)^{16}}\right]$$

11. $$\alpha = \frac{1}{10}\left[1 + \frac{1}{1+k} + \frac{1}{(1+k)^2} + \ldots\ldots + \frac{1}{(1+k)^{9}}\right]$$

12. $\alpha = \frac{1}{6} \left[ 1 + \frac{1}{1+k} + \frac{1}{(1+k)^2} + \ldots + \frac{1}{(1+k)^5} \right]$

since (i) is accommodated by T*, a marginal tax rate discounted for the type (i) lag, and (ii) affects taxes on inflows in the same way as tax relief on outflows assuming that inflows and out-flows occur at the same time within each accounting year.
For plant and machinery and scientific research, $\alpha = 1$, reflecting 100 per cent capital allowances in the year of expenditure as the maximum relief. New industrial buildings receive a 50 per cent allowance in the year of expenditure together with a 4 per cent allowance annually, including one in the first year, until the asset is written off. Hence, the final 2 per cent is written off in the thirteenth year. Expenditure on second-hand industrial buildings is written off over the remaining tax life of the asset. Buildings initially acquired before 6th November 1962 are deemed to have a useful life of 50 years for tax purposes and those initially acquired after 6th November 1962 are deemed to have a life of 25 years. Where a building was built in, say, October 1962 and acquired by another firm in October 1978, then 34 of the 50 years of tax write-offs are still remaining. Where the second-hand purchase price does not exceed the cost to the initial owner, the second-hand price is written off over the remaining life of the asset. However, where the second-hand price exceeds the cost to the initial owner, only the original cost may be written off. This is reflected in a much lower value of $\alpha$. Agricultural buildings and works are written off over ten years, new patents over seventeen years and others over the remaining life of the patent, and know-how is written off over six years.
Under the Capital Allowances Act of 1968 the ineligibility of capital expenditure for tax allowances is applicable to buildings which are either not of "qualifying" type or not of a "qualifying"

trade. Normally there are no capital allowances for investments in buildings not in the manufacturing industry. In these cases $\alpha = 0$, indicating a potentially strong tax disincentive to invest. From Table 3 we see that the partial capital allowances represented by $\alpha < 1$ apply for instance to industrial buildings even of a "qualifying" type and trade. Hence, if we accept the assumptions of the Capital Asset Pricing Model, then we must conclude that the tax system acts as a potential disincentive to expand factory premises and other buildings. But even 100 per cent allowances on plant and machinery cannot always be relieved against taxable income for the year of expenditure or against the three preceding years under section 177(3A) of the Income and Corporation Taxes Act 1970.

2.7 *Conclusion*

Although the real world is more imperfect than the Capital Asset Pricing Model would imply, it does at least provide an insight into the likely "direction" of the effects of taxation on investment decisions, provided of course that the tax system is correctly described by the model. Perhaps the assumptions of immediate relief for losses and the application of taxes to cash inflows and not "accounting" inflows seem at first to be the more obvious violations from reality. Interestingly, if the firm has a large balance of tax losses brought forward then profits from a new capital project may not be taxable with the result that the investment decision is not distorted by the tax system. The analysis presented is relevant, however, for firms with a high level of guaranteed taxable inflows from other projects. As to the assumption of a cash flow tax system on inflows, although the tax base lies within an accrual accounting framework to a large extent the periodic investment in debtors and creditors are self-cancelling, and the increase in inventories is now normally an allowable deduction under the stock relief legislation, a modification away from accrual accounting principles. At least for the purposes of this chapter the approximations are not unreasonable.

We have shown that under the Capital Asset Pricing Model an excess burden on the activities of the private sector may occur through the effect of imperfect tax allowances on capital expenditure. Except in the cases where taxable profits are large enough to absorb 100 per cent tax depreciation on plant and machinery and scientific research, the present system of capital allowances may reduce the expected return to an amount below that required by the post-tax level of risk. Alternatively stated, the fiscal system does not reduce the level of risk by an amount sufficient to compensate for the decline in expected returns. Investments requiring extensions to premises appear to be the hardest hit, and in particular those in the retail industry, since they do not qualify at all for capital allowances on buildings. It would therefore not be unreasonable to assume *a priori* that the tax system has been partly responsible for the low level of investment in this country.

# CHAPTER 3

## The after tax valuation of the levered firm under the asset pricing model

## 8.1 Abstract

The purpose of this chapter is to investigate the effects of the imputation tax system on the optimal capital structure of the corporate enterprise. The valuation procedures derive from the Shape-Lintner-Mossin Asset Pricing Model and so the conclusions are based, in particular, on a capital market which is perfect apart from taxation complexities, and which determines equilibrium prices according to mean and variance.

Although the extension to a risky model differs from that of Modigliani and Miller, not surprisingly the same results of the taxless but otherwise perfect world pertain. However, we develop an important extension to Modigliani and Miller's work under Corporation Tax by determining a solution where the interest on debt capital is not risk-free. Moreover, we incorporate a personal tax framework based on the British system and analyse the effects of dividend policy on optimal capital structure.

In partial equilibrium, with homogeneous tax rates, the requirements of a neutral tax system are violated in the UK situation. By contrast, with heterogeneous tax rates there is a clientele effect and an irrelevant capital structure in general equilibrium.

## 3.2. Notation (for chapter 3 only)

$A$ = proportion of total shares in the levered firm held by an investor,

$b$ = the basic rate of income tax.

$d$ = the net dividend payout rate,

$\mathrm{cov}(r_j, k_m)$ = covariance of the rates of return on security j with those on the efficient market portfolio,

$D$ = the market value of debt capital in a levered firm.

$g$ = the rate of capital gains tax.

$h$ = the (higher) marginal rate of personal tax on investment income.

$h_D$ = higher rate of income tax on interest received,

$h^*_D$ = marginal rate of income tax on interest received for the marginal debentureholder.

$h_S$ = higher rate of income tax on dividends.

$h^*_S$ = marginal rate of income tax on dividends for the marginal shareholder.

$k_d$ = the contractual interest expressed as a proportion of the market value of debt capital, denoted D.

$k_{im}$ = the rate of return on the efficient market portfolio in the ith state of nature.

$\bar{k}_m$ = the mean rate of return on the efficient market portfolio.

$\lambda$ = $(\bar{k}_m - r_F) / \text{var}(k_m)$, where var $(k_m)$ denotes the variance of the rates of return on the efficient market portfolio,

$\sum_{i=1}^{n} p_i$ = the probability that the net operating cash flow is insufficient to cover the debenture interest.

$\sum_{i=1}^{N} p_i$ = the probability that the net operating cash flow will be insufficient to cover in full both the contractual interest and the repayment of debt principal.

$p_i$ = the probabilty of occurrence of the ith state of nature, where i=1 to $\infty$ represents all discrete states.

$\bar{R}_d$ = the mean cash return to the holder of a debt security, after all taxes.

$\bar{R}_e$ = the mean cash return to the holder of an equity security in a levered firm, after all taxes,

$r_F$ = the risk-free rate of interest.

$R_{id}$ = the cash return to the holder of a debt security in the ith state of nature after all taxes.

$R_{ie}$ = the cash return to the holder of an equity security in a levered firm in the ith state of nature, after all taxes.

$R_{ij}$ = the cash return to the holder of security j in the ith state of nature, after all taxes,

$\bar{r}_j$ = the minimum required mean rate of return on an individual security j .

$r_o$ = equilibrium rate of interest on fully tax-exempt bonds ,

$\bar{R}_T$ = the mean tax bill,

$\bar{R}_u$ = the mean cash return to the holder of an equity security in an unlevered firm, after all taxes,

$S_L$ = the market value of equity capital in a levered firm ,

$S_u$ = the market value of equity capital in an unlevered firm ,

$T$ = the full rate of corporation tax ,

$T_{PD}$ = personal income tax rate on income from debt ,

$T^*_{PD}$ = marginal rate of personal income tax on interest from debt for the marginal debentureholder,

$T_{PS}$ = personal tax rate for shareholders ,

$V_j$ = the market value of security j .

$V_L$ = the market value of a levered firm($V_L = S_L + D$).

$V_u$ = the market value of an unlevered firm ($V_u = S_u$) .

$X_i$ = the net operating cash flow before tax in the ith state of nature

### 3.3. Optimal capital structure in the tax-free situation

Under the Capital Asset Pricing Model[1] the proposition by Modigliani and Miller, in their 1958 paper,[2] that "the market value of any firm is independent of its capital structure and is given by capitalising its expected return at the rate appropriate to its class" still holds in the tax-free situation. This may be proved as follows.

The mean rate of return denoted $\bar{r}_j$ of a risky security j may be described by

$$\bar{r}_j = r_F + \lambda \operatorname{cov}(r_j, k_m) \qquad (1)\,,$$

where $r_F$ = the risk-free interest rate

$$\lambda = \frac{\bar{k}_m - r_F}{\operatorname{var}(k_m)}\,,$$

$\bar{k}_m$ = the meanrate of return on the efficient market portfolio, and

$\operatorname{var}(k_m)$ = the variance of the rates of return on the efficient market portfolio.

By definition

$$\operatorname{cov}(r_j, k_m) = \sum_{i=1}^{\infty} p_i \,(r_{ij} - \bar{r}_j)\,(k_{im} - \bar{k}_m) \qquad (2)\,,$$

where $p_i$ = the probability of occurence of the ith state of nature, where i=1 to $\infty$ represents all states.

Expressing our rates of return in terms of cash flows

$$r_{ij} = \frac{R_{ij}}{V_j} - 1 \qquad (3)\,.$$

where $R_{ij}$ = the cash returns to the holder of security j in the ith state of nature. (in a one period model it includes a return of both interest and capital)

$V_j$ = the market value of security j .

Hence $\bar{r}_j = \dfrac{\bar{R}_j}{V_j} - 1$ (4) .

From (2) and (3) and (4)

$$cov\ (r_j, k_m) = \sum_{i=1}^{\infty} p_i \left[\frac{(R_{ij}-1)}{V_j} - \frac{(\bar{R}_j -1)}{V_j}\right] \left[k_{im} - \bar{k}_m\right]$$

$$= \sum_{i=1}^{\infty} p_i \frac{(R_{ij} - \bar{R}_j)\ (k_{im} - \bar{k}_m)}{V_j} \qquad (5)$$

From (1) (4) and (5),

$$\frac{\bar{R}_j}{V_j} - 1 = r_F + \lambda \frac{\sum_{i=1}^{\infty} p_i\ (R_{ij} - \bar{R}_j)\ (k_{im} - \bar{k}_m)}{V_j} .$$

Multiplying by $V_j$ and rearranging:

$$V_j\ (1 + r_F) = \bar{R}_j - \lambda \sum_{i=1}^{\infty} p_i\ (R_{ij} - \bar{R}_j)\ (k_{im} - \bar{k}_m) \qquad (6) .$$

Now, where a firm is unlevered

$R_{ij} = X_i$ ,

where

$X_i$ = the net operating cash flow before tax in the ith state of nature with mean $\bar{X}$ *[There are two assumptions concerning the definition of $X_i$ which need to be made explicit. Firstly given (a) the choice of a one-period valuation model and (b) the assumption that in states i = 1 to N when $(X_i - Dk_d)(1-T) < D$ then shares are worth zero, then

$X_i$ is not net operating cash flow: it is the end of period liquidation value of all the company's assets. This problem makes a difference taxwise. Secondly, it will be stated that in states $i = N+1$ to $\infty$ the proportion $(1-d)$ of the positive excess proceeds $(X_i - Dk_d)(1-T) - D$ will be reinvested. But it is implicitly assumed that retentions earn only the firm's required rate rate of return (i.e. there are no quasi-rents).] *

Hence:

$$V_u \ (1+r_F) = \bar{X} - \lambda \sum_{i=1}^{\infty} p_i \ (X_i - \bar{X}) \ (k_{im} - \bar{k}_m) \qquad (7) \ ,$$

where $V_u$ = the market value of the unlevered firm.

From equation (6)

$$D \ (1+r_F) = \bar{R}_d - \lambda \sum_{i=1}^{\infty} p_i \ (R_{id} - \bar{R}_d) \ (k_{im} - \bar{k}_m) \qquad (8) \ ,$$

* I am extremely indebted to Professor Ken Peasnell for these points within the squared brackets.

and $S_L \quad (1 + r_F) = \bar{R}_e - \lambda \sum_{i=1}^{\infty} p_i \quad (R_{ie} - \bar{R}_e) \quad (k_{im} - \bar{k}_m)$ (9),

where D = the market value of the debt capital in a levered firm,

j = d represents a debt security,

j = e represents an equity security in a levered firm,

$S_L$ = the market value of equity capital in a levered firm.

However, since the cash returns to equity holders are given by the net operating cash flow after payment of interest and debt capital

$$R_{ie} = X_i - R_{id} \tag{10}$$,

$$\text{and } \bar{R}_e = \bar{X} - \bar{R}_d \tag{11}$$.

From (8) (9) (10) and (11)

$$S_L \ (1+r_F) + D \ (1+r_F) =$$

$$\bar{R}_e + \bar{R}_d - \lambda \sum_{i=1}^{\infty} p_i \quad (R_{ie} - \bar{R}_e + R_{id} - \bar{R}_d) \ (k_{im} - \bar{k}_m)$$

$$= \bar{X} - \lambda \sum_{i=1}^{\infty} p_i \quad (X_i - \bar{X}) \quad (k_{im} - \bar{k}_m) \tag{12}$$.

Therefore from (7) and (12)

$$V_u \quad (1+r_F) = \bar{X} - \lambda \sum_{i=1}^{\infty} p_i \quad (X_i - \bar{X}) \ (k_{im} - \bar{k}_m) = S_L \ (1+r_F) + D(1+r_F)$$

$$\text{and } V_u = V_L = \frac{1}{1 + r_F} \left[ \bar{X} - \lambda \text{ cov } (X, k_m) \right] \tag{13}$$,

where $V_L$ = the market value of a levered firm $(S_L + D)$.

Hence, in a tax free situation, the market value of any firm is independent of its capital structure given

(1) homogeneous expectations,

(2) single period wealth maximisation based on mean and variance,

(3) borrowing and lending at a risk free rate of interest,

(4) perfect capital markets.

In particular the market value of debt is interesting. From Equation (8) may be derived an expression for the value of risky debt capital, which covers the situation where it is not certain that claims by debt-holders will be fully met. In the extreme case where the net operating cash flow will be insufficient to meet claims of debt capital and interest in all states of nature, then the returns to debtholders will be exactly matched by the net operating cash flow.

Formally

$$R_{id} = X_i \text{ for } N = \infty \quad ,$$

$$\text{and } D = \frac{\bar{X} - \lambda \text{ cov } (X, k_m)}{1 + r_F} \qquad \text{for } N = \infty \ ,$$

where $\sum_{i=1}^{N} p_i$ = the probability that the net operating cash flow will be insufficient to cover both the contractual interest and repayment of principal.

If the debt claims are never fully met in all states of nature, then debt capital is essentially an equity investment and hence the relevant risk premium relates to the systematic risk of the firm's operating cash flows. However, as the probability of the claims being met increases the risk premium is reduced.

$$\lambda \sum_{i=1}^{\infty} p_i \ (X_i - \bar{X}) \ (k_{im} - \bar{k}_m) \geqslant \lambda \sum_{i=1}^{\infty} p_i \ (R_{id} - \bar{R}_d) \ (k_{im} - \bar{k}_m) \quad .$$

$$\text{i.e. } \lambda \sum_{i=1}^{\infty} p_i \ (X_i - \bar{X}) \ (k_{im} - \bar{k}_m)$$

$$\geqslant \lambda \sum_{i=1}^{N} p_i \ (X_i - \bar{R}_d) \ (k_{im} - \bar{k}_m) + \lambda \sum_{i=N+1}^{\infty} p_i \ (R_{id} - \bar{R}_d) \ (k_{im} - \bar{k}_m) \quad ,$$

since $\quad X_i = R_{id}$ for $i = 1$ to $N$ ,

and $\quad X_i \geqslant R_{id}$ for $i = N+1$ to $\infty$ ,

and $\quad \bar{X} \geqslant \bar{R}_d$ ,

It is, of course, assumed that the firm's net operating cash flows are not negatively correlated with those of the efficient market, i.e. the systematic risk is not negative.

At the other extreme, where the minimum operating cash flow exceeds the repayments of interest and capital then

$$\sum_{i=1}^{N} p_i = 0 \quad ,$$

and $R_{id} = \bar{R}_d$ for all $i$ ,

Hence from equation (8)

$$D(1+r_F) = \bar{R}_d \quad ,$$

giving $r_F = \dfrac{\bar{R}_d}{D} - 1$ (14) .

But from (4) and (14) for $j = d$,

$\bar{r}_d = r_F$ revealing as expected that where repayments of interest and capital are guaranteed with certainty the market rate of interest on debt capital is, in a perfect market, equated with the risk-free rate.

3.4. The effect of corporate taxation on the valuation of the firm

Let us now turn to the effects of a tax system based on operating cash flows with perfect relief for losses and tax relief given on debenture interest paid. We shall begin by analysing the mean returns to debtholders and shareholders and then calculate the covariance of returns to each security holder.

Let n be defined such that for i = 1 to n the net operating cash flow is insufficient to cover the interest and hence there is no taxable income. For i = n+1 to $\infty$ tax is only paid on the net operating cash flow $X_i$ less the interest. Hence the mean tax bill, denoted, $\bar{R}_T$ is given by:

$$\bar{R}_T = \sum_{i=n+1}^{\infty} p_i \quad (X_i - Dk_d)T \tag{15}$$

where $k_d$ = the contractual interest expressed as a proportion of the market value of debt capital, denoted D.

As far as shareholders are concerned they only receive income when the net operating cash flow after interest and tax, $\left[(X_i - Dk_d) \ (1-T)\right]$, exceeds the repayment of debt capital, D. Hence the mean payment to shareholders, denoted $\bar{R}_e$ is given by:

$$\bar{R}_e = \sum_{i=N+1}^{\infty} p_i \left[(X_i - Dk_d) \ (1-T) - D\right] \tag{16}$$

The position with respect to debtholders is more complicated. Where the net operating cash flow, $X_i$ does not exceed the contractual interest payment the firm pays no tax and hence the payment to debtholders is $X_i$, therefore $R_{id} = X_i$ for i to 1 to n (17).

The fact that $Dk_d-R_{id}$ may be positive illustrates that debt may be subject to some risk.

Where the net operating cash flow exceeds the interest payment the firm pays tax of $(X_i-Dk_d)$ T.

But if there is still insufficient to repay the debt capital the returns to debtholders are equal to the net operating cash flow, $X_i$, less the tax bill of $(TX_i - TDk_d)$. Therefore $R_{id} = X_i - TX_i + TDk_d$ for $i = n+1$ to $N$ (18) ,

Finally where the firm has sufficient income after tax to repay the debt interest and capital:

$$R_{id} = Dk_d + D \text{ for } i = N+1 \text{ to } \infty \qquad (19) .$$

Hence the mean payment to debt holders, denoted $\bar{R}_d$, is given by

$$\bar{R}_d = \sum_{i=1}^{n} p_i X_i + \sum_{i=n+1}^{N} p_i \ (X_i - TX_i + TDk_d) + \sum_{i=N+1}^{\infty} p_i \ (Dk_d + D) \quad (20) ,$$

By way of reconciliation we may calculate from equations (15),(16) and (20)

$$\bar{R}_e + \bar{R}_d + \bar{R}_T =$$

$$\sum_{i=n+1}^{\infty} p_i \ (X_i)T - \sum_{i=n+1}^{\infty} p_i (Dk_d)T + \sum_{i=N+1}^{\infty} p_i X_i - \sum_{i=N+1}^{\infty} p_i (X_i)T - \sum_{i=N+1}^{\infty} p_i Dk_d +$$

$$\sum_{i=N+1}^{\infty} p_i \ Dk_d \ T - \sum_{i=N+1}^{\infty} p_i D + \sum_{i=1}^{n} p_i X_i + \sum_{i=n+1}^{N} p_i X_i - \sum_{i=n+1}^{N} p_i X_i T + \sum_{i=n+1}^{N} p_i Dk_d T +$$

$$\sum_{i=N+1}^{\infty} p_i \ Dk_d + \sum_{i=N+1}^{\infty} p_i D \quad ,$$

$$\text{Therefore } \bar{R}_e + \bar{R}_d + \bar{R}_T = \sum_{i=N+1}^{\infty} p_i \ X_i + \sum_{i=1}^{n} p_i X_i + \sum_{i=n+1}^{N} p_i X_i \quad ,$$

$$\text{giving } \bar{R}_e + \bar{R}_d + \bar{R}_T = \bar{X} \qquad (21) ,$$

As expected, the mean net operating cash flow must equal the mean returns to all security holders and the Inland Revenue.

Finally, from equations (15) and (21)

$$\bar{R}_e + \bar{R}_d = \bar{X} - \sum_{i=n+1}^{\infty} p_i \ (X_i - Dk_d) \ T \qquad (22) ,$$

For the moment, this completes our analysis of the mean returns to security holders after corporation tax.

Let us now investigate the covariances.

By definition,

$$\text{cov}\ (R_j, k_m) = \sum_{i=1}^{\infty} p_i\ (R_{ij} - \bar{R}_j)\ (k_{im} - \bar{k}_m)$$

$$= \sum_{i=1}^{\infty} p_i\, R_{ij}\ (k_{im} - \bar{k}_m)\ - \bar{R}_j \sum_{i=1}^{\infty} p_i\, k_{im} + \bar{R}_j \sum_{i=1}^{\infty} p_i\, \bar{k}_m \quad .$$

But since $\bar{R}_j \sum_{i=1}^{\infty} p_i k_{im} = \bar{R}_j \bar{k}_m$ and $\bar{R}_j \sum_{i=1}^{\infty} p_i \bar{k}_m = \bar{R}_j \bar{k}_m$, then

$$\text{cov}\ (R_j\, k_m) = \sum_{i=1}^{\infty} p_i\, R_{ij}\ (k_{im} - \bar{k}_m) \qquad (23) \quad .$$

This will be used as the definition of the covariance.

Now, $R_{ie} = 0$ for $1 = 1$ to $N$ ,

and, from equation (16)

$$R_{ie} = (X_i - Dk_d)\ (1 - T) - D \quad \text{For } i = N + 1 \text{ to } \infty$$

$$= X_i - TX_i + TDk_d - Dk_d - D \quad \text{For } i = N + 1 \text{ to } \infty \qquad (24) \quad .$$

Therefore from equations (23) and (24) for $j = e$,

$$\text{cov}\ (R_{ie}, k_m) = \sum_{i=N+1}^{\infty} p_i\ (k_{im} - \bar{k}_m)\ \ (X_i - TX_i + TDk_d - Dk_d - D) \quad (25) \quad .$$

Equation (23), rewritten for debtholders, is given by

$$\text{cov}\ (R_{id}, k_m) = \sum_{i=1}^{\infty} p_i\, R_{id}\ (k_{im} - \bar{k}_m) \qquad (26) \quad .$$

From equations (17), (18), (19) and (26)

$$\text{cov}\ (R_{id}, k_m) = \sum_{i=1}^{n} p_i\ \ (k_{im} - \bar{k}_m)\ \ (X_i)\ +$$

$$+ \sum_{i=n+1}^{N} p_i\ (k_{im} - \bar{k}_m)\ \ (X_i - TX_i + TDk_d)\ \ + \text{(PTO)}$$

$$+ \sum_{i=N+1}^{\infty} p_i \quad (k_{im} - \bar{k}_m) \quad (Dk_d + D) \tag{27}$$

From equations (25) and (27)

$$\text{cov} \quad (R_{ie}, k_m) + \text{cov} \; (R_{id}, k_m) \quad = \sum_{i=1}^{n} p_i \; (k_{im} - \bar{k}_m) \quad (X_i)$$

$$+ \sum_{i=n+1}^{N} p_i \; (k_{im} - \bar{k}_m) \quad (X_i - TX_i + TDk_d)$$

$$+ \sum_{i=N+1}^{\infty} p_i \quad (k_{im} - \bar{k}_m) \quad (X_i - TX_i + TDk_d)$$

$$= \sum_{i=1}^{n} p_i \quad (k_{im} - \bar{k}_m) \; (X_i) + \sum_{i=n+1}^{\infty} p_i \; (k_{im} - k_m) \; (X_i - TX_i + TDk_d)$$

$$= \sum_{i=1}^{\infty} p_i \; (k_{im} - \bar{k}_m) \quad (X_i) - \sum_{i=n+1}^{\infty} p_i \; (k_{im} - \bar{k}_m) \quad (TX_i - TDk_d) \tag{28}$$

Now the CAPM valuation model shows (equations (9) and (8)), rewritten for convenience,

$$S_L \; (1 + r_F) \quad = \quad \bar{R}_e \quad - \quad \lambda \; \text{cov} \quad (R_e, k_m) \tag{29}$$

$$D \; (1+r_F) \quad = \quad \bar{R}_d \quad - \quad \lambda \quad \text{cov} \quad (R_d, \; k_m) \tag{30}$$

$$\text{giving } V_L \; (1+r_F) = \bar{R}_e + \bar{R}_d \quad - \quad \lambda \quad (\text{cov} \; (R_e, \; k_m) + \text{cov} \; (R_d, k_m)) \tag{31}$$

From equations (22), (28) and (31)

$$V_L \; (1+r_F) = \bar{X} \quad - \sum_{i=n+1}^{\infty} p_i \; (X_i - Dk_d)T - \lambda \; ( \sum_{i=1}^{\infty} p_i \; (k_{im} - \bar{k}_m) \; (X_i) -$$

$$\sum_{i=n+1}^{\infty} p_i \; (k_{im} - \bar{k}_m) \quad (TX_i - TDk_d)) \tag{32}$$

Let us now compare this with the respective equation for an unlevered firm, the covariance for which is, from equation (23)

$$\mathrm{Cov}\ ((1-T)X,k_m) = (1-T) \sum_{i=1}^{\infty} p_i X_i (k_{im} - \bar{k}_m) \qquad (33)$$

Therefore, from equation (6), after corporation tax,

$$V_u (1 + r_F) = \bar{X} (1-T) - \lambda (1-T) \sum_{i=1}^{\infty} p_i X_i (k_{im} - \bar{k}_m) \qquad (34)$$

$$\text{Therefore } \bar{X} = V_u (1+r_F) + T\bar{X} + \lambda \sum_{i=1}^{\infty} p_i X_i (k_{im} - \bar{k}_m) - T\lambda \sum_{i=1}^{\infty} p_i X_i (k_{im} - \bar{k}_m) \qquad (35)$$

Substituting for $\bar{X}$ in Equation (32)

$$V_L (1+r_F) = V_u(1+r_F) + T \sum_{i=1}^{\infty} p_i X_i + \lambda \sum_{i=1}^{\infty} p_i X_i (k_{im} - \bar{k}_m)$$

$$- T\lambda \sum_{i=1}^{\infty} p_i X_i (k_{im} - \bar{k}_m) - T \sum_{i=n+1}^{\infty} p_i X_i + \sum_{i=n+1}^{\infty} p_i Dk_d T$$

$$- \lambda \sum_{i=1}^{\infty} p_i (k_{im} - \bar{k}_m) (X_i) + \lambda \sum_{i=n+1}^{\infty} p_i (k_{im} - \bar{k}_m) (TX_i)$$

$$- \lambda \sum_{i=n+1}^{\infty} p_i (k_{im} - \bar{k}_m) (TDk_d) \quad .$$

Therefore:

$$V_L (1+r_F) = V_u (1+r_F) + T \sum_{i=1}^{n} p_i X_i - \lambda \sum_{i=1}^{n} p_i TX_i (k_{im} - \bar{k}_m)$$

$$+ T \sum_{i=n+1}^{\infty} p_i Dk_d - \lambda \sum_{i=n+1}^{\infty} p_i (k_{im} - \bar{k}_m) TDk_d \quad .$$

$$\text{Therefore } V_L (1+r_F) = V_u (1+r_F) + \sum_{i=1}^{n} p_i TX_i (1 - \lambda (k_{im} - \bar{k}_m))$$

$$+ \sum_{i=n+1}^{\infty} p_i TDk_d (1 - \lambda (k_{im} - \bar{k}_m)) \qquad (36)$$

Hence $V_L = V_u + \frac{T}{1+r_F} \left\{ \sum_{i=1}^{n} p_i X_i (1-\lambda (k_{im} - \bar{k}_m)) \right.$

$$\left. + \sum_{i=n+1}^{\infty} p_i Dk_d (1 - \lambda (k_{im} - \bar{k}_m)) \right\} \qquad (37),$$

Where the interest is risk-free n = 0 and $k_d = r_F$, giving

$$V_L = V_u + \frac{T}{1+r_F} r_F D \sum_{i=1}^{\infty} p_i (1 - \lambda (k_{im} - \bar{k}_m)),$$

But $\sum_{i=1}^{\infty} p_i k_{im} = \bar{k}_m$, hence ,

$$V_L = V_u + \frac{TDr_F}{1+r_F} \qquad (38),$$

This is the CAPM equivalent of the Modigliani-Miller 1963 paper. "The value of the levered firm exceeds that of the unlevered firm by the capitalised value of the tax relief on the interest payments" .... "capitalised at the more favourable certainty rate r rather than at the rate for uncertain streams "....where...." r = the rate at which the market capitalises the sure streams generated by debts."

There is a slight discrepancy in that the MM paper gives

$$V_L = V_u + TD \qquad (39),$$

However this may be reconciled by amending our one period model into perpetuity so that we capitalise the tax relief on the interest payments in future periods also. Hence, from (38) :

$$V_L = V_u + TDr_F \left( \frac{1}{1+r_F} + \frac{1}{(1+r_F)} 2 + \frac{1}{(1+r_F)} 3 + \ldots \right)$$

$$= V_u + TD.$$

Nevertheless the MM1963 paper does not provide a solution where the interest on debt capital is not risk free. But let us consider the implication that since there is a tax advantage to financial leverage the greater the leverage the higher the value of the firm. Fortunately equation (37) provides the general solution. Let us consider the extreme case where the interest is not covered in each state of nature, i.e. $n = \infty$. Hence equation (37) reduces to:

$$V_L = V_u + \frac{T}{1+r_F} \left\{ \sum_{i=1}^{\infty} p_i X_i - \lambda \sum_{i=1}^{\infty} p_i X_i (k_{im} - \bar{k}_m) \right\} \quad (40).$$

From equation (23) and (40)

$$V_L = V_u + \frac{T}{1+r_F} \left\{ \bar{X} - \lambda \text{ cov } (X, k_m) \right\} ,$$

But from equation (13) showing the value of $V_u$ before tax,

$$V_L \text{ (after tax)} = V_u \text{ (after tax)} + T (V_u \text{ (before tax)} ) \quad (41).$$

Given that

$$\bar{X} \geq \lambda \text{ cov } (X, k_m) ,$$

and $\sum_{i=1}^{\infty} p_i X_i \geq \sum_{i=1}^{n} p_i X_i + \sum_{i=n+1}^{\infty} p_i Dk_d$ ,

and by comparison of expressions (40) and (37) we note that the CAPM framework supports the view that with tax deductibility of interest payments, maximum financial leverage is predicted. Equation (41) is a clumsy expression of the relationship between the predicted equilibrium value after tax of the levered firm and the value of the unlevered firm before and after tax.

### 3.5. The effect of personal and corporate taxation on the valuation of the firm

We shall now incorporate into the analysis an imputation tax system such that on payment of a dividend the company is required to pay to the Inland Revenue a proportion b of the gross dividend as an Advance payment of Corporation Tax (ACT) which we shall assume initially to be fully deductible from the mainstream corporation tax charge. Shareholders pay a proportion h of the gross dividend as a higher rate tax on investment income, although they receive a tax credit at the basic income tax rate of b on the gross dividend, i.e. the ACT paid by the company is imputed to the shareholders. We shall assume that of funds available to shareholders a net dividend payout at the rate of d is made. Capital gains tax is at the rate g. Debenture-holders pay a higher rate tax on interest at the rate h.

The amount available for paying dividends is given by the net operating cash flow, less debenture interest, less corporation tax, less the repayment of debt capital:

$$((X_i - Dk_d)\ (1 - T) - D) \text{ for } i = N+1 \text{ to } \infty ,$$

The dividend paid is $d\ ((X_i - Dk_d)\ (1 - T) - D)$ ,

ACT thereon is $\dfrac{b}{1 - b}\ d\ ((X_i - Dk_d)\ (1 - T) - D)$ ,

The higher rate tax on the gross dividend is

$$h\ (1 + \frac{b}{1-b})\ d\ ((X_i - Dk_d)\ (1 - T) - D) .$$

The higher rate tax less the tax deducted at source is

$$\left\{ h\ (1 + \frac{b}{1-b}) - \frac{b}{1-b} \right\} d\ ((X_i - Dk_d)\ (1 - T) - D) = \frac{h - b}{1 - b}\ d\ ((X_i - Dk_d)(1-T) - D) .$$

Hence the dividend received net of personal tax is

$$(1 - \frac{h - b}{1 - b})\ d\ ((X_i - Dk_d)\ (1 - T) - D) = \frac{1 - h}{1 - b}\ d\ ((X_i - Dk_d)\ (1 - T) - D)\ .$$

The mean dividend net of personal tax is

$$\sum_{i=N+1}^{\infty} p_i\ ((X_i - Dk_d)\ (1 - T) - D)\ (d\ \frac{1 - h}{1 - b})\ ,$$

Let the value of the shares in the levered firm at the beginning of the period be $S_L$. For i = 1 to N shareholders receive nothing, and the shares are worth zero. With a capital gains tax rate of g, there is a capital loss for tax purposes worth $gS_L$. For i=N+1 to $\infty$ a retention is made and hence after paying a dividend the shares are worth, assuming a perfect capital market,

$$((X_i - Dk_d)\ (1 - T) - D\ )\ \ (1 - d)\ ,$$

Capital gains tax for i=N+1 to $\infty$ is

$$\{\ ((X_i - Dk_d)\ (1 - T) - D)\ (1 - d) - S_L\ \}\ g\ ,$$

Hence the mean value of the shares after payment of the dividend and after capital gains tax is given by

$$\sum_{i=N+1}^{\infty} p_i \left[ ((X_i - Dk_d)\ (1 - T) - D)\ (1 - d) - g\ \{((X_i - Dk_d)\ (1-T)\ -D)(1-d)\ -S_L\} \right]$$

$$+ \sum_{i=1}^{N} p_i\ gS_L\ ,$$

The mean return to all shareholders in the form of dividends or capital gains, after all personal taxes is therefore

$$\bar{R}_e = \sum_{i=N+1}^{\infty} p_i\ ((X_i - Dk_d)\ (1 - T) - D)\ \frac{(1 - h)}{1 - b}\ d + \sum_{i=1}^{N} p_i gS_L$$

$$+ \sum_{i=N+1}^{\infty} p_i \{((X_i - Dk_d)(1-T) - D)(1-d) - g((X_i - Dk_d)(1-T) - D)(1-d) + gS_L\ \}$$

$$= gS_L + \sum_{i=N+1}^{\infty} p_i\ ((X_i - Dk_d)\ (1 - T) - D)(d\ \frac{(1 - h)}{1 - b} + (1 - d)\ (1 - g)) \qquad (42)$$

By contrast, for a firm financed solely by equity capital i.e. for $N = 0$ and $D = 0$:

$$\bar{R}_u = gS_u + \sum_{i=1}^{\infty} p_i X_i (1 - T) \left(d \frac{(1 - h)}{1 - b} + (1 - d) (1 - g)\right) \quad (43),$$

Therefore

$$\bar{R}_e = \bar{R}_u - gS_u + gS_L - \sum_{i=1}^{N} p_i X_i (1 - T) \left(d \frac{(1 - h)}{1 - b} + (1 - d) (1 - g)\right)$$

$$- \sum_{i=N+1}^{\infty} p_i (Dk_d (1 - T) + D) \left(d \frac{(1 - h)}{1 - b} + (1 - d) (1 - g)\right) \quad (44),$$

Let us now turn to the after-tax returns to debtholders. Firstly, where the net operating cash flow does not exceed the contractual interest payment, the firm pays no tax and the net operating cash flow is fully paid to debtholders. Where there is no repayment of debt capital the value of the tax loss to debtholders is $gD$. Therefore

$$R_{id} = X_i (1 - h) + gD \quad \text{for} \quad i = 1 \text{ to } n \quad (45),$$

Secondly, where the net operating cash flow exceeds the interest payments but where there is insufficient to repay the debt capital then the interest after tax is $Dk_d (1-h)$; the repayment of debt capital is $(X_i - Dk_d) (1 - T)$, and the value of the tax loss to debtholders is $g(D - (X_i-Dk_d) (1 - T))$. Therefore

$$R_{id} = Dk_d (1-h) + (X_i - Dk_d) (1-T) + g(D - (X_i-Dk_d)(1-T)) \quad \text{for } i=n+1 \text{ to } N \quad (46).$$

Finally, where the firm has sufficient income after tax to repay the debenture interest and capital

$$R_{id} = Dk_d (1 - h) + D \quad \text{for} \quad i = N+1 \text{ to } \infty \quad (47),$$

Hence,

$$\bar{R}_d = \sum_{i=1}^{n} p_i (X_i (1-h) + gD) + \sum_{i=n+1}^{N} p_i \Big[Dk_d (1-h) + (X_i - Dk_d)(1-T) + gD$$

$$-g(X_i - Dk_d) (1 - T)\Big] + \sum_{i=N+1}^{\infty} p_i (Dk_d (1-h) + D) \quad (48),$$

From equations (44) and (48)

$$\bar{R}_d + \bar{R}_e = \bar{R}_u - gS_u + gS_L$$

$$+ \sum_{i=1}^{n} p_i \ \{ X_i \ (1-h) + gD - X_i \ (1-T) \ (d\frac{(1-h)}{1-b} + (1-d) \ (1-g)) \ \}$$

$$+ \sum_{i=n+1}^{N} p_i \ \{ Dk_d \ (1-h) + (X_i - Dk_d) \ (1-T) + gD - g \ (X_i - Dk_d)(1-T)$$

$$- X_i \ (1-T) \ (d \ \frac{(1-h)}{1-b} + (1-d) \ (1-g)) \ \}$$

$$+ \sum_{i=N+1}^{\infty} p_i \ \{ Dk_d \ (1-h) + D - (Dk_d \ (1-T) + D) \ (d \ \frac{(1-h)}{1-b} + (1-d) \ (1-g)) \ \} \qquad (49),$$

When d = 1, g = 0, b = 0, h = 0, then from equation (49)

$$\bar{R}_e + \bar{R}_d = \bar{R}_u + T \sum_{i=1}^{n} p_i X_i + T \sum_{i=N+1}^{\infty} p_i \ Dk_d + T \sum_{i=n+1}^{N} p_i \ Dk_d \ .$$

But since, under these parameters,

$$\bar{R}_u = \sum_{i=1}^{\infty} p_i \ X_i \ (1-T) \qquad ,$$

then

$$\bar{R}_e + \bar{R}_d = \bar{X} - \sum_{i=n+1}^{\infty} p_i \ (X_i - Dk_d) \ T \qquad (50),$$

as in equation (22).

Now that we have calculated the mean returns to all security holders let us investigate the covariances. Since by definition

$$\text{cov} \ (R_j, k_m) = \sum_{i=1}^{\infty} p_i \ R_{ij} \ (k_{im} - \bar{k}_m) \qquad ,$$

then from equation (49) and from equation (43), we may derive the covariances (see equations (51) to (55) in the appendix).

Therefore from equations (54), (55), (49) and (51) (See Appendix)

$$V_L\ (1+r_F) = V_u\ (1+r_F) - gS_u + gS_L$$

$$+ \sum_{i=1}^{n} p_i\ (1 - \lambda\ (k_{im} - \bar{k}_m))\ \{ X_i\ (1-h) + gD - X_i\ (1-T)\ (d\ \frac{(1-h)}{1-b} + (1-d)\ (1-g))\}$$

$$+ \sum_{i=n+1}^{N} p_i\ (1 - \lambda\ (k_{im} - \bar{k}_m))\ \{Dk_d\ (1-h) + (X_i - Dk_d)\ (1-T) + gD$$

$$- g\ (X_i - Dk_d)\ (1-T) - X_i\ (1-T)\ (d\ \frac{(1-h)}{1-b} + (1-d)\ (1-g))\ \}$$

$$+ \sum_{i=N+1}^{\infty} p_i\ (1 - \lambda\ (k_{im} - \bar{k}_m))\ \{ Dk_d\ (1-h) + D - (Dk_d(1-T) + D)\ (d\frac{(1-h)}{1-b}$$

$$+ (1-d)\ (1-g))\ \} \qquad (56)$$

The above expression may be rewritten:

$$V_L\ (1+r_F) = V_u\ (1+r_F)$$

$$+ \sum_{i=1}^{n} p_i\ (1 - \lambda\ (k_{im} - \bar{k}_m))\ \{X_i(1-h) + gD + gS_L - X_i(1-T)\ (d\ \frac{(1-h)}{1-b} + (1-d)\ (1-g)) - gS_u\}$$

$$+ \sum_{i=n+1}^{N} p_i\ (1 - \lambda\ (k_{im} - \bar{k}_m))\ \{ Dk_d\ (1-h) + (X_i - Dk_d)(1-T) - g\ (X_i - Dk_d)\ (1-T)$$

$$+ gD + gS_L - X_i\ (1-T)\ (d\ \frac{(1-h)}{1-b} + (1-d)\ (1-g)) - gS_u\ \}$$

$$+ \sum_{i=N+1}^{\infty} p_i\ (1 - \lambda\ (k_{im} - \bar{k}_m))\ \{ Dk_d\ (1-h) + D + gS_L$$

$$-(Dk_d\ (1-T) + D)\ (d\ \frac{(1-h)}{1-b} + (1-d)\ (1-g)) - gS_u\ \} \qquad (57)$$

The rationale of equation (57) may be explained as follows: The first term on the RHS of the equation demonstrates that before tax adjustments the value of an unlevered firm is the same as that of a levered firm in a perfect capital market. The second term represents the risk-adjusted value of the tax effects in those states of nature, $i = 1$ to $n$, where the net operating cash flow is insufficient to cover the contractual interest payments. With a levered firm in such states, debenture holders receive the net operating cash flow, $X_i$, on which they pay personal tax at the rate of h, together with a tax loss on the value of the debt D, at the capital gains tax rate of g; and shareholders receive a tax loss worth $gS_L$. By contrast, for an unlevered firm the net operating cash flow, $X_i$, in such states is subject to corporation tax at the rate T. A proportion of this at the rate of d is paid in dividends and is grossed up by $1/(1-b)$, for the imputed tax credit, but bears tax at the higher rate of h; the remainder of $(1-d)$ is a capital gain and taxed at the rate of g, with tax relief on the value of the shares $S_u$, also at the rate of g. The risk-adjustment is catered for by the expression $(-\lambda \ (k_{im} - \bar{k}_m))$.

The third term represents the risk-adjusted value of the tax effects in those states of nature, $i = n+1$ to N, where the net operating cash flow is sufficient to cover the contractual interest payments but not enough to meet the full repayment of debt capital. Since the interest is paid in full debtholders receive gross interest of $Dk_d$ which is subject to personal tax at the rate of h. A partial repayment of debt capital is made on the net operating cash flow, $X_i$, after interest, $Dk_d$, and after tax at the rate of T. However, this repayment of capital is subject to capital gains tax at the rate of g although there is relief on the value of the debt,D. As far as shareholders are concerned in those states of nature where there is insufficient to make the required payments to debtholders in full, returns to shareholders are nil, apart from the value of the tax loss $gS_L$. By contrast, for an unlevered firm the net operating income after corporation tax, $X_i(1-T)$, is available for dividends at the rate of d on which personal tax is paid at the rate $h/(1-b)$ as explained above, and for capital gains at the rate of $(1-d)$ on which capital gains tax is paid at the rate of g with relief at the rate of g on the original value of the shares, $S_u$.

Finally, the fourth term shows the risk-adjusted value of the tax effects for those states of nature, $i = N+1$, to $\infty$, where obligations to debt-holders are met in full. Debtholders therefore receive the contractual interest $Dk_d$ on which income tax is paid at the higher rate h, and a capital repayment of D. The cash flow available to shareholders is given by $((X_i - Dk_d)(1-T)-D)$ being the net operating cash flow $X_i$, after interest $Dk_d$, after tax at the rate T, and after repayment of debt capital D. A proportion thereof, is paid in dividends at the rate d and is worth after personal tax:

$$d\frac{(1-h)}{1-b}\left((X_i - Dk_d)(1-T) - D\right) \quad ,$$

The remainder is a capital gain and worth after tax relief on the original value $S_L$:

$$(1-d)(1-g)\left((X_i - Dk_d)(1-T) - D\right) + gS_L \quad ,$$

However, had the firm been unlevered $D = 0$ and the returns in the form of dividends and capital gains after all taxes are

$$d\frac{(1-h)}{1-b}\left(X_i(1-T)\right) \quad ,$$

and

$(1-d)(1-g)(X_i(1-T)) + gS_u$ respectively.

Hence before the risk-adjustment the overall gain after tax to the shareholders of a levered firm vis-a-vis those of an unlevered firm are:

$$d\frac{(1-h)}{1-b}\left[(X_i - Dk_d)(1-T)-D\right] + (1-d)(1-g)\left[(X_i - Dk_d)(1-T)-D\right] + gS_L -$$

$$\{d\frac{(1-h)}{1-b}\left[X_i \ (1-T)\right] + (1-d)(1-g)\left[X_i \ (1-T)\right] + gS_u\}$$

$$= gS_L - (Dk_d \ (1-T) + D) \ (d \ \frac{(1-h)}{1-b} + (1-d) \ (1-g)) - gS_u \ ,$$

thus explaining the remainder of the fourth term in equation (57), noting that the $X_i$ terms cancel out.

Observe that where h = 0, b = 0, g = 0, and d = 1, equation (57) simplifies to

$$V_L \ (1+r_F) = V_u \ (1+r_F) + \sum_{i=1}^{n} p_i \ (1 - \lambda \ (k_{im} - \bar{k}_m)) \ X_i \ T$$

$$+ \sum_{i=n+1}^{\infty} p_i \ (1 - \lambda \ (k_{im} - \bar{k}_m)) \ Dk_d \ T \qquad (58)$$

as in equation (36)

## 3.6. Sufficient conditions for a neutral tax system

Let us consider equation (57) in relation to the conditions under the present tax system which would be consistent with the MM irrelevant capital structure.[2] Where gearing is irrelevant, by definition

$$D + S_L = S_u \qquad (59),$$

and

$$gD + gS_L = gS_u \qquad (60).$$

Hence from the second-term on the right hand side of equation (57) a sufficient condition for neutrality is

$$(1-h) = (1-T)\left(d\,\frac{(1-h)}{1-b} + (1-d)(1-g)\right) \qquad (61),$$

Similarly from the third term

$$Dk_d\left((1-h) - (1-T)(1-g)\right) + X_i\left((1-T)(1-g)\right)$$

$$= X_i (1-T)\left(d\,\frac{(1-h)}{1-b} + (1-d)(1-g)\right) \qquad (62),$$

and from the fourth term, by substituting for $D = gD + D(1-g)$

$$Dk_d (1-h) + D(1-g) = Dk_d (1-T)\left(d\,\frac{(1-h)}{1-b} + (1-d)(1-g)\right)$$

$$+ D\left(d\,\frac{(1-h)}{1-b} + (1-d)(1-g)\right) \qquad (63).$$

From equation (61), with no dividend paid, $d = 0$, and

$$1-h = (1-T)(1-g) \quad ,$$

$$\text{i.e.} \quad h = 1 - (1-T)(1-g) \qquad (64).$$

Where there is no retention, $d = 1$, and

$$1-h = (1-T)\,\frac{(1-h)}{1-b} \quad ,$$

giving $T = b$ (65)

From equation (62), we require for tax neutrality when no retention is made, $d = 1$,

$(1-h) = (1-T)\ (1-g)$ as in equation (64),

and $(1-T)\ (1-g) = (1-T)\ \frac{(1-h)}{1-b}$,

giving $h = 1 - (1-g)\ (1-b)$ (66)

When no dividend is paid, from equation (62) for tax neutrality,

$d = 0$ and $(1-h) = (1-T)\ (1-g)$ as before.

Finally, from equation (63) for tax neutrality, when no retention is made, $d = 1$ and from the coefficients of $Dk_d$,

$(1-h) = (1-T)\ \frac{(1-h)}{1-b}$,

giving $b = T$ as in equation (65);

and from the coefficients of $D$,

$(1-g) = \frac{(1-h)}{1-b}$,

giving $h = 1 - (1-b)\ (1-g)$ as in equation (66).

For a full retention, $d = 0$, and from the coefficients of $Dk_d$ in equation (63),

$(1-h) = (1-T)\ (1-g)$ as in equation (64)

Hence, the sufficient conditions* for tax neutrality are

$h = 1 - (1-g)\ (1-b)$ from equation (66),

and

$h = 1 - (1-g)\ (1-T)$ from equation (64),

giving $T = b$ as in equation (65).

* I am very grateful to Professor Ken Peasnell for correcting a recurrent error in my earlier analysis where I had stated that the conditions were necessary rather than sufficient.

The requirements of a neutral tax system depending on dividend policy for alternative states of nature are shown in table 4 below.

Table 4 Conditions for a neutral tax system (partial equilibrium)

| State of nature / Dividend Policy | Insufficient cash profits to pay interest on debentures | Insufficient cash profits, after interest, to repay debt capital | Sufficient cash profits to meet all debt obligations |
|---|---|---|---|
| No dividends | $h=1-(1-T)(1-g)$ | $h=1-(1-T)(1-g)$ | $h=1-(1-T)(1-g)$ |
| Full dividends | $b = T$ | $h=1-(1-T)(1-g)$<br>$h=1-(1-b)(1-g)$ | $b = T$<br>$h=1-(1-b)(1-g)$ |

So far we have assumed a situation of partial equilibrium in that the value given for h, the higher rate of income tax, for the marginal investor was exogenously determined. Furthermore, it was assumed that this value was the same both for the shareholder and the debentureholder. These assumptions will now be relaxed.

## 3.7 Taxes and market equilibrium

In Miller's 1977 paper [4] he has argued against the view that bankruptcy costs and agency costs balance against the tax advantages of debt finance to result in an optimal capital structure. He suggests that the corporate debt ratios in the early 1970's appeared to be only marginally higher than those of the 1920's despite enormously higher tax rates. "And since failure to close the gap cannot convincingly be attributed to the bankruptcy or agency costs of debt financing, there would seem to be only one way left to turn: the tax advantages of debt financing must be substantially less than the conventional wisdom suggests" (p.266) [4]. Furthermore he states that the tax deductibility of interest payments does not change the result that in general equilibrium the value of the firm is independent of its capital structure. He begins his analysis under partial equilibrium and follows Modigliani and Miller's 1969 paper [5]. The returns to the investor who owns a fraction A of the shares in the levered firm are

$$A\ (X-Dk_d)\ (1-T)\ (1-T_{PS}) \quad .$$

Interest at the rate of $k_d$ on debt of D is offset against the uncertain return of X on the firm's real assets. After corporation tax at the rate of T the net income is subject to personal income tax at the rate of $T_{PS}$. The same income after tax could be achieved by investing $AS_u$ in a twin unlevered corporation and borrowing on personal account an amount of

$$A\left[\frac{(1-T)\ (1-T_{PS})}{(1-T_{PD})}\right] D \quad ,$$

where

$T_{PD}$ = the personal income tax rate on income from debt.

Since interest is tax deductible the net cost of borrowing is

$$A \left[ \frac{(1-T)\ (1-T_{PS})}{(1-T_{PD})} \right] Dk_d\ (1-T_{PD}) = A \left[ (1-T)\ (1-T_{PS}) \right] Dk_d \qquad (67) .$$

Together with the income from shares of

$$A\tilde{X}\ (1-T)\ (1-T_{PS}) \qquad ,$$

this yields to the investor in an unlevered firm a net return, after interest on personal borrowing, of

$$AX\ (1-T)\ (1-T_{PS}) - A \left[ (1-T)\ (1-T_{PS}) \right] Dk_d = A\ (X-Dk_d)(1-T)(1-T_{PS}) \qquad (68) ,$$

which is the same as the return to the investor in the levered firm.

The market value of the investor's interest in the levered firm is $AS_L$, whereas the market value of the investor's interest in the unlevered firm, net of personal debt, is

$$AS_u - A \left[ \frac{(1-T)\ (1-T_{PS})}{1-T_{PD}} \right] D \quad .$$

In partial equilibrium

$$AS_L = AS_u - A \left[ \frac{(1-T)\ (1-T_{PS})}{1-T_{PD}} \right] D \qquad (69) ,$$

giving

$$S_u = S_L + \left[ \frac{(1-T)(1-T_{PS})}{1-T_{PD}} \right] D \quad . \qquad (70)$$

But since the value of the levered firm is the sum of the values of equity and debt, or

$$V_L = S_L + D \qquad (71) ,$$

then

$$V_L - V_u = V_L - S_u = S_L + D - S_u$$

$$= S_L + D - S_L - \left[\frac{(1-T)(1-T_{PS})}{1-T_{PD}}\right] D$$

$$= \left\{1 - \left[\frac{(1-T)(1-T_{PS})}{1-T_{PD}}\right]\right\} D \qquad (72)$$

This represents the gain from leverage for the shareholders in a firm holding real assets. Note that this result is dependent on the fact that interest on debt, $Dk_d$, is always less than the uncertain operating income of X, and implies a risk-free interest rate on debt which would be unrealistic particularly for high levels of leverage.

Miller argues (on page 268 [4]) that "any situation in which the owners of corporations could increase their wealth by substituting debt for equity (or vice cersa) would be incompatible with market equilibrium. Their attempts to exploit these opportunities would lead, in a world with progressive income taxes, to changes in the yield on stocks and bonds and in their ownership patterns. These changes, in turn, restore the equilibrium and remove the incentives to issue more debt .."

An assumption is made that $T_{PS}$ is zero, that debt is riskless, that the market is frictionless apart from taxation, that

$r_o$ = the equilibrium rate of interest on fully tax-exempt bonds ,

and

$T^*_{PD}$ = the marginal rate of personal income tax on interest from debt for the marginal investor.

Since a rate of interest of $r_o$ can be achieved after personal tax (of zero), the marginal investor paying tax would only be willing to buy corporate debt of which the gross rate of interest is at least

$$\frac{r_o}{(1-T^*_{PD})} ,$$

or $r_o$ after personal tax.

In market equilibrium

$$\frac{(1-T)(1-T_{PS})}{1-T^*_{PD}} = 1 \qquad (73),$$

$$\text{or} \quad \frac{1-T}{1-T^*_{PD}} = 1 \quad \text{for } T_{PS} = 0, \qquad (74),$$

$$\text{giving} \quad T = T^*_{PD} \qquad (75).$$

Hence the gross rate of interest on corporate debt would be

$$\frac{r_o}{1-T},$$

in market equilibrium. "Market interest rates have to be grossed up to pay the taxes of the marginal bondholder, whose tax rate in equilibrium will be equal to the corporate tax rate" (p.270[4]). Since interest is tax deductible the net cost to the company is $r_o$.

Miller states (p.269[4]) that "there will be an equilibrium level of aggregate corporate debt and hence an equilibrium debt-equity ratio for the corporate sector as a whole. But there would be no optimum debt ratio for any individual firm".

If we accommodate both dividends and capital gains, then following Miller's analysis adapted for the UK situation:

$$1 - T_{PS} \equiv \frac{d(1-h_S)}{1-b} + (1-d)(1-g) \qquad (76),$$

where

$d$ = the pay-out ratio (as in section 3.6),

$h_S$ = the higher rate of income tax on dividends,

$b$ = the basic rate of income tax (as in section 3.6),

$g$ = the rate of capital gains tax (as in section 3.6),

and

$$1 - T_{PD} \equiv 1 - h_D \qquad (77),$$

where

$h_D$ = the higher rate of income tax on interest received.

Hence

$$V_L - V_U = \left\{ 1 - \frac{(1-T)\left[\frac{d(1-h_s)}{1-b} + (1-d)(1-g)\right]}{1-h_D} \right\} D \qquad (78).$$

Now, if we let $h_D^*$ be the value of $h_D$ for the marginal investor, then Miller's analysis, for the UK, would lead to the conclusion that in market equilibrium:

$$1 - h_D^* = (1-T)\left[\frac{d(1-h_s^*)}{1-b} + (1-d)(1-g)\right] \qquad (79).$$

Assuming a full retention of dividends (d=o), this gives :

$$1 - h_D^* = (1-T)(1-g) \quad ,$$

$$\text{or} \quad h_D^* = 1 - (1-T)(1-g) \qquad (80)$$

This is consistent with the analysis of section 3.6 (see equation (64)). Where the firm adopts a dividend policy that can be described by a full payout rate (d=1), then from equation (79)

$$h^*_D = 1 - \frac{(1-T)(1-h^*_S)}{1-b} \qquad (81);$$

From (80) and (81),

$$h^*_S = 1 - (1-b)(1-g) \qquad (82).$$

Again this is consistent with section 3.6 (see equation (66)). However in the preceding section a requirement for tax neutrality was that T=b, which is no longer a constraint under general equilibrium.

An extension will now be made to the CAPM analysis under general equilibrium by substituting $h^*_D$ for h, whenever h represents the tax rate on debenture interest; and $h^*_S$ for h, whenever h represents the marginal tax rate on dividends. It follows that (from equation (57))

$$V_L(1+r_F) = V_u(1+r_F)$$

$$+ \sum_{i=1}^{\wedge} p_i \left[1-\lambda(k_{im} - \bar{k}_m)\right] \left\{ X_i(1-h^*_D) + gD + gS_L \right.$$

$$\left. - X_i(1-T)\left[\frac{d(1-h^*_S)}{1-b} + (1-d)(1-g)\right] - gS_u \right\}$$

$$+ \sum_{i=n+1}^{N} \rho_i \left[1 - \lambda\,(k_{im} - \bar{k}_m)\right] \left\{ Dk_d(1-h^*_D) + (X_i - Dk_d)(1-T) \right.$$

$$- g\,(X_i - Dk_d)(1-T) + gD + gS_L$$

$$\left. - X_i\,(1-T)\left[\frac{d(1-h^*_S)}{1-b} + (1-d)(1-g)\right] - gS_u \right\}$$

$$+ \sum_{i=N+1}^{\infty} \rho_i \left[1 - \lambda(k_{im} - \bar{k}_m)\right] \left\{ Dk_d\,(1-h^*_D) + D + \frac{d\,(1-h^*_S)}{1-b} \right.$$

$$\times \left[(X_i - Dk_d)(1-T) - D\right]$$

$$+ (1-d)(1-g)\left[(X_i - Dk_d)(1-T) - D\right] + gS_L$$

$$\left. - \left\{ \frac{d(1-h^*_S)}{1-b}\left[X_i\,(1-T)\right] + (1-d)(1-g)\left[X_i(1-T)\right] + gS_u \right\} \right\} \quad (83).$$

The latter part of the expression which deals with states of nature, i = N+1 to $\infty$ where obligations to debentureholders are met in full, requires further explanation. Debentureholders receive(i) the contractual interest $Dk_d$ on which income tax is paid at the higher rate of $h^*_D$ for the marginal debentureholder, and (ii) a capital repayment of D. The cash flow available to shareholders is given by $\left[(X_i - Dk_d)(1-T) - D\right]$ being the net operating cash flow $X_i$, after interest $Dk_d$, after tax at the rate T, and after repayment of debt capital, D. A proportion of this is paid in dividends at the rate d and is worth, after personal tax, to the marginal shareholder

$$\frac{\acute{d}(1-h^*_S)}{1-b}\left[(X_i - Dk_d)(1-T) - D\right] .$$

The remainder is a capital gain and worth after tax relief on the original value $S_L$:

$$(1-d)(1-g)\left[(X_i - Dk_d)(1-T) - D\right] + gS_L .$$

However, had the firm been unlevered D = 0 and the returns in the form of dividends and capital gains after all taxes are

$$\frac{d(1-h^*_S)}{1-b}\left[X_i \quad (1-T)\right] ,$$

and

$$(1-d)(1-g)\left[X_i \quad (1-T)\right] + gS_u \quad , \text{ respectively .}$$

Hence the sufficient conditions for an irrelevant capital structure are

$$\text{(i)}\quad 1-h^{*}_{D} = (1-T)\left[\frac{d(1-h^{*}_{S})}{1-b} + (1-d)(1-g)\right] \qquad (84),$$

when there are insufficient cash profits to pay interest on debentures;

$$\text{(ii)}\quad 1-h^{*}_{D} = (1-T)(1-g) \qquad (85),$$

$$\text{(iii)}\quad (1-T)(1-g) = (1-T)\left[\frac{d(1-h^{*}_{S})}{1-b} + (1-d)(1-g)\right] \qquad (86),$$

when there are insufficient cash profits, after interest, to repay debt capital; and

$$\text{(iv)}\quad (1-h^{*}_{D}) = (1-T)\left[\frac{d(1-h^{*}_{S})}{1-b} + (1-d)(1-g)\right] \qquad (87),$$

$$\text{(v)}\quad 1-g = \frac{d(1-h^{*}_{S})}{1-b} + (1-d)(1-g) \qquad (88),$$

when there are sufficient cash profits to meet all debt obligations.

Equations (ii) and (iv) are determined from the coefficients of $Dk_d$, and (i) and (iii) from the coefficients of $X_i$.

In the states of nature $i = N+1$ to $\infty$ the terms in D, $S_L$ and $S_u$ give a sufficient condition for irrelevancy of

$$D + gS_L - gS_u = D\left[\frac{d\ (1-h^{*}_{S})}{1-b} + (1-d)(1-g)\right] \qquad (89).$$

But with an irrelevant capital structure:

$$D + S_L = S_u \tag{90}$$,

or $$gD + g\,S_L = gS_u \tag{91}$$,

or $$gS_L - gS_u = -gD \tag{92}$$,

hence from (89) and (92)

$$D - gD = D\left[\frac{d(1-h^*_S)}{1-b} + (1-d)(1-g)\right] \tag{93}$$,

or $$1-g = \frac{d(1-h^*_S)}{1-b} + (1-d)(1-g) \tag{94}$$,

as in equation (v). The sufficient conditions for an irrelevant capital structure are shown in table 4a.

The conditions may therefore be summarised by

$$h^*_D = 1 - (1-T)(1-g) \tag{95}$$,

and $$h^*_S = 1 - (1-b)(1-g) \tag{96}$$.

These results are the same as Miller's analysis extended for the UK tax system (see equations (80) and (82) ). Hence the marginal debentureholder has a marginal tax rate on investment income of $h^*_D$, as determined by equation (95) ; and the marginal shareholder has a marginal tax rate on investment income of $h^{*}_{S}$ as determined by equation (96). For these values equation (83) reduces to

$$V_{L} = V_{u} \tag{97}$$,

in a perfect capital market in general equilibrium.

Table 4a Conditions for an irrelevant capital structure (general equilibrium)

| STATE OF NATURE | | |
|---|---|---|
| Insufficient cash profits to pay interest on debentures | Insufficient cash profits, after interest, to repay debt capital | Sufficient cash profits to meet all debt obligations |
| NO DIVIDENDS | | |
| $h^*_D = 1-(1-T)(1-g)$ | $h^*_D = 1-(1-T)(1-g)$ | $h^*_D = 1-(1-T)(1-g)$ |
| FULL DIVIDENDS | | |
| $h^*_D = 1 - \frac{(1-T)(1-h^*_S)}{1-b}$ | $h^*_D = 1-(1-T)(1-g)$<br>$h^*_S = 1-(1-b)(1-g)$ | $h^*_D = 1 - \frac{(1-T)(1-h^*_S)}{1-b}$<br>$h^*_S = 1 - (1-b)(1-g)$ |

### 3.8 Conclusions

Even ignoring the effects of personal taxation under the imputation tax system it has been shown how the Modigliani and Miller (1963) paper, revised to accommodate the capital asset pricing model, may provide solutions where the interest on debt capital is not risk-free. This was achieved by splitting the covariance factors for different states of nature categorised according to the different tax effects. It was concluded that in partial equilibrium the CAPM supports the view that with tax deductibility of interest payments, maximum financial leverage is predicted whether debt capital is risk-free or risky.

Inclusion of personal capital gains taxation enabled us to consider capital losses as well and the implications for the valuation ofthe firm. Furthermore with the addition of personal income taxes and imputed tax credits the effects of dividend policy on the valuation model could be analysed.

Two conditions were shown to be sufficient for a neutral tax system in partial equilibrium:

$$h = 1 - (1-T)\ (1-g) \qquad (64),$$

and

$$T = b \qquad (65).$$

It will be recalled from chapter one that for the efficient allocation of resources, tax neutrality is required, assuming the market to be otherwise efficient.

If interest relief were at the basic rate of income tax then equation (65) would hold. Aproblem that arises under the present system is that the basic rate of income tax, denoted b, does not equal the full rate of Corporation Tax, denoted T. Over the past few years the basic rate of income tax has varied between 30 per cent and 35 per cent, whereas the full rate of Corporation Tax has stayed at 52 per cent. Hence there is still a tax preference for debt finance in partial equilibrium. Further tax complexities such as different marginal rates of Corporation Tax under alternative states of nature will be considered in chapter six, together with a discussion of tax time lags. Afurther problem arising is that not all investors pay capital gains tax or higher rates of income tax. Consequently the values of h and g are not constant. If however all investors were to pay capital gains tax at 30 per cent then with a full rate of Corporation Tax at 52 per cent for a neutral tax system (i) the value of h would be 66.4 per cent $[1-(1-0.52)(1-0.30)]$ and (ii) the basic rate of income tax would be 52 per cent.

Unfortunately this would mean that there would be full imputation which under the draft EEC Directive is not allowed. "The draft Directive proposes that there should be imputation systems in operation with a single rate of tax between 45 and 55 per cent. Also, the imputation credit shall be between 45 and 55 per cent of the Corporation tax that would have to be paid on a sum equal to the taxable income out of which the dividend could be paid" (James and Nobes (1978)[6]).

Interestingly, the rate of 30 per cent being 3/7 (≏43%) of the dividend falls outside the guidelines anyway. Moreover, as discussed in chapter 5 the marginal rate of Corporation Tax can be well in excess of 55 per cent. But since the UK has different systems of allowances than in other member states, to equalise the rates of taxation would be more cosmetic than equitable.

Furthermore a higher tax rate of 66.4 per cent for all investors would cause serious problems of equity. A Utilitarian view might be that the tax on each given slice of income should represent an equal loss of personal satisfaction or utility. Hence investors with more wealth or higher incomes ought to pay higher rates of tax progressively. Hence there is a conflict between principles of equity and those of economic efficiency. However, this is endemic and requires a trade off based on social and political judgements outside the scope of this thesis.

The model was extended for the situation of market equilibrium in which the marginal investors in stocks and shares have marginal tax rates such that, for the UK,

$$h^*_D = 1 - (1-T)(1-g) ,$$

and

$$h^*_S = 1 - (1-b)(1-g) .$$

For these tax rates the valuation of a particular firm is independent of its capital structure on an after-tax basis.

The importance of Miller's 1977 paper is to illustrate the equilibrium process for the market as a whole and to reinforce the view that, in the long-run, prices adjust to reflect after-tax yields for the marginal investor. It may be more useful to financial managers however to consider the short-run states of disequilibrium. It will be shown in chapter 6 that a firm's financing decisions may change over time due to interactivities between investment and financing decisions. In particular the capital allowance effects from expenditure may alter the relative attractiveness of new issues, retentions or debt finance. The result may be that the firm may now attract a different clientele of investors. During the period of temporary disequilibrium the value of the firm may not be indifferent with regard to capital structure. Furthermore, if general equilibrium analysis were applied to capital project appraisal all projects which would not be unattractive would have zero net present values since excess returns would already have been reflected in share prices[7]. However, given that the financial managers of the firm have access to valuable inside information regarding the firm's prosperity it may still be in the best interests of the existing shareholders to undertake projects which have positive net present values under partial equilibrium, that is just before the share price increases. Economic analyses in states of market disequilibrium may not only help provide valuable insights into the development of the financial theory of the firm, but may also be of greater operational significance to financial managers in a world in which market values may be regarded as moving towards a general equilibrium position which is continually changing. It can be argued that microeconomic decisions always take place in a state of disequilibrium[7,8]: "....it would be a betrayal of economic analysis to imagine that the equilibrium constructions in the analysis were describing precise states of affairs....In the matter of investment, for example, or in relation to financing decisions....the firm considers undertaking additional expenditures not because it is in some kind of equilibrium situation, but because it explicitly recognises a disequilibrium condition; disequilibrium in the sense that additional profit and income opportunities are seen to exist and investment is contemplated totake advantage of them" (p.375[8])

The models under consideration are based on the assumption of capital markets being perfect apart from tax complexities. In particular bankruptcy costs have been ignored. But apart from the differences between asset scrap values and values in use it is the author's view that there is a loss of social utility resulting from the effects of bankruptcies. These include forinstance the financial and psychological damage caused by redundancies of the employed and self-employed. It was stated in the introduction that the measurement of the loss of social utility was outside the scope of this thesis. Nevertheless its identification is an important social issue. If the tax system were designed such that for each particular firm the price ratio of debt to equity in the absence of taxation were the same as that after tax, then taxation would not interfere with the efficient choice between the two. There would be a nil excess burden and a reduction in the present loss of social welfare.

Schneller[9] has stressed the fact that Miller's 1977 result depends in particular on the assumptions that default considerations are ignored. Where default is a possibility, there may also be an interior solution for the earnings-retaining firm. Also, where investors have different marginal tax rates , financing policies that attempt to maximise value after personal tax may not be operational. This suggests a clientele effect whereby a firm's financing policies attract shareholders and debentureholders with appropriate marginal tax rates. High income shares and debenture stocks are likely to attract a clientele with low or nil marginal rates of income tax, and capital growth shares may attract investors with low marginal rates of capital gains tax.

It can also be seen that Miller's result depends on the tax deductibility of personal interest payments. Although this holds under the UK tax system for interest on loans for the purchase or improvement of land and buildings, it is not normally allowable for non-business purposes. An investor could increase a home mortgage to provide sufficient personal leverage, but by the time the new mortgage arrangements are completed they would probably be out of alignment with the individual's financial needs. Hence even in the long-run there could be permanent disequilibrium. A further discussion of Miller's work is presented in section 4.5 of the next chapter, which devotes itself to a brief survey of the literature.

## 3.9 Appendix Derivation of covariances

From equations (49) and (43), we have

$$\mathrm{cov}\ (R_d, k_m) + \mathrm{cov}\ (R_e, k_m)$$

$$= \sum_{i=1}^{\infty} p_i\ (k_{im} - \bar{k}_m) \left[ X_i\ (1-T)\ (d\ \frac{(1-h)}{1-b} + (1-d)\ (1-g)) \right]$$

$$+ \sum_{i=1}^{n} p_i\ (k_{im} - \bar{k}_m)\ \{ X_i\ (1-h) + gD - X_i\ (1-T)\ (d\ \frac{(1-h)}{1-b} + (1-d)(1-g))\ \}$$

$$+ \sum_{i=n+1}^{N} p_i\ (k_{im} - \bar{k}_m)\ \{ Dk_d (1-h) + (X_i - Dk_d)(1-T) + gD - g\ (X_i - Dk_d)\ (1-T)$$

$$- X_i\ (1-T)\ (d\ \frac{(1-h)}{1-b} + (1-d)\ (1-g))\ \}$$

$$+ \sum_{i=N+1}^{\infty} p_i\ (k_{im} - k_m)\ \{Dk_d\ (1-h) + D - (Dk_d\ (1-T) + D)(d\ \frac{(1-h)}{1-b} + (1-d)(1-g))\} \quad (51)$$

For an unlevered firm, from equation (43)

$$\mathrm{cov}\ (R_u, k_m) = \sum_{i=1}^{\infty} p_i\ (k_{im} - \bar{k}_m)\ \{ X_i\ (1-T)\ (d\ \frac{(1-h)}{1-b} + (1-d)\ (1-g))\ \} \quad (52) .$$

But since

$$V_u\ (1+r_F) = \bar{R}_u - \lambda\ \mathrm{cov}\ (R_u, k_m) \quad (53) ,$$

$$\text{then } \bar{R}_u = V_u\ (1+r_F) + \lambda \sum_{i=1}^{\infty} p_i\ (k_{im} - \bar{k}_m)\ \{ X_i\ (1-T)\ (d\ \frac{(1-h)}{1-b} + (1-d)(1-g))\} \quad (54)$$

Now, equation (31) is rewritten below for convenience,

$$V_L\ (1+r_F) = \bar{R}_e + \bar{R}_d - \lambda\ (\mathrm{cov}\ (R_e, k_m) + \mathrm{cov}\ (R_d, k_m)) \quad (55) .$$

Hence equation (56) may be derived from equations (49),(51),(54) and (55).

# CHAPTER 4

## On the economics of business taxation

## 4.1 Notation (for chapter 4 only)

$a$ = proportion of wealth invested in a risky asset.

$\alpha$ = constant.

$b$ = basic rate of income tax.

$B$ = constant.

$\beta$ = constant.

$c$ = stochastic variable.

$e$ = stochastic variable representing a pre-tax yield on a risky asset.

$E$ = expected value.

$h$ = higher rate of personal income tax.

$I$ = investment outlay.

$k$ = discount rate.

$\bar{m}_2$ = the second central moment of the distribution.

$\bar{m}_3$ = the third central moment of the distribution.

$\mu$ = mean.

$Py$ = probability of y.

$\phi U$ = function of U.

$r$ = yield of risk-free asset.

$R_A$ = absolute risk aversion.

$s$ = standard deviation.

$T$ = corporate tax rate.

$u$ = stochastic variable.

$U$ = utility, $\left[U(Y) = \text{utility of } Y\right]$.

$\nu$ = variable.

$W$ = expected utility of wealth.

$w_o$ = initial wealth.

$x$ = stochastic variable.

$X$ = return (variable).

$Y$ = end of period wealth.

$y$ = random variable.

$z$ = deviation from mean.

4.2. Tax-shifting

The statutory incidence of company taxation may differ from the economic incidence in that companies which are legally liable to pay a tax may alter their patterns of spending and investment with a result that a new distribution of total tax liabilities emerges.

Leakages may occur by passing on a tax increase in the form of increased prices to the consumer, especially to the extent that competitors are in a similar tax position; by substituting inferior materials to reduce costs; by organising production more efficiently, by being more aggressive in wage negotiations; or by substituting one form of financing method for another as relative tax advantages for alternative financial instruments change. However, Musgrave and Musgrave[1] state that the empirical evidence in the United States on the long-run economic incidence of corporate taxation is conflicting and it is uncertain whether or not the tax is shifted. They note that tax changes typically coincide with changes in government expenditure and it is difficult to isolate the effects of the two variables. In the United Kingdom a time-series analysis of short-run shifting of company taxation by Davis[2] has produced results consistent with zero or little shifting of the tax. This is supported by Westlake's questionnaire[3] which indicated that "at the most about 40% of companies have the ability to shift tax increases, and only 8% say they have shifted tax increases in the past". In this analysis, apart from changes in financing methods, tax shifting will be ignored.

## 4.3. Tax Neutrality

In 1948 Brown [4] recommended a corporate tax system based on cash flows so that, with a constant tax rate and full offset for losses, in present value terms the tax relief on cash outflows bears the same proportion to pre-tax cash outflows as the tax on cash inflows bears to the pre-tax cash inflows, with a result that for capital investment decision purposes a shareholder wealth maximising firm may ignore corporate taxation altogether and the government effectively becomes a business partner. However, with a constant time lag between the accounting year-end and the tax payment date, since cash flows occur at variable dates within the accounting period, there are variable time lags between cash flows and tax reliefs or payments thereon. Mellors [5] seems to understand that this necessarily always implies a disincentive - "By increasing the rate at which an asset's cost may be written off against taxable profits the government is not increasing the incentive to invest so much as reducing the disincentive to invest that is built into our tax system. This built-in disincentive effect is attributable to the time factor ... a system of free depreciation represents the smallest departure from tax neutrality that is possible, given the inevitable lags inherent in the tax system." However, it will be shown later that the system may provide an incentive instead. Additionally, Musgrave and Musgrave [6] have suggested that a cash flow tax system would raise no revenue: "An interesting question arises: what happens when depreciation is permitted to be taken in its entirety at the time the investment is made, i.e. when all investment costs may be expensed? Combined with perfect loss offset, this would in fact mean that there is no tax. With a 50 per cent tax rate, investment of $100 would yield an immediate refund of $50 which, if reinvested, would yield a refund of $25, and so forth until a total refund of $100 was obtained. The investor would combine the initial investment of $100 with an additional $100 advanced by the Treasury, and resulting earnings on $200 net of the 50 per cent tax would be the same as the earnings on $100 without tax". The same view is also supported by Swan [7]. However, the statement that there is no tax is misleading in that it is only true if either, the firm invests in projects with zero NPVs so that the discounted tax relief on outflows is equated with the discounted tax payments on inflows; or if the firm keeps reinvesting its earnings into perpetuity and hence never makes a cash return to shareholders in the foreseeable future. The implication of the statement, however, is that such a tax leaves unchanged the rate of return on the investment and hence does not distort the decision whether to invest. This approach is the one adopted in the Meade [8] report. Since the investment decision is not distorted there is by definition no excess tax burden, which is different from saying that there is no tax.

A cash flow tax system has also been criticised by Samuelson: [9] "Fast-depreciation gimmicks in the Swedish, Japanese, German, British, and American tax codes are not a return to just recognition of economic obsolescence ... They are competitive bribes and giveaways, designed to undertax money income ... in order to attract investment from other countries and to stimulate the total of domestic investment growth. If we call spades spades, lets call bribes bribes". His conclusion is perhaps distorted by a confusion over equity and efficiency. If the purpose of a corporation tax is to expand the tax base of shareholders, then undistributed corporate income, not effectively taxed under capital gains tax if the shareholder does not sell the share, is distorted under a cash flow tax. An equitable corporation tax to be consistent with a personal income tax, ought to be based on business income reflecting true economic depreciation. It will later be shown that a personal tax based on income without tax relief on interest distorts investment decisions and is therefore inefficient. However, Samuelson [10] provides an alternative neutral corporate tax base: "If, and only if, true loss of economic value is permitted as a tax deductible depreciation expense will the present discounted value of a cash-receipt stream be independent of the tax rate". The rationale for this may proceed as follows. With no tax an investment is financially attractive if the net receipts (cash inflows less outflows) less the capital loss over the period (true economic depreciation) exceeds the cost of borrowing. Now, if interest is tax deductible and if depreciation for tax purposes is based on economic depreciation then, with taxation, an investment is worthwhile if one minus the tax rate times (net receipts less true economic depreciation) exceeds one minus the tax rate times the cost of borrowing. Hence if the tax rate is a positive fraction the investment decision is not distorted. However, not only does the analysis assume that all capital investment is financed by debt capital, but also perfect certainty is implicitly implied. A more recent paper by King [11] presents a very similar analysis to that of Samuelson, although he admits that "in the context of uncertainty profits taxation may play a different role".

Furthermore, Sumner [12] has argued that "to permit interest deductibility as well as free depreciation would make the corporate tax system a net inducement to invest" He goes on to suggest that "free depreciation is already permitted on machinery; the only changes needed would be in the regulations concerning depreciation of buildings and the withdrawal of interest deductibility .... this would treat debt and equity on an equal basis". This is incorrect since as we have shown in chapter 3, under the imputation system interest relief would have to be at the basic rate of income tax to avoid tax distortions in financing decisions. The capital market inefficiency of free depreciation together with interest deductibility is reiterated by King [13] "if the tax system allows both interest payments and investment expenditure to be tax deductible, which is the current position in the United Kingdom, the introduction of a corporate profits tax would lead to a flow of capital into the corporate sector. In this case the higher the corporate profits tax rate, the higher the level of investment in the corporate sector!" The rationale of his analysis may be explained by use of an arithmetical example, on similar lines to those in the Meade [14] report. Consider an investment in a machine priced at £1m and financed by debt capital requiring a 10% gross rate of interest each year into perpetuity. The pre-tax rate of return on the machine of £1m required to finance the debt capital is 4.8%, with a corporate tax rate of 52%, and 4% with a corporate tax rate of 60%, assuming interest deductibility. Since more investments now become financially attractive, a higher level of investment is predicted. In the table below, this result is contrasted with a system of free depreciation but no interest deductibility, which requires a pre-tax rate of return on the machine of 10%, regardless of the corporate tax rate, and is equated with the rate of interest on debt.

The flaw in the argument, of course, rests on the assumption of full offset for losses after capital allowances, where allowances turn profits into losses. In examples (i) and (ii) since the interest relief fully cancels the annual return, providing nil net taxable income, the capital allowance on the machine is deferred forever. Hence, although the tax statutes may provide for both interest deductibility and free depreciation, the pre-tax rate of return on the machine costing £1m required to finance sufficient debt capital to purchase the machine is still 10%, resulting in a neutral tax system.

Table 5

Free Depreciation and Interest deductibility

| | Interest deductible | | Interest not deductible | |
|---|---|---|---|---|
| | (a) | (b) | (a) | (b) |
| | Corporate tax rate = 52% (i) | Corporate tax rate = 60% (ii) | Corporate tax rate = 52% (iii) | Corporate tax rate = 60% (iv) |
| Rate of interest on debt | 10% | 10% | 10% | 10% |
| Price of Machine (capital outlay) | £1m | £1m | £1m | £1m |
| Cost after 100% instantaneous capital allowance | £480,000 | £400,000 | £480,000 | £400,000 |
| Raise debt capital | £480,000 | £400,000 | £480,000 | £400,000 |
| Required pre-tax rate of return on capital outlay to finance debt capital | 4.8% | 4% | 10% | 10% |
| Required annual pre-tax income | £48,000 | £40,000 | £100,000 | £100,000 |
| less interest relief | (48,000) | (40,000) | NIL | NIL |
| Net taxable income | NIL | NIL | 100,000 | 100,000 |
| Corporation Tax | NIL | NIL | 52,000 | 60,000 |
| After tax income | NIL | NIL | 48,000 | 40,000 |
| Debt interest (=Annual pre tax income less Corp.Tax) | £48,000 | £40,000 | £48,000 | £40,000 |

Moreover, the alternative tax system providing free depreciation but no interest deductibility can be shown to be no longer fiscally neutral when the capital allowance carry-forward provisions are reflected in the analysis. If the company earns a pre-tax return of, say, 10% on the original price of the machine, of outlay I (£s), for the first ten years the capital allowances determine zero corporate tax payments. Consequently, the debt capital required to finance the project is given by I and not I(1-T) where T is the corporate tax rate. If we assume that all net cash flows are paid to debtholders, then the annual return is 10% x I for ten years and 10% x I(1-T) thereafter. From this we may derive an internal rate of return, denoted k, being the effective interest on debt capital:

$$I = 10\%\, I \left(\frac{1}{1+k} + \frac{1}{(1+k)^2} + \ldots\ldots + \frac{1}{(1+k)^{10}}\right)$$

$$+\ 10\%\, I\,(1-T) \left[\frac{1}{(1+k)^{11}} + \frac{1}{(1+k)^{12}} + \ldots\ldots\right] .$$

With a corporate tax rate at 52 per cent the internal rate of return is approximately 7½%, which is significantly less than that of 10% before tax. This results in economic inefficiency since the marginal rate of substitution of present into future income before tax does not equal that after tax.

Hence, with more realistic modelling of the tax provisions the conclusions by King,[15] Sumner,[16] Stiglitz,[17] Swan[18] and Hartman[19] that free depreciation without interest deductibility is equivalent to true economic depreciation with interest deductibility, as far as productive investment decisions are concerned, is misleading. However, even with a negative tax system the two alternatives are not equivalent under inflation; as Sumner[20] points out "whereas with stable prices the calculation of economic depreciation is merely extremely difficult, in the presence of inflation it becomes impossible. Since a costless alternative route to the same end is available, there seems little point in further consideration of the practical problems involved". Similarly, Bierman [21] states that the primary argument in favour of allowing immediate expensing of capital equipment with instantaneous relief is its simplicity.

4.4. Tax allowances as a source of funds

Carsberg and Hope[22] have recognised that firms do not always base investment decisions on discounted cash flow analysis. Also in a survey by Schofield[23] of the equipment replacement decisions of 20 companies between March 1970 and March 1971 it was found that "Payback and Accounting Rate of Return are used except on large projects where Discounted Cash Flow is used ..... Taxation and Investment Incentives are seldom considered except on large projects (i.e. DCF evaluations). Therefore the effect on Government Investment Incentives depends on the ratio of total capital expenditure on small projects to that on large projects". Investment decisions based on accounting rate of return calculations would ignore normally in particular the benefits of accelerated depreciation for tax purposes through the time value of money. The equalisation of profits for accounting purposes and the comparison with actual tax due is shown in the following table.

Table 6

Deferred Taxation (£s)

| | Year | 1 | 2 | 3 | ... | 1-10 |
|---|---|---|---|---|---|---|
| | Profit, after depreciation, before tax | 1,000,000 | 1,000,000 | 1,000,000 | ... | 10,000,000 |
| | Depreciation (added back)* | 10,000 | 10,000 | 10,000 | ... | 100,000 |
| | | 1,010,000 | 1,010,000 | 1,010,000 | | 10,100,000 |
| | Less capital allowance | (100,000) | NIL | NIL | ... | (100,000) |
| | | 910,000 | 1,010,000 | 1,010,000 | | 10,000,000 |
| (a) | Actual tax @ 50% | 455,000 | 505,000 | 505,000 | ... | 5,000,000 |
| | Capital allowance | 100,000 | NIL | NIL | ... | 100,000 |
| | less depreciation | (10,000) | (10,000) | (10,000) | ... | (100,000) |
| | | 90,000 | (10,000) | (10,000) | | NIL |
| (b) | Deferred tax @ 50% | 45,000 | (5,000) | (5,000) | ... | NIL |
| (a)+(b) | Tax charge to Profit and Loss Account | 500,000 | 500,000 | 500,000 | ... | 5,000,000 |

* Note that since depreciation is disallowed for tax purposes it is added back in order to reverse the original deduction in determining profit after depreciation.

Equally important, however, is the capital allowance as a source of funds, as evidenced by Deferred Tax Accounts in company Balance Sheets. This may be demonstrated by considering two projects, A and B, with cash inflows of £1,564-10 at the end of year 4 and £765-50 at the end of year 3, respectively. Assuming indivisibility of outlays, then if project A requires an outlay of £1,000 and B requires £500, then with £1,000 capital rationing and no tax both A and B cannot be undertaken and B would be preferred since it has a higher net terminal value using a 10% reinvestment rate. The net terminal value of project A at time 4 is £1564-10 less £1,000 $(1 + 10\%)^4$ which equals £100. By contrast the net terminal value of project B at time 3 is £765-50 less £500 $(1 + 10\%)^3$ which equals £100, determining a net terminal value at time 4, with £10 interest on reinvestment for a further year, of £110. However, with a cash flow tax at 50% and a one year tax time lag, both projects may be undertaken (see table below), project A now and project B in one year's time, assuming that there are profits from other projects against which to offset the capital allowances.

Table 7 Capital rationing and taxation (bracketed variables are cash outflows)

| Time | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Project A: Pre-tax cash flows | (1000) | | | | 1564-10 | |
| (Taxes) capital allowance | | 500 | | | | (782-05) |
| Project B: Pre-Tax cash flows | | (500) | | | 765-50 | |
| (Taxes) capital allowance | | | 250 | | | (382-75) |
| Net cash flows | (1000) | NIL | 250 | NIL | 2329-60 | (1164-80) |

When both projects are undertaken the net terminal value at time 5 is now £110 after tax, £55 of which relates to A's pre-tax cash flows, taxes and allowances and £55 of which relates to those for B. Since both have positive net terminal values after tax the acceptance of both is preferable to the acceptance of either. By contrast without tax only one project would have been undertaken during the first 3 years. It would have been possible to invest in project A at time 0 and project B at time 4. However, because of the capital allowance project A can be used to finance project B at an earlier date.

Interactivities between projects and the importance of both profitability and liquidity effects have been highlighted by Adelson[24] and Fawthrop[25], and suggest the use of a programming model. On the question of taxation Fawthrop[25] has stated

> " little attention is paid in project appraisal literature to the wide variety of tax situations which potentially face the analyst. The universal assumption seems to be that either the project itself will generate a sufficient taxable surplus, or that adequate taxable profits already exist elsewhere in the company's operations, to mop up those generous initial or other capital allowances which authors and lecturers alike seem almost to imply are the sole prerogative of discounting techniques. The problems of time-plotting such allowances as carry-forwards in loss situations; the programming intricacies of selection and timing which arise in such situations as when several subsidiaries of a group are submitting tax-adjusted evaluations, yet group taxable profits are inadequate to sustain all the potential allowances; the potential inter-dependencies of projects where the realisation of one project's capital allowances is a function of the acceptance or

> rejection of some other project or projects; the realistic treatment of disposal gains or losses accruing subsequent to the end of the appraisal study-period - such issues as these are left to the initiative of the analyst, who (one suspects) too often accepts a convention of '12-months staggering' for want of inspiration to the contrary'.

Also Thomas[26] argued that "both the liquidity and profitability effects of incentives are important, but that the former is more important than the latter". The liquidity position of companies was also found to be a major determinant of investment behaviour in the British Economy according to an econometric study by Agarwala and Goodson[27]. Hence the use of payback as an appraisal technique.

4.5. Taxation, Capital Structure and Dividend Policy

In their famous paper, Modigliani and Miller[28] showed that, assuming firms can be divided into 'equivalent return' classes and that shares are traded in perfect markets under conditions of perfect competition, then the market value of any firm is independent of its capital structure and is given by capitalising its expected return at the rate appropriate to its class, since levered companies cannot command a premium over unlevered companies because investors have the opportunity of putting the equivalent leverage into their portfolio directly by borrowing on personal account. However, Stiglitz[29a,29b] later proved that it is not necessary to assume that there are two or more firms which are otherwise identical, that the argument does not require the existence of risk classes, and that the competitiveness of the capital market is of no importance provided the price paid by one individual (or firm) for a bond or share is the same for all other individuals. He has also noted that the theorem is limited if expectations are a function of financial policy or if individual borrowing is an imperfect substitute for firm borrowing.[29c] Moreover he shows that if there is a chance of bankruptcy, bonds become risky assets and there is no reason to suppose that the nominal rate of interest should be the same function of the debt-equity ratio for all firms or individuals, and that since for a firm with a given market value, a takeover bid is much easier if there is a large debt-equity ratio, the possibility of takeovers probably increases the rate of interest which a firm must pay on its bonds.[30]

Now, even in the absence of these other imperfections, Modigliani and Miller [31] demonstrate that the tax advantage of debt results in a higher level of after-tax income for any given level of before-tax earnings with the result that the value of the levered firm exceeds that of the unlevered firm by the capitalised value of the tax relief on the interest payments, although to capitalise the relief at the rate for a certain income stream as they suggest is in practice unrealistic at high levels of leverage.

Farrar and Selwyn [32] have shown that for the US tax system, personal and corporate, "for any positive operating income, rate of interest, return of equity, shareholding period, level of debt, marginal tax liability (personal or corporate), whatever, the existence of preferential tax treatment for capital gains, guarantees both gross and net personal income can be improved by shifting returns to investors, to the extent possible, from dividends to capital gains". Although the operating income of the firm is treated as an uncertain quantity, the returns to debt capital are treated as constant regardless of the state of the world and hence the analysis is essentially a model under certainty. The equivalent results for the UK imputation system are derived by King [33] although a world of certainty is assumed. Elton and Gruber [34] derived marginal stockholder tax brackets by studying the ex-dividend behaviour of common stocks and showed that these tax brackets are related to a firm's dividend policy. Hence they provided "evidence in support of Modigliani's and Miller's [35] clientele effect, suggesting that a change in dividend policy could cause a costly change in shareholder wealth", and illustrated "one form of market rationality in that stockholders in higher tax brackets show a preference for capital gains over dividend income relative to these on lower tax brackets".

Also, using two variables to represent relative time preferences, and an estimate of the individual's differential tax rate on dividends and capital gains, Pettit [36] was able to explain a significant portion of the observed cross-sectional variation in individual portfolio dividend yields. This US empirical investigation suggested a significant dividend clientele effect. Further US empirical evidence by Galpoor and Zimmerman [37] have shown that investors in higher pre-investment marginal tax rates tend to acquire disproportionate shares of losses in those industries such as Real Estate and Oil and Gas Extraction which receive relatively favourable tax treatment. In contrast, investors with lower pre-investment marginal tax rates tend to acquire disproportionate shares of losses in those industries such as Wholesale and Retail Trade which have relatively less access to favourable tax treatment. They note that this is consistent with economic theory which suggests that assets which receive preferential income tax treatment should be held predominantly by taxpayers in higher marginal tax brackets.

Litzenberger and Van Horne[38] consider a multi-period model based on time-state preference theory and include personal taxes as well as corporate taxes. They assert that in the absence of bankruptcy costs there would be a net advantage associated with debt financing and that the elimination of the double taxation of dividends would reduce the occurrence of bankruptcy and therefore reduce also the social costs associated with bankruptcy. Litzenberger and Ramaswamy[39] present empirical evidence to support a 'tax clientele CAPM'. This is consistent with Elton and Gruber[34] and Litzenberger and Ramaswamy[40].

In chapter 3 Miller's 1977 paper[41] was reviewed and the implications for capital structure under general equilibrium were discussed for the UK tax system. It was shown that after personal and corporate taxation, ignoring bankruptcy costs, capital structure for the firm is irrelevant in market equilibrium. This is consistent with Miller's analysis even though his model was not based on CAPM and even though he assumed a risk-free rate of interest on debt and implied a nil dividend payout rate. Miller's result reveals an equilibrium level of aggregate corporate debt and hence an equilibrium debt-equity ratio for the corporate sector as a whole. For the UK it was determined that with heterogeneous tax rates there is a clientele effect of dividend policy and an irrelevant capital structure for the individual firm. By contrast it was shown that in partial equilibrium the requirements of a neutral tax system were violated in the UK situation. The partial equilibrium model used, however, assumed homogeneous tax rates. In particular the higher rate of income tax on debenture interest was assumed to be the same as the higher rate of income tax on dividends in all cases.

Schweller's work[42] showed that Miller's result depends on the assumption that default considerations are ignored. He suggested a clientele effect whereby a firm's financing policies attract shareholders and debentureholders with appropriate marginal tax rates. De Angelo and Masulis[43] generalise Miller's work using a two-date state-preference model. It is assumed that utility maximising investors are taxed at rates which differ across investors and security classes. They consider the aggregate behaviour of firms supplying securities and the investors' aggregate demand induced by taxation.

"On the supply side, all firms obtain the same constant marginal value of debt for all levels of leverage so that all respond identically by supplying only the debt or equity claim priced at a premium. Debt and equity can be in positive aggregate supply simultaneously only in the absence of a price premium, which implies that each firm is indifferent to leverage. On the demand side, the heterogeneous personal tax treatment of different investors' debt and equity income ensures that some investors will demand the debt or equity claim priced at a discount. Given no price premium, there will be positive aggregate demand for both debt and equity claims. Together ASR (Aggregate Supply Response) and TIPAD (Tax-Induced Positive Aggregate Demand) preclude debt or equity from being a totally dominant form of financing and yield the equilibrium pricing condition....which implies leverage irrelevancy for the individual firm" (p.458[43]).

They also show that the irrelevance of the firm's capital structure holds under alternative personal tax codes.

Taggart[44] examines Miller's model under incomplete capital market conditions, and introduces costs associated with debt. He observes that as the capital structure of one firm changes there must be offsetting changes by other firms. "Value-invariance holds in an 'intra-equilibrium'[45] sense here since we are dealing with the relative values of firms, keeping security supplies and market prices constant. Value-invariance would not hold in an 'inter-equilibrium' sense, by contrast , since a change in capital structure by just onefirm will alter security supplies, thus bringing about a whole new equilibrium with different relative interest rates and firm market values" (p.650[44]). He notes that similar points are made by other authors[46,47]. Taggart considers special costs associated with corporate debt, such as costs to avoid bankruptcy, and costs incurred in the negotiation and enforcement of debt contracts. He points out that to the extent that debt costs are associated with the notion of business risk classes, firms within a given risk class would tend to have relatively similar capital structures. He notes that this is broadly consistent with empirical observations by Schwartz and Aronson[48], Scott[49] and Scott and Martin[50]. Furthermore he argues that under incomplete capital market conditions, where all portfolio combinations are not possible, shareholders' preferences for capital structure policy will not be unanimous.

Chen and Kim[51] and Jensen and Meckling[52] have pointed out that owner-managers will expropriate the wealth of suppliers of outside capital. Where the outside capital is equity, there will be excessive corporate fringe benefits. As to corporate bondholders, Chen and Kim have asserted that an increase in non-pecuniary benefits reduces the coverage in the case of bankruptcy and thus decreases the market value of the bonds. In addition, wealth transfers from outside debtholders may occur through investment decisions, some of which are suboptional. In a well-functioning market for managers, it is argued however that adjustments to their wages cancel out the fringe benefits.

A separate line of thought has been based on asymmetric information between managers and investors. Since managers have inside information then investors require a financial signalling device[51,53,54]. Managers search for an optimal capital structure to maximise their own wealth[51,53]. However, there is a problem that managers may make false signals[51].

## 4.6. Taxation and Risk-taking

An original paper by Domar and Musgrave [55] in 1944 showed that a proportional tax with full offset for losses increases total risktaking. Since the rate of return is reduced through taxation by the same proportion as the reduction in risk, private risktaking remains the same, but the Government becomes a business partner sharing both risk and return. In this way total risktaking, private and public, is increased. Their analysis however, is based on a choice of investments limited to cash and one risky asset. Later, Tobin [56] specified the nature of utility curves which would be consistent with loci of constant expected utility of wealth, which prove the Domar-Musgrave result. He found that normal curves and quadratic curves for the relevant range satisfied the requirements.

Feldstein [57] found that "in the general case in which the investor divides his portfolio between two risky assets, it is impossible to predict the effect of a proportional taxation without further knowledge of the properties of the indifference curves".

An example of a quadratic utility function is given on page 29 of Van Horne [58]

$$u = 2x - 0.05\,x^2 \,.$$

By differentiating utility we derive marginal utility

$$u' = 2 - 0.10x \,,$$

which is positive for $x < \frac{2}{0.10}$ i.e. showing positive marginal utility for the relevant range, and which decreases as $x$ increases - the concept of diminishing marginal utility. If we differentiate again we derive the rate of change of marginal utility:

$$u'' = -0.10 \quad .$$

Now, for a wealthy person we would expect the modulus of the rate of change of marginal utility as a proportion of marginal utility to decline with an increase in wealth since wealthy people can 'afford' to risk more. This is Arrow's [59] property of decreasing absolute risk aversion. However, risk aversion denoted $R_A$, is given by

$$R_A = -\left[\frac{u''}{u'}\right] = 0.10\ (2-.10x)^{-1} \quad .$$

Hence: $$R_A^1 = 0.10\ (-1)\ (2-.10x)^{-2}\ (-.10)$$

$$= \frac{0.01}{(2-.10x)^2} \quad ,$$

which is positive for a quadratic utility function. Feldstein noted this result and concluded that the quadratic utility function is an inappropriate basis for analysing the effects of taxation on risktaking. Feldstein went on to show that using a utility function of the form

$$u(c) = BC^{\alpha} + u_o \quad ,$$

with $$\alpha = \beta + 1, \ \alpha B > o \quad ,$$

the preference ordering of a probability distribution is unchanged by the introduction of a proportional tax. However, although this function satisfies positive marginal utility, diminishing marginal utility and decreasing risk aversion, it assumes a constant elasticity of the marginal utility function, denoted $\beta$. A further drawback of these models is that there exists a riskless yieldless asset, namely cash. Although such models have been extended by Stiglitz[60] to deal with cases where the return to the safe asset is greater than zero, he specifies whether absolute risk aversion is constant, increasing or decreasing. However, Hartman[61] has shown that the effect of some taxes are "unambiguous even when none of the assets available to investors is riskless and yieldless and neither the utility function nor the distribution of returns is specified". Similarly, Stiglitz[62] has recently examined the effects of taxation on risktaking without making specifications as to the utility function, although he assumes that 99.999'% of wealth invested is debt capital. With initial wealth of $\omega_0$, Stiglitz assumes a proportion (1-a) is invested in a risk-free asset with a pre-tax yield at the rate of r. Hence after tax and interest relief the yield is

$$\omega_0(1-a)r \quad ,$$

With a pre-tax yield on the risky asset at the rate of e (a stochastic variable), the return on the risky asset after tax and interest relief is

$$\omega_0 ae\ (1-T) + T.\ r.\ \omega_0 . a \quad ,$$

Hence total wealth (capital plus yield) is

$$\omega_0 \left[1 + (1-a)r + ae(1-T) + T.r.a\right]$$

$$= \omega_0\left[1+ r + a(e-r)\ (1-T)\right] \quad ,$$

as derived by Stiglitz.

Note that we are effectively considering a one period model.

Now, the end of period wealth is

$$Y = w_o \left[1 + r + a\,(e-r)(1-T)\right] .$$

$$E\left[U(Y)\right] = E\left[U \;\{w_o \left[1 + r + a(e-r)(1-T)\right]\}\right] = W .$$

$$\frac{dW}{de} = \frac{\partial E\left[U(Y)\right]}{\partial a} \frac{da}{de} + \frac{\partial E\left[U(Y)\right]}{\partial T} \frac{dT}{de} .$$

$$\frac{dW}{de} = E\left[U'(Y) \frac{\partial Y}{\partial a}\right] \frac{da}{de} + E\left[U'(Y)\frac{\partial Y}{\partial T}\right] \frac{dT}{de} .$$

$$\frac{dW}{de} = w_o\, E\left[U'(Y)\;(e-r)(1-T)\right] \frac{da}{de} + w_o\, E\left[U'(Y)\;(-a(e-r)\right] \frac{dT}{de} .$$

For a maximum, $\frac{dW}{de} = o$ ;

$$o = (1-T)\; E\left[U'(Y)\;(e-r)\right] \frac{da}{de} - a\; E\left[U'(Y)\;(e-r)\right] \frac{dT}{de} .$$

Therefore $\frac{da}{dT} = \frac{a\;E\left[U'(Y)\;(e-r)\right]}{(1-T)\left[E\;U'(Y)(e-r)\right]} = \frac{a}{1-T}$ .

With tax rates between zero and 100 per cent then da/dT is positive. Hence Stiglitz concludes that an increase in tax leads to an increase in the demand for the risky asset.

Let us now extend Stiglitz' analysis to consider

(i) borrowing by a firm at a fixed rate of interest to match fully the investment in a risk free investment,

(ii) equity finance as the sole source of funds for the risky investment, the returns to which are wholly in dividends,

(iii) the interactions of an imputation tax system. At first the tax rates other than T will be constant, but then the analysis will deal with the effects of stochastic personal tax rates and stochastic rates of imputed credits.

The yields on the safe investment, with interest deductibility, is

$w_o (1-a)\ r\ (1-T) + Tw_o (1-a)r = w_o (1-a)r$ , as before.

The profit after Corporation Tax on the risky investment is

$w_o ae\ (1-T)$ .

Since a full dividend payment rate is assumed, this amount, becomes the net dividend, the gross equivalent of which is

$$\frac{w_o\, a.e.(1-T)}{1-b} ,$$

where b = the basic rate of income tax .

After all personal taxes, the dividend is worth

$$\frac{w_o\, a.e.\ (1-T)\ (1-h)}{1-b} ,$$

where h = the higher rate of personal income tax.

Total wealth now becomes

$$w_o\left[1+ (1-a)\ (1-h)r + \frac{a(1-h)\ (1-T)}{1-b}.e\right] .$$

$$E\left[U(Y)\right] = E\left[U\left\{w_o\left[1 + (1-a)(1-h)r + \frac{a(1-h)(1-T)}{1-b}\ e\right]\right\}\right] = W .$$

$$\frac{dW}{de} = \frac{\partial E}{\partial a}\left[U\ (Y)\right]\frac{da}{de} + \frac{\partial E}{\partial T}\left[U(Y)\right]\frac{dT}{de} .$$

For $\frac{dW}{de} = o$ ,

$$o = E\left[U'(Y)w_o\left\{ - (1-h)r + \frac{(1-h)(1-T)e}{1-b}\right\}\right]\frac{da}{de}$$

$$+ E\left[U'(Y)w_o\left\{ - \frac{a\ (1-h)e}{1-b}\right\}\right]\frac{dT}{de} .$$

$$o = E\left[U'(Y)e\right]\left(\frac{(1-h)(1-T)}{1-b}\right)\frac{da}{de} - E\left[U'(Y)\right]\ (1-h)r\ \frac{da}{de}$$

$$- E\left[U'(Y)e\right]\ a\ \frac{(1-h)}{1-b}\ \frac{dT}{de} .$$

$$\frac{da}{de}\left\{ E\left[U'(Y)e\right]\frac{(1-h)(1-T)}{1-b} - E\left[U'(Y)\right](1-h)r\right\} = E\left[U'(Y)e\right]\frac{a(1-h)}{1-b}\ \frac{dT}{de} ,$$

$$\frac{da}{dT} = \frac{E\left[U'(Y)e\right]\frac{a(1-h)}{1-b}}{E\left[U'(Y)e\right]\frac{(1-h)(1-T)}{(1-b)} - E\left[U'(Y)\right]\ (1-h)r} .$$

But $E\left[U'(Y)\right] = o$ .

Therefore

$$\frac{da}{dT} = \frac{\left[a(1-h)/(1-b)\right]\ E\left[U'(Y)e\right]}{\left\{\left[(1-h)(1-T)/(1-b)\right\}\ E\left[U'(Y)e\right]}\ ,$$

or $\frac{da}{dT} = \frac{a}{1-T}$ as before.

With a variable rate of income tax and both T and h constant,

$$\frac{dW}{de} = \frac{\partial E\left[U(Y)\right]}{\partial a}\ \frac{da}{de} + \frac{\partial E\left[U(Y)\right]}{\partial b}\ \frac{db}{de}\ ,$$

For $\frac{dW}{de} = o$ ,

$$o = E\left[U'(Y)\left\{-(1-h)r + \frac{(1-h)(1-T)e}{1-b}\right\}\ w_o\right]\ \frac{da}{de}$$

$$+\ E\left[U'(Y)\left(-\frac{a(1-h)(1-T)(-1)}{(1-b)^2}\right\}e\right]\ \frac{db}{de}\ .$$

For $\frac{dW}{de} = o$ ,

$$\frac{da}{db} = \frac{-E\left[U'(Y)\left\{a\ (1-h)(1-T)/(1-b)^2\right\}\ e\right]}{-E\left[U'(Y)\left\{(1-h)r\ -\ (1-h)(1-T)e/(1-b)\right\}\right.}\ .$$

$$\frac{da}{db} = \frac{E[U'(Y)e] \; a \; (1-T)}{(1-b)\left[E[U'(Y)] \; (1-b)r - (1-T)\; E[U'(Y)e]\right]}$$

Provided $E[U'(Y)](1-b)r < (1-T)\;E[U'(Y)e]$, and

$E[U'(Y)e]$ is positive, then da/db is negative and an increase in the basic rate of income tax leads to a decrease in the demand for the risky asset.

Finally with the higher rate of income tax variable and both T and b constant

$$\frac{dW}{de} = \frac{\partial E[U(Y)]}{\partial a} \frac{da}{de} + \frac{\partial E}{\partial h}[U(Y)] \frac{dh}{de}$$

For $\frac{dW}{de} = o$,

$$o = E\left[U'(Y)\left\{-(1-h)r + \frac{(1-h)(1-T)e}{1-b}\right\} w_o\right] \frac{da}{de}$$

$$+ E\left[U'(Y)\left\{-(1-a)r - \frac{a(1-T)e}{1-b}\right\} w_o\right] \frac{dh}{de}$$

Therefore

$$\frac{da}{dh} = \frac{E[U'(Y)\{(1-a)(1-b)r + a(1-T)e\}]}{E[U'(Y)\{(1-h)(1-T)e - (1-h)(1-b)r\}]}$$

The relationship depends on the basic rate of income tax, the risk-free rate of interest, the probability distribution of e, and the corporate tax rate.

The effect of taxation on both business and financial risktaking has been discussed in terms of Markowitz[63] mean-variance approach extended to the Sharpe[64] Lintner[65] Capital AssetPricing Model. We shall now specify the justification for such an approach. Earlier it had been stated that the mean-variance analysis is appropriate where utility curves are quadratic or returns are normally distributed. Unfortunately, not only does the quadratic utility curve possess the property of increasing absolute risk aversion, but it has a limited range of positive marginal utility and as Hirshleifer[66] notes "We cannot accept the quadratic even as an approximation, however well it may fit in the neighbourhood of the mean return $\mu(X)$, because we are dealing with risky portfolios that require us to evaluate the utility of values for the random variable X diverging considerably from the mean". Moreover, tax systems with either progressive rates, or proportional rates but with carry-forward provisions of allowances, would turn normal probability distributionsinto skewed ones and hence prima facie limit the usefulnessof the mean-variance approach. However, Tsiang[67] has argued that the approach is justified provided the "aggregate risk taken by the individual concerned is small compared with his total wealth, including his physical, financial, as well as human wealth". He expands a utility function into a Taylor series, of the general form, for a convergent series,

$$f(v+x) = f(v) + f'(v)x + f''(v)\frac{x^2}{2!} + f'''(v)\frac{x^3}{3!} + \ldots$$

If y is a random variable and z is the deviation from the mean, then

$$\begin{aligned} U(y) &= U(\bar{y}+z) \\ &= U(\bar{y}) + U'(\bar{y})z + U''(\bar{y})\frac{z^2}{2!} + U'''\frac{(\bar{y})z^3}{3!} + \ldots \end{aligned}$$

He noted that the expected utility is

$$E[U(y)] = \int_{-\infty}^{\infty} U(\bar{y}+z)\ f(z)dz \quad ,$$

where f(z) is the density function of z ,

z is the deviation of y from $\bar{y}$ .

$$E[U(y)] = U(\bar{y})\int_{-\infty}^{\infty} f(z)dz + U'(\bar{y})\int_{-\infty}^{\infty} z\ \ f(z)dz$$

$$+ \frac{U''(\bar{y})}{2!}\int_{-\infty}^{\infty} z^2\ f(z)dz + \frac{U'''(\bar{y})}{3!}\int_{-\infty}^{\infty} z^3\ f(z)dz + \ldots$$

However, $\int_{-\infty}^{\infty} f(z)dz = \sum_y p_y = 1$ ,

where $p_y$ is the probability of $y$ .

Therefore,

$$\int_{-\infty}^{\infty} zf(z)dz = \sum_y (y-\bar{y})p_y = \sum_y yp_y - \bar{y} = \bar{y} - \bar{y} = o \quad ,$$

$$\int_{-\infty}^{\infty} z^2 f(z)dz = \sum_y (y-\bar{y})^2 p_y = \text{ the variance } ,$$

$$\int_{-\infty}^{\infty} z^3 f\ (z)dz = \text{ the third central moment } .$$

Hence

$$E\big(U(y)\big) = U(\bar{y}) + U''(\bar{y})\frac{\bar{m}_2}{2!} + U'''(\bar{y})\frac{\bar{m}_3}{3!} + \ldots .$$

where $\bar{m}_2$ = the second central moment of the distribution ,

$\bar{m}_3$ = the third central moment of the distribution .

Tsiang notes that if risk (variance) is assumed to be infinitesimally small, higher order central moments are assumed to be of even smaller orders. Hence utility may be approximated by

$$E\big(U(y)\big) = U(\bar{y}) + U''(\bar{y})\frac{s^2}{2} \quad ,$$

where s is the standard deviation .

In this way expected utility curves may be described by the mean $\bar{y}$ and variance $s^2$ where the latter is very small in absolute magnitude. However, Tsiang shows that a fair approximation of the expected utility function is obtained if the risk remains small relative to the total wealth of the individual concerned, and gives examples where the standard deviation ranges only from zero up to 10 per cent of the individual's expected value of total wealth.

As to financial risktaking, Stiglitz[68] has shown that following the mean-variance analysis the value of the firm is independent of the debt-equity ratio in the absence of taxation.

In chapter 3 this was re-examined under an imputation tax system, although we ignored the Magill and Constantinides' result[69] that when trading opportunities on the capital market are no longer available costlessly, the investor substantially modifies his concept of an optimal portfolio which now consists of a whole region in the portfolio space. The analysis followed the principle of Bar-Yosef and Kolodny[70] that, under the CAPM, a separation of the covariance into the systematic risk associated with the dividend return and that associated with the capital gain return can provide a basis for showing that investors have a net preference for receiving their return in the form of capital gains.

A difficulty is that, as noted by Elton and Gruber,[71] "given the predominance of income tax rates above capital gains tax rates most investors will tilt their portfolio in favour of stocks with low dividends and it is unlikely that markets will clear at prices determined by the Sharpe-Lintner-Mossin form of the CAPM" They consider the situation of the investor subject to a higher tax rate than the effective tax rate in the market .

For positive Beta stocks, the lower the dividend yield, all other things being equal, the more likely the stock is to be held in more than market proportions. Also, "... if two stocks have identical Betas, residual risk and dividend yields, an investor who pays a tax rate higher than the effective average in the market is more likely to hold more than market proportions of the stock which represents a larger share of the market ... An investor with a tax rate lower than the effective rate in the market will act in the opposite manner."

Stapleton and Burke [72] have considered an imputation tax system under the Capital Asset Pricing Model based on Brennan's model [73] which assumes that the dividend component of the company's total expected end of period total return is known with certainty i.e. the whole of the uncertainty regarding end of period return attaches to the capital gains component. By contrast the model which we used in chapter 3 was more general and encompassed risky dividends as well as risky capital gains.

4.7. Conclusions

It was asserted (chapter one) that tax neutrality was a desirable requirement in a tax system. In this brief survey of the literature we have seen that a number of authors have suggested that a cash flow tax system would fulfil this aim although there has been some debate concerning (i) disincentive effects caused by the arbitrariness of the dates of accounting periods, (ii) the revenue raising capacity of the tax system and (iii) some confusion has arisen between issues of equity and efficiency.

It was shown that to have both instantaneous free depreciation and interest deductibility resulted in a pre-tax rate of return on a capital outlay required to finance debt capital, at a lower figure than the rate of interest on debt. This violates the principle established in chapter one that the marginal rate of substitution of future for present consumption, as valued by consumers or savers, should be equal to the marginal rate of transformation of present into future goods in production. Authors have suggested that if interest deductibility were abolished then with free depreciation the rate of interest on debt would be equal to the pre-tax rate of return on the capital outlay required to finance the debt capital. However with the capital allowance carry forward provisions this was shown to be no longer true.

A number of authors had shown that for the US situation the demand for risky assets is inversely proportional to one minus the corporate tax rate. Not only was this result shown to be inappropriate for the UK imputation tax system but the relationship between taxation and risktaking was more complex than previous models would suggest. In contrast to other models restrictive assumptions were not placed on the form of the utility function.

The Modigliani and Miller papers were briefly reviewed. It was stated that to capitalise the relief at the rate for a certain income stream is in practice unrealistic at high levels of leverage. Other models, which were based on CAPM, have assumed risk-free dividends. The extensions dealing with risky debt, dividends and capital gains have already been dealt with in chapter three under the CAPM. In the present chapter an attempt was made to justify the CAPM approach by assuming wide diversification of shareholder portfolios such that the risk on a shareholder's stake in a firm's investment is very small in relation to the total wealth of the individual concerned. The implication is that an individual firm's bankruptcy, ceteris paribus, may have no material effect on shareholder utility. However the disutilities of society caused by the social and psychological effects of bankruptcies are ignored by such a model.

The lack of consideration of the complexities of tax effects on capital project appraisal procedures both in practice and in the finance literature has been highlighted. In particular (i) to deal with interdependencies of projects through capital allowances and (ii) to reflect the practical importance of both liquidity (capital rationing) as well as profitability effects, a programming model is required to solve capital budgeting problems in nontrivial cases. Such a model is presented in chapter seven. But because of the complexities of the tax system, before incorporating the tax framework into a financial programming model, it is instructive to isolate the effects on investment decisions (chapter 5) and financing decisions (chapter 6) of each major tax rule using numerous simple numerical and algebraic examples.

# CHAPTER 5

## The impact of taxation on capital project appraisal under certainty

5.1. Notation (for chapter 5 only)

$A^{CT}_{q}$ = ACT payable for the accounting period ending at time q ,

$A_N$ = present value of capital allowances under current cost accounting principles with an asset replacement period of N years ,

b = the basic rate of income tax ,

$C^A_j$ = capital allowance in period j ,

$D_j$ = dividend paid at time j ,

$\Delta F_j$ = the change in accounting depreciation in period j of those fixed assets relating to production overheads, resulting from project acceptance ,

$J_j$ = outlay J in period j ,

k = discount rate ,

$k_i$ = the rate of asset price inflation ,

$k_n$ = a nominal required rate of return (constant) ,

$k_r$ = a real rate of interest ,

K = the later of the fiscal periods involved in the accounting period ,

m = the last accounting period in which a capital allowance is claimed ,

N = asset replacement period ,

n = the last time period in which an allowance is claimed ,

$\Delta N^{TI}_{j}$ = the change in net taxable income in period j resulting from project acceptance ,

P = proportion of total taxable income to be taxed at the rate $T_K$ .

q = the delay between the date of capital expenditure and the end of the accounting year in which each allowance is claimed .

$R_j$ = dividend received at time j .

$S^c_j$ = the change in the closing balance of inventory in period j resulting from project acceptance .

$S^o_j$ = the change in the opening balance of inventory in period j resulting from project acceptance .

T = the corporate tax rate .

$u_j$ = the proportion of the volume of unsold goods to the volume of production during the period.

V = present value of net operating cash flows before tax.

$W_j$ = the change in net working capital in period j resulting from project acceptance .

$X_j$ = net cash inflow at time j .

y = the time gap between the end of the accounting period on which the allowance is based and the tax payment date .

z = stock relief percentage (currently z = 0.15) .

## 5.2. Introduction

It needs to be stated at the outset that much of the first half of this chapter is primarily concerned with the mechanics of basic corporation tax computations which are already well established in the professional tax literature. To add salt to the wounds I shall resort to symbolic representation of the tax rules and the reader may indeed now query the rationale for such material being included in a doctoral thesis in this way. It has been stated earlier that one of the criticisms against finance authors, lecturers and financial analysts is that tax complexities are frequently ignored and if such material were now omitted there would be a serious loss in the usefulness of the thesis. Furthermore, the numerous algebraic and arithmetical examples in this chapter should provide the groundwork for the full model to be presented in chapter seven, and without painstakingly progressing through the ABC of the tax system we might soon become lost in a sea of algebra. The symbolic representation of the tax rules in the present chapter should therefore help to provide the groundwork for chapter seven. I really do apologise to the reader for the tediousness of this chapter but believe that its contents nevertheless do serve a useful role.

5.3 A cash flow tax system

It has been well established ((Brown, 1948), (Lawson and Stark, 1975), (Meade, 1978) and in chapter two of this thesis), that a corporate tax system based on cash flows may offer a neutral solution to the capital investment decision. For instance, consider a firm spending £1m in return for £100,000 at the end of each year into perpetuity. With 100 per cent capital allowances and full offset for losses (ignoring the tax lag between the pre-tax cash flows and the incremental tax cash flows thereon) then the firm spends only £480,000 after tax relief at 52 per cent, but receives £48,000 p.a. after tax, thus maintaining the pre-tax rate of return at 10 per cent p.a. With such a system the investment decision is not changed by the imposition of taxation. In present value terms the tax relief on cash outflows bears the same proportion to pre-tax cash outflows as the tax on cash inflows bears to the pre-tax cash inflows. The plus or minus sign of the NPV of a project before tax will be the same as that after tax. The result is that, assuming a discounted dividends share valuation model, for capital invesment decision purposes, a shareholder wealth maximising firm may ignore corporation tax altogether and the government effectively becomes a business partner.

Provided the time lags between pre-tax cash flows and taxes/ allowances thereon are constant, then given the tax system as outlined above, a neutral solution exists if the present value of the capital allowances, ignoring these inherent tax lags, equals the invesment outlay (J). This is the principle of the cash flow tax system.

Consider the numerical illustration in table 8

Table 8 Present value of pre-tax cash flows

| End of Year | Cash Flow (£) | Discount Factor @ 15% | NPV |
|---|---|---|---|
| 0 | -3000 | 1. | -3000 |
| 1 | 1000 | 0.8696 | 869.6 |
| 2 | 2000 | 0.7561 | 1512.2 |
| 3 | 1000 | 0.6575 | 657.5 |
| | | | NPV = £ 39.3 |

The outlay at the end of year 0 reduces the net cash inflows from the portfolio of projects undertaken by the firm of which the above project is one. Hence if inflows are taxed at say, 52% the tax bill based on net cash inflows at the end of year 0 will be reduced by 52% x £3,000 = £1,560. Similarly, the £1,000 cash flow at the end of year one will bear tax of £520, the £2,000 will bear tax of £1,040, and the £1,000 inflow at the end of year 3 will bear tax of £520 also. With a time delay in settling tax bills of say 1 year we show in table 9 the changes in tax payments resulting from project acceptance, and the present value of the tax effects are given in table 10.

Table 9 Tax effects

| End of Year | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Reduction in tax through allowance on capital expenditure (£) | | 52%.3000 = 1,560 | | | |
| Taxes on inflows (£) | | | 52%.1000 = 520 | 52%.2000 = 1,040 | 52%.1000 = 520 |

Table 10 Present value of tax effects

| End of Year | Cash Flow | Discount Factor | NPV |
|---|---|---|---|
| 0 | - | 1.0 | - |
| 1 | 1560 | 0.8696 | 1356.5 |
| 2 | -520 | 0.7561 | -393.2 |
| 3 | -1040 | 0.6575 | -683.8 |
| 4 | -520 | 0.5718 | -297.3 |
| Present value of the tax effects (£) | | | -17.8 |

Hence the Net present value of the project is £39.3 - 17.8 = £21.5

Because of the time value of money the effective tax rate of 52% is reduced. With a 1 year tax time lag and a 15% discount rate the marginal tax rate is effectively reduced to 52% x $\frac{1}{1+15\%}$ = 0.52 x 0.87 = 45.22%. Hence the £39.3 NPV is reduced by 45.22% x £39.3 = £17.8 to £21.5 as above. The effect of a cash flow tax may be shown algebraically as follows:

$$\text{NPV (before tax)} = -J + \frac{X_1}{1+k} + \frac{X_2}{(1+k)^2} + \frac{X_3}{(1+k)^3} + \ldots \quad (1),$$

where J = outlay ,

$X_j$ = net cash inflow at the end of time j ,

k = discount rate .

$$\text{NPV(of tax effects)} = \frac{T.J}{1+k} - \frac{T.X_1}{(1+k)^2} - \frac{T.X_2}{(1+k)^3} - \frac{T.X_3}{(1+k)^4} - \ldots$$

$$= -\frac{T}{1+k}\left[-J + \frac{X_1}{1+k} + \frac{X_2}{(1+k)^2} + \frac{X_3}{(1+k)^3} + \ldots\right] \quad (2),$$

However, the contents of the square brackets are equal to NPV(before tax).

Hence NPV(of tax effects) = $-\frac{T}{1+k}$ x NPV(before tax) .

Since NPV(after tax) = NPV(before tax) + NPV(**of tax effects**) ,

then NPV(after tax) = NPV(before tax) $-\left[\frac{T}{1+k}\right.$ x NPV(before tax)$\left.\right]$ ,

$$\text{NPV(after tax)} = \text{NPV(before tax)} \times \left[1 - \frac{T}{1+k}\right] \quad (3).$$

Where $0<T<1$ and $k>0$ then $\left[1 - \frac{T}{1+k}\right]$ is always positive. Hence the tax effects do not change the sign of the NPV. If the NPV(before tax) is positive, the NPV(after tax) will also be positive; and if the NPV(before tax) is negative, then the NPV(after tax) will also be negative. Finally with a NPV(before tax) equal to zero, the NPV(after tax) will also be zero. This latter case may be demonstrated by considering an investment project which offers an immediate cash outflow of £2,486 in return for cash inflows of £1,000 p.a. at the end of each of the next 3 years.

If we assume a money discount rate of 10% , we note from table 11 that the investment is marginal since it has a net present value of zero (to the nearest pound).

Table 11: Non-tax cash flows

| Year | Cash Flow | Discount Factor | Net Present Value |
|---|---|---|---|
| Now | (2,486) | 1.000 | £(2,486) |
| 1 | 1,000 | 0.909 | 909 |
| 2 | 1,000 | 0.826 | 826 |
| 3 | 1,000 | 0.751 | 751 |
| | | | NIL |

Now, under our simplified model a tax system is considered neutral if it allows at the margin a NPV of taxes on inflows to be equated with a NPV of tax relief on outflows. Let us assume that the time lag between cash outlays and tax allowances is the same as that between cash inflows and taxes thereon. In the extreme theoretical case where this time lag is zero and the rate of tax T is constant, the NPV of the change in the tax bill resulting from acceptance of the marginal project is zero, as demonstrated by table 12.

Table 12: Tax cash flows

| | Year | Cash Flow | D.F. | NPV |
|---|---|---|---|---|
| Capital allowance | Now | £2,486 X T | 1.000 | £2,486 X T |
| Tax on inflow | 1 | (1,000)X T | 0.909 | (909)X T |
| Tax on inflow | 2 | (1,000)X T | 0.826 | (826)X T |
| Tax on inflow | 3 | (1,000)X T | 0.751 | (751)X T |
| | | | | NIL |

With a one year tax time-lag the cash flows in table 12 need to be discounted for a further year. Hence each figure in the final column of table 9 would need to be multiplied by the factor 1/(1+10%) resulting in the same total NPV of zero. In this way with constant tax time lags and constant tax rates, a neutral effect on the investment decision is obtained since the discounted relief on the capital expenditure of the marginal investment fully compensates for the discounted tax on future cash inflows. Provided the cash outlay occurs at the same time during the accounting period as future cash inflows during future periods, then neutrality would still be obtained.

## 5.4. Tax time lags and the accounting period

For companies that began trading before 1965 the accounting year preceding April of year (x) forms the basis for the tax payable on 1 January of year (x+1). For instance, the accounting year ended 31 December 1977 which precedes April 1978/April 1979 forms the basis for the tax payable on 1 January 1979. Otherwise, for companies that began trading after 1965, the tax is payable nine months after the end of the accounting period. The importance of the accounting period can be demonstrated by considering the same project which is marginal before tax. Assume that the investment is made on 1 January 1978 with cash generated on 31st December 1978, 31 December 1979, and 31 December 1980. With an accounting year end also at 31 December, both the expenditure and the first inflow are assessed in the same period. For long established companies tax on the net taxable income for the accounting period ended 31 December 1978 is payable on 1 January 1980. Thus, with a one year time lag from the end of the accounting period the project obtains a value of minus £249 X T when discounted to 1 January 1980 (Table 13). Alternatively, if the investment were made on 31 December 1978 with inflows on 1 January 1980, 1981 and 1982, there would be a positive NPV (Table 14).

Table 13 : Tax cash flows

| | Timing of tax bill | Cash Flow | Discount Factor | NPV at 1.1.80 |
|---|---|---|---|---|
| Capital allowance | 1.1.80 | £2,486 X T | 1.000 | £2,486 X T |
| Tax on inflow | 1.1.80 | (1,000)X T | 1.000 | (1,000)X T |
| Tax on inflow | 1.1.81 | (1,000)X T | 0.909 | (909)X T |
| Tax on inflow | 1.1.82 | (1,000)X T | 0.826 | (826)X T |
| | | | | £(249)X T |

Table 14 : Tax cash flows

| | Timing of tax bill | Cash Flow | Discount Factor | NPV at 1.1.80 |
|---|---|---|---|---|
| Capital allowance | 1.1.80 | £2,486 X T | 1.000 | £2,486 X T |
| Tax on inflow | 1.1.82 | (1,000)X T | 0.826 | (826)X T |
| Tax on inflow | 1.1.83 | (1,000)X T | 0.751 | (751)X T |
| Tax on inflow | 1.1.84 | (1,000)X T | 0.683 | (683)X T |
| | | | | £226 X T |

In this way the timing of expenditure in relation to the accounting period can be important in the marginal case. Under conditions of certainty the tax relief on capital expenditure may be formally valued as follows, assuming no delays between the end of the accounting period and the date when the tax for the period is due:

$$NPV = T\left(\sum_{j=o}^{n} \frac{C_j^A}{(1+k)^j}\right) \qquad (4),$$

where $C_j^A$ = the capital allowance in period j ,

$k$ = the rate of interest ,

$T$ = the corporate tax rate ,

$n$ = the last time period in which an allowance is claimed .

Capital allowances are determined by the date when the asset is brought into use and though the Inland Revenue permits the date of the capital expenditure as a proxy, it is discretionary. In the analysis we shall assume that the two dates are the same. Hence the present value of the capital allowance is given by:

$$NPV = T\sum_{q=o}^{m}\left[\frac{C_q^A}{(1+k)^{q+y}}\right] \qquad (5),$$

where

$q$ = the delay between the date of capital expenditure and the end of the accounting year in which each allowance is claimed ,

$y$ = the time gap between the end of the accounting period on which the allowance is based and the tax payment date,

$m$ = the last accounting period in which a capital allowance is claimed.

With variable rates of corporation tax, we need to take account of the retrospective nature of the legislation in that the Corporation tax rate for the financial year 1976 is not determined until the Finance Act 1977 is passed at the end of July. Therefore in applying the rates of Corporation Tax to taxable income for a company whose year end is 31 December, for instance, any capital expenditure between 1 January and 31 December 1976 reduces the taxable income for that accounting period which in turn is apportioned as to one quarter taxable at the rate for the Financial Year 1975 and three quarters at the rate for the Financial Year 1976. During the early part of 1977 benefits of Capital allowances for the preceding year would still be unknown in relation to three quarters of the allowable capital expenditure. Hence, since accounting periods ending at dates other than 31 March cover more than one Financial Year, taxable profits after capital allowances need to be apportioned over the respective time periods to reflect the different rates of tax for each Financial Year. Since accounting periods for tax purposes are limited to 12 months' duration, each accounting period cannot extend into more than two Financial Years.

Therefore

$$NPV = \sum_{q=o}^{m} \frac{\left[PT_K + (1-P)\ T_{K-1}\right] C^A_q}{(1+k)^{q+y}} \qquad (6),$$

where $P$ = proportion of total taxable income to be taxed at the rate $T_K$, where,

$K$ = the later of the fiscal periods involved in the accounting period,

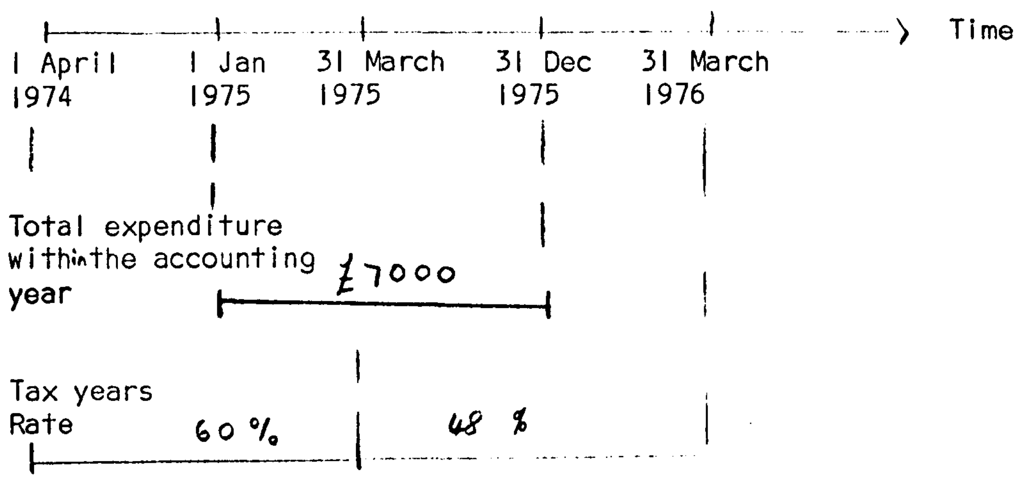
$K-1$ = the earlier of the fiscal periods involved in the accounting period,

$(1-P)$ = proportion of total taxable income to be taxed at $T_{(K-1)}$.

As an example let us assume the following for illustrative purposes only :

| | | Rate |
|---|---|---|
| Financial Year 1974 (April 1974 to March 1975) | : | 60% |
| Financial Year 1975 (April 1975 to March 1976) | ; | 48% |

The accounting year ended December 1975 straddles two tax years:




Since the accounting year straddles two tax years the expenditure is apportioned for tax purposes over 3 months (1 Jan to 31 March) and 9 months (1 April to 31 December):

| | | Reduction in tax bill |
|---|---|---|
| £7,000 x 3/12 @ 60% | = | £1,050 |
| £7,000 x 3/12 @ 48% | = | £2,520 |
| | | £3,570 |

Note that $C^A_q$ = £7,000, $T_k = 0.48$, $T_{k-1} = 0.60$, $P = 9/12$.

## 5.5. Imperfect relief for capital expenditure

The principle was established in chapter two that, even where taxes are paid on net operating cash flows, when the present value of capital allowances from a project is less than the capital outlay then there is a disincentive to invest in a project, of which the NPV before tax is zero. The symbol to represent the present value of capital allowances as a proportion of cost was denoted $\alpha$. Where $\alpha < 1$ there is a potential tax disincentive, where $\alpha > 1$ there is a potential incentive to invest, and a neutral system operates if $\alpha = 1$. For an investment in a new industrial building, then with a discount rate of 10 per cent per annum $\alpha \doteqdot 0.81$, ignoring (i) time lags between the end of each accounting period and the annual tax payment date and (ii) the timing of the expenditure in relation to the date of the accounting year-end. For second hand industrial buildings values of $\alpha$ were also less than one.

Hence, if we accept the assumptions of the NPV model then we must conclude that the tax system offers no incentive to expand factory premises and other buildings. Moreover, with inflation the present value of the tax relief is even further reduced. Consequently, where capital allowances are less than 100% at the time of expenditure, a project which is marginal before taxation becomes financially unattractive. This may be demonstrated by the following example. Assume a long established company with a December 31 year-end is considering spending on 1 July 1978 the sum of £25,000 on a new industrial building, with a nil scrap value at the end of the project's life, in return for 12 annual inflows from 1st July 1979 of £3,668-92. It can be shown from Table 15 that the project is marginal before tax since the present value of the outflow of £25,000 is equated with the present value of inflows.

Tax considerations alone would therefore determine the financial attractiveness of the proposition under our simplified decision model.

Table 15 : Non-tax cash flows

| | Date | Cash Flow | D.F. | N.P.V. at 1.7.78 |
|---|---|---|---|---|
| Expenditure | 1.7.78 | £(25,000) | 1.000 | £(25,000) |
| Inflows | 1.7.79-1.7.90 | 3,668-92 p.a. | 6.814 | 25,000 |
| | | | | NIL |

Since the expenditure on 1 July 1978 falls in the accounting period ended 31.12.78 the initial allowance of £12,500 and the writing-down allowance of £1,000 in the first year reduces the value of the tax bill due on 1 January 1980. The writing down allowance of £500 for the year ended 31.12.90 is equal to the balance of allowances not yet claimed by that date. From table 16 we observe the disastrous tax consequences of the investment in this particular example.

Table 16 : Tax cash flows (discounted to 1.1.80 for convenience)

| Accounting period ended | Tax date | Cash Flow | D.F. | NPV at 1.1.80 |
|---|---|---|---|---|
| Allowances | | | | |
| 31.12.78 | 1.1.80 | £13,500 X T | 1.000 | £13,500 X T |
| 31.12.79-31.12.89 | 1.1.81-1.1.91 | £ 1,000 X T p.a. | 6.495 | 6,495 X T |
| 31.12.90 | 1.1.92 | £ 500 X T | 0.319 | 160 X T |
| | | | | 20,155 X T |
| Taxes on inflows | | | | |
| 31.12.79-31.12.90 | 1.1.81-1.1.92 | £ 3,668-92 X T | 6.814 | (25,000) X T |
| | | | | £(4,845) X T |

Let us ignore (i) time lags between the end of each accounting period and the annual tax payment date and (ii) the timing of the expenditure in relation to the date of the accounting year end.

The NPV (before tax) is given by (V–J)

$$\text{where } V = \sum_{j=o}^{\infty} \frac{X_j}{(1+k)^j} \qquad (7).$$

After tax we have

$$\text{NPV (after tax)} = V(1-T) - J(1-\alpha T) \qquad (8).$$

Where the NPV (after tax) is negative

$$V(1-T) - J + \alpha JT < o$$

or

$$\alpha < \frac{1-(1-T)\frac{V}{J}}{T} \qquad (9).$$

When we have a net present value before tax of zero, i.e. V = J, then

$$\frac{1-(1-T)\frac{V}{J}}{T} = 1 \quad .$$

Hence with 100 per cent capital allowances $\alpha = 1$ and there is a neutral effect on the decision to invest.

By contrast where $\alpha$ is less than one there comes a point where $\alpha$ is sufficiently low that a project which is attractive before tax is no longer attractive after tax. Consider the present value of net cash inflows before tax equal to 20 per cent more than the present value of the outlay, i.e. V = 1.2 J. Where the corporate tax rate is 52 per cent then the NPV (after tax) is negative if

$$\alpha < \frac{1-(1-0.52)\,1.2}{0.52} \quad ,$$

that is (approximately)

$$\alpha \quad < \quad 0.8154 \quad .$$

Where the discount rate is 10 per cent per annum, then for a new industrial building is has already been established that

$\alpha \approx 0.81$. Hence, in this instance, even though the present value of net cash inflows before tax is 20% greater than the present value of the capital outlay, the tax rules for capital allowances result in a disincentive to invest. The general solution is described by inequality (9).

In chapter three the absence of capital allowances for premises in the retail industry was highlighted. Hence, where the NPV (after tax) is negative

$$V(1 - T) \quad - \quad J \quad < 0 \qquad \text{for } \alpha = 0 \quad ,$$

$$\text{giving} \quad \frac{V}{J} \quad < \quad \frac{1}{1 - T} \qquad (10) \; ,$$

$$\text{or} \quad \frac{V}{J} \quad < \quad \frac{25}{12} \qquad \textit{for } T = 52\% \quad .$$

Hence, even if the present value of the net operating cash inflows before tax are as high as twice the present value of the capital expenditure on retail buildings, the tax effects make the project financially unattractive. For every £12,000 of capital expenditure now on retail buildings, the present value of the net operating cash flows before tax need to be at least £25,000 for the project to be acceptable.

## 5.6. Current cost capital allowances

The present value of the total depreciation charged to the profit and loss account, for every pound of outlay incurred at the beginning of the accounting period, will be

$$\alpha = \sum_{q=1}^{m} \frac{1}{m} \frac{(1+k_i)^{q+y}}{(1+k_n)^{q+y}} ,$$

where

$m$ = asset life ,

$k_i$ = rate of asset price inflation ,

$k_n$ = nominal discount rate ,

$y$ = lag between end of accounting period and tax date .

When $k_i = k_n$, $\alpha = 1$, which indicates neutrality.

When $k_i < k_n$, $\alpha < 1$, which represents a potential disincentive to invest.

Finally, for $k_i > k_n$, $\alpha > 1$ and there is a potential incentive to invest.

A more interesting question is to examine tax incentives if backlog depreciation is included in which case total tax allowances represent the future replacement cost of the asset.

In the remainder of this section, CCA depreciation will refer to the depreciation in the balance sheet, which includes backlog depreciation, and not in the profit and loss account.

Table 17 Current Cost Depreciation

| End of year (i) | Price of new asset(£) (ii) | Accumulated CCA depreciation to date (iii) | Accumulated CCA depreciation to previous year (iv) | CCA depreciation for year plus backlog depreciation (v) = (iii) - (iv) |
|---|---|---|---|---|
| 0 | J | - | - | - |
| 1 | $J(1+k_i)$ | $\frac{1}{5}J(1+k_i)$ | nil | $\frac{1}{5}J(1+k_i)$ |
| 2 | $J(1+k_i)^2$ | $\frac{2}{5}J(1+k_i)^2$ | $\frac{1}{5}J(1+k_i)$ | $\frac{1}{5}J\left[2(1+k_i)^2 - (1+k_i)\right]$ |
| 3 | $J(1+k_i)^3$ | $\frac{3}{5}J(1+k_i)^3$ | $\frac{2}{5}J(1+k_i)^2$ | $\frac{1}{5}J\left[3(1+k_i)^3 - 2(1+k_i)^2\right]$ |
| 4 | $J(1+k_i)^4$ | $\frac{4}{5}J(1+k_i)^4$ | $\frac{3}{5}J(1+k_i)^3$ | $\frac{1}{5}J\left[4(1+k_i)^4 - 3(1+k_i)^3\right]$ |
| 5 | $J(1+k_i)^5$ | $J(1+k_i)^5$ | $\frac{4}{5}J(1+k_i)^4$ | $\frac{1}{5}J\left[5(1+k_i)^5 - 4(1+k_i)^4\right]$ |

In table 17 CCA depreciation figures are evaluated for a five year project assuming $k_i$ = the rate of asset price inflation. (This will later be contrasted with $k_n$ a nominal rate, and $k_r$ a real rate, yet to be defined more precisely.) By summing the entries in column (v) we note that ignoring the time value of money the total CCA depreciation over 5 years is $J(1+k_i)^5$, being the future replacement cost of the asset. Since this exceeds the outlay J, then with inflation on asset prices there is an incentive to invest. But since this assumes that the required nominal rate of return, denoted $k_n$, is zero let us now introduce the time value of money and discount the CCA capital allowances in column (v).

The present value of the allowances for the five year project is given by:

$$A_5 = \frac{J}{5}\left[\left(\frac{1+k_i}{1+k_n}\right) + 2\left(\frac{1+k_i}{1+k_n}\right)^2 - \frac{(1+k_i)}{(1+k_n)^2} + 3\left(\frac{1+k_i}{1+k_n}\right)^3 - 2\,\frac{(1+k_i)^2}{(1+k_n)^3} + 4\left(\frac{1+k_i}{1+k_n}\right)^4 - 3\,\frac{(1+k_i)^3}{(1+k_n)^4} + 5\left(\frac{1+k_i}{1+k_n}\right)^5 - 4\,\frac{(1+k_i)^4}{(1+k_n)^5}\right]$$

$$= \frac{J}{5}\left(1 - \frac{1}{1+k_n}\right)\left[\frac{1+k_i}{1+k_n} + 2\left(\frac{1+k_i}{1+k_n}\right)^2 + 3\left(\frac{1+k_i}{1+k_n}\right)^3 + 4\left(\frac{1+k_i}{1+k_n}\right)^4\right] + J\left(\frac{1+k_i}{1+k_n}\right)^5 .$$

Hence in general when the asset is replaced in N years' time:

$$A_N = J\left(\frac{1+k_i}{1+k_n}\right)^N + \frac{J}{N}\left(\frac{k_n}{1+k_n}\right)\left[\frac{1+k_i}{1+k_n} + 2\left(\frac{1+k_i}{1+k_n}\right)^2 + 3\left(\frac{1+k_i}{1+k_n}\right)^3 + \dots + (N-1)\left(\frac{1+k_i}{1+k_n}\right)^{N-1}\right] \qquad (11) \quad .$$

It is instructive to consider whether there is an investment incentive if the rate of price inflation ($k_i$) on the asset happens to be the same as the nominal discount rate of $k_n$. Hence for $k_i = k_n$, from equation (11)

$$A_1 = J \quad ,$$

$$A_2 = J + \frac{J}{2}\left(\frac{k_n}{1+k_n}\right) \quad ,$$

$$A_3 = J + \frac{J}{3}\left(\frac{k_n}{1+k_n}\right)(1+2) \quad ,$$

$$A_4 = J + \frac{J}{4}\left(\frac{k_n}{1+k_n}\right)(1+2+3) \quad ,$$

$$A_5 = J + \frac{J}{5}\left(\frac{k_n}{1+k_n}\right)(1+2+3+4) \quad .$$

From this progression we see that the general rule is

$$A_N = J + J\left(\frac{k_n}{1+k_n}\right)\left(\frac{N-1}{2}\right) \qquad \text{for } k_i = k_n \qquad (12) \quad .$$

With positive rates of interest $k_n > o$ and $k_n/(1+k_n) > o$, and with asset life spans exceeding one year $N>1$ and $(N-1)/2 > o$, hence $A_N > J$ for $k_i = k_n$. Therefore, there exists a tax incentive to invest when the rate of price inflation on the asset equals the discount rate for appraisal purposes.

For instance, consider an investment outlay of £1m, a discount rate of 10 per cent, price inflation on the asset of 10 per cent, and a 5 year life:

$$A_5 = J + J\left(\frac{k_n}{1+k_n}\right)\left(\frac{5-1}{2}\right),$$

$$= 1{,}181{,}818.$$

This is equivalent to a capital allowance of 118.18 per cent in present value terms, and since only a 100 per cent allowance is required to provide a neutral effect on the capital investment decision, then there is a tax incentive to invest with CCA capital allowances (for $k_i = k_n$).

Furthermore, if the rate of asset price inflation exceeds the discount rate then $(1+k_i)/(1+k_n)$ is greater than unity. The first term alone on the right hand side of equation (11) exceeds J, and since the other terms are positive then $A_N > J$ for $k_i > k_n$. Once again this suggests a tax incentive to invest.

Let us now move to the more interesting part of the analysis which is to consider $k_i < k_n$. For this purpose it is useful to introduce a real rate of interest, denoted $k_r$, such that

$$(1+k_n) = (1+k_i)(1+k_r) \tag{13}$$

Note however that although $k_n$ is the required nominal rate of interest for accepting or rejecting the project, $k_r$ is not necessarily the real rate of interest required by shareholders to justify project acceptance. This would only be the case if the rate of asset price inflation were the same as the general rate of inflation on the basket of goods bought by shareholders from cash generated by dividends, assuming a discounted dividends share valuation model. Hence from equations (11) and (13)

$$A_N = \frac{J}{(1+k_r)^N} + \frac{J}{N}\left(\frac{k_n}{1+k_n}\right)\left[\frac{1}{1+k_r} + \frac{2}{(1+k_r)^2} + \frac{3}{(1+k_r)^3} + \ldots + \frac{N-1}{(1+k_r)^{N-1}}\right] \tag{14}$$

For instance for N=3, $k_r = 0.03$, $k_n = 0.10$

$$A_3 = \frac{J}{(1.03)^3} + \frac{J}{3}\left(\frac{0.10}{1.10}\right)\left[\frac{1}{1.03} + \frac{2}{(1.03)^2}\right]$$

$$= J \times 1.0017$$

Under these parameters, in DCF terms the CCA tax system is equivalent to a 100.17 per cent capital allowance which is approximately neutral but offers a slight incentive to invest. However as the asset replacement period extends the incentive is increased. For example with N = 4 and $k_r = 0.03$, $k_n = 0.10$

$$A_4 = \frac{J}{(1.03)^4} + \frac{J}{4}\left(\frac{0.10}{1.10}\right)\left[\frac{1}{1.03} + \frac{2}{(1.03)^2} + \frac{3}{(1.03)^3}\right],$$

$$A_4 = J \times 1.0158 > A_3$$

Moreover, the larger the nominal discount rate, $k_n$, the greater the incentive. Since $k_x/(1+k_x) > k_z/(1+k_z)$ for $k_x > k_z$, then $A_N\ (n=x) > A_N\ (n=z)$ from equation (14). For instance for N = 4, $k_r = 0.03$ and $k_n = 0.20$,

$$A_4\ (k_n = 0.20) = J \times 1.1219 > A_4\ (k_n = 0.10).$$

The question which now poses itself is whether there are circumstances in which a current cost capital allowance would provide a disincentive to invest. By inspection of equation (14) we observe that the greater the real rate of interest, denoted $k_r$, the lower the present value of the capital allowances. For instance for N = 7, $k_n = 0.20$, $k_r = 0.10$ then

$$A_7 = \frac{J}{(1.1)^7} + \frac{J}{7}\,\frac{1}{6}\left[\frac{1}{1.1} + \frac{2}{(1.1)^2} + \frac{3}{(1.1)^3} + \frac{4}{(1.1)^4} + \frac{5}{(1.1)^5} + \frac{6}{(1.1)^6}\right]$$

$$= J \times 0.8475$$

Hence under these circumstances in DCF terms the current cost capital allowances are equivalent to an immediate capital allowance of 84.75 per cent. Since a 100 per cent allowance would provide a neutral solution, the CCA tax base would create tax disincentives for high values of $k_r$. These circumstances exist when the required discount rate for project appraisal purposes exceeds the rate of asset price inflation by a large amount.

## 5.7 An accrual accounting tax system

Although investment decisions under the NPV model are based on cash flows, the payments of tax are determined according to an accrual accounting system. Let us compare the tax based in table 18, with the net cash flows from the project for a particular year, as shown by table 19.

Table 18 : The tax base

| | | |
|---|---|---|
| Sales based on accrual accounting principles | | £200,000 |
| Opening Stock | £13,000 | |
| Purchases based on accrual accounting principles | 130,000 | |
| | 143,000 | |
| less closing stock | (43,000) | |
| Cost of goods sold | | (100,000) |
| The tax base before capital allowances | | £100,000 |

Table 19 : Net Cash flow before tax

| | | |
|---|---|---|
| Opening balance of debtors | | £15,000 |
| Sales based on accrual accounting principles | | 200,000 |
| | | 215,000 |
| Less closing balance of debtors | | (25,000) |
| | | 190,000 |
| Opening balance of creditors and expenses (excluding depreciation) | 10,000 | |
| Purchases based on accrual accounting principles | 130,000 | |
| | 140,000 | |
| less closing balance | (14,000) | |
| less cash expenditure on materials and other expenses | | (126,000) |
| Net cash flow from trading | | £64,000 |

Continuing the illustration, before stock relief the tax base of £100,000 differs from net cash income from trading of £64,000 by the periodic investment in working capital of £36,000 (Table 20)

Table 20 Periodic investment in net working capital

| | | | |
|---|---|---|---|
| Increase in stocks: | | | |
| | Closing balances | | |
| | Raw materials | | |
| | Work in progress | | |
| | Finished goods | | £43,000 |
| less | Opening balances | | |
| | Raw materials | | |
| | Work in progress | | |
| | Finished goods | | (£13,000) |
| | | | £30,000 |
| Increase in debtors: | | | |
| | Closing balance | £25,000 | |
| less | Opening balance | (£15,000) | |
| | | | £10,000 |
| | | | £40,000 |
| less | Increase in creditors and expenses (excluding depreciation) | | |
| | Closing balance | £14,000 | |
| | Opening balance | (£10,000) | ( £4,000) |
| Periodic investment in net working capital | | | £36,000 |

Although the firm is generating £64,000 of cash from trading the tax bill is based on £100,000. With a tax rate of 52% the company is paying £52,000 on £64,000 of net cash flow.

The adverse effects however are partly mitigated through stock appreciation relief which gives a tax allowance on the periodic increase in stocks in excess of a proportion, which we shall denote z, currently at 15%, of trading profits after capital allowances. In the above example, if there are no capital allowances in the period then the tax base is reduced (table 21).

Table 21 Tax after stock relief

| | | |
|---|---|---|
| Tax base before stock relief | | £100,000 |
| Closing stock | £43,000 | |
| Opening stock | 13,000 | |
| Increase | 30,000 | |
| less 15% of £100,000 | (15,000) | |
| less stock appreciation relief | | 15,000 |
| Tax base after stock relief | | £85,000 |
| Tax thereon @ 52% | | £44,200 |

Hence, in this illustration the effective tax rate is 44200/64000 = 69%, ignoring the tax payment time gap and the time value of money.

A numerical illustration of the effect of a £100 capital allowance, when stock relief is claimed, is shown below in table 22. The £100 capital allowance has decreased the tax base by £100 x (1 + 0.15) = £115.

Algebraically $C_q^A = £100$,

$z = 0.15$

Hence, allowing for stock relief, the present value of the capital allowance is

$$NPV = \sum_{q=0}^{m} \frac{\left[PT_k + (1 - P)\, T_{k-1}\right] (1 + z)\, C_q^A}{(1 + k)^{q + y}} \qquad (15)$$

Table 22 Tax base after stock relief and capital allowances

| | | Tax computation before accepting project | | Effect of £100 capital allowance arising from the project |
|---|---|---|---|---|
| Schedule D case I trading profit after capital allowances, but before stock relief: | | £100,100 | | £100,000 |
| Opening stock | £13,000 | | £13,000 | |
| Closing stock | £43,000 | | £43,000 | |
| Increase | £30,000 | | £30,000 | |
| less 15% of £100,100: | (£15,015) | | | |
| 15% of £100,000 | | | (£15,000) | |
| Stock relief | | (£14,985) | | (£15,000) |
| Tax base | | £85,115 | | £85,000 |

We note that the stock relief decreases the tax base by the appreciation of stock less 15% of the base before stock relief. Hence the change in net taxable income in period j resulting from project acceptance is given by

$$\Delta N^{TI}_j = ( X_j + W_j - C^A_j) (1 + z) - (S^c_j - S^o_j) \qquad (16),$$

where $X_j$ = the change in cash flows from trading in period $j$ resulting from project acceptance,

$W_j$ = the change in net working capital in period j resulting from project acceptance,

$S^c_j$ = the change in the closing balance of inventory in period $j$ resulting from project acceptance.

$S^o_j$ = the change in the corresponding opening balance

Although depreciation is replaced by capital allowances as the measure of capital consumption for tax purposes, it is still relevant to the tax calculation since it is included in the valuation of stock. Furthermore, the stock relief provisions do not fully cover the periodic increase in the closing stock. Where stock relief is claimed the effect of depreciation, via its inclusion in $W_j$, $S^c_j$, and $S^o_j$, is to change net taxable income in period j by

$$(u_j \Delta F_j - u_{j-1} \Delta F_{j-1})(1+z) - (u_j \Delta F_j - u_{j-1} \Delta F_{j-1})$$

$$= z(u_j \Delta F_j - u_{j-1} \Delta F_{j-1}) \quad .$$

Where $\Delta F_j$ = the change in accounting depreciation in period j of those fixed assets relating to production overheads, resulting from project acceptance,

and $u_j$ = the proportion of the volume of unsold goods to the volume of production during the period.

Hence, in the first period j in which depreciation is charged $\Delta F_{j-1}$ is zero but $\Delta F_j$ is positive. We note that depreciation arising from the project increases net taxable income in period j = 1 by $(z \times u_j)\Delta F_j$. By contrast where n represents the last period containing depreciation from the investment, $\Delta F_n$ is positive but $\Delta F_{n+1}$ is zero. Hence in period n+1 the effect of depreciation from the project is to decrease net taxable income by $(z \times u_n)\Delta F_n$ if stock relief is claimed; and by $u_n \times \Delta F_n$ if stock relief is not claimed. However, in practice these niceties may not be significant. For instance, if we assume that $z$=15%, u=20%, and the asset is depreciated over ten years on a straight-line basis, then $(z \times u_n)\Delta F_n$ equals 0.3% of the asset cost. With a marginal tax rate of

50 per cent this is worth only 0.15 per cent of the asset cost even ignoring the time value of money. On the other hand, let us take the example of a firm with a very high opening inventory in the initial year of a new project, sufficient to match heavy sales from other projects such that despite the fact that all production of this first year is held in stock, there is insufficient stock appreciation vis-à-vis taxable trading profit to claim stock relief. With a four-year project $u_1 \times \Delta F_1$ = 25% of the asset cost. Hence, with short-term projects and heavy stockbuilding such complexities may be important in the marginal case.

Let us consider our model so far. Excluding Advance Corporation Tax, the Net Present Value Model shows that a capital investment project should be accepted if:

$$\sum_{j=0}^{n} \frac{X_j - J_j}{(1+k)^j} - \sum_{q=0}^{m} \frac{(PT_K + (1-P)T_{K-1})\,\Delta N_q^{TI}}{(1+k)^{q+y}} > 0 \qquad (17)\ ,$$

where $X_j$ = the increase in cash income in period j resulting from project acceptance,

$J_j$ = capital investment outlay in period j,

$n$ = the project horizon date,

$\Delta N_q^{TI}$ = the change in net taxable income in period q resulting from project acceptance.

## 5.8 Net taxable income and the marginal tax rate

For the Financial Year 1978 which accrues from 1 April 1978 to 31 March 1979 the full rate of 52% applies to companies with net taxable income over £100,000 and the small companies rate of 42% applies to net taxable income under £60,000. These rates and limits were legalised by the

Finance Act which received Royal assent in the summer of 1979. Table 23 shows rates and limits which have been determined by previous Finance Acts.

Table 23      Marginal tax rates

| Financial Year | Finance Act which sets the rates for the Financial Year under consider-ation | Full rate of Corp-oration Tax | Small Companies rate | Marginal relief lower limit (£) | Marginal relief upper limit (£) | Marginal relief fraction | Marginal tax rate when marginal small companies relief applies |
|---|---|---|---|---|---|---|---|
| 1974 (1 April 74 31 March 75) | 1975 | 52% | 42% | 25000 | 40000 | 1/6 = .1667 | 68.67% |
| 1975 (1 April 75 31 March 76) | 1976 | 52% | 42% | 30000 | 50000 | 3/20 = .15 | 67% |
| 1976 (1 April 76 31 March 77) | 1977 | 52% | 42% | 40000 | 65000 | 4/25 = .16 | 68% |
| 1977 (1 April 77 31 March 78) | 1978 | 52% | 42% | 50000 | 85000 | 1/7 = .1429 | 66.29% |
| 1978 (1 April 78 31 March 79) | 1979 | 52% | 42% | 60000 | 100000 | 3/20 = .15 | 67% |

The marginal fraction of 3/20 for the Financial Year 1978 applies where net taxable income lies between £60,000 and £100,000. For instance, if NTI were £80,000 the tax payable would be calculated as follows:

| | |
|---|---|
| £80,000 @ 52% | £41,600 |
| <u>less</u> marginal small companies relief | |
| 3/20 (100,000 - 80,000) | (3,000) |
| Tax payable on £80,000 | £38,600 |

However, if NTI were increased to £80,100 then tax payable would increase by £67 determining a marginal tax rate of 67%:

| | |
|---|---|
| £80,100 @ 52% | 41,652 |
| <u>less</u> marginal small companies relief | |
| 3/20 (100,000 - 80,100) | (2,985) |
| Tax payable on £80,100 | £38,667 |

Marginal tax rates under marginal small companies relief are shown in Table 23 for other Financial Years as well.

Note that these rates only apply within the relief limits, the first slice being taxed at 42%. For instance the tax on £10,000 would be £4,200 and the tax on £70,000 would be £60,000 @ 42% plus £10,000 @ 67%, which equals £31,900. At the lower limit the tax may be calculated as follows:-

| | |
|---|---|
| £60,000 @ 52% | £31,200 |
| <u>less</u> marginal small companies relief | |
| 3/20 (100,000 - 60,000) | (6,000) |
| Tax on £60,000 @ 42% | £ 25,200 |

Hence the marginal fractions are determined by the limits in the following way:

$$\text{Marginal fraction} = \frac{\text{Lower Limit}}{\text{(Upper less lower limit)}} \times \text{(full rate less small companies rate)}$$

<u>e.g's</u> Marginal fraction for April 78/March 79 $= \frac{60,000}{100,000-60,000} \times$ (52% - 42%) = 3/20 .

Marginal fraction for April 77/March 78 $\frac{50,000}{85,000 - 50,000} \times$ (52% - 42%) = 1/7 .

The marginal tax rate when marginal small companies relief applies is determined in turn by the addition of the full rate of Corporation Tax and the marginal fraction:

<u>e.g's</u> for April 78/March 79 : 52% + 3/20 = 52% + 15% = 67% ;

for April 77/March 78 : 52% + 1/7 = 52% + 14.29% = 66.29% .

Although a lower limit of £60,000 may seem very small for a sizeable company, the net taxable income is calculated after stock relief, capital allowances and other deductions and hence the small companies rate may be charged on companies with high pre-depreciation profits but substantial capital allowances. Small companies rate is therefore not a tax on small companies as such but a tax on companies with small net taxable incomes.

Note that where an accounting period straddles more than one Financial Year for tax purposes then more than one marginal tax rate may be applied to the same level of capital allowance of the accounting period. For instance, capital expenditure of £10,000 on plant and machinery during the accounting year ended 31 December 1978 will change the tax bill by (see equation (6))

$$£10,000 \quad (T_K \times 3/4 + T_{K-1} \times 1/4) \quad ,$$

where $T_K$ = the marginal tax rate for the Financial Year 1978 ,

$T_{K-1}$ = the marginal tax rate for the Financial Year 1977.

From Table 23 we see that the marginal tax rates could be $T_K$ = 67%, $T_{K-1}$ = 42%, and hence the capital allowance changes the tax bill by £(10,000 x 3/4 x 67%) + £(10,000 x 1/4 x 42%) = £6,075. Hence the marginal tax rate is effectively 60.75% ignoring the time value of money.

5.9 The imputation system

Under Schedule 14 Finance Act 1972 a company is required to make advance payments of Corporation Tax (ACT) on the 14th day of the month following a "quarter" during which dividends paid exceed dividends received. For this purpose "quarters" end on 31st March, 30th June, 30th September, 31st December and on the last day of the accounting period if this falls on another day.

With a basic rate of income tax at 30%, a dividend of £70 has an ACT attached payment of £30. The shareholder is treated as having received £100 gross on which he is liable to income tax at a marginal rate which may be in excess of the 30% basic rate. If he pays tax on investment income at the marginal rate of say 50% the dividend bears a total tax of 50% of £100 = £50. However, since £30 has already been paid by the company he pays the difference of £50 - £30 = £20. In this way, under the imputation system, the tax the company pays on the dividend is "imputed" to the shareholder.

More generally, the ACT on the dividend paid at time j ($D_j$) will be

$$\frac{b}{1 - b} D_j ,$$

where b = the basic rate of income tax. Similarly, the tax credit on a dividend received at time j ($R_j$) will be

$$\frac{b}{1 - b} R_j .$$

Since ACT payable is based on the difference between franked payments and franked investment income during the quarterly return period, the ACT payable for the accounting period ending at time q, say at the end of the month j = 12, will normally be

$$A^{CT}_q = \sum_{j=1}^{12} (D_j - R_j) \frac{b}{1 - b} \qquad (18) .$$

Note that $\sum_{j=1}^{12} D_j$ includes interim dividends both declared and paid between months j = 1 to 12 ( the current year); and also includes final dividends declared in the previous year, but paid within the current year. Since the Corporation Tax for the same period is

$$(P\, T_K + (1 - P)\, T_{K-1})\, N^{TI}_q ,$$

the net mainstream corporation tax after ACT setoff will therefore normally be

$$N^{MCT}_q = (PT_K + (1 - P)\; T_{K-1})N^{TI}_q - \sum_{j=1}^{12} (D_j - R_j) \frac{b}{1 - b} \qquad (19) ,$$

payable at time (q + y), provided

$$A^{CT}_q \leq b.N^{TI}_q ,$$

<u>a requirement of the tax provisions.</u>

Restriction in ACT setoff

Since there is a maximum amount of ACT paid on dividends which is available for offset against the Corporation Tax bill for the year, there is a minimum rate of net mainstream Corporation Tax. With a 34 per cent setoff of ACT against a 52 per cent corporate tax rate, there remains a net mainstream corporation tax rate of 18 per cent, if ACT setoff is restricted.

Hence the immediate tax benefit of capital expenditure in this case is only at the rate of 18 per cent (Buckley - 1975). Therefore where ACT setoff is restricted :

$$N_q^{MCT} = (PT_K + (1-p)\,T_{K-1} - b)\,N_q^{TI} \qquad (20),$$

payable at time $(q + y)$ .

Hence where ACT setoff is restricted throughout the foreseeable future, the decision criterion for project acceptance is modified to:

$$\sum_{j=o}^{n} \frac{X_j - J_j}{(1+k)^j} - \sum_{q=o}^{m} \frac{(PT_K + (1-P)T_{K-1} - b)\,\Delta N_q^{TI}}{(1+k)^{q+y}} > o \qquad (21).$$

With variable rates of income tax, the maximum ACT restriction for the accounting period ending at time q is :

$$b_{k-1}\,(1-P)\,N_q^{TI} + b_K\,P\,N_q^{TI} \,.$$

Hence inequality (13) becomes :

$$\sum_{j=o}^{n} \frac{X_j - J_j}{(1+k)^j} - \sum_{q=o}^{m} \left[ \frac{(\,(T_K - b_K)P + (T_{K-1} - b_{K-1})(1-P))\,\Delta N_q^{TI}}{(1+k)^{q+y}} \right] > o \qquad (22)$$

.

No restriction in ACT setoff

If ACT setoff is not restricted and the portfolio of the firm's investments is treated as one project, then the decision criterion for "acceptance" may be represented by:

$$\sum_{j=o}^{n} \frac{X_j - J_j}{(1+k)^j} - \sum_{q=o}^{m} \frac{\left[PT_K + (1-P)T_{K-1}\right]\Delta N^{TI}_q - \left[D_q - R_q\right]\frac{b}{1-b}}{(1+k)^{q+y}} > o \qquad (23)$$

where $D_q$ and $R_q$ represent respectively dividends paid and received during the accounting period ended at time p.

Clearly, the sensitivity of the test, ceteris paribus, would depend upon the extent to which franked payments exceeded franked investment income. With constant dividends paid and received the decision criterion would still be represented by inequality (17) since the change in net taxable income due to acceptance of the incremental project is equal to the change in the net mainstream corporation tax base. However, the investment decision is one of the principal determinants of the level of future dividends in that future dividends are paid out of the benefits of current and future investments. Returning to our model, let $\Delta$ D represent the increase in the payment of dividend, $\Delta N^{TI}$ the change in net taxable income and $\Delta N^{MCT}$ the change in net mainstream corporation tax, due to acceptance of the incremental project such that in a given period:

$$\Delta N^{MCT} = \Delta N^{TI}.T - \Delta D.\frac{b}{1-b} \qquad (24)$$

Outlined below is a numerical example of the changes in the tax bill of a project for a particular accounting period where there is an explicit dividend policy.

Table 24 Net mainstream corporation tax

| | Before Project £ | After Project £ |
|---|---|---|
| Net taxable income for accounting year ended 31 December 1978 | 100,000 | 100,100 |
| Net dividends paid on 1st January 1978 | 33,000 | 33,033 |
| Advanced Corporation Tax at a tax inclusive rate of 34/100 i.e., at a tax exclusive rate of 34/66 | 17,000 | 17,017 |
| Mainstream Corporation Tax @ 52% | 52,000 | 52,052 |
| Setoff | 17,000 | 17,017 |
| Net Mainstream Corporation Tax (NMCT) | 35,000 | 35,035 |

An example of a possible set of tax payment dates is given below:

1. The extra Dividends of £33 are paid on 1st January 1978.
2. The extra ACT of £17 is paid on 14th April 1978.
3. The extra Net Mainstream Corporation Tax of £35 is payable on 1st October 1979.

Because of the explicit dividend policy, there is an extra dividend of £33. If shareholders are non-taxpayers they also receive a rebate from the Inland Revenue for the ACT of £17. Hence, the dividend plus the ACT is part of the required return to shareholders and taken into account in the time preference rate, denoted k. The tax attributable to the cash flows of the project is therefore the £35 increase in the Net Mainstream Corporation Tax:

$$(\pounds 100{,}100 - \pounds 100{,}000) \times 0.52 - (\pounds 33{,}033 - \pounds 33{,}000) \times \frac{0.34}{0.66} = \pounds 35 .$$

Since $\pounds 35 = \Delta N^{MCT}$,

$\pounds 100{,}100 - \pounds 100{,}000 = \Delta N^{TI}$,

$0.52 = T$ ,

£33,033 - £33,000 = $\Delta D$ and

$0.34 = b$ ,

we derive a general expression as in equation (24):

$$\Delta N^{MCT} = \Delta N^{TI}.T - \Delta D \frac{b}{1-b} \quad .$$

Hence the revised decision criterion would be to accept the incremental project if:

$$\sum_{j=o}^{n} \frac{X_j - J_j}{(1+k)^j} - \sum_{q=o}^{m} \frac{\left[PT_K + (1-P)\, T_{K-1}\right] \Delta N^{TI}{}_q - \Delta D_q \frac{b}{1-b}}{(1+k)^{q+y}} > o \qquad (25)$$

Although this criterion would be based upon the best available information at the time of the appraisal, the effect of future expenditure on other projects may be to reduce taxable income further by extra capital allowances, perhaps to the extent that $bN^{TI}$ becomes less than ACT; with the consequence that the decision criterion for the current project needs to be amended in retrospect to inequality (22) or at least to a hybrid of the two if ACT setoff is restricted for only a part of the project's life.

If inequality (25) corresponds with the financial framework of a particular firm, then the capital investment appriasal team will need guidelines from the board of directors on the extent to which future dividends will be increased in line with higher levels of profits. Without a detailed model the team would have to perform sensitivity analyses on the changes in future dividends as a result of project acceptance, although clearly such tests would also be carried out on the other estimates.

## 5.10. Marginal tax rates for investment decisions

Let us now recapitulate the marginal tax rates relevant to capital investment decisions. Even if we extend 100% depreciation to all capital expenditure and freeze the current rates of taxation, different annual net inflows or outflows may still be subjected to one of at least a dozen marginal rates of taxation. Since capital allowances reduce schedule D Case 1 net profits, let us investigate how the marginal tax rate, based on a £100 change in net taxable income, may depend upon the degree of Advance Corporation Tax setoff, stock appreciation relief,and the level of net taxable income. Using the rates for the Financial Year 1977 we note from table 25 how the marginal tax rate varies with net taxable income.

Table 25 Marginal tax rates

| Examples | (i) | | (ii) | | (iii) | |
|---|---|---|---|---|---|---|
| Trading profits after capital allowances | £100,000 | £100,100 | £10,000 | £10,100 | £71,000 | £71,100 |
| Tax thereon | 52,000 | 52,052 | 4,200 | 4,242 | 34,920* | 34,986.29 |
| Marginal tax rate (%) | | 52 | 42 | | | 66.29 |

* £34,920 = £71,000 x 52% - 1/7 £(85,000 - £71,000)

Furthermore, where Advance Corporation Tax setoff is restricted throughout the life of the project, then the marginal rax rate may be reduced to (T - b), where T is the corporate tax rate and b is the basic rate of income tax. Hence, even though the corporation tax rate may be fixed at 52% for a number of years, changes in the basic rate of income tax may affect investment decisions in the corporate sector: (Table 26)

Table 26 Marginal tax rates

| Examples | (iv) | | (v) | | (vi) | |
|---|---|---|---|---|---|---|
| Net taxable income | £100,000 | 100,100 | 10,000 | 10,100 | 71,000 | 71,100 |
| Mainstream Corporation Tax | 52,000 | 52,052 | 4,200 | 4,424 | 34,920 | 34,986.29 |
| Dividends paid (less dividends received) | 99,000 | 99,000 | 9,900 | 9,900 | 36,300 | 36,300 |
| ACT thereon | 51,000 | 51,000 | 5,100 | 5,100 | 18,700 | 18,700 |
| Setoff | 34,000 | 34,034 | 3,400 | 3,434 | 24,140 | 24,174 |
| Net Mainstream Corporation Tax | 18,000 | 18,018 | 800 | 808 | 10,780 | 10,812.29 |
| Total tax paid | 69,000 | 69,018 | 5,900 | 5,908 | 29,480 | 29,512.29 |
| Marginal tax rate(%) | | 18 | | 8 | | 32.29 |

Since the convention in this country is to charge the usage of stock on a FIFO basis, in a period of inflation part of the accounting profit on sale of stock is related to the rise in its cost from the date of purchase to sale. However, the current stock relief reduces this extra burden of tax on the enterprise by allowing a tax deduction equal to the excess of the increase in stock value during the accounting period over a proportion'z(currently at 15%) of trading profits for tax purposes, with capital allowances already deducted. A £1 increase in trading profit leads to an increase in after tax profits of £(1-T) only if the increase in stock value is less than the given proportion of trading profit before stock relief. The two instances when this occurs is when either stock clawback applies or when no stock adjustment for tax purposes is made. However, when stock relief is claimed, an increase in net trading profits of £1 will reduce stock relief by £0.15, where $z = 15\%$. The resultant marginal rates of corporation tax are therefore increased by the factor (1+z): (Table 27).

Table 27 Marginal tax rates

| Without restrictions in ACT setoff | (vii) | | (viii) | | (ix) | |
|---|---|---|---|---|---|---|
| Trading profit after capital allowances (TP) | £100,000 | £100,100 | 30,000 | 30,100 | 80,000 | 80,100 |
| Increase in stock | 30,000 | 30,000 | 20,000 | 20,000 | 30,000 | 30,000 |
| 15% of TP | 15,000 | 15,015 | 4,500 | 4,515 | 12,000 | 12,015 |
| Stock relief | 15,000 | 14,985 | 15,500 | 15,485 | 18,000 | 17,985 |
| Net taxable income | 85,000 | 85,115 | 14,500 | 14,615 | 62,000 | 62,115 |
| Mainstream Corporation Tax | 44,200 | 44,259.80 | 6,090 | 6,138.30 | 28,954.29 | 29,030.51 |
| Marginal tax rate (%) | | 59.8 | | 48.3 | | 76.2 |

| With restrictions in ACT setoff | (x) | | (xi) | | (xii) | |
|---|---|---|---|---|---|---|
| Net taxable income (as above) | 85,000 | 85,115 | 14,500 | 14,615 | 62,000 | 62,115 |
| Mainstream corporation tax (as above) | 44,200 | 44,259.80 | 6,090 | 6,138.30 | 28,954.29 | 29,030.51 |
| Dividends paid (less received) | 99,000 | 99,000 | 9,900 | 9,900 | 66,000 | 66,000 |
| ACT thereon | 51,000 | 51,000 | 5,100 | 5,100 | 34,000 | 34,000 |
| Setoff | 28,900 | 23,939.10 | 4,930 | 4,969.10 | 21,080 | 21,119.10 |
| Net Mainstream Corporation Tax | 15,300 | 15,320.70 | 1,160 | 1,169.20 | 7,874.29 | 7,911.41 |
| Total tax paid | 66,300 | 66,320.70 | 6,260 | 6,269.20 | 41,874.29 | 41,911.41 |
| Marginal tax rate(%) | | 20.7 | | 9.2 | | 37.1 |

Hence, in the absence of foreign investment we may tabulate the marginal rates of corporation tax as follows: (Table 28). In table 29 the figures are updated for the Financial Year 1978.

Where net taxable income is negative then unless capital allowances may be carried back under section 177(3A) ICTA 1970 or group relief is available under section 258, then the discount factor applied to the capital allowance would be less than unity, resulting in a disincentive to invest. Indeed the growth in success of the leasing industry has been greatly aided by passing the full capital allowance onto the lessor with an appropriate adjustment in the leasing rental. To obtain tax neutrality any losses carried forward would need to be inflated at the firm's reinvestment rate. Even if interest were applied to losses carried forward at a rate laid down by statute, those firms in more risky industries which apply discount rates higher than the statutory rate would be penalised.

Table 28 Marginal tax rates for the Financial Year 1977

| Example | Net taxable income | Stock appreciation Relief | ACT setoff | Marginal tax rate for the Financial Year 1977 |
|---|---|---|---|---|
| i | over £85,000 | not claimed | not restricted | 52% = $T$. |
| ii | under £50,000 but positive | not claimed | not restricted | 42% = $T_s$. |
| iii | Between £50,000 and £85,000 | not claimed | not restricted | 66.29% = $T_m$. |
| iv | over £85,000 | not claimed | restricted | 18% = $T-b$. |
| v | under £50,000 but positive | not claimed | restricted | 8% = $T_s-b$. |
| vi | Between £50,000 and £85,000 | not claimed | restricted | 32.29% = $T_m-b$. |
| vii | over £85,000 | claimed | not restricted | 59.8% = $T(1+z)$. |
| xiii | Under £50,000 but positive | claimed | not restricted | 48.3% = $T_s(1+z)$. |
| ix | Between £50,000 and £85,000 | claimed | not restricted | 76.23% = $T_m(1+z)$. |

| Example | Net taxable income | Stock appreciation relief | ACT setoff | Marginal tax rate for the Financial Year 1977 |
|---|---|---|---|---|
| x | Over £85,000 | claimed | restricted | 20.7% = $\frac{(T-b)(1+z)}{(1+s)}$ , |
| xi | Under £50,000 but positive | claimed | restricted | 9.2% $(T_s - b)(1+z)$ . |
| xii | Between £50,000 and £85,000 | claimed | restricted | 37.13% $(T_m - b)(1+z)$ . |

Notation:

$T$ = the full rate of Corporation Tax.

$T_s$ = Small Companies Rate.

$T_m$ = the marginal rate when marginal small companies relief is claimed, (MSCR = 52% + (50,000/(85,000 - 50,000)) x (52%-42%) = 66.29%) .

$b$ = the basic rate of income tax .

$z$ = the percentage applied in the stock appreciation relief formula.

Table 29 Marginal tax rates for the Financial Year 1978

| Net taxable income | Stock appreciation relief | ACT setoff | Marginal tax rate for the Financial Year 1978 |
|---|---|---|---|
| Over £100,000 | not claimed | not restricted | 52% = $T$. |
| Under £60,000 but positive | not claimed | not restricted | 42% = $T_s$ . |
| Between £60,000 and £100,000 | not claimed | not restricted | 67% = $T_m$ . |
| Over £100,000 | not claimed | restricted | 19% = $T-b$ . |
| Under £60,000 but positive | not claimed | restricted | 9% = $T_s-b$ . |
| Between £60,000 and £100,000 | not claimed | restricted | 34% = $T_m-b$ . |
| Over £100,000 | claimed | not restricted | 59.8% = $T(1+z)$. |
| Under £60,000 but positive | claimed | not restricted | 48.3% = $T_s(1+z)$ . |
| Between £60,000 and £100,000 | claimed | not restricted | 77.05% = $T_m(1+z)$. |
| Over £100,000 | claimed | restricted | 21.85% $(T-b)(1+z)$. |
| Under £60,000 but positive | claimed | restricted | 10.35% = $(T_s-b)(1+z)$. |
| Between £60,000 and £100,000 | claimed | restricted | 39.1% = $(T_m-b)(1+z)$ . |

## 5.11. Foreign investment and the marginal tax rate

Let us now extend the analysis to foreign investments and assume a basic rate of income tax at 34%. Only with a restriction of ACT setoff would we prima facie expect a marginal tax rate of 52% to be reduced to 18%. However, even though Advance Corporation Tax on foreign profits may not be restricted, the double taxation relief restriction under s. 100 Finance Act 1972 reduces the marginal tax rate on UK profits by the basic rate of income tax. This is shown in table 30 where we assume that:

(i) ACT setoff is restricted against UK profits but not against foreign profits (hence 'UK' ACT setoff equals 34% UK profits and 'foreign' ACT setoff equals the balance of 34/66 x Dividends less 34% UK profits).

(ii) Double taxation relief is restricted, i.e. to 52% foreign profits less the 'foreign' ACT setoff.

Table 30 Double taxation

| | |
|---|---|
| Mainstream Corporation Tax (MCT) | = 52% UK profits + 52% foreign profits . |
| ACT setoff | = 34% UK profits + [34/66 x Dividends - 34% UK profits] . |
| Double taxation relief (DTR) | = 52% foreign profits - 34/66 x Dividends +34% UK profits . |
| Net Mainstream Corporation Tax (NMCT) | = 18% UK profits . |
| Total tax | = 34/66 Dividends + 18% UK profits + [foreign tax rate x foreign profits] . |

Note

1. NMCT = MCT — (ACT setoff + DTR) .

2. Total tax paid = 34/66 Dividends + NMCT + foreign tax .

By observing the coefficients in the equation for 'Total tax' we note that the marginal tax rate on UK profits is 18% and that on foreign profits is the foreign tax rate. The same marginal tax rates pertain if ACT setoff and DTR are both fully restricted (table 31), and table 32 demonstrates the position where ACT is fully offset against UK profits (with no offset against foreign profits) and double taxation relief is restricted.

Table 31 Double taxation

MCT = 52% UK Profits + 52% F. Profits (where F= foreign).

ACT setoff = 34% UK Profits + 34 %F. Profits.

DTR = 18% F. Profits.

NMCT = 18% UK Profits.

Total Tax = (34/66) x Dividends + 18% UK Profits + foreign tax rate x F. Profits.

Hence: the marginal tax rate on UK Profits is 18%, and that on foreign profits is the foreign tax rate.

Table 32 Double taxation

MCT = 52% UK Profits + 52% F. Profits.

ACT setoff = 34/66 Dividends.

DTR = 52% F. Profits.

NMCT = 52% UK Profits - 34/66 Dividends.

Total Tax = 52% UK Profits + foreign tax rate x F. Profits.

Hence: the marginal tax rate on UK Profits is 52%, and that on foreign profits is the foreign tax rate.

We are now able to tabulate the marginal tax rates according to whether there are restrictions in

(a) ACT setoff on UK Profits

(b) ACT setoff on foreign Profits

(c) Double taxation relief (Table 33).

TABLE 33 Marginal tax rates

| Restrictions in (a), (b) or (c) | Marginal Tax rates | |
|---|---|---|
| | UK Profits | Foreign Profits |
| NONE (Table 34) | 52% | 52% |
| (a) (Table 35) | 52% | 52% |
| (a), (b) (Table 36) | 18% | 18% |
| (a), (c) (Table 30) | 18% | foreign tax rate |
| (c) (Table 32) | 52% | foreign tax rate |
| (a), (b), (c) (Table 31) | 18% | foreign tax rate |

Assumptions

1. Net taxable income is over £85,000.
2. No relief is claimed for stock appreciation.
3. The full rate of Corporation Tax is 52% on net taxable income over £85,000.
4. The basic rate of income tax is 34%.
5. The Financial Year is 1977.

TABLE 34 Double taxation

| | |
|---|---|
| MCT = | 52% UK profits + 52% foreign profits. |
| ACT setoff fully against UK profits | = 34/66 Dividends. |
| DTR = | foreign tax rate x F. profits. |
| NMCT = | 52% UK profits + 52% F. profits - 34/66 Dividends - foreign tax rate x F. profits. |
| Total tax = | 52% UK profits + 52% F. profits. |

TABLE 35 Double taxation

| | |
|---|---|
| MCT = | 52% UK profits + 52% F. Profits. |
| ACT setoff (restricted against UK profits) = | 34% UK profits + 34/66 Dividends - 34% UK profits. |
| DTR = | foreign tax rate x foreign profits. |
| NMCT = | 18% UK profits + 52% F. profits - 34/66 Dividends + 34% UK profits - foreign tax rate x F. profits. |
| Total tax = | 52% UK profits + 52% F. profits. |

TABLE 36 Double taxation

| | |
|---|---|
| MCT = | 52% UK profits + 52% F. profits. |
| ACT setoff (fully restricted) = | 34% UK profits + 34% F. profits. |
| DTR = | foreign tax rate x F. profits. |
| NMCT = | 18% UK profits + 18% F. profits - foreign tax rate x F. profits. |
| Total tax = | (34/66) x Dividends + 18% UK profits + 18% x F. profits. |

## 5.12 Application of multiple marginal tax rates

Let us now assume the 1977 tax rates to be frozen into the future and examine the effects on capital market efficiency in our basic NPV decision model. With heavy capital investments during one accounting period it is not unlikely that net taxable income may be reduced to below the threshold of £40,000 with the result that dividends paid less dividends received will exceed the basic rate of income tax applied to net taxable income or that double taxation relief will be restricted and the increase in the value of trading stocks become high enough to claim stock relief. The benefit of the capital allowance on the last investment project may very well be 9.2% (eg xi), with future income being taxed at 37.13% (eg xii) and later 20.7% (eg x). Substituting these tax rates in table 12 we have (table 37):

Table 37 : Tax cash flows

| | Year | Cash flow | D.F. | NPV |
|---|---|---|---|---|
| Capital allowance | Now | £2,486 x 9.2% | 1.000 | £229 |
| Tax on inflow | 1 | (1,000) x 37.13% | 0.909 | (338) |
| " | 2 | (1,000) x 20.7% | 0.826 | (171) |
| " | 3 | (1,000) x 20.7% | 0.751 | (155) |
| | | | | £(435) |

In this case we note from table 37 that the marginal investment before tax may result in an after tax NPV which is negative. Even with a one year tax time lag this would still be negative by £396.= £435 x 1/(1+10%), assuming a 10% discount rate.

## 5.13. Requirements of a neutral tax system

We have seen that the present system of corporation tax bears some of the features of a cash flow (or expenditure) tax.

One of the advantages of an expenditure tax base is that the relationship between present and future consumption before tax is the same as that between present and future consumption after tax, which are both equated with the investment discount factor. For instance, if we let the consumption potential without taxes in period 0 be C, then the consumption potential without taxes in period 1 is $C(1+k)$ where $k$ = the reinvestment rate, [following Musgrave and Musgrave (1976)].

Hence the ratio between present and future consumption is

$$\frac{C}{C(1+k)} = \frac{1}{1+k} \quad .$$

Similarly, the present consumption after tax = $C(1-t_e)$ where $t_e$ = the marginal rate of expenditure tax, and the future consumption after tax = $C(1+k)\,(1-t_e)$ determining a ratio between present and future consumption of

$$\frac{C(1 - t_e)}{C(1+k)(1-t_e)} = \frac{1}{1+k} \quad ,$$

equating, once more, with the investment discount factor.

Turning now to an income tax base, let the present income in period 0 without taxes = $I$. With reinvestment the accumulated wealth in period 1 without taxes = $I\,(1+k)$, giving a ratio between present and future consumption potential without taxes of

$$\frac{I}{I(1+k)} = \frac{1}{1+k} \quad \text{as before} \quad .$$

With an income tax, the present income after tax = $I(1-h)$ where $h$ = the marginal rate of income tax. Where the capital of $I(1-h)$ is reinvested the income thereon after tax = $I\,(1-h)k(1-h)$ .

Hence the future wealth in period 1 available for consumption is

$$I(1-h) + I(1-h)\,k\,(1-h) = I(1-h)\left[1+k(1-h)\right] .$$

Therefore the ratio between present and future consumption after income tax =

$$\frac{I(1-h)}{I(1-h)[1+k(1-h)]} = \frac{1}{1 + k\ (1-h)}$$ (Musgrave and Musgrave (1976))

Since the lower the denominator, the higher the ratio, the effect of an income tax is to value present vis-à-vis future income more highly than is warrented by the pre-tax yield of the investment, causing a disincentive to invest. Moreover, where the tax rate is not constant, under both tax systems consumption preferences are altered, resulting in economic inefficiency. Indeed we have highlighted the existence of several marginal rates of corporation tax applicable to the investment decision even if the present tax system and rates were perpetuated throughout the life of a project. Some of this excess burden would be removed by abolishing small companies rate, marginal small companies relief, and ACT setoff restrictions. However, as long as the corporate tax system remains a hybrid based on both income and expenditure principles, and given the multiple tax rates, our complicated analysis of the tax implications for investment decisions will remain.

### 5.14 Conclusions

The main conclusions of the chapter are as follows:

The timing of capital expenditure in relation to the accounting period affects the delay in receiving the benefit of a lower tax bill. Expenditure incurred at the end of an accounting period will attract relief within a shorter time interval, in which case the relief will be even more beneficial the greater the time value of money.

If stock relief is claimed, the marginal tax rate applied to a capital allowance is increased under the present legislation. Expenditure on plant and machinery may effectively be relieved at a marginal tax rate of, say 59.80% ($T(1+z) = 52\% (1+15\%) = 59.8\%$) .

To the extent that tax computations follow historic cost accounting principles, a corporation "income" tax may in some circumstances be applied when the real "income" is negative. It is therefore necessary to predict money cash flows and, after certain adjustments, the tax thereon. The effect of inflation during the time interval between the date of capital expenditure and that of paying a lower tax bill is to devalue the benefit of the allowance. This may be particularly costly where taxable profits before capital allowances are insufficient to fully offset the allowances, which then may have to be carried forward to future accounting periods. Clearly, however, the firm benefits by paying tax on income at a date when the currency will be worth less in real terms.

If it is believed that the corporation tax system should be based on profits then for reasons of equity an historic cost tax base would generally be thought to be inferior to a CCA tax base. But the introduction of 100 per cent capital allowances was a relative incentive to invest compared with the older systems of capital allowances based on a form of historic cost depreciation with relief spread over a number of years. However, assuming constant tax rates, constant tax time lags between cash flows and taxes/allowances thereon and full relief for losses, free depreciation offers neither an incentive nor disincentive to invest, and would therefore be an appropriate basis for promoting economic efficiency. By contrast it has been shown that the CCA capital allowance system would not necessarily have a neutral effect on the investment decision. For this reason it is recommended that the present system of 100 per cent allowances on plant and machinery be retained and that on buildings be changed accordingly.

Although investment decisions under the NPV model are based on cash flows, the tax calculations reflect on accrual accounting system. In addition to forecasting a project's pre-tax cash flow it is necessary to include in the tax base the project's periodic investment in working capital since there is effectively a tax on working capital in addition to a tax on cash flow. However, this is partly mitigated by stock appreciation relief.

Despite the fact that depreciation is replaced by capital expenditure when predicting pre-tax cash flows, and replaced by capital allowances when predicting capital asset consumption for tax purposes, we saw how the depreciation policy of the firm may have a subtle effect on cash flows in that depreciation included in the overheads element of the stock valuation has an influence on taxable profits.

A multiplicity of marginal tax rates was shown to exist. Hence expenditure incurred on some projects may reduce net taxable income to such an extent that the marginal tax rate for the next project being considered is now different. Interdependencies between projects are also affected by the capital allowance carry forward provisions. Furthermore the claiming of stock relief or reductions in stock on one project may cause the marginal tax rate on another project to change. To deal with these interactivities the basic NPV model is inadequate and we require a programming model instead. This will be developed in chapter seven.

Under an imputation tax system it is important to predict whether there may be any restriction in Advance Corporation Tax setoff. Any surplus ACT in an accounting period may result in a lowering of the marginal rate of corporation tax for investment decisions. The dividend policy was therefore shown to have critical tax implications for investment decisions suggesting the need for a simultaneous solution of investment and dividend decisions. A model to cater for such a solution is provided in chapter seven. But to avoid losing sight of the tax complexities of financing decisions, let us next consider financing decisions to some extent in isolation to investment decisions.

# CHAPTER 6

## Corporate financing decisions within the framework of income tax, corporation tax and capital gains tax: a model under certainty

## 6.1. Notation

| | | |
|---|---|---|
| $b$ | = | basic rate of income tax. |
| $B'$ | = | see equation (30) for definition. |
| $d$ | = | dividend payout rate. |
| $D$ | = | expected dividend. |
| $g$ | = | capital gains tax rate. |
| $G'$ | = | see equation (31) for definition. |
| $h$ | = | higher rate of income tax. |
| $h^*_D$ | = | marginal rate of income tax on interest for the marginal debentureholder. |
| $h^*_S$ | = | marginal rate of income tax on dividends for the marginal shareholder. |
| $i_4$ | = | quarterly rate of interest. |
| $I$ | = | gross interest. |
| $J$ | = | investment outlay. |
| $k$ | = | annual rate of interest. |
| $N$ | = | expected net profits after tax. |
| $\phi$ | = | the change in mean shareholder return caused by taxation. |
| $P$ | = | retained profit. |
| $Q$ | = | the value of a retention after personal and corporate tax. |
| $R$ | = | pre-tax return on asset. |
| $t$ | = | personal income tax rate under a non-imputation system. |
| $T$ | = | corporate tax rate. |
| $T'$ | = | see equation (29) for definition. |

V = the time lag between receipt of a net dividend or net interest and the payment of the excess of the higher rate tax on the gross equivalent over the basic rate tax deducted at source.

w = the time lag in years from the time of the capital gain accrual to the capital gains tax payment date on realisation of the gain.

W = expected increase in shareholder wealth.

y = the number of years between the time of the dividend payment and the time that the ACT is setoff against the mainstream corporation tax.

z = the number of years between the time when the debenture interest is paid and when a payment is to be made for the net mainstream corporation tax, against which the interest is claimed.

## 6.2 Introduction

The role of this chapter is to analyse the micro-economic effects of the UK tax system on financing decisions of the company under certainty. The models employed will introduce variables additional to those found in the econometric work by King* to represent, in particular, the following tax characteristics:

(a) Capital gains not being realised immediately, the effective capital gains tax rate being determined by the length of time the asset is held;

(b) the Schedule 20 income tax deductions at source on debenture interest and the inherent tax time lags;

(c) the similar tax time lags resulting from Schedule 14 quarterly deductions for Advance Corporation Tax; and

(d) the existence of multiple marginal rates of corporation tax.

It must be stressed that financial risk and other non-tax considerations will be ignored. It will be shown that excluding (a) to (d), the results are not surprisingly consistent with those of chapter 3 where risk was explicitly recognised.

## 6.3. A non-imputation tax system

It is convenient to begin by analysing a non-imputation tax system along the lines developed by Stiglitz** and make the following assumptions:

(a) a shareholder buys all the shares of a new firm for J units of currency;

(b) the company then invests J in a project obtaining a capital allowance of T x J on an asset yielding a return of R units of currency, greater than J;

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* King (1977) See references

** Stiglitz (1972) See references

(c) at the end of the period the asset is worthless and is sold for scrap at the market price of zero;

(d) the shareholder disposes of the shares at the end of the period at market value;

(e) there are no tax time lags;

(f) there is no time value of money;

(g) there are no special tax rules relating to "close" companies;

(h) capital markets are perfect.

The expected net profits after tax, denoted $N$ is represented by the summation of:

(1) the expected after tax income of the company; and

(2) the tax allowance on the capital investment

$$N = R(1-T) + T.J \tag{1}$$

With a dividend payout rate of d, the expected dividend for the period is given by

$$D = d\left[R(1-T) + T.J\right] \tag{2}$$

Hence, under the assumption of perfect capital markets the retained profit, and proceeds on disposal of the shares are both equal to

$$P = (1-d)\left[R(1-T) + T.J\right] \tag{3}$$

The capital gain is therefore equal to

$$(1-d)\left[R(1-T) + T.J\right] - J \tag{4}$$

Hence the expected increase in shareholder wealth during the period is equal to

$$W = (1-t)d\left[R(1-T) + T.J\right] + (1-g)\{(1-d)\left[R(1-T) + T.J\right] - J\} \tag{5}$$

Where the first term on the right hand side of the equation represents the dividend after the personal income tax at the tax-exclusive rate of t, and the second term represents the capital gain on disposal after the capital gains tax payment at the rate of g. By rearranging equation (5) we have

$$W = R(1-T)d\ (1-t) + T.J.d(1-t) + R(1-T)(1-g)(1-d) + (1-d)(1-g)T.J - J(1-g) \tag{6}$$

Now, let us consider the effect of earnings retention on the shareholder wealth. When a full distribution is made, by substituting for $d = 1$ in equation (5) we have

$$W\ (\text{for } d = 1) = R(1-t)(1-T) + J\left[T(1-t) + g - 1\right] \tag{7}$$

Similarly, for a full retention

$$W\ (\text{for } d = 0) = R(1-g)(1-T) + J\left[T(1-g) + g - 1\right] \tag{8}$$

As expected, if the personal income tax-exclusive rate and capital gains tax rates are identical, $W$ (for $d=0$) = $W$ (for $d=1$) and the shareholder is indifferent to the retention of earnings or the payment of dividends, ceteris paribus. However, if $g < t$, a common feature under the present UK tax system, the shareholder benefits by retention of earnings

$$W\ (\text{for } d = o) \ - \ W\ (\text{for } d = 1)$$

$$= \left[R(1-T) + T_{\bullet} J\right] \left[(1-g) - (1-t)\right]$$

$$= (t-g) \left[R(1-T) + T_{\bullet} J\right]$$

$$= (t-g)N \qquad (9).$$

Hence, by a full retention of earnings the shareholder wealth is increased by the excess of the personal income tax-exclusive rate over the capital gains tax rate applied to the firm's net profit after tax.

Now that we have established that the personal tax system appears biased in favour of internal finance through the ploughback of profits, let us examine how the relationship of the marginal tax rate on dividends versus capital gains affects the investment decision of a capital project which is marginal in the absence of taxation.

$$W\ (\text{without taxes}) = R - J \qquad (10).$$

Hence, the effect of taxation is to change the mean shareholder return by the variable $\phi$, such that

$$\phi = W\ (\text{with taxes}) - W\ (\text{without taxes}) \qquad (11).$$

From equations (6), (10) and (11) we obtain

$$\phi = R(1-T) \left[d(1-t) + (1-d)(1-g)\right] + d(1-t)\ T_{\bullet} J$$
$$+ (1-d)(1-g)\ T_{\bullet} J - (1-g)J - (R-J) \qquad (12).$$

Where

$R = J$, and $\phi < 0$ we have

$$(1-T)d(1-t) + (1-d)(1-g)(1-T) + d(1-t)T + (1-d)(1-g)T - 1 + g < 0$$

$$(13).$$

which simplifies to

$$t > g \tag{14}$$

Hence, in the case of a marginal project, where R=J, the effect of taxation is to reduce the mean shareholder return provided t>g.

## 6.4. The dividend decision under the tax imputation system

So far we have assumed that there is no integration of personal and corporate taxes. Let us now consider the UK imputation system under which a tax payment at the basic rate of income tax, denoted b, is made on the gross equivalent of the dividend and offset against the Mainstream Corporation Tax of the company. The gross equivalent of the dividend is

$$\frac{D}{1-b},$$

$$= D \times \frac{100}{67},$$

where

$$b = 33\%.$$

The total personal tax thereon is

$$\frac{hD}{1-b}.$$

Where h is the shareholder's marginal rate of income tax. Against this is offset the advance payment of Corporation Tax on the dividend of

$$\frac{bD}{1-b} = \frac{33}{67} \times D,$$

where

$$b = 33\%.$$

Hence, the net personal tax is

$$\frac{h-b}{1-b} \cdot D \quad ,$$

that is,

$$t = \frac{h-b}{1-b} \tag{15}$$

From equation (9) we note that by a full retention of earnings, the shareholder wealth is increased by the excess of the adjusted personal income tax rate, denoted

$$t = \frac{h-b}{1-b} \quad ,$$

over the capital gains tax rate, applied to the firm's net profit after tax. Hence the shareholder makes a gain if

$$\frac{h-b}{1-b} > g \tag{16}$$

Under the Finance Act 1978 the basic rate of b is 33 per cent, and the values of h and g are shown in tables 38and 39 . Although the higher rates of income tax are well understood, the 50 per cent marginal rate of tax on capital gains between £5,000 and £9,500 is perhaps not very well known.

Where the basic rate of income tax is 33 per cent and the capital gains tax is 30 per cent, the critical value of h, the higher rate of personal tax, is 53.1 per cent. Under these conditions, shareholders with marginal income tax rates of more than 53.1 per cent would prefer retentions, and those with lower marginal tax rates would prefer distributions, ceteris paribus.

Note that if inequality(16) were an equality, the result would give the same as equation (66) of chapter 3, the latter being a requirement of a neutral tax system under conditions of risk.

A further refinement in the model is to take account of time lags between the payment of Advance Corporation Tax (ACT) and the Net Mainstream Corporation Tax (NMCT). Assuming a 33 per cent basic rate of income tax, a £67 dividend has an attached £33 credit on which the shareholder pays tax at the (usually) higher rate of h on £100, but receives relief of £33. If the ACT is paid in the quarterly period following the dividend payout and the NMCT paid one year after the dividend, then the present value of the extra cost to the company of the dividend vis-a-vis a retention is therefore

$$\pounds 67 \times \frac{33}{67} \times \left[\frac{1}{(1+i_q)} - \frac{1}{(1+i_q)^4}\right] ,$$

where $i_q$ = the relevant quarterly rate of interest.

We may now formally incorporate into the analysis the effect of tax time lags.

Let w = the time lag in years from the time of the capital gain accrual to the capital gains tax payment date on realisation of the gain,

y = the number of years between the time of the dividend payment and the time that the ACT is setoff against the mainstream corporation tax,

z = the number of years between the time when the debenture interest is paid and when a payment is to be made for the net mainstream corporation tax, against which the interest is claimed,

0.25 = the number of years between the payment of a dividend and the ACT thereon and between the payment of interest and the basic rate of income tax thereon, under the quarterly accounting tax system.

Table 38 Marginal rates of income tax = h (Finance Act 1978)

| Net taxable income | | Marginal rate of investment surcharge | Marginal tax rate = h |
|---|---|---|---|
| | | % | % |
| First | £750 | NIL | 25 |
| | - | 10 | 35 |
| | - | 15 | 40 |
| Next | £7,250 | NIL | 33 |
| | - | 10 | 43 |
| | - | 15 | 48 |
| Next | £1,000 | NIL | 40 |
| | - | 10 | 50 |
| | - | 15 | 55 |
| Next | £1,000 | NIL | 45 |
| | - | 10 | 55 |
| | - | 15 | 60 |
| Next | £1,000 | NIL | 50 |
| | - | 10 | 60 |
| | - | 15 | 65 |
| Next | £1,500 | NIL | 55 |
| | - | 10 | 65 |
| | - | 15 | 70 |
| Next | £1,500 | NIL | 60 |
| | - | 10 | 70 |
| | - | 15 | 75 |
| Next | £2,000 | NIL | 65 |
| | - | 10 | 75 |
| | - | 15 | 80 |
| Next | £2,500 | NIL | 70 |
| | - | 10 | 80 |
| | - | 15 | 85 |
| Next | £5,500 | NIL | 75 |
| | - | 10 | 85 |
| | - | 15 | 90 |
| Remainder | | NIL | 83 |
| | - | 10 | 93 |
| | - | 15 | 98 |

Table 39 Marginal rates of capital gains tax = g (Finance Act 1978)

| Gain | | | Tax | Marginal tax rate = g |
|---|---|---|---|---|
| | | | | % |
| First | £1,000 | | NIL | 00 |
| Next | £4,000 | | £600 | 15 |
| Next | £4,500 | | £2,250 | 50 |
| Total | £9,500 | @ 30% = | £2,850 | |
| Remainder | | @30% | | 30 |

With a time lag of w years from the time of the capital gain accrual to the capital gains tax payment on realisation of the gain, the gain is taxed at the effective rate of

$$\frac{g}{(1+k)^w} ,$$

where

k = the relevant rate of interest (= the risk-free rate since risk is ignored in this chapter) .

Therefore, in net of tax terms a dividend is worth to the shareholder

$$\frac{1-h}{1-b} .D ,$$

yet the retention of an amount equal to the cost to the company, of this dividend is worth to the shareholder

$$D \left[ 1 + \frac{b}{(1-b)(1+k)^{0.25}} - \frac{b}{(1-b)(1+k)^y} \right] \left[ 1 - \frac{g}{(1+k)^w} \right] ,$$

where y = the number of years between the time of the dividend payment and the time that the ACT is setoff against the NMCT.

Therefore a retention is preferable to a new issue (ignoring flotation costs) if

$$\frac{1-h}{1-b} < \left[ 1 + \frac{b}{(1-b)(1+k)^{0.25}} - \frac{b}{(1-b)(1+k)^y} \right] \left[ 1 - \frac{g}{(1+k)^w} \right] ,$$

giving

$$h > 1 - \left[ 1 - \frac{g}{(1+k)^w} \right] \left[ 1 - b + \frac{b}{(1+k)^{0.25}} - \frac{b}{(1+k)^y} \right] \qquad (17)$$

The basis for the comparison between retentions and new issues is the same as that presented by Miller and Modigliani (1961). In the absence of taxation then, given the investment decision of the firm, in a perfect frictionless market (i) earnings retentions are financially equivalent to (ii) the payment of dividends and the raising of a new issue to cover the dividends so that the capital projects can be financed.

Illustration 1

If we consider the case of an immediate realisation of the capital gain on retention of profits together with the NMCT payable at the same time as the ACT, then we have $w=0$ and $y=0.25$. With these values inequality (17) simplifies to:

$$h > g(1-b) + b \qquad (18)$$

Hence, for a shareholder paying income tax at the basic rate and capital gains tax at a rate of 30 per cent, a retention of profits is preferred to a dividend distribution if the marginal rate of tax on investment income is greater than approximately 53 per cent. However, this assumes that the capital gain is immediately realised ($w=0$) and that the relief for ACT setoff is achieved at the same time as the tax payment on the dividend.

Under this simplified model, where the shareholder is a basic rate taxpayer then $h=b$ and $t=0$. Therefore with a marginal rate of capital gains tax of zero ($g=0$), a basic rate taxpayer in theory feels indifferent between a dividend and a retention of earnings, but a higher rate taxpayer prefers a retention.

With a marginal rate of capital gains tax at 15 per cent ($g=0.15$), a retention of £67 is worth 85 per cent of £67 = £56.95 net of tax. By contrast, a dividend of £67 is treated as £100 franked investment income and worth £100 x $(1-h)$ net of tax. Hence where $h$ = 43.05 per cent the shareholder is indifferent between a dividend and a capital gain; where $h <$ 43.05 per cent a dividend is preferred; and where $h >$ 43.05 per cent a retention is preferred.

Illustration 2

Let us now relax the assumption of a zero time lag between the payment of ACT and NMCT. The following values will be used in the illustration:

$b = 0.33,\ 0.34$ ,

$w = 0$ ,

$y = 1,\ 2$ ,

$k = 0.07,\ 0.14$ ,

$g = 0.30$ .

With an annual time preference rate of 7 per cent.

$(1+i_v)^4 = 1.07$ ,

$i_v = 0.0171$, and

$(1+i_v)^{-1} - (1+i_v)^{-4} = 0.0486$.

Hence a dividend of £67 costs the company (when y=1):

£67 + 0.0486 × £33 = £68.60.

However, the time lag between the dividend payment and the ACT set-off may be at least two years (y=2). For instance, with a December year-end an old established company paying a dividend in January 1978 will pay ACT in April 1978 and receive ACT setoff relief in January 1980. Hence a dividend of £67 is equivalent to a retention of:

$$£67 + £33\left[(1+i_v)^{-1} - (1+i_v)^{-8}\right] = £70.62.$$

From inequality (17) we derive h > 51 per cent. The same value for h is obtained when b=0.34. Hence, assuming the capital gain to be realised immediately (w=0) and a time preference rate of 7 per cent, retentions are preferred for taxpayers with marginal tax rates on investment income in excess of 51 per cent. A variation in the rate of k only causes a slight variation in the result. For instance, when k=0.14 and b=0.34, at the margin we obtain h=49 per cent.

Illustration 3

Let us now consider deferred realisations of the capital gain of 5 and 10 years:

$w = 5, 10$,

$b = 0.34$,

$g = 0.3$,

$y = 2$,

$k = 0.14$.

With a 5 year time lag between the capital gain accrual and the tax thereon ($w=5$), from inequality (17) we derive $h > 38.6$ per cent. Similarly with a 10 year time lag we obtain $h > 33$ per cent. In the latter situation all taxpayers with a marginal rate of tax on investment income not lower than the basic rate of 34 per cent will prefer a retention to a dividend payment, ceteris paribus. Further sensitivity tests of the retention versus dividend decision are shown in Table 40 and Figure 4.

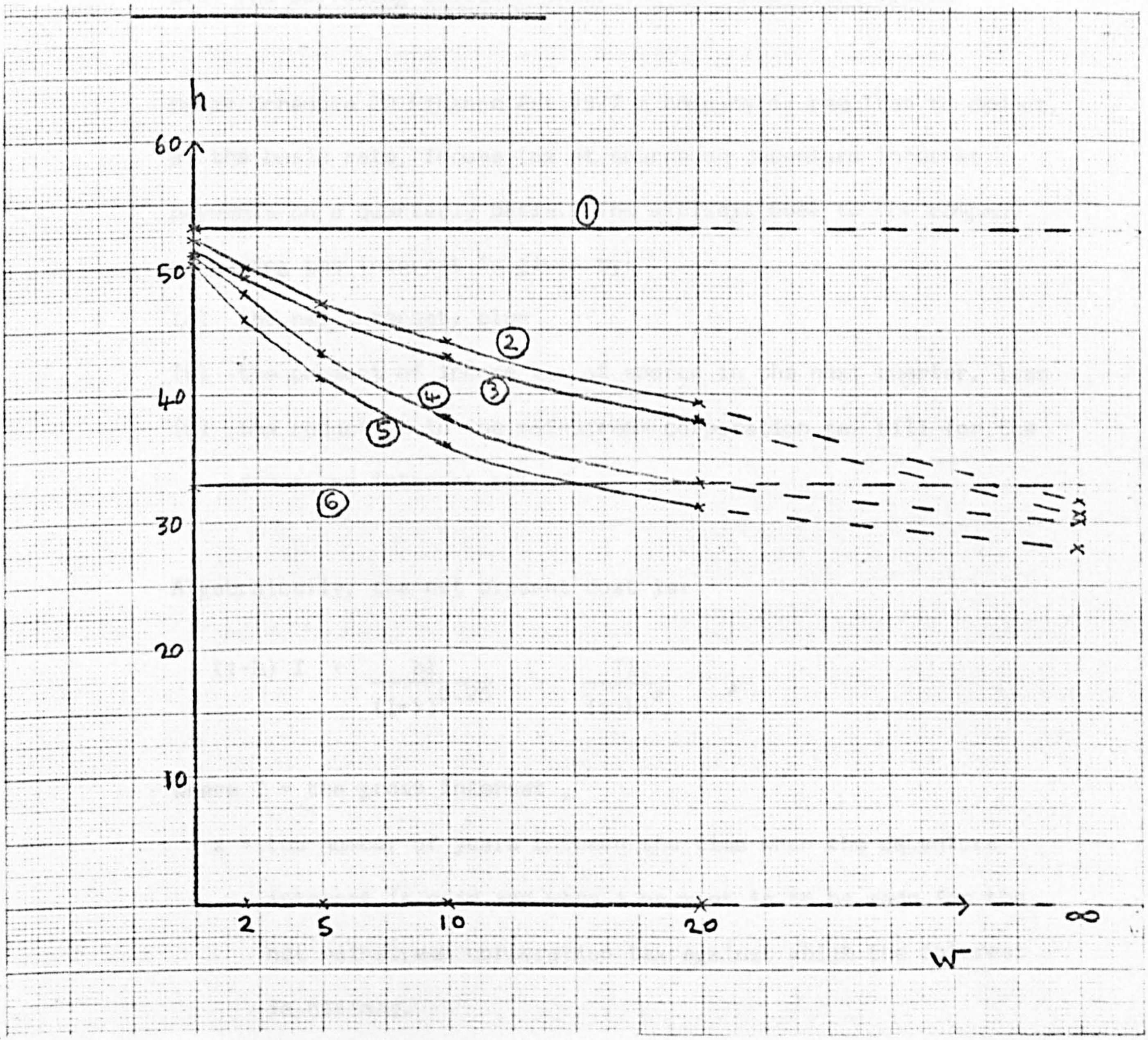
The table illustrates that it does not necessarily follow that if the basic rate of income tax exceeds the capital gains tax rate then from a tax standpoint a shareholder who is a basic rate taxpayer prefers the company not to pay a dividend.

We observe from figure 4 that for very low discount rates ($k \simeq o$) retentions are always preferable if the higher rate of income tax exceeds 51.3 per cent. For higher discount rates the critical value of h is reduced and the longer the shareholding period the bigger the reduction. Delays in ACT setoff of a further year create marginally smaller critical values of h. As $k \rightarrow \infty$ the critical value of h approaches the basic rate of income tax.

Table 40 Critical values of the higher rate of income tax: retentions versus new issues

| g | b | w | y | k | h |
|---|---|---|---|---|---|
| 0.3 | 0.33 | ∞ | 1 | 0.1 | 0.3078 |
| 0.3 | 0.33 | 20 | 1 | 0.1 | 0.3386 |
| 0.3 | 0.33 | 10 | 1 | 0.1 | 0.3878 |
| 0.3 | 0.33 | 5 | 1 | 0.1 | 0.4367 |
| 0.3 | 0.33 | 4 | 1 | 0.1 | 0.4496 |
| 0.3 | 0.33 | 3 | 1 | 0.1 | 0.4638 |
| 0.3 | 0.33 | 2 | 1 | 0.1 | 0.4794 |
| 0.3 | 0.33 | 1 | 1 | 0.1 | 0.4966 |
| 0.3 | 0.33 | 0 | 1 | 0.1 | 0.5154 |
| 0.3 | 0.33 | ∞ | 2 | 0.1 | 0.2805 |
| 0.3 | 0.33 | 20 | 2 | 0.1 | 0.3126 |
| 0.3 | 0.33 | 10 | 2 | 0.1 | 0.3637 |
| 0.3 | 0.33 | 5 | 2 | 0.1 | 0.4145 |
| 0.3 | 0.33 | 2 | 2 | 0.1 | 0.4589 |
| 0.3 | 0.33 | 0 | 2 | 0.1 | 0.5036 |
| 0.3 | 0.33 | ∞ | 1 | 0.05 | 0.3183 |
| 0.3 | 0.33 | 20 | 1 | 0.05 | 0.3954 |
| 0.3 | 0.33 | 10 | 1 | 0.05 | 0.4439 |
| 0.3 | 0.33 | 5 | 1 | 0.05 | 0.4785 |
| 0.3 | 0.33 | 2 | 1 | 0.05 | 0.5038 |
| 0.3 | 0.33 | 0 | 1 | 0.05 | 0.5228 |
| 0.3 | 0.33 | ∞ | 2 | 0.05 | 0.3033 |
| 0.3 | 0.33 | 20 | 2 | 0.05 | 0.3821 |
| 0.3 | 0.33 | 10 | 2 | 0.05 | 0.4316 |
| 0.3 | 0.33 | 5 | 2 | 0.05 | 0.4671 |
| 0.3 | 0.33 | 2 | 2 | 0.05 | 0.4930 |
| 0.3 | 0.33 | 0 | 2 | 0.05 | 0.5123 |
| 0.3 | 0.33 | 1 to ∞ | 1 to ∞ | 0 | 0.5310 |
| 0.3 | 0.33 | 1 to ∞ | 1 to ∞ | ∞ | 0.3300 |

Figure 4: Critical values of the higher rate of income tax: retentions versus new issues




Key

(1) k = o ,

(2) k = 0.05, y = 1 ,

(3) k = 0.05, y = 2 .

(4) k = 0.10, y = 1 ,

(5) k = 0.10, y = 2 .

(6) k = ∞ .

Assumption g = 0.3, b = 0.33 .

## 6.5 The borrowing decision under the tax imputation system

Under Schedule 20 Finance Act 1972 a company is required to deduct, at the basic rate, income tax at source on debenture interest payments on a quarterly basis. The explicit cost to the company of paying the interest is given by:

(a) the net interest, plus

(b) the payment of income tax at source in the next quarter, less

(c) the reduction in the mainstream corporation tax bill for the debenture interest relief.

Algebraically, the net present cost is:

$$(1-b)\,I + \frac{bI}{(1+k)^{0.25}} - \frac{TI}{(1+k)^{z}},$$

where I = the gross interest ,

z = the number of years between the time when the debenture interest is paid and when a payment is to be made for the net mainstream corporation tax against which the interest is claimed.

Similarly, the net present cost to the company of a dividend is:

$$D + \frac{bD}{(1-b)\,(1+k)^{0.25}} - \frac{bD}{(1-b)(1+k)^{y}},$$

If the security holder were to ignore risk, as is assumed in this chapter then we may let

$$D = (1-b)I.$$

Hence borrowing is preferred to dividend payments (i.e., to new issues, ignoring flotation costs) if

$$1 + \frac{b}{(1-b)(1+k)^{0.25}} - \frac{T}{(1-b)(1+k)^{z}} \tag{20}$$

$$< 1 + \frac{b}{(1-b)(1+k)^{0.25}} - \frac{b}{(1-b)(1+k)^{y}} ,$$

where the left-hand side of the inequality represents the explicit cost to the company of paying the interest and the right-hand side reflects the cost of the dividend.

The above expression may be simplified to

$$\frac{T}{(1+k)^{z}} > \frac{b}{(1+k)^{y}} \tag{21}$$

We observe that where z=y, borrowing is preferred if

$$T > b \tag{22}$$

Note that if in inequality (22) were an equality, this would be the same as equation (65) of chapter 3, a requirement of a neutral tax system under conditions of risk.

The relative tax advantages of borrowed versus equity funds may be demonstrated by the use of arithmetical examples. Let us assume that the company has an option whether to pay out £100 in debenture interest (£67 net of income tax deducted at source, assuming a 33% basic rate of income tax) or £100 in dividends (£67 net of Advance Corporation Tax). If the debenture holder has the same marginal tax rate as the shareholder then the £100 gross of tax is worth the same after tax, ignoring risk. However, the position as far as the company is concerned is more interesting.

With Schedule D company profits of, say £100,000 we see below in table 41 that the tax advantage of debenture interest relative to dividends is 19%, assuming a 52% rate of Corporation Tax. It may be thought that where ACT is restricted then the cost of the dividends becomes, relatively, even more expensive than before. This is not true. Although the dividends become more expensive, the same applies to the interest. In table 42 we show a base situation of restricted set-off of ACT with unrelieved ACT carried forward indefinitely. Even if we can relieve the ACT carried forward to the next year, the reduction in the net mainstream corporation tax applies to cases (ii) and (iii) (Table 43).

The relative tax benefit of debenture interest can be increased to 29% if the charge of income is sufficiently great to reduce the net taxable income below £50,000 at which point the small companies rate of 42% becomes effective. This is demonstrated in table 44. Although the tax deductibility of the interest reduced the cost of the debt by 62%, being £31,000 as a percentage of £50,000, the ACT setoff relating to the dividend reduces the cost of the dividend by 33%. Hence the relative tax advantage of debt finance is 62% - 33% = 29%. Note that the generous relief of 62% is made up of relief at the 52% normal rate plus the extra 10% relief from the application of small companies rate at 10 percentage points lower than that of the normal rate.

Furthermore, the relative tax advantage of debt finance may be exceptionally increased to 33.29% if the marginal small companies relief is applied to net taxable income between £50,000 and £85,000 under the Finance Act 1978. Hence, in table 45 we see that the debenture interest is relieved at the effective marginal small companies rate of 66.29%, whereas the gross dividends are relieved at 33%, giving a relative tax advantage of 33.29%.

Not surprisingly, with net taxable income at less than £50,000 the relative tax advantage of debt finance is only 42% - 33% = 9% (see table 46).

Occasionally, the debenture interest may be great enough to spread a number of tax bands. For instance, the marginal tax rate for taxable profits between £50,000 and £85,000 is 66.29%, being 52% + $\left[50{,}000 / (85{,}000 - 50{,}000)\right] \times \left[52\% - 42\%\right]$, under the Finance Act 1978, and the marginal rate for net taxable income below £50,000 is 42%. Hence, if the debenture interest spreads the two bands equally the marginal tax relief on all the debenture interest is $(\frac{1}{2} \times 42\%) + (\frac{1}{2} \times 66.29\%) = 54.145\%$ .

Since debenture interest is relieved at 54.145% compared with ACT setoffs on dividends of 33%, the relative tax advantage of debt finance is 54.145% - 33% = 21.145% (see table 47) .

Finally, if the corporate tax rate were the same as the basic rate of income tax, then the tax preference for debt finance would be removed (see table 48). This situation, however, does not arise under current Revenue Law. For convenience, a summary of the tax advantages of debt versus equity finance is given in table 49.

Interestingly, if bank interest is used to replace debenture interest then a higher tax relief is obtained provided the company is claiming relief on the appreciation of trading stock. Since the stock appreciation rules presently in force allow a deduction equal to the excess of the increase in stock value during the accounting period over a proportion, currently at 15 per cent, of trading profits for tax purposes after bank interest but before charges on income, then bank interest relief is effectively 15 per cent higher than debenture interest relief. The numerical illustration in table 50 shows that by substituting £100 bank interest for debenture interest, the tax bill is reduced by £100 x 52% x 15% = £7.80. In this way if we compared the relative tax advantage of bank finance versus equity finance the figures in the final column of table 49 would all be increased by 15%, to 21.85%, 10.35%, and 38.2835%. Bank finance in the remainder of this analysis is ignored.

Table 41 (£s) After-tax cost of finance

| | Base Situation | Situation with debenture interest of £100 | Situation with gross dividends of £100 |
|---|---|---|---|
| | (i) | (ii) | (iii) |
| Schedule D | 100,000 | 100,000 | 100,000 |
| Charge on income | NIL | 100 | NIL |
| Net taxable income | 100,000 | 99,900 | 100,000 |
| Mainstream Corporation Tax | 52,000 | 51,948 | 52,000 |
| Dividends paid | NIL | NIL | 67 |
| Advance Corporation Tax | NIL | NIL | 33 |
| Net mainstream corporation tax | 52,000 | 51,948 | 51,967 |
| Change in net mainstream corporation tax over base | NIL | (52) | (33) |
| After-tax cost of finance | | 100-52=48 | 67+33-33=67 |

The relative tax advantage of debenture interest is £67 - £48 = £19 being 19% of £100 gross cost.

Table 42(£s) After-tax cost of finance

| | Base Situation | Situation with debenture interest of £100 | Situation with gross dividends of £100 |
|---|---|---|---|
| | (i) | (ii) | (iii) |
| Schedule D | 100,000 | 100,000 | 100,000 |
| Charge on income | NIL | 100 | NIL |
| Net taxable income | 100,000 | 99,900 | 100,000 |
| Mainstream Corporation Tax | 52,000 | 51,948 | 52,000 |
| Dividends paid | 67,000 | 67,000 | 67,067 |
| ACT | 33,000 | 33,000 | 33,033 |
| ACT setoff | 33,000 | 32,967 | 33,000 |
| ACT c/fd | NIL | 33 | 33 |
| Net mainstream corporation tax | 19,000 | 18,981 | 19,000 |
| Change in net mainstream corporation tax over base | | (19) | NIL |
| After-tax cost of finance | | 100-19=81 | 67+33=100 |

The relative tax advantage of debenture interest is £100 - £81 = £19, being 19% of £100 gross cost.

Table 43(£s) After-tax cost of finance

| | Base Situation in year 2 | Situation with debenture interest of £100 in year 1 only | Situation with gross dividends of £100 in year 1 only |
|---|---|---|---|
| | (i) | (ii) | (iii) |
| Schedule D, say , | 200,000 | 200,000 | 200,000 |
| Charge on income | NIL | NIL | NIL |
| Net taxable income | 200,000 | 200,000 | 200,000 |
| Mainstream Corporation tax | 104,000 | 104,000 | 104,000 |
| Dividends paid | 67,000 | 67,000 | 67,000 |
| ACT | 33,000 | 33,000 | 33,000 |
| ACT b/fd | NIL | 33 | 33 |
| ACT setoff | 33,000 | 33,033 | 33,033 |
| Net MCT | 71,000 | 70,967 | 70,967 |
| Change in net MCT over base | | (33) | (33) |
| After tax costs of finance over the 2 years | | 100-19-33=48 | 67+33-33=67 |

The relative tax advantage of debenture interest is £67 - £48 = £19, being 19% of £100 gross cost.

Table 44 (£s) After-tax cost of finance

| | Base Situation | Situation with debenture interest of £50,000 | Situation with gross dividends of £50,000 |
|---|---|---|---|
| | (i) | (ii) | (iii) |
| Schedule D | 100,000 | 100,000 | 100,000 |
| Charge on income | NIL | 50,000 | NIL |
| Net taxable income | 100,000 | 50,000 | 100,000 |
| Mainstream Corporation Tax | 52,000 | 21,000 | 52,000 |
| Dividends paid | NIL | NIL | 33,500 |
| ACT | NIL | NIL | 16,500 |
| ACT setoff | NIL | NIL | 16,500 |
| Net Mainstream Corporation Tax | 52,000 | 21,000 | 35,500 |
| Change in net MCT over base | | (31,000) | (16,500) |
| After-tax cost of finance | | 50,000-31,000=19,000 | 33,500+16,500-16,500=33,500 |

The relative tax advantage of debenture interest is £33,500 - £19,000 = £14,500, being 29% of £50,000 .

## Table 15 (£s) After-tax cost of finance

| | Base Situation | Situation with debenture interest of £10,000 | Situation with gross dividends of £10,000 |
|---|---|---|---|
| | (i) | (ii) | (iii) |
| Schedule D | 60,000 | 60,000 | 60,000 |
| Charge on income | NIL | 10,000 | NIL |
| Net taxable income | 60,000 | 50,000 | 60,000 |
| Tax @ 52% | 31,200 | 26,000 | 31,200 |
| less marginal small companies relief | $\frac{1}{7} \times 25,000$ | $\frac{1}{7} \times 35,000$ | $\frac{1}{7} \times 25,000$ |
| | = 3,571 | = 5,000 | = 3,571 |
| Mainstream Corporation Tax | 27,629 | 42% 50,000 = 21,000 | 27,629 |
| Dividends paid | NIL | NIL | 6,700 |
| ACT | NIL | NIL | 3,300 |
| ACT setoff | NIL | NIL | 3,300 |
| Net Mainstream Corporation Tax | 27,629 | 21,000 | 24,329 |
| Change in net MCT over base | | (6,629) | (3,300) |
| After-tax cost of finance | | 10,000 - 6,629 = 3,371 | 6,700 + 3,300 - 3,300 = 6,700 |

The relative tax advantage of debenture interest is £6,700 - £3,371 = £3,329, being 33.29% of £10,000.

Table46 After-tax cost of finance

| | Base Situation | Situation with debenture interest of £10,000 | Situation with gross dividends of £10,000 |
|---|---|---|---|
| | (i) | (ii) | (iii) |
| Schedule D | 50,000 | 50,000 | 50,000 |
| Charge on income | NIL | 10,000 | NIL |
| Net taxable income | 50,000 | 40,000 | 50,000 |
| Tax at 42% | 21,000 | 16,800 | 21,000 |
| Dividends paid | NIL | NIL | 6,700 |
| ACT | NIL | NIL | 3,300 |
| ACT setoff | NIL | NIL | 3,300 |
| Net MCT | 21,000 | 16,800 | 17,700 |
| Change in net MCT over base | NIL | (4,200) | (3,300) |
| After tax costs of finance | | 10,000 - 4,200 = 5,800 | 6,700+3,300-3,300 = 6,700 |

The relative tax advantage of debenture interest is £6,700 - £5,800 = £900, being 9% of £10,000.

Table 47 (£s) After-tax cost of finance

| | Base Situation | Situation with debenture interest of £20,000 | Situation with gross dividends of £20,000 |
|---|---|---|---|
| | (i) | (ii) | (iii) |
| Schedule D | 60,000 | 60,000 | 60,000 |
| Charge on income | NIL | 20,000 | NIL |
| Net taxable income | 60,000 | 40,000 | 60,000 |
| Mainstream Corporation Tax | 27,629 | 16,800 | 27,629 |
| Dividends paid | NIL | NIL | 13,400 |
| ACT | NIL | NIL | 6,600 |
| ACT setoff | NIL | NIL | 6,600 |
| Net MCT | 27,629 | 16,800 | 21,029 |
| Change in net MCT over base | | (10,829) | (6,600) |
| After-tax cost of finance | | 20,000 - 10,829 = 9,171 | 13,400 + 6,600 - 6,600 = 13,400 |

The relative tax advantage of debenture interest is £13,400 - £9,171 = £4,229, being 21.145% of £20,000.

Table 48 After-tax cost of finance

| | Base Situation | Situation with £100 debenture interest | Situation with gross dividends of £100 |
|---|---|---|---|
| | (i) | (ii) | (iii) |
| Schedule D | 100,000 | 100,000 | 100,000 |
| Charge on income | NIL | 100 | NIL |
| Net taxable income | 100,000 | 99,900 | 100,000 |
| MCT @ 33% | 33,000 | 32,967 | 33,000 |
| Dividends paid | NIL | NIL | 67 |
| ACT | NIL | NIL | 33 |
| ACT setoff | NIL | NIL | 33 |
| Net MCT | 33,000 | 32,967 | 32,967 |
| Change in net MCT over base | | (33) | (33) |
| After-tax cost of finance | | 100 - 33 = 67 | 67 + 33 - 33 = 67 |

The relative tax advantage of debenture interest is £67 - £67 = zero .

Table 49 A summary of the tax advantages of debt versus equity finance under the Finance Act 1978.

| Net taxable income | ACT setoff | Marginal corporate tax relief on gross debenture interest | ACT setoff against net mainstream corporation tax, as a fraction of gross dividends | Relative tax advantage of debt finance |
|---|---|---|---|---|
| | | (i) | (ii) | (i) - (ii) |
| Over £85,000 | not restricted | 52% | 33% | 19% |
| Under 50,000 but positive | not restricted | 42% | 33% | 9% |
| Between £50,000 and £85,000 | not restricted | 66.29% | 33% | 33.29% |
| Over £85,000 | fully restricted | 19% | NIL | 19% |
| Under £50,000 but positive | fully restricted | 9% | NIL | 9% |
| Between £50,000 and £85,000 | fully restricted | 33.29% | NIL | 33.29% |
| Nil or negative | fully restricted | NIL | NIL | NIL |

Table 50 (£s) Debenture versus bank interest

| | Tax computation with £100 debenture interest | | Tax computation with £100 bank interest | |
|---|---|---|---|---|
| Schedule D profit before bank interest and before stock relief | | 100,100 | | 100,100 |
| less bankinterest | | NIL | | (100) |
| Profit before stock relief | | 100,100 | | 100,000 |
| Opening stock | 13,000 | | 13,000 | |
| Closing stock | 43,000 | | 43,000 | |
| Increase | 30,000 | | 30,000 | |
| less 15% of profit before stock relief | (15,015) | | (15,000) | |
| Stock relief | | (14,985) | | (15,000) |
| Tax before debenture interest | | 85,115 | | 85,000 |
| less debenture interest | | (100) | | NIL |
| Net Taxable Income | | 85,015 | | 85,000 |
| Tax thereon @ 52% | | 44,207.80 | | 44,200 |

Perhaps a surprising outcome of the analysis is that the relative tax advantage of debt finance is unaffected by the restricted setoff of ACT, since the marginal tax rate on the interest is reduced to the same extent as the marginal tax rate on the dividend. However, the size of net taxable income is a critical factor. Since high levels of capital allowances may reduce the level of net taxable income to below £85,000 or even £50,000 then capital expenditure decisions create interesting interactivities with respect to marginal tax rates for financing decisions.

In table 51 we illustrate the effects of an increase in gearing on the after-tax returns to those who provide sources of corporate finance. Firms (a) (b) and (c) each have the same total market capitalisation of £lm. and earn a 20% rate of return, determined exogenously by the business risks of the firms' activities. For firm (a) the 20% rate of return before tax becomes 9.6% after tax, i.e. the effective total tax rate, personal and corporate, is

$\frac{20 - 9.6}{20} = 52\%$.

By comparing (a) with (b) we note that with £5,000 debenture interest the total return to all providers of capital increases by £5,000 x 19% = £950.

Hence for firm (b), the effective total tax rate, personal and corporate, is $\frac{20 - 9.695}{20} = 51.525\%$. Similarly, by comparing firm (a) with firm (c) we observe that the total return increases by £10,000 x 19% = £1,900, and that the total tax rate is (20 - 9.79)/20 = 51.5%. In this way as more debt finance is introduced to replace equity capital, the total annual returns available to all providers of capital increase by the debenture interest times the 'relative tax advantage of debt finance' as per table 12.

Table 51 Overall rate of return

| | (a) | (b) | (c) |
|---|---|---|---|
| Value of the firm | £1,000,000 | £1,000,000 | £1,000,000 |
| 20% return before tax | 200,000 | 200,000 | 200,000 |
| Interest | NIL | 5,000 | 10,000 |
| Tax base | 200,000 | 195,000 | 190,000 |
| Tax thereon @ 52% | 104,000 | 101,400 | 98,800 |
| After tax return | 96,000 | 93,600 | 91,200 |
| Dividends (net) | 96,000 | 93,600 | 91,200 |
| ACT | 47,284 | 46,101 | 44,919 |
| Net MCT | 56,716 | 55,299 | 53,881 |
| Returns to equity holders after personal tax @ 33% | 96,000 | 93,600 | 91,200 |
| Returns to debentureholders after personal tax @ 33% | NIL | 3,350 | 6,700 |
| Total return after personal tax | 96,000 | 96,950 | 97,900 |
| Overall rate of return after all taxes | 9.6% | 9.695% | 9.79% |

The foregoing analysis has implied that, ceteris paribus, £1 after tax in the hands of a debenture holder would be equivalent to £1 after tax in the hands of a shareholder; and that a £1 payment after tax for debenture interest would be equivalent, as far as the company is concerned, to a £1 payment after tax for dividends. Clearly this ignores the way both investors and corporate managers view the risk attached to alternative forms of finance. Since debenture interest is paid in priority to dividends it is more certain and hence has a lower rate of interest required by the investor. The effect of financial risk on capital structure has already been considered in chapter 3 within the framework of the Capital Asset Pricing Model.

Let us now analyse the borrowing versus retention decision by comparing a retention worth Q after tax in the hands of a shareholder with interest worth Q after tax in the hands of a debenture holder. Before capital gains tax the retention is worth

$$\frac{Q}{1 - \dfrac{g}{(1+k)^{w}}} .$$

By contrast, the explicit cost to the company of debenture interest is given by

$$I \left[ 1 - b + \frac{b}{(1+k)^{0.25}} - \frac{T}{(1+k)^{z}} \right] .$$

Since the debenture interest after personal tax is worth Q to the debenture holder, then

$$I \; (1-h) = Q \qquad (23),$$

giving

$$I = \frac{Q}{1-h} \qquad (24).$$

Hence, borrowing is preferred to a retention if

$$\frac{Q}{1-h} \left[ 1 - b + \frac{b}{(1+k)^{0.25}} - \frac{T}{(1+k)^{z}} \right] < \frac{Q}{1 - \dfrac{g}{(1+k)^{w}}} \qquad (25),$$

that is, if

$$h < 1 - \left[ 1 - \frac{g}{(1+k)^{w}} \right] \left[ 1 - b + \frac{b}{(1+k)^{0.25}} - \frac{T}{(1+k)^{z}} \right] \qquad (26).$$

The critical values of h are shown in Table 52 and Figure 5.

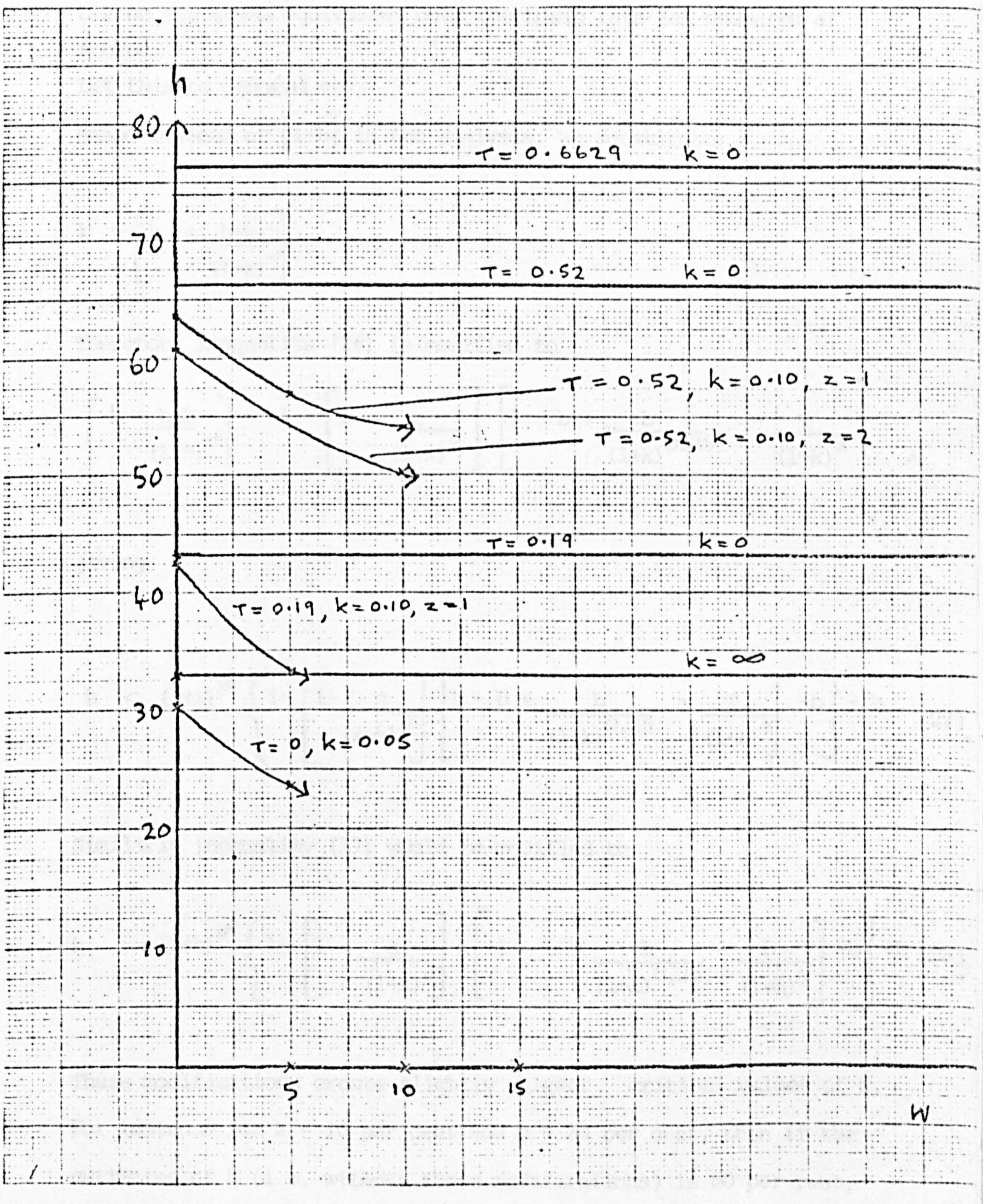
Figure 5 bears similarities to figure 4. It is assumed that capital gains tax is 30 per cent and the basic rate of income tax is 33 per cent

With a marginal corporate tax rate at 52 per cent the critical value of h will be 66.4 per cent when the discount rate is zero. The greater the discount rate, the lower the corporate tax rate, and the longer the shareholding period, then the lower the critical value of h. Where $k \rightarrow \infty$, h approaches 33 per cent. Note that where the company is not paying any corporation tax, perhaps through heavy capital allowances wiping out taxable income before allowances, then a retention can be preferable to borrowing where h is greater or equal to the basic rate of income tax (e.g. $w = 0$, $T = 0$, $k = 0.05$, $b = 0.33$, $g = 0.3$, $h = 0.3028$).

Table 52 Critical values of the higher rate of income tax: retentions versus debt.

| g | b | w | z | T | k | h |
|---|---|---|---|---|---|---|
| 0.3 | 0.33 | 5 | 1 | 0.6629 | 0.05 | 0.7211 |
| 0.3 | 0.33 | 5 | 1 | 0.6629 | 0.10 | 0.6830 |
| 0.3 | 0.33 | 5 | 1 | 0.52 | 0.05 | 0.6169 |
| 0.3 | 0.33 | 5 | 1 | 0.52 | 0.10 | 0.5773 |
| 0.3 | 0.33 | 5 | 1 | 0.42 | 0.05 | 0.5441 |
| 0.3 | 0.33 | 5 | 1 | 0.42 | 0.10 | 0.5033 |
| 0.5 | 0.34 | 0 | 1 | 0 | 0.05 | 0.5021 |
| 0.5 | 0.33 | 0 | 1 | 0 | 0.05 | 0.5020 |
| 0.5 | 0.32 | 0 | 1 | 0 | 0.05 | 0.5019 |
| 0.3 | 0.33 | 5 | 1 | 0.3329 | 0.05 | 0.4806 |
| 0.5 | 0.34 | 1 | 1 | 0 | 0.05 | 0.4784 |
| 0.5 | 0.33 | 1 | 1 | 0 | 0.05 | 0.4783 |
| 0.3 | 0.33 | 5 | 1 | 0.3329 | 0.10 | 0.4388 |
| 0.3 | 0.33 | 5 | 1 | 0.19 | 0.05 | 0.3765 |
| 0.3 | 0.33 | 5 | 1 | 0.19 | 0.10 | 0.3332 |
| 0.3 | 0.34 | 0 | 2 | 0. | 0.04 | 0.3077 |
| 0.3 | 0.33 | 5 | 1 | 0.09 | 0.05 | 0.3037 |
| 0.3 | 0.34 | 0 | 1 | 0 | 0.05 | 0.3030 |
| 0.3 | 0.33 | 0 | 1 | 0 | 0.05 | 0.3028 |
| 0.3 | 0.32 | 0 | 1 | 0 | 0.05 | 0.3027 |
| 0.3 | 0.34 | 1 | 1 | 0 | 0.05 | 0.2887 |
| 0.3 | 0.33 | 1 | 1 | 0 | 0.05 | 0.2886 |
| 0.3 | 0.33 | 2 | 1 | 0 | 0.05 | 0.2750 |
| 0.3 | 0.33 | 3 | 1 | 0 | 0.05 | 0.2621 |
| 0.3 | 0.33 | 5 | 1 | 0.09 | 0.10 | 0.2592 |
| 0.3 | 0.33 | 4 | 1 | 0 | 0.05 | 0.2498 |
| 0.3 | 0.33 | 5 | 1 | 0 | 0.05 | 0.2381 |
| 0.3 | 0.33 | 5 | 2 | 0 | 0.05 | 0.2381 |
| 0.3 | 0.33 | 5 | 1 | 0 | 0.10 | 0.1926 |
| 0.15 | 0.34 | 0 | 1 | 0 | 0.05 | 0.1535 |
| 0.15 | 0.33 | 0 | 1 | 0 | 0.05 | 0.1534 |
| 0.15 | 0.32 | 0 | 1 | 0 | 0.05 | 0.1533 |
| 0.15 | 0.34 | 1 | 1 | 0 | 0.05 | 0.1464 |
| 0.15 | 0.33 | 1 | 1 | 0 | 0.05 | 0.1463 |
| 0 | 0.34 | 0 | 1 | 0 | 0.05 | 0.004 |
| 0 | 0.33 | 0 | 1 | 0 | 0.05 | 0.004 |
| 0 | 0.33 | 1 | 1 | 0 | 0.05 | 0.004 |
| 0 | 0.34 | 1 | 1 | 0 | 0.05 | 0.004 |
| 0 | 0.33 | 2 | 1 | 0 | 0.05 | 0.004 |
| 0 | 0.33 | 3 | 1 | 0 | 0.05 | 0.004 |
| 0 | 0.32 | 0 | 1 | 0 | 0.05 | 0.004 |
| 0 | 0.33 | 5 | 1 | 0.52 | 0.10 | 0.5773 |
| 0.3 | 0.33 | 10 | 1 | 0.52 | 0.10 | 0.5406 |
| 0.3 | 0.33 | 0 | 1 | 0.52 | 0.10 | 0.6364 |
| 0.3 | 0.33 | $W \geq 0$ | $Z \geq 0$ | $0 < \tau < 1$ | $\infty$ | 0.3300 |
| 0.3 | 0.33 | $W \geq 0$ | $Z \geq 0$ | 0.52 | 0 | 0.6640 |
| 0.3 | 0.33 | $W \geq 0$ | $Z \geq 0$ | 0.6629 | 0 | 0.7640 |
| 0.3 | 0.33 | 10 | 2 | 0.52 | 0.10 | 0.5026 |
| 0.3 | 0.33 | 0 | 2 | 0.52 | 0.10 | 0.6063 |
| 0.3 | 0.33 | 5 | 1 | 0.19 | 0.10 | 0.3332 |
| 0.3 | 0.33 | $W \geq 0$ | $Z \geq 0$ | 0.19 | 0 | 0.4330 |
| 0.3 | 0.33 | 0 | 1 | 0.19 | 0.10 | 0.4264 |

Figure 5 Critical values of the higher rate of income tax: retentions versus debt




Assumption g = 0.3, b = 0.33

A further refinement would be to introduce time lags in relation to the delay between the receipt of a net dividend or net interest and the payment of the excess of the higher rate tax on the gross equivalent over the basic rate tax deducted at source.

Let this be denoted $v$

Hence instead of $\left[1-h\right]$ in the analysis, we substitute:

$$1 - \left[b + \frac{h-b}{(1+k)^{v}}\right].$$

Therefore inequality (26) is modified to

$$\left[b + \frac{h-b}{(1+k)^{v}}\right] < 1 - \left[1 - \frac{g}{(1+k)^{\omega}}\right]\left[1 - b + \frac{b}{(1+k)^{0.25}} - \frac{T}{(1+k)^{z}}\right],$$

giving

$$h < (1+k)^{v}\left\{1 - \left[1 - \frac{g}{(1+k)^{\omega}}\right]\left[1 - b + \frac{b}{(1+k)^{0.25}} - \frac{T}{(1+k)^{z}}\right] - b\right\} + b \qquad (27).$$

Similarly inequality (17) would be modified to

$$h > (1+k)^{v}\left\{1 - \left[1 - \frac{g}{(1+k)^{\omega}}\right]\left[1 - b + \frac{b}{(1+k)^{0.25}} - \frac{b}{(1+k)^{y}}\right] - b\right\} + b \qquad (28).$$

These modifications create slightly higher critical values of h. For instance for k = 10 per cent and b = 30 per cent, then if the estimates of h (i.e. without these modififations) is 50 per cent, the true value of h is 50.98 per cent for $v$ = 0.5, and 52 per cent for $v$ = 1.

When the estimate of h has a smaller value the discrepency is reduced. For instance, for k = 10% and b = 30% then if the estimate of h is 40 per cent, the true value of h is 40.49 per cent for $\sigma = 0.5$, and 41 per cent for $\sigma = 1$. However as the discount rate increases the discrepancy is increased. For k = 20% and b = 30%, then if the estimate of h is 50 per cent, the true value of h is 51.9 per cent for $\sigma = 0.5$, and 54 per cent for $\sigma = 1$.

## 6.6. Taxes and market equilibrium

Let us modify the notation to simplify the general equilibrium analysis. Let

$$T' = 1 - \left[1-b + \frac{b}{(1+k)^{0.25}} - \frac{T}{(1+k)^{z}}\right] \qquad (29),$$

$$B' = 1 - \left[1-b + \frac{b}{(1+k)^{0.25}} - \frac{b}{(1+k)^{y}}\right] \qquad (30),$$

and

$$G' = 1 - \left[1- \frac{g}{(1+k)^{w}}\right] \qquad (31).$$

Furthermore let

$h^*_S$ = the marginal rate of income tax on dividends for the marginal shareholder,

and

$h^*_D$ = the marginal rate of income tax on interest for the marginal debentureholder.

Therefore in general equilibrium inequality (17) can be modified for the case of the irrelevance between retentions and new issues, when

$$h^{*}_{S} = 1 - \left[1-G'\right] \left[1-B'\right] \qquad (32);$$

and from inequality (26) there is an irrelevance between borrowing and retentions when

$$h^{*}_{D} = 1 - \left[1-G'\right] \left[1-T'\right] \qquad (33).$$

When the after-tax value of the dividend to the marginal shareholder equals the after-tax value of the interest to the marginal debentureholder then

$$\frac{D(1-h^{*}_{S})}{1-b} = I\,(1-h^{*}_{D}) \qquad (34),$$

or

$$\frac{D(1-h^{*}_{S})}{1-h^{*}_{D}} = I\,(1-b) \qquad (35).$$

The net cost to the company of the interest is

$$(1-b)\ I + \frac{bI}{(1+k)^{0.25}} - \frac{TI}{(1+k)^{z}}$$

$$= I\left[1-T'\right].$$

The net cost to the company of the dividend is

$$D\left[1 + \frac{b}{(1-b)(1+k)^{0.25}} - \frac{b}{(1-b)(1+k)^{y}}\right]$$

$$= \frac{D}{(1-b)}\left[1-b + \frac{b}{(1+k)^{0.25}} - \frac{b}{(1+k)^{y}}\right]$$

$$= \frac{D(1-B')}{(1-b)}.$$

These two costs are the same when

$$I\ (1-T') = \frac{D(1-B')}{1-b} \qquad (36);$$

or

$$I(1-b) = \frac{D(1-B')}{1-T'} \qquad (37).$$

From (35) this gives

$$\frac{D(1-h^*_S)}{1-h^*_D} = \frac{D(1-B')}{1-T'} \qquad (38)$$

Substituting for $h^*_S$ (from equation (32) ) into equation (38) gives

$$h^*_D = 1 - \left[1-G'\right] \left[1-T'\right] \qquad (39)$$

which is the same as equation (33). Hence in general equilibrium the marginal shareholder has a marginal tax rate on dividends of $h^*_S$, as determined by equation (32); and the marginal debentureholder has a marginal tax rate on interest of $h^*_D$ , as determined by equation (33). The result is an irrelevant capital structure for the firm in market equilibrium.

## 6.7 Conclusions

Although the model has been based on certainty, and hence the discount rate k was constant, the conditions for tax neutrality were the same as those of chapter 3, where risk was then explicitly considered, assuming that complexities in relation to tax time lags and multiple marginal tax rates are ignored. This is not to suggest that all the present results will necessarily exactly follow under conditions of risk. Where the tax payments, reliefs and allowances are highly certain then the foregoing analysis with constant k may be generally acceptable for most purposes. A simple adjustment for risk may be to attach subscripts, 1, 2, 3, ... , to the values of k when identifying time lags for different tax effects. Each risk-adjusted discount rate of $k_1$, $k_2$, $k_3$, ... , can therefore be regarded as different, and can in theory be derived, inter alia, according to the covariability of the respective tax cash flow with the rate of return on the efficient market portfolio. The purpose of this chapter however is to demonstrate that even in a certain world, in partial equilibrium (i) the optimal financing decision is a complex issue per se, and (ii) it is not independent of the investment decision. Since this can be achieved without resorting to a more complex risk-adjusted model, a present value framework with constant discount rates was adopted.

Since debenture interest is relieved at the corporate tax rate and ACT is setoff at the basic rate of income tax then there is a general preference for debentures instead of new issues. It was shown that although ACT setoff restrictions increase the cost of the dividends the debenture interest becomes more costly also, such that the general preference for debt finance is maintained. It is possible however for the tax lags, denoted z (re: interest relief) and y (re: ACT setoff), to be different. For instance, ACT may be carried back up to two years whereas unrelieved interest may only be carried forward. If heavy capital expenditure wipes out net taxable income in the current year then the debenture interest relief in present value terms will be reduced. By contrast the ACT on dividends paid now may be setoff immediately to the extent that the previous two years' setoff limits have not been fully utilized. Hence where z is sufficiently greater than y then $T/(1+k)^z$ is not necessarily greater than $b/(1+k)^y$. Furthermore with heavy capital allowances even large companies with high pre-tax accounting profits may pay small companies' rate, which will be applied as the marginal tax rate on debenture interest. The investment decision is therefore not independent of the financing decision.

If the marginal rate of corporation tax on debenture interest were the same as the basic rate of income tax then the borrowing versus retention decision could be modelled in almost the same way as the retention versus new issue decision. In this case the relationship between the critical higher rate of income tax and the capital gains tax lag would be identical to that depicted in figure 4 by substituting z for y. Hence for very low discount rates ($k \doteq o$) retentions would be preferable if the higher rate of income tax exceeded 51.3 per cent. For higher discount rates the critical value of h would be reduced and at the limit ($k = \infty$) it would approach the basic rate of income tax. It has been shown that under the Finance Act 1978 the marginal rate of corporate tax relief on debenture interest is 33.29 per cent if ACT setoff is restricted and net taxable income falls between the marginal relief limits. Since this approximates the basic rate then the above relationship would appear to hold in this instance.

Where ACT setoff is not restricted then, within this band of net taxable income, the marginal tax relief is 66.29 per cent, and for very low discount rates and short shareholding periods the critical value of h can be very high; and for k = o equals 76.4 per cent. Consequently for companies whose net taxable income falls within the marginal relief bands the borrowing or retention decision is very sensitive to whether ACT setoff is restricted or not. It may then be queried how ACT setoff restrictions may arise when the firm is retaining profits. This may result from optimal distribution decisions of previous years which no longer hold. Consider a higher rate of income tax at 40 per cent being representative, and let g = 0.3, b = 0.33, w = 5, y = 1, and k = 10 per cent. From table 40 it can be observed that for these parameters a dividend is preferable to a retention. Now if unforeseen expenditure is then incurred on acquiring plant and machinery (I assume here a disequilibrium situation in an otherwise perfect market) the net taxable income may be wiped out, resulting in a delayed setoff of ACT, if there were ACT restrictions in the previous two years and there is insufficient franked investment income. But even with a two year lag in obtaining the setoff (y = 2), from table 40 the critical value of h changes such that a retention is now preferable to a dividend. Hence investment decisions may affect financing decisions by changing the lag in obtaining the setoff for ACT. Furthermore we have already argued that the investment decisions may result in substantial changes to the marginal tax relief on debenture interest, upsetting previously optimal decision rules. A programming model to deal with inter-activities, caused by taxation, between investment and financing decisions, will be presented in chapter 7.

The earlier models presented in this chapter have been based on homogeneous rates of personal taxation on investment income. It was shown that the conclusions are the same as those of chapter 3 ignoring tax time lags. Similarly, in general equilibrium with heterogeneous tax rates both the one period model of chapter three and the multi-period model here have proved an irrelevant capital structure after tax. It was stated that the firm's financing decisions may change over time due to interactivities between investment and financing decisions. In particular the capital allowance effects fromexpenditure may alter the relative attractiveness of new issues, retentions or debt. The result is that the firm may now attract a different clientele of investors. During the period of temporary disequilibrium the value of the firm may not be indifferent with regard to capital structure.

# CHAPTER 7

## The use of mathematical programming in corporate financial planning under the imputation tax system

## 7.1. NOTATION (for chapter 7 only)

$a_{Bj}$ = net cash outflow of project B at period j (exogenous).

$A^{CT}_{p}$ = total ACT on dividends paid in the accounting period ended at time p (endogenous).

$A^{CTBA}_{p}$ = a variable to help determine $A^{CTB1A}_{p}$ (see model) (endogenous).

$A^{CTB1A}_{p}$ = ACT considered for being carried back at least one year (endogenous).

$A^{CTB2A}_{p}$ = ACT considered for being carried back at least two years (endogenous).

$A^{CTF}_{p}$ = ACT carried forward from period p to p+4 (endogenous).

$A^{CTF1A}_{p}$ = the first of two components constituting ACT carried forward from period p to p+4 (endogenous).

$A^{CTF2A}_{p}$ = the second of two components constituting ACT carried forward from period p to p+4 (endogenous).

$A_p^{CTFB}$ = additional ACT carried forward from period p-4 to period p as a consequence of the carry-back of capital allowances from time p (endogenous).

$A_p^{CTFC}$ = additional ACT carried forward from period p-8 to period p-4 as a consequence of the carry-back of capital allowances from time p (endogenous).

$A_p^{CTFD}$ = additional ACT carried forward from period p-12 to period p-8 as a consequence of the carry-back of capital allowances from time p (endogenous).

$b_j$ = basic rate of income tax at time j (parameter).

B = refers to project B (subscript).

$B'$ = total number of projects (parameter).

$C_p^{AB1}$ = capital allowances considered for being carried back at least one year (endogenous).

$C_p^{AB2}$ = capital allowances considered for being carried back at least two years (endogenous).

$C_p^{AB3}$ = capital allowances considered for being carried back three years (endogenous).

$C_p^{AF}$ = capital allowances carried forward from period p (endogenous).

$C_{Bj}^{AO}$ = the contribution of the Bth project (if accepted) to the capital allowances on items other than plant and machinery arising in period j (exogenous).

$C_{Bj}^{AP}$ = the contribution of the Bth project (if accepted) to the capital allowances on plant and machinery arising in period j (exogenous).

$C_j$ = closing balance of cash in period j (endogenous).

$C_p^{ABIU}$ = a carryback figure ignoring the constraint that it must only relate to plant and machinery (endogenous).

$C^V$ = cash flows extending beyond the horizon of projects,discounted to period n+1 (exogenous).

$D_j$ = (net) dividend paid at time j (endogenous).

$\Delta_{Bj}$ = the change in accounting depreciation in period j of those fixed assets relating to production overheads, resulting from project B's acceptance (exogenous).

$E_{Bj}$ = the contribution of the Bth project (if accepted) to the expenses for period j based on accrual accounting principles (exogenous).

$E^Q_j$ = new equity issue in period j (endogenous).

$E^{Q\ limit}_j$ = limit on amount of new equity issues in period j (parameter).

f = the proportion of the accounting period ending at time p that is covered by the latter tax year involved (parameter).

$g'$ = a tax lag equal to the gap between the date of payment of tax and the end of the accounting period on which the tax is based (parameter).

$G$ = the time gap between the date of payment of the dividend and the ACT thereon (normally 3 months) (parameter).

$I_{Bj}$ = the contribution of the Bth project (if accepted) to the investment outlay at time j (exogenous).

$I_p^{NTBA}$ = interest paid brought forward under section 177(8) ICTA 1970 from period p to p+4 (under a quarterly model) (endogenous).

$I_p^{NTBB}$ = interest paid brought forward under section 177(8) from period p-4 to p, revised on account of the carry-back of capital allowances from time p (endogenous).

$I_p^{NTBC}$ = interest paid brought forward under section 177(8) from period p-8 to p-4, revised on account of the carry-back of capital allowances from time p (endogenous).

$I^{NTBD}_{p}$ = interest paid brought forward under section 177(8) from period p-12 to p-8, revised on account of the carry-back of capital allowances from time p (endogenous)

$I^{NTBQ}_{j}$ = unrelieved income tax on interest brought forward under the quarterly accounting system for income tax from period j to j+1 (endogenous)

$j$ = refers to period number (subscript)

$J$ = refers to time subscript in summation expressions (subscript)

$K_{j}$ = cash flows at period j arising from existing activities (exogenous)

$k_{L}$ = short-term pre-tax lending rate (exogenous)

$k_{WL}$ = long-term pre-tax borrowing rate (exogenous)

$k_{WS}$ = short-term pre-tax borrowing rate (exogenous)

$L_{j}$ = amount of short-term lending incurred at period j (endogenous)

$L^{F}_{p}$ = losses carried forward from period p to p+4 (endogenous).

$L^{HS}_{p}$ = left-hand side of an equation (in my workings the sides of the equations are occasionally switched!!) (endogenous).

$L^{O}_{p}$ = a variable to help determine $L^{F}_{p}$ (see model) (endogenous).

$M$ = a million or such constant high enough to ensure that any constraint containing M is satisfied in theoptimal solution (parameter).

$M^{CTA}_{p}$ = mainstream corporation tax for the accounting period ended at time p (endogenous).

$M^{CT1A}_{p}$ = mainstream corporation tax for the first quarter of the accounting period ended at time p (endogenous).

$M^{CT2A}_{p}$ = mainstream corporation tax for the remaining quarters of the accounting period ended at time p (endogenous).

$M^{CTB}_{p}$ = mainstream corporation tax for the accounting period ended at time p-4 , adjusting for the carry-back of capital allowances from time p (endogenous).

$M^{CT1B}_{p}$ = mainstream corporation tax for the first quarter of the accounting period ended at time p-4, adjusting for the carry-back of capital allowances from time p (endogenous).

$M^{CT2B}_{p}$ = mainstream corporation tax for the remaining quarters of the accounting period ended at time p-4, adjusting for the carry-back of capital allowances from time p (endogenous).

$M^{CTC}_{p}$ = mainstream corporation tax for the accounting period ended at time p-8, adjusting for the carry-back of capital allowances from time p (endogenous).

$M^{CT1C}_{p}$ = mainstream corporation tax for the first quarter of the accounting period ended at time p-8, adjusting for the carry-back of capital allowances from time p (endogenous).

$M_p^{CT2C}$ = mainstream corporation tax for the remaining quarters of the accounting period ended at time p-8, adjusting for the carry-back of capital allowances from time p (endogenous).

$M_p^{CTD}$ = mainstream corporation tax for the accounting period ended at time p-12, adjusting for the carry-back of capital allowances from time p (endogenous).

$M_p^{CT1D}$ = mainstream corporation tax for the first quarter of the accounting period ended at time p-12, adjusting for the carry-back of capital allowances from time p (endogenous).

$M_p^{CT2D}$ = mainstream corporation tax for the remaining quarters of the accounting period ended at time p-12, adjusting for the carry-back of capital allowances from time p (endogenous).

$M_p^{F}$ = marginal fraction at time p (parameter).

$M_p^{RL}$ = marginal relieflower limit at time p (parameter).

$M^{RU}_{p}$ = marginal relief upper limit at time p (parameter).

n = project horizon date (parameter).

$N^{MCTA}_{p}$ = net mainstream corporation tax based on net taxable income for the accounting period ended at time p (endogenous).

$N^{MCTB}_{p}$ = net mainstream corporation tax based on net taxable income for the accounting period ended at time p-4, adjusting for the carry-back of capital allowances from time p (endogenous).

$N^{MCTC}_{p}$ = net mainstream corporation tax based on net taxable income for the accounting period ended at time p-8, adjusting for the carry-back of capitalallowances from time p (endogenous).

$N^{MCTD}_{p}$ = net mainstream corporation tax based on net taxable income for the accounting period ended at time p-12, adjusting for the carry-back of capital allowances from time p (endogenous).

$N^{TIE}_{Bj}$ = net taxable income attributable to project B (if accepted) in period j excluding capital allowances and stock relief (endogenous).

$N^{TIA}_{p}$ = net taxable income in period p (endogenous).

$N^{TIB}_{p}$ = net taxable income in period p-4 adjusting for the carry-back of capital allowances from period p (endogenous).

$N^{TIC}_{p}$ = net taxable income in period p-8, adjusting for the carry-back of aapital allowances from period p (endogenous).

$N^{TID}_{p}$ = net taxable income in period p-12, adjusting for the carry-back of capital allowances from period p (endogenous).

$\phi 1, \phi 2$ etc. = formulae as per appendix 1.

p = refers to accounting period number (subscript).

$p^{1}$ = refers to accounting period in summation expressions (subscript).

$P_{Bj}$ = the contribution of the Bth project (if accepted) to purchases of materials in period j based on accrual accounting principles (exogenous).

$q$ = number of typical projects for inclusion at horizon (parameter).

$r$ = cost of equity capital (exogenous).

$R_{Bj}$ = the contribution of the Bth project (if accepted) to the revenues in period j based on accrual accounting principles (exogenous).

$R^{HS}_{p}$ = right-hand side of an equation (endogenous).

$S^{ECS}_{Bj}$ = schedule D case 1 profits attributable to project B (if accepted) in period j excluding capital allowances and stock relief (endogenous).

$S^{ESA}_{p}$ = schedule D case 1 profits for the accounting period ended at time p excluding stock relief (endogenous).

$S_p^{ESB}$ = schedule D case 1 profits for the accounting period ended at time p-4 excluding stock relief, adjusted for the carry-back of capital allowances from period p (endogenous).

$S_p^{ESC}$ = schedule D case 1 profits for the accounting period ended at time p-8 excluding stock relief, adjusted for the carry-back of capital allowances from period p (endogenous).

$S_p^{ESD}$ = schedule D case 1 profits for the accounting period ended at time p-12 excluding stock relief, adjusted for the carry-back of capital allowances from period p (endogenous).

$S_p^{HDA}$ = schedule D case 1 profits for the accounting period ended at time p (endogenous).

$S_p^{HDB}$ = schedule D case 1 profits for the accounting period ended at time p-4, adjusted for the carry-back of capital allowances from period p (endogenous).

$S_p^{HDC}$ = schedule D case 1 profits for the accounting period ended at time p-8, adjusted for the carry-back of capital allowances from period p (endogenous).

$S_p^{HDD}$ = schedule D case 1 profits for the accounting period ended at time p-12, adjusted for the carry-back of capital allowances from period p (endogenous).

$S_p^{RA}$ = stock increase for the accounting period ended at time p less 15 per cent of schedule D case 1 profits after capital allowances (if $S_p^{RA}$ is positive; otherwise $S_p^{RA}$ is zero) (endogenous).

$S_p^{RB}$ = revised value of $S_{p-4}^{RA}$ to adjust for the carry-back of capital allowances from period p (endogenous).

$S_p^{RC}$ = revised value of $S_{p-8}^{RA}$ to adjust for the carry-back of capital allowances from period p (endogenous).

$S_p^{RD}$ = revised value of $S_{p-12}^{RA}$ to adjust for the carry-back of capital allowances from period p (endogenous).

$T_j$ = the corporate tax rate applied to net taxable income at time j (parameter).

$\theta_i$ = a zero-one variable (endogenous).

$u_j$ = the proportion of the volume of unsold goods to the volume of production during period j (parameter).

$W_j^L$ = amount of long run borrowing at period j (endogenous).

$W_j^{L\ limit}$ = limit on amount of long run borrowing incurred in period j (parameter).

$W_j^S$ = amount of short-run borrowing incurred at period j (endogenous).

$W_j^{S\ limit}$ = limit on amount of short-term borrowing incurred in period j (parameter).

$x_B$ = fractional acceptance of project B ($0 \leq x_B \leq 1$) (endogenous).

$z_1$ = the proportion that the closing balance of accounts receivable in period j bears to the revenues of period j based on accrual accounting principles (parameter) *

$z_2$ = the proportion that the closing balance of accounts payable in period j bears to the purchases of period j based on accrual accounting principles (parameter)

$z_3$ = the proportion that the closing balance of expenses owing in period j bears to the expenses of period j based on accrual accounting principles (parameter)

$z_4$ = the proportion that the closing stock of raw materials in period j bears to the purchases of materials in period j based on accrual accounting principles (parameter)

$z_5$ = the proportion that the closing stock of finished goods in period j bears to the cost of production during period j (parameter)

$z_6$ = the stock appreciation relief proportion, currently at 15 per cent (parameter)

* footnote: e.g. in table 19 $z_1 = 25000/200000 = 0.125$

## 7.2. Introduction

In earlier chapters assumptions have been made regarding perfect frictionless capital markets in which investment and financing decisions can be determined independently. The only imperfection that has been introduced is that the tax system may create potential incentives or disincentives for the firm to invest and that the choice of financing mix may also be distorted by taxation. It has been shown that under a corporate cash flow tax system with full instantaneous relief for losses and constant tax rates, the decision as to whether to proceed with a capital investment proposal is not affected by taxation when the method of appraisal is based on discounted cash flow analysis. Several factors may in practice create an environment in which tax neutrality no longer holds. The carry-forward of capital allowances may result in a present value of capital allowances as a proportion of cost being less than 100 per cent. It has been shown that this may result in a potential disincentive to invest. Yet when several divisions of a firm are appraising projects independently some may be unaware of the extent to which the firm has a sufficiency of profits against which to offset capital allowances. One division may assume that capital allowances have to be carried forward on the basis that it is generating a low level of current taxable profits. Since the present value of capital allowances as a proportion of cost for this division may appear to be less than 100 per cent this division may regard its projects to be unacceptable, yet they could be attractive when account is taken of profits from other divisions. It follows that all combinations of projects be jointly considered, a view supported by Fawthrop (1971), Grundy and Burns (1979), Buckley (1975), Rickwood and Groves (1979), and Berry and Dyson (1979). Further reasons for considering combinations of projects include claims for stock relief. For one division may be running down stocks to such an extent that the firm as a whole may not be able to claim relief for the appreciation of trading stocks. A smaller division that is considering projects, which require an investment in working capital, may have to pay tax not only on its net operating cash flow but also on the periodic investment in net working capital (see chapter 5 ). Since a tax system based on cash flows can have a neutral effect on the investment decision, then a tax based on working capital in addition to operating cash flows can be a disincentive to invest. As the incremental tax effects arising from projects depend on the tax position of the firm as a whole then all combinations of projects, including those representing the existing activities of the firm, need to be considered together. Furthermore, it is the firm's overall tax position which determines the extent to which Advance Corporation Tax is restricted. Investments in plant and machinery may reduce net taxable income to such an extent that part of the tax benefits for the firm, from the setoff of Advance Corporation Tax against its mainstream corporation tax liability, may be reduced in present value terms. It has been shown in chapter 5 that this may reduce the marginal corporate tax rate applied to the benefit of the capital allowance on plant and machinery. But it has been argued in chapter 6 that the delay in the setoff of ACT affects the after-tax cost to the company of paying a dividend and thus may distort the optimal financing mix of the firm.

The reduction in net taxable income as a result of heavy capital allowances may distort too the marginal tax rate at which relief is obtained on debenture interest and where capital allowances fully wipe out taxable profits, the reduction in the mainstream corporation tax liability may be delayed for many years. In present value terms the after-tax cost of interest may be increased substantially. Hence not only do projects need to be appraised simultaneously but also the total investment programme and the firm's financing decision policies need to be analysedtogether.

As a means of achieving this end a mathematical programming model may be used. The mathematical programming methodology has already been applied to corporate financial planning in particular by Weingartner (1974), Chambers (1967 and 1971), Carsberg (1976), Wilkes (1977), Ashton (1978), and Bhaskar (1978). Yet, the provisions of the tax system have either been omitted or grossly over-simplified. Note that although the work by Ashton has suggested that accept or reject decisions for capital projects may be performed fairly efficiently without using complex mathematical programming models, the tax interactivities of the type discussed in this chapter were not considered. A notable paper that has considered tax interactivities between projects has been published by Berry and Dyson(1979) and represents a parallel study which confirms the main argument in this chapter that the tax system can be programmed in an optimisation model. They demonstrate that because of the tax system, investment projects under consideration need to be appraised simultaneously and may be solved by a mathematical programming model normally used for cases of capital rationing. In the present chapter, the situation of potential rationing of capital is included, not because in order to incorporate the tax provisions the firm needs to be in a capital rationing situation, but partly since the resultant model may be of more general interest and capital rationing constraints after tax may, in any case, be excluded if desired. More significantly, the existence of borrowing and lending opportunities is important firstly through the tax deductibility of interest payments, which may be deferred under section 177(8), perhaps as a result of capital allowances on new investments wiping out profits against which to offset the interest, and secondly because of the tax treatment of interest received. Also, in contrast to the Berry and Dyson paper, which incorporated net present values of projects, in this chapter the objective function is based on a discounted dividends approach, which provides an important link with the constraints pertaining to the tax treatment of Advance Corporation Tax. The model will be an extension to that by Bhaskar (1978), whose work will be briefly reviewed shortly.

## 7.3. Model complexity and the purpose and usefulness of the model

A significant characteristic of the model presented in this chapter is the use of integer values for some of the variables. These arise directly from constraints to accommodate the tax rules. In principle one way of finding the optimal solution is by the method of branch and bound (Wagner, 1975), although mixed integer programming problems can be considerably more complex to solve than a linear programming model. Furthermore the tax system is of such complexity that the resultant model is a lengthy one. This is not to say that all constraints should be programmed for a particular firm on every occasion. For instance, if it is obvious that the firm has taxable profits from other existing projects that are so large as to absorb the capital allowances on new projects being considered, and hence the capital allowances carry-back provisions and carry-forward provisions are not needed, then clearly the appropriate constraints for capital allowances can be readily omitted. Similarly, if the periodic investment in trading stocks will always be less than 15% of taxable profits after capital allowances, then appropriate sections of the full model are not required. Also, different firms have different tax profiles with respect to the size of taxable profits compared with the marginal relief limits, and ACT carried forward of backwards. Again, this may permit some reduction in the size of the model under appropriate circumstances.

In an attempt to be realistic , quarterly periods are used to accommodate the quarterly payments for Advance Corporation Tax and income tax deducted at source on debenture interest. For many firms this refinement to the programme may be too costly in running the model compared with the benefits over a smaller model using annual time periods. Nevertheless it would be fairly straightforward to condense the model to ignore the quarterly refinements.

A sceptic may query the purpose of presenting a model which for practical purposes may be too time-consuming to solve. The principle of parsimony, noted by Bhaskar (1978 p.160) may long since have been violated: "A liberal interpretation would define a parsimonious model as being, ceteris paribus, sufficiently simple for the manager to understand, yet sufficiently complicated to embody the most important relationships holding in the real world. If a model becomes too complicated, practical experience would suggest that the model loses its usefulness because the manager cannot have an overall grasp of the model. On the other hand, if the model is too simple, it may omit important financial influences which are found in reality".

But the purpose of this chapter is to explain that the tax complexities <u>can</u> be incorporated into a mathematical programming model that seeks an optimal solution to the selection of capital projects and the determination of an optimal financing mix. The main problem in presenting a general model is that many firms have different tax profiles and so parts of the general model would be redundant for the particular firm. The objective of the model presented here, however, is to demonstrate to financial model builders how to include in the mathematical program, constraints to accommodate the complex tax effects of investment and financing decisions of the firm. The role is more of an educational one and the chapter is addressed to experts in financial modelling who may require some guidelines on the area of taxation. Indeed since the tax system changes rapidly parts of the model as they stand would soon become outdated. . . But once the model builder appreciates the nature of the programming constraints that are described here it should not be too difficult to accommodate anticipated, and actual, changes in the tax legislation. In this way as a guide to taxation and financial model-building, within an optimisation model, this chapter attempts to fill a serious gap in the literature. The model builder may in practice seek a model which, in terms of its efficiency in the determination of an optimal solution, lies somewhere between the more simplistic models of chapters 5 and 6 and the present model which is theoretically more sensitive to the reality of the tax environment, yet operationally very difficult to solve.

In any event a model needs to be adapted to its particular situation and it would be unwise to attempt to apply the model in this chapter without any adaptations. It is left to the discretion of the model-builder to simplify the present general model for the more critical aspects of the tax position of the firm in question.

The resultant solution may be suboptimal, when programming costs are excluded, yet the full model which ought to provide an optimal solution may not warrant the programming and related costs.

7.4 The pre-tax corporate financial planning model by Bhaskar

The main features of the linear programming models used in corporate financial planning are conveniently summarised by Bhaskar (1978) as follows (with some changes in notation)

Maximise:

$$\sum_{j=o}^{n} \frac{D_j - E_j^Q}{\prod_{J=1}^{j} (1+r_J)} + \frac{1}{\prod_{J=1}^{n+1} (1+r_J)} \left[ \sum_{B=1}^{B'+q} C^V x_B \right.$$

$$\left. + (1+k_L)\, L_n - (1 + k_{WS})\, W_n^S - (1 + k_{WL}) \sum_{j=o}^{n} W_j^L \right] ,$$

which represents [discounted (dividends less new equity issues] plus [discounted horizon values of (projects) plus (lending and interest accruing at horizon) less (short-term borrowing and interest owing at horizon) less (principal of long-term borrowed funds and interest at horizon)] .

Subject to

a) $j = o$ - the first capital constraint of the model

$$\sum_{B=1}^{B'} a_{Bo}\, x_B + L_o - W_o^S - W_o^L + D_o - E_o^Q = K_o \quad .$$

b) $j \neq o$ - typical capital constraint of the model

$$j = 1, 2, \ldots n \quad .$$

$$\sum_{B=1}^{B'} a_{Bj} x_B + L_j - (1 + k_L)L_{j-1} - W_j^S + (1 + k_{WS})W_{j-1}^S$$

$$- W_j^L + k_{WL} \sum_{J=O}^{j-1} W_J^L + D_j - E_j^Q = K_j \quad ,$$

representing (project net cash outflows) plus (funds lent) minus (interest plus principal on funds lent in previous period) minus (new short-term borrowing) plus (interest and principal of old short-term borrowing ~~minus~~ (new borrowing) plus (interest on existing borrowing) plus (dividends) minus (new equity issues) equals (capital from existing operations).

c) Upper bound project limits (for all B)

$$x_B \leqslant 1 \quad ,$$

d) Financing limits, for j = o, ... n .

$$W_j^S \leqslant W_j^{S\ \text{limit}} .$$

$$W_j^L \leqslant W_j^{L\ \text{limit}} .$$

$$E_j^Q \leqslant E_j^{Q\ \text{limit}} .$$

e) Non-negativity constraints .

$$x_B \geqslant o \quad \text{for all B} .$$

similarly $D_j$ , $E_j^Q$, $W_j^S$,

$W_j^L$, $L_j$, for all j .

## 7.5 Modifications to the model for personal and corporate taxation

Bhaskar notes that

" one omission in the model is corporate taxation ... However, there is a problem to the incorporation of tax into a general model. First, tax is often peculiar to the organisation arrangement of a firm and the nature of the investment and financing decision. Second, it is difficult to provide an accurate yet precise functional form for corporate taxation. However, it is possible to include corporate taxation in a simplistic and rather unsatisfactory way ... Because this method of including corporate taxation makes a number of assumptions about the sufficiency of profits and the extent of the tax liability ... For practical use the omission of corporate taxation is serious. Assuming that taxable profits are sufficient always to pay tax, then corporate taxation can be added as indicated earlier by defining a project to include all differential cash flows. This implies that in periods of net cash outflows for a project an allowance for possible tax relief would be added to the cash flows. In periods of net cash inflows an additional amount must be added to represent tax payable. Similarly, net interest may be shown as net of corporate taxation relief. In the UK, ACT payable on issued dividends and mainstream corporation tax is more difficult to incorporate. One method is to add the relevant amount of ACT to dividends. An associated tax allowance may then be made in the period in which the associated mainstream tax liability falls due ... However, this method for the inclusion of corporation tax does make a number of assumptions as to the sufficiency of profits and the extent of the existing tax liability."

Bhaskar's model will now be extended to incorporate the main features of the imputation tax system.

A summary of the changes to be made is outlined below:

In the objective function:

(i) the discounted dividends will be adjusted for personal income tax

(ii) the discounted horizon valuation will include the post-horizon net mainstream corporation tax payments and the ACT after the horizon on dividends paid before the horizon.

In the liquidity constraints:

(iii) the net cash outflows of each project will be broken down in terms of accounting flows and working capital constituents to distinguish between the cash flow base for discounting purposes and the accrual accounting base for tax purposes

(iv) advance corporation tax payments will be introduced

(v) adjustments will be made to interest received and paid for income tax deducted at source

(vi) net mainstream corporation tax payments will be dealt with. In particular calculations will be required for

a) Schedule D Case I profits excluding capital allowances and stock appreciation relief

b) Schedule D Case I profits after capital allowances and losses carried forward and backwards

c) Schedule D Case I profits after stock appreciation relief

d) Schedule D Case III interest received

e) Charges on income and the carry-forward provisions

f) The restriction of ACT setoff in the determination of net mainstream corporation tax

g) The relief for income tax on any unfranked receipts.

vii) Quarterly payments for income tax on net unfranked payments will be introduced.

viii) ACT repayments arising from the carry-back provisions will be considered.

7.6. Objective function

From the full model, which is presented in Appendix 2, it can be seen that the first term in the objective function, which relates to dividends, is adjusted for personal tax by multiplying by $(1-h_j)/(1-b_j)$. By dividing by $(1-b_j)$ a net dividend of 70p for instance is grossed up to £1 where the basic rate b at time j, $b_j = 30\%$. The expression of minus $h_j$ reduces the gross dividend to a net dividend after higher rate tax of h at time j. The cost of equity capital r is after personal tax. Two additional expressions are added to Bhaskar's objective function. The first amendment is to insert the net mainstream corporation tax payments, $N_j^{MCT}$, for post-horizon periods $j = n+1, \ldots, \infty$, where n is the horizon date. The other adjustment is for Advance Corporation Tax after the horizon on dividends paid before the horizon. Apart from the first two terms in the objective function the formula represents an horizon valuation. It is implicitly assumed that the cash flows after corporation tax arising after the horizon date, which are contained in the "{ }" brackets, are paid out as dividends and so the resultant figure is multiplied by $(1-h_j)/(1-b_j)$. Problems of restricted setoff of ACT after the horizon are ignored and the lag between the time for the payment of the ACT based on dividends paid after the horizon, and the setoff against the mainstream corporation tax liability is treated as nil .

## 7.7. Liquidity constraints

Constraint (1) corresponds with Bhaskar's capital constraints. The contents of the first squared bracket in constraint (1) relate to the pre-tax net cash inflow of project B at period j , and is the same as Bhaskar's $-a_{Bj}$. The constituent parts of the cash flow are, however, decomposed to take account of debtors, creditors, accruals and prepayments, since tax is to some extent based on profit rather than cash flow ( Lawson and Stark, 1975).* The first constraint shows the funds generated from existing operations, $K_j$, after payments of net mainstream corporation tax of $N^{MCT}_{j-g'}$ after tax rebates of

$(N^{MCTD}_{j-g'} - N^{MCTC}_{j-4-g'})$, $(N^{MCTC}_{j-g'} - N^{MCTB}_{j-4-g'})$, and

$(N^{MCTB}_{j-g'} - N^{MCTA}_{j-4-g'})$ arising from the carry-back of

capital allowances ; after dividends of $D_j$, less equity issues of $E^O_j$; after advance corporation tax of $D_{j-G}(b_{j-G}/(1-b_{j-G}))$; after lending of $L_j$ , less amounts

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* footnote : e.g. see table 19.

receivable from last period's lending of $L_{j-1}$, with interest of $k_L L_{j-1}$, after income tax deducted at source at the rate $b_j$; after appropriate after-tax adjustments for short-run borrowing of $W^S_j$ and long-run borrowing of $W^L_j$; and finally after income tax payments arising from interest paid or received. Note that the final adjustment for income tax is given by the term including $\theta_{p15}$. The $\theta$ variables require special consideration and will be discussed later.

## 7.8. Schedule D Case 1 profits before capital allowances

The Schedule D Case 1 profits attributable to project B (if accepted) in period j, excluding capital allowances and stock appreciation relief is denoted by $S^{ECS}_{Bj}$ and is derived from constraint (2) which is an equality. The terms are similar to the net operating cash flow of $-a_{Bj}$ except that capital expenditure of $I_{Bj}$ is excluded and account is taken of the periodic change in trading stock so that operating cash flows can be converted into profit figures. The model assumes that the closing stock of raw materials in period j is a constant proportion $z_4$ in relation to the purchases of materials in period j, $P_{Bj}$, based on accrual accounting principles.* Similarly, the closing stock of finished

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*

footnote : e.g. where there is no finished stock, from table 18 : $z_4 = 43000/130000 \doteq 0.33$.

goods in period j, bears a constant proportion $z_5$, of the cost of production during period j. Work-in-progress is treated as nil at the beginning and end of each time period.

7.9. A quarterly model and a December year-end

Since there is a quarterly accounting system for both ACT payments and income tax on unfranked payments it is convenient to assume that the standard period of time j used in the model relates to one quarter. If it is assumed that the company's year-end is 31st December then j = 1 might conveniently represent the March quarter for the first planning period. Clearly, alternative year-end dates may be modelled following procedures not too dissimilar to the ones that are about to be explained. Since there are different tax rules for different periods there are various groups of constraints relating to different time periods: sections (a) for $j \neq 2,6,10,14$ ...; (b) for $j = 4,8,12,16$ ...; (c) for $j = 1,5,$...; (d) for $j \neq 1,5$..; and (e) for $j = 2,6,10,14$ ... . With a December year-end, the December quarter relates to $j = 4,8,12,16$ ... . The group of constraints under (b) which determine the net mainstream corporation tax payable, following calendar years, are therefore based on these particular values of j .

7.10. Capital allowances

To the extent that capital allowances exceed Schedule D Case 1 profits (before capital allowances) from the same trade, the balance may be carried back three years under section 177(3A), Income and Corporation Taxes Act 1970 if it relates to a claim for 100 per cent capital allowances on plant and machinery. If $L_{p}^{HS1}$ of equation (9) is negative then it shows that there may be some capital allowances considered for being carried back at least one year. This depends on whether (1) the capital allowances brought forward from period p-4, $C_{p-4}^{AF}$, i.e. from a year ago under a quarterly model; plus (2) the capital allowances on plant and machinery, $C_{\beta J}^{AP}$, and on items other than on plant and machinery, $C_{\beta J}^{AO}$, for the four quarters of the current year J = p-3 to p for all projects B; plus (3) losses brought forward from period p-4, $L_{p-4}^{F}$ exceed (4) $S_{\beta J}^{ECS}$, denoting Schedule D Case 1 profits excluding capital allowances and stock relief .

Consider the constraints (10) to (14) , reproduced below:

$$C_{p}^{ABIU} = L_{p}^{HS1}\,(\theta_{p1} - 1) \qquad (10),$$

$$L_{p}^{HS1} - M\theta_{p1} \leqslant 0 \qquad (11),$$

$$L_{p}^{HS1}.\theta_{p1} \geqslant 0 \qquad (12),$$

$$1 \geqslant \theta_{p1} \geqslant 0 \qquad (13),$$

$$\theta_{p1} \text{ integral} \qquad (14) .$$

M represents a million or a figure so high that any constraint containing M is satisfied in the optimal solution. From (13) and (14), $\theta_{pl}$ is either zero or one. The question is whether $L_p^{HS1}$ is negative . Consider three values for $L_p^{HS1}$ , (i) plus £1,000 , (ii) zero and (iii) minus £1,000.

(i) <u>£1,000</u>

From constraint (11) for $L_p^{HS1} - M\theta_{pl}$ to be negative, $M\theta_{pl}$ must be greater than 1000 . Since $\theta_{pl}$ is either zero or one it must therefore be one. If it takes on the value of one then (12) reads $1000 \geqslant 0$ which is true. From equation (10) $C_p^{ABIU} = 1000(1-1) = 0$. But $C_p^{ABIU}$ represents the carry-back figure , ignoring for the moment the constraint that it must only ralate to plant and machinery , and since this value is zero for $L_p^{HS1}$ positive, it may be correctly inferred that there are no carry-back considerations when there are sufficient profits against which to offset capital allowances.

(ii) <u>zero</u>

From constraint (11) for $\left[L_p^{HS1} - M\theta_{pl}\right]$, to be negative, or zero, $\theta_{pl}$ is either zero or one. . Constraint (12) now reads $0 \geqslant 0$ which is true, and equation (10) becomes $C_p^{ABIU} = 0$. The rationale is that when capital allowances exactly cancel out taxable profits, before capital allowances, the carry-back figure of allowances is zero.

(iii) - £1,000

From constraint (11) $L_p^{HS1} - M\theta_{pl}$ is negative since the first term is negative and the second term is minus M (if $\theta_{pl} = 1$) or zero (if $\theta_{pl} = 0$). The constraint is therefore satisfied for both values of $\theta_{pl}$. But from constraint (12), if $\theta_{pl}$ is one then the constraint reads $-1000 \geqslant 0$ which is untrue, and if $\theta_{pl} = 0$, we obtain $0 \geqslant 0$ which is correct. Hence $\theta_{pl} = 0$. By referring back to constraint(10):

$$C_p^{ABIU} = -1000(0-1) = 1000,$$

which means that the maximum amount considered for carry-back is £1000.

Constraints (15) to (20) ensure that only that part relating to capital allowances on plant and machinery can be carried back. Since the carry-back is allowed for three years, the procedure is repeated three more times via constraints (21) to (30), (31) to (40), and (41) to (50). In particular $C_p^{AF}$ is determined, which represents any remaining capital allowances on plant and machinery that have not been offset against the profits of the three preceding years and therefore are carried forward from period p to p+4 under the quarterly model. To the extent that some losses may be created other than by claiming 100 per cent capital allowances, further constraints are required ( (50i) to (50vi) ).

## 7.11. Stock appreciation relief

Relief for the appreciation of trading stocks may be claimed to the extent that there are sufficient profits against which to offset the relief. Constraints (51a) to (56a) determine whether there is a positive value for $S_p^{ESA}$, denoting Schedule D Case 1 profits for the accounting period ending at time p excluding stock appreciation relief. Stock relief is claimed if the periodic increase in stocks exceeds $z_6$ times Schedule D Case 1 profits after capital allowances but before stock relief [(57) to (67a)]. Stock clawback has been ignored. Indeed it is anticipated that clawback will disappear from the legislation provided the company continues to trade . Where capital allowances are carried back one year, stock relief is recomputed for the preceding year ( (51b) to (67b)), for the second preceding year ( (51c) to (67c) ), and for the third preceding year ( (51d) to (67d) ).

7.12. Interest and the quarterly accounting for income tax

Income tax is deducted at source on interest paid and offset against income tax on interest received on a quarterly basis. At the end of the accounting year any unrelieved tax on net interest received may be offset against the charge for the corporation tax liability. In appendix 2 the group (c) constraints (133) to (137) begin a new calendar year and therefore exclude $I^{NTBQ}_{j-1}$, denoting unrelieved income tax on interest brought forward under the quarterly accounting system from period j-1 to period j, unlike the group (d) constraints (138) to (142).

Consider constraint (133) reproduced below:

$$I^{NTBQ}_{j} = ( b_{j}(k_{L} L_{j-1} - k_{WS} W^{S}_{j-1} - k_{WL} \sum_{J=0}^{j-1} W^{L}_{J} ) )\theta_{p12} \quad .$$

If $\theta_{p12}$ equals one (determined from constraints (134) to (137) ), indicating that there is some interest to be brought forward from the first quarter of an accounting period, then the interest is calculated according to the present basic rate of income tax $b_j$. This rate is applied to (i) the short-term pre-tax lending rate of $k_L$ on one-period lending $L_{j-1}$ from period j-1; less (ii) the short-term pre-tax borrowing rate of $k_{WS}$ on one-period borrowing $W^S_{j-1}$ from period j-1; less (iii) the long-term pre-tax borrowing rate of $k_{WL}$ on the cumulative amount of long-run borrowing $W^L_J$ from time 0 up to the preceding period j-1. Hence for j = 2,6,10,14 ..., there are income tax payments in the liquidity constraints (143) arising from net interest paid in periods j= 1,5,9,13 ... . This is denoted by

$$\left[-k_L L_{j-2} + k_{WS} W^S_{j-2} + k_{WL} \sum_{J=0}^{j-2} W^L_J\right] b_{j-1} \quad .$$

A zero-one variable of $\theta_{p14}$ determines whether this additional term of income tax is zero, i.e. if there is any unrelieved interest carried forward.

## 7.13. Advance Corporation Tax Setoff

The total Advance Corporation Tax on dividends paid in the four quarters ended at time p is $D_J b_J / (1-b_J)$ for J = p-3 to p, where p=4,8,12 ... ( equation (68) ) . A dividend of 70pence ,$(D_J)$, has a gross value of £1, $[D_J/(1-b_J)]$, where the basic rate of income tax $b_J$= 30%.

The ACT on the dividend is 30 pence = $b_J$x £1 = $D_J b_J/(1-b_J)$, as above.

Now, the maximum ACT restriction for the accounting period ending at time p is given by the basic rate of income tax times net taxable income. When the basic rate of income tax changes in the second quarter of the calendar year (from April) there is a need to take a weighted average of net taxable income.

Assume net taxable income of £400,000 and a basic rate of income tax of 30% in period one (January to March) and 32% in periods two to four (April to December). The maximum ACT setoff is therefore restricted to £126,000; being

$$(30\% \;\; (1-\tfrac{3}{4}) + 32\%.\ \tfrac{3}{4})\ £400{,}000 \quad , \qquad \text{or}$$

$$(b_{p-3} \;(1-f) + \; b_p \; f\;)\; N_p^{TIA} \qquad\qquad \text{for } p=4.$$

ACT is carried forwards or backwards if the ACT brought forward of $A_{p-4}^{CTF} + A_p^{CTFB}$ (see footnote) , plus the ACT for the present year of $A_p^{CT}$ exceeds

$$(b_{p-3}(1-f) + b_p\, f)N_p^{TIA} \qquad \text{for } p=4,8,\ldots \;\; .$$

<u>footnote</u>

$A_p^{CTFB}$ represents additional ACT arising from the carry-back of capital allowances.

To simplify the notation this value is denoted $\phi 6A$ (Appendix 1):

$$\phi 6A = A^{CTF}_{p-4} + A^{CTFB}_{p} + A^{CT}_{p} - (b_{p-3}(1-f)+b_{p}f)N^{TIA}_{p} .$$

Constraints (69a) to (72a) can now be written as:

$$A^{CTB1A}_{p} = (\phi 6A)\theta^{A}_{p16}\,\theta^{A}_{p17} + A^{CT}_{p}(1-\theta^{A}_{p17}) \qquad (69a),$$

$$A^{CTBA}_{p} = (\phi 6A)\theta^{A}_{p16} \qquad (70a),$$

$$(\phi 6A) - M\theta^{A}_{p16} \leqslant 0 \qquad (71a),$$

$$A^{CTBA}_{p} \geqslant 0 \qquad (72a).$$

M represents a million (or higher) value as before. The $\theta^{A}_{p16}$ and $\theta^{A}_{p17}$ variables have values of zero or one (constraints (73a) to (75a)). $A^{CTB1A}_{p}$ denotes the ACT considered for being carried back at least one year.

Consider three alternative values of $[\phi 6A]$ at (i) £50,000; (ii) 0; and (iii) -£50,000.

(i) <u>£50,000 surplus ACT</u>

From (71a), $50{,}000 - M\,\theta^{A}_{p16}$ is negative only if $\theta^{A}_{p16}$ is one.

Hence from (70a), $A^{CTBA}_{p} = (\phi 6A)\ 1 = (\phi 6A)$, as required.

(ii) <u>Zero Surplus ACT</u>

From (71a), $\theta^{A}_{p16}$ is one, or zero. From (70a),

$A^{CTBA}_{p} = 0$, as required.

(iii) <u>-£50,000 (and no surplus ACT)</u>

From (71a), $\left[-50{,}000 - M\theta^{A}_{p16}\right]$ is negative no matter whether $\theta^{A}_{p16}$ is zero or one. If $\theta^{A}_{p16}$ is zero, from (70a), $A^{CTBA}_{p} = (-50{,}000)\ 0 = 0$, which satisfies (72a). Conversely, if $\theta^{A}_{p16}$ is one, from (70a), $A^{CTBA}_{p} = (-50{,}000)\ 1 = -50{,}000$, which no longer satisfies (72a). Hence $\theta^{A}_{p16}$ is zero and $A^{CTBA}_{p} = 0$, as required.

A problem arises in that a further requirement is that the ACT considered for being carried back at least one year from period p to p-4 is restricted to $A^{CT}_{p}$, being the ACT for quarters p-3 to p.

Hence $\phi 6A = A^{CTB1A}_{p}$ if $A^{CTBA}_{p} \leqslant A^{CT}_{p}$ ; otherwise

$A^{CTB1A}_{p} = A^{CT}_{p}$.

---

| Example | iv | v |
|---|---|---|
| ACT c/fwd from previous year | 90,000 | 147,600 |
| ACT for present year ($A^{CT}_{p}$) | 64,000 | 6,400 |
| Total ACT | 154,000 | 154,000 |
| Maximum ACT setoff | 126,000 | 126,000 |
| ACT considered for carry back | 28,000 | 6,400 |
| ACT carried forward only | nil | 21,600 |
| $A^{CTB1A}_{p}$ (=($\phi$ 6A) = Total ACT less maximum) | 28,000 | |
| $A^{CTB1A}_{p}$ (= $A^{CT}_{p}$) | | 6,400 |

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From (76a) and (77a) reproduced below ;

$$- (A^{CTBA}_{p} - A^{CT}_{p}) - M\Theta^{A}_{p17} \leqslant 0 \qquad (76a),$$

$$- (A^{CTBA}_{p} - A^{CT}_{p}) \Theta^{A}_{p17} \geqslant 0 \qquad (77a);$$

if $A^{CTBA}_{p}$ exceeds $A^{CT}_{p}$ (example (v)) the left-hand side

of constraint (76a) is negative whether $\Theta^{A}_{p17}$ is zero

or one. But $-\left[A^{CTBA}_{p} - A^{CT}_{p}\right]$ will be negative, and so $\theta^{A}_{p17}$ must be zero for constraint (77a) to be satisfied.

Using $\theta^{A}_{p17} = 0$, we may refer back to equation (69a) and deduce $A^{CTB1A}_{p} = (\phi 6A)\theta^{A}_{p16} . 0 + A^{CT}_{p} (1-0) = A^{CT}_{p}$

$= £6,400$ as in example (v).

Consider $A^{CT}_{p}$ exceeding $A^{CTBA}_{p}$ (example (iv)).

The value $-\left[A^{CTBA}_{p} - A^{CT}_{p}\right]$ is now positive and so

$\theta^{A}_{p17} = 1$ to satisfy (76a). Therefore from equation (69a)

$$A^{CTB1A}_{p} = (\phi 6A)\theta^{A}_{p16} . 1 + A^{CT}_{p} ( 1-1) = (\phi 6A)\theta^{A}_{p16} .$$

But it has been argued earlier that with surplus ACT (consider the previous illustration of +£50,000 surplus), then $\theta^{A}_{p16}$ is one. Hence, $A^{CTB1A}_{p} = (\phi 6A) = £28,000$ as in example (iv).

By similar procedures, constraints (78a) to (96a) determine the ACT rules for the second year of carry-back and any remaining ACT carried forward, $A^{CTF}_{p}$, which is split into two components $A^{CTF1A}_{p}$ and $A^{CTF2A}_{p}$, one for each year.

## 7.14 Net taxable income

Constraints (97a) to (101a) determine net taxable income for the current year, which is based on schedule D profits of $S_p^{HDA*}$, interest on lending at the rate $k_L$, less interest on borrowing at the rate $k_{WS}$ (short-term) and at $k_{WL}$ (long-term), less $I_{p-4}^{NTBA}$. The last term of $I_{p-4}^{NTBA}$ represents interest paid brought forward under section 177 (8) Income and Corporation Taxes Act 1970 from last year (four quarters ago: hence p-4). Where there is no net taxable income unrelieved interest is carried forward further (constraints (102a) to (106a)). Net taxable income for each of the three preceeding years is recalculated for the carry-back of capital allowances (constraints: (97b) to (106b), (97c) to (106c), and (97d) to (106d)).

## 7.15 Marginal tax rates

Berry and Dyson (1979) have already discussed the different marginal tax rates in relation to net taxable income. One of the differences here is that the model divides the accounting year to cater for the possibilities of

* Footnote : $S_p^{HDA}$ is calculated in equation (63a).

tax rates during April to December being different from those during January to March. The constraints are numbered (107a) to (123a), (107b) to (123b), (107c) to (123c), and (107d) to (123d) .

## 7.16 Net Mainstream Corporation Tax

If ACT setoff is restricted, $\theta^{A}_{p30}$ = zero (from constraint (125a)); and equation (124a), which determines the net mainstream corporation tax, is reduced to

$$N^{MCTA}_{p} = M^{CTA}_{p} - N^{TIA}_{p}\left[b_{p-3}(1-f) + b_{p}f\right] - I^{NTBQ}_{p} \quad .$$

The three terms on the fight show respectively the gross mainstream corporation tax; the maximum setoff; and $I^{NTBQ}_{p}$, (unrelieved income tax which can be set off against the net mainstream tax liability).

When ACT setoff is not restricted $\theta^{A}_{p30}$ = 1 and $\theta^{A}_{p31}$ = 1 (constraints (125a) to (132a)). Hence (124a) is reduced to

$$N^{MCTA}_{p} = M^{CTA}_{p} - A^{CTF}_{p-4} - A^{CTFB}_{p} - A^{CT}_{p} - I^{NTBQ}_{p} \quad .$$

The five terms on the right denote respectively (i) the gross mainstream corporation tax, (ii) surplus ACT from last year before taking into account the carry-back of capital allowances, (iii) extra ACT from last year after taking into acoount the carry-back of capital allowances; (iv) ACT for the current year; and (v) unrelieved income tax.

Since capital allowances can be carried back up to three years, previous years' figures for net mainstream corporation tax and extra ACT carried forward are recomputed [constraints: (124b) to (132b), (124c) to (132c), and (124d) to (132d)]. Tax rebates arising from these retrospective calculations have already been included in the program (see the heading entitled 'Liquidity constraints').

## 7.17. Conclusion

The purpose of the model was stated to be to demonstrate that the complex tax effects can be accommodated into a mathematical programming model which seeks an optimal solution to the firm's investment and financing decisions. It was shown that some of the variables are zero or one and hence the model becomes a mixed integer programming model which is very difficult to solve where there are numerous variables and constraints.

Earlier in the chapter it was stressed that the model-builder in practice would wish to condense the present model to suit the tax profile peculiar to the particular firm, which should result in a significant improvement in the time taken to obtain the solution to the [illegible] program. But since different firms have different tax profiles it seemed more appropriate here to present a generalised but more complex model. Implications which follow from the requirement of a joint solution to investment and financing decisions are discussed in the concluding chapter which follows.

APPENDIX 1

$$\phi 1 = \{z_5[z_4 P_{B,J-1} + P_{BJ} - z_4 P_{BJ} + E_{BJ} + \Delta_{BJ}]$$

$$- z_5[z_4 P_{B,J-2} + P_{B,J-1} - z_4 P_{B,J-1} + E_{B,J-1} + \Delta_{B,J-1}]$$

$$+ z_4 P_{BJ} - z_4 P_{B,J-1}\} .$$

$$\phi 2 = k_L L_{j-1} - k_{WS} W^S_{j-1} - k_{WL} \sum_{J=0}^{j-1} W^L_J .$$

$$\phi 3 = b_j \left[k_L L_{j-1} - k_{WS} W^S_{j-1} - k_{WL} \sum_{J=0}^{j-1} W^L_J\right] + I^{NTBQ}_{j-1} ,$$

$$\phi 4 = \left[-k_L L_{j-2} + k_{WS} W^S_{j-2} + k_{WL} \sum_{J=0}^{j-2} W^L_J\right] b_{j-1} .$$

$$\phi 5 = b_{j-1}\left[-k_L L_{j-2} + k_{WS} W^S_{j-2} + k_{WL} \sum_{J=0}^{j-2} W^L_J\right] - I^{NTBQ}_{j-2} .$$

$$\phi 6A = A^{CTF}_{p-4} + A^{CTFB}_{p} + A^{CT}_{p} - \left[b_{p-3}(1-f) + b_p f\right] N^{TIA}_{p} ,$$

$$\phi 7A = A^{CTF}_{p-8} + A^{CTFB}_{p-4} + A^{CT}_{p-4} - A^{CTB1A}_{p-4}$$

$$- A^{CTF1A}_{p-4} + A^{CTFC}_{p} - A^{CTFB}_{p} + A^{CTB1A}_{p}$$

$$- \left[b_{p-7}(1-f) + b_{p-4} f\right] N^{TIB}_{p-4} .$$

$$\phi 8A = A^{CTF}_{p-12} + A^{CTFB}_{p-8} + A^{CT}_{p-8} - A^{CTB1A}_{p-8}$$

$$- A^{CTF1A}_{p-8} - A^{CTFB}_{p-4} + A^{CTB1A}_{p-4} - A^{CTB2A}_{p-4}$$ (cont)

$$+ A^{CTFC}_{p-4} + A^{CTFD}_{p} - A^{CTFC}_{p}$$

$$- \left[b_{p-11}\,(1-f) + b_{p-8}f\right] N^{TIC}_{p-8} \quad .$$

$$\phi 9A = A^{CTF}_{p-4} + A^{CT}_{p} + A^{CTFB}_{p}$$

$$- \left[b_{p-3}\,(1-f) + b_{p}f\right] N^{TIA}_{p} - A^{CTB1A}_{p} \quad .$$

$$\phi 10A = S^{HDA}_{p} + \sum_{J=p-3}^{p} k_L L_{J-1} - \sum_{J=p-3}^{p} k_{WS} W^{S}_{J-1}$$

$$- \sum_{p^1=p-3}^{p} \left[k_{WL} \sum_{J=0}^{p^1-1} W^{L}_{J}\right] - I^{NTBA}_{p-4} \quad .$$

$$\phi 10B = S^{HDB}_{p} + \sum_{J=p-7}^{p-4} k_L L_{J-1} - \sum_{J=p-7}^{p-4} k_{WS} W^{S}_{J-1}$$

$$- \sum_{p^1=p-7}^{p-4} \left[k_{WL} \sum_{J=0}^{p^1-1} W^{L}_{J}\right] - I^{NTBB}_{p-4} \quad .$$

$$\phi 10C = S^{HDC}_{p} + \sum_{J=p-11}^{p-8} k_L L_{J-1} - \sum_{J=p-11}^{p-8} k_{WS} W^{S}_{J-1}$$

$$- \sum_{p^1=p-11}^{p-8} \left[k_{WL} \sum_{J=0}^{p^1-1} W^{L}_{J}\right] - I^{NTBC}_{p-4}$$

$$\phi 10D = S^{HDD}_{p} + \sum_{J=p-15}^{p-12} k_L L_{J-1} - \sum_{J=p-15}^{p-12} k_{WS} W^{S}_{J-1}$$

$$- \sum_{p^1=p-15}^{p-12} \left[k_{WL} \sum_{J=0}^{p^1-1} W^{L}_{J}\right] - I^{NTBD}_{p-4} \quad .$$

$\phi 11$, $\phi 12$ – have been eliminated from the program .

$$\phi 13D = A^{CTF}_{p-16} + A^{CT}_{p-12} + A^{CTFB}_{p-12} + A^{CTFC}_{p-8}$$

$$+ A^{CTB1A}_{p-8} + A^{CTFD}_{p-4} + A^{CTB2A}_{p-4}$$

$$- \left[ b_{p-15} (1-f) + b_{p-12} f \right] N^{TID}_{p} \quad .$$

$$\phi 13C = A^{CTF}_{p-12} + A^{CT}_{p-8} + A^{CTFB}_{p-8} + A^{CTFC}_{p-4}$$

$$+ A^{CTB1A}_{p-4} + A^{CTFD}_{p}$$

$$- \left[ b_{p-11} (1-f) + b_{p-8} f \right] N^{TIC}_{p} \quad .$$

$$\phi 13B = A^{CTF}_{p-8} + A^{CTFC}_{p} + A^{CT}_{p-4} + A^{CTFB}_{p-4}$$

$$- \left[ b_{p-7} (1-f) + b_{p-4} f \right] N^{TIB}_{p} \quad .$$

$$\phi 14B = A^{CT}_{p-4} + A^{CTF}_{p-8} + A^{CTFB}_{p-4}$$

$$- A^{CTB1A}_{p-4} - A^{CTF1A}_{p-4} + A^{CTFC}_{p}$$

$$- \left[ b_{p-7} (1-f) + b_{p-4} f \right] N^{TIB}_{p} \quad .$$

$$\phi 14C = A^{CT}_{p-8} + A^{CTF}_{p-12} + A^{CTFB}_{p-8} - A^{CTB1A}_{p-8}$$

$$- A^{CTF1A}_{p-8} + A^{CTFC}_{p-4} - A^{CTFB}_{p-4}$$

(cont)

$$+ A^{CTB1A}_{p-4} - A^{CTB2A}_{p-4} + A^{CTFD}_{p}$$

$$-\left[b_{p-11}\ (1-f) + b_{p-8}\ f\right] N^{TIC}_{p} \quad .$$

$$\phi 14D = A^{CTF}_{p-16} + A^{CT}_{p-12} + A^{CTFB}_{p-12} - A^{CTB1A}_{p-12}$$

$$- A^{CTF1A}_{p-12} + A^{CTFC}_{p-8} - A^{CTFB}_{p-8}$$

$$+ A^{CTB1A}_{p-8} - A^{CTB2A}_{p-8} + A^{CTFD}_{p-4} - A^{CTFC}_{p-4}$$

$$+ A^{CTB2A}_{p-4} - A^{CTF2A}_{p-4}$$

$$- \left[b_{p-15}\ (1-f) + b_{p-12} f\right] N^{TID}_{p} \quad .$$

Appendix 2

The full model is given below

Maximise

$$\sum_{j=o}^{n} \frac{D_j}{\prod_{J=1}^{j}(1+r_J)} \left[\frac{1-h_j}{1-b_j}\right] - \sum_{j=o}^{n} \frac{E^Q_j}{\prod_{J=1}^{j}(1+r_J)}$$

$$+\left\{\frac{1}{\prod_{J=1}^{n+1}(1+r_J)} \left[\sum_{B=1}^{B^1+q} C^V x_B + (1+k_L)\ L_n\right.\right.$$

$$\left. - (1+k_{WS})\ W^S_n - (1+k_{WL}) \sum_{j=o}^{n} W^L_j\right] - \sum_{j=n+1}^{\infty} \left[\frac{N^{MCT}_{j-g'}}{\prod_{J=O}^{j}(1+r_J)}\right]$$

$$\left. - \sum_{j=n+1}^{n+G} \frac{D_{j-G}\left(\frac{b_{j-G}}{1-b_{j-G}}\right)}{\prod_{J=O}^{j}(1+r_J)}\right\} \frac{1-h_j}{1-b_j}$$

a) Constraints

for $j \neq 2, 6, 10, 14$, etc.

$$-\sum_{B=1}^{B'} \left[z_1 R_{B,j-1} + R_{Bj} - z_1 R_{Bj} - I_{Bj} - z_2 P_{B,j-1}\right.$$

$$\left.-P_{Bj} + z_2 P_{Bj} - z_3 E_{B,j-1} - E_{Bj} + z_3 E_{Bj}\right] x_B$$

$$+ D_{j-G}\left(\frac{b_{j-G}}{1-b_{j-G}}\right) + L_j - L_{j-1} - k_L L_{j-1}(1-b_j)$$

$$- W^S_j + W^S_{j-1} + K_{WS} W^S_{j-1} (1-b_j) - W^L_j$$

$$+ K_{WL} \sum_{J=0}^{j-1} W^L_J \; (1-b_j) + D_j - E^Q_j$$

$$+ \{b_{j-1} [- K_L L_{j-2} + K_{WS} W^S_{j-2} + K_{WL} \sum_{J=0}^{j-2} W^L_J] - I^{NTBQ}_{j-2}\} \; \theta_{p15}$$

$$+ N^{MCTA}_{j-g'} + N^{MCTD}_{j-g'} - N^{MCTC}_{j-4-g'} + N^{MCTC}_{j-g'} - N^{MCTB}_{j-4-g'}$$

$$+ N^{MCTB}_{j-g'} - N^{MCTA}_{j-4-g'} = K_j \qquad (1).$$

$$S^{ECS}_{Bj} = R_{Bj} - P_{Bj} - E_{Bj} - z_4 P_{B,j-1} + z_4 P_{B,j}$$

$$+ z_5 \left[ z_4 P_{B,j-1} + P_{Bj} - z_4 P_{Bj} + E_{Bj} + \Delta_{Bj} \right]$$

$$- z_5 \left[ z_4 P_{B,j-2} + P_{B,j-1} - z_4 P_{B,j-1} + E_{B,j-1} + \Delta_{B,j-1} \right] \qquad (2).$$

$$- I^{NTBQ}_{j-1} - M\theta_{p15} \leq 0 \qquad (3).$$

$$- I^{NTBQ}_{j-1} \; \theta_{p15} \geq 0 \qquad (4).$$

$$\{\phi_5\} \; \theta_{p15} \geq 0 \qquad (5).$$

$$1 \geq \theta_{p15} \geq 0 \qquad (6).$$

$$\theta_{p15} \text{ integral} \qquad (7).$$

b) <u>j = 4, 8, 12, 16 etc</u>

$$j = p \tag{8}$$

$$\sum_{J=p-3}^{p} \sum_{B} S^{ECS}{}_{BJ} x_B - C^{AF}{}_{p-4} - \sum_{J=p-3}^{p} \sum_{B} C^{AP}{}_{BJ} x_B - \sum_{J=p-3}^{p} \sum_{B} C^{AO}{}_{BJ} x_B$$

$$-L^{F}{}_{p-4} = L^{HS1}_{p} \tag{9}$$

$$C^{ABIU}_{p} = L^{HS1}_{p} \quad (\theta_{p^1} - 1) \tag{10}$$

$$L^{HS1}_{p} - M\theta_{p^1} \leqslant 0 \tag{11}$$

$$L^{HS1}_{p} \quad \theta_{p^1} \geqslant 0 \tag{12}$$

$$1 \geqslant \theta_{p^1} \geqslant 0 \tag{13}$$

$$\theta_{p^1} \text{ integral} \tag{14}$$

$$-C^{AB1}{}_{p} = -\left[\sum_{J=p-3}^{p} \sum_{B} C^{AP}{}_{BJ} x_B\right] \theta_{p^2} \; (1-\theta_{p^1}) + C^{ABIU}{}_{p} \; (\theta_{p^2} - 1) \tag{15}$$

$$C^{ABIU}{}_{p} - \sum_{J=p-3}^{p} \sum_{B} C^{AP}{}_{BJ} x_B - M\theta_{p^2} \leqslant 0 \tag{16}$$

$$C^{AB1}{}_{p} \geqslant 0 \tag{17}$$

$$1 \geqslant \theta_{p^2} \geqslant 0 \tag{18}$$

$$\theta_{p^2} \text{ integral} \tag{19}$$

$$\left[C^{ABIU}_{p} - \sum_{J=p-3}^{p} \sum_{B} C^{AP}_{BJ} x_B\right] \theta_{p^2} \geq 0 \qquad (20).$$

$$L^{HS2}_{p} = \sum_{J=p-7}^{p-4} \sum_{B} S^{ECS}_{BJ} x_B - C^{AF}_{p-8} - \sum_{J=p-7}^{p-4} \sum_{B} C^{AP}_{BJ} x_B - \sum_{J=p-7}^{p-4} \sum_{B} C^{AO}_{BJ} x_B$$

$$-L^{F}_{p-8} \qquad (21).$$

$$L^{HS2}_{p} \theta_{p^3} \geq 0 \qquad (22).$$

$$L^{HS2}_{p} - M\theta_{p^3} \leq 0 \qquad (23).$$

$$1 \geq \theta_{p^3} \geq 0 \qquad (24).$$

$$\theta_{p^3} \text{ integral} \qquad (25).$$

$$C^{AB2}_{p} = \left[C^{AB1}_{p} - \theta_{p^3} L^{HS2}_{p}\right] \theta_{p^4} \qquad (26).$$

$$\left[C^{AB1}_{p} - L^{HS2}_{p}\right] - M\theta_{p^4} \leq 0 \qquad (27).$$

$$\left[C^{AB1}_{p} - L^{HS2}_{p}\right] \theta_{p^4} \geq 0 \qquad (28).$$

$$1 \geq \theta_{p^4} \geq 0 \qquad (29).$$

$$\theta_{p^4} \text{ integral} \qquad (30).$$

$$L^{HS3}_{p} = \sum_{J=p-11}^{p-8} \sum_{B} S^{ECS}_{BJ} x_B - C^{AF}_{p-12} - \sum_{J=p-11}^{p-8} \sum_{B} C^{AP}_{BJ} x_B$$

$$- \sum_{J=p-11}^{p-8} \sum_{B} C^{AO}_{BJ} x_B - \left[C^{AB1}_{p-4} - C^{AB2}_{p-4}\right] - L^{F}_{p-12} \qquad (31).$$

$$L^{HS3}_{p} \theta_{p^5} \geq 0 \qquad (32).$$

$$L^{HS3}_{p} - M\theta_{p5} \leq 0 \quad (33).$$

$$1 \geq \theta_{p5} \geq 0 \quad (34).$$

$$\theta_{p5} \text{ integral} \quad (35).$$

$$C^{AB3}_{p} = \left[C^{AB2}_{p} - \theta_{p5} L^{HS3}_{p}\right] \theta_{p6} \quad (36).$$

$$\left[C^{AB2}_{p} - L^{HS3}_{p}\right] - M\theta_{p6} \leq 0 \quad (37).$$

$$\left[C^{AB2}_{p} - L^{HS3}_{p}\right] \theta_{p6} \geq 0 \quad (38).$$

$$1 \geq \theta_{p6} \geq 0 \quad (39).$$

$$\theta_{p6} \text{ integral} \quad (40).$$

$$L^{HS4}_{p} = \sum_{J=p-15}^{p-12} \sum_{B} S^{ECS}_{BJ} x_{B} - C^{AF}_{p-16} - \sum_{J=p-15}^{p-12} \sum_{B} C^{AP}_{BJ} x_{B} - L^{F}_{p-16}$$

$$- \sum_{J=p-15}^{p-12} \sum_{B} C^{AO}_{BJ} x_{B} - \left[C^{AB1}_{p-8} - C^{AB2}_{p-8}\right] - \left[C^{AB2}_{p-4} - C^{AB3}_{p-4}\right] \quad (41).$$

$$L^{HS4}_{p} \theta_{p7} \geq 0 \quad (42).$$

$$L^{HS4}_{p} - M\theta_{p7} \leq 0 \quad (43).$$

$$1 \geq \theta_{p7} \geq 0 \quad (44).$$

$$\theta_{p7} \text{ integral} \quad (45).$$

$$C^{AF}_{p} = \left[C^{AB3}_{p} - \theta_{p7} L^{HS4}_{p}\right] \theta_{p8} \quad (46).$$

$$\left[C^{AB3}_{p} - L^{HS4}_{p}\right] - M\theta_{p8} \leq 0 \qquad (47).$$

$$\left[C^{AB3}_{p} - L^{HS4}_{p}\right]\theta_{p8} \geq 0 \qquad (48).$$

$$1 \geq \theta_{p8} \geq 0 \qquad (49).$$

$$\theta_{p8} \text{ integral} \qquad (50).$$

$$L^{o}_{p} = \sum_{J=p-3}^{p} \sum_{B} S^{ECS}_{BJ} x_B - C^{AF}_{p-4} - L^{F}_{p-4} - \sum_{J=p-3}^{p} \sum_{B} C^{AO}_{BJ} x_B \qquad (50i).$$

$$L^{F}_{p} = L^{o}_{p} (\theta_{p33} - 1) \qquad (50ii).$$

$$L^{o}_{p} - M\theta_{p33} \leq 0 \qquad (50iii)$$

$$L^{o}_{p}\theta_{p33} \geq 0 \qquad (50iv).$$

$$0 \geq \theta_{p33} \geq 1 \qquad (50v).$$

$$\theta_{p33} \text{ integral} \qquad (50vi).$$

$$R^{HS1A}_{p} = \sum_{J=p-3}^{p} \sum_{B} S^{ECS}_{BJ} x_B - C^{AF}_{p-4} - \sum_{J=p-3}^{p} \sum_{B} C^{AP}_{BJ} x_B$$

$$-L^{F}_{p-4} - \sum_{J=p-3}^{p} \sum_{B} C^{AO}_{BJ} x_B \qquad (51a).$$

$$S^{ESA}_{p} = R^{HS1A}_{p} \theta^{A}_{p9} \qquad (52a).$$

$$R^{HS1A}_{p} - M\theta^{A}_{p9} \leq 0 \qquad (53a).$$

$$S^{ESA}_{p} \geq o \qquad (54a).$$

$$1 \geq \theta^{A}_{p\hat{9}} \geq o \qquad (55a).$$

$$\theta^{A}_{p9} \text{ integral} \qquad (56a).$$

$$R^{HS1B}_{p} = \sum_{J=p-7}^{p-4} \sum_{B} S^{ECS}_{BJ} x_B - C^{AF}_{p-8} - \sum_{J=p-7}^{p-4} \sum_{B} C^{AP}_{BJ} x_B$$

$$-L^{F}_{p-8} - \sum_{J=p-7}^{p-4} \sum_{B} C^{AO}_{BJ} x_B - C^{AB1}_{p} \qquad (51b).$$

$$S^{ESB}_{p} = R^{HS1B}_{p} \theta^{B}_{p9} \qquad (52b).$$

$$R^{HS1B}_{p} - M\theta^{B}_{p9} \leq o \qquad (53b).$$

$$S^{ESB}_{p} \geq o \qquad (54b).$$

$$1 \geq \theta^{B}_{p9} \geq o \qquad (55b).$$

$$\theta^{B}_{p9} \text{ integral} \qquad (56b).$$

$$R^{HS1C}_{p} = \sum_{J=p-11}^{p-8} \sum_{B} S^{ECS}_{BJ} x_B - C^{AF}_{p-12} - L^{F}_{p-12}$$

$$- \sum_{J=p-11}^{p-8} \sum_{B} C^{AP}_{BJ} x_B - \sum_{J=p-11}^{p-8} \sum_{B} C^{AO}_{BJ} x_B - C^{AB1}_{p-4}$$

$$-C^{AB2}_{p} \qquad (51c).$$

$$S^{ESC}_{p} = R^{HS1C}_{p} \theta^{C}_{p9} \qquad (52c).$$

$$R^{HS1C}_{p} - M\theta^{C}_{p9} \leq o \qquad (53c).$$

$$S^{ESC}_{p} \geq 0 \qquad (54c).$$

$$1 \geq \theta^{C}_{p9} \geq 0 \qquad (55c).$$

$$\theta^{C}_{p9} \text{ integral} \qquad (56c).$$

$$R^{HS1D}_{p} = \sum_{J=p-15}^{p-12} \sum_{B} S^{ECS}_{BJ} x_B - C^{AF}_{p-16} - L^{F}_{p-16}$$

$$- \sum_{J=p-15}^{p-12} \sum_{B} C^{AP}_{BJ} x_B - \sum_{J=p-15}^{p-12} \sum_{B} C^{AO}_{BJ} x_B$$

$$-C^{AB1}_{p-8} - C^{AB2}_{p-4} - C^{AB3}_{p} \qquad (51d).$$

$$S^{ESD}_{p} = R^{HS1D}_{p} \theta^{D}_{p9} \qquad (52d).$$

$$R^{HS1D}_{p} - M\theta^{D}_{p9} \leq 0 \qquad (53d).$$

$$S^{ESD}_{p} \geq 0 \qquad (54d).$$

$$1 \geq \theta^{D}_{p9} \geq 0 \qquad (55d),$$

$$\theta^{D}_{p9} \text{ integral} \qquad (56d).$$

$$L^{HS5}_{p} = \sum_{J=p-3}^{p} \sum_{B} \{z_5[z_4 P_{B,J-1} + P_{BJ} - z_4 P_{BJ}$$

$$+E_{BJ} + \Delta_{BJ}] - z_5[z_4 P_{B,J-2} + P_{B,J-1}$$

$$- z_4 P_{B,J-1} + E_{B,J-1} + \Delta_{B,J-1}] + z_4 P_{BJ} - z_4 P_{B,J-1}\} \qquad (57).$$

$$\left[L^{HS5}_{p} - z_6 S^{ESA}_{p}\right] - M\theta^{A}_{p10} \leq o \qquad (58a).$$

$$\left[L^{HS5}_{p} - z_6 S^{ESA}_{p}\right]\theta^{A}_{p10} \geq o \qquad (59a).$$

$$S^{RA}_{p} = \left[L^{HS5}_{p} - z_6 S^{ESA}_{p}\right]\theta^{A}_{p10} \qquad (60a).$$

$$1 \geq \theta^{A}_{p10} \geq o \qquad (61a).$$

$$\theta^{A}_{p10} \text{ integral} \qquad (62a).$$

$$S^{HDA}_{p} = \left[S^{ESA}_{p} - S^{RA}_{p}\right]\theta^{A}_{p11} \qquad (63a).$$

$$\left[S^{ESA}_{p} - S^{RA}_{p}\right] - M\theta^{A}_{p11} \leq o \qquad (64a).$$

$$\left[S^{ESA}_{p} - S^{RA}_{p}\right]\theta^{A}_{p11} \geq o \qquad (65a).$$

$$1 \geq \theta^{A}_{p11} \geq o \qquad (66a).$$

$$\theta^{A}_{p11} \text{ integral} \qquad (67a).$$

$$\left[L^{HS5}_{p} - z_6 S^{ESB}_{p}\right] - M\theta^{B}_{p10} \leq o \qquad (58b).$$

$$\left[L^{HS5}_{p} - z_6 S^{ESB}_{p}\right]\theta^{B}_{p10} \geq o \qquad (59b).$$

$$S^{RB}_{p} = \left[L^{HS5}_{p} - z_6 S^{ESB}_{p}\right]\theta^{B}_{p10} \qquad (60b).$$

$$1 \geq \theta^{B}_{p10} \geq o \qquad (61b).$$

$$\theta^{B}_{p10} \text{ integral} \qquad (62b).$$

$$S^{HDB}_{p} = \left[S^{ESB}_{p} - S^{RB}_{p}\right]\theta^{B}_{p11} \quad (63b).$$

$$\left[S^{ESB}_{p} - S^{RB}_{p}\right] - M\theta^{B}_{p11} \leq o \quad (64b).$$

$$\left[S^{ESB}_{p} - S^{RB}_{p}\right]\theta^{B}_{p11} \geq o \quad (65b).$$

$$1 \geq \theta^{B}_{p11} \geq o \quad (66b).$$

$$\theta^{B}_{p11} \text{ integral} \quad (67b).$$

$$\left[L^{HS5}_{p} - z_6 S^{ESC}_{p}\right] - M\theta^{C}_{P10} \leq o \quad (58c).$$

$$\left[L^{HS5}_{p} - z_6 S^{ESC}_{p}\right]\theta^{C}_{p10} \geq o \quad (59c).$$

$$S^{RC}_{p} = \left[L^{HS5}_{p} - z_6 S^{ESC}_{p}\right]\theta^{C}_{p10} \quad (60c).$$

$$1 \geq \theta^{C}_{p10} \geq o \quad (61c).$$

$$\theta^{C}_{p10} \text{ integral} \quad (62c).$$

$$S^{HDC}_{p} = \left[S^{ESC}_{p} - S^{RC}_{p}\right]\theta^{C}_{p11} \quad (63c).$$

$$\left[S^{ESD}_{p} - S^{RC}_{p}\right] - M\theta^{C}_{p11} \leq o \quad (64c).$$

$$\left[S^{ESC}_{p} - S^{RC}_{p}\right]\theta^{C}_{p11} \geq o \quad (65c).$$

$$1 \geq \theta^{C}_{p11} \geq o \quad (66c).$$

$$\theta^{C}_{p11} \text{ integral} \quad (67c).$$

$$\left[L^{HS5}_{p} - z_6 S^{ESD}_{p}\right] - M\theta^{D}_{p10} \leq o \qquad (58d).$$

$$\left[L^{HS5}_{p} - z_6 S^{ESD}_{p}\right]\theta^{D}_{p10} \geq o \qquad (59d).$$

$$S^{RD}_{p} = \left[L^{HS5}_{p} - z_6 S^{ESD}_{p}\right]\theta^{D}_{p10} \qquad (60d).$$

$$1 \geq \theta^{D}_{p10} \geq o \qquad (61d).$$

$$\theta^{D}_{p10} \text{ integral} \qquad (62d).$$

$$S^{HDD}_{p} = \left[S^{ESD}_{p} - S^{RD}_{p}\right]\theta^{D}_{p11} \qquad (63d).$$

$$\left[S^{ESD}_{p} - S^{RD}_{p}\right] - M\theta^{D}_{p11} \leq o \qquad (64d).$$

$$\left[S^{ESD}_{p} - S^{RD}_{p}\right]\theta^{D}_{p11} \geq o \qquad (65d).$$

$$1 \geq \theta^{D}_{p11} \geq o \qquad (66d).$$

$$\theta^{D}_{p11} \text{ integral} \qquad (67d).$$

$$A^{CT}_{p} = \sum_{J=p-3}^{p}\left[D_J\right]\frac{b_J}{1-b_J} \qquad (68).$$

$$A^{CTB1A}_{p} = \{A^{CTF}_{p-4} + A^{CTFB}_{p} + A^{CT}_{p}$$

$$-\left[b_{p-3}(1-f) + b_p f\right] N^{TIA}_{p}\}\ \theta^{A}_{p16}\ \theta^{A}_{p17}$$

$$+ A^{CT}_{p}(1-\theta^{A}_{p17}) \qquad (69a).$$

$$A^{CTBA}_{p} = \{\phi 6A\}\ \theta^{A}_{p16} \tag{70a}$$

$$\{\phi 6A\} - M\theta^{A}_{p16} \leq o \tag{71a}$$

$$A^{CTBA}_{p} \geq o \tag{72a}$$

$$1 \geq \theta^{A}_{p16} \geq o \tag{73a}$$

$$1 \geq \theta^{A}_{p17} \geq o \tag{74a}$$

$$\theta^{A}_{p16}, \quad \theta^{A}_{p17} \text{ integral} \tag{75a}$$

$$-\left[A^{CTBA}_{p} - A^{CT}_{p}\right] - M\theta^{A}_{p17} \leq o \tag{76a}$$

$$-\left[A^{CTBA}_{p} - A^{CT}_{p}\right]\theta^{A}_{p17} \geq o \tag{77a}$$

$$A^{CTB2A}_{p} = \{ A^{CTF}_{p-8} + A^{CTFB}_{p-4} + A^{CT}_{p-4} - A^{CTB1A}_{p-4}$$

$$- A^{CTF1A}_{p-4} + A^{CTFC}_{p} - A^{CTFB}_{p} + A^{CTB1A}_{p}$$

$$- \left[b_{p-7}(1-f) + b_{p-4}\, f\right] N^{TIB}_{p-4} \}\ \theta^{A}_{p18}\ \theta^{A}_{p19}$$

$$+ A^{CTB1A}_{p}\ (1 - \theta^{A}_{p19}) \tag{78a}$$

$$\{\phi 7A\}\ \theta^{A}_{p18} \geq o \tag{79a}$$

$$\{\phi 7A\} - M\theta^{A}_{p18} \leq o \tag{80a}$$

$$1 \geq \theta^{A}_{p18} \geq o \qquad (81a).$$

$$1 \geq \theta^{A}_{p19} \geq o \qquad (82a).$$

$$\theta^{A}_{p18}, \ \theta^{A}_{p19} \ \text{integral} \qquad (83a).$$

$$-\left[\{\phi 7A\} - A^{CTB1A}_{p}\right] - M\theta^{A}_{p19} \leq o \qquad (84a).$$

$$-\left[\{\phi 7A\} - A^{CTB1A}_{p}\right] \theta^{A}_{p19} \geq o \qquad (85a).$$

$$A^{CTF2A}_{p} = \{A^{CTF}_{p-12} + A^{CTFB}_{p-8} + A^{CT}_{p-8} - A^{CTB1A}_{p-8}$$

$$- A^{CTF1A}_{p-8} - A^{CTFB}_{p-4} + A^{CTB1A}_{p-4} - A^{CTB2A}_{p-4}$$

$$+ A^{CTFC}_{p-4} + A^{CTFD}_{p} - A^{CTFC}_{p}$$

$$- \left[b_{p-11}(1-f) + b_{p-8}f\right] N^{TIC}_{p-8} \} \ \theta^{A}_{p20} \qquad (86a).$$

$$\{\phi 8A\} - M\theta^{A}_{p20} \leq o \qquad (87a).$$

$$A^{CTF2A}_{p} \geq o \qquad (88a).$$

$$1 \geq \theta^{A}_{p20} \geq o \qquad (89a).$$

$$\theta^{A}_{p20} \ \text{integral} \qquad (90a).$$

$$A^{CTF1A}_{p} = \{A^{CTF}_{p-4} + A^{CT}_{p} + A^{CTFB}_{p}$$

$$-\left[b_{p-3}(1-f) + b_{p}f\right] N^{TIA}_{p} - A^{CTB1A}_{p} \} \ \theta^{A}_{p21} \qquad (91a).$$

$$\{\phi 9A\} - M\theta^{A}_{p21} \leq o \tag{92a}$$

$$A^{CTF1A}_{p} \geq o \tag{93a}$$

$$1 \geq \theta^{A}_{p21} \geq o \tag{94a}$$

$$\theta^{A}_{p21} \text{ integral} \tag{95a}$$

$$A^{CTF}_{p} = A^{CTF1A}_{p} + A^{CTF2A}_{p} \tag{96a}$$

$$N^{TIA}_{p} = \{S^{HDA}_{p} + \sum_{J=p-3}^{p} k_L L_{J-1} - \sum_{J=p-3}^{p} k_{WS} W^{S}_{J-1} - \sum_{p^1=p-3}^{p} \left[k_{WL} \sum_{J=0}^{p^1-1} W^{L}_{J}\right] - I^{NTBA}_{p-4}\} \theta^{A}_{p22} \tag{97a}$$

$$N^{TIB}_{p} = \{S^{HDB}_{p} + \sum_{J=p-7}^{p-4} k_L L_{J-1} - \sum_{J=p-7}^{p-4} k_{WS} W^{S}_{J-1} - \sum_{p^1=p-7}^{p-4} \left[k_{WL} \sum_{J=0}^{p^1-1} W^{L}_{J}\right] - I^{NTBB}_{p-4}\} \theta^{B}_{p22} \tag{97b}$$

$$N^{TIC}_{p} = \{S^{HDC}_{p} + \sum_{J=p-11}^{p-8} k_L L_{J-1} - \sum_{J=p-11}^{p-8} k_{WS} W^{S}_{J-1} - \sum_{p^1=p-11}^{p-8} \left[k_{WL} \sum_{J=0}^{p^1-1} W^{L}_{J}\right] - I^{NTBC}_{p-4}\} \theta^{C}_{p22} \tag{97c}$$

$$N^{TID}_{p} = \{S^{HDD}_{p} + \sum_{J=p-15}^{p-12} k_L L_{J-1} - \sum_{J=p-15}^{p-12} k_{WS} W^{S}_{J-1} - \sum_{p^1=p-15}^{p-12} \left[k_{WL} \sum_{J=0}^{p^1-1} W^{L}_{J}\right] - I^{NTBD}_{p-4}\} \theta^{D}_{p22} \tag{97d}$$

$$\{\phi 10A\} - M\theta^{A}_{p22} \leq 0 \tag{98a}$$

$$\{\phi 10A\}\ \theta^{A}_{p22} \geq 0 \tag{99a}$$

$$1 \geq \theta^{A}_{p22} \geq 0 \tag{100a}$$

$$\theta^{A}_{p22} \text{ integral} \tag{101a}$$

$$\{\phi 10B\} - M\theta^{B}_{p22} \leq 0 \tag{98b}$$

$$\{\phi 10B\}\ \theta^{B}_{p22} \geq 0 \tag{99b}$$

$$1 \geq \theta^{B}_{p22} \geq 0 \tag{100b}$$

$$\theta^{B}_{p22} \text{ integral} \tag{101b}$$

$$\{\phi 10C\} - M\theta^{C}_{p22} \leq 0 \tag{98c}$$

$$\{\phi 10C\}\ \theta^{C}_{p22} \geq 0 \tag{99c}$$

$$1 \geq \theta^{C}_{p22} \geq 0 \tag{100c}$$

$$\theta^{C}_{p22} \text{ integral} \tag{101c}$$

$$\{\phi 10D\} - M\theta^{D}_{p22} \leq 0 \tag{98d}$$

$$\{\phi 10D\}\ \theta^{D}_{p22} \geq 0 \tag{99d}$$

$$1 \geq \theta^{D}_{p22} \geq 0 \tag{100d}$$

[redacted]

$$- I_p^{NTBA} = \{\phi 10A\} \; \theta_{p23}^{A} \tag{102a}$$,

$$- \{\phi 10A\} \; \theta_{p23}^{A} \geq o \tag{103a}$$.

$$- \{\phi 10A\} \; - M\theta_{p23}^{A} \leq o \tag{104a}$$.

$$1 \geq \theta_{p23}^{A} \geq o \tag{105a}$$.

$$\theta_{p23}^{A} \text{ integral} \tag{106a}$$.

$$- I_p^{NTBB} = \{\phi 10B\} \; \theta_{p23}^{B} \tag{102b}$$.

$$- \{\phi 10B\} \; \theta_{p23}^{B} \geq o \tag{103b}$$.

$$- \{\phi 10B\} \; - M\theta_{p23}^{B} \leq o \tag{104b}$$.

$$1 \geq \theta_{p23}^{B} \geq o \tag{105b}$$.

$$\theta_{p23}^{B} \text{ integral} \tag{106b}$$.

$$- I_p^{NTBC} = \{\phi 10C\} \; \theta_{p23}^{C} \tag{102c}$$.

$$- \{\phi 10C\} \; \theta_{p23}^{C} \geq o \tag{103c}$$.

$$- \{\phi 10C\} - M\theta_{p23}^{C} \leq o \tag{104c}$$.

$$1 \geq \theta_{p23}^{C} \geq o \tag{105c}$$.

$$\theta_{p23}^{C} \text{ integral} \tag{106c}$$.

$$-I^{NTBD}_{p} = \left\{\phi 10D\right\} \theta^{D}_{p23} \qquad (102d).$$

$$-\left\{\phi 10D\right\} \theta^{D}_{p23} \geqslant 0 \qquad (103d).$$

$$-\left\{\phi 10D\right\} - M\theta^{D}_{p23} \leqslant 0 \qquad (104d).$$

$$1 \geqslant \theta^{D}_{p23} \geqslant 0 \qquad (105d).$$

$$\theta^{D}_{p23} \quad \text{integral} \qquad (106d).$$

$$M^{CTA}_{p} = M^{CT1A}_{p} + M^{CT2A}_{p} \qquad (107a).$$

$$M^{CT1A}_{p} = \left[0.52\ N^{T1A}_{p}\ (1-f)\right] \theta^{A}_{p24} + \left[0.40N^{T1A}_{p}\ (1-f)\right] \theta^{A}_{p25}$$

$$+ \left[(1-f) \left\{0.40 \quad M^{RL}_{p-3} + \right.\right.$$

$$\left[0.52 + \frac{M^{RL}_{p-3}}{M^{RU}_{p-3} - M^{RL}_{p-3}} (0.52 - 0.40)\right]$$

$$\times \left( N^{TIA}_{p} - M^{RL}_{p-3} \right)\}\Big] \; \theta^{A}_{p26} \qquad (108a).$$

$$\theta^{A}_{p24} + \theta^{A}_{p25} + \theta^{A}_{p26} = 1 \qquad (109a).$$

$$\theta^{A}_{p24},\ \theta^{A}_{p25},\ \theta^{A}_{p26} \geq 0 \qquad (110a).$$

$$\theta^{A}_{p24},\ \theta^{A}_{p25},\ \theta^{A}_{p26} \quad \text{integral} \qquad (111a).$$

$$\left[ N^{TIA}_{p} - M^{RU}_{p-3} \right] \theta^{A}_{p24} \geq 0 \qquad (112a).$$

$$\left[ N^{TIA}_{p} - M^{RU}_{p-3} \right] - M\theta^{A}_{p24} \leq 0 \qquad (113a).$$

$$\left[ M^{RL}_{p-3} - N^{TIA}_{p} \right] \theta^{A}_{p25} \geq 0 \qquad (114a).$$

$$\left[ M^{RL}_{p-3} - N^{TIA}_{p} \right] - M\theta^{A}_{p25} \leq 0 \qquad (115a).$$

$$M^{CT2A}_{p} = \left[ 0.52\, N^{TIA}_{p} f \right] \theta^{A}_{p27} + \left[ 0.40\, N^{TIA}_{p} f \right] \theta^{A}_{p28}$$

$$+ f \left\{ 0.40\, M^{RL}_{p} + \left[ 0.52 + \frac{M^{RL}_{p}}{M^{RU}_{p} - M^{RL}_{p}} (0.52-0.40) \right.\right.$$

$$\left.\left. \times \left[ N^{TIA}_{p} - M^{RL}_{p} \right] \right\} \theta^{A}_{p29} \tag{116a}$$

$$\theta^{A}_{p27} + \theta^{A}_{p28} + \theta^{A}_{p29} = 1 \tag{117a}$$

$$\theta^{A}_{p27},\ \theta^{A}_{p28},\ \theta^{A}_{p29} \geq 0 \tag{118a}$$

$$\theta^{A}_{p27},\ \theta^{A}_{p28},\ \theta^{A}_{p29} \quad \text{integral} \tag{119a}$$

$$\left[ N^{TIA}_{p} - M^{RU}_{p} \right] \theta^{A}_{p27} \geq 0 \tag{120a}$$

$$\left[ N^{TIA}_{p} - M^{RU}_{p} \right] - M\theta^{A}_{p27} \leq 0 \tag{121a}$$

$$\left[ M^{RL}_{p} - N^{TIA}_{p} \right] \theta^{A}_{p28} \geq 0 \tag{122a}$$

$$\left[ M^{RL}_{p} - N^{TIA}_{p} \right] - M\theta^{A}_{p28} \leq 0 \tag{123a}$$

$$M_p^{CTB} = M_p^{CT1B} + M_p^{CT2B} \tag{107b}$$

$$M_p^{CT1B} = \left[0.52\, N_p^{TIB}\ (1-f)\ \theta_{p24}^{B} + \left[0.40 N_p^{TIB}\ (1-f)\right]\theta_{p25}^{B}\right.$$

$$+ \left[(1-f)\left\{0.40\, M_{p-7}^{RL} + \left[0.52 + \frac{M_{p-7}^{RL}}{M_{p-7}^{RU} - M_{p-7}^{RL}}\ (0.52 - 0.40)\right]\right.\right.$$

$$\left.\left. \times\ (N_p^{TIB} - M_{p-7}^{RL})\right\}\right]\ \theta_{p26}^{B} \tag{108b}$$

$$\theta_{p24}^{B} + \theta_{p25}^{B} + \theta_{p26}^{B} = 1 \tag{109b}$$

$$\theta_{p24}^{B},\ \theta_{p25}^{B},\ \theta_{p26}^{B} \geqslant 0 \tag{110b}$$

$$\theta_{p24}^{B},\ \theta_{p25}^{B},\ \theta_{p26}^{B} \quad \text{integral} \tag{111b}$$

$$\left[N_p^{TIB} - M_{p-7}^{RU}\right]\ \theta_{p24}^{B} \geqslant 0 \tag{112b}$$

$$\left[N^{TIB}_{p} - M^{RU}_{p-7}\right] - M\Theta^{B}_{p24} \leq 0 \qquad (113b).$$

$$\left[M^{RL}_{p-7} - N^{TIB}_{p}\right] \Theta^{B}_{p25} \geq 0 \qquad (114b).$$

$$\left[M^{RL}_{p-7} - N^{TIB}_{p}\right] - M\Theta^{B}_{p25} \leq 0 \qquad (115b).$$

$$M^{CT2B}_{p} = \left[0.52\ N^{TIB}_{p}\ f\right] \Theta^{B}_{p27} + \left[0.40\ N^{TIB}_{p}\ f\right] \Theta^{B}_{p28}$$

$$+ f \left\{ 0.40\ M^{RL}_{p-4} + \left[ 0.52 \right.\right.$$

$$+ \frac{M^{RL}_{p-4}}{M^{RU}_{p-4} - M^{RL}_{p-4}} (0.52 - 0.40)$$

$$\left.\left. \times \left[ N^{TIB}_{p} - M^{RL}_{p-4} \right] \right] \right\} \Theta^{B}_{p29} \qquad (116b).$$

$$\Theta^{B}_{p27} + \Theta^{B}_{p28} + \Theta^{B}_{p29} = 1 \qquad (117b).$$

$$\Theta^{B}_{p27},\ \Theta^{B}_{p28},\ \Theta^{B}_{p29} \qquad \geq 0 \qquad (118b).$$

$$\Theta^{B}_{p27},\ \Theta^{B}_{p28},\ \Theta^{B}_{p29} \qquad \text{integral} \qquad (119b).$$

$$\left[ N^{TIB}_{p} - M^{RU}_{p-4} \right] \Theta^{B}_{p27} \qquad \geq 0 \qquad (120b).$$

$$\left[ N^{TIB}_{p} - M^{RU}_{p-4} \right] - M\Theta^{B}_{p27} \qquad \leq 0 \qquad (121b).$$

$$\left[ M^{RL}_{p-4} - N^{TIB}_{p} \right] \Theta^{B}_{p28} \qquad \geq 0 \qquad (122b).$$

$$\left[ M^{RL}_{p-4} - N^{TIB}_{p} \right] - M\Theta^{B}_{p28} \qquad \leq 0 \qquad (123b).$$

$$M^{CTC}_{p} = M^{CT1C}_{p} + M^{CT2C}_{p} \qquad (107c).$$

$$M^{CT1C}_{p} = \left[ 0.52\ N^{TIC}_{p}\ (1-f) \right] \Theta^{C}_{p24} + \left[ 0.40\ N^{TIC}_{p}\ (1-f) \right] \Theta^{C}_{p25}$$

$$+ \left[ (1-f) \left\{ 0.40\ M^{RL}_{p-11} + \left[ 0.52 \right.\right.\right.$$

$$+ \frac{M^{RL}_{p-11}}{M^{RU}_{p-11} - M^{RL}_{p-11}} \quad (0.52 - 0.40) \Bigg]$$

$$x \quad \left(N^{TIC}_{p} - M^{RL}_{p-11}\right)\Bigg\}\Bigg] \; \theta^{C}_{p26} \qquad (108c).$$

$$\theta^{C}_{p24} + \theta^{C}_{p25} + \theta^{C}_{p26} = 1 \qquad (109c).$$

$$\theta^{C}_{p24}, \; \theta^{C}_{p25}, \; \theta^{C}_{p26}, \quad \geq 0 \qquad (110c).$$

$$\theta^{C}_{p24}, \; \theta^{C}_{p25}, \; \theta^{C}_{p26} \quad \text{integral} \qquad (111c).$$

$$\left[N^{TIC}_{p} - M^{RU}_{p-11}\right] \theta^{C}_{p24} \quad \geq 0 \qquad (112c).$$

$$\left[N^{TIC}_{p} - M^{RU}_{p-11}\right] -M\theta^{C}_{p24} \quad \leq 0 \qquad (113c).$$

$$\left[M^{RL}_{p-11} - N^{TIC}_{p}\right] \theta^{C}_{p25} \quad \geq 0 \qquad (114c).$$

$$\left[M^{RL}_{p-11} - N^{TIC}_{p}\right] - M\theta^{C}_{p25} \quad \leq 0 \qquad (115c).$$

$$M^{CT2C}_{p} = \left[0.52\, N^{TIC}_{p}\, f\right]\theta^{C}_{p27} + \left[0.40\, N^{TIC}_{p}\, f\right]\theta^{C}_{p28}$$

$$+ f\left\{0.40\, M^{RL}_{p-8} + \left[0.52 + \frac{M^{RL}_{p-8}}{M^{RU}_{p-8} - M^{RL}_{p-8}}\,(0.52-0.40)\right]\right.$$

$$\left. \times\left[N^{TIC}_{p} - M^{RL}_{p-8}\right]\right\}\theta^{C}_{p29} \qquad (116c).$$

$$\theta^{C}_{p27} + \theta^{C}_{p28} + \theta^{C}_{p29} = 1 \qquad (117c).$$

$$\theta^{C}_{p27},\ \theta^{C}_{p28},\ \theta^{C}_{p29} \geqslant 0 \qquad (118c).$$

$$\theta^{C}_{p27},\ \theta^{C}_{p28},\ \theta^{C}_{p29} \quad \text{integral} \qquad (119c).$$

$$\left[N^{TIC}_{p} - M^{RU}_{p-8}\right]\theta^{C}_{p27} \geqslant 0 \qquad (120c).$$

$$\left[N^{TIC}_{p} - M^{RU}_{p-8}\right] - M\theta^{C}_{p27} \leqslant 0 \qquad (121c).$$

$$\left[M^{RL}_{p-8} - N^{TIC}_{p}\right]\theta^{C}_{p28} \geqslant 0 \qquad (122c),$$

$$\left[ M^{RL}_{p-8} - N^{TIC}_{p} \right] - M\Theta^{C}_{p28} \leq 0 \qquad (123c).$$

$$M^{CTD}_{p} = M^{CT1D}_{p} + M^{CT2D}_{p} \qquad (107d).$$

$$M^{CT1D}_{p} = \left[ 0.52\ N^{TID}_{p} (1-f) \right] \Theta^{D}_{p24} + \left[ 0.40\ N^{TID}_{p} (1-f) \right] \Theta^{D}_{p25}$$

$$+ \left[ (1-f) \left\{ 0.40\ M^{RL}_{p-15} + \left[ 0.52 + \frac{M^{RL}_{p-15}}{M^{RU}_{p-15} - M^{RL}_{p-15}} (0.52-0.40) \right] \right. \right.$$

$$\left. \left. x\, (N^{TID}_{p} - M^{RL}_{p-15}) \right\} \right] \Theta^{D}_{p26} \qquad (108d).$$

$$\Theta^{D}_{p24} + \Theta^{D}_{p25} + \Theta^{D}_{p26} = 1 \qquad (109d).$$

$$\Theta^{D}_{p24},\ \Theta^{D}_{p25},\ \Theta^{D}_{p26} \geq 0 \qquad (110d).$$

$$\Theta^{D}_{p24},\ \Theta^{D}_{p25},\ \Theta^{D}_{p26} \quad \text{integral} \qquad (111d).$$

$$\left[N^{TID}_{p} - M^{RU}_{p-15}\right] \theta^{D}_{p24} \geq 0 \qquad (112d).$$

$$\left[N^{TID}_{p} - M^{RU}_{p-15}\right] - M\theta^{D}_{p24} \leq 0 \qquad (113d).$$

$$\left[M^{RL}_{p-15} - N^{TID}_{p}\right] \theta^{D}_{p25} \geq 0 \qquad (114d).$$

$$\left[M^{RL}_{p-15} - N^{TID}_{p}\right] - M\theta^{D}_{p25} \leq 0 \qquad (115d).$$

$$M^{CT2D}_{p} = \left[0.52\ N^{TID}_{p}\ f\right] \theta^{D}_{p27} + \left[0.40\ N^{TID}_{p}\ f\right] \theta^{D}_{p28}$$

$$+ f\left\{0.40\ M^{RL}_{p-12} + \left[0.52 + \frac{M^{RL}_{p-12}}{M^{RU}_{p-12} - M^{RL}_{p-12}} (0.52 - 0.40)\right]\right.$$

$$\left. x\left[N^{TID}_{p} - M^{RL}_{p-12}\right]\right\} \theta^{D}_{p29} \qquad (116d).$$

$$\theta^{D}_{p27} + \theta^{D}_{p28} + \theta^{D}_{p29} = 1 \qquad (117d).$$

$$\theta^{D}_{p27},\ \theta^{D}_{p28},\ \theta^{D}_{p29} \geq 0 \qquad (118d).$$

$$\theta^{D}_{p27},\ \theta^{D}_{p28},\ \theta^{D}_{p29} \quad \text{integral} \tag{119d}$$

$$\left[N^{TID}_{p} - M^{RU}_{p-12}\right]\theta^{D}_{p27} \geqslant 0 \tag{120d}$$

$$\left[N^{TID}_{p} - M^{RU}_{p-12}\right] - M\theta^{D}_{p27} \leqslant 0 \tag{121d}$$

$$\left[M^{RL}_{p-12} - N^{TID}_{p}\right]\theta^{D}_{p28} \geqslant 0 \tag{122d}$$

$$\left[M^{RL}_{p-12} - N^{TID}_{p}\right] - M\theta^{D}_{p28} \leqslant 0 \tag{123d}$$

$$N^{MCTA}_{p} = \left[M^{CTA}_{p} - N^{TIA}_{p}\left[b_{p-3}(1-f) + b_{p} f\right]\right.$$

$$\left. - I^{NTBQ}_{p}\right](1-\theta^{A}_{p30}\,\theta^{A}_{p31}) + \left[M^{CTA}_{p} - A^{CTF}_{p-4}\right.$$

$$\left. -A^{CTFB}_{p} - A^{CT}_{p} - I^{NTBQ}_{p}\right]\theta_{p30}\,\theta_{p31} \tag{124a}$$

$$-\left[A^{CTB1A}_{p}\right]\theta^{A}_{p30} \geqslant 0 \tag{125a}$$

$$-\left[A^{CTB1A}_{p}\right] - M\theta^{A}_{p30} \leqslant 0 \tag{126a}$$

$$1 \geq \theta^{A}_{p30} \geq 0 \qquad (127a),$$

$$\theta^{A}_{p30} \quad \text{integral} \qquad (128a).$$

$$-\left[A^{CTF}_{p}\right] \theta^{A}_{p31} \geq 0 \qquad (129a),$$

$$-\left[A^{CTF}_{p}\right] - M\theta^{A}_{p31} \leq 0 \qquad (130a).$$

$$1 \geq \theta^{A}_{p31} \geq 0 \qquad (131a).$$

$$\theta^{A}_{p31} \quad \text{integral} \qquad (132a).$$

$$N^{MCTB}_{p} = \left[M^{CTB}_{p} - N^{TIB}_{p}\left[b_{p-7}(1-f) + b_{p-4}\, f\right]\right.$$

$$\left. - I^{NTBQ}_{p-4}\right] \quad (1-\theta^{B}_{p30})$$

$$+\left[M^{CTB}_{p} - A^{CTF}_{p-8} - A^{CTFC}_{p} - A^{CTFB}_{p-4}\right.$$

$$- A^{CT}_{p-4} - I^{NTBQ}_{p-4} \Big] \; \theta^{B}_{p30} \qquad (124b),$$

$$\left\{ \phi 13B \right\} - M\,(1-\theta^{B}_{p\,30}) \leqslant 0 \qquad (125b).$$

$$\left\{ \phi 13B \right\} (1-\theta^{B}_{p30}) \geqslant 0 \qquad (126b).$$

$$1 \geqslant \theta^{B}_{p30} \geqslant 0 \qquad (127b).$$

$$\theta^{B}_{p30} \text{ integral} \qquad (128b).$$

$$A^{CTFB}_{p} = \left\{ \phi 14B \right\} \theta^{B}_{p32} \qquad (129b).$$

$$\left\{ \phi 14B \right\} - M\theta^{B}_{p32} \leqslant 0 \qquad (130b).$$

$$A^{CTFB}_{p} \geqslant 0 \qquad (131b).$$

$$0 \leqslant \theta^{B}_{p32} \leqslant 1 \; ; \; \theta^{B}_{p32} \text{ integral} \qquad (132b).$$

$$N^{MCTC}_{p} = \Big[ M^{CTC}_{p} - N^{TIC}_{p} \big[ b_{p-11}(1-f) + b_{p-8}\, f \big]$$

$$- I^{NTBQ}_{p-8} \Big] \left(1-\theta^{C}_{p30}\right)$$

$$+ \Big[ M^{CTC}_{p} - A^{CTF}_{p-12} - A^{CTFD}_{p} - A^{CTFB}_{p-8}$$

$$- A^{CTFC}_{p-4} - A^{CTB1A}_{p-4} - A^{CT}_{p-8} - I^{NTBQ}_{p-8} \Big] \theta^{C}_{p30} \qquad (124c)$$

$$\left\{ \phi 13C \right\} - M\left(1-\theta^{C}_{p30}\right) \leq 0 \qquad (125c)$$

$$\left\{ \phi 13C \right\} \left(1-\theta^{C}_{p30}\right) \geq 0 \qquad (126c)$$

$$1 \geq \theta^{C}_{p30} \geq 0 \qquad (127c)$$

$$\theta^{C}_{p30} \quad \text{integral} \qquad (128c)$$

$$A^{CTFC}_{p} = \left\{ \phi 14C \right\} \theta^{C}_{p32} \qquad (129c)$$

$$\left\{\phi 14C\right\} - M\theta^{C}_{p32} \leq 0 \qquad (130c)$$

$$A^{CTFC}_{p} \geq 0 \qquad (131c)$$

$$0 \leq \theta^{D}_{p32} \leq 1 \quad ; \ \theta^{D}_{p32} \ \text{integral} \qquad (132c)$$

$$N^{MCTD}_{p} = \left[ M^{CTD}_{p} - N^{TID}_{p}\left[ b_{p-15}(1-f) + b_{p-12}\, f \right] - I^{NTBQ}_{p-12} \right] (1-\theta^{D}_{p30})$$

$$+ \left[ M^{CTD}_{p} - A^{CTF}_{p-16} - A^{CT}_{p-12} - A^{CTFB}_{p-12} - A^{CTFC}_{p-8} - A^{CTB1A}_{p-8} - A^{CTFD}_{p-4} - A^{CTB2A}_{p-4} - I^{NTBQ}_{p-12} \right] \theta^{D}_{p30} \qquad (124d)$$

$$\left\{\phi 13D\right\} - M(1-\theta^{D}_{p30}) \leq 0 \qquad (125d)$$

$$\left\{\phi 13D\right\} \quad (1-\theta^{D}_{p30}) \quad \geqslant 0 \qquad (126d).$$

$$1 \geqslant \theta^{D}_{p30} \geqslant 0 \qquad (127d).$$

$$\theta^{D}_{p30} \quad \text{integral} \qquad (128d).$$

$$A^{CTFD}_{p} = \left\{\phi 14D\right\} \theta^{D}_{p32} \qquad (129d).$$

$$\left\{\phi 14D\right\} - M\theta^{D}_{p32} \leqslant 0 \qquad (130d).$$

$$A^{CTFD}_{p} \geqslant 0 \qquad (131d).$$

$$0 \leqslant \theta^{D}_{p32} \leqslant 1 \;\; ; \;\; \theta^{D}_{p32} \quad \text{integral} \qquad (132d).$$

c) <u>For j = 1, 5, 9, 13 etc</u>

$$I^{NIBQ}_{j} = \{ b_j \left[ k_L L_{j-1} - k_{WS} W^S_{j-1} - k_{WL} \sum_{J=0}^{j-1} W^L_J \right] \} \theta_{p^{12}} \quad (133)$$

$$b_j \left[ k_L L_{j-1} - k_{WS} W^S_{j-1} - k_{WL} \sum_{J=0}^{j-1} W^L_J \right] - M\theta_{p^{12}} \leq 0 \quad (134)$$

$$\{ b_j \left[ k_L L_{j-1} - k_{WS} W^S_{j-1} - k_{WL} \sum_{J=0}^{j-1} W^L_J \right] \} \theta_{p^{12}} \geq 0 \quad (135)$$

$$1 \geq \theta_{p^{12}} \geq 0 \quad (136)$$

$$\theta_{p^{12}} \text{ integral} \quad (137)$$

d) <u>For j ≠ 1, 5, 9, 13 etc</u>

$$I^{NIBQ}_{j} = \{ b_j \left[ k_L L_{j-1} - k_{WS} W^S_{j-1} - k_{WL} \sum_{J=0}^{j-1} W^L_J \right]$$

$$+ I^{NIBQ}_{j-1} \} \theta_{p^{13}} \quad (138)$$

$$\{ \phi^3 \} - M\theta_{p^{13}} \leq 0 \quad (139)$$

$$\{ \phi^3 \} \theta_{p^{13}} \geq 0 \quad (140)$$

$$1 \geq \theta_{p^{13}} \geq 0 \quad (141)$$

$$\theta_{p^{13}} \text{ integral} \quad (142)$$

e) $\underline{j \neq o}$

$\underline{j = 2, 6, 10, 14 \text{ etc}}$

$$- \sum_{B=1}^{B^1} \Big[ z_1 R_{B,j-1} + R_{Bj} - z_1 R_{Bj} - I_{Bj} - z_2 P_{B,j-1}$$

$$- P_{Bj} + z_2 P'_{Bj} - z_3 E_{B,j-1} - E_{Bj} + z_3 E_{Bj} \Big] x_B$$

$$+ D_{j-G} \left( \frac{b_{j-G}}{1-b_{j-G}} \right) + L_j - L_{j-1} - K_L L_{j-1} (1-b_j)$$

$$- W^S_j + W^S_{j-1} + K_{WS} W^S_{j-1} (1-b_j) - W^L_j$$

$$+ K_{WL} \sum_{J=0}^{j-1} W^L_J (1-b_j) + D_j - E^Q_j$$

$$+ \{ [ -K_L L_{j-2} + K_{WS} W^S_{j-2} + K_{WL} \sum_{J=0}^{j-2} W^L_J ] b_{j-1} \} \theta_{p^{14}}$$

$$+ N^{MCTA}_{j-g'} + N^{MCTD}_{j-g'} - N^{MCTC}_{j-4-g'} + N^{MCTC}_{j-g'} - N^{MCTB}_{j-4-g'}$$

$$+ N^{MCTB}_{j-g'} - N^{MCTA}_{j-4-g'} = K_j \qquad (143).$$

$$- I^{NTBQ}_{j-1} - M\theta_{p^{14}} \leq o \qquad (144).$$

$$- I^{NTBQ}_{j-1} \; \theta_{p^{14}} \geq o \qquad (145).$$

$$\{ \phi^4 \} \; \theta_{p^{14}} \geq o \qquad (146).$$

$$1 \geq \theta_{p^{14}} \geq o \qquad (147).$$

$$\theta_{p^{14}} \text{ integral} \qquad (148);$$

$$S^{ECS}_{Bj} = R_{Bj} - P_{Bj} - E_{Bj} + z_4 P_{Bj} - z_4 P_{B,j-1}$$

$$+ z_5 \left[ z_4 P_{B,j-1} + P_{Bj} - z_4 P_{Bj} + E_{Bj} + \Delta_{Bj} \right]$$

$$- z_5 \left[ z_4 P_{B,j-2} + P_{B,j-1} - z_4 P_{B,j-1} + E_{B,j-1} + \Delta_{B,j-1} \right] \quad (149),$$

f) For $j \neq 4, 8, 12, 16$ etc.

$$N^{MCT}_j = o \quad (150),$$

g) Upper bound limit for projects

$$x_B \leqslant 1 \quad (151),$$

h) Financing limits, for $j = o, \ldots n$

$$W^S_j \leqslant W^{S\ limit}_j \quad (152),$$

$$W^L_j \leqslant W^{L\ limit}_j \quad (153),$$

$$E^Q_j \leqslant E^{Q\ limit}_j \quad (154),$$

i) 'Pre-tax' non-negativity constraints

$$x_B \geqslant o \quad \text{for all B} \quad (155),$$

similarly $D_j$, $E^Q_j$, $W^S_j$, $W^L_j$, $L_j$

for all j .

# CHAPTER 8

## Conclusion

The case for tax neutrality has been supported in that it is consistent with the minimisation of excess burden, which would otherwise arise from a loss of social utility through changes in economic choices distorted by taxation. Since the theory of finance is in turn concerned with the efficient allocation of resources over time by firms and individuals, the effects on corporate investment and financing decisions of the imposition of personal and corporate taxation is of crucial importance to corporate financial management.

In order to isolate the effects of taxation from other imperfections, capital markets were initially assumed to be otherwise perfect. Given perfect capital markets the separation of investment and financing decisions ensue. But under the imputation tax system this separation theorem does not hold. Investment and financing decisions are no longer independent even in capital markets which are perfect apart from tax complexities. The study began at the point where the existing theory of the one-period Capital Asset Pricing Model has presently reached under perfect capital markets. This enabled us to provide insights into the isolated tax effects under conditions of risk on (i) investment decisions and (ii) financing decisions.

It was shown that there exists a potential excess burden on the activities of the private sector through the effect of imperfect tax allowances on capital expenditure. Except in the cases where taxable profits are large enough to absorb 100 per cent tax depreciation on plant and machinery and scientific research, the present system of capital allowances may reduce the expected return to an amount below that required by the post-tax level of risk.

Under the present tax system there exists a multiplicity of marginal tax rates for investment decisions. These depend in particular on the level of allowances, pre-depreciation accounting profits, dividend policy, stock appreciation relief and double taxation of foreign profits. It was stated that expenditure incurred on some projects may reduce net taxable income to such an extent that the marginal tax rate for the next project being considered is now different. Furthermore the claiming of stock relief, or reductions in stock on one project, may cause the marginal tax rate on another project to alter. The solution therefore requires that all combinations of projects be jointly considered.

As to the financing decision, it was demonstrated that ignoring personal taxation the single period CAPM supports the view that with tax deductibility of interest payments, maximum financial leverage is predicted in partial equilibrium; and it was not necessary to assume that debt capital is risk-free. By including personal taxation as well, a complex equation was derived to express the relationship between the after-tax valuation of the levered firm and that of a firm financed wholly by equity, assuming each firm to have the same pre-tax risk attached to the operating cash flows. Two sufficient conditions for a neutral tax system are found. One of these conditions is that the basic rate of income tax should be the same as the full rate of Corporation Tax. It was noted that under the present imputation tax system, this is outside the draft EEC Directive. The second condition requires that the higher rate of income tax should be uniquely determined according to the corporate tax rate and the rate of capital gains tax. However, with heterogeneous tax rates capital structure was found to be irrelevant in market equilibrium.

With perfect capital markets and a world of certainty there exists a basic preference for debt finance vis-à-vis new issues of shares even with ACT set-off restrictions under partial equilibrium. However (i) behavioural restrictions on excessive leverage, perhaps through the increased possibilities of takeovers resulting from a relatively smaller equity base, and (ii) with bankruptcy costs through a short-fall between going concern values and asset scrap values, there would appear to be a limit to excessive leverage.

The borrowing versus retention decision was found to be complex in partial equilibrium. In general, (i) the greater the discount rate, (ii) the lower the corporate tax rate and (iii) the longer the shareholding period, then the lower the value of the higher rate of income tax above which a retention is preferable to borrowing. The decision was found to be very sensitive to the marginal rate of Corporation Tax at which the debenture interest is relieved. Where heavy capital allowances wipe out taxable income before allowances, then a retention can be preferable to borrowing where the higher rate of income tax is greater or equal to the basic rate of income tax.

In periods of disequilibrium it was shown that financing policies should vary over time even when higher and basic income tax rates, shareholding periods and capital gains tax rates are held constant. These arise from the effect of capital investment decisions on (i) Advance Corporation Tax set-off restrictions,

(ii) the determination of the extent of the carry forward of debenture interest relief and (iii) the marginal tax rate at which the relief is obtained. Tax inter-activities between projects and between the total investment programme and the firm's financing decision policies, were subsequently analysed within the framework of a programming model.

It may be observed that the programming constraints to accommodate taxation arose out of the tax laws and not out of specific assumptions on capital rationing. This supports the view expressed earlier that a joint solution to investment and financing decisions is required even if capital markets are perfect in the absence of taxation.

Since the model presented can be reduced in size to eliminate capital rationing constraints, it is felt that it was more useful not to exclude capital rationing from the final model. The main point here is that the rationing due to taxation is essentially that of capital allowances rather than external capital. I am informed that a number of substantial firms, such as Imperial Chemical Industries, General Electric Company, Fisons, Bowaters and Rowntree Mackintosh, are confronted with this real problem of capital allowance rationing. If there are large balances of capital allowances brought forward then it is arbitrary which division or factory may receive the benefits of not paying tax on the profits from new projects. Alternatively it is arbitrary which unit may receive the benefits of being able to offset capital allowances from new projects against guaranteed taxable profits from other existing projects.

For the efficient allocation of resources it would seem that the decision whether to accept a project would need to be made centrally. For many firms this would violate the managerial philosophy of decentralisation of decision-making. Yet with divisional autonomy some projects in one division may be rejected because of capital allowances carried forward whilst other projects from another division may be rejected because of high marginal tax rates on future profits. By contrast the treasurer's department of the group may decide that both projects should be undertaken such that the capital allowances from one division may be offset against the profits from the other division, increasing the present value of the capital allowances for the first division and reducing the marginal tax rate for the second. By not letting divisions borrow capital allowances the value of the business as a whole may be reduced through the rejection of projects which are financially attractive for the group as a whole after tax.

For one division the acceptance of some projects requiring heavy investments in trading stocks may be sensitive to the claiming of stock appreciation relief. Other divisions may be no longer investing in projects with heavy stockbuilding perhaps to the extent that stock levels are substantially falling. Because of this position the group as a whole may be subject to stock clawback. On this basis the decision to accept the projects requiring heavy investments in stocks may become suboptimal.

Since the dividend policy of the firm as a whole, and its borrowing or lending decisions, are affected by investment decisions within the group, then the treasurer's department would need investment plans from the divisions. In turn the divisions require statements regarding, inter alia, Advance Corporation Tax setoff restrictions in order to help determine marginal tax rates for investment decisions. Consequently, an iterative process is required if (i) divisions are to retain a large degree of autonomy and (ii) suboptimal decisions for the group as a whole are to be minimised. Thus the divisions need to submit broad statements concerning projections of capital investment outlays, net periodic adjustments in trading stocks and profits, all on a pre-tax basis. From these plans the treasurer's department may make broad projections primarily concerning dividend payments, ACT setoff restrictions, debenture interest paid and received, new issues of debt and equity, debt repayments, stock relief, capital allowances and taxable profits. Consequently, marginal tax rates for projects may be estimated over the foreseeable future, together with estimates of the extent to which capital allowances may be offsettable against profits from different divisions. When this is conveyed to the divisions, revised investment and profit plans can be submitted to the treasurer's department and so the process may continue until a satisfactory equilibrium is hopefully reached. This does not preclude suboptimal decisions, since there may be tendencies to bias the estimates of costs and profits perhaps to understate profitability to safeguard being criticised in the future for underperformance. Furthermore, projects which appear financially unattractive before tax, and which may later prove to be financially attractive after tax, may never be accommodated in any financial plans submitted to the treasurer's department. They may be eliminated at an earlier stage and not reconsidered.

Finally, it is the author's view that one of the criticisms of the current state of the theory of business finance is that it ignores the complexities of the UK imputation tax system which in turn creates significant relationships between corporate investment and financing decision variables. It is hoped that this piece of research has made some progress in remedying this deficiency.

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