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# Competition in Posted Prices With Stochastic Discounts[*][†]

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## Abstract

We study price competition between firms over public list or posted prices when a fraction of consumers (termed 'bargainers') can subsequently receive discounts with some probability. Such stochastic discounts are a feature of markets in which some consumers bargain explicitly and of markets in which sellers use the marketing practice of couponing. Even though bargainers receive reductions off the posted prices, the potential to discount dampens competitive pressure in the market, thus raising all prices and increasing profits. Welfare falls because of the stochastic nature of the discounts, which generates some misallocation of products to consumers. We also find that stochastic discounts facilitate collusion by reducing the market share that can be gained from a deviation.


**Keywords**: Posted prices; list prices; collusion; bargaining; negotiation; haggling; discounting; coupons; price takers.

**JEL Classification**: C78; D43; L13.

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# 1 Introduction

Firms often offer discounts off public list or posted prices. For example, Busse et al. (2006) document the widespread use of price discounts against fixed list prices by car dealerships in California, while Goldberg (1996), Scott Morton et al. (2011) and the United Kingdom's Competition Commission (2000, Appendix 7.1) document that discounts in the private automobile market vary significantly across consumers, with some consumers receiving no discount at all off the list price. A similar pattern holds for discounts off estate agent (realtor) fees in the United Kingdom: the Office of Fair Trading (2004, Section 4.48) found that almost 50% of house sellers using an estate agent had tried to negotiate fees, with 80% of those receiving a reduction. Public commentators often refer to evidence of bargaining off the list price in retail stores such as jewellers, shoe shops, travel agents, furniture stores and electrical retailers (see, e.g., Sunday Times, 2008, and Daily Telegraph, 2009). In these examples, bargainers secure a reduction with some probability that depends on the specifics of the interaction between the consumer and the given sales representative. Such stochastic discounting is also a feature of the marketing strategy of offering discount coupons, which is widespread: Musalem et al. (2008) report that 286 billion coupons were issued in the United States in 2006. Coupons or vouchers can be delivered by direct mail, or by being placed in media outlets such as newspapers or the Internet; these reductions, however, are discovered by the target consumers only with some probability.

In this paper, we study the effect of discounting on price competition between firms when a fraction of consumers (termed 'bargainers') can be strategically offered discounts off list prices that they receive with some probability. We find that even though bargainers often receive reductions off list prices, stochastic discounts raise all prices and cause a misallocation of goods to consumers that lowers total welfare. Furthermore, when the firms interact repeatedly, discounts facilitate collusion by the firms.

We develop a tractable model of differentiated product price competition followed by strategically chosen stochastic discounts. First, the two firms simultaneously set list prices that become common knowledge. 'Price takers' buy at list prices. After the list prices become known, each firm can also choose to offer a discount price that 'bargainers' secure with some probability less than one. We call this probability the 'discount reliability'. Both categories of consumer are uniformly distributed along a Hotelling line, and so share a common view of product differentiation.

In Section 3.1, we offer two specific interpretations of this model of stochastic discounts, which motivate the assumption that discounts are received stochastically. We provide a summary here:

- Our leading interpretation is that the model captures explicit bargaining in a simple and tractable way. 'Price takers' do not attempt to bargain, while 'bargainers' approach both firms to ask for a better price than the one posted. A bargainer receives a particular firm's reduced price offer with probability less than one. This assumption captures in a simple way the fact that bargaining is uncertain: the psychological costs and tension of bargaining and the danger of frayed emotions lead to the possibility that negotiations between the sales representative and the consumer break down.

- Our second interpretation is that the model captures the use of discount coupons. 'Bargainers' regularly visit websites or newspapers in which firms offer discount coupons, while 'price takers' do not. The assumption that discount price offers are received with probability less than one captures the fact that the bargainers do not always find a firm's coupon (e.g., due to inattention, not visiting the website at the right time, not getting the right issue of the newspaper).

Stochastic discounts affect the optimal pricing strategy in important ways. In particular, we demonstrate that list prices and the stochastic discounts act as strategic complements. The reason is that a firm's list price is relevant not only to price takers, but also to bargainers who fail to receive the firm's own discount price offer (e.g., who fail to discover the firm's discount coupon, or who fail to close a deal with the firm's sales representative). More specifically, the firm's list price will compete with the rival firm's discount price for bargainers who fail to receive the firm's own discount price offer but who do receive the rival's discount price. Thus, when a firm raises its list price, it becomes less of a competitive threat for such bargainers, which in turn gives the rival firm an incentive to increase its discount price (the strategic complementarity). This softening of the competition for bargainers leads to higher equilibrium discount prices at the discounting stage, and thus to higher profits from bargainers. Anticipating these benefits encourages the firms to raise their list prices, and so in equilibrium list prices and discount price offers rise above the standard (Hotelling) level.

The moderating influence on competition brought about by bargainers and stochastic discounts not only increases prices, but also raises profits and lowers both consumer surplus and welfare. Prices are a transfer from consumers to firms: hence higher prices are welfare neutral. However, stochastic discounts also lower welfare: a bargainer who happens to receive a discount price offer from only one firm might be left with the choice between paying a high list price for the product she prefers and a lower discounted price for the less attractive product. The consumer can, in effect, be bribed to accept a less-ideal product. This generates some misallocation of products to consumers, which lowers the efficiency of the market. Furthermore, as the proportion of bargainers in the market increases, all prices rise at an increasing rate (both list prices and discount price offers), and welfare falls at an increasing rate.

We also study the effect of stochastic discounts on the ability of the industry to sustain collusive outcomes, thus extending our work to a dynamic setting. To simplify the analysis, we consider the case where the products are perfect substitutes. However, we also extend our analysis by allowing any number of firms $N \geq 2$ to compete in the market. We find that discounts facilitate collusion. The mechanism is that discounts lower the profits available from deviating on a collusive agreement. If a firm deviates only in the discounts that it offers, it foregoes any increase in the market share of price takers. If, instead, a firm deviates by lowering its publicly posted list price, then the rival firms can respond by discounting more aggressively. In either case the increase in market share available to firms from a deviation is reduced, thus allowing collusion to be sustained at lower discount factors than would be possible without discounting.

To illustrate the insights of our analysis, consider the setting of the Harvard Business School case, *The Toy Game* (Brandenburger, 1995). As the case notes, collecting miniature cars is a game to children, but it is big business for Matchbox and Hot Wheels, two leading brands. Matchbox and Hot Wheels compete in retail prices; they also decide how aggressively to offer rebates via discount coupons. The case simplifies by studying couponing when children have observable idiosyncratic preferences over the cars, which can be used as the basis for price discrimination and targeted coupon delivery. We avoid this simplification in our analysis. Suppose instead that the firms cannot observe the children's preferences and can only imperfectly deliver coupons to the children (e.g., by placing coupons in the press or on the Internet). Some, but not all, children will pick up a coupon for one or even both cars. Our analysis delivers answers to many natural questions that one might ask of couponing competition in this environment. For instance, we will show that couponing raises prices to those children who buy at the list prices, and to those who get a coupon. Furthermore, welfare declines, since children who find only one coupon may buy the car that they prefer least on account of the price differential between list and coupon prices. We will also show that when there is no differentiation between the cars, couponing facilitates collusion between Matchbox and Hot Wheels.

The paper proceeds as follows: Section 2 relates our results to the existing literature; Section 3 sets out the model; Section 4 characterizes the equilibria; Section 5 conducts comparative statics on prices, profits and welfare; Section 6 considers collusion; Section 7 discusses robustness; and Section 8 concludes. All proofs are relegated to the Appendix.

## 2 Relation to the literature

It is common for consumers to receive discounts off list prices: in Section 1 we noted evidence from a number of sources of stochastic discounting in automobile retailing, realtor fees, travel agents and furniture stores, among others. Our model contributes to the economics and marketing literature on differentiated product price competition by characterizing the effect of strategically chosen stochastic discounts off previously posted list prices on those list prices, discount price offers, profits and welfare.

A small and recent literature considers the consequences of discounts and bargaining when some consumers take list or posted prices as given. Korn (2007) and Zeng et al. (2007) consider monopolists, and so are silent about the implications of discounts on competitive outcomes, which is the focus of this study.[1] Desai and Purohit (2004) and Zeng et al. (2007) focus on the marketing decision of whether to permit bargaining or not. In Desai and Purohit (2004), when both firms permit bargaining, the list prices are irrelevant to the bargainers since they are never effective outside options; thus the strategic interaction between list prices and discount prices is severed. Raskovich (2007) finds that a big enough proportion of bargainers causes list prices to jump from marginal cost to the monopoly price. The mechanism is different than ours: Raskovich (2007) assumes the firms that post higher list prices are weaker bargainers and so are more attractive to bargaining consumers. Finally, in complementary work, Gill and Thanassoulis (2009) also find that bargaining can raise prices. However, the setup in Gill and Thanassoulis (2009) is rather different: (i) the firms compete in quantities, with a Cournot auctioneer setting a single public list price, and so the firms are not able to compete directly in list prices; (ii) instead of using stochastic discounts, bargaining is modeled as an application of Burdett and Judd (1983) search in which bargainers differ in the number of firms they approach, giving rise to mixed-strategy equilibria instead of the pure-strategy equilibria that we find here; and (iii) the products are assumed to be homogeneous, thus precluding our analyses of the welfare loss due to product misallocation and of how the effect of bargaining on prices, profits, consumer surplus and welfare changes with the degree of product differentiation. Furthermore, Gill and Thanassoulis (2009) do not study the effects of bargaining on profits or welfare: they were only able to show partial and ambiguous results for consumer surplus alone in special limiting cases; instead, we are able to show that the presence of bargainers always increases profits while lowering consumer surplus and welfare, and we can show that welfare is always falling as the proportion of bargainers increases. Finally, we extend our analysis to repeated interaction and

[1] Kuo et al. (2011) and Kuo et al. (2012) also consider monopolists, with a focus on, respectively, inventory management and supply chain relationships. As well as setting a posted price, in these papers the monopolist is allowed to commit to a lower bound below which it will never sell.

demonstrate that bargaining facilitates firm collusion, while Gill and Thanassoulis (2009) only consider one-shot competition.

Our analysis demonstrates that the presence of bargainers can be bad for welfare and can raise equilibrium list and discount prices. Our results therefore provide a possible justification for the findings of Davis and Holt (1994) and Cason et al. (2003), whose experiments show that when consumers can haggle below a posted price, prices tend to be higher and efficiency lower.

Much of the rest of the literature exploring bargaining in consumer markets examines the choice between committing to a fixed price and allowing consumers to bargain in the absence of a posted price (e.g., Bester, 1993, Wang, 1995, Arnold and Lippman, 1998, Camera and Delacroix, 2004, and Myatt and Rasmusen, 2009.) Chen et al. (2008) estimate a structural model of competing bargained prices in a setting with no posted prices. There is also a small literature on bargaining below a posted price when all consumers bargain (e.g., Chen and Rosenthal, 1996a, 1996b, and Camera and Selcuk, 2009). Although our results apply in the special case when all consumers are bargainers (see Section 7.4), these prior works are not special cases of ours since they include assumptions orthogonal to the ones we build on. In Chen and Rosenthal, (1996a, 1996b), buyers have to incur inspection costs to find out the value of the good, and so the list price acts as a commitment to not exploit the bargainer after inspection costs have been incurred. Chen and Rosenthal (1996b) consider only the case of a monopolist. Chen and Rosenthal (1996a) also consider the case of duopolists who have just one good for sale each; as a result, as soon as one firm trades the other becomes a monopolist. In Camera and Selcuk (2009) the discount price is not chosen strategically, but is a function of the excess demand at any given seller, and so rises if more buyers decide to approach a seller.

Our analysis also sheds light on the marketing practice of couponing, which as noted in Section 1 is widespread. In particular, our contribution is to offer insight into how the opportunity to offer coupons may alter competitive outcomes, both in terms of the depth of discounts and the equilibrium level of the list prices themselves. Many prominent analyses of couponing focus on monopoly settings, and so do not study the impact of coupons on competitive outcomes (e.g., Narasimhan, 1984, Anderson and Song, 2004). A few papers consider competitive couponing. Shaffer and Zhang (1995) assume that coupon reductions are set simultaneously with list prices, thus severing the strategic interaction we study between list and reduced prices. Narasimhan (1988) studies competition mainly in a setting in which firms can set only one price. An extension (Narasimhan, 1988, p.439) considers firms that deliver coupons to a subset of consumers. However, the consumers that the firms compete over (the 'switchers') receive each firm's coupon for sure; thus, the list prices do not form a relevant competitive constraint, and so the strategic interaction between the coupons and list prices does not arise. Dhar et al. (1996) consider the profitability of different types of coupons, but only one firm is allowed to use coupons, and the choice of list price is not considered. Finally, Rao (1991) studies competition in shelf-price reductions off previously chosen 'regular' prices (which act like coupons received by all consumers) between a national brand (preferred by all consumers) and a local brand: the model is used to show that the local brand does not discount in equilibrium.

Discounts off list or posted prices also occur in vertically-related industries. In healthcare, Sorensen (2003) documents that in Connecticut many hospitals negotiate reductions off list prices, with variation in the extent of the discounts. In retail petrol, Cook (1997) notes that discounts are sometimes used as an inducement for the downstream firm to trade exclusively with the upstream firm. Competition in these types of markets is different to the setting we study here, since the buyers are competing among themselves for business downstream; nonetheless, the findings of our study may provide some insight into how bargaining affects competition at the upstream level.

To the best of our knowledge, we are the first to study dynamic repeated competition in markets with discounts and list prices, and so the mechanism by which discounts facilitate collusion in markets with public list prices is new. Our result complements the existing literature that shows how collusion can be facilitated when firms operate in more than one market. Bernheim and Whinston (1990) find that competing in multiple markets can make collusion easier since firms are able to transfer excess punishment capacity from one market to another. This theoretical prediction has been confirmed: the empirical literature provides evidence that multimarket contact helps collusion in numerous industries such as airlines (Evans and Kessides, 1994), telephony (Parker and Röller, 1997) and cement (Jans and Rosenbaum, 1997). Spector (2007) shows that if a firm is a monopolist in one market but competes in another, then bundling can help collusion by shrinking the demand available in the competitive market.[2] In both cases, linkages across markets make collusion more sustainable, while our complementary results show that strategic linkages across segments within a single market can also make collusion easier to sustain. In particular, we find that price deviations aimed at the price taker segment induce a competitive response in the bargainer segment that reduces the incentive to deviate from any collusive agreement.

Our analysis also complements a broader literature in which firms sell to two different types of consumer. In Stahl (1989)'s model of search, consumers have high or low search costs. In Rosenthal (1980), Varian (1980) and Narasimhan (1988), some consumers only consider buying from one firm while others buy at the lowest price. In these papers the firms are unable to

[2]In contrast, Montero and Johnson (2012) identify a setting with multiple markets in which collusion is inhibited by bundling.

price discriminate between consumers, and this leads to mixed-strategy pricing equilibria. In this paper, the competing firms are able to discriminate between consumers (bargainers and price takers) by setting both list and discount prices, and our model yields pure-strategy pricing equilibria in the one-shot game.[3] Our price and welfare results can therefore be interpreted as shedding light on the implications of competitive price discrimination; Section 7.9 develops this point, and explicitly links our findings to the existing literature on price discrimination.

# 3 The model of stochastic discounts

Two competing firms sell a differentiated product and compete in prices. The firms have the same constant marginal cost of production $c \geq 0$, have no fixed costs, and seek to maximize their expected profits. To capture product differentiation, we adopt the standard Hotelling framework: the two firms are located at the opposite ends of a Hotelling line of length 1 with a uniform density of consumers along it, and the consumers have a linear Hotelling 'transport cost' $t > 0$. As in Hotelling (1929), every consumer purchases exactly one unit and the market is always covered. There are two types of consumer. A proportion $\mu \in (0, 1)$ are 'price takers', and the remaining proportion $1 - \mu$ are 'bargainers'. A consumer's type is independent of her location on the Hotelling line, and firms cannot observe a consumer's location. We capture stochastic discounts through the following two-stage game:

1. List-price-setting stage: The firms simultaneously choose publicly posted list prices $l_i \geq 0$ and $l_j \geq 0$. Each price taker purchases at the list price that gives her the highest surplus net of transport costs.

2. Discount stage: The firms simultaneously choose discount prices $p_i \in [0, l_i]$ and $p_j \in [0, l_j]$. Each bargainer receives a particular firm's discount price offer with probability $\beta \in (0, 1)$; we call this probability the 'discount reliability'. If the bargainer does not receive the price offer, she is still able to purchase at the public list price.[4] Each bargainer buys at the available price that gives her the highest surplus net of transport costs.[5]

In Section 7 we discuss the importance of various assumptions of the model for our results, beginning in Section 7.1 with a discussion of the assumption that discounts are stochastically received (i.e., that $\beta \in (0, 1)$).

[3]When we study collusion with perfect substitutes in Section 6, we do find that immediately following a deviation in list prices, and so off the equilibrium path, the firms set discount prices according to a mixed strategy. The mechanism giving rise to mixing for this segment is related to that which gives rise to mixing for the whole market in these earlier papers.

[4]The random draws that determine whether discount price offers are received are independent across firms and bargainers.

[5]All consumers randomize in the event of a tie.

### 3.1 Two interpretations

We offer two specific interpretations of this model of stochastic discounts.

#### 3.1.1 Explicit bargaining

Our leading interpretation is that the model captures explicit bargaining. Reflecting real-world bargaining, bargainers actively approach both firms to ask for a better price than the one posted. The firms simultaneously choose the discount prices to offer to bargainers, which act as final take-it-or-leave-it offers. The game timing and strategic choice over the price to offer bargainers capture that firms can adjust their bargaining policy in response to their rival's choice of list price. The assumption that discount price offers are received with probability $\beta \in (0, 1)$ captures that bargaining may break down or that sales staff may differ in their willingness and ability to negotiate and give discounts.[6] However, the list prices are binding and thus are always available.

Under this interpretation, the difference between price takers and bargainers can be motivated by consumers having either high or low personal costs of bargaining.[7] We can think of price takers as consumers who suffer significant bargaining costs: the costs could be real, e.g., time costs, or psychological, e.g., the embarrassment of starting a negotiation; alternatively, the price takers are not aware that discounts might be available. Bargainers, on the other hand, have low costs of bargaining and are aware that firms are willing to negotiate. Given their low costs of bargaining, it is natural to assume that the bargainers approach both firms for a better price offer.

#### 3.1.2 Discount coupons

Our second interpretation is that the model captures settings in which firms offer discount coupons through media sources, such as websites or newspapers, that are regularly visited by 'bargainers'. Discount coupons allow consumers to buy at a fixed discounted price. The assumption that discount price offers are received with probability $\beta \in (0, 1)$ captures that bargainers who visit the media source do not always find a firm's coupon. Instead of being passive recipients of coupon discounts, we could also think of the bargainers as consumers with a low cost of time who find it worthwhile actively to seek out discount opportunities, while price takers have a high cost of time and so do not pay attention to, or search for, discount coupons. To similar

[6]Bargaining models with an exogenous probability of breakdown have been studied by, for instance, Binmore et al. (1986), Stole and Zwiebel (1996) and de Fontenay and Gans (2005).

[7]This heterogeneity in bargaining costs parallels the heterogeneity in personal search costs adopted in the search literature. For example, in Stahl (1989) search costs are low or high. Note, however, that under this interpretation bargainers are doing more than searching: they are actively inviting sellers to beat their publicly posted list prices.

effect, bargainers may suffer shocks to their available time so that their ability to collect each firm's discount coupon, or to redeem it before it expires, is random.

## 4 Equilibrium analysis

We proceed by backward induction to find the symmetric pure-strategy subgame-perfect Nash equilibria of the two-stage game.[8] In order to prove the existence of equilibrium, we make the action space compact: we do so by restricting list prices to lie at or below marginal cost plus twice the transport cost, that is $l_i, l_j \leq 2t + c$.[9] We also restrict all prices to lie at or above marginal cost, that is $l_i, l_j, p_i, p_j \geq c$.[10] In Proposition 1 we show that there exists a unique pure-strategy Nash equilibrium at the discount stage for any combination of list prices chosen at the list-price-setting stage. Given these equilibrium discount price offers, in Proposition 2 we show that there exists a unique symmetric pure-strategy Nash equilibrium at the list-price-setting stage with list prices different from the standard Hotelling level $t + c$. An equilibrium at the standard Hotelling level also exists, but we explain below why we do not find such an equilibrium compelling. Throughout this section, when we wish to identify the firms according to their list prices, we use firm 1 and firm 2 to denote that $l_1 \geq l_2$.

We start by characterizing the equilibrium discount price offers as a function of the list prices chosen at the list-price-setting stage.

[8] The two list prices are common knowledge; furthermore, in the equilibria we study they are equal. This holds throughout the manuscript. Nonetheless, we need to solve the discount stage given unequal list prices to allow us to consider the effect of deviations at the list-price-setting stage.

[9] Proving existence is challenging because the profit function at the discount stage is the sum of two quasi-concave functions. As a result, best-response functions at the discount stage might be discontinuous, and so equilibrium discount price offers might not be continuous in the list prices chosen at the list-price-setting stage. When we make the action space compact by restricting $l_i, l_j \leq 2t+c$, we are able to show that there exists a unique pure-strategy equilibrium at the discount stage and that the equilibrium discount price offers are continuous in the list prices chosen at the list-price-setting stage. A compact action space might arise naturally if: (i) there exist potential entrants that will enter the market profitably if prices rise too high or (ii) consumer valuations are bounded. Finally, we note that the equilibrium prices are always far from the upper bound: it is straightforward to show that the equilibrium prices in (2) are always closer to the standard Hotelling level of $t + c$ than they are to the upper bound of $2t + c$.

[10] This assumption is without loss of generality, since pricing at marginal cost gives weakly higher profits than pricing below marginal cost.

**Proposition 1** *Given any list prices $l_1 \geq l_2$ such that $l_1, l_2 \in [c, 2t+c]$, there exists a unique pure-strategy Nash equilibrium at the discount stage given by:*

*1.*
$$p_1^* = \frac{1}{2-\beta}(t+c) + \left(1 - \frac{1}{2-\beta}\right)\left(\frac{2l_2 + \beta l_1}{2+\beta}\right)$$
$$p_2^* = \frac{1}{2-\beta}(t+c) + \left(1 - \frac{1}{2-\beta}\right)\left(\frac{2l_1 + \beta l_2}{2+\beta}\right)$$

*when* $\frac{1}{2-\beta}(t+c) + \left(1 - \frac{1}{2-\beta}\right)\left(\frac{2l_1+\beta l_2}{2+\beta}\right) \leq l_2$; *and*

*2.*
$$p_1^* = \min\left\{\frac{t+c+l_2}{2},\ l_1\right\}$$
$$p_2^* = l_2$$

*when* $\frac{1}{2-\beta}(t+c) + \left(1 - \frac{1}{2-\beta}\right)\left(\frac{2l_1+\beta l_2}{2+\beta}\right) > l_2$.

Proposition 1 shows that the equilibrium discount price offers are increasing in the list prices that were set at the list-price-setting stage. To understand this result, we present here a firm's best-response function in the discount stage when the rival sets a discount price at or above the standard Hotelling level (derived in Lemma 6 within the proof of Proposition 1):

$$p_i^* = \min\left\{\frac{(t+c) + (1-\beta)\,l_j + \beta p_j}{2},\ l_i\right\} \quad \textit{when} \quad p_j \geq t+c. \tag{1}$$

Recall that a firm's discount price $p_i$ is bounded above by the firm's list price $l_i$, which was set at the list-price-setting stage. At an interior best response in the discount stage, firm $i$ is competing for: (i) bargainers who receive the rival's discount price offer $p_j$; and (ii) bargainers who do not receive the rival's discount price offer, and so can buy from the rival only at its list price $l_j$. The probability that a bargainer receives the rival's discount price offer is $\beta$, and so in expectation the firm is competing against a price of $(1-\beta)\,l_j + \beta p_j$. The best-response function (1) shows that firm $i$ undercuts this expected price by taking an average of this expected price and the standard Hotelling price of $t+c$.

Thus, if firm $j$ raises its list price at the list-price-setting stage, then firm $i$ has an incentive to increase its discount price in the discount stage, since firm $j$'s list price is less of a competitive threat in the competition for bargainers who do not receive the rival's (that is, firm $j$'s) discount price offer: $dp_i^*/dl_j = (1-\beta)/2 > 0$. This softening of competition at the discount stage induces firm $j$ to increase its own discount price, since $i$'s discount price is less of a competitive threat in the competition for bargainers who do receive the rival's (that is, firm $i$'s) discount price offer: $dp_j^*/dp_i = \beta/2 > 0$. The intersection of the best-response functions yields the equilibrium discount price offers given in Proposition 1. The list prices and discount prices are strategic complements, since an increase in a firm's list price increases both firms' equilibrium discount price offers.

This effect applies to more than just bargaining. In the couponing interpretation of our model, a higher rival list price implies that customers who fail to pick up a rival's discount coupon can be attracted with a less generous coupon, thus pushing down the equilibrium reductions that coupons offer.

This strategic dynamic link between list prices and subsequent discounting is key to the results that we will generate in our analysis. The corollary below describes the equilibrium discount price offers for symmetric list prices.

**Corollary 1** *Given any symmetric list prices $l_1 = l_2 = l \in [c,\ 2t+c]$, there exists a unique pure-strategy Nash equilibrium at the discount stage given by:*

*(a)* $$p_1^* = p_2^* = \frac{1}{2-\beta}(t+c) + \left(1 - \frac{1}{2-\beta}\right) l \in (t+c,\ l)$$

*when* $l > t+c$; *and*

*(b)* $$p_1^* = p_2^* = l$$

*when* $l \le t+c$.

We can see that when the list prices are above the standard Hotelling level of $t + c$, the equilibrium discount prices are a weighted average of the standard Hotelling price and the list prices. Thus, when responding to the bargainers, the firms push their discount price offers away from the standard Hotelling level and up toward the prevailing list prices. How far the equilibrium discount prices are from the list prices depends upon the discount reliability $\beta$. The higher is $\beta$, the greater is the proportion of bargainers that receives the rival's discount price offer. When setting its discount price, a firm trades off the incentive to compete aggressively for bargainers who receive the rival's discount price offer and less aggressively for bargainers who do not receive the rival's discount price offer, and so can buy from the rival only at its list price. As the discount reliability $\beta$ increases, the first set of bargainers grows at the expense of the second set, and so the equilibrium discount prices move toward the standard Hotelling level.

We now turn to the firms' decision as to what list prices to set.

**Proposition 2** *Given $l_i, l_j \in [c,\ 2t+c]$, (i) there exists a symmetric pure-strategy Nash equilibrium at the list-price-setting stage given by:*

$$l^* \;=\; (t+c)+t\left[\frac{2\,(1-\beta)\,(2-\beta)\,\beta}{\left(\frac{\mu}{1-\mu}\right)(4-\beta^2)\,(2-\beta)+2\,(1-\beta)\,(4-2\beta+\beta^2)}\right] \in\; (t+c,\ 2t+c)\,, \quad (2)$$

*and (ii) there are no other symmetric pure-strategy equilibria with $l \neq t+c$.*

If the firms considered only the price takers, prices would be set just as in the standard Hotelling model, and so the only equilibrium list prices would be $l^* = t + c$. However, when thinking about how to set list prices, the firms also anticipate how the chosen list prices will affect competition at the discount stage. We saw above that list prices and discount prices are strategic complements: the higher the list prices, the weaker the competitive challenge for the custom of those bargainers who do not receive the stochastic discount from the rival, and so the higher the discount price offers that can be supported in equilibrium. This effect implies that the presence of the bargainers moderates competitive forces in the market by reducing the incentive to undercut any given rival list price. Proposition 2 describes the unique equilibrium in which this moderating force allows list prices to rise above the standard Hotelling level of $t + c$.

The equilibrium list price (2) is a function of the proportion of price takers $\mu$ and the discount reliability $\beta$, as well as the more frequently studied parameters of product differentiation $t$ and marginal cost $c$. In Section 5, we will explore in detail the comparative statics of the price levels and welfare with respect to these parameters. Among other results, we will show that the equilibrium list prices and discount price offers always increase as the proportion of bargainers $1-\mu$ goes up, and that the equilibrium list prices and discount price offers are quasi-concave in the discount reliability $\beta$.

Proposition 2 further shows that no equilibrium can exist with list prices below the standard Hotelling level, that is with $l^* < t+c$. If such an equilibrium existed, then a firm could increase profits from both bargainers and price takers by deviating and raising its list price toward the standard Hotelling level.

Remark 1 notes that we cannot rule out an equilibrium in which the firms set list prices at the standard Hotelling level, that is with $l^* = t + c$ (in such an equilibrium, by Corollary 1 the discount prices are also at the Hotelling level).

**Remark 1** *The equilibrium price $t + c$ in the standard Hotelling model without bargainers also constitutes an equilibrium at the list-price-setting stage.*

However, this equilibrium is not a compelling one to study or to expect for at least two reasons. First, from the firms' perspective an equilibrium at Hotelling prices is Pareto dominated by the equilibrium with $l^* > t+c$ (profits in this equilibrium are higher since all prices are higher and the market is covered). Second, the equilibrium is not stable in the following sense: if firm $j$ were to set a list price of $l_j = t+c+\varepsilon$ for some arbitrarily small $\varepsilon > 0$, then at this slightly higher list price firm $i$ would have an incentive to increase its list price beyond $l_j = t + c + \varepsilon$, and thus move its list price even further from the Hotelling level. By contrast, the equilibrium offered in Proposition 2 is stable in this sense.[11] This formalizes the idea that there is something knife-edge about prices at the Hotelling level. At any symmetric list prices between the Hotelling level and the equilibrium with $l^* > t + c$, there is upward pressure on the list prices: an increase in a firm's list price raises the rival's discount price, thus softening competition for the bargainers and increasing that firm's total profits. However, exactly at the Hotelling list prices, the discount prices hit their upper bound, and hence a small upward list price deviation cannot induce a corresponding upward shift in the rival's discount price offer. For both of these reasons it appears to us more defensible to focus on the price-increasing equilibrium. Thus, in the next section, we study the more interesting and Pareto-dominant equilibrium with $l^* > t+c$ in which the bargainers do in fact affect competition between the firms.

# 5 Comparative statics of prices, profits and welfare

In this section, we analyze the properties of the equilibrium with list prices above the standard Hotelling level, that is with $l > t + c$, outlined in Proposition 2.[12] Using Part (a) of Corollary 1 in Section 4, the equilibrium discount price offers are given by

$$p = \frac{1}{2-\beta}(t+c) + \left(1 - \frac{1}{2-\beta}\right) l, \tag{3}$$

a weighted average of the list prices and the standard Hotelling price.

**Proposition 3** *Compared to the benchmark with only price takers, the presence of bargainers: (i) raises the list prices and discount price offers; (ii) raises the firms' profits; (iii) lowers consumer surplus; and (iv) lowers total welfare.*

[11]By construction of the symmetric equilibrium $l^* > t + c$, $[d\pi_i/dl_i]_{l_i=l_j=l^*} = 0$. From the proof of Claim 4 in the proof of Proposition 2, setting $l_i = l_j = l > t + c$, $[d\pi_i/dl_i]_{l_i=l_j=l}$ is a linear function of $l$ with a strictly negative slope. This implies that for $l \in (t + c, l^*)$ we must have $[d\pi_i/dl_i]_{l_i=l_j=l} > 0$ and for $l > l^*$ we must have $[d\pi_i/dl_i]_{l_i=l_j=l} < 0$.

[12]Throughout this section and the related proofs, for notational clarity we omit the stars when referring to equilibrium prices.

Proposition 3 is a key result of our study. Once some of the consumers become 'bargainers', that is they are willing to bargain explicitly or collect coupons, the list prices rise. Section 4 described in detail how the presence of bargainers moderates competition: higher list prices allow equilibrium discount prices to rise, since higher list prices weaken the competitive challenge for the custom of those bargainers who do not receive the stochastic discount from the rival. This strategic complementarity between list and discount prices reduces the incentive to undercut the rival's list price, leading to higher equilibrium list prices, which in turn give rise to higher equilibrium discount prices. As a result, profits must rise, and consumer surplus must fall since the market is covered. This delivers results (i), (ii) and (iii) of Proposition 3. The higher prices are a transfer from consumers to firms, and so are welfare neutral using a total surplus criterion.[13] Nonetheless, welfare falls because of the stochastic nature of the discounts. A bargainer who happens to receive a discount price offer from only one firm might be left with the choice between paying a high list price for the product she prefers and a lower discount price for the less-attractive product. This generates some misallocation of products to consumers, which lowers the efficiency of the market, yielding result (iv) of Proposition 3.

So far we have compared a market with a positive fraction of bargainers to the standard case where all consumers buy at the list prices. Next, we consider the effect of marginal changes in model parameters on prices (Section 5.1) and welfare (Section 5.2).

## 5.1 Prices

We start by reporting the effect of marginal changes in the proportion of bargainers on prices.

**Proposition 4** *As the proportion of bargainers increases: (i) the list prices rise; (ii) the discount price offers rise; and (iii) the difference between the list prices and the discount price offers also rises. Furthermore, all three rise at an increasing rate.*

The moderating influence of bargainers on competition means that both list prices and discount price offers go up as we increase the proportion of bargainers in the market: the list prices become increasingly set to allow profits to be made from the bargainers at the discount stage, which in turn allows the discount prices to rise. Proposition 4 further tells us that the list prices go up faster than the discount prices: the gap becomes larger since discount prices are a weighted average of the list prices and the standard Hotelling price (see (3) above), where the weights do not depend on the proportion of bargainers (given the list prices, discount prices only target the bargainers). Finally, we can see that the prices go up at an increasing

[13] Many competition authorities focus mainly on consumer surplus instead of on the sum of consumer and producer surplus. This is the case in the United Kingdom and the European Union for example. For such market regulators an increase in prices as compared to the Hotelling benchmark would be unwelcome *per se*.

rate: as bargainers become more prevalent in the population of consumers, the reduction in the competitive pressure on list prices becomes increasingly powerful, resulting in a convex increase in the list prices. Since the discount prices are a weighted average of the list prices and the standard Hotelling price, the discount prices also inherit this convexity. The left-hand panel of Figure 1 (between Propositions 7 and 8) portrays this convexity graphically for discount reliability $\beta = 1/2$.

Recall that our model assumes that the proportion of bargainers $1 - \mu \in (0, 1)$; however, as the proportion of bargainers tends to zero, the prices tend to the standard Hotelling level, and as the proportion of bargainers tends to 1, the prices tend to their level when all consumers bargain. Section 7.4 explains in detail how our equilibrium and comparative statics results extend to the case when all consumers bargain.

Next we conduct comparative statics of the prices with respect to the discount reliability $\beta$.

**Proposition 5** *The list prices, the discount price offers, and the difference between the list prices and the discount price offers are quasi-concave in the discount reliability $\beta$. Furthermore, the discount price offers peak at a lower discount reliability than do the list prices.*

First, let us recall how discount prices change in the discount reliability $\beta$, holding list prices fixed. As explained in the discussion following Corollary 1 in Section 4, the discount prices given in (3) are a weighted average of the standard Hotelling price and the list prices, with the weight on the standard Hotelling price increasing in the discount reliability $\beta$. Briefly, as bargainers become more likely to receive the rival's discount price offer, competition for the bargainers becomes more intense, driving the discount prices down relative to the list prices.

Now, let us consider the effect of $\beta$ on the list prices. When a firm raises its list price, there are two competing effects on profits. First, holding discount prices fixed, the firm loses market share among price takers and among those bargainers who fail to receive its discount price offer. Second, the higher list price increases the equilibrium discount prices. Overall, the incentive to increase list prices is highest for intermediate values of discount reliability $\beta$. When the discount reliability $\beta$ is low, the first effect dominates, since few bargainers receive the discount price offers. When the discount reliability $\beta$ is high, the first effect dominates once again: since the bargainers are likely to receive both discount price offers, intense competition for the bargainers drives discount prices toward the standard Hotelling level, and so a list price increase has little effect on the equilibrium discount prices.

Given that the discount prices are a weighted average of the standard Hotelling price and the list prices, the quasi-concavity of the discount prices follows from the quasi-concavity of the list prices, as does the quasi-concavity of the difference between the list prices and discount prices.

Furthermore, the discount prices peak at a lower value of $\beta$ than do the list prices because, as noted above, the weight placed on the list prices falls in $\beta$.

We now consider the comparative statics of prices with respect to product differentiation.

**Proposition 6** *As the degree of product differentiation $t$ increases: (i) the list prices rise; (ii) the discount price offers rise; and (iii) the difference between the list prices and the discount price offers also rises. Furthermore, the list prices and discount price offers rise faster than do prices in the benchmark with only price takers.*

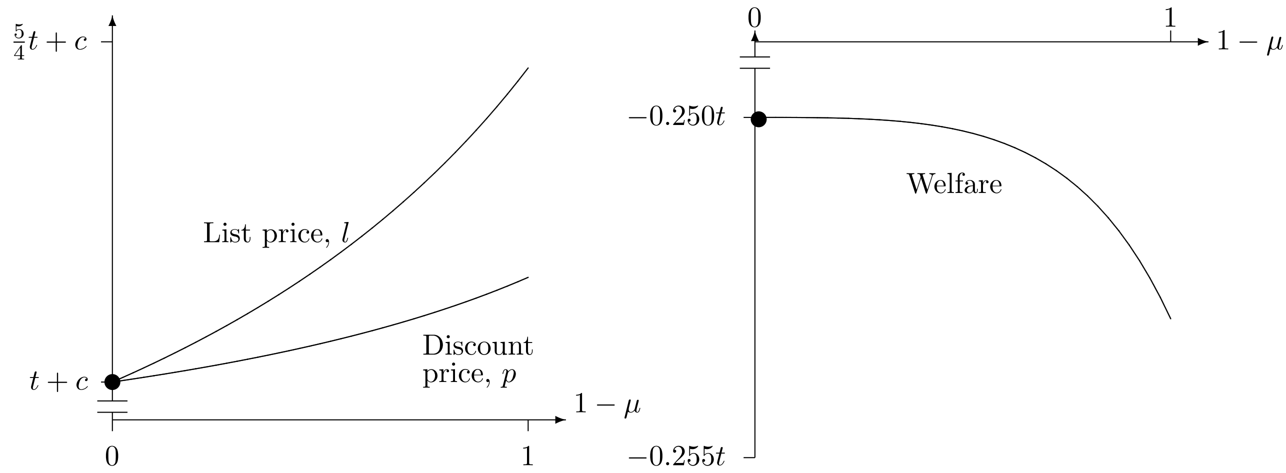
As the degree of product differentiation between the firms increases, price reductions capture a smaller share of consumers, and so competition in both the discount stage and the list-price-setting stage is relaxed. This explains why both prices rise. The strategic complementarity between list prices and discount prices allows prices to rise more rapidly than in the benchmark with only price takers. Finally, the difference between the list prices and discount prices rises since the discount prices are a weighted average of the list prices and the standard Hotelling price, with the weight depending on the discount reliability (see (3) above).

### 5.2 Welfare

We now turn to the comparative statics of welfare, starting with how welfare changes in the proportion of bargainers.

**Proposition 7** *As the proportion of bargainers increases, total welfare falls. Furthermore, total welfare falls at an increasing rate.*

Proposition 7 tells us that welfare is always decreasing in the proportion of bargainers. Since the market is covered, and so prices are just a transfer from consumers to firms, the effect of bargainers on welfare depends only on how the bargainers affect the (mis)allocation of goods to consumers. We noted in Proposition 4(iii) that the greater the proportion of bargainers, the greater the difference between the list prices and the discount prices. This increasing disparity makes bargainers who receive a discount price offer from only one of the firms more likely to settle for a lower-priced but less-attractive product, thus worsening the misallocation of goods. Furthermore, as the proportion of bargainers goes up, some price takers become bargainers who receive only one discount price offer, and who might therefore be tempted to buy a less-attractive but discounted product. Welfare falls at an increasing rate because of the convexity of the difference between list prices and discount prices reported in Proposition 4. Figure 1 shows graphically how prices and welfare change in the proportion of bargainers.




Notes: The proportion of bargainers $(1 - \mu)$ increases from 0 to 1 as the proportion of price takers $\mu$ falls from 1 to 0. The dot in each graph gives the benchmark case of Hotelling competition with only price takers. Both graphs are drawn to scale, with $\beta = 1/2$. The welfare graph normalizes surplus from a product at a consumer's exact location on the Hotelling line to zero.

Figure 1: Prices and welfare as the proportion of bargainers $(1 - \mu)$ changes

Next we explore how welfare is affected by the discount reliability $\beta$.

**Proposition 8** *Total welfare is quasi-convex in the discount reliability* $\beta$.

As noted above, since the market is covered, the effect of bargainers on welfare depends only on how the bargainers affect the (mis)allocation of goods to consumers. This misallocation is increasing in: (a) the difference between the list prices and discount prices, which makes bargainers who receive a discount price offer from only one of the firms more likely to settle for a lower-priced but less-attractive product; and (b) the proportion of bargainers who receive only one discount price offer. We know from Proposition 5 that the difference between the list prices and discount prices is quasi-concave in $\beta$. Furthermore, the proportion of bargainers who receive only one discount price offer, $2\beta(1-\beta)$, is also quasi-concave in $\beta$. Thus, welfare inherits its quasi-convexity in $\beta$.

Proposition 8 implies that a social planner who wished to maximize welfare would prefer extremes of the discount reliability parameter $\beta$. With intermediate discount reliability, many bargainers will receive only one discount price offer, and this price will be significantly below the posted list prices, thus encouraging such bargainers to buy from a seller that is not their best match.

It is not easy to subdivide welfare into the behavior of consumer surplus and profits; however some results are available.

**Proposition 9** *When the discount reliability $\beta$ is not too high, the firms' profits rise and consumer surplus falls as the proportion of bargainers increases.*

List prices, discount prices, and the difference between them, increase in the proportion of bargainers (Proposition 4). The marginal impact on profits and consumer surplus is not clear-cut, since a greater proportion of consumers buy at the discount price. When the discount reliability is not too high, the increase in prices is the dominant factor.

Finally, we turn to the behavior of profits and welfare as a function of product differentiation.

**Proposition 10** *As the degree of product differentiation $t$ increases: (i) the firms' profits rise; (ii) consumer surplus falls; and (iii) total welfare falls. Furthermore, profits rise faster than in the benchmark with only price takers, while consumer surplus and welfare fall faster than in the benchmark.*

We know from Proposition 6 that as the degree of product differentiation increases, the list prices and discount prices rise, and do so faster than in the benchmark with only price takers, while the difference between the list prices and discount prices also goes up. As a result, profits rise faster than in the benchmark. The increase in prices unambiguously increases profits, since the proportion of bargainers buying at the discount prices does not the change: the increased differentiation makes bargainers less sensitive to price differences, and in our model this effect exactly cancels the bigger gap between list prices and discount prices. The higher transport costs cause welfare to fall directly, and also worsen the loss in welfare from misallocated products (even though the extent of misallocation does not change), and so welfare falls faster than in the benchmark. Consumer surplus also falls faster than in the benchmark, given that profits increase faster and welfare falls faster.[14]

# 6 Collusion

Thus far we have analyzed one-shot competition and demonstrated that the presence of bargainers reduces the competitive pressure on firms when setting their list prices, thus allowing list prices, and subsequently discount prices, to rise. In this section we study dynamic competition in markets with both bargainers and price takers. In particular, we study the ability of the firms to collude on a price $z$ that is the same at both the list-price-setting stage and the discount stage (and so $l = p$ under collusion), and we will conclude that the presence of bargainers can facilitate collusion by lowering the critical discount factor above which collusion can be sustained.

[14] The comparative statics with respect to cost are straightforward. Just like in the benchmark with only price takers: (i) increases in marginal cost $c$ lead to a one-to-one increase in prices; (ii) the firms' profits are independent of marginal cost; (iii) consumer surplus falls one-to-one in marginal cost; and (iv) total welfare falls one-to-one in marginal cost. Proofs and intuition can be found in Gill and Thanassoulis (2015) (see Propositions 7 and 12).

## 6.1 Dynamic model of stochastic discounts

To study dynamic repeated competition with both bargainers and price takers, we assume that the firms interact for an infinite number of periods, and that in each period they play the two-stage game described in Section 3. We further assume that the firms apply a per-period common discount factor $\delta \in (0, 1)$ to profits, but that they do not discount within periods (i.e., profits from bargainers at the discount stage are not discounted relative to profits from price takers).[15] We extend our analysis by allowing any number of firms $N \geq 2$ to compete in the market. At the same time, we simplify the analysis by only considering the case $t = 0$, that is perfect substitutes. Unlike the analysis in Section 4, we do not impose any upper bound on list prices.

## 6.2 Equilibrium concept and off-equilibrium punishment

We approach the question of collusion in the standard way of seeking a symmetric subgame-perfect Nash equilibrium in which the firms collude on a common price $z > c$. Since our analysis allows for both list price setting and discounts, we investigate collusion in which the firms collude on the same price $z$ at both the list-price-setting stage and the discount stage.[16] As is common in collusion analyses, we focus on collusive equilibria supported by the threat of reversion to the lowest-payoff non-collusive symmetric equilibrium.

If a firm deviates from the collusive price $z$ at the discount stage, from the next period onward the firms revert to the unique symmetric pure-strategy Nash Equilibrium of the one-shot two-stage game.[17] In this non-collusive equilibrium, by the standard logic of Bertrand price competition with perfect substitutes, all prices are at marginal cost and profits are zero: a firm which undercuts its rivals' list price by an arbitrarily small amount secures the business of all the price takers.

If, instead, a firm deviates from the collusive price $z$ at the list-price-setting-stage, the firms immediately revert within period to the unique non-collusive symmetric equilibrium of the discount stage, and from the next period onward then revert to the zero-profit equilibrium of the one-shot two-stage game. In the within-period non-collusive symmetric equilibrium of the discount stage immediately following a list-price deviation, discount prices are given by a

[15] We abstract from within-period discounting of profits for simplicity. However, all our results would extend if the firms applied a within-period discount factor $\delta_2 \in (0, 1)$ to profits from bargainers, since the firms would then behave as if the proportion of price takers was $\mu_2 = \mu/[\mu + (1 - \mu)\delta_2] \in (0, 1)$ and the proportion of bargainers was $(1 - \mu_2) = (1 - \mu)\delta_2/[\mu + (1 - \mu)\delta_2] \in (0, 1)$.

[16] If we introduce a common maximum willingness to pay $v > c$, then our analysis applies for any collusive price $z \in (c, v]$. When $z = v$, the firms collude on the monopoly price.

[17] If the firms cannot directly observe discount price offers, they can still detect deviations at the discount stage, since the deviant firm captures the proportion $\beta$ of bargainers who receive the deviant firm's discount price offer.

mixed-strategy equilibrium and profits are positive.[18]

### 6.3 Collusive equilibrium with bargaining

Having discussed how deviations are punished, we are now in a position to consider collusion in the infinitely-repeated game. Proposition 11 shows that bargainers facilitate collusion.

**Proposition 11** *Consider a market with $N \geq 2$ firms. When goods are perfect substitutes, the presence of bargainers facilitates collusion compared to the benchmark with only price takers: the critical discount factor that allows collusion to be subgame perfect is strictly lower with bargainers.*

The reason that the presence of bargainers facilitates collusion is that the opportunity to offer discounts to bargainers lowers the profits available from deviating on a collusive agreement. If a firm deviates only in the discount prices that it offers, it foregoes any increase in the market share of price takers. If, instead, a firm deviates by lowering its publicly posted list price, then the rival firms, observing this deviation, would respond by setting discount price offers more aggressively. Which of the two possible deviations is optimal depends upon the parameters. In either case, however, the increase in market share available to firms from a deviation is reduced, allowing collusion to be sustained at lower discount factors than would be possible without discounting. In the penultimate paragraph of Section 2, we discuss how our finding that linkages across segments within a single market can make collusion easier to sustain complements the literature on multi-market contact that finds that linkages across markets helps collusion (Bernheim and Whinston, 1990).

Next we consider how the ease of collusion varies in the proportion of bargainers and in the discount reliability.

**Proposition 12** *There is a threshold proportion of bargainers $1 - \widehat{\mu} > 0$ such that:*

*(i) when the proportion of bargainers lies above the threshold, the critical discount factor that allows collusion to be subgame perfect increases in the proportion of bargainers; and*

*(ii) when the proportion of bargainers lies below the threshold, the critical discount factor decreases in the proportion of bargainers.*

A higher proportion of bargainers increases the profitability of deviating at the discount stage by increasing the profit from undercutting rivals' collusive discount price offers. At the

[18] The details are in Lemma 10 in the proof of Proposition 11. This recourse to mixed strategies is a consequence of the assumption of perfect substitutes: if a firm sets a discount price of $p_i > c$ for sure, then a rival could gain the business of all the bargainers who receive both discount price offers (and no lower offers from other firms) by just undercutting $p_i$, while a firm that sets $p_i = c$ loses profits from bargainers who receive no other discount price offers. In the mixed-strategy equilibrium, a firm trades off the incentive to price high to profit from bargainers who receive few discount price offers against the incentive to price low to increase the probability of selling to bargainers who receive many price offers.

same time, a higher proportion of bargainers reduces the profitability of deviating at the list-price-setting stage by reducing the number of price takers who buy at the deviant list price and increasing the impact on profits of the within-period reversion to competition for bargainers at the discount stage. When the proportion of bargainers is large enough that deviating at the discount stage is the more profitable deviation, a marginal increase in the proportion of bargainers increases the critical discount factor by increasing further the profitability of deviating at the discount stage. (Note that if the discount reliability $\beta$ is low, no proportion of bargainers is large enough to make deviating at the discount stage more profitable: $1-\widehat{\mu} > 1$, and so only case (ii) applies.) When the proportion of bargainers is small enough that deviating at the list-price-setting stage is more profitable, a marginal increase in the proportion of bargainers decreases the critical discount factor by decreasing the profitability of deviating at the list-price-setting stage.

**Proposition 13** *There is a threshold discount reliability $\widehat{\beta} > 0$ such that:*

*(i) when the discount reliability lies above the threshold, the critical discount factor that allows collusion to be subgame perfect increases in discount reliability; and*

*(ii) when the discount reliability lies below the threshold, the critical discount factor decreases in discount reliability.*

A higher discount reliability $\beta$ increases the profitability of deviating at the discount stage by increasing the proportion of bargainers that receive the deviant discount price offer. At the same time, a higher discount reliability reduces the profitability of deviating at the list-price-setting stage by increasing the impact on profits of the within-period reversion to competition for bargainers at the discount stage, since the competition for bargainers becomes more intense. When the discount reliability is large enough that deviating at the discount stage is the more profitable deviation, a marginal increase in discount reliability increases the critical discount factor by increasing further the profitability of deviating at the discount stage. (Note that if the proportion of bargainers $1-\mu$ is low, no level of discount reliability is large enough to make deviating at the discount stage more profitable: $\widehat{\beta} > 1$, and so only case (ii) applies.) When the discount reliability is small enough that deviating at the list-price-setting stage is more profitable, a marginal increase in discount reliability decreases the critical discount factor by decreasing the profitability of deviating at the list-price-setting stage.

# 7 Discussion and robustness

In this section we discuss the importance of various model assumptions for our results. Finally, in Section 7.9, we reinterpret our work as competitive price discrimination and relate our results to the literature in this area.

### 7.1 Stochastic discounts

The stochastic nature of the discounts is captured by the fact that the probability $\beta$ that each bargainer receives a particular firm's discount price offer (the 'discount reliability') lies strictly in $(0, 1)$. Such stochastic discounting has wide applicability; in Section 3.1 we described interpretations for the case of bargaining and the case of couponing.

In Sections 4 and 5, we showed that the stochastic discounts create a strategic link between list and discount prices that allows all prices to increase above the standard Hotelling level. When $\beta = 0$ or $\beta = 1$, however, this strategic link is broken in the one-shot game. When $\beta = 0$, the firms are not able to deliver discounts, and so the equilibrium list prices are the same as in the standard Hotelling model. When $\beta = 1$, equilibrium list prices and discount price offers cannot lie above the standard Hotelling level. Suppose they did: at the discount stage, bargainers would receive both firms' discount price offers with certainty, and so competition for bargainers would drive discount price offers to the standard Hotelling level; thus, the firms would deviate downward at the list-price-setting stage, increasing profits from price takers without any impact on the equilibrium discount price offers.

When $\beta = 0$ in the repeated game, the discount stage is irrelevant since discount price offers are never received, and so the critical discount factor above which collusion can be sustained is the same as in the benchmark with only price takers. When $\beta = 1$, however, the presence of bargainers continues to facilitate collusion. The reason is that the opportunity to offer discounts to bargainers continues to lower the profits available from deviating on a collusive agreement in the way described in the paragraph following Proposition 11. Furthermore, Proposition 12, which shows how the ability to collude varies in the proportion of bargainers, continues to hold.

### 7.2 Exogenous discount reliability

Our model assumes that the discount reliability $\beta$ is common across firms and exogenously given. Firms may have some scope to change their own discount reliability, for instance by training their sales staff in bargaining techniques or by offering discount coupons through a greater number of media sources. A full equilibrium analysis of each firm's choice of list price, discount price and discount reliability is beyond the scope of this paper. Nonetheless, we provide a partial analysis. In particular we study whether, taking as given the equilibrium prices at a common discount reliability $\beta$, the firms have an incentive to deviate by changing their own discount reliability $\beta_i$ at the discount stage of the one-shot game.

**Remark 2** *Taking as given the equilibrium prices at a common discount reliability $\beta$, a firm would increase its profits if it could raise its own discount reliability $\beta_i$ at zero cost at the discount stage of the one-shot game. As the common discount reliability $\beta$ tends to one, the increase in profits tends to zero.*

Remark 2 shows that firms benefit if they can increase their discount reliability compared to the industry level. By increasing the probability with which its discount price offers are received, a firm increases the proportion of bargainers that it sells to, but at the same time loses revenue on bargainers who would have been willing to buy from the firm at its list price. At the equilibrium list and discount price offers, the increase in volume always dominates.

Of course, in practice increasing discount reliability is likely to have a direct cost (e.g., costs of training staff or paying media sources to advertise coupons). The exact specification of the costs will depend on the specific environment. However, Remark 2 tells us that when the common discount reliability approaches one, a firm's incentive to increase its own discount reliability falls towards zero (this happens because the list and discount prices approach the standard Hotelling level). Introducing a well-behaved explicit cost of increasing discount reliability would therefore lead to a stable interior $\beta$ from which the increase in profits from deviating by increasing $\beta_i$ is outweighed by the cost. Thus, our partial analysis provides support for active discounting by firms as a robust feature of markets in which some consumers seek to bargain or a coupon technology is available. It is therefore an important result that welfare is damaged when the proportion of bargainers increases (Proposition 7).

When the firms collude successfully in the repeated game, profits do not depend on the discount reliability. However, the payoff from deviation does depend on the discount reliability. Proposition 13 tells us that when the common discount reliability $\beta$ is low, the firms have a joint incentive to coordinate on a higher $\beta$ in order to reduce the critical discount factor that allows collusion to be subgame perfect.

### 7.3 Two-stage model

We have studied price competition with stochastic discounting in a two-stage model in which firms first set and observe each other's list prices, and then offer discounts that bargainers receive stochastically. In this section, we note that this two-stage setup is crucial to our results.

Suppose that we changed our model so that the firms set their list prices and discount price offers simultaneously, thus collapsing the competition to a single-stage game. In that case, equilibrium list and discount price offers in the one-shot game could not lie above the standard Hotelling level. The reason is that the rival firm would not be able to respond at the discount stage to a deviation in list prices, and so the standard Hotelling undercutting logic would apply.[19]

In the repeated game, the two-stage model is also indispensable to determine that bargainers facilitate collusion on a common price $z$. If list prices and discount price offers were chosen simultaneously in a single stage, then a firm could deviate from a candidate collusive equilibrium by undercutting all prices slightly and so securing the full market demand. Thus, firms would have the same incentive to deviate downward as in the benchmark with only price takers.

### 7.4 Two types of consumer

Our model includes two types of consumer: price takers and bargainers. We include these two types because we think it natural that in many markets with posted prices, some consumers will attempt to bargain down the price, while others will be unwilling to attempt to negotiate with sellers, perhaps due to time constraints, psychological costs of engaging in bargaining, or lack of information about the opportunities for bargaining. Furthermore, including the two types of consumer allows us to conduct comparative statics in Section 5 on the proportion of the bargaining type in the population. These comparative statics help us to understand how bargaining affects competition in markets with posted prices, and they may also prove useful to competition authorities when they consider whether to encourage bargaining in markets.

Nonetheless, we now consider what happens in our model if there are no price takers and only bargainers in the population of consumers, that is if $\mu = 0$. All of our equilibrium results for the one-shot game reported in Propositions 1 and 2 and in Remark 1 continue to hold. More specifically, the equilibrium list and discount prices reported in (2) and (3) continue to share the same functional form, setting $\mu = 0$. The reason is that the dynamic link between the list prices and discount price offers remains, since bargainers who receive only one discount price offer continue to choose between that discount price offer and the other firm's list price. In the case of the repeated game, Proposition 11 also continues to hold: bargainers continue to facilitate collusion because not all bargainers receive a deviant firm's discount price offer, and so a firm that deviates at the discount stage sells to only a fraction of the bargainers.

The comparative statics results in Sections 5 and 6.3 are also robust. In particular, Proposition 3 continues to hold: prices and profits continue to be higher than in the benchmark with only price takers, and consumer surplus and welfare continue to be lower. Furthermore, all the comparative statics results with respect to the discount reliability $\beta$ and the degree of product

[19] If $l^* = p^* > t + c$, a small downward deviation in both $l_i$ and $p_i$ would be profitable just like in the standard Hotelling model. If $l^* > p^* \geq t + c$, a small downward deviation in $l_i$ would be profitable (with respect to both price takers and bargainers who failed to receive the firm's own discount price offer).

differentiation also carry through: that is, Propositions 5, 6, 8, 10 and 13 continue to hold.[20]

### 7.5 Discount prices below list prices

We have assumed throughout this study that the list prices remain available to all consumers, even in the discounting stage. In this sense, the assumption that the discount prices must lie weakly below the list prices is without loss of generality: consumers would always select the lower of the discount price and the list price if purchasing from a seller. The assumption that list prices are always available to consumers is uncontroversial in the couponing interpretation of the model. In the case of bargaining, this is also natural when the list prices are publicly posted, as would be the case in many retail environments.

However it is possible to imagine that list prices could be withdrawn in some markets if bargaining begins, and so the bargained price could be higher than the withdrawn list price. If we changed our model so that list prices were not available to bargainers, discount price offers would no longer be competing with the list prices. The strategic interaction that we have studied between list prices and discount price offers would therefore be completely severed in the one-shot game, and so the presence of bargainers would have no impact on the list prices offered to price takers. Furthermore, bargainers would be exposed to the risk of receiving no price offer at all, and so bargaining is unlikely to be a robust feature in such environments.

### 7.6 Unit demand

Our analysis has used the standard Hotelling model, in which consumers have unit demands and the market is always covered. Thus, market demand is inelastic, although each firm's demand is elastic since consumers can be won or lost to the rival firm. Introducing a downward-sloping market demand function would complicate the analysis substantially. However, we conjecture that downward-sloping market demand would not affect the key intuition that bargainers cause all prices to rise. The reason is that the strategic link between list prices and discount price offers would remain: higher list prices would continue to soften competition at the discount stage. Furthermore, this softening of competition would continue to be profitable for the firms, since the benchmark equilibrium price with only price takers would continue to lie below the monopoly level. Given our conjecture, the welfare loss from bargainers would be reinforced: in addition to the misallocation of products to consumers caused by the gap between list and discount prices, the higher prices would also lower total market demand. We also conjecture

[20] With respect to Proposition 5, when $\mu = 0$ the list prices and the difference between the list prices and the discount price offers are now always increasing in $\beta$, and so no longer have an interior peak. Formally, this means that they remain quasi-concave in $\beta$. The discount prices retain an interior peak, while welfare (Proposition 8) retains an interior trough.

that, with downward-sloping market demand, bargainers would continue to facilitate collusion compared to the benchmark with only price takers, since the opportunity to offer discounts to bargainers would continue to lower the profits available from deviating on a collusive agreement for the reasons described in the paragraph following Proposition 11.

### 7.7 Two competing firms

We extended our analysis to any number of competing firms when considering the effect of bargainers on the ability to collude. However, we restricted our analysis of the one-shot game to duopoly competition on the Hotelling line. Introducing more than two firms, for instance on a Salop circle (Salop, 1979), would complicate the analysis substantially. However, we conjecture that introducing more than two firms would not affect the key intuition that bargainers cause all prices to rise, for the same reasons as given in Section 7.6 in the case of downward-sloping demand. Note that with many firms on a Salop circle, the softening of competition from higher list prices would propagate around the circle: if a firm anywhere on the circle raised its list price, then nearby firms would respond by raising their discount prices; and, anticipating this, firms further along the circle would raise their discount prices as well. We also conjecture that the increase in prices caused by bargainers would weaken as the number of firms increased on a fixed Salop circle, since firms would move closer together on the circle, and we know from Proposition 6 that in the two-firm case a reduction in product differentiation lowers the gap between prices with and without bargainers.

### 7.8 Consumer type independent of location

Our analysis assumes that price takers and bargainers are both uniformly distributed along the Hotelling line, implying that the ratio of bargainers to price takers is independent of location. One might consider differing distributions of bargainers and price takers along the Hotelling line that retain the symmetry around the midpoint so that neither firm is favored. This would imply that the ratio of bargainers to price takers would vary along the Hotelling line. In any symmetric equilibrium, the key characteristic of a consumer density function is the density at the midpoint. In the standard Hotelling paradigm, for example, if consumers are distributed according to the density function $h(x)$, supported on $[0,1]$ and symmetric so that $h(x) = h(1-x)$, then equilibrium prices would be $c + t/h(\frac{1}{2})$ (Shilony, 1981). Hence, non-uniform distributions of consumers can either raise or lower equilibrium prices.

As a result, when bargainers are distributed differently to price takers along the Hotelling line in the one-shot game, we conjecture that the presence of bargainers would have two effects. First, the strategic link between list prices and discount price offers would continue to create an incentive for the firms to push up their list prices in order to soften competition at the discount stage. Furthermore, the strategic link between list prices and discount prices would operate through those bargainers away from the midpoint who are indifferent between one firm's discount price and the other firm's list price: depending on the relative density of such bargainers, the strategic link would have a bigger or smaller effect on prices. Second, as noted above, introducing differing consumer distributions would create a difference in the equilibrium prices that the firms would set for the two groups in the absence of any strategic link. This in turn would influence the equilibrium list and discount prices.

Our analysis of collusion abstracted from product differentiation entirely, and so it does not make sense to allow the ratio of bargainers to price takers to vary with location.

### 7.9 Stochastic discounting as price discrimination

Proposition 3 delivers the result that in the one-shot game, compared to the benchmark with only price takers, the presence of bargainers raises all prices and profits, while lowering consumer surplus and total welfare. Proposition 11 further demonstrates that in the repeated game, collusion is easier to sustain when bargainers are present than in the benchmark with only price takers. Thus, the ability of the firms to deliver discounts damages competition in both the static and dynamic variants of the model. We can interpret the ability to offer stochastic discounts to bargainers, while excluding price takers from the opportunity to receive the discounts, as a form of price discrimination. Viewed in this light, we can interpret our results as saying that banning price discrimination, and so preventing the firms from offering stochastic discounts to a subset of consumers, is welfare improving in the context of our model.

These insights are complementary to the existing research on competitive price discrimination. Our model is close to the paradigm of competitive third-degree price discrimination (see Stole, 2007), since the firms can offer a discount price that only a subset of the consumers (the bargainers) can receive. However, our model maintains an important strategic difference: discounts cannot be delivered to the bargainers with certainty, and so the competition for the two types of consumer are linked even under price discrimination.

Thisse and Vives (1988) study price competition when the firms can price discriminate perfectly according to observable location. They show that such price discrimination leads to price reductions at all points on the Hotelling line, and as a result price discrimination enhances consumer welfare. When symmetric firms sell to two markets with different demand characteristics, Holmes (1989) demonstrates that competitive price discrimination causes prices to rise in the 'strong' market, and fall in the 'weak', leading to welfare predictions that depend upon the relative curvature of the demand functions in each market. When firms are asymmetric, and so differ according to the market segments in which they are strong (such as in a Hotelling model where a consumer is local to one seller, and distant to the other), Corts (1998) shows that competitive price discrimination can raise all prices or lower all prices. Our analysis differs from all of these because of the strategic link between consumer segments created by the imperfect delivery of price reductions. It is this strategic interaction that causes all prices to rise when price discrimination is permitted.

Turning to collusion, prior work has extended Thisse and Vives (1988)'s model of perfect price discrimination to a dynamic setting. Gupta and Venkatu (2002) demonstrate that allowing price discrimination according to observable consumer location harms the ability of firms to collude if punishment is of the traditional grim-trigger form.[21] Liu and Serfes (2007) generalize this analysis by allowing firms to partition the Hotelling line and deliver a single price to each partition: collusion becomes harder to sustain as the partitioning becomes finer. In these studies, the ability to price discriminate perfectly makes deviation more attractive than under uniform pricing and so harms collusion. These analyses differ from ours since they all build upon a model in which price discrimination lowers firms' prices in the one-shot game. The strategic link across consumer segments created by imperfect discounting that we have studied reverses these results: collusion becomes easier to sustain since a deviation in one market (price takers) elicits a competitive response in the second market (bargainers).

# 8 Conclusion

Discounts off public list prices are commonplace. In this paper we have developed and analyzed a model of dynamic price competition between firms when some consumers buy at discount prices while others buy at list prices. We extend the literature by studying strategically chosen stochastic discounts in markets with prior list-price-setting competition. We document the effect of competition in posted prices with stochastic discounts on discount and list prices, profits and welfare.

We demonstrate that the effect of having a greater proportion of consumers in the population who receive discounts with some probability is to raise list prices. The main driver behind our results is the dynamic link between the list prices and subsequent discount prices. Discounts respond to list price deviations – and the list price will be the competing offer for those consumers who do not receive the rival's discount price offer, due to failed bargaining or inattention to coupons. The dynamic nature of the interaction causes the discount prices to become strategic complements to the list prices.

[21]If optimal multi-period penal codes can be used, then this conclusion can be reversed (Miklós-Thal, 2008).

The insights of this model apply in a wide variety of settings. In markets, such as the automobile market, where some consumers buy at list prices and others may receive bargained discounts, the predictions are immediate: the greater the proportion of the population who seek to bargain, the higher list prices will be. Similarly, in the case of couponing: if a large proportion of consumers are targeted by coupons then the competitive pressure on list prices will be reduced and these list prices will rise. This results in a welfare loss since misallocation of consumers to firms results.

We end by discussing briefly how our work might open up a debate about optimal policy towards bargaining and discount coupons in markets. A naive view would argue that since consumers who negotiate reductions off posted prices or use discount coupons pay lower prices than do price takers, bargaining and discount coupons ought to be encouraged. This was certainly the view of the United Kingdom's Office of Fair Trading with regard to the estate agency (realtor) market: "Greater [...] negotiation by consumers will increase competitive pressures on estate agents and result in better value for money in terms of both lower prices and higher service quality [...] We will therefore undertake an information campaign to raise consumer awareness of the benefits [...] of negotiating fee rates." (Office of Fair Trading, 2004, Section 1.12). However such a policy recommendation might be counterproductive: in our model, increasing the proportion of bargainers increases list prices and bargained prices, while reducing welfare. In this light it is interesting to note that policymakers sometimes do try to limit bargaining: until 2001 the *Rabattgesetz* (statute on discounts) and the *Zugabeverordnung* (regulation governing free gifts with sales) severely restricted the ability of German retailers to offer discounts off posted prices (Finger and Schmieder, 2005, Korn, 2007). We hope that our results will encourage further research into the conditions under which policymakers ought to encourage or restrict bargaining and discount couponing in consumer markets.

## Appendix

**Proof of Proposition 1.** There is a proportion $1-\mu \in (0,1)$ of bargainers; here for simplicity we normalize this proportion to 1. Let $[x]^{[0,1]} \equiv \max\{0, \min\{x, 1\}\}$, and let

$$\psi(y_i, y_j) = (y_i - c)\left[\frac{t + y_j - y_i}{2t}\right]^{[0,1]} \tag{4}$$

represent firm $i$'s profits in the standard Hotelling model when firm $i$ sets a price $y_i$ and its rival sets a price $y_j$ (the market share is determined by the indifferent consumer at location $x$ that solves $y_i + tx = y_j + t(1-x)$). Then

$$\pi^{rd}(p_i; p_j) = \beta^2 \psi(p_i, p_j) \tag{5}$$

represents firm $i$'s profits from bargainers who receive both discount price offers (the superscript '$rd$' represents the fact that these bargainers receive the rival firm's discount price offer). Similarly,

$$\pi^{nrd}(p_i; l_j) = \beta(1-\beta)\psi(p_i, l_j) \tag{6}$$

represents profits from bargainers who receive only firm $i$'s offer (the superscript '$nrd$' represents the fact that these bargainers do not receive the rival firm's discount price offer). Given that $p_i$ affects profits only in these two cases, total profits at the discount stage are given by:

$$\pi^{disc}(p_i; p_j, l_j) = (\text{constant}) + \pi^{rd}(p_i; p_j) + \pi^{nrd}(p_i; l_j). \tag{7}$$

We will use the following definitions extensively, where $\phi$ functions represent maxima ignoring the $p_i \leq l_i$ constraint on firm $i$'s discount price offers, but we restrict attention to discount price offers and list prices of the rival that satisfy the $p_j \leq l_j$ constraint:

$$\begin{aligned}
\phi^{rd}(p_j) &\equiv \arg\max_{p_i \in [c,\infty)} \pi^{rd}(p_i; p_j); \\
\phi^{nrd}(l_j) &\equiv \arg\max_{p_i \in [c,\infty)} \pi^{nrd}(p_i; l_j); \\
\phi^{disc}(p_j, l_j) &\equiv \arg\max_{p_i \in [c,\infty)} \pi^{disc}(p_i; p_j, l_j); \\
p_i^* &\equiv \arg\max_{p_i \in [c,l_i]} \pi^{disc}(p_i; p_j, l_j).
\end{aligned}$$

The strategy of the proof is as follows.

First, Lemmas 1-8 determine the properties of the best-response function $p_i^*$. Lemma 1 demonstrates that profits at the discount stage $\pi^{disc}(p_i; p_j, l_j)$ are the sum of two quasi-concave functions in firm $i$'s discount price $p_i$. Lemma 2 determines the maxima of these constituent functions, and Lemma 3 notes that the maximum of $\pi^{disc}(p_i; p_j, l_j)$ must lie between the maxima of the constituent quasi-concave parts. Lemmas 4-7 determine best responses when the rival's discount and list prices both lie on the same side of the standard Hotelling price of $t + c$. Lemma 4 establishes that the profit function $\pi^{disc}(p_i; p_j, l_j)$ is strictly concave over the whole region between the maxima of the constituent quasi-concave parts, which allows us to use first-order conditions to determine the best-response discount prices. Lemma 5 does so ignoring the constraint that discount prices must lie below list prices, Lemma 6 takes the constraint into account, and Lemma 7 shows that the constraint is not binding for the firm with the higher list price if its list price is above $t+c$. Finally, Lemma 8 considers the case in which the rival's discount price is below $t+c$ with no restrictions on the rival's list price.

Next, Claims 1-3 use these best-response functions to establish the equilibrium discount prices and confirm there can be no others. We split the problem into three cases: both firms having set list prices above $t+c$ (Claim 1); both firms having set list prices below $t+c$ (Claim 2); and one firm having set a list price above $t+c$ and the rival below $t+c$ (Claim 3). Together these deliver the proof of Proposition 1. ■

**Lemma 1** *$\pi^{rd}(p_i;p_j)$ and $\pi^{nrd}(p_i;l_j)$ are quasi-concave in $p_i$. In particular, $\pi^{rd}(p_i;p_j)$ is strictly increasing in $p_i$ when $p_i \in [c, p_j - t]$ (if this range exists), strictly concave in $p_i$ when $p_i \in [p_j - t, p_j + t]$ and equal to 0 when $p_i \in [p_j + t, \infty)$, and similarly for $\pi^{nrd}(p_i;l_j)$.*

**Proof.** Immediate from (5) and (6), given that $(t + p_j - p_i)/2t \geq 1 \Leftrightarrow p_i \leq p_j - t$ and $\beta \in (0,1)$. ■

**Lemma 2** *(i) $\phi^{rd}(p_j) = \frac{1}{2}(p_j + t + c) \in (p_j - t, p_j + t)$; (ii) $\phi^{nrd}(l_j) = \frac{1}{2}(l_j + t + c) \in (l_j - t, l_j + t)$; and (iii) $\phi^{rd}(p_j) \leq \phi^{nrd}(l_j)$.*

**Proof.** $(p_i - c)(t + p_j - p_i)/2t$ is strictly concave in $p_i$ and maximized at $\frac{1}{2}(p_j + t + c) \geq c$, given that $p_j \geq c$. Therefore, using (5) and Lemma 1, $\phi^{rd}(p_j) = \frac{1}{2}(p_j + t + c)$, if we can show that $\frac{1}{2}(p_j + t + c) \in (p_j - t, p_j + t)$. We show that this condition holds, in two steps. First, $\frac{1}{2}(p_j + t + c) < p_j + t \Leftrightarrow c < p_j + t$, which holds given $p_j \geq c$ and $t > 0$. Second, $\frac{1}{2}(p_j + t + c) > p_j - t \Leftrightarrow 3t + c > p_j$, which holds given $p_j \leq l_j \leq 2t + c < 3t + c$. An analogous argument gives (ii), and (iii) then follows from $p_j \leq l_j$. ■

**Lemma 3** *$\phi^{disc}(p_j, l_j) \in \left[\phi^{rd}(p_j), \phi^{nrd}(l_j)\right]$.*

**Proof.** Immediate given that $\pi^{rd}$ and $\pi^{nrd}$ are quasi-concave (Lemma 1) and that each has a unique arg max (Lemma 2). ■

**Lemma 4** *If $p_j \geq t + c$ or $l_j < t + c$, then (i) $\left[\phi^{rd}(p_j), \phi^{nrd}(l_j)\right] \subseteq [p_j - t, p_j + t]$ and (ii) $\left[\phi^{rd}(p_j), \phi^{nrd}(l_j)\right] \subseteq [l_j - t, l_j + t]$, and so $\pi^{disc}(p_i; p_j, l_j)$ is strictly concave in $p_i$ when $p_i \in \left[\phi^{rd}(p_j), \phi^{nrd}(l_j)\right]$.*

**Proof.** From Lemma 2, $\phi^{rd}(p_j) = \frac{1}{2}(p_j + t + c) > p_j - t$ and $\phi^{nrd}(l_j) = \frac{1}{2}(l_j + t + c) < l_j + t$. Furthermore:

(i) $\frac{1}{2}(l_j + t + c) \leq p_j + t \Leftrightarrow l_j \leq 2p_j + t - c$; and (ii) $\frac{1}{2}(p_j + t + c) \geq l_j - t \Leftrightarrow p_j + 3t + c \geq 2l_j$.

In both cases, the inequality holds when $p_j \geq t+c$ given $l_j \leq 2t+c$, and holds when $l_j < t+c$ given $p_j \geq c$. From Lemma 1, (5) is strictly concave in $p_i$ when $p_i \in [p_j - t, p_j + t]$ and (6) is strictly concave in $p_i$ when $p_i \in [l_j - t, l_j + t]$. Thus, $\pi^{disc}(p_i; p_j, l_j)$, given by (7), is the sum of two strictly concave functions, and so strictly concave in $p_i$, when $p_i \in \left[\phi^{rd}(p_j), \phi^{nrd}(l_j)\right]$. ■

**Lemma 5** *If $p_j \geq t+c$ or $l_j < t+c$, then $\phi^{disc}(p_j, l_j) = \frac{1}{2}[t + c + (1-\beta) l_j + \beta p_j]$.*

**Proof.** From Lemma 3, $\phi^{disc}(p_j, l_j) \in \left[\phi^{rd}(p_j), \phi^{nrd}(l_j)\right]$. If $p_j \geq t+c$ or $l_j < t+c$, from Lemma 4, (7) is strictly concave in $p_i$ when $p_i \in \left[\phi^{rd}(p_j), \phi^{nrd}(l_j)\right]$. The first-order condition,

$$\beta^2 \left[\frac{(p_i - c)(-1) + (t + p_j - p_i)}{2t}\right] + \beta(1-\beta)\left[\frac{(p_i - c)(-1) + (t + l_j - p_i)}{2t}\right] = 0,$$

is satisfied at $p_i = \frac{1}{2}[t + c + (1-\beta) l_j + \beta p_j]$. Furthermore using Lemma 2, $\frac{1}{2}[t + c + (1-\beta) l_j + \beta p_j] \in \left[\phi^{rd}(p_j), \phi^{nrd}(l_j)\right]$, giving the result. ■

**Lemma 6** *If $p_j \geq t+c$ or $l_j < t+c$, then $p_i^* = \min\left\{\frac{1}{2}[t + c + (1-\beta) l_j + \beta p_j], l_i\right\}$.*

**Proof.** From Lemmas 1 and 2, (7) is strictly increasing in $p_i$ when $p_i \in [c, \phi^{rd}(p_j))$. From Lemmas 3 and 4, (7) is strictly increasing in $p_i$ when $p_i \in [\phi^{rd}(p_j), \phi^{disc}(p_j, l_j))$, if this range exists. Thus, when $l_i < \phi^{disc}(p_j, l_j)$, $p_i^* = l_i$, which together with Lemma 5 gives the result. ■

**Lemma 7** *If $l_1 \geq t+c$ and either $p_2 \geq t+c$ or $l_2 < t+c$, then $p_1^* = \frac{1}{2}[t + c + (1-\beta) l_2 + \beta p_2]$, i.e., firm 1's best response is not constrained by its list price.*

**Proof.** Given that $l_1 \geq l_2 \geq p_2$ and that $l_1 \geq t + c$, $2l_1 \geq t + c + (1-\beta) l_2 + \beta p_2$, and so $l_1 \geq \frac{1}{2}[t + c + (1-\beta) l_2 + \beta p_2]$. The result then follows from Lemma 6. ■

**Lemma 8**

*(i) If $p_j < t+c$ and $l_i > p_j$, then $p_i^* > p_j$.*
*(ii) If $p_j < t+c$ and $l_i \leq p_j$, then $p_i^* = l_i$.*

**Proof.** From Lemma 2, $\phi^{rd}(p_j) = \frac{1}{2}(p_j + t + c) > p_j$ given $p_j < t+c$. From Lemmas 1 and 2, (7) is strictly increasing in $p_i$ when $p_i \in [c, \phi^{rd}(p_j))$, and so $p_i^* > p_j$ when $l_i > p_j$ and $p_i^* = l_i$ when $l_i \leq p_j$. ■

We now complete the proof of Proposition 1 by splitting the list-price space $l_1, l_2 \in [c, 2t+c]$ into three regions.

**Claim 1** *When $l_1 \geq l_2 \geq t+c$, (i) there is a unique pure-strategy equilibrium in which $p_1, p_2 \geq t+c$, given by $(p_1^*, p_2^*)$ in Proposition 1, and (ii) there are no other pure-strategy equilibria.*

**Proof.** (i) For $p_1, p_2 \geq t+c$, the best-response functions are given in Lemma 6. The unconstrained best-response functions $\frac{1}{2}\left[t+c+(1-\beta)\, l_2+\beta p_2\right]$ and $\frac{1}{2}\left[t+c+(1-\beta)\, l_1+\beta p_1\right]$ intersect at

$$\begin{aligned}
\widehat{p}_1 &= \frac{1}{2-\beta}(t+c)+\left(1-\frac{1}{2-\beta}\right)\left(\frac{2l_2+\beta l_1}{2+\beta}\right) \quad \text{and} \\
\widehat{p}_2 &= \frac{1}{2-\beta}(t+c)+\left(1-\frac{1}{2-\beta}\right)\left(\frac{2l_1+\beta l_2}{2+\beta}\right).
\end{aligned} \tag{8}$$

Given $l_1 \geq l_2 \geq t+c$, $2l_1+\beta l_2 \geq 2l_2+\beta l_1 \geq (2+\beta)(t+c)$, and so $\widehat{p}_1, \widehat{p}_2 \geq t+c$. By Lemma 7, only firm $2's$ list-price constraint is relevant. Thus, $(p_1^*, p_2^*) = (\widehat{p}_1, \widehat{p}_2)$ when $\widehat{p}_2 \leq l_2$; and $(p_1^*, p_2^*) = \left(\frac{1}{2}(t+c+l_2), l_2\right)$ when $\widehat{p}_2 > l_2$, substituting $p_2 = l_2$ into $p_1^*$ from Lemma 7. Recalling that $l_2 \geq t+c$, we confirm that $p_1^*, p_2^* \geq t+c$ when $\widehat{p}_2 > l_2$. The equilibrium corresponds to $(p_1^*, p_2^*)$ in Proposition 1, since $\min\left\{\frac{1}{2}(t+c+l_2), l_1\right\} = \frac{1}{2}(t+c+l_2)$ here.

(ii) Suppose there is an equilibrium in which $p_i^*, p_j^* < t+c$. Given $\min\{l_i, l_j\} \geq t+c$, we can apply Lemma 8(i) to show that $p_i^* > p_j^* > p_i^*$, a contradiction. Suppose there is an equilibrium in which $p_j^* \geq t+c > p_i^*$. Given $\min\{l_i, l_j\} \geq t+c$, $\min\left\{\frac{1}{2}\left[t+c+(1-\beta)\, l_j+\beta p_j^*\right], l_i\right\} \geq t+c$, and so by Lemma 6 $p_i^* \geq t+c$, a contradiction. ■

**Claim 2** *When $t+c > l_1 \geq l_2$, there is a unique pure-strategy equilibrium, given by $(p_1^*, p_2^*)$ in Proposition 1.*

**Proof.** Given $t+c > l_1 \geq l_2$, the best-response functions are given by Lemma 6. The unconstrained best-response functions $\frac{1}{2}\left[t+c+(1-\beta)\, l_2+\beta p_2\right]$ and $\frac{1}{2}\left[t+c+(1-\beta)\, l_1+\beta p_1\right]$ intersect at $(\widehat{p}_1, \widehat{p}_2)$ given in (8). Once firm $i$'s list-price constraint binds, $i$'s best-response function becomes flat, and so the best-response functions intersect a single time with the intersection weakly to the south-west of $(\widehat{p}_1, \widehat{p}_2)$, giving a unique equilibrium.

Note that $\widehat{p}_2 > l_2$, given that $t+c > l_2$ and $2l_1+\beta l_2 \geq (2+\beta)\, l_2$. Thus, at least one constraint must bind. We cannot have $p_1^* = l_1 < t+c$ and $p_2^* < l_2$, since then from Lemma 6, $p_2^* = \frac{1}{2}(t+c+l_1) < l_2$, a contradiction given $l_1 \geq l_2$ and $t+c > l_2$; instead, we must have $p_2^* = l_2$, and therefore $p_1^* = \min\left\{\frac{1}{2}(t+c+l_2), l_1\right\}$. ■

**Claim 3** *When $l_1 \geq t+c > l_2$, there is a unique pure-strategy equilibrium, given by $(p_1^*, p_2^*)$ in Proposition 1.*

**Proof.** Suppose first that there is an equilibrium with $p_2^* = l_2$. Given $l_1 \geq t+c > l_2$, by Lemma 7, $p_1^* = \frac{1}{2}(t+c+l_2)$, and therefore $p_1^* < t+c$ and $l_2 < p_1^*$. Thus, Lemma 8(ii) shows that $p_2^* = l_2$, and hence we have an equilibrium at $(p_1^*, p_2^*) = \left(\frac{1}{2}(t+c+l_2), l_2\right)$. This corresponds to $(p_1^*, p_2^*)$ in Proposition 1, since $\widehat{p}_2 > l_2$ (given (8), $t + c > l_2$ and $2l_1 + \beta l_2 > (2 + \beta)\, l_2$), and $\min\left\{\frac{1}{2}(t + c + l_2), l_1\right\} = \frac{1}{2}(t + c + l_2)$ here.

There are no other equilibria. The only candidates have $p_2^* < l_2$. Suppose such an equilibrium exists. Given $l_1 \geq t+c > l_2$, $p_2^* < t+c$ and $l_1 > p_2^*$, and so we can apply Lemma 8(i) to show that $p_1^* > p_2^*$. Suppose first that $p_1^* < t + c$. We can apply Lemma 8 to show that $p_2^* > p_1^*$ or $p_2^* = l_2$, either of which gives a contradiction. Suppose second that $p_1^* \geq t + c$. Given $l_1 \geq t + c > l_2$, $\min\left\{\frac{1}{2}\left[t + c + (1 - \beta)\, l_1 + \beta p_1^*\right], l_2\right\} = l_2$, and so by Lemma 6 $p_2^* = l_2$, a contradiction. ■

**Proof of Proposition 2.** From Proposition 1, there exists a unique pure-strategy Nash equilibrium at the discount stage given by $(p_1^*, p_2^*)$. These discount prices are themselves functions of the list prices $\{l_i, l_j\}$ set in the first stage. Recalling (4), profits at the list-price-setting stage are given by

$$\begin{aligned}
\pi_i^{list}\left(l_i; l_j, p_i^*, p_j^*\right) &= \left[\mu + (1-\mu)(1-\beta)^2\right] \psi(l_i, l_j) \\
&+ (1-\mu)\, \beta\, (1-\beta)\, \psi(p_i^*, l_j) \\
&+ (1-\mu)(1-\beta)\, \beta \psi(l_i, p_j^*) \\
&+ (1-\mu)\, \beta^2 \psi(p_i^*, p_j^*).
\end{aligned} \tag{9}$$

We add the subscript '$i$' to denote the firm's identity, since for notational convenience we exclude the arguments of $\pi_i^{list}$ in the remainder of the proof. The first line gives firm $i$'s profits from the proportion $\mu$ of price takers, and from the bargainers who receive neither discount price offer. The second (third) line gives profit from the bargainers who receive only firm $i$'s (firm $j$'s) offer. The final line gives profits from the bargainers who receive both offers.

The strategy of the proof is as follows. First, Lemma 9 establishes that the discount prices are continuous functions of the list prices, thus demonstrating that each firm's first-stage payoff function, $\pi_i^{list}$, is continuous in its own list price $l_i$. Claim 4 demonstrates that, starting from symmetric list prices above the standard Hotelling level of $t + c$, $\pi_i^{list}$ is locally concave in $l_i$. This allows us to characterize a unique candidate equilibrium, given by (2) in the statement of Claim 4. Finally, Claim 5 establishes existence of this candidate equilibrium by showing that $\pi_i^{list}$ is concave on a number of distinct ranges, and then using the continuity from Lemma 9 to stitch these ranges together to show global quasi-concavity of the payoff function. ■

**Lemma 9** *$\pi_i^{list}$ is continuous in the firm's own list price $l_i$.*

**Proof.** (9) is clearly continuous in $p_i^*$, $p_j^*$ and the direct effect of $l_i$. We also need to show continuity in the indirect effect of $l_i$ via the discount prices $p_i^*$ and $p_j^*$. First, we show continuity of $p_i^*$ and $p_j^*$ in $l_i$ for $l_i \geq l_j$. We need to show that $(p_1^*, p_2^*)$ in Proposition 1 are continuous in $l_1$. Clearly, we have continuity within case 1 and within case 2. As we move from case 1 to case 2, $p_2^*$ changes continuously. From Claims 1-3, it is always the case that either $p_2 \geq t+c$ or $l_2 < t+c$ in equilibrium; thus, using Lemma 6, $p_1^*$ is also continuous given that $p_2^*$ is continuous. Second, we show continuity of $p_i^*$ and $p_j^*$ in $l_i$ for $l_i \leq l_j$. We need to show that $(p_1^*, p_2^*)$ in Proposition 1 are continuous in $l_2$. Again, $p_2^*$ is clearly continuous, and therefore $p_1^*$ is also continuous by the same argument as above for the first range. We have shown continuity on two ranges that share a common boundary, and so we have continuity on the union of those ranges. ■

**Claim 4** *Any symmetric pure-strategy Nash equilibrium at the list-price-setting stage in which $l^* \in [c, 2t+c]$ and $l^* \neq t+c$ must be given by $l^* =$ (2) $\in (t+c, 2t+c)$ in Proposition 2.*

**Proof.** First, we consider $l^* \in (t+c, 2t+c]$. At $l^* > t+c$, using Proposition 1 and Corollary 1, $p_i^*$ and $p_j^*$ are given by case 1 with

$$p_i^* = p_j^* = \frac{1}{2-\beta}(t+c) + \left(1 - \frac{1}{2-\beta}\right) l^* \in (t+c, l^*).$$

For small deviations, $p_i^*$ and $p_j^*$ continue to be given by case 1 in Proposition 1, with

$$\frac{dp_i^*}{dl_i} = \frac{\beta(1-\beta)}{4-\beta^2} \text{ and } \frac{dp_j^*}{dl_i} = \frac{2(1-\beta)}{4-\beta^2}. \tag{10}$$

Thus, referring back to (9), we are in the interior at $l^* \in (t+c, 2t+c]$, since $|l_j - l_i| < t$, $|l_j - p_i^*| < t$, $\left|p_j^* - l_i\right| < t$ and $\left|p_j^* - p_i^*\right| < t$, and we remain in the interior for small deviations in $l_i$. Locally, the first derivative is thus given by:

$$\begin{aligned} 2t\frac{d\pi_i^{list}}{dl_i} &= \left[\mu + (1-\mu)(1-\beta)^2\right](t + l_j - 2l_i + c) \\ &+ (1-\mu)\beta(1-\beta)(t + l_j - 2p_i^* + c)\frac{dp_i^*}{dl_i} \\ &+ (1-\mu)(1-\beta)\beta\left[\left(t + p_j^* - 2l_i + c\right) + (l_i - c)\frac{dp_j^*}{dl_i}\right] \\ &+ (1-\mu)\beta^2\left[\left(t + p_j^* - 2p_i^* + c\right)\frac{dp_i^*}{dl_i} + (p_i^* - c)\frac{dp_j^*}{dl_i}\right]. \end{aligned} \tag{11}$$

Furthermore, $\pi_i^{list}$ is locally strictly concave in $l_i$. Using (10) and (11), some algebra gives:

$$2t\frac{d^2\pi_i^{list}}{d(l_i)^2} = -2\mu - 2(1-\mu)(1-\beta)\left(\frac{\beta^3 - 8\beta + 16}{(4-\beta^2)^2}\right) < 0.$$

Substituting (10) into the first-order condition given by setting (11) = 0, and then solving for

$l^*$, gives (2). We can check that (2) $\in (t+c, 2t+c)$. For all $\mu \in (0,1)$,

$$t+c < (2) < (t+c)+t\left[\frac{(2-\beta)\beta}{(4-2\beta+\beta^2)}\right] = (t+c)+t\left[\frac{(2-\beta)\beta}{(4-4\beta+2\beta^2)+(2-\beta)\beta}\right] < (t+c)+t.$$

(2) is our unique candidate equilibrium in which $l^* \in (t+c, 2t+c)$. At $l = l_i = l_j = 2t+c$, upward deviations are not permitted, given $l_i \leq 2t+c$. Thus, we need to check that (11) $< 0$, to ensure that there is an incentive to deviate downward. By inspection, (11) is linear in $l = l_i = l_j$ and is strictly positive for $l = l_i = l_j$ close enough to $t+c$ (so that $p_i^* = p_j^*$ are close to $t+c$). Furthermore, from above (11) $= 0$ at $l^* = (2) < 2t+c$, and so (11) $< 0$ at $l = l_i = l_j = 2t+c$.

Second, we consider $l^* < t+c$. Using Proposition 1 and Corollary 1, $p_i^*$ and $p_j^*$ are given by case 2 with $p_i^* = p_j^* = l^*$. Consider a small upward deviation to $l_1 = l^* + \varepsilon$. From Proposition 1, we remain in case 2, $p_2^*$ remains unchanged at $l^*$, and $p_1^*$ rises to $l_1 = l^* + \varepsilon < \frac{1}{2}(t+c+l^*)$. Thus, given $l^* < t+c$ and so $l^*$ lies below the standard Hotelling equilibrium level, the deviation is strictly profitable for the same reason as in the standard Hotelling model, yielding the desired contradiction. ■

**Claim 5** *The candidate symmetric pure-strategy Nash equilibrium at the list-price-setting stage given by $l^* =$ (2) $\in (t+c, 2t+c)$ in Proposition 2 exists.*

**Proof.** Given $l^* > t+c$, using Proposition 1 and Corollary 1, $p_i^*$ and $p_j^*$ are given by case 1 with

$$p_i^* = p_j^* = \frac{1}{2-\beta}(t+c) + \left(1 - \frac{1}{2-\beta}\right)l^* \in (t+c, l^*). \tag{12}$$

Given $l^* < 2t+c$, and referring back to (9), we are in the interior at $l^*$, since $|l_j - l_i| < t$, $|l_j - p_i^*| < t$, $\left|p_j^* - l_i\right| < t$ and $\left|p_j^* - p_i^*\right| < t$. We prove existence by showing that when $l_j = l^*$, $\pi_i^{list}$ is quasi-concave in $l_i$ with a maximum at $l^*$, and so there is no incentive to deviate.

(i) First, consider upward deviations to $l_1 > l^*$. Let $\widetilde{l_1}$ solve

$$\frac{1}{2-\beta}(t+c) + \left(1 - \frac{1}{2-\beta}\right)\left(\frac{2l_1 + \beta l^*}{2+\beta}\right) = l^*. \tag{13}$$

From (12), the left-hand side of (13) $< l^*$ at $l_1 = l^*$, and so $\widetilde{l_1} > l^*$.

(i)(a) When $l_1 \in \left[l^*, \widetilde{l_1}\right]$, $p_1^*$ and $p_2^*$ are given by case 1 in Proposition 1. We remain in the interior, given that $2t + c \geq l_i \geq p_i$, that $p_1^*$ and $p_2^*$ are increasing in $l_1$ from Proposition 1, and that $p_i^*, p_j^* > t+c$ at $l_1 = l^*$ from (12). Given that we remain in the interior, the proof of Claim 4 thus shows that $\pi_1^{list}$ is strictly concave in $l_1$ with a maximum at $l^*$, noting that (10) holds for all $l_1 \in \left[l^*, \widetilde{l_1}\right]$ since we are in case 1, and so (11) is linear in $l_1$.

(i)(b) When $l_1 > \widetilde{l_1}$, $p_1^*$ and $p_2^*$ are given by case 2 in Proposition 1, and so $p_1^* = \frac{1}{2}(t+c+l^*) <$

$l_1$ and $p_2^* = l^*$. Again, we remain in the interior, since $p_i^*, p_j^* > t + c$. Thus, using (9),

$$\pi_1^{list} = (constant) + \left[\mu + (1-\mu)(1-\beta)^2 + (1-\mu)(1-\beta)\beta\right](l_1 - c)\left(\frac{t + l^* - l_1}{2t}\right),$$

which is strictly concave in $l_1$ and maximized at $\frac{1}{2}(t + c + l^*) < \widetilde{l_1}$, given $\frac{1}{2}(t + c + l^*) < l^*$ and $l^* < \widetilde{l_1}$ from above.

(i)(c) Using (i)(a), (i)(b) and the continuity of $\pi_i^{list}$ from Lemma 9, $\pi_1^{list}$ is quasi-concave in $l_1$ when $l_1 \geq l^*$, with a maximum at $l^*$, and so there is no incentive to deviate upward.

(ii) Second, consider downward deviations to $l_2 < l^*$. Let $\widetilde{l_2}$ solve

$$\frac{1}{2-\beta}(t+c) + \left(1 - \frac{1}{2-\beta}\right)\left(\frac{2l^* + \beta l_2}{2+\beta}\right) = l_2,$$

which gives

$$\widetilde{l_2} = \left(\frac{2+\beta}{4-\beta}\right)(t+c) + \left(1 - \frac{2+\beta}{4-\beta}\right)l^* \in (t+c, l^*)\,. \tag{14}$$

Note further that

$$\widetilde{l_2} \in \left(l^* - t, \frac{t + c + l^*}{2}\right), \tag{15}$$

since $\widetilde{l_2} > l^* - t$ given $\widetilde{l_2} > t + c$ from (14) and $t + c > l^* - t$ from (2), and $\widetilde{l_2} < \frac{1}{2}(t + c + l^*)$ using (14), $l^* > t + c$ and $\frac{2+\beta}{4-\beta} > \frac{1}{2}$.

(ii)(a) When $l_2 \in \left[\widetilde{l_2}, l^*\right]$, $p_1^*$ and $p_2^*$ are given by case 1 in Proposition 1. Note that $p_i^*, p_j^* > t + c$ at $l_2 = \widetilde{l_2}$, since $\widetilde{l_2} > t + c$ from (14) and so $2\widetilde{l_2} + \beta l^* > (2+\beta)(t+c)$. Given that $2t + c \geq l_i \geq p_i$, that $p_1^*$ and $p_2^*$ are increasing in $l_2$ from Proposition 1, and that $p_i^*, p_j^* > t + c$ at $l_2 = \widetilde{l_2}$, we remain in the interior. An analogous argument to that in case (i)(a) above therefore shows that $\pi_2^{list}$ is strictly concave in $l_2$ with a maximum at $l^*$.

(ii)(b) When $l_2 \in \left(l^* - t, \widetilde{l_2}\right)$, $p_1^*$ and $p_2^*$ are given by case 2 in Proposition 1, and so $p_1^* = \frac{1}{2}(t + c + l_2) < l^*$ and $p_2^* = l_2$. We remain in the interior, since $\left|\frac{1}{2}(t + c + l_2) - l_2\right| = \left|\frac{1}{2}(t + c - l_2)\right| < t$ (given that $c \leq l_2 < l^* < 2t + c$) and $|l^* - l_2| < t$ here. Thus, using (9),

$$\begin{aligned} \pi_2^{list} &= \left[\mu + (1-\mu)(1-\beta)^2 + (1-\mu)\beta(1-\beta)\right](l_2 - c)\left(\frac{t + l^* - l_2}{2t}\right) \\ &+ \left[(1-\mu)(1-\beta)\beta + (1-\mu)\beta^2\right](l_2 - c)\left(\frac{t + \frac{1}{2}(t + c + l_2) - l_2}{2t}\right). \end{aligned} \tag{16}$$

The first line of (16) is strictly concave and maximized at $\frac{1}{2}(t + c + l^*)$, but $\frac{1}{2}(t + c + l^*) > \widetilde{l_2}$ from (15). The second line of (16) is strictly concave and maximized at $\frac{1}{2}(3t + 2c)$, but $\frac{1}{2}(3t + 2c) > \widetilde{l_2}$ since $\frac{1}{2}(3t + 2c) > \frac{1}{2}(t + c + l^*)$ (given that $2t + c > l^*$) and $\frac{1}{2}(t + c + l^*) > \widetilde{l_2}$ from (15). Summing together, $\pi_2^{list}$ is strictly concave in $l_2$ with a maximum strictly greater than $\widetilde{l_2}$.

(ii)(c) When $l_2 \leq l^* - t$, recalling that $\widetilde{l_2} > l^* - t$ from (15), all the analysis in (ii)(b) continues to apply, except that $l^* - l_2 \geq t$, and so $(l_2 - c)(t + l^* - l_2)/2t$ in the first line of (16) is replaced by $(l_2 - c)$ since market shares are bounded at 1. Summing a linearly increasing function and a strictly concave function with a maximum above $\widetilde{l_2}$, it continues to be the case that $\pi_2^{list}$ is strictly concave in $l_2$ with a maximum strictly greater than $\widetilde{l_2}$, and so strictly greater than $l^* - t$.

(ii)(d) Using (ii)(a), (ii)(b) and (ii)(c) and the continuity of $\pi_2^{list}$ from Lemma 9, $\pi_2^{list}$ is quasi-concave in $l_2$ when $l_2 \leq l^*$, with a maximum at $l^*$, and so there is no incentive to deviate downward. ■

**Proof of Remark 1.** From Proposition 1 and Corollary 1, at $l = t + c$, $p_i^* = p_j^* = t + c = l$, and so all prices are the same as in the standard Hotelling model. Upward deviations to $l_1 > l$ give $p_1^* = \frac{1}{2}(t + c + l) = l < l_1$ and $p_2^* = l$ (case 2 in Proposition 1), and so discount price offers do not change. Thus, the deviation is not profitable for the same reason as in the standard Hotelling model. Downward deviations to $l_2 < l$ give $p_1^* = \frac{1}{2}(t + c + l_2) < l$ and $p_2^* = l_2$ (again case 2 in Proposition 1, since $\widehat{p}_2$ from (8) falls less fast than $l_2$ does). Given downward deviations induce a competitive response by the rival for some of the customers, they are less profitable than in the standard Hotelling model, and so they are not profitable since they are not in the standard Hotelling model. ■

**Proof of Proposition 3.** From (2) and (3),

$$p = (t + c) + t \left[ \frac{2(1-\beta)^2 \beta}{\left(\frac{\mu}{1-\mu}\right)(4 - \beta^2)(2 - \beta) + 2(1-\beta)(4 - 2\beta + \beta^2)} \right]. \tag{17}$$

In the benchmark with only price takers, $l = t + c$. From (2) and (17), $l > t + c$ and $p > t + c$ given $\beta \in (0,1)$, $\mu \in (0,1)$ and $t > 0$. Since the market is covered, every consumer buys at a price above $t + c$; thus each firm's profits are strictly higher than in the benchmark given the equilibrium is symmetric. Since the market is covered, total welfare falls linearly in transport costs $T$. Transport costs are given by:

$$\begin{aligned} T &= \left\{\mu + (1-\mu)\left[(1-\beta)^2 + \beta^2\right]\right\}\left(\frac{t}{4}\right) \\ &+ (1-\mu) 2\beta (1-\beta) \left( \int_0^{\frac{1}{2} + \frac{l-p}{2t}} tx dx + \int_{\frac{1}{2} + \frac{l-p}{2t}}^1 t(1-x) dx \right). \end{aligned}$$

The first line captures the average transport cost of those who choose between two equal prices. The second line captures the transport cost of the bargainers who receive only one discount price offer. Note that $\frac{1}{2} + \frac{l-p}{2t} \in (0,1)$, since the equilibrium is interior from the proof of Claim 5.

Integrating gives:

$$
\begin{aligned}
T &= \left\{\mu + (1-\mu)\left[(1-\beta)^2 + \beta^2\right]\right\}\left(\frac{t}{4}\right) + (1-\mu)\,2\beta\,(1-\beta)\left[\frac{t}{4} + t\left(\frac{l-p}{2t}\right)^2\right] \\
&= \frac{t}{4} + (1-\mu)\,\beta\,(1-\beta)\,\frac{(l-p)^2}{2t}. \qquad (18)
\end{aligned}
$$

Using (2) and (17), $l - p > 0$. Thus, $T$ is strictly higher than the benchmark level of $t/4$, and hence total welfare is strictly lower. Finally, since total welfare is strictly lower and profits are strictly higher, consumer surplus must be strictly lower. ■

**Proof of Proposition 4.** The proportion of bargainers rises as the proportion $\mu$ of price takers falls. Let

$$
g \equiv \mu\left(4-\beta^2\right)(2-\beta) + 2\,(1-\mu)\,(1-\beta)\left(4 - 2\beta + \beta^2\right).
$$

Given $\mu \in (0,1)$ and $\beta \in (0,1)$, $g > 0$. Using (2),

$$
\begin{aligned}
\frac{dl}{d\mu} &= -2\beta\,(1-\beta)\,(2-\beta)^2\left(4-\beta^2\right)\left(tg^{-2}\right) < 0; \qquad (19) \\
\frac{d^2 l}{d\mu^2} &= 4\beta^2\,(1-\beta)\,(2-\beta)^2\left(4-\beta^2\right)\left[3\beta^2 + 8\,(1-\beta)\right]\left(tg^{-3}\right) > 0. \qquad (20)
\end{aligned}
$$

(19) gives (i), while the convexity of list prices shown in (20) implies that the list prices rise at an increasing rate. Using (3), (19) and (20),

$$
\frac{dp}{d\mu} = \left(\frac{1-\beta}{2-\beta}\right)\frac{dl}{d\mu} < 0 \;\text{ and }\; \frac{d^2 p}{d\mu^2} = \left(\frac{1-\beta}{2-\beta}\right)\frac{d^2 l}{d\mu^2} > 0; \qquad (21)
$$

hence (ii) holds and the discount price offers rise at an increasing rate. Finally, using (2), (3), (19) and (20),

$$
\begin{aligned}
\frac{d\,(l-p)}{d\mu} &= \frac{d}{d\mu}\left(\frac{1}{2-\beta}\left[l - (t+c)\right]\right) = \left(\frac{1}{2-\beta}\right)\frac{dl}{d\mu} < 0; \qquad (22) \\
\frac{d^2\,(l-p)}{d\mu^2} &= \left(\frac{1}{2-\beta}\right)\frac{d^2 l}{d\mu^2} > 0. \qquad (23)
\end{aligned}
$$

Therefore (iii) holds and the difference rises at an increasing rate. ■

**Proof of Proposition 5.** (i) Let $q \equiv \left(\frac{l-(t+c)}{t}\right)^{-1}$. Using (2):

$$\begin{aligned}
q &= \frac{1}{2}\left(\frac{\mu}{1-\mu}\right)\frac{\left(4-\beta^2\right)}{(1-\beta)\,\beta} + \frac{\left(4-2\beta+\beta^2\right)}{(2-\beta)\,\beta}; \\
\frac{\partial q}{\partial \beta} &= \frac{1}{2}\left(\frac{\mu}{1-\mu}\right)\frac{\left(-4+8\beta-\beta^2\right)}{(1-\beta)^2\,\beta^2} + \frac{-8\,(1-\beta)}{(2-\beta)^2\,\beta^2}; \qquad (24)\\
\frac{\partial^2 q}{\partial \beta^2} &= \left(\frac{\mu}{1-\mu}\right)\frac{\left[4-12\beta\,(1-\beta)-\beta^3\right]}{(1-\beta)^3\,\beta^3} + \frac{8\,[4-3\beta\,(1-\beta)-3\beta]}{(2-\beta)^3\,\beta^3} > 0.
\end{aligned}$$

Note that $\frac{\partial^2 q}{\partial \beta^2} > 0$, since $\beta\,(1-\beta) \leq 1/4$, that $\lim_{\beta \downarrow 0} \frac{\partial q}{\partial \beta} = -\infty$, since both terms in (24) tend to $-\infty$, and that $\lim_{\beta \uparrow 1} \frac{\partial q}{\partial \beta} = +\infty$, since the first term in (24) tends to $+\infty$ and the second to 0. Thus, $q$ is strictly convex in $\beta$ with a trough at $\widetilde{\beta} \in (0,1)$; and hence $q$ is strictly quasi-convex in $\beta$. Since $q > 0$, it follows that $q^{-1}$ is strictly quasi-concave in $\beta$ with a peak at $\widetilde{\beta} \in (0,1)$ (see, e.g., Floudas, 1995, Section 2.3.2(iii)). Thus, $l = q^{-1}t + (t+c)$ is also strictly quasi-concave in $\beta$ with a peak at $\widetilde{\beta} \in (0,1)$. Note that $\widetilde{\beta} > 4 - 2\sqrt{3}$, since the first term in (24) $= 0$ at $\beta = 4 - 2\sqrt{3} \simeq 0.54$ (the unique root of $-4+8\beta-\beta^2$ in $(0,1)$), while the second term is negative for all $\beta \in (0,1)$, and so $\frac{\partial q}{\partial \beta} < 0$ for all $\beta \in (0, 4-2\sqrt{3}]$.

(ii) Let $r \equiv \left(\frac{p-(t+c)}{t}\right)^{-1}$. Using (17):

$$\begin{aligned}
r &= \frac{1}{2}\left(\frac{\mu}{1-\mu}\right)\frac{\left(4-\beta^2\right)(2-\beta)}{(1-\beta)^2\,\beta} + \frac{\left(4-2\beta+\beta^2\right)}{(1-\beta)\,\beta}; \\
\frac{\partial r}{\partial \beta} &= \left(\frac{\mu}{1-\mu}\right)\frac{\left(-4+12\beta-5\beta^2\right)}{(1-\beta)^3\,\beta^2} + \frac{\left(-4+8\beta-\beta^2\right)}{(1-\beta)^2\,\beta^2}; \qquad (25)\\
\frac{\partial^2 r}{\partial \beta^2} &= \left(\frac{\mu}{1-\mu}\right)\frac{\left[8-32\beta\,(1-\beta)+15\beta^2\,(1-\beta)+\beta^2\right]}{(1-\beta)^4\,\beta^3} + \frac{2\left[4-12\beta\,(1-\beta)-\beta^3\right]}{(1-\beta)^3\,\beta^3} > 0.
\end{aligned}$$

By a similar argument to that in part (i), $p$ is strictly quasi-concave in $\beta$ with a peak at $\widetilde{\widetilde{\beta}} \in (0,1)$. The only difference is that both terms in (25) tend to $+\infty$ as $\beta$ tends to 1. Note that $\widetilde{\widetilde{\beta}} \in \left(\frac{2}{5}, 4-2\sqrt{3}\right)$, since the first term in (25) $= 0$ at $\beta = 2/5$ (the unique root of $-4+12\beta-5\beta^2$ in $(0,1)$), while the second term in (25) $= 0$ at $\beta = 4-2\sqrt{3}$ (the unique root of $-4+8\beta-\beta^2$ in $(0,1)$), and so $\frac{\partial r}{\partial \beta} < 0$ for all $\beta \in (0, \frac{2}{5}]$ and $\frac{\partial r}{\partial \beta} > 0$ for all $\beta \in [4-2\sqrt{3}, 1)$. Thus, $\widetilde{\widetilde{\beta}} < \widetilde{\beta}$, since $\widetilde{\beta} > 4-2\sqrt{3}$ from part (i).

(iii) Let

$$\begin{aligned}
M &\equiv \left(\frac{\mu}{1-\mu}\right)\left(4-\beta^2\right)(2-\beta) + 2\,(1-\beta)\left(4-2\beta+\beta^2\right) > 0; \qquad (26)\\
A &\equiv \left(2\beta^3 - 6\beta - 3\beta^3\mu + 4\right).
\end{aligned}$$

Using (2) and (17):

$$l - p = \frac{2t\beta\,(1-\beta)}{M}. \qquad (27)$$

After some manipulation, we can show that

$$(1-2\beta)\,M-\beta\,(1-\beta)\,\frac{\partial M}{\partial\beta}=\frac{(2-\beta)}{(1-\mu)}A,$$

which in turn gives

$$\frac{d\,(l-p)}{d\beta}=\frac{2t\,(2-\beta)}{M^2\,(1-\mu)}A. \tag{28}$$

Note that $\frac{2t(2-\beta)}{M^2(1-\mu)}>0$, and so (28) shares the same sign as $A$. At $\beta=0$, $A=4>0$. At $\beta=1$, $A=-3\mu<0$. Furthermore,

$$\frac{\partial A}{\partial\beta}=-6\left(1-\beta^2\right)-9\beta^2\mu<0.$$

Thus, there is a $\overset{\approx}{\tilde{\beta}}\in(0,1)$ such that $A>0$ when $\beta<\overset{\approx}{\tilde{\beta}}$ and $A<0$ when $\beta>\overset{\approx}{\tilde{\beta}}$. Since the sign of (28) matches that of $A$, this implies that $l-p$ is strictly quasi-concave in $\beta$ with a peak at $\overset{\approx}{\tilde{\beta}}\in(0,1)$. ■

**Proof of Proposition 6.** From (2), $\partial l/\partial t>1$. From (17), $\partial p/\partial t>1$. Using (26) and (27), $\partial\,(l-p)\,/\partial t>0$. In the benchmark with only price takers, $l=t+c$, and so $\partial l/\partial t=1$. ■

**Proof of Proposition 7.** Recall that, since the market is covered, welfare falls linearly in transport costs $T$. Differentiating (18),

$$\begin{aligned}\frac{dT}{d\mu}&=\frac{\beta\,(1-\beta)}{t}\left[-\frac{(l-p)^2}{2}+(1-\mu)\,(l-p)\,\frac{d\,(l-p)}{d\mu}\right];\\ \frac{d^2T}{d\mu^2}&=\frac{\beta\,(1-\beta)}{t}\left\{\left[-2\,(l-p)+(1-\mu)\,\frac{d\,(l-p)}{d\mu}\right]\frac{d\,(l-p)}{d\mu}+(1-\mu)\,(l-p)\,\frac{d^2\,(l-p)}{d\mu^2}\right\}.\end{aligned}$$

From (26) and (27), $l-p>0$. Thus, using (22), $dT/d\mu<0$, and so total welfare falls in the proportion of bargainers. Using (22) and (23), $d^2T/d\mu^2>0$, and so total welfare falls at an increasing rate. ■

**Proof of Proposition 8.** Recall that, since the market is covered, welfare falls linearly in transport costs $T$. Using (18), (26) and (27):

$$T=\frac{t}{4}+\frac{2t\,(1-\mu)\,\beta^3\,(1-\beta)^3}{M^2}. \tag{29}$$

Let

$$B\equiv 24-60\beta+42\beta^2-6\beta^3+\mu\left(8\beta-24\beta^2+7\beta^3\right).$$

After some manipulation, we can show that

$$(1-\mu)\left[3(1-2\beta)M-2\beta(1-\beta)\frac{\partial M}{\partial \beta}\right]=B,$$

which in turn gives

$$\frac{dT}{d\beta}=\frac{2t\beta^2(1-\beta)^2}{M^3}B. \tag{30}$$

Note that $\frac{2t\beta^2(1-\beta)^2}{M^3}>0$, and so (30) shares the same sign as $B$. At $\beta=0$, $B=24>0$. At $\beta=1$, $B=-9\mu<0$. Furthermore,

$$\frac{\partial^2 B}{\partial \beta^2}=6(14-6\beta-8\mu+7\beta\mu)>0.$$

Thus, there is a $\overline{\beta}\in(0,1)$ such that $B>0$ when $\beta<\overline{\beta}$ and $B<0$ when $\beta>\overline{\beta}$. Since the sign of (30) matches that of $B$, this implies that $T$ is strictly quasi-concave in $\beta$ with a peak at $\overline{\beta}\in(0,1)$. Thus, welfare is strictly quasi-convex in $\beta$ with a trough at $\overline{\beta}\in(0,1)$. ■

**Proof of Proposition 9.** Using (9), and noting that $\frac{1}{2}+\frac{l-p}{2t}\in(0,1)$ and $\frac{1}{2}-\frac{l-p}{2t}\in(0,1)$ (since the equilibrium is interior from the proof of Claim 5), each firm's profits are given by

$$\begin{aligned}
\pi &= \frac{1}{2}[\beta\mu+(1-\beta)](l-c)+\frac{1}{2}(1-\mu)\beta(p-c)-\frac{1}{2}(1-\mu)(1-\beta)\beta t^{-1}(l-p)^2, \text{ and}\\
\frac{d\pi}{d\mu} &= \frac{1}{2}\beta(l-c)+\frac{1}{2}[\beta\mu+(1-\beta)]\frac{dl}{d\mu}-\frac{1}{2}\beta(p-c)+\frac{1}{2}(1-\mu)\beta\frac{dp}{d\mu}\\
&\quad+\frac{1}{2}(1-\beta)\beta t^{-1}(l-p)^2-(1-\mu)(1-\beta)\beta t^{-1}(l-p)\frac{d(l-p)}{d\mu}.
\end{aligned}$$

Using (2), (3), (19), (21) and (22), and after some manipulation,

$$\frac{\frac{d\pi}{d\mu}}{\frac{dl}{d\mu}}=\frac{1}{2}+\left(\frac{1-\mu}{2-\beta}\right)\frac{\beta}{2}\left\{1-\left[\underbrace{\mu+(1-\mu)\frac{(2-2\beta)(4-2\beta+\beta^2)}{(2-\beta)(4-\beta^2)}+2}_{(i)}\right]\left[1+(1-\beta)\left(\frac{l-p}{t}\right)\right]\right\}.$$

Note $(i)>0$ and, from (26) and (27), $\frac{l-p}{t}>0$. Since $2-\beta>2-2\beta$ and $4-\beta^2>4-2\beta+\beta^2$, $(i)<3$. From (3), $l-p=\frac{l-(t+c)}{2-\beta}$, and hence using (2) $\frac{l-p}{t}<\frac{\beta}{4-2\beta+\beta^2}$. Thus we can determine a bound:

$$\begin{aligned}
\frac{\frac{d\pi}{d\mu}}{\frac{dl}{d\mu}} &> \frac{1}{2}+\left(\frac{1-\mu}{2-\beta}\right)\frac{\beta}{2}\left[-2-3\frac{(1-\beta)\beta}{(4-2\beta+\beta^2)}\right]\\
&= \frac{8-16\beta+5\beta^2+\mu\beta\left(8-\beta-\beta^2\right)}{2(2-\beta)(4-2\beta+\beta^2)}.
\end{aligned} \tag{31}$$

Now $(31)>0$ for any $\mu>0$ if $8-16\beta+5\beta^2>0$, which in turn requires $\beta<\left(8-2\sqrt{6}\right)/5\simeq$

0.6202. From (19) $dl/d\mu < 0$, and so for $\beta \in \left(0, \left(8 - 2\sqrt{6}\right)/5\right]$ we have $d\pi/d\mu < 0$, and hence profits increase in the proportion of bargainers. From Proposition 7, total welfare always falls in the proportion of bargainers, and hence consumer surplus falls when profits increase. ■

**Proof of Proposition 10.** Using (9), and noting that $\frac{1}{2} + \frac{l-p}{2t} \in (0,1)$ and $\frac{1}{2} - \frac{l-p}{2t} \in (0,1)$ (since the equilibrium is interior from the proof of Claim 5), each firm's profits are given by:

$$\pi = (l-p)\left[\underbrace{\frac{\mu + (1-\mu)(1-\beta)^2}{2} + (1-\mu)(1-\beta)\beta\left(\frac{1}{2} - \frac{l-p}{2t}\right)}_{(a)}\right] + \frac{p-c}{2}. \tag{32}$$

Using (26) and (27), $\frac{l-p}{2t}$ is independent of $t$. From the proof of Proposition 6, $\partial(l-p)/\partial t > 0$ and $\partial p/\partial t > 1$. Thus, $\partial\pi/\partial t > \frac{1}{2}$, since $(a) > 0$ given $\frac{1}{2} - \frac{l-p}{2t} > 0$ from above. In the benchmark with only price takers,

$$\pi = \frac{1}{2}(l-c) = \frac{1}{2}(t+c-c) = \frac{1}{2}t, \tag{33}$$

and so $\partial\pi/\partial t = \frac{1}{2}$.

Recall that, since the market is covered, welfare falls linearly in transport costs $T$. Using (26) and (29), $\partial T/\partial t > \frac{1}{4}$. In the benchmark with only price takers, $T = \frac{1}{4}t$, and so $\partial T/\partial t = \frac{1}{4}$.

Since the market is covered, consumer surplus falls linearly in $T + 2\pi$. From above, $T$ and $\pi$ rise faster than in the benchmark, and so consumer surplus falls faster than in the benchmark.

■

**Proof of Proposition 11.** Lemmas 10 and 11 establish equilibrium behavior when the two-stage game is played only once. Claim 6 then completes the proof, recalling Section 6.2, which describes the equilibrium concept and off-equilibrium punishments. ■

**Lemma 10** *Suppose that the list prices are given by* $\{l_j : j \in \{1, 2, ..., N\}\}$ *and suppose that* $n \geq 1$ *firms, including firm* $i$*, set the lowest list price* $\underline{l} \equiv \min\{l_j\} > c$*. Then there is a unique symmetric Nash equilibrium of the discount stage, in which all firms offer prices*

$$p \in \left((1-\beta)^{N-1}(\underline{l} - c) + c\ ,\ \underline{l}\ \right)$$

*drawn from the distribution*[22]

$$F(p) = \frac{1}{\beta} - \left(\frac{1-\beta}{\beta}\right)\left(\frac{\underline{l} - c}{p-c}\right)^{\frac{1}{N-1}}. \tag{34}$$

[22]It makes no difference to the analysis whether (i) for every bargainer a firm draws a price from the pricing distribution or (ii) a firm draws a single price from the pricing distribution, which it then offers to all bargainers.

*Firm $i$ makes expected profits from the bargainers of:*

$$\pi_i = (l_i - c)(1-\beta)^{N-1}\left(\frac{1-\beta}{n} + \beta\right).$$

**Proof.** We start by showing that any symmetric equilibrium must be mixed with: (i) $F(\underline{l}) = 1$; (ii) no mass points in the density function; and (iii) $F(p) < 1$ for $p < \underline{l}$.

(i) Recall from Section 3 that $p_i \leq l_i$, and by assumption firm $i$ is one of the $n \geq 1$ firms setting the lowest list price $\underline{l}$, so $p_i \leq \underline{l}$. Thus, in a symmetric equilibrium $F(\underline{l}) = 1$.

(ii) If there were a mass point at price $p > c$, a firm could deviate profitably by lowering its discount price to $p - \varepsilon$ just below the mass point whenever it would have offered $p$. This increases sales by a discrete amount (when the bargainer also receives an offer of $p$ from a rival firm and receives no lower offers) in return for a vanishingly small loss and so is a profitable deviation. If there were a mass point at $p \leq c$, a firm would deviate upward to sell at a strictly positive profit to bargainers who receive its offer and not any of the rivals'.

(iii) Suppose that the support of $F$ is bounded above at $p < \underline{l}$. From (ii), the probability that any of the rival firms offers this highest price to the bargainers is zero. Thus any firm offering the highest discount price could deviate profitably by raising price towards $\underline{l}$ since the firm will continue to sell at the offered price if and only if the bargainer receives its offer and not any of the rivals'.[23]

Now suppose that firm $i$ offers a discount price $p < \underline{l}$, and the rival firms draw their discount prices from the same distribution $F$. If a bargainer does not receive firm $i$'s offer, then the firm sells at $l_i$ with probability $1/n$ when the bargainer also fails to receive any of the other firms' offers. If, instead, a bargainer does receive the offer, then the firm sells at $p$ when $p$ is below any other offers received by the bargainer. The probability that a bargainer receives $k$ of the $N-1$ rival firms' offers is

$$\beta^k (1-\beta)^{N-1-k} \begin{pmatrix} N-1 \\ k \end{pmatrix}$$

where the binomial coefficient counts the number of (unordered) combinations of $k$ rivals that can be constructed out of a set of $N-1$. Combining, we can write firm $i$'s expected profit from the bargainers at any offered price $p < l_i$ as

$$\pi_i(p) = (l_i - c)(1-\beta)^N \frac{1}{n} + (p-c)\beta \sum_{k=0}^{N-1} \beta^k (1-\beta)^{N-1-k} \begin{pmatrix} N-1 \\ k \end{pmatrix} (1 - F(p))^k .$$

[23] Even if the density is zero at the highest price in the support of the distribution, by continuity profit at this price must be the same as for prices in the interior of the mixing distribution.

Using the Binomial Theorem (e.g., Kreyszig, 1993, p. 1165), we have

$$\begin{aligned}\pi_i(p) &= (l_i - c)(1-\beta)^N \frac{1}{n} + (p-c)\beta\left[1-\beta+\beta(1-F(p))\right]^{N-1} \\ &= (l_i - c)(1-\beta)^N \frac{1}{n} + (p-c)\beta(1-\beta F(p))^{N-1}. \qquad (35)\end{aligned}$$

For firm $i$ to be willing to randomize, its profit must be constant at all points in the support of $F$. To find $\pi_i$, we consider firm $i$'s profit from setting a price which tends to the upper bound of the support of $F$, that is $l_i = \underline{l}$:

$$\begin{aligned}\pi_i &= \lim_{p \uparrow l_i} \pi_i(p) = (l_i - c)(1-\beta)^N \frac{1}{n} + (l_i - c)\beta(1-\beta)^{N-1} \qquad (36) \\ &= (l_i - c)(1-\beta)^{N-1}\left(\frac{1-\beta}{n} + \beta\right).\end{aligned}$$

Equating (35) and (36) yields

$$\left(\frac{l_i - c}{p - c}\right)(1-\beta)^{N-1} = (1-\beta F(p))^{N-1}.$$

This can be solved to yield (34). The lower bound of the support $\underline{p}$ can then be determined by setting $F(\underline{p}) = 0$.

Clearly, firm $i$ has no incentive to deviate downward from $\underline{p}$: from (ii) there can't be a mass point at $\underline{p}$, so the firm would continue to sell to the same proportion of bargainers. It is readily confirmed that any firm $j$ whose list price is above $\underline{l}$ has the same pricing distribution since the profit for such a firm $j$ is the same as for firm $i$, except that firm $j$ makes no profit when its offer is not received. We also have to check that such a firm $j$ has no incentive to deviate to $p_j \in [\underline{l}, l_j]$: at $p_j = \underline{l}$ the firm would sell to only $1/(n+1)$ of the bargainers who receive only its offer; at $p_j > \underline{l}$ the firm would fail to sell to any of the bargainers when its offer is received. ∎

**Lemma 11** *The unique symmetric pure-strategy Nash Equilibrium of the one-shot two-stage game has all prices at marginal cost and profits of zero.*

**Proof.** Suppose first that there is a symmetric equilibrium with list prices $l^* > c$. A given firm $i$ could deviate profitably by lowering its list price to $l^* - \varepsilon$. The firm would then sell to all the price takers. From Lemma 10, profits from bargainers would also rise, since there would then be $n = 1$ firms with the lowest list price as opposed to $n = N$. Symmetric list prices $l^* = c$ form an equilibrium, since profits from price takers and bargainers are zero at $l_i \geq \left\{l_j^* : j \neq i\right\} = c$. ∎

**Claim 6** *The critical discount factor that allows collusion to be subgame perfect* $\delta^{\dagger} < 1 - 1/N$, *and so is strictly lower in the presence of bargainers.*

**Proof.** Recall that we are looking for subgame-perfect Nash equilibria in which the firms collude on a price $z$ at the list-price-setting stage and the discount stage, supported by the threat of reversion to the lowest-payoff non-collusive symmetric equilibrium. In the benchmark case with only price takers, it is well-known that such collusion can be sustained by the threat of reversion to the zero-profit one-shot equilibrium when the discount factor $\delta \geq 1 - 1/N$.

Lemma 10 with $n = 1$ gives profits in the unique non-collusive symmetric equilibrium in the discount stage during a period in which a deviation from $z > c$ occurred at the list-price-setting stage. By Lemma 11, the lowest-payoff non-collusive symmetric equilibrium of the one-shot two-stage game must give profits of zero in the periods after a deviation occurred. Deviating at the list-price-setting stage to $l_i = z - \varepsilon$ wins all the price takers. Using Lemmas 10 and 11, total profit from this deviation is given by

$$\pi^{\text{dev1}} \simeq \mu (z - c) + (1 - \mu)(z - c)(1 - \beta)^{N-1} = (z - c)\left[\mu + (1 - \mu)(1 - \beta)^{N-1}\right]. \tag{37}$$

An alternative deviation would be to deviate at the discount stage to $p_i = z - \varepsilon$, instead of deviating at the list-price-setting stage. The deviant firm would capture all the bargainers whenever its price offer was received and $1/N$ of the bargainers otherwise. Thus, total profit from this deviation (including profit at the list price-setting-stage preceding the deviation) is given by

$$\pi^{\text{dev2}} \simeq \mu\left(\frac{z - c}{N}\right) + (1 - \mu)\left[\beta (z - c) + (1 - \beta)\left(\frac{z - c}{N}\right)\right] = \left(\frac{z - c}{N}\right)\left[1 + (1 - \mu)\beta (N - 1)\right]. \tag{38}$$

Hence the collusion can be sustained at all discount factors $\delta \geq \delta^{\dagger}$, where

$$\begin{aligned} \frac{z - c}{N(1 - \delta^{\dagger})} &= \left(\frac{z - c}{N}\right)\max\left\{N\left[\mu + (1 - \mu)(1 - \beta)^{N-1}\right], 1 + (1 - \mu)\beta (N - 1)\right\}, \text{ i.e.,} \\ \delta^{\dagger} &= 1 - \frac{1}{\max\left\{N\left[\mu + (1 - \mu)(1 - \beta)^{N-1}\right], 1 + (1 - \mu)\beta (N - 1)\right\}}. \end{aligned} \tag{39}$$

Clearly, $\delta^{\dagger} < 1 - 1/N$, since

$$\max\left\{N\left[\mu + (1 - \mu)(1 - \beta)^{N-1}\right], 1 + (1 - \mu)\beta (N - 1)\right\} < N$$

given $\mu \in (0, 1)$ and $\beta \in (0, 1)$. ■

**Proof of Proposition 12.** Deviating at the list-price-setting stage gives $\pi^{dev1}$ given in (37). Deviating at the discount stage gives $\pi^{dev2}$ given in (38). From (37), (38) and (39), the critical discount factor that allows collusion to be subgame perfect is

$$\delta^{\dagger} = 1 - \frac{z-c}{N \max\{\pi^{dev1}, \pi^{dev2}\}}. \tag{40}$$

We can see that:

$$\begin{aligned}
\frac{\partial \pi^{dev1}}{\partial \mu} &= (z-c)\left[1-(1-\beta)^{N-1}\right] > 0; \\
\frac{\partial \pi^{dev2}}{\partial \mu} &= \left(\frac{z-c}{N}\right)\left[-\beta\,(N-1)\right] < 0.
\end{aligned}$$

Thus, $\pi^{dev1}$ increases linearly in $\mu$ while $\pi^{dev2}$ decreases linearly. Furthermore, when $\mu = 1$, from (37) and (38) $\pi^{dev1} > \pi^{dev2}$, and so $\pi^{dev1} \gtreqless \pi^{dev2} \Leftrightarrow \mu \gtreqless \widehat{\mu}$, with $\widehat{\mu} < 1$ or equivalently $1 - \widehat{\mu} > 0$. Note that when $\mu = 0$ and $\beta = 1$, $\pi^{dev1} < \pi^{dev2}$, and when $\mu = 0$ and $\beta = 0$, $\pi^{dev1} > \pi^{dev2}$; thus, $\widehat{\mu} > 0$ for $\beta$ close to but smaller than 1, and $\widehat{\mu} < 0$ for $\beta$ close to but larger than 0.

If $\widehat{\mu} > 0$ or equivalently $1-\widehat{\mu} < 1$, then when $\mu < \widehat{\mu}$ or equivalently $1-\mu > 1-\widehat{\mu}$, using (40) $\delta^{\dagger} = 1-(z-c)/\left(N\pi^{dev2}\right)$. Thus, $\delta^{\dagger}$ increases in $\pi^{dev2}$, which in turn decreases in $\mu$, and hence $\delta^{\dagger}$ increases in the proportion of bargainers $1-\mu$. When $\mu > \widehat{\mu}$ or equivalently $1-\mu < 1-\widehat{\mu}$, $\delta^{\dagger} = 1-(z-c)/\left(N\pi^{dev1}\right)$. Thus, $\delta^{\dagger}$ increases in $\pi^{dev1}$, which in turn increases in $\mu$; hence $\delta^{\dagger}$ decreases in the proportion of bargainers $1-\mu$. ■

**Proof of Proposition 13.** Deviating at the list-price-setting stage gives $\pi^{dev1}$ given in (37). Deviating at the discount stage gives $\pi^{dev2}$ given in (38). The critical discount factor that allows collusion to be subgame perfect is given by (40). We can see that:

$$\begin{aligned}
\frac{\partial \pi^{dev1}}{\partial \beta} &= (z-c)\left[(1-\mu)(N-1)(1-\beta)^{N-2}(-1)\right] < 0; \\
\frac{\partial \pi^{dev2}}{\partial \beta} &= \left(\frac{z-c}{N}\right)\left[(1-\mu)(N-1)\right] > 0.
\end{aligned}$$

Thus, $\pi^{dev1}$ decreases in $\beta$ while $\pi^{dev2}$ increases linearly. Furthermore, when $\beta = 0$, from (37) and (38) $\pi^{dev1} > \pi^{dev2}$, and so $\pi^{dev1} \gtreqless \pi^{dev2} \Leftrightarrow \beta \lesseqgtr \widehat{\beta}$ with $\widehat{\beta} > 0$. Note that when $\beta = 1$ and $\mu = 0$, $\pi^{dev1} < \pi^{dev2}$, and when $\beta = 1$ and $\mu = 1$, $\pi^{dev1} > \pi^{dev2}$; thus, $\widehat{\beta} < 1$ for $\mu$ close to but larger than 0, and $\widehat{\beta} > 1$ for $\mu$ close to but smaller than 1.

If $\widehat{\beta} < 1$, then when $\beta > \widehat{\beta}$, using (40) $\delta^{\dagger} = 1-(z-c)/\left(N\pi^{dev2}\right)$. Thus, $\delta^{\dagger}$ increases in $\pi^{dev2}$, which in turn increases in $\beta$. When $\beta < \widehat{\beta}$, $\delta^{\dagger} = 1-(z-c)/\left(N\pi^{dev1}\right)$. Thus, $\delta^{\dagger}$ increases in $\pi^{dev1}$, which in turn decreases in $\beta$. ■

**Proof of Remark 2.** Using (9), noting that $\frac{1}{2}+\frac{l-p}{2t}\in(0,1)$ and $\frac{1}{2}-\frac{l-p}{2t}\in(0,1)$ (since the equilibrium is interior from the proof of Claim 5), and taking as given the equilibrium prices at a common level of $\beta$, each firm's profits are a linear function of their own discount reliability $\beta_i$:

$$\begin{aligned}\pi(\beta_i) &= (\text{constant})+(1-\mu)\beta_i(1-\beta)\left[\underbrace{-\left(\frac{l-p}{2}\right)+(p-c)\left(\frac{l-p}{2t}\right)}_{(i)}\right] \\ &+ (1-\mu)\beta_i\beta\left[\underbrace{-\left(\frac{l-p}{2}\right)+(l-c)\left(\frac{l-p}{2t}\right)}_{(ii)}\right].\end{aligned}$$

Note that

$$(i)=\left(\frac{l-p}{2}\right)\left(\frac{p-(t+c)}{t}\right)>0$$

since $l-p>0$ from (26) and (27) and $p>t+c$ from (17). Furthermore, $(ii)>(i)$, since $l>p$. Thus, $\frac{\partial\pi}{\partial\beta_i}>0$. From (2) and (17), $\lim_{\beta\uparrow 1} l=\lim_{\beta\uparrow 1} p=t+c$. Thus, $\lim_{\beta\uparrow 1}\frac{\partial\pi}{\partial\beta_i}=0$. ■

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