Multi-Hop Relaying Using Energy Harvesting

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***Abstract*—In this letter, the performance of multi-hop relaying using energy harvesting is evaluated. Both amplify-and-forward and decode-and-forward relaying protocols are considered. The evaluation is conducted for time-switching energy harvesting as well as power-splitting energy harvesting. The largest number of hops given an initial amount of energy from the source node is calculated. Numerical results show that, in order to extend the network coverage using multi-hop relaying, time-switching is a better option than power splitting and in some cases, decode-and-forward also supports more hops than amplify-and-forward.**

***Index Terms*—Energy harvesting, multi-hop relaying, power-splitting, time-switching.**

I. Introduction

Energy harvesting improves the performance of wireless relaying by relieving the energy consumption burden at the relays. Several works have been conducted in this area. In their seminal paper [1], the authors proposed two methods of signal relaying based on energy harvesting using time-switching (TS) and power-splitting (PS). The harvested energy was then used to forward the received signal to the destination. In [2], assuming that the relay does not have any energy storage capacity, the optimum tradeoff between harvesting time and relaying time has been studied. In [3], assuming multiple energy harvesting relays are available, the allocation policies of the total energy harvested from multiple sources have been studied among different relays. Reference [4] used PS to study the outage performance and the averaged harvested energy for both non-cooperative and cooperative systems suffering from large-scale network interference. Reference [5] used stochastic geometry to study the effect of random relay location on the performance of decode-and-forward (DF) relaying. In [6], the throughput was analyzed for a DF relaying system. In [7], a joint power splitting and antenna selection scheme was considered to maximize the achievable rate for amplify-and-forward (AF) relaying.

All the aforementioned works have given useful guidance on the performance of energy-harvesting-based relaying to allow system designers to make informed decisions. However, most of them have focused on dual-hop relaying instead of multi-hop relaying. From a practical point of view, one of the main benefits of relaying is to extend the network coverage without additional infrastructure, for which multi-hop relaying plays an important role, as the extended coverage often increases with the number of hops used. From a theoretical point of view, it is also of great interest to know how many times the initial energy carried by the signal from the source node can be harvested or how many relays this initial energy can support to satisfy certain performance criterion.

In this letter, the performance of multi-hop relaying using energy harvesting is evaluated. Both AF and DF relaying protocols are investigated. For each protocol, TS and PS harvesting strategies are applied at the relays. Subject to a fixed throughput restriction for each hop, the largest numbers of hops are studied, where the initial energy carried by the signal from the source node is harvested by one relay after the other until the remaining energy is not sufficient to support the required throughput. Numerical results show that, for the purpose of extending network coverage using multi-hop relaying, TS is better than PS. Also, DF and AF have similar performances for TS, but DF is much better than AF for PS.

II. Derivations

Consider a relaying system with one relaying link from the source node to the destination node via several hops. All the nodes are half-duplex and have a single antenna. The signal from the source node is relayed by relaying nodes in multiple hops, one node in each hop, until it arrives at the destination node. In each hop, the relaying node first harvests the energy of the signal from the previous hop and the harvested energy is then used to relay the signal to the next hop. Assume that in each hop it takes T seconds to transmit the signal and that the initial transmission power of the source is P_0.

A. Time-switching

In the TS method, the relay spends a portion of the relaying time on harvesting the energy of the signal from the source node or the previous hop. Denote α_m as this portion at the m-th relay in the m-th hop, with $0 \leq \alpha_m \leq 1$ and $m = 1, 2, \cdots$. The throughput in this hop is

$$R_0 = (1 - \alpha_m) \log_2(1 + \gamma_m) \tag{1}$$

where γ_m will be determined later.

1) AF: In the AF protocol, the received signal from the previous hop is amplified and forwarded to the next hop directly. Thus, the received signal at the m-th relay is given by

$$y_m[k] = \sqrt{P_{m-1}} g_{m-1} \frac{u_m}{\sqrt{d_m^v}} y_{m-1}[k] + n_{ma}[k] + n_{mc}[k] \tag{2}$$

where $m = 1, 2, \cdots$ index the hops, P_{m-1} is the transmission power of the $(m-1)$-th node and P_0 is the transmission power of the source, u_m is the fading coefficient of the m-th hop assumed constant in each block, d_m is the node distance in the m-th hop, v is the path loss exponent, g_{m-1} is the

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amplification factor at the $(m-1)$-th relay, $y_{m-1}[k]$ is the received signal in the $(m-1)$-th hop and $y_0[k] = s[k]$ is the transmitted binary phase shift keying (BPSK) signal with equal *a priori* probabilities for $s[k] = 1$ and $s[k] = -1$, $n_{ma}[k]$ is the additive white Gaussian noise (AWGN) introduced by the RF front at the m-th relay, and $n_{mc}[k]$ is the AWGN introduced in the RF to baseband conversion at the m-th relay [1]. Furthermore, assume that the fading coefficient u_m is known. For simplicity, in the following, we set the distance d_m to 1. If it is required to model the distances explicitly, one can simply replace u_m with $\frac{u_m}{\sqrt{d_m^v}}$ in the following results. Also, assume that $n_{mc}[k]$ is a Gaussian random variable with mean zero and variance σ_{mc}^2, and $n_{ma}[k]$ is a Gaussian random variable with mean zero and variance σ_{ma}^2. Using (2), one further has

$$\begin{aligned} y_m[k] &= \prod_{i=1}^{m} u_i \prod_{i=1}^{m-1} g_i \prod_{i=0}^{m-1} \sqrt{P_i} s[k] \\ &+ \sum_{i=1}^{m} [\prod_{j=i+1}^{m} u_j \prod_{j=i}^{m-1} g_j \prod_{j=i}^{m-1} \sqrt{P_j}(n_{ia}[k] + n_{ic}[k])]. \end{aligned} \tag{3}$$

In this work, two types of AF protocol are considered: fixed-gain and variable-gain [8]. For fixed-gain AF, the amplification factor is set to $g_i = \frac{1}{\sqrt{P_{i-1}\Delta_i + \sigma_{ic}^2 + \sigma_{ia}^2}}$, where $\Delta_i = E\{|u_i|^2\}$ [9]. For variable-gain AF, the amplification factor is set to $g_i = \frac{1}{\sqrt{P_{i-1}|u_i|^2 + \sigma_{ic}^2 + \sigma_{ia}^2}}$. Also, in (3), the product of $\prod_{i=1}^{m-1} g_i$ in the signal term and the products of $\prod_{j=i+1}^{m} u_j$, $\prod_{j=i}^{m-1} g_j$ and $\prod_{j=i}^{m-1} \sqrt{P_j}$ in the noise term are mathematically not well-defined when the lower limit is larger than the upper limit. For example, when $m = 1$, $\prod_{i=1}^{0} g_i$ is not defined. If this happens, the product is set to 1 to avoid this notational problem.

From (2), the energy harvested at the m-th relay is

$$E_{hm} = \eta \prod_{i=0}^{m-1} P_i \prod_{i=1}^{m-1} g_i^2 \prod_{i=1}^{m} u_i^2 \alpha_m T \tag{4}$$

where η is the conversion efficiency of the energy harvester. Since each hop has a total transmission time of T, the transmitted power at the m-th node can be calculated as

$$P_m = \frac{E_{hm}}{T} = \eta \prod_{i=0}^{m-1} P_i \prod_{i=1}^{m-1} g_i^2 \prod_{i=1}^{m} u_i^2 \alpha_m \tag{5}$$

The normalization in (5) is with respect to T, as T is the total transmission time that determines the transmission power.

One sees from (4) that the harvested energy at each hop decreases at an accelerated rate when the hop number increases. At some hop, the energy will not be sufficient for transmission based on certain criterion. In this case, γ_m in (1) is given by

$$\gamma_m = \frac{\prod_{i=1}^{m} u_i^2 \prod_{i=1}^{m-1} g_i^2 \prod_{i=1}^{m-1} P_i (1-\alpha_m) T}{\sum_{i=1}^{m} [\prod_{j=i+1}^{m} u_j^2 \prod_{j=i}^{m-1} g_j^2 \prod_{j=i}^{m-1} P_j (\sigma_{ia}^2 + \sigma_{ic}^2)]}. \tag{6}$$

It is of great interest to find the first value of m for which $(1-\alpha_m)\log_2(1+\gamma_m) < R_0$. Before this happens, each hop has a throughput of R_0 such that the TS coefficient α_m can be calculated as

$$\alpha_m = 1 - \frac{(2^{R_0/(1-\alpha_m)} - 1)W}{\prod_{i=1}^{m} u_i^2 \prod_{i=1}^{m-1} g_i^2 \prod_{i=1}^{m-1} P_i T}, 0 \le \alpha_m \le 1 \tag{7}$$

for the m-th relay, where $W = \sum_{i=1}^{m} [\prod_{j=i+1}^{m} u_j^2 \prod_{j=i}^{m-1} g_j^2 \prod_{j=i}^{m-1} P_j (\sigma_{ia}^2 + \sigma_{ic}^2)]$. This equation does not lead to a closed-form expression of α_m but it can be easily solved using mathematical software.

2) DF: In the DF protocol, the received signal from the previous hop is first decoded and then the decoded information is forwarded to the next hop. Thus, the received signal at the m-th relay is given by

$$y_m[k] = \sqrt{P_{m-1}} u_m s_{m-1}[k] + n_{ma}[k] + n_{mc}[k] \tag{8}$$

where $s_{m-1}[k] = sign\{y_{m-1}[k] u_{m-1}^*\}$ is the decoded and forwarded information at the relay, $sign\{x\} = 1$ when $x > 0$ and $sign\{x\} = -1$ when $x < 0$ is the signum function, and all other symbols are defined as before. For DF, the harvested energy at the m-th relay can be derived as

$$E_{hm} = \eta P_{m-1} u_m^2 \alpha_m T. \tag{9}$$

Similarly, using the throughput criterion for DF, one has γ_m in (1) as

$$\gamma_m = \frac{P_{m-1} u_m^2 (1-\alpha_m) T}{\sigma_{ma}^2 + \sigma_{mc}^2}. \tag{10}$$

The TS coefficient α_m can be calculated as

$$\alpha_m = 1 - \frac{(2^{R_0/(1-\alpha_m)} - 1)(\sigma_{ma}^2 + \sigma_{mc}^2)]}{P_{m-1} u_m^2 T}, 0 \le \alpha_m \le 1 \tag{11}$$

for the m-th relay, until the throughput drops below the threshold as $(1-\alpha_m)\log_2(1+\gamma_m) < R_0$, for which the value of m is determined as the largest number of hops supported by the initial power of P_0.

B. Power-splitting

The previous subsection discussed the TS method. On the other hand, in the PS method, the relay splits a portion of the received signal power for the whole relaying time T as harvested energy. Denote ρ_m as the splitting factor at the m-th relay, with $0 \le \rho_m \le 1$ and $m = 1, 2, \cdots$. The throughput in this case is

$$R_0 = \log_2(1 + \gamma_m) \tag{12}$$

where γ_m is given later.

1) AF: In this case, the received signal at the m-th relay is given by [1]

$$\begin{aligned} y_m[k] &= \sqrt{(1-\rho_m) P_{m-1}} g_{m-1} u_m y_{m-1}[k] \\ &+ \sqrt{1-\rho_m} n_{ma}[k] + n_{mc}[k] \end{aligned} \tag{13}$$

for T seconds, where all the symbols are defined as before. One further has

$$\begin{aligned} y_m[k] &= \prod_{i=0}^{m-1}\sqrt{(1-\rho_{i+1})P_i}\prod_{i=1}^{m}u_i\prod_{i=1}^{m-1}g_i s[k] \\ &+ \sum_{i=1}^{m}[\prod_{j=i}^{m-1}\sqrt{(1-\rho_{j+1})P_j} \\ &\cdot \prod_{j=i+1}^{m}u_i\prod_{j=i}^{m-1}g_j((1-\rho_i)n_{ia}[k]+n_{ic}[k])] \end{aligned} \quad (14)$$

The same received signals are also used for energy harvesting. Using the harvested energies, the transmission power at the m-th relay can be calculated as

$$P_m = \eta\prod_{i=0}^{m-1}P_i\prod_{i=1}^{m-1}g_i^2\prod_{i=1}^{m}u_i^2\prod_{i=1}^{m-1}(1-\rho_i)\rho_m. \quad (15)$$

Compared with (5) for the TS method, one sees that the harvested power in (15) has an additional term of $\prod_{i=1}^{m-1}(1-\rho_i)$. Since this term is smaller than 1 and decreases quickly as m increases, the harvested power using the PS method is smaller than that using the TS method, under similar conditions. As a result, the largest number of hops using the PS method is smaller than that using the TS method, which is not desirable for network coverage extension. This will be verified by numerical results later.

Again, applying the throughput per hop restriction, one has γ_m in (12) for PS as

$$\gamma_m = \frac{\prod_{i=0}^{m-1}(1-\rho_{i+1})P_i\prod_{i=1}^{m}u_i^2\prod_{i=1}^{m-1}g_i^2T}{U} \quad (16)$$

$U = \sum_{i=1}^{m}[\prod_{j=i}^{m-1}\sqrt{(1-\rho_{j+1})P_j}\prod_{j=i+1}^{m}u_i^2\prod_{j=i}^{m-1}g_j^2((1-\rho_i)\sigma_{ia}^2+\sigma_{ic}^2)]$. The PS splitting factor ρ_m can be calculated as

$$\rho_m = 1-\frac{(2^{R_0}-1)\sigma_{mc}^2}{\prod_{i=0}^{m-1}(1-\rho_{i+1})P_i\prod_{i=1}^{m}u_i^2\prod_{i=1}^{m-1}g_i^2T-V} \quad (17)$$

where $0 \le \rho_m \le 1$ and $V = \{\sum_{i=1}^{m-1}[\prod_{j=i}^{m-1}\sqrt{(1-\rho_{j+1})P_j}\prod_{j=i+1}^{m}u_i^2\prod_{j=i}^{m-1}g_j^2((1-\rho_i)\sigma_{ia}^2+\sigma_{ic}^2)]+\sigma_{ma}^2\}(2^{R_0}-1)$. Unlike the TS coefficient α_m, the PS splitting factor ρ_m does have a closed-form expression.

2) DF: For DF, the received signal at the m-th relay is

$$\begin{aligned} y_m[k] &= \sqrt{(1-\rho_m)P_{m-1}}u_m s_{m-1}[k] \\ &+ \sqrt{1-\rho_m}n_{ma}[k]+n_{mc}[k] \end{aligned} \quad (18)$$

for the whole relaying period T seconds, where all the symbols are defined as before. Using the PS method, the transmission power at the m-th relay can be calculated as

$$P_m = \eta\rho_m P_{m-1}u_m^2. \quad (19)$$

Then, applying the throughput restriction to each hop, one has γ_m in (12) as

$$\gamma_m = \frac{(1-\rho_m)P_{m-1}u_m^2T}{(1-\rho_m)\sigma_{ma}^2+\sigma_{mc}^2}. \quad (20)$$

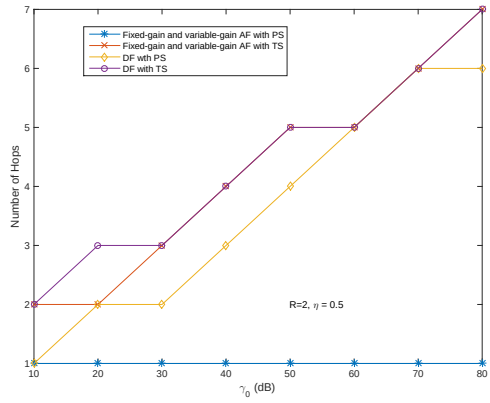

Fig. 1. The maximum number of hops vs. γ_0 for $d=1$, $R=2$ and $\eta=0.5$.

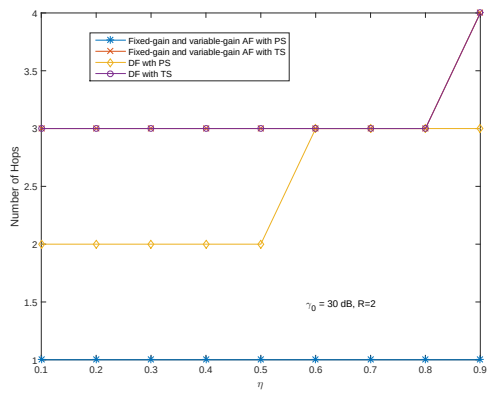

Fig. 2. The maximum number of hops vs. η for $d=1$, $R=2$ and $\gamma_0=30\ dB$.

Using this restriction, the PS splitting factor ρ_m can be calculated as

$$\rho_m = 1-\frac{(2^{R_0}-1)\sigma_{mc}^2}{P_{m-1}u_m^2T-\sigma_{ma}^2(2^{R_0}-1)}, 0\le\rho_m\le 1. \quad (21)$$

In the next section, numerical examples of these calculations will be shown.

III. Numerical Results and Discussion

In this section, numerical examples will be presented to show the dependence of the largest number of hops on different relaying protocols, different harvesting strategies as well as different system parameters. To do this, in the calculations, the parameters are set as $\sigma_{ma}^2=\sigma_{mc}^2=0.01$, $u_m^2=0.1$ and $\Delta_m=0.1$ for $m=1,2,\cdots$, $v=3$ and $T=1$. Also, let $\gamma_0=\frac{a_1^2P_0T}{\sigma_{1c}^2+\sigma_{1a}^2}$ be the initial signal-to-noise ratio (SNR) in the first relay, which is directly related to the initial amount of energy P_0T from the source node.

Figs. 1 - 3 show the largest number of hops for different values of γ_0, η and d. From these figures, several observations can be made. Firstly, under the same conditions, the value of the largest number of hops generally increases when the initial SNR γ_0 and the conversion efficiency η increase or when the distance d decreases. This means that one may extend the network coverage by either increasing the amount

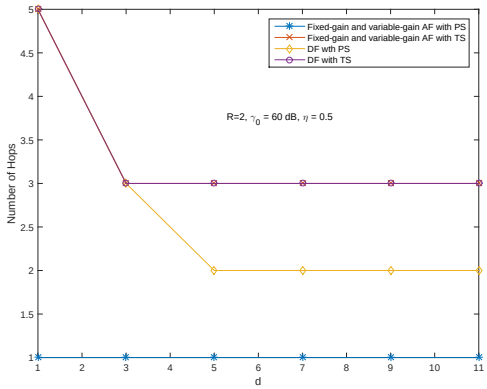

Fig. 3. The maximum number of hops vs. d for $\eta = 0.5$, $R = 2$ and $\gamma_0 = 60\ dB$.

of energy transferred from the source node and subsequently harvested by all relaying nodes, or improving the efficiency of the energy harvester. This is expected. Among the three parameters examined, one can also see that the initial SNR γ_0 has the largest impact on the network coverage extension. On the other hand, the curves are relatively flat when η changes, indicating that it may not be worth improving the design of the energy harvester to extend network coverage when the initial amount of energy is low to medium. This is important, as the improvement of energy harvester often requires a considerable amount of time and effort.

Secondly, comparing different relaying protocols, one sees that AF relaying and DF relaying have similar performances when TS is used, as they have similar harvested energies in this case. When PS is used, DF relaying is much better than AF relaying. This can be explained by comparing (15) with (19), where one sees that the harvested power of AF is much smaller than that of DF due to the extra term of $\prod_{i=1}^{m-1}(1-\rho_i)$ in the product and thus, the energy transferred from the source node can be exhausted quickly in AF relaying. For fixed-gain AF relaying and variable-gain AF relaying, they have the same performances, as we have set $a_m^2 = \Delta_m = 0.1$ such that their amplification factors are actually the same.

Thirdly, comparing the TS method with the PS method, one sees that the TS method can achieve many more hops than the PS method in most cases. This is especially true for AF relaying, where the largest number of hops for PS is always 1, while the largest number of hops for TS could reach 7. This is explained as follows. For the TS method in AF relaying, the powers of the signals for harvesting and for relaying are the same, and only the relaying time is switched. Thus, the first relay can choose to take a small amount of time and therefore harvest a small amount of energy (power is fixed during this), conserving the majority of the energy for later use by the following relays. However, for the PS method in AF relaying, the powers of the signals for harvesting and relaying are different. Even if the first relay chooses to split a small portion of power for relaying and therefore harvest a small amount of energy (time is fixed during this), with the good intent of passing on the majority of energy for later use by the following relays, according to AF relaying, the first relay is in fact only using a large transmission power (conserved for the following relays) to transmit a very weak signal (due to a small portion of power splitted) to the next relay. This is as undesirable as using a small transmission power (harvest most energy from the source node at the first relay) to transmit a strong signal (due to a large portion of power splitted for the first hop). In both situations, the received power at the next relay will be very small, wasting a huge amount of conserved energy. Thus, the PS method may not be effective for network coverage extension based on harvesting.

In summary, the best way of extending the network coverage using multi-hop relaying with energy harvesting is to use the TS method, followed by DF relaying using the PS method. One should avoid using AF with PS to extend the network coverage. Note that it is not possible to optimize the power-splitting factor and the time-switching coefficient in our system model where the throughput for each hop is fixed. In fact, they are also fixed and calculated directly by using (7), (11), (17) and (21). If one uses a random value of α_m or ρ_m, the throughput restriction may not be satisfied, which is against the design purpose in this work. On the other hand, when the throughput is not fixed, it is possible to find the optimum values of α_m and ρ_m that maximize the throughput, which is beyond the scope of this work. Note also that our derivation simplifies the analysis by setting the distances to 1. Ultimately, it is the signal-to-noise ratio that matters, not the fading coefficient or the path loss. Finally, for multi-hop communications, protocols with better spectral efficiency may be available. For example, if the node in the first hop is far away from the node in the last hop, they could transmit signal simultaneously. However, this requires designs of new relaying protocols and is not the focus of this work. All the results are for a fixed channel gain of $u_m^2 = 0.1$ and not averaged over different channel realizations.

References

[1] A.A. Nasir, X. Zhou, S. Durrani, R.A. Kennedy, "Relaying protocols for wireless energy harvesting and information processing," *IEEE Transactions Wireless Communications*, vol. 12, pp. 3622 -3636, July 2013.

[2] I. Krikidis, G. Zhang, B. Ottersten, "Harvest-use cooperative networks with half/full-duplex relaying,"*Proceedings of the 2013 IEEE Wireless Communications and Networking Conference*, Shanghai, China, 2013, pp. 4256 - 4260.

[3] Z. Ding, S.M. Perlaza, I. Esnaola, H.V. Poor, "Power allocation strategies in energy harvesting wireless cooperative networks," *IEEE Transactions Wireless Communications*, vol. 13, pp. 846 - 860, Feb. 2014.

[4] I. Krikidis, "Simultaneous information and energy transfer in large-scale networks with/without relaying," *IEEE Transactions Communications*, vol. 62, pp. 900 - 912, Mar. 2014.

[5] Z. Ding, I. Krikidis, B. Sharif, H.V. Poor, "Wireless information and power transfer in cooperative networks with spatially random relays," *IEEE Transactions Wireless Communications*, vol. 13, pp. 4440 - 4453, Aug. 2014.

[6] Y. Gu, S. Aissa, "Interference aided energy harvesting in decode-and-forward relaying system," *Proc. ICC'14*, 5378 - 5382, 2014.

[7] Z. Zhou, M. Peng, Z. Zhao, Y. Li, "Joint power splitting and antenna selection in energy harvesting relay channels," *IEEE Signal Processing Letters*, vol. 22, pp. 823 - 827, July 2015.

[8] M. Di Renzo, F. Graziosi and F. Santucci, "A comprehensive framework for performance analysis of dual-hop cooperative wireless systems with fixed-gain relays over generalized fading channels," *IEEE Transactions on Wireless Communications*, vol. 8, pp. 5060-5074, Oct. 2009.

[9] R.U. Nabar, H. Bolcskei, and F.W. Kneubuhler, "Fading relay channels: Performance limits and space-time signal design, *IEEE J. Sel. Areas Commun.*, vol. 22, pp. 1099-1109, Aug. 2004.