WAGE DETERMINATION WITH ASYMMETRIC INFORMATION

by

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PREFACE

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Summary

This thesis contains 5 independent chapters together with an Introduction and a General Conclusion. All five chapters consider the problem of wage determination in an economy characterized by asymmetric information. The solution which is implemented, for example a pair consisting of the wage and the level of employment, is restricted to elicit all possible relevant information. This forces some additional constraints upon the optimization problem of the agents.

Chapters 2 and 3 demonstrate that since the firm does not voluntarily share its information with other agents, the level of employment is not efficient. In both a separating and a pooling equilibrium, underemployment is the case. Note here that the equilibrium obtained changes qualitatively from Chapter 2 to Chapter 3. We return to this in the General Conclusion.

Chapter 4 elaborates upon Chapter 2. It is shown that in an otherwise competitive economy, employment and investment are lowered since they are used as signalling devices, compared to the case of symmetric and perfect information. In a model characterized by monopoly, this conclusion is no longer true. The effect upon investment is no longer unambiguous. We also return to this in the General Conclusion.

Chapters 5 and 6 consider economic policy in the case of a separating, respectively, pooling equilibrium. It is shown that in the case of a separating equilibrium, taxation can improve upon the situation. For a pooling equilibrium we show the existence of multipliers.

General for these models is that the introduction of asymmetric information certainly does have an effect, but also that the results are possibly non-robust to assumptions with respect to the market form.

CHAPTER 1

INTRODUCTION

I. General Introduction.

Traditionally microeconomics, or perhaps more precisely value theory, has been characterized by rigorous models describing the interaction amongst atomistic agents: producers and consumers. The behavior of these agents are derived from primitives such as preferences and production possibilities (endowments and technology) and the analysis provides a coherent view of the economy. The conclusions reached in classical value theory are based upon assumptions of maximizing behavior, maximization taking place under idealised conditions such as for example fully flexible prices, perfect coordination, perfect information, absence of externalities, just to draw attention to a few of the simplifying assumptions made. This very general class of models is characterized in particular by the full employment of resources. Emphasis is upon the determination of relative prices and hence the allocation of scarce resources.

In contrast to this, traditional macroeconomic theory focuses upon the level of utilization of resources - perhaps especially the level of as well as the dynamics of employment - and the general level of prices (Branson (1979)). It is by now generally accepted, and has been for some time, that this traditional approach to macroeconomic issues has been proved to perform in an unsatisfactory way. Theoretically the traditional models are poorly founded. The essential flaw in traditional Keynesian macroeconomic theory is the absence of a consistent foundation based upon the choice theoretic framework of microeconomic theory. This was forcefully demonstrated by Friedman (1968) and Lucas (1976). Empirically Keynesian theory performs badly, especially the inflationary tendencies experienced during the sixties and seventies indicated the importance of expectations (cf. Laidler (1982)) and the supply shocks demonstrated the fallacy of relying on models completely demand determined (Klein (1978)).

This once established distinction between microeconomics as the study of the allocation of scarce resources and macroeconomics as the study of the level and dynamics of economic aggregates has been recognized to be unduly restrictive, perhaps due to the poor performance of macroeeconomics since the sixties. One of the consequences of this is the increased interaction between micro- and macroeconomic theory. This interaction has proved successful since replacing some of the very strict assumptions traditionally adhered to in microeconomics by weaker assumptions, studies of fix-price economics (Dreze (1975)), temporary equilibrium models (Hicks (1939), Grandmont (1977)), uncertainty and imperfect information (Radner (1968)), contracts (Hart and Holmstrom (1987)) and moral hazard (Prescott and Townsend (1981)) have been allowed for within the general equilibrium construct. All of these developments, in addition to being major achievements in economic theory, serve to illustrate that broadly interpreted microeconomic theory has an important role in macroeconomic theory. The insistence on optimizing behavior on the part of the agents is now as common to macroeconomics as it is a central feature of classical value theory. The question not yet resolved in macroeconomic theory is whether competitive equilibrium suffices to explain basic macroeconomic facts or whether it is the case that deviations from the competitive equilibrium paradigm is necessary (Blanchard and Fischer (1990), Kydland and Prescott (1990)). It is not controversial that imperfections are present. It is, however, controversial that imperfections are important at the aggregate level. It will be argued below that with respect to the labour markets in the Nordic countries, the institutional setting allow for an analysis emphasizing imperfections relative to the perfectly competitive paradigm.

The theme of this thesis is the determination of the wage and the level of employment in economies characterized by asymmetric information. Since the focus is upon the level of employment the theme of the thesis can be said to be rooted in macroeconomic theory. However, as the process of wage determination is explicitly considered, and considered in a model firmly embedding the idea of optimizing agents, this thesis is also firmly rooted in microeconomic theory. Thus, although the models presented in the following are not general equilibrium models, the method used in addressing a macroeconomic issue is by now widely accepted as a method involing the core ideas of microeconomic theory (cf. Blanchard and Fisher (1990)). Also, note in passing, that the insistence upon asymmetric information places this thesis within the branch of literature which attempts to explain macroeconomic phenomena by deviations from a fully competitive paradigm. Let us now turn attention to the questions which we attempt to address in this thesis.

Consider two agents, a producer of goods and a supplier of labour, who are engaged in a bargain over the wage which is to dictate the exchange of income for labour. The questions addressed here all evolve around the restriction on such a bargaining outcome which may arise due to the presence of asymmetric information. Information is asymmetric in the sense that the producer is better informed about the value of the marginal product of labour compared to the supplier of labour. We can offer two reasons why asymmetric information can arise in the relationship between an employer and an employee. First, if employees change their job by going from one employer to another, they may be badly informed about the characteristics of the new firm, as to, for example, the level of demand, price of raw materials, production function. Such a source of asymmetric information structures, despite being obviously present, appears to be relatively unimportant. In the long run agents will learn about these characteristics. Also, employees already with the firm may have this information and share it with newcomers. A second source of asymmetric information arises if it is accepted that the economy is inherently stochastic. It is fully conceivable that both demand and cost are subject to some stochastic innovation. For example the level of demand for a product of a particular firm may be state dependent. It is not unrealistic to argue that this is not so much of a problem to the firm who directly observes demand, whereas to the employees this poses a problem since the level of demand is observed only indirectly by the demand for labour.

At a general level a study emphasizing asymmetric information appears to be interesting since compared to the classical theory of value, additional restrictions upon the feasible set of allocations are added. The feasible set in the classical theory of value is unambiguously determined by technological constraints jointly with endowments. Under asymmetric information additional restrictions arise due to the revelation principle (Myerson (1979), Laffont (1980), Chapter 1, Radner (1982)). To be more precise as to the question which we want to address here: let a bargain between an employer and an employee take place under symmetric information. This bargain dictates one pair of nominal wage and level of employment from the set of feasible pairs of nominal wage and employment. What is addressed in this thesis is whether or not the presence of asymmetric information enforces any restrictions upon this set, so that for any given wage the level of employment is lowered relatively to the situation under symmetric information. The idea behind such an approach is that that with asymmetric information the actual actions of the agents also have the role of eliciting information (on this, see Radner (1982), Laffont & Maskin (1982)). For example, assume that the value of the marginal product of labour is state dependent and that only the employer is informed about the state. The outcome of the bargain between the employer and the employee is dependent upon the announcement of which state has actually occurred. This announcement is made by the better informed agent, i.e., the employer. To implement a bargaining solution, that is for the solution to elicit all relevant information and thus confirm the announcement to the lesser informed party, it may well be that the level of employment serves as a signal as to the realized state of nature (for a relevant discussion, see Townsend (1987), Section 6). It is these problems which are studied here.

The questions as we phrased them above emphasize the allocative role of the wage. Consider the model of the classical value theory. In that model a price system exists which allocates the scarce amount of resources to achieve an equilibrium which is Pareto efficient. In the current context the important aspect of the price system is that prices in a very precise meaning elicit all relevant information. Consider the following simple situation. Let a consumer divide his income between two goods. It is trivial that in a competitive environment the optimal choice depends upon the relative price between the two goods. In general equilibrium it is also the case that the relative price of these two goods reflects that production is efficient. Thus, once the consumer observes the price, he is observing the technology and he realizes that he can do no better. This is also so for the producer. Observing market prices he is observing the preferences of the consumers and he realizes that he can do no better. Consequently, in equilibrium all agents know that no gains can be made by adhering to a different strategy. This, of course, also applies to the labour market equilibrium. Thus, one aspect of the general equilibrium is that the resources of the individual employee are divided efficiently between working hours and leisure. In this sense the classical value theory supports full employment. Such a notion of full employment, that is employment related to efficiency considerations, is alien to traditional

Keynesian theory (see for example the textbook by Branson (1979)). This is not so, however, for newer approaches to macroeconomics since these are based upon an explicitly choice theoretic framework (see for example Mankiw (1989)). The questions asked in this thesis and the methods used in the attempt to provide an answer, clearly place our approach within the latter of these two traditions. Thus, in the current study of labour market equilibrium efficiency considerations have an explicit role to play. Thus, concern is not so much for the level of employment, but rather for the level of employment compared to a first best situation. Here the first best is identified with the case of symmetric information.

The main motivation for the current study is to be found within the realms of macroeconomic theory, not so much as to the method of analysis but as to the questions asked. Since it became apparent that the basic Keynesian paradigm was not appropriate for the understanding of the experiences of the seventies and onwards, two major traditions have developed within macroeconomic theory.

The stage for one of these two traditions is based upon the work of Friedman (1968) and Lucas (1976). The central thesis is that business cycles are best understood within neoclassical theory. In this respect, note that the classical general equilibrium construct can be extended to a stochastic setting (Kydland and Prescott (1990)). This attempt to understand business cycles is summarised by the real business cycle model (Plosser (1989), Kydland and Prescott (1990). The simplest real business cycle model is the neoclassical model of consumption and investment (Ramsey (1928), Allais (1947), Samuelson (1958), Diamond (1965)). This model is explicitly dynamic and can thus be concerned with changes in the levels, and if the otherwise static model is extended to embed productivity shocks, it will generate fluctuations which are claimed to resemble observed facts (arguments in favour of this are given in Plosser (1989), arguments against are given in Mankiw (1989) and Pagan).

The second major tradition emphasizes the necessity of deviating from the competitive paradigm in the attempt to understand business cycles. The basic postulate is not that real business cycle models are irrelevant, but that they do not suffice to explain what is observed. Mostly the models found within this tradition are partial in nature and focus attention upon one market at a time and they are primarily static (a good example of this is the recent textbook by Blanchard & Fischer (1989), Chapters 8 and 9). Thus, advances have been made within the price setting behavior of firms under monopolistic competition. Problems of moral hazard and adverse selection are introduced to models of the financial sector. These approaches have illustrated that despite the presence of optimizing agents, the economy may perform inefficiently. Also, attention has increasingly been focused upon the labour market. The theory of implicit contracts and search is discussed in relation to macroeconomic theory by Frank (1986) and the insider-outsider theory is well established (Lindbeck & Snower (1989)) with an eye to macroeconomics.

There seems to be good reason to be concerned with the labour market. This is one of the markets which by casual observation appear to diverge the most from the competitive assumption of atomistic non-strategic agents. This is confirmed for the Nordic countries where union density (union members as percent of the total labour force) is in Sweden 90, in Finland 80, in Denmark 75 and in Norway 60 (all figures are approximate, see Calmfors (1989)). Hence, in this respect it is fully justified to analyse the implications of labour market imperfections. However, a high union density is not a sufficient reason for claiming that thorough deviations from the competitive framework have occurred. The organization as well as the behavior of unions must be taken into account. Most models of trade union behavior support a level of employment lower than the competitive (see Oswald (1985) for a survey). Also empirical support is found for the fact that a high union density is important. Based on comparative studies of the OECD countries (Calmfors & Driffill (1988), Freeman (1988)) it is safe to conclude that in countries with a high union density bargaining will certainly have an effect upon macroeconomic performance (see also Calmfors (1989)).

In the literature concerned with bargaining and macroeconomic performance a distinction is often made between centralized and decentralized bargaining (Calmfors & Driffill (1988), Freeman (1988)). It is normally assumed that bargaining is a centralized process in the sense that one all encompassing union bargains with one representative firm over wages or profits. This is the idea behind for example the literature deriving time consistent policies for a government facing an active trade union. The current study is presumably best interpreted as a theoretical investigation into the effects of decentralized bargaining with asymmetric information. That is, the bargain takes place between a specific firm and the employees of this firm. That such a study is warranted derives from the fact that firstly centralized and decentralized bargaining may well coexist, and indeed does so at least in the Nordic countries. These are the countries normally claimed to have the most centralized bargaining process (Freeman (1988), Calmfors (1989)). In fact, approximately half of the increases in the money wage rate for the Nordic countries are accounted for by such local settlements (Flanagan (1988). Secondly, in nearly all countries the tendency in recent years has been towards more decentralization (Elvander (1988), Calmfors (1989)). In conclusion, it is potentially interesting also in a macroeconomic context to study the implications of local bargains between employers and employees as will be done here.

Also the issues addressed in this thesis are potentially interesting also from a microeconomic point of view. Recently, several successful attempts have been made to model the labour market so as to escape some of the traditional conclusions of microeconomic theory. These include the theory of implicit contracts, search, efficiency wages and the insider-outsider theory. The current study can be seen as yet another attempted contribution to modelling the economics of the labour market.

In particular, a comparison with the literature on implicit contracts provides in its own right a motivation for this study. Abandoning the assumption that workers are passive will reverse the conclusion with respect to the level of employment obtained in the contracts literature. In Chapter 2 and in the first section of Chapter 3 it is shown that a separating equilibrium is characterized by underemployment. In contrast, the more realistic of the implicit contracts models support overemployment.

The main difference between the models adhered to in this thesis and those of the implicit contracts literature is to be found in the description of the labour market. The latter approaches to the labour market assumes that workers are passive and let firms offer a contract subject to the constraint that it must secure the reservation level of utility to workers. In this thesis workers are assumed to participate actively in the determination of the wage.

In most chapters (2, 3, 4, 6) the monopoly union model is applied. That is, the union alone determines the wage. However, Chapter 5, being only an example, suggests that as long as the union has a bargaining strength strictly bounded from below at zero, the level of employment is inefficiently low.

Finally, since the emphasis in this thesis is on the implications of asymmetric information, the analysis can be seen as an illustration and exemplification of the rather abstract exposition of the diverse ways in which imperfect information affects the allocation of resources given by Laffont (1980).

Although not a topic of this thesis, note here that the results presented in the following chapters suggest that the effects of asymmetric information may depend upon the market form. This is so qualitatively (that is, the nature of the equilibrium may change (see Chapter 3)) as well as quantitatively (that is, the effect upon the quantities may change (see Chapter 4)).

II. This Thesis.

II.a. Methodological considerations.

The aim of this section is to give the argument supporting unemployment; a theme which runs throughout this thesis. Furthermore, it is discussed whether one should interpret the models presented here so as to allow command-like equilibrium or whether a decentralized equilibrium should be applied. This is done with reference to the theory of incentives. Finally, we discuss the limitations to the results due to the choice of specific functional forms.

In this thesis an analysis of wage and employment determination under asymmetric information is set forth. In all of the models analysed in the following the wage is determined by a bargain between a firm and a trade union. The decision with respect to the level of employment is left to the firm, thus the model is a right to manage model (Andrews & Nickel (1983)). The novel aspect of these models is that it is assumed that the firm is better informed with respect to a parameter which (jointly with other variables) determines the value of the bargain. Let this parameter be denoted Ξ and let the non-vanishing distribution of Ξ be denoted by $\Psi(\cdot)$. The support is $[\underline{\Xi},\bar{\Xi}]$.

The information structure can then be described as follows. Let the actual realization of Ξ be drawn from $\Psi(\cdot)$ which jointly with the support is known by the firm as well as the union. This ensures that the subjective and the objective distributions coincide, i.e., the equilibrium which is studied is in full agreement with the rationality of expectations. The firm has superior information compared to the union assuming that the firm has direct access to the actual realized value of Ξ. The kind of eqilibria which is studied here is equilibria contingent upon the announcement made by the firm with respect to the value of Ξ. This will in general impose restrictions upon the equilibrium level of employment since this serves as signal that the firm (the better informed party) has actually made a truthful announcement. As will be demonstrated in Chapter 2, in a sequential equilibrium (two periods) this will imply an insufficiently low level of labour demand. In a one period model the equilibrium is characterized by an insufficiently low level of labour supply, as is demonstrated by the first part of Chapter 3. Apart from the exercise of the first part of Chapter 3, only sequential equilibria are examined. This allows the use of the Perfect Baysian Equilibrium, or Sequential Equilibrium (Kreps & Wilson (1982))[1)], and is presumably the most reasonable setting for a problem of asymmetric information (on this, see Radner (1982)).

To be more specific, assume that the production function of the firm is defined over labour and capital, where labour and capital are complements. The trade union as well as, of course, the firm has perfect knowledge as to the functional form. However, the trade union may be less than perfectly informed as to the value of the production, that is, the demand function, or as to the level or quality of the stock of capital. Examples of both situations are given in this thesis. During a first period the wage is taken to be exogenously given[2]. The firm has to announce the value of Ξ which is realized. Given this announcement and the actions of the firm the union draws inferences as to the real value of Ξ. Based upon this inference the trade union and the firm enter a bargain. Mostly we take the simplest possible bargaining process which can be imagined, namely that the union dictates the money wage which is to rule in the second period. In consequence the profit to the firm in the second period is related to its actions during the first period. Thus, some strategic considerations are imposed upon the firm due to this "dynamics" in an otherwise static economy.

In standard (and static) microeconomic theory a firm employs labour up to the point where the wage (in a competitive framework) is equalled by the value of the marginal product of labour. Amongst other things this ensures efficiency, or to put it a little differently: full employment. All of the chapters to follow are concerned with the question of whether this notion of full employment can be supported in an economy characterized by asymmetric information, and thus strategic considerations of the agents.

The reason why unemployment can be expected to occur is that in a setting like that described just above the firm's second period profit is, ceteris paribus, higher the lower is the perception of the union of the value of the marginal product of labour. If then there is a strictly increasing relationship between the value of the marginal product of labour and the level of employment, this suffices to establish a result supporting unemployment. To be more precise, assume that the first period level of employment is an increasing function of the announced value of Ξ, call this $\bar{\Xi}$. Thus we have $L_1=\Theta(\bar{\Xi})$, where $\Theta'(\cdot)>0$. Furthermore, assume that the second period wage is an increasing function of $\bar{\Xi}$, that is, $w_2=T(\bar{\Xi}), T'(\cdot)>0$. Assume that the second period profit of the firm be a strictly decreasing function of w_2, i.e., $\pi=\pi(w_2)$, $\pi'(\cdot)<0$. Can the firm during the first period announce truthfully and be trusted, that is, can $\bar{\Xi}=\Xi$ be a feasible announcement? This cannot be an equilibrium since if it was, then the firm could announce $\bar{\Xi}=\Xi-\varepsilon$. Doing so, the firm suffers a loss of profit in the first period since it has to behave in accordance with its announcement, this is the assumption on L_1. This loss is of the order of ε^2. However, if the firm is trusted, then in the second period it realizes an increase of profits of the order of ε since the second period wage is lower than otherwise. Thus, the firm always has an incentive to claim that the realized value of Ξ is lower than what is actually the case. Or to put it differently, if the firm announces Ξ, then the union will infer $\bar{\Xi}=\Xi+\varepsilon$. Hence, to support an announcement $\bar{\Xi}=\Xi$, the firm essentially has to announce

$\Xi-\varepsilon$ which gives rise to an inference of $\tilde{\Xi}=(\Xi-\varepsilon)+\varepsilon=\Xi$. With the assumption made upon L_1, this is equivalent to the use of employment as a signal. The signalling role of employment dictates an inefficiently low level of employment.

Let us concentrate for a moment on separating equilibria. In Chapters 2 and 3, especially, it is establishe that the level of employment serves as a quantity signal. The role is to elicit the relevant information. But as noted in these chapters, this is done at a cost. In the two period model of Chapter 2 the firm loses some first period profit in order to seveal the truth. In the one period model of Chapter 3 the union sacrifices some utility in order to extract the truth. Thus, the economy is caught in a "catch-22". If the better informed agent credibly could pass on his information to the lesser informed party, a welfare gain would arise. What are then the reasons for the fact that this information cannot be transmitted?

Consider the result of Chapter 3 first. The firm is better informed compared to the union. Ex-ante the only information in addition to common knowledge must be provided by the firm which has an incentive to misrepresent the truth. Since this is a one-period model, the firm cannot be punished if caught lying and in consequence it is the actual behavior if the firm and the union which will have to elecit the relevant information. Thus, the economy cannot escape the "catch-22" situation.

If we turn attention to the equilibrium discribed in Chapter 2, the situation may look more promising with respect to the transfer of information. This is so since this model is a two-period model. Consider the case when the firm behaves according to the realized value of Ξ. One way that the firm could avoid being identified as a $\Xi+\varepsilon$ type would be to "open its books" to the union. Despite the fact that profit in a real world context is less precisely defined than here, this raises another problem. If the union were to believe the "books", then the firm could just set up another firm. The role of this new firm is to buy the products of the producing firm such as to make production look as little profitable as possible (according to common knowledge). If the union realizes this, then it cannot put any trust into the "books". Thus, the quantity signal is still needed.

A different route to take would be to put union representatives on the board of directors. However, if there still is a conflict between the strive for profit and reward of labour such a solution is subject to the same remarks as above. We can imagine only one case in which workers' representativeson the board of directors would solve the incentive problem. And this is the case of labour-managed firms since in this case there will be no conflict between workers and management since they all have the same preferences.

Above the argument supporting underemployment was sketched and reference was made to the competitive solution. In all of the models presented in the following, a command equilibrium is analysed. Since the models presented in this thesis are perhaps best interpreted as models of local bargaining, this may appear objectionable. This is so since both the firm and the union may enjoy monopoly power. In defence of such an approach based upon the notion of a command optimum, note that this, since it rules out monopoly effects, gives efficiency the best chance. Thus, analysing the command optimum, we ask the question of which kind of inefficiencies arises in an otherwise competitive economy characterized by asymmetric information. At a more formal level, consider the problems analysed here as a problem within the theory of incentives[3)]. This theory is concerned with the problems faced by a planner[4)] when the objectives of the planner are different from those of the individual agents and when the actions of the planner depend upon the behavior or information of the agents. If we consider the firm and the union as the agents of the economy and the planner as society itself, the problems addressed in this thesis are readily interpreted as problems within the theory of incentives. In such a setting the planner's choice of action involves what may be called a double maximization: the planner maximizes his own utility[5)] subject to the constraint that once the planner has dictated an incentive scheme[6)], the agents will maximize their own objective functions. However, despite these arguments in favour of analysing a command equilibrium, we have also analysed the corresponding decentralized equilibrium which allows for monopoly effects. In two cases the distinction between a command equilibrium and a decentralized equilibrium are important in the sense that conclusions change qualitatively[7)].

The current analysis is restricted in its level of generality for two reasons. Firstly, the models applied are all partial equilibrium models. Secondly, the current analysis is based upon the introduction of specific functional forms.

TABLE 1.		
	Utility Function	Production Function
Chapter 2	wL	$y = f(L,K)$
Chapter 3	$wL + v(L)$	$y = f(L,K)$
Chapter 4	$U(wL,L)$, $U_{12} < 0$	$y = f(L,K)$
Chapter 5	$wL(1-L)$	$y = L^{1/2}$
Chapter 6	$wL - \alpha L^2$	$y = (LK)^{1/2}$

Let us comment on Chapters 5 and 6 first. These chapters are concerned with policy issues. For this reason extremely simple functional forms have been chosen in order to obtain results which allow for a reasonably simple policy analysis. Chapters 2, 3 and 4 are all more general in scope, and it has been possible to obtain results with quite general production structures. The restrictions on these results are found in the specification of the utility function describing the preferences of the union. Results with respect to the level of employment are established by examination of first order differential equations. Not surprisingly, to establish unambiguous results we require a reasonably simple structure. This, at least in a first study, implies that we have to be fairly selective in the choice of model specification.

II.b. Summary of Chapters 2-6.

Chapter 2 considers, as all of the chapters, a bargaining process between a firm and a union. The simplest possible bargain is imagined, namely that the trade union in a two period relationship dictates the second period wage. The firm is supposed to sell its product in a fully competitive market. The competitive price is common knowledge. The production function is defined over the input of labour and capital, or capacity. The firm has full knowledge of capacity. It is assumed that the trade union is imperfectly informed about capacity in the sense described earlier. Thus, the knowledge of the union is described as a support $[k_l, k_u]$ and a non vanishing distribution $\psi(k)$. This situation can be interpreted as the trade union either cannot observe the quantity of the capital stock or is poorly informed about its quality. The equilibrium described allows for an interpretation of a decentralized as well as of a command equilibrium. The main result of Chapter 2 is that only separating equilibria exist, that is, in equilibrium all relevant information is elicited. A second important result is the fact that first period employment serves as a signal and that this dictates underemployment and, of course, underproduction, compared to the case of full information. Since all relevant information is elicited during the first period, the second period equilibrium is not disturbed relative to the case of symmetric information. Thus, the only effect is that first period employment is lowered. Hence, this result argues that a welfare loss occurs. The chapter is divided into two parts. The first part analysis the case of a two-point distribution. The second part generalizes this to the case of a continuum of types.

The result of Chapter 2 is subject to further considerations. It is natural to analyse the implications with respect to the level of investment (cf. Grout (1984)). This is the subject of Chapter 4. Also, since the result can be interpreted as if the shadow wage of labour exceeds the money wage it is natural to consider taxation as the proper policy towards underemployment. This is the theme of Chapter 5. However, before we turn to an analysis of the equilibrium established in Chapter 2, we consider in Chapter 3 if we may have a different kind of equilibrium.

Narrowing down the range of possible firms, that is, in a sense letting information be more precise, one perhaps would believe that the equilibrium would become more efficient. This is analysed in Chapter 3. Chapter 3 again analyses a bargaining situation. In this model the firm is facing a downward sloping demand curve which is subject to shock. The firm is perfectly informed as to the realized value of this shock whereas the union only knows the process generating the stochastic injection. The first part of this chapter argues that in a one-period relationship a separating equilibrium is obtained. Two propositions are offered; one for the case of a command equilibrium and one for the case of equilibrium in a decentralized economy. It is shown that the level of supply of labour is inefficiently low in both the command and the decentralized equilibrium. The second part of Chapter 3 is devoted to dynamic considerations. It is shown that in a dynamic setting the separating equilibrium cannot be supported if the range of firms is sufficiently narrowed down and if the firm with the lowest possible profit is a "zero-profit" firm. In this case a pooling equilibrium obtains[8]. However, in the case of a decentralized economy the sequential equilibrium does not cease to exist. This is so because the firm with the lowest monopoly profit enjoys a profit strictly bounded from below by a number greater than zero.

In a pooling equilibrium different types of agents behave alike. This may have important macroeconomic implications and, of course, implications for macroeconomic policy. This is the theme of Chapter 6.

Chapter 4 analyses the role of employment and investments when the firm enjoys information advantages. Compared to the previous chapters, the firm now has to choose not only employment but also its level of investments. Two situations are analysed. Firstly, one in which a planner dictates the second period wage. Under these circumstances none of the two parties enjoy first-mover advantages. It is shown that not only is the level of investments low compared to a situation of symmetric information, but for any level of investment it is underutilized. This result cannot be shown in a decentralized economy. It is still the case that underemployment occurs, as is to be expected from Chapter 2.

However, the effect on investments is ambiguous. The fact that employment is lowered has two opposing effects. The value of the marginal product of capital is lowered (at least with a neo-classical production function). This tends to reduce investments. On the other hand, since the level of employment is lowered, the effect of capital formation upon the wage (through the value of the marginal product of labour) becomes less important. This tends to increase the level of investments. To show that this ambiguity is not due to the generality of the model, an example is offered. Using a Cobb-Douglas production function it is confirmed that we can-not expect to obtain an unambiguous result.

Chapter 5 restates the results of Chapter 2 employing less general functional forms but applying a more general outcome of the bargaining process, the Nash bargaining solution. It is shown that underemployment occurs, unless the firm has all of the bargaining strength. Also it is shown that employment decreases with union strength. But the focus of this chapter is the interpretation of the underemployment result as an externality. This points to use of taxes. In this respect the results are mixed. The following kind of taxes are considered profit taxes, revenue taxes, output taxes and wage taxes. It is argued that only wage and output taxation can improve upon the inefficient equilibrium.

The final chapter returns to the case of a pooling equilibrium. In the first part it is discussed under which circumstances a pooling equilibrium obtains. It is argued that if the two types considered are sufficiently alike, the resulting equilibrium is a pooling equilibrium. The second part focuses upon the possible effects of economic policy as well as of a stochastic injection to the economy. Contrary to the preceding chapter, the economic policy measures introduced in this chapter can be interpreted as traditional fiscal policy. The second part of the chapter is divided into three subsections of which the first two consider stochastic injections or policy measures which are specific to the state of nature. It is seen that there are circumstances in which the economy will not respond immediately to either a fully anticipated shock or to a known policy intervention. The third part is concerned with ideosyncratic shocks or general policy intervention (i.e., state independent injections to the economy). In particular, these demand changes can be interpreted as traditional fiscal policy measures and are followed by adjustments in employment and production (both if the pooling equilibrium is still supported after the demand change or if the equilibrium changes from a pooling to a separating equilibrium), like in traditional Keynesian models. However, this effect is not immediately related to the traditional Keynesian multiplier story since within the current framework this is just the optimal response by firms to a publicly known demand increase.

Finally, note that some of the results presented here point to the fact that it is important whether we analyse the consequences of asymmetric information in a general equilibrium setting or in a setting allowing for monopoly effects. The underemployment result of Chapter 2 remains valid in both a planned and a decentralized economy, where the planned economy is intended to minic a competitive economy. However, this is not so for the results of Chapters 3 and 4. In Chapter 3 it was the zero-profit condition which generated a pooling equilibrium. In Chapter 4, unless monopoly effects were suppressed, it was not possible to obtain unambiguous results. Thus care must be taken when interpreting the results of an analysis of problems relating to asymmetric information. If it is felt that the economy is characterized by imperfections relative to the competitive paradigm so that neither the zero-profit condition nor the zero-elasticity is fulfilled, then the partial equilibrium presented here will be appropriate. Alternatively, but this is outside the scope of this thesis and a topic in its own right, the consequences of asymmetric information must be analysed in a general equilibrium setting, for example a model of monopolistic competition.

III. Notes.

1. Strictly speaking these two equilibria concepts are not identical. Any equilibrium which is a Sequential Equilibrium is also a Perfect Baysian Equilibrium but not vice versa. This is so since a Sequential Equilibrium in addition to possessing the features of a Perfect Baysian Equilibrium also encompasses the idea of a Trembling Hand Equilibrium.

2. This actually implies that instead of studying a Perfect Baysian Equilibrium we study a Continuation Equilibrium.

3. The following is based upon the extensive survey by Laffont and Maskin (1982).

4. The planner is often thought of as a government or as society itself.

5. In the current setting utility is maximized when efficiency obtains in the exchange of income for labour.

6. In this setting an incentive scheme is triple consisting of $\{\Xi, L_1, w_2\}$; that is, given the announcement as to the realized value of Ξ, the level of employment in the first period the planner dictates the second period wage.

7. On this, see the summary of the different chapters.

8. Strictly speaking another, very complicated, equilibrium may coexist with the simple pooling equilibria.

CHAPTER 2

WAGE DETERMINATION IN A MODEL OF SEQUENTIAL BARGAINING

I. Introduction.

In this chapter, a model of wage determination is set forth. The principal feature of the model is that the wage is determined as the outcome of a bargaining process involving a trade union and a firm. Our objective is to analyse the problems, if any, arising, when the firm, compared to the trade union, is better informed with respect to some exogenous variables determining the outcome of the bargaining process. It is assumed that the trade union has the opportunity to draw inferences over time about these variables conditional upon the actions of the firm. A natural setting is one of sequential bargaining. The bargaining strength of the parties involved is given exogenously, i.e., it is not possible by (strategic) commitments to change the division of the surplus arising from the production process.

To be more specific, assume that the production function is defined over labour and capacity. The trade union knows the functional form of the production function. However, its knowledge about the capacity is only probabilistic. The firm has perfect knowledge about capacity. During a time spell, period 1, the union observes for given values of nominal wage and price, the actions of the firm. In between the 1st and 2nd period, the trade union and the firm negotiate a wage. The revenue of the bargaining to the trade union is partly determined by the beliefs (conditional upon the first period action) about capacity. The strategic interaction, arising because of different information sets between the trade union and the firm, is the concern of this chapter. To facilitate the analysis the following simple outcome of the bargaining situation is postulated. After period 1, the trade union draws its inferences about capacity and conditional upon these, announces the wage which maximizes some postulated utility function. This is the simple monopoly union model. Many more (and also more complicated) bargaining processes may be imagined, but in order to keep the analysis simple, the above outcome of the bargaining process is maintained throughout the paper.

The markets for goods are all assumed to be perfectly competitive. Consequently, real income is maximized by maximizing nominal income. In most of the paper, the utility of the trade union is assumed to equal real income. Only specific examples (cf. Lemma 1 and 2) consider disutility of work specifically.

For given wages and prices, firms normally employ up to a level of employment for which the value of the marginal product equals the going wage rate. The attempted contribution is to analyse whether this equilibrium can be sustained, in the framework described above. Firstly, the paper analyses what type of equilibrium result, i.e., whether a separating or pooling equilibrium are obtained. A main result of this chapter is that only separating equilibria exist. Given this, it is of interest to characterize such an equilibrium. A second main result of this chapter is that any separating equilibrium involves less production and employment (unless the firm has the highest capacity), in the first period compared to standard results.

Thus, the mere presence of a wage bargaining process tends to support underemployment.

The analysis deserves attention for at least two reasons. Firstly, the labour market is the market which, in its institutional settings, varies perhaps mostly from the core assumptions of the Walrasian model; i.e., price taking and lack of strategic interactions. Thus, to formulate and analyse explicitly some of the strategic considerations which may arise in this market can be seen as yet another contribution to the growing literature on microfoundations of macroeconomic theory. The result for the level of employment tends to support this supposition.

Secondly, the analysis provided in this paper is related to the theory of implicit contracts. The theory of implicit contracts under asymmetric information suggest that, only if firms are more risk averse than are workers, underemployment results. It does seem likely that it is the case that firms are less averse to risk than workers are (firms may better diversify their risk). In the case of risk neutrality contract theory suggests full employment. The result obtained in this paper does not rely upon assumptions regarding the parties attitude towards risk and yet, we obtain underemployment.

The difference between this approach and that of the literature on implicit contracts is that in the current setting, it is the trade union who offers a contract (perfectly elastic supply of labour at some wage), whereas in the contracts literature, it is the firm who is offering the workers a contract (making the firm's income a residual)[1].

Section II analyses the case where the capacity can take on only two values. The analytically more exciting case, in which capacity is distributed according to a continuous distribution function, is analyzed in Section III. A summary is offered in Section IV, where the results are related to other results given in the literature on trade unions.

II. A Two-Point Distribution Function.

This section of the paper analyses the strategic behavior arising as a consequence of the bargaining process going on between the trade union and the firm, in the case where capacity k takes on only two values; k_u and k_l, respectively. It is assumed that $k_u > k_l$ where u, respectively l, refers naturally to upper, respectively lower. Also, once capacity is fixed at k_u or k_l (according to a two-point probability distribution), it is invariant over the two periods. It is shown that only separating equilibria result, that the k_l-type may well be distorted, and that the first period produces less than the Walrasian output. Contrary to this, a k_u-type firm is never distorted and produces, in the first period, an amount equal to the Walrasian output.

The results put forward here partly serve as an illustration of the results for the continuous distribution case considered in Section III. However, the results of this section are also of interest on their own because they, partly, are contrary to the results given in Freixas et al. (1985), where this line of reasoning is also used. Contrary to the results obtained in Freixas et al. (1985), the current analysis support only separating equilibria. The reason for this is that in Freixas et al. (1985), an absolute lower bound upon the actions of the agents is an inherent part of the structure of the economy.

At times purely technical analysis is carried out. This serves only the purpose of supporting the equilibrium. It is therefore worthwhile to give a brief account of the following. Central to the analysis presented here and in the next section is the idea that the higher the value of k is, as perceived by the trade union, the higher the second period wage is. An example of sufficient technical conditions, for this to be the case, is given in Lemma 1 and Lemma 2[2]. These conditions are concerned with the properties of the production function and the sufficient conditions given here rule out, for example, the Cobb-Douglas production function. However, an example, following Lemma 1 and Lemma 2, shows that also for the Cobb-Douglas production function it will be the case that the higher is the perceived value of k, the higher is the second period wage, if the disutility of labour is given by some convex function.

Proposition 1 characterizes the equilibrium production. The proof of this proposition is instructive as the level of production is determined exactly, depending upon type. The proof of this proposition rests upon two technical lemmas; Lemma 3 and Lemma 4.

These lemmas describe for each type of firm, i.e., $k=k_l$ or $k=k_u$, the feasible levels of production. Proofs of these lemmas are based on the sequential nature of the bargaining process.

Denote the prior probability of $k=k_u$, by $Pr(k=k_u)=\upsilon_1$ and in a similar way $Pr(k=k_l)=1-\upsilon_1$. During the first period, the price and wage are p and w_1, respectively. The firm produces according to $f(l,k)$, where $k=k_u$ or $k=k_l$. l is the level of employment. The trade union knows the functional form $f(.,.)$. Observing actual output, as well as actual employment, the trade union knows whether production takes place as with capacity k_u or k_l.

Let $\Phi_2(k,p,w_2)$ be the second period demand for labour. Then the following 2 lemmas give some structure to the model.

<u>Lemma 1</u>. A sufficient condition for the second period wage to increase in υ_2, the updated probability that $k=k_u$, i.e., $\partial w_2/\partial\upsilon_2>0$ is that

i) for a given labour demand schedule the maximization of second period income by the trade union is solved for a strictly positive and finite value of w_2[3)]

ii) $\frac{\partial}{\partial k}(\Phi_2(k,p,w_2)+w_2\frac{\partial\Phi_2(k,p,w_2)}{\partial w_2}) > 0$

<u>Proof</u>: see Appendix.

Condition i) of the lemma is quite usual, which means that second period income can be maximized, that is, the function giving second period income is concave or quasi-concave. Condition ii) ensures that the wage claim is actually increasing in k.

Immediately from this we have, when π_2^F is second period profit to the firm:

Lemma 2. Conditions i) and ii) of Lemma 1 are sufficient for the high capacity firm to have an incentive to act as if it is a low capacity firm, i.e., $\partial\pi^F{}_2/\partial\upsilon_2<0$.

Proof: See Appendix.

Lemma 2 offers an insight into the basic characteristic of this model. It states that the higher the ex-post probability that a firm has $k=k_u$, the lower the profit received by the firm (irrespective of its actual type). In a world with only two types of firms, this provides an incentive to the high capacity firm to pretend that it is a low capacity firm. That is, a firm with realized value $k=k_u$ has an incentive to claim that the realized value of k equals k_l. The firm is restricted to behave according to its claim. If the firm reveals that $k=k_u$, the updated value of υ_1, called υ_2, is 1. If, on the other hand, the firm claims that $k=k_l$, the updated value of υ_2 is $\upsilon_1<1$, which according to Lemma 2 results in higher profit[4]. Note, at this point nothing has been said to indicate how a k_u-type firm does mimic a k_l-type firm. This will be discussed shortly.

Lemmas 1 and 2 may well be thought to be unduly restrictive with respect to assumptions made on f. In particular, the Cobb-Douglas production functions are ruled out by assumption i) of the lemmas. However, also for the Cobb-Douglas production function exhibiting decreasing returns, it is the case that the second period profit is decreasing in υ_2, once the disutility of working is given by a strictly convex function. Consider

$$y_i = l_i^{\alpha} k_i^{\beta} \qquad i = 1,2 \qquad \alpha+\beta < 1$$

Profits are[5]

$$\pi_i = p \cdot y_i - w_i l_i$$

Hence, for any given w_2 labour demand in period 2 is

$$l^d{}_2 = [\frac{p^{\alpha} k^{\beta}}{w_2}]^{\frac{1}{1-\alpha}}$$

where k takes on one of the two values k_l or k_u. Assume that the trade union maximizes real income less disutility of working. Consider the case where the disutility of working is given by $b(l)=l^2$, a strictly convex function[6]. The expected utility is given by

$$\upsilon_2[\alpha p k_u^{\beta})^{\frac{1}{1-\alpha}} w_2^{\frac{-\alpha}{1-\alpha}} - (\alpha p k_u^{\beta})^{\frac{2}{1-\alpha}} w_2^{\frac{-2}{1-\alpha}} +$$

$$[1-\upsilon_2][\alpha p k_l^{\beta})^{\frac{1}{1-\alpha}} w_2^{-\frac{\alpha}{1-\alpha}} - (\alpha p k_l^{\beta})^{\frac{2}{1-\alpha}} w_2^{\frac{-2}{1-\alpha}}]$$

Manipulation of first order conditions gives

$$w_2^{\frac{-2+\alpha}{1-\alpha}} \left(\frac{2(\alpha p)}{\alpha}\right)^{\frac{1}{1-\alpha}} = \frac{\upsilon_2 k_u^{\frac{\beta}{1-\alpha}} + (1-\upsilon_2)k_1^{\frac{\beta}{1-\beta}}}{\upsilon_2 k_u^{\frac{2\beta}{1-\alpha}} + (1-\upsilon_2)k_1^{\frac{2\beta}{1-\alpha}}}$$

From this:

$$\left[-\frac{2+\alpha}{1-\alpha} w_2^{\frac{-2+\alpha}{1-\alpha}} \left(\frac{2(\alpha p)}{\alpha}\right)^{\frac{1}{1-\alpha}}\right]dw_2 = \frac{k_u^{\frac{\beta}{1-\alpha}} k_1^{\frac{\beta}{1-\alpha}} \left(k_1^{\frac{\beta}{1-\alpha}} - k_u^{\frac{\beta}{1-\alpha}}\right)}{\left(\upsilon_2 k_u^{\frac{2\beta}{1-\alpha}} + (1-\upsilon_2)k_1^{\frac{2\beta}{1-\alpha}}\right)^2} d\upsilon_2$$

Hence, $dw_2/d\upsilon_2 > 0$.

Also, second period profit is written as

$$\pi_2^F = p l_2^\alpha k_k^\beta - w_2 l_2$$

Using the Envelope Theorem

$$\frac{d\pi_2^F}{dw_2} = -\, l_2 < 0$$

The discussion offered above took as exogenous υ_2; the ex-post probability that $k=k_u$. The aim is to give a fully dynamic analysis and, hence, to make υ_2 an endogenous variable.

Denote actual output in any period by x_i, $i=1,2$. Observing x_i, the trade union indirectly observes the value of k. The value of k, inferred by the trade union, or more precisely the value of υ_2, impinges upon the future stream of profits of the firm. Hence, the firm may engage in strategic behavior so as to affect

υ_2. Before considering the solution to this problem, some further reflections on the strategic behavior is needed.

Since both the trade union and the firm are active decision makers, it is not clear how a k_u-type firm mimics a k_l-type firm. If the firm believes that the trade union, by observing a high output, infers that the value of k is low and if, on the other hand, the trade union believes that the firm has the above beliefs, then an equilibrium, in which firms overproduce, is perfectly viable. However, in the following, it is assumed that a focal point of the game is that both the trade union and the firm associates falling output with a lower value of k. In the next section, it is seen that within the current model, the sign of the derivatives of the strategies giving first period output and second period wage can only be signed pairwise. Alternatively, k_u can be given the interpretation of an absolute upper bound upon production. Thus, if the k_u-type firm is to deviate, then it must do so by producing less than $f_1(k_u, \Phi_1(k_u, p, w_1))$. Hence, only "under-production" is the result of strategic behavior.

To sum up, a situation is considered in which it may be to the advantage of a k_u-type firm to "under-produce" in order to set $\upsilon_2=\upsilon_1$. That is, a situation materializes in which the trade union observing x_1, obtains no more information than what is contained in υ_1. If the k_u-type firm produces $f_1(k_u, \Phi_1(k_u, p, w_1))$, then $\upsilon_2=1$. Denote the wage claim put forward and accepted for the second period by w_2. This is the so-called monopoly union model (see Nickel and Andrews (1982)). The union unilaterally determines the wage and it is left to the firm to decide the level of employment.

To be more precise, consider the following two-period game. During the first period, the firm chooses to produce as if it is a low capacity firm, irrespective of its type. Prices and wages are assumed to be given. In between the first and the second period, the trade union puts forward a wage claim, w_2, which is taken as the exogenous wage ruling in the second period. The price is unchanged. In order to put forward an optimal wage claim, the trade union faced with an output corresponding to a low capacity firm has to decide upon the ex-post probability $\upsilon_2(x_1)$, a function of $x_1(k,l)=f_1(k_l,\Phi(k_l,p,w_1))$, where $x_1(.,.)$ denotes actual production and Φ_1 denotes first period labour demand. In this case $\upsilon_2(\cdot)=\upsilon_1$. On the other hand, the trade union may be confronted with an output $x_1(k,l)= f_1(k_u,\Phi_1(k_u,p,w_1))$. In this case $\upsilon_2=1$.

In the following, only a subset of the Perfect Baysian Equilibrium (PBE) is considered. For any one equilibria to be a PBE it is required (cf. Freixas et al. (1985)), given any first period wage, that:

P.1 $l_2 = \Phi_2(k,p,w_2)$ is a maximizer for $\pi_2{}^F$

P.2 $w_2 = w(f_1(k,\Phi_1(k,p,w_1))$ is a maximizer for
$E[w_2 \cdot \Phi(k,p,w_2) | f_1(k,\Phi_1(k_u,p,w_1))]$

P.3 $l_1 = \Phi_1(k,p,w_1)$ is a maximizer for $\pi_1^F + \pi_2^F$ given $l_2 = \Phi_2(k,p,w_2)$

P.4 w_1 is a maximizer of $E[w_1 \cdot \Phi_1(k,p,w_1)] + E[w_2 \cdot \Phi_2(k,p,w_2) | f_1(k,\Phi_1(k_u,p,w_1))]$

BC υ_2 the up-dated probability of $k=k_u$ is Bayes-consistent with the prior probability υ_1 and $f_1(k,\Phi_1(k,p,w_1))$.

The above conditions are common and are nothing but a kind of "dynamic rationality" constraints. Conditions P.1-P.4 state that each party, given what is going to come and what has been passing, at any given point in time has to choose optimally. Finally, the condition BC requires that the forecast of what is going to come is consistent with current and past actions.

The equilibrium described by P.1-P.3 and BC and any w_1 is called a continuation equilibrium. Thus, continuation equilibrium is induced by a PBE and a specific first period wage.

<u>Lemma 3</u>. If $\partial f/\partial k>0$, then $x_1(k_1,1) \leq f_1(k_1,\Phi_1(k_1,p,w_1))$.

<u>Proof</u>. Consider $x_1(k_1,1) > f_1(k_1,\Phi_1(k_1,p,w_1))$ where $\partial f/\partial k>0$. Hence, from the higher output it is inferred that the value of k is higher than what it actually is, i.e., υ_2 is higher than what it would be if $x_1(k_1,1) \leq f_1(k_1,\Phi_1(k_1,p,w_1))$. This reduces second period profit and nothing is gained in first period profit.

Q.E.D.

Whether $x_1(k_1,\Phi_1(k_1,p,w_1))=f_1(k_1,\Phi_1(k_1,p,w_1))$ or $x_1(k_1,\Phi_1(k_1,p,w_1))<f_1(k_1,\Phi_1(k_1,p,w_1))$ is resolved in the proof of Proposition 1.

Lemma 4. If $\partial f/\partial k>0$, then $x_1(k_u,1)\varepsilon[x_1(k_1,1),f_1(k_u,\Phi_1(k_u,p,w_1))]$.

Proof. By assumption $\partial f/\partial k>0$. Using Lemma 3: $x_1(k_1,1)\leq f_1(k_1,\Phi_1(k_1,p,w))<f_1(k_u,\Phi_1(k_u,p,w))$. Now if a k_u type firm plays anything else but $x_1(k_1,1)$, it is identified as a k_u-type firm and, hence, profits are maximized choosing $f_1(k_u,\Phi_1(k_u,p,w))$. The only alternative is to choose to play $x_1(k_1,1)$.

Q.E.D.

Lemmas 3 and 4 restrict the strategies to be employed by the firm in the continuation equilibrium. In Proposition 1, the characteristics, the existence and uniqueness of a continuation equilibrium are considered. Only one type of continuation equilibrium is viable. The equilibrium is unique.

Proposition 1 considers three types of equilibria. In a pooling equilibrium, the two different firms produce the same output in the first period. In a semi-separating equilibrium, the k_u-type firm randomizes between $f_1(k_u,\Phi_1(k_u,p,w_1))$ and $x_1(k_1,1)$, whereas a k_1-type firm always produces $x_1(k_1,1)$. Finally, in a separating equilibrium, the k_u-type firm always produces $f_1(k_u,\Phi_1(k_u,p,w_1))$. The k_1-type firm produces $x_1(k_1,1)\leq f_1(k_1,\Phi_1(k_1,p,w_1))$. The proof is based upon the sequential nature of the game, the fact that deviations from the simple static solution to the profit maximization problem is costly, and finally, the fact that a trade union observing $f_1(k_u,(\Phi_1 k_u,p,w_1))$, adjusts its belief according to $\upsilon_2=1$. If production is $x_1(k_l,l)$, no information relative to υ_1 is obtained and consequently $\upsilon_2=\upsilon_1$.

Proposition 1. For $\partial f/\partial k>0$, the only continuation equilibrium possible is a separating equilibrium, where a k_u-type firm plays $f_1(k_u,\Phi_1(k_u,p,w))$ and a k_l-type firm plays $x_1(k_l,l)\leq f_1(k_l,\Phi_1(k_l,p,w))$.

Proof. Consider the conditions for either a pooling or a semi-separating equilibrium to exist. It must be so that for the k_l-type firm it is optimal to play some $\underline{x}_1(k_l,l)$ which the k_u-type firm also chooses to produce, at least probabilistically. If both types choose the same output, then $\upsilon_2=\upsilon_1$ and, hence, the second period wage faced by a k_u-type firm, respectively k_l-type firm, is lower, respectively higher, than if a separating equilibrium obtains. Now, if $\underline{x}_1(k_l,l)$ is optimal to a k_l-type firm, then $\partial\pi(k_l,\underline{x}_1)/\partial x_1$ is of second order smallness[7)] and consequently by deviating and playing $x_1<\underline{x}_1(k_l,l)$ the k_l-type firm may identify itself as a k_l-type firm (here we use the assumption $\partial f/\partial k>0$). Therefore, in the second period, this increases profits by a factor of first order smallness through the wage claim. Hence, whatever the decision of the k_u-type firm the k_l-type firm always chooses to produce slightly below the k_u-type firm. Anticipating this, the k_u-type firm always produces

$f_1(k_u, \Phi_1(k_u, p, w))$. Why is this so? The reason is that, whatever the k_u-type firm produces, the k_l-type firm will chose to deviate due to the argument above. Thus, no matter what the costs the k_u-type firm imposes upon itself, it is always identified as a k_u-type firm, and thus the second period gain is zero.

Whether the k_l-type firm produces exactly $f_1(k_l, \Phi_1(k_l, p, w_1))$ or $\underline{x}_1(k_l, l) < f_1(k_l, \Phi_1(k_l, p, w_1))$, depends upon the incentive for a k_u-type firm to produce $f_1(k_l, \Phi_1(k_l, p, w_1))$. If the cost of producing $f_1(k_l, \Phi_1(k_l, p, w_1))$ to a k_u-type firm is small relative to the potential gain in second period profit $x_1(k_l, l) <$ $f_1(k_l, \Phi_1(k_l, p, w_1))$, otherwise the k_l-type produces $f_1(k_l, \Phi_1(k_l, p, w_1))$.

Q.E.D.

Proposition 1, established that only separating equilibria exist. The central idea is that the k_u-type firm always produces $f_1(k_u, \Phi_1(k_u, p, w))$ and the k_l-firm $\underline{x}_1(k, l) \leq f_1(k_l, \Phi_1(k_l, p, w))$ in the incentive compatible solution.

The situation which is formally described in Proposition 1 can be given two interpretations. Firstly, the situation imagined can be that a union is facing one of many firms not able to identify which one. Thus, the firm must (possibly) deviate in order to allow the union to make the correct inference (otherwise the firm will meet a higher wage claim in the second period). Alternatively, there may be only one firm but the characteristic of this firm is unknown to the union. Again the firm, in order to avoid excessive wage claim, needs to separate itself out from the other type. Which one of these interpretations are given is immaterial to the formal argument.

These results are a manifestation of the fact that signals are costly to send and the fact that the model operates over two consecutive time periods. Since the model operates over more than one period, it is possible due to the argument in the proof of the proposition for a k_l-type firm to separate itself out. And it is indeed, at the margin, always profitable to do so. If the model had only one period, the trade union would have to choose a wage based upon the mere announcements as to what type of firm it faces. If the anouncements are free to make, then of course a type k_u-firm always announces k_l. Thus, in this setting (with non-binding contracts) it is crucial that the model is one of two periods.

The next section considers the continuous distribution case and it will be quite clear how considerations, based on the sequential nature of the game, may be used to characterize the differential equation giving the strategy of a typical firm.

III. The Continuous Distribution Case.

Consider now the case in which a continuum of firms exists. As before, firms are parameterized by capacity. Capacity is distributed according to $\psi(k)$, with support $[k_l, k_u]$. It is shown that given some regularity conditions only separating equilibria exist, and that the model predicts a unique separating equilibrium. Having only separating equilibria, the second period welfare loss due to strategic behavior becomes zero. However, as the firms have to fulfil the incentive compatibility constraints, put upon this model, there will be a welfare loss in the first period.

The formal structure of the analysis is similar to that of Milgrom and Roberts ((1980) and (1982)) and based upon the idea of sequential equilibria (Kreps and Wilson (1982), Freixas et al. (1985)). However, the results obtained here are somewhat stronger than those offered in, for example, Milgrom and Roberts ((1980) and (1982)). It is possible to strengthen the results due to analysis by Mailath (1987).

The following exposition is rather involved and it is useful here to give a brief summary of this section. Three main results will be established. The most important is Proposition 3 and Theorem 1.

Proposition 3 states that the equilibrium strategy is a separating strategy, i.e., different firms behave differently during the first period. Furthermore, the separating strategy is unique, as is shown in Theorem 1. These are central results. Also are Proposition 2, Proposition 3 and Lemma 6. Firstly, these results account for the fact that, even though the equilibrium strategy is separating, a welfare loss occurs as the first period production is lower than the first best production. Secondly, these taken together form a rationale for the 'assumptions' regarding the sign of the derivative of the strategies. These assumptions are important in the argument that first period production is less than the first best. Finally, Proposition 4 and Proposition 5 give, respectively, second order conditions and identification of the unique solution.

In addition, Lemma 5 gives, quite trivially, the second period strategy. Theorem 2 is not important to the analysis in its own right, but it does facilitate a deeper understanding of the main result of Proposition 3.

III.a. The Game and Its Solution.

Before concentrating upon results, the game is described formally. Consider the firm. During the first period a firm of a given type decides upon an output, knowing that this decision affects the wage claim, and, hence, profits in the second period. Formally, the mapping t_1 from R_+ to $R_+U\{0\}$, describes production in the first period: $x_1=t_1(k)$. Production is subject to some technical constraints $x_1=f(k,l)$. The trade union responds to x_1 demanding some wage given by the strategy s: $w_2=s(x_1)$, where s is a mapping from $R_+U\{0\}$ to $R_+U\{0\}$. Finally, the firm responds to t_1 and s by producing $x_2=t_2(x_1,w_2)$, hence, t_2 is a mapping from $R_+U\{0\}xR_+U\{0\}$ to $R_+U\{0\}$.

The set of optimal strategies t_2^*, s^*, t_1^* must satisfy where a bar denotes a conjecture:

$$\pi_2^F(t_2^*)\geq\pi_2^F(t_2)\ \forall\ t_2\neq t_2^*$$

$$s^*[\bar{t}_1(k)]\Phi_2[s^*[\bar{t}_1(k),\bar{t}_2]]\geq s[\bar{t}_1(k)]\Phi_2[s[\bar{t}_1(k),\bar{t}_2]] \qquad s\neq s^*$$

$$\pi^F[t_1^*,\bar{s},\bar{t}_2]\geq\pi^F[t_1,\bar{s},\bar{t}_2]\ \forall\ t_1\neq t_1^* \qquad \pi^F=\pi_1^F+\pi_2^F$$

$$\bar{t}_1=t_1^* \qquad \bar{s}=s^* \qquad \bar{t}_2=t_2^*$$

These conditions are similar to P.1-P.3 and BC of Section II. They are, however, stated somewhat more formally in terms of strategies here. This exposition brings out the problems of signing the derivatives of t_1 and s. Clearly, any equilibrium satisfying the above conditions is a PBE. Picking out one (conditional upon a given first period wage), results in a continuation equilibrium.

III.b. Characterization of Equilibrium Strategies

In this section, some results are proven which concerns the derivatives of the equilibrium strategies. It is argued that $\partial t_2^*/\partial w_2<0$, $\partial s^*/\partial x_1>0$ and $\partial t_1^*/\partial k>0$. Furthermore, the strategy t_1^* is differentiable given some rather unrestrictive regularity conditions.

Starting in the second period, it is easily seen that Φ_2 is nothing but the ordinary demand for labour. Thus, the following lemma is immediate.

Lemma 5. The higher the second period wage is, the lower the output is in the second period, i.e.,

$$\frac{\partial t_2^*}{\partial w_2} < 0$$

The higher the capacity is, the higher the second period output is for any given wage w_2, i.e.,

$$\frac{\partial t_2^*}{\partial k} > 0$$

Proof. See Appendix.

Although it is easy to characterize the equilibrium strategies t_2, it turns out to be considerably more complicated to offer results concerning t_1 and s.

Let us focus, therefore, upon a restricted class of strategies, those which are strictly monotonic and continuous. This is not so much of a restriction as it, perhaps, appears to be. The reason is that it can be shown that if $s'>0$, then $t_1'>0$ and t_1 is continuous. Given that $t_1'>0$ and t_1 continuous then $s'>0$ and s is continuous. Recent results, due to Mailath (1987)[8)], are heavily used.

The following proposition gives the sign of the derivative of s.

Proposition 2. If the strategy t_1 is strictly increasing in k, then the strategy s is strictly increasing in the first period production, hence,

$$\frac{\partial t_1}{\partial k_1} > 0 \Rightarrow \frac{\partial s}{\partial x_1} > 0$$

Proof. See Appendix.

The proof of Proposition 2 relies upon the existence of differentiable strategies t_1; this is fortunately not unduly restrictive as will be seen shortly.

Write the payoff to the firm as

$$(1) \quad p(k,t_1^{-1}(x_1),x_1) = \pi_1^F(k,x_1) + \pi_2^F(k,t_1^{-1}(x_1))$$

Denote by k^*, the inferred value of k, that is, $t_1^{-1}(x_1)$. Write (1) as

$$(2) \quad p(k,k^*,x_1) = \pi_1^F(k,x_1) + \pi_2^F(k,k^*)$$

For some separating strategy, assume that strict incentive compatibility is fulfilled.

$$(3) \quad t_1(k) = \underset{x_1 \varepsilon t_1([k_l,k_u])}{\text{Argmax}} \; p(k,k^*,x) \quad \forall \; k \varepsilon [k_l,k_u]$$

Some regularity conditions on the payoff function are needed. Two follow from the earlier discussion. Clearly from (2)

$$p_2 = \frac{\partial \pi_2^F}{\partial k^*} < 0 \qquad \text{(R1)}$$

$$p_{13} = \frac{\partial^2 \pi_1^F}{\partial x_1 \partial k} > 0 \tag{R2}$$

Condition (R1) follows immediately from (2) by partial differentiation, with respect to k*. What is stated in (R1) is that, ceteris paribus, an increase in the value of k, as perceived by the trade union, decreases second period profit. This is quite natural in this model, as a higher perceived value of k results, in a higher second period wage. This decreases profits.

Consider condition (R2). This condition requires that for a fixed value of k* (and in consequence w_2) the marginal profit of output is increasing in k. This is the case, for example, for the Cobb-Douglas production function. This condition is, in essence, a ranking condition. Assume that at a certain value of k, the marginal profit starts to decrease, with increases in k, call this value $\tilde{k}$. Hence, for $k<\tilde{k}$ output increases as k increases, as output (given k*) is found equaling the marginal profit of output to zero. For $k>\tilde{k}$, output decreases with increases in k. Thus, for a fixed value of k* different firms at best cannot be ranked and at worst may behave similarly. Such a situation is very irregular and is ruled out.

Furthermore, assume

$$p_{33} < 0 \text{ for } p_3(k,k^*,x_1) = 0, \quad x_1 \varepsilon R_+ \tag{R3}$$

The (strategic) optimization problem faced by the firm is obviously quite complex. Consequently, it is not possible a priori, to exclude the case that the solution to the first order conditions actually yield a minimum rather than a maximum. Also corner solutions cannot be excluded. Restricting attention to strategies satisfying (R3) ensures an interior solution. The restriction put upon the strategy t_1, in order to make the payoff function quasi concave, is discussed in Proposition 4.

Finally, define

$$X_1 = \{x_1 \varepsilon R_+ \cup \{0\} \;|p(k,k,x_1) \geq p(k,k^W,t_1(k^W))\}$$

X_1 is the set of relevant signals and defined relative to the worst point estimate (to the firm). This is clearly k_u, as this results in the higest second period wage. Since $X_1 \varepsilon R_+ \cup \{0\}$, X_1 is bounded from below, at zero. Consider a firm wanting to deviate, in order to be identified as $k-\Delta k$, where of course $(k-\Delta k)\varepsilon[k_l, k_u]$. If this is possible to achieve with a finite production, then of course X_1 is also bounded from above. Alternatively, if any firm k can be defined as $k-\Delta k$ only by letting $x_1 \rightarrow \infty$[9], then the first period loss is approaching ∞ and consequently no deviation is profitable. Thus, in conclusion:

X_1 is bounded (R4)

As an alternative to this regularity condition, a more complicated but also more general version, can be offered (see Mailath (1987), Section 4):

$$p_{33}(k,k^*,x_1) \geq 0 \Rightarrow \left|p_3(k,k^*,x_1)\right| > T \qquad \text{(R4')}$$

where T is positive.

This condition ensures that p, the payoff function, cannot asymptote any constant function of x_1 (the actual output), as x_1 approach either $(-\infty)$ or $(+\infty)$. Taken together (R3) and (R4') implies that X_1 is bounded[10].

The following proposition gives some structure to the equilibrium.

Proposition 3. The strategy giving first period output, as a function of capacity, i.e., t_1, is a continuous differentiable strategy. Assume that the strategy used by the trade union, s, is strictly increasing, then t_1 is also strictly increasing.

Proof. See Appendix.

This result is considerably stronger than those obtained in Milgrom and Roberts (1982), where a similar analysis of a different problem was carried out. The existence of any equilibria, other than separating equilibria, has been ruled out using two assumptions. Firstly, the assumption $s'>0$, a quite reasonable assumption.

The second assumption needed is (R4). In the present setting X_1

is bounded, as argued in the proof of Proposition 3. The idea of the argument is based upon the sequentiality of the game which guarantees an upper bound upon x_1. Assumption (R3) gives a lower bound. To support a case where a fully separating equilibrium does not exist it would have to be the case that several firms would want to produce the same level of output, for example zero, or at the lowest level conceivable (see Chapter 3 on this).

Figure 1 (see next page)

An alternative to assumption (R4) was provided by (R4'). Assumption (R4') and (R3) together ensure that x_1 is bounded and in this case the set of admissable solutions is R. Consequently, no solution hits the boundary of this set.

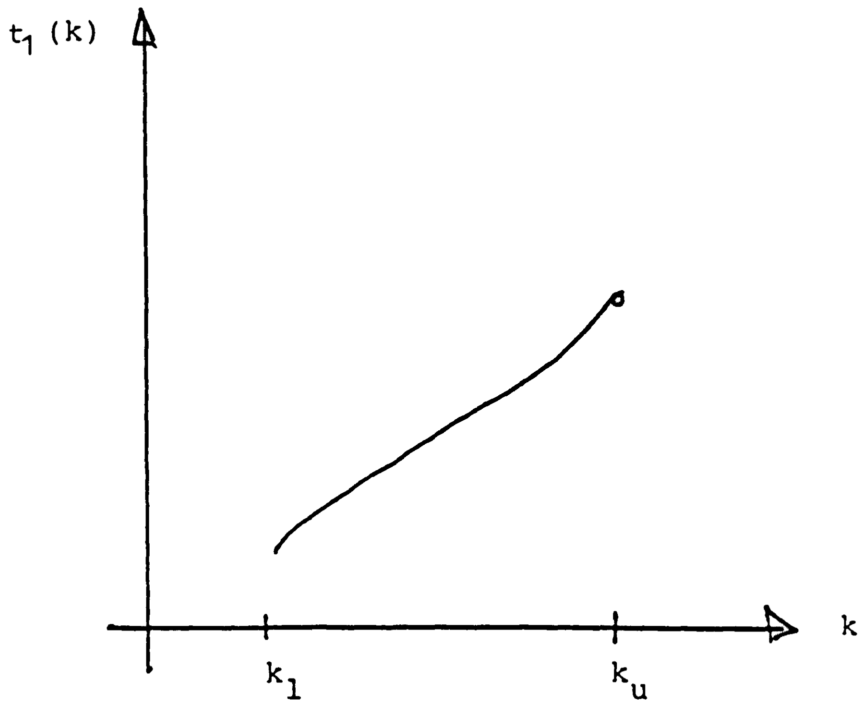
Finally, the following result is easily established.

<u>Lemma 6</u>. For t_1 continuous and $t_1'>0$, s is continuous.

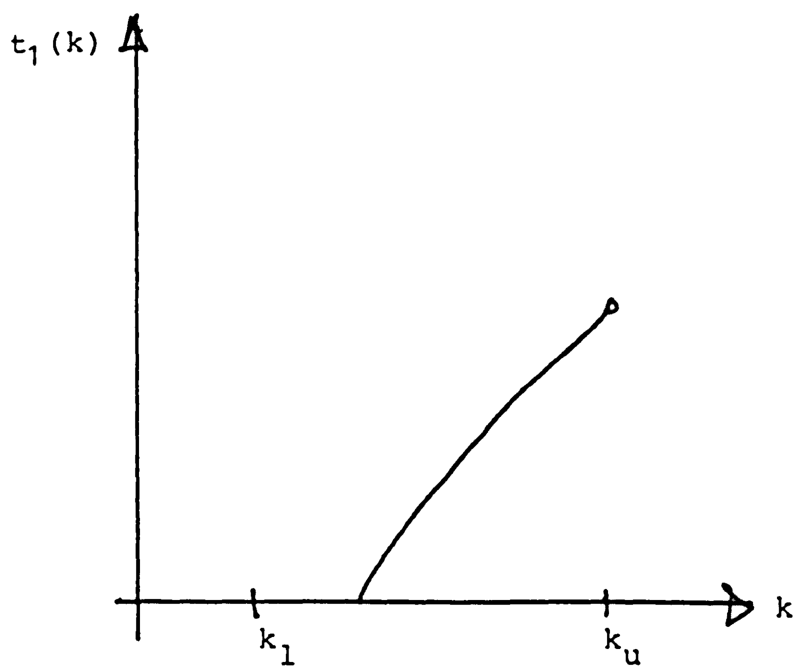
<u>Proof</u>. See Appendix.

Now, for $s'>0$ then t_1 is differentiable and, hence, continuous and also $t_1'>0$ (Proposition 3). On the other hand for $t_1'>0$ then $s'>0$ (Proposition 2). Finally for t_1' continuous s is continuous. This does rationalize the assumptions made, at least to some degree.

Figure 1



Example of a fully separating strategy.



Example of a "not-fully" separating strategy.

III.c. Uniqueness.

This subsection of the analysis is concerned with the uniqueness of the separating equilibrium. Not surprisingly uniqueness is established by an initial value condition, since in essence such a condition serves as a boundary value for the differential equation giving t_1 (for an introduction see Boyce and Diprima (1977)). The uniqueness result is offered in Theorem 1. Uniqueness of the equilibrium is closely related to the property of p_3/p_2. The reason for this is that $-p_3/p_2$ gives the slope of the isoprofit locus in the (k,x_1) space. Theorem 2 establishes that p_3/p_2 is decreasing in k for k^* in the domain of $t_1(k)$. This result demonstrates graphically the result of Theorem 1.

Let $\tilde{k}$ be any arbitrary inferred value of k within the range of feasible signals. Then Theorem 1 reads:

Theorem 1. If $t_1{}^*$ satisfies the incentive compatibility constraint and the initial value condition is valid, then assuming that $p_2(.,.,.)$ is finite any other strategy $\tilde{t}_1$ doing so is identical to $t_1{}^*$, i.e., $t_1{}^*$ is unique.

Proof. In Mailath (1987) it is proved that if

$$t_1(k^W) = f(k^W)$$

when k^w is the worst point estimate to the firm and $f(\cdot)$ the Walrasian output, and if

$$p_{13} \cdot t_1 > 0$$

then the solution to

$$\frac{dt_1}{dk} = - \frac{p_2(.,.,.,.)}{p_3(.,.,.,.)}$$

is unique if $|p_2(.,.,.,.)|$ is bounded.

Since $p_2(.,.,.,.)$ is assumed to be finite, $p_2(.,.,.,.)$ is clearly bounded.

Furthermore, since the payoff function is continuous and (R1)-(R4) apply, we use Theorem 1 of Mailath (1987) to establish that

$$\frac{dt_1}{dk} = - \frac{p_2(.,.,.,.)}{p_3(.,.,.,.)}$$

solves (3).

Q.E.D.

Hence, if it can be shown that a solution exists, then it is unique. The unique solution will be identified in Proposition 5. Before considering the existence problem, the following illustrative result deserves attention.

Theorem 2. Assume that the incentive compatibility constraint is satisfied. Then[11]

(4) $$\frac{p_3(k,\tilde{k},x_1)}{p_2(k,\tilde{k},x_1)}$$

is strictly decreasing in k for $\tilde{k}$ in the domain of t_1..

Proof. Using Theorem 3 of Mailath (1987) and the fact that $p_2<0$ (as t_1 is a strictly increasing continuous function which by assumption satisfies the incentive compatibility constraint).

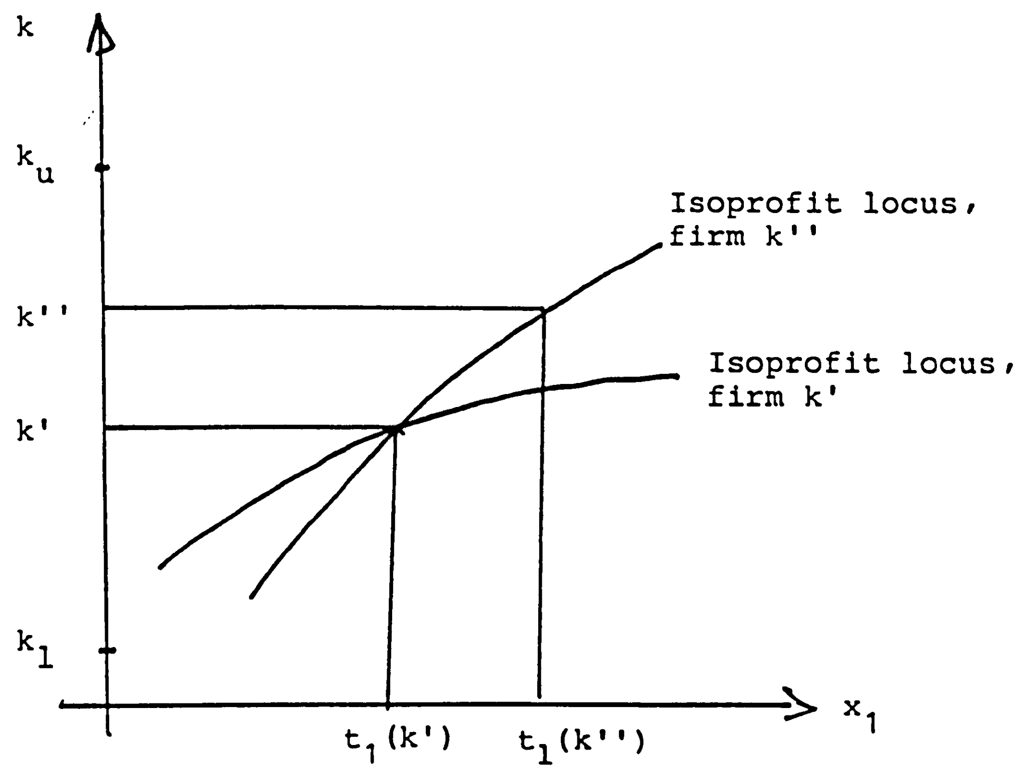
Q.E.D.

The property which has been shown to be valid, with respect to (4), is known as the single crossing property. This property arises naturally in some cases (Rothschild and Stiglitz (1976)) or it may be assumed directly (Riley (1979)). Understanding of the separating equilibrium is enhanced if we focus on this property. Consider the isoprofit locus of some firm k', say, in the $(\tilde{k},x_1)$ space. The slope of the isoprofit locus is

$$\left.\frac{d\tilde{k}}{dx_1}\right|_{d\pi=0,k'} = -\frac{p_3}{p_2} > 0$$

for $p_3>0$, which is a reasonable assumption here[12]. Now, consider the slope of the isoprofit curve of some other firm k''>k'; from Theorem 2 the slope increases. This is depicted in Figure 2 (next page). Clearly for t_1 to be a separating strategy, the agent k' must prefer $(k',t_1(k'))$ to $(k',t_1(k''))$ and vice versa. Such conditions are clearly satisfied if the single crossing property is valid.

Figure 2


k
k_u
k''
k'
k_l
Isoprofit locus, firm k''
Isoprofit locus, firm k'
$t_1(k')$
$t_l(k'')$
x_1

To expand on the issue of the single crossing property return to the example given in Section II. Since $t_1{}'>0$ then for any signal k^* the second period wage is found to be (p except for p(.,.,.,.) denotes a price):

$$w_2^* = 2^{\frac{1-\alpha}{2-\alpha}}\alpha^{\frac{\alpha}{2-\alpha}}p^{\frac{1}{2-\alpha}}k^{*\frac{\beta}{1-\alpha}}$$

Hence, the payoff to a firm with capacity k, signalling a capacity k^*, can be written as (after a few manipulations)

$$p(k,k^*,x_1) = px_1 - w_1\left(\frac{x_1}{k^\beta}\right)^{\frac{1}{\alpha}} +$$

$$(1-\alpha)p^{\frac{2}{2-\alpha}}k^{\frac{\beta}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}}2^{\frac{-\alpha}{2-\alpha}}k^{*\frac{-\beta\alpha}{(2-\alpha)(1-\alpha)}}$$

For this example, it is easily verified that both (R1) and (R2) are valid. The slope along any isoprofit locus is found to be

$$\left.\frac{dk^*}{dx_1}\right|_{\pi=\bar{\pi},k=\bar{k}} = -\frac{p-\frac{1}{\alpha}\, w_1 k^{-\frac{\beta}{\alpha}} x_1^{\frac{1-\alpha}{\alpha}}}{(1-\alpha)p^{\frac{2}{2-\alpha}}k^{\frac{\beta}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}}2^{\frac{-\alpha}{2-\alpha}}\frac{-\beta\alpha}{(2-\alpha)(1-\alpha)}k^{*\frac{-\beta\alpha}{(2-\alpha)(1-\alpha)}-1}}$$

Consider the slope of the isoprofit locus for two values of $k, k_2>k_1$.

$$\text{sign}\left[\frac{d}{dk}\left(\left.\frac{dk^*}{dx_1}\right|_{\pi=\bar{\pi}}\right)\right] = \text{sign}\left[\frac{w_1k^{-\frac{\beta}{\alpha}}x_1^{\frac{1-\alpha}{\alpha}}}{\alpha} - p\cdot x\right]$$

It is not possible to sign the right hand side. However, by Theorem 2 the slope is decreasing. Referring back to Figure 2, it is easily established that if this is the case, then t_1 is indeed an increasing function of k. On the other hand, assume t_1 to be decreasing, then a graphical argument easily establishes a contradiction of Theorem 2. Hence, the existence of a monotone strategy and the sign of its derivative is intimately related to the preferences of the agents, as described by the isoprofit loci.

III.d. Existence

Turning to the existence proof or rather the necessary conditions for existence, let us consider in more detail the solutions to the optimization problems faced by the agents. The optimal strategy of the trade union solves

$$\text{Max } w_2\Phi_2(w_2, t_1^{-1}(x_1))$$

Hence, $s^*(x_1)$ satisfies

$$\Phi_2(w_2, t_1^{-1}(x_1)) + w_2\frac{\partial\Phi_2(w_2, t_1^{-1}(x_1))}{\partial w_2} = 0$$

The function $s^*(x_1)$ is defined by

$$(5) \quad s^*(x_1) = \gamma(t^{*-1}(x_1))$$

Noting that $w_2 = s^*(x_1)$ then $\gamma' = (s^{*\prime}/t^{-1\prime})$ is found to be

$$\gamma' = -\frac{\partial\Phi_2/\partial k + w_2 \frac{\partial^2\Phi_2}{\partial w\partial k}}{2\frac{\partial\Phi_2}{\partial w_2} + w_2\frac{\partial^2\Phi_2}{\partial w_2^2}}$$

The numerator is positive by assumption ii) of Lemma 1 and Lemma 2. Assuming second order conditions to be satisfied for the maximization problem facing the trade union, the denominator is negative. Consequently, $\gamma'>0$.

As a consequence, the firm solves its optimization problem by choosing $x_1 = t_1{}^*(k)$, such that

$$\max_{x_1} \pi_1^F(k,x_1) + \pi_2^F(k,\gamma(t_1^{*-1}(x_1)))$$

The first order condition to this problem reads

$$(6) \quad \frac{\partial\pi^F}{\partial x_1} + \frac{\partial\pi_2^F}{\partial w_2}\frac{d\gamma}{dk}\frac{dt^{*-1}}{dx_1} = 0$$

Thus, write

$$(7) \quad \frac{\partial\pi_1^F}{\partial x_1} + \frac{\partial\pi_2^F(k,\gamma(k))}{\partial w_2}\frac{\gamma'}{t_1^{*\prime}} = 0$$

Equation (7) is a differential equation in t_1'. As it stands, this equation has a family of solutions which is characterized by a boundary condition. However, only one solution is viable, the one for which $t_1{}^*(k_u) = f^*(k_u)$. By $f^*(\cdot)$ is denoted the true Walrasian output corresponding to the capacity in question.

The equilibrium is described by some t_1 satisfying (7) and by the initial value condition implied by sequentiality. As $\partial\pi_2^F/\partial w_2<0$ and both $\gamma'>0$ and $t_1'>0$, then $\partial\pi_1^F/\partial x_1>0$. Hence, in the equilibrium dictated by (7), the firm deviates from the simple competitive solution (which is given by $\partial\pi_1^F/\partial x_1=0$). Less is produced in the first period in equilibrium compared to the competitive equilibrium. This may at first sight seem counterintuitive. The reason, of course, is that as t_1^* is strictly monotonic, the wage set in the second period will not be different from the one set under symmetric information, i.e., when the union perfectly well knows the type of the firm. Why then should the firm engage in an activity when apparently the only result is a loss of profits.

Consider the possibility that the firm chooses its first period output according to $d\pi_1^F/dx_1=0$; this strategy we have denoted $f^*(k)$. If the firm is assumed to pursue this strategy as an equilibrium strategy, then upon deviating to $f^*(k)-\varepsilon$, the trade union (incorrectly) infers that capacity is $f^{-1}(f^*(k)-\varepsilon)$, and accordingly the wage set, in the second period, is lower than it would otherwise be by the order of ε. However, the cost of deviating from $f^*(k)$ is of the order ε^2 since $\partial\pi_1^F/\partial x=0$ at the point of deviation. Or to put it differently, the strategy f^* does not satisfy the incentive compatibility constraints laid down in the structure of the model. Thus, the following claim has been substantiated.

Claim. Any solution to the bargaining problem involves under-employment, except for $k=k_u$.

To ensure the existence of a pair of equilibrium strategies (t_1^*, s^*), as given in (7) and (5), the following Proposition 4 is offered. Once t_1^* is a separating strategy s^*, as defined by (5), is optimal against t_1^* in so far as it has been assumed that second order conditions are satisfied. The main concern of Proposition 4 is to give conditions upon t_1 such that the payoff function is quasiconcave in the relevant interval for x_1. If this is so, then t_1 as given in (7) is optimal against s^* as given by (5). Thus, the result of proposition 4 may take the place of the regularity condition (R3). The reason for offering this result, not just assuming that second order conditions are satisfied for the optimization problem of the firm, is that this optimization problem is rather complex as x_1 enters both π_1^F and π_2^F.

Proposition 4. Let t_1 satisfy (7) and s satisfy (5). Then (t_1, s) is an equilibrium strategy if for some capacity $z \varepsilon [k_l, k_u]$ playing the capacity k it is true that

$$t'(z) > \inf_{\dot{k}\varepsilon[0,\bar{k}]} \gamma'(z)\left\{\frac{\dfrac{\partial\pi_2^F(z,\gamma(\tilde{k}))}{\partial w_2} - \dfrac{\partial\pi_2^F(\dot{k},\gamma(\tilde{k}))}{\partial w_2}}{\dfrac{\partial\pi_1^F(\dot{k},k)}{\partial x_1} - \dfrac{\partial\pi_1^F(z,k)}{\partial x_1}}\right\}$$

where $t_1(\bar{k})=f(k_u)$

Proof. See Appendix.

Proposition 4 gives conditions upon t_1 which ensure that the equilibrium strategies, discussed earlier in this paper, exist. In particular, for equilibrium strategies satisfying the inequality constraint of Proposition 4, the regularity condition (R3) which may be thought of as somewhat artificial is fulfilled. Hence, so far it has been established that strategies exist that satisfy strict incentive compatibility. The unique strategy satisfying the differential Equation (7) is identified in the next proposition.

Proposition 5. Let $k=\sigma(x_1)$ solve the differential equation $\sigma'=f(x_1,\sigma)$ where

$$(8)\quad d(x,\sigma) = [\frac{d\pi_2^F(\bar{k},w_2(\sigma))}{dw_2} s']^{-1}(\sigma - \frac{\partial\pi_1^F}{\partial x_1})$$

σ equals the solution to the problem (3) and is unique given a boundary condition.

Proof. See Appendix.

Q.E.D.

Proposition 5 narrows down the family of solutions identified in Proposition 4, to only one solution curve characterized by $x_1{}^*(k_u,1)$. Hence, existence is established, as well as a unique solution is identified.

IV. Conclusion

In this chapter a simple version of the monopoly-union model is used to analyse if it is the case that a competitive firm bargaining with a trade union over wage deviates from the static first best with respect to output of goods and input of labour. Two results were produced. In an environment where information is asymmetric, in the sense that only the firm knows the true value of the marginal product of labour, all firms except the one at "the top" deviated downwards in the first period. Thus, underemployment arise out of the bargaining process. As the equilibrium shown to exist is a separating equilibrium, second period production and employment is not disturbed compared to the situation under full information. Hence, in this sense, the strategic behavior unambiguously lowers welfare.

Secondly, the results suggest that an inefficient level of employment in the form of underemployment may result from labour market contracts, even if the agents involved are risk-neutral. This goes contrary to the results found in the implicit contracts literature. Hopefully, this issue will be explored at length later. Finally, note that the analysis presented can be taken to represent a model of first mover disadvantages (Gal-Or (1987)). As such, the analysis is also related to some of the industrial organization models. Although the attention is focused directly upon the relationship between a trade union and a firm, the current paper is not very similar in scope to other papers discussing this relationship (for a survey see Calmfors (1985)).

This is because we study directly the consequences for the restriction on the strategies, if these are to be incentive compatible. The result of this paper is of primary interest, in the context of economies characterized by large trade unions, where the wage is set by local offices of the trade union and firms in a bargaining context. Thus, the negotiation process is firm specific.

We took as given the value of k. One interpretation is to assume that k is composed of some investment $\tilde{k}$ and a quality parameter, ε. Thus, $k=\tilde{k}+\varepsilon$. In such a framework $\tilde{k}$ may be known precisely and ε is interpretated as a random variable. This random variable is then observed by the firm after deciding upon $\tilde{k}$, but known only probabilistically to the union. This interpretation, perhaps, allows for an extension of this (short-run) model into a long run model of employment, wages and investment along the lines of Grout ((1985) and (1984)).

V. Notes.

1. This is why the assumption of risk aversion is needed. The first-best can be implemented of the risk-neutral agent has the private information by making his income a residual claim.

2. A full characterization involves signing third order derivatives. In general, this is not possible within standard assumption.

3. This is not the case, if the production function is of the Cobb-Douglas type, for example. Here $w_2=0$ maximizes second period income.

4. At this point, it may well be suggested that it is relevant to discuss the case that trade unions ask for some kind of profit sharing. This possibility is excluded for two reasons. Firstly, this is not as common as the simple employer-employee relationship. Secondly, unless it is assumed that the concept of profit in reality is strictly well defined and observable to the trade union this raises a problem on its own.

5. Capital cost is fixed.

6. Unless we introduce $b(l)=l^2$ or some other strictly convex function, we find that the optimal wage claim is $w_2=0$. Alternatively, introduce some upper limit upon l, e.g. l max. Then l max determines the wage.

7. This is nothing but the Envelope Theorem.

8. However, at this point it must be acknowledged that the equilibrium set of strategies suffers from a basic non-robustness. One can show that if $s'<0$, then $t_1'<0$ and vice versa. This, of course, gives results exactly opposite to

what is obtained in the following. However, it seems very natural in this setting to consider $s'>0$ and $t_1'>0$ as the focal points of the game and consequently rule out the possibilities that $d'<0$ and $t_1'<0$.

9. Actually, this goes contrary to the signs of s' and t_1', however, this argument is designed only to show that X_1 is bounded.

10. The regularity condition (R4') thus is needed in the case of the range of types increasing indefinitely ($k_u \rightarrow \infty$).

11. By p_3 is meant $(\partial p(k,\tilde{k},x_1))/\partial x_1$.

12. The firm would choose to expand output for $p_3>0$, thus the region in which $p_3<0$ is not realized.

VI. Appendix

Proof of Lemma 1. For any given price p and announced wage w_2, the firm maximizes profits. Hence, demand for labour is a derived demand. This is realized by the trade union and taken into account when earnings are maximized

$$\max_{l} \; p\cdot f(l,k) - w_2\cdot l \qquad f_l>0,\; f_{ll}<0$$

$$f_k>0,\; f_{kk}<0$$

$$f_{lk}=f_{kl}>0$$

First and second order conditions are

(A.1) $p\frac{\partial f}{\partial l}(l,k) - w_2 = 0$

(A.2) $\frac{\partial^2 f}{\partial l^2}(l,k) < 0$

Equation (A.1) defines (implicitly) a labour demand function (in the following the subscript 2 is dropped from Φ_2)

(A.3) $l^d = \Phi(k,p,w_2)$

Substitute (A.3) into (A.1)

$$(A.4) \qquad p\frac{\partial f[\Phi(k,p,w_2),k]}{\partial l} - w_2 \equiv 0$$

From (A.4)

$$p\frac{\partial^2 f}{\partial l^2}\frac{\partial \Phi}{\partial w_2} - 1 \equiv 0 \Rightarrow$$

$$(A.5) \qquad \frac{\partial \Phi}{\partial w_2} = \frac{1}{p\frac{\partial^2 f}{\partial l^2}} < 0$$

From (A.4)

$$(A.6) \qquad \frac{\partial \Phi}{\partial k} = -\frac{\frac{\partial^2 f}{\partial k \partial l}}{\frac{\partial^2 f}{\partial l^2}} > 0$$

Now, consider the maximization problem faced by the trade union

$$\max_{w_2} \upsilon_2(w_2\cdot\Phi(k_u,p,w_2)) + (1-\upsilon_2)(w_2\Phi(k_l,p,w_2))$$

First and second order condition reads

$$\text{FOC} \qquad \upsilon_2[\Phi(k_u,p,w_2) + w_2\frac{\partial\Phi(k_u,p,w_2)}{\partial w_2}] +$$

$$(1-\upsilon_2)[\Phi(k_l,p,w_2) + w_2\frac{\partial\Phi(k_l,p,w_2)}{\partial w_2}] = 0$$

SOC $$\upsilon_2[2\frac{\partial\Phi(k_u,p,w_2)}{\partial w_2}] + w_2\frac{\partial^2\Phi(k_u,p,w_2)}{\partial w_2^2}] +$$

$$(1-\upsilon_2)[2\frac{\partial\Phi(k_1,p,w_2)}{\partial w_2} + w_2\frac{\partial^2\Phi(k_1,p,w_2)}{\partial w_2^2}] < 0$$

The second order condition is assumed to be satisfied, i.e., for all values of k

$$2\frac{\partial\Phi(k,p,w_2)}{\partial w_2} + w_2\frac{\partial^2\Phi(k,p,w_2)}{\partial w_2^2} < 0$$

From the first order condition

$$d\upsilon_2[\Phi(k_u,p,w_2) + w_2\frac{\partial\Phi(k_u,p,w_2)}{\partial w_2} - (\Phi(k_1,p,w_2) +$$

$$w_2\frac{\partial\Phi(k_1,p,w_2)}{\partial w_2}] + [\text{S.o.C.}]\cdot dw_2 = 0 \Rightarrow \frac{dw_2}{d\upsilon_2} = \frac{[A]}{-[\text{S.O.C.}]}$$

where

$$A=\Phi(k_u,p,w_2)+w_2\ \frac{\partial\Phi(k_u,p,w_2)}{\partial w_2} - (\Phi(k_1,p,w_2)+w_2\frac{\partial\Phi(k_1,p,w_2)}{\partial w_2}$$

A gives the value of

$$\frac{\partial}{\partial\upsilon_2}(\frac{\partial w_2\Phi_2(.,.,.,.)}{\partial w_2})$$

Using assumption ii) of Lemma 1 stating that $\Phi(.,.,.,.)+(w_2(\partial\Phi(.,.,.,.)/\partial w_2))$ is an increasing function of k, it is clear that $A>0$.

Hence,

$$\frac{\partial w_2}{d\upsilon_2} > 0$$

Q.E.D.

<u>Proof of Lemma 2</u>. Show that

$$\frac{\partial \pi_2^F}{\partial w_2} < 0 \quad \text{as} \quad \frac{\partial w_2}{\partial \upsilon_2} > 0$$

Write the second period profit as

$$\pi_2^F = p \cdot f(k,l) - w_2^* l$$

Using the Envelope Theorem

$$\frac{\partial \pi_2^F}{\partial w_2^*} = -\Phi < 0$$

as $p \cdot (\partial f / \partial l) - w_2 = 0$ is the ordinary first order condition for the optimal level of employment.

Q.E.D.

<u>Proof of Lemma 5</u>. Consider

$$\underset{l_2}{\text{Max}}\ pf(l_2,k) - w_2 \cdot l_2$$

The first order condition prescribes a labour demand function of the form $\Phi_2(w_2,k)$. Hence, production $t_2{}^* = f(\Phi_2(w_2,k),k)$. Thus,

$$\frac{\partial t_2^*}{\partial w_2} = \frac{\partial f}{\partial l_2} \frac{\delta \Phi_2(.,.)}{\partial w_2} < 0$$

and

$$\frac{\partial t_2^*}{\partial k} = \frac{\partial f}{\partial l_2} \frac{\partial \Phi_2(.,.)}{\partial k} + \frac{\partial f}{\partial k} > 0$$

Q.E.D.

Proof of Proposition 2. Labour demand is given by $\Phi_2(w_2,k)$. Assume that for some w_2 $w_2\Phi_2(w_2,k)$ is maximized, thus first and second order conditions are satisfied. Assume $t_1'>0$. Thus any observed x_1 reveals the correct value of k. Consequently, w_2 as some function of k, $\gamma(k)$ is given by

$$\Phi_2(w_2,k) + w_2\frac{\partial \Phi_2(w_2,k)}{\partial w_2} = 0$$

Hence,

$$w_2 = \gamma(k) = \gamma(t^{-1}(x_1)) = s(x_1)$$

From the first order condition

$$\Phi_2(\gamma(k),k) + \gamma(k)\frac{\partial \Phi_2(\gamma(k),k)}{\partial w_2} \equiv 0$$

Hence,

$$\gamma' \equiv -\frac{\frac{\partial \Phi_2}{\partial k} + w_2 \frac{\partial^2 \Phi_2}{\partial w \partial k}}{2\frac{\partial \Phi_2}{\partial w_2} + w_2 \frac{\partial^2 \Phi_2}{\partial w_2^2}}$$

Using assumption ii) Lemma 1 and assuming that second order conditions are satisfied it is concluded that $s'>0$.

Q.E.D.

Proof of Proposition 3. To show that t_1 is a continuous differentiable strategy use Theorem 2 of Mailath (1987) which states that if an appropriate initial value condition is satisfied, then the strategy is continuous. Note that

$$t_1(k^w) = f^*(k^w) \quad \text{where} \quad \text{if } p_2<0 : k^w=k_u$$

$$\text{and} \qquad p_2>0 : k^w=k_l$$

This initial value condition is implied by sequentiality. Note that $t_1(k) \leq f^*(k)$ where f^* denotes the competitive solution. Suppose that $t_1([k_l,k_u]) \to [0,f^*(k_u)]$ is one-to-one incentive compatible and that $t_1(k_u)=f^*(k_u)$. If the firm deviates from $t_1(k_u)$ to $f^*(k_u)$ in the first period, he increases first period profits. If, furthermore, some $k \varepsilon [k_l,k_u]$ exists so that k is inferred from $f^*(k_u)$, then the wage claim will be lower than it would if it was known that $k=k_u$. Hence, second period profits increase. However, if $f^*(k_u)=t_1([k_l,k_u])$, the union behaves according to $\psi(k)$ and the support $[k_l,k_u]$. Hence, the following maximization problem is solved.

$$\int_{k_l}^{k_u} w_2 \Phi_2(w_2,k)\psi(k)dk$$

The first order condition is

$$\int_{k_1}^{k_u} [\Phi_2(w_2,k)\psi(k)+w_2 \frac{\partial\Phi_2(w_2,k)}{\partial w_2} \psi(k)]dk = 0$$

This defines a function giving the wage claim as

$$w_2 = \gamma[\int_{k_1}^{k_u} k\psi(k)dk]$$

which is certainly less than the maximum wage ever faced by any firm: $w_2^{max}=\gamma(k_u)$. Hence, in conclusion $k^w=k_u$, so the initial value condition is satisfied.

To prove that $t_1'>0$ note that $p_{13}>0$ (this in effect is due to the assumption that $s'>0$).

Proof of Lemma 6. Assume that s is discontinuous (Figure A.1)

Figure A.1 (see next page)

Hence, for $\varepsilon \to 0$

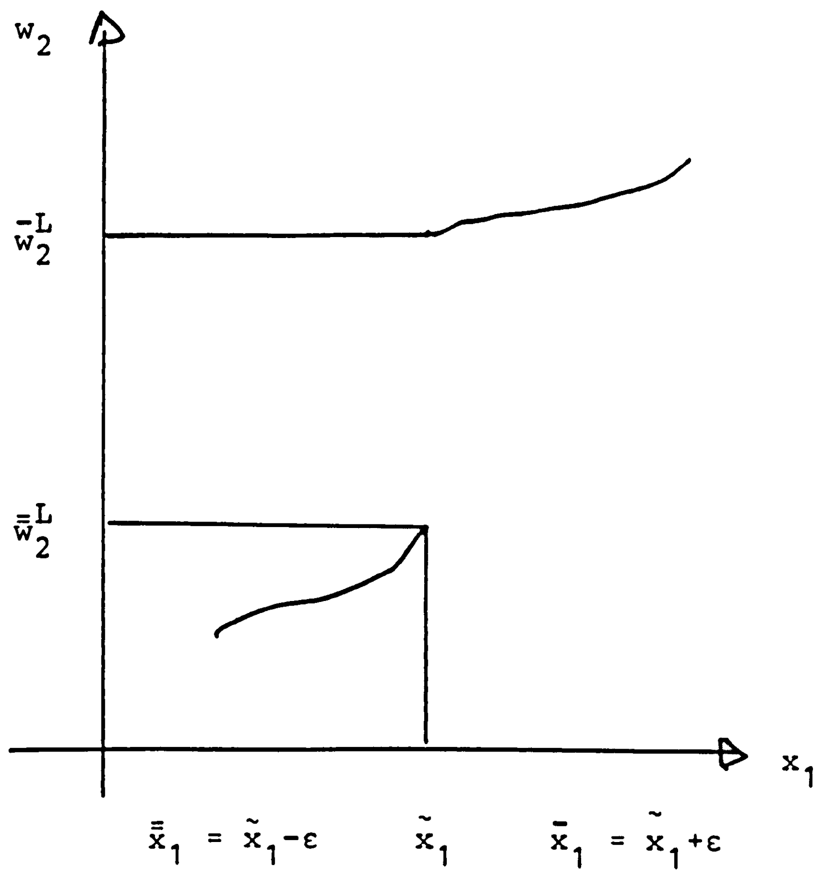
$$\bar{x}_1 \to \tilde{x}_1$$

$$\bar{\bar{x}}_1 \to \tilde{x}_1$$

$$w_2 \to \bar{w}_2^L$$

$$w_2 \to \bar{\bar{w}}_2^L$$

Figure A.1



w_2
$\bar{w}_2^L$
$\bar{\bar{w}}_2^L$
x_1
$\bar{\bar{x}}_1 = \tilde{x}_1 - \varepsilon$
$\tilde{x}_1$
$\bar{x}_1 = \tilde{x}_1 + \varepsilon$

Demand in the second period is given by $\Phi_2(w_2,k)$ which is continuous in both arguments. For any given k some value w_2 exists which maximizes $w_2\Phi_2(w_2,k)$. This function $w_2=\gamma(k)$ is also continuous.

Define $G(k_1,k_2)=\gamma(k_1)\Phi_2(\gamma(k_1),k_1)-\gamma(k_2)\Phi_2(\gamma(k_2),k_2)$. Clearly from the continuity of Φ_2 and γ

$$\lim_{k_1\to k_2} G(k_1,k_2) = 0$$

For $t_1'>0$

(A.8) $G[t_1^{-1}(\bar{x}_1),t_1^{-1}(\bar{\bar{x}}_1)] =$

$$\gamma(t_1^{-1}(\bar{x}_1))\Phi_2(\gamma(t_1^{-1}(\bar{x}_1)),\bar{k}_1)-\gamma(t_1^{-1}(\bar{\bar{x}}_1))\Phi_2(\gamma(t_1^{-1}(\bar{\bar{x}}_1)),\bar{\bar{k}}_1) =$$

$$s(\bar{x}_1)\Phi_2(s(\bar{x}_1),\bar{k}_1 - s(\bar{\bar{x}}_1)\Phi_2(s(\bar{\bar{x}}_1),\bar{\bar{k}}_1)$$

We know that $\lim_{\bar{k}_1\to\bar{\bar{k}}_1} G = 0$. However, as $\lim_{\bar{x}_1\to\tilde{x}_1} s(x_1) = w_2{}^L$ and $\lim_{\bar{\bar{x}}_1\to\tilde{x}_1} s(\bar{\bar{x}}_1) = \bar{w}_2{}^L$, from (A.8) $\lim_{\bar{k}_1\to\bar{\bar{k}}_1} G(k_1,k_2) \neq 0$ which is a contradiction, hence, s is continuous.

Q.E.D.

<u>Proof of Proposition 4</u>. s as given in (5) is optimal against t_1, hence, it is left to prove that t_1 satisfying (7) is optimal against s.

For any given value of k, the firm will never consider producing more than $f^*(k)$, the competitive output. Also an absolute lower bound is obtained in $t_1(0)$, hence, $t_1(0) \le t_1(k) \le f^*(k)$. We have to show that the optimization problem is quasiconcave on $[t_1(0), t_1(\bar{k}))$. Then the first order condition (11) is sufficient for local optimality. Since any outcome of the strategy t_1 belongs to $[t_1(0), f(k_u)]$, this also ensures global optimality.

Let the expected value of the profit of a firm with capacity z playing k be $\pi^F(z,k)$ and let $\partial\pi^F/\partial x_1$ be evaluated at (z,k). The differential equation (7) is written

$$\frac{\partial\pi}{\partial x_1}(z, f^{-1}(t,(k))) \equiv 0$$

Consider $x_1 = f(k)$.

The strategy now is that we prove that if any other signal than x_1

$$\tilde{x}_1 = f(\tilde{k})$$

is chosen, then the firm prefers to return to the original signal.

Let us first give an expression for $\partial\pi^F/\partial x_1$

$$\frac{\partial\pi^F(z,k)}{\partial x_1} = \frac{\partial\pi_1^F(z,k)}{\partial x_1} + \frac{\partial\pi_2^F(z,\gamma(k))}{\partial x_1}\,\frac{\gamma'}{t_1'}$$

Now consider

$$\frac{\partial\pi^F(z,\tilde{k})}{\partial x_1} - \frac{\partial\pi^F(z,k)}{\partial x_1}$$

That is, we examine the first order condition if we change the signal from k to $\tilde{k}$.

We have

$$\frac{\partial\pi_1^F}{\partial x_1}(z,\tilde{k}) - \frac{\partial\pi_1^F}{\partial x_1}(z,k) + \left(\frac{\partial\pi_2^F(z,\gamma(\tilde{k}))}{\partial w_2} - \frac{\partial\pi_2^F(z,\gamma(k))}{\partial w_2}\right)\frac{\gamma'}{t_1'}$$

Consider $\tilde{k}>k$ then

$$\frac{\partial\pi_1^F(z,\tilde{k})}{\partial x_1} - \frac{\partial\pi_1^F(z,k)}{\partial x_1} > 0$$

$$\frac{\partial\pi_2^F(z,\gamma(\tilde{k}))}{\partial w_2} - \frac{\partial\pi_2^F(z,\gamma(k))}{\partial w_2} < 0$$

Hence,

$$t_1'(z) > \inf \gamma' \frac{\dfrac{\partial\pi_2^F(z,\gamma(\tilde{k}))}{\partial w_2} - \dfrac{\partial\pi_2^F(z,\gamma(k))}{\partial w_2}}{\dfrac{\partial\pi_1^F(z,k)}{\partial x_1} - \dfrac{\partial\pi_1^F(z,k)}{\partial x_1}} \Rightarrow \frac{\partial\Pi^F}{\partial x_1}$$

Hence, $\partial\pi^F/\partial k$ for $k>\bar{k}$. Thus, the firm never chooses a signal $\bar{k}<k$. Alternatively, for $\bar{k}<k$.

$$\frac{\partial\pi_1^F(z,\tilde{k})}{\partial x_1} - \frac{\partial\pi_1^F(z,k)}{\partial x_1} < 0$$

$$\frac{\partial\pi_2^F(z,\gamma(\tilde{k}))}{\partial w_2} - \frac{\partial\pi_2^F(z,\gamma(k))}{\partial w_2} > 0$$

Hence, for

$$t_1'(z) > \inf \gamma' \frac{\dfrac{\partial\pi_2^F(z,\gamma(\tilde{k}))}{\partial w_2} - \dfrac{\partial\pi_2^F(z,\gamma(k))}{\partial w_2}}{\dfrac{\partial\pi_1^F(z,\tilde{k})}{\partial x_1} - \dfrac{\partial\pi_1^F(z,k)}{\partial x_1}} \Rightarrow \frac{\partial\pi^F}{\partial x_1} < 0$$

Thus, $\partial\pi^F/\partial k<0$. This ensures optimality

Q.E.D.

<u>Proof of Proposition 5</u>. If a solution to (8) exists, Proposition 4 ensures that σ is identical to the t_1 identified in that proposition and, hence, $t_1=\sigma^{-1}$ solves (3).

Let $f(x_1,\sigma)$ be continuous in x_1 and differentiable in σ. From a theorem in the theory of differential equations (Pontryagin (1962)), there exists some maximal value x_1^* such that

(A.9) $\quad \sigma(t_1(k_u)) = k_u$

such that (12) is satisfied on $[0,x_1^*]$. In our case we actually know the value of x_1^*, it is nothing but $x_1^*=f^*(k_u)$; the initial value condition following from sequentiality.

Consider the case where x_1^* is finite (this is obviously the case in our problem). We have that as $x_1 \to (x_1^*) \Rightarrow \sigma \to -\infty$, hence,

$$\sigma(x_1) \leq [k_l, k_u]$$

Thus, range of σ; the solution to the differential equation includes $[k_l, k_u]$ and $\sigma' > 0$ is clealy seen from the figure and also follows from (12). Thus, $t_1 \equiv \sigma^{-1}$ exists, is unique and solves (11).

CHAPTER 3

WAGE DETERMINATION AND POOLING EQUILIBRIA IN A UNIONIZED ECONOMY

I. Introduction

Recently much effort has been directed at the question of wage formation in unionized economies[1] There are good reasons for this. In some countries, in particular the Scandinavian and some western European countries, the union participation rate is very high indeed (Calmfors 1989) and the traditional Walrasian assumption is far from reasonable. Furthermore, from a macroeconomic point of view it is unfortunate that dominant lines of thought have failed to incorporate obvious characteristics of modern economies.

Hitherto, efforts have been directed in principally two directions. One is the study of microeconomic behavior of trade unions (see Oswald (1985)). Primarily two competing models are on the scene, the monopoly-union model, in which it is assumed that the trade union dictates the wage unilaterally, and the efficient bargaining model where efficiency may prevail if for example the parties bargain over wage but the employment decision is left to the firm (Oswald (1985)). The second major concern is the macroeconomic implication of the existence of trade unions, or to be more precise the interaction between policy measures and wage formation. These problems are typically formulated as a game and the aim is to identify the causes of unemployment in economies with centralized wage setting (see for example Driffil (1986)).

The approach of this chapter falls mainly within the last of these two lines of research but is, however, somewhat different.

With respect to macroeconomic dynamics it is of great interest to analyse the effects exclusively due to the fact that a bargaining process take place under asymmetric information. In these circumstances the level of output and employment may serve as a signalling device. In otherwise static models the resultant equilibrium may be inefficient in the sense that output and employment deviate from the simple static solutions. One such case was analysed in Chapter 2. It was established that a welfare loss occur. However, the equilibrium obtained there possessed some salient features, given that information is asymmetric. In particular, different firms behave differently, i.e. the equilibrium is separating. In the current chapter the possibility of obtaining pooling equilibria is discussed.

We discuss wage determination in a model of sequential equilibrium. An ill-informed planner or auctioneer attempts to set a wage which clears the market. Before the start of the first period the wage for this period is fixed. Based upon this wage the firm reacts by choosing an equilibrium level of employment. Prices on goods are assumed to be exogenous. Depending upon observation of the first period level of employment and prior knowledge the planner dictates the wage ruling in the second period. We impose the following restrictions upon the behavior of the firm. The firm recognizes the fact that any first period choice of labour and output is used by the planner in order to dictate the second period wage. Hence, the firm may deviate from its normal first best behavior. Assume that the firm may claim that the level of demand is, say, lower than what is actually the case. To behave consistently the firm has to adjust employment to a level corresponding to the announced level of demand. Assume, furthermore, that after the end of the first period the price is observed. Hence, also output and input of capital, which we assume to be unobserved, have to adjust to the level of demand ruling. From this the planner draws inferences about demand and dictates a second period wage.

During the second period the firm is restricted to choose a level of employment consistent with the second period wage and the postulated level of demand. Otherwise workers have the right to default and are assumed to do so. However, the price is no longer any restriction since it is revealed to the planner and the workers only after the second period when the game has ended. It is shown, under the circumstances postulated in this paper, that a planning equilibrium cannot be a fully revealing equilibrium. Furthermore, if we want to restrict ourselves to "nice" equilibria, we end up with a pooling equilibrium. This is of potential interest to macroeconomic dynamics and will be investigated in a less complicated setting in Chapter 6.

It is, however, also shown that if the idea of a planning equilibrium is given up and replaced by a decentralized economy, then due to monopoly profits only a separating equilibrium exists.

Before proceeding to the analysis it is worthwhile to discuss the circumstances which are favourable to pooling equilibria. In models encompassing some dynamics it is natural to use the idea of sequential equilibrium (Kreps and Wilson (1982)). If the transmission of information is costly, for example by deviation from an otherwise first best situation, a separating equilibrium will normally result. An exception to this is that the set of possible signals are bounded (see Mailath (1988)). An alternative to this is to narrow down the range of types so that no matter what action is observed by the less informed part this action contains virtually no information (Laffont and Tirole (1986)). This is the approach taken in this paper.

In the next section the model is set forth and a static analysis is given. Next, in Section III the dynamic case is analysed and concluding remarks are found in Section IV. Proofs are given in an appendix.

II. One Period Analysis

II.1 The Model[2)]

Demand is given stochastically by

$$(1) \quad p = \theta D(y)$$

where $\theta \epsilon [\underline{\theta}, \bar{\theta}]$ and $f' < 0$, $f'' > 0$. Production takes place according to a commonly known production function defined over employment and capital stock. In the following, we assume capital stock to be invariant.

(2) $y = F(L,K)$

Labour is supplied according to the following separable utility function

(3) $U = u(wL) + \upsilon(L)$

We may prefer to interpret (3) as representing the utility of any individual worker and hence, require that the equilibrium reflects, under uncertainty, the degree of risk aversion as introduced in (3). Alternatively, and to give the competitive solution its best chance L in (3) can be thought of as composed by n identical workers, and if n is large (approaching infinity) argue that the wage has to equal expected marginal disutility of labour (cf. Arrow & Lind (1970)). Conclusions are given for both of these interpretations.

II.2. The Solution under Symmetric Information

As a reference consider first the solution under the assumption that θ is known by both parties. We consider the decentralized economy first and as a special case the planned economy.

Equilibrium requires

(4) $\theta F_L D(1+E_{p,y}) = w$

$$(5)\quad E_{p,y} = \frac{dD(y)}{dy}\,\frac{y}{D(Y)}$$

Using (4) and assuming that central costs of capital are fixed and, furthermore, assuming that second order conditions are satisfied

$$(6)\quad L^d = \psi(\tfrac{w}{\theta}), \qquad \psi'<0$$

Now, consider the optimization problem of the union

$$\max_{w}\ u(w\psi(\tfrac{w}{\theta})) + \upsilon(\psi(\tfrac{w}{\theta}))$$

The first order condition is

$$(7)\quad u'(\cdot)\{\psi(\cdot) + \tfrac{w}{\theta}\psi'(\cdot)\} + \tfrac{1}{\theta}\,\upsilon'(\cdot)\psi'(\cdot) = 0$$

In equilibrium we have

$$(8)\quad \frac{dw}{d\theta} = -\,\frac{\frac{\partial}{\partial\theta}\{u'(\cdot)\{\psi(\cdot) + \frac{w}{\theta}\,\psi'(\cdot)\} + \upsilon'(\cdot)\frac{1}{\theta}\,\psi'(\cdot)\}}{\frac{\partial}{\partial w}\{u'(\cdot)\{\psi(\cdot) + \frac{w}{\theta}\,\psi(\cdot)\} + \upsilon'(\cdot)\frac{1}{\theta}\,\psi'(\cdot)\}}$$

Assuming that (7) actually is an optimum, we have

$$(9)\quad \frac{\partial}{\partial w}\{u'(\cdot)\{\psi(\cdot) + \tfrac{w}{\theta}\,\psi'(\cdot)\} + \upsilon'(\cdot)\tfrac{1}{\theta}\,\psi'(\cdot)\} < 0$$

Thus,

$$(10)\quad \mathrm{Sign}\,[\tfrac{dw}{d\theta}] = \mathrm{Sign}[\tfrac{\partial}{\partial\theta}\,\{u'(\cdot)\{\psi(\cdot) + \tfrac{w}{\theta}\,\psi(\cdot)\} + \upsilon'(\cdot)\tfrac{1}{\theta}\,\psi'(\cdot)\}]$$

We show the following

Lemma 1: If the indifference curve defined from $U=u(wL)+\upsilon(L)$ in the (L-w) space is characterized by $\partial L/\partial w>0$ and $\partial^2 L/\partial w^2<0$, then

(11) $u'(\cdot) + u''(\cdot)wL > 0$

Proof: See Appendix.

Figure 1 (see next page)

Now we can show

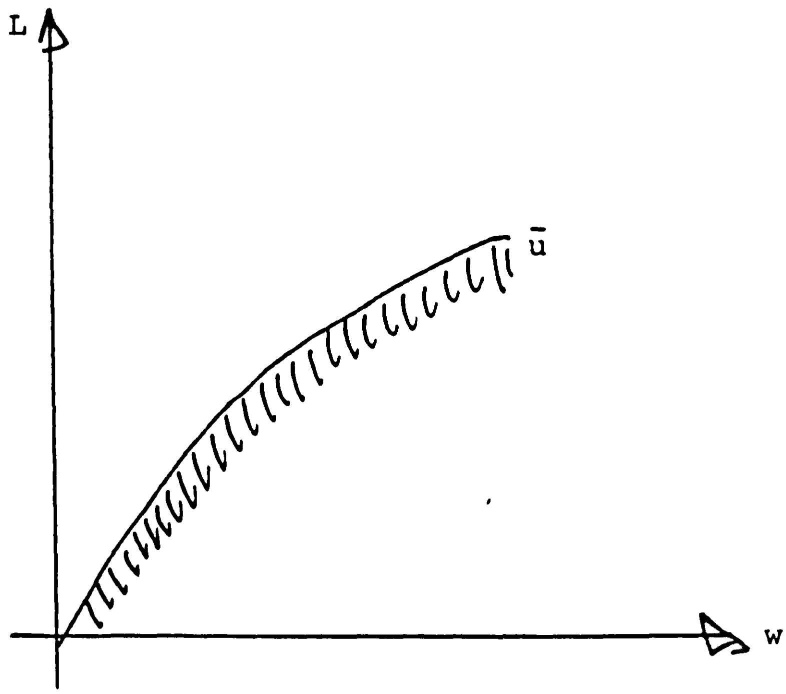
Lemma 2: If $\partial L/\partial w>0$ and $\partial^2 L/\partial w^2<0$, then

(12) $\frac{dw}{d\theta} > 0$

Proof: See Appendix.

Equation (12) contains the basic idea behind the dynamic analysis. In a continuing relationship between two parties bargaining over wage, (12) taken together with the assumption of uncertainty offers an incentive on the part of one of the parties, the firm, to make an attempt to persuade its opponent, the planner that the value of θ is lower than what is actually the case. Alternatively, consider the firm faced with some $\theta\epsilon[\underline{\theta},\bar{\theta}]$. This firm announces some θ, perhaps the true one, to the trade union. However, if such an announcement is costless, the

Figure 1



$$\frac{\partial L}{\partial w} > 0 \qquad \frac{\partial^2 L}{\partial w^2} < 0$$

trade union cannot put any thrust in it. The reason, of course, is found in (12) saying that wages are increasing in θ. Thus, the firm, if signalling cost is zero, announces $\underline{\theta}$. This has to be recognized in any solution of the dynamic case. This idea is similar to those found in the planning literature (e.g. Freixas, Guesnerie, Laffont (1984) and the literature on implicit contracts (e.g. Hart & Holmström (1987)). Before we consider the one period equilibrium under asymmetric information, let us briefly concern ourselves with the agents attitude towards risk.

However, let us first see that our results are qualitatively unchanged for a competitive or planning economy. The planned economy is characterized by the fact that the planner dictates all prices that is to say $dD(y)/dy=0$. Thus, in this case the solution is characterized by (compare with (4))

(4') $\theta F_L \cdot D = w$

Also, labour demand is given by (compare with (6))

(6') $L^d = \tilde{\psi}(\frac{w}{\theta}) \quad \tilde{\psi}' < 0$

Equilibrium in the labour market requires

(7') $u'(\cdot)w + \upsilon'(\cdot) = 0$

Finally, also in a Walrasian equilibrium

$$(12') \quad \frac{dw}{d\theta} > 0$$

Thus, if the economy consists of firms, workers and a planner (implementing a command equilibrium), then, unless the planner is just as well informed as the firm, the same problem arises as in a free economy.

II.3. Risk Considerations

Since the utility function of the union is concave, they will, in general, hedge against risk in the wage claim put forward. In the following we analyse pairs of the wage and level of employment, enforcing a fully revealing equilibrium. However, let us here just assume that the planner or trade union (depending upon whether we study a command equilibrium or a decentralized equilibrium) put forward a wage claim not contingent upon the realization of employment.

Assume that total labour supply is made up of the labour supply of n identical individuals.

If n is large (in principle as n approaches infinity), using an argument due to Arrow and Lind (1970), the planner optimizes by choosing the wage so as to equate the expected marginal utility of labour to the expected marginal disutility of labour, i.e.

$$(13) \int_{\underline{\theta}}^{\bar{\theta}} u'(\cdot)wg(\theta)d\theta = -\int_{\underline{\theta}}^{\bar{\theta}} \upsilon'(\cdot)g(\theta)d\theta$$

The idea behind (13) is as follows. Even though the decisions made by the trade union have to reflect the preferences of union members, it can be the case that the union should ignore the attitude towards risk as given in (3). This will be the case if it is assumed that total income is distributed (evenly) among union members. The reason is that for a large number of members the income and associated risk for any member is insignificant compared to total income (Arrow and Lind (1970) pp. 373-374). In such circumstances the cost of risk-bearing to each individual approaches zero as the number of members approach infinity (this is the result of Section II of Arrow and Lind (1970)). Consider the static optimization problem. The role of the planner, accordingly, is to choose a wage which clears the market. This is also the wage which obtains overall economic efficiency. Best of all, of course, would be the case where the chosen wage equals the marginal disutility of labour. Having chosen this particular wage the equilibrium conditions on the factor market implies production efficiency and in addition equilibrium as well as efficiency on the goods market. However, when θ is unknown, or rather known only probabilistically, the planner acts according to (14) or in general according to the utility function given in (3).

II.4. The Solution under Asymmetric Information

Now, consider the situation arising under asymmetric information. Consider the structure of the model. As noted in the introduction the equilibrium is separating for the one period version of the model. Thus, the uncertainty with respect to θ is resolved in the solution to be found. However, even though θ is revealed to the union, it is the case that at the time when the wage is dictated by the trade union θ is known only probabilistically. Since the utility function given in (3) is concave, reflecting risk aversion, the union - in a planning equilibrium - has to maximize by choosing L appropriately

(14) $E(U) = E(u(wL)) + E(\upsilon(L))$

The wage, w, is dictated by the planner, L^d is given by

$$L^d = \psi(\frac{w}{\theta}) \qquad \psi' < 0$$

respectively,

$$L^d = \tilde{\psi}(\frac{w}{\theta}) \qquad \tilde{\psi}' < 0$$

The parameter θ belongs to the support $[\underline{\theta},\bar{\theta}]$ and is distributed according to dG=g where $g(\cdot)$ is strictly bounded from below at zero.

Given any wage claim the firm chooses some labour input. The first propositi considers the restrictions upon the strategy choosing labour input.

<u>Proposition 1</u>. For any given wage w, for $l^*(\theta)$ to be implementable $l^*(\theta)$ is a fully revealing strategy where $\partial l^*/\partial\theta > 0$ and

hence, differentiable almost everywhere.

Proof: See Appendix.

The idea of the proof is simple. The argument given is that for $l^*(\theta)$ to be implementable it must be preferred to some other strategy, in particular the strategy $l^*(\theta'|\theta)$[3], i.e., $\pi(l^*(\theta)) > \pi(l^*(\theta'|\theta))$. The restrictions on the form of the profit function as well as the assumptions made with respect to the information set ensures the result.

In Proposition 1 only the sign of $\partial l^*/\partial\theta$ is given. Note that Proposition 1 allows for the static first best level of labour demand. This is not surprising considering the structure of the model. Given that the trade union have decided upon some wage according to (13) or (14), the firm is a residual claimant and the firm behaves according to the first best.

Proposition 1 does not offer a full characterization of the one period or static equilibrium since we are concerned here with pairs of $\{w,L\}$ supporting fully revealing equilibria. To obtain such a characterization the labour supply must be recovered for all values of $\theta\epsilon[\underline{\theta},\bar{\theta}]$. The labour supply is derived given the restriction that the trade union recognizes the incentive on the part of the firm to misrepresent θ. To be more precise; labour supply is some functional relationship between w and L and θ. This relationship is decided upon prior to any exchange of wage for labour. Thus, in announcing θ the firm may misrepresent θ in order to obtain a more favourable wage. The labour supply function is restricted to enforce the firm to reveal θ truthfully. The labour supply function is characterized in Proposition 2 and Corollary 1. Let the wage dictated by the planner be given as a function of profits: $T(\pi)$.

Proposition 2. The supply of labour implicit in the planning equilibrium is given by the equation

$$u'(\cdot)T(\pi) + \upsilon'(\cdot) = (Z(\bar{\theta}) - Z(\theta))(D'y+D)y_L$$

$$Z(\bar{\theta}) - Z(\theta) > 0 \;\forall\; \theta \neq \bar{\theta}$$

where $Z(\cdot)$ is an increasing function (described in the Appendix).

Proof: See Appendix.

Corollary 1: For $\theta=\bar{\theta}$ the first best solution is realized.

The above result need a few comments. The term $T(\pi)$ which gives the wage which is contingent upon profits since a change in "profits opportunities" changes the behavior of the firm. The term on the right hand side is due to the incentive compatibility restrictions added to the problem.

Corollary 1 is known in the literature as the "no distortion at the top" result. Intuitively we have proven the following. Consider a firm faced with $\theta=\bar{\theta}$. This firm can do no better than

/

play $L^*(\theta)=L^*(\bar{\theta})$ because $L^*(\bar{\theta})$ is played by the firm facing the worst wage claim. Such a firm can only do worse choosing some $L^*(\theta)$ indicating that the realized value of $\theta>\bar{\theta}$. On the other hand, choosing some $L^*(\theta)$ indicating $\theta<\bar{\theta}$ the firm is not believed and it will still face the maximum wage.

From Proposition 2

Claim 1. For $\theta<\bar{\theta}$ the wage is less than the marginal disutility of $L^*(\theta)$.

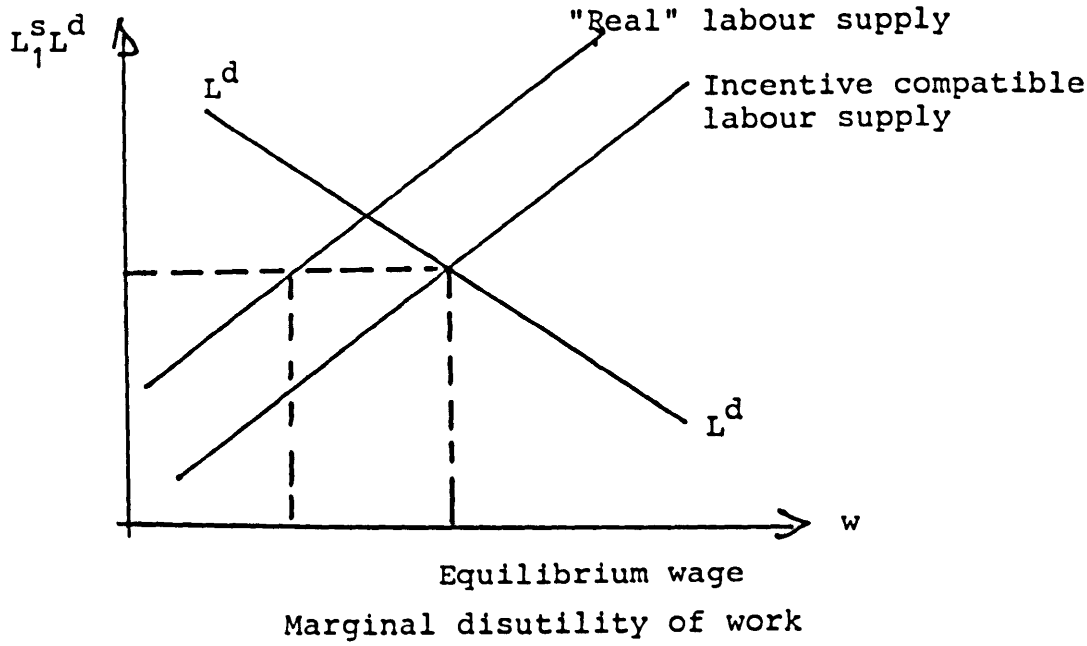
This result may easily be illustrated (cf. Figure 2).

Figure 2 (see next page)

In essence this result is due to the fact that any firm $\theta<\bar{\theta}$ is confronted with a labour supply scheme that support the announcement θ. This labour supply schedule is given in Proposition 2 and lies below the one for the case of symmetric information. This result reflects the cost to workers of obtaining full information.

Furthermore, since labour demand is undisturbed and labour supply is lowered, the level of employment is inefficiently low. This result is akin to some of the results obtained in the literature on contracts under asymmetric information (cf. Hart & Homström (1987)) although the current results point towards underemployment as opposed to overemployment.

Figure 2



$L_1^s L^d$
"Real" labour supply
L^d
Incentive compatible
labour supply
L^d
w
Equilibrium wage
Marginal disutility of work

At this point perhaps some comments on the result obtained so far are needed. Given that the wage is determined according to (14) why is it not the case that firms hire the Walrasian level of employment? The explanation of this apparantly counterintuitive result is found in the structure of the economy envisaged in the analysis. For the wage given by (14) all the labour that is needed will be supplied, with one caveat however. That is, workers will only enter into a relationship with firms if they learn the true value of θ. This imposes some costs to the economy. These costs are described by the relationship between the level of employment and wage and θ, as given in Proposition 2 and Corollary 1.

The institutional setting characterizing the economy analysed perhaps seem somewhat artificial. For this at least two apologies can be made. Firstly, one of the main points to be illustrated in this paper is that even if the simple static equilibrium in a planned economy is a separating one, it is the case that in a dynamic setting pooling equilibria occur. Thus, the analysis of the current section serves as a reference to later results. Secondly, if it is accepted that a conflict of interest exists in the labour market, it seems to be reasonable to accept only incentive compatible solutions, i.e. solutions which truthfully reflect the parameter θ.

In the proof of Proposition 2 and Corollary 1 use was made of the formulation given in (13), that is, risk aversion on part of the workers is ignored. This is not a restriction if adopting the following assumption.

Assumption: $G(\theta)$ is strictly increasing.

This assumption gives separation its best chance. It is not a restriction since the aim is to illustrate that despite the fact that separation is the outcome in the static case this is not so when the model is phrased in a dynamic context.

Proposition 3: Proposition 2 and Corollary 1 remain valid using (14) instead of (13).

Proof: See Appendix.

The result of Proposition 3 can be shown to be valid also for the case of a decentralized economy, that is, the union now takes into consideration the effect of L upon w.

Corollary 2: The results of Proposition 3 remain valid in the decentralized economy.

Proof: See Appendix.

Let us summarize the findings before the dynamic model is considered. We take the command optimum as a benchmark since this gives the first best solution the best chance of surviving. Under certainty the planner dictates a wage which ensures efficiency. In the corresponding decentralized economy inefficiencies occur. However, these arise because of first mover advantages. Introducing uncertainty, the results are modified. In a command equilibrium the labour supply schedule is based upon expectations with respect to the realization of demand. Such a labour supply schedule ensures that the firm truthfully reveal the realized value of θ, but does not, in general, ensure economic efficiency. This result is also valid in a decentralized economy. In the case of what we may term a "static countinuing relationship", i.e. where only incentive compatible choices may be made, the well known "no distortion" at the top result was established. Hence, for $\theta=\bar{\theta}$ the wage set by agency support efficiency in production. For $\theta<\bar{\theta}$ this is no longer the case. Any incentive compatible solution is a separating equilibrium, that is, the labour input chosen by the firm reveals the realized value of θ. But employment is inefficiently low.

III. Dynamic Analysis

Proposition 1 identified the sign of $\partial l^*/\partial w$. Proposition 2 and Corollary 1 together with labour demand characterized the level of employment. The previous analysis was confined to one period only. The aim of the currrent section is to extend the model to two periods. This allow for a study of the determination of the inference of the union with respect to θ.

Define normal profit as

(15) $\pi_N = py - wL - rK - R$

where R is economic rent. In the standard neo-classical world economic rent arises due to, for example, differences in the quality of land. We allow for monopoly rent in Section III.2, and we will see that this changes the results dramatically.

We proceed by characterizing the solution to the dynamic programming problem faced by the firm and the agency. In particular an equilibrium is described by $w_1(\tilde{\theta}_1)$, $w_2(\tilde{\theta}_2)$ where $w_i(\tilde{\theta}_i)$, i = 1,2 is the wage dictated by the agency in the first and second period, respectively. $\tilde{\theta}_i$, i = 1,2 denotes the value of θ as inferred by the agency, and obviously in the first period this is based only on prior knowledge. The value of $\tilde{\theta}_2$ may either equal the true value of θ, this is the case in a separating equilibrium, or may be based upon prior knowledge. This is the case in a pooling equilibrium. To complete the description of the equilibrium add the vector (L_1,x_1), (L_2,x_2), i=1,2. L_i describes the decision of the firm with respect to the level of employment, and x_i the decision whether to stay in the market or not (x_1 takes the value 0 or 1).

Only Perfect Baysian Equilibria (PBE) are considered. This equilibrium concept captures the idea of dynamic programming and its features are described by P1) to P4) in addition to B).

P1) L_2,x_2 maximizes π^2 given $w_2(\cdot)$

P2) $w_2(\cdot)$ is optimal given $\tilde{\theta}_2$

P3) L_1, x_1 max $\pi_1 + \delta\pi_2$ given $w_1(\cdot)$ and $\tilde{w}_2(\cdot)$ where $\tilde{}$ denotes a conjectured value.

P4) $w_1(\cdot)$ max $u_1 + \delta u_2$ given $\tilde{L}_1$, $\tilde{L}_2$, $\tilde{w}_2$

B) θ is derived from the support $[\underline{\theta},\bar{\theta}]$, distribution function, P3) and L_1 using Bayes' rule.

In the following attention is focused upon continuation equilibria: a set of strategies satisfying P1), P2), P3) and B) given any strategy $w_1(\cdot)$.

III.1. Command Equilibrium

In our world with only one (representative) firm, the standard Walras factor market equilibrium conditions are

$$w = \theta D(F(L,K))F_L(\cdot,\cdot)$$

$$r = \theta D(F(L,K))F_K(\cdot,\cdot)$$

Assuming that the production function is of the CRS type, these conditions result in R=0. However, if p is unknown, as is the case here, and due to be inferred by some imperfectly informed agency, then in a repeated relationship (here two periods) the firm has an incentive to misrepresent the value of the parameter θ in order to be confronted with a second period wage which is lower than the wage which obtains if θ is revealed. In order to misrepresent the value of θ the firm may have to deviate from the static first best solution. As a matter of fact this was the case also in the static analysis. What is shown here is that in the dynamic repeated relationship with small uncertainty pooling equilibria are a robust feature of the model.

The following result is obtained (cf. Laffont and Tirole (1986)).

Proposition 4. For any w_1 (such that $x_1=1$) there exists no fully separating continuation equilibrium.

Proof. See Appendix.

The intuition behind the argument, given rigorously in the proof, is the following (Laffont and Tirole (1986)). As the agent in a separating equilibrium obtains a second period profit of zero, he can do no better than maximize his first period profit. Take this to be an equilibrium. Any deviation results in a second order loss of profit. But because the planner is now convinced wrongly that θ has taken on a value $\tilde{\theta}<\theta$ more favourable to the firm, the firm will enjoy a first order increase in profits. This is so for all firms except the one facing $\theta=\underline{\theta}$. Thus, it is to be expected that for θ in the neighborhood of $\underline{\theta}$ a pooling equilibrium results in which $L^*(\underline{\theta})$ is played irrespective of type.

To be a bit more precise. Consider a situation where a firm faced with a realized value says θ deviates slightly from $L^*(\theta)$. Thus, $L^*(\theta-d\theta)<L^*(\theta)$. If this were to be a separating equilibrium, the planner would, erroneously, infer a value of $\theta-d\theta$. The loss in profits is of order ε^2. In the second period the firm faces a wage $w(\theta-d\theta)<w(\theta)$. The price of capital is exogenous and given by r. The optimal factor input combination respects $F_L/F_K=w/r$. And furthermore, in order to behave consistently the firm chooses to equate $w(\theta-d\theta)$ to the value of marginal product of labour evaluated at the price for the realization $(\theta-d\theta)$. Thus, we have (with a little abuse of notation $w(\theta-d\theta)L(\theta-d\theta)=(\theta-d\theta)D(\theta-d\theta)\cdot F_L(\theta-d\theta)l(\theta-d\theta)$. Similary for the input of capital. Of course, for any output on the market the demand function must be respected. Hence, second period profit is

$$\begin{aligned}
\pi_2 &= \theta D(F(L(\theta-d\theta),K(\theta-d\theta)))F(L(\theta-d\theta),K(\theta-d\theta)) \\
&\quad - (\theta-d\theta)D(F(L(\theta-d\theta),K(\theta-d\theta)))F_L(L(\theta-d\theta),K(\theta-d\theta)) \\
&\quad - (\theta-d\theta)D(F(L(\theta-d\theta),K(\theta-d\theta)))F_K(L(\theta-d\theta),K(\theta-d\theta)) \\
&= d\theta D(F(L(\theta-d\theta),K(\theta-d\theta)))F(L(\theta-d\theta),K(\theta-d\theta))
\end{aligned}$$

which is of order $d\theta$, strictly larger than the loss which was of order $(d\theta)^2$). Obviously, if it is not possible to deviate downwards (the bottom) then $d\theta=0$. Hence, for $\theta\in B(\underline{\theta},\varepsilon)$ pooling equilibrium obtains.

Proposition 4 does not exclude the possibility that over some range of the support $[\underline{\theta},\bar{\theta}]$ there may exist separating equilibria. It does, however, exclude the possibility that over the whole range $[\underline{\theta},\bar{\theta}]$ the continuation equilibrium may be separating. The reason why we do not obtain a fully separating equilibrium which will be the case in most circumstances (Kreps and Cho (1987)) is the fact that playing a strategy which fully reveals the value of θ results in a payoff of zero in the second period. This makes it profitable to deviate. If all firms, i.e. firms faced with different values of θ, were to deviate this could perhaps restore the separating equilibrium. This is not so in our analysis. Consider a firm of type $\underline{\theta}_n$, the new lowest support for θ. This firm realizes a super normal profit of 0 and in consequence a profit of 0. Thus, this firm will never deviate during the first period. We have then established an absolute lower bound upon the level of employment. Hence, any firm with $\theta \geq \underline{\theta}_n$ cannot deviate to a level of employment below the one just established.

We may now give a more precise characterization of the equilibrium. An equilibrium is said to exhibit infinite reswitching (Laffont and Tirole (1986)) if for some θ_0 and θ_1 there exists an infinite ordered sequence in $[\underline{\theta},\bar{\theta}]$, call this $\{\theta_k\}_{k\varepsilon N}$[4],such that it is optimal to play θ_0 for a realization θ_{2k}, and it is optimal to play θ_1 for θ_{2k+1} for all k. An equilibrium is said to exhibit pooling over a large scale $(1-\varepsilon)$ (Laffont and Tirole (1986)) if for some value θ and $\theta_1 < \theta_2$, we have $(\theta_2-\theta_1)/(\bar{\theta}-\underline{\theta}) > (1-\varepsilon)$ and it is an optimal strategy for θ_1 and θ_2 to play θ.

Consider now three types of equilibria. The simplest is the full pooling equilibrium. Note that the full pooling equilibrium is a subcase of an almost full pooling equilibrium (i.e. $\varepsilon=0$). The second and third type of equilibria are the almost full pooling equilibrium and the infinite reswitching equilibrium. If there is to be found an equilibrium offering the government higher utility than the full pooling equilibrium, then this equilibrium either has almost full pooling <u>or</u> infinite reswitching and much pooling.

Consider for a fixed value of $\bar{\theta}$ a sequence of economies with lower bound $\underline{\theta}_n$ and the (truncated) density function $f_1(\theta)=f_1(\theta)/[1-F_n(\underline{\theta}_n)]$ defined on $[\underline{\theta}_n,\theta]$. That is, if the initial range $[\underline{\theta},\bar{\theta}]$ is large, the result of Proposition 5 is valid only for the narrower range $[\underline{\theta}_n,\bar{\theta}]$.

<u>Proposition 5</u>. Consider any given first period wage inducing a separating equilibrium in the one period game. For this wage and any $\varepsilon>0$ there exists some $\theta_\varepsilon<\bar{\theta}$ such that for all n for which $n\varepsilon\{n|\underline{\theta}_n>\theta_\varepsilon\}$ any equilibrium dominating the full pooling equilibrium involves either

i) $(1-\varepsilon)$ of the firms hires L

or

ii) has some firms exhibiting infinite reswitching and the rest pooling over a large scale.

<u>Proof</u>: See Appendix.

This result is illustrated in Figure 3.

Figure 3 (see next page)

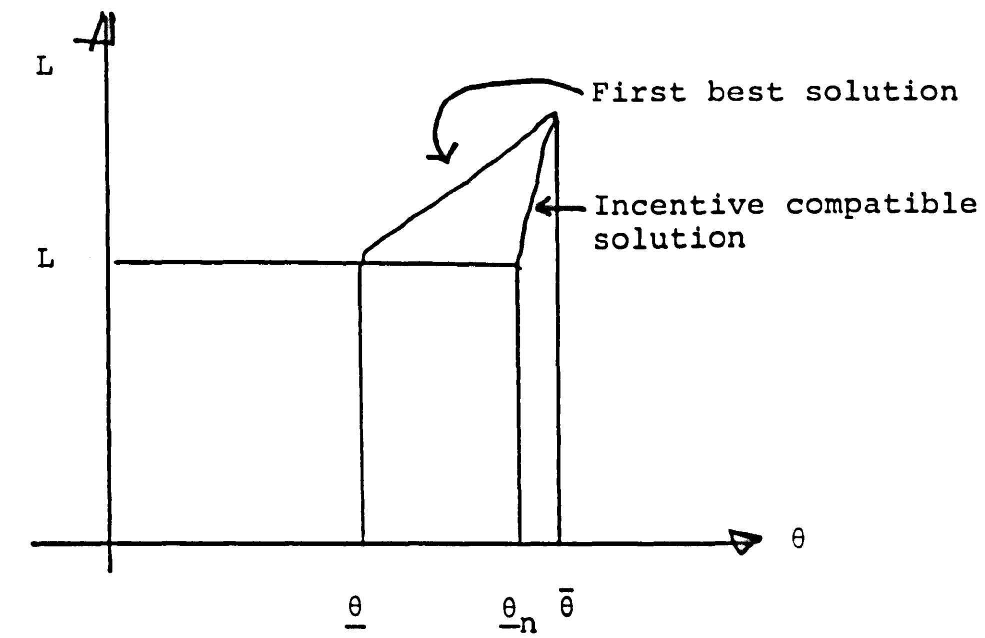
It is unfortunate that two types of equilibria occur. Following the arguments given in Laffont and Tirole (1986), it is reasonable to postulate that the full pooling equilibrium is preferred to its complex contender. The basic idea behind their argument is that since one of the features of the equilibrium is to elicit information, the simple equilibrium should be preferred. To extract information in the case of an infinite reswitching equilibrium requires an enormous amount of knowledge and sophistication on the part of the lesser informed party. For the agency to implement a strategy supporting infinite reswitching requires an enormous amount of knowledge of the game, in particular with respect to the description of uncertainty. A unique imployment target is, in contrast, more robust to mistakes in the description of the game.

If it is assumed that the agency is allowed to use only simple rules, then of course he will choose a pooling equilibrium.

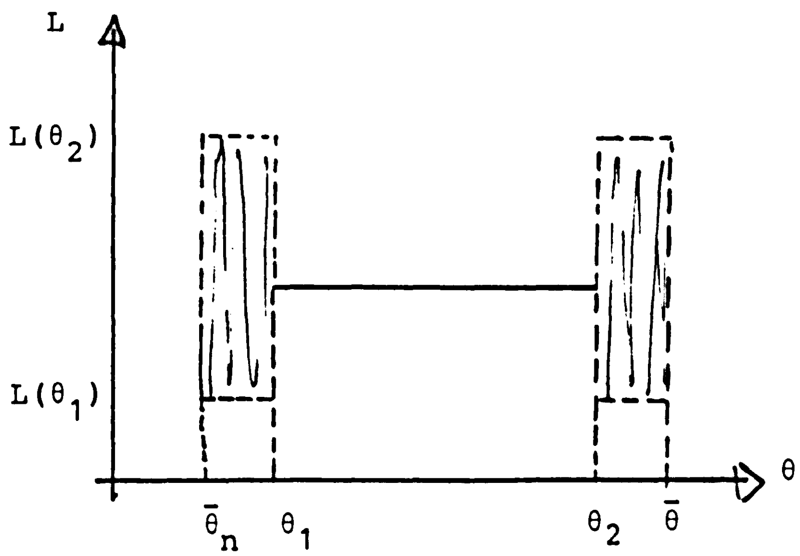
III.2. Decentralized Equilibrium

Perhaps the strongest feature of the pooling equilibrium analysed in Section III.2 is the fact that it arises naturally (after narrowing down the range of types). Or put differently even for the initial range of types the fully separating equilibrium could

Figure 3



Example of equilibrium i) of Prop. 5.



Example of equilibrium ii) of Prop. 5.

be excluded. Normally the sequential nature of the economy in addition to a second period payoff strictly greater than zero will support a fully separating equilibrium (in general, see Cho and Kreps (1987), for a similar analysis, see Chapter 2). In this section we will argue why a pooling equilibrium cannot be supported in a decentralized economy.

Since in this case the firm take advantages of its monopoly power, the factor market equilibrium conditions are[5]

$$w = \theta(D'(\cdot)F(\cdot,\cdot) + D(\cdot))F_L(\cdot,\cdot)$$

$$r = \theta(D'(\cdot)F(\cdot,\cdot) + D(\cdot))F_K(\cdot,\cdot)$$

Thus, for any given wage second period profits are

$$\pi = \theta D(\cdot)F(\cdot,\cdot) - \theta(D'(\cdot)F(\cdot,\cdot) + D(\cdot))(LF_L(\cdot,\cdot) + KF_K(\cdot,\cdot))$$

$$= \theta D(\cdot)F(\cdot,\cdot) - \theta(D'(\cdot)F(\cdot,\cdot) + D(\cdot))F(\cdot,\cdot)$$

$$= - \theta D'(\cdot)F^2(\cdot,\cdot) > 0$$

In consequence, for any given second period wage rate all firms will realize a super normal profit strictly bounded from below at zero. This, of course, is also so for the firm of type $\theta=\underline{\theta}$. Hence, if such a firm during the first period behaves as if $\theta=\underline{\theta}$, it wil be idéntified as a firm of type $\theta=\underline{\theta}+d\theta$. This carries with it a loss of second period profits of the order of $d\theta$. However, assuming that playing $\theta=\underline{\theta}$ is optimal in the first period, the cost to the firm of behaving as if $\theta=\underline{\theta}-d\theta$ is of the order of $(d\theta)^2$. But in doing so, the firm is identified as a $\theta=\underline{\theta}$ type, contrary to a $\theta=\underline{\theta}+d\theta$ type. The overall win is $d\theta-(d\theta)^2=(1-d\theta)d\theta >0$. Consequently, once the firm of type $\theta=\underline{\theta}$ stands to make a profit during the second period, it is not possible to establish a lower bound upon the range of signals where this lower bound dictates "no deviation from the first best". For this reason neither the argument given together with Proposition 4 nor the formal proof carries through here. The situation arising here is analysed thoroughly in the preceding chapter, where the existence of fully separating equlibria is established by an argument based on the sequentiality of the economy. Naturally, if for some reason the super normal profit of some firm, $\underline{\theta}_n$, is 0, then the analysis of the last section applies.

IV. Summary

This essay has reconsidered the simple neoclassical analyses of (partial) equilibrium on a single market and the adjoining factor markets. Normally we find that efficiency obtains and that factor rewards equal the value of marginal product with respect to the relevant factor. Also, the standard analysis results in a separating equilibrium.

In what has been termed a static continuing relationship the equilibrium is for any given wages still separating. However, for all values $\theta\varepsilon[\underline{\theta},\bar{\theta}]$ except $\theta=\bar{\theta}$ the equilibrium is inefficient as

$w > VMP_L(\theta)$, that is the level of employment, is not as high as it would otherwise be in a first best world.

In a dynamic continuing relationship, at least for small uncertainty, things are even worse. The only simple equilibrium which can be obtained is a pooling equilibrium. This pooling equilibrium has all firms playing $L(\underline{\theta}_n)$. Hence, inefficiency, in particular including underemployment, results in both periods.

The findings suggest that when the assumption of perfect and symmetric information are relaxed to one of probabilistic and asymmetric information the usual neo-classical findings have to be modified. And this is so, not because of any monopolistic elements or assumptions of risk aversion (cf. Equation (14)) but only because strategic considerations result from the relaxed assumption with respect to information. As a matter of fact, monopoly elements tend to restore a fully separating equilibrium.

Secondly, once the firm is no longer a residual claimant (as is normally the case in the contracts literature) we obtain strong inefficiency even though none of the agents are risk averse.

We may point to at least two interesting aspects of this analysis which deserves to be the subject of further considerations. The result of the analysis may have consequences for the relevant policy consideration. In particular, introducing employment taxes or subsidies may prove to make deviation sufficiently costly to avoid the existence of the equilibrium described in Proposition 4 and 5. Also, it would be interesting to extend the analysis to deal with changes in between periods of the parameter θ if, for example, a firm with a value θ attemts to mimic a firm faced with $\underline{\theta}$ and θ suddenly decreases, it has only become easier to obtain a full pooling equilibrium. Hence, the equilibrium is maintained. If, however, θ suddenly increases, it may prove too expensive to maintain the pooling equilibrium and a separating equilibrium results. As a consequence, at least in our model, the wage, price, and production increase. Hence, we obtain a theory of downward stickiness in prices as well as quantities. We hope to analyse this in subsequent papers.

V. Notes

1. See for example Scandinavian Journal of Economics 87.2.

2. In Chapter 4 a model of the following form is discussed

$$p = \theta\bar{p}$$

$$\bar{p} = 1$$

$$y = f(L,k)$$

$$\lambda y = f(\lambda L,\lambda k)$$

$$U = U(wL,L)$$

$$U_1 > 0 \quad U_{11} < 0$$

$$U_2 < 0 \quad U_{22} < 0$$

$$U_{12} < 0$$

With respect to wage and employment in equilibrium similar results obtain.

3. By this we mean to behave as if θ' is realized when the truth is that θ is realized.

4. N denotes positive integers.

5. Clearly, if in a decentralized economy we assume that $p=\theta D(Y)$ is exogenous to the firm, we return to the case of Section III.2. However, here we analyse the case in which $P=\theta D(y)$ is endogenous. This is partly because our specification may appear to be at variance with $P=\theta D(y)$ exogenous and partly for completeness.

VI. Appendix

Proof of Lemma 1

We analyse

$$U = u(wL) + \upsilon(L)$$

We have for $U=\bar{U}$ and $dL/dw>0$

$$u'Ldw + (u'w + \upsilon')dL = 0 \Rightarrow$$

$$\frac{dL}{dw} = -\frac{u'L}{u'w + \upsilon'} > 0$$

We require the indifference curve to be concave and the "as good as" set is convex, that is, we assume $\partial^2 L/\partial w^2 < 0$.

As we assume $\partial^2 L/\partial w^2 < 0$

$$\left|\frac{d^2L}{dw^2}\right| = \frac{\partial}{\partial w}\left\{\frac{u'L}{u'w + \upsilon'}\right\} > 0 \quad \Rightarrow$$

$$\frac{\partial}{\partial w}\left\{\frac{u'(w\cdot\Psi(w))\psi(w)}{u'(w\psi(w))w + \upsilon'(\psi(w))}\right\} > 0$$

Now,

$$\frac{\partial}{\partial w}\left\{\frac{u'(w\psi(w))\psi(w)}{u'(w\psi(w))w + \upsilon'(\psi(w))}\right\} =$$

$$[\frac{1}{(u'w + \upsilon')}]^2\{u'\psi'\frac{1}{\theta} + u''\psi(\psi + \frac{w}{\theta}\psi'))(u'w + \upsilon') -$$

$$u'\psi((u' + u''w\{\psi + \frac{w}{\theta}\psi'\}) + \upsilon'\frac{1}{\theta}\psi')\}$$

From the first order condition (7), and assuming that $(u'+w\psi u'')>0$

$$(\frac{\psi'}{\theta}(u' + w\psi u'') + u''\psi^2)(u'w + \upsilon') =$$

$$(\frac{\psi'}{\theta}(u' + w\psi u'') + u''\psi^2)(- \theta\frac{u'\psi}{\psi'}) < 0$$

Thus, for $\frac{\partial}{\partial w}\{\frac{u'(w\psi(w))\psi(w)}{u'(w\psi(w)w + \upsilon'(\psi(w)}\} > 0$ we require

$$- [u'\psi((u' + w\psi u'') + u''w^2\psi_w + \upsilon''\psi_w)] > 0$$

which is not possible for $u'+u\psi u''>0$. Hence, $u'+w\psi u''<0$.

<u>Proof of Lemma 2</u>

We have $u'+\psi wu''<0$. Examine the optimization problem of the union.

$$U = u(wL) + \upsilon(L)$$

$$U = u(w\psi(\frac{w}{\theta})) + \upsilon(\psi(\frac{w}{\theta}))$$

F.O.C.

$$(A.1) \quad u'(\cdot)\{\psi(\cdot) + (\tfrac{w}{\theta})\psi'(\cdot)\} + \upsilon'(\cdot)\tfrac{1}{\theta}\psi'(\cdot) = 0$$

Note since

$$u'(\cdot) > 0,\ \upsilon'(\cdot), \psi'(\cdot) > 0:\ \psi(\cdot) + \tfrac{w}{\theta}\psi'(\cdot) < 0$$

Define

$$E_{\psi,w} = \frac{\partial\psi(\frac{w}{\theta})}{\partial w} \ \frac{w}{\psi(\cdot)}$$

Thus,

$$\psi(\cdot)(1 + E_{\psi,w}) < 0 \quad \text{in equilibrium}$$

Assume that second order condition is satisfied

$$\frac{\partial}{\partial w}\ \{u'(\cdot)\{\psi(\cdot) + \frac{w}{\theta}\ \psi'(\cdot)\} + \upsilon'(\cdot)\frac{1}{\theta}\ \psi'(\cdot)\} < 0$$

Rewrite (A.1)

$$(A.2) \quad u'(\cdot)\{\psi(\cdot)(1 + E_{\psi,w}) + \upsilon'(\cdot)\tfrac{1}{\theta}\psi' = 0$$

Using (A.1)

$$\frac{\partial}{\partial w}\ \{u'(\cdot)\{\psi(\cdot) + \frac{w}{\theta}\ \psi'(\cdot)\} + \upsilon'(\cdot)\frac{1}{\theta}\ \psi'(\cdot)\}dw +$$

$$\frac{\partial}{\partial \theta}\ \{u'(\cdot)\{\psi(\cdot) + \frac{w}{\theta}\ \psi'(\cdot)\} + \upsilon'(\cdot)\frac{1}{\theta}\ \psi'(\cdot)\}d\theta = 0 \Rightarrow$$

$$\frac{dw}{d\theta} = - \frac{\frac{\partial}{\partial\theta}\ \{u'(\cdot)\{\psi(\cdot) + \frac{w}{\theta}\ \psi'(\cdot)\} + \upsilon'(\cdot)\frac{1}{\theta}\ \psi'(\cdot)\}}{\frac{\partial}{\partial w}\ \{u'(\cdot)\{\psi(\cdot) + \frac{w}{\theta}\ \psi'(\cdot)\} + \upsilon'(\cdot)\frac{1}{\theta}\ \psi'(\cdot)\}}$$

Evaluate S.O.C.

$$\frac{\partial}{\partial w}\ \{u'(\cdot)\{\psi(\cdot) + \frac{w}{\theta}\ \psi'(\cdot)\} + \upsilon'(\cdot)\frac{1}{\theta}\ \psi'(\cdot)\} < 0 \quad \Rightarrow$$

$$u'(\cdot)\{2\frac{1}{\theta}\ \psi' + w\frac{1}{\theta^2}\ \psi''\} + u''(\cdot)\{\psi + \frac{w}{\theta}\ \psi'\}^2 +$$

$$\upsilon'(\cdot)\ \frac{1}{\theta^2}\ \psi''\} + \upsilon''(\cdot)(\frac{1}{\theta}\ \psi')^2 < 0 \quad \Rightarrow$$

(A.3) $(\frac{1}{\theta})^2[u'(2\psi'\theta + w\psi'') + u''(\psi\theta + w\psi')^2 + \upsilon'\psi'' + \upsilon''\psi'^2] < 0$

Now evaluate

$$\frac{\partial}{\partial\theta}\ \{u'(\cdot)\{\psi(\cdot) + \frac{w}{\theta}\ \psi'(\cdot)\} + \upsilon'(\cdot)\frac{1}{\theta}\ \psi'(\cdot)\} =$$

$$u'\{\psi' - \frac{w}{\theta^2} - \frac{w}{\theta^2}\psi' - \frac{w}{\theta}\psi''\ \frac{w}{\theta^2}\} + u''\{w\psi'(- \frac{w}{\theta^2})\}\{\psi(\cdot) + \frac{w}{\theta}\psi'\}$$

$$+ \upsilon'\{-\frac{1}{\theta^2}\ \psi' - \frac{1}{\theta}\ \psi''\frac{w}{\theta^2}\} + \upsilon''\psi'\{-\frac{w}{\theta^2}\}\ \frac{1}{\theta}\ \psi'(\cdot) =$$

$$- \frac{w}{\theta^2}\ \{u'\{2\psi' + \frac{w}{\theta}\ \psi''\} + u''w\psi'\{\psi + \frac{w}{\theta}\ \psi'\} +$$

$$\upsilon'\{\frac{1}{w}\ \psi' + \frac{1}{\theta}\ \psi''\} + \upsilon''\psi'\frac{1}{\theta}\ \psi'\} =$$

$$- \frac{w}{\theta^3}\ \{u'\{2\psi'\theta + w\psi''\} + u''w\psi'\{\psi\theta + w\psi'\} +$$

$$\upsilon'\{\frac{\theta}{w}\ \psi' + \psi''\} + \upsilon''\psi'^2\}$$

Assume $\partial w/\partial\theta<0$, thus,

$$\frac{\partial}{\partial\theta}\ \{u'(\cdot)\{\psi(\cdot)\ +\ \frac{w}{\theta}\ \psi'(\cdot)\}\ +\ \upsilon'(\cdot)\frac{1}{\theta}\ \psi'(\cdot)\}\ <\ 0$$

Hence,

$$\{u'\{2\psi\theta\ +\ w\psi''\}\ +\ u''w\psi'\{\psi\theta\ +\ w\psi'\}\ +$$

$$\upsilon'\{\frac{\theta}{w}\ \psi'\ +\ \psi''\}\ +\ \upsilon''\psi'^2\}\ >\ 0\ =>$$

$$u''w\psi'\{\psi\theta\ +\ w\psi'\}\ >$$

(A.4) $$-\{u'\{2\psi'\theta\ +\ w\psi''\}\ +\ \upsilon'\{\frac{\theta}{w}\ \psi'\ +\ \psi''\}\ +\ \upsilon''\psi'^2\}$$

Using (A.3)

(A.5) $$-\{u'\{2\psi'\theta\ +\ w\psi''\}\ +\ \upsilon'\psi''\ +\ \upsilon''\psi'^2\}\ >\ u''\{\psi\theta\ +\ w\psi'\}^2$$

Using (A.5) in (A.4)

$$u''w\psi'\{\psi\theta\ +\ w\psi'\}\ >\ u''\{\psi\theta\ +\ w\psi'\}^2\ -\ \upsilon'\frac{\theta}{w}\ \psi'\ =>$$

$$u''\{\psi\theta\ +\ w\psi'\}\{w\psi'\ -\ \psi\theta\ -\ w\psi'\}\ >\ -\upsilon'\frac{\theta}{w}\ \psi'$$

$$-\ u''\{\psi\theta\ +\ w\psi'\}\psi\theta\ >\ -\ \upsilon'\frac{\theta}{w}\ \psi'$$

Using (A.1)

$$- u''\{\psi\theta + w\psi'\}\psi\theta > - u'\frac{\theta}{w}\psi' \Rightarrow$$

$$- u''\{\psi\theta + w\psi'\}\psi > \frac{1}{w}(- u'\frac{\theta}{w}\psi') \Rightarrow$$

$$- u''\{\psi\theta + w\psi'\}\psi > \frac{1}{w}u'\{\psi\theta + w\psi'\} \Rightarrow$$

$$- u''\psi w < u' \Rightarrow$$

$$u' + u''\psi w > 0$$

which is a contradiction.

<u>Proof of Proposition 1</u>

We have

$$\pi = \theta D(y)y - wl$$

where

$$l^d = \psi(\frac{w}{\theta})$$

Thus,

$$\pi(\theta'|\theta) = \theta D(F(\psi(\frac{w}{\theta'}),k))F(\psi(\frac{w}{\theta'}),k) - w\psi(\frac{w}{\theta'})$$

Assume that $\theta>\theta'$. If $l^d=\psi(w/\theta)$ is to be implementable, then

$$\pi(\theta|\theta) > \pi(\theta'|\theta)$$

and

$$\pi(\theta'|\theta') > \pi(\theta|\theta')$$

Consequently,

(A.6) $$\theta D(F(\psi(\tfrac{W}{\theta}),k))F(\psi(\tfrac{W}{\theta}),k) - w\psi(\tfrac{W}{\theta}) >$$

$$\theta D(F(\psi(\tfrac{W}{\theta'}),k))F(\psi(\tfrac{W}{\theta'}),k) - w\psi(\tfrac{W}{\theta'})$$

(A.7) $$\theta' D(F(\psi(\tfrac{W}{\theta'}),k))F(\psi(\tfrac{W}{\theta'}),k) - w\psi(\tfrac{W}{\theta'}) >$$

$$\theta' D(F(\psi(\tfrac{W}{\theta}),k))F(\psi(\tfrac{W}{\theta}),k) - w\psi(\tfrac{W}{\theta})$$

Combining (A.6) and (A.7), we have

$$\theta D(F(\psi(\tfrac{W}{\theta}),k))F(\psi(\tfrac{W}{\theta}),k) - w\psi(\tfrac{W}{\theta}) +$$

$$\theta' D(F(\psi(\tfrac{W}{\theta'}),k))F(\psi(\tfrac{W}{\theta'}),k) - w\psi(\tfrac{W}{\theta'}) >$$

$$\theta D(F(\psi(\tfrac{W}{\theta'}),k))F(\psi(\tfrac{W}{\theta'}),k) - w\psi(\tfrac{W}{\theta'}) +$$

$$\theta' D(F(\psi(\tfrac{W}{\theta}),k))F(\psi(\tfrac{W}{\theta}),k) - w\psi(\tfrac{W}{\theta}) \Rightarrow$$

$$(\theta-\theta')\{D(F(\psi(\tfrac{W}{\theta}),k))F(\psi(\tfrac{W}{\theta}),k)\} >$$

$$(\theta-\theta')\{D(F(\psi(\tfrac{w}{\theta'}),k))F(\psi(\tfrac{w}{\theta'}),k)\} \quad =>$$

$$D(F(\psi(\tfrac{w}{\theta}),k))F(\psi(\tfrac{w}{\theta}),k) > D(F(\psi(\tfrac{w}{\theta'}),k))F(\psi(\tfrac{w}{\theta'}),k)$$

Let us consider the function $H=D(F(\psi(w/\theta),k))F(\psi(w/\theta),k)$. We have

$$\frac{\partial H}{\partial F} = (D'(\cdot,\cdot)F(\cdot,\cdot) + D(\cdot,\cdot))F_1(\cdot,\cdot) > 0$$

Thus, $\psi(w/\theta)>\psi(w/\theta)$, and hence, $\psi'<0$ since $\theta>\theta'$.

If $0 < \varepsilon < \underline{\theta} - \bar{\theta} < T$ where T is finite, then the function $l^*(\theta)$ is of bounded variation. This taken together with $\partial l^*(\theta)/\partial\theta > 0$ ensures that $\psi(w/\theta)$ is differentiable almost everywhere (Taylor (1973) Section 9.1).

Proof of Proposition 2 and Corollary 1

We have (Proposition 1 and Equation (7)) that labour demand is given by the following almost everywhere differentiable equation:

$$\text{(A.8)} \quad L^d(w) = \tilde{\psi}(\tfrac{\theta}{w})$$

In the current case the trade union seeks to maximize

$$U = u(wL) + \upsilon(L)$$

by the appropriate choice of L. It is known that the labour demand follows (A.8). Hence, for a given K a particular choice of w results in a level of employment given by (A.8). Once w and L have been determined the value of π is given. The trade union takes these subsequent actions into consideration when choosing w.

Consider a point of differentiability of the profit function:

$$\pi = p \cdot y - wL - rk$$

Maximizing with respect to L

$$\frac{d\pi}{dL} = \theta(D'y + D)y_L - w = 0$$

Differentiating with respect to θ

$$\frac{d\pi}{d\theta} = D \cdot y + (\theta(D'y + D)y_L - w)\frac{dL}{d\theta}$$

Hence,

$$\frac{d\pi}{d\theta} = D \cdot y$$

The planner dictates some wage, taking into consideration the subsequent actions of the firm and the union as well as the incentive on part of the firm to misrepresent the truth. Thus, $w=T(\pi)$ (cf. $\pi_\theta=Dy$).

The optimisation problem of the union can now be considered as a control problem in L. Note that once L is chosen, w is given. The choice of L affect w as well as total profits. Thus, the trade union faces a feed back from the firm with respect to labour demand in its choice of L (and thus, implicitly w). The above problem is written sligthly different to make clear that it is a control problem (see Kamien and Schwartz (1981) Chapter 15).

$$\text{Max } u(T(\pi)L) + \upsilon(L)$$

$$\text{s.t.} \quad \pi_\theta = Dy$$

$$\pi \geq 0$$

The Hamiltonian function H is

(A.9) $\quad H = u(T(\pi L) + \upsilon(L) + \gamma(\theta)D\cdot y$

First order conditions are

(A.10) $\quad (u'(\cdot)T(\pi) + \upsilon'(L) + \gamma(\theta)(D'y + D)y_L)\frac{dL}{dw} = 0$

(A.11) $\quad u'(\cdot)L\frac{dT}{d\pi}\,\pi_\theta + \gamma_\theta Dy = 0$

We have (from $\pi_{\pi=\bar{\pi}}$ = py-wL-rk) that

$$\frac{\partial T}{\partial \pi} = \frac{-1}{L}$$

Hence, from (A.11).

(A.12) $-u'(\cdot) + \gamma_\theta = 0$ where $u'(\cdot) > 0$

Let the solution to (A.12) be

(A.13) $\gamma(\theta) = Z(\theta) + \text{constant}$

where $Z'(\theta)=u'(\theta)>0$.

Combining (A.13) and (A.10)

(A.14) $U'(\cdot)T'(\pi) + \upsilon'(\cdot) = -(Z(\theta) - \text{constant})(D'y + D)y_L$

Since the worst point estimate is $\theta=\bar{\theta}$, we have: constant $= -Z(\bar{\theta})$. Hence,

(A.15) $u'(\cdot)T(\pi) + \upsilon'(\cdot) = (Z(\bar{\theta}) - Z(\theta))(D'y + D)y_L$

Figure A.1 (see next page)

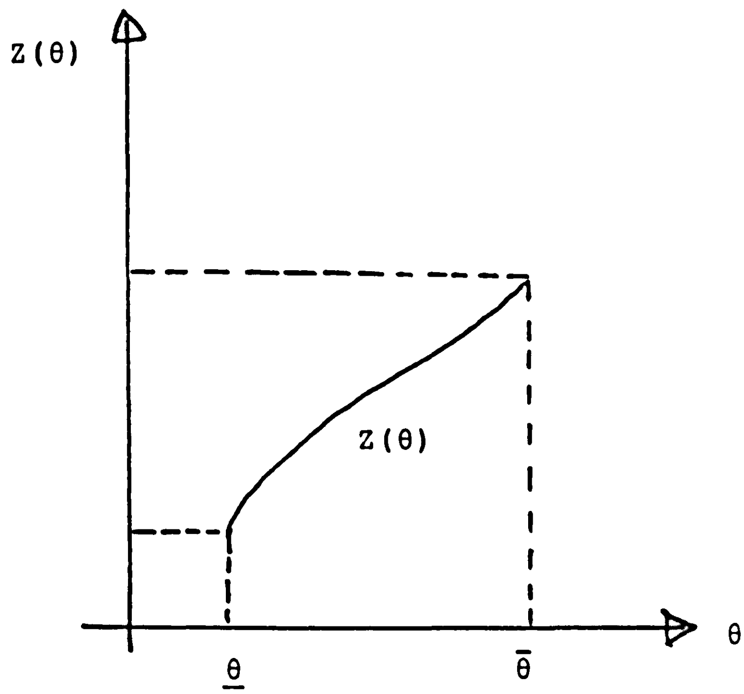
Since $Z'(\cdot)=u'(\cdot)>0$, we have $Z(\bar{\theta})-Z(\theta)$ as illustrated by Figure A.1. Also, using the first order condition of the equilibrium of the firm (cf. Equation (4)), we have $(D'y+\theta)>0$.

Thus, underemployment occurs.

<u>Proof of Proposition 3</u>.

The maximization problem of the union, now reflecting the

Figure A.1



attitude towards risk, reads

$$\max_{L} \int_{\underline{\theta}}^{\bar{\theta}} (u(wL) + \upsilon(L))dG(\theta)d\theta$$

where $w=T(\pi)$ is given by (A.8) and the restrictions are as in the proof of Proposition 2.

The Hamiltonian is

$$H = (u(T(\pi)L) + \upsilon(L))\frac{dG(\theta)}{d\theta} + \gamma(\theta)Dy$$

First order conditions are

(A.16) $$(u'(\cdot)T(\pi) + \upsilon'(\cdot))\frac{dG(\theta)}{d\theta} + \gamma(\theta)(D'y + D)y_L = 0$$

(A.17) $$- u'(\cdot)L\frac{dT}{d\pi}\pi_\theta\frac{dG(\theta)}{d\theta} + \gamma_\theta Dy = 0$$

Thus,

$$-u'(\cdot)\frac{dG(\theta)}{d\theta} + \gamma_\theta = 0$$

Hence,

$$\gamma_\theta = u'(\cdot)\frac{dG(\theta)}{d\theta}$$

Let the solution be

(A.18) $$\gamma(\theta) = Z(\theta) + \text{constant } Z_\theta = u' \frac{dG(\theta)}{d\theta} > 0$$

Combining (A.18) and (A.16)

$$\{u'(\cdot)T(\pi) + \upsilon'(\cdot);0\text{xå}g(\theta) =$$

$$-(Z(\theta) + \text{constant})(D'y + D)y_L$$

Thus, constant = $-Z(\bar{\theta})$

Hence, the solution is

$$\{u'(\cdot)T(\pi) + \upsilon'(\cdot)\}g(\theta) =$$

$$(D'y + D)y_L(Z(\bar{\theta}) - Z(\theta))$$

<u>Proof of Corollary 2</u>

The maximization problem, now reflecting the attitude towards risk, reads

$$\max \int_{\underline{\theta}}^{\bar{\theta}} (u(wL) + \upsilon(L))dG(\theta)d\theta$$

where L is given by (A.8) and the restrictions are as in the proof of Proposition 2.

The Hamiltonian is

$$H = (u(T(\pi,L)L) + \upsilon(L))\frac{dG(\theta)}{d\theta} + \gamma(\theta)y$$

First order conditions are

(A.16) $(u'(\cdot)(T(\pi,L) + T_L(\cdot)L) + \upsilon'(\cdot))\frac{dG(\theta)}{d\theta} + \gamma(\theta)(D'y + D)y_L$

(A.17) $- u'(\cdot)L\frac{dT}{d\pi}\pi_\theta\frac{dG(\theta)}{d\theta} + \gamma_\theta Dy = 0$

Thus,

$$-u'(\cdot)\frac{dG(\theta)}{d\theta} + \gamma_\theta = 0$$

Hence,

$$\gamma_\theta = u'(\cdot)\frac{dG(\theta)}{d\theta}$$

Let the solution be

(A.18) $\gamma(\theta) = Z(\theta) + \text{constant } Z_\theta = u' \frac{dG(\theta)}{d\theta} > 0$

Combining (A.18) and (A.16)

$$\{u'(\cdot)\{T(\pi,L) + T_L(\cdot)L\} + \upsilon'(\cdot)\}g(\theta) =$$

$$- (Z(\theta) + \text{constant})(D'y + D)y_L$$

Thus, constant $= - Z(\bar{\theta})$

Hence, the solution is

$$\{u'(\cdot)\{T(\pi,L) + T_L(\cdot)L\} + \upsilon'(\cdot)\}g(\theta) =$$

$$(D'y + D)y_L(Z(\bar{\theta}) - Z(\theta))$$

Proof of Proposition 4.

$L^*(\theta)$ is increasing of bounded variation and hence, it is a differentiable strategy.

Consider $\theta > \theta^*$. In a separating equilibrium if θ is drawn from $[\underline{\theta},\bar{\theta}]$ labour input and wage is some function of θ: $(L(\theta),w(\theta))$. If θ' was drawn, we would have $(L(\theta'),w(\theta'))$. In a separating equilibrium $\pi_i^N(\theta) = \pi_i^N(\theta') = 0$, i=1,2 that is profits in both periods are zero.

Now if in a separating equilibrium θ deviates to play θ' and was believed as it would (erroneously) be, it would be faced with a wage $w(\theta')<w(\theta)$ and hence, the firm is able to obtain supernormal rent $SRN_2(\theta'|\theta)=(\theta-\theta')D(F(L(\theta'),K(\theta'))F(L(\theta'),K(\theta'))>0$ in the second period. That is $\pi_2(\theta'|\theta)>\pi_2^N(\theta)$. On the other hand, if some firm facing θ' deviates to play θ, we have $\pi_2(\theta|\theta')<\pi_2^N(\theta)$.

For a continuation equilibrium to be fully revealing:

$$\pi_1(l^*(\theta)) + \delta\pi_2^N(l^*(\theta)) > \pi_1(l^*(\theta-d\theta) + \delta\pi_2(l^*(\theta-d\theta)) \Rightarrow$$

$$\pi_1(l^*(\theta)) + \delta\pi_2^N(l^*)(\theta)) > \pi_1(l^*(\theta-d\theta)) + \delta(\pi_2^N(l^*(\theta))$$

$$+ \delta SNR_2(\theta-d\theta)) \Rightarrow$$

$$\pi_1(l^*(\theta)) > \pi_1(l^*(\theta-d\theta)) + \delta SNR_2(l^*(\theta-d\theta))$$

Analogously

$$\pi_1(l^*(\theta)) > \pi_1(l^*(\theta+d\theta)) + \delta SNR_2(l^*(\theta+d\theta))$$

Thus, we have

$$\pi_1(l^*(\theta)) - \pi_1(l^*(\theta-d\theta)) > \delta SNR_2(l^*(\theta-d\theta))$$

$$- \delta SNR_2(l^*(\theta+d\theta)) > \pi_1(l^*(\theta+d\theta)) - \pi_1(l^*(\theta))$$

As l^* and hence, π and SNR_2 are differentiable

(i) $$\frac{d\pi_1}{dl} l^*_\theta(\theta) > \lim_{d\theta \to 0} \delta SNR_2(l^*(\theta-d\theta))$$

(ii) $$\lim_{d\theta \to 0} -\delta SNR(l^*(\theta+d\theta)\theta) > \frac{d\pi_1}{dl} l^*_\theta(\theta)$$

As $\lim_{d\theta \to 0} SNR(l^*(\theta-d\theta)) = \lim_{d\theta \to 0} SNR(l^*(\theta+d\theta)) = 0$

(i) and (ii) contradict the continuity of $l^*(\theta)$.

<u>Proof of Proposition 5</u>.

The full pooling equilibrium scheme must satisfy

$$\frac{d}{dw} \int_{\underline{\theta}}^{\bar{\theta}} (U(wL(\theta,w),L(\theta,w))g(\theta)d\theta = 0$$

where $l(\theta,w)$ as in the text. This results in some wage $w(\underline{\theta},\bar{\theta})$ or if we narrow down the range of the support to $[\underline{\theta}_n,\theta]$ the wage is $w(\underline{\theta}_n,\bar{\theta})$.

$$\Delta_2 = U(w(\theta,\bar{\theta})l(\theta,w(\theta,\bar{\theta}),l(\theta,w(\theta,\bar{\theta})) -$$

$$U(w(\underline{\theta}_n,\bar{\theta})L(\underline{\theta}_n,w(\underline{\theta}_n,\bar{\theta}),L(\underline{\theta}_n,w(\underline{\theta}_n,\bar{\theta}))$$

where $\theta \varepsilon [\underline{\theta}_n,\bar{\theta}]$. We have, trivally, from continuity

$$\lim_{\underline{\theta}_n \to \bar{\theta}} \Delta_2 = 0$$

Similarly, consider some wage $\bar{w} = \psi(l^*(\bar{\theta}))$. The first period distortion to the union by enforcing some strategy $l^*(\theta)$ is

$$\Delta_1 = U(w\psi(\theta),\psi(\theta)) - U(\bar{w}\psi(\bar{\theta}),\psi(\bar{\theta}))$$

Also here we have

$$\lim_{\underline{\theta}_n \to \bar{\theta}} \Delta_1 = 0$$

Hence, for $\underline{\theta}_n \to \bar{\theta}$ the first period distortion to the union relative to a full information and full pooling scheme tends to zero. Hence, if we can show that the distortion remains finite for continuation equilibria not satisfying i) and ii), we have finished since then only equilibria satisfying i) or ii) may dominate a full pooling equilibrium.

Consider some first period wage w and two distinct values of l: l_0 and l_1. Both of these strategies are assumed to belong to the equilibrium path for some n. Let θ_i, i=0,1, be the supremum of types θ for which firms participate (i.e., those firms for which $x_i=1$ (cf. the text))

$$\theta_i = \sup\{\theta | \pi(l_i^*(\theta)) \geq \Pi(\ ) \mid x_i = 1\}$$

Clearly,

$$SNR_2(l_i, \theta_i) = 0$$

For $\theta_1 > \theta_0$ and hence, $l(\theta_1) > l(\theta_0)$ we have

$$SNR_2(\theta_0 | \theta_1) < 0$$

$$SNR_2(\theta_1 | \theta_0) > 0$$

We have

$$\pi_1(l(\theta_0)|\theta_0) + \delta\pi_2(l(\theta_0)|\theta_0) >$$

$$\pi_1(l(\theta_1)|\theta_0) + \delta\pi_2^N(l(\theta_1)|\theta_0) + \delta SNR_2(\theta_1|\theta_0)$$

and

$$\pi_1(l(\theta_1)|\theta_1) + \delta\pi_2(l(\theta_1)|\theta_1) >$$

$$\pi_1(l(\theta_0)|\theta_1) + \delta\pi_2^N(l(\theta_0)|\theta_1) + SNR(\theta_0|\theta_1)$$

Combining these two expressions (cf. the proof Proposition 1) we have

$$0 > (\theta_0 - \theta_1)D(F(K(\theta_1),L(\theta_1)))F(K(\theta_2),L(\theta_1)) +$$

$$(\theta_1 - \theta_0)D(F(K(\theta_0),L((\theta_0)))F(K(\theta_0),L(\theta_0)) \Rightarrow$$

(A.19) $$D(F(K(\theta_1),L(\theta_1)))F(K(\theta_1),L(\theta_1)) -$$

$$D(F(K(\theta_0),L(\theta_0)))F(K(\theta_0),L(\theta_0)) > 0$$

Now consider a sequence of economies $(\theta_n{}^0,\theta_n{}^1,l(\theta_n{}^0),l(\theta_n{}^1))$, that is a sequence of realizations of θ and their supremum. We will show that in the limit $l(\theta_n{}^0)$ and $l(\theta_n{}^1)$ are "far" apart.

If (A.19) is to be satisfied in the limit, a necessary condition is

(A.20) $$\lim_{\theta_n^0 \to \bar{\theta}} l(\theta_n{}^0) \neq \lim_{\theta_n^1 \to \bar{\theta}} l(\theta_n{}^1)$$

or

$$D(F(K(\theta_1),L(\theta_1)))F(K(\theta_1),L(\theta_1)) -$$

$$D(F(K(\theta_0),L(\theta_0)))F(K(\theta_0),L(\theta_0)) \to 0$$

for $\theta_n{}^0 \to \bar{\theta}$ and $\theta_n{}^1 \to \bar{\theta}$. Thus, (A.20) since it is at variance with (A.14) implies that only one of $l(\theta_n{}^0)$ and $l(\theta_n{}^1)$ can belong to the equilibrium path.

A prerequisite for the equilibrium we study now is that it dominates the full pooling equilibrium. Hence, the distortion to the union relative to the employment target must converge to zero. That is, only a negligible number of firms can deviate from this employment target. What is said in (A.20) is that for a given n there exists for some n a value θ_n and a set of employment targets L_n (since (A.20) is only comparing two types) such that the corresponding suprema to all employment targets is θ_n ($\theta^1{}_n$ and $\theta^2{}_n$ both converge to θ_n), and that these employment targets are chosen by $(1-\varepsilon)$ of firms. Since all employment targets have the same supremum there is at least one employment level which is optimal to firms sufficiently far apart.

Finally, consider the case where the equilibrium does not exhibit infinite reswitching. Then for $l(\theta_0), l(\theta_1) \varepsilon L_n, l(\theta_1)$ is strictly preferred to $l(\theta_0)$ in some interval (θ_n, θ_1) and θ_n cannot be a supremum for $l(\theta_0)$. Hence, there exists only one employment level in L_n and the equilibrium is, upto ε, a full pooling equilibrium.

CHAPTER 4

WAGE SETTING, INVESTMENT AND ASYMMETRIC INFORMATION

I. Introduction

This chapter analyses the role of employment and investment as a signal when the information structure of the economy is asymmetric. We analyse a situation in which the equilibrium wage rate is assumed to be dictated by a planner who attempts to clear the market. The planner dictates a wage scheme regulating the exchange of wages for employment between a firm and its workers. The wage scheme is restricted to elicit all relevant information. Thus, it is contingent upon some parameter (or a sufficient statistic for this parameter), characterizing the economy. Here it is assumed for convenience that this parameter is the price, and the price is known probabilistically by the planner and the workers, but precisely by the firm. Thus, the decision of the planner is subject to uncertainty, and consequently, the wage dictated by the planner is based upon the price (or a sufficient statistic) as announced by the firm. Any wage suggested by the planner must be incentive compatible, that is, this wage jointly with observable variables must ensure that the firm reveals the true price. Or to phrase it differently: workers will accept for a given wage a level of labour supply only if this amount of labour exchanged ensures that they are "not fooled".

As a further exercise, the economy is analysed but without assuming the presence of a planner.

Such an exercise allows for market imperfections due to monopolistic behavior on the part of the two agents. This may be the more realistic case to consider in the case of a firm specific bargain.

The role of employment and investments, in this model, is to support the announcement made by the firm with respect to the price, thus, supporting the contract agreed upon.

The result of the particular institutional framework analysed in the present setting indicates inefficiencies. A command equilibrium possesses two characteristics. Once the level of investment is decided upon, the contract results in underemployment relative to the outcome under perfect (and symmetric) information. Secondly, the level of investments is less than what is the case under symmetric information. These results are only partly true in a decentralized economy. Employment is still inefficiently low compared to the situation under symmetric information. However, the effect upon capital is no longer unambiguous. Thus, even qualitatively, there is not an equivalence between the command optimum and a free market economy.

As an alternative interpretation, this chapter can be thought of as modelling the role of investment as a signal when the equilibrium wage rate is the result of an explicit or implicit bargain between workers and their employers. (This is essentially Section IV). The value of the bargain is subject to some stochastic shock, which is known only by the firm. Thus, compared to other models of investment and wage formation (e.g., the

Weitzman share model), some further incentive compatibility constraints are added to the problem. The results presented here, thus, modify the positive conclusions reached in the share model. This is just one reason why we should concern ourselves with the issue of this paragraph.

The topics presented in this paper are interesting for several other reasons. The labour market appears to be the market which diverges most from the Walrasian assumption of exogenous prices and lack of strategic behavior. Thus, an analysis of the possible process determining wages and its impact upon the level of investment is appropriate in the attempt to understand modern economic issues, including the idea of a microfoundation for macroeconomics.

Furthermore, this chapter supplements several papers on related issues. Grout (1984) shows that in the absence of binding contracts, a Nash bargaining solution to the wage determination problem supports lower levels of investment. However, for a given level of investment, input of labour and the share of profits are unchanged. The results obtained here modify these results.

Finally, the results of this analysis can be compared with those of Azariadis (1983) and Grossman & Hart (1983) who show that implicit contracts under asymmetric information result in underemployment in adverse states of nature. Contrary to this Chari (1983) and Green & Kahn (1983) obtain high employment in favourable states of nature. Differences in results are due to differences in the specification of the preference structure. This issue will be discussed further in the conclusion. At any rate, there is agreement that under asymmetric information the level of employment diverges from the Walrasian level. The analysis presented here can be seen as an extension of the above papers, since the level of investments is introduced into the model as a signalling device.

II. The Model

This section specifies and discusses the model. Furthermore, an alternative interpretation of the results to come is given. This interpretation consider the problem as a problem of implementing a contract. Thus, the form of this contract is discussed informally. In particular, attention is drawn to the fact that the contract used in the analysis is what is termed self-enforcing. This implies that the specific contractual agreement envisaged here can be thought of as an implicit contract as well.

A firm can sell at some given price all of its output. This price is subject to some shock, i.e.,

(2.1) $P = \theta\bar{P}$

$\bar{P}$ normalized to 1

$\theta \in [\theta_l, \theta_u]$, $dG = g(\theta) > 0 \ \forall \ \theta$

The shock is distributed according to $g(\cdot)$ which is non-vanishing everywhere, with a support $[\theta_l, \theta_u]$, $\theta_u > \theta_l$ and by assumption $\int_\theta g(\theta)\cdot d\theta = 1$. Denote by Ω_F, Ω_u, and Ω_p, the information set of the firm, the trade union and the planner, respectively . It is assumed $\Omega_F = \{\theta, g(\cdot), [\theta_l, \theta_u]\}$ and $\Omega_i = \{g(\cdot), \{\theta_l, \theta_u]\}$ i=u,p; that is, the firm knows the realized value of θ, whereas the union and the planner only know the form $g(\cdot)$ and the support $[\theta_l, \theta_u]$. The fact that $\Omega_p = \Omega_u \neq \Omega_F$ accounts for the incentive compability restrictions added to the optimization problem.

Let y be output and L and K inputs of labour and capital. The production function, F(.,.), exhibits constant returns to scale[1], thus,

$$(2.2) \qquad \begin{cases} y = F(L,K) \\ \lambda y = F(\lambda L, \lambda K) \qquad \lambda \in R_+ \end{cases}$$

Combining (2.1) and (2.2) the profit function is

$$(2.3) \qquad \pi = \theta F(L,K) - wL - rK$$

The model is closed assuming that (2.3) possesses an optimum, for example assuming that w and r are increasing in L, respectively K.

We will now offer an interpretation in terms of implementing a contract. Consider the following scenario: let the trade union[2] have some objective function (to be specified shortly). The firm announces some value of θ, call this $\tilde{\theta}$, and decides upon a level of investment. Assuming that, once used in production, the investment deteriorates completely, the stock of capital equals investment. Investment takes place in the first period. This is also the period in which the planner has to implement the self-enforcing contract (note that due to this, it is irrelevant whether the contract is binding or not). Given the announcement $\tilde{\theta}$ and the stock of capital, which is affected by the contract, the planner seeks to implement a contract which clears the market and elicits all information. The planner does so by dictating a wage. The restriction on the planner is that the contract must be self-enforcing or incentive compatible.

The assumption made here with respect to Ω_F and $\Omega_u = \Omega_p$ complicates the decisions of the firm as well as that of the planner. Assume that the firm knows the utility function of the trade union and, thus, the objective of the planner. Hence, the firm can reproduce the optimal decision of the planner. Thus, if the firm moves first, the choice of K and $\tilde{\theta}$ is based upon knowledge of subsequent actions. These subsequent actions are the decision with respect to the wage and the decision, given θ and K, and in period 2, with respect to labour demand. If the planner moves first, the opposite is of course true. The basic idea of the model is that once the firm has announced θ, the planner suggests a contract, hence the planner moves first. Thus, it is clear that any first mover advantage to the firm (since the wage depends upon the stock of capital) cannot be realized, since the planner aims at efficiency. The incentive compatibility restrictions are, in essence, that the contract has to be acceptable to the trade union.

At this point, a short digression is necessary. Assume that the game is played between the firm and the trade union. In this case, the wage dictated by the trade union would depend upon K. Thus, even if the value of θ, drawn according to $g(\cdot)$ and $[\theta_l, \theta_u]$, was known to both parties, i.e., $\Omega_u = \Omega_F$, the level of investment would differ from the Walrasian level. This is so because the firm would enjoy the classical Stackelberg leader position. This issue will be discussed in Section IV.

Let the utility function have positive but decreasing marginal utility of income and a negative and decreasing marginal utility of labour supply. Furthermore, assume that the cross derivative is negative, that is, the marginal utility of income is decreasing as more labour is supplied (an example is given in Note 3). This may, perhaps, find some justification in the consumption theory suggested by Becker (e.g., Becker (1971), Chapter 3).

(2.4) $U = U[wL, L]$

$$U_1 > 0 \quad U_{11} < 0$$

$$U_2 < 0 \quad U_{22} < 0$$

$$U_{12} < 0$$

The planner has to decide upon a wage such that $l^d(\cdot)$ is equal to $l^s(\cdot)$. This problem is subject to two restrictions. The level of employment is given by maximization of the profit, once K is fixed. Secondly, only a wage which forces the firm to reveal truthfully the realized value of θ is accepted by the union (and thus by the planner). That is, the contract has to support an announcement $\tilde{\theta}=\theta$.

It is now possible to discuss the contractual arrangement which we envisage in more detail but at a rather informal level. For any given choice of K and announcement $\tilde{\theta}$ the contract specifies the wage and the labour demand.

$$(2.5) \quad C = \{(\omega(K,\tilde{\theta}),l^d(\omega(K,\tilde{\theta}))(K,\tilde{\theta})\}$$

Note, the contract C is defined over $\tilde{\theta}$, not θ. This implies that the firm has the possibility of making an announcement $\tilde{\theta}\neq\theta$.

The specification of C is important because even though the firm is a residual claimant with respect to θ once the wage is fixed the firm is <u>not</u> a residual claimant with respect to the anouncement $\tilde{\theta}$ made in the first period meaning that it may be to the advantage of the firm to make some announcement $\tilde{\theta}\neq\theta$. Why is this so? Consider the contract C*, with an announcement $\tilde{\theta}=\theta$[4-5], i.e., for some reason the announced value equals the true value.

$$(2.5') \quad C^* = \{\omega^*(K,\tilde{\theta}),l^d(\omega^*(K,\tilde{\theta}))(K,\tilde{\theta})\}$$

If this situation is optimal to the firm, the resultant wage is

$w^*(\theta)$. Consider an announcement $\tilde{\theta}=\theta'\neq\theta$ for which it is the case that $w^*(\theta')<w^*(\theta)$ and θ' close to θ[6]. Deviating from the announcement $\tilde{\theta}=\theta$ to $\tilde{\theta}=\theta'$, the firm suffers a loss since the level of employment must correspond to the annoucement θ'. However, as by assumption $\tilde{\theta}=\theta$ is first best this loss is of order ε^2. The gain, on the other hand, is of order ε, since $w^*(\theta')<w^*(\theta)$. Hence, the overall gain is $(1-\varepsilon)\varepsilon>0$. Consequently, announcing $\tilde{\theta}=\theta$ is not an incentive compatible solution. However, note also that a contract C^* and $\tilde{\theta}=\theta'$, although preferred by the firm to C^* and $\tilde{\theta}=\theta$ is not time consistent (or self-enforceable), since the firm in the second period would like to deviate from $l^d(\omega,K,\theta')$ to $l^d(\omega,K,\theta)$. Both of these problems will be dealt with by an appropriate design of C, given in (2.5).

Consider the restrictions to be put on l^d by the restriction to incentive compatible contracts. Clearly this is interesting since any restriction to be put on l^d may account for deviations from the Walrasian outcome. Since the incentive compatibility restrictions on the firm are that they behave according to their announcement[7] $l(\theta'|\theta)=l(\theta'|\theta')$, $w(\theta'|\theta)= w(\theta'|\theta')$ and vice versa and that they announce truthfully the realized value of θ, the following lemma is easily established[8].

<u>Lemma 1</u>: If l^d is to be incentive compatible then $\partial l^d/\partial\theta>0$.

<u>Proof</u>: (The proof proceeds as the proof of Proposition 1 in Chapter 3 and a sketch suffices).

The idea of the proof is simple. Pick out any two θ and θ' with, say, $\theta>\theta'$. If l^d is to satisfy incentive compatibility, then $\pi(\theta|\theta)>\pi(\theta'|\theta)$ and $\pi(\theta'|\theta)>\pi(\theta|\theta')$. Using this, the results follow.

Lemma 1 allows for many demand functions, including the Walrasian demand function. It is not surprising that Lemma 1 is unrestrictive. The lemma is concerned with the second period decisions of the firm and in this period the firm is a residual claimant, since only non-binding contracts are considered. Consequently, the firm has no incentive to misrepresent the value of θ. As a matter of fact, for a time consistent, incentive compatible contract the labour demand is the Walrasian labour demand. Thus, if a deviation from the Walrasian level of employment is the result of the contract (2.5), then this is because the wage deviates from the Walrasian wage. The reason, as it emerged from the discussion of the design of the contract, is that the trade union accepts only a wage which is different from that under symmetric information because the trade union, in accepting a contract, must ensure that for the contract accepted, the firm will truthfully reveal the realized value of θ. Observing that the firm, in the first period, is not yet a residual claimant, it is not surprising that the Walrasian solution is infeasible.

Before proceeding to the analysis of the model, consider the contract (2.5) and the decision of the planner. Strictly speaking, the planner suggests a supply schedule for labour and not a perfectly elastic supply of labour at some wage rate. Thus, the model is apparently in disagreement with the monopoly union model. However, for a time consistent and incentive compatible contract the perception of the trade union and the planner with respect to l^d is correct. Thus, once the planner has decided upon a supply schedule which is acceptable to the trade union, the equilibrium employment rate is implicitly determined. Hence, the contract (2.5) could as well be replaced with

$$(2.5'') \quad C = \{(K,\tilde{\theta}),\bar{w},l^d(\bar{w},K,\tilde{\theta})\}$$

where $\bar{w}$ solves $l^s(w,K,\tilde{\theta})=l^d(w,K,\tilde{\theta})$.

III. Wages, Employment and Investment

In this section a command-like equilibrium is analysed. The following, perhaps somewhat restrictive, story may justify this. We consider an economy made up of a great number of firms all of which are alike. The firms are subject to some stochastic injection known exclusively by themselves and only probabilistically by the two other agents in this economy, the planner and the union (representative worker). In such an economy the role of the planner is to dictate a wage which clears the market and which elicits all relevant information. Alternatively, proceeding from Section II we can think of the planner as announcing a contract specifying the wage and the labour demand contingent upon the firm's announcement of the "value" of the stochastic injection and the level of investments they choose. The contract is restricted to support a truth telling behavior of the firms.

This section argues that the economy is inefficient in the sense that investment is less than under symmetric information and is underutilized. This is so despite the fact that the interpretation given just above is extremely favourable to attaining efficiency. An alternative interpretation is that this model describes union-specific bargaining. An analysis of this is deferred until the next section.

Let us first, however, as a reference analyse the case of symmetric investment.

III.1. Wages, Employment and Investment with Symmetric Information.

This section of the paper analyses, in the case of symmetric information, the demand and supply decisions with respect to labour of the firm and the trade union, respectively, and the action of the planner. This is a prelude and reference to the analysis for the case of asymmetric information.

Consider the demand for labour. For a time-consistent contract the optimal demand for labour is

$$(3.1) \quad \theta F_L(L,K) = w$$

Clearly (3.1) is nothing but the Walrasian demand for labour. Assuming that the value of the marginal product of labour is decreasing in L and increasing in K then l^d is given by[9)]

(3.2) $L^d = g(\theta, K, w)$

$g_\theta > 0,\ g_K > 0,\ g_w < 0$

If information is symmetric, then the planner faces, with certainty, a labour demand schedule as given in (3.2). Consequently the planner chooses a level of employment by dictating a wage which equates the marginal disutility of working to the marginal utility of working (for a formal exposition see Appendix (A.I)).

(3.3) $U_1\{\cdot\}w + U_2\{\cdot\} = 0$

Hence, equilibrium in the labour market possesses the characteristic that the wage set equates the marginal disutility of labour to its marginal utility.

Let the solution to (3.3) be $h(\cdot)$ defined implicitly by (3.3).

(3.3') $L^S = h(w)$

In Appendix (A.I) it is argued that the solution to (3.2) and (3.3') can be written as

(3.4) $w = \omega^*(K, \theta)$

$\omega^*_\theta > 0,\ \omega^*_K > 0$

Hence, profit is written as

$$(3.5) \quad \pi(K,\theta) = \theta F[g(\theta,K,\omega^*(K,\theta)),K] - \omega^*(K,\theta)g(\theta,K,\omega^*(K,\theta))-r\cdot K$$

Since the wage is dictated in period 1 by the planner, the wage is parametric to the firm, $\omega^*(K,\theta)=\bar{\omega}^*(K,\theta)$. Consequently, the first order condition is

$$(3.6) \quad \theta F_K[g(\theta,K,\omega^*(K,\theta)),K] = r$$

Hence, the value marginal product of K is equalled to r. Thus, the Walrasian solution with respect to K is obtained.

III.2. Wages, Employment and Investment with Asymmetric Information.

We now consider the case in which the planner will have to make its decisions contingent upon the announcement $\tilde{\theta}$ made by the firm and the observable level of capital. The role of the planner is to decide on some wage $\omega(K,\tilde{\theta})$, so that given this wage the announcement made by the firm can be trusted, i.e., $\tilde{\theta}=\theta$. To obtain a time consistent, incentive compatible solution the planner in deciding upon $\omega(K,\theta)$ needs to take into consideration the subsequent actions of the firm.

Note again that for a time consistent, incentive compatible solution to obtain the labour demand is given by

(3.7) $\theta F_L(L,K) = w$

Now, consider the effect on profits, π, of a change in θ, once K has been decided upon.

$$\frac{\partial \pi}{\partial \theta} = (F_L(L,K) - w)\frac{\partial l^d}{\partial \theta} + F(L,K)$$

Using (3.7)

(3.8) $\pi_\theta = F(L,K)$

Consequently, when the planner or auctioneer announce some wage scheme, this wage scheme and the subsequent actions of the firm and the union must have two characteristics: firstly, the markets must be cleared and secondly, the specific information possessed by the firm must be elicited. Thus, note in particular that the planner must take into account the relationship described by (3.8). Equation (3.8) describes the value to the firm of lying and being believed: If the firm announces θ-dθ when the actual value is θ, the firm makes an extra profit of the order Ydθ.

The following proposition is proved in the Appendix.

<u>Proposition 1</u>. In a planning equilibrium with asymmetric information the equilibrium wage, T(Y), satisfies

(3.9) $U_1\{T(Y) + L\frac{\delta T}{\delta Y}\frac{\delta Y}{\delta L}\} + U_2 = K, \quad K > 0$

except for $\theta=\theta_u$ in which case

$$U_1\{T(Y) + L\frac{\delta T}{\delta Y}\frac{\delta Y}{\delta L}\} + U_2 = 0$$

Since the second order condition dictates $\delta^2 U/\delta w^2 < 0$, we have that the wage is such that the marginal disutility of labour is less than the marginal utility. Thus, the supply of employment is inefficiently low (see Figure 1, see next page).

Let the solution to (3.9) and (3.7) be denoted ω[10] (this is also derived formally in Appendix (A.II)).

(3.10) $w = \omega(K,\theta)$

$$\omega_K > 0, \ \omega_\theta > 0$$

Comparing (3.7), (3.9) to (3.3) it is clear that (this is actually also proved in the proof of Proposition 1).

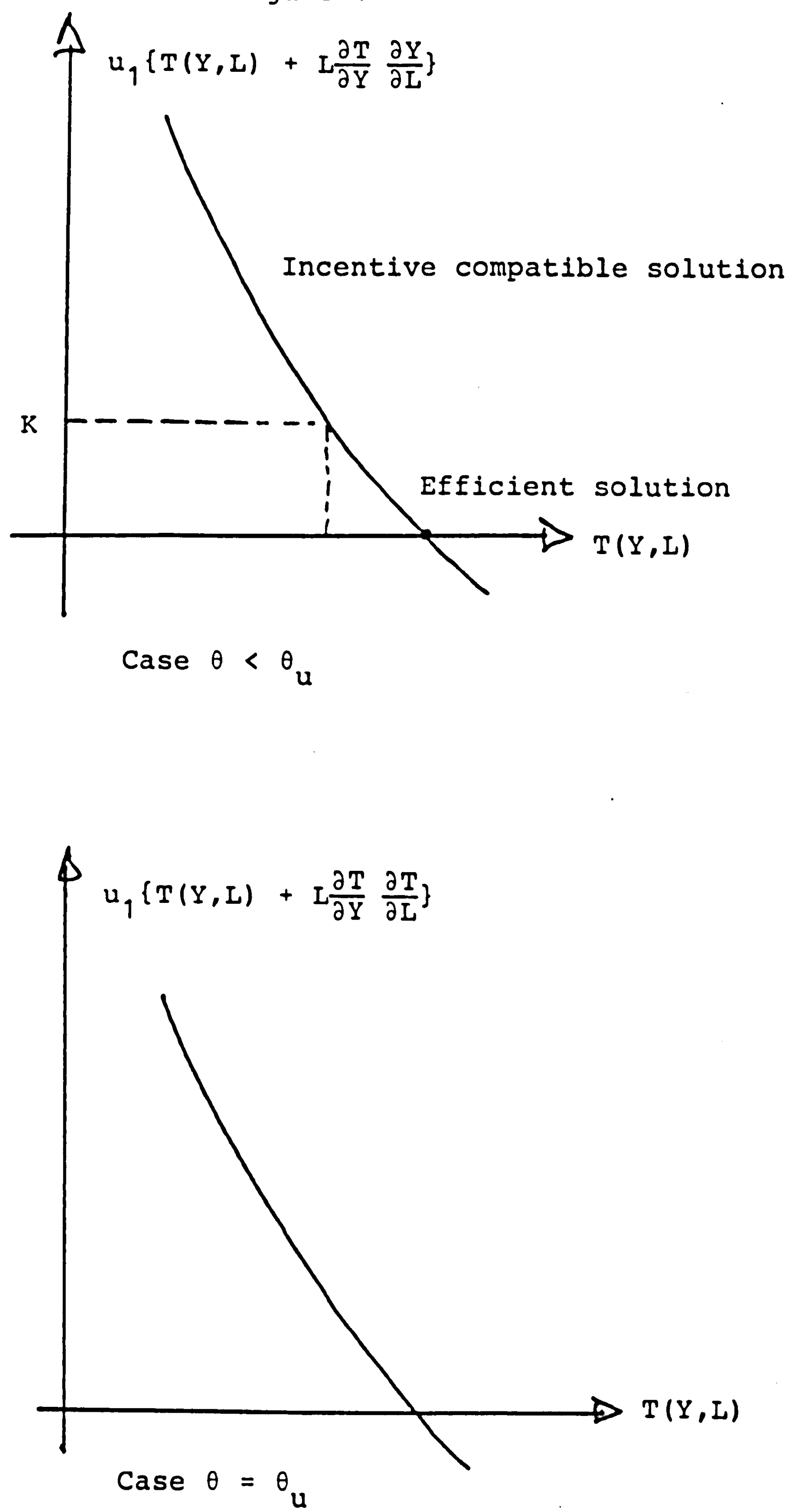
$$\omega(K,\theta) > \omega^*(K,\theta)$$

Turning attention to the choice of the stock of capital write profits as

(3.11) $\pi(K,\theta) = \theta F(g(\theta,K,\omega(K,\theta)),K) - \omega(K,\theta)g(\theta,K,\omega(K,\theta)) - r\cdot K$

Since the wage is dictated before K is chosen, the first order

Figure 1


$u_1\{T(Y,L) + L\frac{\partial T}{\partial Y}\frac{\partial Y}{\partial L}\}$
Incentive compatible solution
K
Efficient solution
T(Y,L)
Case θ < θ_u
$u_1\{T(Y,L) + L\frac{\partial T}{\partial Y}\frac{\partial T}{\partial L}\}$
T(Y,L)
Case θ = θ_u

condition is

$$(3.12) \quad F_K[g(\theta,K,\omega(K,\theta)),K) = r$$

Comparing (3.12) to (3.6) we have

Proposition 2. A planning equilibrium under asymmetric information involves less investment and for a given level of investment less employment.

The mechanism behind Proposition 2 is that since employment is used as a signal, asymmetric information lowers the level of employment. Thus, the value of the marginal product of capital is lowered. This in turn is followed by less investments.

IV. A Game Played between the Firm and the Union

In this section we analyse the implication of firm specific bargaining. The interaction between the firm and the union is direct and no longer through a planner. Thus, we consider an economy in which a local trade union is linked to a specific firm. The interpretation here may be that the union may face one of many firms not being able to tell precisely which one. An alternative interpretation and perhaps more realistic is that the union is not as well informed as the firm with respect to some value affecting the profitability of the firm. It may be that this interpretation does not seem to be much at variance with the one given in the previous section. However, once attention is focused upon a firm specific bargaining problem, the assumption of a planner cannot reasonably be sustained. This allows the two parties, the firm, and the union to exploit first mover advantages. An implication of this is that the presence of asymmetric information may (with the kind of bargaining process assumed here) actually support a higher level of investments. This contrasts with the findings of Grout (1984).

Let us consider first the situation arising under symmetric information.

Consider the decision with respect to employment. We have

$$\theta F_L(K,L) = w$$

Consequently, the labour demand is given by

$$L^d = g(\theta,K,w)$$

$$g_\theta > 0 \quad g_K > 0 \quad g_\omega < 0$$

Since the trade union will use its monopoly power, we find the following maximization problem

$$\max_w U(wg(\theta,K,w),\ g(\theta,K,w))$$

$$\text{s.t. } \pi = F(L,K) - wL - rK$$

$$F_L(L,K) = w$$

$$\pi_\theta = F(L,K)$$

The first order condition is

$$U_1(\cdot)\{g(\cdot,\cdot,\cdot) + wg_w(\cdot,\cdot,\cdot)\} + U_2(\cdot)g_w(\cdot,\cdot) = 0$$

Let the solution of this first order condition be $w=\tilde{\omega}^*(K,\theta)$ (opposed to $\omega^*(K,\theta)$, see (3.4)).

Since we do not know the sign of g_{wK} and $g_{w\theta}$, it is not possible to see if the sign of $\tilde{\omega}^*_K$, $\tilde{\omega}^*_\theta$ is unambiguous.

Denote the solution by $\tilde{\omega}^*(\theta,K)$. Thus, profits are

$$\pi = \theta F[g(\theta,K,\tilde{\omega}^*(\theta,K)),K] - \tilde{\omega}^*(\theta,K)g(\theta,K,\tilde{\omega}^*(\theta,K) - rK$$

First order conditions (dictating the choice of K) are

$$\theta F_L\{g_K + g_w\tilde{\omega}^*_K\} + \theta F_K - \{\tilde{\omega}^*_K g + \tilde{\omega}^*(g_K + g_w\tilde{\omega}^*_K)\} - r = 0$$

Using $\theta F_L(K,L)=\tilde{\omega}^*$, this reduces to

$$\theta F_K(\cdot) = r + \tilde{\omega}^*{}_K g(\cdot)$$

The sign of $\tilde{\omega}^*_K$ is unknown, hence, it is not possible to see if $\tilde{K}^* \gtrless K^*$, that is, if the stock of capital in a decentralized equilibrium is greater than or less than that in a command optimum.

Focusing on the case of asymmetric information, we offer the following proposition.

Proposition 3. Assuming that second order conditions are satisfied: When monopoly effects are present, employment is "inefficiently" low under asymmetric information.

Proof: See Appendix (A.III).

The intuition behind this result is given in Milgrom & Roberts (1982). Suppose that the firm announced its true value and was believed. Would this be an optimum? Clearly no, since if the firm is beleived to be telling the truth, it could deviate (at a cost of ε^2 since it is at its static oiptimum) and announce at value slightly lower than the one actually realized. This would imply an increase in profits of the order of ε through a low wage claim. Thus, the total gain is $\varepsilon(1-\varepsilon)>0$. This effect is recognized by the union and in order to pre-emt an inference dictating an excessively high wage, the firm deviates downward (see also Chapter 1).

Consider the effect upon investment: clearly since employment falls so will the value of the marginal product of capital. This tends to reduce the level of investment. However, in this setting, as opposed to that of earlier section, additional effects are present. One effect present is the direct effect upon the wage claim of the choice of investment, cf. $\tilde{\omega}_K^*$ and $\tilde{\omega}_K$. It is a priori not possible to decide upon the relationship between these two variables. Also present is an effect upon the sensitivity of labour demand of the wage to be set (cf. $g_K(\tilde{\omega}^*)$ and $g_K(\tilde{\omega})$). Again it is not a priori possible to decide on the relationship between these effects since they depend upon the third derivative of the production function.

We can, however, offer the following result. Let $\omega(\tilde{\cdot})$ denote the optimal wage claim under asymmetric information (derived in the proof of Proposition 3).

<u>Proposition 4</u>. A sufficient condition for investments to be lower compared to a situation under symmetric information is:

$$\tilde{\omega}_K g(\cdot) > \tilde{\omega}_K^* g(\cdot)$$

Proof: See Appendix (A.IV).

The result is readily explained. Since, in equilibrium the level of employment is lower than compared to the situation under symmetric information, the value of the marginal product (F_K) is lowered, say $F_K{}^0$ shifts down to $F_K{}^1$ due to the fall in the level of employment. This is the effect identified in the previous section. However, a change in the value of K has also other indirect effects. There is an effect upon the wage bill, since the wage changes. This effect is given by

$$\tilde{\omega}_K(\cdot)g(\cdot)$$

Before we comment on this effect, let us briefly discuss the effect upon employment due to the effect upon the wage. Profits are changed since the level of employment is changed.

$$\left.\frac{\partial \pi}{\partial \tilde{\omega}}\right|_K = (\theta F_L(\cdot) - w)\frac{\partial l}{\partial \tilde{\omega}}$$

But this is 0 since it is nothing but the first order condition for the choice of employment. Hence, the effects upon profits apart from the direct effect $(\theta F_K(\cdot)-r)$ are restricted to $\tilde{\omega}_K g(\cdot)$.

We have argued that the marginal value of an additional unit of capital is given by $\theta F_K{}^1$ as opposed to $\theta F_K{}^0$. What about costs? The marginal cost of acquisition of one more unit of capital is

$$r + \tilde{\omega}_K g(\cdot)$$

Since $g_w<0$, and since $\tilde{\omega}>\tilde{\omega}^*$, we find that $g(\tilde{\omega})<g(\tilde{\omega}^*)$. This effect tends to reduce the marginal cost of capital. The functions $\tilde{\omega}^*(\theta,K)$ and $\omega(\theta,K)$ are two different functions and as such it is not possible to compare their curvature. Hence, we would not know whether $\tilde{\omega}_K(\theta,K)>\tilde{\omega}_K{}^*(\theta,K)$ or $\tilde{\omega}(\theta,K)<\tilde{\omega}^*(\theta,K)$. Thus, it is in general not possible to say whether the cost of acquiring capital increases or decreases with the introduction of asymmetric information. The effects discussed are illustrated in Figure 2 (see next page). In consequence, we consider a specific example.

Let the production function be given as

$$y = L^{\alpha}K^{1-\alpha}$$

Assume that the utility function is of the following simple form

$$u = wL - 1/2\ L^2$$

In Appendix (A.V) this example is solved for the case of symmetric as well as asymmetric information. It is found that the marginal benefit of one additional unit of capital is given by

$$MB(K)_{SI} = \theta(1-\alpha)(\alpha\theta(1+\alpha))^{\frac{\alpha}{2-\alpha}}K^{-\frac{\alpha}{2-\alpha}}$$

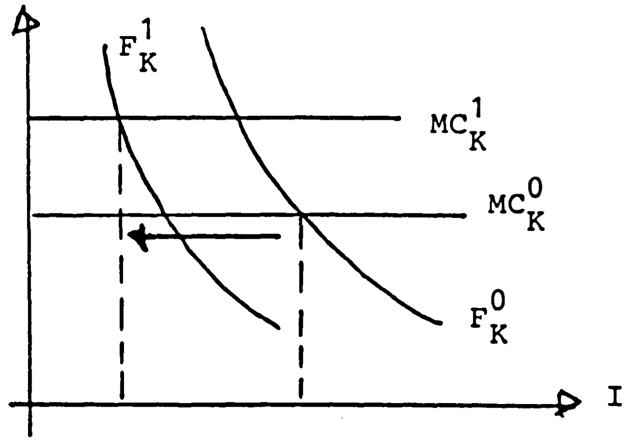
$$MB(K)_{ASI} = \theta(1-\alpha)(\alpha[\theta(1+\alpha) - (\bar{\theta}-\theta)])^{\frac{\alpha}{2-\alpha}}K^{-\frac{\alpha}{2-\alpha}}$$

The marginal cost of acquisition of capital is

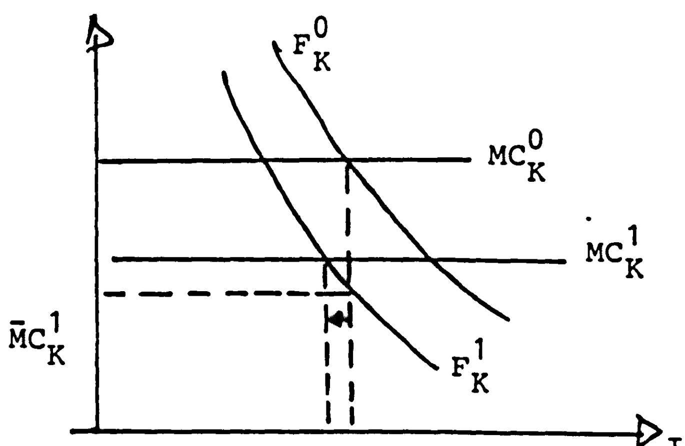
$$MC(K)_{SI} = r + \alpha(\alpha\theta(1+\alpha))^{\frac{\alpha}{2-\alpha}}(1-\alpha)\frac{3-\alpha}{2-\alpha}\ K^{2-\alpha}$$

$$MC(K)_{ASI} = r + \alpha(\alpha[\theta(1+\alpha) - (\theta-\theta)])^{\frac{\alpha}{2-\alpha}}(1-\alpha)\frac{3-\alpha}{2-\alpha}\ K^{2-\alpha}$$

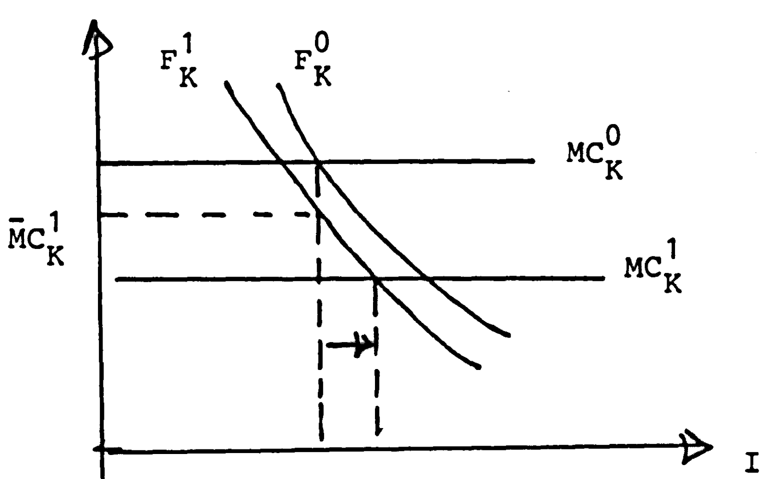
Figure 2



$MC_K^1 > MC_K^0$



$MC_K^1 < MC_K^0$ but $MC_K^1 > \bar{M}C_K^1$



$MC_K^1 < MC_K^0$ and $MC_K^1 < \bar{M}C_K^1$

Hence, we see first of all that in this example $MC(K)_{ASI} < MC(K)_{SI}$. The presence of asymmetries in the information sets of the two parties actually decreases the marginal cost of capital. Secondly, it is clear the $MB(K)_{SI} > MB(K)_{ASI}$. The last of these two results confirm our earlier general results. The first of the results seems to suggest that ambiguity in the general setting arises because two opposing effects are at work. The ambiguity does not arise because the specification is too general to allow conclusions to be drawn. Hence, it may well be the case that the presence of asymmetric information sets actually increases the level of capital. This in turn may increase employment. However, note that for a given level of capital underemployment will still occur.

The result that under asymmetric information a bargaining process (in the extreme form introduced here) may actually increase the level of investments (capital) contrasts with the finding of Grout (1984). Results are not directly comparable, however. In Grout (1984) a situation is analysed in which a firm realizes that upon an irreversible investment decision the union may exploit the different threat point (compared to that of no investment). It is shown that investment without binding contracts decreases investment compared to the case of binding contracts. Informational aspects are not considered in Grout (1984) and further comparisons are outside the scope of this paper.

V. Conclusion

This paper is concerned with the formation of wages and the subsequent decisions on employment and investment. Not surprisingly, if the economy is characterized by perfect and symmetric information, a social planner may, by dictating a proper wage, induce a contract which results in a wage, level of employment and investment which equal that obtained in a competitive economy.

However, under uncertainty and asymmetric information this result is no longer valid. If the value of the price of goods is known perfectly by the firm, but only probabilistically by the planner and the union, only incentive compatible contracts will be accepted by the union. In such circumstances, the planner cannot enforce a result which equals that of a competitive economy. Wages, respectively employment will be higher, respectively lower compared to an economy with perfect and symmetric information. Also investment will be lower. These results are modified in the case of a decentralized economy. Employment is still inefficiently low. However, investments may well increase compared to the case of symmetric information.

Grout (1984) also obtains results similar to the ones we present for the case of a command equilibrium. In Grout (1984), wages and input levels, as well as profits, are determined as the outcome of a Nash bargain. In the absence of binding contracts, the wage and the level of employment must be consistent with potential bargains made after the purchase of capital. Given that the union has any power at all (an apparently reasonable assumption), inputs are not employed efficiently. The reason for this result is that the real price of capital increases beyond the simple price of capital, since the firm has to add to the cost price the spill over effect upon the total wage bill. However, note that the results obtained here in the case of a decentralized economy our results contrast those of Grout (1984), at least partially, since investments may increase.

In this chapter, such spill-over effects (externalities) have been ruled out. The result obtained here is due only to strategic considerations. Note also that the discussion of whether contracts are binding or not is immaterial, since the contracts set forth in this paper are self-enforceable.

It has been argued here that underemployment is the outcome of a contractual agreement between the firm and the union. Thus, the work presented here is close to that of the implicit contracts literature (for a survey see Azariadis and Stiglitz (1983)).

In Azariadis (1983) and Grossman and Hart (1983), underemployment is the rule, whereas in Chari (1983) and Green and Kahn (1983), overemployment is the rule. However, to obtain underemployment in this class of models, it is noteworthy that firms have to be more risk averse at the margin than workers are. In the present setting,
no assumptions were made with respect to the degree of risk aversion of the two parties. In addition, the current paper also

analyses the implication with respect to the level of investments. Thus, it can be seen as an extension of models of implicit contracts. An obvious extension of the current work would be to incorporate the idea of insurance against loss of income, which is the central idea of the implicit contracts literature.

VI. Notes.

1. This is of importance only in Section IV and is not used in the current section.

2. In this section, the phrase "trade union" and "worker" is used freely since planner is introduced.

3. For example, let the utility function be of the Cobb-Douglas type $U=(wL)^{\alpha}(\frac{1}{L})^{+\alpha}$. Hence, $U_{12}=\alpha(\alpha-1)(wL)^{\alpha-1}(\frac{1}{L})^{-\alpha}<0$.

4. By * is denoted the value resulting under symmetric information.

5. Note $l^{*d} = l^{d}$.

6. $\theta'\epsilon B(\theta,\varepsilon)$.

7. By $l(\theta'|\theta)$ is meant $\tilde{\theta}=\theta'$ and the realized value is θ, etc.

8. Note the Lemma is not concerned with time consistency.

9. Insert $g(\theta,K,w)$ into (3.1); g_{θ}, g_{K}, g_{w} are obtained immediately.

10. Insert $\omega(K,\theta)$ into (3.9) and (3.7) and using second order conditions the derivatives of ω_{K} and ω_{θ} are obtained.

VII. Appendix

A.1. Derivation of the Command Equilibrium

Consider a situation where we look at an economy consisting of only three players: a producer (and thus demander of labour), a consumer (and supplier of labour), and an auctioneer or planner. These are endowed with the following pay-off functions:

$$\pi = \theta F(L,K) - wL - rK$$

$$U = U(wL,L)$$

$$W = -\tfrac{1}{2}(L^d - L^S)^2, \quad W(0) = 0, \quad W(x) < 0, \quad \forall x \neq 0$$

The planner chooses first, dictating w, the wage. The remaining players act subsequently. Thus, the planner in choosing w has to take into consideration the behavior of these players.

Solving for the behvior of the producer, we have

$$\theta F_L(L,K) = w \Rightarrow$$

$$L^d = g(\theta,K,w)$$

where (see note 9) $g_\theta>0$ $g_K<0$ and $g_w<0$. This is Equation (3.2) in the text, which describes the behavior of the demanders of labour.

Solving for the behavior of the producers, we have

$$U_1(\;,\;)w + U_2(\cdot,\cdot) = 0 \Rightarrow$$

$$L^S = h(w)$$

where we assume that $h'(w)>0$. This is Equation (3.3) in the text describing the behavior of the suppliers of labour. Now, the planner faces the following problem

$$\underset{w}{\text{Max}} \; - \frac{1}{2}(g(\theta,K,w) - h(w))^2$$

We find the first order condition

$$- [g(\theta,K,w) - h(w)][g_w - h_w] = 0$$

Since $g_w - h_w < 0$ this requires that the equilibrium wage satisfies

$$g(\theta,K,w) \equiv h(w)$$

From this we have the solution

$$w = \omega^*(K,\theta)$$

where $\omega^*_K > 0$, $\omega^*_\theta > 0$.

A.II. Proof of Proposition 1

The proof of this proposition proceeds as in the derivation of the command equilibrium. Thus, three agents are considered: the demander of labour, the supplier of labour, and the planner. Since the firm has to choose the level of employment, once the wage is set and a capital stock is decided upon it is trivial that the demand for labour is described by:

$$L^d = g(\theta, K, w)$$

$$g_\theta > 0 \quad g_K > 0 \quad g_w < 0$$

The planner maximizes a utility function taking into consideration the subsequent actions of the agents. We assume that the union for a given wage (dictated by the planner) will accept only levels of employment that support truthful revelation of θ. Now let the wage dictated by the planner be $T=T(Y)\theta$. The fact that the planner has to dictate a wage scheme ($T(Y)$) and not just a wage is that the resolution of the game must elicit all information and the firm has an incentive to lie (cf. 3.8), an effect which must be reflected in the planner's choice of "the wage".

Thus, let us focus upon the choice of labour supply. For a given wage scheme, the maximization problem of the $T=T(Y)$ union can be written as

(A.P.2) $\underset{L}{\text{Max}}\ U(T(Y)L,L)$

$$\pi_\theta = Y$$

T(Y) captures the fact that for some given profit level, $\theta F(L,K)-wL-rK$, L is chosen according to the standard condition $w=\theta F_L(L,K))$ and, thus, is a function of total production.

The Hamiltonian, written in terms of L, for (A.P.2) is

$$\text{(A.1)} \quad H = U(T(Y)L,L) + \gamma(\theta)Y$$

The first order conditions are found to be

$$\text{(A.2)} \quad U_1\{T(Y) + L\frac{\delta T}{\delta Y}\frac{\delta Y}{\delta L}\} + U_2 + \gamma(\theta)\frac{\delta Y}{\delta L} = 0$$

$$\text{(A.3)} \quad U_1\{T(Y)\frac{dL}{d\theta} + L\frac{\delta T}{\delta Y}\frac{\delta Y}{\delta L}\frac{dL}{d\theta}\} + U_2\frac{dL}{d\theta} + \gamma(\theta)\frac{\delta Y}{\delta L}\frac{dL}{d\theta} + \gamma'(\theta)Y + U_1L\frac{dT}{dY}\frac{\delta Y}{\delta \pi}\pi_\theta = 0$$

Clearly (A.2), (A.3) reduce to

$$\text{(A.4)} \quad U_1\{T(Y) + L\frac{\delta T}{\delta Y}\frac{\delta Y}{\delta L}\} + U_2 + \gamma(\theta)\frac{\delta Y}{\delta L} = 0$$

$$\text{(A.5)} \quad \gamma'(\theta)Y + U_1L\frac{\delta T}{\delta Y}\frac{\delta Y}{\delta \pi}\pi_\theta = 0$$

Using $\pi \equiv PY-wL-rK$ and noting that $w=T(Y,L)$, it is seen that

$$0 \equiv PdY - L\cdot dT \Rightarrow$$

$$\frac{\delta T}{\delta Y} = \frac{P}{L}$$

Also $\quad \delta\pi = P\delta Y \Rightarrow$

$$\frac{\delta Y}{\delta\pi} = \frac{1}{P}$$

Using this, (A.4) and (A.5) reduce to

$$\text{(A.6)} \qquad U_1\{T(Y) + L\frac{\delta T}{\delta Y}\frac{\delta Y}{\delta L}\} + U_2 + \gamma(\theta)\frac{\delta Y}{\delta L} = 0$$

$$\text{(A.7)} \qquad \gamma'(\theta) - U_1 = 0$$

Let the solution to (A.7) be

$$\gamma(\theta) = Z(\theta) + \text{constant}$$

Thus, the first order condition reduces to

$$U_1\{T(Y) + L\frac{\delta T}{\delta Y}\frac{\delta Y}{\delta L}\} + U_2 = (-Z(\theta) - \text{constant})\frac{\delta Y}{\delta L}$$

The worst point estimate to the firm is $\theta = \theta_U$, thus, in this case the first best solution obtains (see Mailath (1988)). That is, for $\theta=\theta_U$ we have $U_1\{\cdot\}+U_2=0$. This is the boundary condition dictating "constant". Hence, "constant = $-Z(\theta_U)$".

Thus, we have

$$\text{(A.8)} \qquad U_1\{T(Y) + L\frac{\delta T}{\delta Y}\frac{\delta Y}{\delta L}\} + U_2 = (Z(\theta_U) - Z(\theta))\frac{\delta Y}{\delta L}$$

Since $Z(\cdot)$ is an increasing function ($Z'(\cdot)=\gamma'(\cdot)=U_1(\cdot)>0$), this concludes the proof.

Q.E.D.

Let us, however, formally show that the wage is higher and the level of employment lower compared to the case of symmetric information. (This is only shown loosely in the text).

Since $Z'(\theta)>0$, the right hand side is for $\theta\neq\theta_U$ strictly bounded from below at zero and, hence,

$$U_1\{T(Y) + L\frac{\delta T}{\delta Y}\frac{\delta Y}{\delta L}\} + U_2 > 0$$

In consequence for any given scheme T(Y), the amount of labour supplied is, ceteris paribus, lower. Let the solution to (A.8) be

$$L^S = \tilde{h}(w,\theta)$$

Thus, since all information is elicited, it remains for the planner to maximize $w=-(1/2)(L^d-L^s)^2$.

The first order condition is

$$- [g(\theta,K,w) - \tilde{h}(w,\theta)][g_w - \tilde{h}_w] = 0$$

This reduces to

$$g(\theta, K, w) = \tilde{h}(w, \theta)$$

Since $\tilde{h}(w,\theta) < h(w,\theta)$, the equilibrium wage is higher and the level of employment lower compared to the case of symmetric information.

A.III. Proof of Proposition 3

This proof proceeds different from the predecessor. The reason being that only two agents are involved here: the demander of labour and the supplier of labour. The union now has to dictate the wage. Thus, a monopoly effect is introduced.

Consider w to be some function of Y and L: w=T(Y,L). Thus, w can be interpretated as a control variable, whereas Y and L are state variables. Thus, the maximization problem of the union can be written as

$$\text{(A.P.2)} \quad \max_{w} U(T(Y,L)L, L)$$

$$\pi_\theta = \bar{Y}$$

T(Y,L) captures the fact that for some given profit level F(L,K)-wL-rK), L is chosen according the standard condition $F_L(L,K))=w$.

The Hamiltonian, written in terms of L, for (A.P.2) is

$$(A.9) \quad H = U(T(Y,L)L,L) + \gamma(\theta)Y$$

The first order conditions are found to be

$$(A.10) \quad U_1\{T(Y,L)\frac{dL}{dw} + L\frac{\delta T}{\delta Y}\frac{\delta Y}{\delta L}\frac{dL}{dw} + L\frac{\delta T}{\delta L}\frac{dL}{dw}\} + U_2\frac{dL}{dw} + \gamma(\theta)\frac{\delta Y}{\delta L}\frac{dL}{dw} = 0$$

$$(A.11) \quad U_1\{T(Y,L)\frac{dL}{d\theta} + L\frac{\delta T}{\delta Y}\frac{\delta Y}{\delta L}\frac{dL}{d\theta} + L\frac{\delta T}{\delta L}\frac{dL}{d\theta}\} + U_2\frac{dL}{d\theta} + \gamma(\theta)\frac{\delta Y}{\delta L}\frac{dL}{d\theta}$$

$$+ \gamma'(\theta)Y + U_1L\frac{dT}{dY}\frac{\delta Y}{\delta \pi}\pi_\theta = 0$$

Clearly (A.10), (A.11) reduce to

$$(A.12) \quad U_1\{T(Y,L) + L\frac{\delta T}{\delta Y}\frac{\delta Y}{\delta L} + L\frac{\delta T}{\delta L}\} + U_2 + \gamma(\theta)\frac{\delta Y}{\delta L} = 0$$

$$(A.13) \quad \gamma'(\theta)Y + U_1L\frac{\delta T}{\delta Y}\frac{\delta Y}{\delta \pi}\pi_\theta = 0$$

Using $\pi \equiv pY-wL-rK$ it is easily seen that $\delta T/\delta Y=p/L$ and $\delta Y/\delta\pi=1/p$. Thus, (A.12) and (A.13) reduce to

$$(A.14) \quad U_1\{T(Y,L) + L\frac{\delta T}{\delta Y}\frac{\delta Y}{\delta L} + L\frac{\delta T}{\delta L}\} + U_2 + \gamma(\theta)\frac{\delta Y}{\delta L} = 0$$

$$(A.15) \quad \gamma'(\theta) - U_1 = 0$$

Let the solution to (A.15) be

$$\gamma(\theta) = Z(\theta) + \text{constant}$$

Thus, the first order condition reduces to

$$U_1\{T(Y,L) + L\frac{\delta T}{\delta Y}\frac{\delta Y}{\delta L} + \frac{\delta T}{\delta L}\} + U_2 = (-Z(\theta) + \text{constant})\frac{\delta Y}{\delta L}$$

The worst point estimate to the firm is $\theta = \theta_U$, thus, in this case the first best solution obtain (see Mailath (1988)). Hence, "constant = $-Z(\theta_U)$".

Thus, we have

$$\text{(A.16)} \quad U_1\{T(Y,L) + L\frac{\delta T}{\delta Y}\frac{\delta Y}{\delta L} + L\frac{\delta T}{\delta L}\} + U_2 = (Z(\theta_U) - Z(\theta))\frac{\delta Y}{\delta L}$$

Since the left hand side is declining in L to satisfy second order conditions, we see that as $Z(\theta_U)-(Z(\theta)(\delta Y/\delta L))>0$, the amount of labour employed decreases under asymmetric information.

Q.E.D.

A.IV. Proof of Proposition 4

Using the first order condition of the labour market, we have to compare, for the case of symmetric and asymmetric information, respectively:

$$\theta F^*_K(\cdot) = r + \tilde{\omega}^*_K g(\cdot)$$

$$\theta F_K(\cdot) = r + \tilde{\omega}_K g(\cdot)$$

where $\tilde{\omega}>\tilde{\omega}^*$. Since $F_K{}^*(\cdot)>F_K(\cdot)$, all K, a sufficient condition for

K to decrease in the presence of asymmetric information is that $\tilde{\omega}_K * g(\cdot) < \tilde{\omega}_K * g(\cdot)$, when $\tilde{\omega}_K$ denotes $\partial\tilde{\omega}/\partial K$.

A.V. Solution of example

We have

(A.16) $y = L^{\alpha}K^{1-\alpha}$

(A.17) $U = wL - 1/2\ L^2 \qquad U_1 = 1 \qquad U_2 = -1$

$\pi = \theta y - \omega l - rK$

Using (A.16), we have that (A.14) simplifies to

(A.18) $\gamma'(\theta) = 1$

Hence,

(A.19) $\gamma(\theta) = \theta$

In equilibrium $w = \alpha\theta L^{\alpha-1}K^{1-\alpha} = \alpha\theta\,\frac{Y}{L}$. Thus,

(A.20) $T(y,L) = \alpha\theta\,\frac{Y}{L}$

Using (A.17), (A.19), and (A.20) in (A.15), the general first order condition

$$\theta\alpha\ \frac{Y}{L} + L(\frac{\theta\alpha}{L})(\frac{\alpha Y}{L}) - L = (\bar{\theta}-\theta)\alpha\ \frac{Y}{L}$$

This reduces to

$$\text{(A.21)}\quad L = (\alpha[\theta(1+\alpha) - (\bar{\theta}-\theta)]K^{1-\alpha})^{\frac{1}{2-\alpha}}$$

Thus, in equilibrium

$$\text{(A.22)}\quad w = \alpha(\alpha[\theta(1+\alpha) - (\bar{\theta}-\theta)]K^{1-\alpha})^{\frac{\alpha-1}{2-\alpha}}K^{1-\alpha}$$

In the case of symmetric information, these two conditions reduce to

$$\text{(A.21')}\ L = (\alpha\theta(1+\alpha)K^{1-\alpha})^{\frac{L}{2-\alpha}}$$

$$\text{(A.22')}\ W = \alpha(\alpha\theta(1+\alpha)K^{1-\alpha})^{\frac{\alpha-1}{2-\alpha}}K^{1-\alpha}$$

The profit of the firm is given by

$$\pi = \theta L^{\alpha}K^{1-\alpha} - wL - rK$$

where L and w are given by (A.21)-(A.22) in the case of asymmetric information and by (A.21')-(A.22') if information is symmetric. Using this, the result in the text is easily found.

CHAPTER 5

WAGE BARGAINING IN SEQUENTIAL EQUILIBRIUM.
DEVIATIONS FROM THE FIRST BEST AND WELFARE IMPROVING TAXES

I. Introduction.

In economies characterized by wage determination by bargaining it is of interest to analyse the consequences of the incentive comnpatibility constraints arising due to asymmetric information. This we did in a predecessor Chapter 2 to the current chapter.

The wage is determined as the outcome of some bargaining process involving a trade union and a firm. Attention is focused upon the problems arising if the firm compared to the trade union is better informed with respect to some exogenous variable partly responsible for the outcome of the bargaining process. The bargaining strength of the two parties involved is assumed to be given exogenously, and at the same time it is assumed that the trade union has the opportunity to draw inferences over time about the variable conditioning the outcome of the bargaining. Inferences are conditional on past actions as well as common prior knowledge. Within the two period model the natural equilibrium to look for is a sequential equilibrium. Hence, actions by the firm taken in the first period are used by the trade union to draw inferences about the unknown variable and are thus reflected in the outcome of the bargain and consequently in second period profits. This is realized by the firm and taken into account when deciding upon first period actions.

In Chapter 2 the bargaining process was taken to be very simple. The trade union simply announces the wage which is to rule in the second period. Given this wage firms adjust input of labour until equality between the marginal product of labour and the wage is reached. This is the by now well-known monopoly union model.

In an environment characterized by asymmetric information in the sense that only the firm knows the true value of the marginal product of labour it was shown in Chapter 2 that the incentive compatibility constraints enforces inefficiency. Within the first period all firms except the one at "the top" deviates downward in the separating equilibrium, i.e., different types of firms (values of marginal product of labour) choose different actions (level of labour input) and consequently the trade union correctly identifies the firm. Naturally, the second period production and employment are undisturbed compared to the situation under full information. However, due to the first period deviation the overall level of welfare is lowered compared to an economy characterized by full information.

The assumption that the labour market can be described as simply as is done in the monopoly union model is objectionable in so far as this description simply does not agree with common practice in a process of centralized wage setting (Nickel & Andrews (1983)). Also, the assumption turns out to be restrictive. This is, perhaps, not unexpected. The incentive to deviate from the first best during the first period does, at least partly, depend upon the gain from doing so. Intuitively, the larger the gain the stronger the incentive to deviate. And the monopoly union model, as it is so favourable to the trade union, presumably provides a strong incentive for deviation. Implicitly, the monopoly union model involves a bargaining strength of 1 on the part of the union. It is to be expected that as the bargaining strength of the trade union falls the incentive to deviate falls and the deviation becomes smaller, in the limit approaching zero.

The aim of this chapter is to provide an analysis of this issue. Also of interest is the possibilities of introducing welfare improving taxes. We report some positive results on this issue.

In Section 2 the model is presented and some preliminary results are offered. Section 3 deals with a precise characterization of the optimal strategy and some properties of this strategy. In section 4 the issue of taxation is analysed. Conclusions are given in Section 5.

II. The Model

The economy considered here consists of a range of different firms, each firm tied up to a trade union. Each firm produces according to $y = \sqrt{l}$ but differs with respect to the evaluation of the output. This value we denote p, which is distributed according to f(p) with support $[p_l, p_u]$. This is the only way in which firms differ.

The wage faced by any firm is w_1 during the first period. That is, whatever is the type of the firm, it will face the same wage. This is not necessarily so in the second period. If the equilibrium is a separating one, then different firms will face different wages. This construction can be interpretated literally, that is, there exists a whole range of firms indexed by the price. Alternatively, the situations is one in which there is only one firm, but its type is unknown to the union (on this see Milgrom and Roberts (1982)).

It is assumed that the trade union's prior belief about the type of firm with which it is faced is also given by the distribution f(p) and support $[p_l, p_u]$. Second period beliefs are the updated first period beliefs.

Consider now the following scenario. The range of firms are faced, in the first period, with the common wage w_1. This wage is left unexplained in this context but may be, for example, the wage which maximized the expected value of the bargain (to be defined later). After the end of the first period, but before the start of the second the trade union and the firm become involved in a bargaining process determining the wage which is to rule in the second period. Once agreement is reached, it is assumed to be binding for both parties. It is assumed that the bargaining process maximizes the expected value of the bargain <u>conditional</u> upon prior knowledge <u>as well as</u> the information revealed by the firm during the first period. That is, any decision taken by the firm during the first period does, perhaps only probabilistically, reflect the value of p, which is before period 1 known only according to f(p) and $[p_l, p_u]$ to the trade union. Consequently, insofar as the value of p is reflected in the outcome of the bargaining process, any first period decision influences the profits obtained by the firm in the second period.

Before the sequential equilibrium under imperfect information is analysed let us consider the equilibrium under perfect information. A firm producing a product of some value $p \in [p_l, p_u]$ maximizes overall profits. Hence,

$$\max_{l_1, l_2} p\sqrt{l_1} - w_1 l_1 + p\sqrt{l_2} - w_2 l_2 \qquad p \in [p_l, p_u]$$

The wage rates w_1 and w_2 are taken to be exogenous variables. Hence, the first order conditions read

$$(1) \quad l_i^d = \frac{p^2}{4w_i^2} \qquad i = 1,2$$

Consequently, the Walras' output in the two periods is given by

$$(2) \quad y_i = \frac{p}{2w_i} \qquad i = 1,2$$

Let us now turn attention to the dynamic equilibrium. The interest of this paper is in separating strategies only. This turns out to be unrestrictive given the structure analysed here. Let us assume that the strategy t maps R_+ into $R_+ U\{0\}$ prescribing for some value of $p \in \{p_l, p_u\}$ a value of y, hence, $t(p)=y_1$. Given that the equilibrium we are looking for is a separating equilibrium it is known that t is strictly monotonic[1]. Hence, for any output, y, belonging to the range of possible outputs, $r[t[p_l, p_u]]$, the trade union infers the value of p correctly. At this point note that even if the equilibrium strategy is separating and, consequently, the strategic behavior does not result in a level of profit in the second period different from that under perfect information, the optimal strategy can still dictate deviations from the first best solution. The reason for this is that the first best is not incentive compatible. The argument has been given in Chapter 1 in the General Introduction as well as in more detail in Chapter 2 and we refer the reader to those paragraphs.

In order to proceed we need to determine the sign of t'. Assume that the firm chooses the strategy $t(p)=y$ where t is a separating strategy, that is, $t^{-1}(y)=p$. The wage ruling in the second period is determined by a bargaining process. Here we take the bargain to be defined over the welfare of the union and profits. The welfare of the union is given by:

$$U(w_2,l_2) = w_2 l_2 (1-l_2) \qquad l \leq 1$$

This function may be justified by again referring to the consumption theory of Becker (cf. Becker (1971)). Consider a competitive solution. Clearly with the above specification of the utility function we find that the supply of labour for any given wage is: $l^s=1/2$. Using (9) we have $l^d=p^2/(4w_1{}^2)$. Assume now that employment is demand determined, hence output is by $y_1=p/(2w_1)$. This first best output for the first period should be contrasted to production under asymmetric information (see (14)).

Using that $l_2=(p/2w_2)^2$, we find

$$U(w_2, l_2) = \frac{p^2}{4w_2} - \frac{p^4}{4w_2^3}$$

Profits are given as

$$p\sqrt{l} - wl = p\frac{t^{-1}(y)}{2w_i} - \frac{(t^{-1}(y))^2}{4w_i} =$$

$$\frac{t^{-1}(y))^2}{2w_i} - \frac{(t^{-1}(y))^2}{4w_i} = \frac{t^{-1}(y)^2}{4w_i}$$

It is assumed that in the case of no agreement each party obtains a level of utility equal to zero. Thus, the value of the bargain is

$$(3) \quad \Omega = (\frac{p^2}{4w_2} - \frac{p^4}{16w_2^3})^{\beta}(\frac{p^2}{4w_2})^{1-\beta}$$

The parameter $\beta(1-\beta)$ gives the bargaining strength of the union (firm) and is dependent for example upon the time preference of the two parties (see (Binmore, Rubinstein, Wolinsky (1986)). Note at this point that the expression Ω assumes that the threat point of both parties is zero. Thus, interpretation of the case $\beta \rightarrow 0$ must be carried out with great care. The exclusion of threat points different from zero is the price for obtaining a (nearly) explicit solution.

Differentiating with respect to w_2, the agreed upon wage is

$$w_2 = t^{-1}(y)G(\beta), \qquad G(\beta) = (\frac{1+2\beta}{4})^{1/2}$$

In the current framework we consider w_2 to be given by some strategy, call this s, which maps $R_+ \cup \{0\}$ into $R_+ \cup \{0\}$. Hence, we write

(4) $w_2 = s(y) = t^{-1}(y)G(\beta)$

Clearly, we have

Lemma 1: The wage resulting in the second period is an increasing function of first period production if first period production is increasing in the value of the good produced by the firm, i.e., in p, by the firm.

Also we have

Lemma 2: If the strategy prescribing the first period output is strictly incentive compatible, then it is also continuous and differentiable. Furthermore, if the strategy dictating the wage ruling in the second period is increasing in first period output, then the strategy t is strictly increasing.

Before we proceed to the proof of this lemma let us introduce some terminology. By a strictly incentive compatible strategy is meant a strategy $t(p)=y$ which fully reveals the value of $p \in [p_l, p_u]$ to the trade union. In the proof it is necessary to be concerned with the payoff function to the firm. Denote this by $\pi(p,\tilde{p},y)$ where $\tilde{p}$ denotes the value of p as it is inferred by the trade union. We have

$$\pi(p,t^{-1}(y),y) = \pi(p,\tilde{p},y)$$

Hence,

$$\pi(p,\tilde{p},y) = \pi_1(p,y) + \pi_2(p,p)$$

We easily establish

(R1) $\pi_2 = \dfrac{\delta\pi_2}{\delta\tilde{p}} < 0$

This is so since

$$\frac{\partial\pi}{\partial\tilde{p}} = \frac{\partial\pi_2}{\partial w_2}\ \frac{\partial w_2}{\partial\tilde{p}} < 0$$

(R2) $\pi_{13} = \dfrac{\delta^2\pi_2}{\delta p\delta y} > 0$

(R2) following immediately from differentiating $\pi(p,\tilde{p},y)$.

We can proceed to the proof of this lemma.

<u>Proof of Lemma 2</u>: See Appendix.

<u>III. The Equilibrium</u>

We are now in a position to consider the characterization of the optimal strategy t(p)=y. The firm maximizes overall profits taking into consideration that the wage ruling in the second period is given by (4). Total profits comprise of first and second period profits. First period profits are given simply by $p \cdot y - w_1 y^2$. Now, second period profits are

$$\pi_2 = py_2 - w_2 y_2^2$$

Since y_2 is chosen so as to maximize π_2, we have $y_2 = p/(2w_2)$. Using the expression for w_2 we have

$$y_2 = \frac{p}{2}(t^{-1}(y)G(\beta))^{-1}$$

Hence, we obtain

$$\pi_2 = py_2 - w_2 y_2^2$$

$$= y_2(p - w_2 \frac{p}{2w_2}$$

$$= \frac{1}{2} py_2$$

$$= \frac{p^2}{4}(t^{-1}(y)G(\beta))^{-1}$$

Consequently, total profits are

$$\pi = py - w_1 y^2 + \frac{p^2}{4}(t^{-1}(y)G(\beta))^{-1}$$

The first order condition is

$$(5) \quad (p - 2w_1 t) + \frac{p^2}{4} \frac{1}{G(\beta)} \frac{-1}{(t^{-1}(y))^2} \frac{1}{t'(p)} = 0$$

Note that $(t^{-1}(y))^2=p^2$, $y=t$ and $(1/t'(p))=(dp/dt)$. Thus, (5) reduces to

$$(6) \quad (p-2w_1t)dt + \frac{-1}{4}\frac{1}{G(\beta)}dp = 0$$

Thus, the optimal strategy is given by the non-exact differential Equation (6). In general, non-exact differential equations are extremely difficult to solve but in this case an integrating factor is easily seen to be (see Boyce & Diprima (1977))

$$\mu = \exp^{-4\ G(\beta)t}$$

Thus, multiplying Equation (6) by μ an exact differential equation is obtained and such equations have implicit solutions. We have

$$(7) \quad \exp^{-4\ G(\beta)t}\ (p-2w_1t)dt + \exp^{-4\ G(\beta)t}\ \frac{-1}{4}\frac{1}{G(\beta)}\ dp = 0$$

Consider the solution to this differential equation which is given by $\varphi(p,t)=0$ where φ is to be determined. The constant c is to be determined by some initial value conditions. From Equation (7) and the theory of exact differential equations we have

$$\varphi_p = \exp^{-4\ G(\beta)t}\ (-\frac{1}{4})\ \frac{1}{G(\beta)}$$

Consequently $\varphi(p,t)$ is given by

(8) $\varphi = \exp^{-4\,G(\beta)t}\,(-\frac{1}{4})\,\frac{1}{G(\beta)}\,p + h(t)$

From this we find

(9) $\varphi_t = \exp^{-4\,G(\beta)t}\;p + h'(t)$

But also from Equation (7) and the theory of exact differential equations we have

(10) $\varphi_t = \exp^{-4\,G(\beta)t}\,(p-2w_1t)$

Combining (9) and (10)

$$h'(t) = -\,\exp^{-4\,G(\beta)t}\;2w_1t$$

Integrating by parts

(11) $h(t) = \exp^{-4G(\beta)t}\;\frac{1}{4}w_1\,\frac{1}{G(\beta)}\,(t + \frac{1}{4}\,\frac{1}{G(\beta)})$

Combining Equations (8) and (11) the solution to the differential Equation (6) is given by a set of (p,t,c) where c is determined by initial value conditions

(12) $\exp^{-4G(\beta)t}\;[-\frac{1}{4}\,\frac{1}{G(\beta)}\,p + \frac{1}{2}w_1\,\frac{1}{G(\beta)}\,(t + \frac{1}{4}\,\frac{1}{G(\beta)})] = c$

Referring to Lemma 2 of the previous section, t is increasing in p. Furthermore, from the proof of this lemma $t(p_u) = p_u/2w_1$.

Hence, the constant which identifies the unique solution (on this see Mailath (1987) and Chapter 2 for an application) is given by

$$c = \exp^{(-4G(\beta)(p_u/2w_1))}\left[\frac{-1}{4}\frac{1}{G(\beta)}\, p_u + \frac{1}{2}\, w_1\frac{p_1}{1G(\beta)}\left(\frac{u}{2w} + \frac{1}{4}\frac{1}{G(\beta)}\right)\right]$$

Hence, the unique solution is given by

$$(13)\quad -\frac{1}{4}\, p + \frac{1}{2}w_1\left(t + \frac{1}{4}\frac{1}{G(\beta)}\right) = \frac{1}{2}\exp^{4G(\beta)\left(t - \frac{p_u}{2w_1}\right)}\frac{1}{4}\, w_1\frac{1}{G(\beta)})$$

Rewriting (13) slightly we have

$$(14)\quad -\frac{p}{2w_1} + t = [\exp^{4G(\beta)\left(t - \frac{p_u}{2w_1}\right)} - 1]\frac{1}{4}\frac{1}{G(\beta)}$$

The term on the left hand side would equal 0 under perfect information (compare with $t=p/2w_1$ which is the solution under symmetric information). However, the term on the right hand side is bounded from above at 0. From this we have:

<u>Proposition 1</u>

For any value of $p\varepsilon[p_1, p_u]$ except p_u, the corresponding production is lower than it is under perfect information.

In addition to Proposition 1 a A comparative static result for $dt/d\beta$ can be obtained. Using (14) it is seen that

$$(15)\quad \frac{dt}{d\beta} = \frac{G'(\beta)}{4G^2(\beta)}(1 - \exp^{4G(\beta)(t-\frac{p_u}{2w_1})})^{-1} \cdot$$

$$\{1 - \exp^{4G(\beta)(t-\frac{p_u}{2w_1})}) + 4G(\beta)(t-\frac{p_u}{2w_1})\}$$

Thus, the sign of $dt/d\beta$ depends upon the sign of

$$\{1 - \exp^{4G(\beta)(t-\frac{p_u}{2w_1})}) + 4G(\beta)(t-\frac{p_u}{2w_1})\}$$

Consider equivalently the function $1-\exp^x+x$, where $x=4G(\beta)(t-(p_u/2w_1))<0$. Thus, it is clear that $1-\exp^x+x<0$. Hence, in conclusion $dt/d\beta<0$, that is, output is increasing as β, the bargaining strength of the trade union is decreasing. We state this as

<u>Proposition 2</u>

Output is decreasing in the bargaining strength of the union.

Consider now the welfare loss. This is written as

$$W = \int_{p_1}^{p_u}(y^* - t(p,\beta))f(p)dp$$

where y^* is the competitive output $(p/2w_1)$.

Using Proposition 2 we state the following lemma

Lemma 1

$$\frac{dW}{d\beta} = \int_{p_l}^{p_u} - \frac{dt(p,\beta)}{d\beta} f(p)dp > 0$$

Thus, as β is increased, the welfare loss of the economy, as measured by the deviation from the first best output, is increased.

In the next section we consider the possibilities for introducing welfare improving taxes.

IV. Welfare Improving Taxes

We established in the last section that during the first period any firm except the one at the top deviates from its first best. In this section, our aim is to analyse whether it is the case that simple tax schedules can be designed, which restore the first best solution. We propose four taxes: a profit tax, a revenue tax, an output tax, and finally a wage tax. It is to be expected that we can suggest simple linear output taxation as well as revenue taxation. Such tax rules change the shape of the profit function. A simple linear profit tax on the contrary does not change the shape of the profit function and thus, we expect that in this case non-linear taxes are needed. This is also the case for wage taxation, unless the second period wage is taxed away so that the union simply does not care about the result.

Obviously, for a planner to introduce a tax scheme, some information is required. At this point, we may adhere to one of two assumptions. It can be assumed that the taxing authority know the realized value of the price from the beginning of the game. Thus, taxes can be based upon this knowledge. However, we have no reason to assume that the taxing authority has the same information as the firm. And if it did, why not just pass on this information to the union? Furthermore, taxes which are based upon prior knowledge as well as updated beliefs, must be expected to raise even greater problems since this presumably strengthens the incentive to deviate. If such taxes are to be introduced, they must be designed so as to punish the firm for producing anything different from the inferred first best output. This issue will not be addressed here but a (safe) conjecture will be that this necessarily require marginal tax rates higher than 100%. Thus, in the current context, in the case of firms we will be looking for taxes which can be based on parameters and endogenous values which can be observed. This restriction does not apply for taxation of wage or income in the second period since in a separating equilibrium the government knows the price after the first period.

On this basis and given the exclusion of tax schemes with marginal taxes above 100%, we conclude that a non-linear income tax will do the job. This tax scheme is based on knowledge of β and τ. The other tax schemes considered are found unacceptable either because they require knowledge of the price or because they involve marginal tax rates above 100%.

IV.1. Profit Taxation

Consider the linear profit taxation scheme $\mu\cdot\tau$. In this case, as labour demand is unaltered compared to the situation without taxation, the value of the bargaining is

$$\Omega = \left(\frac{t^{-1}(y)^2}{4w} - \frac{t^{-1}(y)^4}{16w^3}\right)^{\beta}\left((1-\mu)\frac{t^{-1}(y)}{4w}\right)^{1-\beta}$$

The first order condition to this optimization problem is identical to that with no taxation. Thus, a simple linear profit tax does not change the behavior of the agents.

Let us consider the simple non-linear profit tax $\mu\cdot\pi$. In this case the first order condition of the static problem reads[2)]

$$(16)\quad \left(\frac{p}{2\sqrt{l}} - w\right)(1 - 2\mu(p\sqrt{l} - wl)) = 0$$

Let the solution to (16) be $l_2=\psi(w_2,p)$. If $\psi(w_2,p)$ is the optimal choice, then second period profits are

$$\pi_2 = (p\sqrt{\psi(w_2,p)} - w_2\psi(w_2,p))(1-\mu(p\sqrt{\psi(w_2,p)} - w_2\psi(w_2,p)))$$

Thus, the value of the bargain is given by

$$\Omega = \left(w_2\frac{p^2}{4w_2} - \frac{p^4}{16w^3}\right)^{\beta}(\pi_2 - \mu\pi_2^2)^{1-\beta}$$

where $p=t^{-1}(y)$ and π_2 as given above. Assume that the solution to this problem is $w_2=\varphi(y,\mu)$ with $\varphi_1>0$.

Hence, the firm faces the following payoff

$$\pi = (py-wy^2)(1-\mu(py-wy^2)) + \pi_2(\varphi(y,\mu),\mu)$$

where

$$\pi_2(\varphi(y,\mu),\mu) = (p\int\psi(\varphi(y,\mu),p) - w_2\psi(\varphi(y,\mu),p)) \cdot$$

$$(1-\mu(p\int\psi(\varphi y,\mu),p) - w_2\psi(\varphi(y,\mu),p)))$$

Thus, first order conditions are

$$p - 2w_1t(1-2\mu(pt-w_1t^2)) + \frac{\partial\pi_2}{\partial\varphi}\,\frac{\partial\varphi}{\delta t^{-1}(y)}\,\frac{1}{t'} = 0$$

Thus, for $t=p/2w_1$ to be a solution, we require

$$\frac{\delta\pi_2}{\delta w}\,\frac{\partial\varphi}{\partial t^{-1}(y)} = 0$$

Thus, presumably, only very complicated taxation schemes exist. The schemes depend upon p, the value of which the firm has to communicate to the taxing authority. This on its own may change the structure of the problem faced by the firm, unless the taxing authority has full knowledge of the value of p.

IV.2. Revenue Taxation

With respect to a revenue taxation scheme it is reasonable to expect that a simple linear scheme will do the job. The reason, of course, is that for a given price, revenue taxation, if the tax rate is negative, will increase the value of the marginal product of labour for any level of employment. Thus, the negative effects of increased employment upon second period profits can be neutralized by an appropriate revenue tax rate.

If revenue is taxed with a rate of μ, labour demand is given by

$$l_2^d = \left(\frac{p(1-\mu)}{2w_2}\right)^2$$

and profits by

$$\pi_2 = \frac{(p(1-\mu))^2}{4w_2}$$

Thus, the value of the bargaining is

$$\Omega = \left(w_2\left(\frac{\pi(1-\mu)}{2w_2}\right)^2\left(1 - \left(\frac{p(1-\mu)}{2w_2}\right)^2\right)\right)^{\beta}\left(\frac{(p(1-\mu))^2}{4w_2}\right)^{1-\beta}$$

which is written as

$$\Omega = (1-\mu)^2\left(\frac{p^2}{4w_2} - \frac{p^4(1-\mu)^2}{16w_2^3}\right)^{\beta}\left(\frac{p^2}{4w_2}\right)^{1-\beta}$$

where $p=t^{-1}(y)$. The first order condition to this problem is

$$\beta\left[-\frac{p^2}{4w_2^2} + \frac{3p^4(1-\mu)^2}{16w_2^2}\right]^{\beta}\left[\frac{p^2}{4w_2}\right] -$$

$$[1-\beta]\left[\frac{p^2}{4w_2} - \frac{p^4(1-\mu)^2}{16w_2^3}\right]\left[\frac{p^2}{4w_2^2}\right] = 0$$

This reduces to

$$w_2 = p(1-\mu)\sqrt{\frac{1+2\beta}{4}}$$

The payoff to the firm is given by

$$py(1-\mu) - w_1y^2 + \frac{p^2(1-\mu)^2}{4t^{-1}(y)(1-\mu)^2}\sqrt{\frac{4}{1+2\beta}}$$

Thus, the solution is given by the differential equation

$$(p(1-\mu)-2w_1t)\frac{dt}{dp} - \frac{1-\mu}{4}\sqrt{\frac{4}{1+2\beta}} = 0$$

If $t=p/2w_1$ is to be a solution, we easily find

$$-\frac{\mu p}{2w_1} - \frac{1-\mu}{4}\sqrt{\frac{4}{1+2\beta}} = 0$$

It is a problem that the tax scheme introduced here depends upon p. However, if we allow such a tax scheme, then the tax rate

$$\mu = \left(\frac{1}{2}\sqrt{\frac{1}{1+2\beta}} - \frac{p}{2w_1}\right)^{-1}\frac{1}{2}\sqrt{\frac{1}{1+2\beta}}$$

implement the first best solution.

Alternatively, assume that separate tax rates are in use for each period. Not surprisingly, one concludes that a first period tax, $\mu_1=0$, and a second period tax, $\mu_2=1$, implement the first best solution.

Thus, if we allow a tax scheme as given implicitly here, the first best solution can be implemented. It is disturbing, however, that the choice of a tax rate applicable to both periods μ depends upon p. Only if the agency enforcing the taxation scheme has the same information as the firm, it is possible to choose μ as above.

IV.3. Output Taxation

Introducing output taxation at a rate μ, we find that profits and labour demand, respectively, are given by $(p-\mu)^2/4w$ and $((p-\mu)/2w)^2$, respectively. Hence, the value of the bargain is

$$\Omega = (w_2(\frac{p-\mu}{2w_2})^2(1-(\frac{p-\mu}{2w_2})^2))^\beta(\frac{(p-\mu)^2}{2w_2})^{1-\beta}$$

First order condition are

$$\beta(-\frac{(p-\mu)^2}{4w_2} + \frac{3(p-\mu)^4}{16w_2^4})^\beta(\frac{(p-\mu)^2}{4w_2}) - [1-\beta][\frac{p^2}{4w_2} - \frac{p^4(1-\mu)^2}{16w_2^3}]\ [\frac{p^2}{4w_2^2}] = 0$$

Thus, the second period wage is seen to be

$$w_2 = (t^{-1}(y)-\mu)\sqrt{\frac{1+2\beta}{4}}$$

Consequently, the payoff to the firm is given by

$$(p-\mu)y - w_1y^2 + \frac{(p-\mu)^2}{t^{-1}(y)-\mu}\sqrt{\frac{4}{1+2\beta}}$$

The solution t satisfies

$$(p-\mu-2w_1t)\frac{dt}{dp} - \sqrt{\frac{4}{1+2\beta}} = 0$$

For $t=p/2w_1$ to be a solution, we find

$$\frac{-\mu}{2w_1} - \sqrt{\frac{4}{1+2\beta}} = 0$$

We conclude that a simple subsidy of output, depending only upon w_1 and β, enforces a solution identical to the first best solution. Note that this tax cheme is introduced without difficulty as the choice of μ is not dependent upon p.

IV.4. Wage Taxation

Consider now wage taxation which in principle also can take the form of a subsidy. The utility function of the trade union now reads $u=wl(1-\mu)(1-l)$. The expression for profit as well as labour demand is unchanged. Hence, the value of the bargain is

$$\Omega = ((1-\mu)(\frac{p^2}{4w_2} - \frac{p^4}{16w_2^3}))^{\beta}(\frac{p^2}{4w_2})^{1-\beta}$$

The first order condition to this problem is

$$\beta(1-\mu)(\frac{p^2}{4w_2^2} + \frac{3p^4}{16w_2^4})(\frac{p^2}{4w_2}) -$$

$$(1-\beta)(1-\mu)(\frac{p^2}{4w_2} - \frac{p^4}{16w_2^3})(\frac{p^2}{4w_2^2}) = 0$$

This condition is identical to the one for the case without taxation.

Consider now the tax scheme

$$T(wl) = \mu(wl)^2$$

Thus, the bargaining problem is

$$\max_{w} \Omega = (wl(1-l) - \mu(wl)^2)^{\beta}(\frac{p^2}{4w_2})^{1-\beta}$$

This reduces to

$$\max_{w} \Omega = (\frac{p^2}{4w_2} - \frac{p^4}{16w_2^3} - \mu \frac{p^4}{16w^2})^{\beta}(\frac{p^2}{4w_2})^{1-\beta}$$

The first order conditions

$$\beta(-\frac{p^2}{4w_2^2} + \frac{3p^4}{16w_2^4} + \mu\frac{2p^4}{16w^3})\frac{p^2}{4w_2} -$$

$$(1-\beta)(\frac{p^2}{4w_2} - \frac{p^4}{16w_2^3} - \mu\frac{p^4}{16w_2^2})\frac{p^2}{4w_2^2} = 0$$

This reduces to

$$\beta(-1 + \frac{3p^2}{4w_2^2} + \mu\frac{2p^2}{4w_2}) -$$

$$(1-\beta)(1 - \frac{p^2}{4w_2^2} - \mu\frac{p^2}{4w_2}) = 0$$

This again is written

$$(17)\quad \frac{p^2}{4w_2^2}(1+2\beta) + \mu\frac{p^2}{4w_2}(1+\beta) = 1$$

At this point, note that if taxes can be designed so as to make w_2 independent of p, then the first best solution can be implemented. Let w_2* be the optimal wage from (17). Rewriting and differentiating we find

$$[2w_2 - \frac{\mu}{4}p^2(1+\beta)]dw + [w_2\frac{\mu p}{2}(1+\beta) + \frac{p}{2}(1+2\beta)]dp = 0$$

Hence,

$$\mu = -\frac{1}{w}\,\frac{1+2\beta}{1+\beta}$$

ensures that the choice made by the firm in this period leaves the second period profit undisturbed.

Hence, using this tax rate the trade union asks for a wage which results in the first best solution. The idea is that any excessive wage claim is "taxed" very hard and thus even if it is accepted by the firm, the final result is not very beneficial to the union.

The tax scheme found here is appealing because of its simplicity: it does not depend upon p. Hence, introducing this tax scheme, we do not add the incentive problems in the sense that a further need to signal the value of p (from the firm to the taxing authority) arises.

Let us therefore offer some further remarks on this tax scheme which actually turns out to be a subsidy. First of all, it must be noted that since no explicit threat point is contained in Ω (the value of the bargain), it makes no sense to analyse the consequences of $\beta \to 0$.

What is the virtue of this tax scheme? If we consider an increase in the wage rate, it is seen that this increase is taxed in the sense that the subsidy decreases. We have

$$T = \mu(wl)^2$$

$$T = -\frac{1}{w}\, l^2 \;\; \frac{1+2\beta}{1+\beta}$$

It is seen that the increase in the wage decreases T - the subsidy - dramatically since $1/w$ decreases, but also since l^2 decreases. This effect is designed so that it dominates the positive effect (through wl) of increasing the wage rate and thus makes the union ask for the "right" wage.

V. Conclusion

Centralized wage determination in economies characterized by differential information does, in general, result in deviations from the first best. The aim of this paper was to analyse the importance of bargaining strength. One prior was confirmed. If the trade union has no bargaining power at all, the (strategic) behavior of the firm coincides with the first best. If the bargaining strength of the trade union is strictly bounded from below at zero, the (strategic) behavior of the firm involved deviations from the first best. In the present case the firms produce less compared to production under perfect information. Also the output (dictated by the strategy) is decreasing in β, the bargaining strength of the trade union.

With respect to welfare improving taxes, the results were mixed. It is possible to introduce simple taxes in the case of output taxation and wage taxation. However, taxing revenue as well as profit creates problems as the tax scheme is dependent upon p, the value known only (precisely) by the firm. In these cases it must be assumed that p is known by the agency imposing the tax scheme or the tax scheme creates incentive compatibility constraints on its own.

VI. Notes

1. Using (IV.1)

$$(1-\mu)(p-2w_1y) + \frac{p^2}{2}\int\frac{\beta\tau}{(1-\beta)(1-\mu) + \beta(1-\mu)}\frac{-1}{(t^{-1}(y))^2}\frac{dp}{dt} = 0$$

where $dp/dt = 1/t'(\varphi)$. Use that $y=t$ and $t^{-1}(y)=\varphi$. Then we arrive at

$$(1-\mu)(p-2w_1y)\frac{dt}{dp} - \frac{1}{2}\int\frac{\beta\tau}{(1-\beta)(1-\mu) + \beta(1-\mu)} = 0$$

2. The profit accruing to the firm during the second period is for a given wage given by

$$\pi_1 = (py_1 - w_2l_2)(1-\mu(py_2-w_2l_2))$$

$$= (p\surd l_2-w_2l_2)(1-\mu(p\surd l_2-w_2l_2))$$

First order conditions are seen to be

$$(\frac{p}{2\surd l}-w_2)(1-\mu(p\surd l_2-w_2l_2)) -$$

$$\mu(p\surd l_2-w_2l_2)(\frac{p}{2\surd l_2}-w_2) = 0 \Rightarrow$$

$$(\frac{p}{2\surd l_2}-w_2)(1-2\mu(p\surd l_2-w_2l_2)) = 0$$

VII. Appendix

Proof of Lemma 2

By assumption the strategy is one-to-one and incentive compatible. The worst point estimate to be held by the union is $P_w = P_u$. This follows from the sequentiality of the game played between the union and the firm (for details, see Chapter 2, Proof of Proposition 3). Since $\pi_2 < 0$, using Theorem 2 of Mailath (1987)

$$t(P_u) = f^*(P_u)$$

where $f^*(P_u)$ is the Walrasian output, since $\pi_{13} > 0$, again using Mailath (Theorem 2, 1987), then

$$t' > 0$$

Since $t' > 0$, t is also continuous and differentiable.

CHAPTER 6

POOLING EQUILIBRIA AND MULTIPLIERS

I. Introduction.

An important theme in Keynesian economic theory is that of a price and wage mechanism fundamentally different from the one underlying the Arrow-Debreu economy. In particular, following a shock adjustments take place at least partly and in some models exclusively in quantitites as opposed to prices. An example of this is the simple multiplier in the naive IS-LM model. Once the assumptions of price and wage rigidity are abandoned, the Keynesian conclusions are no longer valid. And perhaps worse, research focusing on Keynesian economic theory using models pertaining to the neoclassical school seems to indicate that the central results obtained in Keynesian theory are embraced in these neoclassical models. This is the so-called neoclassical synthesis. Hence, it seems as though there is no conceptual difference between neoclassical and Keynesian models.

However, due to the works of Clower (1965) and Leijonhufud (1968) a different approach was taken. The classical assumption of an instantaneous price adjustment is abandoned in favour of an assumption of fixed prices (Hicks (1965)). In the short run prices are completely invariable whereas they may adjust to demand shocks in the longer run. The allocation in this short run can be achieved by a rationing mechanism (see, e.g., Dreze (1975), Grandmont (1977)). This is the basic thrust of the temporary equilibrium concept. Such models produce Keynesian results but are firmly based on agents optimizing behavior. As such this line of thought provides a foundation for macroeconomics. However, left unexplained is the crucial question of price formation. The formation of prices are not explained and consequently it is impossible within the confinement of temporary equilibrium models to explain the rigidity of prices. The aim of this chapter is to explore the possibility of establishing existence of pooling equilibria and analyse the resulting dynamics.

In a paper analysing the ratchet effect, Laffont and Tirole (1986) show that if a model is characterized by small parameter uncertainty, then it may well be relevant to focus attention upon pooling equilibria, that is, equilibria where agents characterized by some stochastic parameter take the same action irrespective of the realized value of this parameter. In the analysis due to Laffont and Tirole (1986) there was assumed to be a continuum of types. Less favourable to the existence of a full pooling equilibria is the case of only a finite number of types. Such a situation is analysed by Freixas, Guesnerie and Tirole (1985) and it is established that for some parameter configurations, a pooling equilibrium may obtain. Central to the existence of a pooling equilibrium in both the discrete and the continuous case, is the fact that there is one type of agent who will always realize a second period pay off of zero. Such an agent has no incentive to deviate in the first period to improve the second period pay off.

This idea was used in Chapter 3 where the neo-classical partial equilibrium of the goods and adjoining factor market was reconsidered. The idea of the analysis is as follows. In a repeated relationship lasting for two periods, bargaining over wage takes place. The value of marginal product of labour is known only probabilistically by the agency but with certainty by the firm. In particular it is assumed that the agency may dictate the wage it would like to rule in the second period of the relationship. It is then left to the firm to dictate the level of employment. This level of employment has to correspond to the level of the value of marginal product of labour as announced in the first period, otherwise workers may default and quit working. However, workers are unable to verify the value of the marginal product of labour during this period. This model can be seen as a version of the monopoly union model. Obviously, whatever the objective of the agency the decision made by the agency is, under uncertainty, not independent of the first period actions taken by the firm. Consequently, any first period decision made by the firm is taken with the knowledge that any private information revealed by this decision will be used later by the agency. Assuming that the firm during the first period behaves competitively and that the agency attempts to equate the marginal disutility of labour to the value of the marginal product of labour through the wage, we studied the resulting type of equilibrium. In Chapter 3 for the case of a continuum of agents, we found that for small uncertainty the only simple[1] equilibrium which was possible was a full pooling equilibrium. We suggested that the existence of pooling equilibria would have interesting implications for price and wage dynamics. As such pooling equilibria can provide for a foundation of macroeconomics. An analysis of this hypothesis is the scope of this chapter. In particular, we focus attention on the possibility of multiplier effects. In order to keep the analysis simple, we adhere to the case of only two types (see Freixas, Guesnerie and Tirole (1985)). This does not seem to entail any restriction apart from the one that very complex equilibria are ruled out[2]. The condition of small uncertainty for a full pooling equilibrium to exist arises naturally. We return to this with a few remarks in the conclusion. In the next section we set forth the model and discuss the types of equilibria which may arise. In Section III we offer some comments on the implication with respect to the dynamic resulting from a pooling equilibrium. Finally, concluding comments are given in Section IV.

II. The Model.

We concentrate upon a continuing relationship between a firm and an agency. These two parties are related through some process in which the agency, in between the periods defining the continuing relationship, dictates some wage. The wage dictated is based on some utility function. The reason that the agency interferes in between periods is due to the fact that over time the agency, by observing the actions of firm, may update the priors upon which the initial wage was set. In particular we assume that there are three periods T_1, T_2 and T_3. Periods T_1 and T_2 define the first period of the relationship. During this period factor renumeration is taken as some exogenous variable. In particular, the wage may be thought of as a result of some labour market agreement and for this reason it is not unreasonable to take wage as an exogenous variable. During this first period demand conditions may well change. The change occurs in between periods T_1 and T_2. After period T_2 and before period T_3, the wage is renegotiated. This structure may be justified referring to institutional facts, for example the tradition that agreements are made for some time and typically do not reflect changes in, say, market demand (such contracts are in practice inoperable). Alternatively, if it is costly to monitor the market all time, it may be optimal to agree upon a wage in anticipation of what will happen in the relevant period and then after a certain time period agree upon another wage taking into account the experience gained in previous time periods.

The economy is described by the following set of equations

(1) $p=f(y)+\Theta \qquad f_y<0 \qquad f_{yy}>0$

(2) $y=\sqrt{l}\sqrt{k}$

(3) $u=wl-\alpha\frac{l^2}{2}$

Consider now the workings of the economy. Before period T_1, Θ is drawn according to some known probability distribution and is then unchanged over all of the three time periods. We consider a two point distribution described by the obejctive and subjective probabilities $Pr(\Theta=\Theta_1)=v_1$ and $Pr(\Theta=\Theta_2)=1-v_1$ and $\Theta_1>\Theta_2$. Before date T_1 an agency interferes to dictate a wage which will rule in periods T_1 and T_2. This wage is based on the agency's knowledge of the structure of the economy as well as knowledge of v_1, Θ_1 and Θ_2. We assume that the agency attempts to dictate a wage such as to equate the wage to the expected marginal disutility of labour. This is strictly speaking not in accordance with (3) but assuming that the resulting 1 is divided upon many workers, this ensures economic efficiency at very little cost (in terms of risk, cf. Arrow & Lind (1970); see also the discussion in Chapter 3.)

Given a wage ruling in the first period the firm chooses a set of actions of which only a subset is observed by the agency. The actions taken by the firm and observed by the agency are used by the agency to dictate the wage ruling in the subsequent period T_3. In particular, assume that wage and labour input are observed during the period by the agency whereas input of capital as well as output remains unobserved in period T_1 and T_2. The price ruling in period 1, respectively period 2 is observed by the end of these periods. Hence, we have assumed that any agent may react to a price only in a subsequent period. Hence, when the firm chooses labour input and price, it does take into account the affect this will have upon subsequent periods' profit. Before we describe the resulting sequential equilibrium in detail, let us offer some insight into the basic incentive mechanism characterizing this model. If the wage in period T_3 is set according to some probability derived from v_1, we call this revised (ex-post) probability v_2, we have in a pooling equilibrium:

Proposition 1 $\partial\pi_3(\Theta_1)/\partial v_2<0$ and $\partial\pi_3(\Theta_2)/\partial v_2=0$.

Proof: see Appendix.

The idea of proof rests upon two ideas, and is simple. The higher is v_2 the higher is the expected value of marginal product of labour and this is reflected in the wage claim. Once the wage is dictated, the supply of labour is given as perfectly elastic until some limit $l(w)$ is reached. In particular the total wage bill will be less than what obtains if $v_2=1$. In particular, for $v_2=1$ we have $\stackrel{\approx}{w}(v_2=1)l(\stackrel{\approx}{w}(v_2=1))=\frac{1}{2}p(\Theta_1)y(\Theta_1)$ and hence profit equals zero. But for v_2 decreasing the discrepancy between $\stackrel{\approx}{w}(v_2=1)l(\stackrel{\approx}{w}(v_2=1))$ and $\stackrel{\approx}{w}(v_2)l(\stackrel{\approx}{w}(v_2))$ increases as v_2 decreases. From this the result follows.

So far the discussion offered has taken as given the value of v_2; the ex-post probability that $\Theta=\Theta_1$. We will now consider the repeated relationship somewhat closer. The aim is to give a fully dynamic analysis and hence to make v_2 endogenous. The firm was supposed to be restricted to choose Walrasian levels of employment and prices during the first two periods. Denote these variables by $(l(\Theta_1)$ and $p(\Theta_1)$, respectively. Consider the following three period games. In the first and second period the agency puts forward a wage claim. The firm, whether faced with a realization of $\Theta=\Theta_1$ or $\Theta=\Theta_2$, chooses some labour input and a corresponding price. Faced with $l(\Theta_1)$ and $p(\Theta_1)$ the agency has to decide upon the ex-post probability that $\Theta=\Theta_1$ and $\Theta=\Theta_2$. Obviously, in making its decision the firm takes this into consideration. This structure is identical to the one analysed in Fudenberg and Tirole (1983), for which the Kreps-Wilson (1982) notion of Perfect Baysian Equilibrium is relevant.

For any equilibria to be a Perfect Baysian Equilibrium we require (cf. Freixas et al (1985)):

P.1 $l_3=l(w_3)$ is a maximizer for p_3^F and is consistent with the announced value of Θ

P.2 w_2 is a maximizer for u given subsequent actions

P.3 $l_i=l(w_i)$, i=1,2 is a maximizer for $(\pi_1^F+\pi_2^F+\pi_3^F$ given the subsequent actions

P.4 w_1 is a maximizer for $u+\delta u$ given the subsequent actions

BD v_2 the updated probability of $\Theta=\Theta_1$ is Bayes-consistent with prior probability v_1 and $l_1(w_1)$

We restrict attention even further to continuation equilibrium. A continuation equilibrium is a PBE with an exogenous w_1. Hence, for any given value w_1 a continuation equilibrium is a vector (l_1, w_2, l_2, v_2) satisfying P1,P2,P3 and BC. The idea of perfectness requires that the continuation equilibrium is induced by the equilibrium of the initial game. We consider the existence and uniqueness of a continuation equilibrium. First we derive the following two lemmas. In particular Lemma 1 is necessary for the existence of pooling equilibria.

Lemma 1 $l_i^*(\Theta_2 I)=l_i(\Theta_2)$ i=1,2

Proof.[3)] A firm facing $\Theta=\Theta_2$ will never deviate and play $l^*(\Theta_2) > l_i(\Theta_2)$, i=1,2 as the agency then infers a value of $\Theta > \Theta_2$ and hence the third period wage increases. If such a firm were to deviate and play $l_i^*(\Theta_2) > l_i(\Theta_2)$, i=1,2, then it would incur a loss in the first two periods. It would still, however, be met by the minimum wage claim in the third period. Hence, $\pi_F{}^3$ is the same as the firm did not deviate and in period T_1 and T_2 the first best solution is realized.

Q.E.D.

Lemma 2. $l_i^*(\Theta_1) \varepsilon \{l_i(\Theta_2), l_i(\Theta_1)\}$ i=1,2

Proof. If $l_i^*(\Theta_1)$ i=1,2 belongs to the support of the firm faced with $\Theta=\Theta_1$, then unless $l_i^*(\Theta_1)=l_i(\Theta_2)$, i=1,2, $v_2 = 1$. For $l_i^*(\Theta_1) \neq l_i(\Theta_2)$, i=1,2, hence $v_2=1$ and third period profits are zero. In the first two periods the first-best solution $(l_i(\Theta_1))$ i=1,2 is realized.

Q.E.D.

Hence, we may obtain three types of continuation equilibria. Referring to Lemmas 1 and 2 we see that in a pooling equilibrium both firms employ a labour force equal to that of the firm faced with $\Theta=\Theta_2$. In a separating equilibria the firms employ $l_i(\Theta_1)$ and $l_i(\Theta_2)$, i=1,2, respectively. In a semi-separating equilibrium a firm faced with $\Theta=\Theta_1$ randomizes between $l_i(\Theta_1)$ and $l_i(\Theta_2)$, i=1,2.

Proposition 2. There exists a continuation equilibrium, in particular.

i) for Θ_1 such that $\pi^F(v_2=v_1)\geq\pi^F(v_2=1)$ we have a pooling equilibrium

ii) for Θ_1 such that $\pi^F(v_2=v_1)\leq\pi^F(v_2=1)$ we have a separating equilibrium.

iii) for Θ_1 such that $\pi^F(v_2=v_1)>\pi^F(v_2\varepsilon(v_1,1)>\pi^F(v_2=1)$ we have a semi-separating equilibrium.

Proof. See Appendix.

We concentrate in the following on pooling equilibria. We note that the existence of pooling equilibria arises because the firm with the lowest value of Θ always faces a second period profit of zero. Hence, in our model the existence of pooling equilibria is not pathological. We briefly discuss the likeliness that the continuation equilibrium turns out to be a pooling equilibrium. Normally pooling equilibria occur only if the different types are not too "far apart". This we may illustrate, and hence we provide some insight into the proporties, we must be satisfied for a pooling equilibrium to exist.

Using (1)-(3) we find the competitive solution, given the wage ruling in the first two periods and the value of Θ to be

(4) $l_w^d=\surd(\frac{r}{w})f^{-1}(2\surd(wr)-\Theta)$

(5) $k_w^d=\surd(\frac{w}{r})f^{-1}(2\surd(wr)-\Theta)$

(6) $y_w = f^{-1}(2\int(wr)-\Theta)$

(7) $p_w = 2\int(wr)$

Now, a firm faced with $\Theta=\Theta_1$ may choose to pool, i.e. it attempts to make the agency believe that it faces $\Theta=\Theta_2$. The profit to a firm with this behavior is, in each of the periods T_1 and T_2

(8) $\pi_i = \int(wr)[f^{-1}(2\int(wr)-\Theta_1)-f^{-1}(2\int(wr)-\Theta_2)]^2 i=1,2$

In a pooling equilibrium the wage ruling in period T_3 is decided upon with knowledge of the priors v_1, Θ_1 and Θ_2. Call this wage $\approx{w}$. This wage is assumed to be known by the firm. That is, we have assumed that the firm has perfect knowledge of the preferences of the agency. To evaluate third period profits note that in a pooling equilibrium the wage, $\overset{\approx}{w}$, dictated by the agency solves.

(9) $\overset{\approx}{w} = \alpha(v_1\int(\frac{r}{\overset{\approx}{w}})f^{-1}(2\int(\overset{\approx}{w}r)-\Theta_1)+(1-v_1)\int(\frac{r}{\overset{\approx}{w}})f^{-1}(2\int(\overset{\approx}{w}r)-\Theta_2))$

Now consider the competitive solution to the optimization problem of the firm with $w=\overset{\approx}{w}$ and $\Theta=\Theta_1$. We have

(4') $l_w^d = \int(\frac{r}{\overset{\approx}{w}})f^{-1}(2\int(\overset{\approx}{w}r)-\Theta_1)$

(5') $k_w = \int(\frac{\overset{\approx}{w}}{r})f^{-1}(2\int(\overset{\approx}{w}r)-\Theta_1)$

Comparing $l_w{}^d$ with the right hand side in (9) we conclude $l_w{}^d >$ $l_e{}^d$. But as $\tilde{w}$ is set to equate $l_e{}^d$ to l^s, labour supply, we see that the firm in the third period faces a quantity constraint. Only l_s can be employed and at a wage $\tilde{w}$ which is not subject to negotiation within the period. The equilibrium condition for the market for capital reads.

$$(10)\quad r=(f(\int l \int k)+\Theta_1)\frac{1}{2}\sqrt{\int}\frac{l}{k}$$

From (10) we find $dk/dl<0$. Hence $k>k\tilde{w}(\Theta_1)$. Using the first order condition for the optimization problem for $\Theta=\Theta_1$,

$$(11)\quad \tilde{w}=(f(y)+\Theta_1)\frac{y}{2l}$$

$$(12)\quad r=(f(y)+\Theta_1)\frac{y}{2k}$$

we have $r\tilde{k}>rk(\Theta_1)=\int(\tilde{w}r)f\quad(w\int(\tilde{w}r)-\Theta\ )$. Furthermore, for any fixed wage $\tilde{w}$ we have that $\tilde{l}\tilde{w}=\tilde{w}^2/\alpha$. Using (8) we obtain $\tilde{w}\tilde{l}=v_1\int(\tilde{w}r)f^{-1}$ $(w\int(\tilde{w}r)-\Theta_1)+(1-v_1)\int(\tilde{w}r)f^{-1}(2\int(\tilde{w}r)-\Theta_1$.

Thus, we obtain

$$(13)\quad \pi_3>(1-v_1)\int(\tilde{w}r)(f^{-1}(2\int(\tilde{w}r)-\Theta_1)-f^{-1}(2\int(\tilde{w}r^9-\Theta_2))$$

Hence, using (8) and (12) pooling occurs if

$$(14)\quad (1-v_1)>2(f^{-1}(2\int(\tilde{w}r)-\Theta_1)-f^{-1}(2\int(\tilde{w}r)-\Theta_2)$$

Hence for $\Theta_1-\Theta_2$ sufficiently small, that is, if different types are sufficiently alike, pooling obtains. This is quite in line with results available elsewhere. Furthermore, due to the convexity of f^{-1} we have that for $\Theta_1-\Theta_2$ constant but for a different level $(\Theta_1+\varepsilon)-(\Theta_2-\varepsilon)$ the cost of pooling increases. In the following we consider only pooling equilibria and the dynamics and quantities.

III. Towards a Dynamic Analysis.

We now consider changes in the level of demand occurring in between T_1 and T_2. We assume that these changes are unexpected but observable to both parties. We may consider a change in either Θ_1 or Θ_2 or, perhaps most interesting, a change in both Θ_1 and Θ_2. This last possibility can be interpreted as a fully acknowledged increase in public demand.

Before we continue, let us briefly recapitulate the structure set out in the proceeding section. During periods T_1 and T_2, defining the first period of the continuing relationship, workers observe the input of labour but not the input of capital and total output. After period T_1, respectively period T_2, p_1, respectively p_2, is observed. Based upon observation of prices, as a proxy for demand, a wage ruling in T_3, the second period in the continuing relationship, is dictated by the trade union. Employment during this period has to be in accordance with the announced level of demand and a competitive solution. Otherwise workers may quit. However, only after this period when the game ends, the price ruling in this period is observed but is obviously not relevant.

Assume that for some values Θ_1 and Θ_2 (14) are satisfied, and hence a pooling equilibrium obtains, that is, a firm faced with $\Theta=\Theta_1$ behaves as if $\Theta=\Theta_2$. How, consider the effects of a change in Θ_1, so we have $\Theta=\Theta_1+\varepsilon$ in T_2 where $\Theta=\Theta_2$ remains unchanged in T_2. First of all we have to be concerned with the nature of the equilibrium. To have a pooling equilibrium after the change of Θ_1 to $\Theta_1+\varepsilon$, the equilibrium condition reads (this is a sufficient condition only).

$$(15)\ \int(wr)\{[f^{-1}(2\int(wr)-\Theta_1)-f^{-1}(w\int(wr)-\Theta_2)]^2+$$

$$[f^{-1}(2\int(wr)-{}_1\Theta-\varepsilon)-f^{-1}(2\int(wr)_2-\Theta)]^2<$$

$$(1-v_1)\int(\overset{\approx}{wr})[f^{-1}(2\int(wr)-\Theta_1-\varepsilon)-f^{-1}(2\int(\overset{\approx}{wr})-\Theta_2)]$$

where (15) is evaluated at the start of period T_2 when the change in the intercept of demand has become known. In the case that the inequaility in (15) is satisfied, the firm continues to pool in T_2. In this case a change in demand occurring in between T_1 and T_2 results only in an increased input of capital and consequently an increased output. In the pooling equilibrium the input of labour as well as the price remains unchanged (given by (4) and (7), respectively). We have the following multipliers.

$$\frac{dk}{d\varepsilon} = - \frac{k}{f'(y)\cdot y}$$

$$\frac{dy}{d\varepsilon} = \frac{-1}{f'(y)}$$

$$\frac{dl}{d\varepsilon} = 0$$

$$\frac{dp}{d\varepsilon} = 0$$

Before the start of period T_3 a new wage is dictated. This wage is based upon knowledge of $\Theta_1+\varepsilon$ and Θ_2. In particular, it is reasonable to assume that the wage ruling in T_1 and T_2 is given by

$$(16)\quad \tilde{\tilde{w}}=\alpha(v_1\int\frac{r}{\tilde{\tilde{w}}}f^{-1}(2\int(\tilde{\tilde{w}}r)-\Theta_1)+(1-v_1)\int(\frac{r}{\tilde{\tilde{w}}})f^{-1}(2\int(\tilde{\tilde{w}}r)-\Theta_2$$

whereas the wage in T_3 is

$$(17)\quad w_0=\alpha(v_1\int(\frac{r}{w_0})f^{-1}(2\int(w_0r)-\Theta_1-\varepsilon)+(1-v_1)\int(\frac{r}{w_0})f^{-1}(2\int(w_0r)-\Theta_2)$$

Using (16) and (17) we see that $w_0>w$, but w_0 is still lower than what would be contained in a separating equilibrium, that is, the wage set in period T_3 is not efficient as it fails to equate marginal disutility of working to the value of marginal product of labour. A result of this is that the use of capital is expanded compared to what is the case in a first-best world.

If ε is sufficiently big, then (15) is violated; call this value ε_0. In case that (15), devaluated before T_2, is no longer true, the firm changes its strategy and a separating equilibrium obtains in period T_2. Hence, production, input of labour, and capital and the price are given by (4)-(7). Notice that compared to the previous case the use of capital declines, whereas the use of labour increases. Output as well as prices are unchanged compared to the previous case. In period T_3 we obtain a wage satisfying

$$(18)\quad w_1=\alpha\int\left(\frac{r}{w_1}\right)f^{-1}(2\int(w_1 r)-\Theta_1-\varepsilon)$$

w_1 and the price $p_w=\int(w_1 r)$ imply that efficiency is obtained as these values result in equality between the marginal disutility of labour and the value of marginal product of labour.

Consider a change so that the upper value of the intercept is $\Theta_1-\varepsilon$. If Θ_1 and Θ_2 are values resulting in a pooling equilibrium, then for $\Theta_1-\varepsilon>\Theta_2$, $\varepsilon>0$, $\Theta_1-\varepsilon$ and Θ_2 will also result in a pooling equilibrium. In period T_2 production and use of capital is adjusted downwards. Price and use of labour remain the same. We have the following set of multipliers

$$\frac{dk}{d\varepsilon} = -\frac{k}{f'(y)\cdot y}$$

$$\frac{dy}{d\varepsilon} = \frac{-1}{f'(y)}$$

$$\frac{dl}{d\varepsilon} = 0$$

$$\frac{dp}{d\varepsilon} = 0$$

The price which obtains in period T_3 is given by

$$(19)\quad w_2=\alpha(v_1\int(\frac{r}{w_2})f^{-1}(2\int(w_2r)-\Theta_1+\varepsilon)0$$

$$(1-v_1)\int(\frac{r}{w_2})f^{-1}(2\int(w_2r)-\Theta_2))$$

This wage is lower than the one which ensures efficiency, which in this case is given by $w=\alpha\int(\frac{r}{2})f^{-1}(2\int(wr)-\Theta_1+\varepsilon)$. Also in this case we do not obtain equality between marginal disutility of labour and the value of marginal product of labour.

Consider now a change in Θ_2. First, if Θ_2 decreases to $\Theta_2-\varepsilon$ where ε is such that the pooling equilibrium is maintained, then a firm characterized by $\Theta=\Theta_1$ has to behave as if $\Theta=\Theta_2-\varepsilon$. Consequently, the use of labour is adjusted downwards. However, as prices remain unchanged (in this particular model) and demand has to be fulfilled, actual production remains the same. Hence, more capital is used in period T_2. Using (4)-(7) we find the following multipliers.

$$\frac{dl}{d\varepsilon} = \int(\frac{r}{w})\ \frac{1}{f'(2\int(wr)-(\Theta_2-\varepsilon))}$$

$$\frac{dp}{d\varepsilon} = 0$$

$$\frac{dk}{d\varepsilon} = \frac{k}{l}\ -\int(\frac{r}{2})\ \frac{1}{f'(2\int(wr)-(\Theta_2-\varepsilon))}$$

$$\frac{dy}{d\varepsilon} = 0$$

Comparing the wage ruling in period T_1 and T_2 to that of period T_3, we have that the former is given by (16) whereas the latter is given by

$$(20)\quad w_3=\alpha(v_1\int(\tfrac{r}{w_3})f^{-1}(2\int(w_3r)-\Theta_1)+$$

$$(1-v_1)\int(\tfrac{r}{2_3})f^{-1}(2\int(w_3r)-\Theta_2+\varepsilon))$$

Again, as $\Theta_2-\varepsilon>\Theta_1$ we have that $\bar{w}_3<w$, where $\bar{w}$ is the wage which solves $\bar{w}=\alpha\int(\frac{r}{w})f^{-1}(2\int(\bar{w}r)-\Theta_1)$. Hence, the wage dictated in period T_3, although it adjusts the shock ε falling upon demand in bad states of nature, is too low to ensure equality between the marginal disutility of labour and the value of marginal product of labour. Obviously, if Θ_2 changes to $\Theta_{2-\varepsilon}$ such that from period T_2 donwards it is unprofitable to maintain the pooling equilibrium, then in period T_2 production use of capital and labour is adjusted according to equations (4)-(6). In period T_3 the wage is given by $w=\alpha\int(\frac{r}{w})f^{-1}(2\int(rw)-\Theta_1)$. This is the wage which guarantee efficiency.

If Θ_2 changes to $\Theta_2\pm_\varepsilon$ such that $\Theta_2+_\varepsilon<q_1$, the pooling equilibrium is maintained in period T_2. However, production is unchanged whereas labour input increases and the use of capital decreases in period T_2 compared to period T_1. We have the following multipliers

$$\frac{dl}{d\varepsilon} = \int(\frac{r}{w})\ \frac{1}{f'(2\int(wr)-(\Theta_2-\varepsilon))}$$

$$\frac{dp}{d\varepsilon} = 0$$

$$\frac{dk}{d\varepsilon} = -\frac{k}{I}\int\left(\frac{r}{w}\right)\frac{1}{f'(2\int(wr)-(A_2-\varepsilon))}$$

$$\frac{dy}{d\varepsilon} = 0$$

The wage which obtains in period T_3 is given by

$$(21)\quad w_4 = \alpha(v_1\int(\frac{r}{w_4})f^{-1}(2\int(w_4r)-\Theta_1)+$$

$$(1-v_1)\int(\frac{r}{w_4})f^{-1}(w\int(w_4r)-\Theta_2-\varepsilon))$$

Obviously, as $\Theta_2+\varepsilon<\Theta_1$ this wage is still too low to guarantee efficiency.

Finally, let us consider the case where both Θ_1 and Θ_2 change. This case may be considered as the situation where the government adheres to general expansive or contractive policy. Consider an increase, so that the support is now $\Theta_2+\varepsilon$ and $\Theta_1+\varepsilon$ respectively. The change takes place from period T_1 to T_2 and is unexpected. Obviously, from the curvature of f we see that the possibility of maintaining a pooling equilibrium declines with further increa- increases. If the pooling equilibrium cannot be maintained, we return to the case where Θ_1 changes to $\Theta_{1+\varepsilon}$, Θ_2 is the same and a separating equilibrium obtains in T_2. If the pooling equilibrium is maintained, we obtain the results from the case where Θ_2 changes to $\Theta_{2+\varepsilon}$ and Θ_1 is unchanged. Obviously, the wage ruling in T_3 is adjusted to reflect the new value of the support.

If we have a general decine, then surely, if we start off from a pooling equilibrium, this is maintained, and we refer to the results given for the case when Θ_2 changes to $\Theta_{2-\varepsilon}$, Θ_1 remains unchanged and the pooling equilibrium is maintained. Also in this case the wage obtaining in period T_3 is adjusted to reflect the new value of the support.

The above-mentioned analysis is amended only slightly if we introduce perfect foresight. In this case the profitability of pooling is evaluated from the start, that is, if for example Θ_1 changes to $\Theta_{1+\varepsilon}$, the relevant condition for pooling equilibrium to occur is (15) evaluated before T_1 whereas with myopic foresight the relevant condition is (14). Clearly the change is here that if a pooling equilibrium cannot profitably be maintained, a separating equilibrium results right from the start. Otherwise the analysis is similar to the one presented above.

IV. Conclusion.

In this chapter we gave some conditions under which a pooling equilibrium obtains when demand is characterized by small uncertainty. In such a pooling equilibrium, the level of employment chosen by the firm is too small if economic efficiency is the hallmark. Whether this is important or not, is not to be judged within the confinements of this model. The reason is, of course, that in a general context a competitive firm is small (of measure zero) compared to the economy. However, in the case where a continuum of firms is present, we have shown (Chapter 3, also Laffont and Tirole (1986)) that a non-degenerate subset of firms deviates. Hence, the deviation will certainly be finite. It is obviously unreasonable to attempt to give similar arguments in a model characterized by a two-point distribution.

In Section III we analysed quantity adjustments as well as wage adjustments. We found that changes in demand, in both directions and in both states of nature, are followed by wage adjustment. In a pooling equilibrium, however, if we assume that the economy is in a good state of nature, the wage was, even after the adjustment, too low to ensure economic efficiency. This is purely a result of the fact that the wage is based only upon probabilistic knowledge. The results with respect to quantity adjustments are perhaps more interesting. In the case that demand in the good state changes, this is not reflected in the level of employment as it is the case in a bad state of nature. Conversely, if demand in a bad state of nature is changed, this is reflected in the level of employment. If demand in both states of nature changes, this is also reflected in the level of employment.

V. Notes.

1. Two types of equilibria obtain, pooling and infinite reswitching equilibria. The pooling equilibrium is the simple one.

2. Infinite reswitching equilibria are ruled out.

3. This proof relies upon the assumption that $l(\theta)$ is the Walrasian level of employment for θ, i.e., l is increasing in θ. This may be somewhat of a restriction. On this, see the discussion in Chapter 2.

V. Appendix

Proof of Proposition 1

Consider the wage claim. Firms maximize third period profit

$$\pi_3 = p \cdot y - wl - rk$$

We have

$$w = p \cdot \frac{y}{2l}$$

$$r = p \cdot \frac{y}{2k}$$

Hence $\frac{w}{r} = \frac{k}{l}$

We have

$$w = (\Theta + f(\sqrt{(\tfrac{w}{r})}l))\ \frac{\sqrt{(w/r)}}{2} \Rightarrow$$

$$2\sqrt{(wr)} = \Theta + f(\sqrt{(\tfrac{w}{r})}l)$$

Hence, $(2\sqrt{(\frac{r}{w})}-f'(\sqrt{(w/r)}l)\frac{1}{2}\sqrt{(\frac{1}{wr})}l)dw = f'(\sqrt{(\frac{w}{r})}l)\sqrt{(\frac{w}{r})}dl$

We have $\frac{dl}{dw} = < 0$

Also $-f'(\sqrt{(\frac{w}{r})}l_1\Theta)\sqrt{(\frac{w}{r})}dl = d\Theta$

We have $\frac{dl}{d\Theta} > 0$

Hence $\quad l^d = \psi(w,\Theta) \quad \psi_w < 0 \quad \psi_\Theta > 0$

Let v_2 to the updated belief that $\Theta = \Theta_1$. The wage is set according to

$$w = (v_2\psi(w,\Theta_1) + (1-v_2)\psi(w,\Theta_2)$$

Hence $\quad dw(1-(v_2 \frac{d\psi}{dw}(w,\Theta_1)+(1-v_2)\frac{d\psi}{dw}(w,\Theta_2))) =$

$$(\psi(w,\Theta_1) - \psi(w,\Theta_2))\, dv_2 \qquad \Rightarrow$$

$$\frac{dw}{dv_2} = \frac{(\psi(w,\Theta_1) - \psi(w,\Theta_2))}{1-(v_2 \frac{d\psi}{dw}(w,\Theta_1)+(1-v_2)\frac{d\psi}{dw}(w,\Theta_2))} > 0$$

Consider the wage which is established by the first order condition. Call this wage $w=\tau(v_2)$. We have just shown that $\tau' > 0$. Profits are written

$$p = py - wl - rk$$

Consider two values of Θ; $\Theta_1 > \Theta_2$. Now, for the competitive solution we have

$$w = p(\Theta_i)\frac{y(\Theta_i)}{2l(\Theta_i)}$$

$$i=1,2$$

$$r = p(\Theta_i)\frac{y(\Theta_i)}{2k(\Theta_i)}$$

Hence, $\pi(\Theta_i) = 0$

Consider what happens to a firm of type Θ_1 if the wage is given by $\tau(1/2=v_1)$. Since $\tau'>0$ we have

$$w(1/2 = 1) > \tau(v_2 = v_1) > w(v_2 = 0)$$

As the firm is of type Θ_1, labour demand exceeds labour supply at the wage $\tau(v_2=v_1)$. Consequently, the firm chooses an input $l=\tau(v_2=v_1)$.

The firm also optimizes with respect to the level of capital. We find

$$r = p(\Theta_i)\,\frac{y(\Theta_i)}{2k(\Theta_i)}$$

Hence profits

$$\pi_3 = p(\Theta_1)y(\Theta_1) - (\tau(v_2 = v_1))^2$$

$$\pi_3 = \frac{p(\Theta_1)y(\Theta_1)}{2} - (\tau(v_2 = v_1))^2$$

$$\pi_3 = \frac{p(\Theta_1)y(\Theta_1)}{2} - (\tau(v_2 = v_1))^2$$

Since we have $\tau(v_2=v_1)>w(\Theta_1)$ and $l(\tau(v_2=v_1)<l(\Theta_1)$, we find that $\tau(v_2=v_1)l(\tau(v_2=v_1))<p(\Theta_1)\,\frac{y(\Theta_1)}{2}$ and in consequence

$$\pi_3(\Theta_1,\tau(v_2 = v_1) > 0$$

In particular, the lower is v_2 and hence $\tau(v_2=v_1)l(\tau(v_2=v_1))$ the higher is π_3.

Consider a firm of type Θ_2. We have that labour demand at the going wage rate is less than labour supply. In consequence profits to such a firm is zero. Hence, profits to a type Θ_2 firm does not change with changes in v_2.

Proof of Proposition 2

Ad i). Consider the possibility that a firm with $\Theta=\Theta_1$ plays $l^*(\Theta_1)=l(\Theta_2)$ against $l^*(\Theta_1)=(\Theta_1)$. The gain by doing so is

(A.1) $\quad \pi^F(v_2 = v_1) - \pi^F(v_2 = 1)$

Hence, a necessary condition for playing $l(\Theta_2)$ is

(A.2) $\quad \pi^F(v_2 = v_1) > \pi^F(v_2 = 1)$

Consider now if (A.2) is a sufficient condition. We specify the following out of equilibrium beliefs.

(A.3) $\quad l^*(\quad) \neq l(\Theta_2)\colon v_2 = 1$

Hence, if a firm employs anything but $l(\Theta_2)$, it is believed that this firm is of the high capacity type, i.e., $v_2=1$. But if (A.2) is satisfied, it obviously pays such a firm to employ $l(\Theta_2)$ as this results in the ex-post probability $v_2=v_1$.

It is trivial (cf. (A.3)) that a firm faced with $\Theta=\Theta_2$ chooses $l^*(\Theta_2) = l(\Theta_2)$.

Ad ii). Consider again, the possibility that a firm faced with $\Theta=\Theta_1$ plays $l^*(\Theta_1)=l(\Theta_2)$ against $l^*(\Theta_1)=l(\Theta_1)$. The gain by doing so is

(A.4) $\quad \pi^F(v_2 = v_1) - \pi^F(v_2 = 1)$

Hence, a necessary condition for playing $l(\Theta_1)$ is

(A.5) $\quad \pi^F(v_2 = v_1) < \pi^F(v_2 = 1)$

Consider now, if (A.5) is sufficient condition also. We specify the following out of equilibrium beliefs

(A.6) $\quad l^*(\quad) \neq l(\Theta_2) : v_2 = 1$

(A.7) $\quad l^*(\quad) \neq l(\Theta_1) : v_2 = 0$

From (A.6), respectively (A.7) we see that a firm faced with $\Theta=\Theta_1$, respectively $\Theta=\Theta_2$, optimizes by playing $l(\Theta_1)$, respectively $l(\Theta_2)$.

Ad iii). Referring to Lemma 1 and Lemma 2 for $\Theta=\Theta_2$ $l^*(\Theta_2)=l(\Theta_2)$ with probability 1. Hence, if $l(\Theta_2)$ is observed $v_2 \in (v_1, 1)$. Consider a firm for which $\Theta=\Theta_1$. The gain of playing $l(\Theta_2)$ against $l(\Theta_1)$ is

(A.8) $\pi^F(v_2) - \pi^F(v_2 = 1)$

Hence, a necessary condition for the existence of a semi-separating equilibrium is

(A.9) $v_2^* \varepsilon (v_1, 1) : \pi^F(v_2^*) = \pi^F(v_2 = 1)$

From Proposition 1 we know that π^F is strictly decreasing in v_2. Hence, for $\tilde{v}_2$.

(A.10) $\pi^F(v_1) > \pi^F(v_2^*) > \pi^F(v_2^* = 1)$

Assume that (A.10) is satisfied. As π^F is decreasing in v_2, there exists one and only one real number $v_2 E(v_1, 1)$ such that (A.9) is satisfied. Hence, there exists a real number x such that if $l(\Theta_1)$ is employed with probability, x and $l(\Theta_2)$ are employed with probability 1-x the posterior of $v_1 : v_2 = (v_1 O(1-x)(1-v_2)))^{-1} v_1$ when $\varphi_1 (k, \Phi(k,p,w_1))$ is produced whtn v_2 satisfy (A.9).

Consider the following out of equilibrium beliefs

(A.11) $l^*(\cdot) = l(\Theta_2) : v_2 = 1$

(A12) $l^*(\cdot) = l(\Theta_1) : v_2 = 0$

Clearly, from Lemma 2, (A.11) and (A.12) a low capacity firm prefers to produce $l(\Theta_2)$. Consider then a high capacity firm.

Taking the beliefs as given in (A.11) and (A.12) any out of equilibrium production $l^*(\ \ )=l(\Theta_1)$ and $l^*(\ \ )=l(\Theta_2)$ is inferior to the equilibrium strategy.

Finally, we have to show that for any given value of Θ_1 and Θ_2, only one of the conditions in the proposition is satisfied. Consider the conditions (A.2), (A.5) and (A.10). These conditions are mutually exclusive, hence the continuation equilibrium is unique.

CHAPTER 7

GENERAL CONCLUSION

Despite the, admittedly, weak empirical content of this thesis som general conclusions emerge. It has been demonstrated that if the wage is determined in a bargaining process between a firm and a trade union, the information structure characterizing the economy is important. This is so in the sense that if firms are better informed about the value of the marginal product of labour compared to the union, then in addition to price signals also quantity signals are needed to support an incentive compatible solutin. The conclusion to emerge in this respect is that underemployment is needed to support the revelation of relevant information.

Other general conclusions to emerge are that the nature of the equilibrium is not unambiguous. This is seen clearly comparing Chapters 2 and 3. Introducing a zero profit constraint in an otherwise separating equilibrium can lead to a pooling equilibrium. Thus, the qualitative nature of the equilibrium is very much dependent upon whether one has in mind a very partial model or say a monopolistic competition model. These results suggest that this is an issue which should be addressed more generally.

Also, and partly following from this, Chapters 5 and 6 illustrate the potential effects of eocnomic policy in respectively a separating and a pooling equilibrium. Since Chapters 2 and 3 demonstrated a non-robustness of the nature of the equilibrium, policy conclusions should be drawn with great care.

Finally, Chapter 4 illustrated (with respect to investments) that when monopoly effects are present, the introduction of asymmetric information and thus incentive constraints do not have an unambiguous effect either increasing or reducing the level of investment. This again points to the fact that when evaluating the effect of asymmetric information, care must be taken in the specification of the model, in particular with respect to assumption regarding the market form.

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