# Demographics and The Behavior of Interest Rates

**Abstract**

Interest rates are very persistent. Modelling the persistent component of interest rates has important consequences for forecasting. Factor models of the term structure are restricted VAR models that project over a long-horizon the one-period risk free rate to obtain yields at longer horizon as the sum of the expected future monetary policy and the term premia. The included factors are typically mean reverting and the equilibrium real rates are considered constant (think, for example, of the standard Taylor-rule), partial adjustments to equilibrium yields are then used to rationalize the persistence in observed data. As a result the empirical models feature a very high level of persistence that makes long-horizon predictions inherently inaccurate. This paper relates the common persistent component of the U.S. term structure of interest rates to the age composition of population. The composition of age structure determines the equilibrium rate in the monetary policy rule and therefore the persistent component in one-period yields. Fluctuations in demographics are then transmitted to the whole term structure via the expected policy rate components. We build an affine term structure model (ATSM) which exploits demographic information to capture the dynamics of yields and produce useful forecasts of bond yields and excess returns that provides economic value for long-term investors.

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# 1 Introduction

Recent evidence shows that the behavior of interest rates is consistent with the decomposition of spot rates as the sum of two processes, (i) a very persistent long term expected value and (ii) a mean-reverting component (Fama, 2006; Cieslak and Povala, 2015). Traditionally, factor models of the term structure are restricted VAR models that project over a long-horizon the one-period risk free rate to obtain yields at longer horizon as the sum of the expected future monetary policy and the term premia. In the macro-finance literature the included factors are typically mean reverting and the equilibrium real rates are considered constant (think, for example, of the Taylor-rule, the most successful model of the risk free rate, i.e., the monetary policy rate in the recent empirical literature where the two factors are the (expected) output-gap and deviation of (expected) inflation from the target), partial adjustments to equilibrium yields are then used to rationalize the persistence in observed data (see Figure 1 for both nominal interest rates and the real counterparts, that is, nominal interest rates net of expected inflation). As a result the empirical models feature a very high level of persistence that makes long-horizon predictions inherently inaccurate. The recent policy debates around "secular stagnation" revive the idea that demographics -through low population growth and increasing life expectancy- is one of the key determinants of the declining real interest rates (Hansen, 1938; Summers, 2014; Eggertsson and Mehrotra, 2014; Eichengreen, 2015; Aksoy and others, 2015). Others are more sceptical about secular stagnation and argue that there are other trends and cyclical forces behind low interest rates (Goodhart and Erfurth, 2014; Hamilton and others, 2015). This paper offers a novel interpretation for the persistent long-term component of US interest rates by relating it to the age composition of the population.

Modelling the persistent component of interest rates has important consequences for forecasting. Consider Affine Term Structure Models (ATSM), this class includes models with macro-finance factors but also models with only term structure factors (see,for example, the no-arb Nelsen and Siegel model proposed by Christensen et al.(2011)). In this framework, given the dynamics of the short term rate, a VAR representation for the factors is used to project the entire term structure. The risk premia are identified by posing a linear (affine) relation between the price of risk and the factors. The no-arbitrage assumption allows to pin down the dynamics of the entire term structure by imposing a cross-equation restrictions structure between the coefficients of the state model (the VAR for the factors), and the measurement equation that maps the factors in the yields at different maturities (Ang, Dong and Piazzesi, 2007; Dewachter and Lyrio, 2006). The potential problem with this general structure is that the resulting restricted VARs are very persistent and generate, by their nature, very imprecise forecasts at long-horizons (Giannone, Lenza and Primiceri, 2014). This feature might therefore explain the, somewhat disappointingly, mixed results from the forecasting performance of affine term structure models (Duffee, 2002; Favero, Niu and Sala, 2012; Sarno, Schneider and Wagner, 2014). Interestingly, enlarging the information set by explicitly considering a large number of macroeconomic variables as factors (Moench, 2008) has generated some clear improvement. We propose a model for the nominal term structure in which the equilibrium real rate is slowly evolving as a function of a demographic variable. To this end we include in the specification for the risk free rate a demographic factor and we then model the term structure of nominal rates by augmenting the standard VAR used in the literature with the (exogenous) dynamics of the demographic factor. When the monetary policy authorities set the policy rate, they mainly react to cyclical swings reflected in the transitory (expected) variations of output from its potential level and of (expected) inflation from its target, but the success of the policy action critically hinges on the uncertainty around the equilibrium real interest rate (Laubach and Williams, 2003; Hamilton and others, 2015). Therefore the policy makers closely follow the slowly evolving changes in the economy, that is, trends, which take place at lower frequency (see, for example, Bernanke, 2006).[1] In

[1] "*... adequate preparation for the coming demographic transition may well involve significant adjustments in our patterns of consumption, work effort, and saving* ..." Chairman Ben S. Bernanke, Before The Washington Economic Club, Washington, D.C., October 4, 2006. Also, research agenda questions (Theme 5) stated on Bank of England website stresses the importance of secular trends, in particular demographics, in determining equilibrium interest rates.

particular, the target for the policy rate is set by implicitly taking into account the life-cycle savings behavior of the population to determine the equilibrium policy rate. Linking the target policy rates to demographics makes Taylor-type rule of monetary policy capable of generating observed persistence in interest rates in presence of a much lower coefficient on lagged policy rates (Diebold and Li, 2006; Diebold and Rudebush, 2013).[2]

Yields at different maturities depend on the sum of short rate expectations and the risk premium. While it is less plausible to consider the risk premium as a non-mean reverting component (e.g., Dai and Singleton, 2002), the presence of a persistent component related to demographics can be rationalized in terms of smooth adjustments in short-rate expectations that take decades to unfold. In particular, we consider a demographic variable MY, a proxy for the age structure of the U.S. population originally proposed by Geneakopoulos, Magill and Quinzii, 2004 (GMQ from now onwards), and defined as the ratio of middle-aged (40-49) to young (20-29) population in the U.S. as the relevant demographic variable to determine the persistent component of interest rates.[3]

First, we illustrate our permanent-transitory decomposition using demographic information. Then we propose an affine term structure model (ATSM) which parsimoniously incorporates demographic channel in one-period yield via the central bank reaction function, and all yields at longer maturities as the sum of future expected policy rates and the term premium. The advantage of an ATSM is that the term premia are explicitly modeled using both observable and unobservable factors.[4] This framework provides a the natural complement to the Taylor rule. In this specification, given the dynamics of the short term

[2]When young adults, who are net borrowers, and the retired, who are dissavers, dominate the economy, savings decline and interest rates rise. The idea is certainly not a new one as it can be traced in the work of Wicksell (1936), Keynes (1936), Modigliani and Brumberg (1954), but it has received relatively little attention in the recent literature.

[3]In principle there are many alternative choices for the demographic variable, MY. However, using a proxy derived from a model is consistent with economic theory (Giacomini and Ragusa, 2014) and reduces the risk of a choice driven by data-mining. Importantly, MY is meant to capture the relative weights of active savers investing in financial markets. Our results are robust to an alternative specification of the demographic variable, middle-aged to old (MO) ratio, another variable consistent with the GMQ model.

[4]The literature is vast, few related examples are Ang and Piazzesi (2003), Diebold, Rudebusch, and Aruoba (2006), Gallmeyer, Hollifield, and Zin, (2005), Hordahl, Tristani, and Vestin (2006), Rudebusch and Wu (2008), Bekaert, Cho, and Moreno (2010).

rate, a more quickly mean reverting and stationary VAR representation for the factors is used to project the entire term structure. We show that the demographic ATSM not only provides improved yield forecasts with respect to traditional benchmarks considering statistical accuracy (Carriero and Giacomini, 2011), but it also provides economic gains for long term investors in the context of portfolio allocation (Sarno, Schneider and Wagner, 2014; Gargano, Pettenuzzo and Timmermann, 2014).

To our knowledge, the potential relation between demographics and the target policy rate in a reaction function has never been explored in the literature. This analysis is relevant for two reasons. First, the persistence of policy rates cannot be modeled by the mainstream approach to central bank reaction functions that relate monetary policy exclusively to cyclical variables. Second, putting term structure model at work to relate the policy rate to all other yields requires very long term projections for policy rates. In a monthly model, 120 step ahead predictions of the one-month rate are needed to generate the ten-year yield. However, long-term projections are feasible in a specification where the persistent component of the policy rates is modelled via demographics while macroeconomic factors capture the cyclical fluctuations. For instance, a standard VAR could be used to project the stationary component, while the permanent component is projected by exploiting the exogeneity of the demographic variable and its high predictability even for a very long-horizon.[5]

The trend-cycle decomposition of interest rates has been also recently investigated by Fama (2006) and Cieslak and Povala (2015), who argue that the predictive power of the forward rates for yields at different maturities could be related to the capability of appropriate transformations of the forward rates to capture deviations of yields from their permanent component. These authors propose time-series based on backward looking empirical measures of the persistent component; in particular Fama (2006) considers a five-year backward looking moving average of past interest rates and Cieslak and Povala

[5]The Bureau of Census currently publishes on its website projections for the age structure of the population with a forecasting horizon up to fifty years ahead and historical Census reports back to 1950 are available to avoid look-ahead bias.

(2015) consider a ten-year discounted backward-looking moving average of annual core CPI inflation. We propose instead a forward looking measure for which reliable forecasts are available for all the relevant horizons. Figure 2 illustrates the existence of a persistent component both in nominal and real (net of expected inflation) interest rates by relating it to different measures of slowly evolving trends. The Figure reports the yield to maturity of one-Year US Treasury bond, along with the persistent components as identified by Fama (2006) and Cieslak and Povala (2015), and the demographic variable, MY.

The Figure shows that MY not only strongly comoves with the alternative estimates of the persistent component, but it is also capable of matching exactly the observed peak in yields at the beginning of the eighties. The very persistent component of yields is common to the entire term structure of interest rates: Figure 1a and 1b illustrate this point by reporting both the US nominal and real interest rates at different maturities. The trend is visible both in nominal and real short rate.[6] The visual evidence reported in Figures 1-2 motivates the formal investigation of the relative properties of the different observable counterparts for the unobservable persistent component of the term structure.

Our framework brings together four different strands of the literature: i) the one analyzing the implications of a persistent component on spot rates predictability, ii) the empirical literature modeling central bank reaction functions using the rule originally proposed by Taylor (1993), iii) the one linking demographic fluctuations with asset prices, and iv) the term structure models with observable macro factors and latent variables.

The literature on spot rates predictability has emerged from a view in which forecastability is determined by the slowly mean-reverting nature of the relevant process. Recently, it moved to a consensus that modeling a persistent component is a necessary requirement for a good predictive performance (Bali, Heidari and Wu, 2009; Duffee, 2012).

[6] While the nominal short rate is directly observable, the real rate depends on how the expected inflation is modelled. Here we predict the inflation from an autoregressive model using growth rate of GDP deflator. Our main analysis will be based on observable nominal rates, since the demographic channel affects the nominal rates via the (unobserved) real rates regardless of different inflation specifications (Gozluklu and Morin, 2015).

Early literature attributes this predictability to the mean reversion of the spot rate toward a constant expected value. This view has been recently challenged; the predictability of the spot rate captured by forward rates is either attributed to a slowly moving, yet still stationary, mean (Balduzzi, Das, and Foresi, 1998) or to the reversion of spot rates towards a time-varying very persistent long-term expected value (Fama, 2006; Cieslak and Povala, 2015).[7]

Our choice of the variable determining the persistent component in short term rate is funded in the literature linking demographic fluctuations with asset prices, and in the empirical approach to central bank reaction functions based on Taylor's rule. Taylor rule models policy rates as depending on a long term equilibrium rate and cyclical fluctuations in (expected) output and inflation. The long term equilibrium rate is the sum of two components: the equilibrium real rate and equilibrium inflation, which is the (implicit) inflation target of the central bank. Evans (2003) shows that over longer horizons, expectation of the nominal and real yields rather than the inflation expectations dominate in the term structure. The long-term equilibrium is traditionally modeled as a constant reflecting the discount factor of the representative agent (Eggertsson and Mehrotra, 2014).[8] However, Woodford (2001) highlights the importance of a time-varying intercept in the feedback rule to avoid excess interest rate volatility while stabilizing inflation and output gap. This paper allows for a time-varying target for the equilibrium policy rate by relating it to the changing age composition of population instead of population growth. The use of a demographic variable allows us to explicitly model the change of regime in the spot rate proving an alternative to regime-switching specifications, e.g., Gray (1996); Ang and Bekaert (2002).[9]

[7] There are other alternative views in the literature which argue for a unit root in the spot rates (De Wachter and Lyrio, 2006; Christensen, Diebold and Rudebusch, 2011, Hamilton and others, 2015) or suggest a near unit root process to model the persistent component (Cochrane and Piazzesi, 2008; Jardet, Monfort and Pegoraro, 2011; Osterrieder and Schotman, 2012).

[8] Theoretical models suggest a link between equilibrium rate and growth in the economy, measured either via output or consumption growth. However, empirical evidence on economic fundamentals driving equilibrium rate is at best weak (Hamilton and others, 2015).

[9] Earlier papers (Bai and Perron, 2003; Rapah and Wohar, 2005) document structural breaks in the mean

Our approach to monetary policy rule has an important difference from the one adopted in the monetary policy literature. In this literature monetary policy has been described by empirical rules in which the policy rate fluctuates around a constant long-run equilibrium rate as the central bank reacts to deviations of inflation from a target and to a measure of economic activity usually represented by the output gap. The informational and operational lags that affect monetary policy (Svensson, 1997) and the objective of relying upon a robust mechanism to achieve macroeconomic stability (Evans and Honkapohja, 2003), justify a reaction of current monetary policy to future expected values of macroeconomic targets. As the output-gap and the inflation-gap are stationary variables, this framework per se is not capable of accommodating the presence of the persistent component in policy rates. One outstanding empirical feature of estimated policy rules is the high degree of monetary policy gradualism, as measured by the persistence of policy rates and their slow adjustment to the equilibrium values determined by the monetary policy targets (Clarida, Gali and Gertler, 2000; Woodford, 2003; Orphanides and Williams, 2007). Rudebusch (2002) and Soderlind, Soderstrom and Vredin (2005) have argued that the degree of policy inertia delivered by the estimation of Taylor-type rules is heavily upward biased. In fact, the estimated degree of persistence would imply a large amount of forecastable variation in monetary policy rates at horizons of more than a quarter, a prediction that is clearly contradicted by the empirical evidence from the term structure of interest rates.[10] Rudebusch (2002) relates the "illusion" of monetary policy inertia to the possibility that estimated policy rules reflect some persistent shocks that central banks face. The introduction of demographics allows to model this persistent component of the policy rate as the time-varying equilibrium interest rate is determined by the age-structure of the population. Hence we estimate the natural rate implied by the U.S. demographics that is consistent with no-arbitrage in the term structure

real interest rates.

[10] In a nutshell, high policy inertia should determine high predictability of the short-term interest rates, even after controlling for macroeconomic uncertainty related to the determinants of the central bank reaction function. This is not in line with the empirical evidence based on forward rates, future rates (in particular federal funds futures) and VAR models.

of interest rates (Pescatori and Turunen, 2015; Hamilton and others, 2015).

The idea of using demographics to determine the persistent component of the whole term structure complements the existing literature that uses demography as an important variable to determine the long-run behavior of financial markets (Abel, 2001). While the literature agrees on the life-cycle hypothesis[11] as a valid starting point, there is disagreement on the correct empirical specification and thus the magnitude of demographic effects (Poterba, 2001; Goyal, 2004). Substantial evidence is available on the impact of the demographic structure of the population on long-run stock-market returns (Ang and Maddaloni, 2005; Bakshi and Chen, 1994; Goyal, 2004; Della Vigna and Pollet, 2007, Favero, Gozluklu and Tamoni, 2011). However, the study of the empirical relation between demographics and the bond market is much more limited, despite the strong interest for comovements between the stock and the bond markets (Lander, Orphanides and Douvogiannis, 1997; Campbell and Vuoltenaho, 2004; Bekaert and Engstrom, 2010, Gozluklu and Morin, 2015).

GMQ (2004) consider an overlapping generation model in which the demographic structure mimics the pattern of live births in the U.S., that have featured alternating twenty-year periods of boom and busts. They conjecture that the life-cycle portfolio behavior (Bakshi and Chen, 1994) plays an important role in determining equilibrium asset prices. Consumption smoothing by the agents, given the assumed demographic structure requires that when the MY ratio is small (large), there will be excess demand for consumption (saving) by a large cohort of retirees (middle-aged) and for the market to clear, equilibrium prices of financial assets should adjust, that is, decrease (increase), so that saving (consumption) is encouraged for the middle-aged (young). The model predicts that the price of all financial assets should be positively related to MY and it therefore also predicts the negative correlation between yields and MY. Note that we use the results of the GMQ model to rationalize the target for policy rate at generational frequency, in this framework there is no particular reason why the ratio of middle-aged to young population should be directly linked

[11] Life cycle investment hypothesis suggests that agents should borrow when young, invest for retirement when middle-aged, and live off their investment once they are retired.

to aggregate risk aversion.[12] Following this intuition, we take a different approach from the available literature that studies the relationship between real bond prices and demographics through the impact on time-varying risk (Brooks, 1998; Bergantino, 1998; Davis and Li, 2003).

We concentrate on the relation between equilibrium real interest rate and the demographic structure of population as we consider the target inflation rate as set by an independent central bank who is not influenced by the preferences of the population. However, a possible relation between the population age structure and inflation has been investigated in other studies (Lindh and Malberg, 2000, Juselius and Takats, 2015) which show evidence on the existence of an age pattern of inflation effects. However, Gozluklu and Morin (2015) show that in an overlapping generations model with cash-in-advance constraints, the equilibrium relation between real interest rates and population age structure is robust to monetary shocks. Our approach is consistent with McMillan and Baesel (1988) who analyze the forecasting ability of a slightly different demographic variable, prime savers over the rest of the population. Our work is also related to Malmendier and Nagel (2013), who show that an aggregate measure that summarizes the average life-time inflation experiences of individuals at a given point in time is useful in predicting excess returns on long-term bonds.

We shall implement the formal investigation in four stages. First, we illustrate the potential of the temporary-permanent decomposition to explain fluctuations of the term structure using the demographic information. Second, we introduce a formal representation of our simple framework, by estimating a full affine term structure model (ATSM) with time varying risk premium. Third, we run a horse-race analysis between a random walk benchmark, standard Macro ATSM and proposed demographic adjusted ATSM. We consider several measures of statistical accuracy and economic value for different investment horizon. Fourth, we investigate the relative performance of MY and other backward looking measures

[12] Recent literature also shows that consumption smoothing across time rather than the risk management across states is the primary concern of the households (Rampini and Viswanathan, 2014).

proposed in the literature to model the persistent component of interest rates. Finally, after assessing the robustness of our empirical findings, the last section concludes.

# 2 Demographics and the Structure of Yield Curve

We motivate our analysis with a simple framework, in which the yield to maturity of the 1-period bond, $y_t^{(1)}$, is determined by the action of the monetary policy maker and all the other yields on n-period (zero-coupon) bonds can be expressed as the sum of average expected future short rates, that is, expectations hypothesis (EH), and the term premium, $rpy_t^{(n)}$:[13]

$$y_t^{(n)} = \frac{1}{n}\sum_{i=0}^{n-1} E_t[y_{t+i}^{(1)} \mid I_t] + rpy_t^{(n)} \tag{1}$$

$$y_t^{(1)} = y_t^* + \beta(E_t\pi_{t+k} - \pi^*) + \gamma E_t x_{t+q} + u_{1,t+1}$$

In setting the policy rates, the Fed reacts to variables at different frequencies. At the high frequency the policy maker reacts to cyclical swings reflected in the output/unemployment gap, $x_{t+q}$, that is, transitory discrepancies of output from its potential level, and in deviation of inflation, $\pi_{t+k}$, from the implicit target ($\pi^*$) of the monetary authority. Monetary policy shocks, $u_{1,t+1}$,also happen. As monetary policy impacts macroeconomic variables with lags, the relevant variables to determine the current policy rate are k-period ahead expected inflation and q-period ahead expected output gap. However, cyclical swings are not all that matter to set policy rates. We posit that the monetary policy maker also takes into account the equilibrium level of interest rates $y_t^*$ (which is determined by the sum of a time varying real interest rate target and the inflation target $\pi^*$) accordingly to the slowly evolving changes in the economy that take place at a generational frequency, that is, those spanning several decades. We relate this to the age structure of population,

[13] We adopt Cochrane and Piazzesi (2005) notation for log bond prices: $p_t^{(n)}$ = log price of n-year discount bond at time t. The continuously compounded spot rate is then $y_t^{(n)} \equiv -\frac{1}{n}p_t^{(n)}$

$MY_t$ as it determines savings behavior of middle-aged and young population.

The relation between the age structure of population and the equilibrium real interest rate is derived by GMQ in a three-period overlapping generation model in which the demographic structure mimics the pattern of live births in the US. Live births in the US have featured alternating twenty-year periods of boom and busts (GMQ, 2004; Gozluklu and Morin, 2015). Let $q_o$ ($q_e$) be the bond price and $\{c_o^y, c_o^m, c_o^r\}$ ($\{c_e^y, c_e^m, c_e^r\}$) the consumption stream (young, middle-aged, retired) in two consecutive periods, namely odd and even. GMQ assume that in odd (even) periods a large (small) cohort N(n) enters the economy, therefore in every odd (even) period there will be {N,n,N}({n,N,n}) cohorts living.[14] In the simplest deterministic setup, following the utility function over consumption

$$U(c) = E(u(c^y) + \delta u(c^m) + \delta^2 u(c^r))$$
$$u(x) = \frac{x^{1-\alpha}}{1-\alpha} \qquad \alpha > 0$$

The agent born in an odd period then faces the following budget constraint

$$c_o^y + q_o c_o^m + q_o q_e c_o^r = w^y + q_o w^m \tag{2}$$

and in an even period

$$c_e^y + q_e c_e^m + q_o q_e c_e^r = w^y + q_e w^m \tag{3}$$

Moreover, in equilibrium the following resource constraint must be satisfied

$$Nc_o^y + nc_o^m + Nc_o^r = Nw^y + nw^m + D \tag{4}$$
$$nc_e^y + Nc_e^m + nc_e^r = nw^y + Nw^m + D \tag{5}$$

where D is the aggregate dividend for the investment in financial markets.

[14] Note that under the assumption of perfectly stationary demographic structure, the relative cohort size, middle-aged over young population is the same as the middle-aged over retired population.

In this economy an equilibrium with constant real rates is not feasible as it would lead to excess demand either for consumption and saving. When the MY ratio is small (large), that is, an odd (even) period, there will be excess demand for consumption (saving) by a large cohort of young (middle-aged) and for the market to clear, equilibrium prices of financial assets should adjust, that is, decrease (increase), so that saving (consumption) is encouraged for the middle-aged. Thus, letting $\mathrm{q}_t^b$ be the price of the bond at time t, in a stationary equilibrium, the following holds

$$\begin{aligned} \mathrm{q}_t^b &= \mathrm{q}_o \text{ when period odd} \\ \mathrm{q}_t^b &= \mathrm{q}_e \text{ when period even} \end{aligned}$$

together with the condition $\mathrm{q}_o < \mathrm{q}_e$. In the absence of risk, the substitutability of bond and equity together with the no-arbitrage condition implies that

$$\frac{1}{q_t^b} = 1 + y_t = \frac{D + q_{t+1}^e}{q_t^e}$$

where $q_t^e$ is the real price of equity and $t \in \{odd, even\}$.

So, since the bond prices alternate between $\mathrm{q}_o^b$ and $\mathrm{q}_e^b$, then the price of equity must also alternate between $\mathrm{q}_t^e$ and $\mathrm{q}_t^e$. Hence the model predicts a positive correlation between real asset prices and MY, and a negative correlation between MY and bond yields; in other words the model implies that a bond issued in odd (even) period and maturing in even (odd) period offers a high (low) yield, since the demographic structure is characterized by a small (large) cohort of middle-aged individuals, hence low MY ratio in odd (even) periods.[15]

Therefore, the main prediction of the model is that real interest rates should fluctuate with the age structure of population. Unfortunately real interest rates are not observable for most of our sample. Inflation-indexed bonds (TIPS, the Treasury Income Protected Securities) have traded only since 1997 and the market of these instruments

[15] The implications of the evidence for stock market predictability are further investigated in Favero, Gozluklu and Tamoni (2011).

faced considerable liquidity problem in its early days.[16] Following the literature, we obtain ex-ante real rates by estimating an autoregressive model for inflation and generating inflation expectations for the relevant horizon. In Figure 3 we plot the time series behavior of both the ex-ante real short (3-month) and long (5-year) rates together with MY, and see that both series closely comove with the demographic variable capturing the age structure of the U.S. population with the exception of the recent quantitative easing (QE) period. The evidence is similar if we extract ex-ante real rates from a more sophisticated econometric model for inflation using stochastic volatility with drifting coefficients (D'Agostino and Surico, 2012) or a structural model (Ang, Bekaert and Wei, 2008).[17]

Consistently with the GMQ model we consider the following permanent-transitory decomposition for the 1-period policy rates:

$$\begin{aligned}
y_t^{(1)} &= P_t^{(1)} + C_t^{(1)} = \rho_0 + \rho_1 MY_t + \rho_2 X_t \\
P_t^{(1)} &\equiv \rho_0 + \rho_1 MY_t = y_t^* \\
C_t^{(1)} &\equiv \beta(E_t \pi_{t+k} - \pi^*) + \gamma E_t x_{t+q} + u_{1,t+1} = \rho_2 X_t
\end{aligned}$$

and, assuming that the inflation gap and the output gap can be represented as a stationary VAR process, yields at longer maturity can be written as follows

$$\begin{aligned}
y_t^{(n)} &= \rho_0 + \frac{1}{n}\sum_{i=0}^{n-1} \rho_1 MY_{t+i} + b^{(n)} X_t + rpy_t^{(n)} \qquad (6) \\
y_t^{(n)} &= P_t^{(n)} + C_t^{(n)} \\
P_t^{(n)} &= \rho_0 + \frac{1}{n}\sum_{i=0}^{n-1} \rho_1 MY_{t+i} \\
C_t^{(n)} &= b^{(n)} X_t + rpy_t^{(n)}
\end{aligned}$$

[16] Ang, Bekaert and Wei (2008) have solved the identification problem of estimating two unobservables, real rates and inflation risk premia, from only nominal yields by using a no-arbitrage term structure model that imposes restrictions on the nominal yields. These pricing restrictions identify the dynamics of real rates (and the inflation risk premia).

[17] Over a longer sample period 1900Q1-2013Q4, the demographic variable MY explains about 15 percent of the variation in real short rates once we use a AR(1) model with stochastic volatility to extract the expected inflation.

The decomposition of yields to maturity in a persistent component, reflecting demographics, and a cyclical components reflecting macroeconomic fluctuations and the risk premia, is consistent with the all the stylized facts reported so far documenting the presence of a slow moving component common to the entire term structure. Moreover, the relation between the permanent component and the demographic variable is especially appealing for forecasting purposes as the demographic variable is exogenous and highly predictable even for very long-horizons.[18] No additional statistical model for $MY_{t+i}$ is needed to make the simple model operational for forecasting, as the bureau of Census projections can be readily used for this variable, as it can be safely considered strongly exogenous for the estimation and the simulation of the model to our interest.

# 3 Demographic adjusted ATSM

We now propose an ATSM which parsimoniously incorporates demographic channel in one-period yield via the central bank reaction function and models all yields at longer maturities as the sum of future expected policy rates and the term premium. We take as a benchmark the standard model for the nominal yield in which the long-term equilibrium real yield is a constant and we augment them with MY that is included to capture the slowly moving mean of real yields. Hence we consider the role of demographics within a more structured specification that explicitly incorporates term premia. In particular, we estimate the following Demographic ATSM:

$$
\begin{aligned}
y_t^{(n)} &= -\frac{1}{n}\left(A_n + B_n' X_t + \Gamma_n \mathrm{MY}_t^n\right) + \varepsilon_{t,t+1} \qquad \varepsilon_{t,t+n} \sim N(0, \sigma_n^2) \qquad (7) \\
y_t^{(1/4)} &= \delta_0 + \delta_1' X_t + \delta_2 \mathrm{MY}_t \\
X_t &= \mu + \Phi X_{t-1} + \nu_t \quad\quad \nu_t \sim i.i.d.N(0, \Omega)
\end{aligned}
$$

[18] One can conjecture a world with endogenous fertility choice (Barro and Becker, 1989; Wang et al., 1994). However, in our sample a Granger causality test between real interest rates and the MY ratio suggests that the demographic variable Granger causes real bond yields, and not the other way around.

where $\Gamma_n = \left[\gamma_0^n, \gamma_1^n \cdots, \gamma_{n-1}^n\right]$, and $\mathrm{MY}_t^n = [\mathrm{MY}_t, \mathrm{MY}_{t+1} \cdots, \mathrm{MY}_{t+n-1}]'$, $y_t^{(n)}$ denotes the yield at time $t$ of a zero-coupon government bond maturing at time $t+n$, the vector of the states $X_t = [f_t^o, f_t^u]$, where $f_t^o = [f_t^\pi, f_t^x]$ are two observable factors extracted from large-data sets to project the inflation and output gap using all relevant output and inflation information which the Fed uses to set the monetary policy rate in a data-rich environment (Bernanke and Boivin, 2003; Ang, Dong and Piazzesi, 2005), while $f_t^u = \left[f_t^{u,1}, f_t^{u,2}, f_t^{u,3}\right]$ contain unobservable factor(s) capturing fluctuations in the unobservable interest rate target of the Fed orthogonal to the demographics fluctuations, or interest rate-smoothing in the monetary policy maker behavior. Consistently with the previous section and recent literature (e.g., Ang and Piazzesi, 2003; Huang and Zhi, 2012; Barillas, 2013), we extract the two observable stationary factors from a large macroeconomic dataset following Ludvigson and Ng (2009) to capture output and inflation information (see Appendix B).

Our specification for the one period-yield is a generalized Taylor rule in which the long-term equilibrium rate is related to the demographic structure of the population, while the cyclical fluctuations are mainly driven by the output gap and fluctuations of inflation around the implicit central bank target. Note that in our specification the permanent component of the 1-period rate is modelled via the demographic variable and the vector of the states $X_t$ is used to capture only cyclical fluctuations in interest rates. Hence, it is very natural to use a stationary VAR representation for the states that allows to generate long-term forecasts for the factors and to map them into yields forecasts. $\mathrm{MY}_t$ is not included in the VAR as reliable forecasts for this exogenous variable up to very long-horizon are promptly available from the Bureau of Census. The model is completed by assuming a linear (affine) relation between the price of risk, $\Lambda_t$, and the states $X_t$ by specifying the pricing kernel, $m_{t+1}$, consistently and by imposing no-arbitrage restrictions (see, for example, Duffie and Kan, 1996; Ang and Piazzesi, 2003). We solve the coefficients $A_{n+1}$, $B_{n+1}'$ and $\Gamma_{n+1}$ recursively (see Appendix A). We study the modified affine term structure model in assuming the more general case of time varying risk premium, that is, the market prices of risk are affine in five state variables $\lambda_0 = \begin{bmatrix} \lambda_0^{\pi} & \lambda_0^{x} & \lambda_0^{u,1} & \lambda_0^{u,2} & \lambda_0^{u,3} \end{bmatrix}$ where $\lambda_0$ is a non-zero vector and $\lambda_1$ is a diagonal matrix;

$$
\begin{aligned}
\Lambda_t &= \lambda_0 + \lambda_1 X_t \\
m_{t+1} &= \exp(-y_t^{(1/4)} - \frac{1}{2}\Lambda_t' \Omega \Lambda_t - \Lambda_t \varepsilon_{t+1})
\end{aligned}
$$

$$
\begin{aligned}
A_{n+1} &= A_n + B_n' \left(\mu - \Omega\lambda_0\right) + \tfrac{1}{2} B_n' \Omega B_n + A_1 \\
B_{n+1}' &= B_n' \left(\Phi - \Omega\lambda_1\right) + B_1' \\
\Gamma_{n+1} &= [-\delta_2, \Gamma_n]
\end{aligned}
$$

Note that the imposition of no-arbitrage restrictions allows to model the impact of current and future demographic variables on the term structure in a very parsimonious way, as all the effects on the term structure of demographics depend exclusively on one parameter: $\delta_2$. Our structure encompasses traditional ATSM with macroeconomic factors, and no demographic variable, labelled as Macro ATSM, as this specification is obtained by setting $\delta_2 = 0$. In other words, the traditional Macro ATSM, which omits the demographic variable, is a restricted version of the more general Demographic ATSM. The no arbitrage restrictions guarantees that when $\delta_2 = 0$ also $\Gamma_n = 0$ : as demographics enter the specification of yields at longer maturities only via the expected one-period yield, the dynamics of yields at all maturities become independent from demographics if $\mathrm{MY}_t$ does not affect the one-period policy rate. However, when the restriction $\delta_2 = 0$ is imposed, the structure faces the problem highlighted in the previous section of having no structural framework for capturing the persistence in policy rates in the case of using only stationary state variables. In fact, to match persistence in the policy rates, some of the unobservable factors must be persistent as the observable factors are, by construction, stationary. Then, the VAR for the state will include a persistent component which will make the long-term forecasts of policy rates, necessary to model the long-end of the yield curve, highly uncertain and unreliable. In the limit case of a non-stationary VAR, long-term forecast become useless as the model is non-mean reverting and the asymptotic variance diverges to infinite.

## 3.1 Model Specification and Estimation

We estimate the model on quarterly data by considering the 3-month rate as the policy rate. The properties of the data are summarized in Table 1. The descriptive statistics reported in Table 1 highlight the persistence of all yields, both nominal (Table 1.1) and real (Table 1.2) yields, which is not matched by the persistence of the macroeconomic factors extracted from the large data-set and it is instead matched by the persistence of the demographic variable MY.

We evaluate the performance of our specification with MY against that of a benchmark discrete-time ATSM obtained by imposing the restriction $\delta_2 = 0$ on our specification. Following the specification analysis of Pericoli and Taboga (2008), we focus on a parsimonious model including three latent factors and only contemporaneous values of the macro variables. We use the Chen and Scott's (1993) methodology; given the set of parameters and observed yields latent variables are extracted by assuming that number of bonds which are priced exactly is equal to the number of unobserved variables. Hence we assume that 3-month, 2-year and 5-year bond prices are measured without error and estimate the model with maximum likelihood. We assume the state dynamics to follow a VAR(1). We impose the following restrictions on our estimation (Favero, Niu and Sala, 2012):

i) the covariance matrix $\Omega$ is block diagonal with the block corresponding to the unobservable yield factor being identity, and the block corresponding to the observable factors being unrestricted, that is,

$$\Omega = \begin{bmatrix} \Omega^o & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$$

ii) the loadings on the factors in the short rate equation are positive, $0 \leq$ -$A_1$

iii) $f_0^u = 0$.

We first estimate the model for the full sample 1964Q1-2013Q4, the estimated results are reported in Table 2. The results show significant evidence of demographics in the reaction function. The additional parameter $\delta_2$ in the Demographic ATSM is highly significant with the expected negative sign. Moreover, we notice that while the unobservable level factor picks up the persistence in the Macro ATSM specification, the demographic variable dominates the level factor which becomes negligible in the Demographic ATSM. This observation is especially relevant in the context of out-of-sample forecasting. The omission of the demographic variable results in overfitting of the restricted model. Such a restricted model may be useful in explaining the in-sample patterns of the data, but does not reflect the true data generating process of bond yields (Duffee, 2011). We also notice that the estimated dynamics of the unobservable factors, especially the level factor, is very different when the benchmark model is augmented with MY. In fact, in the Macro ATSM model the third factor is very persistent and the matrix $(\Phi - I)$ describing the long-run properties of the system is very close to be singular, while this near singularity disappears when the persistent component of yields at all maturities is captured by the appropriate sum of current and future age structure of the population. In this case the VAR model for the states becomes clearly stationary and long-term predictions are more precise and reliable.

## 3.2 Out-of-Sample Forecasts

We complement the results of full sample estimation by analyzing the properties of out-of-sample forecasts of our model at different horizons. The key challenge facing ATSM models is that they are good at describing the in-sample yield data and explain bond excess returns, but often fail to beat even the simplest random walk benchmark, especially in long horizon forecasts (Duffee, 2002; Guidolin and Thornton, 2011; Sarno, Schneider and Wagner, 2014). In our multi-period ahead forecast, we choose iterated forecast procedure, where multiple step ahead forecasts are obtained by iterating the one-step model forward

$$\widehat{y}^{(n)}_{t+h|t} = \widehat{a}_n + \widehat{b}_n \hat{X}_{t+h|t} + \widehat{c}_n \mathrm{MY}^n_{t+h}$$
$$\hat{X}_{t+h|t} = \sum_{i=0}^{h} \hat{\Phi}^i \widehat{\mu} + \hat{\Phi}^h \hat{X}_t$$

where $\widehat{a}_n = -\frac{1}{n}\widehat{A}_n, \widehat{b}_n = -\frac{1}{n}\widehat{B}_n$ and $\widehat{c}_n = -\frac{1}{n}\widehat{\Gamma}_n$ are obtained by no-arbitrage restrictions. Forecasts are produced on the basis of rolling estimation with a rolling window of eighty observations. The first sample used for estimation is 1961Q3-1981Q2. We consider 5 forecasting horizons (denoted by h): one to five years. For example, for the one year forecasting horizon, we provide a total of 126 forecasts for the period 1982Q2 - 2013Q4, while the number of forecasts reduces to 111 for 5-year ahead forecasts.

Forecasting performance is measured by the ratio of the root mean squared forecast error (RMSFE) of the Demographic ATSM to the RMSFE of a random walk forecast and to the RMSFE of the benchmark Macro ATSM without the demographic variable. In parentheses, we report the p-values of the forecasting test due to Giacomini and White (2006) which is a two-sided test of the equal predictive ability of two competing forecasts. In addition, we compute the Clark and West (2006, 2007) test statistics and associated p-values testing the forecast accuracy of nested models. The additional Clark and West statistics are useful in evaluation the forecasting performance, because it corrects for finite sample bias in RMSFE comparison between nested models. Without the correction, the more parsimonious model might erroneously seem to be a better forecasting model if we only consider the ratio of RMSFE. Forecasting results from different models are reported in Table 3. Panel A compares the forecasts of Demographic ATSM against the random walk benchmark, while Panel B uses the restricted Macro ATSM ($\delta_2 = 0$) as the benchmark.

The evidence on statistical accuracy using different tests shows that the forecasting performance of the Demographic ATSM dominates the traditional Macro ATSM, especially in longer horizon starting from 2 years. Including demographic information in term structure models seems decisive to generate a better forecasting performance. By using an affine structure to model time-varying risk one can impose more structure on the yield dynamics and still improve on the forecasting performance of a simpler model once demographics is incorporated into the model to project future bond yields. The finding is striking in light of earlier evidence from the above cited literature which highlights the difficulty of forecasting future yields using ATSM specification. In online Appendix Table A.1, we generate out-of-sample forecasts, replacing the MY variable with the MO variable, where MO is constructed as ratio of the middle-aged cohort, age 40– 49, to the old-age cohort, age 60–69. The results show that the model using MO perform better than the benchmark models, yet is outperformed by the original model including MY.

In order to demonstrate the importance of a common demographics related component to explain the common persistent component in the term structure, we conduct the following dynamic simulation exercise: using the full-sample estimation results, both Macro ATSM and Demographic ATSM are simulated dynamically from the first observation onward to generate yields at all maturities. The simulated time series in Figure 4 show that, while the model without demographics converges to the sample mean, the model with demographics feature projections that have fluctuations consistent with those of the observed yields, except the recent period of quantitative easing whose start is indicated by the vertical line in 2008Q3. These simulations confirm that fitting a persistent level factor does not necessarily result in accurate out-of-sample forecasts.

## 3.3 Forecast Usefulness and Economic Value

Out-of-sample forecasting results reported in Table 3 suggest that the random walk model which does not impose any structure on yield dynamics and risk premium is still a valid benchmark, especially for short horizon forecasts up to one year. So, in the context of out-of-sample forecasting, the question is whether to choose a completely parsimonious model with no economic structure or a full fledged ATSM specification which models risk dynamics while capturing the persistence in interest rates via common demographic component. In this section, we follow the framework proposed by Carriero and Giacomini (2011) which is flexible enough to allow for forecast combination and assess the usefulness of two competing models, by both using a statistical and an economic measure of forecast accuracy. In particular, in the former case given a particular type of loss function, e.g., quadratic loss, the forecaster finds the optimal weight $\kappa^*$ which minimizes the expected out-of-sample loss of the following combined forecast

$$y_{t+h|t}^{(n),*} = \widehat{y}_{t+h|t}^{(n),RW} + (1-\kappa)(\widehat{y}_{t+h|t}^{(n),DATSM} - \widehat{y}_{t+h|t}^{(n),RW})$$

where $\widehat{y}_{t+h|t}^{(n),RW}$ ($\widehat{y}_{t+h|t}^{(n),DATSM}$) is the h-period ahead yield forecast at time t of the random walk model (Demographic ATSM) of a bond maturing in n periods.

If estimated $\kappa^*$ is close to one, then it suggests that only the random walk models is useful in forecasting bond yields. If on the other hand estimated $\kappa^*$ is close to zero, than Demographic ATSM model dominates the random walk benchmark in out-of-sample forecasting. Estimated $\kappa^*$ close to 0.5 implies that both models are equally useful in forecasting. In Table 4 Panel A, we provide estimated $\kappa^*$, and t-statistics $t^{\kappa=0}$ and $t^{\kappa=1}$ to test the null hypotheses $\kappa = 0$ and $\kappa = 1$, respectively. Results are broadly in line with the evidence reported in Table 3; while the parsimonious random walk model is useful for 1-year ahead forecasts, more structured Demographic ATSM provides more useful long horizon yield forecasts.

So far the evidence is limited to statistical forecast accuracy, but recent literature finds that statistical accuracy in forecasting does not necessarily imply economic value in portfolio choice, especially for bond excess returns (Thornton and Valente, 2012; Sarno, Schneider and Wagner, 2014; Gargano, Pettenuzzo and Timmermann, 2014). Carriero and Giacomini (2011) framework can be extended to find the optimal portfolio weight $w^*$ as a function $\kappa^*$ by minimizing the utility loss of an investor with quadratic utility who has to choose among $m$ risky bonds. We implement this test for 1-year and 2-year holding periods. In the first case, m=4, namely the investor chooses among 2-year to 5-year bonds. In the second case, the investment opportunity set consists of 3 bonds given the data we use in our forecasting exercise. Let the bond excess returns (net of 3-month spot rate) be a 4x1 vector, $\mathrm{rx}_{t+1} = [rx^{(2)}, rx^{(3)}, rx^{(4)}, rx^{(5)}]$ in case of 1-year holding period and a 3×1 vector $\mathrm{rx}_{t+2} = [rx^{(3)}, rx^{(4)}, rx^{(5)}]$ for 2-year holding period. Given our yield forecasts we can compute the bond excess returns

$$
\begin{aligned}
rx_{t+1} &= -n\ y_{t+1}^{(n)} + (n+1)y_t^{(n+1)} - y_t^{(1/4)} \\
rx_{t+2} &= -n\ y_{t+2}^{(n)} + (n+2)y_t^{(n+2)} - y_t^{(n/4)}
\end{aligned}
$$

and using our forecasting models we obtain excess return forecasts

$$
\begin{aligned}
\widehat{rx}_{t+1} &= -n\ \widehat{y}_{t+1|t}^{(n)} + (n+1)y_t^{(n+1)} - y_t^{(n/4)} \\
\widehat{rx}_{t+2} &= -n\ \widehat{y}_{t+2|t}^{(n)} + (n+2)y_t^{(n+2)} - y_t^{(1/4)}
\end{aligned}
$$

Panel B in Table 4 reports the estimated forecast combination weight $\kappa^*$, and associated t-statistics $t^{\kappa=0}$ and $t^{\kappa=1}$ to test the null hypotheses $\kappa = 0$ and $\kappa = 1$, respectively. As before, we consider the random walk specification as the benchmark model and compare the forecast combination weight $\kappa^*$ of either the Demographic ATSM or Macro ATSM models against the random walk benchmark. For 1-year holding period, the random walk model clearly dominates Macro ATSM model in line with earlier evidence. However, the optimal weight is not statistically different from 0.5 if we combine the random walk model with Demographic ATSM, suggesting that both models are equally relevant for an investor with 1-year horizon. On the other hand, for long term investors it is evident that the Demographic ATSM is the only model that is useful for forecasting bond excess returns.

## 3.4 Long Term Projections

One of the appealing features of the Demographic ATSM specification is that the availability of long-term projections for the age-structure of the population which can be exploited to produce long-term projections for the yield curve. In our specification, yields at time $t+j$ with maturities $t+j+n$ are functions of all realization of MY between $t+j$ and $t+j+n$. The exogeneity of the demographic variable and the availability of long term projections is combined in the affine model with a parsimonious parameterization generated by the no-arbitrage restrictions that allow to weight properly all future values of MY with the estimation of few coefficients. As a result future paths up to 2045 can be generated for the entire term structure, given the availability of demographic projections up to 2050.[19] In Figure 5, we compare the in-sample estimation and out-of-sample forecasts for both the 3-month spot rate and 5-year bond yield. While the in-sample estimation results are very similar, the long term projections reveal that the Macro ATSM is not able to capture the persistence in true data generating process. In particular, spot rate forecasts of the Macro ATSM model converge to the unconditional mean within 2 quarters, while it takes approximately 15 years (around 2030) for the Demographic ATSM forecasts to reach the unconditional mean.

While it is difficult to generate accurate inflation forecasts into distant future using either model (note that both models use a stationary inflation factor exctracted from a large macro dataset), the projections of the Demographic ATSM and AR(1) forecasts for expected inflation suggest that the real spot rates will remain negative only for the next few years. Hence the Demographic ATSM predicts a gradual recovery of the real spot rates as opposed to a secular decline. Clearly, forecasts about the distant future natural rate involve high uncertainty (Hamilton and others, 2015), nonetheless the forecasts of the Demographic ATSM are broadly consistent with the findings of Pescatori and Turinen (2015).

[19] The Bureau of Census websites provides projections for demographics variable up to 2050 and the current 5-year yield depends on the values of MY over the next five years.

# 4 Alternative Specifications of Permanent Component

The existence of a permanent component in spot rates has been identified in the empirical literature by showing that predictors for return based on forward rates capture the risk premium and the business cycle variations in short rate expectations. Fama (2006) explains the evidence that forward rates forecast future spot rates in terms of a mean reversion of spot rates towards a non-stationary long-term mean, measured by a backward moving average of spot rates. Cieslak and Povala (2015) explain the standard return predictor based on the tent-shape function of forward rates proposed by Cochrane and Piazzesi (2005) as a special case of a forecasting factor constructed from the deviation of yields from their persistent component. The latter is measured by a discounted moving-average of past realized core inflation.

In this section we use the standard framework to assess the capability of MY to capture the permanent component of spot rates against that of the different proxies proposed by Fama (2006) and Cieslak and Povala (2015). This framework is designed to compare the forecasting ability of the spot rates deviations from their long term expected value and forward spot spreads. We implement it by taking three different measures of the permanent component: our proposed measure based on the age composition of population, the measure adopted by Fama based on a moving average of spot rates, and the measure proposed by Cieslak-Povala based on a discounted moving average of past realized core inflation.

Given the decomposition of the spot interest rates, $y_t^{(1)}$in two processes: a long term expected value $P_t^{(1)}$, that is subject to permanent shocks, and a mean reverting component $C_t^{(1)}$:

$$y_t^{(1)} = C_t^{(1)} + P_t^{(1)}$$

The following models are estimated

$$
\begin{aligned}
y_{t+4x}^{(1)}\text{-}y_t^{(1)} &= a^x + b^x D_t + c^x[f_{t,t+4x}^{(1)}\text{-}y_t^{(1)}] + d^x[y_t^{(1)}\text{-}P_t^{(1),i}] + \varepsilon_{t+4x} \\
P_t^{(1),1} &= \frac{1}{20}\sum_{i=1}^{20} y_{t-i-1}^{(1)} \qquad (8) \\
P_t^{(1),2} &= \frac{\sum_{i=1}^{40} \upsilon^{i-1}\pi_{t-i-1}}{\sum_{i=1}^{40}\upsilon^{i-1}} \qquad (9) \\
P_t^{(1),3} &= e^x \frac{1}{4}\sum_{i=1}^{4} MY_{t+i-1} \qquad (10)
\end{aligned}
$$

where $f_{t,t+4x}^{(1)}$ is the one-year forward rate observed at time $t$ of an investment with settlement after $3x$ years and maturity in $4x$ years, $y_t^{(1)}$is the one-year spot interest rate, $\pi_t$ is annual core CPI inflation from time $t-4$ to time $t, \upsilon$ is a gain parameter calibrated at 0.96 as in Cieslak and Povala, and $MY_t$ is the ratio of middle-aged (40-49) to young (20-29) population in the US, $D_t$ is a step dummy, introduced by Fama in his original study, taking a value of one for the first part of the sample up to August 1981 and zero otherwise. This variable captures the turning point in the behavior of interest rates from a positive upward trend to a negative upward trend occurred in mid-1981.

The specification is constructed to evaluate the predictor based on the cyclical component of rates against the forward spot spread. In his original study, Fama found that, conditional on the inclusion of the dummy in the specification, this was indeed the case. This evidence is consistent with the fact the dominant feature in the spot rates of an upward movement from the fifties to mid-1981 and a downward movement from 1981 onwards is not matched by any similar movement in the forward-spot spread which looks like a mean reverting process over the sample 1952-2004. We extend the original results by considering alternative measures of the permanent component over a sample up to the end of 2013[20]. The results from estimation on quarterly data are reported in Table 5.

[20] 1-year Treasury bond yields are taken from Gurkaynak et al. dataset. Middle-young ratio data is available at annual frequencies from Bureau of Census (BoC) and it has been interpolated to obtain quarterly series.

We consider forecasts at the 2, 3, 4 and 5-year horizon. For each horizon we estimate first a model with no cyclical component of interest rates but only the forward spot spread, then we include the three different proxies for the cyclical components of interest rates. The estimation of the model with the restriction $d^x = 0$ delivers a positive and significant estimate of $c^x$ with a significance increasing with the horizon $x$. However, when the restriction $d^x = 0$ is relaxed, then the statistical evidence on the significance of $c^x$ becomes much weaker. In fact, this coefficient is much less significant when the cycle is specified using the demographic variable to measure the permanent component and when any measure of the cycle in interest rates is introduced in the specification. The inclusion of the dummy is necessary only in the case of the Fama-cycle, while in the cases of the inflation based cycle and the demographic cycle the inclusion of the dummy variable is not necessary anymore to capture the turning points in the underlying trend. This confirms the capability of demographics and smoothed inflation of capturing the change in the underlying trend affecting spot rates. The performance the demographic cycle, however, dominates the inflation cycle at each horizon. The estimated coefficient on the demographic variable is very stable at all horizons, while the one on the discounted moving average of past inflation is more volatile.In online Appendix Table A.2, we also provide the ATSM model out-of-sample forecasts substituting MY with the CP trend. The model does not perform better than the benchmarks reported in Table 3.

In the ATSM models we only considered so far stationary state variables. The arbitrage-free dynamic Nelson-Siegel framework of Christensen, Diebold, and Rudebusch (2011) allows for a unit root on the dynamics of the latent level factor. In principle one can assume a non-stationary dynamics for the spot rates, however we argue in this paper that a VAR for the stationary factors augmented with the exogenous demographic variable is a valid alternative strategy for long-term yield forecasting. In order to test this claim, we compare the forecasting performance of the Demographic ATSM and the independent-factor AFNS model (Christensen et al., 2011) and report the results in Table A.3

of online Appendix. The table shows that while both models provide comparable forecasts for one year forecasting horizon, the Demographic ATSM performs better (in terms of RMSFE) than the independent-factor AFNS model beyond one year forecasting horizon, in particular for the short rates.

# 5 Robustness

This section examines the robustness of our results along three dimensions. First, we extend our results to international data, since all the empirical results reported are based on US data. Second, in all forward projections we have implemented so far we have treated $\mathrm{MY}_{t+i}$ at all relevant future horizons as a known variable. Predicting MY requires projecting population in the age brackets 20-29, and 40-49. Although these are certainly not the age ranges of population more difficult to predict[21] the question on the uncertainty surrounding projections for MY is certainly legitimate. Therefore, we consider projections under different fertility rates and consider foreign holdings of US debt securities. Third, one might object that our statistical evidence on $\mathrm{MY}_t$ and the permanent component of interest rates is generated by the observation of a couple of similar paths of nonstationary random variables. Although the spurious regression problem is typical in static regression and all the evidence reported so far is based on estimation of dynamic time-series model, some simulation based evidence might be helpful to strengthen our empirical evidence.

## 5.1 International Evidence

We provide international evidence to evaluate the evidence so far on a larger and different dataset. In particular, the demographic variable $\mathrm{MY}_t$ is constructed for a large panel of 35 countries over the period 1960-2011 (unbalanced panel)[22]. We consider the

[21] Improvement in mortality rates that have generated over the last forty years difference between actual population and projected population are mostly concentrated in older ages, after 65.

[22] The results are robust when we contruct a smaller panel with balanced data. The demographic data is collected from Worldbank database.

performance of augmenting autoregressive models for nominal bond yields [23] against the benchmark where the effects of demographics is restricted to zero.

The results from the estimation are reported in Table 6. The evidence on the importance of MY in capturing the persistent component of nominal yields is confirmed by the panel estimation. Note that the coefficient on MY is significant with the expected sign even if once we control for the autoregressive component. However, recall that the use of the demographic variable in the ATSM is motivated in an OLG model specifically designed for the U.S. population age structure and calibrated using U.S. data. While the model in principle can be amended for modeling the term structure of other countries with a similar population pyramid, there is an important challenge for small countries with less developed financial markets. [24]

## 5.2 The Uncertainty on Future MY

To analyze the uncertainty on projections on MY we use the evidence produced by the Bureau of Census 1975 population report, which publishes projections of future population by age in the United States from 1975 to 2050.[25] The report contains projections based on three different scenarios for fertility, which is kept constant and set to 1.7, 2.1 and 2.7, respectively. All three scenarios are based on the estimated July 1, 1974 population and assume a slight reduction in future mortality and an annual net immigration of 400,000 per year. They differ only in their assumptions about "future fertility". Since there is only 5-year forecasts from 2000-2050, we interpolate 5-year results to obtain the annual series. Then we construct $MY_t$ ratio by using this annually projection results of different fertility rates from 1975 to 2050.

[23] Bond yield are collected from Global Financial data. Long term bond yields are 10-year yields for most of the countries, except Japan (7-year), Finland, South Korea, Singapore (5-year), Mexico(3-year), Hong Kong(2-year).

[24] We thank an anonymous referee for pointing out that for some countries the domestic bond markets may be less important for life-cycle investment behavior, e.g., households from the emerging markets may instead demand foreign assets, in particular U.S. securities (dollarization).

[25] The report provides annual forecasts from 1975 to 2000 and five-year forecasts from 2000 to 2050.

To evaluate the uncertainty surrounding projections for our relevant demographic variable, Panel A in Figure 6 reports plot actual $\mathrm{MY}_t$ and projected $\mathrm{MY}_t$ in 1975.

The actual annual series of $\mathrm{MY}_t$ is constructed based on information released by BoC until December, 2010, while, for the period 2011 to 2050 we use projections contained in the 2008 population report. The figure illustrates that the projections based on the central value of the fertility rate virtually overlaps with the observed data up to 2010 and with the later projections for the period 2011-2050 (Davis and Li, 2005). Different assumptions on fertility have a rather modest impact on MY.

Another concern about the uncertainty on future MY is regarding the foreign holdings of US debt securities. The theoretical justification of the demographic effect comes from a closed economy model, that is, it assumes segmented markets. As long as the foreign demographic fluctuations do not counteract the US demographic effect, this assumption should be innocuous. Therefore we compute a demographic variable which takes into account the foreign holdings of US securities, in particular total debt and US Treasury holdings. Following the last report by FED New York published in April 2013, we identify the countries with most US security holdings and compute the middle age-young ratio for those countries, namely Japan, China, UK, Canada, Switzerland, Belgium, Ireland, Luxembourg, Hong Kong.[26] We compute the MY ratio adjusted for foreign holdings; the MY ratio is a weighted average of the MY ratios of those countries with most US security holdings. The weights are computed based on the relative US security holdings reported in Table 7 of the report. In our estimation, we keep the weights fixed at 2012 holdings.

As we see from Panel B in Figure 6, the shape of the demographic variable does not change substantially once we take into account either total debt or treasury holdings. We observe that during the early 2000s, for a short period, the predictions of the original MY variable, and the MY variable adjusted for foreign treasury holdings differ. However, the

[26] We do not have age structure data for Cayman Islands, Middle East countries and rest of the world. So we account for 60% of foreign bond holdings as of June 2012. Source: Demographic data 1960-2000 from World Bank Population Statistics, Data 2011-2050 from US Census International Database.

discrepancy between the two series is temporary and the variables start to comove again in the out-of-sample period. While foreign holdings of US Treasuries have been increasing during the last decade, there is no reason to think that the trend will continue forever, e.g., Feldstein (2011). In online Appendix Tables A.4 and A.5, we provide the ATSM model out-of-sample forecasts substituting MY with the MY adjusted for foreign treasury holdings and a global MY, an equally-weighted cross-sectional average of foreign MY variables. The models perform better than the benchmarks, but they are outformed by the demographic ATSM reported in Table 3.

## 5.3 A Simulation Experiment

To assess the robustness of our results we started from the estimation of a simple autoregressive model for both nominal and real 3-month rates over the full sample. By bootstrapping the estimated residuals we have then constructed five thousand artificial time series for the short-rates. These series are very persistent (based on an estimated AR coefficient of 0.963 and 0.932 for nominal and real rates, respectively) and generated under the null of no-significance of MY in explaining the 3-month rates. We have then run one thousand regression by augmenting an autoregressive model for the artificial series with $\mathrm{MY}_t$.

Figure 7 shows that the probability of observing a t-statistics of -2.60 on the coefficient on $\mathrm{MY}_t$ is 0.070 for the nominal rate and a t-statistics of -2.67 on the coefficient on $\mathrm{MY}_t$ is 0.039 for the real rate.[27] In line with the OLG model prediction it is less likely that the demographic variable captures the persistence of the real rate by chance. The results more striking if we limit the sample to pre-crisis period, up to 2008Q3.[28] This small fraction of simulated t-statistics capable of replicating the observed results provides clear evidence against the hypothesis that our statistical results on demographics and the permanent

[27] The t-statistics $\mathrm{MY}_t$ coefficient in the OLS regression of the 3-month rate on its own lag and the demographic variable.

[28] The probability of observing a t-statistics of -3.02 on the coefficient on $\mathrm{MY}_t$ is 0.032 for the nominal rate and a t-statistics of -3.12 on the coefficient on $\mathrm{MY}_t$ is 0.018 for the real rate.

component of interest rates are spurious.

# 6 Conclusion

The entire term structure of interest rates features a common persistent component. Our evidence has shown that such a persistent component is related to a demographic variable, to ratio of middle-aged to young population, $MY_t$. The relation between the age structure of population and the equilibrium real returns of bonds is derived in an overlapping generation model in which the demographic structure mimics the pattern of live births in the US. The age composition of the population defines the persistent component in one-period yields as it determines the equilibrium rate in the central bank reaction function. The presence of demographics in short-term rates allows more precise forecast of future policy rates, especially at very long-horizon, and helps modeling the entire term structure. Term structure macro-finance models with demographics clearly dominate traditional term-structure macro-finance models and random walk benchmarks. When demographics are entered among the determinants of short-term rates, a simple model based on a Taylor rule specification for yields at longer maturities outperforms in forecasting traditional term structure models. Better performance is not limited to statistical accuracy, but also confirmed by utility gains using the demographic information. There is a simple intuitive explanation for these results: traditional Taylor-rules and macro finance model do not include an observed determinant of yields capable of capturing their persistence. Linking the long-term central bank target for interest rates to demographics allows for the presence of a slowly moving target for policy rates that fits successfully the permanent component observed in the data. Rudebusch (2002) relates the "illusion" of monetary policy inertia to the possibility that estimated policy rules reflect some persistent shocks that central banks face. Our evidence illustrates that such persistent component is effectively modeled by the age structure of the population. The successful fit is then associated to successful out-of-sample predictions because the main driver of the permanent component in spot rates is exogenous and predictable. Overall, our results show the importance of including the age-structure of population in macro-finance models. As pointed out by Bloom, Canning and Graham (2003) one of the remarkable features of the economic literature is that demographic factors have so far entered in economic models almost exclusively through the size of population while the age composition of population has also important, and probably neglected, consequences for fluctuations in financial and macroeconomic variables. This paper has taken a first step in the direction of linking fluctuations in the term structure of interest rates to the age structure of population.

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Table 1.1

Summary Statistics of the Data in Term Structure Models

| | Central Moments | | | | Autocorrelations | | |
|---|---|---|---|---|---|---|---|
| | mean | Stdev | Skew | Kurt | Lag 1 | Lag 2 | Lag 3 |
| 3-month | 5.0031 | 3.0855 | 0.6914 | 4.1167 | 0.9351 | 0.8874 | 0.8642 |
| 1-year | 5.4697 | 3.1833 | 0.5313 | 3.5486 | 0.9499 | 0.9087 | 0.8784 |
| 2-year | 5.6861 | 3.1087 | 0.4494 | 3.3606 | 0.9553 | 09208 | 0.8931 |
| 3-year | 5.8577 | 3.0221 | 0.4371 | 3.2724 | 0.9597 | 0.9290 | 0.9024 |
| 4-year | 6.0020 | 2.9374 | 0.4605 | 3.2360 | 0.9628 | 09343 | 0.9084 |
| 5-year | 6.1273 | 2.8589 | 0.4999 | 3.2282 | 0.9650 | 0.9377 | 0.9123 |
| LN output factor | 0.0674 | 0.9899 | −0.4323 | 5.7257 | 0.2506 | 0.0835 | 0.1342 |
| LN inflation factor | 0.0504 | 1.0065 | −0.2071 | 8.7346 | 0.0638 | 0.0228 | −0.0302 |
| Fama Trend | 0.0530 | 0.0303 | 0.0519 | 2.5980 | 0.9953 | 0.9882 | 0.9788 |
| CP Trend | 0.0393 | 0.0177 | 0.4548 | 2.1044 | 0.9989 | 0.9958 | 0.9908 |
| MY | 0.8620 | 0.2023 | −0.2075 | 1.5614 | 0.9974 | 0.9936 | 0.9887 |

Notes. This table reports the summary statistics. 1, 4, 8, 12, 16, 20 quarter yields are annualized (in percentage) zero coupon bond yields from Fedral Reserve Board (Gurkaynak, Sack and Wright(2006)). LN Inflation and real activity refer to the price and output factors extracted from large dataset using extended time series according to Ludvigson and Ng (2009). Fama trend is the 5-year moving average of 1-year Treasury bond yield and CP trend is the 10-year moving average of core inflation (Cieslak and Povala, 2015). MY is the middle-aged to young ratio. Quarterly sample 1961Q3-2013Q4.

Table 1.2

Summary Statistics of Ex-ante Real Yields and Expected Inflation

| | Central Moments | | | | Autocorrelations | | |
|---|---|---|---|---|---|---|---|
| | mean | Stdev | Skew | Kurt | Lag 1 | Lag 2 | Lag 3 |
| Real Yields | | | | | | | |
| 3-month | 2.6862 | 2.7205 | 0.4109 | 3.6694 | 0.9202 | 0.8655 | 0.8442 |
| 1-year | 3.1313 | 2.7912 | 0.2486 | 3.2275 | 0.9384 | 0.8912 | 0.8603 |
| 2-year | 3.3474 | 2.7131 | 0.1639 | 3.1526 | 0.9447 | 0.9052 | 0.8772 |
| 3-year | 3.5187 | 2.6225 | 0.1570 | 2.1417 | 0.9499 | 0.9146 | 0.8875 |
| 4-year | 3.6626 | 2.5327 | 0.1913 | 3.1653 | 0.9534 | 0.9204 | 0.8937 |
| 5-year | 3.7873 | 2.4484 | 0.2446 | 3.2057 | 0.9557 | 0.9239 | 0.8973 |
| Expected Inflation | | | | | | | |
| 3-month | 2.3169 | 0.5927 | 0.7562 | 3.7250 | 0.9878 | 0.9636 | 0.9337 |
| 1-year | 2.3385 | 0.6041 | 0.7704 | 3.7599 | 0.9878 | 0.9635 | 0.9335 |
| 2-year | 2.3386 | 0.6036 | 0.7700 | 3.7532 | 0.9874 | 0.9630 | 0.9326 |
| 3-year | 2.3390 | 0.6046 | 0.7774 | 3.7724 | 0.9863 | 0.9615 | 0.9301 |
| 4-year | 2.3394 | 0.6059 | 0.7875 | 3.8015 | 0.9854 | 0.9600 | 0.9276 |
| 5-year | 2.3399 | 0.6074 | 0.7981 | 3.8326 | 0.9846 | 0.9587 | 0.9254 |

Notes. This table reports the summary statistics. 1, 4, 8, 12, 16, 20 quarter ex-ante real yields are obtained by substracting the expected inflation from equivalent nominal bond yields. Expected inflation is the predicted inflation from an autoregressive model using growth rate of GDP deflator. Quarterly sample 1961Q3-2013Q4.

TABLE 2
ATSM Full-Sample Estimates

| | Demographic ATSM | | | | | Macro ATSM | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Companion form $\Phi$** | | | | | | | | | | |
| | −0.125 | 0.137 | −0.153 | −0.253 | 0.165 | −0.133 | 0.134 | 0.067 | −0.311 | 0.240 |
| | (0.082) | (0.123) | (0.140) | (0.135) | (0.111) | (0.095) | (0.104) | (0.105) | (0.132) | (0.192) |
| | −0.057 | 0.348 | 0.147 | 0.079 | −0.220 | −0.054 | 0.380 | −0.092 | 0.066 | −0.279 |
| | (0.073) | (0.087) | (0.090) | (0.125) | (0.112) | (0.072) | (0.104) | (0.110) | (0.168) | (0.119) |
| | −0.028 | 0.041 | 0.764 | −0.251 | 0.101 | −0.015 | 0.059 | 0.981 | 0.036 | −0.087 |
| | (0.040) | (0.026) | (0.142) | (0.040) | (0.068) | (0.009) | (0.041) | (0.023) | (0.120) | (0.112) |
| | −0.017 | 0.057 | −0.178 | 0.622 | 0.060 | −0.015 | 0.075 | −0.039 | 0.608 | 0.172 |
| | (0.028) | (0.021) | (0.040) | (0.174) | (0.032) | (0.039) | (0.024) | (0.060) | (0.141) | (0.043) |
| | −0.002 | 0.001 | 0.240 | 0.189 | 0.754 | 0.020 | −0.044 | 0.018 | 0.305 | 0.681 |
| | (0.018) | (0.021) | (0.075) | (0.076) | (0.095) | (0.041) | (0.052) | (0.107) | (0.034) | (0.127) |
| **Short rate parameters** | | | | | | | | | | |
| $\delta_1$ | −0.006 | 0.157 | 0.000 | 0.000 | 2.739 | −0.007 | 0.263 | 2.321 | 0.000 | 1.544 |
| | (0.039) | (0.121) | (0.000) | (0.000) | (0.372) | (0.059) | (0.119) | (0.588) | (0.000) | (0.957) |
| $\delta_2$ | −0.010 | | | | | 0 | | | | |
| | (0.004) | | | | | | | | | |
| **Price of risk $\lambda_0$ and $\lambda_1$** | | | | | | | | | | |
| $(\lambda_0)^T$ | −0.004 | −0.004 | 0.003 | 0.004 | −0.003 | −0.108 | −0.008 | −0.002 | −0.002 | 0.008 |
| | (0.014) | (0.002) | (0.003) | (0.002) | (0.001) | (0.261) | (0.013) | (0.009) | (0.010) | (0.014) |
| $\lambda_1$ | −0.045 | | $\cdots$ | | **0** | −0.000 | | $\cdots$ | | **0** |
| | (0.325) | | | | | (0.004) | | | | |
| | | −0.685 | | | | | −0.012 | | | |
| | | (0.297) | | | | | (0.046) | | | |
| | $\vdots$ | | −0.017 | | $\vdots$ | $\vdots$ | | −0.016 | | $\vdots$ |
| | | | (0.053) | | | | | (0.067) | | |
| | | | | −1.162 | | | | | −1.129 | |
| | | | | (0.565) | | | | | (0.619) | |
| | **0** | | $\cdots$ | | −0.972 | **0** | | $\cdots$ | | 0.000 |
| | | | | | (0.708) | | | | | (0.000) |
| **Innovation covariance matrix $\Omega^o \times 10^5$** | | | | | | | | | | |
| | 0.537 | 0.033 | | | | 0.535 | 0.046 | | | |
| | | 0.487 | | | | | 0.494 | | | |

Notes. This table reports the maximum likelihood estimation results for the system (7) with time-varying risk premium. The left panel contains estimated results for the unrestricted model which includes the demographic variable MY. The right panel reports estimated results of the system with the restriction $\delta_2$ equal to zero. Standard errors are provided within parentheses. Quarterly sample 1964Q1-2013Q4.

Table 3

Affine Model Out-of-Sample Forecasts

| Panel A. Random walk Benchmark | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| h | 4 | | 8 | | 12 | | 16 | | 20 | |
| | RMSFE (GW) | CW (p-val) | RMSFE (GW) | CW (p-val) | RMSFE (GW) | CW (p-val) | RMSFE (GW) | CW (p-val) | RMSFE (GW) | CW (p-val) |
| $\widehat{y}^{(1/4)}_{t+h\|t}$ | 1.224 (0.016) | 0.814 (0.208) | 0.941 (0.001) | 5.624 (0.000) | 0.813 (0.000) | 8.118 (0.000) | 0.832 (0.000) | 7.057 (0.000) | 0.932 (0.000) | 5.803 (0.000) |
| $\widehat{y}^{(1)}_{t+h\|t}$ | 1.158 (0.010) | 0.338 (0.368) | 0.923 (0.006) | 5.188 (0.000) | 0.821 (0.001) | 7.466 (0.000) | 0.839 (0.000) | 6.359 (0.000) | 0.935 (0.001) | 5.391 (0.000) |
| $\widehat{y}^{(2)}_{t+h\|t}$ | 1.158 (0.034) | −0.145 (0.558) | 0.951 (0.000) | 4.317 (0.000) | 0.874 (0.000) | 6.088 (0.000) | 0.897 (0.001) | 5.083 (0.000) | 0.991 (0.013) | 4.281 (0.000) |
| $\widehat{y}^{(3)}_{t+h\|t}$ | 1.158 (0.008) | −0.337 (0.632) | 0.982 (0.393) | 3.649 (0.000) | 0.926 (0.001) | 4.890 (0.000) | 0.948 (0.113) | 4.070 (0.000) | 1.036 (0.258) | 3.341 (0.000) |
| $\widehat{y}^{(4)}_{t+h\|t}$ | 1.154 (0.000) | −0.397 (0.654) | 1.008 (0.065) | 3.126 (0.001) | 0.969 (0.390) | 3.892 (0.000) | 0.990 (0.002) | 3.286 (0.001) | 1.070 (0.090) | 2.651 (0.004) |
| $\widehat{y}^{(5)}_{t+h\|t}$ | 1.147 (0.000) | −0.387 (0.651) | 1.027 (0.002) | 2.705 (0.003) | 1.003 (0.075) | 3.076 (0.001) | 1.023 (0.172) | 2.689 (0.004) | 1.096 (0.016) | 2.182 (0.015) |
| Panel B. Macro ATSM Benchmark | | | | | | | | | | |
| h | 4 | | 8 | | 12 | | 16 | | 20 | |
| | RMSFE (GW) | CW (p-val) | RMSFE (GW) | CW (p-val) | RMSFE (GW) | CW (p-val) | RMSFE (GW) | CW (p-val) | RMSFE (GW) | CW (p-val) |
| $\widehat{y}^{(1/4)}_{t+h\|t}$ | 1.060 (0.000) | 1.496 (0.067) | 0.894 (0.000) | 8.410 (0.000) | 0.778 (0.000) | 9.673 (0.000) | 0.747 (0.001) | 9.203 (0.000) | 0.756 (0.002) | 6.962 (0.001) |
| $\widehat{y}^{(1)}_{t+h\|t}$ | 1.014 (0.002) | 2.531 (0.006) | 0.859 (0.011) | 9.218 (0.000) | 0.761 (0.010) | 10.689 (0.000) | 0.744 (0.005) | 9.757 (0.000) | 0.760 (0.001) | 7.158 (0.000) |
| $\widehat{y}^{(2)}_{t+h\|t}$ | 0.989 (0.000) | 2.967 (0.002) | 0.837 (0.001) | 9.487 (0.000) | 0.752 (0.001) | 11.280 (0.000) | 0.743 (0.000) | 10.080 (0.000) | 0.766 (0.001) | 7.485 (0.000) |
| $\widehat{y}^{(3)}_{t+h\|t}$ | 0.975 (0.000) | 3.181 (0.001) | 0.825 (0.000) | 9.596 (0.000) | 0.749 (0.000) | 11.476 (0.000) | 0.745 (0.000) | 10.122 (0.000) | 0.773 (0.001) | 7.687 (0.000) |
| $\widehat{y}^{(4)}_{t+h\|t}$ | 0.965 (0.000) | 3.394 (0.000) | 0.817 (0.000) | 9.713 (0.000) | 0.746 (0.000) | 11.491 (0.000) | 0.748 (0.000) | 10.083 (0.000) | 0.778 (0.001) | 7.829 (0.000) |
| $\widehat{y}^{(5)}_{t+h\|t}$ | 0.959 (0.000) | 3.598 (0.000) | 0.811 (0.000) | 9.801 (0.000) | 0.745 (0.000) | 11.387 (0.000) | 0.751 (0.000) | 9.980 (0.000) | 0.784 (0.001) | 7.906 (0.000) |

Notes. This table provides yield forecast comparison of Demographic ATSM against the Random Walk model (Panel A) and Macro ATSM (Panel B) benchmarks. We use the in-sample estimators, from 1961Q3 to 1981Q2, to generate out-of-sample forecasts until 2013Q4. h indicates 4, 8, 12, 16, 20 quarter out-of-sample forecasts. We measure forecasting performance as the ratio of the root mean squared forecast error (RMSFE) of our model against the benchmarks. We report in parentheses the p-values of the forecasting test due to Giacomini and White (2006) in the columns with FRMSE. A p-value below 0.01 (0.05, 0.10) indicates a significant difference in forecasting performance at the 1% (5%, 10%) level. We also measure forecasting performance using Clark and West (2006, 2007) test. We report the test statistics in the columns CW for each horizon together with p-values in parentheses below. Quarterly sample 1981Q3-2013Q4.

TABLE 4

Out-of-Sample Forecast Usefulness

| Panel A. Bond Yields - Quadratic Loss | | | | | |
|---|---|---|---|---|---|
| h | 4 | 8 | 12 | 16 | 20 |
| | $\widehat{\kappa}_{(t^{\kappa=0}),[t^{\kappa=1}]}$ | $\widehat{\kappa}_{(t^{\kappa=0}),[t^{\kappa=1}]}$ | $\widehat{\kappa}_{(t^{\kappa=0}),[t^{\kappa=1}]}$ | $\widehat{\kappa}_{(t^{\kappa=0}),[t^{\kappa=1}]}$ | $\widehat{\kappa}_{(t^{\kappa=0}),[t^{\kappa=1}]}$ |
| $\widehat{y}^{(1/4)}_{t+h\|t}$ | 0.816<br>(4.60***)<br>[−1.04] | 0.098<br>(0.56)<br>[−5.16***] | −0.238<br>(−1.19)<br>[−6.17***] | −0.035<br>(−0.15)<br>[−4.36***] | 0.307<br>(2.02**)<br>[−4.55***] |
| $\widehat{y}^{(1)}_{t+h\|t}$ | 0.708<br>(3.61***)<br>[−1.49] | −0.040<br>(−0.30)<br>[−7.83***] | −0.232<br>(−1.17)<br>[−6.19***] | −0.016<br>(−0.07)<br>[−4.59***] | 0.316<br>(2.26**)<br>[−4.88***] |
| $\widehat{y}^{(2)}_{t+h\|t}$ | 0.726<br>(3.17***)<br>[−1.20] | −0.076<br>(−0.59)<br>[−8.35***] | −0.134<br>(−0.73)<br>[−6.18***] | 0.112<br>(0.59)<br>[−4.70***] | 0.413<br>(3.51***)<br>[−4.98***] |
| $\widehat{y}^{(3)}_{t+h\|t}$ | 0.744<br>(2.97***)<br>[−1.02] | −0.068<br>(−0.50)<br>[−7.74***] | −0.019<br>(−0.11)<br>[−6.04***] | 0.226<br>(1.34)<br>[−4.60***] | 0.490<br>(4.63***)<br>[−4.81***] |
| $\widehat{y}^{(4)}_{t+h\|t}$ | 0.754<br>(2.83***)<br>[−0.92] | −0.035<br>(−0.23)<br>[−6.80***] | 0.091<br>(0.57)<br>[−5.68***] | 0.320<br>(2.04**)<br>[−4.34***] | 0.548<br>(5.48***)<br>[−4.53***] |
| $\widehat{y}^{(5)}_{t+h\|t}$ | 0.755<br>(2.71***)<br>[−0.88] | 0.011<br>(0.07)<br>[−5.93***] | 0.188<br>(1.20)<br>[−5.16***] | 0.395<br>(2.62***)<br>[−4.01***] | 0.590<br>(6.05***)<br>[−4.20***] |

| Panel B. Bond Excess Returns - Portfolio Utility Loss | | |
|---|---|---|
| holding period | 1-year | 2-year |
| | $\widehat{\kappa}_{(t^{\kappa=0}),[t^{\kappa=1}]}$ | $\widehat{\kappa}_{(t^{\kappa=0}),[t^{\kappa=1}]}$ |
| Demographic ATSM | 0.595<br>(1.57)<br>[−1.07] | 0.316<br>(1.85*)<br>[−4.00***] |
| Macro ATSM | 0.611<br>(2.61***)<br>[−1.67*] | 0.707<br>(1.94*)<br>[−0.80] |

Notes. This table provides results on forecasting usefulness according to Carriero and Giacomini (2011) test. Panel A shows yield forecast comparison of Demographic ATSM against the Random Walk benchmark. Panel B shows bond excess return forecast comparison of Demographic and Macro ATSM against the Random Walk benchmark. We use the in-sample estimators, from 1961Q3 to 1981Q2, to generate out-of-sample forecasts until 2013Q4. h indicates 4, 8, 12, 16, 20 quarter out-of-sample forecasts. We report $\widehat{\kappa}$, the weight on the restricted (random walk) model, and the test statistics associated with $\kappa = 0$ and $\kappa = 1$ in the parentheses below. Quarterly sample 1981Q3-2013Q4.

Table 5
Predictive Regressions for the 1-year Spot Rate

$y^{(1)}_{t+4x}$ -$y^{(1)}_t$ $= a^x$+$b^x D_t$+$c^x[f^{(1)}_{t,t+4x}$ -$y^{(1)}_t]$+$d^x[y^{(1)}_t$-$P^{(1),i}_t]$+$\varepsilon_{t+4x}$

$P^{(1),1}_t$=$\frac{1}{20}\sum_{i=1}^{20} y^{(1)}_{t-i-1}$ $(FAMA)$, $P^{(1),2}_t$=$\frac{\sum_{i=1}^{40} 0.96^{i-1}\pi_{t-i-1}}{\sum_{i=1}^{40} 0.96^{i-1}}$$(CP)$, $P^{(1),3}_t$=$e^x\frac{1}{4}\sum_{i=1}^{4} MY_{t+i-1}$

| | $a^x$ (s.e.) | $b^x$ (s.e.) | $c^x$ (s.e.) | $d^x$ (s.e.) | $e^x$ (s.e.) | $R^2$ | |
|---|---|---|---|---|---|---|---|
| no cycle with dummy | −1.99 (0.26) | 2.36 (0.134) | 1.29 (0.17) | | | 0.28 | |
| Fama cycle with dummy | −1.88 (0.25) | 3.30 (0.38) | 0.42 (0.24) | −0.54 (0.12) | | 0.35 | |
| Fama cycle | −0.74 (0.25) | | 0.87 (0.28) | −0.01 (0.11) | | 0.11 | $x = 2$ |
| CP cycle | 0.78 (0.38) | | −0.17 (0.27) | −0.63 (0.13) | | 0.20 | |
| MY cycle | 6.83 (1.16) | | 0.11 (0.20) | −0.54 (0.08) | −0.093 (0.009) | 0.27 | |
| no cycle with dummy | −3.04 (0.26) | 3.50 (0.33) | 2.01 (0.16) | | | 0.50 | |
| Fama cycle with dummy | −2.93 (0.25) | 4.35 (0.37) | 1.20 (0.24) | −0.50 (0.11) | | 0.54 | |
| Fama cycle | −1.42 (0.27) | | 1.79 (0.31) | 0.20 (0.13) | | 0.22 | $x = 3$ |
| CP cycle | 0.22 (0.44) | | 0.45 (0.32) | −0.58 (0.15) | | 0.26 | |
| MY cycle | 8.13 (1.26) | | 0.49 (0.21) | −0.65 (0.09) | −0.095 (0.010) | 0.39 | |
| no cycle with dummy | −3.56 (0.25) | 4.18 (0.32) | 2.23 (0.16) | | | 0.59 | |
| Fama cycle with dummy | −3.46 (0.24) | 4.90 (0.136) | 1.55 (0.23) | −0.43 (0.11) | | 0.62 | |
| Fama cycle | −1.75 (0.29) | | 2.21 (0.32) | 0.36 (0.13) | | 0.25 | $x = 4$ |
| CP cycle | −0.17 (0.49) | | 0.77 (0.34) | −0.47 (0.17) | | 0.25 | |
| MY cycle | 9.36 (01.30) | | 0.50 (0.22) | −0.75 (0.09) | −0.094 (0.010) | 0.43 | |
| no cycle with dummy | −3.57 (0.26) | 4.37 (0.33) | 2.00 (0.16) | | | 0.56 | |
| Fama cycle with dummy | −3.46 (0.25) | 5.15 (0.37) | 1.27 (0.24) | −0.46 (0.11) | | 0.59 | |
| Fama cycle | −1.65 (0.30) | | 1.98 (0.32) | 0.36 (0.14) | | 0.18 | $x = 5$ |
| CP cycle | −0.41 (0.53) | | 0.77 (0.36) | −0.33 (0.018) | | 0.17 | |
| MY cycle | 10.48 (1.35) | | 0.18 (0.23) | −0.83 (0.010) | −0.092 (0.010) | 0.41 | |

Notes. This table shows predictive regressions with alternative permanant components. $f^{(1)}_{t,t+4x}$ is one-year forward rate observed at time $t$ of an investment with settlement after $3x$ years and maturity in $4x$ years, $y^{(1)}_t$is 1-year spot rate, $\pi_t$ is annual core CPI inflation, $MY_t$ is the middle aged to young ratio, $D_t$ is a time dummy ($D_t = 1$ from 1961Q3 to 1981Q2). Standard errors are Hansen-Hodrick (1980) adjusted. Quarterly sample 1961Q3-2013Q4.

Table 6

International Panel

Benchmark model: $R_{lt} = \alpha_0 + \alpha_1 R_{lt-1} + \varepsilon_t$

Augmented model: $R_{lt} = \beta_0 + \beta_1 R_{lt-1} + \beta_2 \mathrm{MY}_t + \varepsilon_t$

| Specification | $R_{lt-1}$ | $\mathrm{MY}_t$ | $\bar{\mathrm{R}}^2$ |
|---|---|---|---|
| (1) | 0.729 (8.39***) | | 0.55 |
| (2) | 0.676 (7.29***) | −0.044 (−3.78***) | 0.58 |

Notes. This table reports international evidence. Pooled regression coefficients account for country fixed effects. $R_{lt}$ is the nominal bond yield. Specification (1) is the benchmark model and specification (2) is the augmented model with $\mathrm{MY}_t$. The reported t-statistics are based on Driscoll-Kraay (1998) standard errors robust to general forms of cross-sectional (spatial) and temporal dependence. Asterisks *, ** and *** indicate significance at the 10 percent, 5 percent and 1 percent levels, respectively. Last column report within group $\mathrm{R}^2$. There are 35 countries, and 1530 observations in an (unbalanced) panel. Annual sample 1960-2011.

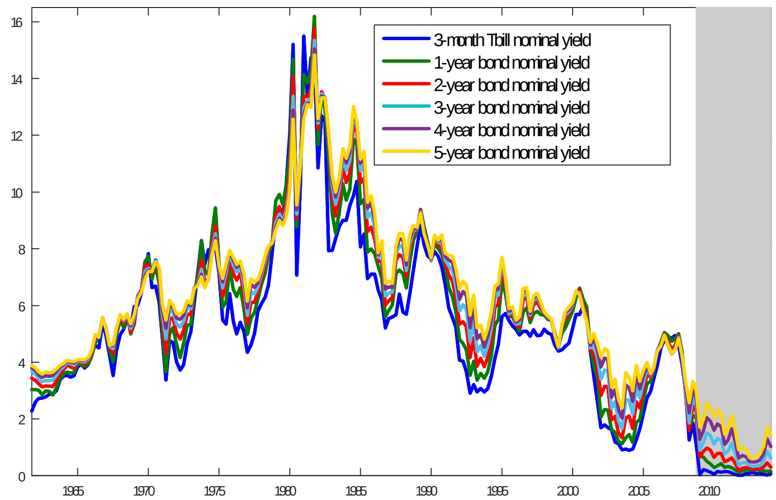


Figure 1.1. Nominal Bond Yields.

Notes. This figure shows the US post-war nominal yields. The grey shaded area covers from the beginning of the first round of quantitative easing (2008Q3) to the end of the sample. Quarterly sample 1961Q3-2013Q4.

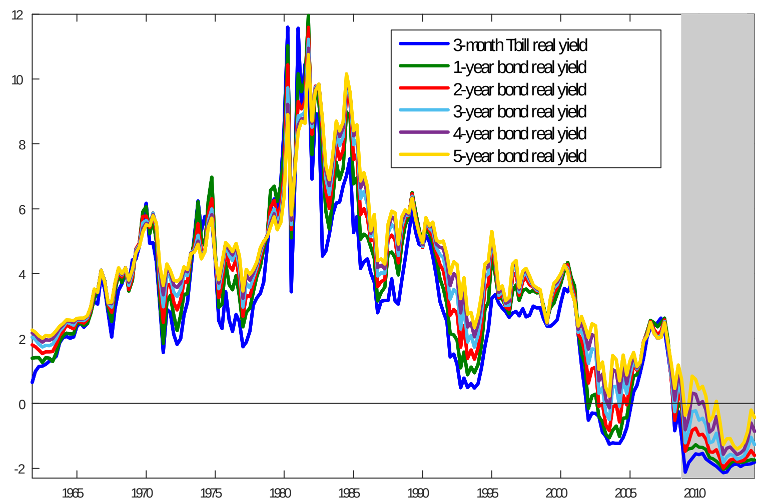


Figure 1.2. Real Bond Yields.

Notes. This figure shows the US post-war real yields. Real yields are computed as the nominal yields minues expected inflation, that is, the predicted inflation from an autoregressive model using growth rate of GDP deflator. The grey shaded area covers from the beginning of the first round of quantitative easing (2008Q3) to the end of the sample. Quarterly sample 1961Q3-2013Q4.

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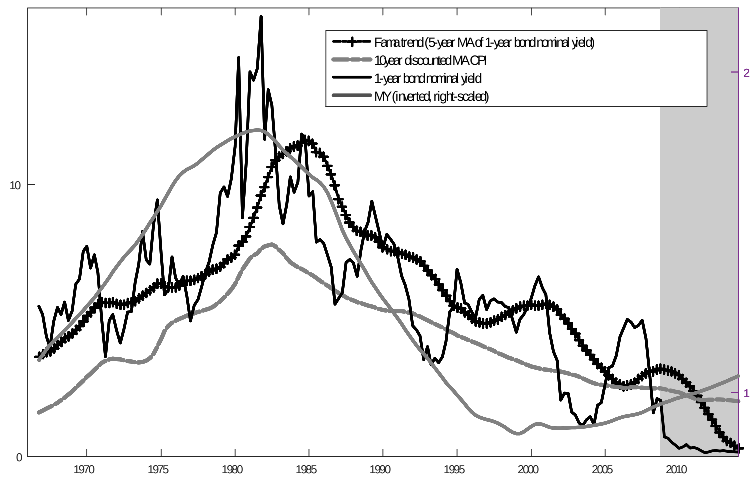


Figure 2.1. 1-Year US Treasury bond nominal yields and the permanent component.

Notes. This figure compares the middle-aged to young ratio, MY (inverted, right-scaled, solid dark grey line), Fama trend (dashed grey line with plus), that is, 5-year moving average of 1-year Treasury bond yield, CP trend, that is,10 year moving average of core inflation (dashed light grey line) with 1-year Treasury bond nominal yield (solid black line). The grey shaded area covers from the beginning of the first round of quantitative easing (2008Q3) to the end of the sample. Quarterly sample 1966Q3-2013Q4.

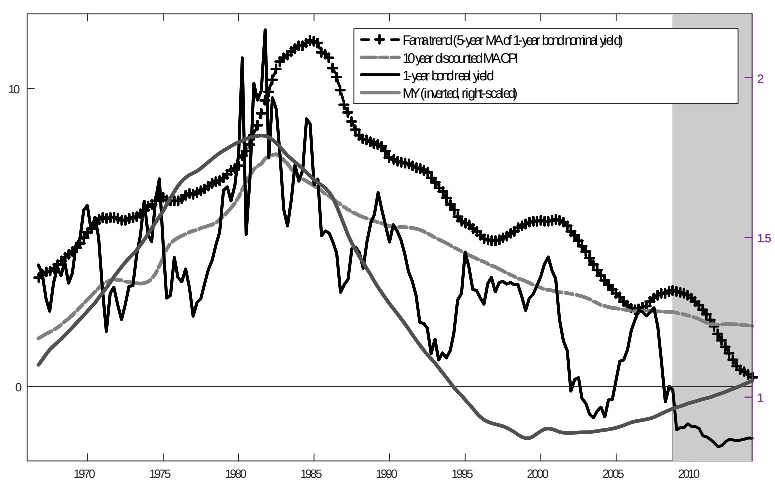


Figure 2.2. 1-Year US Treasury bond real yields and the permanent component.

Notes. This figure compares the middle-aged to young ratio, MY (inverted, right-scaled, solid dark grey line), Fama trend (dashed grey line with plus), i.e., 5-year moving average of 1-year Treasury bond yield, CP trend, i.e.,10 year moving average of core inflation (dashed light grey line) with 1-year Treasury bond real yield (solid black line). The grey shaded area covers from the beginning of the first round of quantitative easing (2008Q3) to the end of the sample. Quarterly sample 1966Q3-2013Q4.

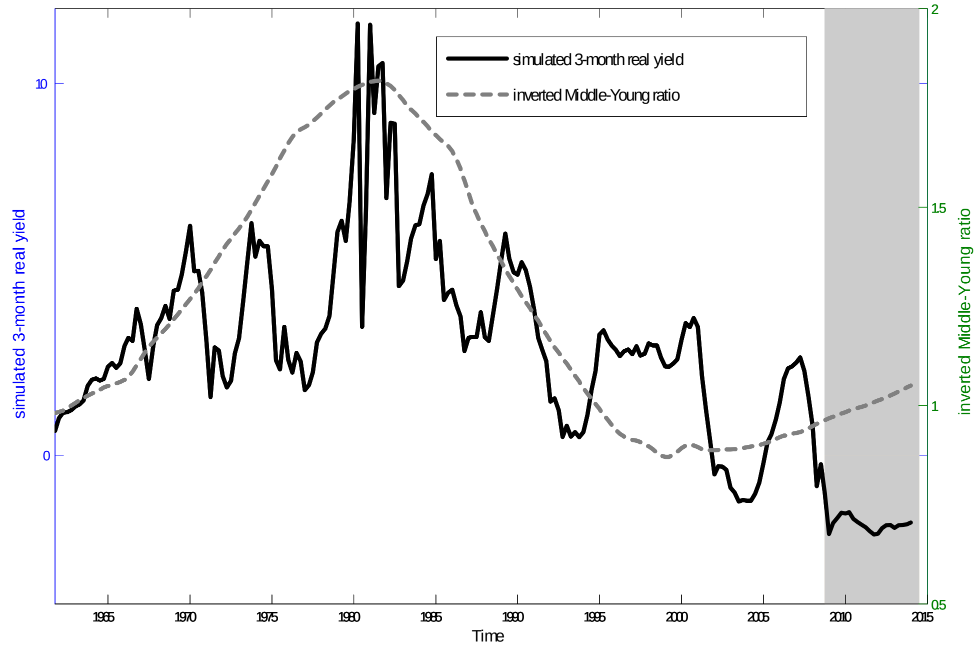


Figure 3.1. U.S. ex-ante real short rate (3-month) and MY (inverted, right-scale).

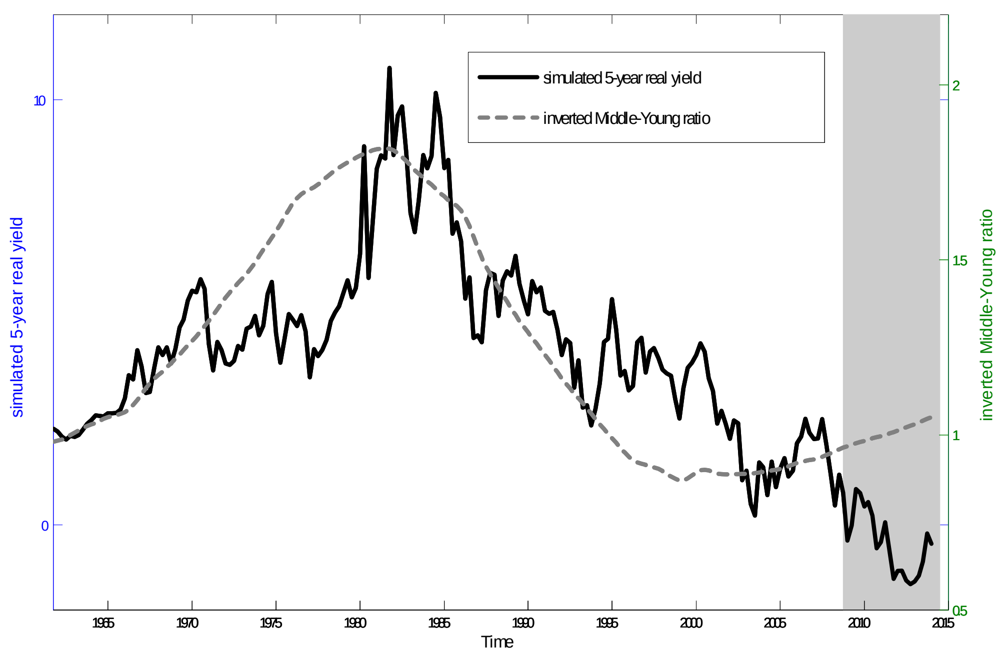


Figure 3.2. U.S. ex-ante real long rate (5-year) and MY (inverted, right-scale).

Note: The grey shaded area covers from the beginning of the first round of quantitative easing (2008Q3) to the end of the sample. Quarterly sample 1961Q1-2013Q4.

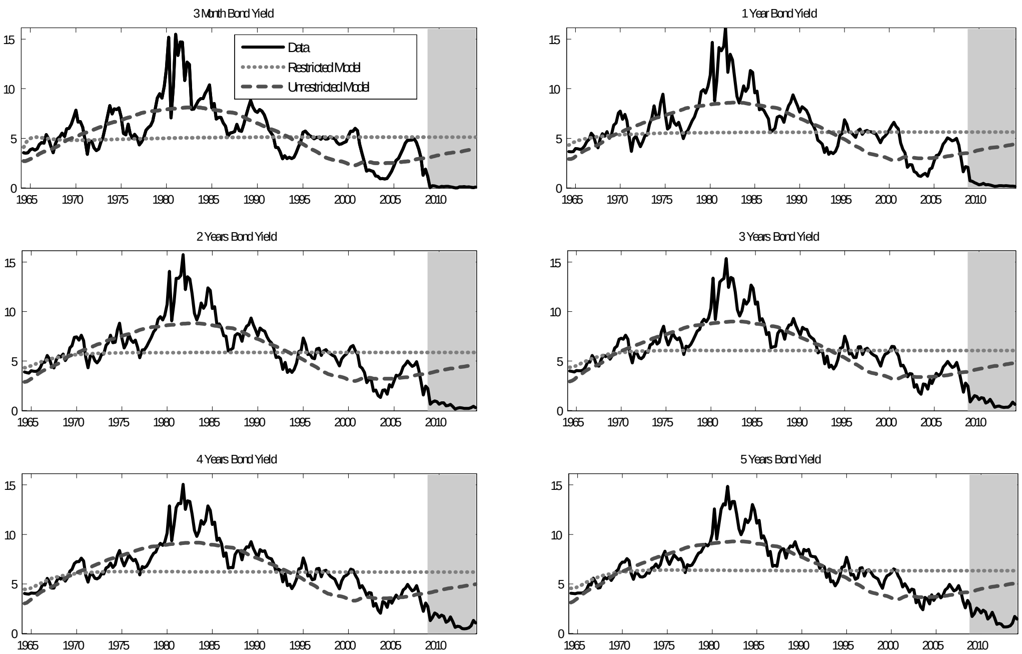


Figure 4. Dynamic Simulations.

Notes. This figure plots the time series of bond yields (maturity: 3m, 1y, 2y, 3y, 4y, 5y) along with those dynamically simulated series from the benchmark Macro ATSM (dashed light grey line) and Demographic ATSM (solid dark grey line). The affine models with time-varying risk premia are estimated over the full sample and dynamically solved from the first observation onward. The grey shaded area covers from the beginning of the first round of quantitative easing (2008Q3) to the end of the sample. Quarterly sample 1964Q1-2013Q4.

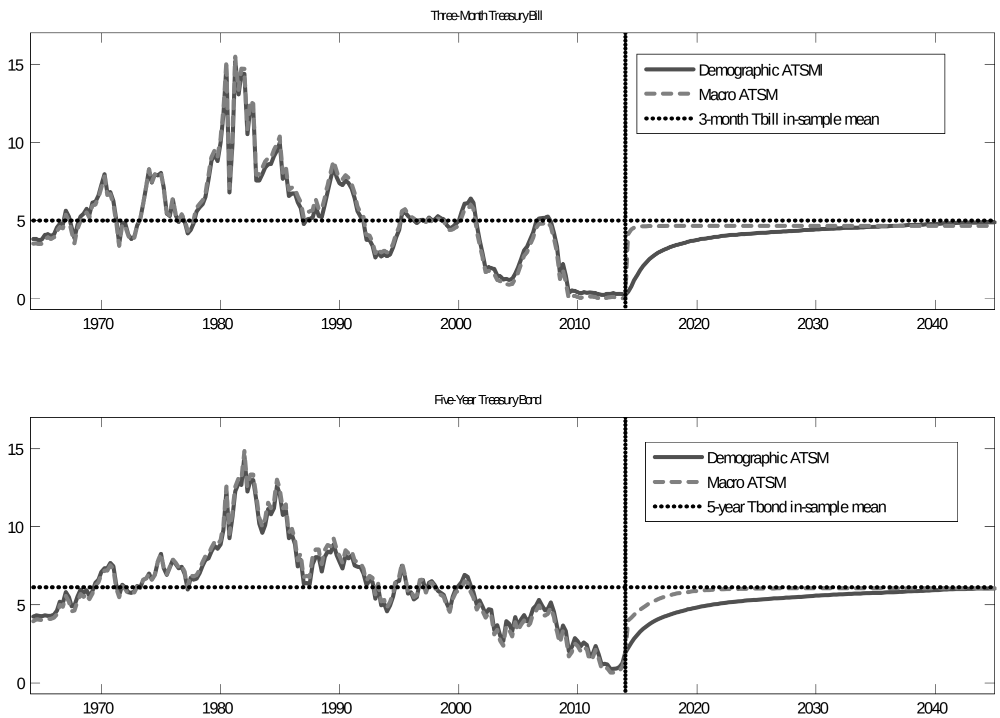


Figure 5. In-sample fitted values and dynamically simulated out-of-sample predictions.

Notes. This figure plots the in-sample estimated values (1964Q1-2013Q4) and out-of-sample predictions (2014Q1-2045Q4) of 3-month (reported in the upper panel) and 5-year (reported in the lower panel) yields. The Demographic ATSM (solid dark grey lines) and Macro ATSM (dashed light grey lines) are estimated over the whole sample 1964Q1-2013Q4. Using the estimated model parameters, models are solved dynamically forward starting from 1964Q1. The black dash lines are in-sample mean of associated yields, and the vertical dash line shows the end of in-sample estimation period. Quarterly sample 1964Q1-2013Q4.

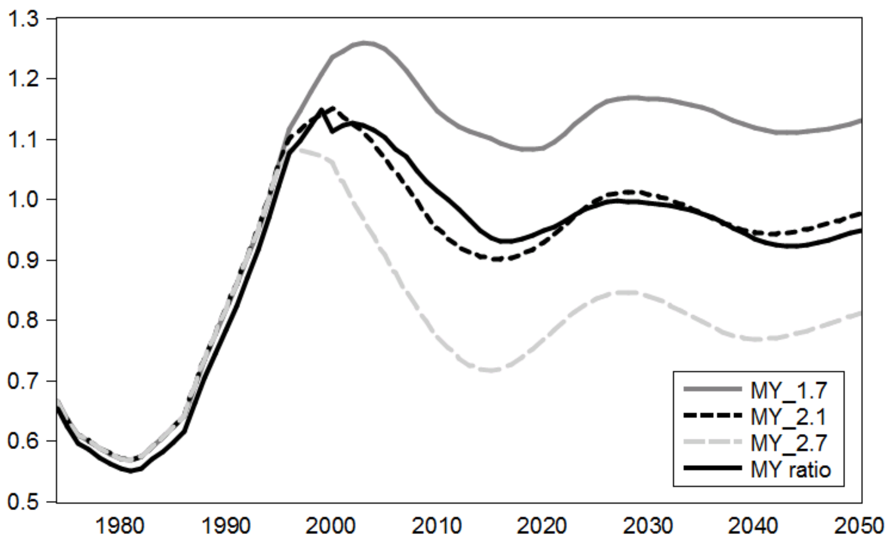


Figure 6.1. MY projections and fertility rates.

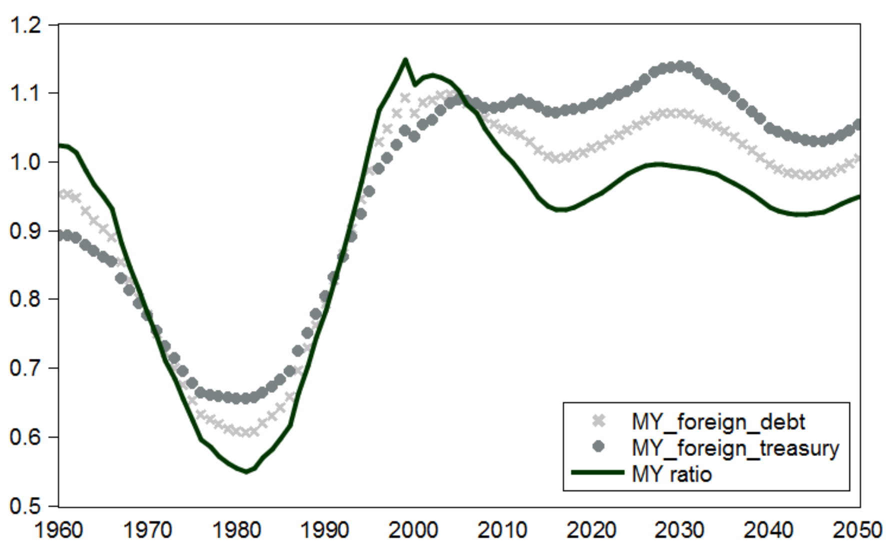


Figure 6.2. MY projections and foreign holdings.

Notes. This figure plots the middle-aged young (MY) ratio and its long run projections based on alternative scenarios for the fertility rate and foreign holdings. The MY ratio (solid black line) is based on annual reports of BoC while MY_1.7 (solid grey line), MY_2.1 (dashed black line) and MY_2.7 (dashed grey line) in Panel A are predicted in 1975 under 1.7, 2.1 and 2.7 fertility rates, respectively. All the projection information in Panel A is from BoC's 1975 population estimation and projections report. Panel B projections are based on authors' calculation from New York Fed's report on foreign portfolio holdings of U.S. Securities (April 2013).

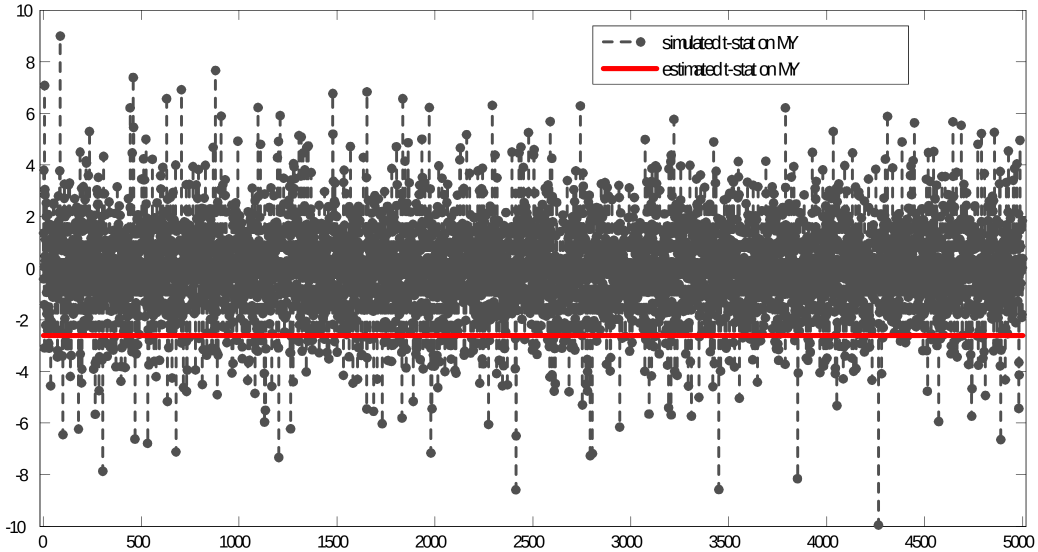


Figure 7.1. Simulated vs. estimated t-statistics, norminal 3-month yield.

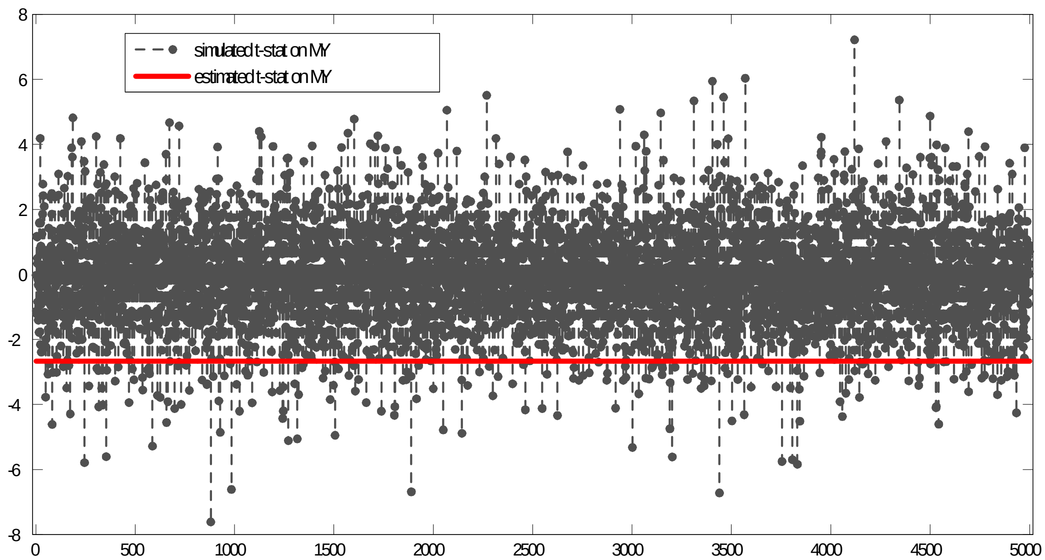


Figure 7.1. Simulated vs. estimated t-statistics, real 3-month yield.

Notes. This figure shows simulated t-statistics on MY ratio which is obtained from an autoregressive model where the dependent variable is an artificial series bootstrapped (5000 simulations) from an autoregressive model for both nominal and real 3-month rate. The estimated t-statistics is the observed value of the t-statistics on MY ratio in an autoregressive model for the actual nominal or real 3-month rate augmented with MY ratio.

## APPENDIX A. Derivation of Demographic ATSM

We consider the following model specification for pricing bonds with macro and demographic factors:

$$\begin{aligned}
y_t^{(n)} &= -\frac{1}{n}\left(A_n + B_n' X_t + \Gamma_n \mathrm{MY}_t^n\right) + \varepsilon_{t,t+1} \qquad \varepsilon_{t,t+n} \sim N(0, \sigma_n^2) \\
X_t &= \mu + \Phi X_{t-1} + \nu_t \qquad \nu_t \sim i.i.d.N(0, \Omega) \\
y_t^{(1/4)} &= \delta_0 + \delta_1' X_t + \delta_2 \mathrm{MY}_t \\
\Lambda_t &= \lambda_0 + \lambda_1 X_t \\
m_{t+1} &= \exp(-y_t^{(1)} - \frac{1}{2}\Lambda_t' \Omega \Lambda_t - \Lambda_t' v_{t+1}) \\
P_t^{(n)} &\equiv \left[\frac{1}{1 + Y_{t,t+n}}\right]^n, \; y_t^{(n)} \equiv \ln\left(1 + Y_t^{(n)}\right) \\
\Gamma_n \mathrm{MY}_t^n &\equiv \left[\gamma_0^n, \gamma_1^n \cdots, \gamma_{n-1}^n\right] \begin{bmatrix} \mathrm{MY}_t \\ \mathrm{MY}_{t+1} \\ \vdots \\ \mathrm{MY}_{t+n-1} \end{bmatrix} \quad X_t = \begin{bmatrix} f_t^{\pi} \\ f_t^{x} \\ f_t^{u,1} \\ f_t^{u,2} \\ f_t^{u,3} \end{bmatrix}
\end{aligned}$$

Bond prices can be recursively computed as:

$$\begin{aligned}
P_t^{(n)} &= E_t[m_{t+1} P_{t+1}^{(n-1)}] = E_t[m_{t+1} m_{t+2} P_{t+2}^{(n-2)}] \\
&= E_t[m_{t+1} m_{t+2} \cdots m_{t+n} P_{t+n}^{(0)}] \\
&= E_t[m_{t+1} m_{t+2} \cdots m_{t+n} 1] \\
&= E_t[\exp(\sum_{i=0}^{n-1}(-y_{t+i,t+i+1} - \frac{1}{2}\Lambda_{t+i}' \Omega \Lambda_{t+i} - \Lambda_{t+i}' \nu_{t+i+1}))] \\
&= E_t[\exp(A_n + B_n' X_t + \Gamma_n' \mathrm{MY}_t^n)] \\
&= E_t[\exp(-n y_t^{(n)})] \\
&= E_t^Q[\exp(-\sum_{i=0}^{n-1} y_{t+i}^{(1)})]
\end{aligned}$$

where $E_t^Q$ denotes the expectation under the risk-neutral probability measure, under which the dynamics of the state vector $X_t$ are characterized by the risk neutral vector of constants $\mu^Q$ and by the autoregressive matrix $\Phi^Q$

$$\mu^Q = \mu - \Omega \lambda_0 \text{ and } \Phi^Q = \Phi - \Omega \lambda_1$$

To derive the coefficients of the model, let us start with $n = 1$:

$$P_t^{(1)} = \exp(-y_t^{(1/4)}) = \exp(-\delta_0 - \delta_1' X_t - \delta_2 \mathrm{MY}_t)$$

$A_1 = -\delta_0$, $B_1 = -\delta_1$ and $\Gamma_1 = \gamma_0^1 = -\delta_2$, Then for $n+1$,we have $P_t^{(n+1)} = E_t[m_{t+1}P_{t+1}^{(n)}]$

$$= E_t[\exp(-y_t^{(1/4)} - \frac{1}{2}\Lambda_t'\Omega\Lambda_t - \Lambda_t'\nu_{t+1})\exp(A_n + B_n'X_{t+1} + \Gamma_n \mathrm{MY}_{t+1}^n)]$$
$$= \exp(-y_t^{(1/4)} - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n)E_t[\exp(-\Lambda_t'\nu_{t+1} + B_n'X_{t+1} + \Gamma_n \mathrm{MY}_{t+1}^n)]$$
$$= \exp(-y_t^{(1/4)} - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n + \Gamma_n \mathrm{MY}_{t+1}^n)E_t[\exp(-\Lambda_t'\nu_{t+1} + B_n'(\mu + \Phi X_t + \nu_{t+1}))]$$
$$= \exp[-\delta_0 - \delta_1'X_t - \delta_2 MY_t - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n + \Gamma_n \mathrm{MY}_{t+1}^n + B_n'(\mu + \Phi X_t)]E_t[\exp(-\Lambda_t'\nu_{t+1} + B_n'\nu_{t+1})]$$
$$= \exp[-\delta_0 - \delta_1'X_t - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n - \delta_2 \mathrm{MY}_t + B_n'(\mu + \Phi X_t)$$
$$+ \Gamma_n \mathrm{MY}_{t+1}^n]\exp\{E_t[(-\Lambda_t' + B_n')\nu_{t+1}] + \frac{1}{2}var[(-\Lambda_t' + B_n')\nu_{t+1}]\}$$
$$= \exp[-\delta_0 - \delta_1'X_t - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n + B_n'(\mu + \Phi X_t)$$
$$+ \left[-\delta_2, \gamma_0^n,\ \gamma_1^n \cdots, \gamma_{n-1}^n\right] \mathrm{MY}_t^{n+1}]\exp\{\frac{1}{2}var[(-\Lambda_t' + B_n')\nu_{t+1}]\}$$

To simplify the notation we define $[-\delta_2, \Gamma_n] \equiv \left[-\delta_2, \gamma_0^n,\ \gamma_1^n \cdots, \gamma_{n-1}^n\right]$

$$= \exp\left\{-\delta_0 - \delta_1'X_t - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n + B_n'(\mu + \Phi X_t) + [-\delta_2, \Gamma_n]\mathrm{MY}_t^{n+1}\right\}$$
$$\exp\left\{\frac{1}{2}E_t[(-\Lambda_t' + B_n')\nu_{t+1}\nu_{t+1}^{'}(-\Lambda_t + B_n)]\right\}$$
$$= \exp\left\{-\delta_0 - \delta_1'X_t - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n + B_n'(\mu + \Phi X_t) + [-\delta_2, \Gamma_n]\mathrm{MY}_t^{n+1}\right\}$$
$$\exp\left\{\frac{1}{2}[\Lambda_t'\Omega\Lambda_t - 2B_n'\Omega\Lambda_t + B_n'\Omega B_n)]\right\}$$
$$= \exp\left\{-\delta_0 + A_n + B_n'\mu + (B_n'\Phi - \delta_1')X_t - B_n'\Omega\Lambda_t + \frac{1}{2}B_n'\Omega B_n + [-\delta_2, C_n]MY_t^{n+1}\right\}$$
$$= \exp\left\{-\delta_0 + A_n + B_n'\mu + (B_n'\Phi - \delta_1')X_t - B_n'\Omega(\lambda_0 + \lambda_1 X_t) + \frac{1}{2}B_n'\Omega B_n + [-\delta_2, \Gamma_n]\mathrm{MY}_t^{n+1}\right\}$$
$$= \exp\left\{A_1 + A_n + B_n'(\mu - \Omega\lambda_0) + \frac{1}{2}B_n'\Omega B_n + (B_n'\Phi - B_n'\Omega\lambda_1 + B_1')X_t + [-\delta_2, \Gamma_n]\mathrm{MY}_t^{n+1}\right\}$$

Then we can find the coefficients following the difference equations

$$A_{n+1} = A_1 + A_n + B_n'(\mu - \Omega\lambda_0) + \frac{1}{2}B_n'\Omega B_n$$
$$B_{n+1}' = B_n'\Phi - B_n'\Omega\lambda_1 + B_1'$$
$$\Gamma_{n+1} = [-\delta_2, \Gamma_n]$$

## APPENDIX B: Data Description

**Demographic Variables:** The U.S. annual population estimates series are collected from U.S. Census Bureau and the sample covers estimates from 1900-2050. Middle-aged to young ratio, $\mathrm{MY}_t$ is calculated as the ratio of the age group 40-49 to age group 20-29. Past $\mathrm{MY}_t$ projections for the period 1950-2013 are hand-collected from various past Census reports available at http://www.census.gov/prod/www/abs/p25.html. MY projections under different fertility rates are based on BoC's 1975 population estimation and projections report.

**Spot rate**: 3-Month Treasury Bill rate is taken from Goyal and Welch (2008) extended collecting data from St. Louis FRED database.

**Bond yields:** Bond yields are collected from Gurkaynak, Wright and Sack (2007) dataset, end of month data.

**Core Inflation**: Time-series of core inflation are collected from St. Louis FRED database.

**International data**: International bond yields are collected from Global Financial Data up to 2011. Benchmark bond yield is the 10-year constant maturity government bond yields. For Finland and Japan, shorter maturity bonds, 5-year and 7-year, respectively, are used, since a longer time-series is available. International $\mathrm{MY}_t$ estimates for the period 1960-2008 are from World Bank Population estimates and projections from 2009-2050 are collected from International database (US Census Bureau).

**Macro factors**: Stationary output and inflation factors are constructed following the data appendix of Ludvigson and Ng (2009). Data series of Group 1 (output) and Group 7 (prices) are extended up to 2013Q4 using data from Bureau of Economic Analysis (BEA) and St. Louis FRED databases.