Model and Survey Estimates of the Term Structure of US Macroeconomic Uncertainty.

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Abstract

Survey data on macro-forecasters suggests their assessments of future output growth and inflation uncertainty are too high. We find that model estimates of the term structure of *ex ante* or perceived macro uncertainty are more in line with *ex post* RMSE measures than the survey respondents' perceptions. At shorter horizons the models' assessments of the uncertainty characterising the outlook is lower than that indicated by the survey data histograms, and closer to the RMSE estimates. Recent developments in econometric modelling ensure that the models' information sets line up with the timing of information available to the survey respondents, thus enabling a fair comparison.

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1 Introduction

The effects of macroeconomic uncertainty on economic activity has long been of interest to economists, including whether surprises in uncertainty cause declines in output, or vice versa.[1] It is common to measure general uncertainty about the macroeconomic outlook using option-implied volatility estimates from stock market or exchange rate data, or survey-based data on the dispersion of forecasts or on consumer confidence.[2] Rather than attempting to measure general macroeconomic uncertainty our interest is in uncertainty more narrowly defined: uncertainty about the future course of inflation, and uncertainty about future output growth. This is because direct estimates of inflation and output growth uncertainty are provided by survey respondents' reported histograms, and our aim is to compare survey measures of uncertainty with model-based estimates.[3]

Recently, Rossi and Sekhposyan (2015) proposed a measure of macroeconomic uncertainty based on comparing the realized forecast error to the historical distribution of the past forecast errors made by the US Survey of Professional Forecasters (SPF) respondents. Their measure is *ex post* in the sense that the realization of the variable (output growth) is required for the computation. The SPF also provides respondents' forecast distributions of the annual rate of output growth and the inflation rate, in the form of histograms. These histograms can be employed to compute survey estimates of *ex ante* uncertainty. Clements (2014a) computes these *ex ante* uncertainty measures from survey forecasts, and compares them with measures based on past forecast errors (see, e.g., Reifschneider and Tulip, 2007 and Knüppel, 2014), which are typically expressed in terms of Root Mean Squared Error (RMSE). He finds that *ex ante* uncertainty exceeds RMSE 'realized uncertainty' for both inflation and output growth at within-year horizons.

In this paper, we aim to better understand the mismatch between the *ex ante* and *ex post* survey estimates of uncertainty. A natural question to ask is whether the mismatch would have arisen had the SPF respondents based their probability assessments and point predictions on macroeconomic forecasting models. To this end, we estimate *ex ante* and *ex post* uncertainty using models that could in principle have been used by the respondents, in the sense that the models are real-time and use only information available at the times that the corresponding survey forecasts were made. For the model estimates to shed light on the mismatch between the *ex ante* and *ex post* survey estimates, we need the model forecasts to be close to the survey forecasts in terms of their forecast accuracy (i.e., *ex post* uncertainty). This leads us to consider MIDAS models, so that the information set used by the model is similar to that available to the survey respondents in terms of timeliness. We

[1] For example, Carroll (1996) considers the effects of uncertainty about labour income on households spending decisions, and Dixit and Pindyck (1994) and Bloom (2009) consider the effects on firms and their investment plans.

[2] Bloom (2009, Table 1, p.629) shows that stock market volatility is correlated with cross-sectional measures of uncertainty: the cross-sectional standard deviation of firms' pre-tax profit growth; a cross-sectional stock-return measure; the cross-sectional spread of industry productivity growth; and the dispersion of the Livingstone half-yearly survey forecasts of GDP.

[3] Of course the survey respondents may well base their forecasts on models, so the distinction is between mechanical model-based forecasts and forecasts which make use of model(s) and judgment to varying degrees.

find that MIDAS models are close to being as accurate as the survey forecasts at short horizons, if not at all longer horizons. We then consider whether the models' *ex ante* forecasts of uncertainty are more closely attuned with RMSE estimates.[4]

The comparison of model and survey forecasts is made in terms of the term structure of uncertainty, that is, how uncertainty is resolved as the forecast horizon shortens. The forecasts underlying the survey uncertainty estimates are fixed-event (see, e.g., Nordhaus, 1987, Clements, 1995), that is, repeated forecasts made at different origins of a given target (the year-on-year calendar growth rate of output or prices in a particular year). This characteristic of the survey data determines the nature of the uncertainty estimates required from the models for a fair comparison. The importance of data timeliness when comparing survey and model forecasts has been stressed by Faust and Wright (2009), *inter alia*, and motivates the use of mixed-frequency forecasting models. Such models can be setup to draw on data up to the point in time at which the corresponding survey return was made, so that the model and survey information sets are closely aligned in the time dimension. The models' outputs are carefully designed to match the quantities which can be calculated from the survey responses. For example, the survey measures of forecast uncertainty relate to calendar-year annual inflation and output growth made at horizons of (approximately) one-quarter up to eight quarters ahead. We show how estimates of these quantities can be obtained from the forecasting models' outputs. In addition, the models are specified and estimated using the data which would have been available in real time, to match the surveys which are by their nature real time. That is, we use only vintages or maturities of data that would have been available at the point in time each forecast was made (see, e.g., Croushore, 2011a, 2011b on real-time data analysis).

In calculating the term structure, we average over forecast origins, so that time variation in the uncertainty levels ought to largely cancel out. As a consequence, our benchmark model estimates the term structure of calendar-year output growth and inflation uncertainty without modelling time-varying heteroscedasticity, but including both monthly and daily predictors to match the model information sets with those available to the survey respondents. As a robustness check, we also consider the model proposed by Pettenuzzo, Timmermann and Valkanov (2015) that incorporates time-varying heteroscedasticity in models with mixed-frequency data.

Notice that the RMSEs are unconditional measures, in that they capture average performance (for a given horizon). *Ex ante* uncertainty is instead a conditional notion, as it measures uncertainty at each point in time. However, the averaging of the *ex ante* estimates over time - to generate the term structure of uncertainty - results in estimates which are essentially unconditional, and so are comparable to the RMSE estimates in this respect. The *ex ante* assessments would be expected to be broadly in line with the RMSEs if they are well calibrated.

Finally, one of our underlying assumptions for many of the calculations is that survey forecasters

[4]In the context of assessing DSGE model forecasts, Herbst and Schorfheide (2012) similarly assess whether the realized RMSEs are commensurate with what would be expected given the DSGE model's predictive distribution.

are targeting an early-vintage release of GDP growth or inflation, such as the official estimate released shortly after the reference quarter: specifically, two quarters later. This is common practice in the real-time forecasting literature, because it seems reasonable to assume that the three rounds of annual revisions and the occasional benchmark revisions which are known to occur will result in changes which are largely unpredictable (see, e.g., Landefeld, Seskin and Fraumeni, 2008 and Fixler, Greenaway-McGrevy and Grimm, 2014 on the Bureau of Economic Analysis data revisions releases). Moreover, rankings between competing forecasting models may not be sensitive to the vintage used for the actual values, and the best forecasting model for predicting early-release data may remain the best for predicting fully-revised data, albeit that all the models' forecasting performances would be expected to deteriorate. However, the choice of early-release versus fully-revised data is shown to be less benign for comparisons of *ex ante* uncertainty and RMSE.

To anticipate our main finding, our model *ex ante* measures are markedly less than the survey *ex ante* estimates at within-year horizons, and are in fact less than the model and survey RMSE estimates at one and two quarter horizons. Had the survey respondents used such a model to generate *ex ante* uncertainty estimates they would have tended to under-estimate *ex post* uncertainty at the shortest two horizons.

Our paper is related to Patton and Timmermann (2011), who estimate the degree of predictability of state variables over different horizons by fitting unobserved component models to survey forecasts of annual GDP growth and inflation. We instead use reported histogram forecasts to estimate uncertainty, and compare these with model estimates of *ex ante* uncertainty and RMSEs.

The plan of the rest of the paper is as follows. Section 2 outlines how we compute the survey data estimates and describes the available data, as this determines the nature of the forecast uncertainty estimates we require from the models. Our survey-based measures are constructed in such a way as to mitigate the tendency towards over-dispersion of the aggregate SPF histogram, and the inflation of the survey *ex ante* measure. Section 3 describes the models, how we obtain measures of uncertainty from them which are comparable to the survey estimates, and presents the baseline estimates. A number of extensions to the models are also considered, including allowing for stochastic volatility and for longer-term changes in volatility, such as the onset of the Great Moderation (see, e.g., McConnell and Perez-Quiros, 2000). Section 4 concludes.

2 Measuring survey uncertainty

Direct estimates of forecast uncertainty are provided by the standard deviations (or variances) of the survey respondents' reported histograms. From the individual respondents' histograms, an aggregate or consensus histogram can be obtained by simple equal-weighting. A consensus *ex ante* uncertainty measure could be calculated as the standard deviation of the aggregate histogram (denoted by σ_h^{agg}), or as the average of the individuals' histogram standard deviations (denoted by $\overline{\sigma}_{h,ea}$). Gneiting and Ranjan (2013, section 3.1) show that if the individual distributions are probabilistically calibrated (see, e.g., Dawid, 1984 or Gneiting and Ranjan, 2013), then the aggregate distribution will be over-dispersed. In our setting, this implies that even if the *ex ante* and *ex post* uncertainty were equal for each individual, the consensus *ex ante* uncertainty, defined as the standard deviation of the aggregate distribution, would exceed the root mean squared error (RMSE) of the consensus point forecast (defined as the mean of the individuals' point forecasts), suggesting under-confidence (*ex ante* uncertainty in excess of *ex post*). Consequently, we take the average of the individual histogram standard deviation estimates as the survey *ex ante* uncertainty measure, EAU ($\overline{\sigma}_{h,ea}$). This is defined as:

$$\overline{\sigma}_{h,ea} = N^{-1} \sum_n \left(N_{n,h}^{-1} \sum_i \sigma_{i,n|n-h} \right) \tag{1}$$

where $\sigma_{i,n|n-h}$ is the estimated standard deviation for respondent i of a histogram forecast for annual growth (inflation) in calendar year n, where n is in years, made h-quarters ahead ($h = 1, \ldots, 8$).[5] Hence we average over individuals (i) and forecast targets (n), where $N_{n,h}$ is the number of forecasters of target n (for a horizon h), and N is the number of years. We calculate the standard deviations of the individual histograms by first fitting normal distributions when probability is assigned to three or more intervals (following Giordani and Söderlind, 2003), and fitting triangular distributions for one and two interval histograms (as explained in Engelberg, Manski and Williams (2009, p.37-8)). The results were qualitatively unchanged if we fitted generalized beta distributions instead of normal distributions, as in Clements (2014a). We also calculated standard deviations directly from the histograms.[6]

For purposes of comparison, we also calculate σ_h^{agg}:

$$\sigma_h^{agg} = N^{-1} \sum_n \sigma_{n|n-h} \tag{2}$$

where $\sigma_{n|n-h}$ denotes the standard deviation of the aggregate histogram for year n at a horizon h (again based on fitting a normal distribution).

In addition to estimating average EAU using the surveys, we can obtain an *ex post* measure from the forecast errors once the actual values become known. Note that the *ex ante* estimates ((1) and (2)) are made in advance at time $n-h$, whereas the RMSE offers an *ex post* assessment of how uncertain the outcomes were given the forecasts. The first estimate of 'realized' uncertainty is

[5] In principle a comparison of the *ex ante* and *ex post* measures for each individual might be fairer, but to obtain a single survey measure we use the average of the individual *ex ante* measures as the survey measure, on the grounds that this relates to the 'average' forecaster.

[6] Letting x_s denote the midpoint of interval s, with probability p_s, the mean is $\overline{x} = \sum_s x_s p_s$, and the variance is $\sum_s (x_s - \overline{x})^2 p_s - w^2/12$, where w is the interval length and the last term is Sheppard's correction, commonly applied when variances are calculated from histograms.

the sample standard deviation of the consensus forecast errors at horizon h ($h = 1, \ldots, 8$), namely:

$$\hat{\sigma}_{h,ep} = \sqrt{N^{-1} \sum\nolimits_n \left(e_{n|n-h} - e_h \right)^2} \tag{3}$$

where $e_{n|n-h} = z_n - z_{n|n-h}$, $e_h = N^{-1} \sum_t e_{n|n-h}$, and $z_{n|n-h}$ is the cross-section median of the individual point[7] forecasts. Here $z_{n|n-h}$ refers to the forecast of the annual growth rate in calendar *year* n made h *quarters* earlier. The second is the consensus RMSE given by:

$$RMSE_{h,ep} = \sqrt{N^{-1} \sum\nolimits_n e_{n|n-h}^2}. \tag{4}$$

It might be argued that the consensus forecast will under-estimate the uncertainty faced by the average individual, i.e., the average individual RMSE. This would be the case if individual survey respondents have access to private information, and the consensus forecasts were markedly more accurate than those of an individual forecaster. Hence in comparing *ex ante* measures such as $\overline{\sigma}_{h,ea}$, say, with past *consensus* forecast error RMSEs it might be argued we are not comparing like with like. Hence we also report average individual RMSEs, and forecast error standard deviations (following Reifschneider and Tulip, 2007). In terms of choosing between the RMSE and the standard deviation of the forecast errors, our preferred measure is the RMSE. The standard deviation is included to see the size of the (squared) bias: generally this is relatively small.

In order to facilitate the comparison of the model predictions to the survey forecasts of the calendar-year growth rates, it will prove useful to approximate the annual growth rate by a weighted average of quarter-on-quarter growth rates. Denoting by y_t the quarterly growth rate, i.e., $y_t = 100 \ln(Y_t/Y_{t-1})$ where Y is the quarterly level (the price deflator, or level of output), then the annual growth rate, z_t, can be written as $z_t = \sum_{j=0}^{6} w_j y_{t-j}$ where $w_j = \frac{j+1}{4}$ for $0 \leq j \leq 3$, and $w_j = \frac{7-j}{4}$ for $4 \leq j \leq 6$. z_t is a quarterly variable and approximates the calendar-year growth rate when t refers to a fourth quarter of the year.

Hence for a $h = 1$ quarterly horizon forecast of z_t, the values $\{y_{t-1}, \ldots, y_{t-6}\}$ are known, so that the variance of z_t conditional on this information set is $Var\left(z_t \mid \mathcal{I}_{t-1}\right) = Var\left(w_0 y_t \mid \mathcal{I}_{t-1}\right)$. Alternatively, for a $h = 7$ horizon forecast, for example, all the quarters (y_t to y_{t-6}) need to be forecast: $Var\left(z_t \mid \mathcal{I}_{t-7}\right) = Var\left(\sum_{j=0}^{6} w_j y_{t-j} \mid \mathcal{I}_{t-7}\right)$.

For the survey forecasters, we are able to evaluate the accuracy of this approximation for the *ex post* uncertainty measures. This provides some indication as to the likely accuracy of the approximation for the model forecasts. As an example, consider an $h = 2$ forecast (i.e., a forecast

[7] An alternative would be to use the cross-section median of the means of the individual histograms. The choice between the two may appear innocuous, but Engelberg *et al.* (2009) and Clements (2010, 2014b) show that the SPF respondents' point predictions and measures of central tendency derived from the histograms are not always consistent one with another. The use of the histogram means gives rise to marked increases in the $\hat{\sigma}_{h,ep}$ and RMSE measures at $h = 1$ for both output growth and inflation.

made in response to a survey in the third quarter of the target year). The exact method of calculating the calendar year growth rate forecasts adds the (point) forecasts of the current and next quarter levels to the data for the first two quarters of the year, and calculates the percentage change relative to the previous year. The approximation weights the point forecasts of (100 times) the quarterly log differences for the current and next quarter with actual quarterly log differences (for the previous five quarters). In both cases, forecast errors are calculated using as actual values the percentage changes in the annual levels for year n (average/sum of the values of the four quarters in the year n) and year $n-1$.

Our survey data is from the US Survey of Professional Forecasters (SPF) as it spans a long historical period compared to other surveys which provide similar information.[8] It is a quarterly survey of macroeconomic forecasters of the US economy from 1968 to the present day, initially administered by the American Statistical Association (ASA) and the National Bureau of Economic Research (NBER) as the ASA-NBER Survey, and since June 1990 by the Philadelphia Fed as the SPF: see Zarnowitz (1969), Zarnowitz and Braun (1993) and Croushore (1993).

The SPF provides respondents' forecast distributions of the annual rate of output growth and the inflation rate, in the form of histograms. The histograms refer to the annual change from the previous year to the year of the survey, as well as of the survey year to the following year. As an example, consider the histogram forecasts of the annual output growth in 2005. We have eight histograms of this target, beginning with a forecast made in the first quarter of 2004 (a forecast of the following year relative to the survey quarter year), and ending with a forecast made in the fourth quarter of 2005 (a forecast of the survey quarter year on the previous year). We term these 8 to 1 step ahead forecasts. The histograms are reported by the middle of the middle month of the quarter, so this defines the cut off point for information for the models.

Hence we will need to construct forecasts of the uncertainty from the models that relate to the annual growth rate made at horizons of 1 to 8 quarters in advance. It is the form of the survey uncertainty estimates - namely, that they relate to the annual growth in a calendar year - that dictates the form of the model-based estimates we calculate in section 3.

2.1 Empirical Results

These results are based on the $N = 32$ annual targets from 1983 to 2014, and forecast horizons, $h = 1, \ldots, 8$. The real-time data on GDP and the GDP deflator is taken from the Real Time Data Set for Macroeconomists (RTDSM) maintained by the Federal Reserve Bank of Philadelphia (see Croushore and Stark, 2001).

The results in Table 1 are broadly in line with Clements (2014a). From Table 1 it is apparent that for both variables EAU declines as h declines but remains high relative to RMSE, regardless

[8] The Bank of England Survey of External Forecasters provides similar information for the UK, but only from beginning in 1996, and since 1999 the ECB Survey of Professional Forecasters (SPF) covers the euro area.

of how EAU is calculated, and regardless of whether the comparison is to the RMSE or the forecast error standard deviation. There are differences between variables. For output growth, the *ex ante* measures are less than the *ex post* for horizons in excess of a year, whereas for inflation the two are more closely aligned at these longer horizons. Hence the survey forecasters over-state the uncertainty about the outlook for inflation and output growth at within-year horizons.

We present formal tests of the equality of the EAU and RMSE estimates (last column of the table), specifically between the EAU - $\overline{\sigma}_{h,ea}$ - and the RMSE. The p-values indicate that the EAU and RMSE are statistically different for both variables at short horizons.

As expected, $\overline{\sigma}_{h,ea}$ lies below σ_h^{agg}, but qualitatively our findings are unaffected whichever of the two is used. Further, it is not the case that the use of the consensus forecast errors to calculate the RMSE is the reason for the RMSE lying below the *ex ante* measure at within-year horizons. If we use the average of the individual respondents' forecast standard deviations, or RMSEs (denoted by $\overline{\hat{\sigma}}_{i,ep}$ and $\overline{RMSE_i}$ respectively, in the table) the overall picture is unchanged. Particularly for inflation, the average of the respondents' RMSEs is higher than the RMSE of the consensus forecast errors, but the *ex ante* measures are markedly higher still.

We find that using the quarterly growth rate approximation to the annual growth rate for the survey forecasts for the two *ex post* measures yields virtually identical estimates in most cases (see the last two columns of the table). It appears unlikely that the use of this approximation to facilitate the calculation of both *ex ante* and *ex post* measures for the model forecasts will matter, although this remains a conjecture.

2.2 Are survey forecasters targeting true values?

The survey respondents assign probabilities to inflation and output growth falling in certain intervals. Hitherto we have assumed they are implicitly targeting an early release of the outcome variable, such as the quarterly vintage value released two quarters after the reference quarter. It may be that the 'true value' is being targeted, which we measure by a recent estimate of the data point (here, the 2016:Q3 data vintage). Relative to targeting an early release, the reported histograms would incorporate additional uncertainty to reflect the cumulative effect of the uncertain data revisions between the early release and the final release.

Table 2 compares the survey *ex ante* measure (the average of the individual standard deviations from table 1) with the survey RMSE assuming the forecasters target the final release, so that the RMSE is calculated from final-vintage actuals (we also report the early-release actuals RMSE for ease of comparison). Note that the histograms and hence survey *ex ante* uncertainty are unchanged: all that has changed is the interpretation, and hence the correct *ex post* comparator is the RMSE calculated by comparing the point forecasts to the final-vintage data.

For output growth, *ex ante* uncertainty falls between the two RMSE measures (one using second-release actuals, the other final actuals) for the shortest 3 horizons, but for inflation, *ex ante* uncertainty exceeds the higher RMSE measure for within-year horizons. These findings suggest that the assumption that forecasters target true values is not able to explain the apparent under-confidence in forecasting inflation. But under-confidence would turn into over-confidence for the within-year output growth forecasts if it were the case that forecasters' uncertainty assessments are of true values.

3 Models for inflation and output growth uncertainty

We consider Mixed Data Sampling (MIDAS) regressions to exploit monthly and daily information in addition to the past quarterly values of the series being forecast.[9] Our baseline models do not permit time-variation in the model's forecast-error variance. This might be expected to be of secondary importance for estimating the term structure. However, some recent MIDAS models do allow second moment dynamics (see, e.g., Pettenuzzo *et al.*, 2015), and in section 3.4 we contrast the results using such models to those obtained using the model discussed in this section.

MIDAS models allow us to exploit the information content of monthly and daily data when computing model-based uncertainty estimates. MIDAS models have been used by a number of authors to exploit daily and monthly data, including Ghysels and Wright (2009), Andreou, Ghysels and Kourtellos (2013) and Clements and Galvão (2008, 2017). The choice of monthly explanatory variables is guided by economic calendars such as Bloomberg[10], which describes a set of data releases identified as 'market moving'. Most of these data releases refer to monthly measures of economic activity, such as industrial production, nonfarm payroll (employment), PMI (purchasing managers index), retail sales, and housing activity. We elect to use the monthly predictors labeled as 'market moving' which are related to economic activity and which are available as real-time data vintages from 1982. The variables are listed in Table 3. Of these, only PMI is not subject to revisions. We also include daily equity index (SP500) returns. This variable has been shown to incorporate the effect of macroeconomic news during the first month of the quarter (Gilbert, Scotti, Strasser and Vega, 2015), and to have predictive ability for future output growth (Andreou *et al.*, 2013) and output growth data revisions (Clements and Galvão, 2017).

We use real-time data throughout, to match the intrinsically real-time nature of the surveys. Given that both the deflator and output are revised over time, this means using the then available data vintages. The timing of the surveys is such that advance estimates of the previous quarter values of output and inflation are known. To be precise, consider an $h = 1$ survey forecast. This is made in the middle of the fourth quarter of the year, when the advance estimates of the national

[9] Andreou, Ghysels and Kourtellos (2011) and Foroni and Marcellino (2013) provide recent reviews of MIDAS models. See also Ghysels, Santa-Clara and Valkanov (2006), Ghysels, Sinko and Valkanov (2007) and Clements and Galvão (2008).

[10] http://www.bloomberg.com/markets/economic-calendar/

accounts for the third quarter have been issued. In our notation, this implies the model is estimated on data through $n-h$ (the third quarter of the year) from the $n+1-h$ quarterly vintage (the Q4 vintage). So for the first forecast target (the annual rate of inflation in 1983), for $h=1$, we estimate the model on quarterly data up to and including 1983:Q3 from the data vintage available at 1983:Q4.

3.1 MIDAS Specification

Our target variable is the calendar-year growth rate z_n. At time $n-h$, a number of quarterly values of y_t will need to be forecasts. To be able to use the same forecasting model to compute both a point forecast $\hat{z}_{n|n-h}$ and an EAU measure $var^{ea}(z_{n|n-h})$, we estimate a MIDAS model for q_t^h, defined below as the weighted 'future' quarters for a forecast horizon h:

$$\begin{aligned} q_t^h &= \sum_{i=0}^{h-1} \frac{1+i}{4} y_{t-i} \text{ for } h = 1, ..., 4 \\ q_t^h &= \sum_{i=0}^{3} \frac{1+i}{4} y_{t-i} + \sum_{i=4}^{h} \frac{7-i}{4} y_{t-i} \text{ for } h = 5, 6 \\ q_t^h &= z_t \text{ for } h \geq 7. \end{aligned} \tag{5}$$

As before, y_t is the quarterly growth rate, and z_t is the annual growth rate computed quarterly[11] and equal to the calendar-year value when t is the last quarter of a year. The q_t^h is the LHS variable in the MIDAS models described below. For a given horizon h, (5) gives the quarters that need to be forecast, weighted by their importance in approximating z_t. For $h=7$, for example, there are no relevant quarterly growth rate data at $t-7$ required to compute z_t so $q_t^7 = z_t$.

To explain the MIDAS models we introduce some notation. Let x_t^M denote a variable available at the monthly frequency, where x_t^M is the last month in quarter t, $x_{t-1/m}^M$ the penultimate month in quarter t, etc., and $m^M = 3$ (the number of months in a quarter). Typically data for the first month of the survey quarter will be in the agents' information set, because respondents file their forecasts around the middle of the middle month of the quarter. This is indicated in our models by a one month 'lead', i.e., $l^M = 1/3$ (one month). As a illustration, consider a model with a single monthly indicator variable and lags of the growth rate of the quarterly dependent variable:

$$\begin{aligned} q_t^h &= \beta_0 + \beta_Q \sum_{i=0}^{p-1} w(\theta_Q, i) y_{t-h-i} \\ &\quad + \beta_M \sum_{i=0}^{p^M-1} w(\theta_M, i) x_{t-h-(i/3)+l^M}^M + \varepsilon_t^h, \end{aligned} \tag{6}$$

[11] Recall $z_t = \sum_{j=0}^{6} w_j y_{t-j}$ where $w_j = \frac{j+1}{4}$ for $0 \leq j \leq 3$ and $w_j = \frac{7-j}{4}$ for $4 \leq j \leq 6$.

where we may use $p^M = p \times m^M$, where p is the maximum number of lags in quarters of the quarterly dependent variable. In the empirical application, we use $p = 8$ and estimate weighting functions for both the lags on the quarterly dependent variable and for the monthly indicator. As an example, suppose $l^M = 1/3$ and $h = 1$. Then (6) implies the use of data on the first month of the quarter t ($x_{t-2/3}$), as this would have been available to the survey respondents who file their returns by the middle of the middle month of quarter t.

For the lag weighting functions we use the beta function:

$$\begin{aligned} w(\theta; i) &= \frac{f(\boldsymbol{\theta}; i)}{\sum_{j=1}^{K} f(\boldsymbol{\theta}; j)} \\ f(\boldsymbol{\theta}; i) &= \frac{(k)^{\theta_1 - 1}(1-k)^{\theta_2 - 1}\Gamma(\theta_1 + \theta_2)}{\Gamma(\theta_1)\Gamma(\theta_2)}; \quad k = i/(K+1). \end{aligned}$$

In the case of quarterly lags, $K = p$, and in the case of monthly lags $K = pm^M$.

The model with both monthly and daily indicators is:

$$\begin{aligned} q_t^h &= \beta_0 + \beta_Q \sum_{i=0}^{p-1} w(\theta_Q, i) y_{t-h-i} \\ & \quad \beta_M \sum_{i=0}^{p^M-1} w(\theta_M, i) x^M_{t-h-i/3+l^M} \\ & \quad + \beta_D \sum_{i=0}^{m^D-1} w(\theta_D, i) x^D_{t-h-i/60+l^D} + \varepsilon_t^h, \end{aligned} \tag{7}$$

where $m^D = 60$ (approximately number of business days in a quarter), so we use one quarter of daily data. Because there is no delay on the release of financial data, and forecasts are computed in the middle of the quarter, we use $l^D = 20/60 = 1/3$ (where 20 is the approximate number of business days in a month). This implies the use of daily information on the first month. (The convention is that x_t^D is the last day of quarter t. Therefore when $h = 1$, $i = 0$ and $l^D = 1/3$, $x^D_{t-h-i/60+l^D} = x^D_{t-2/3}$, indicating the last day of the first month of quarter t).

To incorporate the information of the set of monthly indicators in Table 3, we substitute x_t^M by the factor f_t^M in equation (7). The factor f_t^M is obtained by principal components using the five monthly series (see, e.g., Marcellino and Schumacher (2010)). Before the estimation of the factor, we transformed observed monthly levels to monthly quarterly growth rates at annual rates: $x_t^M = 400(\log(X_t^M) - \log(X^M_{t-m^M}))$, where X_t^M is the variable in levels. In the case of daily data, we apply a similar transformation to the original daily values in levels, namely $x_t^D = 400(\log(X_t^D) - \log(X^D_{t-m^D}))$.

MIDAS models are estimated by nonlinear least squares using a numerical optimization algorithm and a grid search for the initial values of the parameters of the weighting functions.

3.1.1 MIDAS EAU

The model EAU estimates are the estimated standard errors of the models for a given h, computed as described in the Appendix to take into account parameter uncertainty. Empirically, the contribution of parameter uncertainty is small: it raises the EAU measure by 5 to 10% depending on the forecasting horizon. Notice however that no allowance is made for the estimation error associated with the extraction of the factors.

3.1.2 MIDAS RMSE

The calculation of RMSE requires point forecasts of the calendar-year growth rates z_n, for 1983 to 2014. Using the forecasts, $q^h_{n|n-h}$, of the unknown quarterly components, q^h_n, we compute the forecasts of the calendar-year growth rates, $z_{n|n-h}$, as:

$$z_{n|n-h} = q^h_{n|n-h} + \sum_{j=h}^{6} w_j y_{n-j}, \tag{8}$$

for $h = 1, ..., 6$ and as before $w_j = \frac{j+1}{4}$ for $0 \leq j \leq 3$, and $w_j = \frac{7-j}{4}$ for $4 \leq j \leq 6$. The RMSE is the square root of the mean of the squared forecast errors over the $N = 32$ observations of the out-of-sample period for each h, i.e., $\sqrt{N^{-1} \sum_{n=1}^{N} \left(z_n - z_{n|n-h}\right)^2}$.

3.2 Benchmark MIDAS Empirical Results

As previously mentioned, we estimate the MIDAS regressions using the vintages that would have been available to the survey respondents at each point in time. Details of the data sources for the monthly predictors in the MIDAS models are given in table 3. As in the survey evaluation, these results are based on the $N = 32$ annual targets from 1983 to 2014, and forecast horizons, $h = 1, \ldots, 8$.

Table 4 presents results for the MIDAS model in (7) but substituting x^M_t by the monthly factor f^M_t. The one-quarter-ahead MIDAS model EAU estimates are around a quarter and a sixth of the survey estimates, for output growth and inflation. By way of contrast, the one-quarter-ahead RMSE model estimates are similar to the survey values for both variables, and we do not reject the null of equal accuracy using conventional significance levels. The results of the tests for whether the model EAU and the model RMSE are equal in Table 4 suggest that we are unable to reject the null at the 5% level, including at $h = 1$ (in stark contrast to the results for surveys in Table 1).

The model reverses the relative magnitudes of EAU and RMSE at short horizons compared to the findings for the surveys. In a sense, the finding that the model fares less well out-of-sample compared to the in-sample fit is unsurprising: out-of-sample performance has often been championed as the gold standard for model evaluation because of the tendency for in-sample fit to prove an unreliable guide to out-of-sample performance (see, e.g., Clements and Hendry, 2005 for a discussion). The key finding here is that survey respondents over-estimate forecasting uncertainty at short horizons, while the same is not true for a real-time mixed frequency model that exploits the information of monthly and daily indicators.

As a check on our use of a rolling estimation window, the last two columns of Table 4 show the effect of using an expanding window of data (i.e., a recursive forecasting scheme). We report the ratio of the model EAU and RMSE estimates from the recursive and rolling schemes. The results show that the RMSEs are largely unaffected at $h = 1$ for both variables, but that the EAU estimates are increased by around 10-20% for all h when the recursive scheme is used. This is consistent with the rolling scheme offering a degree of adaptability to the lower levels of economy-wide volatility following the Great Moderation (see, e.g., McConnell and Perez-Quiros, 2000), whereas using the recursive scheme keeps the earlier observations. Section 3.5 reports on an alternative to rolling window estimation designed to capture abrupt changes in volatility.

3.3 Benchmark Model - Robustness Checks

Table 5 checks the robustness of the results to the specification of the number of lags at the different frequencies. In the benchmark model, we set $p = 8$ quarterly lags, 24 monthly lags of the factor, and 60 daily lags (3 months of daily data). Because of the nature of the MIDAS model, large numbers of lags can be accommodated without the number of parameters to be estimated increasing. Hence little attention tends to be paid to the choice of the maximum number of lags. As a robustness check, we set $p = 4$ quarterly lags, and use 12 monthly lags and one month of daily data. The results in Table 5 indicate the results are largely unchanged in key respects. We still find that at the short horizons the model EAU are markedly smaller than the survey EAU.

The second check replaces q_t^h in equation (7) by the annual growth rate, $z_t = 100\left[\left(\sum_{i=0}^{3} Y_{t-i}\right) / \left(\sum_{i=4}^{7} Y_{t-i}\right)\right.$ $\sum_{j=0}^{6} w_j y_{t-j}$, where Y_t is the quarterly GDP (or the GDP deflator) in levels, and y_t are quarterly growth rates, and otherwise keeps the same specification of the right-hand-side regressors as in the benchmark model ($p = 8, m^D = 60$). As argued above, q_t^h only includes the future quarterly growth rates unknown at $t - h$, weighted according to the approximation described in section 2, whereas z_t does not vary with h, that is, the quarterly information already available. Table 6 records the results, which again are largely unchanged from the results using q_t^h. However, if we reduce the maximum lag orders, as in Table 5, the RMSEs and EAU estimates increase markedly (not shown to save space). This suggests using z_t with long lags yields reasonable results, but this approach is not as robust as using q_t^h.

3.4 MIDAS models and stochastic volatility

In this section we employ a Bayesian approach to estimate MIDAS models, allowing for stochastic volatility. The approach closely follows Pettenuzzo *et al.* (2015). A Bayesian approach allows more complicated models to be estimated, including models in which the variance equation is allowed to depend on high-frequency data. By and large these extensions and refinements do not qualitatively affect our findings for the term structure relative to the simple MIDAS, although as shown by Pettenuzzo *et al.* (2015), would likely produce more accurate time-series predictive densities.

The Bayesian estimation and subsequent extensions are facilitated by a small change in the MIDAS model given in equation (7). Instead of using a beta function to constrain the coefficients on the high-frequency lags, an Almon function is used instead, namely:

$$w(\theta, i) = \sum_{i=1}^{q} \theta_i k^i,$$

where q is the polynomial order. The main advantage of this function is that it is linear on the parameters θ_i, simplifying the Bayesian algorithm employed for estimation. We define vectors of monthly and daily lags:

$$\begin{aligned} X_{t-h}^{(M)} &= [x_{t-(3h/m^M)+l^M}^M, ..., x_{t-(3h/m^M)-(p^M-1)/m^M+l^M}^M]' \\ X_{t-h}^{(D)} &= [x_{t-(60h/m^D)+l^D}^D, ..., x_{t-(60h/m^D)-(m^D-1)/m^D+l^D}^D]' \end{aligned}$$

and corresponding 'Q matrices' as in Pettenuzzo *et al.* (2015, eqn. 6), that is, a $(q+1) \times p^M$ matrix denoted by Q^M and a $(q+1) \times m^D$ denoted by Q^D. Q^M and Q^D contain no unknowns. If we define $\widetilde{X}_t^{(M)} = Q^M X_t^{(M)}$ and $\widetilde{X}_t^{(D)} = Q^D X_t^{(D)}$, then the MIDAS regression can be written as:

$$q_t^h = \beta_0 + \sum_{i=0}^{p-1} \beta_{Q,1+i} y_{t-h-i} + \boldsymbol{\theta}'_M \widetilde{X}_{t-h}^{(M)} + \boldsymbol{\theta}'_D \widetilde{X}_{t-h}^{(D)} + \varepsilon_t^h. \tag{9}$$

and the unknown parameters $\beta_{Q,1+i}$, $i = 0, \dots, p-1$, $\boldsymbol{\theta}_M$ and $\boldsymbol{\theta}_D$ can be estimated directly. More compactly, equation (9) can be written as:

$$q_t^h = Z_{t-h}\Psi + \varepsilon_t^h \tag{10}$$

where:

$$\begin{aligned} Z_{t-h} &= [1, y_{t-h}, ..., y_{t-h-p-1}, \widetilde{X}_{t-h}^{\prime(M)}, \widetilde{X}_{t-h}^{\prime(D)}]' \\ \Psi &= [\beta_0, \beta_1, ..., \beta_q, \boldsymbol{\theta}_M, \boldsymbol{\theta}_d]. \end{aligned}$$

If we assume that $\varepsilon_t^h \sim N(0, \sigma_\varepsilon^2)$, and assume normal priors for the parameters β_i, $\boldsymbol{\theta}_M$ and $\boldsymbol{\theta}_D$, and an inverted-gamma prior for σ_ε^2, we can use a Gibbs sampler to obtain the posterior distribution of the coefficients, and the predictive density for $q^h_{n|n-h}$, that is, $q_n|Z_{n-h}$, having integrated out Ψ. Bauwens, Lubrano and Richard (2000, p.61 and p.138) show that, by using these priors, the predictive density of one-step-ahead forecasts is a Student t-distribution, with moments which have closed form solutions.[12] Following Pettenuzzo *et al.* (2015), the predictive density for $q^h_{n|n-h}$ is computed using Gibbs sampler draws (this is required for the case we allow time variation in the parameter σ_ε^2). After disregarding the initial burn-in draws from the conditional distributions, we compute the j^{th} draw from the predictive density as:

$$
\begin{aligned}
q_n^{h,(j)} &= Z_{n-h}\Psi^{(j)} + \sigma_\varepsilon^{(j)}\eta^{(j)};\ \eta^{(j)} \sim N(0,1) && (11)\\
\Psi^{(j)}|\sigma_\varepsilon^2 &\sim N(\bar{b}, \bar{V}) && (12)\\
\sigma_\varepsilon^{2(j)}|\Psi &\sim IG(\bar{s}^2/2, \bar{v}/2), && (13)
\end{aligned}
$$

where the posterior means and variances $(\bar{b}, \bar{V})$ for the conditional distribution $\Psi|\sigma_\varepsilon^2$ are computed as Pettenuzzo *et al.* (2015), equations (16) and (20). Prior means and variances are also set as in Pettenuzzo *et al.* (2015), and the value of the degrees of freedom of the inverse gamma distribution $\bar{v}$ is computed as:

$$\bar{s}^2 = s_y^2(\underline{v}^0 * (n-h)) + \sum_{t=1}^{n-h}\left(q_t^{(h)} - Z_{t-h}\Psi\right)^2$$

where s_y^2 is computed as in Pettenuzzo *et al.* (2015, eqn. (16)) and $\underline{v}^0 = 0.005$.[13] Note that we are using observations up to $n-h$ to compute the conditional draws, but the predictive density targets observation n. The *ex ante* standard error is calculated as:

$$EAU_{n|n-h} = \sqrt{\frac{1}{M}\left[\sum_{j=1}^{M}\left(q_n^{h,(j)} - \left(1/M\sum_{j=1}^{M} q_n^{h,(j)}\right)\right)^2\right]}, \qquad (14)$$

where M is the total number of draws of the predictive density obtained from the Gibbs sampler algorithm summarized in equations (11) to (13).

Relative to the MIDAS model of section 3, we have outlined a model that employs a different weighting function to deal with the dimensionality issue arising from the large number of lags, and by implementing a Gibbs sampler to compute the *ex ante* variance. This model can be extended in a number of ways. For example, we could allow the variance of the disturbances to change over

[12] The 'direct forecasting' approach is natural for MIDAS models, as otherwise the high-frequency explanatory variables would need to be modelled and forecast. Direct forecasting entails a different set of coefficient estimates for each forecasting horizon h, but that means we always have essentially 'one-step' predictive densities. This implies that we can avoid the nonlinearities that arise from iterating one-step forecasts: see Bauwens *et al.* (2000, p.138).

[13] We draw from the inverse gamma distribution by drawing first from the standard normal, and then applying the required transformations based on both parameters as described in Blake and Mumtaz (2012).

time. A MIDAS model with stochastic volatility would be:

$$\begin{aligned} q_t^h &= Z_{t-h}\Psi + \exp(\upsilon_t^h/2)\eta_t;\ \eta_t \sim N(0,1) \\ \upsilon_t^h &= \lambda_0 + \lambda_1 \upsilon_{t-h}^h + \xi_t; \xi_t \sim N(0,\sigma_\xi^2), \end{aligned} \tag{15}$$

where υ_t is the log standard deviation of the disturbances to the conditional mean at time t. Here the equation for the log standard deviation follows an AR(1) process with normal disturbances, although it is sometimes specified as a random walk, especially for forecasting inflation. The above model is a simplified version of that of Pettenuzzo *et al.* (2015), which also allows the high-frequency observables to directly impact conditional volatility υ_t. Such a model could be specified as:

$$\upsilon_t^h = \lambda_0 + \lambda_1 \upsilon_{t-h}^h + \boldsymbol{\theta}'_{M,v}\widetilde{X}_{t-h}^{(M)} + \boldsymbol{\theta}'_{D,v}\widetilde{X}_{t-h}^{(D)} + \xi_t. \tag{16}$$

As before, we rewrite the above equation more compactly as:

$$\upsilon_t^h = V_{t-h}\Lambda + \xi_t$$

where:

$$\begin{aligned} V_{t-h} &= [1, \upsilon_{t-h}, \widetilde{X}_{t-h}^{\prime(M)}, \widetilde{X}_{t-h}^{\prime(D)}]' \\ \Lambda &= [\lambda_0, \lambda_1, \boldsymbol{\theta}_{M,v}, \boldsymbol{\theta}_{d,v}]. \end{aligned}$$

Even though all the disturbances are normally distributed, the form of the predictive densities $q_{n|n-h}^{(h)}$ and $\upsilon_n^{(h)}$ is not known, but can be calculated using the Gibbs sampler.[14] Note that the required predictive density is $q_n|Z_{n-h}$, that is, we need to integrate out the effects of estimating the unobservables $\upsilon_1, ..., \upsilon_{n-h}$,(which has to be done numerically), as well as of the parameters, Λ. The j^{th} draw of the predictive density is obtained as:

$$\begin{aligned} q_n^{h,(j)} &= Z_{n-h}\Psi^{(j)} + \exp(\upsilon_n^{h,(j)}/2)\eta^{(j)};\ \eta^{(j)} \sim N(0,1) \\ \upsilon_n^{h,(j)} &= V_{n-h}^{(j)}\Lambda^{(j)} + \sigma_\xi^{(j)}\eta^{(j)} \\ \Psi^{(j)}|\sigma_\xi^2, \upsilon^{n-h}, \Lambda &\sim N(\bar{b}, \bar{V}) \text{ where } \upsilon^{n-h} = (\upsilon_1, ..., \upsilon_{n-h}) \\ \upsilon^{n-h,(j)}|\ \Psi, \Lambda, \sigma_\xi^2 &\sim \text{Mixture of Normals Algorithm} \\ \sigma_\xi^{2(j)}|\Psi, \upsilon^{n-h}, \Lambda &\sim IG(\bar{s}_\xi^2/2, \bar{v}/2) \\ \Lambda^{(j)}|\sigma_\xi^2, \upsilon^{n-h}, \Psi &\sim N(\bar{m}, \bar{V}_m) \end{aligned} \tag{17}$$

[14] We are able to compute moments of the conditional distributions $q_n|Z_{n-h}, \upsilon_n$ or even $q_n|Z_{n-h}, \upsilon_{n-h}$, but is it hard to get $q_n|Z_{n-h}$, that is, integrating out the uncertainty on unobserved volatilities.

where $\bar{s}_{\xi}^{2} = \underline{s} + \sum_{t=1}^{n-h} \left(\upsilon_t^{(h)} - Z_{t-h}\Psi \right)^2$. The posterior mean and variance $\bar{m}$ and $\bar{V}_m$ are obtained from Pettenuzzo *et al.* (2015, eqns. (31) and (32)). All priors are set as in Pettenuzzo *et al.* (2015). The algorithm to draw the time series of the unobserved log standard deviations υ_t is from Chan and Hsiao (2014) who provide a time-efficient implementation of the mixture of normals algorithm proposed by Kim, Shephard and Chib (1998).[15] We implement the Gibbs sampler by obtaining the conditional draws in the order described above.

Based on M predictive density draws of $q_n^{h,(j)}$, we can obtain the *ex ante* variance via equation (14).

Table 7 presents EAUs and RMSEs for five MIDAS specifications using 90-quarters rolling windows. Bayesian specifications are computed employing 15,000 Gibbs draws and removing the initial 1000 ($M = 14,000$). The first column reproduces the MIDAS results for the model of section 3, eqn. (7). The MIDAS model draws on both daily and monthly predictors, and is estimated by NLS with beta weighting functions. The second column shows the results for the B_MIDAS, that is, the model described in equation (9). The B_MIDAS model has Almon weighting function instead and it is estimated by Bayesian methods. The third column presents results for the B_MIDAS but with stochastic volatility as in equation (15). The last two columns present results for the Double-MIDAS specification (eqn. (16)) with just monthly data (column 4) or both monthly and daily data (column 5). All these specifications substitute x_t^M with a monthly factor f_t^M to exploit information on a set of monthly series.

It appears that the Bayesian approach (column 2) increases the estimates of EAU at horizons beyond $h = 2$ for both variables, but that this is largely offset for inflation once we consider the Double-MIDAS model. Nevertheless, the differences between the set of MIDAS models are small relative to the difference between the models' and survey estimates, and do not affect the key qualitative findings. Differences between the models' RMSEs across h are generally small as expected since the inclusion of stochastic volatility normally does not improve the accuracy of point forecasts (as, for example, in Pettenuzzo *et al.* (2015)). We should note, however, that the Bayesian MIDAS specifications are normally more accurate forecasters of inflation at long horizons than the MIDAS of section 3, reducing the already small RMSE gap between model and survey.

3.5 Modelling long-run volatility changes

Our use of both rolling estimation and stochastic volatility enables the model EAU estimates to adapt to the reduction in underlying volatility documented by McConnell and Perez-Quiros (2000) and Sensier and van Dijk (2004), *inter alia.* [16] However, for abrupt, one-off changes in volatility,

[15] We modified the matlab code available at http://people.anu.edu.au/joshua.chan/code.html for our purposes. In the equations given by (17) we have ignored the required draws of the mixture sampler described in Chan and Hsiao (2014).

[16] There are other modelling possibilities, such as the Unobserved Component Stochastic Volatility (UCSV) model of Stock and Watson (2007) applied to US inflation, which allows the variances of permanent and transitory components

approaches other than rolling windows may be preferable, and we use the testing procedure of Sensier and van Dijk (2004) to identify breaks in the conditional variance, and to estimate the date of the break. Their supWald statistic for a break in the variance is applied to the MIDAS disturbances, using p-values computed as in Hansen (1997).[17] If we find a break at the 5% significance level, we only use the observations after the break to compute the *ex ante* uncertainty measure. This strategy made little difference to the shorter-horizon estimates, and the results are not reported.

4 Conclusions

We have used MIDAS models to generate uncertainty measures that are comparable to the survey estimates in terms of the target and the forecast horizon, and are intended to closely approximate the information available to the survey respondent. The MIDAS model *ex ante* measures are markedly smaller than the survey *ex ante* estimates at short horizons, even though the RMSEs of the models and the surveys are broadly comparable at short horizons. The application of Double-MIDAS specifications, which allow for volatility changes depending on macroeconomic variables and stock returns, result in estimates which are qualitatively inkeeping with these findings.

Although other models could be used, it would be surprising if the resulting RMSE estimates were radically different, at least at the short horizons. This is because the MIDAS model's RMSEs for both output growth and inflation are 'close' to those for the survey forecasts at $h = 1$, and arguably the survey forecasts constitute an approximate upper bound on the accuracy with which a model might be expected to forecast in practice. There are a number of papers attesting to the good short-term performance of survey forecasts (see, e.g., Ang, Bekaert and Wei, 2007 and Faust and Wright, 2009).

The key anomaly is that the survey *ex ante* estimates suggest much greater uncertainty at short horizons than the model estimates (both *ex ante* and *ex post*) and the survey *ex post* values. For example, approximate 95% intervals for calendar year output growth and inflation made in the 4^{th} quarter of the year are $\pm 1\%$ of the central projection, whereas intervals based on model or survey RMSE estimates are closer to $\pm 0.6\%$ and to $\pm 0.4\%$ for output growth and inflation respectively.

A possible explanation is if we assume that survey forecasters *ex ante* uncertainty reflects the outlook for the fully-revised values of the variables, including the rounds of annual revisions and benchmark revisions. In that case, we find evidence that the survey *ex ante* uncertainty under-estimates the *ex post* uncertainty of output growth, but our assessment for inflation is unchanged. This alternative assumption on the behaviour of professional forecasters is necessarily speculative because the surveys themselves provide no information on the vintages of data the histograms refer

to change over time. Cogley, Sargent and Surico (2012) fit a model fo this type to the US for the period 1791–2011, and capture the overall decline in inflation volatility since the mid 1980s as a decline in the variance of the permanent component.

[17] We remove the first and last 15% of the observations from the grid used to search for break dates.

to.

Otherwise professional forecasters typically over-estimate the uncertainty surrounding within-year forecasts of output growth and inflation when forecasting horizons are shorter than one year. Our key empirical discovery is that models - such as the MIDAS model - which are able to match the survey forecasts accuracy (on RMSE), would suggest *ex ante* uncertainty well below the survey *ex ante* measure at short horizons. These results are robust to the use of different *ex post* measures (such as the average across individual RMSEs and standard deviations of forecast errors; the RMSE and forecast standard deviation of the consensus forecast), and to the different measures of professional forecaster *ex ante* uncertainty considered in this paper.

However, an important caveat is that the matching of the (unconditional) *ex ante* and *ex post* assessments does not necessarily coincide with improved density forecast accuracy.[18] As an example, suppose $y_t \sim N\left(0, \sigma_t^2\right)$, and either $\sigma_t^2 = 1$ or $\sigma_t^2 = 4$, both with probability $\frac{1}{2}$. A forecast density M_1: $D\left(0, 2\frac{1}{2}\right)$ is correctly calibrated unconditionally with equal *ex ante* and *ex post* uncertainty. This is not true of a second forecast density, M_2: $D\left(0, \sigma_t^2 + 0.1\right)$, but M_2 would be preferred to M_1 in terms of accuracy. Hence we conclude that the survey short-horizon forecasts are over-dispersed compared to the model estimates, and compared to RMSE estimates, but acknowledge that less dispersed forecast densities will not necessary be more accurate on score-based forecast density measures.

5 Appendix

5.1 Computation of EAU with estimated MIDAS models

Consider the generic MIDAS model, written for quarterly, monthly and daily regressors, as:

$$q_t^h = G\left(\boldsymbol{\kappa}, \mathbf{x}_{t-h}\right) + \varepsilon_t^h$$

where:

$$\mathbf{x}_{t-h} = \begin{pmatrix} y_{t-h}, ..., y_{t-h-p+1}, x^M_{t-(3h/m^M)+l^M}, \ldots, x^M_{t-(3h/m^M)-(p^M-1/m^M)+l^M}, \\ x^D_{t-(60h/m^D)+l^D}, ..., x^D_{t-(60h/m^D)-(m^D-1/m^D)+l^D} \end{pmatrix},$$

and $\boldsymbol{\kappa} = \left(\theta^Q, \theta^M, \theta^D, \beta_Q, \beta_M, \beta_D\right)$.

Then the forecast error is given by:

$$e_{n|n-h} = q_n - q_{n|n-h}$$

[18] We are grateful to an anonymous referee for making this point, and for the illustrative example we give in the text.

where $q_{n|n-h}$ is the forecast using the estimated model with observations up to $n-h$, so the forecast error is:

$$e_{n|n-h} = \varepsilon_n^h + G(\boldsymbol{\kappa}, \mathbf{x}_{n-h}) - G(\hat{\boldsymbol{\kappa}}, \mathbf{x}_{n-h}) \tag{18}$$

Using a Taylor series expansion when $\hat{\boldsymbol{\kappa}}$ is close to $\boldsymbol{\kappa}$ (assuming a reasonable sample size):

$$\begin{aligned} G(\hat{\boldsymbol{\kappa}}, \mathbf{x}_{t-h}) &\approx G(\boldsymbol{\kappa}, \mathbf{x}_{t-h}) + \frac{\partial G(\boldsymbol{\kappa}, \mathbf{x}_{t-h})}{\partial \hat{\boldsymbol{\kappa}}} (\hat{\boldsymbol{\kappa}} - \boldsymbol{\kappa}) & (19) \\ &= G(\boldsymbol{\kappa}, \mathbf{x}_{t-h}) + \mathbf{x}_{t-h}(\hat{\boldsymbol{\kappa}}) (\hat{\boldsymbol{\kappa}} - \boldsymbol{\kappa}) & (20) \end{aligned}$$

Substituting from (19) into (18) gives:

$$e_{n|n-h} = \varepsilon_n^h + \mathbf{x}_{n-h}(\hat{\boldsymbol{\kappa}}) (\hat{\boldsymbol{\kappa}} - \boldsymbol{\kappa})$$

so that

$$var\left(e_{n|n-h}\right) = \sigma_{\varepsilon,n-h}^2 + \mathbf{x}_{n-h}(\hat{\boldsymbol{\kappa}}) var(\hat{\boldsymbol{\kappa}} - \boldsymbol{\kappa}) \mathbf{x}_{n-h}(\hat{\boldsymbol{\kappa}})' \tag{21}$$

where $Var\left(\varepsilon_n^h\right) = \sigma_{\varepsilon,n-h}^2$.

So assuming an estimator $\widehat{var(\hat{\boldsymbol{\kappa}})}$, we can calculate the second term in the above expression, which is the contribution of parameter estimation uncertainty to the model's *ex ante* uncertainty. Write the full sample $(t = 1, 2, .., n-h)$ gradient as $\mathbf{x}(\hat{\boldsymbol{\kappa}}) = \partial G(\hat{\boldsymbol{\kappa}}, \mathbf{x}) / \partial \hat{\boldsymbol{\kappa}}$. Then $\widehat{var(\hat{\boldsymbol{\kappa}})}$ is computed using a sandwich variance-covariance matrix by applying the usual Newey-West formulae to the full sample gradient $\mathbf{x}(\hat{\boldsymbol{\kappa}})$ and the residuals $\hat{\boldsymbol{\varepsilon}} = \mathbf{q}^h - G(\hat{\boldsymbol{\kappa}}, \mathbf{x}_{t-h})$. We compute the required gradients using numerical derivatives.

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Table 1: Results for Survey Forecasts: *Ex Ante* and RMSE Forecast Uncertainty.

	Ex ante			*Ex Post*						Test p-values
	$\overline{\sigma}_{h,ea}$	σ_h^{agg}	$\overline{\sigma}_{h,ea,np}$	$\widehat{\sigma}_{h,ep}$	RMSE	$\overline{\widehat{\sigma}}_{i,ep}$	$\overline{RMSE_i}$	$\widehat{\sigma}_{h,ep,app}$	RMSE_{app}	$\overline{\sigma}_{h,ea} = RMSE$
h					Annual Output Growth					
8	1.00	1.15	1.09	1.54	1.55	1.51	1.65	.	.	0.14
7	0.98	1.14	1.05	1.50	1.51	1.43	1.53	.	.	0.09
6	0.94	1.05	1.02	1.31	1.31	1.42	1.49	.	.	0.15
5	0.88	0.99	0.95	1.08	1.09	1.09	1.15	.	.	0.15
4	0.83	0.94	0.90	0.83	0.86	0.79	0.89	0.87	0.87	0.79
3	0.75	0.79	0.81	0.52	0.52	0.57	0.57	0.54	0.54	0.00
2	0.63	0.65	0.69	0.37	0.37	0.39	0.39	0.37	0.37	0.00
1	0.46	0.50	0.54	0.27	0.27	0.27	0.26	0.28	0.28	0.01
h					Annual Inflation					
8	0.83	1.01	0.88	0.82	0.93	0.83	0.95	.	.	0.57
7	0.85	1.01	0.89	0.72	0.85	0.80	0.96	.	.	0.78
6	0.82	0.92	0.85	0.60	0.70	0.74	0.88	.	.	0.03
5	0.76	0.86	0.79	0.52	0.56	0.66	0.77	.	.	0.00
4	0.71	0.81	0.76	0.40	0.40	0.50	0.55	0.39	0.39	0.00
3	0.68	0.75	0.72	0.27	0.29	0.37	0.40	0.27	0.29	0.00
2	0.60	0.66	0.65	0.20	0.21	0.28	0.29	0.20	0.21	0.00
1	0.48	0.54	0.52	0.18	0.18	0.19	0.19	0.17	0.17	0.00

Notes. The estimates are based on the surveys from 1982:1 to 2014:4, of annual output growth and inflation in 1983 to 2014 (32 years). $\overline{\sigma}_{h,ea}$ is the average of the individual standard deviations. σ_h^{agg} is the standard deviation of the aggegate distribution. The standard deviations of the aggregate and individual histograms are calculated by fitting normal distributions when three or more intervals are given non-zero probability, and by fitting triangular distributions otherwise. $\overline{\sigma}_{h,ea,np}$ instead calculates the standard deviations from the individual histograms directly (without fitting normal distributions) and then takes the cross-section average. The standard deviation $\widehat{\sigma}_{h,ep}$ and the RMSE ('RMSE') use the second-release real-time data series to calculate forecast errors using the median of the point forecasts as the consensus forecast. $\overline{\widehat{\sigma}}_{i,ep}$ and $\overline{RMSE_i}$ are the averages of the individual forecast-error standard deviations and RMSEs, respectively. The two measures augmented by the subscript 'app' use the approximation involving the weighted sum of log quarterly differences to calculate the forecast of the annual percentage change. (Not available for the longer-horizon forecasts due to nature of survey). The final column records the p-values of the test of equality between the EAU ($\overline{\sigma}_{h,ea}$) and RMSE.

Table 2: Survey Estimates: Forecasting Early Release Data or True Values

	Output Growth			Inflation		
h	EAU	RMSE Early Vint.	RMSE Fully Rev.	EAU	RMSE Early Vint.	RMSE Fully Rev.
8	1.00	1.55	1.79	0.83	0.93	1.05
7	0.98	1.51	1.71	0.85	0.85	1.02
6	0.94	1.31	1.52	0.82	0.70	0.87
5	0.88	1.09	1.35	0.76	0.56	0.73
4	0.83	0.86	1.28	0.71	0.40	0.57
3	0.75	0.52	0.93	0.68	0.29	0.51
2	0.63	0.37	0.87	0.60	0.21	0.41
1	0.46	0.27	0.81	0.48	0.18	0.36

The Early Vintage RMSE is calculated using the quarterly-vintage available two quarters after the reference quarter, as in table (1). The Fully-Revised RMSE uses actual values from the 2016:Q3 data vintage.

Table 3: Monthly Indicator Variables

Variable	Description	Vintages Available	Source
Ind. Prod.	Total Industrial Production	1966:M1-2015:M6	RTDSM
Empl.	Employees on non-agricultural payrolls	1966:M1-2015:M6	RTDSM
Sales	Retail and Food Services Sales; Retail Sales before vintage 1992:M1.	1966:M1-2015:M6	ALFRED
Housing	New Privately Owned Houses Started	1968:M2-2015:M6	RTDSM
PMI	Purchasing Managers Index: ISM since 2002, but previously NAPM.	Obs: 1959:M1-2015-M6 (not subject to revisions)	Datastream

The RTDSM is maintained by the Philadelphia Fed: see Croushore and Stark (2001). ALFRED is maintained by the St Louis Fed.

Table 4: Estimates of Macro Uncertainty: MIDAS Relative to Survey Forecasts

h	Output Growth						
	EAU$_m$	EAU$_{m/s}$	RMSE$_m$	RMSE$_{m/s}$	EAU$_m$=RMSE$_m$	EAU$_m$ rec/roll	RMSE$_m$ rec/roll
8	1.91	1.90	1.64	1.06	0.18	1.08	1.16
7	1.88	1.91	1.65	1.09	0.27	1.09	1.05
6	1.72	1.84	1.53	1.17**	0.31	1.11	1.03
5	1.46	1.66	1.37	1.25**	0.39	1.12	0.93
4	1.05	1.27	0.90	1.04	0.04	1.12	1.00
3	0.63	0.84	0.64	1.23*	0.96	1.11	0.97
2	0.32	0.51	0.41	1.11	0.14	1.08	0.96
1	0.13	0.28	0.29	1.04	0.10	1.08	0.99
h	Inflation						
	EAU$_m$	EAU$_{m/s}$	RMSE$_m$	RMSE$_{m/s}$	EAU$_m$=RMSE$_m$	EAU$_m$ rec/roll	RMSE$_m$ rec/roll
8	0.99	1.19	1.13	1.21	0.72	1.19	1.27
7	0.94	1.10	1.00	1.17	0.97	1.17	1.18
6	0.83	1.01	0.79	1.13	0.51	1.19	1.21
5	0.70	0.92	0.65	1.18	0.32	1.19	1.18
4	0.54	0.76	0.55	1.40*	0.68	1.17	1.17
3	0.33	0.48	0.37	1.28**	0.64	1.17	0.98
2	0.17	0.29	0.24	1.12	0.32	1.17	1.01
1	0.07	0.15	0.18	1.03	0.27	1.11	0.99

The annual targets are from 1983 to 2014 as in Tables 1 and 2.
The 'm' subscript denotes the MIDAS model with a monthly data factor and daily equity price returns, and 's' the survey, where EAU$_s$ is $\overline{\sigma}_{h,ea}$. The MIDAS EAU incorporates a term in parameter estimation uncertainty. The MIDAS model results in the left panel are based on estimation of the model using rolling windows of 90 observations, over forecasting origins from 1982:1 to 2014:4. The model has $p = 8$ quarterly lags, 24 monthly lags, and 60 daily lags.
*, **, *** denote the rejection of the null that model is as accurate as the survey in favour of the alternative that the model is statistically worse than the survey at respectively 10%, 5% and 1% significance levels.
The column headed 'EAU$_m$ = RMSE$_m$' records the p-values of the test of equality between EAU and RMSE.
The two columns in the right panel report the results of using a recursive forecasting scheme for the MIDAS model. The model is estimated on expanding windows of data. The right panel columns give the ratio of the recursive and rolling estimates of EAU and RMSE.

Table 5: Robustness check I: MIDAS model with $p = 4$ and only a month of daily data

	Output growth					
	EAU_m	$\text{EAU}_{m/s}$	RMSE_m	$\text{RMSE}_{m/s}$	Equal RMSEs	EAU_m=RMSE_m
8	2.00	1.99	1.73	1.11	0.21	0.21
7	1.89	1.93	1.70	1.13	0.07	0.34
6	1.75	1.87	1.52	1.16	0.05	0.17
5	1.50	1.71	1.07	0.98	0.58	0.00
4	1.09	1.31	0.85	0.99	0.69	0.01
3	0.66	0.88	0.61	1.18	0.04	0.40
2	0.33	0.52	0.40	1.07	0.19	0.21
1	0.13	0.28	0.28	1.03	0.39	0.10
	Inflation					
	EAU_m	$\text{EAU}_{m/s}$	RMSE_m	$\text{RMSE}_{m/s}$	Equal RMSEs	EAU_m=RMSE_m
8	1.11	1.33	0.93	1.00	0.74	0.04
7	1.14	1.22	0.86	1.01	0.47	0.04
6	0.90	1.10	0.65	0.94	0.81	0.00
5	0.75	0.98	0.61	1.09	0.18	0.02
4	0.56	0.79	0.49	1.23	0.15	0.05
3	0.34	0.50	0.34	1.18	0.01	0.60
2	0.18	0.30	0.25	1.18	0.04	0.26
1	0.07	0.15	0.18	1.02	0.34	0.29

Relative to the benchmark MIDAS model in Table 4 with $p = 8$ quarterly lags, 24 monthly lags, and 60 daily lags, we set $p = 4$ quarterly lags, and use 12 monthly lags and one month of daily data. The last two columns are the p-values from testing the equality of RMSEs between the surveys and models, and the equality of the two model-based uncertainty measures.

Table 6: Robustness check II: Using annual growth rates as the dependent variable in the MIDAS model

					Ratios to Survey			
	Output		Inflation		Output		Inflation	
	EAU	RMSE	EAU	RMSE	EAU	RMSE	EAU	RMSE
8	1.95	1.64	1.05	1.20	1.94	1.06	1.25	1.29
7	1.93	1.66	0.99	1.06	1.96	1.10	1.16	1.24
6	1.76	1.50	0.88	0.84	1.88	1.14	1.07	1.21
5	1.51	1.33	0.75	0.69	1.72	1.22	0.98	1.24
4	1.08	0.98	0.57	0.58	1.31	1.13	0.80	1.46
3	0.66	0.65	0.35	0.40	0.89	1.25	0.52	1.39
2	0.34	0.39	0.19	0.27	0.54	1.06	0.31	1.28
1	0.15	0.27	0.09	0.20	0.34	1.00	0.18	1.11

The specification of the right-hand-side regressors is as in the benchmark MIDAS of Table 4. The left-hand-side variable is z_t.

Table 7: Alternative MIDAS models

h	MIDAS	B_MIDAS	B_MIDAS_SV	D_MIDAS (M)	D_MIDAS (M+D)
			Output Growth		
			Ex ante uncertainty		
8	1.91	2.17	2.04	1.95	2.84
7	1.88	2.10	1.96	1.97	2.37
6	1.72	1.95	1.95	2.06	2.52
5	1.46	1.69	1.64	1.80	1.98
4	1.05	1.26	1.18	1.34	1.63
3	0.63	0.76	0.67	0.69	0.71
2	0.32	0.40	0.35	0.38	0.40
1	0.13	0.16	0.13	0.14	0.14
			RMSE		
8	1.64	1.67	1.76	1.73	1.67
7	1.65	1.68	1.83	1.84	1.84
6	1.53	1.56	1.55	1.80	1.65
5	1.37	1.34	1.24	1.23	1.27
4	0.90	1.02	1.06	0.95	1.11
3	0.64	0.76	0.71	0.65	0.70
2	0.41	0.38	0.37	0.39	0.37
1	0.29	0.29	0.28	0.28	0.29
			Inflation		
			Ex ante uncertainty		
8	0.99	1.12	1.03	1.09	1.25
7	0.94	1.02	0.91	0.97	1.10
6	0.83	0.92	0.82	0.85	0.92
5	0.70	0.79	0.71	0.71	0.80
4	0.54	0.60	0.56	0.52	0.60
3	0.33	0.36	0.33	0.33	0.33
2	0.17	0.19	0.18	0.20	0.20
1	0.07	0.08	0.07	0.07	0.08
			RMSE		
8	1.13	1.15	0.94	0.89	0.98
7	1.00	0.98	0.84	0.82	0.88
6	0.79	0.79	0.67	0.64	0.66
5	0.65	0.68	0.53	0.54	0.54
4	0.55	0.54	0.50	0.46	0.51
3	0.37	0.36	0.37	0.38	0.38
2	0.24	0.25	0.25	0.24	0.25
1	0.18	0.19	0.19	0.18	0.19

The first column 'MIDAS' refers to the model in Table 4. The other models are, respectively: the Bayesian MIDAS, the Bayesian MIDAS with SV, and the 'Double' MIDAS with monthly data (higher frequency-data in the mean and variance equations), and and the 'Double' MIDAS with monthly and daily data (higher frequency-data in the mean and variance equations).