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# The Development of Mathematical Resilience in KS4 Learners

by

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A thesis submitted in partial  
fulfilment of the requirement for  
the degree of Doctor of Education.

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## Table of Contents

Table of Figures.....	7
Declaration .....	9
Abstract.....	10
Section 1: Introduction and Background to Research Project.....	11
Section 2: Literature Review .....	16
2.1 The problems with the teaching of mathematics .....	16
2.2 Research into the causes of mathematical anxiety .....	21
2.3 Research into mathematical resilience .....	32
2.4 Research into the effect gender has on mathematical success.....	37
2.5 Research into teaching for mathematical thinking and understanding.....	41
2.6 Research into the use of group work in mathematics.....	47
2.7 Research into developing independent learners .....	51
2.8 Conclusion.....	56
Section 3: The planning stage.....	57
3.1 The Research questions .....	57
3.2 Methodology.....	57
3.2.1 Making my research meaningful .....	60
3.2.2 Use of examination questions.....	64
3.2.3 Use of learner journals to record feelings about the tasks.....	66
3.2.4 Use of group interviews.....	69
3.2.5 Use of researcher journal to record observations .....	70

3.3 Validity of the research findings .....	72
3.4 Ethical considerations.....	74
3.5 The research group .....	76
3.6 Analysing my Data .....	77
3.7 Conclusion .....	79
Section 4: The teaching episodes .....	80
4.1 The reconnaissance stage .....	80
4.2 Intervention one: plotting coordinates.....	83
4.3 Intervention two: planning a trip .....	87
4.4 Intervention three: drawing straight line graphs.....	94
4.5 Intervention four: investigating straight line graphs .....	103
4.6 Intervention five: the data handling cycle .....	116
4.7 Intervention six: statistical investigation .....	122
4.8 Intervention seven: exploring the number grid .....	128
4.9 Intervention eight: the history of mathematics .....	139
4.10 Intervention nine: Real life problems .....	142
4.11 Summary of interventions .....	151
Section 5: Learner discussions .....	155
5.1 Responses from learner one, Martin.....	157
5.2 Response from learner two, James .....	162
5.3 Response from learner three, Polly .....	166

Section 6: Analysis of findings.....	171
6.1 Learners demonstrating resistance to changing the culture of teaching and learning within the classroom.....	172
6.2 Being Stuck .....	173
6.2.1 How learners initially dealt with being stuck .....	173
6.2.2 Learners learning to use the support available to them .....	175
6.2.3 The use of scaffolding the help build learner confidence in dealing with being stuck.....	178
6.2.4 Making links with other mathematical topics to deal with being stuck	180
6.2.5 Using strategies from other subject areas to help make progress.....	181
6.3 Building confidence in tackling problems.....	184
6.3.1 Use of estimation to boost confidence.....	184
6.3.2 Adding an element of choice to build confidence .....	185
6.4 The motivation for learning mathematics.....	187
6.5 Accepting a new classroom culture .....	188
Section 7: Conclusions .....	190
7.1 Findings .....	190
7.2 Successes and limitations of the project.....	195
7.3 The next steps .....	195
Bibliography .....	197
Appendices .....	210
Appendix 1: Ethical approval.....	210

## Table of Figures

Figure 1: A simplified conceptual map describing the interplay between learners' attitudes and academic performance (OECD, 2016). .....	25
Figure 2: The failure cycle (Ernest, 2015, p189).....	29
Figure 3: The success cycle (Ernest, 2015). .....	31
Figure 4: The Growth Zone Model (Johnston-Wilder et al., 2013). .....	34
Figure 5: The Learning Zone model (Senninger, 2000). .....	35
Figure 6: The action research cycle (Lewin, 1948). .....	59
Figure 7: Learner journal prompt sheet .....	68
Figure 8: An example question used in intervention one. ....	85
Figure 9: The first section of the investigation used in intervention four. ....	105
Figure 10: The first section of the investigation used in intervention four. ....	106
Figure 11: Some equations used by Val and Emily as part of task two. ....	108
Figure 12: Question taken from Edexcel November 2012 GCSE mathematics paper. ....	114
Figure 13: The data handling cycle. ....	116
Figure 14: Match the picture with the graph. ....	118
Figure 15: The number grid investigation explanation sheet. ....	129
Figure 16: Table of results.....	134

Figure 17: An example question taken from Edexcel Higher Paper 2 June 2014.	
.....	143
Figure 18: Example question taken from Edexcel GCSE mathematics A Linear	
Higher: Pupil book. ....	144
Figure 19: Question taken from Edexcel GCSE mathematics A Linear Higher:	
Pupil book.....	146
Figure 20: Summary of the findings from each action research cycle.....	152

## **Declaration**

I declare that this thesis is my own work and has not been submitted for a degree at another university.



## **Abstract**

This action research project focussed on the key components of the construct mathematical resilience and how mathematical resilience can be developed in learners who are working towards their GCSE in mathematics. Split-screen lesson objectives, one related to a mathematical skill and the other related to a learning skill, were used to focus the learner's attention onto each skill. These learning skills were chosen to encourage a particular group of learners to gain the confidence, persistence and perseverance to allow them to work inside the Growth Zone. The overall aim of this action research project was to improve the attainment of learners in their GCSE mathematics examination.

## **Section 1: Introduction and Background to Research Project**

This action research project explores methods of improving outcomes in mathematics with a specific focus on learners who are currently predicted to just miss out on a ‘good GCSE pass’ which is defined as a ‘grade 5’ in the new scale or a ‘grade C’ in the old scale. Overcoming mathematical anxiety, which is well documented in literature (e.g. Ashcraft, 2002, Callan, 2015), is one of the foci that I consider, although the main focus of the research was to change the way the learners approach learning, in particular, the development of ‘mathematical resilience’. This construct, defined by Johnston-Wilder et al. (2013), describes the ways in which learners can develop positive approaches to the learning of mathematics that give them strategies to overcome any difficulties they may face.

Before I consider the barriers that many learners have toward learning mathematics, I will explain how learners’ attainment is measured at age sixteen in England (at the time of writing) and the impact that this has for learners and schools.

Since Summer 2016, English schools are ranked on performance tables issued by the Department of Education based on the new ‘progress 8’ measure (DfE, 2016). For this new performance measure, every learners’ eight best GCSE examination results are used to measure the progress they have made between the end of year six (age eleven) and the end of year eleven (age sixteen). These eight subjects must include English and mathematics, three EBAC subjects (the sciences, modern or ancient languages, computing, history and geography) and three other approved qualifications. English, if studied alongside literature, and mathematics

are double weighted within this measure. A learner's score is calculated using a formula and the mean value of all of the learner's scores is taken to give an overall school score. A 'progress 8' score of zero indicates that, on average, the learners are making the progress expected. A score of one means learners are making, on average, one grade higher than expected progress and a score of minus one means they are making, on average, one grade lower than expected. The definition of expected progress was defined by the DfE after the summer 2016 examination results were collated. With 'progress 8' being based on the progress made instead of the percentage reaching a certain level of attainment, it is intended that schools will reduce focussing on groups of learners whose predicted attainment is at key borderlines and instead focus on every learner (DfE, 2016).

The two main audiences of this new progress measure are expected to be parents of new learners and Ofsted, the official body which reports on school performance to the Department for Education via a written report that is available in the public domain. As part of this Ofsted report, schools are graded as one of 'Outstanding', 'Good', 'Requires Improvement' or 'Unsatisfactory'. If a school's 'progress 8' measure falls below the floor target of minus zero point five, the school is likely to be graded either 'Requires Improvement' or 'Unsatisfactory'. The consequences of this are significant for schools, with subsequent progress being monitored closely by Ofsted until they are deemed to be 'Good' (DfE, 2016).

Since 2014, there has also been the requirement that learners who fail to achieve a grade C or above in GCSE English or mathematics continue working towards this qualification in post-16 education (age 16-19) (DfE, 2016b). If a school or

college fails to ensure this happens, funding for these learners is withheld for all courses they are studying. Good passes in English and mathematics are also the minimum standard required for entry into University and many careers options.

The learners who are part of the research group were following the new Mathematical Curriculum for Key Stage 4, which prepares them for their GCSE examination. I shall briefly discuss this curriculum here to clarify both its different content and how the research interventions fit into the research group's studies.

The new mathematics Curriculum for Key Stage 4 (DfE, 2014) introduced three overarching skills that relate to all subject content. These are 'Develop Fluency', 'Reason Mathematically', and 'Solve Problems'. The main focus of this action research project is predominantly on 'Solve Problems', although the problems solved include elements from the other two areas. The DfE states that learners are expected to be able to do the following under the 'Solve Problems' heading:

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial contexts
- make and use connections between different parts of mathematics to solve problems
- model situations mathematically and express the results using a range of formal mathematical representations, reflecting on how their solutions may have been affected by any modelling assumptions

- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems; interpret their solution in the context of the given problem

(DfE, 2014).

My central question for my research is “How do I change my teaching practice so that my learners become mathematically resilient?” I carried out my research as an action research project based around one of the classes I taught. The research took place over an eighteen-month period and consisted of nine key interventions that took place alongside normal timetabled lessons. Each intervention made use of split-screen objectives, which focused on a learning skill and a mathematical skill. I hoped that by developing these learning skills the learners would become more mathematically resilient and as result increase their ability to tackle unfamiliar problems as met on examinations and improve their engagement with mathematics to increase attainment. Each intervention was planned based on the needs of the learners identified in the previous interventions. Data were collected by making use of teacher researcher observations and field notes, learner journals, learner interviews and outcomes in GCSE questions to monitor the success of the research cycle.

In section 2, I discuss the key literature related to the action research project to help give a deeper understanding of the reasoning behind, and the rational of, the design of the interventions. I start by considering the causes of mathematical anxiety and looking at the construct of mathematical resilience before discussing suggested strategies that can be used to help develop mathematical resilience.

In the following section, I expand on my central research question and discuss the rationale behind the choice of my research design and the methods I used for data collection. I discuss how I planned to analyse the data and how I ensured my research remained ethical.

In sections 4 and 5, I describe the interventions that took place in detail and the observations made during them. Relevant comments made in student journals and mentioned in interviews were used alongside the outcomes in GCSE examination questions to create a thick description of the action. These sections provide the evidence for section 6 where I present an analysis of my findings split into the different categories that emerged from my analysis. These findings were used to provide the evidence for my conclusions related to my research questions in section 7.

## **Section 2: Literature Review**

In this literature review, I consider what research has found to be some of the problems related to mathematical teaching before exploring in more detail some of the barriers faced by learners of mathematics. It is only with an understanding of what these problems and barriers are that I can then consider ways of addressing them in my teaching. I review the literature based around the construct mathematical resilience and consider the role this could play in helping to address the needs of my learners before researching in detail some teaching strategies that I could use to overcome the barriers faced by these learners.

### **2.1 The problems with the teaching of mathematics**

Personal experience has shown that many learners who fail to gain a ‘good’ pass in GCSE mathematics do not enjoy the subject. Many of these learners are very successful in other areas of the school curriculum but lack the motivation to put in the effort required to succeed in mathematics. In this section, I shall consider some of the research around why learners do not enjoy mathematics, with the aim of understanding better what steps could be taken to overcome this issue.

One of the areas of major concern about mathematics teaching is that over time it has become more didactic as teachers feel they need to ‘teach to the test’ in order to cover the curriculum content in time for external examinations on which the whole school is judged (Ofsted, 2008). A research project by Nardi and Steward (2003) investigated some of the possible reasons for a decrease in the number of

learners studying mathematics beyond the compulsory age of sixteen. Although some learners feel happier being taught what they need to know for the test, these findings suggested learners had become disaffected because some mathematics teaching has become T.I.R.E.D: tedious, isolated, rote learning, elitist and depersonalised.

Nardi and Steward said ‘learners do not like irrelevant, decontextualized, textbook based mathematical tasks (formal tasks)’ (Nardi and Steward, 2003, p351). They further suggested that learners disliked mathematics because they find it difficult to engage in mathematical activities that they find too abstract, meaningless or without aim and find doing so tedious and boring. Furthermore, Brown, Brown and Bibby (2008) state that some of the factors that have an impact on the uptake of mathematics at A-level are because maths is perceived to be ‘hard’, ‘boring’ and ‘useless’.

Nardi and Steward (2003) also found that due to the demands of the curriculum, many teachers opt out of justifying the methods they advocate and teach algorithms or procedures that give correct answers if applied correctly without making connections or encouraging their learners to seek understanding. The result of this is that the learners have what Skemp (1987) refers to as an instrumental understanding of the mathematics. They can use the rules but do not know why the rules work. In order to achieve such understanding, the teacher gives examples on the board and the learners spend the lesson working through many examples, all using the same method. Nardi and Steward (2003) interviewed many learners who thought that in order to be successful in mathematics you must



be good at remembering rules. An instrumental understanding can work well for examination purposes for some learners but as soon as the learner comes across a problem that is slightly different from the norm, they no longer know what to do. An alternative is to develop relational understanding (Skemp, 1987). Relational understanding is achieved when the learners understand the rules they are using and how they fit into the bigger picture. Nardi and Steward's (2003) research suggested that many learners had become disaffected and suggest that this was because they wanted to understand what they were doing, not just be able to apply what appear to be arbitrary rules. Boaler (1998, 2013) also found this also to be the case, especially with female learners of mathematics.

Von Glasersfeld (1985) suggests that the primary goal of mathematics instruction has to be the learner's conscious understanding of what he or she is doing and why it is being done. In order for this to be the case, learners need to gain a relational understanding of the mathematical content of the lesson; not only do they need to be able to carry out the calculation but be able to explain why it works. Brown (1997) describes the concept of 'thinking like a mathematician'. He defines this as placing greater emphasis on problem solving heuristics, encouraging learners to make use of a wide variety of inductive and deductive skills to work on incomplete knowledge. Wheeler (1982) discusses a similar view to Brown when he described the concept of 'mathematizing'. Wheeler suggests that it is more useful to know how to mathematize than to know a lot of mathematics and is unsure why many teachers do not encourage their learners to function like a mathematician. Both Brown (1997) and Wheeler's ideas (1982), that knowing and understanding how to solve mathematical problems is more important than

knowing lots of mathematical processes, are essential elements to meeting Von Glasersfeld's (1985) goal. My personal observations of mathematics teaching in four different schools has shown very little evidence of learners being taught to understand the methods but instead being taught to follow algorithms, often with very little understanding of what they are doing.

Evidence suggests those teachers who choose to 'teach to the test' are having a negative impact on learners' experiences of mathematics and, in most cases, are not giving learners the mathematical understanding that is required to succeed in external examinations (Ernest, 2015). Perhaps a way to help change learners' perceptions of mathematics being T.I.R.E.D, is to consider the learning of mathematics from a constructivist position. Kilpatrick (1987) identifies the consequences for learning mathematical processes that arise from a constructivist point of view. The first consequence he discusses is that teaching, which he defines as using procedures that aim to generate understanding, should be distinguished from the commonly seen practice of training, which he describes as using procedures that aim at repetitive behaviour. Kilpatrick's definition of teaching can be argued to lead learners to develop an understanding that Skemp (1985) would describe as relational, whereas Kilpatrick's definition of training is consonant with Skemp's idea of instrumental understanding. As part of his ideas, Kilpatrick discussed the idea that communication between teacher and learner should move away from the transfer of knowledge and instead become a process of guiding learning. Many of the findings raised by Nardi and Steward (2003) and Boaler (1998, 2013) suggest that this approach may help overcome many of the negative attitudes learners have towards mathematics.

A common theme in all of the research reviewed so far is that the issues affecting learner motivation and achievement are a consequence of the styles of teaching being adopted in classrooms. The research often refers to didactic teaching that focuses on instrumental understanding as a quick reward for less effort to get learners through their next examination. I have heard maths teachers argue that for many learners an instrumental understanding of the material is sufficient for their future careers. Although I have witnessed this form of teaching, it is most certainly not all like this. Over the past few years, there have been many initiatives at both national and regional level, for example National Strategies and NCETM, designed to equip teachers with the skills and confidence to teach the mathematics curriculum in a way that is expected to enable learners to gain a relational understanding of the content and to develop their problem solving skills.

The work of Goodall, Johnston-Wilder and Russell (2016) compares mathematical learning in the home with that generally experienced in school. They found that learners typically experienced real mathematical learning at home (not homework) as being Accessible, Linked, Inclusive, Valued and Engaging or A.L.I.V.E. compared to views that mathematical learning in schools was often T.I.R.E.D. (Nardi and Steward 2003). Exploring ways of making learning mathematics in school A.L.I.V.E. could help change the negative views that many learners of mathematics share.

## **2.2 Research into the causes of mathematical anxiety**

I consider that one of the largest barriers to overcome in the teaching that is planned for this research project is that of mathematical anxiety. Here I explain my understanding of what mathematical anxiety is and the impact it has on learners of mathematics in order to deduce potential ways to reduce the level of mathematical anxiety experienced by learners while carrying out mathematical problem solving. I also consider the impact that motivation for learning has on the outcomes of mathematical tasks.

Ashcraft (2002) defines mathematical anxiety as ‘a feeling of tension, apprehension or fear that interferes with math performance’ (p181). Interestingly, his research indicates that there is only a very weak correlation (-0.17) between mathematical anxiety and intelligence based on IQ tests. He found that in the US, a higher percentage of females scored highly on a mathematical anxiety scale compared to males. This idea is developed further in section 2.4.

One thing that Callan (2015) makes clear is that mathematical anxiety is not the same as being ‘bad’ at mathematics. Levels of mathematical anxiety can be measured using different rating scales e.g. Math Anxiety Scale (Betz, 1978), the Maths Anxiety Rating Scale (Richardson and Suinn, 1972). Research carried out by Harms (2014) has shown that the area of the brain that triggers when someone experiences mathematical anxiety is that same area that is affected by self-harm.

One of the common strategies for coping with mathematical anxiety is avoidance (Ashcraft, 2002 and Callan, 2015). The major consequence with this coping

strategy is that those that avoid mathematics are exposed to less mathematics at school and do not retain much of what they have covered. This results in lower levels of attainment and achievement in external examinations. Recent data from the Programme of International Student Assessment (OECD, 2016) indicates that mathematical anxiety has an equivalent effect of being behind by almost one year of school. Callan (2015) suggests that mathematical anxiety can be developed as a consequence of the way mathematics is taught in schools. She argues that mathematical anxiety can be picked up from teachers who themselves suffer from mathematical anxiety and also if learners are taught in a way that makes learners believe there is only one correct method to solve a particular problem.

Many authors, for example Callan (2015) and OECD (2016), consider negative attitudes towards mathematical learning to be a contributory factor in mathematics anxiety. Consequently, a consideration of literature relating to attitudes to mathematics is likely to provide insight into the barriers that some learners find when learning mathematics. As discussed earlier, Nardi and Steward (2003) discovered that the learners found their mathematical education to be tedious, isolated, rote learning, elitist and depersonalised. They found that learners who prefer variety, community, learning for understanding, inclusions and personalisation in their learning of mathematics are unlikely to persevere when faced with any difficulties and thus are likely to quickly find themselves dissociated from progressively difficult mathematical concepts.

Nardi and Steward (2003) also commented that one of the reasons that learners dislike mathematics is that they find it difficult to engage in mathematical

activities that they find too abstract, meaningless or without aim and find doing so tedious and boring. These comments imply that the aims of many educators are different to those expressed by Von Glasersfeld (1985), which were discussed earlier, and possibly more related to ‘teaching to the test’ resulting from the pressure, which is put upon teachers to improve examination results.

Linked in with the ideas from Nardi and Steward (2003) is the concept of purpose and utility as defined by Ainley, Pratt and Hansen (2006). They define a purposeful task as a task which has a meaningful outcome for the learner. This could be an actual product or a solution of an engaging problem. The utility of mathematical ideas is defined by learning mathematics in a way that is not just about the ability to use a technique or an idea but also for the learner to construct their own meaning about why these techniques and ideas are useful. Ainley et al. (2006) suggest that the typical content of a school’s curriculum does not give opportunities for learning the utility of mathematical ideas, even within tasks that attempt to relate to real-life settings.

The views expressed by Von Glasersfeld (1985) and the findings of Nardi and Steward (2003) and Ainley et al. (2006) all suggest that there is a need to contextualise mathematics to help learners to see the relevance of mathematical ideas and therefore why they need to understand and use the ideas that are contained in the mathematical curriculum. Over the past decade there have been many attempts to contextualise mathematics through the ‘Using and Applying’ strand of the National Curriculum (DfEE, 1999) and more recently the introduction of the ‘Functional Skills’ (DfE, 2014) element to the GCSE. Despite

these efforts, it has been suggested (e.g. Nardi and Steward (2003) and Ainley et al. (2006)) that learners find the contextualisation used too abstract and they are still seeing mathematics as tedious exercise without any relevance to their lives. Boaler (1998, 2013) provided a possible solution to abstract contextualisation through a case study of the curriculum in an English school. The mathematics curriculum discussed in her research is completely investigation based. Learners are given a real life problem to work on for an extended period and are allowed to choose what to investigate. The mathematics taught comes directly from needing a particular mathematical idea in order to solve the learners' problem, thus any mathematics comes out of the context. A similar suggestion is made by Ernest (2015) in his fourth aim of his visionary goals for school mathematics, which is discussed later in this section. Boaler (2003) comments on how this approach greatly changed the learners' attitudes towards mathematics by seeing the relevance of what they were learning or, using the ideas of Ainley et al. (2006), they developed a sense of the utility of the mathematical ideas. The outcomes described by Boaler (2013) indicated that the learners were also able to mathematize (Wheeler, 1982). Mathematizing was evident in the learners being able to adapt what they already knew in one area of mathematics and link it to other topics, which on the surface may appear unrelated.

Another finding from Nardi and Steward (2003) was that the learners saw mathematics classrooms as a place where you work independently and in silence, which they termed 'isolated'. On the few occasions when learners were allowed to discuss what they were learning with their peers, they commented on the usefulness of collaborative learning for helping them both to pick up the

techniques and to understand what they were doing. Since the nature of mathematics is to be abstract, learners need to be able to share their views and thoughts so that they can develop their own understanding of the topics and begin to apply mathematical ideas (Swan, 2008). This idea is developed further in section 2.6.

As mentioned earlier in this section, many researchers (e.g. Callan (2015) and OECD (2016)), believe that negative attitudes and experiences of mathematics teaching can lead to mathematical anxiety. OECD (2016) has constructed a simplified conceptual map describing the interplay between learners' attitudes and academic performance (see Figure 1 below).

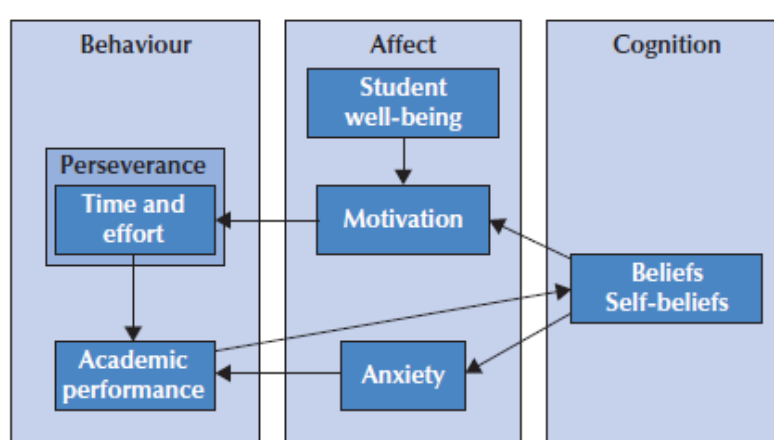


Figure 1: A simplified conceptual map describing the interplay between learners' attitudes and academic performance (OECD, 2016).

This framework suggests that one way to increase academic performance is by decreasing anxiety, which stems from learners' self-beliefs. Another way is through increased perseverance by putting in more time and effort. The framework suggests that this increase in perseverance comes from an increase in motivation.



Ryan and Deci (2000) define 'to be motivated' as 'to be moved to do something' (p54). In their work on Self-Determination Theory (Deci and Ryan, 1985), they separate motivation into two types, intrinsic motivation and extrinsic motivation, based upon the reasons behind what is causing the motivation. In its simplest form, they define intrinsic motivation as motivation caused by doing something because it is enjoyable or interesting. Extrinsic motivation is defined as motivation caused by doing something because it results in another positive outcome.

Eccles and Wigfield (2002) suggest that learners are more likely to increase the time and effort put into a task when they are motivated to do so. They suggest that in terms of an educational context, intrinsic motivation can be because the learners are 'hooked' by the task and mathematical activity being done and by the pleasure of continuing until the task is complete. Extrinsic motivation can be related to learners knowing that getting better at mathematics increases the chance of achieving a 'good pass' in external examinations or give them the skills required for future careers. It is believed and accepted by many authors such as OECD (2015) and Voss and Schauble (1992), that intrinsic motivation plays the largest role in increasing engagement in a task and as a result increasing academic performance. Despite this, personal observation across four different schools has seen a much greater emphasis on promoting extrinsic motivation related to gaining a 'good pass' being used to encourage learners to engage with mathematical learning. Ryan and Deci (2000) split extrinsic motivation up into two different cases. The first relates to the classical case of extrinsic motivation. In this case learners can be motivated to carry out the task with resentment, resistance and

disinterest. My own observations have shown that this happens when learners are extrinsically motivated by the thought of gaining a 'good pass'. However, in the other case, learners carry out the task with an inner acceptance of the utility of the task.

In the framework above (OECD, 2016), an increase in time and effort put in by the learner is linked with an increase in perseverance. It is important not to confuse perseverance with persistence. Thom and Pirie (2002) describe perseverance as 'the sense in knowing when to continue with and ... knowing when to abandon a particular strategy or action in search of a more effective or useful one (p2)'.

Williams' (2014) research found that learners who were not perseverant often avoided going outside their Comfort Zone, even when these learners were very confident mathematicians. This idea of a change of approach becomes important when looking at overcoming the barriers faced by learners in mathematics; they must not be afraid to change direction when solving a problem if they are not making progress. Williams' (2014) findings also suggest that the combination of perseverance, persistence and confidence is required to become resilient, something that is essential in problem solving so will be important for this research.

In the PISA framework above, self-belief is shown to have an impact on both level of mathematical anxiety and motivation. Bandura (1997) discusses the idea of self-efficacy, the extent to which a learner believes in their own ability to solve a

particular mathematical problem, and self-concept, which is a learner's belief in their own ability in mathematics. He also discusses the idea of resilient self-efficacy, which indicates learners who overcome barriers through perseverant effort. His research has led him to believe that these ideas have a large impact on learners' perseverance with, and motivation to work on, a problem. OECD research (2015) has found that learners who have low levels of self-efficacy and self-concept believe that putting in extra effort makes no any difference to their mathematical ability. This leads to a negative impact on academic success as a result of being disengaged during lessons in mathematics.

Ernest (2015) describes what he believes are some of the unintended outcomes of school mathematics or the 'hidden curriculum' These are:

- a) mathematics is intrinsically difficult and inaccessible to all but a few.
- b) success in mathematics is due to fixed inherited talent rather than to effort.
- c) mathematics is a male domain, and is incompatible with femininity.
- d) mathematics is a European science, to which other cultures have contributed little.
- e) mathematics is an abstract theoretical subject disconnected from society and day-to-day life.
- f) mathematics is abstract and timeless, completely objective and absolutely certain.
- g) mathematics is universal, value-free and culture-free.

(Ernest, 2015, p188)

Although he acknowledges there is now research that disproves these beliefs, he feels they are still views that are held by many learners and parents. This has also been documented by others (e.g. Callan, 2015) who comment that it is considered acceptable to publicly declare that ‘you are not good at mathematics’. Ernest comments that this negative image is also displayed in many mathematics classrooms.

Ernest (2013, 2015) discusses the impact that a learner’s own perceived competence and self-efficacy has on their attainment in external examinations. He has developed a failure cycle that shows that a low self-concept becomes a self-fulfilling prophecy. This failure cycle is shown below in Figure 2.

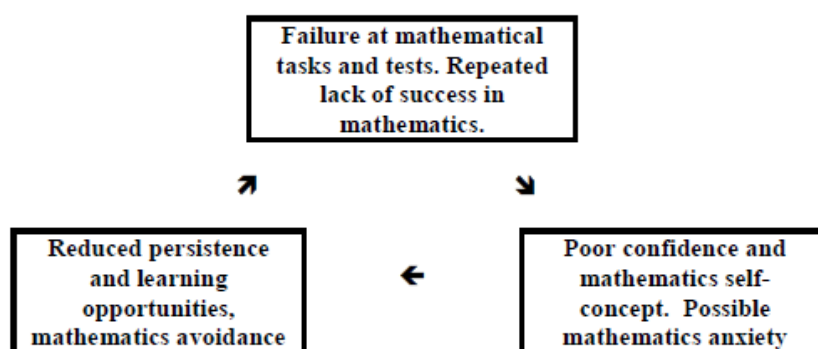


Figure 2: The failure cycle (Ernest, 2015, p189).

This cycle shares similarities with the conceptual map described by OECD (2016) although makes reference to the impact that repeated failure can have. Those with poor confidence often do not know how to cope with failure and use avoidance strategies as discussed previously. Maslow’s Hierarchy of Needs theory (1987) suggests that a learner does whatever they can to avoid threats (perceived or real)

to their personal self-esteem. Once a learner has been subsumed within this failure cycle, it can become hard to escape. To combat this failure cycle, Ernest (2015) has visionary goals for school mathematics. He wants to move away from defining the curriculum by content and move towards a set of higher level orientations and capability related to mathematics. These are:

1. mathematical confidence,
2. mathematical creativity through problem posing and solving,
3. social empowerment through mathematics (critical citizenship),
4. broader appreciation of mathematics.

(Ernest, 2015, p189)

He feels that the first strand, developing mathematical confidence, is perhaps the most important of all these. This aligns with the findings of other researchers (e.g. Bandura, 1997; OECD, 2013; OECD, 2015). He argues that ‘effective knowledge and capabilities rest on freedom from negative attitudes to mathematics, and build on feeling of enablement, empowerment as well as enjoyment in learning and using mathematics’ (p190). He has found that this results in learners approaching learning mathematics with a positive attitude, being prepared to accept and persist with challenging problems. If these orientations are in place the failure cycle is inverted and becomes the upwardly spiralling success cycle, which can be seen below in Figure 3.

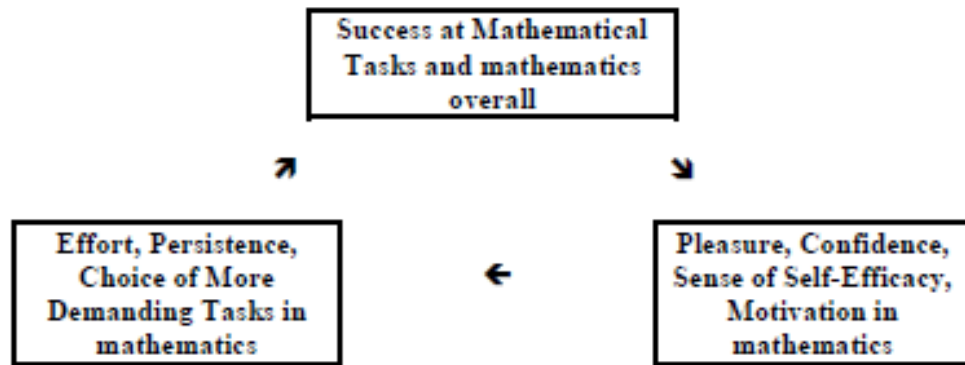


Figure 3: The success cycle (Ernest, 2015).

This cycle demonstrates the effect that intrinsic motivation can have on attainment in examinations. An increase in self-efficacy and self-concept can lead to more effort and persistence, which in turn leads to a reduction in mathematical anxiety and a greater success with mathematics, reflecting the findings of OECD (2016). Ernest advises schools that by focussing on the quality of the learning experience of the learner and reducing the importance of examinations and testing then the outcome in these examinations and tests are likely to be higher.

In this section, I looked at some of the research into the causes of mathematical anxiety and explored the links between learner's beliefs and academic performance. These links should be considered when planning the approach of the interventions used as part of this research project.

### **2.3 Research into mathematical resilience**

In this section I will initially look at resilience before discussing the construct of mathematical resilience. The Growth Zone model is introduced; the Growth Zone model plays a key part in the development of the interventions.

Hernandez-Martinez and Williams, 2011, discuss how the definitions of resilience have changed over time. They describe how the early definitions of resilience were based around the individual characteristics of the learner before moving towards a definition that expanded to include external social factors such as poverty. Based on their research, they defined resilience as ‘a dynamic process of interaction between sociocultural contexts and the agency of developing individuals’ (Hernandez-Martinez and Williams, 2011, p3).

Lerman’s (1996) research is based around the sociocultural view of learning. In this view of learning, the learners make meanings first in the socio-cultural domain which is then internalised by the learner within discursive practices. These socio-cultural factors can include the schooling and classroom cultures that learners may have become accustomed to working within. Being resilient may require the learner to cope outside of the culture they have become accustomed to, something which the Growth Zone model (Johnston-Wilder et al., 2013) can support.

Mathematics resilience is a construct described by Johnston-Wilder et al. (2013) as ‘a positive stance towards mathematics’. Becoming mathematically resilient is about learning to approach mathematics rather than avoiding mathematics,

replacing fear and negative thoughts with observations based on curiosity and awareness, reflecting on thoughts and feelings using notation and labelling (Siegel, 2007). Siegel believes that resilience can be learnt through experience. Although resilience is needed for learning in all subjects, the problems discussed in section 2.1 have resulted in learners who are able to demonstrate resilience in other subjects but are not able to demonstrate the same resilience when learning mathematics (Johnston-Wilder et al., 2013). The research of Johnston-Wilder et al. found that learners were not using this behaviour in mathematics because they either felt that these behaviours would not work for mathematics or because they had been discouraged from using them.

Extending this definition following a review of research on resilience and their own work, Johnston-Wilder et al. (2013) believe that mathematical resilience is multi-dimensional with connected factors including understanding the personal value of mathematics, having an understanding of how to work at mathematics and having awareness that support is available from peers, other adults, ICT, internet etc. The last two factors reflects the findings of Dweck (2000) who suggests that learners who have high levels of resilience know that it is worth persevering when they encounter failure and have many different strategies for dealing with this, they work collaboratively with their peers and have the language skills needed to express their understanding of mathematical learning'. The focus here was on creating a positive environment for mathematical learning to take place.



Further development of the construct mathematics resilience has led to a development from Vygotsky's (1978) Zone of Proximal Development into consideration of the affective domain, which Johnston-Wilder et al. (2013) call the Growth Zone Model and can be seen below in Figure 4.



Figure 4: The Growth Zone Model (Johnston-Wilder et al., 2013).

The Comfort Zone in the centre of the diagram, also known as the Safe Zone, represents the area where learners can perform tasks and activities independently with minimal support required from their teacher or peers. This is often confidence building repetitive tasks. Little new learning takes place within the Comfort Zone.

The Growth Zone is where new learning happens; this could be related to a new topic, a new approach or a adapting to a new classroom culture. In this zone, learners need some guidance and support. Johnston-Wilder et al. (2013) suggest that in order for learners to avoid developing anything above mild anxiety the learning environment must be one of collaboration, trust, courage, persistence and perseverance. To enter the Growth Zone, learners need to feel motivated by the task or mathematics and feel appropriately supported. The learner needs to be encouraged to work logically, make mistakes and recruit support when needed or they may start displaying some of the signs of mild mathematical anxiety.

The Growth Zone shares similarities to a learner's 'Zone of Proximal Development' as described by Vygotsky (1978). This zone refers to the gap between what the learner can do alone and what the learner can do with suitable support. More knowledgeable or skilled people are considered by Vygotsky (1978) to be able to help bridge the gap with the aid of suitable scaffolding. Once the learning process has taken place, the scaffolding or support can be removed.

The outer zone is the Anxiety or Danger Zone. When learners are in this zone the task is beyond their capabilities even with support. This can lead to more signs of mathematical anxiety and can lead to the learners making use of avoidance strategies. No effective learning takes place within this zone. Repeated exposure to the Anxiety Zone can result in the development of persistent mathematical anxiety.

A similar model has been developed by Senninger (2000). This model consists of three zones and can be seen in Figure 5.

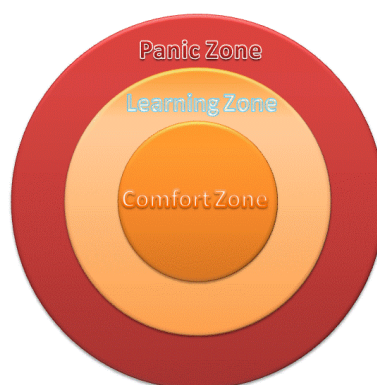


Figure 5: The Learning Zone model (Senninger, 2000).

Senninger (2000) describes the Comfort Zone as a safe haven; an area where things are familiar and there is no need to take risks. He argues that it is only in the Learning Zone where learners can make new discoveries. His research suggests that when new discoveries are made the Comfort Zone slowly expands. Similar to Johnston-Wilder et al. (2013), he does warn about taking care in encouraging learners to move from the Comfort Zone and into the Learning Zone. If they are pushed too far, they move into the Panic Zone, which he describes as the zone where any learning is blocked by the sense of fear. Once in this zone all energy is moved away from learning and used to control anxiety.

Johnston-Wilder et al. (2013) propose that ‘positive mathematical experiences, together with a language of risk awareness and risk management can help learners to develop risk taking and risk management processes, which then lead to mathematical growth’ (p 3). They feel that overtime the learners need to have exposure to the positive factors of learning mathematics. To help decide how learning could be improved, Johnston-Wilder et al. (2013) suggest increasing the learner’s voice.

In planning the interventions, it is essential that learners are given the chance to feel sufficiently supported and encouraged to enter the Growth Zone. Care must be taken to avoid pushing them too far into this zone because confidence and willingness to learn could decrease greatly if they enter the Anxiety Zone, limiting future progress.

## **2.4 Research into the effect gender has on mathematical success**

In this section, I consider the impact that gender has on mathematical success, most notably on levels of anxiety. Higher levels of mathematical anxiety are seen in females in the UK (OECD, 2015).

In Summer 2015, the percentage of girls gaining a ‘good pass’ at GCSE (grade C or above) was one percent ahead of boys (68% compared with 67%, Raiseonline, 2016). Similarly, the percentage of girls making expected progress (three levels of progress between the end of year six and the end of year eleven) was five percent ahead of boys (67% compared with 62%, Raiseonline, 2015). Thus girls now achieve more highly in mathematics compared to boys. Despite this, there is still a larger percentage of boys who continue studying mathematics beyond age sixteen, which is when mathematics was traditionally no longer a compulsory subject, compared to girls. Truss (2013) has found that even the girls who achieve well at GCSE are less likely to go on to study mathematics at A-level. This has an impact on all STEM subjects (science, technology, engineering and mathematics). Dunn (2014) found that eighty percent of girls who achieved an A\* at GCSE physics, which is strongly related to mathematics in content, did not go on to study physics at A-level.

As discussed previously, self-efficacy is the belief of a learner that they can achieve mastery to the required level in an area and impacts greatly on attainment in tests (OECD, 2015). Since having low levels of self-efficacy in mathematics can be equivalent to almost losing one year of schooling (OECD, 2015), girls’ attainment may be seriously compromised as research has shown that in general, girls have lower levels of mathematical self-efficacy than boys (Callan, 2015).

Conversely, enabling girls to build their self-efficacy with regard to mathematics may impact positively on a school's mathematics results. Ernest (2015) agrees with Callan but argues that the gap is perhaps reported as being larger than it actually is, due to the willingness of girls to be more honest about their feelings towards their ability to work with mathematics. Ernest's (2015) view leads to the conclusion that building self-efficacy may be vital for boys as well and may again lead to increased examination scores.

In Boaler's research (1997, 2013) on girls in mathematics, she takes an alternative viewpoint to a large part of the research in the twenty years preceding her study that discusses why girls underachieve in mathematics. Up until her study, a lot of research put the blame for underachievement upon the girls themselves. Attribution theory has been used to explain why girls may underachieve; putting the blame on the anxiety of the girls and explaining their own failure on their own perceived lack of ability. Many initiatives have been used to overcome this problem and improve the girls' achievement but Boaler believes the problem is not with the girls, but in the way mathematics is currently taught in schools. Ernest (2015) also believes one of the unintended outcomes of the school mathematics is that mathematics is seen as a male domain and is incompatible with femininity. Although gender gaps have now closed in attainment, girls continue to lack mathematical self-efficacy and self-concept, which he feels is why girls are not choosing to study mathematics beyond the compulsory school age.

Boaler (1997, 2013) became aware early on in her research that the approach to teaching in an exam driven setting was having a negative effect on the girls. The

boys adapted to this style of teaching by becoming competitive and seeing it as a challenge to get more questions correct than others in the group. The girls, however, found that this approach did not allow them to gain the understanding of the work they considered needful. They felt the pace was too fast at times and that they ended up being moved onto a new topic without getting to grips with the previous one. Scott-Hodgetts (1986) discussed how she puts learners on a spectrum ranging from serialists to holists. A serialist is someone who prefers to develop knowledge as a set of instructions that they build up one by one, understanding each as they go along, whereas a holist likes an overview of the ‘big picture’ to map out their learning. In the work of Chinn and Ashcroft (2007), they describe serialists as inchworms and holistic learners as grasshoppers. They claim that most learners’ preferences lie somewhere between these two extremes. As mathematical learning requires flexibility in thinking, they think a blend of the two is the most successful style for succeeding in mathematics.

Although Scott-Hodgetts (1986) conjectured that in general boys are holists while girls are serialists, it was the work of other researchers, such as Bevan (2004), who found this was generally the case. Baron-Cohen’s research (2003) has provided evidence that the average female brain is empathizing and the average male brain is systemizing. He describes empathizing as the desire to respond to someone else’s emotions with an appropriate emotion and systemizing as the desire to work out how something works and understanding why it works. These ideas, especially for the average male brain, link in with the work of other researchers, e.g. Bevan (2004), who have found that in general boys are holist learners and girls are serialists. In Boaler’s research (1997, 2013) the lessons in one school were very

much steered towards the holist way of thinking, with all techniques being explained at the start of the lesson and then spending the remaining part of the lesson practising the techniques. The girls at this setting said they preferred to work at their own pace either through self-study booklets or coursework, which exemplifies serialist behaviours, wanting to understand each step or technique before they move onto the next one. Scott- Hodgetts (1986) came across similar finding with girls excelling in their coursework compared with boys. Callan (2015) believes one of the key reasons for girl's disengagement with mathematics is that it is often taught as detached activities, unrelated to any social context to give them meaning.

Boaler (2013) compared how teaching styles affected learners' confidence in the subject at both of her research schools. When asked if they thought they were good at mathematics 6% of girls who followed an exam-driven curriculum responded yes, compared to 23% of those who followed an investigative curriculum. The boys responded 32% and 22% respectively. These results show that for this small sample, the open-ended approach greatly improved the impression girls have of their self-efficacy but the boys seemed to suffer in this type of environment. Research carried out by Carrington (2015) in which she interviewed year nine girls (age fourteen) and year twelve girls (aged seventeen) had similar outcomes with regard to the girls' choice of learning style. When speaking to the year twelve learners, many of them said that they would prefer to have had a more practical GCSE, which looked at how mathematics would be used in everyday life and that this would be more useful to them for their future.

Callan (2015) states that girls demonstrate higher levels of mathematical anxiety compared to boys (in the UK) even when the girls are performing well. Research carried out by Beilock et al. (2010) in the USA shows that girls taught by mathematically anxious female teachers adopt their mathematical anxiety whereas boys do not. In a similar way, mathematical anxiety can be passed on from mother to daughter (Beilock et al., 2010). To ensure that more girls opt to study mathematics beyond the compulsory age, strategies need to be considered to overcome mathematical anxiety and consequential mathematical self-exclusion.

While the interventions were planned and implemented in the research project, it was important to monitor carefully the needs of the female members of the group. Many of the females in the group are very successful elsewhere in the school curriculum but struggle to achieve similar grades in their mathematics. It was interesting to explore further why this is the case.

## **2.5 Research into teaching for mathematical thinking and understanding**

This section considers ways of developing a relational understanding to allow learners to develop their mathematical understanding.

In their work on mathematical thinking, Mason et al. (1985) describe four processes that underlie mathematical thinking and understanding: specialising, generalising, conjecturing and convincing.



Specialising is when learners turn to specific examples to help them understand what is happening. Mason et al. (1985) recommend using specialising when a learner is unable to proceed with a mathematical problem. Very often looking at actual cases of the problem allows a pattern to be spotted and thus enable a move to generalising. Generalising happens when a learner notices and expresses a pattern forming. Once generalisation has taken place, the learner can attempt to conjecture. A conjecture is a statement that seems reasonable but whose truth has yet to be established. Convincing is the step that involves the learner believing the conjecture to be true. This step can give rise to an argument that is often the basis of a formalised mathematical proof.

In Boaler (1998, 2013), at the school which followed an investigational approach to learning, the learners were reported to go through a similar process of mathematical thinking in their mathematics lessons. When it came to the final exam, they were able to use their mathematical knowledge and problem solving skills to explore unknown contexts and devise a correct solution. Learners from the other school, which followed a more traditional chalk-and-talk approach, found the problem solving process harder. Instead of looking at the problem and working out what techniques were appropriate, they tried the selection of techniques they had practised until one of them worked or they decided that none of them worked. There was no evidence to suggest they had used any of the four processes that underlie mathematical thinking discussed by Mason et al. (1985).

Mason et al. (1985) describe mathematical thinking as a developmental process. They suggest that the only way to improve mathematical thinking is for the learner

to practise and reflect on what they did and how. In lesson planning, teachers need to ensure that learners are given the opportunity to question and reflect, and provide ample time and space for this to occur. Wright and Taverner (2008) also emphasise the necessity to reflect. They describe the importance of being able to metacognize, which can be described as the ability to identify and evaluate strategies that have been used and use this knowledge to identify strategies to solve other problems in different contexts. With the pressures of delivering the curriculum in a given time, it is very easy for a teacher to become objective led and stop discussions once the objectives of a lesson have been met (Wright and Taverner, 2008). However, doing this results in the learners having insufficient time to reflect on what they have done and, as a result, not developing their mathematical thinking as far as they could have done.

The type of learning that takes place in a classroom is often dependant on the scheme of work being used. Swan (2005) discusses two main types of schemes of work. The first is a target-defined scheme which provides a list of contents put in an order with dates by which the learning should have taken place. The second type is an activity-based scheme. An activity-based scheme is an organised list of learning activities and problems, which are cross-referenced to learning objectives. Swan (2005) discusses how in an activity-based scheme of work there may appear to be less content covered but he found in his studies that the learning becomes more permanent as the mathematical knowledge is understood rather than imitated. This is the sort of understanding that Skemp (1987) refers to as relational, the type of understanding that Nardi and Steward (2003) suggest may be one of the ways of changing the learners' views that mathematics is TIRED.

Swan (2005) also describes three different methods of teaching, transmission, discovery and challenging teaching, which can result from the type of scheme of work used within the school. In transmission teaching, learners are taught through teacher-led examples followed by textbook questions. Generally, each method is explained one step at a time and learners are required to practise these methods on many similar types of question. In discovery teaching, the teacher presents tasks that expect learners to explore and discover mathematical ideas for themselves in an investigative way. Swan (2005) says challenging teaching has similarities to discovery teaching but describes the main features of challenging teaching as teaching that ‘emphasises the interconnected nature of the subject and it is ‘challenging’ because it confronts common difficulties through careful explanations rather than attempts to avoid them’ (Swan, 2005, p 4).

An idea publicised through the NRICH website to promote challenging teaching is the idea of ‘low threshold high ceiling’ tasks. McClure (2011) defines a low threshold high ceiling task as a mathematics task which everyone in a given group can access and begin to work on, whatever their previous experience or knowledge. Such a task also has many possibilities for the learners to complete more challenging activities. Similar to this is an idea of a rich task. Swan (2005) describes a task as rich if it is accessible to all but yet extendable to the more experienced or knowledgeable, allowing learners to make their own decisions. He says that a rich task needs to encourage originality and invention and prompt questions such as ‘what if?’ and ‘what if not?’ In summary, a rich task is a task that allows all learners to find something challenging to work on, no matter what

their previous attainment. Although these two types of tasks sound similar, McClure (2013) describes the main difference between the two as their starting point. She argues that rich tasks have a defined starting point but then a lot of different possibilities whereas in a low threshold high ceiling task there is not defined starting point.

Another key component in the development of learners' mathematical thinking is their ability to visualise. Cuoco et al. (1996) describe many areas of mathematics where visualisation can be used to develop mathematical understanding. These include geometry, visualising data and visualising change. The last example, visualising change, is a powerful tool to help with generalising, allowing the learner to develop the skills required to see what remains invariant under an operation.

Johnston-Wilder and Mason (2005) recommend three strategies to assist teachers in developing learners' visualisation powers. The first of those is 'say what you see'. This is when learners are given a geometric image or worked example and are asked to describe what they see. Someone describing something that others are unclear about gives the learner the opportunity to reinforce conjectures made and encourages learners to question each other to deepen their mathematical understanding. The second strategy given is 'same and different'. This is when learners are given two or more objects and are asked what they see that is the same and what they see that is different. This technique allows the learners to develop their noticing skills and assists them in making conjectures. The third strategy given is 'another and another'. This is when the learner is given some initial

conditions and is asked to give an example that fits those conditions. They are then asked to create a more challenging example and then another. This third strategy is also recommended by Bills et al. (2004).

Again, it must be emphasised that teaching that enables learners to develop their thinking skills, in the ways advocated by Cuoco et al. (1996), Johnston-Wilder and Mason (2005) and Bills et al. (2004) is not a process that takes place in one lesson. It could take years for both the learners and the teachers to become experts in working in this way. However, the more exposure learners get to developing thinking skills at school level, the more their Comfort Zone expands and the more prepared they are to take risks (Senninger, 2000). Ofsted (2012) backed up the need for teachers to emphasize problem solving across the curriculum and make more use of tasks and activities that foster learners' deeper understanding by using more practical resources and visual images.

Watson et al.'s work on deep learning (2003) focuses on learners who were low attaining. The aim was to look at strategies for these learners to make deep progress in mathematics. Watson et al. said that if these learners made deep progress they would learn more mathematics, become better learners of mathematics and feel more confident in being a learner of mathematics. These ideas link in with the factors of mathematical resilience (Johnston-Wilder et al., 2013). Watson et al. (2003) found that those in lower sets, similar to the research group, find it difficult to know when they are making progress with their learning. During the Deep Learning project, the research was focussed around teachers using a variety of different learning styles that resulted in the development of

helpful learning habits and as a result led to the learners making deep progress or becoming more mathematically resilient.

One of the aims of this action research project is to develop the learners' ability to tackle the problem solving questions on the GCSE examination paper. When designing the learning activities and deciding the role of the teacher- researcher, it was essential to consider the different strategies above. One of the factors discussed earlier for developing mathematical resilience is having an understanding of how to work at mathematics (Johnston-Wilder et al., 2013). Tasks were carefully designed to allow learners the chance to develop their mathematical thinking skills whilst being careful that the support they can access keeps them within the Growth Zone and out of the Anxiety Zone.

## **2.6 Research into the use of group work in mathematics**

One of the factors for developing mathematical resilience is about having an awareness of the support that is available from peers and other resources (Johnston-Wilder et al., 2013). Thus working with peers as pairs or in groups was going to be an important feature within this action research project. For this reason, it was useful to consider the literature focussed on group work in mathematics.

Wright and Taverner (2008) have found that in many subjects in UK, group work is commonly used as part of a lesson. However, in mathematics, group work is rarely used by many teachers. Ofsted (1999) said that in order to improve

mathematics further, schools should give learners adequate opportunities to discuss mathematics. More recently, Ofsted (2012) reported that in the very best lessons, learners were extensively working collaboratively with each other, using this learning opportunity to enhance their understanding of mathematics.

The theory of learning known as Constructivism emphasises that learners must take an active role in their own learning. The later literature on this theory grew from the work of Piaget and Vygotsky. Piaget (2001) conjectured that one of the main influences on child development was maturation, a term used to describe the changes that take place as a child grows older. He suggested that learning occurs in four stages: the sensori-motor stage (approximately birth to two years old), the pre-operational stage (approximately two years old to seven years old), the concrete operational stage (seven to twelve years old) and the formal operation stage (twelve years old and upwards). The last of these stages is most relevant to this research because in the school in which the research took place all the learners were fourteen years old or above. At this stage, Piaget (2001) states learners are able to see that their personal experience is only one possibility and they are able to generate systematically different scenarios for any situation. Essentially, Piaget (2001) is saying that as a child grows older, the tools they use to think extend, allowing them to have a different view of the world.

Muijs (2004) amongst others argues that although Piaget's theory has been highly influential, the stages given are far too rigid. He criticises Piaget for seeing learning as being largely dependent on their stage of development and does not take into account social interactions with other learners. Vygotsky (1978) also did

not think that maturation on its own was enough and his research led him to believe that children's development came through interaction with others of both similar intellectual levels and those of higher intellectual levels. One of his main ideas is that the learner has a 'Zone of Proximal Development' which was discussed in section 2.3.

Muijs (2004) felt that while Vygotsky's work filled in some of the gaps in Piaget's research, it lacked reference to the links between a child's natural development and the effects it had on their learning. He states that Piaget's research needs to be complemented by more recent research that has developed in the field of brain functions. However, Vygotsky's ideas have influenced classroom practice and the development of many of the ideas behind collaborative learning (Muij, 2004).

Based on personal experience, mathematics teachers have mentioned that they have found collaborative learning or group work to be an ineffective use of lesson time, stating that their learners have spent a large proportion of the lesson off task with only a few learners undertaking the work. Personal research has found that this is not usually the case in other subjects where learner routinely work effectively as groups, which results in the conjecture that this could be caused by a lack of certain skills in many mathematics teachers, potentially caused by their own school experience of mathematical teaching.

Wright and Taverner (2008) and Swan (2005) have both identified that many mathematics teachers consider collaborative learning as a possible problem and both give teachers similar advice about their role during group work. They both



emphasize the importance of having a clear purpose for the task. Learners must know what they are trying to achieve and must realise that it is often not the final answer that is important but the method used to reach the final answer. It is also important for teachers to listen to learners before any intervention takes place. Poor interventions could divert learners' attention away from what they are discussing and attempting to learn. This is true not only for group work but for mathematical learning in general. Any intervention by the teacher should be about asking learners to describe, explain and interpret what they are discussing and encouraging them and the group as a whole to think about their ideas in more depth. Kilpatrick (1987) showed that teaching needs to become more about using procedures to generate understanding. In this case, the carefully planned interventions are used to generate the learners' understanding and assist the learning process. Wright and Taverner (2008) report that poor intervention into the discussion from the teacher can often lead to learners stopping talking when they realise that the teacher is listening so it is vital when using group work to try and only 'eaves drop' on conversations at least some of the time.

It is also important to remember that a learner's ability to work in groups takes time to develop. Teachers need to devote time within their lessons to discussing effective group work and allow time to develop team working skills. A problem they may need to overcome is convincing learners that mathematics is not a subject where you always work independently, something that they may have believed for the majority of their time at school. Once learners are confident in working collaboratively when necessary it seems that they are more likely to stay

in the Growth Zone (Johnston-Wilder et al., 2013) by making use of available support to keep them away from the Anxiety Zone.

## **2.7 Research into developing independent learners**

One of the key intentions in my research phase was to develop the learner's ability to work more independently and become less reliant on the support of the teacher. For this reason, I looked at some research on developing independent learners, in particular Guy Claxton (2004)'s Building Learning Power model.

One of the difficulties learners face when moving on from school mathematics (Hoyles et al., 2001) is that they are not independent learners and rely on the input of teachers to allow them to further their learning. It is my understanding that this is seen across all subjects and not just mathematics. As part of the government's 'personalisation of learning' initiatives, Hargreaves (2005) proposed there were nine gateways to personalising education, one of these being 'learning to learn'. This gateway looks at developing the skills in learners of independent learning, something which may support another factor of mathematical resilience, the ability to make use of external support to aid learning.

To support the development of mathematical learners this research project is going to aim to embed elements of a 'learning to learn' model into daily mathematics lessons. Following a review of the different models available, I decided to make use of Guy Claxton's Building Learning Power model.

Claxton (2004, 2006) examined the research on 'learning to learn' closely and found that, although there were a number of models which schools could adopt to develop learning to learn, he was not convinced they were as powerful as they could be. He built upon the contemporary research and developed his own model for learning to learn, which he called the 'Building Learning Power' model (Claxton 2004, 2006, Chambers et al., 2004). Claxton suggests that to become a successful learner there are four key learning dispositions that a learner must master. These are resilience, resourcefulness, reflectiveness and reciprocity. These are broken up into a further seventeen learning capacities, which are designed to stretch and strengthen the 'learning muscles'.

Claxton (2006, 2010) suggests that resilience is about the learner being ready, willing and able to lock on to learning. He describes a resilient learner as a learner who likes a challenge and enjoys learning with understanding and who knows making mistakes is part of the difficult process of learning. A teacher can develop the skills that their learners need to become a resilient learner in many ways. The primary intention is to give learners the opportunity to 'get stuck' on a problem. Mason et al. (1985) and others state that learning does not begin until the learner becomes stuck. For many learners this is uncomfortable place to be and the most common reaction I see is for them to either give up or ask the teacher to tell them what to do. By providing prompt sheets of different strategies to use when 'stuck', the learners have ideas to hand to begin to deal with their difficulty. This approach is similar to that of Johnston-Wilder et al. (2013) who describe the importance of carefully planning the support of learners while in the Growth Zone. Over time, this process becomes habitual to learners, allowing them to take control of their

own learning more readily and easily. With practice, a learner develops a range of mathematical skills that allows them to move on from being ‘stuck’.

According to Claxton (2004), being resourceful is about having a range of techniques and attitudes for dealing with uncertainty. These include the ability to ask effective questions, making links between different situations, capitalising on the different resources available to them, imagining the possibilities and being able to think in a rigorous way. Teachers can encourage resourcefulness by recognising and rewarding good questions and answers given by the learners. They can also ensure the questions that they ask are designed to prompt their learners’ thinking. For example, asking questions starting with ‘how come ....?’, ‘what if ...?’ allows learners to access and use higher order thinking skills and explore topics in a greater depth. This again echoes the aspect of recruiting support in mathematical resilience described by Johnston-Wilder et al. (2013).

The next learning disposition is reflectiveness. This is about being ready, willing and able to become more strategic about learning. Learners need to have an understanding about how they learn in order to improve it. Developing this area requires learners to be able to plan their learning for each context and to adapt their plan if they encounter unexpected difficulties. Encouraging learners to think about possible difficulties in advance and writing a learning log can help learners progress. According to Claxton (2006), the learning log encourages the learner to reflect on learning processes and refer back to previous learning experiences to better tackle similar problems encountered in the future. In a similar way to Claxton, both Mason et al. (1985) and Wright and Taverner (2008) emphasises

the importance of being reflective when thinking mathematically. The use of journal to support the development of mathematical resilience is discussed further in section 3.2.4.

The last learning disposition is reciprocity. This is about being willing and able to work alone or with others. In order to become an effective learner, the learners should be able to decide when it is best to work alone or with others. When working with others, the aim is to listen to each other and pick up on the good habits of others, including the teacher. To develop the skills of reciprocity in a classroom, it is suggested by Claxton (2004, 2006) that the teacher asks the learners to develop a code of conduct for working as a group. Once this has been established, setting a problem that requires each group member to work on a different part of the answer together and then combine their work to come up with a full solution can encourage learners to develop skills in this area. The advice of Claxton on group work closely relates to that of Swan (2005) and Wright and Taverner (2008) although Claxton is keen to allow learners to choose whether to work independently or collaboratively depending on the situation. Mathematics is traditionally seen as a subject worked on independently; thus Swan (2005) and Wright and Taverner (2008) fear that learners will stick with the isolationist path they are familiar with if given the choice of independent or group work. They see learning both how to work together as a group in mathematics and the benefits that accrue from doing so as important rather than being given the choice of how to work. Based on Vygotsky's (1978) theories about the 'Zone of Proximal Development', learners become uncomfortable with the unfamiliar until they are given the support or scaffolding to allow them to develop the new skills.

One of the underlying themes of the Building Learning Power model is that teachers present themselves to their class as learners. One of the main objectives of the model is to create lifelong learners and so it is important that teachers are seen to be learning. Sharing their learning progress with the class and the problems they have to overcome is an invaluable resource for your classroom. If the learners see the teacher ‘getting stuck’, it can help change the common misconception that ‘being stuck’ implies you are stupid. Learners can see the adage that ‘being stuck means you are about to start learning’ modelled in the learning work of a respected teacher. Teachers sharing their own mathematical thinking skills with learners may give them the scaffolding to support their journey towards being able to think mathematically by allowing them to mimic the teachers’ skills.

Claxton (2008) discusses one of the biggest reservations teachers have, that using this model embedded into the curriculum instead of as a distinct lesson takes time away from curriculum content. With increased pressures from the government to boost results and published league tables, many feel that adopting this model would have a negative effect on examination results. Claxton disagrees with this idea and suggests using what he calls ‘split-screen lessons’ to allow teachers to do both. This is when the lesson is based on the usual curriculum content but the activities used to deliver the content allow learners to develop one of the seventeen learning dispositions.

The idea of split-screen learning objectives was adopted throughout the planned interventions. It was felt that a focus on this strategy helps support the

development of the learners to be able tackle problem solving questions more independently.

## **2.8 Conclusion**

Having considered the relevant literature, I reflected on the key messages to take into account when planning the research interventions. In order to develop learners who are more mathematically resilient, they need to be encouraged to leave the Comfort Zone and move towards the Growth Zone (Johnston-Wilder et al., 2013). I took care to ensure that learners were supported within this zone to avoid them entering the Anxiety Zone. If pushed into the Anxiety Zone, they risk an increase in mathematical anxiety, which reduces the progress they make towards becoming resilient. I carefully considered the strategies I used to support the learners in the Growth Zone; too much support encourages dependency but too little support increases the risk of entering the Danger Zone. The research indicates that working collaboratively and developing an active voice can help support learners work in the Growth Zone.

## **Section 3: The planning stage**

### **3.1 The Research questions**

The central research question for an action research project is “How do I improve my practice?” (McNiff, 2013). In this case, my central question is “How do I change my teaching practice so that my learners become mathematically resilient?”

Following my review of the field of literature, I developed the following research questions, which helped me answer my central question:

Will changing my practice to encourage more resilient behaviour enable the learners:

1. to develop sufficient confidence to work in the ‘Growth Zone’?
2. to use resilient behaviours during learning so that they improve their ability to answer examination questions correctly?
3. to increase their engagement in learning mathematics?

When planning this action research project, I aimed to find evidence to help me answer these three questions.

### **3.2 Methodology**

As a practising teacher, my focus over recent years has been on working with learners who are close to, but likely to just miss out on, gaining a ‘good’ pass in their GCSE mathematics examinations. Over the years, I have tried many different strategies to allow these learners to develop the skills and knowledge to raise their



attainment in the examinations in order to gain a 'good' pass. However, I am becoming increasingly aware that one of the key issues in them failing to achieve this grade is their inability to persevere when they encounter difficulty. I believe that overcoming this lack of perseverance and gaining resilience is the key to allowing them to successfully achieve a 'good' pass in their GCSE examinations.

In researching the different approaches that could be used to carry out this research, I came across the following two definitions of action research that I felt suited the nature of the research I wanted to undertake. McNiff (2013) says 'action research is always to do with improving learning, and improving learning is always to do with education and personal and professional growth, many people regard action research as a powerful form of educational research' (p24). Cohen et al. (2011) define action research as 'a small-scale intervention in the functioning of the real world and a close examination of the effects of such an intervention' (p226). For this reason, I decided to carry out my research as a teacher researcher using action research.

Although the model has been refined in recent years, this research project was based around the action research cycle which is summarised in Figure 6 below which was first described by Lewin in 1948. This is a cyclical model in that the outcome of one intervention impacts on the next. Trip (2005) suggests any action research project should begin with a reconnaissance. This stage allows the researcher to find out more about the starting points and needs of the research participants. The next step is to carefully plan the first intervention based on the findings during the reconnaissance. The intervention is implemented and

observations made of the outcomes. These observations can take a variety of different forms. Once these observations are made, the researcher reflects on what has happened and uses these reflections to plan the next cycle.

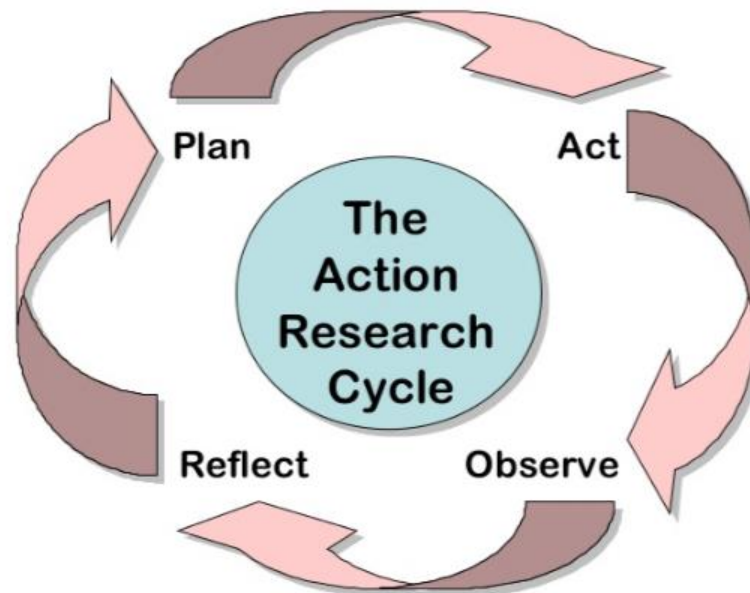


Figure 6: The action research cycle (Lewin, 1948).

Although I refer to the original model, the work of Kemmis and McTaggart (1992) expanded on the definition of action research describing it as a professional practice development tool, emphasizing the importance of all of the participants having a voice when undertaking the research, especially in the review stage. For the purpose of my research, this meant that when I reviewed the success of the intervention I needed to include a collection of the learners' own reflections.

Having decided on action research, the next stage was to decide upon the data collection methods to be used to gather evidence. I considered what data collection methods would give the greatest insight into what was actually happening in the classroom, whether changes are superficial or long lasting and whether learning is

actually improved. Thus, my decisions about whether to use a qualitative, quantitative or mixed methods approach depended on the research questions.

### **3.2.1 Making my research meaningful**

Since the millennium, educational research has been criticised internationally with regards to its usefulness, validity and relevance (Hartas, 2010). Oancea (2005) drew upon the vast number of written articles that criticised educational researchers from Europe and the USA to summarise and group the concerns people had. The most relevant finding is related to the methodologies that educational researchers use. It was felt that many methods employed were not reliable and were inconclusive due to lack of rigour. Concerns were raised over bias in interpreting results as well as concerns with the subjective qualitative methods being used without sufficient meaningful triangulation (Hartas, 2010). Creswell (2009) suggests that the mixed methods approach allows the researcher to use a combination of qualitative and quantitative methods to allow more in-depth analysis and triangulation of results, helping to address some of these concerns.

The nature of action research requires sufficient data to be collected to allow reflection on the impact of each intervention. The nature of my research questions required observations to be recorded by the teacher researcher, the opinion and views of learners to be captured, as well as data of learner performance in examination questions. For these reasons, a mixed method approach was the most appropriate methodology.

After researching the different research designs available, I felt that making use of a quasi-experimental research design was the most appropriate for this research project. One of the major advantages of using a quasi-experimental research design is that it allowed me to work with my own class within their normal timetabled mathematics lessons. Other research designs which rely on random allocation, such as experimental designs, would not allow this essential aspect. There are many different structures of quasi-experimental designs; some involve the use of pre- and post-intervention tests and observations and some involve using the idea of a control group. Since quasi-experimental research design does not use random allocation to these groups, the experimental and control groups are unlikely to be equivalent, so the control group in this type of research is called the comparison group (Hartas, 2010). When selecting the comparison group, I need to try to make it as similar as possible to the experimental group. For example, many researchers opt to study two parallel groups of similar ability. This however, may not result in an equal balance of other factors, such as socio-economic factors. Such differences can cause selection bias and make it harder to justify that any change seen during the research project is a result of the intervention and not a result of other factors that were not equivalent between the groups.

However, the fact that the research is taking place in a natural setting can result in better external validity and increase the transferability of the research because it has taken place in an actual classroom. The disadvantage of this setting is that it becomes harder to control external factors (Muij, 2008). For example, during the

time-scale of a research project, some learners in one of the groups may undertake some other form of intervention that is going on in the school. These factors are harder to control than they would be in an artificial situation such as a specially designed laboratory. If the experimental and comparison groups are within the same school, friends could share what they have been doing in their lesson and influence the progress being made by learners in the comparison group. This lack of control for external variables makes it less effective than experimental research for establishing causality. Validity, reliability and transferability of the research project is discussed in more detail later on.

I expect that over time learners improve without any additional intervention on top of their usual learning experiences. Therefore, in order to measure the additional progress made with the intervention I chose to use a comparison group. The research and comparison groups undertook the same pre and post testing or observation and experienced the same conditions of learning with the exception of the specific intervention. The intervention was only given to the research group. With all other variables being kept constant, I would then be able to suggest that any extra progress made by the research group was a result of the intervention.

It can be the case that, because participants know they have had some type of intervention, they improve because of a belief in the intervention. This is termed the 'Hawthorne effect'. This could increase their confidence in their ability and as a result, the outcomes of the post-intervention observation or testing. To combat this, Muijs (2008) suggests providing the comparison group with a 'placebo intervention' to help account for this factor. However, in education this can be

difficult to design as any 'placebo intervention' has unpredictable results. One way around this is to have two comparison groups, one that receives the placebo intervention and one that receives no intervention at all. This seems too complicated to use over a long time in a real school.

Although I had access to a parallel comparison group of similar ability as the research group, I felt that ethically I should not use them for a 'placebo intervention' or other comparison group. Working in an open faculty, we often share lesson resources and ideas and I could not guarantee that the methods I used were not used by the comparison group and would feel it unethical to deliberately deny a group access to anything their teacher feels would be helpful to them. However, I was able to use a similar group I taught last year as a comparison group. Although I do not have measures of how their resilience developed over the year, I have progress data based on benchmarked national tests. This class was taught using my 'normal' style of teaching so any significant improvement of progress in the research group is likely to indicate the intervention had an impact.

The interventions were planned to take place over an eighteen-month period, starting in the January of year 10, one term into their GCSE course. This was to allow me time to fully understand the learners' needs and the focus of the needs for the interventions. The interventions were spread across eighteen months, the timing of which was based on the topics being covered in the scheme of work and the needs of the group. As the class teacher for these learners, I taught the group for four hours per week. Whenever possible, lessons focussed on similar learning

skills to the interventions to allow the skills to become embedded. The research project finished just before the GCSE examination.

Throughout the research, the following methods of data collection were used to collect the evidence to enable me to answer my research questions:

- Past examination questions on problem solving in mathematics
- Learners own journals recording their observations and feeling from tasks (semi-structured)
- My own journal recording field notes from the planning and delivering the interventions
- Focus groups to follow up learner journal observations

### **3.2.2 Use of examination questions**

One of the desired outcomes of the action research was to improve the performance in GCSE assessments, specifically in the problem solving questions. One way the improvement was measured was by the progress made during the year on the learners' ability to answer GCSE questions.

Feldt and Brennan (1993) warn of four different types of threat to the reliability of the use of pre and post-tests. These are:

- Individuals
- Situational factors
- Test maker factors

- Instrument variables.

The first of these refers to the participants' motivation, concentration and their related skills. The learners in this group have regular assessments based on GCSE questions and are accustomed to the procedure and protocols of testing. Learners were aware that I was undertaking research but to reduce the impact of the Hawthorne effect I explained that it is part of our regular assessment schedule in the school. In line with school policy, learners who are entitled to extra time or the use of a reader and scribe in external assessments were entitled to this in the tests. This attempted to ensure that the outcomes of the tests were a measure of their mathematical ability and not their reading ability.

The second threat is situational factors. The tests took place in the classroom, conducted in the same way as previous tests to help minimise stress.

The third identified threat is related to test marker factors. To ensure consistency in the marks given to each learner on both tests, I marked both tests using the mark schemes and grade boundaries issued by the examination board, taking time to mark each test carefully to ensure as consistent marking as possible.

The final threat is related to the instrument variables. By making use of past GCSE exam papers tested by the examination board, this threat was reduced.



### **3.2.3 Use of learner journals to record feelings about the tasks**

As suggested by Claxton (2006), one of the key tools I used for getting feedback on the learner's feelings about a task was through recording this information in their own journal. Due to the importance of this tool, I discuss some of the key pieces of literature which have led to me choosing this as a research tool and supported its design.

The idea that writing can positively impact on learning stems from the Vygotskian view that language and thoughts can both be transformed in the act of representation (Vygotsky, 1962). Borasi and Rose (1989) suggest that 'Writing to Learn' is an approach that can be used to give a personalised and deeper understanding of mathematics. One form of writing that they research is in the use of mathematics journals. They describe this as a log or personal notebook in which learners can share their views and thoughts on a mathematics course.

Fried and Amit (2003) discuss the use of a learner's workbook in lessons, a book that is traditionally used to complete class tasks. They describe this workbook as a public document that contains finished work and can be inspected by teachers at any time. They conjecture that the lack of privacy in this workbook results in learners failing to reflect on their learning and reducing their ability to grasp the mathematical ideas. In their paper, they discuss the use of a learner journal, which is a private document where learners can record their own reflections and thoughts on the mathematics being studied. The authors argue that this journal should remain private to the learners to give them a place for uninhibited reflection. They

claim that as soon as the journal is viewed by a teacher, the learners feel that their ability is being assessed, which can result in them not engaging properly in the reflection process. Borasi and Rose (1989) share the opinion that a journal that gives learners the chance to reflect on their learning is an essential part of the learning process, but they argue that making it a public document that is shared between the learners and the teacher brings many benefits to both the learner and the teacher. One of the key benefits they discuss is the dialogue that can be formed between the learner and the teacher. The comments made may prompt the teachers to change their own practice by giving teachers a greater insight into the personalised needs of the learners. Borasi and Rose (1989) found that this increased knowledge of the learner could help develop relationships and break down some of the barriers to learning mathematics.

Although Borasi and Rose (1989) suggest that learners should have the freedom to choose what they write in their journals, they feel that being encouraged to reflect on the class work and recording their reflections and reactions to the topics being covered in lessons can improve the benefits of these journals. Borasi and Rose found when piloting the use of journals in their research that learners were not aware of what to write in a mathematical journal when it was left open-ended. To combat this, the researchers produced a sheet of prompts for the learners. The sample for their research project was first year undergraduates. Since I worked with younger learners, I felt that prompts were essential to encourage effective reflection.

I devised the prompts given in Figure 7. Learners were asked to answer questions one and two, and any other two questions, to allow choice of the most relevant questions based upon the way they progressed with the task.

Questions to consider	
1.	Describe what you did today.
2.	How do you feel about the problem?
	- What went well?
	- What would be even better if ...?
	- What did you learn?
3.	How did you go about tackling the problem?
	- Did you have a go? What did you try?
	- Did you carry on when others gave up?
	- Did you make suggestions?
	- Did you try to think of different things that might help?
	- What did you learn about solving problems?
4.	If you got stuck
	- What did you get stuck on?
	- Did you think you might have made a mistake?
	- How did you unstuck yourself?
	- How did you feel when you were stuck and how did you feel when you became unstuck?
5.	If you helped someone else
	- What did you help them with?
	- How did you help them?
	- How did you feel about helping?
	- How do you feel about the maths you helped someone with?

Figure 7: Learner journal prompt sheet

I piloted these prompts with a group of learners of similar ability levels and age to the research group. Learners were given an open-ended problem solving task to do, then asked to respond to some of the prompts. Afterwards their responses were analysed to check that their responses matched with the prompt given. I felt that

the questions used suitable language that was accessible to the learners and that their responses were in-line with the responses I expected.

During the action research project learners were asked to complete a journal entry on a regular basis including after interventions took place so I gained specific feedback on the task. I gave each learner a copy of the questions to keep in the journal to ensure they had easy access to the prompts. Following each entry, I considered a selection of journals and responded to any questions and concerns they wrote down. I used these to inform the planning of future tasks.

The journals remained private between the learners and myself. I made the learners aware that, unlike their class book, the journals were not included in my assessment of their mathematical ability. In line with ethical guidelines, learners had the option to opt out of having their journals used as evidence as part of my research project, but they still needed to undertake the activity as part of their timetabled lessons.

#### **3.2.4 Use of group interviews**

Groups of learners were invited to a group interview to give them the opportunity to expand on their views and allow me to ask probing questions to find out more about their views, behaviours and progress. These took place at different times during the research project.

As part of the in-school monitoring and evaluation procedures, groups of learners were regularly interviewed to get their views of different aspects of the school. The familiarity of this type of group discussion, as opposed to one-to-one interviews, which are generally used in school for investigating poor behaviour, make it more likely that the learners gave honest responses in this situation. During these group interviews, I had a set of questions to ask, based on the recent tasks covered in class, my own observations of the learners participating in the tasks and feedback from learner journals. Thus the questions varied between the different interviews.

Alongside these formal group interviews, I conducted informal interviews during observation of the learners completing the tasks. This allowed me to get immediate views of learners' thoughts and gain a deeper understanding of the processes they are following.

Notes were taken during these interviews to record the data that were elaborated on immediately after the interview to provide more detail about what was said and any relevant information from their body language. These notes were analysed for key themes.

### **3.2.5 Use of researcher journal to record observations**

When carrying out survey research or interviews, it is assumed that the views given by the respondents are consistent with their actions. However, this is often not the case (Hartas, 2010). This difference can affect the validity of a survey and

lead to response bias. To counteract this, I triangulated the findings from the interviews and learner journals with observation research that were carried out when learners were taking part in the interventions.

Observation research allows the researcher to watch the research participants in their natural environment (Muijs, 2008). The benefits of this are it allows the researcher to make their own decisions about the participants' actions, which can lead to greater consistency across different participants. With there being a wide variety of observation methods, the researcher has more flexibility and is able to focus on areas of interest. As a result of the observations taken place in a natural environment, it is easier to transfer the findings to other similar settings.

Cohen et al. (2011) suggest one of the main drawbacks of observational research is the Hawthorne effect. With the observer being a teacher researcher and observations taking place in timetabled lessons, the impact of this effect on the findings was minimised. However, being a teacher researcher does mean that during these interventions I carried out more than one role and thus my time was split between ensuring the learners are able to make progress and observing and recording the behaviours and discussions in the room. To reduce the impact of this, the tasks were carefully designed and structured to require minimal teacher input.

The styles of observation taking place depended on the type of intervention. These included descriptive observation records, behaviour counts and time sampling. I recorded observation notes during and after the intervention for analysis.

Alongside this, I kept a reflective journal on how I perceived the success of the intervention. This helped me plan and design further interventions to meet the needs of the learners.

### **3.3 Validity of the research findings**

Hartas (2010) suggests that there are four main types of validity commonly examined in educational research. These are internal validity, construct validity, external validity and ecological validity. Although many of these have already been addressed in section 3.2, I shall briefly look at each one now.

Internal validity aims to show that the findings from a piece of research is actually shown in the data (Cohen et. al. 2011). The interviews and learner journals helped me explore further what was observed and thus achieve high levels of internal validity.

Construct validity is related to whether the test or measurement tool effectively measures the construct it is meant to measure. To minimise threats to this form of validity I have ensured I have carried out a thorough literature research into mathematical resilience and have used tested tools to measure it.

External validity relates to the ability to recreate the results of the research in another setting. This research is action research involving only one class. The interventions were developed around the needs of this group. However, as we have seen in the literature review, the problems faced by this group are commonly faced

by learners across the world. During the write up, I included details of the interventions used and how they were implemented to allow others to replicate this study.

Ecological validity is related to how much the findings from the research reflect people's everyday experiences and their views. By undertaking this project as action research and ensuring the learners' views form a large part of the review stage, I developed their interventions based upon the views they shared with me. I carried this out in a usual timetabled lesson, which helped reflect their normal school day experience.

To ensure the reliability of the findings, I used a variety of measurement tools to allow triangulation to take place in an ethical and practical way. The nature of the research and the methods being used were carefully selected to minimise the impact of the Hawthorne effect, which is a common threat to validity and reliability in education research (Cohen et al., 2011, Hartas, 2010).

An important part of any action research project is to ensure transparency and transferability (Popplewell and Hayman, 2012). During the reporting of the action research project, I ensured that I gave sufficient information about the participants, the interventions and the outcomes to allow others to transfer a similar action research project to their own setting.



### **3.4 Ethical considerations**

Lindsay (2010) suggests there are four ethical principles related to educational research, which I used as a starting point for considering the ethical implication for my research project. They are

- The principle of respect for a person's rights and dignity
- The principle of competence
- The principle of responsibility
- The principle of integrity.

The first ethical principle is focused on the participant. It covers the researcher giving appropriate respect for the participant's fundamental rights, dignity and their right to privacy, confidentiality, self-determination and autonomy. The second ethical principal is related to the competence of the researcher. This principle suggests that researchers should recognise the boundaries of their work and work with techniques in which they have been trained. The third ethical principle is related to the professional and scientific responsibilities that the researcher has to the participant, the research community and the society in which they work. It states that researchers should avoid doing harm and take responsibility for their own actions. The final ethical principle is related to the responsibilities that the researcher has to the research community. It suggests that researchers seek to promote integrity in educational research and are honest, fair and respectful to each other.

As a teacher researcher, my main priority had to be ensuring that the learners are making progress in mathematics; each intervention was designed with the goal of improving their mathematical learning. Lack of resilience and confidence in their ability to progress had been identified as an issue that was holding my learners back by their current and previous teachers and so undertaking the research could be justified as necessary to ensure their future success.

I informed all the participants that they were involved in an action research project and an explanation about the aims of the research was shared. Due to the nature of this action research project, all interventions took place in the normal timetabled lessons. This means that learners were not able to opt out of the intervention or journal recording but they had the right to request that their data not be used as part of the research and to not take part in the interview groups.

To ensure the confidentiality of the learners, no names were recorded on any formal documentation or records. Although their learning journals contained their name, records of the names were destroyed after the data has been collected from them.

As discussed previously, I did not use another class as a comparison group and instead used the results for a group of similar ability but did not take part in the specific interventions this group will have done. This allowed discussion about the research and resources to be shared with colleagues teaching similar groups.

Ethical approval was obtained from the University of Warwick for this action research to take place (appendix 1) following a discussion of the research methods and procedures being used. All work was regularly shared with my supervisors who worked with me to ensure that I worked within my competences.

### **3.5 The research group**

The research group consisted of twenty-seven learners, sixteen females and eleven males, all aged fourteen or fifteen (Year Ten learners). Sixty-six percent of these learners are defined, under Department of Education definitions (Raiseonline, 2015) as low attaining learners in mathematics based on their end of Key Stage Two National Assessment scores, an examination taken at the end of their primary education. The remaining thirty-four percent are defined as middle attaining learners. One learner has a statement of special educational needs, six learners appear on the special education register at the school action level (mostly due to dyslexia, low reading age and poor numeracy skills), one learner has English as their second language and one learner is a traveller, resulting in low attendance to lessons. Six learners in the group are eligible for free school meals, a factor which, based on national examination results, is said to link to underachievement. A government initiative is in place to close the gap between those eligible for free school meals and those who are not, which provides an additional nine hundred pounds of funding per learner.

Based on guidance issued from the Department of Education, these learners will be on the borderline of a ‘good’ pass in their GCSE examinations if they make

‘expected progress’. For this cohort, a grade C is the expected standard for a sixteen-year-old learner and is the minimum grade required for many courses beyond GCSE level. For these reasons, combined with accountability measures that the DfE has in place, which the school must be aware of, the mathematics department are under pressure to ensure that these learners achieve the grade C and not a grade D.

### **3.6 Analysing my Data**

Throughout my analysis, I strove to make the research transparent. Eddy, Hollingworth, Caro, Tsevat, McDonald, and Wong (2012) described transparency as ‘clearly describing the model structure, equations, parameter values, and assumptions to enable interested parties to understand the model’ (2012, p733). According to Tong, Flemming, McInnes, Oliver and Craig (2012), there are five areas in which transparency can be achieved, ‘introduction, methods and methodology, literature search and selection, appraisal, and synthesis of findings’ (2012, p183). I intend to give enough information to enable understanding of the ‘accuracy, limitations, and potential applications’ (Eddy et al., 2012, p734) of this research.

The analysis happened in two phases. The initial analysis happened after each intervention when I reflected back on what had happened and made plans for future interventions that would address the issues that I uncovered. This level of analysis was recorded in my field notes. The second phases of analysis began after

some of the interventions were completed and continued after the end of the eighteen-month intervention period.

In the second analysis phase, I first constructed a narrative of the interventions using my field notes and the learners' journals. My intention in this activity was to make plain what happened and to give a thick description (Geertz, 1977) allowing the meanings that I ascribed to the data to be transparent. However, I also recognise that forming the narrative is a process of analysis in itself, in which I decide what is important to include and what can be left out. I therefore made a conscious effort in this process to present as full a narrative as possible, given word length restrictions, and to include data that reflected less well on my professional practice and that I may have preferred not to have happened. In my design of the narratives, I kept in mind the need for my reader to trust that the lessons happened in the way I described, thus making the meanings ascribed trustworthy.

I then examined the narratives constructed from the data and applied thematic analysis. 'Thematic analysis is a method for identifying, analysing and reporting patterns (themes) within data' (Braun and Clarke, 2006, p.79). The codes I used defined the nature of the data and gave some indication of how they are related to the research questions and to each other (Braun and Clarke, 2006). Since the data comes from an extensive potential sphere, there was no importance placed on the prevalence of any or all of the coded data (Braun and Clarke, 2006). This type of analysis could also support discovery of additional features that had not been previously considered.

### **3.7 Conclusion**

To summarise, I undertook an action research project making use of a mixed method approach based on a quasi-experimental design. In order to produce a transparent analysis, I collected data from each intervention using a variety of methods to allow triangulation to take place. All interventions involved teacher researcher observations and discussions with learners, which were recorded in my field notes and the use of learner journals. When appropriate, learners were given questions to complete from past GCSE papers. This allowed me to compare their outcomes with the outcomes of the comparison group. On top of this, interviews allowed me to further explore observations made or comment written by different learners. By collecting data using a variety of methods, I was able to triangulate results to maximise the reliability of my findings.

## **Section 4: The teaching episodes**

In this section, I initially discuss my findings from the reconnaissance stage before discussing the key action research cycles that took place during the research phase. Following the initial reconnaissance or the outcomes of the previous intervention, the next cycle was planned, undertaken, analysed and reviewed before the planning of the next cycle took place. The outcome of one action research cycle affected the development of the next. For each action research cycle, or intervention, I shared the objectives for the lesson, the lesson activities and the rationale behind the design of the activity followed by a description of my observations and the feedback given by the learners in their journals. A full summary of the findings of each action research cycle can be found in section 4.11.

### **4.1 The reconnaissance stage**

As discussed in section 3.5, I had been teaching the research group for a term before I started the cycles of interventions for my action research. The aim of waiting a term before starting the study was to give me the opportunity to gain an insight into what barriers to learning I needed to address through the interventions to support these learners. The first term was my reconnaissance stage.

In line with school policy, the research group sat a baseline assessment during their second week of year 10 at the school. This baseline assessment was a GCSE

examination paper from a previous year. I marked these papers using the official mark scheme issued by the examination board and analysed them to get an insight into their current mathematical attainment. Looking at the marks achieved for different topics, I found that particular of strengths of the group were addition, subtraction and multiplication of positive integers, calculating simple probabilities as fractions and interpreting simple bar charts and pictograms. The topics for development included algebra (all elements), percentages, fractions and area. I also looked at the different styles of questions, and found that the learners were more successful with the short one or two mark questions, especially those that made it easy to see what they were required to do. Those questions that contained a lot of text or involved multi-step problems were not answered well and often were not attempted. When I spoke to some of the learners afterwards, they said that they did not attempt a question if it looked difficult. During a discussion with one of these learners, I was able to get them to correctly explain the steps involved in solving such a question verbally even though they did not attempt it.

During these initial weeks, I also asked the class to complete a mini-investigation based on investigating how many squares were on a chessboard. Although many learners could see that there were more than sixty-four squares, I observed that they found it difficult to get an accurate answer for the number of squares because they were approaching the problem without a plan. This resulted in them missing many squares and counting some twice. When they did get the correct answers, the majority did not present their work in a way that aided them in finding a pattern. From these observations, I knew that I needed to focus on approaching



investigations in a systematic way and encouraging them to present their findings in a format that would support them in generalising their findings.

The second term started with a focus on number work. During this unit, I saw many examples of student lacking the confidence to persevere when the structure of the question changed. For example, when looking at column subtraction they were able to confidently subtract two positive integers of any size but as soon as I asked them to use the same technique with decimals, they were reluctant to attempt that question because they said it looked different. During the discussions I had with them following this, they mentioned that they did not want to attempt the different question in case they got it wrong. I came across more evidence of this happening during the remainder of the first term.

Following the reconnaissance stage, I found that this particular group of learners lacked confidence in their mathematical ability and preferred to remain within their comfort zone, focussing on routine questions that were all very similar. As soon as they came across something that looked slightly different, they felt as though they could not do it, despite discussion with me afterwards revealing that they could do it. I also observed that they were poor at reading the information given to them in questions and that they would use avoidance techniques to avoid having to work on questions they felt looked challenging. The only way I saw them accessing support when stuck was by asking the teacher for help with the hope that the teacher would do it for them. During discussion with learners, I also found that they did not see mathematics as a subject that develops skills they will

require for their future. I aimed to address these barriers to learning through the interventions.

During this stage, I also spoke to the learners about their learning in their previous school. They revealed that their previous education in mathematics followed the same routine. Each lesson began with the teacher introducing the topics followed by a number of examples. Following this, they would work from a textbook or worksheet attempting many similar questions, progressively getting more difficult. Each question was similar to that of the example. Although the textbooks often had problem solving questions in each exercise, the learners told me that they often missed them out 'because they looked difficult'. This style of lesson is what I witnessed when I went to visit them in their previous school as part of our transition visits. Observations during the reconnaissance phase indicated that the learners had become encultured into practices of rote learning, procedural competency and the idea of there either being a correct or wrong answer. In order to address their lack of resilience I needed to carefully consider how I would change this culture of learning.

#### **4.2 Intervention one: plotting coordinates**

The first intervention lesson was based around plotting coordinates in all four quadrants. For this lesson, split-screen objectives were used, as explained in the literature review section 2.8. One objective related to the learning of a mathematical technique and the other related to a Building Learning Power outcome. For this lesson the mathematical objective was to be able to plot

coordinates in all four quadrants and the Building Learning Power objective was about being resourceful, with the emphasis on carefully reading instructions and using different support mechanisms when stuck and therefore not relying on the teacher telling you what to do. These objectives were based on what I perceived to be some of the barriers to learning found during my initial reconnaissance. I had briefed the learning support assistant in advance to not to give direct support but instead to ask questions designed to encourage the learners to think about the different ways they could access support.

Following the sharing of the lesson objectives and a discussion about what the learners were being asked to achieve, I issued each learner with written instructions and a number of questions with a list of coordinates. The instructions were as follows:

‘Plot the points below and join them up in the order shown. Where you see a gap or begin a new line, start a new part of the shape. Do not join this to the previous coordinate.’

These instructions were read out to the class before the task began and I reminded the learners about what they could do if they got stuck. Following these instructions, learners were asked to complete the questions in any order. An example question is given below in Figure 8.

1).  $(-4,-2) (-1,2) (-1,1) (0,2)$   $(-6,6) (-6,5) (-5,5) (-5,4) (-3,4) (-3,2)$   $(4,10) (6,7) (6,6)$   
 $(2,7) (3,7) (3,8) (2,7)$  - shade in.  $(-1,-2) (-1,-5) (1,-3)$   $(-1,6) (-2,4) (0,5) (4,3)$   
 $(-3,-9) (2,-9) (0,-7)$   $(4,5) (5,6) (5,7) (4,8) (4,5)$   $(-2,0) (-1,0) (-1,-1) (-2,0)$  - shade in.  
 $(2,6) (2,5) (3,5) (3,6) (2,6)$   $(-6,6) (-5,6) (-5,5) (-4,5) (-4,3) (-2,3) (-2,2) (-3,2)$   
 $(2,6) (3,6) (3,5) (2,6)$  - shade in.  $(-2,0) (-2,-1) (-1,-1)$   $(0,-7) (2,-5) (3,-2) (0,2)$   
 $(4,10) (0,8) (-2,9) (-1,7) (0,6\frac{1}{2}) (-1,6)$   $(2,7) (2,8) (3,8)$   $(-4,-2) (-3,-5) (-1,-7) (-3,-9)$   
 $(4,3) (6,6)$   $(1,8\frac{1}{2}) (1,10) (3,9\frac{1}{2})$   $(1,4\frac{1}{2}) (1,3) (3,3\frac{1}{2})$ .

Figure 8: An example question used in intervention one.

Within two minutes of starting the task, three learners had their hand up asking for help. They all claimed they did not know what they had to do. On speaking to the first learner, it became clear he knew how to plot the coordinates but did not know what he had to do once he had plotted the points. When I suggested that he read the instructions at the top, his response was ‘why can’t you just tell me what to do?’. I asked him what else he could do if he was stuck and he eventually said ‘ask a friend’. He did this and was able to continue but did so stubbornly and without his usual pace.

The other two learners had the same problem. They held the view that it was the teacher’s job to tell them what to do and it was not their job to read the instructions. One learner commented that if I did not tell them what to do then they would not do the task. This attitude towards their learning was also evident in the comments made in their learner journals that each learner completed afterwards. One learner wrote ‘The teacher would not help me when I got stuck and made me ask another student. I wasted lots of time waiting for help so I didn’t get the picture finished. The worksheet was boring and a waste of time because I will never need to use coordinates in real life.’ In this comment, I saw the learner making use of avoidance techniques, perhaps to shield his uncertainty over how to cope when

outside his Comfort Zone. He was clearly annoyed at not getting the task completed and tried to pass the blame onto someone else. There is also evidence that he did not find the task useful because it seemed to him to lack any obvious real life relevance and he indicated that as a consequence he became demotivated in completing the task. The lack of real life applications is something that Nardi and Steward (2003) suggest is adding to learners' dislike of mathematics.

One observation I made from this comment that supported me in my future planning of interventions was that I needed to look at ways of managing the change of expectations of these learners. Their past expectation of being able to ask the teacher for help when they became 'stuck' was no longer the way I wanted them to operate.

The learners' journals indicated that many learners appeared to enjoy the challenge of the task. One commented that 'the rules on the sheet made it a little confusing' but 'working with a partner and discussing the task made it easier and helped us understand more and we could compare our work and see where we had done wrong'. My observation records indicated that I did see learners working together and being proactive by dealing with problems themselves instead of asking an adult for help.

By monitoring the requests for help, just over eighty percent of the class were seen to move away from relying on teacher help and instead using peer support when required. Having this as a lesson objective appeared to make a difference. One area that I felt was missing in this task were the links to real life applications of

mathematics, as mentioned by some of the learners in their journals. The initial reconnaissance indicated that the majority of the class did not see the relevance of mathematics in life so this is something I ensured was more evident in the next intervention.

### **4.3 Intervention two: planning a trip**

Based on my reflections from the first intervention, I wanted to make sure the second intervention was based around a real life application of mathematics. The mathematical learning objective was to carry out calculations involving time and reading information from timetables. The Building Learning Power objective was about selecting the most relevant resources available to solve the problem. These resources would all be available from the internet. I told the learners that they should work in pairs and should make use of each other's skills if they became stuck. I felt that this task would give me the opportunity to observe the levels of perseverance that the different learners demonstrated.

Following the introduction of the objectives, I explained what I expected them to achieve and set them the following task.

‘On the Wednesday during the half term holiday the learning support assistant and I would like to travel to London to see a matinee performance of a musical. We will be traveling from school using public transport. Following the show, we would like to have dinner before returning home. We need a schedule for the day,

telling us how we are going to get there, the times of transport and details of how much it will cost us’.

For this intervention I told the class that I would be observing what they were doing and would be ‘less helpful’, which means that I would not be available to answer questions. The Learning Support Assistant was briefed only to answer questions on difficulties related to the mathematics, for example not knowing how to read a bus timetable.

I decided to focus my attention for the first part of the session on a pair of boys, one of the pair being the learner, Tom, who commented that the first intervention lacked relevance to real life. The other learner, William, was a learner who I felt demonstrated low levels of self-concept. The pair randomly decided we would leave the school car park at 11am and by referring to the bus timetable online, identified the bus we needed to catch to the station. Both learners were confident in reading the online timetable.

Their next step was finding a website that displayed train times. Tom said he had used this website before but William was unsure how it worked so Tom supported him. Tom was smiling and talking with confidence when helping William. He later said that he often got the train to London to visit his uncle and often helps with planning the journey. Following the previous negativity from Tom, I was pleased to see that he was engaged with this task. I surmised this may be due to the task relating closely to experiences he had in his life and he was confident that he could successfully complete it. He later commented in his journal ‘It was good

planning a trip to London. It was something that I have done before although I am not sure what it has got to do with maths. William didn't get it but I was able to tell him what to do.' His comments link in with the findings of Nardi and Steward (2003) and Goodall et al. (2016) who suggest that learners are more engaged if they see the relevance of the task. Although he was progressing confidently through the activity, I found no evidence of him being challenged by the task.

Once they had written up the first stage of the journey, William asked Tom where about in London they needed to get to. At this point, Tom said he did not know and put his hand up to ask me. I reminded them of the name of the musical and they looked up the venue. On looking up the venue they saw that it started at 2.30pm with doors opening at 2pm and realised that they had not left enough time to get from the train station to the theatre. Both learners reacted badly to this discovery with Tom commenting 'I don't know why we need to do this stupid task anyway, clearly they aren't really going to see it.' William quickly calmed down and realised that it would not be a difficult task to change what they had already done. He suggested that we took the earlier train and get an earlier bus to the train station. Tom was still not happy and left William to amend the travel plans to get to London. This reaction from Tom showed a relative lack of resilience. As soon as he made an error when working in the Growth Zone, he became critical of the task, perhaps as a way of masking his perceived failure. This type of behaviour is predicted by Maslow (1987), who says a learner does whatever they can to avoid threats to their personal self-esteem; in this case, Tom was avoiding these threats by refusing to do more work.



The next stage of the journey involved detailing how to get from the train station to the theatre using the underground. William brought up a map of the underground and asked Tom where the train station was. He responded with 'I don't know'. William spent a few minutes looking at it while Tom ripped up a bit of paper that was lying beside his computer. William was struggling so he searched online and found a website that planned the route for him. He copied this route across to his document without speaking to Tom.

Following the discovery of an error, Tom no longer wanted to complete the activity. He went from being positive and enjoying the task to seeing it as pointless. I spoke to both of them at this point. I asked them what problem they had come across. William, who had lacked confidence at the start of the task responded and stated 'We realised that once we got to London we didn't have enough time to get to the show. We had to change the times of the train and the bus to make sure we arrived on time.' I asked him if he would do it any differently next time and he responded 'we should have found out what time the show started and then found the best train and did the bus bit last.' I then asked Tom directly how they corrected the mistake. His response was 'I don't know; I don't get it. I don't see how doing this is going to help me with my exam', suggesting that he had entered the Anxiety Zone. The slightest hint that Tom 'had got it wrong' seemed to have destroyed both Tom's confidence and good mood. He was enjoying the task and progressing well until he realised he had made a mistake. From this point onwards he became annoyed with the task, blaming the mistake on the task. Even though the mistake was quickly fixed by William, Tom wanted no further involvement in the task. Writing in his journal the following lesson, he

was able to reflect more rationally on what had happened. He commented that he was enjoying the task, especially being able to use his past experiences of planning and doing similar journeys to help someone else. However, once he found out he made the mistake he wrote 'when I saw we had done it wrong it was too much effort to go back and fix it. I just let William do all of the work'.

William, however, reacted in a different way. As a learner who has demonstrated that he had low levels of self-concept, he often asks for help due to him wanting reassurance that what he is doing is correct. This lack of confidence came across in the initial stages of the task when he asked Tom many questions and let him lead the task. Following the discovery of the error, William changed role and became confident in leading the task when Tom was refusing to get involved. William commented in his journal 'I found the task tricky at first but then it got easier. Tom had to help me with getting the websites but once I knew what to do it was easy. We made a mistake ... but it was easy to sort'. William is an able learner who often demonstrates good progress but I feel he relies too heavily on teacher support. Tom is also a hard working learner who attempts any task given to him. Getting a good grade seems to be important to Tom and he always does his best to complete a task, using the quickest way possible. My observations of him indicate that he aims for what Skemp (1988) describes as an instrumental understanding of the concepts being studied; he wants to know how to do something not why it works. With a little bit of support from Tom, William showed that he was able to persevere when things went wrong. This little bit of support allowed him to remain within the Growth Zone until he reached a solution.

For the second part of the intervention I observed two girls, Jane and Paula. Both are middle attaining learners who behave well in school and attempt every task given to them. Despite this, their levels of self-efficacy and self-concept appear to be low and, like a large proportion of learners within the group, they both often ask for reassurance that they are doing the task correctly.

When I started the observation, they had just completed their route from the train station to the theatre using an underground map. I asked them how they found the task so far. They commented that the activity was easy once they found the correct website to help them. In describing their work, they mentioned in their own words that they had read timetables, carried out calculations involving time, made estimations when deciding how long to allow for lunch, interpreted a network (the underground map) and scheduled activities. When I asked them what mathematics they had been using they said 'none'. They did not see how this task related to the topics we had been studying as part of the GCSE programme of study. It was only through further discussion that they realised they had been using a wide variety of mathematical skills.

Through the discussions that took place with the learners, it appeared that many were unaware that they were using mathematics, perhaps because it was hidden in a real-life context. To verify this theory, I asked each pair to print out their schedules and list at the bottom what mathematics they had used that lesson. Over ninety percent of the pairs managed to list time as a topic but only three pairs (out of twelve) listed reading timetables. No other topics were listed. When I looked at the learner journals, over ninety percent said that they enjoyed the activity with

many of those saying the reason behind this was that it related to something they may need to do in the future. It was clear they could see the relevance of completing such a task. Four of the learners commented on the activity being enjoyable but they were concerned they were not doing 'proper' mathematics. One learner wrote 'Planning the day out was good fun but we have mock exams in a month so we should be doing proper maths. Doing things like this would be good for the last week of term'. After reading this journal I spoke to this learner to get a deeper understanding of what he meant by this comment. I asked him what 'proper maths' was and he said 'proper maths' was when he did questions out of a textbook. I asked him if it was only questions out of a textbook that were proper maths and he said that doing jigsaws were fine but when we do activities where they need to discover the method for themselves, instead of them being told the method, they waste time that could be spent doing the questions. I asked him if he found it difficult when I set them tasks like this and he said a little but he gets annoyed with them because he can sit for fifteen minutes without making any progress. Again there is evidence to suggest that this learner has a lack of resilience. He wants to do well in mathematics but he does not like it when he becomes 'stuck'. It appears that he does not have experience of how to carry out mathematical investigation and lacks strategies to help him when stuck. He is also showing his motivation to work is extrinsic; he is interested in passing the examination, not gaining an understanding of the mathematics. Another aspect of this that provides evidence of resistance to a cultural change is that he displayed learned helplessness. He wanted to be told how to carry out a particular process, rather than working out the process himself, which he feels is wasting his time.

During this task, the majority of learners were seen to be engaged in their learning and I was encouraged to see them bringing together the different mathematical skills we had been studying. By looking at their work, the first objective related to the mathematical content was met. Selecting appropriate resources online was expected to have presented a challenge for the group, which is why it was set as the Building Learning Power objective. However, this did not seem to be a challenge for them, with no learners commenting on this objective in their journals or any observed difficulty in selecting a suitable website.

The main observations of negativity in this task were that when learners got stuck they demonstrated a lack of perseverance; one setback moved some learners from being within Ernest's success cycle to being within his failure cycle. For the next intervention, I planned to explore this idea of learners 'getting stuck' and starting to explore strategies for helping them to become 'unstuck' and persevering when faced with difficulty.

#### **4.4 Intervention three: drawing straight line graphs**

The third intervention was based around an investigation into drawing straight line graphs. Based on result analysis provided by the examination boards, this is a topic many learners in the past have found challenging.

For this intervention I wanted to achieve two things on top of the curriculum objective. The first was to begin to develop strategies for how to deal with 'getting stuck' and to encourage the learners to persevere when they encounter difficulties.

The second was to encourage them to spot patterns and use this to help them reach a solution.

The intervention started with a brainstorming session on the different strategies that could be used if ‘stuck’. After I discussed the learning objectives, I explained I would present a solution to the question ‘Draw the line  $y=2x-1$  for  $-3\leq x\leq 3$ ’ on the board but that there would be no explanation about what I had done. They would need to analyse my solution and look for patterns to enable them to answer similar questions in pairs. After the task I said we would return to the list of strategies created for dealing with being stuck to see if we could expand it. I made it clear that this list was intended to be used to support learners in the future.

Asking the learners to brainstorm different strategies for dealing with ‘getting stuck’ was carried out using the ‘think, pair, share’ technique (Wolff et al., 2015). This technique was used to encourage learners who lack confidence in sharing their ideas to discuss them with a partner. Initially learners have a couple of minutes to think about their response on their own; they then have a couple of minutes to discuss ideas with a partner so they can practise sharing their ideas and get some feedback before sharing with the whole class. Having observed that a large proportion of the group lacks confidence in their mathematical ability, I have used this technique frequently and seen it improve the number of learners who are willing to volunteer a response to questions asked to the class.

When I asked the class to share their views the first response was ‘ask the teacher for help’, which was closely followed by ‘ask a friend’. When I asked if anyone

had anything else to contribute no-one volunteered. At this point, I asked them to think about ways they could help themselves when they were stuck. After a short time, one boy suggested 'look at your class notes or read a textbook'. Another learner added 're-read the question'. I added these to the Stuck Poster and placed it at the side of the board so they could refer to it during the course of the lesson.

I started working through the solution to the question 'Draw the line  $y=2x-1$  for  $-3 \leq x \leq 3$ '. Whilst completing the table of values, Kylie put her hand up to say 'I don't get it'. I reminded her of the aim of the lesson and her response was 'How am I meant to do it if I don't get what you have done?' I explained that that was the aim of the task. As I moved on, I could hear her complaining to the girl beside her. When I started plotting the points from the table Tom, who demonstrated the negativity when becoming stuck in intervention 2, shouted out 'Where have you got those from?' I reminded him that he would be given time to look carefully at what I had done in a few minutes. I heard him mutter 'this is pathetic' under his breath. After completing the example, I reminded the class of the learning objectives and the Stuck Poster and then gave them two questions to complete.

The class quickly got into pairs. After five minutes, levels of engagement were seen to be low, more than half the class was off task and the remaining pairs were struggling. Comments similar to 'can you not just tell us how to do' had been heard many times. With such a large proportion of the class not progressing, I felt it was appropriate to provide some additional guidance or scaffolding to help them complete the table of values. This scaffolding appeared to help the majority of the pairs, who now started talking about the task and thinking about the guidance.

The first pair I focussed my attention on was Kylie and Katie. Kylie is a challenging learner who has shown that she has very low levels of mathematical self-concept by always assuming she is doing each question incorrectly. She frequently says she has never been good at mathematics and cannot wait until she can stop studying the subject. In her previous school, she was in the lowest set and since starting at this school, she has been in the third set out of four. She coped successfully with the demands of the set, but felt initially that she had been put in the set by mistake. I have seen her in lessons panicking when other learners grasp a new topic quicker than she does and she has in the past covered her panic and difficulties by behaving badly to get sent out of the lesson. Although Katie is of a similar ability to Kylie, she seems to cover up her difficulties by becoming over-reliant on teacher support and often will not proceed without getting clarification from the teacher that what she is about to do is correct. Her work is always immaculately presented and she often rips a page out of her exercise book if she makes a mistake.

When I approached them, Kylie said to Katie ‘how are we meant to answer the question if he (the teacher) doesn’t explain it?’ Katie suggested looking at the example carefully and working out how to do it. I reminded them of the scaffolding I gave to the class and asked them if they could see the link between the inequality and the top line of the table. Kylie quickly pointed out that the numbers were the same in both. I suggested they started drawing the table and fill in the gaps. Kylie drew the table quickly while Katie took her time, asking Kylie how many boxes were needed and then, using a ruler, made sure each box was the



same size. Katie was clearly proud of how neat the table looked compared to Kylie's quick attempt. I asked her why it mattered to her so much that the table was neatly drawn and she could not give a definite answer and only gave the response that she did it because she could.

By this point I had observed that the majority of the pairs had encountered a difficulty and were once again off-task. I decided to bring the class back together to share what progress each group had made. It became apparent that the problem was in working out how to find the  $y$ -value from the equations. This was addressed through a class discussion.

Once the class was again on-task, I sat down near two girls, Abi and Alice. Both Abi and Alice were working towards the top end of the group. Alice does not often ask for help from me but uses the support of peers when needed. Abi tries to solve a problem on her own first but asks for help when she feels she can get no further. Both these girls were stuck with the calculation of the  $y$ -values before the additional support given to the class and were now discussing how I got the  $y$ -values. They correctly calculated the  $y$ -values when  $x$  was positive but when  $x$  was negative they made an error; I decided not tell them about the error and let them proceed. Once they plotted the points they realised something had gone wrong. I asked them why they thought that and they said the points should form a straight line. I asked them which points they thought were incorrect and Alice said the negatives  $x$ -values because she 'finds negative numbers hard and always gets them wrong', demonstrating a lack of self-efficacy in one area of the curriculum. Abi got her calculator out and found out that the negative values were indeed incorrect.

They changed their values, re-plotted the points and got the straight line they were after. This observation indicated that the girls have thought in advance what they expected the outcome to be; when this was not the case they knew they had made a mistake.

Before they moved on to the second question, I asked them how they found the task. Alice said that she found it difficult to start with but once I provided them with the scaffolding it was easier. Abi said she did not know how they would have completed the task without this help. Alice said ‘everything we needed to know was there but it was hard to know which bit to look at to help us out’. She said it was similar to their history lessons when they are given different sources to look at and had to use the sources to answer different questions. On discussion with a history teacher this is a skill that is introduced in year seven (age eleven) and is constantly being built upon. In the teacher’s opinion it is a skill that takes many years to develop. Perhaps this was the first observation I had made of someone showing some form of mathematical resilience. Alice was able to link the skills she was doing in mathematics to those used in history and, although she relied on scaffolding, she persevered with the problem and did not let finding out she had made a mistake stop her progressing.

While circulating the class, I observed that the majority of the learners were making progress and had made the required links to complete the questions. Tom, who was observed in a previous intervention, was working with Joey. They did not appear to be working on the task. When I spoke to them about how they were getting on Tom admitted ‘We’ve given up. I don’t know what we need to do. I’ll

get it when you go over it later'. I asked him what strategies he had tried to help him and he said 'I asked the learning support assistant and she wouldn't tell me what to do. I looked at what you had done but it didn't make any sense'. I asked him whether he thought that if he persevered with the problem he would learn from the experience and get better at solving problems. His response was 'my teacher at High School tried to do things like this with us but we never got them so she gave up .... I used to try hard but I will never be good at it, so I don't see what the point is in doing them'. He said he liked it more when we did 'proper maths' referring to what Swan (2005) describes as transmission teaching, when the teacher gives many examples followed by a worksheet full of similar questions to complete. Once again, his responses appeared to indicate that he wants to return to the classroom culture he was familiar with. Similar to the findings of Williams (2014), his lack of ability to persevere meant he would do whatever he could to avoid leaving his Comfort Zone.

At the end of the lesson, I asked the class to reflect on the progress they made during the lesson, specifically what they thought went well and what they could have done to make their learning better. The first two responses were positives about how good it felt when they managed to answer the questions. One said that he nearly gave up after getting nowhere but when I gave the class the hint, it helped him make the final connection to complete the task. For the second part of the questions there were about four comments saying that it would have been better if I had told them how to do it. One of the more interesting comments was that it would have been better if I had some hint cards they could collect if they needed them. He said that he was close to working out how to get the  $y$ -value when I gave

the class a hint, which ‘spoilt the fun’ for him. If they had to come and collect a hint card, those that wanted to spend longer thinking about it could have done. This strategy interested me and it is something I would like to try the future. He also hinted that his motivation for completing the task was intrinsic; he wanted the pleasure of completing the task to improve his learning.

Following this I returned to the stuck list. At this point our list was as follows:

- Ask the teacher for help
- Ask a friend for help
- Look at examples in class notes or a textbook

After the task, one learner suggested adding ‘looking for patterns’ onto it.

The following lesson was delivered by a cover teacher. I explained to the learners that in the lesson I wanted them to go to the computer room to make use of a web based learning resource. Learners were to work through an online-lesson on drawing straight-line graphs before trying the online assessment. The online assessment asks them to complete two or three exam style questions on the computer and gives immediate feedback to the learners on the accuracy of their work.

By setting this lesson, the learners learned the majority of these skills independently through online learning. This allowed me to compare the outcomes of questions on this topic in the end of year assessment compared to the previous cohort who answered the same questions in the exam. The previous cohort was taught using transmission teaching (Swan, 2005).

The four-mark question is given below.

On the grid, draw the graph of  $y = \frac{1}{2}x + 5$  for values of  $x$  from  $-2$  to  $4$

(From Edexcel Paper 1MA0/1F June 2013)

Underneath the question, the learners were given the grid with axes to draw their answer on but they were not given a table of values. The results analysis from the examination board gave the following information

	National Results	Comparison Group Results	Research Group Results
Mean mark out of 4	0.61	0.45	2.3

Lack of data on the standard deviation of these results did not allow statistical tests for significance to be carried out. Although there were other factors such as time scale between teaching the topics and completing the question that may have had an impact on the results, the mean mark for the research group when compared to the comparison group was 57.5% compared to 11.3%. This indicates that teaching the topic in this way was highly likely to be one of the factors that resulted in a positive outcome in their results.

Observations and journal entries for this intervention gave evidence for the potential presence of a lack of mathematical resilience within the group. The large majority of learners actually seemed to give up. They clearly stated that they

wanted to be told how to do the tasks so they could successfully complete a large number of similar questions during the lesson. My hypothesis is that they said they ‘couldn’t be bothered’ or ‘this is pointless’, to try to hide their difficulties, as predicted by Maslow (1987). Only four learners made positive comments about the activity and these were related to the excitement of successfully making the connections required to complete the task, a suggestion that their motivation was intrinsic.

On reflection, this intervention was perhaps the first step in starting to move the class forward in creating a positive stance to mathematical problem solving. Through observations, discussions and reading journal entries I gained a good understanding of the weaknesses of learners within this group. The use of scaffolding to support mathematical thinking would be a major part of future interventions as a way of keeping learners engaged and working on problem. The intention would be to remove this scaffolding gradually over time so learners were completing more of the task independently.

#### **4.5 Intervention four: investigating straight line graphs**

The fourth intervention followed on directly from the third intervention and involved a semi-structured investigation into straight-line graphs using a dynamic graphing ICT package. The aims of this intervention were to develop learners’ skills in spotting patterns, generalising and testing out their generalisations to convince themselves and others that their findings were correct. Following the findings from intervention three, scaffolding would be provided to support their

mathematical thinking. Alongside this I wanted them to gain an understanding of what the 'm' and 'c' in the equation  $y=mx+c$  represented.

Following a brief demonstration on how to use the software, I introduced the aims of the investigation to them making sure they understood exactly what they needed to achieve. Before moving on with the task, I took the opportunity to remind them of our 'stuck list'. I explained that I wanted them to spot patterns, come up with a rule for what they think they have found and then test that their rule works. I introduced the terminology 'conjecture' and 'testing their conjecture' to them. Martin, who is often very negative about investigation work, said it would be better if I taught them what it did instead of 'wasting time working it out for ourselves'. I ignored this comment and moved on to remind them about our Stuck Poster and what strategies they could use if they got stuck.

Each learner was issued with a guidance sheet that gave them prompts to support them through the investigation. On this sheet, I briefly explained what they were doing in the task. The first part of the guidance was looking at the effects of changing 'c'. An extract of this is given below in Figure 9.

Task 1:

Set the axes as follows  $-5 \leq x \leq 5$  and  $-10 \leq y \leq 10$ .

Draw the following graphs on the same page:

- $y=2x+1$
- $y=2x+2$
- $y=2x+3$
- $y=2x+4$

What do you notice about the graphs? Write down what you think the rule is

On a new page draw the following graphs:

- $y=3x-1$
- $y=3x-2$
- $y=3x-3$
- $y=3x-4$

Does your rule still work? If not, write down what you think the new rule is.

Draw enough lines to convince yourself that your new rule is correct

Does it still work?

Predict what  $y=5x-1$  will do.

Were you correct?

Predict what  $y=6x+2$  will do.

Were you correct?

Explore what happens with graphs like  $y=2x$  and  $y=5x$ .

Why does this happen?

Figure 9: The first section of the investigation used in intervention four.

This part of the task purposely had a lot of scaffolding to help model one possible way of tackling this investigation.



For the second task, there was less scaffolding. I hoped that they would approach it in a similar way to the first task. An extract from the task sheet is given below in Figure 10.

<p><b>Task 2</b></p> <p>For the second task you are going to explore what happens when you change the value of '<math>m</math>' in the equation. By drawing a selection of graphs on the same page explore what happens as '<math>m</math>' is changed.</p> <p>Hint: Keep '<math>c</math>' the same in each graph you draw.</p> <p>What do you think the rule is?</p> <p>How are you going to test it?</p> <p>What happens if '<math>m</math>' is negative?</p>
---

Figure 10: The first section of the investigation used in intervention four.

When piloting this task with another group (of a slightly higher prior attainment) they were able to come up with a conjecture about what happened to the line when ' $m$ ' was changed but very few learners in the group considered what would happen if ' $m$ ' was negative. For this reason, I added in an extra prompt at the bottom of the page.

For the first part of the task, I observed a hard-working and relatively high achieving learner within the class, Val. Val entered the first four equations to

produce four graphs. I asked her what she noticed about the graphs. Her first observation was that the four lines were all parallel and had a positive correlation. I asked her why she thought they had a positive correlation and she said 'because the lines go up'. I made the decision not to correct her on the use of the word correlation here to avoid disrupting her thought process but I did speak to her at the end of the session to explain the difference between correlation and gradient. To help her see the link between the graphs and 'c' I suggested she added a text box with the equation of the line by each one. Soon after doing this, she noticed that the 'c' value was the same number that the line passed through on the y-axis. She wrote this down and tested it using the next four lines. Her conjecture worked for those lines so she was able to correctly predict what would happen with the other lines and test that her predictions were correct.

As she started task two I could tell from her body language and facial expressions that she was becoming less confident. She read the task two or three times, opened a new two-dimensional graphing page but entered no equations. She asked Emily, another learner of similar ability to her, what lines they needed to put in. Emily was also unsure so I suggested they looked over the structure of task one to see if that would give them any ideas. Between them they decided to keep 'c' as two and change 'm' so that it was one, two three and four. They plotted these graphs and added their equations onto the end as seen below in Figure 11.

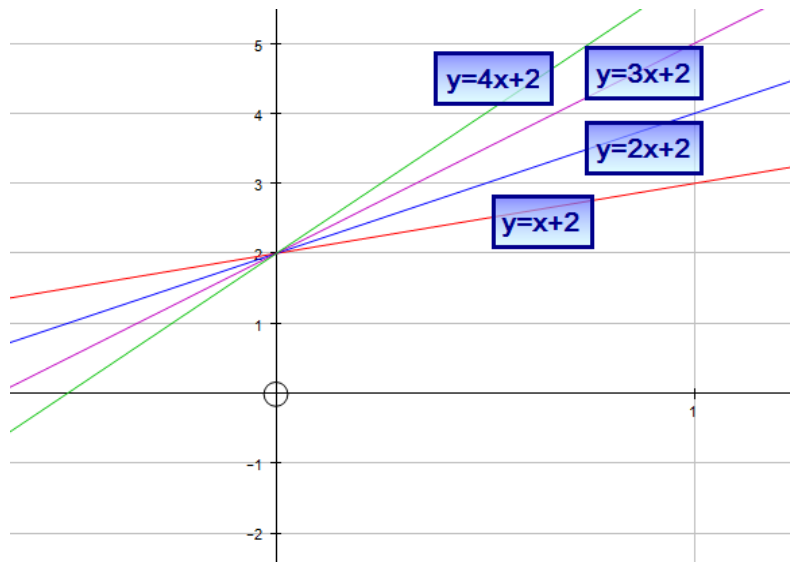


Figure 11: Some equations used by Val and Emily as part of task two.

They were both quick to spot that all the lines intercepted the y-axis at two but struggled to describe in words what else was happening to the graphs. Emily said that to the right of the y-axis the graph of  $y=4x+2$  was the highest and that to the left of the x-axis it was the lowest. Val said the graph of  $y=x+2$  did the opposite so that to the right of the x-axis it was the lowest but the highest graph to the left of the x-axis. Emily then mentioned that the graph of  $y=4x+2$  was steeper than the graph of  $y=x+2$ . After a moment of silence, I asked them to consider what difference the value of ‘m’ made to the line. They both thought for a while then Val mentioned that ‘the bigger the number the higher up the graph is’. Emily interrupted and rephrased Val’s comment as ‘the bigger the number the steeper the line is’. I suggested they wrote down their observation and tested it. They tested it with another four lines.

For the next part, they entered four more graphs with a negative value of 'm'. They had forgotten to open a new page so the graphs were plotted on the same screen as the previous ones. This worked to their advantage and Emily quickly spotted that the new graphs were the mirror image of the other ones. Val added that the lines had a negative correlation because they were going down. She concluded that if the 'm' is positive then the lines have a positive correlation and go upwards and, if 'm' is negative, the lines have a negative correlation and go downwards. Emily agreed with her and they made a note of their idea before testing it further.

Both learners were able to follow the structured part of the investigation with ease. Val was good at asking for help when she needed it but was happy to persevere when she felt confident in doing so. After the lesson when I spoke to Val about her unexpected use of the word correlation she said seeing the lines going up reminded her of when we studied scatter graphs and how the lines she drew were like the lines of best fit added to scatter graphs so she assumed the terminology would be the same. Using this link provides evidence to suggest that she is starting to try to see links between the different topics we are studying, something that the Building Learning Power model promotes.

When the structure of the investigation was removed in task two, both girls momentarily seemed to lose confidence in being able to proceed. It appeared as though they panicked because the second part of the task appeared to be more challenging. The resilience demonstrated in task one had gone. Following the intervention encouraging them to look at the structure of task one they quickly realised the task was accessible to them and continued until they had confidently

come up with a conclusion. The only difficulty they faced was putting their findings into words, a common problem observed with this group. By making use of this intervention, the girls were kept within the Growth Zone and avoided entering the Anxiety Zone. Johnston-Wilder et al. (2013) suggest that one of the attributes of mathematical resilience is being able to struggle with appropriate support, which they were starting to demonstrate.

The next pair I observed was Martin and Billy. Martin was the learner who previously displayed dissatisfaction at having to discover something for himself instead of the information being given to him directly from the teacher. Martin is on track to achieving a good pass in his GCSE in mathematics but seems to lack confidence in his own ability. In a ‘settling in discussion’ I had with him when he first joined my group Martin talked about his previous experience of education in mathematics at his last school. He was in a ‘bottom set’ with learners who were ‘not good’ at mathematics. In his final year at his last school, his teacher was off on long term sick leave and they were given a series of supply teacher who lacked the classroom management skills to control the behaviour of the class. As a consequence, a large proportion of their learning time was lost due to low level disruption. Lessons consisted purely of teacher talk and examples followed by working from a textbook. No effort was made to give learners a relational understanding (Skemp, 1987) of the work and, although Martin did get on with the work, he did not see the relevance of mathematics in his future life.

Martin and Billy decided to work together from the beginning and were quick to get a two-dimensional graphing page opened and managed to set the axes to the

given range with no difficulty. They entered the first four equations and plotted the graphs on the screen. They both went quiet and kept on looking at the sheet and the screen. After a while, they were starting to lose interest in the task so I asked them what they noticed. After an initial response of 'I don't know' Martin mentioned, they are all going in the same direction and are the same distance apart. Billy added, this means they are parallel. I asked if they could see the link between the value of 'c' and the graph. To make it easier I suggested they added a textbox to their graphs so they could clearly see which one was which. This did not help them so I asked them to trace  $y=2x+1$  with their pen starting at the left hand side of the monitor, saying what the line was doing as they went along. Billy traced the line and said that it crosses the  $x$ -axis at  $-0.5$  and the  $y$ -axis at  $1$ . They repeated this for  $y=2x+2$  and  $y=2x+3$ . They were both struggling to see a link so I suggested they drew a table on a mini-whiteboard with three columns: equation; where it crosses the  $x$ -axis and where it crosses the  $y$ -axis. They completed the table and spotted that the value of 'c' was where it crosses the  $y$ -axis. I said that they now needed to test their rules worked with some other points. Martin questioned why he had to do this because he could see it worked for these points. After a bit of persuasion, he grudgingly drew four more lines to check that it worked. The next part was drawing the lines  $y=2x$  and  $y=3x$ . Both Martin and Billy spotted they crossed at the origin but were not sure why. I suggested they looked at the table they had on the mini-white boards and add  $y=2x$  to the bottom of their list. They struggled to understand that the equation was really  $y=2x+0$ .

When Martin and Billy started task two they read the sheet then started talking about something unrelated to the lesson. When I challenged them and tried to

motivate them to tackle the next task, they said they 'could not be bothered'. I asked them why and Billy responded with 'just because.' Based on my knowledge of these two boys my suspicion was that they often pretended they could not be bothered to cover up low levels of self-concept. I suggested some lines for them to start with. They slowly entered the equations and plotted the lines. I asked them what they noticed and Martin responded with 'not a lot'. He could tell I was not impressed with his response so he added that all of the lines crossed the  $y$ -axis at two. I asked them why they thought this was and Billy confidently mentioned that it was because the number at the end of the equation was two. Martin added that this time the lines were not parallel but got wider apart the further away they got from where they crossed the  $y$ -axis. From their actions and comments it was clear their motivation was starting to increase so I suggested that they labelled the lines using textboxes to make it clear which line was which. Once they had done this, they drew a table without prompting and added on where each line crossed the  $x$ -axis and the  $y$ -axis. They looked at the table but could come up with no observations. I was concerned that their motivation would decrease so I suggested they opened a new page and added to it only the graphs on  $y=x+2$  and  $y=2x+2$ . I asked them what was the same and what was different about the two lines. They said the things that were the same were that they were both straight lines and they went through the point 2 on the  $y$ -axis. They were not so confident about describing what was different about the lines. Eventually Martin said the line  $y=2x+2$  is steeper than  $y=x+1$ . I asked if it worked both sides of the  $x$ -axis and Martin said it did because to the left of the  $x$ -axis the line was going down more steeply. I asked them what they thought the line  $y=3x+2$  would do and they both said it would be steeper. I suggested they tested this out. They concluded that it

was working and that the bigger the value of 'm' the steeper the line would be. They then explored what happens when 'm' is negative and came up with a conclusion.

Martin and Billy demonstrated here that with a bit of scaffolding and encouragement they could work in their Growth Zone and become involved in mathematical investigations and thinking although, without this extra support, they quickly give up as soon as they encountered any difficulty. When reviewing the journal entry following this intervention, Billy talked about how the task was easy with me supporting them but he was not sure that he would not have been able to do it without my support. From observations and discussions with Billy it is becoming clearer that he does not like to be taken out of his Comfort Zone and into the Growth Zone. This is perhaps down to low self-concept or fear of making a mistake.



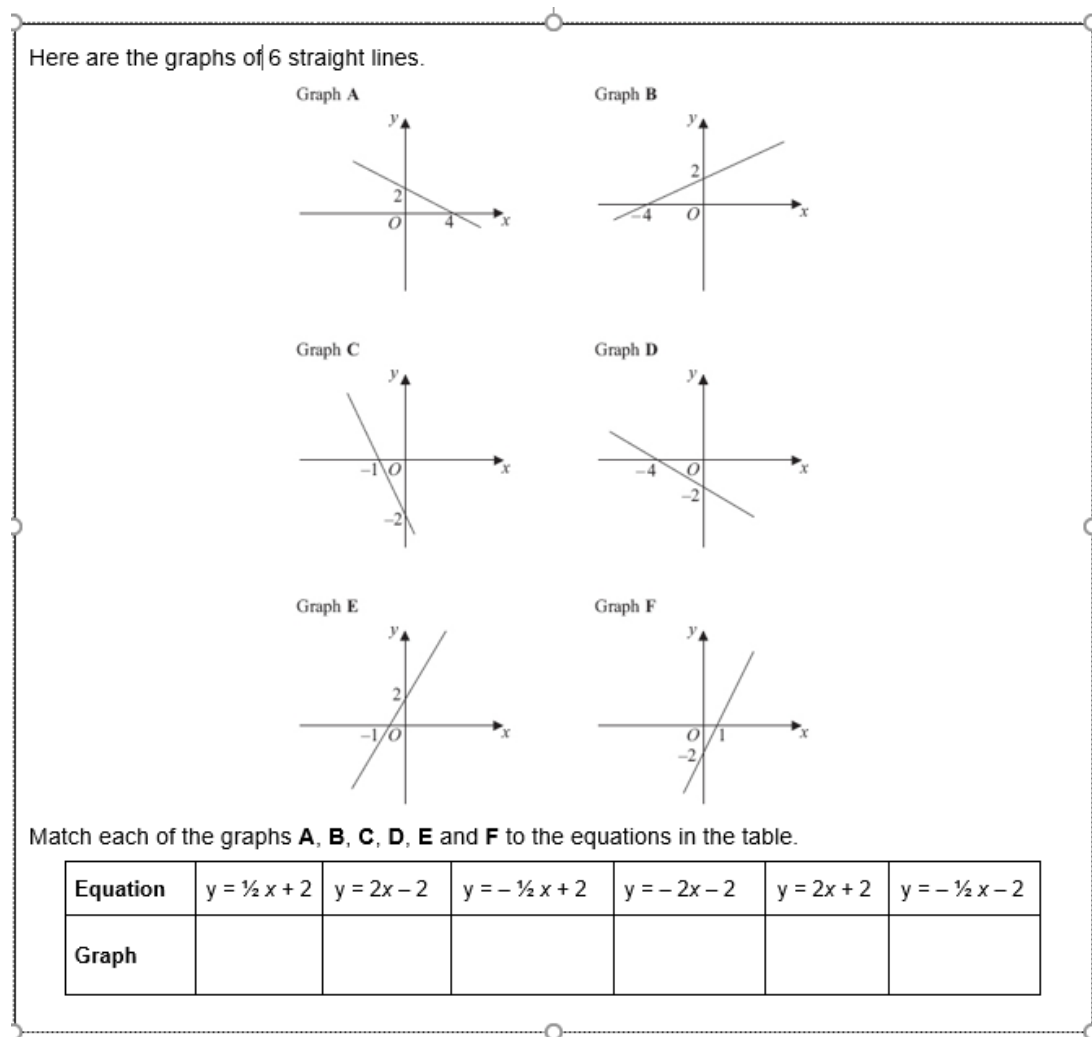


Figure 12: Question taken from Edexcel November 2012 GCSE mathematics paper.

Near the end of the lesson the class were given a question taken from a past paper (Figure 12, above). This was done before we had discussed the findings of their investigation as a class.

They were given time to complete the examination question independently. The papers were marked after the lesson and the mean mark calculated. The mean mark for the class was 2.2 out of 3 (73%) compared with the comparison group's mean mark of 1.2 (40%). An analysis of the standard deviation of the national data was not available so statistical tests could not be carried out. Furthermore, the time scale between teaching the topic and assessing the topic was different for this group compared to the comparison group who sat this questions as part of their GCSE examination during the previous summer, which would impact on the results. Thirteen learners in the class mentioned in their journals how easy they found the examination paper, despite it being a question targeted at grade C. No one mentioned the question being difficult and the lowest mark was one out of three, with many learners gaining full marks.

Overall, I observed that more learners remained engaged with this task, especially the structured task, compared to previous tasks. No one was able to complete the second task without some extra scaffolding provided by the teacher or a peer. When I spoke to the learners when they were off task a large proportion of them said it was because they were stuck and unsure what to do. Further prompting established that they did actually know what to do but they did not think their idea was correct. This resulted in me believing that the biggest barrier that still needed to be overcome was to increase levels of self-efficacy so that the learners were not afraid to try something and to reduce the fear of failing.

#### 4.6 Intervention five: the data handling cycle

As part of the GCSE course in mathematics, learners need to have an awareness of the data handling cycle. This cycle (Figure 13) models the approach to solving a problem or carrying out a statistical investigation. In my experience, learners start the GCSE course in mathematics having learnt how to calculate averages from a set of data and how to represent data but have not gained an understanding of how to interpret these values and graphs and how to use them as part of a statistical investigation.

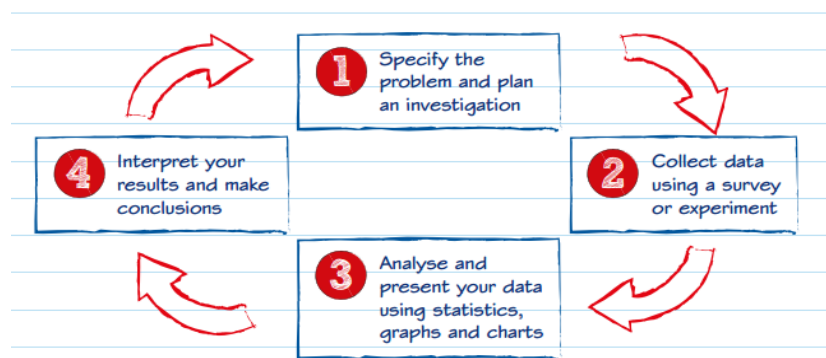


Figure 13: The data handling cycle.

In the previous intervention, I surmised that one of the barriers for the learners was low levels of self-efficacy. To help overcome this I wanted to give the learners the chance to discuss their thinking and logic in pairs so, for this intervention, learners were asked to analyse graphs and give reasons for their conclusions. As part of the GCSE examinations, learners are often required to give reasons behind their answer, something they are not always very confident in doing. My hypothesis is that in the past they have been given an instrumental understanding of these topics and as a result they are unable to mathematize (Wheeler, 1982).

Thus explaining their thinking is challenging because it requires an understanding of the concepts and techniques, not just instrumental application.

For this task, they were asked to match the photograph with the corresponding graph that represented the heights of the people in the picture (Figure 14). They then had to explain, in written form, the reasons behind their decision. Following this, some pairs would be asked to come to the front of the classroom and explain their answers to the class.



Picture 1



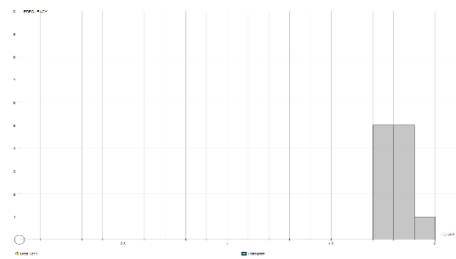
Picture 2



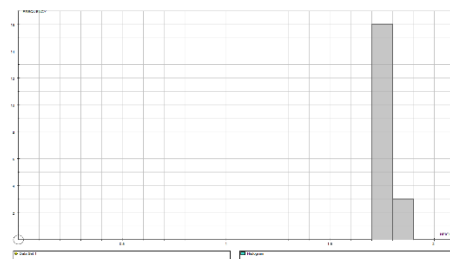
Picture 3



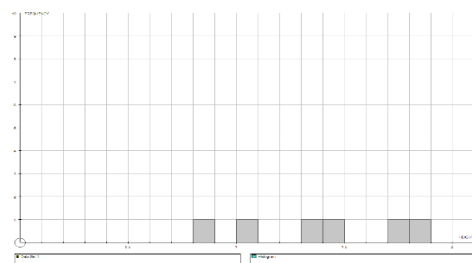
Picture 4



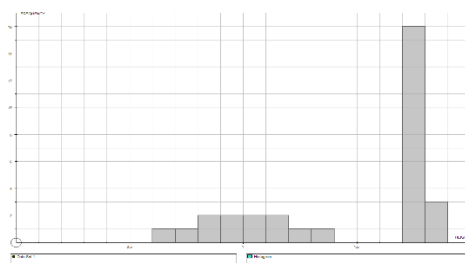
Graph 1



Graph 2



Graph 3



Graph 4

Figure 14: Match the picture with the graph.

For the first part of the task, I positioned myself so I could observe two higher attaining and two lower attaining learners within the class.

Abi and Alice are two hardworking learners who participate well in lessons and attended additional support sessions when they feel extra help is required. They are learners who achieved well in examinations as a result of hard work and memorising techniques; they did not seem to want to know why something works, only how to get marks in the exam. From discussions I have had with them they know that with hard work you can get better at mathematics.

After examining the sheet, Abi asked Alice for clarification that the  $x$ -axis represented height. Alice agreed with Abi and said that because of this one and two must match up with picture one and three. Abi was quick to spot that the sumo-wrestlers were all about the same height but the England footballers had four people on the back row who looked taller than everyone else. With similar logic they matched up the other two photographs although it did require more discussion to decide on the matchings.

During the observation both girls remained engaged on the task and were able to conclude which cards they thought matched through a logical process. They were able to communicate their understanding to each other and were not afraid to question the other person's logic. When questioned, they listened to the reasons behind the other person's disagreement with them. Reflecting on this task in their journals, Abi wrote that she had found the task easy to do. She mentioned that

once they had worked out what each axis was they could look at the people and look at the heights of people to visualise what the graph would look like. Alice also commented that the task was easy although it was a bit challenging for a few of the pictures because some people were sitting down and some were standing up so they had to predict what their height was by looking at their size.

At the same time as this, Barbara and Joey were working together on the task. They are lower attaining learners within the group. They have a different approach to each other in their work in mathematics. Joey will give up quickly when he encounters a difficulty whereas Barbara will always put down an answer, even if it is a guess.

Very soon after the sheet was given out, Barbara asked me what they had to do. I reminded her, then she quickly asked how they were meant to do it. I suggested they both had to decide what the axes on the graph represented and use this to help them. Joey said that the  $x$ -axis must be the height and the  $y$ -axis the number of people of that height. Barbara pointed to graph one and said that this must be mostly tall people. Joey then said that graph two is also most tall people. They looked at the pictures and decided that they must match up to picture one and picture three because the other two pictures have lots of children in them. Joey said that the sumo-wrestlers are pretty much all the same height but some of the footballers are taller than the rest so the footballers must be matched with graph two. Barbara paused for a moment then questioned Joey why it was not graph one. She said that there a wider variety of heights with the footballers but with the sumo-wrestlers they were all the same height except a few so it needed one big

bar with a small bar just above it. Joey agreed, without debate, and they wrote their answers down.

When it came to deciding on the last two Joey had lost interest in the task and left Barbara to complete it. She initially tried to discuss it but his response was ‘I don’t know’. She decided that picture two must match with graph four and picture four must match with graph three. At this point, I intervened and asked Joey if he agreed with Barbara’s reasoning. He quickly glanced over and said yes. He did not want to get involved. After the lesson, I looked at Joey’s journal entry. He has said the task was hard and he did not understand what they had to do. He made no other comment. He did not make reference to how successful he was at the start of the task and only reported on his negative attitude towards the task that started after he was questioned by Barbara on his response.

On reflection, Joey initially appeared to be willing to have a go and was demonstrating some aspects of accepting a cultural change within the classroom. However, as soon as he felt he had made a mistake he wanted to return to culture he was familiar with and gave up on completing the task, possibly as a way of avoiding a threat to his personal self-esteem (Maslow, 1987). This lack of persistence limited the time he remained in the Growth Zone, which in turn limited his mathematical development. This behaviour from Joey had been observed on other occasions. He can be encouraged to work hard on a problem but he demonstrates a fear of not getting the ‘correct’ answer and would rather not take part than risk not completing the problem.



To me, the biggest barrier at this point remained learners giving up when they encountered difficulties; they seemed to lack the ability to persevere and look for alternative ways to solve a problem. The majority were able to start positively on a task and seemed to want to improve but as soon as they came across failure, or perceived failure, they lost interest in the task and stopped. For this reason, I planned the next intervention so that it allowed learner to succeed no matter how far they got with the task.

#### **4.7 Intervention six: statistical investigation**

In the design of this intervention, I wanted to give learners the opportunity to complete a low threshold high ceiling task (McClure, 2001) that allowed them to choose their own success criteria. I wanted to make sure that everyone could succeed and that no one gave up on this task as a result of not being able to get the ‘correct’ solution. I planned the introduction to be teacher led to ensure everyone understood the task and gave them the choice of working either in pairs or individually. As part of the task, they were collecting, analysing and presenting a data set. I reminded them about making use of the ‘Stuck Poster’ if they became stuck.

This intervention followed a unit of work on handling data. In this unit, the class had covered how to calculate the averages from frequency tables and looked at different ways to represent data. They all had access to class notes and class work that covered each of these topics for reference.

After a brief recap of the handling data cycle (Figure 13), I explained to the learners we were going to look at the difference in word length between a broadsheet and a tabloid newspaper, and provide statistical evidence to either prove or disprove a hypothesis. After a short discussion, the learners came up with the hypothesis that the average word length in a broadsheet paper would be higher than the average word length in a tabloid paper. We collaboratively decided that we would collect a sample of two hundred words from a randomly selected page. I gave the learners time to select their sample and complete their data collection sheets. At the end of the lesson, we discussed that our work related to the first two stages of the handling data cycle.

During the next lesson, I introduced the third stage of the data handling cycle, which is about analysing and presenting the data. We brainstormed different ways we could analyse and present the data as a class and I reminded them of the Stuck Poster and ways they could access help if they needed help. The big emphasis was on referring back to their lesson notes and using each other. There was confusion over which techniques and graphs were suitable for use with discrete and continuous data but I resolved this through a class discussion.

Once the class started to work, I sat down next to Piers and Monica who were working together on this task. Both of these learners were towards the upper end of the class based on attainment but both lacked confidence and felt the need to ask for reassurance along the way. When I started observing, they had decided to start by working out the mean, median and mode for each sample. They were able to calculate the mode with ease. They completed the table correctly for calculating

the mean but could not remember which two values they needed to divide. When they asked me, I suggested they think about what value they expected the mean to be close to and then try the division both ways and consider which answer was the most sensible. They did this and were confident they had used the correct calculation.

When it came to the median, they could not remember how to calculate it. They asked for help so I asked them what strategies they could use when they got stuck. They quickly remembered about the Stuck Poster on the wall and decided they would look in their class notes to help remind them of the technique. However, instead of working out the cumulative values of the frequency they worked out the cumulative values of the products of the values and their frequencies. This gave them large values in the cumulative frequency columns. I decided not to intervene and let them continue. They worked out the total frequency was two hundred so that the median must be the one-hundredth value. Using this they worked out the median word length to be two. I asked them at this point if it sounded like a sensible answer and Piers responded with yes because it is a value on their table. I asked them to define the median value and Monica said the middle number in an ordered list. I asked if two looked like the middle number in the list. Piers looked at the frequency column and realised the error they had made.

When they had done all three calculations for each table, I asked them which average was the most appropriate to use. Their initial reaction was to see if there were any outliers, which is normally the case in GCSE examination questions, but they soon realised that this was not appropriate for the data they had. They had

spotted that the mode was the same for both so felt that this was not a good average to use because it would not prove their hypothesis but could not decide whether the mean or median would be the better average to use.

On reflection a few interesting points came up here. The first is that learners are not good at estimating what size their answers should be. If this became common practice with them then they may become more confident that their answers are correct without the need for reassurance from the teacher. In her journal, Monica commented that they worked out the median wrong but if they had checked their answer was sensible and as expected they would have spotted this mistake.

The other point was related to over-familiarity with GCSE examination questions. If they are asked in an examination which average is the most appropriate to use, the sample usually contains a very large value so they are able to rule out the mean. This was not the case here so they were unsure what to do. This is perhaps a sign that learners' mathematical experience is being controlled by what they need to know for the 'test'. Monica mentioned in her journal that working out which average was the best one to use was the hardest part of the lesson. She said that she was unsure whether it was the median or mean because in both cases there were no outliers in her table. This seems to demonstrate her instrumental understanding of this topic (Skemp, 1987)

When I approached Kylie and Katie, two of the weaker learners in the group, they were debating whether their bar chart should have gaps between the bars or not. They had their books open and saw that in one example we had left gaps between

the bars and in the other we had not. Based on previous observations I was aware that these two learners often gave up when faced with difficulties so I was pleased to see they were still on working on the task and had referred back to their class notes, perhaps a sign that the Stuck Poster was starting to have an impact. They told me they wanted to draw a bar chart for each set of data but were not sure if they needed to leave gaps or not. They said in their books they had one example of shoe size where we left gaps and height when we did not leave gaps. I asked them to think about what the difference was between these two types of data. They spotted quickly that we had collected height data on a grouped frequency table and shoe size on a frequency table but could not see any other link. I asked them to read what we had written. They did and said that we needed to leave gaps if it is discrete data. After a short period of time, they said that it was discrete data because you could only have a whole number of words so they would need to leave gaps between their bars.

On speaking to Kylie after this intervention, she mentioned that she finds it useful to work with someone when completing difficult work but once she knows what she is doing she prefers to work alone. This was evident in this lesson. One of the areas of the Building Learning Power model is for learners to know when to work alone and when best to work with others. This is a skill shown by Kylie.

In the last intervention Joey lost focus on the task when he realised he had made a mistake. This intervention was planned to allow learners to select techniques they were happy to use to help avoid this situation. When I approached Joey, he was working with Billy. They had drawn bar charts for each set of data. Each of

the charts had the 'tallest' bar annotated as the mode value. I asked them what they were going to do next. They said they were going to draw a pie chart and then work out the mean, median and range. Billy had his book open on the page that had the examples of how to draw pie charts. They added a column to the table and calculated the angle required for each sector. Once they had drawn the pie chart, I asked them which has the most number of four letter words. Joey initially said you cannot tell because one of the pie charts may be out of more than the other. Billy added that you can compare them because they are both out of two hundred words. Joey agreed with him and then updated his answers to say that the pie chart showed that the tabloid had the most four letter words. After this, observations showed they both continued to work out the mean and the median without too much difficulty.

Despite giving the incorrect answer and being corrected, Joey continued with the task. He had made links with previous work covered and happily completed the task until he had produced a poster, which proved the hypothesis to be correct. Interestingly he did not comment about his error in his journal (unlike on previous occasions) and instead went on to describe how he enjoyed being able to choose what techniques they wanted to use to prove their hypothesis to be correct.

The majority of the class chose to work out the mean, median, mode and range and drew bar charts and pie charts to back up their evidence; a large majority enjoyed the task stating it was because they could choose how they proved their hypothesis. One went further to say that they liked it because they could choose

not to draw a step cumulative frequency polygon, which was something they found challenging when we covered it in lessons.

Although observations of this task did show a perceived confidence boost to the learners, it also emphasized they are happy when the level of challenge is within their Comfort Zone and when they can avoid using techniques that they feel may struggle with, something the previous classroom culture promoted. However, as mentioned previously, some learners were starting to estimate what they expected the answer to look like, which is useful skill to use when working at mathematics (Johnston-Wilder et al., 2013,).

For the next intervention, I felt it would be useful for the learners' development to attempt another task where they chose their own success criteria; I wanted them to see that they can succeed in mathematics. However, I also wanted to remove the option of them being able to avoid techniques that they found challenging.

#### **4.8 Intervention seven: exploring the number grid**

Based on my reflections on the previous intervention, I decided for this intervention to make use of the number grid problem. The introduction to this is given below in Figure 15.

Look at this number grid:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- A box is drawn round four numbers
- Find the product of the top left number and the bottom right number in this box
- Do the same with the top right and bottom left numbers
- Calculate the difference between these products

Figure 15: The number grid investigation explanation sheet.

By carrying out this investigation, I wanted learners to be able to approach the investigation in a logical way, thinking carefully about how they were going to present their findings. Alongside this, the Building Learning Power objective was to spot patterns and use these patterns to make generalisations. Many of the previous investigations we had completed in class had a structure so the learners could follow a framework to guide them through it. The aim would be to provide as little guidance as possible. To provide a scaffold, I planned to give one example using the grid and then ask them to try other squares and see what they noticed. After five minutes of investigation, we would discuss what they had found and brainstorm ways we could extend the investigation. Before they started on this final part, they were told that at the end of the task they would share their findings



with the class so they would need to think carefully about how they were going to present their results so it would be easy for the rest of the class to understand their method.

Once the class had been reminded what the terms difference and product meant, they quickly worked out that for any square they drew on the grid the difference was always equal to ten. When asked how they could extend the investigation, I was met with silence so I made use of the think, share and pair technique. This resulted in the suggestion that we could use the rule on bigger squares. Another suggestion was that we could use rectangles instead of squares to allow us to get more variety. All except two learners decided to investigate larger squares.

For the first part of the activity, I stood back and observed the engagement of learners. By scanning the room, I could see that all learners were on task. Two learners were discussing the work while the remaining learners were working in silence. This continued for the first five minutes of the task.

I went across and sat next to Billy. Billy had worked out the difference for six different sizes of squares. He was able to tell me the difference for each of these squares. I asked how he knew the difference was always the same for each sized square. His initial response was 'because it is'. Then he realised what I was thinking and said that we know it is always ten for a two-by-two square because he had calculated it for many two-by-two squares. So, to be certain that the difference is the same for other sizes of square, he would need to try at least two other squares in difference parts of the grid. I asked him if this was enough to

know that the difference would always be this value. He initially responded with yes but then he paused and changed his answer to no. He said that to be certain he would need to try every two-by-two square on the grid and every three-by-three square on the grid. He said that it would take ages but that way he would be certain.

This interaction was the first time I had witnessed some rigorous mathematical thinking from Billy. He had realised when I was speaking to him that he needed to be more rigorous and had to be certain before he made a generalisation of his findings. Although proof by exhaustion was not the quickest method of proving his claim, it would work successfully in showing what he wanted to prove. In earlier interventions, for example when exploring the equation of a straight line, Billy was seen making statements but was unwilling to justify them. In this intervention he nearly reverted to his usual 'because it is' answer but realised that mathematical explanation require justification.

To avoid him losing the momentum he had gained, I suggested that although it would be better to try every possible square, he should check it worked for three different squares and then if they were the same he could make the statement that he thought they would all be the same. He then continued and remained focussed on the task.

For the rest of the lesson I visually monitored levels of engagement. In general, learners were on task. The majority worked on their own and pretty much in silence. I did speak to some other learners during this time but there were no significant observations made.

As we approached the end of the lesson, two of the higher attaining learners, Piers and James, had their results ready for presentation. The others still had more work to do, so during the next lesson the teaching assistant supported the class and I worked with Piers and James. I opened the discussion by asking how they knew the difference is always the same for any two-by-two square. Piers said that he tried more than one square of each size so the difference must be the same. I suggested they could both say that they ‘believe the difference is always the same’ but if they wanted to say ‘the difference is always the same’ they would need to prove it. James suggested ‘we could prove it using algebra’. I asked him to expand on this and he said ‘we need to use an  $x$ ’. He could not elaborate on this any further.

To help them out I drew a two-by-two grid on the board and wrote ‘ $x$ ’ in the top left column. I asked them what I should write in the second box. Piers said ‘ $x+1$ ’. By referring to a number grid for support he worked out that we needed  $x+10$  and  $x+11$  on the bottom line. I asked them what they had to do next and they said multiply opposite corners then subtract the answer. Piers was quick to point out that it would mean expanding double brackets. They carried this out on mini-whiteboard and subtracted the answers and were amazed when they got an answer of ten. Following this, they returned into the main lesson and completed this process for larger squares on the grid.

On reading their journals, both learners were clearly excited by the fact that they had managed to use algebra to work out the difference. Piers had written ‘We did the task with numbers and always got the same answer. To prove it would

be the same answer for all squares on the grid we used algebra. Although we added  $x$  in we still got the same answers as we did for numbers.’ James’ comment was similar but he also mentioned that it was nice being able to use algebra to help us solve a number problem. When I spoke to him later about this comment he said that normally we only do algebra when we are studying a topic that involves algebra and he does not remember using it when doing a number problem before. He said it was clever how the use of algebra made the number problem so much quicker to solve.

Towards the end of the lesson, I brought the class back together so we could share their findings. Observation of their work showed that eighty-four percent of the class had used a table to present their findings with working out to back up their results. When completing their journals, I asked them to write down why they presented their findings the way they did. Many of the learners commented that it was the most logical way to present it and that putting their findings in a table was a good way to summarise the many pages of working they had. Two learners referred back to when we studied trial and improvement to solve cubic equations and mentioned that when they did not present their solutions in a table they got confused and made silly mistakes. Two of the positive observations seen here were that they were able to use lessons learnt in other topics to support them with this task and they were thinking about the best way to present their solutions to help them see patterns.

Every learner in the class had managed to complete their table of results. These are shown in Figure 16 below.

Size of square	Difference
2 x 2	10
3 x 3	40
4 x 4	90
5 x 5	160
6 x 6	250
7 x 7	360
8 x 8	490
9 x 9	640
10 x 10	810

Figure 16: Table of results.

I asked the class what the answer would be if I selected an  $n$ -by- $n$  square from the grid. I explained what I meant and one learner asked if I wanted them to find the  $n$ th term of the sequence of the numbers in the difference column. This learner was starting to see links between this investigation and a previous topic. This is one of the elements of the Building Learning Power model.

Learners were asked to try to find this pattern for homework. They were reminded about the ‘Stuck Poster’ and we discussed the different strategies they could use to help themselves if they got stuck. I suggested that they wrote down anything they spotted and tried to explain it, if they could, in a mathematical way.

During the following lesson I asked the class for any observations they had made. The first learner to respond pointed out that because they all end in zero they must

be multiples of ten. On collecting in their homework, I observed that twenty-one out of the twenty-five learners who had submitted work had made a similar observation to this. The second observation mentioned, which was noted down by eighteen out of the twenty-five learners, was that each number in the columns started with a square number. Twelve learners made the link that the numbers were all square numbers multiplied by ten.

Later on in the lesson I asked one of the learners about the different strategies he used when doing his homework. He said that initially he worked out the difference between each number in the table to see if it was constant. He linked this to the work we had done on finding the  $n$ th term of a linear sequence. When he noticed they were not the same he said he looked for other patterns. He said he spotted the square numbers straight away and that they had a zero after them so he concluded that they must be square numbers multiplied by ten. He commented that in many investigations we do in mathematics the answers often involve square numbers, making reference to the ‘how many squares are there on a chess board?’ problem that we did during their induction period into year ten. I asked him if he found it useful to think back over previous work and investigations that we had explored to help him with new problems. He said ‘I never used to, but a lot of the things we now do are quite similar so it makes it easier, especially in tasks like this, to see if you can use solutions from other problems to help you out. For this learner, there is some evidence that he is beginning to make links between different tasks, which is one of the aims of the Building Learning Power model.

Apart from the above two observations, no-one had been able to see any further links. I made the decision to give them some hints to see if they could make a link between the size of the grid and the size of the difference.

Initially I sat with Jonathan and James who were working together. When I arrived, they had written each number in the difference column as a square number multiplied by ten. i.e., they had written  $10 = 1^2 \times 10$  and  $40 = 2^2 \times 10$ . They then noticed that the number squared is one less than the size of the side of the square. They then went on to write that for a two-by-two square the difference would be  $2^2 - 1 \times 10$ . I asked them if this gave them the answer of ten. James quickly worked out that it did not and added brackets so it read  $(2^2 - 1) \times 10$ . After some thinking time Jonathan said that it was still wrong because the answer would be thirty. James agreed but they could not see where they had gone wrong. I asked them to explain what they wanted to write. James said 'you need to take one away from the size of the square, square it then times it by ten.' With support, they managed to write down the algebra to reflect this statement.

I returned to see Jonathan and James five minutes later. They had written all values in the table in the format above and had written the difference for an  $n \times n$  square as  $(n-1)^2 \times 10$ . They were not confident that this was correct. I asked them how they could check and Jonathan suggested that they could replace  $n$  by the numbers one to ten and check they got the same answers. They did this using their calculator and were then confident that their solution was correct. It was clear that both learners were proud of their achievement and they later went on to present their solution to the class. They demonstrated the ability to spot a pattern, write it

mathematically, make a conjecture about the rule and then test it for the data they had. Although they did need prompting to write the algebra they demonstrated resilience with this task.

During this session, I also observed Olivia and Polly. Both of these girls are able learners who are predicted to achieve a GCSE grade in mathematics at least two grades lower than those in their other subjects. Most of their other subjects are predominately essay based and rely on them analysing text or sources and writing about them, often having to give their own opinion. I have spoken to them previously about their views on mathematical learning. What they dislike about learning mathematics is that they cannot get better at it by just reading. If they want to get better at history or sociology they need to read more articles or books to find out more about the topic they are studying. In their opinion, it is easy to get high marks if you do some extra reading on the topic being studied. They feel that mathematics is different. They have tried reading revision guides and textbooks but have found it does not have the same impact that it does in their other subjects. They say that they usually understand a topic in lessons but as soon as they go home they have forgotten what to do. They look at their notes but they mean nothing and they cannot follow what they have written. They say they have accepted that they will never be good at mathematics. Interestingly, during a discussion at parent's evening both of Olivia's parents mentioned how they could not do mathematics at school and found it quite amusing. Perhaps the mother's anxiety has been passed on as predicted by Beilock et al. (2010).



When I approached the girls their first comment was that they could not do it and it was too hard. It was clear from their paper that they had made little effort in trying to solve the problem. I asked them what they had found so far and they repeated the observation made by the class. I asked them how they knew it had been multiplied by ten. Olivia was quick to say it was because they ended in a zero. With a bit of prompting they reluctantly wrote down that each number was a square number multiplied by ten. I asked them if they could see a link between a size of the square and the number that was squared and after a bit of thinking time Polly said that it was the square size take away one that was squared. They were quick to come up with a rule and wrote down ‘To get the difference you take one away from the size of the square, square it and then multiply it by ten’. I asked them if it worked and they confidently checked it for each sized square on the table. I then asked them what the difference would be for an  $n$ -by- $n$  square. They did not understand what I was asking them so I posed the question ‘How would you work out the difference for a two-by-two square?’ They confidently responded with ‘you would take one away from two, square the answer and then multiply it by ten’. We repeated the same for a three-by-three, a four-by-four and a five-by-five square. They confidently gave an answer each time. I then said ‘What about an  $n$ -by- $n$  square?’ They both paused for a bit before Polly said ‘I don’t get al.gebra’. After a bit of encouragement, they were able to give the rule for a  $n$ -by- $n$  square using words but struggled to write it down using algebraic notation.

On reflection, there were many positive moves towards learners developing some aspects of mathematical resilience seen in this intervention. A large proportion of

the class were able to persevere with the task. To do this they were using links with other investigations and topics we had worked on, looking for patterns and starting to check their conjectures were working for the data they had. Very few learners managed to get to the rule  $(n-1)^2 \times 10$  without support but they were willing to try. When learners got stuck, the majority now had some strategies to support them; for many this was working with others.

The area where I saw the least progress being made towards being mathematically resilient was with Polly, Olivia and two other girls, Kerry and Laura, who all faced similar barriers towards mathematics. Discussions and their actions showed that they had low levels of self-efficacy in mathematics, which I believe is down to a fear of mathematical notation. They are good, when prompted, at explaining their findings verbally but struggle when asked to use mathematical notation to explain the same thing. To help address this I came up with a further intervention, which focussed on their needs. It was designed to build their confidence and allow them to see that they can successfully complete a mathematical task by using their skills from other areas of the curriculum.

#### **4.9 Intervention eight: the history of mathematics**

Although the whole group took part in this intervention, it was primarily aimed at addressing the needs of the four girls mentioned in the previous intervention who lack confidence in mathematics. Previous lessons indicated that they are good at thinking through mathematical problems but struggle to convert their thinking from words into mathematical notation. Despite this, I do believe that if we can

overcome their fear of working with mathematical notation they will make significant progress in this subject.

For this intervention, I wanted to utilise the strength and perseverance demonstrated by the girl in their other subjects to help them see that they can do mathematics. To do this each learner chose a mathematician and wrote about three different things. These were to:

- talk about their life,
- give an overview of the work they did in mathematics,
- describe one piece of mathematics they worked on in detail.

Before the task was introduced, I spoke about the GCSE mathematics examination and how on some questions, marks would be awarded for the quality of their written communication. This is something this class find challenging. I explained that during this task they would have to research a mathematician using a variety of online resources (some web links were given to them as a starting point) and then consider the three points given above in their own words. I emphasized it was important that they understood what they had written and that they would need to be able to describe the piece of mathematics described in bullet point three to the class.

Many learners were reluctant to begin with many comments heard about this being pointless but the girls were quick to select their mathematicians and get researching. Interestingly, out of the twenty-seven learners that were present for

this task, twenty-five selected Pythagoras as their mathematician, with many writing in their journal that they had selected him because they had already heard of him.

The work submitted was of varying quality. The four girls all submitted a good piece of work that was detailed and well researched. All four had selected Pythagoras and were able to describe his theorem clearly and explain it using diagrams. They had given examples of how to use the theorem, which were clearly annotated explaining what they were doing at each stage.

On reading their journals, I found that all of the girls were positive about the task. Three of them commented that they were not sure how this was going to help them in their GCSE exam but they had enjoyed researching about Pythagoras and by knowing a bit more about his background it made them appreciate and understand his theorem better than they did when we covered it in class. One of the girls spoke about how she did not realise that Pythagoras' theorem could be used so much in real life. She commented about how her dad, who was a builder, could use the theorem to work out the length of fascia boards without climbing up a ladder.

Informal conversations after the task confirmed that the girls enjoyed doing the task. Polly was very positive about the fact that she could choose the mathematician herself and enjoyed reading about his life and his work. She said she found this type of work easy because it is what they needed to do a lot in history and sociology and she said it was good that the class were learning mathematics in a different way to normal.

Some of the other learners were not so positive about the experience when writing in their journals. Many commented that they did not see the point in doing this and would rather be doing ‘proper’ mathematics. One learner, whose GCSE English grade is predicted to be two grades lower compared to his mathematics grade, did not enjoy the task because he struggled with reading and writing and mathematics is usually the most accessible subject for him. He said that all other subjects have too many words in the questions, so he finds it difficult to understand what he needs to do. He said he finds reading and working with numbers and symbols easier.

This intervention received a mixed response from the group but it had the desired impact on the four girls. It confirmed to me that they had the ability to understand and explain mathematical concepts effectively.

#### **4.10 Intervention nine: Real life problems**

This intervention followed directly after the previous intervention and was designed around the needs of the four girls. The intervention looking at the history of mathematics had given them a confidence boost so I wanted to use this boost to encourage them to focus on working with multi-step problems covering different areas of the syllabus. An example of this type of question is given below in Figure 17.

Here is a plan of Martin's driveway.



Martin is going to cover his driveway with gravel.  
The gravel will be 6 cm deep.

Gravel is sold in bags.  
There are  $0.4 \text{ m}^3$  of gravel in each bag.  
Each bag of gravel costs £38

Martin gets a discount of 30% off the cost of the gravel.

Work out the total amount of money Martin pays for the gravel.

Figure 17: An example question taken from Edexcel Higher Paper 2 June 2014.

This question is worth five marks. Learners are required to carry out conversions between centimetres and meters, work out the volume of a cuboid, carry out division with suitable rounding, work out cost and apply a thirty percent discount. From past experience, I know that learners find this type of question challenging, possibly due to the lack of structure in the question. This was evident for the research group in the mock examination that they completed toward the end of their first year of the course. Many learners failed to score more than one mark in a similar question because they found it difficult to identify which techniques they had to use. My hypothesis is that this is down to the tendency of mathematics resources to work with only one skill at a time and avoid questions that lack structure and require the use of techniques from different ‘chapters’. Although many textbooks do now have this style of question towards the end of individual exercises, I have seen the learners use avoidance tactics such as slowing down on

‘easier’ questions so they can avoid having to complete these questions because experience has shown this style of question to be difficult.

Before I introduced this intervention, we spent some time recapping the different strategies we could use when we got stuck. Learners were able to refer to the ‘Stuck Poster’ on the wall and it was clear from the discussion that different learners were developing an understanding that they should use this poster as their first reference point when stuck. Following this discussion, I presented a problem on the board. This is shown below in Figure 18.

**Example**

Adam runs a coach company. He has 6 small coaches, 4 medium coaches, 3 large coaches and 1 double-decker coach.

The table gives information on how many passengers each coach can seat, the cost of hiring the coach and a driver for a day, and how many of these coaches Adam owns.

Adam's Coach Company			
Coach type	Number of seats	Cost of hire	Number owned
<b>Small</b>	25	£100	6
<b>Medium</b>	38	£110	4
<b>Large</b>	54	£120	3
<b>Double-decker</b>	78	£140	1

Rachel wants to hire some coaches from Adam to take 222 people out for the day.

What is the cheapest way for Rachel to do this?

Figure 18: Example question taken from Edexcel GCSE mathematics A Linear Higher: Pupil book.

When I shared the question, I made it clear that at this stage I did not want them to carry out any calculations but instead think about they were going to tackle this problem. Following this, I asked them to share any ideas they had. Billy put his

hand up first and suggested that it would be best to use the double-decker buses first followed by the large, then medium and small. He said this would keep the cost down because the larger the bus, the cheaper it is per person. Abi put her hand up and added to Billy's comment. She said that although you need to start with the biggest bus possible you need to ensure you do not have too many empty seats so you may need to have a double-decker and a small bus. Billy acknowledged that he agreed with her.

Following this Tom put his hand up and said you need to be careful about rounding your answer. He said that if you only had large buses you would need just over four to seat all two hundred and twenty-two people. If you rounded this down to four buses like you normally would then you would not have enough seats so you always need to round up. This comment from Tom demonstrated that he was thinking carefully about taking care when using mathematics for real life applications. He was able to see that a rule we use for rounding is not always appropriate to apply in all contexts.

Following this discussion, we worked through the problem as a class. Learners were happy to give ideas as we progressed through it. As we worked through the solution I talked about the requirement for the 'quality of written communication' marks in the examination and we looked at different ways we could present our solution so it was easy to follow and had all that was required to gain these marks. The first question I gave them is shown below in Figure 19.



Here are the rates charged for Mr Pitkin's telephone.

Line rental	£29.36
Daytime cost	4p for each minute
Evening and weekend	3p for each minute
To mobiles	11p for each minute
International rate (anytime)	8p for each minute

Here are the details of calls made by Mr Pitkin in one quarter.

Type of call	Minutes
Daytime	78
Evening	312
To mobiles	42
International rate	25

Calculate Mr Pitkin's telephone bill for that quarter.

Figure 19: Question taken from Edexcel GCSE mathematics A Linear Higher:  
Pupil book.

I purposely chose this question because it was related to something that the class may need to calculate for themselves in the near future.

At the start of the intervention, I sat next to Kerry and Laura who were working together, two of the girls who enjoyed the intervention on the history of mathematics. From marking their mock examinations, I knew this style of question was something they struggled with. Kerry read the questions aloud. Laura had a set of highlighters out and was highlighting what she felt were key bits of information. She had highlighted the costs. Kerry asked what line rental was. Quite a few other learners were asking this question, so I spoke to the whole class explaining a bit about how phone bills were calculated. It appeared that no-one really understood how phone bills were calculated and they assumed that you just paid a fixed fee each month as you did on a mobile phone.

After this whole class input, Kerry suggested that they put the information into a table to make it easier to see how to calculate the cost for each different type of call. Laura got her calculator out and, for each different type of call, multiplied the number of minutes with the cost to get the total cost. She added it up and said he would be billed £1910. I asked Kerry is she was happy with what Laura had done and she initially said yes but then realised that she had missed out the line rental. They added this on and came up with the answer of £1939.36. I then asked if they were happy with the answer and they both said yes. I suggested they read through the question again to double check they were happy with it all. They did this, decided they were correct having checked all of the calculations then went onto the next questions. I stopped them and asked them if £1939.36 was the size of answer they were expecting. They acknowledged that it seemed expensive but they said they had checked their calculations so it must be correct. I asked them to read carefully what they had put in the table. Kerry then realised they were working in pence so she divided the answer by one hundred to get a bill of £19.3936. Laura said this must be wrong so they went back to the table, recalculated everything in pence then came up with the correct answer.

Although they had made a mistake, both girls persevered until they got the correct answer. They were able to find the key information, presented their solution in a logical way and, with a bit of prompting, identified what error they had made. The error they made did not stop them from moving onto the next question, which they successfully attempted. In the past, these girls have become quite negative about mathematics whenever they got stuck. In their journals, they both commented on

the mistakes they made but instead of describing it in a negative way, they reported it as a silly mistake. Kerry commented that it was quite funny that they had thought the phone bill would be nearly £2000.

We spent two lessons working on problems similar to this one. Progress was not always fast but I found that by reminding learners to read the question carefully, and by asking them to discuss what the question was telling them, they were able to have a good attempt at the question. The key issue I found was that the learners were not checking their answers were of a sensible size. A large majority did not notice on a least one question that the units were not consistent (for example some measurements were given in meters and others were given in centimetres), which resulted in very large or very small answers. Three different learners in the group commented on this in their journals. They said that they kept on getting the wrong answer because they did not check the units were consistent. However, all three also mentioned that because they made this mistake so often in the lesson they started checking the units when they were discussing the question to help avoid them making this mistake.

Two weeks after this intervention, learners were given a past examination question to complete independently. The question was worth five marks and I marked it using the mark scheme provided by the examination board. The mean percentage for the class was forty-four percent, which was slightly below the national mean of forty-six percent of all learners who sat this paper and twenty-four percent above the mean percentage achieved by the comparison group. Lack of data on the standard deviation of these results means I am unable to carry out any

statistical test on the data and the conditions under which the examination question was completed were different, which makes formal comparisons invalid. However, the journal entries were generally positive about this question and no-one scored zero marks, everyone having made a good attempt at the question. The most common error made was the learners not ensuring units were consistent.

After the learners sat the examination question and had their results back, I spoke to Kerry to discuss in more detail her views on this intervention. The first question I asked her was how she found this type of question before we looked at them in class. She mentioned that she did not like them and tried to avoid them as much as possible. I asked her how she did this and she said she would spend longer on the questions towards the start of the exercise so she did not have time to do them or would skip them and move onto the next questions that looked easier. When asked what made a question look easy she said that an easy question was one that you knew what to do straight away, saying that ' $24 + 34$ ' is easier than 'There are twenty-four people on a bus. At the bus stop another thirty-four people get on. How many people are on the bus?' She said the more words there are, the harder it usually is because you need to think harder about what you need to do. She also mentioned that in English, questions are often based on opinions and interpretations so provided you have read the novel you are usually able to come up with an answer but in mathematics, if you cannot work out what you need to do, you need to leave it blank. She also mentioned that in English once you understand the plot and characters it is easy to answer questions but in mathematics, it is all about memorising procedures to follow. If you forget the next step, you are stuck. I asked her how she learnt mathematics at home. She said

she usually learns how to do it from a revision guide but knows that it is really better to do many similar questions to help you learn how to use the technique. Interestingly she knew how to work at mathematics, one of the aspects of mathematical resilience, but chose not to. Perhaps this was down to her feeling you cannot make a mistake when reading a revision guide but can when doing questions.

I then returned the focus of the informal discussion back to the intervention. I asked Kerry how helpful she found the intervention. She said that when she found out what we were going to do she was dreading it because she could not do this type of questions. However, she did say that reading the questions over carefully and discussing it did help her a lot. She said she felt she was getting better at doing this style of question as the lesson progressed. She said that the most important thing she learnt was to check the units were all the same otherwise you would get an answer that was not sensible. I asked her if her view of this type of question had now changed. She said she finds them easier now but still does not like them. She would still prefer a question that told you exactly what to do.

Although Kerry worked hard, she felt that she would never be good at mathematics. It became clear to me in this informal discussion that she would much rather remain within her Comfort Zone. However, she is beginning to show that she is willing to leave this Comfort Zone to allow her to improve, perhaps a sign that she is starting to move towards accepting elements of the classroom culture I am trying to create. I personally feel one of her biggest barriers that she needs to overcome is to move away from trying to gain an instrumental

understanding of the work and try to achieve a relational understanding of the topics. It is clear that this is how she operates in other subjects. In English she mentions the need to understand the plot and characters to answer questions but in mathematics she tried to memorise procedures. This is perhaps why she is currently working at two grades lower in mathematics compared to English. The interventions over the eighteen-month period have tried to address this but maybe this time is not long enough to change the classroom culture that has been embedded over the previous nine years.

#### **4.11 Summary of interventions**

Each of these nine interventions was planned as a cycle of action research. The outcome of the previous intervention gave a focus for the next one and the learning objectives and the tasks were planned with the aim of meeting the new desired outcome. Each intervention was tailored to the needs of the group, which included covering the skills and content required for the GCSE exam. These interventions took place over an eighteen-month period. In between these nine key interventions, I continued to teach the group for four hours per week. During this time, I also made use of split-screen objectives with a similar focus to the interventions, mostly in line with 'being stuck'.

The table below summarises the key findings from each action research cycle or intervention.

Action research cycle/ intervention	Key findings
1 Plotting coordinates	In this cycle, I learnt that making use of the split-screen learning objectives could focus the learners' attention on specific skills. In this case, learners changed the way they behaved when they needed support. I found that this intervention was missing applications to real-life, something that I prioritised when planning the next action research cycle.
2 Planning a trip	In this cycle, I learnt that learners often do not notice when they are using mathematics in real-life scenarios. I also discovered that many of the learners lacked perseverance when they became stuck; one setback could result in them not wanting to continue the task. This is something that I aimed to address in the next action research cycle.
3 Drawing straight line graphs	In this cycle I learnt that the careful use of scaffolding can support learners to develop their noticing skills and perseverance. By having to think about the technique and how to apply it independently, there appeared to be a positive impact on their ability to answer GCSE questions. One of the key concepts I wanted to explore in the next action research cycle was the use of scaffolding to support the learner whilst carrying out an investigation.
4 Investigating straight line graphs	In this cycle, I learnt that scaffolding can help keep the learners engaged in an investigation. When the scaffolding was removed in

	<p>the second task, the learners lost engagement in the task. Although it appeared that they were unsure what to do, this was not the case. Discussion revealed that the learners did know what they had to do but lacked confidence in their own ability. In the next action research cycle, one of the aims will be to look at ways of increasing levels of self-efficacy.</p>
5 The data handling cycle	<p>In this action research cycle I learnt that working in pairs can support the increase levels of self-efficacy when learners are able to support each other but once they encounter difficulties they quickly lose the motivation to continue the task. For this reason, I decided that during the next intervention cycle there would be an element of choice as what techniques learners could use to solve the problem.</p>
6 Statistical investigation	<p>In this action research cycle I learnt that making use of low threshold/high ceiling tasks that allow learners to select their own success criteria , giving them the choice over which techniques to use, can help increase their confidence. However, it does give them the opportunity to avoid anything that may move them into the Growth zone. For the next action research I decided to use a similar idea to this cycle but without the opportunity for learners to avoid techniques they found challenging.</p>
7 Exploring the number grid	<p>In this action research cycle I learnt that by constantly reminding the learner about the Stuck Poster they were starting to refer to it automatically to support them when stuck. My focus now moved onto a small group of girls who were achieving significantly lower in mathematics compared to their other, essay-based subjects. I</p>



	needed to find another way of increasing their self-efficacy in mathematics.
8 The history of mathematics	In this action research cycle I learnt that for some students, making use of skills they could successfully apply in other subjects could help with the learning of a mathematical topic, in this case, by researching a topics and presenting it in the form of an essay. As a result of this task, the learners involved could confidently use the mathematical skill learnt to solve exam style questions. For the next action research cycle, I wanted them to use their newly found confidence to solve multi-step problems.
9 Real life problems	In this last action research cycle I learnt that by persevering as a teacher and continually encouraging the learner to read the question carefully and checking their answer is of a sensible size, their confidence and ability in carrying out multi-step question can grow.

Figure 20: Summary of the findings from each action research cycle.

Having now reported on what I observed during the intervention, the next section will report on the key points of the discussions that took place with students during the period of the action research project.

## **Section 5: Learner discussions**

Before, during and after the research period, I asked learners to take part in discussions. Prior to the research beginning, I selected four different learners who agreed to have informal interviews with me before the intervention started, at the end of the first year of research and after the research was completed. Each learner was selected for a different reason. I reminded all learners of their right not to take part. The fourth learner's data has not been used; despite reminders that I wanted her true feelings, I felt that the answers she was giving during the discussions did not match observations or what she was writing in her learning journal.

The first learner I selected was Martin. When I started teaching him, he was very vocal about how much he disliked mathematics and felt it was wrong that he was forced to study it at GCSE. He did not see how it was relevant to his future. Although he would complete most of the work set, he was at his happiest in lessons when working on consolidation exercises where he had to complete many questions that were all similar.

The second learner I selected was James. James' predicted GCSE grade in mathematics was two grades higher than in any of his other subjects. Testing showed that he had a reading age of a seven-year-old, eight years younger than his age at the start of the research project. James is confident in mathematics and can explain his ideas well orally but struggles in showing his working out, something that is required for many of the marks that are available in the GCSE examination.

The third learner I selected was Polly. Polly is a learner who is currently predicted to achieve mostly grade As in her other GCSE subjects but is only likely to achieve a grade C in her mathematics. She is a hard working learner who will always try with tasks but often seems stuck and has in the past started crying when unable to complete a task in class or for homework. At the 'settling in' parents evening that took place just before the research period began, her mother explained that she becomes very anxious about her mathematics homework and gets frustrated when she feels stuck.

The discussions were semi-structured with some set questions asked in every conversation with the intention of adding in other questions that I felt were relevant to help me understand further the progress they were making to become a resilient learner. The set questions were as follows:

1. What progress do you feel you are making in mathematics?
2. Can you give me an example of a recent lesson/ activity where you feel you made a lot of progress. Why do you think this was the case?
3. Can you give me an example of a recent lesson/activity where you feel you did not make a lot of progress. Why do you think this was the case?
4. What do you think you need to do to get better at mathematics?
5. What do you think are the most effective strategies to use when you 'get stuck'?
6. What is your motivation for doing well in mathematics?

The key findings from the conversations had with these learners are detailed below. I did speak to other learners after interventions and any relevant feedback from these discussions is included alongside the interventions in section 5.4.

### **5.1 Responses from learner one, Martin**

The first conversation with Martin took place when he was relatively new to the school so his views were mostly based on his experiences at his previous school. When questioned about the progress he was making he mentioned it was poor and passed the blame onto the ‘rubbish teacher who couldn’t control the class’. He said because he was in the bottom set, they could do what they wanted, the best teachers were teaching the higher sets. He mentioned that they did nothing new in lessons and only recapped things that they had done in previous years. The two main topics he cited were solving equations and adding fractions, which he said were both pointless because he will not need to use them once he left school. Perhaps this was a way of making himself feel better about not understanding these topics.

During the second interview, he was a bit more positive about the progress he was making. He was pleased that he was no longer in the ‘bottom set’ and liked how we had been looking at new topics that would be required for him to get the grade C he needed for the next steps in his career. This desire to study new topics is one aspect of the culture I was trying to create. He still said many of the things we were doing were, in his opinion, pointless. He said that he now understood how to

calculate percentages without a calculator, which is something he had struggled with since he moved to key stage 3.

Once again, Martin mentioned setting as one of the factors that affected his progress in mathematics, although this time, it was in a positive way. His move to set three out of four in his half-year group had made a difference to his attitude towards mathematics. Later he brought up the fact that he liked covering new topics instead of going over the same topics again. He commented that if he did not understand a topic in year seven and eight then he would be unlikely to understand it in later years so he would rather learn about something new that he had a chance of understanding. From my observations of Martin during lessons, my view is that moving away from the bottom group did have a big impact on his confidence and achievement. Whenever there was a class test or mock exam he was always concerned that he would be 'moved back down' a set if his result was bad. To prevent this happening he started attending after-school sessions and, in my opinion, secreting it from his friends, making use of online learning resources at home, which I saw by using the teacher login to the site that monitored usage.

By time the final interview came, his first comment when asked about his progress was that he was shocked, in a positive way, that he got a high grade D in his mock exam. He said that he now felt he had a good chance of getting a grade C, which would allow him to get on the college course he wanted. He said that he felt a lot more confident with algebra now and that although he thinks the questions are still difficult, he is no longer scared of tackling the 'wordy problems'. It was interesting to note that, by this point, discussions and his actions revealed that his motivation

for doing well was extrinsic, coming from the grades he needed to get onto his college course.

When first asked about a recent lesson he felt allowed him to make a lot of progress, he referred to a lesson taken by a supply teacher at his previous school. He said that the lesson was good because the teacher gave lots of examples to the class at the start of the lesson and then gave them a worksheet to go through, which was straightforward and he managed to get them all correct. This was evidence of his enjoyment of staying in his Comfort Zone. He felt that he was making good progress because he got them all correct, even though they lacked any challenge.

During the second interview, his response was very similar, however during the third he referred to a different style of lesson. He said he now liked a topic that was difficult to start with, but became easier with practise. The example he gave was when we introduced trigonometry in right-angled triangles. He said to start with there was so much to remember, but by the end, it became easy and he was able to solve problems linked to the construction industry, the course he was going to study at college. He indicated that he enjoyed moving outside of his Comfort Zone and although challenging at first, he identified that this is what allowed him to make progress. He also commented positively on the link with real-life applications. Formerly, he has displayed negative views towards mathematics because of its perceived irrelevance to his future life and career.

When asked about an activity that he did not make good progress in, he initially talked about lessons when the teacher did not give enough examples and he felt

the task got too hard too quickly. When this was the case, I saw his motivation and effort towards the task decrease quickly. In the second round of discussion, he talked about investigations when he had to discover a rule himself. He referred to the lesson on investigating equations of straight lines. He felt that he wasted a lot of time working it out for himself, when I could have told them the rule and given them more time to practise questions on it. This is perhaps, related to his extrinsic motivation towards learning mathematics; his motivation is to get a good pass at GCSE and not related to getting better at mathematics. In the final interview, he moved away from describing the structure of a lesson and more towards topics. He stated that he did not make as much progress as he would like when we were looking at probability trees without using a calculator, because he struggled with his times tables. This discussion and observations in class indicated that he is beginning to realise that lessons that allow him to stay in his Comfort Zone were not the most effective for maximising the progress he made. He was able to identify why he did not make progress rather than passing the blame onto the teaching. This was the first time I had seen Martin as a responsible and reflective learner.

For the fourth question, which related to what he had to do to get better at mathematics, his comments in the first discussion were about setting and how being in a higher set would make it easier for him to get better. In each of the following discussions, he was able to pinpoint some topics that he knew he had to work on.

The last of the main questions was about strategies he used when he got stuck. In the first interview, he related being stuck to poor teaching. In the second discussion, he mentioned the Stuck Poster and was able to refer to some of the strategies we had listed. In the final discussion, he was able to elaborate much more and gave examples of when he had become stuck and how he had managed to become unstuck. He did say the most effective technique for him was working with a peer because he found that when he discussed a problem with someone it became clearer what the problem was asking him to do. Evidence of this was also seen in his learning journal where he initially commented that when stuck he would ask for help whereas later there was evidence that he had tried many of the strategies given on the Stuck Poster. From my observations which took place in lessons, Martin was not very good at reading the question or problem carefully which resulted in him being unable to understand the task. When working with someone he had to discuss his ideas, which meant he had read the problem more carefully. This improved his understanding of what he was required to do, which in turn meant he was able to have a better attempt at providing a solution. I also observed that he usually had good ideas about strategies to use, but often needed the reassurance from someone else that these could work before he would try them out for himself.

When asked about motivation, there was also a clear change in his answer over the period of the three discussions. In the first discussion, he made it clear he did not want to study the subject and said being made to study mathematics was wrong. By the second discussion, he said that it was important that he did well because it was important for his future but was unsure how it would be useful. In



the last discussion, he was motivated to do well but this seemed to be down to him needing to get a grade C or above for his college course. He could see that a couple of topics such as trigonometry and Pythagoras' theorem would be useful to him in his career in construction but struggled to see the relevance of many other topics.

During the research phase, I did observe a large change in the attitude and motivation of Martin. Because he was one of the louder learners in the group who was not afraid to say what was on his mind, I felt that I gained a deep understanding of how he was learning at each stage of the project. Toward the end of the research phase, he was starting to show elements of a becoming mathematically resilient. He was seen to be persevering more when he became stuck and he had realised the need to move into the Growth Zone in order to progress.

## **5.2 Response from learner two, James**

As mentioned in the introduction to this section, James is a learner who was predicted to achieve at least two grades higher in GCSE mathematics compared to his other subjects. Discussions with his other subject teachers indicated that this is down to him having a reading age of a seven-year-old. Although he had a scribe and reader to support him in his examinations, he did not get the support needed with reading and writing outside of school where he was expected to carry out his homework and background reading for subjects. This was most notable in English, history and science.

James is a quiet learner and my main concern about the discussion with him was that he would not be honest and instead tell me what he thought I wanted to hear. To help reduce the impact of this, I reminded James at the start of each discussion that I wanted honest opinions and I made sure there was evidence of what I report here in his journal and observed in lessons. It is worth noting that the majority of his journal entries were very brief, and many were partially illegible.

During all three discussions, James indicated that he felt he was making good progress in mathematics. However, the way he described good progress changed. Initially he said he was making good progress in mathematics because he was doing better in mathematics compared to his other subjects. During the second discussion, he based good progress on understanding new topics. He talked about how quickly he had picked up new topics such as trigonometry and histograms, which he knew were grade B and grade A topics. Because he was targeted a grade C in his GCSE mathematics he felt this was the best indication of his progress. In the final discussion, he referred purely to predicted grades when describing progress. When asked if there were any specific topics in which he felt he had achieved well, he mentioned that he had done well in some of the grade B topics. He knew this because of the marks achieved in his homework tasks. It is clear from discussion and observations that James enjoys the challenge of working on topics that are above a grade C and I have seen an increased satisfaction when, in his opinion, he has mastered one of these topics.

When asked the second question, about a lesson in which he feels he made good progress, he initially talked about lessons where he is able to work through a set

of questions that go from easy to hard. To him, good progress was about getting all or most of the answers correct. In the second discussion, he gave a similar response. I asked him if he felt he had made good progress in the intervention we had recently done which investigated straight line graphs using dynamic graphing software. He said he enjoyed it and did manage to discover what the 'm' and 'c' stood for in the lesson but felt it took him a long time to discover so would not describe it as good progress, implying that he believes working quickly is a sign of being good at mathematics. In the final discussion, he mentioned that he thought he had made good progress when we were exploring the number grid and he was able to use his skills in algebra to prove that the difference was always the same for any square on the grid. During the number grid intervention, his reaction and excitement when he found he could use algebra to solve a number problem made this a 'eureka' moment for him. He was seen to be genuinely amazed that he could use his skills in algebra to solve a number problem and was very keen to share what he had found with me.

When asked about a lesson or activity in which he felt he made the least progress, his answer was consistent in all three discussions. To him, the lessons in which he made the least progress were the ones that involved the most reading and writing. He felt this was a barrier to his learning that was holding him back in his other subjects and got frustrated when his poor reading skills impacted on his mathematics. This was seen in observations, especially when we looked at the history of mathematics and worded problems and in mock examinations.

The next question was about what he felt he needed to do to get better at mathematics. In all three discussions, he mentioned the 'wordy' questions. He said when he works with someone, he is usually able to work out what he needs to do but if he is working independently, he finds it difficult. He said that main problem was that these questions often had words that confused him, for example ethnic names. He found he spent so long trying to understand what was being asked that he did not have enough time to answer. As a consequence, he often left out worded problems in examinations.

In the second discussion, I asked him what strategies he had for when he got stuck. He mentioned that he has tried highlighting the key bits of information but he struggles to identify what these are. He did try to read over the problem a few times to help him understand what the question was asking, but often struggled. He also said he had worked through the Stuck Poster and found that working with a peer was the most effective strategy when stuck.

In the discussion that took place soon after I had done the intervention that looked at real life problems, I asked him if this had made a difference. He said he felt more confident in solving worded questions but said that he still struggled at times to work out the key bits of information. He did find it useful when the question was read out to him two or three times, which is what will happen in examinations, but found that the discussions about possible strategies was the most effective way to stop being stuck, something that will not be allowed in examinations.

Discussions with James, observations of him working in lessons and looking at his work showed that James' motivation for doing well in mathematics was because it was, in his opinion, the only subject he was going to get a good grade in, due to him being able to succeed despite his low literacy levels. He mentioned on more than one occasion that mathematics was his favourite subject except when the topics or task involved a lot of reading.

### **5.3 Response from learner three, Polly**

Polly is a learner who shows many of the signs of someone suffering from mathematical anxiety such as avoidance strategies, spending excessive time working on neatness, writing out the full question into her book before answering it and being absent on assessment days. Prior to the research beginning I was made aware of Polly's fear of mathematics and how she would often end up crying at home because she was unable to do something. A discussion with her history teacher suggested that Polly is a resilient learner in history but in mathematics she is not. Here, she prefers to focus on revision of work covered previously and enjoying working on consolidation tasks when she has to complete many similar questions. She is expected to gain a C in mathematics despite most of her other subjects predicting an A or above.

In the first discussion, her response was she was making 'very little' progress in mathematics. She said she tried really hard and always read through class notes and revision guides but felt it made no difference to her ability in mathematics. She was frustrated because she was using the same strategies to progress her

learning in mathematics that were successful in other subjects. Her response during the second discussion was very similar. During the third discussion, she said she felt she had made excellent progress when we looked at the history of mathematics. She had chosen to research and write about Pythagoras and she said that following this, she understood Pythagoras' theorem for the first time. She said she found writing the theorem in her own words helped and she liked the visualisation of the theorem she found on the internet. In this visualisation, there was a square drawn on each side of the right-angled triangle and the two smaller squares were broken down so that they fitted exactly into the larger squares showing that the sum of their areas was the same as the large one. She did comment that she remembered doing something similar in class when we cut up the squares to show they were equal but she admitted that at the time she did not see the link between this and the theorem. This ability to visualise can help develop mathematical thinking (Cuoco et al., 1996).

From early in the research project, I had suspicions that she tried to learn mathematics through developing an instrumental understanding of the topics. Evidence of this came out in all three of the discussions I had with her. When asked what style of lesson she felt allowed her to make the most progress, she described consolidation lessons where transmission teaching was used (Swan, 2005) and they were given lots of examples, then given a worksheet to complete with many similar questions. She said by getting the answer correct she knew that she was making good progress. When asked what style of activity resulted in her not making much progress, she mentioned investigation work and questions when they had to make use of more than one topic, especially one that we had not

covered for a while. This discussion added evidence that she preferred to gain an instrumental understanding of the topic through rote learning. It also gave evidence to support my observation that she enjoyed being in the Comfort Zone. When she left the Comfort Zone, she felt very uneasy, perhaps entering the Anxiety Zone almost immediately, which resulted in her becoming anxious about the work.

In a follow up question, I asked her if she felt rote learning helped her understand topics better. She commented that it helped her make good progress in lessons but admitted that when she came to do homework or revise for a test on the topics she would look back at her class notes and they would not make sense to her. Further questioning indicated that in her other subjects she would carry out research online to deepen her understanding and synthesise different sources together to give her a good understanding of the concept or theory. She did admit that it would be better to do this in mathematics but she did not know how; this is something we have been aiming to address in this research project. Although she knew her method of learning mathematics was not effective, she appeared scared to try to change it. She knew that her method of learning mathematics would usually allow her to succeed in her class work, so stuck with it, even though she suspected it was holding her back in the long term.

Polly knew that her grade in mathematics was important but did not appear to understand how she would use mathematics in the future. She often mentioned that she needed a C in GCSE mathematics to study A-levels and get into university

but seemed unaware about the usefulness of the skills developed by studying mathematics.

When asked about 'getting stuck', she became quite nervous. In the first discussion, she talked about the strategies she used. As mentioned earlier, she was unaware of why these strategies worked in others subjects but not in mathematics. During the second discussion, she talked about the 'Stuck Poster' we had on the wall of the classroom and said she had tried quite a few of them. She found working with a peer to be the most effective way to get through a problem. She did mention she found it easier when she worked with someone who is not in her group of friends. Observations of her group of friends and looking at their exercise books and assessments showed that many of them worked and thought in the same way to Polly and faced similar levels of mathematical anxiety.

In the final discussion, she referred to the lesson on the history of mathematics and said that it did give her a confidence boost and she would quite like to try writing about key topics in this way to help her understand what the topic is saying. I suggested she tried this with other topics although I saw no evidence of this happening.

Throughout the research period, I could clearly see that Polly was frustrated that she was not attaining as well in mathematics compared to her other subjects. Although she often joked about this, often referring to the fact that her mother was also 'rubbish' at mathematics, she seemed frustrated by her lack of progress. However, what I have observed and heard during the research indicated that her



fear of failing was stopping her from changing the way she has always worked even though she felt it was not effective. My hypothesis was that her methods were likely to get her the grade C she needed for her future. Any change could have put this at risk, which was not something she was willing to do.

## **Section 6: Analysis of findings**

In this section, I synthesise the findings from the discussions, observations and personal journals and consider how the evidence relates to the construct of mathematical resilience.

My personal motivation for undertaking this action research project was to help me further my understanding of how my practice could be improved to better develop mathematical resilience in my learners. In order to do this, I first had to understand the barriers faced by learners, which may prevent them from achieving a grade C in the GCSE mathematics examination so that I could explore strategies to overcome them. The barriers seemed to include members of the research group being unwilling to attempt questions that look unfamiliar and often using strategies to avoid encountering difficulties in their work. At the start of the research, I was unclear which aspects of mathematical resilience presented the biggest barrier for the learners and thus needed to be prioritised in my practice. Early on in the research project, I came across evidence in observations, learning journals and discussions that many different elements were barriers. However, the most dominant issues that the learners seemed to face were their inability to make use of appropriate support when struggling with mathematics and their fear of not being able to successfully complete a question. This was perhaps a result of them being encultured in practices of rote learning, procedural competency where they were not often required to think mathematically.

## **6.1 Learners demonstrating resistance to changing the culture of teaching and learning within the classroom**

Throughout the early stages of the intervention, I found evidence of the majority of the class resisting changes to the culture within the classroom. An example of this was seen in discussions when Martin said if he did not understand a topic in year seven and eight then he was unlikely to understand it in year 10. It also occurred in intervention three, when Kylie said ‘I can’t do maths’ and ‘I have never been good at it’ and Joey said ‘I used to try hard but I will never be good at it (problem solving) so I don’t see what the point is in doing them’.

Alongside this, there was evidence that when learners became ‘stuck’ or feared they may not be able to complete the task they avoided responsibility by passing the blame onto the strategies I was making use of. An example of this was in intervention one when Tom said ‘I wasted lots of time waiting for help so I didn’t get the picture finished. The worksheet was boring and a waste of time because I will never need to use coordinates in real life’. Again, in intervention two, Tom was progressing well but when he found out he had made a mistake he stopped working and ripped up the paper he was writing his solution on. In intervention four, only four learners in the class commented positively about the investigation. The rest gave reasons such as ‘couldn’t be bothered’ or ‘this is pointless’, which I interpret as an attempt to cover up their inability to complete the task and work within the classroom culture that I was trying to create.

The biggest aspect of resistance toward changing the culture away from rote learning and procedural competency seen during the early stages of the action research project was the expectation from learners that they should be able to complete a task with ease (Stigler and Hiebert, 1999). Consequently, when this was not possible, they used different strategies to cover up their inability to complete the task with ease. This observation follows the Growth Zone model developed by Johnston-Wilder et al. (2014), which was discussed in section 2.2. Learners showed they were happy in the ‘Comfort Zone’ but once they enter the Growth Zone, the area when learning occurs, they panic and try to return to and remain within the Comfort Zone.

## **6.2 Being Stuck**

My initial analysis of the group indicated to me that one of the biggest barriers to overcome was their view that being stuck was a negative thing. The evidence seen showed the majority of learners giving up when they encountered any difficulties thus removing the chance for them to work within their Growth Zone, something that is essential for their future mathematical development.

### **6.2.1 How learners initially dealt with being stuck**

In the first intervention, there was clear evidence on the learners’ over-reliance on asking the teacher or learning support assistant when they became stuck. Discussions and observations suggested that the learners had become accustomed to seeking the support of an adult as soon as they left their Comfort Zone.

Observation of learners working and discussions showed that by not having an awareness of different strategies to use, they moved directly from their Comfort Zone into their Anxiety Zone when the teacher did not tell them directly what they had to do next or exactly which step to take. Once in this Anxiety Zone many of the learners avoided continuing with the task.

During the interventions, I attempted to remove direct teacher support to encourage them to explore other ways of seeking help. An example of impact of this on progress was seen when Tom said ‘the teacher would not help me when I got stuck and made me ask another student. I wasted lots of time waiting for help so I didn’t get the picture finished. The worksheet was boring and a waste of time because I will never need to use coordinates in real life.’ In this comment, the learner was trying to blame his difficulties on the teacher and the task itself, questioning how it would help him improve his mathematical skills or be useful beyond school. More evidence of this was seen in intervention three when Kylie said ‘how are we meant to answer the question if he (the teacher) doesn’t explain it?’ and in intervention four when Martin said it would be better if I taught them what it did instead of ‘wasting time’ working it out for themselves. Both of these comments indicated that they had become over-reliant on teacher support and were not adapting well to being asked to use unfamiliar strategies to support themselves.

It was clear to me that one of the barriers to overcome was managing the learners’ change in expectation. After many years of asking for teacher support as soon as

they left their Comfort Zone, they would need support in being resourceful and developing other strategies they could use when this strategy was removed.

### **6.2.2 Learners learning to use the support available to them**

One of the key strategies I used throughout the research project, during the interventions and normal timetabled lessons, was the idea of using split-screen lesson objectives. These were discussed in section 2.7 as part of Claxton's Building Learning Power model (2004). The main idea behind split-screen lessons was to have two lesson objectives for each lesson; the first objective related to the mathematical content or techniques and the second related to a learning capacity. The main significance in this case was that the lesson objective related to the learning capacity dealt with knowing what to do when a learner finds that they are stuck. Both the mathematical and the learning capacity objectives were shared at the start of lessons and revisited at the end of lessons.

The strategy of using split-screen lesson objectives was first used in intervention one. The learning capacity objective for intervention one and intervention two was related to being resourceful with the emphasis on reading carefully the information given and thinking about different resource mechanisms you could use when stuck to avoid relying on the teacher telling you what to do. To give the learners the opportunity to make use of these strategies, for the intervention I selected tasks which I expected would take the learners out of their Comfort Zone.

Despite putting a lot of emphasis on being resourceful during the introduction to these intervention lesson, I saw very little evidence of this having an impact. In the third intervention, Kylie struggled to change the way she worked despite knowing the aim of the lesson. While in the early stages of explaining the task, she said ‘I don’t get it’. When reminded that I was not going to be helping them she said ‘How am I meant to do it if I don’t get what you have done?’ Similar negativity towards changing ways of dealing with ‘being stuck’ was seen during the first two interventions from Martin and other learners as discussed in section 6.2.1.

To help support this change, I introduced the creation and use of a ‘Stuck Poster’, a technique suggested by John Mason (2010). This Stuck Poster was created using suggestions given by the learners. Its aim was to provide visual reminders to learners about ways of coping with being stuck. When relevant, this was referred to when the learning capacity objective was shared with the class. I also made sure that the learning support assistant and I referred learners to the poster when they became stuck and asked for help. Mason (2010) suggests that this strategy can be effective provided that the poster is removed after a period of time to avoid learners becoming over-reliant on it.

When I first introduced it in intervention three, I gave time at the end of the intervention and the following class lesson for learners to add to it. Over time the poster grew in size. The final Stuck Poster contained:

- Ask the teacher for help
- Ask a friend for help

- Look back at your class notes
- Read a textbook
- Think about similar investigations we have done
- Read the question carefully
- Highlight key information
- Think about what maths is needed
- Can we use algebra to help?
- Don't give up- keep on going!

Evidence collected from my observations of the class indicated that the 'Stuck Poster' was the turning point in changing the learners' expectations of the role of the teacher in supporting learners when stuck. Prior to using the poster, learners had to think on their own about what other strategies they could use when the teacher would not tell them the answer. With the poster, they could be reminded to review it, which would give them ideas. Evidence of this was seen in intervention six, when Monica and Piers had forgotten to how to calculate the median. When I reminded them to look at the Stuck Poster, they looked at it and managed to select a strategy that allowed them to progress. In the same intervention Kylie, who reacted negatively in intervention three when I would not tell her the answer, was seen referring to the Stuck Poster, which reminded her that she could look at her class notes to help her out. I observed a similar change to Martin, who was also negative about the changing role of the teacher in lessons. In the interview I had with him and in his journal, he made reference to the 'Stuck Poster'. When asked what strategies he used when stuck, his response of 'ask the teacher for help' in the first interview had changed by the second interview to him



being able to discuss the different strategies on the ‘Stuck Poster’ and how useful he found them. In the interview, both James and Polly also mentioned how they had used the ‘Stuck Poster’ to help them out. Interestingly, all three of the learners who were interviewed referred to this ‘Stuck Poster’ without prompting during the second interview, with all of them saying that support from a peer was the most effective way of dealing with being stuck during mathematical tasks.

Although the poster remained on the wall for the majority of the research period, against the advice of Mason (2010), learners demonstrated that they had become familiar with the ideas on the poster and could refer to them in more formal examination, which took place away from the poster and also in discussions I had with them. The intention of the ‘Stuck Poster’ was to continually add to it, which was an adaption of the way Mason originally intended the Stuck Poster to be used.

### **6.2.3 The use of scaffolding the help build learner confidence in dealing with being stuck**

In intervention three and during normal lessons, Katie was seen to have an over-reliance on teacher input. Instead of needing support when stuck, she wanted teacher reassurance that what she was doing was correct before she would progress onto the new part of the problem. When this was not available, she was observed making use of strategies such as spending a long time drawing a neat table or copying out the question to avoid the need to move on. Similar strategies were observed in lessons when learners would slow down on the ‘easier’ questions in textbooks so they would not have enough time to tackle the ‘harder’ questions

that appeared near the end of the exercise. When it came to investigation work, I observed in many of the interventions that learners would ‘give up’ on the task when they were unsure. In intervention four, Billy and Martin had given up on the investigation soon after the support had gone. Discussion with them implied that this was because they were unsure about what to do next. By providing them with some prompting and reassurance, they were able to proceed to the end. In his journal afterwards, Billy wrote ‘the task was easy when the teacher was supporting me but I am not sure I would have been able to do it without his support’. This indicated that the task was located in this Growth Zone and that he did not think to use other strategies for support in lieu of the teacher.

To give learners confidence when working in the Growth Zone, I designed several interventions to provide scaffolding to support them when working. As the research progressed, the amount of scaffolding given decreased. In many cases, despite discussions after the event indicating that the learners had good ideas that would have allowed them to progress, reducing the scaffolding removed the learner’s confidence and stopped them from continuing with the task. Evidence for this was seen in intervention four with Val and Emily. Despite doing well in the first part of the investigation by making use of scaffolding that was available to them, when the structure of the investigation was removed in task two both girls momentarily seemed to lose confidence in being able to proceed. It appeared as though they panicked because the second part of the task was felt to be more challenging.

Although the level of support in the scaffolding decreased during the interventions, I was not able to successfully remove it completely in the time scale available for this research project. Although I have no evidence to back up my claim, I feel that having any amount of scaffolding was enough to give the learners the confidence to begin work. The change I did see was that they were starting to find support away from the teacher. In intervention three, one learner suggested having the idea of hint cards that were available if learners needed them. Although I did not follow up this idea at the time, it would be interesting to see the impact on the learners' confidence of having these available, perhaps with a limit on how many cards someone can have during a lesson. One of the elements of the Growth Zone model is that some form of help is available. Perhaps just knowing that it is available if necessary would be enough to move some of the learners forward.

#### **6.2.4 Making links with other mathematical topics to deal with being stuck**

Another strategy I saw being used to support learners when being stuck was learners starting to make links between different mathematical topics. The first example of a learner trying to make links that I witnessed was Val; in intervention four, she used her knowledge of positive and negative correlation to help her reach a conclusion on gradients of straight lines. Although the link was unexpected, she was able to use the concept of correlation and lines of best fit to help her describe straight lines with a positive gradient. More examples of learners making links with other topics and investigation were seen in intervention seven. When talking to Peter, he described how he tried to use his knowledge of finding the  $n^{\text{th}}$  term of a linear sequence to help him spot the pattern in the number grid problem. When

he realised the difference between terms was not the same, he realised it was not an appropriate technique. He then tried to see other links. He commented ‘in many investigations we do in mathematics the answers often involve square numbers’, which helped him spot the pattern in this investigation. He said he was referring to the investigation we had done looking at the number of squares on a chessboard. I asked him if he found it useful to think back over previous work and investigations to help him with new problems. He replied, ‘I never used to but a lot of the things we now do are quite similar so it makes it easier, especially in tasks like this, to see if you can use answers for other problems to help you out.’ This shows evidence that he is beginning to understand how to learn mathematics. This is one of the aspects of mathematical resilience described by Johnston-Wilder et al. (2013).

#### **6.2.5 Using strategies from other subject areas to help make progress**

Another interesting finding relates to learners developing strategies to succeed in mathematics. Understanding how to work at mathematics is one of the factors of mathematical resilience that I wanted to use the research to develop.

In intervention three, one of the learners made a link between the technique she used with a mathematics task and a technique that is commonly used in history. In this intervention, the group members were presented with a full solution on paper but with no explanation given by the teacher. They had to look over the solution carefully and work out what was happening at each step. The majority of the class found this challenging. Afterwards, Abi commented that what they had

to do was similar to tasks set in history lessons where they would be given a series of primary and secondary resources and they would be required to synthesise the information to reach conclusions and answer questions about the event. When I spoke to the history teacher, she said that this is a skill they start developing in primary school and, for many learners, it becomes well embedded during the later years of secondary school. This observation made me consider how in mathematics we could make greater use of the skills that are already well developed in other subject areas.

Evidence about this aspect of the research comes predominately from four girls. These girls were selected because they were attaining at least two grades lower in GCSE mathematics compared to the other subjects they were studying. Observations of their work and discussion with them indicated that they appeared to show resilience in many of their other subjects but not in mathematics. This was seen in discussions when all four girls separately admitted that they have accepted that they will never be good at mathematics and when discussing Olivia's progress with her mother they found it quite amusing that she could not 'do mathematics'. During classroom conversations and formal discussions there was evidence that these learners wanted to remain in their Comfort Zone. For example, in the discussion with Polly, she said that she felt the style of lesson that allowed her to make the most progress was when they were given many examples and asked to complete a worksheet with numerous similar questions. She said she knew she made lots of progress in these lessons because she could check the answers and usually got the majority correct. Conversely, the style of lesson she said that she made the least progress in was when they were doing investigational work. Early

discussion with these girls revealed that in other subjects they would read a variety of extra sources to support their learning and this would give them the understanding required to succeed in the subject. They found that reading mathematics textbooks and revision guides was not having the same impact that they found it to have in their other subjects. It became clear that in other subjects they aimed for relational understanding of the topics they were studying, which they gained through extra reading. However, in mathematics they just wanted to know how to follow a procedure, not understand why it worked. Their desire for an instrumental understanding in mathematics meant that they felt rote learning was most useful. Despite stating a distinct preference for rote learning, they admitted that it was not working for them. They said that they understood what to do in lessons, but when they had to complete homework or revision, their notes and class work made no sense to them. Observation showed that this resulted in a build-up of mathematical anxiety.

I was interested to find whether I could use their success in other subjects to support their learning in mathematics so I designed intervention eight to allow them to use their skills from these other subjects. The four girls all submitted a piece of work that was detailed and well written. They commented in discussions and in their learning journals that they enjoyed learning in this way and found it useful. All four girls chose to write about Pythagoras and said that having researched it in this way, they now understood what they struggled to understand in class. Polly commented that she enjoyed this task because it was using the same techniques she used in history and sociology, two subjects in which she achieved well.

There were many negative comments about this style of learning from others in the group. Further investigation showed that the five most negative journal entries came from learners who did not study subjects like history and sociology or were predicted grades in these subjects, which were lower than mathematics. One learner, James, commented that this was a result of him struggling to put his ideas into writing. He felt his literacy skills held him back in the majority of his other subjects with the exception of mathematics. The literacy element of this mathematics intervention meant that he felt he now struggled in mathematics where he usually succeeds as well as the other areas of the curriculum.

### **6.3 Building confidence in tackling problems**

Many of the strategies discussed above to help overcome the barrier of being stuck were seen to increase the learners' confidence in tackling problems. During the research other strategies were used to build confidence.

#### **6.3.1 Use of estimation to boost confidence**

In intervention six, Piers and Monica encountered a difficulty in calculating the mean. They were unsure which of the two numbers the divisor was. By reminding them how to estimate the answer, they were able to work it out for themselves. Afterwards Monica commented in her journal 'We worked out the mean wrong but if we had thought about what we expected the size of the answer to be we would have spotted this mistake ourselves'.

The use of estimation as a tool was used successfully again in intervention eight when Kerry and Laura were calculating the phone bill. With a bit of prompting to estimate the size of the expected answer, they identified an error and were able to spot that their calculation had been carried out with inconsistent units. In their journals, both girls commented on the error they made describing it as “silly”. Kerry went onto say how she found it funny that they did not spot their answer was not sensible.

Although I did not explore this idea further, I found some evidence that estimating answers in this way can increase the learners’ confidence. The four learners discussed above had used estimation to identify errors, but instead of giving up with the problem as many learners did in the first three interventions, they were able to persevere and, in the case of Kerry and Laura, they found it quite amusing that they had made a mistake.

### **6.3.2 Adding an element of choice to build confidence**

According to Lee and Johnston-Wilder (2013), an important part of developing mathematics resilience is increased agency, more specifically being able to choose which techniques to use to solve a problem. In intervention six, I allowed learners to choose which type of analysis to use to either prove or disprove a statistical hypothesis.



As predicted by Lee and Johnston-Wilder (2013), observations showed the learners to be more motivated by having this choice. They commented on this in discussions and journals, for example in Joey's journal. The learners liked being able to choose which techniques to use, mainly because they were able to avoid techniques that they were not confident about. From these observations, I inferred that perhaps this increase in confidence was not completely because they were able to choose which techniques to use, but more about being able to avoid the techniques that moved them out of their Comfort Zone. However, it was visible through scrutinising their work that they were more willing to tackle Growth Zone concepts of their own choice.

In intervention seven, learners were also given a choice of which direction to take with the number grid investigation. However, in this case there was less opportunity to avoid using specific techniques. Observations of the learners in the room showed that the majority of learners were on task. Although everyone chose to start by following the same path, the end points for the learners were all different. Some learners proved the results using algebra while others opted for a numerical justification. In my opinion, the advantage of this was learners could move into the Growth Zone without being forced towards the Anxiety Zone. I found, for example with Jonathan and James, that a little bit of scaffolding provided as part of the task planning was all that was required to keep them confident and working in the Growth Zone.

## **6.4 The motivation for learning mathematics**

The third aspect of mathematical resilience discussed by Johnston-Wilder et al. (2013) was understanding the personal value of mathematics.

For many, the lack of motivation for learning mathematics seems to be caused by failing to see how it relates to their life. Many authors (e.g. Von Glasersfeld (1985), Nardi and Steward (2003), Goodall et al. (2016)) suggest there is a need to contextualise mathematical ideas to help the learners see their relevance and therefore why they need to understand and use the ideas that are contained in the mathematical curriculum. I found evidence during my research to back up these suggestions. For example, in intervention one, Tom made a comment about coordinates being pointless because he will never need to use them in real life. I also saw the converse of this, when in intervention seven, Olivia spoke about how she did not realise that Pythagoras' theorem could be used so much in real life. She commented about how her dad, who was a builder, could use the theorem to work out the length of fascia boards without climbing up a ladder. For her, this appeared to be motivational.

Trying to link the mathematics to real life was not as easy as expected. The aim of intervention two was to show the learners how much mathematics could be used in planning a trip to London. However, the learners failed to recognise the link between this task and mathematics. For example, Tom said 'It was good planning a trip to London. It was something that I done before, although I am not sure what it has got to do with maths.' When I asked Jane and Laura what

mathematics they had been using, both of them said none. Another learner wrote in her journal 'Planning the day out was good fun but we have mock exams in a month so we should be doing proper maths.' At the end of the lesson, I asked learners to list what mathematics they had used and they could only come up with one or two ideas. In intervention nine, I also tried to make the task relevant to their lives by basing the mathematics on mobile phones bills. Again, they did not see this as being relevant to them. When I asked one of the learners what 'proper maths' was, he said it was when we did questions out of a textbook just as they need to do in their exams.

Discussion with learners indicated that their main motivation came from getting a grade C in their GCSE examination. For many, for example Martin, this was the grade they needed to get into college or to do A-levels. This motivated them to attend extra sessions and do more revision outside of lessons. Once Martin had his college offer, he was motivated to get his grade C, but appeared to have no interest in becoming good at mathematics or getting above a grade C. My hypothesis is that this is a common extrinsic motivation for learners and perhaps why learners are seeking an instrumental understanding of the material required for a grade C. I have also witnessed many teachers sharing the view that their role as teacher is to get the learners the C required for their next step.

## **6.5 Accepting a new classroom culture**

During interventions seven and nine, I observed that many of the learners who initially avoided investigation work were now persevering and starting to accept

elements of the new culture I was trying to embed. They showed that they had some strategies to use when stuck, predominately collaborating with a partner, and many were starting to gain an understanding of how to work on a mathematical investigation. For example, in intervention seven, when I asked Billy how he knew the difference was always the same for each sized square, his initial response was ‘because it is’ but then he realised he needed to justify this claim. Although there was more evidence of learners working within the new culture towards the end of the research period, there was still evidence of the same learners wanting to return to the old classroom culture. It became clear to me that trying to change the classroom culture during their mid-teenage years was not going to be an easy task, although I did find evidence to suggest that over a period of just over a year small changes can be made to their approach to their mathematical learning.

## Section 7: Conclusions

### 7.1 Findings

When planning this action research project, I aimed to find evidence to help me answer the following three questions.

Will changing my practice to encourage more resilient behaviour enable the learners:

- 1 to develop sufficient confidence to work in the ‘Growth Zone’?
- 2 to use resilient behaviours during learning so that they improve their ability to answer examination questions correctly?
- 3 to increase their engagement in learning mathematics?

The action research consisted of cycles of carefully planned and reviewed interventions for a particular group of learners. The findings from one intervention were used to support the design for the next. The key theme running through all of the interventions was the use of split-screen objectives, which I observed helping to focus the learners on developing resilient behaviours. Perhaps the most significant focus was on helping the learners cope with being ‘stuck’. This was found to be one of the biggest barriers to their mathematical learning, especially when it came to problem solving.

**Research question 1:** Will changing my practice to encourage more resilient behaviour enable the learners to develop sufficient confidence to work in the ‘Growth Zone’?

Based on the needs of the group, the main focus of the interventions, became developing strategies for dealing with struggling and being stuck. Initial findings revealed that learners were hesitant to work in the Growth Zone as a consequence of the fear of ‘getting stuck’ and not knowing what to do when stuck. Many were seen to give up on a task if the teacher did not tell them exactly what they had to do next.

As the research progressed, I observed learners spending longer periods of time working in the Growth Zone. Although they were seen to struggle, I saw that the learners were not giving up as easily when the learning got more challenging indicating to me that they were more willing to push themselves. My observations indicated that making use of the split-screen lesson objectives allowed the class to focus on certain behaviours more easily in lessons. The majority of learners within the group referred to the usefulness of the ‘Stuck Poster’ in allowing them to proceed when stuck, either in discussion or in journal entries. I believe the reason for its success was because it was created by the learners, it was visible at all times and it was referred to on a regular basis. When learners were stuck and unsure what to do, I reminded them to make use of the poster before a teacher would help them. During observations, I saw them becoming more independent in their learning, asking the teacher for help less frequently and instead supporting themselves through difficulties. Consequently, observations and discussion indicated that they had become more confident to work in the Growth Zone. The learners were seen persevering more frequently with problems, trying different strategies to solve a problem instead of giving up.

Another example of resilient behaviours that helped some of the learners remain in the Growth Zone was when learners started seeing links with different mathematical topics and learning in other subjects. Looking for links is part of learning how to work at mathematics. One of the successful interventions for girls who were underachieving in mathematics compared to their other subjects was when they were asked to research and write an essay about the work of a famous mathematician. They were able to use skills learned in history to successfully research and write an essay on their mathematical topic. All commented on their enjoyment of the task and how it helped them understand the mathematics involved. I observed high levels of self-efficacy after this task with these girls showing more confidence that they could solve problems and examination questions based around the topic.

**Research question 2:** Will changing my practice to encourage more resilient behaviour enable the learners to use resilient behaviours during learning so that they improve their ability to answer examination questions correctly?

The overall aim of this action research project was to increase the levels of attainment for my own class. Due to unexpected changes in the style of GCSE examinations, I was unable to compare outcomes of this group with the comparison group who sat the GCSE in a previous year. I was, however, able to compare the performance of the research group with the comparison group on the outcomes of specific questions. Despite both groups having similar baseline data, the mean mark gained by the research group was higher.

During the research, many of the interventions focussed on the skills of how to work at mathematics. Many of the skills were introduced using the idea of split-screen objectives, for example, spotting patterns and by the end of the research, learners were demonstrating higher levels of perseverance. On many occasions, I saw them change their strategy if their current one was not working. Another focus was on presenting their solutions; a clear solution that progresses in a logical way is required to achieve many of the method and quality of written communication marks in the GCSE paper. Through their class work and their homework, I witnessed an improvement in their ability to organise their work in a way that would allow them to achieve higher marks in the exam. Comparing their work with that of the comparison group showed that on average, they were better at presenting logical solutions to questions than the comparison group were.

**Research question 3:** Will changing my practice to encourage more resilient behaviour enable the learners to increase their engagement in learning mathematics?

Initially I thought that an understanding of the personal value of mathematics for this group, and as a result increased levels of engagement in learning mathematics, would be gained by making use of real-life scenarios in activities. Although many learners did say they wanted to see how mathematics was used in real-life, they seemed to not realise they were using mathematics when faced with real-life applications. I expected these real-life applications to intrinsically motivate



learners to complete the tasks but this was not the case. Instead, I found that as learners secured places in sixth form colleges their motivation became extrinsic, based around getting the required grade in mathematics for entry on their course. I observed an increase in engagement in learning mathematics but discussion with the learners resulted in establishing they were they were only interested in learning enough to get the required grade, which would not help them to use mathematics in their day to day lives, an aim of mathematical resilience.

In conclusion, from my findings I now know that:

- Using split-screen learning objectives can focus learner's attention on becoming a better learner whilst learning mathematics.
- Having a learner generated 'Stuck Poster' on display in the classroom can help support learners to become unstuck.
- Encouraging learners to estimate the size of the answer in all calculations can help boost their confidence to believe what they are doing is correct.
- For some learners, researching and writing a summary of a carefully chosen mathematical concept can help increase their understanding and confidence in working with the topic.
- Giving learners an element of choice within a task can help encourage them to work in the Growth Zone.
- The use of learner journals allows the teacher to gain a better insight into the learner's understanding and feeling towards a topic or skill.
- Changing embedded practice takes time- with perseverance from the teacher, change can happen.

## **7.2 Successes and limitations of the project**

As an action research project there is only the intention to improve my own practice, the findings cannot, by definition, be generalizable. However, the depth of data that I have been able to source and analyse gives strength to my conclusions that would not be attainable in any other way. In setting out the data in sections four and five, I have aimed for transparency so that the conclusions that I have drawn may be seen as trustworthy.

The findings of this action research project have already been discussed in my own faculty and plans are in place to fully disseminate these findings to allow approaches to develop mathematical resilience to be shared across all teaching groups. This action research project only focused on the needs of a particular group of learners who were expected to attain a mark around the borderline for good pass in the GCSE so it will be interesting to observe the impact these approaches have on higher and lower attaining students

## **7.3 The next steps**

Although my research focussed around three main research questions, there were a number of other themes that were starting to emerge that were not fully focussed on within this action research project. One of these themes was related to the impact that gender has on the development of mathematical resilience. Researchers such as Bevan (2004) and Baron-Cohen (2003) have explored the difference in the way different genders learn. I observed some differences starting

to emerge related to the way males and females reacted and dealt with being stuck and struggling such as the females making more links with other subjects. It would be interesting to explore this idea further.

Another idea that I explored was related to the learners making estimations of their answers to help increase their confidence in deciding whether their answer was sensible. When learners did this, it helped them spot errors and, as a result, they commented on an increase in confidence to continue. Again, it would be interesting to explore this idea fully in the future and the impact it might have on encouraging learners to spend longer working in the Growth Zone.

During the research, there was evidence that learners found working with peers the most effective strategy for dealing with 'being stuck'. It would be interesting in an action research project in the future to explore further the impact that articulating their mathematical thinking has on the development of learner's mathematical resilience. Indirect findings from this action research project indicate that exploring peer support further could be a key factor in the development of mathematical resilience.

## Bibliography

Ainley, J., Pratt, D. & Hansen, A. (2006) Connecting engagement and focus in pedagogic task design, *British Educational Research Journal*, Vol, 32 No.1, pp23-38.

Ashcraft, M.H., (2002) Math anxiety: Personal, educational, and cognitive consequences. *Current directions in psychological science*, Vol 11, No.5, pp.181-185.

Bandura, A., (1997) *Self-efficacy: The exercise of control*. New York: Freeman.

Baron-Cohen S (2003) *Essential difference: men, women and the extreme male brain*. London: Allen Lane.

Beilock, S.L., Gunderson, E.A., Ramirez, G. and Levine, S.C., (2010) Female teachers' math anxiety affects girls' math achievement. *Proceedings of the National Academy of Sciences*, Vol. 107 No. (5), pp.1860-1863.

Betz, N.E., (1978). Prevalence, distribution, and correlates of math anxiety in college students. *Journal of counselling psychology*, Vol. 25 No. 5, p.441.

Bevan, R. (2004) *Gender and mathematics: what can research tell us about how we teach mathematics to boys and girls?* Paper presented to the Teacher Research Conference by the National Teacher Research Panel, Birmingham, England.

Bills, C. Bills, L. Watson, A. Mason, J. (2004) *Thinkers*. Derby, UK: Association of Teachers of mathematics.

Boaler, J. (1997) Reclaiming School mathematics: the girls fight back', *Gender and Education*, Vol 9, No 3, pp 285-30.

Boaler, J (1997b) Setting, Social class and the survival of the quickest, *British Educational Research Journal*, Vol. 23 No. 5 pp.575-595.

Boaler, J. (1998) Open and Closed Approaches: Pupil Experiences and Understanding. *Journal for Research in mathematics Education*, Vol 29, No 1, pp41-62.

Boaler, J., (2013) March. Ability and mathematics: the mindset revolution that is reshaping education. *Forum*, Vol. 55, No. 1, pp. 143-152.

Borasi, R. and Rose, B. J. (1989) Journal Writing and mathematics Instruction. *Educational Studies in mathematics*, Vol. 20 No. 1, pp 347 – 365.

Braun, V. and Clarke, V., 2006. Using thematic analysis in psychology. *Qualitative research in psychology*, Vol. 3 No. 2, pp.77-101.

Brown, M. Brown, P. Bibby, T. (2008) "I would rather die": reasons given by 16-year-olds for not continuing their study of mathematics. *Research in mathematics Education*, Vol. 10, No. 1, pp 3-18.

Brown, S. (1997) Thinking like a mathematician: A problematic perspective. *For the Learning of mathematics*, Vol. 17, No. 2 pp 36-38.

Callan, S. (2015) *The Fear Factor: Maths anxiety in girls and women*, London: Martsan Press Ltd.

Carrington, D. (2015) *What's it got to do with me?* London: Martsan Press Ltd.

Chambers, M. Powell, G. Claxton. G. (2004) *Building 101 Ways to Learning Power*. Bristol: TLO Limited.

Chinn, S and Ashcroft, R (2007) *mathematics for Dyslexics Including Dyscalculia*, Chichester: Wiley and Sons.

Claxton, G. (2004) *Learning to Learn- The Fourth Generation: Making Sense of Personalised Learning*, Bristol, England: TLO Limited.

Claxton, G. (2006) *Building Learning Power*, Bristol, England: TLO Limited

Cohen, L. Manion, L. Morrison K. (2011) *Research Methods in Education* (6<sup>th</sup> edition), London: Routledge Falmer.

Creswell, J. (2009) *Research Design: Qualitative, Quantitative and Mixed Method Approaches*, Los Angeles: Sage.

Cuoco, A., Goldenberg, E.P. and Mark, J. (1996) Habits of mind: An organizing principle for mathematics curricula. *The Journal of Mathematical Behaviour*, Vol. 15 No.4, pp.375-402.

Deci, E. L., & Ryan, R. M. (1985). The general causality orientations scale: Self-determination in personality. *Journal of research in personality*, Vol. 19 No. 2, pp109-134.

DfEE (1999) mathematics: *The National Curriculum for England* London, UK, DfEE.

DfE (2014) 'National curriculum in English: mathematics programme of study UK' (Online publication), Available at <https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study/national-curriculum-in-england-mathematics-programmes-of-study> Accessed on 26<sup>th</sup> July 2016.

DfE (2016) 'Progress 8 Measure in 2016,2017 and 2018, UK' (Online publication), Available at [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/536052/Progress\\_8\\_school\\_performance\\_measure.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/536052/Progress_8_school_performance_measure.pdf) Accessed 26th July 2016.

DfE (2016b) '16 to 19 funding: Maths and English condition of funding' (Online publication) Available at <https://www.gov.uk/guidance/16-to-19-funding-maths-and-english-condition-of-funding> Accessed 26th July 2016.

Dweck, C.S., (2000). *Self-theories: Their role in motivation, personality and development*, New York: Psychology Press.

Eccles, J.S. and Wigfield, A., (2002). Motivational beliefs, values, and goals. *Annual review of psychology*, Vol. 53 No.1, pp.109-132.

Eddy, D.M., Hollingworth, W., Caro, J.J., Tsevat, J., McDonald, K.M. and Wong, J.B. (2012). Model transparency and validation a report of the ISPOR-SMDM Modelling Good Research Practices Task Force–7. *Medical Decision Making*, Vol. 32 No.5, pp.733-743.

Ernest, P. (2013). *The Psychology of mathematics*, Amazon Kindle.

Ernest, P. (2015). The Social Outcomes of Learning mathematics: Standard, Unintended or Visionary? *Journal of Education in mathematics, Science and Technology*, Vol. 3, No. 3, pp187-192.

Geertz, C. (1977) 'Thick Description: Toward an Interpretive Theory of Culture' New York: Basic Books.



Goodall, J., Johnston-Wilder, S., Russell, R. (2016) 'The Emotions Experienced whilst Learning mathematics at Home' in Xolocotzin, U (Ed.) *Understanding Emotions in Mathematical Thinking and Learning*, Massachusetts: Elsevier Academic Press.

Feldt, L.S. and Brenna, R.L. (1993) Reliability. In Linn, R. (ed.) *Education Measurement*. New York: Macmillian Publishing Co., pp105-146.

Fitz-Gibbon, C.T. (1997) *The value added national project. Final report*. London: School Curriculum and Assessment Authority.

Fried, M.N. and Amit, M., 2003. Some reflections on mathematics classroom notebooks and their relationship to the public and private nature of student practices. *Educational Studies in mathematics*, Vol. 53 No.2, pp.91-112.

Hargreaves, D. (2005) *Personalising Learning 3- Learning to Learn and the new technologies*, London: Specialist Schools and Academies Trust.

Hartas, D. (2010) 'Educational Research and Inquiry: Key Issues and Debates' in Hartas, D. (Ed) *Educational Research and Inquiry: Qualitative and Quantitative Approaches*, London: Continuum.

Hernandez-Martinez, P. & Williams, J. (2011) Against the odd: resilience in mathematics students in transition, *British Educational Research Journal*, Vol. 39 No.1 ,pp.45-59

Hoyles, C. Newman, K. Noss, R. (2001) Changing patterns of transition from school to university mathematics *International Journal of Maths Education in Science and Technology*, Vol 32, No 6, pp 828-845/

Johnston-Wilder, S., Lee, C., Brindley, J. and Garton, E., (2015). Developing mathematical resilience in school-students who have experienced repeated failure. In *Proceedings for the 8<sup>th</sup> International Conference of Education, Research and Innovation*, Seville, Spain.

Johnston-Wilder, S., Lee, C., Garton, E., Goodlad, S. and Brindley, J., (2013). Developing coaches for mathematical resilience. In *Proceedings for the 6<sup>th</sup> International Conference of Education, Research and Innovation*, Seville, Spain. pp.2326-2333.

Johnston-Wilder, S. and Mason, J. (2005) *Developing thinking in Geometry*, London, UK: Open University.

Kemmis, S. and McTaggart, R. (Eds) (1992) *The action research planner* (second edition) Victoria, Australia: Deakin University Press.

Kilpatrick, J., (1987) What constructivism might be in mathematics education. In *Proceedings of the eleventh conference of the international group for the psychology of mathematics education*, Vol. 1, pp. 3-27. Montreal, Universidad de Montreal.

Lee, C. and Johnston-Wilder, S., (2013) Learning mathematics—letting the pupils have their say. *Educational Studies in mathematics*, Vol.83 No. 2, pp.163-180.

Lerman, S. (1996) Intersubjectivity in Mathematics Learning: A Challenge to the Radical Constructivist Paradigm? *Journal for Research in Mathematics Education*, Vol. 27 No. 2, pp.133-150

Lewin, R. (1948) *Resolving Social Conflicts*, New York: Harper.

Lindsay, G. (2010) 'Ethical Considerations and Legal Issues in Educational Research' in Hartas, D. (Ed) *Educational Research and Inquiry: Qualitative and Quantitative Approaches*, London: Continuum.

Maslow, A., (1987). *Maslow's hierarchy of needs*, New York: Salenger Incorporated.

Mason, J. Burton, L. Stacey, K. (1985) *Thinking Mathematically*, Essex: Pearson Education Limited.

McNiff, J., (2013) *Action research: Principles and practice*, Abington: Routledge.

Muijs, D. (2004) *Understanding how learners learn: Theories of Learning and Intelligence*. In Brooks, V., Bills, L., Abbot, I. (Ed) *Preparing to Teach in the Secondary Schools*. Glasgow: Bell & Bain Ltd, pp 45-59.

Muijs, D. (2008) *Doing Quantitative Research in Education with SPSS*, London: Sage

Nardi, E. Steward, E (2003) Is mathematics T.I.R.E.D? A quest of quiet disaffection in the secondary mathematics classroom, *British Educational Research Journal*, Vol. 29 No. 3 pp 345-367.

Oancea, A. (2005) 'Criticisms of educational research: key topics and levels of analysis', *British Educational Research Journal*, Vol. 31, No.2, pp.6-11.

OECD (2015) *The ABC of Gender Equality in Education: Aptitude, Behaviour, Confidence*, Paris: OECD Publishing.

OECD (2016), *Low-Performing Students: Why They Fall Behind and How to Help Them Succeed*, Paris: OECD Publishing.

Ofsted (1999) *Ofsted Subject Reports 1999-2000, Secondary Mathematics*. London: Ofsted.

Ofsted (2008). Mathematics: Understanding the score. London: Ofsted.

Ofsted (2012) *Mathematics made to measure from*  
<https://www.gov.uk/government/publications/mathematics-made-to-measure>  
*accessed 26th July 2016.*

Piaget, J. (2001) *The Child's Concept of Physical Causality*, New Brunswick,  
USA: Transaction Publishers.

Popplewell, R. & Hayman, R (2012). *Where, how and why are Action Research*  
*approached used by international develop non-governmental organisations?* from  
[http://www.intrac.org/data/files/resources/752/Briefing-Paper-32-Where-how-](http://www.intrac.org/data/files/resources/752/Briefing-Paper-32-Where-how-and-why-are-Action-Research-approaches-used-by-international-development-non-governmental-organisations.pdf)  
[and-why-are-Action-Research-approaches-used-by-international-development-](http://www.intrac.org/data/files/resources/752/Briefing-Paper-32-Where-how-and-why-are-Action-Research-approaches-used-by-international-development-non-governmental-organisations.pdf)  
[non-governmental-organisations.pdf](http://www.intrac.org/data/files/resources/752/Briefing-Paper-32-Where-how-and-why-are-Action-Research-approaches-used-by-international-development-non-governmental-organisations.pdf) accessed 26th July 2016

Richardson, F. C., & Suinn, R. M. (1972). The mathematics rating scale:  
Psychometric data. *Journal of Counselling Psychology*, Vol. 19 No. 6, pp551–  
554.

Ryan, R. M., & Deci, E. L. (2000). Self-determination theory and the facilitation  
of intrinsic motivation, social development, and well-being. *American*  
*psychologist*, Vol. 55 No. 1, p68.

Siegel, D. J. (2007). Mindfulness training and neural integration: Differentiation of distinct streams of awareness and the cultivation of wellbeing. *Social Cognitive and Affective Neuroscience*, Vol 2, pp259 –263.

Senninger, T. (2000). Abenteuer leiten ' in *Abenteuern lernen* (Facilitating adventures in learning in adventures), Münster: Ökotopia Verlag.

Scott-Hodgetts, R. (1986). Girls and mathematics: the negative implications of success. *Girls into Maths Can*, London: Holt, Rinehart and Winston.

Skemp, R (1987) *The Psychology of Learning mathematics*, Bloomberg, USA: Lawrence Erlbaum Associates Inc.

Stigler, J.W. and Hiebert, J., (1999). *Understanding and Improving Mathematics Instruction*. In Jaworski, B., Phillips, D. (Ed) Comparing standards internationally: Research and practice in mathematics and beyond: Oxford: Symposium Book, pp 119-134.

Swan, M. (2005) *Improving Learning in mathematics*, London: DfES.

Swann, M. (2008) *mathematics Matters: final Report*, London: NCETM.

Taverner, S. (1997) Modular Courses- A drip feed approach to teaching and learning? *Teaching mathematics and its applications*, Vol 16, No 4. pp196-199.

Thom, J.S. and Pirie, S.E., 2002. Problems, perseverance, and mathematical residue. *Educational Studies in mathematics*, Vol. 50 No. 1, pp.1-28.

Tong, A., Flemming, K., McInnes, E., Oliver, S. and Craig, J., (2012). Enhancing transparency in reporting the synthesis of qualitative research: ENTREQ. *BMC medical research methodology*, Vol. 12 No.1 pp 1-8.

Trip, D. (2005). *Action Research: a methodological introduction*, Perth, Australia: Murdoch University.

Truss, E. (2013) *A gender gap that simply doesn't add up*, London, England: Telegraph Publishing.

Von Glasersfeld, E (1987) *Learning as a Constructive Activity, Problems of Representation in the teaching and learning of mathematics*, New Jersey: Lawrence Erlbaum.

Voss, J. F., and Schauble, L. (1992). Is interest educationally interesting? An interest-related model of learning. In Renninger, A., Hidi, S., and Krapp, A. (eds.), *The Role of Interest in Learning and Development*, Erlbaum, Hillsdale, NJ, pp. 101–120.

Vygotsky, L. (1978) *Mind in Society*. Cambridge, MA, USA: Harvard University Press.

Watson, A., De Geest, E. and Prestage, S., (2003). *Deep Progress in mathematics- The Improving Attainment in mathematics Project*, Oxford: University of Oxford, Department of educational studies.

Wheeler, D (1982) "Mathematization Matters" *For the learning of mathematics*, Vol. 3, No.1 pp 45-47.

Williams, G., (2014). Optimistic problem-solving activity: enacting confidence, persistence, and perseverance. *ZDM*, Vol. 46 No. 3, pp.407-422.

Wolff, M., Wagner, M.J., Poznanski, S., Schiller, J. and Santen, S., (2015) Not another boring lecture: engaging learners with active learning techniques. *The Journal of emergency medicine*, Vol. 48 No.1, pp.85-93.

Wright, D. Taverner, S. (2008) *Thinking through mathematics*. Wakefield, UK: Optimus Education.



## Appendices

### Appendix 1: Ethical approval



#### Application for Ethical Approval for Research Degrees (MA by research, MPHIL/PhD, EdD)

Name of student  
Christopher Chisholm

EdD

Project title: The development of mathematical resilience in KS4.

Supervisor - Clare Lee and Sue Johnston-Wilder

Funding Body (if relevant)

Please ensure you have read the Guidance for the Ethical Conduct of Research available in the handbook.

#### Methodology

Please outline the methodology e.g. observation, individual interviews, focus groups, group testing etc.

The research will involve:

- Delivering an intervention to my year 10 C/D borderline class - I will be looking at strategies to develop mathematical resilience in problem solving
- Pre and post intervention questionnaires
- Use of student journals for recording their thoughts on the intervention (using a prompt sheet)
- Use of journal for recording my own thoughts and observation on the lesson
- Focus group interviews for selected students discussing their views on the intervention

#### Participants

Please specify all participants in the research including ages of children and young people where appropriate. Also specify if any participants are vulnerable e.g. children; as a result of learning disability.

I will be acting as a teacher-researcher during my research. The research will be based on my year 10 C/D borderline GCSE maths class. These students are age 14-15. One

student has a statement of special needs (mobility) and some appear on the SEN register as SA and SAP. An HLTA is present to support these student.

#### Respect for participants' rights and dignity

How will the fundamental rights and dignity of participants be respected, e.g. confidentiality, respect of cultural and religious values?

All students will have to take part in the intervention because it will form a normal part of their timetabled lessons. The intervention will be looking at problem solving in mathematics and the same protocols will be followed as I would for a standard lesson. Any data collected will be recorded anonymously and the student's names will not form part of the formally recorded evidence. Student journals will have names on them in the class to allow them to be handed out- the covers will be removed after the research has finished to avoid the students being identified. I have the headteacher's permission to use the data generated from this intervention. The students will be informed that I will be studying this intervention as part of my research for this doctoral thesis and of the steps that will be taken to ensure confidentiality.

The pupils will have the right not to take part in the focus group if they do not wish to do so

#### Privacy and confidentiality

How will confidentiality be assured? Please address all aspects of research including protection of data records, thesis, reports/papers that might arise from the study.

My school or local authority will not be mentioned in the thesis or papers resulting from the research. Any research evidence collected will be stored anonymously- original copies with names on them will be destroyed once the project has been completed and passed external examination.

Using the internet and my name, my school can be found. However, I teach two year 10 groups every year so it will be difficult for them to find out which class was involved in the research project.

#### Consent

- will prior informed consent be obtained?

- from participants? Partially from others? Yes

- explain how this will be obtained. If prior informed consent is not to be obtained, give reason:

Students will be told I am undertaking a research project looking at ways of increasing their chance of gaining a grade C or above in their GCSE. They will need to take part in the intervention due to it taking place in their timetabled lesson. However, they will be given the option not to have their journal entries or results from questionnaires included in the analysis and write up.

- will participants be explicitly informed of the student's status?

I will inform them that I have undertaken research as part of my doctorate studies.

#### Competence

How will you ensure that all methods used are undertaken with the necessary competence?

Methods used have been discussed with my supervisor. The questionnaire used has been used and tested by other researchers. Recent literature on the research methods used have been used and form part of the thesis. Regular contact via e-mail and skype with my supervisor will take place during the research and any potential issues will be discussed.

#### Protection of participants

How will participants' safety and well-being be safeguarded?

All school health and safety and safeguarding policies will be followed.

#### Child protection

Will a DBS (Disclosure and Barring Service formerly CRB) check be needed?

No- I am working in my normal lessons which will be covered by the school DSB.

#### Addressing dilemmas

Even well planned research can produce ethical dilemmas. How will you address any ethical dilemmas that may arise in your research?

These will be addressed by liaising with my supervisor, other teachers at school and referring to the BERA guidelines.

#### Misuse of research

How will you seek to ensure that the research and the evidence resulting from it are not misused?

Only anonymised data will be issued to my supervisor and other relevant academics from the university.

#### Support for research participants

What action is proposed if sensitive issues are raised or a participant becomes upset?

School policy will be followed at all times. Any issues of concern will be raised with the school's child protection officer.

#### Integrity

How will you ensure that your research and its reporting are honest, fair and respectful to others?

All participants will have their opportunity to give their views about the impact the intervention has had on their learning.

Triangulation of data will help ensure it is valid and data analysis will be discussed on a regular basis with my tutor. She will support me in the identification of misinterpretation of data or where further data needs to be collected.

What agreement has been made for the attribution of authorship by yourself and your supervisor(s) of any reports or publications?

Any publications that are authored by me but added to by my supervisor will be attributed to both of us, the names will be ordered according to the level of contribution from each party.

Other issues?

Please specify other issues not discussed above, if any, and how you will address them.

Signed

Research student



Date: 30<sup>th</sup> January 2014

Supervisor



Date 30<sup>th</sup> January 2014

Action

Please submit to the Research Office (Louisa Hopkins, room WE132)

Action taken

☒

Approved

☐

Approved with modification or conditions – see below

☐

Action deferred. Please supply additional information or clarification – see below

Name

G. LINDSAY

Date

20/2/14

Signature



Stamped

Notes of Action