Essays in Contracting and Liquidity Provision

By

JunJie Liu

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Contents

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Declarations

I declare that any material contained in this thesis has not been submitted for a degree to any other university.

I further declare that all three chapters are products of joint work with Dr Kostas Koufopoulos.

JunJie Liu

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Abstract

The first essay considers a model in which an entrepreneur develops a technology and seeks to sell a stake of her asset for diversification purposes. In our model, output depends on both the quality of the asset and the non-contractible effort made by investors. In this case, signalling the asset type and motivating the investors are two conflicting objects. Our model shows that by applying a mechanism with endogenous commitment, entrepreneurs can achieve the second best allocation. Moreover, when the proportion of high type entrepreneurs is high, our model predicts that low-quality entrepreneurs will sell all of their shares above the fair price (overpricing) whereas the high-quality entrepreneurs may retain a fraction of their shares and sell their share below the fair price (underpricing).

The second essay illustrates a model in which an entrepreneur intends to securitize her risky asset to invest in a new project. In contrast to the settings of pecking order theory, outside investors are able to acquire the type information of the asset by making an effort. This new assumption allows the entrepreneurs to signal their types by motivating the investors' information searching behaviour. Moreover, our model also endogenizes the existence of intermediaries in the issuing process. We conclude that if intermediaries are allowed to offer a menu of contracts to the entrepreneurs, a second best allocation can be sustained as an equilibrium.

The third essay considers a general model in which agents with different production technologies insure each other by entering a futures contract. However, unobservable risk exposure and strategic default of counterparty may prevent agents from fully hedging their risk. In this paper, we compare the market efficiency using two different trading mechanisms—OTC market and centralised clearinghouse. Our model shows that without any aggregate uncertainty, both mechanisms can achieve the second best allocation. Nevertheless, when aggregate uncertainty is introduced into the market, a centralised clearinghouse may dominate the OTC market by including more market participants and diversifying the risk more widely. Additionally, our model predicts that when aggregate uncertainty is extremely high, the central

clearinghouse may have the incentive to provide extra liquidity to certain types of market participants for risk control purposes.

Chapter I:

Overview

This thesis consists of three theoretical essays focusing on the optimal mechanism design for liquidity provision. Briefly speaking, Chapter 2 and Chapter 3 consider models with active investors. The key difference is that the investors are active in different respects. In Chapter 2, investors are active in the production process, which means that they can affect the final output by privately choosing their effort level. In Chapter 3, investors are active in information acquisition. In another word, investors endow with access to the quality information of the project in which they invest. Chapter 4 illustrates a model in which investors with different liquidity demand insure each other with futures contracts. In this model, contracts are unobservable to third parties, and counterparties of the contract can strategically default. Across the whole thesis, we attempt to answer the following two questions: How should we modify our trading mechanism to adapt to these new characteristics of investors? What roles do the centralised intermediaries plays in this new market?

The answer to the first question results from the research on liquidity provision under asymmetric information (for example, Leland and Pyle (1977) and Myers and Majluf (1984)). In these papers, investors are usually assumed to be homogeneous and passive. However, the speed of financial innovation (such as online trading platforms and derivative exchanges) has far exceeded the expectation of previous researchers. The effect brought by innovation has strongly modified investors' behaviour in recent years. For example, Kaplan and Stromberg (2000) and Hellmann and Manju (2000) noted that institutional investors such as venture capitalists might play an active role in a firm's operation. They provide professional services such as management group formation, network building, and product design. Moreover, Fulghieri and Lukin (2001) suggest that previous research on optimal securities design ignore the information production abilities of outside investors. If investors are allowed to identify the quality of assets by exerting effort, high-quality entrepreneurs may choose to issue risky securities to motivate investors' monitoring activities.

To cover the new characteristics of investors, in Chapter 2, we consider models in which privately informed entrepreneurs issue securities to investors with advising capacity. In this Chapter, we find that the conventional methods of signalling (such as equity retention and issuing information-insensitive securities), may not guarantee separation unless we introduce a mechanism with the endogeneity of commitment (Koufopoulos 2010). The main advantage of this mechanism is that it allows cross-subsidies to be introduced, which can solve the new conflicts associated with active investors. In Chapter 2, when the advising services offered by investors become valuable, the unilateral dilemma about equity retention can be extended into a bilateral one. On one hand, entrepreneurs may face a more serious choice in the issuing process compared to the model described by Leland and Pyle, because equity retention for signalling may not only prevent entrepreneurs from diversifying their portfolio but also undermine the incentive for investors to contribute effort to their firm. On the other hand, investors must identify the high-quality projects that are worthy of their contribution. Therefore, for separation purposes, equity retention becomes a necessary method to signal entrepreneurs' type about their projects. However, higher equity retention may also mean less value to be shared by investors, which makes them reluctant to exert effort. As mentioned above, a mechanism with "endogeneity of commitment" must be introduced to make the second best allocation sustainable. Under this mechanism, the investors can offer contracts with or without commitment. It converts a normal signalling game into a screening game and effectively restricts the existence of deviating contracts offered by rivals. As a result, the separating menu can be sustained as an equilibrium in a wider parameter range and the social efficiency can be improved.

With regard to Chapter 3, we consider another securities issuing model with entrepreneurs and speculative investors. In this model, we define speculative investors as those who can acquire signals correlating to the quality of the project. In that case, high type entrepreneurs may issue information-sensitive securities (such as equity) to motivate investors' information acquisition behaviour to separate themselves from the low types. However, as the cost of doing so, high-quality entrepreneurs could suffer from the adverse selection procedure when investors' signals are imperfect. To cope with the dilemma above, in this model we endogenize the existence of the intermediaries. We show that speculative ability of the investors may force the intermediaries to offer a separating menu, which in return improve the social efficiency.

Chapter 4 illustrates a model in which investors attempt to offer liquidity to each other through a forward contract. Departing from the other two chapters, contracts offered by agents are not backed by any tangible asset, and the number of contracts taken by individuals is unobservable. Therefore, strategic default is possible in our model. Investors who participate in the forward contracts must face counterparty risk. In this case, our model shows that both a voluntary report system (Leitner 2012) and a central clearinghouse system can achieve the second best allocation in an economy without aggregate uncertainty. When aggregate uncertainty is introduced, the central clearinghouse may become dominant in risk control and liquidity provision.

As mentioned at the beginning of this part, our essay also attempts to illuminate the role of central financial intermediaries—institutional investors and central clearinghouses. First, for institutional investors, previous research has shown that firms backed by institutional investors with venture capital and private equity are more innovative (Kortum and Lerner 2000) and profitable (Gompers and Lerner 1999a,b, 2001). However, most previous research attributed this effect to the superior information acquisition ability and monitoring skill of institutional investors (Winton 2001, DeMarzo 2005). None of them attempted to explain the impact of the mechanism itself. Hence, in Chapter 2 and Chapter 3, we explain this efficiency gain with the mechanism design instead of the ability of institutional investors. By introducing the intermediaries, the separating menu with cross-subsidies can be sustained as an equilibrium. In the circumstance we discuss, this separation improve the social efficiency by revealing the necessary information for securities buyers to react optimally in the following steps.

Secondly, with regard to the central clearinghouse, Bernanke (1990) empirically analysed the role played by the clearinghouses in the futures market. He concluded that by substituting itself as a seller to every buyer and a buyer to every seller, a clearinghouse becomes an official ‘party to every trade’. This party substitution function effectively reduces the counterparty risk and improves market efficiency. Nevertheless, Duffie and Zhu (2011) suggested that adding a central clearing platform (CCP) to each certain deviated market may reduce netting efficiency. This in turn leads to an increase in average exposure to counterparty risk. In Chapter 4, we find that the superiority of a central clearinghouse may stand out when aggregate uncertainty is introduced into the system, because the optimal allocation may not necessarily be collusion-proof. Conversely, participants of the futures market may always have the incentive to exclude other participants with the same liquidity demand to increase their position. However, this incentive may lead to overexposure of systemic risk and higher ex-post defaulting rate. Additionally, people who are excluded from the futures market may have to insure themselves with their savings. This limits the systematic risk to be efficiently shared among the whole society. Our model illustrates that social welfare can be improved by including all agents in the market. However, for this contract to be implemented the clearinghouse must have the power to standardise all tradable contracts. In addition, the clearinghouse should be authorised to charge agents with high-risk exposure and subsidise other agents with an anti-business circle cash flow. All of these functions rely on the bargaining power of the clearinghouse, so we conclude that a market with a central clearinghouse may have a higher resistance to aggregate uncertainty.

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Chapter II:

Optimal Financing and Advising Under Asymmetric Information

1. Introduction

In most theoretical models of financial contracting under asymmetric information, financiers are assumed to be passive and risk neutral, whereas entrepreneurs are either risk neutral or risk averse. In models with risk-neutral entrepreneurs, different types of entrepreneurs issue different securities (or combinations of securities) to convey information about their types[1]. If entrepreneurs are assumed to be risk averse, high-quality entrepreneurs (firms) can signal their type by retaining a sufficiently high fraction of their firm's equity. The retention of shares conveys a credible signal because it implies a utility cost from under-diversification. This utility cost is even higher for entrepreneurs with low-quality firms. When the retention becomes sufficiently high, a separating equilibrium arises (Leland and Pyle, 1997).

Recently, several empirical studies have illustrated that venture capital plays an active role in a firm's operation and influences the firm's financing choices[2]. In response, theoretical models have incorporated the role of financiers as advisors who improve the operational efficiency of the company they finance. In this paper, we construct a model in which the financier (venture capitalist) also works as an advisor. The model is similar to that of Baldenius and Meng (2010). Entrepreneurs are assumed to be risk averse and financiers to be risk neutral. Financiers can improve the firm's value by exerting effort (advising). However, the key difference between our model and theirs is that we use the three-stage screening game suggested by Koufopoulos (2010)[3] instead of the two-stage signaling game. The main characteristic of this screening game is that the financiers optimally decide whether they commit or not to

[1]For example, Brennan and Kraus (1987), Constantinides and Grundy (1989), and Stein (1992).

[2]For example, Kaplan and Stromberg (2000), and Hellmann and Puri (2000).

[3]More specifically, we consider a three-stage game where at stage 1, financiers offer menus of contracts and specify which of them they are committed to and which not; at stage 2, entrepreneurs apply for (at most) one menu from one financier; and at stage 3, the financiers decide which of the menu offered at stage 1 without commitment will be withdrawn and which not. Of course, after stage 3, provided the financier has not withdrawn the menu and the menu is signed by both parties, the financier becomes committed to it.

the contracts they offer. They can also offer menus of contracts which allows for cross-subsidization across types (contracts).

In this context, the equilibrium always exists, it is (second-best) efficient and unique. More specifically, the possibility of cross-subsidization across contracts is, in general, necessary for the achievement of efficient allocations. Also, the ability of the financier (uninformed) to commit to a menu of contracts makes the decision of the entrepreneurs of whether to take it independent of their beliefs about who else takes it. As a result, starting from any inefficient allocation, it is always possible for a financier to offer a menu with commitment (off-the-equilibrium path) that profitably attracts both types. That is, the combination of cross-subsidization across types and the ability of a financier to commit to a menu (off-the-equilibrium path) eliminates any inefficient allocation as an equilibrium. Also, efficient allocations other than the one which maximizes the expected utility of the good type cannot be supported as equilibria. A deviant firm can offer with commitment a menu which makes the good type better off and profitably attract either only the good type or both types. Finally, the ability of a financier to offer a menu without commitment (on-the-equilibrium path) acts a threat for a potential entrant and supports this allocation as an equilibrium.

Finally, our model has some interesting empirical implications. First, if the operating risk and the proportion of high-quality firm are high, then the equity of high-quality firms is underpriced and that of the low-quality overpriced. Second, there is a negative correlation between equity retention and operating risk and proportion of high-quality firm.

2 Literature review

Our model is related to many strands of the literature. First, on theoretical grounds, our paper is related to Wilson (1977). Wilson proposes a non-Nash-type equilibrium concept (**Anticipatory Equilibrium**) under which an equilibrium always exists. If some feasible (no loss-making) pooling allocations Pareto-dominate the Rothschild-

Stiglitz separating allocation, the zero-profit pooling allocation which maximizes the expected utility of the low risk is the unique “Wilson equilibrium.” The key in Wilson’s solution is that a firm may withdraw a contract if the introduction of a new contract by another firm makes the original contract loss-making. Anticipating this reaction, the new entrant will introduce his contract only if the potential withdrawal of the original contract does not render his contract loss-making. The threat of withdrawal supports as an equilibrium the pooling allocation which maximizes the expected utility of the good type (provided this pooling allocation Pareto-dominates the Rothschild-Stiglitz (1976) separating allocation). Hellwig (1987) proposed a three-stage game which provides game-theoretic foundations to Wilson pooling equilibrium. However, because insurers cannot commit to the contracts they offer at Stage 1, the choice of a contract by an insuree depends on his beliefs about the other insurees’ choices. This leads to a coordination failure and so to multiple equilibria (which, in general, are inefficient).

Miyazaki (1977) employs the Wilson equilibrium concept and allows for the offer of menus of contracts. He shows that the equilibrium allocation is separating, it is always (second-best) efficient and, in general, involves cross-subsidization across types. The Miyazaki equilibrium allocation maximizes the expected utility of the good type given incentive compatibility and feasibility. In this sense, our equilibrium allocation is similar to Miyazaki. Furthermore, our three-stage game provides game-theoretic foundations to Miyazaki equilibrium by extending Hellwig’s game in two ways: First, we allow for the offer of menus of contracts. Second, we allow the firm to optimally decide whether they commit or not to the menu they offer in the first stage of the game (endogenous commitment).

The paper is also closely related to Baldenius and Meng (2010). The key difference between our paper and theirs stems from the usage of the three-stage screening game instead of the two-stage signaling game. In the former game, owing to the endogeneity of commitment, a particular efficient allocation involving cross-subsidies can be sustained as an equilibrium. This equilibrium is unique and interim incentive efficient (second best), and it does not rely on refinements to restrict beliefs off the equilibrium path. In contrast, the two-stage signaling game may end up with multiple equilibria if no refinement is used to restrict beliefs off the equilibrium path. Alternatively, the application of the "intuitive criterion" leads to a unique equilibrium that, in general, is inefficient[4].

Inderst (2001) also considers a model similar to that of Baldenius and Meng but with universal risk neutrality and a continuous effort level. He also uses the standard signaling game and applies the "intuitive criterion" to reduce the set of equilibrium. He suggests that there exists a unique separating equilibrium if the proportion of high type is low whereas there exists a pooling equilibrium if the proportion is high. Nevertheless, our model shows that a separating menu with cross-subsidy can dominate his pooling contract and make a Pareto improvement to his allocation. In another related paper, Martimort and Sand-Zantman (2006) construct a model in which a principal with private information intends to delegate an agent who can contribute unobservable effort to improve the output. Their findings show that projects with higher quality may end up with a contract with a higher share of operating risk retained by the principle. In their model, the effects of adverse selection and moral hazard are separated. Parameter values are restricted to obtain a unique separating equilibrium. To generalize their findings, we introduce the interaction between adverse selection and moral hazard by setting up a dependence relationship between the type of project and the productivity of effort. We also extend our discussion to all parameter conditions to reach a comprehensive conclusion

Additionally, there is another strand of literature that analyses venture capitalist financing in a double moral hazard framework. For example, Casamatta (2003) and Schmidt (2003) considered models in which both the entrepreneurs and the financiers are risk neutral, and the firm's payoff depends bilaterally on the unobservable (non-contractible) effort level chosen by both parties. Their results show that without fund provision, the outside financier can be excluded from the project when the effort from

[4]In adverse selection environments, efficiency generally requires cross-subsidization across types. However, allocations involving cross-subsidies cannot be supported as equilibria in signaling games when the "intuitive criterion" is applied.

entrepreneurs is more efficient. They further suggested that convertible securities may be the optimal financial contract under this environment.

We organize the paper as follows. Section 3 describes the basic assumptions of the model. Section 4 discusses the essential lemmas about the separating equilibrium. In Section 5, we will drive the separating equilibrium of the game and discuss the existence of pooling equilibrium. More specifically, we will derive the conditions under which different equilibria arise and provide the underlying intuition. Section 6 provides the empirical implication of our model. Finally, Section 7 concludes.

3. The model

Our model illustrates the case in which the entrepreneur has developed a technology and seeks to bring it to the market. To do so, he plans to sell a stake of his firm to the active financiers. The underlying two motivations are as follows: First, the entrepreneurs in our model are assumed to be risk averse and expect to diversify the specific risk of their portfolios. Second, some professional service (such as advising and management group formation) or special resource (such as network and marketing channels) provided by the financiers may be required by the entrepreneurs for the firm's operation. More specifically, entrepreneurs may sell a stake of their firm to the venture capitalists (VC) not only to share their specific risk but also to obtain advising and monitoring service from qualified financiers. In comparison to Baldenius and Meng's model, our model allows the commitment to contracts to be determined endogenously. As a result, investors can offer a menu of contracts with or without commitment, and entrepreneurs can maximize their utility by choosing the optimal contract in the market. In response to the contracts offered by their rival, financiers could withdraw their uncommitted contracts that are expected to incur losses, and entrepreneurs will deviate to other contracts remaining in the market.

Technologies in our setting can be classified into two types: low (L) or high (H) quality, which will be referred as the type of entrepreneurs in the following discussion.

The technology of entrepreneurs can affect the output of the firm in two ways: fixed earnings θ_i $(i = L, H)$ and marginal productivity of financiers' effort ϕ_i $(i = L, H)$. The entrepreneur privately knows the type information of each firm, whereas both financiers and entrepreneurs commonly know the prior proportion of each type. Furthermore, no costless communication channel is available to transfer information from entrepreneurs to financiers in our model.

As mentioned above, the value-adding activities taken by the financier generate earnings for the firm, which we define as "investor effort." We use k to denote the cost incurred by these activities and let $k \in \{0, K\}$, where K represents the potential of investor effort. Therefore, the firm's liquidation value will be

$$x_i = \theta_i + \phi_i k + \varepsilon,$$

where $\varepsilon \sim N(0, \sigma^2)$ denotes the operating risk of the project. Additionally, we assume that $\theta_H > \theta_L$ and $\phi_H > 1 > \phi_L > 0$. It means that compared to the low-quality firm, the high-quality firm can generate higher fixed earnings and higher marginal productivity of investors' effort. In addition, the second inequality also illustrates that only the high-quality firm can generate positive net value from investor effort ($\phi_H > 1$). We further define $\Delta\theta \equiv \theta_H - \theta_L$ and $\Delta\phi \equiv \phi_H - \phi_L$ as the differential of fixed earnings and marginal productivities, respectively. Taking p as the proportion of high type in the whole population, we define $\Theta \equiv p\theta_H + (1 - p)\theta_L$ and $\Phi \equiv p\phi_H + (1 - p)\phi_L$ as the expectation of θ and ϕ in the pooling contract. We first discuss the case in which $\Phi < 1$ and extend our discussion to the opposite case in Section 6.[5]

For the financiers, the effort level k they offer depends on both the type of the firm and the fraction of equity $1 - a$ they acquire from the contract. We define the market

[5]If $\Phi > 1$ there may exist a pooling equilibrium if the potential of financiers' effort is sufficiently high. And actually, this assumption works as a restriction of proportion of high type which ensures we will always have separating equilibrium.

price of the whole firms to be P. Departing from Baldenius and Meng's model, we allow the financiers to offer a menu of contracts instead of a single contract to both types of entrepreneurs. Under this setting, the competition between financiers may lead to zero economic profit for the whole menu. However, it is not necessary that each contract included in the menu earns zero profit. In other words, financiers may offer a loss-making contract to a certain type of entrepreneur to make a profit from the other type. These profits and losses are defined as "cross-subsidies" in our model because they altogether work as a kind of subsidy between different types, which we denote as S.

Hence, for any single contract offered by the risk-neutral financiers, we have

$$(1\text{-}a)(x^e - P) - k = S,$$

where x^e represents the expectation of x, and $x^e = \theta + \phi k$.

We suppose that the entrepreneurs have constant risk aversion. Their preferences are described by the utility function $(-\exp\{-\rho W\})$, where ρ stands for the risk aversion coefficient and W stands for the wealth held by the entrepreneur, which can be expressed as $W = (1 - a)P + a(\theta + \phi k + \varepsilon)$. Therefore, the expected utility of entrepreneurs will be:

$$U = (1 - a)P + a(\theta + \phi k) - R(a)$$

In the equation above, $R(a) \equiv \frac{\rho}{2} a^2 \sigma^2$, where σ^2 is by definition the variance of ε and can be viewed as a measure of the operating risk of the firm. $R(a)$ denotes the disutility due to equity retention. All above settings are similar to those in Baldenius and Meng's model. However, the key differences are: (1) We adopt a screening game instead of a signaling game. (2) We endogenize the commitment of contracts in our model. As a result, the game structure of our model becomes:

1. The competitive financiers offer a menu of contracts to the entrepreneurs with or without commitment.
2. The entrepreneurs signal their types by choosing the contract that maximizes their utility.
3. Observing rival's contracts, the financiers withdraw their uncommitted contracts that are expected to be loss-making.
4. Entrepreneurs take the optimal contract remaining on the market.
5. Financiers choose the effort level k.
6. Finally, the firm generates output, and the payoff to financiers and entrepreneurs is realized.

4. Equilibrium

In the following discussion, we will consider only the pure-strategy Bayesian-Nash equilibrium. We assume free entry and exit of the financial market. Therefore, for any menu or contract to be sustained as equilibrium, the two following basic conditions must be satisfied:

1. No menu of contracts offered by financiers in the equilibrium will be loss-making.
2. Any deviation from the equilibrium menu will not increase the financiers' expected payoff.

The first condition underlines the participation constraint of the financiers. If the whole menu generates negative incomes, financiers can leave the market, and we end up with a non-trading equilibrium. A natural result of the conditions above is that the trading surplus between entrepreneurs and financiers is nonnegative ($\mathrm{R(a)} \geq 0$), at least for the low type. Thus, the allocation without trading cannot be an equilibrium. The second condition sources from the definition of Bayesian-Nash equilibrium. If there exists any deviation that can make the financiers better off, they will deviate, resulting in the collapse of the original allocation.

Suppose that a menu containing contracts C_H and C_L is offered in the market and that these two contracts are taken by high type and low type, respectively. With the formula listed in section 3, the price of the firm (P) can be expressed as a function of the entrepreneur's retaining equity a and cross-subsidies S:

$$P_i = \theta_i + \left[\phi_i - \frac{1}{1-a_i}\right] k_i - \frac{S_i}{1-a_i} \quad (i = H \text{ or } L) \tag{1}$$

Under the assumption of full competition between financiers, the economic profit of financiers has to be zero in the equilibrium. Otherwise, competitors can easily attract both types of entrepreneurs by cutting down the profit level. Thus, this context requires:

$$S_H p + (1-p) S_L = 0 \rightarrow S_L = -\frac{p}{1-p} S_H.$$

If $S_i = 0$ $(i = H \text{ or } L)$, formula (1) becomes the zero-profit condition, when both types of equity are fairly priced:

$$P_i = \theta_i + \left[\phi_i - \frac{1}{1-a_i}\right] k_i \quad (i = H \text{ or } L) \tag{2}$$

With the two basic conditions, we can derive some general results, which will be useful for establishing and characterizing the separating equilibria of our model.

Lemma 1: In the separating equilibrium, bad type entrepreneurs will seek to sell the whole firm, and no effort will be contributed to their firm by the financiers.

Proof: In any separating equilibrium, the financiers can infer the type information by observing the contract taken by entrepreneurs. Based on the contract they take, the utility function of low type entrepreneur can be expressed as:

$$U_L = (1 - a_L)P_L + a_L(\theta_L + \phi_L k_L) - R(a_L)$$

The first part represents the fund received by the entrepreneur in the sales of equity. It also equals the fraction of equity sold to financiers $(1 - a_L)$ multiplied by the price of the whole firm. The second part represents the true value of the equity retained by the entrepreneurs. The third part represents the disutility of insufficient diversification as we defined in the last section. By substituting the price equation (1), we can simplify the utility function as:

$$P_L = \theta_L + \left[\phi_L - \frac{1}{1 - a_L}\right] k_L - \frac{S_L}{1 - a_L}$$

$$\rightarrow U_L = \theta_L + (\phi_L - 1)k_L + \frac{p}{1 - p} S_H - R(a_L)$$

From the expression above, we can infer that the sum of the first and second parts of the utility function is independent of a_L. Thus, the equity retention can affect only the level of disutility by $R(a) \equiv \frac{\rho}{2} a^2 \sigma^2$. To maximize her utility, the low type entrepreneur will choose the contracts of $a_L = 0$ if her type has been revealed. On the other hand, because $\phi_L < 1$, it can be inferred that $k_L = 0$. Because the value generated by the effort can never cover the corresponding cost, the financiers will not waste their effort on a bad firm once its type is identified.

Given the discussion above, the utility of the low type entrepreneur is $U_L = \theta_L + \frac{P}{1-P} S_H$ if a low type entrepreneur truly reveals her type. For a separating menu to be sustainable, contracts in the menu must be designed to guarantee that the low type cannot be better off by choosing the contract for the high type. Therefore, the following incentive comparison condition of the low type must be satisfied:

$$(1 - a_H)\theta_H + [\phi_H(1 - a_H) - 1]k_H - S_H + a_H(\theta_L + \phi_L k_H) - R(a_H) \\ \leq \theta_L + \frac{p}{1-p} S_H$$

The inequity above can be rewritten as:

$$(1 - a_H)(\theta_H - \theta_L) + \left[\phi_H(1 - a_H) + \phi_L a_H - 1\right]K - \frac{1}{1-p} S_H - R(a_H) \leq 0$$

In the following section, we will show that this condition is always binding in our separating equilibrium. Because both equity retention and cross-subsidy cause disutility to higher type entrepreneurs, they would never prefer a contract with an unnecessary provision of them. Because contracts with extra equity retention or cross-subsidy do not provide further useful information for financiers to make their decision.

Lemma 2: In any separating equilibrium involving cross-subsidies, high types cross-subsidize low types.

Proof: From the incentive comparison condition above, we can see that the high type entrepreneurs can provide cross-subsidies to prevent the low types from mimicking. Because mispricing from pooling is harmful only to the high types, they are the only type with an incentive to offer cross-subsides. Otherwise, suppose that there exists a separating equilibrium in which the low type offers a subsidy to the high type. In other words, in this equilibrium, financiers are offering a loss-making contract to high types and a profit-making contract to low types ($S_H < 0$, and $S_L > 0$). A new competitor then enters the market and offers a more attractive contract C_L^* to the low type entrepreneurs by cutting down the profit she makes from them. Doubtlessly, the price of equity that low type entrepreneurs get from the new contract will be higher, and they will thus deviate to the new contract. If the original menu of contracts is offered with commitment, the incumbent financier must fulfill her commitment to the high types and holds only the loss-making contracts in her hand. If it is offered without commitment, the incumbent financier will withdraw the contract in stage 3, and both types will take the new contract. Given $\theta_H > \theta_L$, this means that the new entrant can purchase the high-quality equity at a low price and make an even higher profit[6]. In conclusion, the deviation from the original menu can increase financiers' expected payoff. This conclusion contradicts the definition of Bayesian-Nash equilibrium, so the original menu cannot be an equilibrium.

In a separating equilibrium, financiers can infer the true value of θ and ϕ depending on the contract chosen by entrepreneurs. The price offered depends on the type of the entrepreneurs. From formula (1), we have:

$$P_H = \theta_H + \left[\phi_H - \frac{1}{1 - a_H}\right] k_H - \frac{S_H}{1 - a_H} \qquad (i)$$

and

$$P_L = \theta_L + \left[\phi_L - \frac{1}{1 - a_L}\right] k_L - \frac{S_L}{1 - a_L} = \theta_L + \frac{p}{1 - p} S_H \quad (a_L = 0) \qquad (ii)$$

[6]Conversely, in the case of a separating equilibrium in which the high type subsidizes the low type, a contract that makes the high type better off if taken by both types will result in losses.

Notice that for the high type, the share retained by the entrepreneurs, a_H, is negatively correlated with the price of equity. This may be caused by two effects: First, from the second part of (i), it can be seen that if the entrepreneurs retain a higher fraction of equity, then less benefit comes to the financiers from their value-adding activity. Secondly, from the third part of (i), we can infer that keeping the total value of cross-subsidies unchanged, financiers will require a higher underpricing for each share if fewer shares are sold to them. On the other hand, by comparing the price functions of different types, we can show the effect of cross-subsidies on the incentive comparison constraint: It decreases the equity price of high types and increases that of low types, which encourages the low type entrepreneurs to reveal their type truly. In another word, the price adjustment of both types' equity work as "subsidization" from high type to low type. Although the high type entrepreneurs do not directly pay the low type entrepreneurs, the whole system leads to a similar result.

Lemma 3: Among all feasible zero-profit menus, only the one that maximizes the utility of high type can be sustained as an equilibrium.

Proof: Suppose that a menu of contracts C_H and C_L becomes the equilibrium menu of the market. As the result of perfect competition, this menu must satisfy the zero-profit condition, $S_H p + (1-p)S_L = 0$. Otherwise, any rival can simply attract both types of entrepreneurs by offering a more competitive menu. Consider that a new entrant attempts to offer a profit-making menu containing C_H^+ and C_L^+, which satisfies the incentive comparison constraint ($U_L(C_L^+) > U_L(C_H^+)$) and makes the high type better off ($U_H(C_H^+) > U_H(C_H)$). In this case, the high type entrepreneurs deviate to the new menu. If the original menu involves a cross-subsidy, it becomes loss-making when it attracts only the low type. Thus, if the incumbent financiers offer their menu with commitment, they will incur a loss. If the incumbent financiers offer their menu without commitment, they will withdraw the menu in step 3, and then the low type entrepreneurs will have to take C_L^+. However, the new entrant can still end up with positive profit from the whole menu. Otherwise, if the original menu does not involve cross-subsidies, the incumbent financiers will not withdraw their menu, so the new entrant attracts only the high type and makes a positive profit. Because a deviation can increase the expect payoff to the financiers, the original menu C_H and C_L could not be an equilibrium.

5. Equilibrium with non-contractible investor effort

5.1 Investors' effort and Separating equilibrium

One of the most interesting parts of our model is that investors can contribute to the firm value by exerting costly effort. Practically, investors' effort is contractible only when it is tangible and can be valued accurately. However, services such as advising and monitoring can hardly satisfy these two conditions. To avoid the moral hazard problem, the entrepreneurs would prefer to choose a contract that offers sufficient motivation for financiers to exert effort. Under our assumptions, any unit of effort taken to a firm has a constant marginal cost 1 and can generate a marginal benefit of $(1 - a_i)\phi_i$ to the financiers. Hence, we can infer that the effort taking condition is $(1 - a_i)\phi_i \geq 1$ or ($a_i \leq \frac{\phi_i - 1}{\phi_i}$). As mentioned above, because $\phi_L < 1$, no effort will be distributed to the low type firms if their type is revealed truly. For the high types, the trade-off is more complicated. First of all, to motivate financiers to exert effort, high-quality entrepreneurs must signal their type effectively. As we have discussed above, share retention is a credible signal of entrepreneurs' confidence in their project quality. However, retaining too much equity $\left(a_H > \frac{\phi_H - 1}{\phi_H}\right)$ can confound investors' incentive to exert effort. Therefore, in extreme cases, conserving investors' incentive to exert effort and signaling high type entrepreneurs become two contradictive objectives in our model. To solve this problem, we introduce "cross-subsidies" as an extra method for signaling. By providing cross-subsidies, high type entrepreneurs can guarantee the satisfaction of both incentive constraints.

As the cost of doing so, entrepreneurs of high type may have to undertake the price cut. In other words, high type entrepreneurs make a trade-off between the value created by financiers and the price of their securities. If the price cut caused by cross-subsidies exceeds the whole value of the financiers' service, high type entrepreneurs may prefer to retain a sufficiently large share of equity for signaling and make the financiers passive.

As a result, the optimization program can be decomposed into two subprograms (with or without investors' effort), and only the allocation that provides higher utility to the high type entrepreneurs can be sustained as an equilibrium:

Subprogram I: ($k_H = K$)

$$\max_{\{S_H, a_H\}} \theta_H + (\phi_H - 1)K - S_H - R(a_H)$$

Subject to:

$$(1 - a_H)(\theta_H - \theta_L) + [\phi_H(1 - a_H) + \phi_L a_H - 1]K - \frac{1}{1-p} S_H - R(a_H) \leq 0$$

$a_H \geq 0$

$a_H \leq \dfrac{\phi_H - 1}{\phi_H}$

$S_H \geq 0$

Subprogram II: ($k_H = 0$)

$$\max_{\{S_H, a_H\}} \theta_H - S_H - R(a_H)$$

Subject to:

$$(1 - a_H)(\theta_H - \theta_L) - \frac{1}{1-p} S_H - R(a_H) \leq 0$$

$$a_H > \frac{\phi_H - 1}{\phi_H}$$

$$S_H \geq 0$$

Subprogram I illustrates the case in which high type entrepreneurs conserve investors' incentive, so the effort contributed to the high type firm is equal to its maximum K. Subprogram II describes the case in which entrepreneurs abandon investors' effort ($k_H = 0$). We define the optimal contract in subprogram I as C_K and that in subprogram II as C_0. Thus, the separating equilibrium will be C_K when $U_H(C_K) \geq U_H(C_0)$ or C_0 when $U_H(C_K) < U_H(C_0)$.

Proposition 1a

When investors' effort is non-contractible and cross-subsidies are involved in the menu ($S_H > 0$ and $\widetilde{\sigma^2} < \overline{\sigma^2}$):

1. For $\sigma^2 > \widehat{\sigma^2}$, there exists a unique separating equilibrium in which $a_H = a^*$, $S_H = S_H^*$ and investors' effort is efficiently distributed (k_H=K).

2. For $\overline{\sigma^2} < \sigma^2 < \widehat{\sigma^2}$, there exists a unique separating equilibrium in which $a_H = \frac{\phi_H - 1}{\phi_H}$, $S_H > S_H^*$ and investors' effort is efficiently distributed (k_H=K).

3. For $\widetilde{\sigma^2} < \sigma^2 < \overline{\sigma^2}$, there exists a unique separating equilibrium in which $a_H > \frac{\phi_H - 1}{\phi_H}$, $S_H > 0$ and no effort is distributed to the high type (k_H=0).

Proof: See Appendix A1 for the proof.

Firstly, to check the sustainability of the equilibrium above, we need to check that given the incumbent separating menu, whether a competitive investor can introduce other profit-making separating menu that attracts both types of entrepreneurs. As we have defined in the maximization problem above, our menu maximizes the utility of high type entrepreneur under the break-even condition and the incentive constraints. Therefore, it is impossible for the new entrant to offer another separating menu which effectively attracts the high type entrepreneurs and still makes a positive profit. Moreover, since our separating menu with cross-subsidy makes a loss to the low type entrepreneurs, any other separating menu that attracts only the low types will make a loss. As a result, we can conclude that there does not exist any other separating menu that can break our equilibrium.

Secondly, we are going to check whether the competitor can offer a single contract that breaks the separating equilibrium. As we have just mentioned, in this separating menu the high type entrepreneurs subsidize the low type ones. Therefore, the securities of low type are overpriced in the menu. In that case, any single contract that only attracts the low type will be loss-making since it must offer an even higher price to attract the low type entrepreneurs. Besides, if the competitors attempt to offer a "cream-skimming" contract which only attracts the high type, it can be inferred that the incumbent investors will withdraw their menu after observing this new contract. As a result, entrepreneurs of both types may take the new contracts at the same time. If the new contract does not satisfy the break-even condition of pooling, the new entrant may end up with loss-making. Finally, our last question is whether the new entrant can benefit by offering a pooling contract which attracts both types of entrepreneurs. One intuitive way to answer this question is to analysis the whole break-even pooling contract set and check whether there exists any pooling contract that can attract the high type entrepreneurs. However, based on our finding in Section 5.2 and the maximization problem above, the answer is negative. In section 5.2, we have proved that when $\Phi<1$ for any break-even pooling contract (C_P), there always exists a separating menu (C_s) that breaks-even and makes the high type entrepreneurs better-off ($U_H(C_s) \geq U_H(C_P)$). In order word, when the separating menu C_s is offered in the market, the new pooling contract C_P will not be able to attract the high type entrepreneurs. Furthermore, as we have mentioned at the beginning of the previous paragraph, among all the break-even separating menu, our optimal menu (C_S^*) maximizes the utility of high type entrepreneurs. Therefore we can have $U_H(C_S^*) \geq U_H(C_s)$. Simply, it can be inferred that $U_H(C_S^*) \geq U_H(C_s) \geq U_H(C_P)$, so when the optimal separating menu is offered, there is no break-even pooling contract that can effectively attract the high type entrepreneurs. In conclusion, the optimal separating menu can be sustained as an equilibrium.

As for the intuition of the equilibrium, the first case illustrates the situation in which the operating risk is relatively high. In that case, a small fraction of equity retained can credibly convey entrepreneurs' type information. Therefore, the incentive for financiers to exert effort is not undermined by equity retention. On the other hand, owing to the high operating risk, the disutility associated with equity retention is relatively high, and cross-subsidization becomes a more efficient way of signalling. From the proposition above, the necessary condition for the cross-subsidy to be introduced in the equilibrium menu is $\sigma^2 > \widetilde{\sigma^2}$, where $\widetilde{\sigma^2}$ is negatively correlated with the proportion of high type p. Given no change in the whole population, an increase in high type proportion means more providers and fewer receivers of the cross-subsidies. Therefore, the cross-subsidies undertaken by each high type entrepreneur will decrease. As a result, high type entrepreneurs may prefer a separating menu with higher cross-subsidies rather than higher equity retention under this environment.

As the operating risk decreases, the signal from equity retention becomes less informative. In other words, keeping the level of cross-subsidy unchanged, entrepreneurs may have to retain a higher fraction of equity for separation. However, retaining a fraction of equity higher than $\frac{\phi_H - 1}{\phi_H}$ can destroy investors' incentive to exert effort. Conversely, entrepreneurs can choose to hold a fraction no more than $\frac{\phi_H - 1}{\phi_H}$ to motivate the investors. However, this significantly weakens the power of their signal. To compensate for the lost power, high type entrepreneurs may have to offer extra cross-subsidies to prevent low types from mimicking. This dilemma between preserving financiers' effort and paying extra cross-subsidy is well described by sub-cases 2 and 3 of Proposition 1a. The fundamental factors of this trade-off are the operating risk $\boldsymbol{\sigma^2}$ (related to the cost) and the potential of investors $\mathbf{K}$ (related to the benefit). When the operating risk is relatively moderate, high type entrepreneurs may trend to reserve financiers' incentive to exert effort because the amount of extra cross-subsidies remains acceptable to them. Otherwise, when the operating risk is extremely low, the cost of offering extra cross-subsidy will be sufficiently high, which exceeds the value that can be created by the financier. In that case, high-quality entrepreneurs choose to retain a fraction of equity larger than $\frac{\phi_H - 1}{\phi_H}$ and cause the financiers to be passive. In addition, the operating risk should be higher than $\widetilde{\boldsymbol{\sigma}^2}$ (which is defined in the proof of Proposition) for the introduction of cross-subsidy. If this condition is not satisfied, our model will lead to a similar result as Baldenius and Meng's low effort equilibrium, which will also be discussed in the proposition below.

Proposition 1b

When the investor effort is non-contractible, and cross-subsidies are not involved in the menu: ($\mathbf{S_H = 0}$, and $\widetilde{\boldsymbol{\sigma}^2} > \overline{\boldsymbol{\sigma}^2}$)

1. For $\widetilde{\boldsymbol{\sigma}^2} > \boldsymbol{\sigma}^2 \geq \overline{\boldsymbol{\sigma}_2^2}$, there exists a unique separating equilibrium in which $\mathbf{a_H} \leq \frac{\boldsymbol{\phi}_H - 1}{\boldsymbol{\phi}_H}$, and the investor effort is efficiently distributed ($\mathbf{k_H}$=K).

2. For $\boldsymbol{\sigma}^2 < \overline{\boldsymbol{\sigma}_2^2}$, there exists a unique separating equilibrium in which $\mathbf{a_H} > \frac{\boldsymbol{\phi}_H - 1}{\boldsymbol{\phi}_H}$, and no effort is distributed to the high type ($\mathbf{k_H}$=0).

Proof: All proofs will be shown in Appendix A1.

Similarly, we need to check the sustainability of this equilibrium. With similar argument as in Proposition 1a, we can simply prove that there is not another deviating menu which can dominate our optimal separating menu. Given the maximization problem and the break-even constraints, the new menu may either fail to attract the high type entrepreneurs or fail to generate a positive profit. Besides, it is also impossible for the rival to offer a profit-making contract which attracts a certain type of entrepreneurs. Different from the previous discussion, the cross-subsidies are not introduced into the separating menu in this sub-case. In other word, contracts offered to different types satisfy their own break-even condition separately. Since in this separating menu, the low type entrepreneurs have fully securitized their asset at a fair price, the new entrant can never offer a deviating contract which makes the low types better-off without making a loss. On the other hand, as we can see in the proof, the non-mimic condition of low type entrepreneurs is binding in our separating menu. Hence, if the new entrant intends to attract the high type entrepreneurs by decreasing their equity retention or increasing their securities price, the new contract will inevitably attract the low type entrepreneurs as well. Finally, based on our discussion in last proposition, it can also be inferred that any polling contract that makes the high type entrepreneurs better-off will become loss-making. Therefore, the separating menu in Proposition 1b can also be sustained as an equilibrium.

Different from the case in Proposition 1a, Proposition 1b illustrates a case where the proportion of high type is sufficiently low. Hence, equity retention becomes a preferable way of signaling, and high type entrepreneurs may abandon the usage of cross-subsidies when $\sigma^2 < \widetilde{\sigma^2}$ or $p < \tilde{p}$.

$$\sigma^2 < \widetilde{\sigma^2} = \frac{(1-p^2)(\Delta\theta + \Delta\phi K)^2}{2\rho p^2[\Delta\theta + (\phi_H - 1)k_H]} \Leftrightarrow$$

$$p < \tilde{p} = \frac{(\Delta\theta + \Delta\phi K)}{\sqrt{(\Delta\theta + \Delta\phi k_H)^2 + 2\rho\sigma^2[\Delta\theta + (\phi_H - 1)k_H]}}$$

In this case, the trade-off becomes more straightforward. By comparing the cost of retaining equity and the cost of mispricing from pooling with low type entrepreneurs, high type entrepreneurs decided whether to hold a fraction higher than $\frac{\phi_H - 1}{\phi_H}$. However, regardless of entrepreneurs' choice, the financiers will not exert effort to their firm $(\Phi < 1)$. In other words, investors' effort plays no role in entrepreneurs' decision process in this extreme case.

In summary, we find that high type entrepreneurs make a two-dimensional trade-off in the securities issuing process when investors' effort is non-contractible. On one hand, by comparing σ^2 to $\widetilde{\sigma^2}$, which is related to the proportion of high type p, high type entrepreneurs decide whether to introduce cross-subsidies as a way of signaling. On the other hand, by comparing σ^2 to $\overline{\sigma^2}$, which is related to the potential value of investor effort K, high type entrepreneurs decide whether they should introduce investors' effort. However, one of the most interesting findings is that even if high type entrepreneurs do not intend to introduce investors' effort, they would still prefer to separate from the low type. In the following section, we prove that the pooling allocation cannot be sustained as an equilibrium when $\Phi < 1$.

5.2 Pooling Equilibrium

5.2.1 Characteristics of possible pooling equilibrium

In our model, we assume that the expected marginal productivity of effort is below its cost. Thus, financiers exert no effort to any firm if their type is not identified. Under this assumption, we find that the pooling equilibrium cannot be sustained in our model. To be more precise, with $\Phi < 1$, our model always ends up with a unique separating equilibrium. To disprove the existence of a pooling equilibrium, we start by analyzing the characteristics of the potential pooling contracts and then discuss whether any of these contracts can be sustained as an equilibrium.

First, in any pooling equilibrium, investors' break-even constraint must be binding because financiers cannot be suffering a loss in the equilibrium (free participation) and the perfect competition forces financiers to lower their profit to zero. Thus:

$$P_{pol} = \Theta + \left[\Phi - \frac{1}{1 - a_{pol}}\right] k_{pol}$$

Similar to our previous definition, P_{pol}, a_{pol} and k_{pol} represent the price of equity, equity retention and investors' effort in the pooling contract, respectively. As we mentioned above, we assume that $\Phi < 1$. Therefore, financiers will be passive if the type information is not revealed. In other words, $k_{pol} = 0$ and

$$P_{pol} = \Theta = p\theta_H + (1 - p)\theta_L$$

The expected utility functions of the two types of entrepreneurs will then become:

$$U_{Hp} = (1 - a_{pol})P_{pol} + a_{pol}\theta_H - R(a_{pol})$$

$$U_{Lp} = (1 - a_{pol})P_{pol} + a_{pol}\theta_L - R(a_{pol})$$

In the equations above, U_{Hp} and U_{Lp} represent the expected utility of high type and low type, respectively, under the pooling contract. Because $\theta_L < \Theta < \theta_H$, by retaining a fraction of share, entrepreneurs can prevent more of their underpriced securities from being sold. In other words, even if we ignore the effect of signaling, equity retention is still an optional way to alleviate the loss due to adverse selection.

Similar to Lemma 3, we can easily prove that among all the zero-profit pooling contracts, only the one that maximizes the utility of high type can be sustained as an equilibrium. The proof is as follows: Suppose that a pooling contract C_{pol} is sustained as an equilibrium and a new entrant attempts to offer another pooling contract C^*_{pol}, where $U_{Hp}(C^*_{pol}) > U_{Hp}(C_{pol})$. C^*_{pol} can be designed to ensure that the financier can still make a strictly positive profit if both types take it. (The existence of such a contract can be guaranteed if there exists another point C''^*_{pol} that satisfies $U_{Hp}(C''^*_{pol}) > U_{Hp}(C_{pol})$ along the zero-profit line of the financier.) Identical to the previous discussion, high type entrepreneurs deviate to the new contract, and this deviation can cause the incumbent financiers to incur losses if the original contract is offered with commitment. Otherwise, if the original contract is offered without commitment, both types will finally take C_{pol}^{*} and the new entrant ends up with positive profit. In that case, the original pooling contract C_{pol} cannot be an equilibrium.

In conclusion, to find a potential pooling contract, we must solve the following optimization program:

$$\max_{a_{pol}} U_{Hp} = (1 - a_{pol})P_{pol} + a_{pol}\theta_H - R(a_{pol})$$

Subject to:

$$P_{pol} = p\theta_H + (1 - p)\theta_L$$

We solve this program by calculating the first-order condition:

$$\rightarrow \frac{\partial U_{Hp}}{\partial a_{pol}} = -p\theta_H - (1 - p)\theta_L + \theta_H - \rho a_{pol}\sigma^2 = 0$$

$$\rightarrow a_{pol} = \frac{(1 - p)(\theta_H - \theta_L)}{\rho\sigma^2}$$

$$\text{And } U_{Hp} = [p\theta_H + (1 - p)\theta_L] + \frac{(1-p)^2\Delta\theta^2}{2\rho\sigma^2}$$

5.2.2 Nonexistence of pooling equilibrium when $\Phi < 1$

With the discussion above, we pin down a potential pooling contract. For a pooling contract to be sustainable as an equilibrium, it must be able to dominate any separating menu. In other words, there should not exist any separating menu $C_H(a_H, S_H)$ and $C_L(a_L, S_L)$ that satisfies the following conditions:

$$(1-a_H)(\theta_H-\theta_L)+\left[\phi_H(1-a_H)+\phi_L a_H-1\right]K-\frac{1}{1-p}S_H-R(a_H)\leq 0 \qquad (1)$$

(Incentive comparison condition)

$$\left(\phi_H-1\right)K+\theta_H-S_H-R(a_H)\geq[p\theta_H+(1-p)\theta_L]+\frac{(1-p)^2\Delta\theta^2}{2\rho\sigma^2} \qquad (2)$$

(The menu of separating contracts can make good firm better off than the pooling contract)

$$0\leq a_H\leq 1 \qquad (3)$$

(Good firm will retain a nonnegative fraction of equity)

$$S_H\geq 0 \qquad (4)$$

(High types subsidize the low types)

$$a_L=0 \qquad (5)$$

(In separating equilibrium, the low type will sell the whole firm)

$$pS_H+(1-p)S_L=0 \qquad (6)$$

(Zero-profit still holds when including the cross-subsidies)

However, we find that regardless of the parameter value, there always exists at least one menu of separating contracts satisfying all the conditions above. For example:

$$a_H = a_{pol} = \frac{(1-p)(\theta_H - \theta_L)}{\rho\sigma^2}$$

$$S_H = (1-p)(1-a_H)(\theta_H - \theta_L) - (1-p)R(a_H) + \{\omega(\phi_H - 1)K + (1-\omega)(1-p)[\phi_H(1-a_H) + \phi_L a_H - 1]K\} \qquad (\omega \in [0,1])$$

$$a_L = 0$$

$$S_L = -\frac{p}{1-p} S_H$$

Proof: See Appendix A2 for the proof.

Our proof shows that this argument can hold regardless of investors' potential K, because even in the extreme case where $a_H > \frac{\phi_H - 1}{\phi_H}$ and K=0, the above menu still satisfies all the necessary conditions. Therefore, the optimal pooling allocation could not be sustained as an equilibrium. In other words, there is no pooling equilibrium in our model when $\Phi < 1$.

5.2.3 Existence of pooling equilibrium when $\Phi > 1$

Pooling equilibrium can be sustainable only when $\Phi > 1$. In this case, type revealing is no longer the necessary condition for investors' effort to be contributed. This result weakens the incentive for high type entrepreneurs to signal themselves. More specifically, this pooling equilibrium can only arise when operating risk is extremely low. In that case, the equity retention requirement for signaling will be relatively high, and this high retention could undermine the incentive for financiers to make an effort, as we have discussed. Moreover, because investors' effort can be attained even when pooling with the low type, high type entrepreneurs may have no incentive to offer extra cross-subsidies for signaling. Therefore, both methods of signaling can be abandoned by the high type entrepreneurs, and the pooling equilibrium can be sustained.

Proof: See Appendix A4 for the proof

6. Empirical implications

In section 5, we showed that there can exist a separating equilibrium in which the equity issued by the high-quality entrepreneurs are underpriced whereas those of the low-quality entrepreneurs are overpriced. We also show that this equilibrium is more likely to arise when both the proportion of high-quality firms and the operating risk are high. Empirically, a proxy for the proportion of the high type could be the economy-wide conditions. More specifically, we expect that the proportion of the high-quality firms would be higher when the economy is in a boom. With regard to the operating risk of a firm or industry, it can be measured by the variance of its cash flow. Therefore, our model gives the prediction that there should be a positive correlation between the equity mispricing and the two factors above.

Moreover, we showed that the operating risk affects the fraction of equity retained by the high-quality firms. As we proved in the model, the fraction of the equity required for signaling is negatively correlated with the operating risk. The more equity is retained by the entrepreneurs, the less remains for the investors. As a result, financiers may have no incentive to exert effort when the operating risk is extremely low. In other words, our model predicts that in mature industries with low operating risk, financiers may behave passively. Correspondingly, active investors such as venture capitalists may tend to invest in firms with growth opportunity and high operating risk. This prediction is consistent with the empirical finding of Sapienza et al. (1996). They found that project and environment uncertainty does play a role in financiers' value-add activity, and they also observed that venture capital fund managers become more involved in the early stages of projects with higher risk assessment.

7. Conclusions

In this paper, we considered a model in which the firm performance depends on both the firm's (project's) inherent quality and the effort exerted by the venture capitalist. We show that the market equilibrium is closely linked with two factors: i) the proportion of high-quality entrepreneurs and ii) the operating risk of the firms. When both are sufficiently high, there exists a separating equilibrium that includes cross-subsidies. In this equilibrium, the low-quality entrepreneurs sell all of their shares above the fair price (overpricing) whereas the high-quality entrepreneurs retain a fraction of their shares and sell their equity below the fair price (underpricing). If both the proportion of high-quality firms and the operating risk is sufficiently low, the high-quality entrepreneurs will separate themselves from the low-quality ones by retaining a sufficiently high fraction of shares, and the shares sold by both types of entrepreneurs are fairly priced.

We also found that the operating risk affects the fraction of shares retained by the high-quality firms. As the operating risk decreases, the cost of equity retention decreases as well. Therefore, high-type entrepreneurs may have to retain a higher fraction of equity to separate themselves from the low-type firms. However, this also implies that the financiers will receive a lower fraction of equity and have less incentive to exert effort.

Finally, we discuss the necessary condition for a pooling equilibrium to arise. We show that the existence highly depends on the general quality of the whole industry Φ. In this paper, we have proved that when the marginal productivity of the whole industry is less than the marginal cost of financiers' effort ($\Phi < 1$), the separating equilibrium is always dominant. Otherwise, when $\Phi > 1$, which means that, when the financier exerts effort even without type information, a pooling allocation could be sustained as an equilibrium.

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Appendix

A.1 Separating equilibrium with non-contractible effort

As illustrated in the main text of the paper, when the fraction of equity retained by entrepreneur $\mathrm{a_H}$ exceeds $\frac{\phi_h-1}{\phi_H}$, the incentive for investors to contribute effort will be destroyed. Therefore, to simplify our analysis, we use $\mathrm{a_H}=\frac{\phi_h-1}{\phi_H}$ as the cut-off point to decompose our program into two subprograms. In subprogram I, when $\mathrm{a_H}\leq\frac{\phi_h-1}{\phi_H}$, investors continue to increase their effort level until it reaches its maximum k=K. In subprogram II, when $\mathrm{a_H}>\frac{\phi_h-1}{\phi_H}$, investors refuse to exert any effort and k=0. All subcases can be summarized as in the table below:

	Subprogram I: (k=K)		Subprogram II: (k=0)
	$\mathrm{a_H}<\dfrac{\phi_h-1}{\phi_H}$	$\mathrm{a_H}=\dfrac{\phi_h-1}{\phi_H}$	$\mathrm{a_H}>\dfrac{\phi_h-1}{\phi_H}$
$\mathrm{S_H}>0$	Case 1	Case 2	Case 5
$\mathrm{S_H}=0$	Case 3	Case 4	Case 6

Subprogram I: (k=K)

$$\max_{\{\mathrm{a_H},\mathrm{S_H}\}} \theta_\mathrm{H}+(\phi_\mathrm{H}-1)\mathrm{K}-\mathrm{S_H}-\mathrm{R}(\mathrm{a_H})$$

Subject to:

$$(1-\mathrm{a_H})(\theta_H-\theta_L)+[\phi_H(1-\mathrm{a_H})+\phi_L\mathrm{a_H}-1]\mathrm{K}-\frac{1}{1-\mathrm{p}}\mathrm{S_H}-R(\mathrm{a_H})\leq 0$$

$\mathrm{a_H} \geq 0$

$\mathrm{a_H} \leq \frac{\phi_H - 1}{\phi_H}$

$\mathrm{S_H} \geq 0$

Similarly, we solve the program with the Lagrangian method:

$$L = \theta_H + (\phi_H - 1)K - S_H - R(\mathrm{a_H}) - \lambda\left\{(1 - \mathrm{a_H})(\theta_H - \theta_L) + [\phi_H(1 - \mathrm{a_H}) + \phi_L \mathrm{a_H} - 1]K - \frac{1}{1-p}S_H - R(\mathrm{a})\right\} + \mu \mathrm{a_H} + \eta\left(\frac{\phi_H - 1}{\phi_H} - \mathrm{a_H}\right) + \varphi S_H$$

$$\frac{\partial \mathrm{L}}{\partial \mathrm{a_H}} = -(1-\lambda)\rho \mathrm{a_H}\sigma^2 + \lambda(\theta_H - \theta_L) + \lambda(\phi_H - \phi_L)K + \mu - \eta = 0 \quad (1)$$

$$\frac{\partial L}{\partial S_H} = -1 + \frac{\lambda}{1-p} + \varphi = 0 \quad (2)$$

$$\lambda\left\{(1 - \mathrm{a_H})(\theta_H - \theta_L) + [\phi_H(1 - \mathrm{a_H}) + \phi_L \mathrm{a_H} - 1]K - \frac{1}{1-p}S_H - R(\mathrm{a_H})\right\} = 0 \quad (3)$$

$$\lambda \geq 0$$

$$(1 - \mathrm{a_H})(\theta_H - \theta_L) + [\phi_H(1 - \mathrm{a_H}) + \phi_L \mathrm{a_H} - 1]K - \frac{1}{1-p}S_H - R(\mathrm{a_H}) \leq 0$$

$$\mathrm{a_H}\mu = 0 \quad (4)$$

$a_H \geq 0 \quad \mu \geq 0$

$$\varphi S_H = 0 \tag{5}$$

$\varphi \geq 0 \quad S_H \geq 0$

$$\eta\left(\frac{\phi_H - 1}{\phi_H} - a_H\right) = 0 \tag{6}$$

$\eta \geq 0 \quad a_H \leq \dfrac{\phi_H - 1}{\phi_H}$

Case 1

If $\varphi = 0, S_H \geq 0$ *and* $\eta = 0,\ a_H \leq \frac{\phi_h - 1}{\phi_H}$

From (2): $\lambda = 1 - p \rightarrow 0 < \lambda < 1$

So

$$(1 - a_H)(\theta_H - \theta_L) + [\phi_H(1 - a_H) + \phi_L a_H - 1]K - \frac{1}{1 - p} S_H - R(a_H) = 0$$

Then if $a_H = 0, \mu > 0$

From (1): $\lambda(\theta_H - \theta_L) + \lambda(\phi_H - \phi_L) + \mu > 0$ (fail to hold)

So $a_H > 0, \mu = 0$

Then from (1):

$$a_H = a^* = \frac{(1-p)[(\theta_H - \theta_L) + (\phi_H - \phi_L)K]}{p\rho\sigma^2}$$

$$S_H = S_H^* = (1-p)\{(1-a_H)[\theta_H - \theta_L - (\phi_H - \phi_L)K] + (\phi_L - 1)K - R(a_H)$$

Define $\Delta\theta = \theta_H - \theta_L$ and $\Delta\phi = \phi_H - \phi_L$

From $a_H \leq \frac{\phi_h - 1}{\phi_H}$, we have

$$p \geq \frac{\phi_H(\Delta\theta + \Delta\phi K)}{(\phi_H - 1)\rho\sigma^2 + \phi_H(\Delta\theta + \Delta\phi K)}$$

Or

$$\sigma^2 \geq \widehat{\sigma^2} = \frac{(1-p)(\Delta\theta + \Delta\phi K)\phi_H}{\rho p(\phi_H - 1)}$$

And from $S_H \geq 0$, we have

$$p \geq \frac{(\Delta\theta + \Delta\phi K)}{\sqrt{(\Delta\theta + \Delta\phi k_H)^2 + 2\rho\sigma^2[\Delta\theta + (\phi_H - 1)K]}}$$

Or

$$\sigma^2 \geq \widetilde{\sigma^2} = \frac{(1-p^2)(\Delta\theta + \Delta\phi K)^2}{2\rho p^2[\Delta\theta + (\phi_H - 1)K]}$$

In conclusion, this case exists when

$$p > \max\left(\frac{\phi_H(\Delta\theta+\Delta\phi K)}{\left(\phi_H-1\right)\rho\sigma^2+\phi_H(\Delta\theta+\Delta\phi K)}, \frac{(\Delta\theta+\Delta\phi K)}{\sqrt{(\Delta\theta + \Delta\phi k_H)^2+2\rho\sigma^2\left[\Delta\theta+\left(\phi_H-1\right)K\right]}}\right)$$

Or

$$\sigma^2 > max(\frac{(1-p)(\Delta\theta+\Delta\phi K)\phi_H}{\rho p(\phi_H-1)}, \frac{(1-p^2)(\Delta\theta+\Delta\phi K)^2}{2\rho p^2[\Delta\theta+(\phi_H-1)K]})$$

Case 2

If $\varphi = 0, S_H \geq 0$ *and* $\eta > 0, a_H = \frac{\phi_h\text{-}1}{\phi_H}$

Since $a_H = \frac{\phi_h\text{-}1}{\phi_H} > 0$, $\mu = 0$

From (2): $\lambda = 1 - p \rightarrow 0 < \lambda < 1$

$$(1-a_H)(\theta_H - \theta_L) + [\phi_H(1-a_H) + \phi_L a_H - 1]K - \frac{1}{1-p}S_H - R(a_H) = 0$$

$$S_H = (1-p)\{\left(1-\frac{\phi_H-1}{\phi_H}\right)[\theta_H-\theta_L-(\phi_H-\phi_L)K] + (\phi_L-1)K - R(\frac{\phi_H-1}{\phi_H})$$

$$\eta = (1-p)(\Delta\theta+\Delta\phi) - p\rho\sigma^2\frac{\phi_H-1}{\phi_H}$$

From $\eta > 0$ we have,

$$p < \frac{\phi_H(\Delta\theta+\Delta\phi K)}{(\phi_H-1)\rho\sigma^2+\phi_H(\Delta\theta+\Delta\phi K)}$$

Or

$$\sigma^2 < \widehat{\sigma^2} = \frac{(1-p)(\Delta\theta+\Delta\phi K)\phi_H}{\rho p(\phi_H-1)}$$

And from $S_H \geq 0$, we have

$$\sigma^2 \leq \overline{\sigma^2} = \frac{2\phi_H(\Delta\theta + \Delta\phi K) + 2(\phi_L - 1)K\phi_H^2}{\rho(\phi_H - 1)^2}$$

Case 3

If $\varphi > 0, S_H = 0 \; and \; \eta = 0, a_H < \frac{\phi_h - 1}{\phi_H}$

From (2): $\lambda = (1 - p)(1 - \varphi)$

If $a_H = 0, \mu > 0$

Substitute it into

$$(1 - a_H)(\theta_H - \theta_L) + [\phi_H(1 - a_H) + \phi_L a_H - 1]K - \frac{1}{1 - p} S_H - R(a_H) \leq 0$$

$$\rightarrow (\theta_H - \theta_L) + (\phi_H - 1)K \leq 0 \text{ (fail to hold)}$$

So

$$a_H > 0, \; \mu = 0$$

Substituting this conclusion into (1), we have

$$-(1 - \lambda)\rho a_H \sigma^2 + \lambda(\Delta\theta + \Delta\phi K) = 0$$

If $\lambda = 0 \rightarrow -\rho a_H \sigma^2 = 0$ (fail to hold)

So $\lambda > 0$

And

$$(1 - a_H)(\theta_H - \theta_L) + [\phi_H(1 - a_H) + \phi_L a_H - 1]K - R(a_H) = 0$$

$$\rightarrow \rho\sigma^2 a_H^2 + 2(\Delta\theta + \Delta\phi K)a_H - 2[\Delta\theta + (\phi_H - 1)K] = 0$$

$$\rightarrow a_H = \frac{-(\Delta\theta + \Delta\phi K) + \sqrt{(\Delta\theta + \Delta\phi K)^2 + 2\rho\sigma^2[\Delta\theta + (\phi_H - 1)K]}}{\rho\sigma^2}$$

$$S_H = 0$$

From $a_H < \frac{\phi_h\text{-}1}{\phi_H}$ we have

$$\sigma^2 > \overline{\sigma^2} = \frac{2\phi_H(\Delta\theta + \Delta\phi K) + 2(\phi_L - 1)K\phi_H^2}{\rho(\phi_H - 1)^2}$$

And from $\varphi > 0$ we have

$$p < \frac{(\Delta\theta + \Delta\phi K)}{\sqrt{(\Delta\theta + \Delta\phi k_H)^2 + 2\rho\sigma^2[\Delta\theta + (\phi_H - 1)K]}}$$

Or

$$\sigma^2 < \widetilde{\sigma^2} = \frac{(1 - p^2)(\Delta\theta + \Delta\phi K)^2}{2\rho p^2[\Delta\theta + (\phi_H - 1)K]}$$

Case 4

$\varphi > 0, S_H = 0$ *and* $\eta > 0, a_H = \dfrac{\phi_h - 1}{\phi_H}$

From (2): $\lambda = (1 - P)(1 - \varphi)$

Since $a_H = \frac{\phi_h - 1}{\phi_H} > 0$, $\mu = 0$

If $\lambda = 0$, substitute it into (1),

$$\rightarrow -\rho\sigma^2 \frac{\phi_h - 1}{\phi_H} - \eta = 0 \text{ (fail to hold)}$$

So $\lambda > 0$ and

$$(1-\mathrm{a_H})(\theta_H-\theta_L)+[\phi_H(1-\mathrm{a_H})+\phi_L\mathrm{a_H}-1]K-R(\mathrm{a_H})=0$$

$$\rightarrow \mathrm{a_H}=\frac{-(\Delta\theta+\Delta\phi\mathrm{K})+\sqrt{(\Delta\theta+\Delta\phi\mathrm{K})^2+2\rho\sigma^2[\Delta\theta+(\phi_\mathrm{H}-1)\mathrm{K}]}}{\rho\sigma^2}$$

We also have $\mathrm{a_H}=\frac{\phi_h\text{-}1}{\phi_H}$

Thus, case 4 is actually a special case of case 3, when $\frac{\phi_h\text{-}1}{\phi_H}$ becomes a sufficiently large fraction of signaling.

Subprogram II: (k=0)

$$\max_{\{\mathrm{a_H},\mathrm{S_H}\}} \theta_\mathrm{H}-\mathrm{S_H}-\mathrm{R}(\mathrm{a_H})$$

Subject to:

$(1-\mathrm{a_H})(\theta_H-\theta_L)-\dfrac{1}{1-\mathrm{p}}\mathrm{S_H}-R(\mathrm{a_H})$

$\mathrm{a_H}>\dfrac{\phi_\mathrm{H}-1}{\phi_\mathrm{H}}$

$\mathrm{S_H}\geq 0$

$$L=\theta_H-S_H-R(\mathrm{a_H})-\lambda\left\{(1-\mathrm{a_H})(\theta_H-\theta_L)-\frac{1}{1-p}S_H-R(\mathrm{a_H})\right\}+\eta\left(\mathrm{a_H}-\frac{\phi_H-1}{\phi_H}\right)+\varphi S_H$$

$$\frac{\partial \mathrm{L}}{\partial \mathrm{a_H}} = -(1-\lambda)\rho \mathrm{a_H}\sigma^2 + \lambda(\theta_\mathrm{H} - \theta_\mathrm{L}) + \eta \tag{7}$$

$$\frac{\partial L}{\partial S_H} = -1 + \frac{\lambda}{1-p} + \varphi = 0 \tag{8}$$

$$\lambda\left\{(1-\mathrm{a_H})(\theta_H - \theta_L) - \frac{1}{1-p}S_H - R(\mathrm{a_H})\right\} = 0 \tag{9}$$

$$\lambda \geq 0$$

$$(1-\mathrm{a_H})(\theta_H - \theta_L) - \frac{1}{1-p}S_H - R(\mathrm{a_H}) \leq 0$$

$$\varphi \mathrm{S_H} = 0 \tag{10}$$

$$\varphi \geq 0 \qquad \mathrm{S_H} \geq 0$$

Since $\eta\left(\mathrm{a_H} - \frac{\phi_H - 1}{\phi_H}\right)$=0 and $\mathrm{a_H} > \frac{\phi_\mathrm{H} - 1}{\phi_\mathrm{H}}$, we have $\eta = 0$

Case 5

If $\varphi = 0, \mathrm{S_H} \geq 0$

From (8): $\lambda = 1 - p \to 0 < \lambda < 1$

From (7): $-(1-\lambda)\rho a_H \sigma^2 + \lambda(\theta_H - \theta_L) = 0 \to a_H = \frac{(1-P)(\theta_H - \theta_L)}{p\rho\sigma^2}$

And since $\lambda > 0$

$$\to (1 - a_H)(\theta_H - \theta_L) - \frac{1}{1-P} S_H - R(a_H) = 0$$

$$\to S_H = (1-p)[(1-a_H)(\theta_H - \theta_L) - R(a_H)]$$

From $a_H > \frac{\phi_H - 1}{\phi_H}$ we have

$$p < \frac{\Delta\theta\phi_H}{\rho\sigma^2(\phi_H - 1) + \Delta\theta\phi_H}$$

Or

$$\sigma^2 < \frac{(1-p)\Delta\theta\phi_H}{\rho p(\phi_H - 1)}$$

And from $S_H \geq 0$ we also have

$$p \geq \frac{\Delta\theta}{\sqrt{\Delta\theta^2 + 2\Delta\theta\rho\sigma^2}}$$

Or

$$\sigma^2 \geq (\frac{1}{2\rho p^2} - \frac{1}{2\rho})\Delta\theta$$

This case also requires that

$$(\frac{1}{2p^2} - \frac{1}{2})\Delta\theta < \rho\sigma^2 < \frac{(1-p)\Delta\theta\phi_H}{p(\phi_H - 1)}$$

$$\rightarrow (\frac{1}{2p^2} - \frac{1}{2})\Delta\theta < \frac{(1-p)\Delta\theta\phi_H}{p(\phi_H - 1)}$$

$$\rightarrow p > \frac{\phi_H - 1}{\phi_H + 1}$$

To determine the parameter condition under which the menus with investors' effort may dominate menus without effort, we compare the results of case 2 to those of case 5. Define the optimal contract in case 2 as C_K and that in case 5 as C_0. If $U_H(C_K) > U_H(C_0) \rightarrow$

$$\theta_H + (\phi_H - 1)K - (1-p)\left[\left(1 - \frac{\phi_H - 1}{\phi_H}\right)(\Delta\theta - \Delta\phi K) + (\phi_L - 1)K\right] - pR\left(\frac{\phi_H - 1}{\phi_H}\right)$$
$$> \theta_H - (1-p)\left[1 - \frac{(1-p)(\theta_H - \theta_L)}{p\rho\sigma^2}\right]\Delta\theta - pR\left(\frac{(1-p)(\theta_H - \theta_L)}{p\rho\sigma^2}\right)$$

From this inequality, we can deduce that it will be satisfied when:

$$K > \overline{K} \text{ or } \sigma^2 > \overline{\sigma^2}$$

Since we also have $\frac{(1-p)(\theta_H - \theta_L)}{p\rho\sigma^2} > \frac{\phi_H - 1}{\phi_H}$

$$\rightarrow \sigma^2 < \widehat{\sigma^2}$$

Case 6

If $\varphi > 0, S_H = 0$

From (8): $\lambda = (1 - \mathrm{p})(1 - \varphi)$

If $\lambda = 0$ substitute it into (7),

$$\rightarrow -\rho \mathrm{a_H} \sigma^2 = 0 \text{ , and } \mathrm{a_H} > \frac{\phi_\mathrm{H} - 1}{\phi_\mathrm{H}} > 0 (\text{fail to hold})$$

So $\lambda > 0$ and

$$(1 - \mathrm{a_H})(\theta_H - \theta_L) - R(\mathrm{a_H}) = 0$$

$$\rightarrow \mathrm{a_H} = \frac{\sqrt{\Delta\theta^2 + 2\Delta\theta\rho\sigma^2} - \Delta\theta}{\rho\sigma^2}$$

And from (7)

$$\lambda = \frac{\rho \mathrm{a_H} \sigma^2}{\Delta\theta + \rho \mathrm{a_H} \sigma^2}$$

$$\rightarrow \varphi = 1 - \frac{\rho \mathrm{a_H} \sigma^2}{(1 - \mathrm{p})(\Delta\theta + \rho \mathrm{a_H} \sigma^2)}$$

From $\mathrm{a_H} > \frac{\phi_\mathrm{H} - 1}{\phi_\mathrm{H}}$, we have

$$\sigma^2 < \frac{2\phi_\mathrm{H}\Delta\theta}{\rho(\phi_\mathrm{H} - 1)^2}$$

And from $\varphi > 0$ we also have

$$\mathrm{p} < \frac{\Delta\theta}{\sqrt{\Delta\theta^2 + 2\Delta\theta\rho\sigma^2}}$$

Or

$$\sigma^2 < (\frac{1}{2\rho \mathrm{p}^2} - \frac{1}{2\rho})\Delta\theta$$

Similarly, to identify the parameter conditions for introducing investors' effort, we compared the results of case 4 to those of case 6. Define the optimal contract in case 4 as $\mathrm{C'_K}$ and that in case 6 as $\mathrm{C'_0}$. If $\mathrm{U_H}(\mathrm{C_K}) > \mathrm{U_H}(\mathrm{C_0}) \rightarrow$

$$\theta_H + (\phi_H - 1)K - R(\frac{\phi_H - 1}{\phi_H}) > \theta_H - R(\frac{\sqrt{\Delta\theta^2 + 2\Delta\theta\rho\sigma^2} - \Delta\theta}{\rho\sigma^2})$$

It will be satisfied only if $K > \overline{K}_2$ and $\sigma^2 > \overline{\sigma_2^2}$

A2. Discussion about the pooling when $\Phi < 1$

In our model, the financier offers a menu of contracts to the entrepreneurs, after which the entrepreneurs choose the contracts that maximize their utility. In the pooling case, identical contracts will be offered to both types of entrepreneurs. It can be inferred that this contract must be the one that maximizes the utility of the high type. Suppose that if a contract of this kind is offered by one of the financiers; other competitors can infer that any deviating contracts will be accepted by no type or low type only. As a result, to pin down the sustainable pooling contracts, the objective function of our program should remain sourced from the utility function of high type entrepreneurs.

With regard to the effort contributed by the investors, because we assume that the proportion of high type and low type satisfied $p\phi_H + (1 - p)\phi_L < 1$, the marginal productivity of the effort would not be able to cover the corresponding cost. Thus, no effort will be contributed to any firm if a pooling equilibrium is sustained. The price of equity offered to the entrepreneurs will then become:

$$P_{pol} = p\theta_H + (1 - p)\theta_L$$

Additionally, with a pooling contract, no cross-subsidy can be implemented ($S_H = S_L = 0$). Summarizing all the results above, the utility function of the high type entrepreneurs taking the pooling contract will be:

$$U_{Hp} = (1 - a)P_{pol} + a\theta_H - R(a)$$

Using this as the objective function, we derive the first order condition and calculate the optimal equity retention:

$$\frac{\partial U_{Hp}}{\partial a} = -p\theta_H - (1 - p)\theta_L + \theta_H - \rho a\sigma^2 = 0$$

$$\rightarrow a_{pol} = \frac{(1 - p)(\theta_H - \theta_L)}{\rho\sigma^2}$$

$$\text{And } U_{Hp} = [p\theta_H + (1 - p)\theta_L] + \frac{(1-p)^2\Delta\theta^2}{2\rho\sigma^2}$$

For a pooling contract to be sustainable as an equilibrium, the pooling contract must be able to dominate any separating menu. In other word, there should not exist any separating menu (a_H, S_H) and (a_L, S_L) that satisfies the following conditions:

$$(1 - a_H)(\theta_H - \theta_L) + [\phi_H(1 - a_H) + \phi_L a_H - 1]K - \frac{1}{1 - p}S_H - R(a_H) \leq 0 \qquad (1)$$

(Incentive comparison condition)

$$\left(\phi_H - 1\right)K + \theta_H - S_H - R(a_H) \geq [p\theta_H + (1-p)\theta_L] + \frac{(1-p)^2\Delta\theta^2}{2\rho\sigma^2} \quad (2)$$

(The menu of contracts can make the good firms better off than the pooling contract)

$$0 \leq a_H \leq 1 \quad (3)$$

(A good firm retains a nonnegative fraction of equity)

$$S_H \geq 0 \quad (4)$$

(Cross-subsidies can just be from good type to bad type)

$$a_L = 0 \quad (5)$$

(In separating equilibrium, the bad type sells the whole firm)

$$pS_H + (1-p)S_L = 0 \quad (6)$$

(Zero-profit condition of the whole menu when including the cross-subsidies)

Otherwise, if we can prove the existence of such a separating menu, the pooling equilibrium would collapse. Because $a_L = 0$ and $S_L = -\frac{P}{1-P}S_H$, the key point is whether there exists a nonnegative a_H and S_H to satisfy inequalities (1) and (2).

From (1)

$$[\phi_H(1-a_H) + \phi_L a_H - 1]K + (1-a_H)(\theta_H - \theta_L) - R(a_H) \leq \frac{1}{1-p}S_H$$

$$\rightarrow (1-p)\left[\phi_H(1-a_H) + \phi_L a_H - 1\right]K + (1-p)(1-a_H)(\theta_H - \theta_L) - (1-p)R(a_H) \leq S_H$$

From (2)

$$(\phi_H - 1)K + (1-p)(\theta_H - \theta_L) - \frac{(1-p)^2\Delta\theta^2}{2\rho\sigma^2} - R(a_H) \geq S_H$$

So it is required that

$$(1-p)\left[\phi_H(1-a_H)+\phi_L a_H - 1\right]K+(1-p)(1-a_H)(\theta_H-\theta_L) - (1-p)R(a_H)$$
$$\leq (\phi_H - 1)K+(1-p)(\theta_H-\theta_L) - \frac{(1-p)^2\Delta\theta^2}{2\rho\sigma^2} - R(a_H)$$

$$(\phi_H - 1)K - (1-p)\left[\phi_H(1-a_H)+\phi_L a_H - 1\right]K + \{[(1-p)(\theta_H-\theta_L) - \frac{(1-p)^2\Delta\theta^2}{2\rho\sigma^2}$$
$$- R(a_H)] - [(1-p)(1-a_H)(\theta_H-\theta_L) - (1-p)R(a_H)]\} \geq 0$$

This inequality can be decomposed into two sufficient conditions:

$$(\phi_H\text{-}1)K - (1-p)\left[\phi_H(1-a_H)+\phi_L a_H - 1\right]K \geq 0 \qquad (7)$$

And

$$(1-p)(\theta_H-\theta_L) - \frac{(1-p)^2\Delta\theta^2}{2\rho\sigma^2} - R(a_H) - (1-p)(1-a_H)(\theta_H-\theta_L)+(1$$
$$-p)R(a_H) \geq 0 \qquad (8)$$

In (7), since $\phi_H > \phi_L$, for any $a_H \in [0,1]$ the first inequality always holds.

In (8), let us set $a_H = a_{pol} = \frac{(1-p)(\theta_H-\theta_L)}{\rho\sigma^2}$

The inequality becomes

$$a_H(1-p)(\theta_H-\theta_L)-\frac{(1-p)^2\Delta\theta^2}{2\rho\sigma^2}-pR(a_H)\geq 0$$

$$\rightarrow \frac{(1-p)^2\Delta\theta^2}{\rho\sigma^2}-\frac{(1-p)^2\Delta\theta^2}{2\rho\sigma^2}-\frac{p(1-p)^2\Delta\theta^2}{2\rho\sigma^2}\geq 0$$

$$\rightarrow \frac{(1-p)^3\Delta\theta^2}{2\rho\sigma^2}\geq 0$$

Which will hold when $p \leq 1$.

So by setting $a_H = a_{pol} = \frac{(1-p)(\theta_H-\theta_L)}{\rho\sigma^2}$, there exists a S_H to satisfy (1) and (2).

The next step is to prove the existence of a nonnegative S_H ($S_H \geq 0$) that satisfies all the conditions above.

To do so, another inequality must be introduced into the proof:

$$U_{Hp} = [p\theta_H + (1-p)\theta_L] + \frac{(1-p)^2\Delta\theta^2}{2\rho\sigma^2} < \theta_h + (\phi_h - 1)K \text{ (for any K)}$$

This inequality means that high type entrepreneurs cannot be better off in the pooling than in complete information. The left-hand side is the utility of the high type entrepreneur in optimal pooling equilibrium under asymmetric information, whereas the right-hand side is the utility of high type entrepreneurs under complete information. Because the information rent is always nonnegative to the high type entrepreneurs, this inequality will hold economically. For any nonnegative K, the inequality must strictly hold. Extremely, when K=0, we have:

$$[p\theta_H + (1-p)\theta_L] + \frac{(1-p)^2\Delta\theta^2}{2\rho\sigma^2} < \theta_h$$

Clearly, even when the financiers are passive, the high type will be better off to separate from the low type than pooling. This inequality is also plausible in our model because in the pooling equilibrium, no effort will be made by the financiers toward any kind of firm.

$$[p\theta_H + (1-p)\theta_L] + \frac{(1-p)^2\Delta\theta^2}{2\rho\sigma^2} < \theta_H$$

$$\rightarrow \frac{(1-p)\Delta\theta}{2\rho\sigma^2} < 1$$

$$\rightarrow (1-p)\Delta\theta < 2\rho\sigma^2$$

With the inequality above, we can show that $(1-p)(1-a_H)(\theta_H-\theta_L)-(1-p)R(a_H) > 0$, when $a_H = a_{pol} = \frac{(1-P)(\theta_H-\theta_L)}{\rho\sigma^2}$

Thus, we can set

$$S_H = \widetilde{S_H} = (1-p)(1-a_H)(\theta_H-\theta_L)-(1-p)R(a_H) + \{\omega(\phi_H-1)K + (1-\omega)(1-p)[\phi_H(1-a_H)+\phi_L a_H - 1]K\}$$

Since $(\phi_H - 1)K > 0$, there always exist a "ω" that causes the following equation to be positive:

$\omega(\phi_H-1)K + (1-\omega)(1-p)[\phi_H(1-a_H)+\phi_L a_H - 1]K\}$

And $(1-p)(1-a_H)(\theta_H-\theta_L)-(1-p)R(a_H) > 0$, so $\widetilde{S_H} \geq 0$.

It can also be proved that $\widetilde{S_H}$ satisfies the two conditions (1) and (2) above.

In conclusion, by setting:

$$a_H = a_{pol} = \frac{(1-p)(\theta_H - \theta_L)}{\rho\sigma^2}$$

$$S_H = (1-p)(1-a_H)(\theta_H - \theta_L) - (1-p)R(a_H) + \{\omega(\phi_H - 1)K + (1 - \omega)(1-p)[\phi_H(1-a_H) + \phi_L a_H - 1]K\}$$

$$a_L = 0$$

$$S_L = -\frac{p}{1-p} S_H$$

The separating menu can always dominate the optimal pooling contract. In other words, a beneficial deviation always exists, and the pooling contract can never be sustained as an equilibrium when $\Phi \leq 1$.

A3. Discussion about the pooling when $\Phi > 1$

In the following part, we come to the discussion about the existence of pooling equilibrium where $\Phi > 1$:

Since we have:

$$\Phi = p\phi_H + (1-p)\phi_L > 1 \text{ and } \Theta = p\theta_H + (1-p)\theta_L$$

The financiers will make effort if and only if the retaining fraction α_p satisfies the following condition:

$$(1-\alpha_p)\Phi \geq 1 \Rightarrow \alpha_p \leq \frac{\Phi - 1}{\Phi} = \alpha_p^c$$

In that case, the participation constraint of investors becomes:

$$\left(1-\alpha_p\right)\left(x_p-P_p\right)-k_p \geq 0$$

This constraint will be binding under full competition among investors, and the equity price offered to entrepreneurs can be driven as:

$$\left(1-\alpha_p\right)\left(\Theta+\Phi k_p-P_p\right)-k_p=0 \Rightarrow P_p=\Theta+[\Phi-\frac{1}{1-a_p}]k_p$$

Substituting the price equation, the payoff to the high type entrepreneurs can be rewritten as:

$$\begin{aligned}
&\left(1-\alpha_p\right)P_p+\alpha_p\left(\theta_H+\phi_H k_p\right)-R(\alpha_p) \\
&=\left(1-\alpha_p\right)\Theta+\left[\left(1-a_p\right)\Phi-1\right]k_p+\alpha_p\left(\theta_H+\phi_H k_p\right)-R\left(\alpha_p\right) \\
&=\left(1-\alpha_p\right)p\theta_H+\left(1-a_p\right)(1-p)\theta_L \\
&\qquad +\left[\left(1-\alpha_p\right)p\phi_H+\left(1-\alpha_p\right)(1-p)\phi_L-1\right]k_p+\alpha_p\left(\theta_H+\phi_H k_p\right) \\
&\qquad -R(\alpha_p)
\end{aligned}$$

As illustrated above, only the pooling equilibrium that maximizes the payoff to the high type can be sustained. We solve the optimal pooling contract by deriving the first-order condition:

$$\frac{\partial u_H}{\partial \alpha_p}=-p\theta_H-(1-p)\theta_L-p\phi_H k_p-(1-p)\phi_L k_p+\left(\theta_H+\phi_H k_p\right)-\alpha_p\rho\sigma^2=0$$

$$\Rightarrow (1-p)\theta_H-(1-p)\theta_L+(1-p)\phi_H k_p-(1-p)\phi_L k_p=\alpha_p\rho\sigma^2$$

$$\Rightarrow \alpha_p = \frac{(1-p)\Delta\theta + (1-p)\Delta\phi k_p}{\rho\sigma^2} k_p \in [0, K]$$

In the following discussion, we focus only on the case where $\alpha_p \le \frac{\Phi-1}{\Phi}$, which means that the financiers will make effort when the pooling contract is chosen, because when $\alpha_p > \frac{\Phi-1}{\Phi}$, all the analysis is similar to the case where $\Phi < 1$.

Subcase I

When $\alpha_p^* = \frac{(1-p)\Delta\theta+(1-p)\Delta\phi k_p}{\rho\sigma^2} \le \frac{\Phi-1}{\Phi}$

Then α_p^* will be the optimal contract conditional on $\rho\sigma^2 \ge \frac{[(1-p)\Delta\theta+(1-p)\Delta\phi K]\Phi}{\Phi-1}$

In this case, the utility of the high type will be:

$$U_{Hp} = (1-\alpha_p)\Theta + [(1-\alpha_p)\Phi - 1]K + \alpha_p(\theta_H + \phi_H K) - R(\alpha_p)$$

$$= \left[1 - \frac{(1-p)(\Delta\theta + \Delta\phi K)}{\rho\sigma^2}\right]\Theta + \left\{\left[1 - \frac{(1-p)(\Delta\theta + \Delta\phi K)}{\rho\sigma^2}\right]\Phi - 1\right\}K + \frac{(1-p)(\Delta\theta + \Delta\phi K)}{\rho\sigma^2} \times (\theta_H + \phi_H K) - \frac{(1-p)^2(\Delta\theta + \Delta\phi K)^2}{2\rho\sigma^2}$$

$$=\Theta + (1-p)\alpha_p\Delta\theta + [\Phi - 1 + \alpha(1-p)\Delta\phi]K - \frac{(1-p)^2(\Delta\theta+\Delta\phi K)^2}{2\rho\sigma^2}$$

$$=[\Theta + (\Phi-1)K] + (1-p)\alpha_p(\Delta\theta + \Delta\phi K) - \frac{(1-p)^2(\Delta\theta+\Delta\phi K)^2}{2\rho\sigma^2}$$

$$=[\Theta + (\Phi-1)K] + \frac{(1-p)^2(\Delta\theta+\Delta\phi K)^2}{2\rho\sigma^2}$$

Subcase II

When $\frac{(1-p)(\Delta\theta+\Delta\phi K)}{\rho\sigma^2} \geq \frac{\Phi-1}{\Phi}$ and $\frac{(1-p)\Delta\theta}{\rho\sigma^2} \leq \frac{\Phi-1}{\Phi}$,

The results of this subcase will be similar to the part where investors' effort is introduced in subcase III. Thus, we combine the discussion in the following part.

Subcase III

When $\frac{(1-p)\Delta\theta}{\rho\sigma^2} > \frac{\Phi-1}{\Phi}$

The entrepreneurs face a dilemma about whether to introduce the effort from investors. On one hand, a high type entrepreneur can reduce the loss from security mispricing by retaining a higher fraction of equity. On the other hand, he can introduce investors' effort and lower the disutility from insufficient diversification by retaining less equity.

If the optimal pooling contract is designed to introduce investors' effort, then the incentive constraint $\alpha_p \leq \frac{\Phi-1}{\Phi}$ must be satisfied. Since $\frac{(1-p)(\Delta\theta+\Delta\phi K)}{\rho\sigma^2} \geq \frac{(1-p)\Delta\theta}{\rho\sigma^2} > \frac{\Phi-1}{\Phi}$, the high type entrepreneur will choose $\alpha_p = \alpha_c = \frac{\Phi-1}{\Phi}$.

In this case, the utility of high type will be

$$U'_{Hp} = (1-\alpha_p)P_p + \alpha_p(\theta_H + \phi_H k_p) - R(\alpha_p)$$

$$= \frac{\Theta}{\Phi} + \left[\frac{1}{\Phi}\times\Phi - 1\right]K + \frac{\Phi-1}{\Phi}(\theta_H + \phi_H K) - \frac{1}{2}\rho\sigma^2\frac{(\Phi-1)^2}{\Phi^2}$$

Otherwise, if the pooling contract is designed to exclude investors' effort $\alpha_p \geq \frac{\Phi-1}{\Phi}$, the optimal fraction retained will be $\alpha_p = \frac{(1-p)\Delta\theta}{\rho\sigma^2}$.

$$U''_{Hp} = \Theta + \frac{(1-p)^2(\Delta\theta + \Delta\phi K)^2}{2\rho\sigma^2} = \Theta + \frac{(1-p)^2\Delta\theta^2}{2\rho\sigma^2}$$

To analyse the sufficient conditions for investors' effort to be introduced, we derive from the inequality $U''_H > U'_H$.

$$\Theta + \frac{(1-p)^2\Delta\theta^2}{2\rho\sigma^2} > \frac{\Theta}{\Phi} + \left[\frac{1}{\Phi}\times\Phi - 1\right]K + \frac{\Phi-1}{\Phi}(\theta_H + \phi_H K) - \frac{1}{2}\rho\sigma^2\frac{(\Phi-1)^2}{\Phi^2}$$

$$K < \frac{\rho\sigma^2(\Phi-1)}{2\phi_H\Phi} + \frac{(1-p)^2\Delta\theta\Phi}{2\rho\sigma^2(\Phi-1)} - \frac{(1-p)\Delta\theta}{\phi_H}$$

Summary

Summarising the results of all three cases listed above, we end up with the following conclusion:

1. When $K \leq \frac{(\Phi-1)\rho\sigma^2}{\Phi\Delta\phi(1-p)} - \frac{\Delta\theta}{\Delta\phi}$ or $\rho\sigma^2 \geq \frac{[(1-p)(\Delta\theta+\Delta\phi K)]\Phi}{\Phi-1}$

$$\Rightarrow \frac{(1-p)\Delta\theta + (1-p)\Delta\phi k_p}{\rho\sigma^2} \leq \frac{\Phi-1}{\Phi}$$

The optimal pooling contract will be

$$\alpha_p^* = \frac{(1-p)\Delta\theta + (1-p)\Delta\phi k_p}{\rho\sigma^2}$$

And the utility of the high type will become

$$U_{Hp} = [\Theta + (\Phi - 1)K] + \frac{(1-p)^2(\Delta\theta + \Delta\phi K)^2}{2\rho\sigma^2}$$

2. When $K \geq \frac{(\Phi-1)\rho\sigma^2}{\Phi\Delta\phi(1-p)} - \frac{\Delta\theta}{\Delta\phi}$ or $\frac{(1-p)\Delta\theta\Phi}{\Phi-1} \leq \rho\sigma^2 \leq \frac{[(1-p)(\Delta\theta+\Delta\phi K)]\Phi}{\Phi-1}$

The optimal pooling contract will be

$$\alpha_p' = \frac{\Phi - 1}{\Phi}$$

And the utility of the high type will become

$$U'_{Hp} = \frac{\Theta}{\Phi} + \frac{\Phi - 1}{\Phi}(\theta_H + \phi_H K) - \frac{1}{2}\rho\sigma^2\frac{(\Phi - 1)^2}{\Phi^2}$$

3. When $\rho\sigma^2 \leq \frac{(1-p)\Delta\theta\Phi}{\Phi-1}$, two subcases will be possible.

1) If the optimal pooling contract is designed to introduce investors' effort:

The optimal contract will be

$$\alpha_p' = \frac{\Phi-1}{\Phi} \text{ and } U_{Hp} = U'_{Hp} = \frac{\Theta}{\Phi} + \frac{\Phi-1}{\Phi}(\theta_H + \phi_H K) - \frac{1}{2}\rho\sigma^2\frac{(\Phi-1)^2}{\Phi^2}$$

2) If the pooling contract is designed to exclude investors' effort:

The optimal contract will be

$$\alpha_p^* = \frac{(1-p)\Delta\theta}{\rho\sigma^2} \text{ and } U_{Hp} = U''_H = \Theta + \frac{(1-p)^2\Delta\theta^2}{2\rho\sigma^2}$$

The first pooling contract will be dominant when $U''_H < U'_H$

$$\Theta + \frac{(1-p)^2\Delta\theta^2}{2\rho\sigma^2} < \frac{\Theta}{\Phi} + \frac{\Phi-1}{\Phi}(\theta_H + \phi_H K) - \frac{1}{2}\rho\sigma^2\frac{(\Phi-1)^2}{\Phi^2}$$

$$\Rightarrow K > \frac{\rho\sigma^2(\Phi-1)}{2\phi_H\Phi} + \frac{(1-p)^2\Delta\theta\Phi}{2\rho\sigma^2(\Phi-1)} - \frac{(1-p)\Delta\theta}{\phi_H}$$

The second pooling contract will be dominant when $U''_H > U'_H$

$$K < \frac{\rho\sigma^2(\Phi-1)}{2\phi_H\Phi} + \frac{(1-p)^2\Delta\theta\Phi}{2\rho\sigma^2(\Phi-1)} - \frac{(1-p)\Delta\theta}{\phi_H}$$

$$\Rightarrow \frac{(\Phi-1)\rho\sigma^2}{\Phi\Delta\phi(1-p)} - \frac{\Delta\theta}{\Delta\phi} < K < \frac{\rho\sigma^2(\Phi-1)}{2\phi_H\Phi} + \frac{(1-p)^2\Delta\theta\Phi}{2\rho\sigma^2(\Phi-1)} - \frac{(1-p)\Delta\theta}{\phi_H}$$

As illustrated in previous section, the pooling equilibrium will be sustainable only when it strictly dominates all possible separating equilibria. In other words, proving the existence of a pooling equilibrium is equivalent to proving that no other separating menu can dominate the optimal pooling contracts. The method to prove this will be similar to what we have shown in Appendix A3. Because we have three different subcases, we will discuss them one by one in the following.

Subcase I : (the optimal pooling contract can introduce investors' effort)

$$\alpha_p^* = \frac{(1-p)\Delta\theta + (1-p)\Delta\phi k_p}{\rho\sigma^2}$$

$U_{Hp} = [\Theta + (\Phi - 1)K] + \frac{(1-p)^2(\Delta\theta + \Delta\phi K)^2}{2\rho\sigma^2}$

And $K \leq \frac{(\Phi-1)\rho\sigma^2}{\Phi\Delta\phi(1-p)} - \frac{\Delta\theta}{\Delta\phi}$

The object is to prove that there does not exist any separating menu that satisfies the inequities below.

$$[\phi_H(1-\alpha_H) + \phi_L\alpha_H - 1]k_t + (1-\alpha_H)\theta_H - S_H + \alpha_H\theta_L - R(\alpha_H) \leq \theta_L + \frac{p}{1-p}S_H \quad (1)$$

(Incentive comparison constraint of the separating menu)

$$(\phi_H - 1)k_t + \theta_H - S_H - R(R_H) \geq [\Theta + (\Phi - 1)K] + \frac{(1-p)^2(\Delta\theta + \Delta\phi K)^2}{2\rho\sigma^2} \quad (2)$$

(Condition for the separating menu to dominate the pooling contract)

$$0 \leq \alpha_H \leq 1 \quad (3)$$

(Short-selling constraint of entrepreneurs)

$$S_H \geq 0 \quad (4)$$

(Cross-subsidy constraint)

And if $\alpha_H \geq \frac{\phi_H - 1}{\phi_H} \Rightarrow k_t = 0$

If $\alpha_H \leq \frac{\phi_H - 1}{\phi_H} \Rightarrow k_t = K$

(Incentive constraints of investors' effort)

Inequality (1) can be rewritten as

$$\Delta\theta + (\phi_H - 1)k_t - \alpha_H(\Delta\theta + \Delta\phi k_t) - \frac{1}{2}\rho\sigma^2\alpha_H^2 \leq \frac{1}{1-p}S_H$$

Whereas inequality (2) can be rewritten as

$$(\phi_H - 1)k_t - (\Phi - 1)K + (1-p)\Delta\theta - \frac{1}{2}\rho\sigma^2\alpha_H^2 - \frac{(1-p)^2(\Delta\theta + \Delta\phi K)^2}{2\rho\sigma^2} \geq S_H$$

$0 \leq \alpha_H \leq 1$

$S_H \geq 0$

Based on the incentive constraints of investors' effort, the whole program can be decomposed into two sub-programs—with and without investors' effort.

①

$$\Delta\theta + (\phi_H - 1)K - \alpha_H(\Delta\theta + \Delta\phi K) - \frac{1}{2}\rho\sigma^2\alpha_H^2 \leq \frac{1}{1-p}S_H \qquad (a1)$$

$$(1-p)(\Delta\theta + \Delta\phi K) - \frac{1}{2}\rho\sigma^2\alpha_H^2 - \frac{(1-p)^2(\Delta\theta + \Delta\phi K)^2}{2\rho\sigma^2} \geq S_H \qquad (a2)$$

$$0 \leq \alpha_H \leq \frac{\Phi - 1}{\Phi} \qquad (a3)$$

$$S_H \geq 0 \qquad (a4)$$

②

$$\Delta\theta - \alpha_H\Delta\theta - \frac{1}{2}\rho\sigma^2\alpha_H^2 - \frac{(1-p)^2(\Delta\theta + \Delta\phi K)^2}{2\rho\sigma^2} \le \frac{1}{1-p}S_H \qquad (b1)$$

$$(1-p)\Delta\theta - (\Phi - 1)K - \frac{1}{2}\rho\sigma^2\alpha_H^2 - \frac{(1-p)^2(\Delta\theta + \Delta\phi K)^2}{2\rho\sigma^2} \ge S_H \qquad (b2)$$

$$\frac{\Phi - 1}{\Phi} \le \alpha_H \le 1 \qquad (b3)$$

$$S_H \ge 0 \qquad (b4)$$

Case ①

From ① condition (a1) and (a2)

$$(1-p)\Delta\theta + (1-p)(\phi_H - 1)k_t - (1-p)\alpha_H(\Delta\theta + \Delta\phi k_t) - \frac{1}{2}(1-p)\rho\sigma^2\alpha_H^2$$
$$\le (1-p)(\Delta\theta + \Delta\phi K) - \frac{1}{2}\rho\sigma^2\alpha_H^2 - \frac{(1-p)^2(\Delta\theta + \Delta\phi K)^2}{2\rho\sigma^2}$$

$$(1-p)(\phi_L - 1)K \le (1-p)\alpha_H(\Delta\theta + \Delta\phi K) - \frac{1}{2}p\rho\sigma^2\alpha_H^2 - \frac{(1-p)^2(\Delta\theta + \Delta\phi K)^2}{2\rho\sigma^2}$$

Set $\alpha_H = \alpha_p = \frac{(1-p)(\Delta\theta + \Delta\phi)}{\rho\sigma^2} \le \frac{\Phi - 1}{\Phi} \le \frac{\phi_H - 1}{\phi_H}$

$$(1-p)(\phi_L - 1)K$$
$$\le \frac{(1-p)^2(\Delta\theta + \Delta\phi K)^2}{\rho\sigma^2} - \frac{p(1-p)^2(\Delta\theta + \Delta\phi K)^2}{2\rho\sigma^2}$$
$$- \frac{(1-p)^2(\Delta\theta + \Delta\phi K)^2}{2\rho\sigma^2}$$

$$(1-\mathrm{p})(\phi_L-1)K \leq \frac{(1-\mathrm{p})}{2} \times \frac{(1-\mathrm{p})^2(\Delta\theta+\Delta\phi K)^2}{\rho\sigma^2}$$

Because the left-hand side of the inequality $(1-\mathrm{p})(\phi_L-1)K \leq 0$ and the right-hand side $\frac{(1-\mathrm{p})}{2} \times \frac{(1-\mathrm{p})^2(\Delta\theta+\Delta\phi K)^2}{\rho\sigma^2} \geq 0$. Thus, there exists a nonempty set ω_1 of S_H that satisfies both (a1) and (a2).

To satisfy both conditions (a2) and (a4), it is required that

$$(1-\mathrm{p})(\Delta\theta+\Delta\phi K) - \frac{1}{2}\rho\sigma^2\alpha_H^2 - \frac{(1-\mathrm{p})^2(\Delta\theta+\Delta\phi K)^2}{2\rho\sigma^2} > 0.$$

When $\alpha_H = \frac{(1-\mathrm{p})(\Delta\theta+\Delta\phi)}{\rho\sigma^2}$, it becomes

$$(1-\mathrm{p})(\Delta\theta+\Delta\phi K) - \frac{(1-\mathrm{p})^2(\Delta\theta+\Delta\phi K)^2}{2\rho\sigma^2} - \frac{(1-\mathrm{p})^2(\Delta\theta+\Delta\phi K)^2}{2\rho\sigma^2}$$

$$= (1-\mathrm{p})(\Delta\theta+\Delta\phi K) - \frac{(1-\mathrm{p})^2(\Delta\theta+\Delta\phi K)^2}{\rho\sigma^2}$$

$$= (1-\mathrm{p})(\Delta\theta+\Delta\phi K) \times (1 - \frac{(1-\mathrm{p})(\Delta\theta+\Delta\phi K)}{\rho\sigma^2})$$

$$= (1-\mathrm{p})(\Delta\theta+\Delta\phi K) \times (1-\alpha_H) > 0$$

Thus, there also exists a nonempty set ω_2 of S_H that satisfies both (a2) and (a4).

By combining both inequalities above, we have:

$$(1-p)(\Delta\theta+\Delta\phi K)-\frac{1}{2}\rho\sigma^2\alpha_H^2-\frac{(1-p)^2(\Delta\theta+\Delta\phi K)^2}{2\rho\sigma^2}$$
$$>\max\{(1-p)\Delta\theta+(1-p)(\phi_H-1)k_t-(1-p)\alpha_H(\Delta\theta+\Delta\phi k_t)$$
$$-\frac{1}{2}(1-p)\rho\sigma^2\alpha_H^2,0\}$$

Define

$$(1-p)\Delta\theta+(1-p)(\phi_H-1)k_t-(1-p)\alpha_H(\Delta\theta+\Delta\phi k_t)-\tfrac{1}{2}(1-p)\rho\sigma^2\alpha_H^2=\Upsilon$$

Then $\Upsilon>0$ or $\Upsilon<0$ or $\Upsilon=0$

$\Rightarrow \omega_1 \ni \omega_2$ or $\omega_1 \ni \omega_2$ or $\omega_1=\omega_2$

And since $\omega_1,\omega_2 \neq \emptyset$

$\Rightarrow \omega=\omega_1\cap\omega_2\neq\emptyset$

In other words, there exists a nonempty set ω of S_H that satisfies the all three restrictions (a1), (a2) and (a4), and when $\alpha_H=\alpha_p=\frac{(1-p)(\Delta\theta+\Delta\phi)}{\rho\sigma^2}$, (a3) is satisfied as well.

Because all four conditions can be satisfied at the same time, there exists a separating equilibrium menu dominating the pooling contract. Based on lemma 3, the pooling equilibrium collapses. In summary, when the separating menu and the pooling contract can both introduce investors' effort, the pooling contract cannot be sustained as an equilibrium.

Case ② discusses the separating menu where the entrepreneurs choose not to introduce investors' effort. However, it implies that the separating menu without effort would dominate that with investors' effort. Because, in the discussion above, we have proved that separating the menu with investors' effort can dominate the pooling contract, there is no need to check case ② under this parameter condition when $\alpha_p = \frac{(1-p)(\Delta\theta+\Delta\phi)}{\rho\sigma^2}$.

Subcase II : (the optimal pooling contract excludes investors' effort)

Let us move to the case where $\alpha_p = \frac{\Phi-1}{\Phi}$ and $U_{Hp} = U'_{Hp} = \frac{\Theta}{\Phi} + \frac{\Phi-1}{\Phi}(\theta_H + \phi_H K) - \frac{1}{2}\rho\sigma^2\frac{(\Phi-1)^2}{\Phi^2}$.

For the pooling contract above to become optimal, it requires: $K > \frac{\rho\sigma^2(\Phi-1)}{2\phi_H\Phi} + \frac{(1-p)^2\Delta\theta\Phi}{2\rho\sigma^2(\Phi-1)} - \frac{(1-p)\Delta\theta}{\phi_H}$

By similar argument, the discussion is decomposed into two based on whether the optimal separating menu introduces investors' effort:

①

$$(1-p)\Delta\theta + (1-p)(\phi_H - 1)K - (1-p)\alpha_H(\Delta\theta + \Delta\phi K) - \frac{1}{2}(1-p)\rho\sigma^2\alpha_H^2 \leq S_H \tag{a1}$$

$$\frac{\theta_H}{\Phi} - \frac{\Theta}{\Phi} + \frac{\phi_H}{\Phi}K - K + \frac{1}{2}\rho\sigma^2\frac{(\Phi-1)^2}{\Phi^2} - \frac{1}{2}\rho\sigma^2\alpha_H^2 \geq S_H \tag{a2}$$

$$0 \leq \alpha_H \leq \frac{\Phi-1}{\Phi} \tag{a3}$$

$$S_H \geq 0 \tag{a4}$$

②

$$\Delta\theta-\alpha_H\Delta\theta-\frac{1}{2}\rho\sigma^2\alpha_H^2\leq\frac{1}{1-p}S_H \tag{b1}$$

$$\frac{\theta_H}{\Phi}-\frac{\Theta}{\Phi}+\frac{\phi_H}{\Phi}K-K+\frac{1}{2}\rho\sigma^2\frac{(\Phi-1)^2}{\Phi^2}-\frac{1}{2}\rho\sigma^2\alpha_H^2\geq S_H \tag{b2}$$

$$\frac{\Phi-1}{\Phi}\leq\alpha_H\leq 1 \tag{b3}$$

$$S_H\geq 0 \tag{b4}$$

One can easily show that it is impossible to eliminate the pooling equilibrium by following the method above. Thus, we suspect that a pooling equilibrium does exist under this case. To prove this, we first assume that there is no pooling equilibrium under all the parameter conditions. In other word, we can always find a separating menu that strictly dominates the optimal pooling contract.

Then, let us start with case ①

$$\begin{aligned}(1-p)\Delta\theta+(1-p)(\phi_H-1)K-(1-p)\alpha_H(\Delta\theta+\Delta\phi K)-\frac{1}{2}(1-p)\rho\sigma^2\alpha_H^2\\ \leq\frac{\theta_H}{\Phi}-\frac{\Theta}{\Phi}+\frac{\phi_H}{\Phi}K-K+\frac{1}{2}\rho\sigma^2\frac{(\Phi-1)^2}{\Phi^2}-\frac{1}{2}\rho\sigma^2\alpha_H^2\end{aligned}$$

$$\begin{aligned}\alpha_H^2-\frac{2(1-p)(\Delta\theta+\Delta\phi K)}{p\rho\sigma^2}\alpha_H+\frac{2(1-p)(1-\frac{1}{\Phi})\Delta\theta}{p\rho\sigma^2}+\frac{2(1-\frac{1}{\Phi})\phi_H K}{p\rho\sigma^2}\\ -\frac{2(\phi_H-1)K}{\rho\sigma^2}-\frac{(\Phi-1)^2}{p\Phi^2}\leq 0\end{aligned}$$

$$\begin{aligned}p\rho\sigma^2\alpha_H^2-2(1-p)(\Delta\theta+\Delta\phi K)\alpha_H+2(1-p)\left(1-\frac{1}{\Phi}\right)\Delta\theta+2\left(1-\frac{1}{\Phi}\right)\phi_H K\\ -2p(\phi_H-1)K-\rho\sigma^2\frac{(\Phi-1)^2}{\Phi^2}\leq 0\end{aligned}$$

We define $a = p\rho\sigma^2$, $b = 2(1-p)(\Delta\theta + \Delta\phi K)$, and $c = 2(1-p)\left(1-\frac{1}{\Phi}\right)\Delta\theta + 2\left(1-\frac{1}{\Phi}\right)\phi_H K - 2p(\phi_H - 1)K - \rho\sigma^2\frac{(\Phi-1)^2}{\Phi^2}$.

It can be seen from the inequality above that the left-hand side of the inequality is a quadratic form of α_H. For there to be an α_H that satisfies the inequality above, it requires that $b^2 - 4ac \geq 0$. Otherwise, if $b^2 - 4ac < 0$, the inequality cannot be satisfied, and the pooling equilibrium becomes sustainable. Thus, from $b^2 - 4ac < 0$, we have:

$$\begin{aligned}(1-p)^2(\Delta\theta + \Delta\phi K)^2 \\ &- p\rho\sigma^2\left[2(1-p)\left(1-\frac{1}{\Phi}\right)\Delta\theta + 2\left(1-\frac{1}{\Phi}\right)\phi_H K - 2p(\phi_H - 1)K\right. \\ &\left. - \rho\sigma^2\frac{(\Phi-1)^2}{\Phi^2}\right] < 0\end{aligned}$$

$$\begin{aligned}\frac{(\Phi-1)^2}{\Phi^2}(\rho\sigma^2)^2 \\ &- \rho\sigma^2\left[2p(1-p)\left(1-\frac{1}{\Phi}\right)\Delta\theta + 2p\left(1-\frac{1}{\Phi}\right)\phi_H K - 2p^2(\phi_H - 1)K\right] \\ &+ (1-p)^2(\Delta\theta + \Delta\phi K)^2 > 0\end{aligned}$$

This inequality is sufficient for the existence of pooling equilibrium. Given the quadratic form of $\rho\sigma^2$, this condition can be satisfied only if:

$$\begin{aligned}&[2p(1-p)\left(1-\frac{1}{\Phi}\right)\Delta\theta + 2p\left(1-\frac{1}{\Phi}\right)\phi_H K - 2p^2(\phi_H - 1)K]^2 \\ &- 4\frac{(\Phi-1)^2}{\Phi^2}p(1-p)^2(\Delta\theta + \Delta\phi K)^2 > 0\end{aligned}$$

$$[\mathrm{p}(1-\mathrm{p})\Delta\theta + \mathrm{p}\phi_{\mathrm{H}}\mathrm{K} - \frac{\mathrm{P}^2(\phi_{\mathrm{H}}-1)}{\Phi-1}\Phi\mathrm{K}]^2 - \mathrm{p}(1-\mathrm{p})^2(\Delta\theta+\Delta\phi\mathrm{K})^2 > 0$$

$$\begin{aligned}
&\mathrm{p}^2(1-\mathrm{p})^2\Delta\theta^2 + \mathrm{p}^2\phi_{\mathrm{H}}^2\mathrm{K}^2 + \mathrm{p}^4\frac{(\phi_{\mathrm{H}}-1)^2}{(\Phi-1)^2}\Phi^2\mathrm{K}^2 + 2\mathrm{p}^2(1-\mathrm{p})\Delta\theta\phi_{\mathrm{H}}\mathrm{K} \\
&\quad - 2\mathrm{p}^3(1-\mathrm{p})\Delta\theta\frac{\phi_{\mathrm{H}}-1}{\Phi-1}\Phi\mathrm{K} - 2\mathrm{p}^3\frac{\phi_{\mathrm{H}}-1}{\Phi-1}\phi_{\mathrm{H}}\Phi\mathrm{K}^2 \\
&\quad > \mathrm{p}^2(1-\mathrm{p})^2(\Delta\theta^2 + 2\Delta\theta\Delta\phi\mathrm{K}) + \Delta\phi^2\mathrm{K}^2
\end{aligned}$$

$$\begin{aligned}
&\{\mathrm{p}^2\phi_{\mathrm{H}}^2 + \mathrm{p}^4\frac{(\phi_{\mathrm{H}}-1)^2}{(\Phi-1)^2}\Phi - 2\mathrm{p}^3\frac{\phi_{\mathrm{H}}-1}{\Phi-1}\phi_{\mathrm{H}}\Phi - \mathrm{p}(1-\mathrm{p})^2\Delta\phi^2\}\mathrm{K}^2 \\
&\quad > p(1-\mathrm{p})^3\Delta\theta^2 + [2\mathrm{p}^2(1-\mathrm{p})\Delta\theta\phi_{\mathrm{H}} - 2\mathrm{p}^3(1-\mathrm{p})\Delta\theta\frac{\phi_{\mathrm{H}}-1}{\Phi-1}\Phi \\
&\quad + 2\mathrm{p}(1-\mathrm{p})^2\Delta\theta\Delta\phi]\mathrm{K}
\end{aligned}$$

The left-hand side of the inequality above is a quadratic form of K, whereas the right-hand side is a constant. Therefore, when the coefficient of the quadratic term is positive, the inequality will be satisfied when K is sufficiently large.

$$\begin{aligned}
&\mathrm{p}^2\phi_{\mathrm{H}}^2 + \mathrm{p}^4\frac{(\phi_{\mathrm{H}}-1)^2}{(\Phi-1)^2}\Phi - 2\mathrm{p}^3\frac{\phi_{\mathrm{H}}-1}{\Phi-1}\phi_{\mathrm{H}}\Phi - \mathrm{p}(1-\mathrm{p})^2\Delta\phi^2 \\
&\quad = \left\{\mathrm{p}^2\left(\phi_{\mathrm{H}} - \frac{\mathrm{p}(\phi_{\mathrm{H}}-1)}{\Phi-1}\Phi\right)^2 - \mathrm{p}(1-\mathrm{p})^2\Delta\phi\right\}
\end{aligned}$$

When $\Delta\phi \to 0$ or $\mathrm{p} \to 1$, K is relatively large and the operating risk $\rho\sigma^2$ is under a certain area, the pooling equilibrium exists.

Using a similar method, the same argument can be proved in case ② as well.

Chapter III:

Optimal Mechanism Design of Liquidity Provision under Asymmetric Information

1. Introduction

The impact of asymmetric information on securities trading has long been discussed by theoretical research. Similar to the "lemon" market discussed by Akerlof (1970), private information depraves the trading process and reduces liquidity provision. With asymmetric information, high-quality asset owners will tend to retain their asset as the result of the mispricing in the market. To minimise the negative impact, previous researchers have written many articles to discuss the optimal mechanism design under asymmetric information.

In recent years, more and more articles focus on the essential role played by intermediaries in asymmetric information environment (for example Winton (2001) and DeMarzo (2005)). However, these articles assume intermediaries have either a better ability or a stronger incentive to monitor entrepreneurs. To distinguish our study from these previous works, we show that by allowing intermediaries to offer a menu of contracts, the social efficiency and liquidity provision can be improved even without outstanding monitoring ability. In our model, when the separating menu is sustained as an equilibrium, the type information can be obtained by observing the contract chosen by the entrepreneurs. Therefore, by introducing a screening mechanism and designing the menu optimally, the intermediary can fill this information gap without paying any extra cost of effort. This advantage can finally benefit both the investors and the entrepreneurs. On the investors' side, this "free" signal saves their time and cost for information collecting and analysing. On the other hand, it also prevents the entrepreneurs from being misidentified by any inaccurate or misleading information.

In addition, this paper also discusses the optimal securities design, under a different market environment. Previous research related to pecking order theory (Myer and Majluf, 1984; Myers, 1984) suggests that a high-quality asset owner should issue securities with low information sensitivity to alleviate the mispricing. However, Fulghieri and Lukin (2001) point out that their results ignore the information production ability of outside investors. If investors are allowed to identify the quality of an asset by exerting effort, the high-quality owner may choose to issue risky securities to motivate the investors' monitoring behaviours. Incorporating this idea, we also assume that investors are equipped with access to identifying the private information by exerting costly effort. However, our model shows that it is not always on the entrepreneurs' benefit to motivate investors' speculation behave. Firstly, in order to motivate the investors, the selling price of the securities may be cut to compensate for investors' cost of effort. Secondly, information acquired by the investors is imperfect. Therefore, it is associated with a risk of misidentification. Due to this two effects, we are going to show that without complete information, this speculation power can decrease the attractiveness of the directly issuing process and leave more motivation for entrepreneurs to issue through intermediaries.

Additionally, to make our model general, we also endogenize the existence of intermediaries by allowing entrepreneurs to choose to issue securities through intermediaries or directly to investors. As a result, we find that intermediaries are more likely to be introduced in a market where the cost of information is relative high. More importantly, when intermediaries are introduced, and the proportion of High type is sufficiently high, intermediaries can implement "cross-subsidy" to improve social efficiency. Our model also predicts that when the economic environment is good, we are more likely to observe the issuing of risky securities and there will be a price adjustment in the reselling process. Otherwise, when the economic environment is less favourable, we may observe that only the riskless securities will be issued.

The rest of this paper proceeds as follows: Section 2 reviews the important results from the previous literature. We will describe the basic setting and assumptions in our model in Section 3. Then, in Section 4, we will firstly discuss the equilibrium with intermediaries, while the equilibrium without intermediaries will be discussed in Section 5. Finally, we will come to the conclusion and empirical implications in Section 6.

2. Literature Review

One of the most important papers discussing the securities issuing process with asymmetric information is Leland and Pyle (1977). With the risk aversion assumption of the entrepreneurs, they show that the seller of high-value securities can signal their type by partly retaining their risky asset. Although in our model we also have the case where entrepreneurs of High type partly retain their risk asset, the incentive of doing so is different. In our model, entrepreneurs are assumed to be risk neutral and endowed with a short-term investment opportunity. Since the short-term programme has positive NPV, entrepreneurs have an incentive to sell their risky asset to finance this programme. Moreover, the decision to retain an asset is based on the trade-off of underinvestment and mispricing of the asset.

Later, Diamond (1984) creates an intermediation model with ex-post asymmetric information where the cash flow of the firm is unobservable. To cope with the moral hazard problem and ensure the repayment, investors are allowed to monitor the firm at a cost. In the case of multiple investors, he/she finds it becomes efficient to delegate the monitoring to an intermediary. Different from their standpoint, our model mainly discusses the effect of ex-ante asymmetric information. Besides, in our model, we also find there is an efficiency gain by introducing the intermediaries. However, our efficiency gain is mainly sourced from implementing a more efficient separating menu instead of coping with the dilemma of the public good.

By introducing a market making setting, Gorton and Pennachi (1990) build up another model of intermediation with two different kinds of investors, informed and uninformed. In their model, uninformed investors will be exploited by the informed ones at the time when they are in need of liquidity. As a result, an intermediary should be set up by the uninformed investors to split cash flows into riskless debt and equity. In that case, uninformed investors will only trade the riskless securities to satisfy their liquidity demand. With similar ideas but different arguments, Chemla and Hennessy (2011) design a model with uninformed investors and speculators. They point out that there will be an equilibrium where only riskless debt is issued. In addition, they also suggest that when the demand for liquidity is high, an issuer may pool at the structure optimal to the High type so as to introduce the effort from speculators. To distinguish our study from their model, in this article investors are homogeneously endowed with access to the type information, but the information they get is not perfect. Therefore, in our model, the introduction of investor’s effort is not always helpful in decreasing the information rent, rather imperfect information may lead to a risk of misidentification. Our model also shows that this risk may even offer a higher incentive for entrepreneurs to issue their securities through intermediaries.

Additionally, Winton (2001) considers a model where intermediaries have incentives to acquire the quality information about the underlying asset so as to minimise the agency cost. With the existence of liquidity shock, intermediaries may be forced to issue securities backed by their holding. Similarly, DeMarzo (2005) suggests another model in which financial intermediaries have superior ability in valuing assets. Therefore, they can benefit from purchasing the underpriced securities and holding them to maturity. On the other hand, to make the most of their leverage capacity, they may have an incentive to partly resell their asset and reinvest the proceeds. However, due to their superior information, they may face an adverse selection problem in the reselling process. As a result, they suggest that it may become optimal for an intermediary to construct an asset pool and issue securities based on the income of the whole pool. Similar to their model, intermediaries in our mechanism are required to purchase the risk asset from entrepreneurs and resell the securities to investors. However, the ability to acquire information is granted to investors instead of to intermediaries. We are going to show that by offering an efficient separating menu, intermediaries can acquire the type information even without the special ability. Moreover, by introducing intermediaries, the cost of information acquisition can be saved, and we can end up with a higher social efficiency.

On the other hand, our model also discusses the question about optimal security design. As illustrated in pecking order theory (Myer and Majluf, 1984; Myers, 1984), under a trading environment with asymmetric information, high-quality firms may issue less information-sensitive securities to alleviate the mispricing. Later, DeMarzo and Duffie (1999) also examine the problem faced by a firm involved in fundraising under asymmetric information. Their paper shows that when the firm issues securities, the private information will lead to a liquidity loss that has to be traded off against the cost of retention. Similarly, Narayanan (1988) also supports the implication of pecking order theory. He suggests that firms of high quality should minimise the dilution cost by issuing low information sensitive securities. However, all these papers rely on the assumption that the information gap between entrepreneurs and investors is fixed. In other words, they do not consider the effect of information acquisition by investors.

Modifying this assumption, Fulghieri and Lukin (2001) suggest that if outside investors are allowed to produce information, high-quality firms may have an incentive to issue information-sensitive securities to motivate the information acquisition. Moreover, their paper shows that securities of low information sensitivity do not always dominate ones of high information sensitivity. Whether entrepreneurs prefer debt or equity depends on the cost of information acquisition and the precision of the information. Consistent with their argument, in our model we also allow outside investors to detect the quality of the firm by exerting costly effort. However, our model gives entrepreneurs the option to issue their securities indirectly through the intermediaries. In that case, entrepreneurs can signal their types by accepting different contracts offered by intermediaries without introducing the costly effort of investors.

In recent years, several papers also show us some new findings on this topic. Yang and Zeng (2014) suggest that if investors can acquire quality information about the project and screen it through the financing process, the real production of entrepreneurs could depend on investors' information acquisition. Besides, based on the strength of this dependence, either debt or convertible debt could be optimal. Further, Yang (2015) suggests that the optimality of debt may stand out when investors are allowed to acquire information from different aspects of the fundamental. Because investors could benefit from their information at the cost of entrepreneurs through adverse selection. With rational anticipation, entrepreneurs may prefer to issue securities which minimise investors' incentive to get information. The main difference between Yang and Zeng (2014), Yang (2015) and ours is that in their papers, bargain power, and information advantage are separated into two parties: entrepreneurs and investors. In other words, entrepreneurs have the whole ownership of their asset while investors could have a better understanding about the quality of it. Therefore, the ignorant entrepreneurs have no incentive to signal the quality of their asset and investors' information acquisition always becomes harmful to entrepreneurs. Conversely, in our paper, information advantage is endowed by entrepreneurs, so how to transfer information convincingly becomes the main conflict of our model. Under this environment, investors' information acquisition could be valuable to entrepreneurs in certain cases. Also, their paper mainly focuses on a case where aggregate cash flow is fixed, whereas our paper devotes more attention to a case where it is variable. As a result, the impact of information acquisition and optimal securities on social welfare will also be included in our paper.

Other related papers include Boot and Thakor (1993), Allen and Gale (1988, 1994), Madan and Soubra (1991), Zender (1991) and Kalay and Zender (1997). In Boot and Thakor (1993), they illustrate that the optimal strategy for a securities seller is to split the claim on the cash flow into an information-sensitive security and a second claim that is less information-sensitive. Nevertheless, in their model, the seller offers the entirety of their asset for sale, and as a result, the cost of dilution is eliminated. Allen and Gale (1988, 1994), and Madan and Soubra (1991) discuss the optimal securities design problem in risk-sharing while in our model both entrepreneurs and investors are assumed to be risk neutral. Also, we do not consider the securities design based on corporate control as did Zender (1991) and Kalay and Zender (1997).

3. The Model

We consider a three-date (0, 1, 2) model where all agents are risk neutral. There are three different types of agents: entrepreneurs, investors and intermediaries.

Entrepreneurs

At Date 0, an entrepreneur owns a single (long-term) asset with type $\tau \in \{L, H\}$ and he/she privately knows the type of the asset. This asset will deliver τ units of the consumption good at Date 2 with probability one and L is assumed to be less than H. The prior probability that $\tau = H$ is $\lambda \in (0,1)$. The final payoff of the asset is verifiable, and courts can only enforce any contract contingent upon this final payoff at Date 2. Entrepreneurs also have access to a short-term linear technology which allows them to convert each dollar invested in it at Date 1 into β dollar ($\beta > 1$) at Date 2. However, in contrast to the payoff of the long-term asset, the payoff of the short-term technology is not verifiable and so no contract can be written based on the payoff of this technology. The entrepreneurs have no wealth, and so the funds invested in the short-term technology at Date 1 have to be raised by securitizing the long-term asset.

Investors

Investors are competitive and endow with Y>H units of funds which allow them to purchase the entire asset from an entrepreneur. More importantly, investors can receive a signal regarding τ by exerting effort which costs c. The signal is denoted by $\Pi \in \{\Pi_H, \Pi_L\}$ and the probability that the signal is correct by $\sigma \equiv \Pr(\Pi = \Pi_\tau)$. To make the model interesting, we assume that the signals received by investors are informative, that is $\sigma > 1/2$.

Financial Intermediaries (Investment Banks)

Intermediaries are also competitive and can purchase the asset by the entrepreneurs and resell it to investors. More specifically, intermediaries can offer menus of contracts to entrepreneurs. In our model, a contract specifies the price paid by the intermediary to the entrepreneur and the contingent repayment to the intermediary and is denoted by $C_i\ (B_i^j, P_i)$. C_i stands for a contract offered to an entrepreneur who has announced to be of type "i". B_i^j stands for the contingent payment to the intermediary when the asset (entrepreneur) is ex-post proved to be of type "j" while the entrepreneur has announced to be of type "i". P_i stands for the price intermediaries pays to an entrepreneur who has announced to be of type "i" in exchange for the asset. After the entrepreneurs choose their contract, the intermediaries purchase all the securities issued by the entrepreneurs and resell them to investors (possibly after securitizing them).

Game structure

In the first stage of the game, the entrepreneurs decide whether they will issue their securities directly to final investors or through intermediaries.

The Game with Direct Issue of Securities

If an entrepreneur decides to issue her securities directly to investors, the game is:

Stage 2: The entrepreneur decides both the form and the price of her securities.

Stage 3: Given the form of the securities, competitive investors will update their beliefs about the quality of the asset and decide whether they should take the effort to obtain the informative signal about the asset quality.

Stage 4: Investors decide whether they will purchase the securities issued by any of the entrepreneurs.

Stage 5: If some securities are not sold, the entrepreneurs adjust their price and relist them for sale.

Stage 6: Finally, the asset delivers the consumption good, and the payoff to investors is determined by the portfolio of securities they have chosen.

The Game with Intermediated Issue of Securities

If an entrepreneur decides to issue her securities through intermediaries, the game is:

Stage 2: Competitive intermediaries offer menus of contracts to entrepreneurs.

Stage 3: Each entrepreneur chooses one of the contracts offered.

Stage 4: The intermediary securitizes the assets they purchased from entrepreneurs and offer the securities to final investors.

Stage 5: After observing the contracts signed between the entrepreneurs and the intermediaries and the securities issued by intermediaries, investors will update their beliefs about the asset quality and decide whether to exert effort to obtain the informative signal. Given their information set and the resulting beliefs, investors decide whether to purchase the securities issued by intermediaries.

Stage 6: Finally, the asset delivers the consumption good, and the payoff to investors is determined by the portfolio of securities they have chosen.

Two points should be made here: First, we allow for cross-subsidization across contracts within the menu offered by each intermediary. That is, competition among intermediaries will lead to zero profits for the whole menu but not necessarily for each contract. Losses on some contracts are compensated by profits on the other contracts. Second, the intermediaries cannot acquire the informative signal about the asset quality (and do not have any prior information about it). If an intermediary offers a pooling contract (degenerate menu), it does not obtain any information about the asset quality. As a result, if the investors (speculators) acquire the informative signal at the reselling stage, the intermediary will be at an informational disadvantage and may make losses. This suggests that, in order to avoid this informational disadvantage, intermediaries will seek to obtain information about asset quality by offering separating menus.

4. Equilibrium without intermediaries

As we have mentioned above, if intermediaries are not introduced, the entrepreneurs will sell their securities directly to the investors. Based on the securities issued by entrepreneurs, investors update their belief and decide whether they should make an effort to acquire the signal. Incorporating all the information they own, the investors finally make their purchasing decision. To clarify the preference of different types, we firstly analysis potential separating equilibrium in the direct issuing:

Lemma 1: In the direct issuing market, both a separating equilibrium and a pooling equilibrium can be sustained if the High type entrepreneurs issue riskless debt.

Proof:

We start our proof on the separating equilibrium. Suppose that the high type entrepreneurs offer a contract $C_H\ (B_H^H, B_H^L, P_H)$ while the low type entrepreneurs offer a contract $C_L\ (B_L^H, B_L^L, P_L)$:

Correspondingly, the utility functions of high type and low type are

$$U_H = \beta P_H + H - B_H^H$$

$$U_L = \beta P_L + L - B_L^L$$

Besides, since the type can be inferred by the separating contracts, the price of the securities will be equal to its true value. Therefore, we have the following break-even conditions from the investors:

$$P_H \leq B_H^H$$
$$P_L \leq B_L^L$$

Moreover, with the separating contract, neither type should have the incentive to mimic the other type. Hence the following incentive constraints should be satisfied:

$$\beta P_H + H - B_H^H \geq \beta P_L + H - B_L^H$$
$$\beta P_L + L - B_L^L \geq \beta P_H + L - B_H^L$$

Finally, due to limit liability, the entrepreneurs cannot commit a higher payment than their output. Therefore:

$$B_H^H, B_L^H \leq H$$
$$B_H^L, B_L^L \leq L$$

To prevent the Low type entrepreneurs from mimicking, the High type entrepreneurs may increase B_H^L to its maximum. Because the High type entrepreneurs know that their output will never be L, while it may happen to Low type for sure. Hence issuing securities with high payment in low output state can effectively signal their type without incurring any extra cost.

By substituting the $B_H^L = L$ into the incentive constraint of Low type, we can have:

$$\beta P_L + L - B_L^L \geq \beta P_H + L - L \Rightarrow \beta P_L + L - B_L^L \geq \beta P_H$$

Since the output of the Low type entrepreneurs is lower than that of the High types

(L<H), it can be inferred that the Low type entrepreneurs may have a higher incentive to mimic the High types. Therefore, the incentive constraint should be binding:

$$\beta P_L + L - B_L^L = \beta P_H$$

Besides, it can also be proved that the Low type entrepreneurs will fully securitize their asset to maximize their utility.

Suppose that there exists a separating menu where $B_L^L = L - \Delta < L$. Base on the break-even condition, it can be inferred that $P_L \leq B_L^L \Longrightarrow P_L \leq L - \Delta$. As a result, the utility of the Low type entrepreneurs is $U_L = \beta P_L + L - B_L^L \leq (\beta - 1)(L - \Delta) + L$. In that case, if one of the Low type entrepreneurs suggests a new deviating contract where $B_L^H = B_L^L = P_L = L$, it can be inferred that the utility of the Low type in the deviating contract becomes $U'_L = (\beta - 1)L + L > U_L$. Doubtlessly, the Low type entrepreneurs have an incentive to deviate to the new contract. Moreover, this deviating contract is going to be accepted by the investors regardless whether the High type entrepreneurs mimic or not. Therefore, we can conclude that in any separating equilibrium with direct issuing, it must be:

$$B_L^L = P_L = L$$

By substituting $B_L^L = P_L = L$ back to the incentive constraint of Low type entrepreneurs:

$$\beta P_L + L - B_L^L = \beta P_H \Longrightarrow P_H = P_L = L$$

Finally, by substituting $P_H = L$ back to the utility function of High type entrepreneurs, we can infer that the High type entrepreneurs will choose $B_H^H = L$ as well. Because the price of their securities is fixed by the incentive constraint of Low type, any increase on their contingent payment B_H^H cannot generate a higher income. As a result, in the separating equilibrium, the High type entrepreneurs have no incentive to commit $B_H^H > L$. In conclusion, with the direct issuing process, the separating equilibrium can arise only if the High type entrepreneurs issue riskless debt $B_H^H = B_H^L = L$, while the Low type entrepreneurs can issue risky or riskless securities $B_L^L = L$ *and* $B_L^H \epsilon [L, H]$.

From the proof above, we can infer that a pooling allocation where both types issue riskless debt can also be sustained as an equilibrium. This allocation is sustained when investors hold a belief that the one who issue risky securities ($B_i^H > L$) must be the Low type. This belief can satisfy the intuitive criterion, when the return of the investment β is sufficiently low. In that case, the loss of mispricing may overcome the benefit of a higher investment. Therefore, the High type entrepreneurs have no incentive to increase B_H^H. Neither do the Low type entrepreneurs have the incentive. Because under the belief of the investors, increasing B_L^H cannot generate a higher income to them. Besides, since the pay-out of the riskless debt is fixed to be L, the investors may find it worthless to exert any effort. In conclusion, none of the participants can become better-off by deviating to other strategies, and the pooling allocation can be sustained as a Bayesian equilibrium.

However, when the return of short-term investment β becomes sufficiently high, the High type entrepreneurs may have the incentive to issue riskier securities and motivates the investors to "identify" the types by exerting effort. To be more specific, when High type entrepreneurs choose to issue risky securities for a higher selling price, the incentive constraints will be violated. In another word, if the High type entrepreneurs issue any risky securities, the Low type entrepreneurs will always mimic.

Reviewing the simplified incentive constraints of the Low type in the proof of Lemma 1:

$$\beta L + \mathrm{L} - \mathrm{L} \geq \beta \mathrm{B}_{\mathrm{H}}^{\mathrm{H}} + \mathrm{L} - B_H^L$$

$$\beta L \geq \beta \mathrm{B}_{\mathrm{H}}^{\mathrm{H}} + \mathrm{L} - B_H^L \Rightarrow \beta(L - \mathrm{B}_{\mathrm{H}}^{\mathrm{H}}) \geq \mathrm{L} - B_H^L$$

Given the assumption of limit liability, entrepreneurs cannot commit a payment higher than their full income ($B_H^L \leq L$). Therefore, if the High type entrepreneurs commit a $\mathrm{B}_{\mathrm{H}}^{\mathrm{H}} > L$, the incentive constraint of Low type will be violated. In another word, the

Low type entrepreneurs will always have the incentive to mimic the High type if $\mathrm{B}_{\mathrm{H}}^{\mathrm{H}} > L$. As a result, no prior information could be acquired by observing the contract chosen by the entrepreneurs and all the securities are sold at an initial pooling price (P_p).

From the structure of the sequential game above, we can infer that the equilibrium can be solved by implementing backward induction. As the starting point of our discussion, we firstly focus on the small sub-game where the investors make their effort-taking and purchasing decision. At the beginning of this sub-game, the pooling contract (B_p^H, B_p^L, P_p) are given by the entrepreneurs' previous move.

If the investors choose not to acquire any signal, the expected revenue of the securities are:

$$\overline{P} = \lambda B_p^H + (1 - \lambda) B_p^L$$

Therefore, we can infer that the reservation price of the investors will be $P_p = \overline{P}$ without any signal.

The game could become more complicated when the investors exert effort to acquire the signal. In our model, we assume that the signal of investors is informative but not perfect. Since the probability for the signal to match the type is $\sigma \in (\frac{1}{2}, 1)$, securities with positive signal (Π_{H}) are more likely to be the high type. Otherwise, if a negative signal (Π_{L}) is acquired, investors may decrease their expectation to the securities. However, the signal acquired by investors is imperfect. Hence, with a certain probability (1-σ), the signal may fail to reflect the true type of the securities. High type securities could be marked as "low" type by the investors and rejected in the first sales. In that case, the High type entrepreneur may have to decrease their securities price in the second sales. This risk of misidentification may restrict the incentive for High type entrepreneurs to issue their securities directly to the market, especially when the information is relatively inaccurate (σ).

To start a more detail discussion, we summarize all four matching situations about the type and signal with the following table:

Type	Signal	Ex-post Probability	Ex-anti Probability
H	π_H	σ	λσ
H	π_L	$1-\sigma$	$\lambda(1-\sigma)$
L	π_H	$1-\sigma$	$(1-\lambda)(1-\sigma)$
L	π_L	σ	$(1-\lambda)\sigma$

As we can conclude from the table above, for the securities associated with good signal (π_H), the expected value is:

$$P_g = \frac{\lambda\sigma}{\lambda\sigma + (1-\lambda)(1-\sigma)} B_p^H + \frac{(1-\lambda)(1-\sigma)}{\lambda\sigma + (1-\lambda)(1-\sigma)} B_p^L$$

While for the securities associated with bad signal (π_L), the expected value will be:

$$P_b = \frac{\lambda(1-\sigma)}{\lambda(1-\sigma) + (1-\lambda)\sigma} B_p^H + \frac{(1-\lambda)\sigma}{\lambda(1-\sigma) + (1-\lambda)\sigma} B_p^L$$

Based on our assumptions that the signal is informative ($\sigma > 1/2$) and the pay-out of the securities is positive correlated with the output ($B_p^H \geq B_p^L$), it can be inferred that $P_g \geq P_b$.

Given the ex-post expectation of the securities value, we turn to the discussion about the investors' optimal strategy. As we have mentioned in previous paragraphs, the ex-ante proportion of high type entrepreneurs is publicly known as λ. As a result, if the investors purchase all the securities regardless of the signal, the expected value of their asset pool is:

$$\mathrm{E(V)} = \lambda B_p^H + (1-\lambda) B_p^L$$

As we can see, this value can be rationally predicted before the signal acquisition. Therefore, we can infer the following lemma:

Lemma 2: With rational expectation, it can never be an equilibrium for the investors to exert effort and choose to purchase all the securities or reject all the

securities afterward.

Proof: Assume that the entrepreneurs set their initial price equal to P_p in the first sales. If the investors purchase all the securities without taking any effort, their pay-off is:

$$\lambda B_p^H + (1-\lambda)B_p^L - P_p$$

However, if the investors remain to purchase all the securities after exerting effort, their pay-off becomes:

$$\lambda B_p^H + (1-\lambda)B_p^L - P_p - C$$

Given a strictly positive cost of effort C, investors who have chosen the later strategy will become better-off by deviating to the previous one. Therefore, the later strategy cannot be an equilibrium. With similar argument, it can also be found that the investors would choose to be passive when they intend to reject all the offers.

In these two special cases, the payoff to the investors can be expected before the effort-taking decision and is irrelevant to the signal acquired. In another word, the investors cannot benefit from the signal they acquire if they decide to purchase all or reject all. Nevertheless, to obtain this signal, the investors have to pay C as the cost of their effort. Therefore, if the investor expects to purchase all of the securities or leave the market, she will never choose to exert effort.

The only way in which the investors can benefit from their signal is through securities selection. Since the signal acquired by the investors is informative, the investors may generate positive profit by purchasing the securities with good signal and reject those with bad signal. Based on our previous discussion, if the investors exert effort, a fraction equal to $\lambda\sigma + (1-\lambda)(1-\sigma)$ of the securities is going to be identified as “good”. Hence, by purchasing these securities with good signal, the investors can acquire:

$$[\lambda\sigma + (1-\lambda)(1-\sigma)] * (P_g - P_p)$$

In ex-ante, if the investors choose to take effort and make the securities selection, their payoff is:

$$E_{ES} = [\lambda\sigma + (1-\lambda)(1-\sigma)] * \left(P_g - P_p\right) - \mathrm{C}$$

Otherwise, if the investors do exert any effort, there will be no signal which they can use to make their selection. Therefore, they either purchase all the securities or reject to purchase any. By purchasing all the securities, the investors' payoff is:

$$E_{IP} = \lambda B_p^H + (1-\lambda) B_p^L - P_p$$

Otherwise, if the investors reject all the offers from the entrepreneurs, their payoff is:

$$E_{I0} = 0$$

In summary, after excluding some dominated or impossible strategy combination, three strategies are remaining to be compared by the investors: 1, investors exert effort and select their securities with the signal – (effort, selection). 2, investors do not exert any effort and purchase all the securities – (ignorance, purchase). 3, investors do not exert any effort and leave the market – (ignorance, reject). In the following discussion, we are going to compare all these strategies two by two under different pricing area. Given the price set up by the entrepreneurs, a strategy is going to be chosen by the investors if and only if it can generate higher payoff than the other two.

Define $\tilde{P}$ as the reservation price at which the investors feel indifferent between choosing (effort, selection) and (ignorance, reject). From the definition of $\tilde{P}$ and investors' pay-off function, it can be inferred that:

$$[\lambda\sigma + (1-\lambda)(1-\sigma)] * \left(P_g - \tilde{P}\right) - \mathrm{C} = 0$$

$$\Rightarrow \tilde{P} = \frac{\lambda\sigma B_p^H + (1-\lambda)(1-\sigma) B_p^L}{\lambda\sigma + (1-\lambda)(1-\sigma)} - \frac{C}{\lambda\sigma + (1-\lambda)(1-\sigma)}$$

Due to the linearity of the pricing function, if the entrepreneurs choose a price above $\tilde{P}$, the investors would prefer (ignorance, reject) to (effort, selection). Otherwise, if the price is below $\tilde{P}$, obtaining the signal and doing securities selection, may generates higher profit to the investors.

Similarly, define $\bar{P}$ as the reservation price at which the investors feel indifferent between choosing (ignorance, purchase) and (ignorance, reject). We can have:

$$\lambda B_p^H + (1-\lambda)B_p^L - \bar{P} = 0$$

$$\Rightarrow \bar{P} = \lambda B_p^H + (1-\lambda)B_p^L$$

In the case without any signal, if the entrepreneurs set a price $P_p > \bar{P}$, the investors may trend to reject entrepreneurs' offer. Because the initial price of the securities has exceeded their ex-ante expectation. Therefore, purchasing all the securities is a loss-making decision to the investors. In the opposite, if the entrepreneurs set a price $P_p \leq \bar{P}$, the investors may purchase all the securities without any signal and still generate positive profit.

Finally, define $\dot{P}$ as the reservation price at which the investors feel indifferent between choosing (effort, selection) and (ignorance, purchase). Similarly, by definition we have:

$$[\lambda\sigma + (1-\lambda)(1-\sigma)] * \left(P_g - \dot{P}\right) - \mathrm{C} = \lambda B_p^H + (1-\lambda)B_p^L - \dot{P}$$

$$\Rightarrow \dot{P} = P_b + \frac{C}{(1-\lambda)\sigma + \lambda(1-\sigma)}$$

Intuitively, if the price of securities is extremely low, the benefit of excluding the low type securities may decrease. As a result, when the benefit of doing so cannot cover the cost of effort ($P_p \leq \dot{P}$), the investors may choose to purchase all the securities blindly instead of taking any effort. Otherwise, when the potential loss of purchasing low quality securities becomes sufficiently high ($P_p > \dot{P}$), the investors may have incentive to acquire information and select their securities more causally.

To clarify the optimal strategy of investors, we compare the reservation prices under different parameter conditions and end up with the following conclusion:

Proposition 1:

When $\widetilde{\boldsymbol{P}} \leq \overline{\boldsymbol{P}}$, there exist a pooling equilibrium where the entrepreneurs set their initial price $\boldsymbol{P_p} = \overline{\boldsymbol{P}}$ and the investors purchase all the securities without taking any effort.

When $\widetilde{\boldsymbol{P}} > \overline{\boldsymbol{P}}$ and $\boldsymbol{\sigma}\widetilde{\boldsymbol{P}} + (\mathbf{1} - \boldsymbol{\sigma})\boldsymbol{P_b} > \overline{\boldsymbol{P}}$, there exist a pooling equilibrium where the entrepreneurs set their initial price $\boldsymbol{P_p} = \widetilde{\boldsymbol{P}}$, second sales price $\boldsymbol{P_{ps}} = \boldsymbol{P_b}$ and the investors exert effort to do the securities selection.

When $\widetilde{\boldsymbol{P}} > \overline{\boldsymbol{P}}$ and $\boldsymbol{\sigma}\widetilde{\boldsymbol{P}} + (\mathbf{1} - \boldsymbol{\sigma})\boldsymbol{P_b} \leq \overline{\boldsymbol{P}}$, there exist a separating equilibrium where the entrepreneurs issue their securities through intermediaries.

Proof:
The whole proof can be departed into two cases depending on the relationship between $\tilde{P}$ and $\bar{P}$:

Case 1: When $\tilde{P} \leq \bar{P}$.

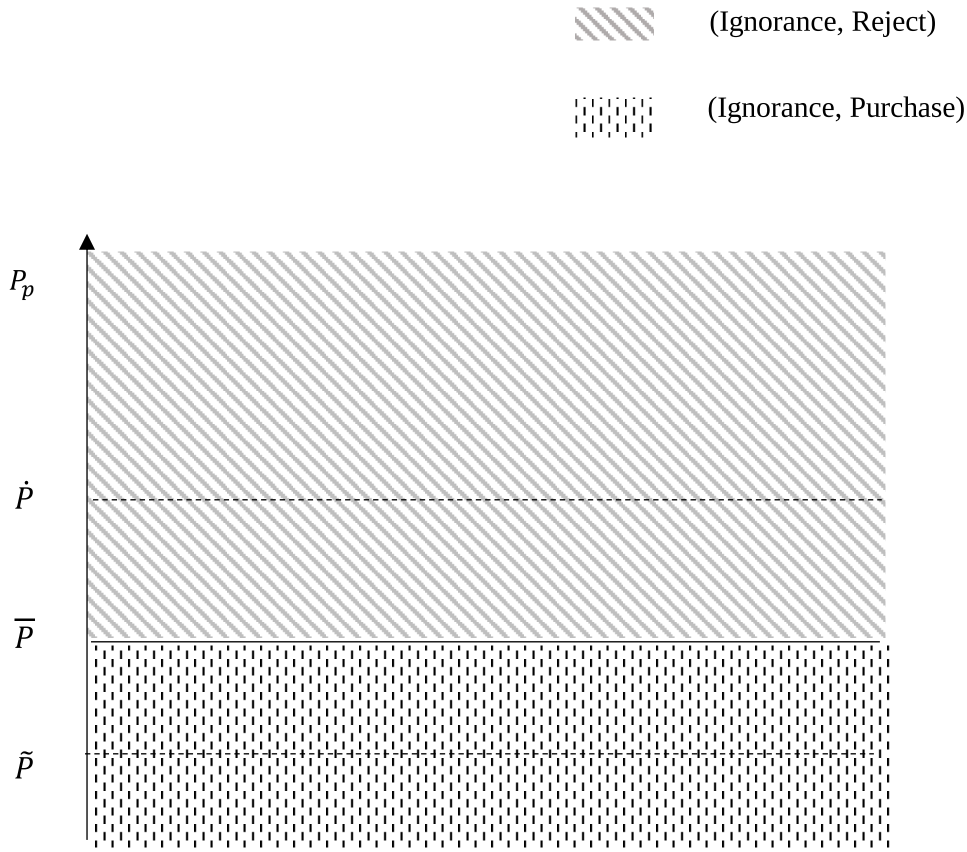
$$\tilde{P} \leq \bar{P} \Rightarrow \frac{\lambda\sigma B_p^H + (1-\lambda)(1-\sigma)B_p^L}{\lambda\sigma + (1-\lambda)(1-\sigma)} - \frac{C}{\lambda\sigma + (1-\lambda)(1-\sigma)} \leq \lambda B_p^H + (1-\lambda)B_p^L$$

$$\Rightarrow \mathrm{C} \geq \lambda(1-\lambda)(2\sigma - 1)(B_p^H - B_p^L)$$

With the same parameter condition, it can be easily proved that $\dot{P} \geq \bar{P}$ in this case.

To simplify our analysis, we conclude the relative relationship between $\tilde{P}$, $\bar{P}$ and $\dot{P}$

with the following graph:



As we can see from the graph above, the whole pricing area can be divided into two. When the entrepreneurs set an initial price above $\bar{P}$, investors' optimal strategy will be (ignorance, reject). As we have defined in the previous section, when the price of securities is above $\bar{P}$, (ignorance, reject) can bring higher pay-off to the investors than (ignorance, purchase). Similarly, when the price is higher than $\tilde{P}$, the investors prefer (ignorance, reject) to (effort, selection). As we can see in this case, any price level above $\bar{P}$ will also be above $\tilde{P}$, which means that, when the entrepreneurs set their price above $\bar{P}$, (ignorance, reject) becomes a better strategy than both (ignorance, purchase) and (effort, selection). However, if the entrepreneurs cut the price below $\bar{P}$, (Ignorance, Purchase) will become the dominant strategy amount the three. As we have defined, when the price is below $\dot{P}$, the investor would become better-off by choosing (ignorance, purchase) instead of (effort, selection). Since any price below $\bar{P}$ is also below $\dot{P}$, it can be inferred that when the price is lower than $\bar{P}$, (ignorance, purchase) becomes a better strategy than both (ignorance, reject) and (effort, selection).

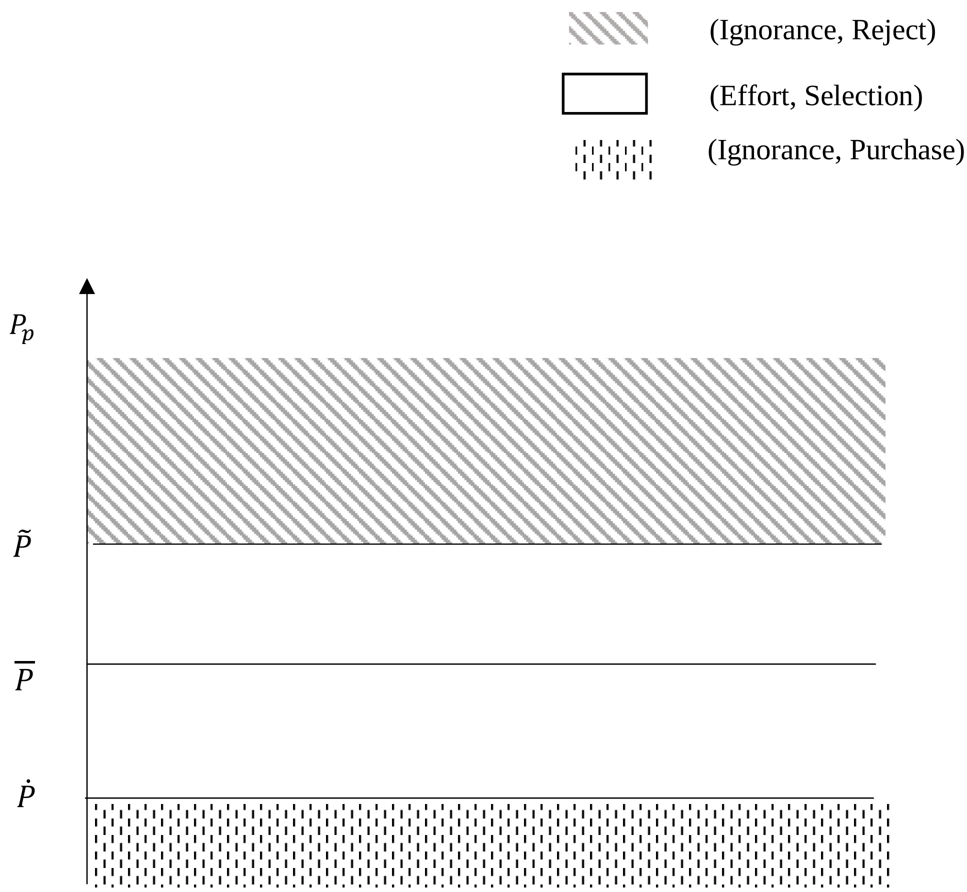
Intuitively, the above case arises when the cost of speculation is sufficiently high. In that case, the investors may not be able to generate sufficient profit from acquiring a signal to cover the corresponding cost. Therefore, when all the securities are sold at a pooling price, the investors may either purchase all or reject all after comparing the market price to their ex-ante expectation about the securities value. Similarly, when the cost of effort is sufficiently high, the intermediaries can also purchase all the securities at $\bar{P}$ and resell them all to the investors at the same price. Given that the investors have no incentive to exert effort, the intermediaries are not going to suffer from the adverse selection cost. Therefore, this equilibrium can be sustained with or without intermediaries. However, our paper mainly focus on the case where the investors exert effort, so in the following discussion we simply assume that $\mathrm{C} < \lambda(1-\lambda)(2\sigma-1)(B_p^H - B_p^L)$.

Case 2: When $\tilde{P} > \bar{P}$.

$$\tilde{P} > \bar{P} \Rightarrow \frac{\lambda\sigma B_p^H + (1-\lambda)(1-\sigma)B_p^L}{\lambda\sigma + (1-\lambda)(1-\sigma)} - \frac{C}{\lambda\sigma + (1-\lambda)(1-\sigma)} > \lambda B_p^H + (1-\lambda)B_p^L$$

$$\Rightarrow \mathrm{C} < \lambda(1-\lambda)(2\sigma-1)(B_p^H - B_p^L)$$

Similar to the last case, it can also be proved that $\dot{P} < \bar{P}$ under the same parameter condition. As a result, the investors' optimal strategy can be concluded by the following graph:



As we can see from the graph above, when the price of securities $P_p > \tilde{P}$, rejecting entrepreneurs' offers becomes the optimal strategy to the investors. Since any price above $\tilde{P}$ is also above $\bar{P}$, the investors can hardly generate a positive profit by purchasing any of the securities. However, when the price of securities decreases to the area between $\tilde{P}$ and $\bar{P}$, the strategy combination (effort, selection) start to become dominating. Because when the securities price is below $\tilde{P}$, the investors can generate a positive expected profit by exerting effort and do the securities selection. However, if the investors attempt to purchase all the securities blindly, they may make a loss because the initial price is above $\bar{P}$.To move one step further, from the location of $\dot{P}$, we can infer that the (effort, selection) strategy can still make a higher profit than (ignorance, purchase) even when the price is between $\bar{P}$ and $\dot{P}$. Finally, in the extreme case where entrepreneurs set a price below $\dot{P}$, the (ignorance, purchase) strategy becomes the optimal one to the investors. As we have discussed in previous sections, when the securities price is extremely low, the benefit of excluding low type securities may not be able to cover the cost of effort.

To summarise our findings in case 2, we find that the investors' incentive for taking effort is undermined when the price of securities is extremely high or extremely low, because the high price of the securities may prevent the investors from purchasing the securities fundamentally. If there is no hope for the investors to break even under any possible trading strategy, they may simply leave the market without even bothering the signal channel. In the opposite, when the securities price is extremely low, the investors may find it unnecessary to exclude the low-quality securities by exerting effort. Excepting the two extreme cases above, investors are motivated to exert effort when the price of securities locates between $\dot{P}$ and $\tilde{P}$. In that case, the benefit from securities selection covers the corresponding cost, thus the (effort, selection) becomes the optimal strategy to the investors. However, in the following discussion, we are going to show that it is not always on the high type entrepreneurs' behalf to motivate the investors. Because the signal acquired by the investors is imperfect, the high type entrepreneurs can be misidentified as "low" type ones. Once it is the case, the high type entrepreneurs may be rejected by the investors in the first sales and suffers a price cut in their reselling process.

With the optimal strategy of the investors, we can discuss the pricing strategy of the entrepreneurs by moving one step backward. Our discussion starts with the case 1 where $C \geq \lambda(1-\lambda)(2\sigma-1)(B_p^H - B_p^L)$. Based on our result in last section, any price above $\bar{P}$ will be rejected by the investors while any price below $\bar{P}$ will not satisfy entrepreneurs' incentive of utility maximization. Therefore, the High type entrepreneurs will choose their initial price equal to $\bar{P}$ in the equilibrium. Correspondingly, the investors will purchase all the securities without taking any effort.

Then we come to the discussion about case 2. Firstly, with similar argument as the previous case, the high type entrepreneurs have no incentive to suggest a price higher than $\tilde{P}$, because it is above the investors' reservation price under any strategy combination. If the high type entrepreneurs set their initial price between $\tilde{P}$ and $\bar{P}$, it can be inferred that the investors may exert effort and do securities selection base on their signal. Since the Low type entrepreneurs always have the incentive to mimic the High type, we may focus on the optimal pricing strategy of the High type. As we have discussed in previous section, the signal acquired by the investors is informative but imperfect. Hence, only with probability σ, the investors may truly identify the High type entrepreneurs and purchase their securities at the initial price P_p. Otherwise, with probability $1 - \sigma$, they are misidentified as "Low type" and rejected by the investors in the first sales. As we have discussed at the beginning of this section, for the securities associated with bad signal, their expected value is:

$$P_b = \frac{\lambda(1-\sigma)}{\lambda(1-\sigma)+(1-\lambda)\sigma} B_p^H + \frac{(1-\lambda)\sigma}{\lambda(1-\sigma)+(1-\lambda)\sigma} B_p^L$$

Therefore, in the second sales, the investors are not going to accept any price above P_b. To maximize their efficient investment, the entrepreneurs are going to set their second sale price at P_b and the investors will purchase all the remaining securities at that price. As a result, if the high type entrepreneurs set their initial price between $\tilde{P}$ and $\bar{P}$, their expected selling price becomes:

$$\mathrm{EP}(P_p) = \sigma P_p + (1-\sigma) P_b$$

Given that the investors are going to exert effort, it is optimal for the high type entrepreneurs is to set their initial price at $\tilde{P}$ in the first sales. Summarizing our conclusions in the first and second sale, the expected selling price of high type securities becomes:

$$\mathrm{EP}(\tilde{P}) = \sigma \tilde{P} + (1-\sigma) P_b$$

Finally, it is also possible for the entrepreneurs to deactivate the investors by setting an initial price below $\dot{P}$. The potential motivation of doing so could be avoiding the risk of misidentification. However, in our model, setting an extremely low price equal or below $\dot{P}$ is never the optimal strategy to the high type entrepreneurs. Because the investors are also allowed to sell their securities through intermediaries in our settings. As we can see from the graph above, $\dot{P} < \bar{P}$ when $C < \lambda(1-\lambda)(2\sigma-1)(B_p^H - B_p^L)$. Following our discussion in the separating equilibrium through intermediaries, the high type entrepreneurs can sell their securities at a price no less than $\bar{P}$ when intermediary is introduced. In other word, setting a price below $\dot{P}$ in the direct issuing process is strictly dominated by the strategy of introducing the intermediaries which is going to be shown in the later section. Therefore, in the following discussion about direct issuing process we may only focus on the case where the entrepreneurs set their initial price $P_p = \tilde{P}$.

When $P_p = \tilde{P}$, the expected price of high type entrepreneurs' securities will be :

$$\mathrm{EP}\big(\tilde{P}\big) = \sigma\tilde{P} + (1-\sigma)P_b$$

Alternatively, if the high entrepreneur do not sell their securities directly to the investors and turn to the intermediaries, regardless of the form of securities, the price is going to be $\bar{P}$. Therefore, the direct issuing process is preferred when $\mathrm{EP}\big(\tilde{P}\big) > \bar{P}$. Otherwise when $\mathrm{EP}\big(\tilde{P}\big) \leq \bar{P}$, the high type entrepreneurs may turn to the intermediaries. To further analysis the necessary condition for introducing the intermediaries, we drive:

$$\sigma\tilde{P} + (1-\sigma)P_b < \bar{P}$$

$$\Rightarrow \mathrm{C} > \dot{C} = \frac{\lambda\sigma + (1-\lambda)(1-\sigma)}{\sigma}\{\frac{\lambda\sigma^2 H + (1-\lambda)(1-\sigma)\sigma L}{\lambda\sigma + (1-\lambda)(1-\sigma)} + \frac{\lambda(1-\sigma)^2 H + (1-\lambda)\sigma(1-\sigma)L}{\lambda(1-\sigma) + (1-\lambda)\sigma} - [\lambda H + (1-\lambda)L]\}$$

Finally, with backward induction, we substitute the expect price in direct issuing $\mathrm{EP}\big(\tilde{P}\big)$ into the utility function of high type entrepreneur to drive the optimal securities :

$$EU_H = \beta\big(\sigma\tilde{P} + (1-\sigma)P_b\big) + H - B_p^H$$

$$EU_H = \beta\left(\frac{\lambda\sigma^2 B_p^H + (1-\lambda)(1-\sigma)\sigma B_p^L}{\lambda\sigma + (1-\lambda)(1-\sigma)} - \frac{\sigma C}{\lambda\sigma + (1-\lambda)(1-\sigma)} + \frac{\lambda(1-\sigma)^2 B_p^H + (1-\lambda)\sigma(1-\sigma)B_p^L}{\lambda(1-\sigma) + (1-\lambda)\sigma}\right) + H - B_p^H$$

To solve the optimal securities:

$$\frac{\partial EU_H}{\partial B_p^L} = \beta\left(\frac{(1-\lambda)(1-\sigma)\sigma}{\lambda\sigma + (1-\lambda)(1-\sigma)} + \frac{(1-\lambda)\sigma(1-\sigma)}{\lambda(1-\sigma) + (1-\lambda)\sigma}\right) > 0$$

$$\Rightarrow B_p^L = L$$

$$\frac{\partial EU_H}{\partial B_p^H} = \beta\left(\frac{\lambda\sigma^2}{\lambda\sigma + (1-\lambda)(1-\sigma)} + \frac{\lambda(1-\sigma)^2}{\lambda(1-\sigma) + (1-\lambda)\sigma}\right) - 1$$

If $\left(\frac{\lambda\sigma^2}{\lambda\sigma+(1-\lambda)(1-\sigma)} + \frac{\lambda(1-\sigma)^2}{\lambda(1-\sigma)+(1-\lambda)\sigma}\right) < 1 \Rightarrow B_p^H = \mathrm{L}$ and $B_p^L = L$. However, this sub-case cannot be sustained as an equilibrium. Because the result we obtained above contradicts to the precondition for case 2 to arise ($\mathrm{C} < \lambda(1-\lambda)(2\sigma-1)(B_p^H - B_p^L) \Rightarrow C < 0$). When the high type entrepreneurs issue riskless securities ($B_p^H = B_p^L = L$), it can be inferred that $P_G = P_B = \bar{P} = \mathrm{L}$, which drives our discussion back to case 1.

Therefore the remaining case is $\beta\left(\frac{\lambda\sigma^2}{\lambda\sigma+(1-\lambda)(1-\sigma)} + \frac{\lambda(1-\sigma)^2}{\lambda(1-\sigma)+(1-\lambda)\sigma}\right) \geq 1 \Rightarrow B_p^H = H$ and we end up with a pooling equilibrium where both types of entrepreneurs issue risky securities ($B_p^H = H$ and $B_p^L = L$).

5. Separating Equilibrium when Intermediaries are Introduced

Following our discussion in the last section, we show that the intermediaries are more likely to be introduced when the cost of the signal is relatively high or the accuracy of the signal is relatively low. To clarify the role played by the intermediaries in securities issuing process, we turn to analysis the equilibrium when the intermediaries are introduced. Distinguishing from the direct issuing process, the intermediaries can offer a menu of contracts to the investors, which allows them to introduce "cross-subsidies." As we mentioned in the game setting part, competition among intermediaries will lead to zero profits for the whole menu but not necessarily for each contract. Losses on some contracts can be compensated by profits on the other contracts. This loss and profit are defined as cross-subsidies in our model, which is defined as S_i.

Therefore, for any separating menu offered by the intermediaries, we should have the following equations:

$$P_H = B_H^H - S_H \text{ and } P_L = B_L^L - S_L$$

In another word, the price of securities only depends on their true value and the corresponding "cross-subsidy." Intuitively, in a separating menu, the authenticity of the announcement is guaranteed by the incentive constraint. Hence the price of securities is not affected by entrepreneurs' commitment in which the type is different from their announcement (B_H^L and B_L^H). However, this commitment can still play an important role in signalling. As we can see, High type entrepreneur can increase their commitment in Low state B_H^L to prevent Low type entrepreneurs from mimicking without incurring any extra cost to themselves.

Moreover, when intermediaries are introduced into the issuing process, "cross-subsidy" is another important way to separate the Low type entrepreneurs from the High type ones. However, the intermediary cannot become loss-making after the "subsidy." Therefore, the following break event condition must be satisfied:

$$\lambda S_H + (1-\lambda)S_L = 0 \Rightarrow S_L = -\frac{\lambda}{1-\lambda}S_H$$

In the special case where cross-subsidies are not introduced into the menu, we simply have $S_L = S_H = 0$. In another word, each separating contract in the menu will satisfy their break-even condition.

By substituting the price equations back into the utility functions, we can have:

$$U_H = \beta P_H + H - B_H^H = \beta\left(B_H^H - S_H\right) + H - B_H^H$$
$$= (\beta - 1)B_H^H - \beta S_H + H$$

$$U_L = \beta P_L + L - B_L^L = \beta(B_L^L - S_L) + L - B_L^L$$
$$= (\beta - 1)B_L^L - \beta S_L + L$$

With the utility functions above, we can now derive some general results which can be useful for establishing and characterising the separating equilibrium for our model.

Lemma 3: In any separating equilibrium with or without cross-subsidy, intermediaries will sell the securities at a fair price to investors.

Proof: As we explained above, for separating purpose, the intermediaries introduce "cross-subsidies" when they purchase securities from the entrepreneurs. By decreasing the price of High type securities and increasing those of the Low type, they strengthen the incentive for the Low type entrepreneurs to report their type truly. However, in the reselling process, it becomes not only unnecessary but also unbeneficial for the intermediaries to introduce "cross-subsidies" in the sales contract. Since contracts are observable in our model, the investors can infer the true type of the securities by observing the contract between entrepreneurs and intermediaries. Suppose that an intermediary resells his/her securities with "cross-subsidies." After inferring the true value of each security, the investors will only purchase the underpriced securities and leave the overpriced ones unsold, which makes the intermediaries loss-making. Hence, when types are revealed in the purchasing process, the intermediaries are going to set their resell price equal to the securities' true value.

Lemma 4: In any separating equilibrium, Low type entrepreneurs will seek to sell the whole claim of their risky asset by choosing a contract with $B_L^L = L$.

Proof:
Similar to our discussion in the previous section, in any separating equilibrium, intermediaries can infer entrepreneurs' type by observing the contract chosen by the entrepreneurs. Therefore, the utility function of Low type entrepreneurs will be:

$$U_L = (\beta - 1)B_L^L - \beta S_L + L$$

To solve the optimal contract chosen by the Low type entrepreneur, we calculate the first order derivative of the utility function on B_L^L and obtain:

$$\frac{\partial U_L}{\partial B_L^L} = \beta - 1 > 0$$

When the type is revealed in the separating menu, the Low type entrepreneurs always have the incentive to fully securitize their risky asset. Because the short-term investment has a strictly positive return, increasing the scale of investment can generate a higher utility to the Low type. However, due to limit liability, a Low type entrepreneur cannot commit more L. Therefore, the Low type entrepreneurs will set $\mathrm{B}_{\mathrm{L}}^{\mathrm{L}} = \mathrm{L}$.

Besides, with a separating menu, neither type of the entrepreneurs should become better off by mimicking the other. In other words, the incentive constraints must be satisfied:

$$\beta\left(\mathrm{B}_{\mathrm{H}}^{\mathrm{H}} - \mathrm{S}_{\mathrm{H}}\right) + \mathrm{H} - \mathrm{B}_{\mathrm{H}}^{\mathrm{H}} \geq \beta\left(\mathrm{B}_{\mathrm{L}}^{\mathrm{L}} - \mathrm{S}_{\mathrm{L}}\right) + \mathrm{H} - \mathrm{B}_{\mathrm{L}}^{\mathrm{H}}$$
$$\beta\left(\mathrm{B}_{\mathrm{L}}^{\mathrm{L}} - \mathrm{S}_{\mathrm{L}}\right) + \mathrm{L} - \mathrm{B}_{\mathrm{L}}^{\mathrm{L}} \geq \beta\left(\mathrm{B}_{\mathrm{H}}^{\mathrm{H}} - \mathrm{S}_{\mathrm{H}}\right) + \mathrm{L} - B_H^L$$

The first one is the incentive constraint for the High type while the second one is the incentive constraint for the Low type. Take the first inequity as an example: the left-hand side represents the utility level if the High type entrepreneur truly reveals his/her type, while the right-hand side of the inequity represents the utility level if the High type entrepreneur pretends to be a Low type one. Similar to the discussion without intermediaries, the Low type entrepreneurs may have a higher incentive to mimic the High type ones. Therefore, by decreasing the price paid to the High type entrepreneurs and increasing that to the Low types, the intermediaries can weaken the motivation for the Low type entrepreneurs to mimic the other.

Lemma 5: In any separating equilibrium involving cross-subsidies, High type subsidises the Low type.

Proof: Suppose that there exists a separating equilibrium where the Low type entrepreneurs offer a subsidy to the High type. In other words, intermediaries are making a profit in the Low type contract and are making a loss in the High type contract ($S_H < 0$ *and* $S_L > 0$). Consider that a new entrant offers a contract, which makes the Low type better-off and implies a strictly positive profit, for example C_L^* ($B_L^{*H} = H, B_L^{*L} = L$ and $P_L^* = L - \Delta$ where $\Delta < S_L$). Obviously, the price paid to the Low type entreprenerus in the new contract is higher than in original one ($P_L = L - S_L$), and the Low type entrepreneurs will deviate to the new contract. In that case, since the incumbent menu intends to generate profit from the Low types and subsidize the High types, it will become loss-making. Therefore, the incumbent menu cannot be sustained as an equilibrium.

Lemma 6: Among all the feasible zero-profit menus, only the one which maximises the utility of the High type can be sustained as equilibrium.

Proof: Suppose that a menu of contracts C_H and C_L becomes the equilibrium in the market. As the result of full competition, this menu must satisfy the zero-profit condition, $\lambda S_H + (1 - \lambda) S_L = 0$. Otherwise, the rival can simply attract both types of entrepreneurs by offering a more competitive menu. Consider that a new entrant attempts to offer a profit-making menu containing C_H^+ and C_L^+, which satisfies the incentive constraint ($U_L(C_L^+) > U_L(C_H^+)$) and makes the High type better-off ($U_H(C_H^+) > U_H(C_H)$). As a result, the High type will deviate to the new menu. If the original menu involves cross-subsidy, as proved in Lemma 2 and 3, the incumbent menu will become loss-making when it is taken only by the Low type. Otherwise, if the original menu does not involve cross-subsidies, the new entrant may only attract the High types and still makes a positive profit. Due to the existence of profitable deviation, the original menu C_H and C_L could not be an equilibrium.

Given the lemmas we have listed previously, the separating equilibrium with intermediaries can be simplified as the solution to the following maximisation problem:

$$\max_{\{B_H^H, S_H\}} U_H = \beta P_H + H - B_H^H = \beta(B_H^H - S_H) + H - B_H^H$$

Subject to:

The Incentive Comparability Constraints:

$$\beta(B_H^H - S_H) + H - B_H^H \geq \beta(B_L^L - S_L) + H - B_L^H$$
$$\beta(B_L^L - S_L) + L - B_L^L \geq \beta(B_H^H - S_H) + L - B_H^L$$

Limited liability constraints:

$$B_L^L, B_H^L \leq L \text{ and } B_H^H, B_L^H \leq H$$

Cross-subsidy must be from high type to low type:

$$S_H \geq 0$$

Break event condition of cross-subsidy:

$$\lambda S_H + (1-\lambda)S_L = 0 \Rightarrow S_L = -\frac{\lambda}{1-\lambda} S_H$$

By solving the maximisation problem above, we obtain the following conclusion:

Lemma 7:

When intermediaries are introduced into the issuing process:

If $\boldsymbol{\beta\lambda - 1 \geq 0 \Rightarrow \lambda \geq \frac{1}{\beta}}$ or $\boldsymbol{\beta \geq \frac{1}{\lambda}}$, there exists a separating equilibrium with cross-subsidy where $\mathbf{B_H^H = H}$ and $\boldsymbol{S_H = (1-\lambda)(H-L)}$.

If $\boldsymbol{\beta\lambda - 1 < 0 \Rightarrow \lambda < \frac{1}{\beta}}$ or $\boldsymbol{\beta < \frac{1}{\lambda}}$, there exists a separating equilibrium without cross-subsidy where $\mathbf{B_H^H = B_H^L = B_L^L = L}$ and $\mathbf{B_L^H \neq L}$.

In the first sub-case, when the proportion of High type entrepreneurs is sufficiently large or the return of short-term investment is sufficiently high, High type entrepreneurs will have the incentive to introduce cross-subsidy for signalling. Since the cost of cross-subsidy is equally shared by all the High type entrepreneurs, a higher proportion can end up with a lower cost for each High type entrepreneur: $(\lambda S_H + (1-\lambda)S_L = 0)$. On the other hand, a high return on investment can compensate High type entrepreneurs for their loss of cross-subsidy. As a result, when either of them is sufficiently high, high type entrepreneurs will choose the separating menu with cross-subsidy. Otherwise, if the proportion of High type is small or the return of short-term investment is low, the High type entrepreneurs may not be well compensated for offering cross-subsidies. In that case, High type entrepreneurs may become better-off by issuing riskless debt and restrict their investment. Similar to the separating equilibrium without intermediaries, High types entrepreneurs will issue riskless debt and the equilibrium price will be $P_H = P_L = L$.

Another noticeable point in the separating equilibrium with intermediaries is that regardless of the parameter conditions, it is always the case:

$$P_H = P_L = \bar{P} = \lambda B_H^H + (1-\lambda)B_L^L$$

This conclusion can be easily inferred in case II, because $P_H = P_L = B_H^H = B_L^L = L$. As for the case I, by substituting $B_H^H = H$, $B_L^L = L$ and $S_H = (1-\lambda)(H-L)$ back into the pricing function, we can have:

$$P_H = B_H^H - S_H = H - (1-\lambda)(H-L) = \lambda H + (1-\lambda)L$$

$$P_L = B_L^L - S_L = L + \frac{\lambda}{1-\lambda} S_H = \lambda H + (1-\lambda)L$$

This result proves our previous suggestion that the selling price of the securities can never be below $\bar{P}$ when intermediaries are introduced into the issuing process. Therefore, the intermediaries are more likely to be introduced when $\sigma\tilde{P} + (1-\sigma)P_b \leq \bar{P}$.

However, by substituting $P_H = P_L = \bar{P}$, $B_H^H = H$ and $B_L^L = L$ back into the incentive constraints in case I, we can see that both incentives constraints become binding at the same time. Hence, to check the sustainability of the separating equilibrium, we need to show that the intermediaries cannot become better-off by offering a pooling contract. Firstly, to attract the entrepreneurs without violating the break-even condition, the intermediaries can only offer a pooling contract C_p: $\mathrm{P_p} = \bar{P}$, $\mathrm{B_p^H} = \mathrm{H}$ and $\mathrm{B_p^L} = L$. If we only focus on the take-over process from the entrepreneurs to the intermediaries, this pooling contract end up with the same pay-off as our separating menu. However, if we also take the following reselling game into consideration, this pooling contract can end up with a loss to the intermediaries. Because if the intermediaries purchase all the securities at a pooling price, they may not be able to identify the type of the entrepreneurs. Therefore, in the reselling process they can only choose to sell all the securities at a pooling price. Given our assumption that $\mathrm{C} < \lambda(1-\lambda)(2\sigma-1)(B_p^H - B_p^L)$, the investor may have incentive to speculate on their signal in the reselling process – by purchasing the securities with good signal and leave the securities with bad signal unsold. Taking this potential cost of adverse selection into consideration, the intermediaries can never break-even by purchasing and reselling at the same price. However, the intermediaries cannot pass this cost back to the entrepreneurs by cutting down their purchasing price below $\bar{P}$, because it will become dominated by the separating menu. Moreover, it is also impossible for the intermediaries to fully pass their cost back to the investors by increasing their reselling price when $\sigma\tilde{P} + (1-\sigma)P_b \leq \bar{P}$, since the cost of effort may partly be internalized into the price, which has been discussed in direct issuing process. With the optimal pricing strategy, the expected income of the intermediary will be: $\lambda\mathrm{H} + (1-\lambda)\mathrm{L} - \mathrm{C} < \bar{P}$, which is insufficient to cover the purchasing cost. In conclusion, the deviation to the pooling contract may decrease the pay-off to the intermediaries, so only the separating menu can be sustained as an equilibrium. Intuitively, the separating menu protect intermediary from speculation by equalizing the information between intermediary and investors. More importantly it can achieve a higher social efficiency by saving investors' cost of effort. Therefore, our model shows that the introduction of intermediaries can separate different types of entrepreneurs more efficiently.

6. The Introduction of Intermediaries and Market Equilibrium

To summarise our findings in the previous two sections, we reach the following propositions.

Proposition 2

When $\lambda \geq \frac{1}{\beta}$ and $\lambda(1-\lambda)(2\sigma-1)(B_p^H - B_p^L) > C > \dot{C}$, there exists a separating equilibrium with intermediaries, where $B_H^H = H$ and $S_H = (1-\lambda)(H-L) > 0$.

As we have mentioned above, the separating equilibrium with cross-subsidy is more likely to arise when the proportion of High type is sufficiently large. With cross-subsidisation, we can observe that the price of High type securities will be underpriced while Low type securities will be overpriced in the take-over process. Moreover, as we have discussed in Lemma 3, all securities will be sold from intermediaries to investors at a fair price. Therefore, we should observe that the price of high (low) quality securities increases (decreases) in the reselling process.

To check the sustainability of the equilibrium, we firstly check whether an intermediary can benefit by offering a deviating menu or deviating contract. From the maximization problem we solved in the previous section, it can be inferred that, given the separating menu we offered, there is no other break-even menu which can make the high type entrepreneurs better-off. Besides, in our separating menu, the incumbent intermediary is making a profit from the high type entrepreneurs and making a loss from the low type ones. Therefore, other separating menus which only attracts the low types will be loss-making. In summary, the intermediary cannot benefit from deviating to another menu. Then we check whether a new intermediary can benefit by offering a single deviating contract. Firstly, since the separating menu is making a loss on the low types, it can be inferred that a single contract that only attracts the low types is loss-making. Secondly, as we can see in the separating menu, the incentive constraints of Low type is binding. In this case, if the intermediaries intend to attract the High type entrepreneurs with a more beneficial contract, the new contract will also attract the Low type. Because in our model, entrepreneurs are assumed to be risk neutral. Hence, a higher purchasing price or a lower real payment may bring equal marginal benefit to both types. In another word, it is unbeneficial to offer a contract that only attracts the High type entrepreneur. Finally, we need to check whether the new entry can benefit by offering a pooling contract which attracts both types of entrepreneurs. As we have discussed in the previous sections, when $\mathrm{C} < \lambda(1-\lambda)(2\sigma-1)(B_p^H - B_p^L)$, Intermediary who offer pooling contract may suffer a loss due to the speculation of the investors. In a word, the new intermediary cannot offer a beneficial pooling contact which attracts both types of entrepreneurs. Therefore, the intermediary has no incentive to deviate in this equilibrium.

Guaranteed by the incentive constraint, neither type of the entrepreneurs may have the incentive to mimic the other inside the menu. Therefore, the deviation remains to be checked is the issuing process. Suppose that there exists an entrepreneur who deviates to the direct issuing process and offer a more beneficial contract. As we have discussed in the section about incentive constraint, any deviating contract that is beneficial to High types will also be beneficial to the Low types. If the new contract is beneficial only to the Low type, then the investors may rationally infer that the deviation is made by the Low type and will not accept any price above L. In that case, the Low type entrepreneurs may worse-off after deviation. On the other hand, if the new contract can benefit both types, the investors may infer that the deviation could be made both types of entrepreneurs. When $C < \lambda(1-\lambda)(2\sigma-1)(B_p^H - B_p^L)$, they may seek to acquire signal by exerting effort. In that case, the maximization price obtained by the entrepreneurs will be $\sigma\tilde{P} + (1-\sigma)P_b$. Under the parameter condition that $C > \dot{C}$, we can have $\sigma\tilde{P} + (1-\sigma)P_b < \bar{P}$. In conclusion, the new deviation does not make the entrepreneurs better-off and the original separating menu through intermediary can be sustained as an equilibrium.

Intuitively, Proposition 2 also shows that the entrepreneurs are more likely to introduce intermediaries when the cost of speculation is relatively high, and the accuracy of the signal is relatively low. Based on our previous discussion, the cost of effort is internalized into the initial price in the first sales. Therefore, a high cost of effort may drive down the expected price in direct issuing process. On the other hand, the introduction of cross-subsidies allows the high type entrepreneurs to sell their securities at a guaranteed price without cutting down their investment level. Hence the benefit of issuing securities through intermediaries becomes more significant when the return of short-term investment β is sufficiently high. More importantly, the introduction of intermediaries improves the social welfare by saving investors' cost of effort which in return benefit the High type entrepreneurs. By endogenizing the existence of intermediary, our mechanism guarantees the separation in the takeover process which fills the information gap between intermediaries and investors. As a result, the intermediaries can sell the securities of both types at their fair price without being manipulated by the investors.

Proposition 3

When $C < \dot{C}$ and $\beta\left(\frac{\lambda\sigma^2}{\lambda\sigma+(1-\lambda)(1-\sigma)}+\frac{\lambda(1-\sigma)^2}{\lambda(1-\sigma)+(1-\lambda)\sigma}\right) \geq 1$, there exist a pooling equilibrium where entrepreneurs sell their risky securities ($B_p^H = H$ and $B_p^L = L$) directly to the investors. And the investors exert effort to select their securities.

Proposition 3 summarizes our findings in the discussion about direct issuing process. On one hand, a relatively high return in short term investment motivate the high type entrepreneurs to issue risky securities. On the other hand, a low cost of effort makes the high type entrepreneurs tend to issue their securities through direct market. In our model, the cost of effort is internalized into the initial price. Hence a decrease in cost of effort can also mean a lower cost for high type entrepreneurs to motivate the investors. As we have assumed at the beginning of the model, the signal acquired by investors is informative which means that high type entrepreneurs are more likely to sell their securities at their initial price than the low types. As a result, when the signal is sufficiently accurate, the high type entrepreneur may end up with a higher expected price of their securities than that from intermediaries.

Since we use backward induction to solve the sequential game in direct issuing process, it can be inferred that the equilibrium in Proposition 3 is sub-game perfect. The only step remains to be checked is whether the entrepreneurs the have the incentive to deviate their issuing channel. Suppose that a high type entrepreneur intends to deviate from the direct issuing market to the intermediary market. Similar to the discussion in the previous section, if the intermediaries offer a new contract that can make the High type entrepreneurs better-off, it will inevitably attract the Low type as well. Therefore, if the intermediary holds a belief that the deviating entrepreneurs are the High types and offers a price equal to H. The new contract will attract the Low types as well. Hence, the intermediaries will become loss-making. Besides, if the intermediaries hold a belief that the deviating entrepreneurs can be both High types and Low types, they will never offer a price higher than $\bar{P}$. When $C < \dot{C}$, we can have $\sigma\tilde{P} + (1-\sigma)P_b < \bar{P}$. In that case, the new deviating contract can never attract the High type but the Low type. Hence, the remaining belief is that the deviating entrepreneurs must be Low type. Hence, the intermediary will only offer a price equal to L for the deviating entrepreneurs and neither types can benefit from the deviation. In conclusion, no deviation in issuing channel can generate higher pay-off to either the high type or the low type, the original equilibrium can be sustained.

Proposition 4

When $\boldsymbol{\lambda < \frac{1}{\beta}}$, there exists a separating equilibrium where $\mathbf{B_H^H = B_H^L = B_L^L = L}$, and $\mathbf{B_L^H \neq L}$, and this separating equilibrium can be sustained with or without the intermediary.

Proposition 5

When $\boldsymbol{\lambda < \frac{1}{\beta}}$, there exists a pooling equilibrium where both types of entrepreneurs issue riskless debt $\mathbf{B_p^H = B_p^L = L}$, and this pooling equilibrium can be sustained with or without the intermediary.

The above two propositions illustrate the special case discussed in Leland and Pyle (1977). When the proportion of High type entrepreneurs and the return from the short-term investment is relatively low, High type entrepreneurs may find it costly to issue risky securities. In that case, High type entrepreneurs may choose securities which minimise mispricing in the market.

As we have mentioned in the proposition, this equilibrium can be sustained with or without intermediaries. Because in both cases, the market price of securities will be L for both types, which is equal to their true value. In that case, investors will never make any effort to identify the asset quality because the pay-out of the securities is independent of the type. Therefore, given that there is no risk of mispricing in the market, the choice of whether to introduce the intermediaries becomes irrelevant to the price of the asset.

7. Conclusion and Empirical Implications

In this paper, we discuss the optimal mechanism design under ex-ante asymmetric information. We show that the introduction of intermediaries can improve the efficiency and liquidity provision by allowing cross-subsidy. Additionally, we illustrate that the usage of cross-subsidy is more likely to be observed when the proportion of High type entrepreneurs is relatively large. Furthermore, our model shows that the existence of speculative investors may offer the intermediaries higher incentive to implement the separating menu. Since the separating menu provides a simple way to transparentize the quality of the asset, it guarantees the efficiency in the resell market and saves investors' cost of effort. In contrast to conventional models of intermediaries, we prove that the efficiency gain of intermediaries relies on the change of mechanism design and contract menu instead of on superior monitoring technology. We also show that intermediaries are more likely to be introduced into the securities issuing process when the cost of speculation is relatively high. Otherwise, entrepreneurs will decide to sell the securities directly to investors at pool price.

More interestingly, we derive the following empirical implications from the model which can be tested in future empirical research: Firstly, our model predicts that risky securities are more likely to be issued when the economic environment is good (a larger proportion of High type entrepreneurs and high return on short-term investment). When the economy is in the boom, we may even observe that the price of High (Low) type securities will increase (decrease) in the reselling process. Conversely, we may observe that only riskless securities will be issued in the market when the economic environment is less favorable, and there will be a lack of liquidity in the market. Secondly, entrepreneurs have a higher incentive to sell risky securities through intermediaries in an inefficient market, while in a mature market with a low cost of information, the introduction of intermediaries may become less beneficial to entrepreneurs.

Dynamically, our model also predicts that the recovery speed of intermediaries after depression will be faster for a less efficient market. This is because in a depressed market, both types of entrepreneurs may issue riskless securities in equilibrium and the introduction of intermediaries then becomes irrelevant. As a result, we may not observe any diversification in the behaviour of High type entrepreneurs in different markets. However, when the economy starts to recover, the existence of better investment opportunity will encourage entrepreneurs to issue riskier securities. At that time, the behaviour of entrepreneurs with high-quality assets begins to diversify. For High type entrepreneurs in an inefficient market, they will prefer to issue their risky securities through intermediaries. In the case of an efficient market, the High type entrepreneurs may feel more beneficial to issue directly to the investors. As a result, a higher growth rate of intermediary usage can be observed in a developing market than in a developed market.

Finally, some of the assumptions in our model can be modified to be included in a more general discussion. For example, in our model, we discuss only a market with multiple entrepreneurs and risky assets. However, to discuss the market of merger and acquisition, this assumption should be replaced by an assumption of unique entrepreneurs and risky assets with a certain probability to be the High type. Further, we also assume that the contracts between entrepreneurs and intermediaries are observable. In certain markets, this assumption may not be realistic. It may be appropriate to assume that the contract setting is observable while the choices of entrepreneurs are not. All these modifications can be looked on as an important future extension of our research.

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Appendix:

A 1: Separating equilibrium with investment bank and cross-subsidies

In this case, the utility function of high type entrepreneurs is:

$$U_H = \beta P_H + H - B_H^H,$$

And that of low type is:

$$U_L = \beta P_L + L - B_L^L.$$

The first term on the left-hand side is the return of the short term investment. The second term represents the output of entrepreneurs' risky asset.

By substituting the price function, the utility functions can be rewritten as:

$$\begin{aligned} U_H &= \beta P_H + H - B_H^H = \beta\left(B_H^H - S_H\right) + H - B_H^H \\ &= (\beta - 1)B_H^H - \beta S_H + H \end{aligned}$$

$$\begin{aligned} U_L &= \beta P_L + L - B_L^L = \beta\left(B_L^L - S_L\right) + L - B_L^L \\ &= (\beta - 1)B_L^L - \beta S_L + L \end{aligned}$$

Thereby the incentive comparability conditions can be inferred:

$$\begin{aligned} \beta\left(B_H^H - S_H\right) + H - B_H^H &\geq \beta\left(B_L^L - S_L\right) + H - B_L^H \\ \beta\left(B_L^L - S_L\right) + L - B_L^L &\geq \beta\left(B_H^H - S_H\right) + L - B_H^L \end{aligned}$$

By definition the payout cannot exceed the income of asset, which is defined as limited liability constraints:

:

$$\mathrm{B_L^L}, \mathrm{B_H^L} \leq \mathrm{L} \text{ and } \mathrm{B_H^H}, \mathrm{B_L^H} \leq \mathrm{H}$$

In any separating menu with cross-subsidies, the high types subsidize the low types. This lemma can be represented by a non-negative constraint of $\mathrm{S_H}$:

$$\mathrm{S_H} \geq 0$$

To further simplify the Incentive comparability constraint, we take the utility of the low type and drive the first order derivative on B_L^L:

$$U_L = (\beta - 1)B_L^L - \beta S_L + L$$
$$\Rightarrow \frac{\partial U_L}{\partial B_L^L} = \beta - 1 > 0$$

To maximize her utility, low type entrepreneurs would like to increase B_L^L as much as possible. The amount of payout is restricted by limit liability $B_L^L \leq L$

Therefore, we have

$$B_L^L = L$$

and

$$U_L = (\beta - 1)L - \beta S_L + L.$$

By substitution, the incentive comparability constraint can be simplified as:

$$\beta \mathrm{L} - \beta S_L \geq \beta\left(\mathrm{B_H^H} - \mathrm{S_H}\right) + \mathrm{L} - B_H^L$$

Intuitively, to increase the cost for low types to mimic, entrepreneurs of high type would like to increase B_H^L to its upper bound. Base on the quality of her own asset, high entrepreneur knows that her output will never be L. So higher payout commitment on Low state generates no real cost to the high type. Similarly, due to limit liability, high type can only set $B_H^L = L$.

Hence the incentive comparability condition becomes:

$$\beta L - \beta S_L \geq \beta\left(B_H^H - S_H\right)$$
$$\beta L - \beta S_L \geq \beta B_H^H - \beta S_H$$

Substituting $S_L = -\frac{\lambda}{1-\lambda} S_H$ into the inequity, we have

$$\beta L + \beta \frac{\lambda}{1-\lambda} S_H \geq \beta B_H^H - \beta S_H$$

$$\beta \frac{1}{1-\lambda} S_H \geq \beta(B_H^H - L)$$

$$\beta S_H \geq \beta(1-\lambda)(B_H^H - L)$$

$$S_H \geq (1-\lambda)(B_H^H - L)$$

As we have discussed in the main text, high type entrepreneurs offer cross-subsidies to prevent low types from mimic them. However, as the cost of doing so, the price of theirs securities becomes lower. Therefore, high type entrepreneurs are reluctant to offer any extra cross-subsidy that is unnecessary for signalling. In another word, the incentive comparability constraint should be binding:

$$S_H = (1-\lambda)(B_H^H - L)$$

By substituting the equation above, we further simplify the utility function of high types:

$$U_H = (\beta - 1)B_H^H - \beta S_H + H$$

$$U_H = (\beta - 1)B_H^H - \beta(1-\lambda)(B_H^H - L) + H$$

To solve for the optimal securities of high types, we drive the first order derivative of the utility function:

$$\frac{\partial U_H}{\partial B_H^H} = (\beta - 1) - \beta(1-\lambda) = \beta - 1 - \beta + \beta\lambda = \beta\lambda - 1$$

From the equation above, we can see that:

If $\beta\lambda - 1 < 0 \Rightarrow \lambda < \frac{1}{\beta}$ or $\beta < \frac{1}{\lambda}$ the Separating equilibrium without cross subsidy will dominate

If $\beta\lambda - 1 \geq 0 \Rightarrow \lambda \geq \frac{1}{\beta}$ or $\beta \geq \frac{1}{\lambda}$ the Separating equilibrium with cross subsidy will exist where $B_H^H = H$

In that case

$$U_H = (\beta - 1)H - \beta(1-\lambda)(H - L) + H$$

$$U_H = \beta[\lambda H + (1-\lambda)L]$$

In this case, both incentive constraints are binding and we end up with a separating equilibrium.

A 2: Separating equilibrium when investment bank is included while the cross-subsidy does not

As we mentioned above the utility function of both entrepreneurs are:

$$U_H = \beta P_H + H - B_H^H$$

$$U_L = \beta P_L + L - B_L^L$$

When $S_H, S_L = 0$, the price of securities of both types are $P_H = B_H^H$ and $P_L = B_L^L$ respectively, and the utility function of High and Low type entrepreneurs become:

$$U_H = (\beta - 1)B_H^H - \beta S_H + H$$

$$U_L = (\beta - 1)B_L^L - \beta S_L + L$$

The incentive comparability constraints of High type and Low type are:

$$(\beta - 1)B_H^H + H \geq \beta B_L^L + H - B_L^H$$

$$(\beta - 1)B_L^L + L \geq \beta B_H^H + L - B_H^L$$

The limited liability constraints are:

$$\mathrm{B_L^L}, \mathrm{B_H^L} \leq \mathrm{L} \text{ and } \mathrm{B_H^H}, \mathrm{B_L^H} \leq \mathrm{H}$$

By similar argument above, in separating equilibrium Low type will issue $\mathrm{B_L^L} = L$. High type entrepreneurs may prefer to set $B_H^L = L$ in order to minimize Low types' incentive to mimic.

Hence the incentive comparability constraint of Low type entrepreneurs can be simplified as:

$$(\beta - 1)L + L \geq \beta B_H^H + L - L$$
$$\beta \mathrm{L} \geq \beta B_H^H$$
$$\beta \mathrm{L} \geq \beta B_H^H$$

In separating equilibrium, this constraint is binding, so we have

$$\beta \mathrm{L} = \beta B_H^H \Rightarrow B_H^H = L$$

Then the utility of H type will be:

$$U_{HW} = (\beta - 1)L + H$$

In this equilibrium, the High type entrepreneur issues the riskless debt with face value L. As a result, the incentive comparability constraint of H type becomes:

$$(\beta - 1)\mathrm{L} + \mathrm{H} \geq \beta \mathrm{L} + \mathrm{H} - B_L^H$$

Obviously, by choosing a face value $B_L^H > L$, the above condition can be strictly satisfied. So the separating equilibrium can be sustained.

Chapter IV:

Clearinghouse and Liquidity Provision under Aggregate Uncertainty

1. Introduction

One interesting topic in financial academic after crisis is whether we should substitute the current OTC trading mechanism with a central clearinghouse in order to better cope with multiple uncertainties? More importantly, by reviewing last financial crisis, we find that the lack of counterparty's information, unexpected large exposure to systemic risk and sequentially default together drive the disaster in financial markets. Acharya and Basin (2014) have shown that the introduction of central clearinghouse can improve the transparence of the market which in return decreases the counterparty risk. While Duffie and Zhu (2011) point out that the introduction of a central counterparty may reduce the efficiency of netting and increase the inefficient social collateral. This paper attempts to show the importance of central clearinghouse in a different way. When contracts are allowed to be designed without any exogenous restriction, the introduction of central counterparty becomes a necessary condition to implement the optimal contract under aggregate uncertainty. The reason is that market participants may have the incentive to increase the scale of their contract which can lead to inefficient risk sharing.

We consider a general model where agents with different production technology insure each other by entering a future contract. In our model, agents of different types may face a state contingent shock in the middle of their investment procedure. At that time, they may demand extract liquidity to continue their investment. To hedge this risk and guarantee their investment to go through, agents may have the incentive to participate in a future contract with a counterparty who can provide liquidity in the state when they need it. By entering a future contract, agents can reduce their inefficient saving for self-insurance and increase the scale of their investment. However, we are going to show that three incentive conflicts may prevent the implementation of the first-best allocation. Firstly, agents can default at the cost of all their future income. Given that the scale of investment is private information of agents, they may consume all their endowment and default sequentially. Secondly, the number of contracts entered by a certain agent is unobservable to the others, which means agents may enter too many contracts that exceed their solvency and default strategically. Thirdly, the introduction of aggregate uncertainty is into our model may further distort the efficient allocation. This key difference distinguishes our work from Leitner's (2012). With aggregate uncertainty, we show that agents may have the motivation to squeeze out other agents with identical technology so as to have better insurance. Given the assumption that agents are allowed to design the contracts they use without any exogenous restrictions, this motivation will prevent the social optimal contract to be chosen. As a result, to implement the optimal contract we need the central clearinghouse to work as the unique counterparty in every trade and "standardize" the contract to be optimal.

To move one step further from previous researches such as Leitner (2012) and Acharya and Basin (2014) our paper shows that the interaction between aggregate uncertainty and incentive of defaulting will restrict the usage of the optimal contract. In their paper, they either assume no aggregate uncertainty (Leitner (2012)) or take the optimal contract as exogenously given (Acharya and Basin (2014)). As a result, they only focus on central clearinghouse's information revelation function, but neither of them successfully point out the necessity of counterparty substitution and contract standardization. In fact, if the market is completely transparent, there is no difference between central trading and separate trading in their model. However, in the following, we will show that both of their papers are just special cases in our generalized model. By relaxing the two conditions above we prove that central trading becomes crucial in the implementation of the social optimal contract and information transparency is no longer a sufficient condition to ensure market efficiency.

Besides, our model attempts to throw light on the relationship between systemic risk and social optimal contract. Our paper shows that a higher margin requirement is necessary to limit the risk exposure of a certain type of agents when aggregate uncertainty is high. Moreover, our model also shows that in the extreme case, the central clearinghouse may even have the incentive to provide liquidity to their counterparty in order to guarantee the trading volume of the market.

In practice, our model implies that a central clearinghouse is more likely to be set up in a market with high aggregate uncertainty (for example market of agriculture product). This implication is consistent with the history of Chicago Board of Trading, which start to use standardized future contracts to substitute forward contract in agriculture product market. In addition, our model also implies that fixing the margin requirement to unique and nonnegative to all types of agents may lead to an inefficient resource allocation and low level of social investment. Therefore, the settings of margin system should be more flexible to match the volatile market situation.

The rest of this paper proceeds as follows. In Section 2 we will briefly review the previous literature about the central counterparty. Section 3 discusses the basic setting and assumptions in our model. And as a comparison, the first best allocation will be illustrated in Section 4. Then, in Section 5, we will analysis the second best allocation when default is allowed. Moreover, we compare our mechanism with OTC bilateral trading mechanism in Section 6. Finally, we will come to the conclusion and empirical implications in Section 7.

2. Literature Review

In 1989, Bernanke empirically analyzed the role played by clearinghouses in the exchange of future market. He concluded that by substituting itself as a seller to every buyer and a buyer to every seller, clearinghouse becomes an official "party to every trade." By interposing itself in this way, the clearinghouse legally assumes the obligation of guaranteeing the execution of each trade to other clearing members. The main purpose is to enable investors to trade without concern about the creditworthiness of the individuals with whom they are dealing.

More importantly, in this paper, he also discusses the optimal method for clearinghouses to deal with different sources of risk. For idiosyncratic risk of agents, clearinghouse should standardize the future contract to efficiently promote the trading between agents and diversify this part of risk. However, for the systematic risk, it becomes impossible to cope with it completely. In that case, it might seem desirable to limit the exposure of the clearinghouse to large, systematic shock explicitly. Nevertheless, our model shows that the standardization of contracts is also of great help in dealing with systematic risk. By limiting the scale of contract, central clearinghouses effectively control the exposure of systematic risk of different agents and achieve a higher social investment.

Although the important characteristics have been showed by Bernanke empirically, the real role played by the central counterparty is still unclear in theoretical discussion. Some papers strongly suspect the introduction of central counterparty (Duffie and Zhu (2009) and Stephens and Thompson (2011)), while others highlight the potential efficiency improvement from CCP (Acharya and Basin (2012) and Carapella and Mill (2012)). Kroszner (1999) illustrates that three functions are essential for the clearinghouse to improve the efficiency of exchange: (1) the standardization of contracts, (2) construction of margin system and (3) introduction of a central counterparty. Although the importance of margin requirement has been long discussed by previous papers, seldom of them accurately point out the driving factors of it. This paper contributes to the literature on market microstructure design by incorporating the three functions above and highlights the role of margin system in dealing with aggregate uncertainty. In our setting, the asset/project returns are nonverifiable and contracts are unobservable (and nonexclusive). As a result, the agents' history of transactions is private information, and this encourages agents to sign contracts with multiple counterparties and subsequently default. To solve this problem, we need to introduce a mechanism to extract this information from agents. The key feature of our mechanism is that there is an intermediary who is the unique trading counterparty for all agents (traders). Since the intermediary is the unique counterparty which agents can trade with, it can extract information about agents' trading histories. Moreover, by setting contract specific margin requirement optimally, the clearinghouse can help to share the systemic risk more efficiently and increase the social investment. More interestingly, our model shows that in the extreme case, when the aggregate uncertainty is sufficiently high, the clearinghouse may have the incentive to set up a negative margin requirement to a certain type of asset owner. To be more precise, during the crisis, the central intermediary can encourage the social investment by collecting the margin from the majority and transferring it to the minority. Thus, our mechanism relaxes the incentive constraints of repayment and increases the investment to its efficient (second-best) level.

In fact, under the discussion about optimal clearing system, previous papers mainly focus on the information sharing role of the platform. (For example Leitner (2012), Bennardo, Pagano and Piccolo (2013) and Goldstein and Leitner (2013)). Amount them, the most closely-related paper to ours is Leitner (2012). In his framework, intermediaries can provide an easy way to monitor agents' positions by collecting agents' voluntary report. And he shows that by setting position limits and revealing the name of agents who hit the limits, intermediaries can effectively overcome the problem of multiple counterparties and subsequent default. However, Leitner's paper does not cover the situation where aggregate uncertainty is also introduced into the economy. In fact, when there is aggregate uncertainty, Leinter's unique optimal margin requirement may no longer fit different types of agents. Our model shows that margin requirement should be contract specific to cope with the systemic risk. And the contract relations between agents are not necessarily to be direct and one-to-one. In our model, we show that indirect and one-to-multiple contract relations can be helpful in dealing with different exposure of systemic risk. Because Leitner's model exogenously assumes that bargain power of different agents should be equal given identical proportions of different types. Bilateral trading and central clearing system can end up with the same result in Leitner's model. However, our model concludes that when the proportions of types become different, the choice of different trading mechanism can become essential. We prove that if agents are allowed to set up any one to one contract without restrictions, agents may tend to increase the "scale" of the contract and drive out partly of other agents from the market. All these incentives may boost up the inefficient social saving and lead to a lower aggregate investment. As shown in the following, in that case, a central clearinghouse in required to work as the unique counterparty to build up indirect contract relation between agents and limit the exposure of a certain type of agents by contract standardization.

Another related paper is Acharya and Basin (2014). In this paper, they focus on a market where agents are risk aversion and intend to share their risk as much as possible. Similar to Leitner's and our settings, agents have the incentive to default with their trading position unobservable. They show that a lack of position transparency can lead to counterparty risk externalities. This means that the market may end up with too many short positions that collect premium upfront but default ex-post, which in return causes inefficient level of risk-sharing, deadweight cost of bankruptcy, and productive inefficiency. To cope with this inefficiency problem, Acharya and Basin (2014) suggest a centralized clearing mechanism that provides transparency of trade positions or a centralized counterparty that observes all trades and set price competitively. However, in their model, asset and contracts are exogenously designed, and the central clearinghouse is just characterized by its information incorporation function. In another word, any other mechanisms with information transparency can achieve the same result in their settings. (for example Leitner's volunteer reporting mechanism). Hence, their model still did not show the key differences between central "trading" and central "information revealing". In fact, their model does not point out the necessity of contract standardization and counterparty substitution. Distinguishes from their paper, contract design is endogenouslized as a part of the game in our model. And we are going to show that contract standardization and counterparty substitution can play essential roles in restricting the risk exposure of a single agent and allowing all the agents to share their risk more efficiently. Therefore, we are going to show that information transparency is not sufficient to guarantee efficiency and central clearinghouse can do more by working as market participants instead of only an information recorders. Besides, in their model agents' future income is given by exogenous endowment which can become observable to the central clearinghouse, while in our model, agent's future income depends on his current unobservable investment level. This assumption allows us to explore the relationship between the aggregate uncertainty and social investment. More importantly, it allows us to compare the advantages and disadvantages between the OTC market and central clearing market. As a result, we find that the central counterparty may have the incentive to provide liquidity to the minority when systemic risk is severe. In fact, by collecting margin from a certain type of agents and lend this fund to their counterparties, clearing house can improve the insurance capacity of the whole system and encourage efficient investment.

As for the discussion about optimal clearing system under aggregate risk, Biais, Heider and Hoerova (2012 and 2015) also give their comparison between central clearing and OTC market. In the paper published in 2012, they suggest that CCP can improve the efficiency when dealing with idiosyncratic risk. By the advantage of risk mutualization, CCP can offer complete insurance to protection buyers and end up with a first best allocation. However, CCP is no longer optimal, when aggregate risk is introduced and the effort of counterparty searching becomes unobservable. Because the full insurance offered by CCP can undermine the incentive for protection buyer to search for a better counterparty, which in return increases the defaulting rate and affects the insurance against systemic risk. Therefore, OTC could dominate CCP by retaining the searching incentive of protection buyer when the systemic risk becomes significant. This result is consistent with Arnold (2014), which suggests that the insurance offered by the CCP may undermine the incentive for banks to screen their counterparty. To make it even worse, the standardized contract traded in CCP may make it impossible to signal the loan quality by imposing different contracts. However, in the recent working paper of Biais, Heider and Hoerova (2015), they construct another model where the protection seller can control their risk exposure after they obtain a signal about their counterparty's future income. They show that when the bad signal arrives, the protection seller may have a higher incentive to increase their risk exposure since the contract is looked on as a liability to the sellers. Thus, they may intend to shift their risk to the buyers. In that case, they prove that margin call from CCP after bad signal can improve sellers' incentive and enhance the ability to share risk. Besides, the introduction of CCP also decreases the margin requirement through risk mutualization. The key difference between our paper and theirs is that we put more attention to the interaction between systemic risk and default. In their models, risk exposure and defaulting are separating to protection buyers and protection sellers correspondingly, while all the trading agents are facing the risk and can choose to default at the same time in our model. In another word, we are applying a hedging setting instead of a convention insurance one as Biais, Heider and Hoerova (2012 and 2015). Moreover, in their model, protection sellers are excluded from the aggregate risk and their probability of defaulting depends only on their type. While in our setting, aggregate risk can impact all the trading agents and it could be impossible for any agents to insure the others fully. With this assumption, we are going to show that by applying the optimal contract, CCP can share the risk more widely and efficiently than the OTC market.

Besides, Duffie and Zhu (2009) suggests that adding an independent central clearing platform (CCP) to each derivative market may reduce the netting efficiency, which in return increase agents' average exposure to counterparty risk. The intuition is that when agent trade with CCP, they will not be able to benefit from bilateral netting between a pair of counterparties across different underlying assets. As a result, CCP will require extra margin for each kind of derivative products, which may affect the efficiency of resource allocation and increase the probability of defaulting. Basing on this idea, Duffie and Zhu point out that netting across a large number of products is necessary for a central counterparty to reduce counterparty risk in a non-transparent market setting. Different from their framework, in our model, we consider an economy with only one source of risk the and one financial asset. And since we endogenize agents' incentive of defaulting in this environment, our model concludes that the introduction of central clearing counterparty can effectively reduce the margin requirement for each contract and improve social welfare.

Our paper also contributes to the recent discussion about mechanism efficiency under aggregate uncertainty. Menkveld (2016) construct a model which endogenize the existence of "arbitrageurs," who benefits from the leverage embed in the margin system, and "standby investors," who benefits from the extremely bad state when the CCP need to unwind the position it inherits from the arbitrageurs in default. He concludes that by offering fire sale premium and building up a default fund, the CCP can balance the proportion of two types of traders and achieve social efficiency. In addition, Acharya, Iyer and Sundaram (2015) point out that the introduction of hedging contract decreases agents' necessary saving for self-insurance, which helps agents to cope with the idiosyncratic risk at lower cost. However, it also decreases their resistance to the systemic risk when the correlated failure takes place. Hence they prove that by imposing a margin requirement and ex-post transfer, the CCP can improve social efficiency and increase aggregate investment. Nevertheless, in their papers, the probability of default is exogenous and depends only on the risk of projects and their correlation. In comparison, our model incorporates the incentive of strategic default and focus on the role played by CCP in coping with systemic risk and avoiding market failure.

Finally, as an extension, our centralized trading mechanism is also helpful to cope with other trading frictions: For example, the matching cost. (Duffie, Garleanu and Pedersen, 2005). Since the intermediary is the unique counterparty agents can trade with, all the potential seller and buyer can contact the intermediary directly instead of matching in the market. In another aspect, it could also be helpful to accelerate the payout from clearinghouses to creditors when trading firm fail in the financial crisis

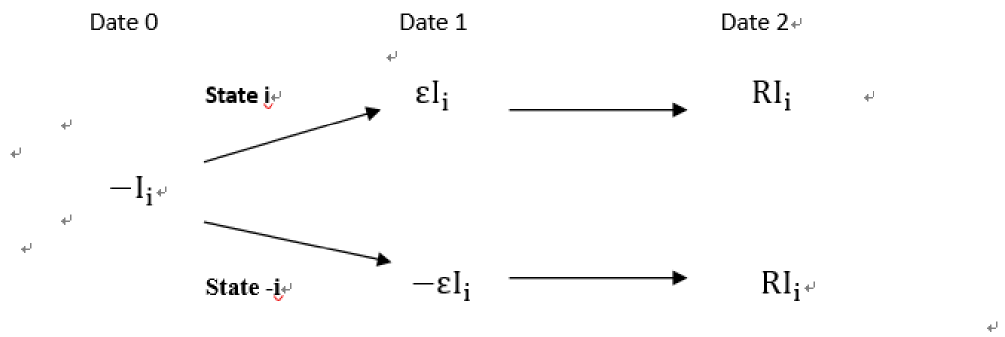
(Squire (2012)). Through netting, clearinghouse provides a quicker intermediate payout to creditors which increase the liquidity provision and decreases uncertainty. However, our model does not incorporate this kind of effect, since we do not consider about the discount factor and agents are assumed to be risk neutral.

3. Game Structure and Basic Assumption

The model describes a market where a continuum of agents enter bilateral hedging contracts for mutual insurance purposes. More specifically, there are two types of agents (Type-1 and Type-2) who may have different income base on the realization of state in date 1. Different from Leitner's model where the contract is a Type-1 agent and a Type-2 agent, in our model we have a central intermediary who works as the unique counterparty to trade with both types of agents. In fact, we find that this mechanism can achieve higher social efficiency than Leitner's mechanism under the condition of aggregate uncertainty. And we will prove this finding in the following section. Agents and the intermediary enter the contract at Date 0, in which they specify payment at Date 1 contingent on the realization of the state. An agent can default strategically, but the cost of doing so will be all her future income. In Leither's model, they assume that contracts are non-exclusive and agents cannot observe the historical and future contracts of their counterparties. In that case, agents may have the incentive to enter multiple contracts and default strategically. To cope with this problem, Leither introduces a central institution which can extract all relevant information about contracts that agents enter by inducing them to report one another. However, in our setting, the intermediary is the only counterparty that agents can trade with, so the it can solve this problem by simply collecting the identity information from the agents.

Besides the trading mechanism, we also generalize Leitner's model by allowing the existence of aggregate uncertainty. That is, instead of assuming equal proportions of both agents, we assume that the proportion of one type of agent is larger than that of the other type. In that case, it is impossible for both types of agents to be completely insured. Without losing the generality, we assume that the proportion of type 1 "p" is larger than the proportion of type 2 "$1-p$", or $p \geq \frac{1}{2}$. Then the remaining settings will be similar to Leitner's model: There are three dates, t=0, 1, 2, and one divisible good -- cash. Uncertainty sources from the random realization of future states, State 1 and State 2, one of which will realize at Date 1. Agents are risk neutral and obtain an expect utility of $\mathrm{E}(c_0 + c_1 + c_2)$ from consuming $c_0, c_1,$ and c_2 dollars at Dates 0, 1, and 2, respectively. Agents are protected by limited liability, so $c_t \geq 0$ at each date.

At Date 0, agents are endowed with one unit of cash and an investment opportunity. Moreover, this investment opportunity requires the human capital of the agent. In another word, taking over the project from the agents will destroy all the future value of the project. Each project lasts for two periods, and the corresponding cash flow can be summarized in the following figure.



Take the project of type 1 as an example: At Date 0, type 1 agent chooses the scale of her investment $I_1 \in [0,1]$ which is unobservable to the intermediaries and other agents. When it comes to Date 1, if state 1 realizes, the project will yield εI_1 ($\varepsilon > 0$) and continue to Date 2 automatically. Otherwise, if state 2 realize, the project requires an additional investment εI_1 to continue. If the additional investment is not paid in full, the project will be forced to liquidate, and the liquidation value of the project is assumed to be 0. If the project continues to maturity, it yields RI_1 dollars at Date 2. With similar structure, the project of a Type 2 agent yields εI_2 in State2 but requires εI_2 in State 1. If the project continues to maturity, it yields RI_2 dollars at Date 2.

We normalize the risk-free rate to be 0% and assume that $R > 1 > \varepsilon$. From the game structure mentioned above, we can see that $R > \varepsilon$ guarantees the efficiency of making additional investment in Date 1. While $R > 1$ implies that each type of the projects has a strictly positive NPV in a world without friction. And finally, $\varepsilon < 1$ ensures the satisfaction of rational constrain.

Following Leitner's structure, we also have the following assumptions:

Assumption 1. An agent cannot commit to paying out of the project's final cash flows (RI_i).

Assumption 2. An agent cannot commit to paying out of the project's interim cash flows (εI_i)

Assumption 1 excludes the possibility that agents self-finance for her project by selling part of her future income in Date 2. Without this assumption, agents can satisfy their liquidity demand by simply securitize their future claims. In that case, there is little incentive to entry the insurance contract. Assumption 2 allows agents to default in Date 1 even when she has the fund to continue. If an agent defaults, her project will be terminated, which means that the agent will lose all her future income. Moreover, since the project requires agents' human capital, intermediary or other agents will not accept the projects' asset as collateral, instead only cash is acceptable as collateral. Specifically, agents and intermediary can open an escrow account through a third party so that money placed in escrow account can be observed by both the agents and intermediary.

Assumption 3. The amount that an agent invests in his project (I_i) and the amount that an agent consumes are private information.

Assumption 3 introduces the risk of strategic default: agents who intend to default can consume all her cash and invest 0 to the project. Without this assumption, agents can simply avoid the counterparty risk by setting up a minimum investment level. Due to the post efficiency constraint $R > \varepsilon$, increasing initial investment can strengthen the motivation for agents to follow their commitment. However, under Assumption 3, cash collateral become necessary to control the potential counterparty risk even when contracts are exclusive.

Assumption 4. An agent cannot observe contracts that other pairs of agents enter (either in the past or the future)

Since all the past and future contracts are not observable to the current counterparty, agents may have the incentive to enter multiple contracts and default strategically. Specifically, agents can consume all their endowment in Date 0 ($I_i = 0$), and enter multiple contracts. As a result, when the "bad" state (state –i) realizes they get payment from their counterparty or the intermediary. While the good (state i) realizes, they default and pay nothing. In other word, they benefit from the "bad" state by entering multiple contracts and substituting their investment with consumption. To prevent the implementation of this strategy, we need a supervision mechanism which effectively reveals the trading history information. As mentioned at the beginning of this section, in Leitner's mechanism, he allows agents to trade directly to each other and sets up a central intermediary who collects and reveals the voluntary trading reports from agents. Due to the potential threats of default form their counterparties, agents have sufficient incentive to report truthfully. In a word, he constructs a "peel monitoring" system to cope with this problem. Different from his mechanism, we introduce a central intermediary to solve the problem above. As we mentioned above, central intermediary works as the only counterparty which agents can trade with. Hence, she has an access to the trading history of all the agents and can simply detect agents who attempts to enter overmuch contracts. In the case without aggregate uncertainty, both mechanisms can solve the problem properly. However, we will show in the following that our mechanism will become superior if aggregate uncertainty is introduced.

Utility Function of Agents

In order to compare the social efficiency between two mechanisms, we firstly assume that there is a social planner who designs the optimal contract to maximize the social welfare. In fact, maximizing the social efficiency is equivalent to maximizing the ex-ante expect utility of the agents. Therefore, the most efficiency mechanism can still be endogenized even in the case without a social planner. For a contract ψ, it illustrates

(k_1, k_2, x_1, x_2) where k_i denotes the margin of agent i and x_i denotes the payout for agent i when state i realizes. Besides, the intermediary is allowed to set different position limits to different types of agents depending on their potential solvency. Think of the case when the intermediary allows the type-1 agents to take" α" contracts and allows the type-2 agents to take "β" contracts. If the incentive constrains are satisfied, no default will actually take place and the utility function of two types of agents will be:

$$U_1(\psi) = 1 - I_1 - \alpha k_1 + \frac{1}{2}(\alpha k_1 + \varepsilon I_1 - \alpha x_1) + \frac{1}{2}(\alpha k_1 + \alpha x_2 - \varepsilon I_1) + RI_1$$
$$= 1 + (R-1)I_1 + \frac{1}{2}\alpha(x_2 - x_1)$$
$$U_2(\psi) = 1 - I_2 - \beta k_2 + \frac{1}{2}(\beta k_2 + \varepsilon I_2 - \beta x_2) + \frac{1}{2}(\beta k_2 + \beta x_1 - \varepsilon I_2) + RI_2$$
$$= 1 + (R-1)I_2 + \frac{1}{2}\beta(x_1 - x_2)$$

Take type 1 agents as an example, $1 - I_1 - \alpha k_1$ denotes the consumption in Date 0 after the investment and margin requirement. Term $\alpha k_1 + \varepsilon I_1 - \alpha x_1$ illustrates the consumption of type 1 agent if state 1 realizes and agent 1 fulfills her commitment. In that case, she firstly received fund from her investment εI_1. But base on the contract, she will pay x_1 for each contract she holds which multiplied by the number of contracts α. After the payment, the margin at Date 0, αk_1 will be returned to agent 1. While the term $\alpha k_1 + \alpha x_2 - \varepsilon I_1$ stands for the consumption of agent 1 in state 2. When state 2 realize, agent 1 is required to make additional investment εI_1 to continue the project. Depending on the contract, she will receive the fund αx_2 from the intermediary. And the margin αk_1 will also be returned to the agent, since it will be irrational to default when she can benefit from the contract. Finally, RI_1 represents the final payoff from the investment if the project is successfully maintained to Date 2. Similarly, agent 2 consumes $1 - I_2 - \beta k_2$ in Date 0, $\beta k_2 + \varepsilon I_2 - \beta x_2$ in Date 1 if state 2 realizes and $\beta k_2 + \beta x_1 - \varepsilon I_2$ if state 1 realizes.

Since all agents are protected by limited liability, regardless of date and states, their consumption must be non-negative:

$$\begin{cases} 1 - I_1 - \alpha k_1 \geq 0 \\ \alpha k_1 + \varepsilon I_1 - \alpha x_1 \geq 0 \\ \alpha k_1 + \alpha x_2 - \varepsilon I_1 \geq 0 \end{cases} \text{ and } \begin{cases} 1 - I_2 - \beta k_2 \geq 0 \\ \beta k_2 + \varepsilon I_2 - \beta x_2 \geq 0 \\ \beta k_2 + \beta x_1 - \varepsilon I_2 \geq 0 \end{cases}$$

To make sure that agents will have the incentive to enter the insurance contract offered by the intermediary, we need to analysis agents' outside option as well. Alternatively, agents who do not enter the contract can maximize their utility by self-insurance. To be more specifically, agents can save partly of their endowment in Date 0 and use the money to satisfy their liquidity demand in Date 1 (autarky). In that case, we denote the saving of the agents by "s", and the optimal allocation will be:

$$\begin{matrix} s = 1 - I \\ s = \varepsilon I \end{matrix} \Rightarrow I = \frac{1}{\varepsilon + 1} \Rightarrow U_A = s + RI = \frac{R + \varepsilon}{1 + \varepsilon}$$

Therefore, in order to attract agents to enter the insurance contract, the social planner must make sure that the rational constraint will be satisfied. In another word, by entering the contract, agents must become better-off than autarky:

$$U_i(\psi) \geq U_A = \frac{R + \varepsilon}{1 + \varepsilon}$$

Finally, as we have mentioned above, in our model we allow for aggregate uncertainty by put different proportion to different types of agents. In that case, it is impossible to offer complete insurance to all the majority agents (type-1 agents in our model). Given the continuity and monotony of the utility function, the optimal allocation can be either offering incomplete insurance to all the type-1 agents or offering complete insurance to part of the type-1 agents (complete insurance with quota) and letting the remaining type-1 agents end up with autarky (ration). We will compare the equilibrium allocation under these two different cases in the following, to see which of them can achieve higher social efficiency.

4. First Best Allocation without Potential Default

Before we start our discussion, to set up a benchmark of efficiency, we firstly solve the case where both the investment level and the number of contracts agents enter are observable (First Best). In that case, there is no potential default risk, and we have the following proposition:

Proposition 1. When the investment level and the number of contracts agents enter are observable, offering incomplete insurance to the entire majority or offering complete insurance to part of the majority can both achieve efficiency.

The proof will be in the following. We will start from the all-participating case and then come to the complete insurance with quota case.

All-Participating Contract

In the case where the intermediary chooses to offer incomplete insurance to all the majority agents, the intermediary must make sure that she will have enough funds to fulfill her commitment. In state 1, the intermediary will collect αx_1 from each type 1 agent and pay βx_1 to each type 2 agent. Whereas in state 2, the intermediary will get βx_2 from each type 2 agent and pay αx_2 to each type 1 agent. To ensure that the cash inflow and outflow are able to break even, intermediary will adjust the position limit"α" and "β" depending on the proportion of agents "p" and "$1-\mathrm{p}$":

$$\begin{matrix} \mathrm{p}\alpha x_1 - (1-p)\beta x_1 = 0 \\ (1-\mathrm{p})\beta x_2 - p\alpha x_2 = 0 \end{matrix} \Rightarrow p\alpha = (1-p)\beta \Rightarrow \beta = \frac{p}{1-p}\alpha$$

Moreover, as we mentioned above we assume that there are more type 1 agents than type 2 agents:

$$\mathrm{p} > 1/2 \Rightarrow \beta > \alpha$$

And to simplify our result, without losing generality, we normalize $\alpha = 1$, so:

$$\beta = \frac{p}{1-p}$$

Base on the assumptions above, we can acquire the first best allocation of the model by solving the following:

$$\max: pU_1(\psi) + (1-p)U_2(\psi)$$

Subject to:

$$\begin{cases} 1 - I_1 - k_1 \geq 0 & \dots \dots (1) \\ k_1 + \varepsilon I_1 - x_1 \geq 0 & \dots \dots (2) \\ k_1 + x_2 - \varepsilon I_1 \geq 0 & \dots \dots (3) \\ 1 - I_2 - \beta k_2 \geq 0 & \dots \dots (4) \\ \beta k_2 + \varepsilon I_2 - \beta x_2 \geq 0 & \dots \dots (5) \\ \beta k_2 + \beta x_1 - \varepsilon I_2 \geq 0 & \dots \dots (6) \end{cases}$$

In another word, the social planner's objective is to maximize the social welfare subject to the limited liability constraints. By substituting the utility function into the object function above, we have:

$$\begin{aligned} &pU_1(\psi) + (1-p)U_2(\psi) \\ &= p + p(R-1)I_1 + \frac{1}{2}p(x_2 - x_1) + (1-p)(R-1)I_2 + \frac{1}{2}\beta(1-p)(x_1 - x_2) \\ &= 1 + (R-1)[pI_1 + (1-p)I_2] \end{aligned}$$

From the equation above, we can easily see that the social welfare $pU_1(\psi) + (1-p)U_2(\psi)$ is strongly related to the social investment $pI_1 + (1-p)I_2$. Since $R > 1$, to maximize the social welfare is equivalent to maximize the social investment. With the same reason, the constraint (1) and (4) should be binding. Thus:

$$I_1 = 1 - k_1 \text{ and } I_2 = 1 - \frac{p}{1-p}k_2$$

If we substitute them back into the objective function, we will get:

$$
\begin{aligned}
&1 + (R-1)[pI_1 + (1-p)I_2] \\
&= 1 + (R-1)[p - pk_1 + (1-p) - pk_2] \\
&= 1 + (R-1)[1 - p(k_1 + k_2)]
\end{aligned}
$$

As we can see from the equation above, the most efficient mechanism must the one which minimize the aggregate collateral (margin requirement) of the society. Since the NPV of the project is positive, it will be more efficient for us to substitute one unit of margin with one unit of investment.

As I mentioned above, under aggregate uncertainty it is impossible to offer complete insurance to both types of agents especially the majority (type-1 agent). However, in order to maximize the social investment, complete insurance should be offered to the minority (type-2 agent). In that case, we should imply that (5), (6) and (3) should be binding while (2) is not. As a result, the optimal contract will be:(see Appendix A1)

$$
\begin{cases}
x_{FI1} = x_{FI2} = \dfrac{\varepsilon}{\beta} \\
k_{FI2} = 0 \\
k_{FI1} = 1 - \dfrac{1}{(1+\varepsilon)} - \dfrac{(1-p)\varepsilon}{p(1+\varepsilon)}
\end{cases}
$$

With this optimal contract, the investment of type 1and type 2 will be:

$$
I_{FI1} = \frac{1}{1+\varepsilon} + \frac{(1-p)\varepsilon I_2}{p(1+\varepsilon)} \ and \ I_{FI2} = 1
$$

The utility of type 1and type 2 will be:

$$
U_{FI1}(\psi) = 1 + (R-1)I_{FI1} = 1 + \frac{R-1}{\varepsilon+1} + \frac{(1-p)(R-1)\varepsilon}{P(1+\varepsilon)}
$$

$$
U_{FI2}(\psi) = 1 + (R-1)\times 1 = R
$$

And the social welfare:

$$pU_{FI1}(\psi) + (1-p)U_{FI2}(\psi) = p + \frac{(R-1)p}{\varepsilon+1} + \frac{(1-p)(R-1)\varepsilon}{p(1+\varepsilon)} + (1-p)R$$

From the result above, we can see that the margin requirement of the minority (type-2) is 0 and she can make a full investment $I_{FI2} = 1$, under the protection of complete insurance. It may be meaningful to find that the margin of type-1 agent is not 0. However, the margin of agent 1 here does not play any role in preventing agent from default. Since both the investment and the number of contracts agents enter are observable, there is no incentive for type-1 agent to do that. In fact, the margin k_{FI1}here just works as "saving" which is similar to the autarky case. Because, when state 2 realizes, it is impossible to satisfy the liquidity demand of agent-1 only with the money from agent-2, the remaining must be complemented by the saving of agent-1 themselves. In order word, intermediary requires agents 1 to save partly of her endowment so as to cope with the aggregate uncertainty. And due to this saving requirement, the investment of type-1 agents is less than optimal level 1. Moreover, we can see that if there is no aggregate uncertainty ($1-p = p\ and\ \beta = 1$), Leitner's most efficient case where both types of agents can make full investment is achievable.

One-to-One Complete Insurance with Quota Case

In this case, to offer complete contract, the intermediary will set up an indirect one-to-one insurance relationship between agents of different kinds. However, because there are more type-1 agents than type-2 agents, the remaining type-1 agents who are not offered the contract will have to end up with autarky. In detail, $1-p$ of the type-1 agent will be offered with the complete insurance contract while the remaining $2\text{p}-1$ type-1 agent will have to satisfy their own liquidity demand by self-insurance.

For agents who are offered with quota, the optimal contract will be:

$$\alpha = \beta = 1$$
$$k_{FC1} = k_{FC2} = 0$$
$$I_{FC1} = I_{FC2} = 1$$
$$x_{FC1} = x_{FC2} = \varepsilon$$

Intuitively, since these agents are offered with a complete insurance contract, their future liquidity demand will be satisfied fully by the payment from the intermediary. Therefore, both types of agents will make a full investment to maximize their utility. Besides, the margin of both types of agents will be 0 because there is no incentive for agents to default under this setting.

As a result, the utility of agents with complete insurance contract will be:

$$U_{FC1}(\psi') = U_{FC2}(\psi') = 1 + (R-1)\times 1 = R$$

So the social welfare will be:

$$(1-p)U_1(\psi') + (1-p)U_2(\psi') + (2p-1)U_A$$

$$= p + \frac{(R-1)p}{\varepsilon+1} + \frac{(1-p)(R-1)\varepsilon}{p(1+\varepsilon)} + (1-p)R$$

Comparing this result with the previous case, we can easily find that the social welfares of two different contracts are identical. Hence, Proposition 1 is proved.

Finally, it is important to point out the relationship between this complete insurance with quota case and Leinter's mechanism. In this specific case, the optimal contract is identical to Leinter's. Different from our mechanism, Leintner allows agents to trade directly with each other while intermediary is only delegated to collect and monitor the voluntary report from them. As a result, under his mechanism, for a contract to be supported as equilibrium, it must be collusion-proof. And Leinter also proves that the complete insurance contract is the unique optimal collusion-proof contract. Therefore, the optimal partial insurance contract which we get in the first case is not implementable in Leinter's mechanism. To see this, we can compare the utility of counterparties who enter the different contracts:

$$U_{FI1}(\psi) = 1 + \frac{R-1}{\varepsilon+1} + \frac{(1-p)(R-1)\varepsilon}{P(1+\varepsilon)} < U_{FC1}(\psi') = R$$

$$U_{FI2}(\psi) = U_{FC2}(\psi') = R$$

In this case, it means each type-1 agent may have the incentive to form a one-to-one collusion by offering a more attractive contract to agent 2 and kicking out other type-1 agents. In another word, the incomplete insurance contract is not collusion prevented. Hence, this contract is implementable only in our mechanism where the intermediary works as the unique counterparty.

More importantly, since Leinter's mechanism requires the contract to be collusion-proof, the complete insurance contract will still be their optimal even with under aggregate uncertainty. However, the efficiency of this bilateral-complete-insurance contract is guaranteed by the assumption of equal proportion and equal bargaining power between counterparty, which is not realistic when there are more type-1 agents than type-2 agents. Obviously, if the propositions of two types of agents are not equal, the minority should have higher bargaining power than the majority due to their scarcity. In the following section, we are going to show that the effect of this imbalance of scarcity could be magnified when incentive constraints are introduced into the model. And the trading mechanism with central intermediary will show her superiority under this environment.

5. Second Best Allocation Allowing Potential Default

In this section, we start our discussion for the second best allocation by introducing the participation constraints (IR_i) and the incentive constraints (IC_i). Since we have solved the outside option for the agents, the participation constraints can simply be written as:

$$U_i(\psi) \geq U_A = \frac{R + \varepsilon}{1 + \varepsilon}$$

As we have mentioned in the game settings, the level of investment I_i is private information of the agent. Therefore, an agent can substitute her investment with consumption and subsequently default at Date 1 as she has no cash flows to pay from.

Hence, the incentive constraints can be described as:

$$U_i(I_i|\psi) \geq U_i(I'_i|\psi), for\ every\ i \in [0, k_i]$$

Simply, this inequity means that the agents will choose an investment level which maximizes her utility. Thus, an optimal contract should be set up so that any deviation from the equilibrium strategy cannot be beneficial. Proved by Leinter, this constraint can be replaced with $U_i(I_i|\psi) \geq U_i(0|\psi)$. In other words, it is enough to focus on the case of full investment substitution due to the linearity of the utility function. Intuitively, an agent who plans to default is better off consuming all her initial endowment rather than investing it and losing it upon default.

Hence, the incentive constraints reduce to:

$$\frac{1}{2}(\mathrm{x}_1 - \mathrm{k}_1) \leq (\mathrm{R} - 1)\mathrm{I}_1 + \frac{1}{2}(\mathrm{x}_2 - \mathrm{x}_1)$$

$$\frac{\beta}{2}(\mathrm{x}_2 - \mathrm{k}_2) \leq (\mathrm{R} - 1)\mathrm{I}_2 + \frac{\beta}{2}(\mathrm{x}_1 - \mathrm{x}_2)$$

With this incentive constraint, we firstly get the following lemma:

Lemma 1: For an agent who is offered an insurance contract and does not plan to default, she can fully substitute her private saving with her collateral.

Comparing the functions of margin and saving in our model, we can see that both methods can transfer fund from period 0 to period 1. The only difference is that margin will be returned to agents if and only if agents fulfill their commitments. From the incentive constraints above, we can see that higher margin requirement (k_1 and k_2) can strengthen the incentive for agents to follow the contract. As a result, a higher margin requirement can support a contract with higher insurance coverage which in return increases the investment of agents. Therefore, for agents who intend to fulfill their contracts, they will prefer to “save” fund using the margin account instead of the private saving one. Besides, given this incentive, by intuitive criterion, agents with private saving can be expected to be the ones who are going to default in Date 1 because they are the only ones who can benefit from this deviation. Doubtlessly, agents of this kind will not be offered an insurance contract by the central counterparty.

With the Lemma 1 above, we will discuss the social welfare under different contract settings: all-participating contracts and complete insurance with quota. And we will find the following result:

Proposition 2. When $R \geq 1 + \frac{1}{2}\varepsilon$, the incentive constraints will not be binding. Both all-participating contracts and complete insurance contract with quota can achieve the first social welfare.

Proposition 3. When $R < 1 + \frac{1}{2}\varepsilon$, incentive constraints of Type 2 agents will become binding. The all-participating contracts will become more efficient than complete insurance contract with quota.

Proposition 2 is relatively straight forward. When the final return of the project is sufficiently high, the opportunity cost of defaulting will become high enough to prevent agents from doing so. Since agents who default in Date 1 will lose all her future income in Date 2, they will find it more beneficial to follow their commitment. As a result, the incentive constraints play no role in the maximization problem. And the result will be identical to what we get in Proposition 1.

In comparison, Proposition 3 is more interesting and complicated. It describes a case where the final return itself become no longer sufficient to guarantee the contract fulfillment. As a result, a margin requirement will be imposed to agents to control the risk of defaulting. And we will have to discuss it case by case:

- **All-Participating Case**

In the case where the intermediary offers all-participating contract, the maximization problem will become:

$$\max: pU_1(\psi) + (1-p)U_2(\psi)$$

Subject to:

$$\begin{cases} 1 - I_1 - k_1 \geq 0 \ldots\ldots ① \\ k_1 + \varepsilon I_1 - \alpha x_1 \geq 0 \ldots\ldots ② \\ k_1 + \alpha x_2 - \varepsilon I_1 \geq 0 \ldots\ldots ③ \end{cases} \text{ and } \begin{cases} 1 - I_2 - \beta k_2 \geq 0 \ldots\ldots ④ \\ \beta k_2 + \varepsilon I_2 - \beta x_2 \geq 0 \ldots\ldots ⑤ \\ \beta k_2 + \beta x_1 - \varepsilon I_2 \geq 0 \ldots\ldots ⑥ \end{cases}$$

$$U_1(\psi) \geq U_A = \frac{R+\varepsilon}{1+\varepsilon} \ldots\ldots ⑦$$

$$U_2(\psi) \geq U_A = \frac{R+\varepsilon}{1+\varepsilon} \ldots\ldots ⑧$$

$$\frac{1}{2}(x_1 - k_1) \leq (R-1)I_1 + \frac{1}{2}(x_2 - x_1) \ldots\ldots ⑨$$

$$\frac{\beta}{2}(x_2 - k_2) \leq (R-1)I_2 + \frac{\beta}{2}(x_1 - x_2) \ldots\ldots ⑩$$

By solving the maximization problem above given $R < 1 + \frac{1}{2}\varepsilon$, we find that two different sub-cases can arise.

Proposition 4. When $R < 1 + \frac{1}{2}\varepsilon$ and $\beta > \frac{8(1+\varepsilon)\varepsilon}{(2R-\varepsilon)(2R-2+\varepsilon)} - \frac{2R+3\varepsilon}{2R-\varepsilon}$, the optimal all-participating contract will exhaust the liquidity provided by Type 2 agents and only the incentive constraint of Type 2 agents will become binding.

Proposition 5. When $R < 1 + \frac{1}{2}\varepsilon$ and $\beta \leq \frac{8(1+\varepsilon)\varepsilon}{(2R-\varepsilon)(2R-2+\varepsilon)} - \frac{2R+3\varepsilon}{2R-\varepsilon}$, both incentive constraints will become binding in the optimal all-participating contract, and there will be liquidity surplus for Type 2 agents in state 2.

Case I

As illustrated in Proposition 4, the first case arises when constraint ⑤ is binding while constraint ⑨ is not. It means that when state 2 arises, the liquidity provided by type 2 agents will be exhausted in state 2. In that case, the optimal contract will be:

$$x_1 = \frac{2 + 3\varepsilon - 2R}{\beta(2R - \varepsilon)}$$

$$x_2 = \frac{2R - 2 + \varepsilon}{\beta(2R - \varepsilon)}$$

$$I_1 = \frac{2R + 2\beta R + \varepsilon - \beta\varepsilon - 2}{\beta(2R - \varepsilon)(1 + \varepsilon)}$$

$$I_2 = \frac{2}{2R - \varepsilon}$$

$$k_1 = 1 - \frac{2R + 2\beta R + \varepsilon - \beta\varepsilon - 2}{\beta(2R - \varepsilon)(1 + \varepsilon)}$$

$$k_2 = \frac{2R - \varepsilon - 2}{\beta(2R - \varepsilon)}$$

Case II

The second case arises when constraint ⑨ is binding and constraint ⑤ is not. It means that the maximization of social investment is restricted by type 1 agents' incentive constraints of defaulting. In that case, the optimal contract will become:

$$x_1 = \frac{(2\beta + 1)\varepsilon^2 + (4\beta R + 4R - 2\beta - 4)\varepsilon + (4R\beta - 8R + 4R^2 + 4 - 4\beta)}{\beta(\varepsilon - 2R + 4)(3\varepsilon + 2R)}$$

$$x_2 = \frac{(\beta + 2)\varepsilon^2 + (4R + 4R\beta - 4\beta - 2)\varepsilon + (4R + 4\beta - 4 - 8\beta R + 4\beta R^2)}{\beta(\varepsilon - 2R + 4)(3\varepsilon + 2R)}$$

$$I_1 = \frac{(4\beta + 2)\varepsilon + 4R - 4 + 4\beta}{\beta(\varepsilon - 2R + 4)(3\varepsilon + 2R)}$$

$$I_2 = \frac{(2\beta + 4)\varepsilon + 4 - 4\beta + 4\beta R}{(\varepsilon - 2R + 4)(3\varepsilon + 2R)}$$

$$k_1 = \frac{3\beta\varepsilon^2 + (8\beta - 4\beta R - 2)\varepsilon + (8\beta R - 4\beta R^2 - 4R + 4 - 4\beta)}{\beta(\varepsilon - 2R + 4)(3\varepsilon + 2R)}$$

$$k_2 = \frac{3\varepsilon^2 + (8 - 4R - 2\beta)\varepsilon + (8R - 4R^2 - 4 + 4\beta - 4\beta R)}{\beta(\varepsilon - 2R + 4)(3\varepsilon + 2R)}$$

Comparing these results with the first best allocation, we can easily see that both types of agents will have to put higher margin in order to satisfy the incentive constraints. In that case, the level of efficient investment will decrease. And it is also not surprising to find that type 2 agent will have a higher investment level than a type 1 agent since they are better insured.

6. Discussion about Social Welfare and Aggregate Investment

Basing on the optimal all-participating contract we have above, we find that the social investment can be maximized by including all the agents into the market. In another word, the all-participating contract (Clearinghouse contract) can achieve a higher social efficiency than the one-to-one contract (Bilateral contract). The underlying intuition is that by introducing more Type 1 agents into the contract, it saves partly of the inefficient saving and lowers down the cost of fulfilling the contracts for the majority type (Type 1). In that case, the all-participating contract lowers down the necessary social collateral for contract fulfillment which in turn leaving more fund available for investment.

In the first case the social investment will be:

$$pI_{SI1} + (1-p)I_{SI2} = \frac{2R + 3\varepsilon - 4p\varepsilon}{(2R-\varepsilon)(1+\varepsilon)}$$

And in the second case the social investment will be:

$$pI_{SI1} + (1-p)I_{SI2} = \frac{2}{\varepsilon - 2R + 4}$$

While with bilateral contract, the social investment will be:

$$pI_{FC1} + (1-p)I_{FC2} = \frac{4(1-p)}{\varepsilon - 2R + 4} + \frac{2p-1}{\varepsilon + 1}$$

We have proved that, under two different cases, the optimal all-participating contracts can always end up with higher aggregate investment than the one-to-one contracts. (See Appendix A2) In another word, a central counterparty trading mechanism is more efficient than the bilateral OTC market. However, in the following section, we also show that this efficient contract setting is not always sustainable if agents are allowed to set up their contract and decide the size of collusion freely. That is to say, in order to achieve the social efficiency, the contract traded in the market must be standardized and the level of insurance offered by a single contract must be limited so as to cover all the market participants.

Collusion-Proof of the Equilibrium

Now suppose that we start with the all-participating equilibrium with incomplete insurance contracts. As we can see, to get better insurances from type 2 agents, type 1 agents may have the incentive to exclude other agents of the same type. To analysis this deviation incentive, we assume that partly of the type 1 agents offer a derivative contract to attract the type 2 agents to form a smaller trading group in order to get better insurance. We define that the proportion of the type 1 agents in new collusion is P' and that of type 2 agents is 1-P'. Since partly of the type 1 agents are excluded from the original market, we know that $\mathrm{P}' < P$ and $1 - \mathrm{P}' > 1 - P$. Moreover we also define that $\beta' = \frac{\mathrm{P}\prime}{1-\mathrm{P}\prime}$. For this deviation to be preferred, it must be the case that with certain resource reallocation, both types of agents inside the collusion become better-off. Under the risk neutral assumption, one necessary condition is that inside the collusion the welfare level should be no less than what they had from the original equilibrium. Since we have shown that our equilibrium is the second best, for any given P' in a new collusion, the highest welfare level of the new collusion should be:

In Case I:

$$\mathrm{P}'\mathrm{U}_{\mathrm{SI1}}(P') + (1 - \mathrm{P}')\mathrm{U}_{\mathrm{SI2}}(P') = 1 + (R-1)\frac{2R + 3\varepsilon - 4P'\varepsilon}{(2R-\varepsilon)(1+\varepsilon)}$$

In Case II:

$$\mathrm{P}'\mathrm{U}_{\mathrm{SI1}}(P') + (1 - \mathrm{P}')\mathrm{U}_{\mathrm{SI2}}(P') = 1 + (R-1)\frac{2}{\varepsilon - 2R + 4}$$

While the social welfare level of the same collusion with original all-participating contract will be:

In Case I:

$$\mathrm{P}'\mathrm{U}_{\mathrm{SI1}}(P) + (1 - \mathrm{P}')\mathrm{U}_{\mathrm{SI2}}(P)$$

$$= 1 + \left(\frac{P'}{P} - P'\right)(R-1)\frac{(2R + 2\beta R + \varepsilon - \beta\varepsilon - 2)}{(2R - \varepsilon)(1 + \varepsilon)} + \left(\frac{P'}{P} - P'\right)\frac{2R - 2 - \varepsilon}{2R - \varepsilon} + (R-1)\frac{2(1 - P')}{2R - \varepsilon} + \frac{(1 - P')(2 - 2R + \varepsilon)}{2R - \varepsilon}$$

In Case II:

$$\mathrm{P}'\mathrm{U}_{\mathrm{SI1}}(P) + (1 - \mathrm{P}')\mathrm{U}_{\mathrm{SI2}}(P)$$

$$= 1 + (R-1)\left(\frac{P'}{P} - 1\right)\frac{(4\beta + 2)\varepsilon + 4R - 4 + 4\beta}{(\varepsilon - 2R + 4)(3\varepsilon + 2R)} + (R-1)\frac{2(1 - P')}{(1 - P)(\varepsilon - 2R + 4)} + \left(\frac{P'}{P} - 1\right)\frac{(\beta - 1)(\varepsilon^2 + 2\varepsilon + 12R - 4R^2 - 8)}{2(\varepsilon - 2R + 4)(3\varepsilon + 2R)}$$

Under our assumption if the deviation is beneficial to both types of agents, we should have:

$$\mathrm{P}'\mathrm{U}'_{\mathrm{SI1}} + (1 - \mathrm{P}')\mathrm{U}'_{\mathrm{SI2}} > \mathrm{P}'\mathrm{U}_{\mathrm{SI1}} + (1 - \mathrm{P}')\mathrm{U}_{\mathrm{SI2}}$$

As we have proved, this condition is satisfied by any parameter value of Case II. While in Case I, it can also be satisfied when the return of investment R is sufficiently high. In that case, Type 1 agents will always find it beneficial to form a smaller trading group until the proportion of both types becomes equal. Since all the Type 1 agents in the new collusion do not consider about the externality they brought to others, they will make the deviation if the benefit of doing so cover the corresponding cost. As we have suggested above, Type 1 agents can benefit from a higher coverage and a higher investment level in a smaller trading group. Therefore when the return of investment R becomes high, the incentive of forming a smaller trading group will be stronger. On the other hand, the cost of excluding other Type 1 source from a higher “cross-subsidy.” As we have mentioned above, Type 2 agents are liquidity provider in the bad state and their producing technology is relatively scarce in the market. Therefore, Type 1 agents have to subsidize Type 2 agents for this scarcity by offering better terms in the contracts, for example, lower contingent payment x_2 and lower margin requirement k_2. So, excluding some of the Type 1 agents means that the remaining Type 1s have to share higher cross subsidy per person. That is why the all-participating contract can be collusive proof when β is sufficiently high (Case I) and the return of investment R is sufficiently low. In summary:(See Appendix A3)

Proposition 6. The all-participating equilibrium will be collusive proof iff $\beta > \frac{8(1+\varepsilon)\varepsilon}{(2R-\varepsilon)(2R-2+\varepsilon)} - \frac{2R+3\varepsilon}{2R-\varepsilon}$ and $R < \frac{(2-3\varepsilon)+\sqrt{17\varepsilon^2+20\varepsilon+4}}{4}$

In conclusion, our model suggests that contracts need to be standardized exogenously in order to maximize social investment when the market is facing aggregate uncertainty. As we have proved above, if agents are allowed to decide the contract freely, the majorities will tend to exclude other agents of the same type and form a smaller trading group which in return increases the amount of inefficient social saving and restricts aggregate investment. As a result, a central counterparty is required to be set up to decide the optimal contract and this contract should be the unique one traded in the market. In another word, our model shows that the existence of OTC contract may lead to social inefficiency in a market with aggregate uncertainty.

Besides, our model also predicts that central clearinghouses are more likely to be set up in a market with high aggregate uncertainty and low return. Because the all-participating contract will become collusive proof, in that case, the equilibrium is more likely to be sustained.

Discussion about the Margin Requirement

In this section, we focus on the margin requirement of different types of agents. From the equilibrium in Case I, we have:

$$k_1 = 1 - \frac{2R + 2\beta R + \varepsilon - \beta\varepsilon - 2}{\beta(2R - \varepsilon)(1 + \varepsilon)}$$

$$k_2 = \frac{2R - \varepsilon - 2}{\beta(2R - \varepsilon)}$$

And from that of Case II, we have:

$$k_1 = \frac{3\beta\varepsilon^2 + (8\beta - 4\beta R - 2)\varepsilon + (8\beta R - 4\beta R^2 - 4R + 4 - 4\beta)}{\beta(\varepsilon - 2R + 4)(3\varepsilon + 2R)}$$

$$k_2 = \frac{3\varepsilon^2 + (8 - 4R - 2\beta)\varepsilon + (8R - 4R^2 - 4 + 4\beta - 4\beta R)}{\beta(\varepsilon - 2R + 4)(3\varepsilon + 2R)}$$

From the equations above, we can simply find that $k_2 < k_1$, which means that the margin requirement for the minority (Type 2) is lower than that of the majority (Type 1). Because Type 2 agents own a relatively scarce production technology, it is no surprise that they get a contract with better terms. With identical reason, one can see that Type 2 agents end up with a higher level of investment $I_{SI2} > I_{SI1}$ and lower contingent payment $x_{SI2} < x_{SI1}$. Both of these conditions decrease the incentive for Type 2 agents to default. More importantly, this difference of incentive is strongly related to the proportion of two types. In extreme cases, when the proportion of type 2 agents is sufficiently small, the margin requirement for Type 2 agents can even become negative. Because when the shock from aggregate uncertainty becomes huge, to offer higher insurance to Type 1 agents, the central intermediary may have motivations to lend fund to Type 2. This method can improve the social efficiency, especially in the case when the liquidity provided by Type 2 is exhausted in bad state (Case I):

$$k_2 = \frac{2R - \varepsilon - 2}{\beta(2R - \varepsilon)}$$

Since $R < 1 + \frac{1}{2}\varepsilon$, we have

$$2R - \varepsilon - 2 < 0 \Rightarrow k_2 < 0$$

Correspondingly we have

$$I_2 = \frac{2}{2R - \varepsilon} > 1$$

Proposition. 7 When$\beta > \frac{8(1+\varepsilon)\varepsilon}{(2R-\varepsilon)(2R-2+\varepsilon)} - \frac{2R+3\varepsilon}{2R-\varepsilon}$ the margin requirement of Type 2 agents will become negative and the investment of Type 2 I_2 will be higher than 1.

In our setting, the capacity for agents to fulfill their commission depends heavily on their scale of investment. Take type 2 agents as an example, when state 2 realizes, they will receive εI from their investment and this contingent revenue are linearly increasing with investment. However, when the proportion of Types 2 agents is sufficiently small, their aggregate payment capacity and scale of investment will be limited by their endowment. As a result, to maximize the social welfare, the central intermediary can collect the margin from Type 1 agents and lend the fund to Type 2 agents in order to increase their aggregate investment. Therefore, in this case, the incentive for Type 2 agents to fulfill their commission will purely rely on their high level of investment.

In general, when aggregate uncertainty is large in the market, the central clearinghouse should limit the holding of the high market-correlated asset by increasing the margin requirement. On the other hand, central clearinghouse should also provide liquidity to agents who work as the real counterparty to the market majority.

7. Conclusion and Empirical Implication

In this paper, we discuss a bilateral insurance trading model with unobservable investment and non-exclusive contract. Different from Leinter's setting, we consider a trading mechanism where intermediary works as the unique counterparty. Under aggregate uncertainty, we discuss two different contract setting: all-participating contract and one-to-one contract. We find that the previous one is more efficient when the incentive of strategic default is incorporated into the model. However, our model also shows that to achieve the optiamal allocation, the trading contract must be standardized and restricted exogenously. Otherwise, agents may have the incentive to form a smaller trading group and increase the scale of their contracts. In another word, if agents are allowed to decide their contracts, the bilateral OTC market is more likely to be formed although it leads to inefficient risk sharing and social investment. Different from previous studies where margin requirement is unique to all the market participant, our model shows that the optimal margin level should be contract specific and adjusted basing on the exposure of aggregate uncertainty. More importantly, our model shows that, when the proportion of a certain type of agents becomes sufficiently high, the central clearinghouse may be motivated to provide liquidity to the counterparties of the majority so as to increase the capacity of insurance and efficient social investment.

In practice, our model predicts that this kind of centralized trading mechanism is more likely to be observed in markets with high aggregate uncertainty. While in the market with low aggregate uncertainty, both the central clearinghouse and one-to-one direct trading market may exist. Take Chicago Board of Trading (CBOT) as an example. CBOT is the most representative agriculture trading board in the world. In 1865, CBOT use standardized future contract to substitute forward contract and successful implement margin system where it works as the counterparty of each agent. Since agriculture is an industry that strongly affected by climate, this trading mechanism increases their capability to cope with this systematic risk. While in recent years, it also extends its business to oil, metal and financial assets. For a market with lower aggregate uncertainty, for example financial asset, our model explains the co-existence of standardized central trading contract and specific one-to-one forward contract.

Moreover, our model also predicts that when the aggregate economy is in the boom (state 1), there exists an excess supply of liquidity. While the economy is in depression (state 2), the liquidity provision will be exhausted when the ratio between different types is sufficiently high. And as proved above, when the proportions of different types are relatively close, the incentive constraints will become binding. This means that the potential risk of defaulting will limit the investment of agents which in return decrease the probability of liquidity crisis in the extreme cases.

Additionally, our model also implies that central clearinghouse should be allowed to provide liquidity to certain types of agents when the market is exposed to a high level of aggregate uncertainty. In another word, fixing the margin requirement to unique and nonnegative to all types of agents may lead to an inefficient resource allocation and low level of social investment. Therefore, the settings of margin system should be more flexible to match the volatile market demand.

As an extension, we also think this centralized trading mechanism may be a constructive attempt to cope with some other problems in OTC market, for example, the cost of matching and moral hazard in monitoring. By including these trading frictions, we expect to make our model more general, and this should be taken as the next step of our research. Additionally, our model leads to several empirical implications which remain to be examined in the future.

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Appendix

A1:

Case I

$$1 - I_1 - k_1 = 0 \ldots \ldots (1)$$

$$k_1 + \varepsilon I_1 - x_1 > 0 \ldots \ldots (2)$$

$$k_1 + x_2 - \varepsilon I_1 = 0 \ldots \ldots (3)$$

$$1 - I_2 - \beta k_2 = 0 \ldots \ldots (4)$$

$$\beta k_2 + \varepsilon I_2 - \beta x_2 = 0 \ldots \ldots (5)$$

$$\beta k_2 + \beta x_1 - \varepsilon I_2 = 0 \ldots \ldots (6)$$

$$U_1(\psi) \geq U_A = \frac{R + \varepsilon}{1 + \varepsilon} \ldots \ldots (7)$$

$$U_2(\psi) \geq U_A = \frac{R + \varepsilon}{1 + \varepsilon} \ldots \ldots (8)$$

$$\frac{1}{2}(x_1 - k_1) < (R - 1)I_1 + \frac{1}{2}(x_2 - x_1) \ldots \ldots (9)$$

$$\frac{\beta}{2}(x_2 - k_2) = (\mathrm{R} - 1)I_2 + \frac{\beta}{2}(x_1 - x_2) \ldots \ldots (10)$$

Take (5),(6) and (10)

From (5)+(6)

$$2\beta k_2 + \beta(x_1 - x_2) = 0$$

From (10)

$$\beta(x_2 - k_2) - 2(R - 1)I_2 = \beta(x_1 - x_2)$$

$$2\beta k_2 + \beta(x_2 - k_2) - 2(R-1)I_2 = 0$$

$$\beta(x_2 + k_2) = 2(R-1)I_2$$

$$x_2 = \frac{2}{\beta}(R-1)I_2 - k_2$$

From (4)

$$k_2 = \frac{1}{\beta}(1 - I_2)$$

$$x_2 = \frac{2}{\beta}(R-1)I_2 - \frac{1}{\beta}(1 - I_2)$$

$$\beta x_2 = 2(R-1)I_2 - (1 - I_2)$$

$$\beta x_2 = 2RI_2 - I_2 - 1$$

$$I_2 = \frac{\beta x_2 + 1}{2R - 1}$$

From (1) and (3)

$$1 - I_1 - k_1 = 0 \Rightarrow k_1 = 1 - I_1$$

$$k_1 + x_2 - \varepsilon I_1 = 0$$

$$1 - I_1 + x_2 - \varepsilon I_1 = 0$$

$$1 + x_2 = I_1 + \varepsilon I_1$$

$$I_1 = \frac{1 + x_2}{1 + \varepsilon}$$

From (4) and (5)

$$1 - I_2 + \varepsilon I_2 - \beta x_2 = 0$$

$$1 - \beta x_2 = I_2 - \varepsilon I_2$$

$$I_2 = \frac{1 - \beta x_2}{1 - \varepsilon}$$

And together with

$$I_2 = \frac{\beta x_2 + 1}{2R - 1}$$

We have

$$\frac{1 - \beta x_2}{1 - \varepsilon} = \frac{\beta x_2 + 1}{2R - 1}$$

$$2\mathrm{R} - 2\mathrm{R}\beta x_2 - 1 + \beta x_2 = \beta x_2 + 1 - \varepsilon\beta x_2 - \varepsilon$$

$$2\mathrm{R} - 2 + \varepsilon = (2\mathrm{R}\beta - \varepsilon\beta) x_2$$

$$x_2 = \frac{2R - 2 + \varepsilon}{\beta(2R - \varepsilon)}$$

And

$$I_2 = \frac{1 - \beta x_2}{1 - \varepsilon} = \frac{2}{2R - \varepsilon}$$

$$I_1 = \frac{1 + x_2}{1 + \varepsilon} = \frac{2R + 2\beta R + \varepsilon - \beta\varepsilon - 2}{\beta(2R - \varepsilon)(1 + \varepsilon)}$$

From

$$2\beta k_2 + \beta(x_1 - x_2) = 0$$

$$\Rightarrow 2(1 - I_2) + \beta(x_1 - x_2) = 0$$

$$\Rightarrow \beta(x_1 - x_2) = 2(I_2 - 1)$$

$$x_1 - x_2 = \frac{2}{\beta}(I_2 - 1)$$

$$x_1 = \frac{2}{\beta}(I_2 - 1) + x_2$$

$$\beta x_1 = 2(I_2 - 1) + \beta x_2$$

$$\beta x_1 + 1 = 2(I_2 - 1) + \beta x_2 + 1$$

$$\frac{\beta x_1 + 1}{2R - 1} = \frac{2(I_2 - 1)}{2R - 1} + \frac{\beta x_2 + 1}{2R - 1}$$

$$\frac{\beta x_1 + 1}{2R - 1} = \frac{2(I_2 - 1)}{2R - 1} + I_2$$

$$\beta x_1 + 1 = 2(I_2 - 1) + (2R - 1)I_2$$

$$I_2 = \frac{\beta x_1 + 3}{2R + 1}$$

$$\Rightarrow \frac{2}{2R - \varepsilon} = \frac{\beta x_1 + 3}{2R + 1}$$

$$2\mathrm{R}\beta x_1 - \varepsilon\beta x_1 + 6R - 3\varepsilon = 4R + 2$$

$$\beta(2\mathrm{R} - \varepsilon)x_1 = 2 + 3\varepsilon - 2R$$

$$x_1 = \frac{2 + 3\varepsilon - 2R}{\beta(2R - \varepsilon)}$$

In summary, we have:

$$x_1 = \frac{2 + 3\varepsilon - 2R}{\beta(2R - \varepsilon)}$$

$$x_2 = \frac{2R - 2 + \varepsilon}{\beta(2R - \varepsilon)}$$

$$I_1 = \frac{2R + 2\beta R + \varepsilon - \beta\varepsilon - 2}{\beta(2R - \varepsilon)(1 + \varepsilon)}$$

$$I_2 = \frac{2}{2R - \varepsilon}$$

$$k_1 = 1 - I_1 = 1 - \frac{2R + 2\beta R + \varepsilon - \beta\varepsilon - 2}{\beta(2R - \varepsilon)(1 + \varepsilon)}$$

$$k_2 = \frac{2R - \varepsilon - 2}{\beta(2R - \varepsilon)}$$

Then we need to check whether our solution satisfies (2)

$$k_1 + \varepsilon I_1 - x_1 > 0 \dots \dots (2)$$

Given (1) $k_1 = 1 - I_1$,

$$1 - I_1 + \varepsilon I_1 - x_2 > 0$$

$$1 - x_1 > (1 - \varepsilon) I_1$$

$$1 - x_1 > (1 - \varepsilon)(1 + \frac{x_2 - \varepsilon}{1 + \varepsilon})$$

$$1 - x_1 > 1 + \frac{x_2 - \varepsilon}{1 + \varepsilon} - \varepsilon - \varepsilon \frac{x_2 - \varepsilon}{1 + \varepsilon}$$

$$(1 + \varepsilon)(\varepsilon - x_1) > (x_2 - \varepsilon)(1 - \varepsilon)$$

$$\varepsilon - x_1 + \varepsilon^2 - x_1 \varepsilon > x_2 - \varepsilon x_2 - \varepsilon + \varepsilon^2$$

$$\varepsilon(x_2 - x_1) > (x_1 + x_2) - 2\varepsilon$$

$$\varepsilon(x_2 - x_1) > \frac{4\varepsilon}{\beta(2R - \varepsilon)} - 2\varepsilon$$

$$(x_2 - x_1) > \frac{4}{\beta(2R - \varepsilon)} - 2$$

$$\frac{4R - 4 - 2\varepsilon}{\beta(2R - \varepsilon)} > \frac{4}{\beta(2R - \varepsilon)} - 2$$

$$\frac{4R - 8 - 2\varepsilon}{\beta(2R - \varepsilon)} > -2$$

$$4R - 2\varepsilon - 8 > -2\beta(2R - \varepsilon)$$

$$2(2R - \varepsilon) + 2\beta(2R - \varepsilon) - 8 > 0$$

$$(\beta + 1)(2R - \varepsilon) > 4$$

Since $\beta > 1$ and $R < 1 + \frac{1}{2}\varepsilon \Rightarrow 2R - \varepsilon < 2$

The above inequity will be satisfied iff

$$\beta > \frac{4}{2R - \varepsilon} - 1$$

Then we turn to (9) and check under what condition it will be satisfied:

$$\frac{1}{2}(x_1 - k_1) < (R - 1)I_1 + \frac{1}{2}(x_2 - x_1)$$

$$x_1 - k_1 < 2(R - 1)I_1 + (x_2 - x_1)$$

$$x_1 - 1 + \frac{1 + x_2}{1 + \varepsilon} < (2R - 2)\frac{1 + x_2}{1 + \varepsilon} + (x_2 - x_1)$$

$$x_1 - 1 + (x_1 - x_2) < (2R - 3)\frac{1 + x_2}{1 + \varepsilon}$$

$$2x_1 - x_2 - 1 < (2R - 3)\frac{1 + x_2}{1 + \varepsilon}$$

$$(1 + \varepsilon)(2x_1 - x_2 - 1) < (2R - 3)(1 + x_2)$$

$$2x_1 - x_2 - 1 + 2\varepsilon x_1 - \varepsilon x_2 - \varepsilon < 2R - 3 + 2Rx_2 - 3x_2$$

$$2(1 + \varepsilon)x_1 - (1 + \varepsilon)x_2 - (1 + \varepsilon) < (2R - 3) + (2R - 3)x_2$$

$$2(1 + \varepsilon)x_1 - (1 + \varepsilon + 2R - 3)x_2 < (2R - 3 + 1 + \varepsilon)$$

$$2(1 + \varepsilon)x_1 - (2R + \varepsilon - 2)x_2 < (2R + \varepsilon - 2)$$

$$2(1 + \varepsilon)x_1 + 2(1 + \varepsilon)x_2 - (2R + 3\varepsilon)x_2 < (2R - 2 + \varepsilon)$$

$$2(1 + \varepsilon)(x_1 + x_2) - (2R + 3\varepsilon)x_2 < (2R - 2 + \varepsilon)$$

$$2(1+\varepsilon)\frac{4\varepsilon}{\beta(2R-\varepsilon)}-(2R+3\varepsilon)\frac{2R-2+\varepsilon}{\beta(2R-\varepsilon)}<2R-2+\varepsilon$$

$$8(1+\varepsilon)\varepsilon-(2\mathrm{R}+3\varepsilon)(2\mathrm{R}-2+\varepsilon)<\beta(2R-\varepsilon)(2R-2+\varepsilon)$$

$$\beta>\frac{8(1+\varepsilon)\varepsilon}{(2R-\varepsilon)(2R-2+\varepsilon)}-\frac{2R+3\varepsilon}{2R-\varepsilon}$$

And it can be proved that

$$\frac{8(1+\varepsilon)\varepsilon}{(2R-\varepsilon)(2R-2+\varepsilon)}-\frac{2R+3\varepsilon}{2R-\varepsilon}>\frac{4}{2R-\varepsilon}-1$$

As a result, the above case will arise when:

$$\beta>\frac{8(1+\varepsilon)\varepsilon}{(2R-\varepsilon)(2R-2+\varepsilon)}-\frac{2R+3\varepsilon}{2R-\varepsilon}$$

Case II

$$1-I_1-k_1=0\ \dots\dots(1)$$

$$k_1+\varepsilon I_1-x_1>0\ \dots\dots(2)$$

$$k_1+x_2-\varepsilon I_1=0\ \dots\dots(3)$$

$$1-I_2-\beta k_2=0\ \dots\dots(4)$$

$$\beta k_2+\varepsilon I_2-\beta x_2>0\ \dots\dots(5)$$

$$\beta k_2+\beta x_1-\varepsilon I_2=0\ \dots\dots(6)$$

$$U_1(\psi)\geq U_A=\frac{R+\varepsilon}{1+\varepsilon}\ \dots\dots(7)$$

$$U_2(\psi)\geq U_A=\frac{R+\varepsilon}{1+\varepsilon}\ \dots\dots(8)$$

$$\frac{1}{2}(x_1 - k_1) = (R-1)I_1 + \frac{1}{2}(x_2 - x_1) \dots\dots(9)$$

$$\frac{\beta}{2}(x_2 - k_2) = (\mathrm{R}-1)I_2 + \frac{\beta}{2}(x_1 - x_2) \dots\dots(10)$$

From (1) and (3), we have

$$1 - I_1 + x_2 - \varepsilon I_1 = 0$$

$$1 - (1+\varepsilon)I_1 + x_2 = 0$$

$$x_2 = (1+\varepsilon)I_1 - 1$$

From (4) and (6), we have

$$1 - I_2 + \beta x_1 - \varepsilon I_2 = 0$$

$$1 - (\varepsilon + 1)I_2 + \beta x_1 = 0$$

$$\beta x_1 = (1+\varepsilon)I_2 - 1$$

From (9)

$$\frac{1}{2}(x_1 - k_1 - x_2 + x_1) = (R-1)I_1$$

$$2x_1 - x_2 - k_1 = 2(R-1)I_1$$

$$2\beta x_1 - \beta x_2 - \beta k_1 = 2\beta(R-1)I_1$$

$$2(1+\varepsilon)I_2 - 2 - \beta(1+\varepsilon)I_1 + \beta - \beta(1 - I_1) = 2\beta(R-1)I_1$$

$$2(1+\varepsilon)I_2 - 2 = \beta(2R - 2 + \varepsilon)I_1$$

From (10)

$$\beta(x_2 - k_2) = 2(R-1)I_2 + \beta(x_1 - x_2)$$

$$\beta(2x_2 - x_1 - k_2) = 2(R-1)I_2$$

$$2\beta x_2 - \beta x_1 - \beta k_2 = 2(R-1)I_2$$

As a result, we have

$$2(1+\varepsilon)I_2 - 2 = \beta(2R-2+\varepsilon)I_1$$

$$2\beta(1+\varepsilon)I_1 - 2\beta = (2R-2+\varepsilon)I_2$$

⇒

$$\beta(2R-2+\varepsilon)I_1 = 2(1+\varepsilon)I_2 - 2$$

$$(2R-2+\varepsilon)I_2 = 2(1+\varepsilon)\beta I_1 - 2\beta$$

$$I_2 = \frac{(2R-2+\varepsilon)\beta I_1 + 2}{2(1+\varepsilon)}$$

$$\frac{[(2R-2+\varepsilon)\beta I_1 + 2](2R-2+\varepsilon)}{2(1+\varepsilon)} = 2(1+\varepsilon)\beta I_1 - 2\beta$$

$$(2R-2+\varepsilon)^2\beta I_1 + 2(2R-2+\varepsilon) = 4(1+\varepsilon)^2\beta I_1 - 4\beta(1+\varepsilon)$$

$$[(2R-2+\varepsilon)^2 - 4(1+\varepsilon)^2]\beta I_1 = -4\beta(1+\varepsilon) - 2(2R-2+\varepsilon)$$

$$I_1 = \frac{(4\beta+2)\varepsilon + 4R - 4 + 4\beta}{\beta(\varepsilon - 2R + 4)(3\varepsilon + 2R)}$$

With

$$\beta(2R-2+\varepsilon)I_1 = 2(1+\varepsilon)I_2 - 2$$

$$(2R-2+\varepsilon)I_2 = 2(1+\varepsilon)\beta I_1 - 2\beta$$

$$(2R-2+\varepsilon)(\beta I_1 + I_2) = 2(1+\varepsilon)(\beta I_1 + I_2) - 2 - 2\beta$$

$$(2\mathrm{R}-4-\varepsilon)(\beta I_1+I_2)=-2-2\beta$$

$$\beta I_1+I_2=\frac{2(1+\beta)}{\varepsilon+4-2R}$$

$$I_2=\frac{(2\beta+4)\varepsilon+4-4\beta+4\beta R}{(\varepsilon-2R+4)(3\varepsilon+2R)}$$

Then from

$$x_2=(1+\varepsilon)I_1-1$$

$$\beta x_1=(1+\varepsilon)I_2-1$$

$$k_1=1-I_1$$

$$\beta k_2=1-I_2$$

We can have

$$x_2=\frac{(\beta+2)\varepsilon^2+(4R+4R\beta-4\beta-2)\varepsilon+(4R+4\beta-4-8\beta R+4\beta R^2)}{\beta(\varepsilon-2R+4)(3\varepsilon+2R)}$$

$$x_1=\frac{(2\beta+1)\varepsilon^2+(4\beta R+4R-2\beta-4)\varepsilon+(4R\beta-8R+4R^2+4-4\beta)}{\beta(\varepsilon-2R+4)(3\varepsilon+2R)}$$

$$k_1=\frac{3\beta\varepsilon^2+(8\beta-4\beta R-2)\varepsilon+(8\beta R-4\beta R^2-4R+4-4\beta)}{\beta(\varepsilon-2R+4)(3\varepsilon+2R)}$$

$$k_2=\frac{3\varepsilon^2+(8-4R-2\beta)\varepsilon+(8R-4R^2-4+4\beta-4\beta R)}{\beta(\varepsilon-2R+4)(3\varepsilon+2R)}$$

As what we have done in the last section, we need to check under what conditions (5) and (2) will be satisfied:

From (5)

$$\beta k_2 + \varepsilon I_2 - \beta x_2 > 0$$

$$(\varepsilon^2 - 2\varepsilon + 4R - 4R^2)\beta > (4R^2 - 4R + 8\varepsilon R - 14\varepsilon - 5\varepsilon^2)$$

$$\beta < \frac{8(1+\varepsilon)\varepsilon}{(2R-\varepsilon)(2R-2+\varepsilon)} - \frac{2R+3\varepsilon}{2R-\varepsilon}$$

Since $\beta > 1$, it requires $4R^2 - 4R + 8\varepsilon R - 14\varepsilon - 5\varepsilon^2 < 0$ and $4R^2 - 4R + 8\varepsilon R - 14\varepsilon - 5\varepsilon^2 < \varepsilon^2 - 2\varepsilon + 4R - 4R^2$. And given our original assumptions $0 < \varepsilon < 1$ and $1 < R < 1 + \frac{\varepsilon}{2}$, all the above requirements can be satisfied when both ε and R are close to 1. As a result, it can be inferred that case II can arise when ε is sufficiently large and R is sufficiently small.

While from (2)

$$k_1 + \varepsilon I_1 - x_1 > 0$$

$$(5\varepsilon^2 + 14\varepsilon - 8\varepsilon R + 4R - 4R^2)\beta + (\varepsilon^2 - 2\varepsilon + 4R + 4R^2) > 0$$

Given the discussion in (5) $4R^2 - 4R + 8\varepsilon R - 14\varepsilon - 5\varepsilon^2 < 0$ which means $5\varepsilon^2 + 14\varepsilon - 8\varepsilon R + 4R - 4R^2 > 0$. And because $\varepsilon < 1 < R$, it can be proved that $\varepsilon^2 - 2\varepsilon + 4R + 4R^2 > 0$. Hence we can have:

$$\beta > \frac{-\varepsilon^2 + 2\varepsilon - 4R - 4R^2}{5\varepsilon^2 + 14\varepsilon - 8\varepsilon R + 4R - 4R^2}$$

Since $\frac{-\varepsilon^2+2\varepsilon-4R-4R^2}{5\varepsilon^2+14\varepsilon-8\varepsilon R+4R-4R^2} < 0$ and $\beta > 1$, the above constrain can always be satisfied.

In conclusion, case II will arise when ε is sufficiently large and R is sufficiently small.

A2:

Then we come to the comparison of social investment (efficiency) between full-participation contracts and one-to-one contracts.

Firstly in the one-to-one contract:

$$\mathrm{I}_1 = \mathrm{I}_2 = 1 - \frac{\varepsilon - 2(\mathrm{R}-1)}{\varepsilon - 2(\mathrm{R}-1) + 2}$$

$$\mathrm{k}_1 = \mathrm{k}_2 = \frac{\varepsilon - 2(\mathrm{R}-1)}{\varepsilon - 2(\mathrm{R}-1) + 2}$$

$$\mathrm{x}_1 = \mathrm{x}_2 = \varepsilon - (1+\varepsilon)\frac{\varepsilon - 2(\mathrm{R}-1)}{\varepsilon - 2(\mathrm{R}-1) + 2}$$

So the social investment of the one-to-one contract:

$$2(1-p)\left[1 - \frac{\varepsilon - 2(\mathrm{R}-1)}{\varepsilon - 2(\mathrm{R}-1) + 2}\right] + (2p-1)\frac{1}{\varepsilon+1} = \frac{4(1-p)}{\varepsilon - 2R + 4} + \frac{2p-1}{\varepsilon+1}$$

While in our all-participation contract, if case I arise:

$$x_1 = \frac{2 + 3\varepsilon - 2R}{\beta(2R - \varepsilon)}$$

$$x_2 = \frac{2R - 2 + \varepsilon}{\beta(2R - \varepsilon)}$$

$$I_1 = \frac{2R + 2\beta R + \varepsilon - \beta\varepsilon - 2}{\beta(2R - \varepsilon)(1+\varepsilon)}$$

$$I_2 = \frac{2}{2R - \varepsilon}$$

$$k_1 = 1 - I_1 = 1 - \frac{2R + 2\beta R + \varepsilon - \beta\varepsilon - 2}{\beta(2R - \varepsilon)(1+\varepsilon)}$$

$$k_2 = \frac{2R - \varepsilon - 2}{\beta(2R - \varepsilon)}$$

So the social investment:

$$\mathrm{p}I_1 = p\times\frac{1-p}{p}\times\frac{2R+2\beta R+\varepsilon-\beta\varepsilon-2}{(2R-\varepsilon)(1+\varepsilon)}$$
$$=\frac{(1-p)(2R+2\beta R+\varepsilon-\beta\varepsilon-2)}{(2R-\varepsilon)(1+\varepsilon)}$$

$$(1-\mathrm{p})I_2 = \frac{2(1-p)}{2R-\varepsilon}$$

$$\mathrm{p}I_1 + (1-\mathrm{p})I_2 = \frac{1-p}{2R-\varepsilon}\times(\frac{2R+2\beta R+\varepsilon-\beta\varepsilon-2}{1+\varepsilon}+2)$$

$$\mathrm{p}I_1 + (1-\mathrm{p})I_2 = \frac{(1-p)(2R+2\beta R+3\varepsilon-\beta\varepsilon)}{(2R-\varepsilon)(1+\varepsilon)}$$

$$\mathrm{p}I_1 + (1-\mathrm{p})I_2 = \frac{2(1-p)R+2pR+3(1-p)\varepsilon-p\varepsilon}{(2R-\varepsilon)(1+\varepsilon)}$$

$$\mathrm{p}I_1 + (1-\mathrm{p})I_2 = \frac{2R+3\varepsilon-4p\varepsilon}{(2R-\varepsilon)(1+\varepsilon)}$$

Then we need to show that

$$\frac{2R+3\varepsilon-4p\varepsilon}{(2R-\varepsilon)(1+\varepsilon)} > \frac{4(1-p)}{\varepsilon-2R+4}+\frac{2p-1}{\varepsilon+1}$$

$$\frac{2R+3\varepsilon-4p\varepsilon}{(2R-\varepsilon)(1+\varepsilon)}-\frac{2p-1}{\varepsilon+1} > \frac{4(1-p)}{\varepsilon-2R+4}$$

$$\frac{2R+3\varepsilon-4p\varepsilon-(2p-1)(2R-\varepsilon)}{(2R-\varepsilon)(1+\varepsilon)} > \frac{4(1-p)}{\varepsilon-2R+4}$$

$$\frac{2R+3\varepsilon-4p\varepsilon-4pR+2p\varepsilon+2R-\varepsilon}{(2R-\varepsilon)(1+\varepsilon)} > \frac{4(1-p)}{\varepsilon-2R+4}$$

$$\frac{4R+2\varepsilon-2p\varepsilon-4pR}{(2R-\varepsilon)(1+\varepsilon)} > \frac{4(1-p)}{\varepsilon-2R+4}$$

$$\frac{2(2R+\varepsilon)(1-p)}{(2R-\varepsilon)(1+\varepsilon)} > \frac{4(1-p)}{\varepsilon-2R+4}$$

$$\frac{(2R+\varepsilon)}{(2R-\varepsilon)(1+\varepsilon)} > \frac{2}{\varepsilon-2R+4}$$

$$(2\mathrm{R}+\varepsilon)(\varepsilon-2\mathrm{R}+4) > 2(2R-\varepsilon)(1+\varepsilon)$$

$$2\mathrm{R}\varepsilon-4R^2+8R+\varepsilon^2-2R\varepsilon+4\varepsilon > 2(2R+2R\varepsilon-\varepsilon-\varepsilon^2)$$

$$\varepsilon^2+4\varepsilon-4R^2+8R > 4R+4R\varepsilon-2\varepsilon-2\varepsilon^2$$

$$3\varepsilon^2+6\varepsilon-4R\varepsilon+4R-4R^2 > 0$$

$$3\varepsilon^2+6\varepsilon-6R\varepsilon > 4R(R-1-\frac{1}{2}\varepsilon)$$

$$3\varepsilon(\varepsilon+2-2\mathrm{R}) > 4R(R-1-\frac{\varepsilon}{2})$$

In this case $\mathrm{R} < 1+\frac{\varepsilon}{2}$, $\varepsilon+2-2\mathrm{R} > 0$.

$$3\varepsilon > 4R\times(-\frac{1}{2})$$

$$3\varepsilon > -2R$$

Given $\varepsilon > 0, R > 0$ the above inequity will always be satisfied.

On the other hand, when case II arise

$$\mathrm{p}I_1 = \frac{(1-p)[(4\beta+2)\varepsilon+4R-4+4\beta]}{(\varepsilon-2R+4)(3\varepsilon+2R)}$$

$$(1-\mathrm{p})I_2 = \frac{(1-p)[(2\beta+4)\varepsilon+4-4\beta+4\beta R]}{(\varepsilon-2R+4)(3\varepsilon+2R)}$$

$$\mathrm{p}I_1+(1-\mathrm{p})I_2 = \frac{(1-p)[(6\beta+6)\varepsilon+4R+4\beta R]}{(\varepsilon-2R+4)(3\varepsilon+2R)}$$

$$\mathrm{p}I_1 + (1-\mathrm{p})I_2 = \frac{(1-p)[6(\beta+1)\varepsilon + 4R(\beta+1)]}{(\varepsilon - 2R + 4)(3\varepsilon + 2R)}$$

$$\mathrm{p}I_1 + (1-\mathrm{p})I_2 = \frac{(1-p)(\beta+1)(6\varepsilon + 4R)}{(\varepsilon - 2R + 4)(3\varepsilon + 2R)}$$

$$\mathrm{p}I_1 + (1-\mathrm{p}) = \frac{2}{\varepsilon - 2R + 4}$$

Because $p \geq \frac{1}{2}$ and $R > 1$,

$$(2p-1)(\varepsilon + 2R - 2) \geq 0$$

$$\Rightarrow 4p\varepsilon + 4p - 2\varepsilon - 2 \geq 2p\varepsilon - 4Rp + 8p - \varepsilon + 2R - 4$$

$$\Rightarrow (4p-2)(\varepsilon + 1) \geq (2p-1)(\varepsilon - 2R + 4)$$

$$\Rightarrow \frac{2}{\varepsilon - 2R + 4} \geq \frac{4(1-p)}{\varepsilon - 2R + 4} + \frac{2p-1}{\varepsilon + 1}$$

A3:

Proof of Collusive Proof

Now suppose that we start with the second-best equilibrium with all agents participate in the contracts. As we can see, to get better insurance from type 2 agents, type 1 agents may have the incentive to exclude other agents of the same type. To analysis this deviation incentive, suppose that partly of the type 1 agents offer a deviate contract to attract the type 2 agents to form a smaller trading group in order to get higher protection. We define that the proportion of the type 1 agents in new collusion is P' and that of type 2 agents is 1-P' correspondingly. Since partly of the type 1 agents are excluded from the original market, we know that $\mathrm{P}' < P$ and $1 - \mathrm{P}' > 1 - P$. For this deviation to break down the original equilibrium, it must be the case that with certain resource reallocation, both types of agents must become better-off. And one necessary condition is that inside the collusion the welfare level should be no less than what they get from the original equilibrium. Since we have shown that our equilibrium is the second best, for any given P' in a new collusion, the highest welfare level can be defined as:

$$\mathrm{P}'\mathrm{U}_{\mathrm{SI1}}(P') + (1-\mathrm{P}')\mathrm{U}_{\mathrm{SI2}}(P')$$

And the welfare level in the original all participating contract

$$\mathrm{P}'\mathrm{U}_{\mathrm{SI1}}(\mathrm{P}) + (1 - \mathrm{P}')\mathrm{U}_{\mathrm{SI2}}(P)$$

As a result, the original all participating equilibrium is collusive proof if:

$$\mathrm{P}'\mathrm{U}_{\mathrm{SI1}}(\mathrm{P}) + (1 - \mathrm{P}')\mathrm{U}_{\mathrm{SI2}}(P) \geq \mathrm{P}'\mathrm{U}_{\mathrm{SI1}}(P') + (1 - \mathrm{P}')\mathrm{U}_{\mathrm{SI2}}(P')$$

Otherwise, it may become beneficial for Type 1 agents to form a smaller trading group and enjoy higher insurance level from Type 2 agents.

Case I:

$$x_1 = \frac{2 + 3\varepsilon - 2R}{\beta(2R - \varepsilon)}$$

$$x_2 = \frac{2R - 2 + \varepsilon}{\beta(2R - \varepsilon)}$$

$$I_1 = \frac{2R + 2\beta R + \varepsilon - \beta\varepsilon - 2}{\beta(2R - \varepsilon)(1 + \varepsilon)}$$

$$I_2 = \frac{2}{2R - \varepsilon}$$

$$U_1 = 1 + (R - 1)I_1 + \frac{1}{2}(x_2 - x_1)$$

$$U_2 = 1 + (R - 1)I_2 + \frac{\beta}{2}(x_1 - x_2)$$

$$x_2 - x_1 = \frac{4R - 4 - 2\varepsilon}{\beta(2R - \varepsilon)}$$

$$x_1 - x_2 = \frac{4 - 4R + 2\varepsilon}{\beta(2R - \varepsilon)}$$

$$U_1 = 1 + (R-1)\frac{2R + 2\beta R + \varepsilon - \beta\varepsilon - 2}{\beta(2R-\varepsilon)(1+\varepsilon)} + \frac{1}{2}[\frac{4R-4-2\varepsilon}{\beta(2R-\varepsilon)}]$$

$$U_2 = 1 + (R-1)\frac{2}{2R-\varepsilon} + \frac{\beta}{2}[\frac{4-4R+2\varepsilon}{\beta(2R-\varepsilon)}]$$

$$\mathrm{P}U_1 = P + (R-1)\frac{(1-P)(2R + 2\beta R + \varepsilon - \beta\varepsilon - 2)}{(2R-\varepsilon)(1+\varepsilon)} + \frac{(1-P)(4R-4-2\varepsilon)}{2(2R-\varepsilon)}$$

$\Rightarrow$

$$\mathrm{P}U_1 = P + (R-1)\frac{2R + \varepsilon - 2P\varepsilon - 2 + 2P}{(2R-\varepsilon)(1+\varepsilon)} + \frac{(1-P)(2R-2-\varepsilon)}{(2R-\varepsilon)}$$

$$(1-\mathrm{P})U_2 = (1-P) + (R-1)\frac{2(1-P)}{2R-\varepsilon} + \frac{P(4-4R+2\varepsilon)}{2\beta(2R-\varepsilon)}$$

$\Rightarrow$

$$(1-\mathrm{P})U_2 = (1-P) + (R-1)\frac{2(1-P)}{2R-\varepsilon} + \frac{(1-P)(2-2R+\varepsilon)}{2R-\varepsilon}$$

$$\mathrm{P}U_1 + (1-\mathrm{P})U_2 = 1 + (R-1)\frac{2R + \varepsilon - 2P\varepsilon - 2 + 2P}{(2R-\varepsilon)(1+\varepsilon)} + (R-1)\frac{2(1-P)}{2R-\varepsilon}$$

$$\mathrm{P}U_1 + (1-\mathrm{P})U_2 = 1 + (R-1)\frac{2R + \varepsilon - 2P\varepsilon - 2 + 2P}{(2R-\varepsilon)(1+\varepsilon)} + (R-1)\frac{2(1-P)(1+\varepsilon)}{(2R-\varepsilon)(1+\varepsilon)}$$

$$\begin{aligned}&\mathrm{P}U_1 + (1-\mathrm{P})U_2 \\ &= 1 + (R-1)\ (2R + \varepsilon - 2P\varepsilon - 2 + 2P + 2 + 2\varepsilon - 2P - 2P\varepsilon)/(2R \\ &- \varepsilon)(1+\varepsilon)\end{aligned}$$

$$\mathrm{P}U_1 + (1-\mathrm{P})U_2 = 1 + (R-1)\frac{2R + 3\varepsilon - 4P\varepsilon}{(2R-\varepsilon)(1+\varepsilon)}$$

$$\mathrm{P}'\mathrm{U}_{\mathrm{SI1}}(P') + (1-\mathrm{P}')\mathrm{U}_{\mathrm{SI2}}(P') = 1 + (R-1)\frac{2R + 3\varepsilon - 4P'\varepsilon}{(2R-\varepsilon)(1+\varepsilon)}$$

$$\mathrm{P}'\mathrm{U}_{\mathrm{SI1}}(P) = P' + (R-1)\frac{P'(2R + 2\beta R + \varepsilon - \beta\varepsilon - 2)}{\beta(2R-\varepsilon)(1+\varepsilon)} + \frac{P'}{2}[\frac{4R-4-2\varepsilon}{\beta(2R-\varepsilon)}]$$

$$(1-\mathrm{P}')\mathrm{U}_{\mathrm{SI2}}(P)=(1-P')+(R-1)\frac{2(1-P')}{2R-\varepsilon}+\frac{\beta(1-P')}{2}[\frac{4-4R+2\varepsilon}{\beta(2R-\varepsilon)}]$$

$$\begin{aligned}\mathrm{P}'\mathrm{U}_{\mathrm{SI1}}(P)&+(1-\mathrm{P}')\mathrm{U}_{\mathrm{SI2}}(P)\\&=1+\left(\frac{P'}{P}-P'\right)(R-1)\frac{(2R+2\beta R+\varepsilon-\beta\varepsilon-2)}{(2R-\varepsilon)(1+\varepsilon)}\\&+\left(\frac{P'}{P}-P'\right)\frac{2R-2-\varepsilon}{2R-\varepsilon}+(R-1)\frac{2(1-P')}{2R-\varepsilon}+\frac{(1-P')(2-2R+\varepsilon)}{2R-\varepsilon}\end{aligned}$$

$$\mathrm{P}'\mathrm{U}_{\mathrm{SI1}}(\mathrm{P})+(1-\mathrm{P}')\mathrm{U}_{\mathrm{SI2}}(P)>\mathrm{P}'\mathrm{U}_{\mathrm{SI1}}(P')+(1-\mathrm{P}')\mathrm{U}_{\mathrm{SI2}}(P')$$

$$\begin{aligned}1+(\mathrm{R}-1)&\frac{2\mathrm{R}+3\varepsilon-4\mathrm{P}'\varepsilon}{(2R-\varepsilon)(1+\varepsilon)}\\&<1+\left(\frac{P'}{P}-P'\right)(R-1)\frac{(2R+2\beta R+\varepsilon-\beta\varepsilon-2)}{(2R-\varepsilon)(1+\varepsilon)}\\&+\left(\frac{P'}{P}-P'\right)\frac{2R-2-\varepsilon}{2R-\varepsilon}+(R-1)\frac{2(1-P')}{2R-\varepsilon}+\frac{(2-2R+\varepsilon)(1-P')}{2R-\varepsilon}\end{aligned}$$

$$\begin{aligned}1+(\mathrm{R}-1)&\frac{2\mathrm{R}+3\varepsilon-4\mathrm{P}'\varepsilon}{(2R-\varepsilon)(1+\varepsilon)}\\&<1+\left(\frac{P'}{P}-P'\right)(R-1)\frac{(2R+2\beta R+\varepsilon-\beta\varepsilon-2)}{(2R-\varepsilon)(1+\varepsilon)}\\&+\left(\frac{P'}{P}-P'\right)\frac{2R-2-\varepsilon}{2R-\varepsilon}+(R-1)\frac{2(1-P')}{2R-\varepsilon}+(P'-1)\frac{(2R-2-\varepsilon)}{2R-\varepsilon}\end{aligned}$$

$$\begin{aligned}(\mathrm{R}-1)&\frac{2\mathrm{R}+3\varepsilon-4\mathrm{P}'\varepsilon}{(2R-\varepsilon)(1+\varepsilon)}\\&<\left(\frac{P'}{P}-P'\right)(R-1)\frac{(2R+2\beta R+\varepsilon-\beta\varepsilon-2)}{(2R-\varepsilon)(1+\varepsilon)}+\left(\frac{P'}{P}-1\right)\frac{2R-2-\varepsilon}{2R-\varepsilon}\\&+(R-1)\frac{2(1-P')}{2R-\varepsilon}\end{aligned}$$

$$\begin{aligned}(\mathrm{R}-1)&\frac{2\mathrm{R}+3\varepsilon-4\mathrm{P}'\varepsilon}{(2R-\varepsilon)(1+\varepsilon)}\\&<P'\times\frac{1-P}{P}\times(R-1)\frac{(2R+2\beta R+\varepsilon-\beta\varepsilon-2)}{(2R-\varepsilon)(1+\varepsilon)}+\left(\frac{P'}{P}-1\right)\frac{2R-2-\varepsilon}{2R-\varepsilon}\\&+(R-1)\frac{2(1-P')}{2R-\varepsilon}\end{aligned}$$

$$(\mathrm{R}-1)\frac{2\mathrm{R}+3\varepsilon-4\mathrm{P}'\varepsilon}{(2R-\varepsilon)(1+\varepsilon)}$$
$$<\frac{P'}{P}(R-1)\frac{(2R+\varepsilon-2P\varepsilon+2P-2)}{(2R-\varepsilon)(1+\varepsilon)}+\left(\frac{P'}{P}-1\right)\frac{2R-2-\varepsilon}{2R-\varepsilon}+(R-1)\frac{2(1-P')}{2R-\varepsilon}$$

$$(\mathrm{R}-1)\frac{2R+\varepsilon-2P'\varepsilon-2+2P'}{(2R-\varepsilon)(1+\varepsilon)}$$
$$<\frac{P'}{P}(R-1)\frac{(2R+\varepsilon-2P\varepsilon+2P-2)}{(2R-\varepsilon)(1+\varepsilon)}+\left(\frac{P'}{P}-1\right)\frac{2R-2-\varepsilon}{2R-\varepsilon}$$

$$(\mathrm{R}-1)\frac{2PR+P\varepsilon-2PP'\varepsilon-2P+2PP'}{(2R-\varepsilon)(1+\varepsilon)}$$
$$<(R-1)\frac{(2P'R+P'\varepsilon-2P'P\varepsilon+2PP'-2P')}{(2R-\varepsilon)(1+\varepsilon)}+(P'-P)\frac{2R-2-\varepsilon}{2R-\varepsilon}$$

$$(\mathrm{R}-1)\frac{(P-P')(2R+\varepsilon-2)}{(1+\varepsilon)}<(P'-P)(2R-2-\varepsilon)$$

$$(\mathrm{R}-1)(2\mathrm{R}+\varepsilon-2)<(\varepsilon+2-2R)(1+\varepsilon)$$

$$2R^2-\varepsilon^2+3\varepsilon R-2R-4\varepsilon<0$$

$$2R^2+(3\varepsilon-2)R-\varepsilon^2-4\varepsilon<0$$

Since $\frac{2-3\varepsilon}{4}<1$ and $R\in(1,1+\frac{\epsilon}{2})$, we can see that $2R^2+(3\varepsilon-2)R-\varepsilon^2-4\varepsilon$ is increasing in $(1,1+\frac{\epsilon}{2})$, so we can check the extreme case where R=1 and $R=1+\frac{\epsilon}{2}$ to see whether the above inequity will be satisfied.

When R=1

$$2R^2+(3\varepsilon-2)R-\varepsilon^2-4\varepsilon=-\varepsilon^2-3\varepsilon<0$$

While R=$1+\frac{\epsilon}{2}$

$$2R^2+(3\varepsilon-2)R-\varepsilon^2-4\varepsilon=\varepsilon^2>0$$

As a result when R<R*, the above inequity will be satisfied which means the all participating contract can be collusive proof. While R>R* the above inequity will not be satisfied which means that it becomes possible for agents to form a smaller trading group and increase the utility level inside the group.

Case 2

$$I_1 = \frac{(4\beta + 2)\varepsilon + 4R - 4 + 4\beta}{\beta(\varepsilon - 2R + 4)(3\varepsilon + 2R)}$$

$$I_2 = \frac{(2\beta + 4)\varepsilon + 4 - 4\beta + 4\beta R}{(\varepsilon - 2R + 4)(3\varepsilon + 2R)}$$

$$x_1 = \frac{(2\beta + 1)\varepsilon^2 + (4\beta R + 4R - 2\beta - 4)\varepsilon + (4R\beta - 8R + 4R^2 + 4 - 4\beta)}{\beta(\varepsilon - 2R + 4)(3\varepsilon + 2R)}$$

$$x_2 = \frac{(\beta + 2)\varepsilon^2 + (4R + 4R\beta - 4\beta - 2)\varepsilon + (4R + 4\beta - 4 - 8\beta R + 4\beta R^2)}{\beta(\varepsilon - 2R + 4)(3\varepsilon + 2R)}$$

$$\mathrm{P}U_1 + (1 - \mathrm{P})U_2 = 1 + (R - 1)\frac{(1 - P)[(6\beta + 6)\varepsilon + 4R + 4\beta R]}{(\varepsilon - 2R + 4)(3\varepsilon + 2R)}$$

$$\mathrm{P}U_1 + (1 - \mathrm{P})U_2 = 1 + \frac{(R - 1)(1 - P)(\beta + 1)(6\varepsilon + 4R)}{(\varepsilon - 2R + 4)(3\varepsilon + 2R)}$$

$$\mathrm{P}U_1 + (1 - \mathrm{P})U_2 = 1 + (R - 1)\frac{2}{\varepsilon - 2R + 4}$$

$$\mathrm{P}'\mathrm{U}_{\mathrm{SI1}}(P') + (1 - \mathrm{P}')\mathrm{U}_{\mathrm{SI2}}(P') = 1 + (R - 1)\frac{2}{\varepsilon - 2R + 4}$$

$$
\begin{aligned}
&\mathrm{P}'\mathrm{U}_{\mathrm{SI1}}(P) + (1-\mathrm{P}')\mathrm{U}_{\mathrm{SI2}}(P) \\
&\qquad = P' + (R-1)\frac{P'[(4\beta+2)\varepsilon+4R-4+4\beta]}{\beta(\varepsilon-2R+4)(3\varepsilon+2R)} \\
&\qquad + \frac{P'(\beta-1)(4R^2-12R+8-2\varepsilon-\varepsilon^2)}{2\beta(\varepsilon-2R+4)(3\varepsilon+2R)} + (1-P') \\
&\qquad + (R-1)\frac{(1-P')[(2\beta+4)\varepsilon+4-4\beta+4\beta R]}{(\varepsilon-2R+4)(3\varepsilon+2R)} \\
&\qquad + \frac{(1-P')(\beta-1)(\varepsilon^2+2\varepsilon+12R-4R^2-8)}{2(\varepsilon-2R+4)(3\varepsilon+2R)}
\end{aligned}
$$

$$
\begin{aligned}
&\mathrm{P}'\mathrm{U}_{\mathrm{SI1}}(P) + (1-\mathrm{P}')\mathrm{U}_{\mathrm{SI2}}(P) \\
&\qquad = 1 + (R-1)\left(\frac{P'}{P}-1\right)\frac{(4\beta+2)\varepsilon+4R-4+4\beta}{(\varepsilon-2R+4)(3\varepsilon+2R)} \\
&\qquad + (R-1)\frac{2(1-P')}{(1-P)(\varepsilon-2R+4)} \\
&\qquad + \left(\frac{P'}{P}-1\right)\frac{(\beta-1)(\varepsilon^2+2\varepsilon+12R-4R^2-8)}{2(\varepsilon-2R+4)(3\varepsilon+2R)}
\end{aligned}
$$

$$
\mathrm{P}'\mathrm{U}_{\mathrm{SI1}}(\mathrm{P}) + (1-\mathrm{P}')\mathrm{U}_{\mathrm{SI2}}(P) > \mathrm{P}'\mathrm{U}_{\mathrm{SI1}}(P') + (1-\mathrm{P}')\mathrm{U}_{\mathrm{SI2}}(P')
$$

$$
\begin{aligned}
&1 + (R-1)\left(\frac{P'}{P}-1\right)\frac{(4\beta+2)\varepsilon+4R-4+4\beta}{(\varepsilon-2R+4)(3\varepsilon+2R)} + (R-1)\frac{2(1-P')}{(1-P)(\varepsilon-2R+4)} \\
&\qquad + \left(\frac{P'}{P}-1\right)\frac{(\beta-1)(\varepsilon^2+2\varepsilon+12R-4R^2-8)}{2(\varepsilon-2R+4)(3\varepsilon+2R)} \\
&\qquad > 1 + (R-1)\frac{2}{\varepsilon-2R+4}
\end{aligned}
$$

$$
\begin{aligned}
&(\mathrm{R}-1)\left(\frac{P'}{P}-1\right)\frac{(4\beta+2)\varepsilon+4R-4+4\beta}{(\varepsilon-2R+4)(3\varepsilon+2R)} \\
&\qquad + \left(\frac{P'}{P}-1\right)\frac{(\beta-1)(\varepsilon^2+2\varepsilon+12R-4R^2-8)}{2(\varepsilon-2R+4)(3\varepsilon+2R)} \\
&\qquad > (R-1)[1-\frac{(1-P')}{(1-P)}]\frac{2}{\varepsilon-2R+4}
\end{aligned}
$$

$$(\mathrm{R}-1)\left(1-\frac{P'}{P}\right)\frac{(4\beta+2)\varepsilon+4R-4+4\beta}{(3\varepsilon+2R)} + \left(1-\frac{P'}{P}\right)\frac{(\beta-1)(\varepsilon^2+2\varepsilon+12R-4R^2-8)}{2(3\varepsilon+2R)} < 2(R-1)[\frac{(1-P')}{(1-P)}-1]$$

$$(\mathrm{R}-1)\frac{P-P'}{P}\times\frac{(4\beta+2)\varepsilon+4R-4+4\beta}{3\varepsilon+2R} + \frac{P-P'}{P}\times\frac{(\beta-1)(\varepsilon^2+2\varepsilon+12R-4R^2-8)}{2(3\varepsilon+2R)} < 2(R-1)\frac{P-P'}{1-P}$$

$$(\mathrm{R}-1)\frac{P-P'}{P}[(8\beta+4)\varepsilon+8R-8-8\beta] + \frac{P-P'}{P}(\beta-1)(\varepsilon^2+2\varepsilon+12R-4R^2-8) < (R-1)\frac{P-P'}{1-P}\times 4(3\varepsilon+2R)$$

$$(\mathrm{R}-1)\frac{P-P'}{P}\{[8P+4(1-P)]\varepsilon+8(1-P)R-8(1-P)+8P\} + \frac{P-P'}{P}(2P-1)(\varepsilon^2+2\varepsilon+12R-4R^2-8) < 4(R-1)(P-P')(3\varepsilon+2R)$$

$$(\mathrm{R}-1)\{[8P+4(1-P)]\varepsilon+8(1-P)R-8(1-P)+8P\} + (2P-1)(\varepsilon^2+2\varepsilon+12R-4R^2-8) < 4(R-1)(3P\varepsilon+2RP)$$

$$(\mathrm{R}-1)(4\varepsilon+4\mathrm{P}\varepsilon+8\mathrm{R}-8\mathrm{RP}-8+16\mathrm{P}-12\mathrm{P}\varepsilon-8\mathrm{PR}) < (2P-1)(8+4R^2-12R-\varepsilon^2-2\varepsilon)$$

$$(\mathrm{R}-1)(4\varepsilon-8P\varepsilon+8R-16RP-8+16P) < (2P-1)(8+4R^2-12R-\varepsilon^2-2\varepsilon)$$

$$(\mathrm{R}-1)(2\mathrm{P}-1)(8-8\mathrm{R}-4\varepsilon) < (2P-1)(8+4R^2-12R-\varepsilon^2-2\varepsilon)$$

$$(\mathrm{R}-1)(8-8\mathrm{R}-4\varepsilon) < (8+4R^2-12R-\varepsilon^2-2\varepsilon$$

$$8\mathrm{R}-8R^2-4\varepsilon R-8+8R+4\varepsilon < 8+4R^2-12R-\varepsilon^2-2\varepsilon$$

$$28\mathrm{R}-12R^2-16-4\varepsilon R+6\varepsilon+\varepsilon^2 < 0$$

$$(\mathrm{R}-1)(4\varepsilon-16+8\mathrm{R}) > (\varepsilon+2-2R)(\varepsilon+2R)$$

Since $\mathrm{R} < 1+\frac{\varepsilon}{2}$ and $\varepsilon < 1$, we can infer that $\mathrm{R} < 1.5$. So $4\varepsilon-16+8\mathrm{R} < 0$, $\varepsilon+2-2\mathrm{R} > 0$, $\mathrm{R}-1 > 0$ and $\varepsilon+2\mathrm{R} > 0$. As a result, the above inequity will not be satisfied. Which means it is possible for agents to form a smaller collusion and increase the utility level inside the group.