Three Essays on the Housing Market

by

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Thesis

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Declarations

I confirm that this work was done wholly while in candidature for a research degree at the University of Warwick. Where I have consulted the published work of others, this is always clearly attributed. Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work. I have acknowledged all main sources of help. None of this work has been published before submission.

Abstract

Chapter 1 is an overview of the thesis in which I explain why work on housing markets merits attention, discuss two broad questions that motivated the research, emphasise the particular avenue I have chosen to pursue, and summarise the new insights to be learned. I also include a short discussion on the methodologies that are used.

In Chapter 2, I introduce information heterogeneity into a user-cost house pricing model. I use the model to shed light on two empirical regularities in the housing market: the predictability of housing return and the positive relationship between rent volatility and housing prices. The model also has predictions on over-pricing and housing price excess volatility.

In Chapter 3, I study a Real Business Cycle model with borrowing constraints and incomplete information. I show that in such an environment noises in signals may have real impacts on the macroeconomy; the effects are induced by learning and amplified and propagated by the collateral effects. Noises may generate sizeable and persistent fluctuations on consumption, credit, asset price, and output.

In Chapter 4, I implement a new strategy to identify shocks that drive the co-movements between housing price and consumption. My results show that, in the United Kingdom, productivity shocks and especially news shocks about future productivity explain most of the co-movements. I also show that more than half of the changes in housing price growth were not related to the changes in consumption growth, which casts doubt on the importance of housing wealth effects on consumption.

Chapter 1

Introduction and Motivation

1.1 Why the Housing Market?

Housing is an asset class central to the households; in many countries, it makes up the largest component of wealth. In the United States (U.S.), for instance, housing wealth alone is nearly equal to all non-housing wealth for households. The housing market lies at the heart of economic and financial crises across the world. Banking crises triggered by housing market collapses have occurred in many developed countries as well as in emerging countries.[1] In the United States, nine of the eleven recessions since World War II were preceded by sustained and substantial problems in the housing market.

To the extent that the housing market has been playing so prominent a role in the economy, it is surprising that it had been of little interest to the mainstream macroeconomists before the Great Recession. As Leamer (2007) observed, no macroeconomics textbook placed "real estate" or "housing" front and centre—even the National Bureau of Economic Research (NBER) macroeconomics data had largely missed housing too. A similar observation comes from Iacoviello (2010), one of whose papers on housing market was once rejected by a macro field journal, because the editor thought that the paper "*focuses on a small niche—the housing market—with limited evidence that this market has the significance that is implied for real economic activity (July 17, 2001)*."

The fact that mainstream economists largely ignored the housing market suggests that the market, and in particular its interactions with the macroeconomy, could not have been understood well. The inadequate attention paid to—and hence the limited understanding of—the housing market, may have led to the widespread failure of the profession to foresee the recent burst of the housing bubble in the United States (as well as in many other countries), and its long-lasting effects on

[1] Examples are the United Kingdom (U.K.) in 1973, Spain in 1977, the United States in 1986, Norway in 1987, Finland and Sweden in 1991, Japan in 1992, and some Asian countries in 1997-1998.

the broader economy.[2] Thanks to the 2008-2009 crisis, however, more research attention has now been directed to the housing market; and substantial progress in understanding the market, as well as its relationship with the macroeconomy, has been made since then. We are, nevertheless, still left with many questions open for more satisfactory answers.

1.2 Two Broad Questions

Among those questions, there are two very fundamental ones: (1) *what determines the market price of housing?* (2) *how does the housing market affect the wider economy?*

The first question is always intriguing, particularly when there has been a continuous appreciation of housing prices in a market but the "fundamentals," such as strong economic growth and rigid supply, seem not enough to justify the boom. In such cases, housing is often considered as being overvalued, or more arguably, there exist "housing bubbles."[3] An understanding of the nature of such booms is desirable, especially by the policymakers, since it helps to predict whether a boom is sustainable or not.

In general, there might be more than one factor behind each boom, and the main driving forces may also vary in different times and markets. Taking the 2000-2006 housing boom in the United States for example, some blame the relaxed standards for mortgage loans or monetary policy, whereas others attribute it to the predatory lending and securitisation or even irrational exuberance. If the mechanisms and factors that induce undesirable housing cycles—presumably harmful insofar as they give rise to imbalances in the economy—can be figured out, micro-prudential and macro-prudential regulations could be imposed. To achieve this, however, we need both empirical research and theoretical reasoning.

[2] In a timely staff working paper that was prepared for Federal Reserve Bank of Kansas City's 2007 Jackson Hole Symposium, Miskin (2007) reviewed the role of housing in the monetary transmission mechanism that had been known to the economists. Despite that the market had already shown significant deteriorations at that moment, he seemed still optimistic about the prospect of the economy, as he wrote in the introduction that "Fortunately, the overall financial system appears to be in good health, and the U.S. banking system is well positioned to withstand stressful market conditions." Even Leamer (2007), who had recognised and forcefully emphasised the importance of housing market in the business cycle, also underestimated the impacts, as he thought "this time troubles in housing will stay in housing."(Leamer, 2007, p. 155). Bezemer (2009) identifies twelve analysts who predicted the crisis but none of them is from the mainstream.

[3] Researchers do not often agree on whether the housing prices in a market are overvalued. For example, during the run-up phase of the recent housing cycle in the United States, Shiller (2005) argued that the run-up was unprecedented and represented a housing bubble, whereas McCarthy and Peach (2004) and Himmelberg, Mayer, and Sinai (2005) argued that the home valuations were mostly in line with fundamentals. With hindsight, it seems Shiller was correct. But the fact that the prices after the collapse have been lingering around the level of the year 2004 also alludes that, the judgements in the two latter papers were not incorrect by that time.

One might argue that there is no need to have specific pricing theories for housing because it is just one type of assets and we have already had many well-established asset pricing theories. Though housing shares many similarities with other assets such as stocks, it has many distinct features that make it deserve special attention. For example, housing is not only an asset but also a consumption good that provides shelter services—a necessity to the households. Moreover, the size of a purchase of a home is generally very large and, as a consumer durable, it usually has very long life; hence housing often comprises a large and potentially volatile share of the household balance sheet. These distinctions can make the participants in the housing market very different from those in other asset markets, with respect to the risk-aversion and incentives, for example. Housing is also tangible and immobile. As a result, the costs involved in housing transactions, both economic and psychological, are large, and the location is very important in price determination. Housing is also different from other assets such as equity in the supply process. The supply of housing incurs land-buying and construction work. Because of the rigid supply of land in many countries nowadays, the relatively long construction process, as well as the slow depreciation, changes in housing supply in response to the changes of market conditions tend to be sluggish.

These and other features, which I will discuss in due course, can make the characteristics of housing market quite different from those of other asset markets. Figure 1.1 shows the historical evolution of prices of homes and stocks in the United States since 1975. It can be seen that housing prices are less volatile than stock prices in the short run; this may reflect the large cost involved in the transaction, which prevents the buying and selling from being too active. The figure also shows that, while closely related, the booms and busts of stock market and those of housing market do not always happen at the same time; there was a stock market crash in 1987—though from today's perspective it was more like a blip—the housing markets in many cities in the U.S. nevertheless only fell two years later, about the time the stock market collapsed again. Another more significant "out-of-step" episode is around the year 2000 when the stock market dot-com bubble burst, but housing prices did not decline at all. Differences in the dynamics of housing and stock prices can also be found in many other countries.

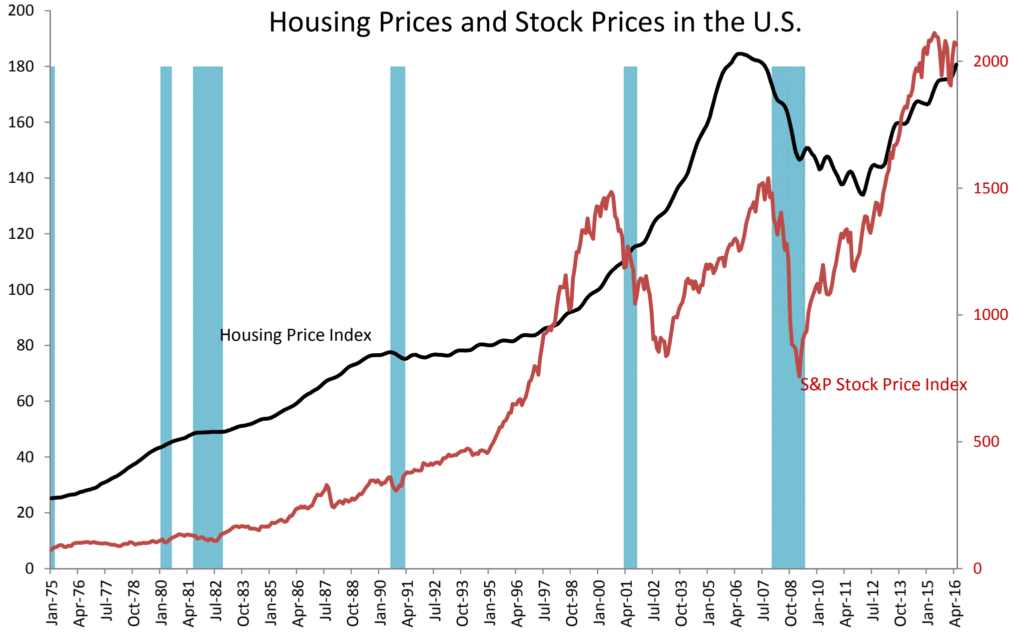


Figure 1.1: Housing Prices and Stock Prices in the United States

Notes: Grey areas indicate NBER recession dates (see www.nber.org/cycles.html). Housing price index and Standard & Poor's stock price index are from http://irrationalexuberance.com/. Both indices are in nominal terms.

The second fundamental question arises in the aftermath of the recent housing market bust. The recession induced by the bust and the slow recovery have led many to believe that developments in the housing sector might not be just a passive representation of macroeconomic activity but instead one of the driving forces of business cycles.

Previous research has identified at least three ways that housing market may contribute to the macroeconomic fluctuations. First, residential investment itself is part of the Gross Domestic Product (GDP). In the United States, for instance, the contribution of residential investment to the weakness before recessions and to the recovery after recessions are found to be substantial (Leamer, 2007), despite of its relatively small share in GDP.[4] The mechanisms behind this have not been completely understood, and current models, both econometric and structural, cannot fully account for the dynamics of residential investment.[5] Second, the housing

[4]The share of housing investment in GDP has been constant around 5 percent throughout the 1952-2008 period (Iacoviello, 2010).

[5]Several models that are employed by the Federal Reserve Board in the U.S. take into account many mechanisms but still do a poor job (Mishkin, 2007). Iacoviello and Neri (2010)'s estimation using a dynamic stochastic general equilibrium (DSGE) model improves the result significantly. In

may affect business investment. Applying a Bayesian vector autoregression (BVAR) model to the data from the U.S., Liu, Wang, and Zha (2013) find that there is a salient co-movement between land prices and business investment after a shock to the former.[6] They propose a collateral mechanism to explain this result, based on the facts that real estate is an important collateral asset for both small firms and large corporations,[7], and the value of land holdings affects firms' borrowing capacity and thereby their business investment and production. Finally, the housing may affect consumer spending. The strong co-movement between housing prices and consumption has long be found, and there is a very large literature in estimating the marginal propensity to consume out of housing wealth, but no firm conclusion has been reached. The theoretical mechanism behind this co-movement remains controversial as well; while some argue for the housing wealth effect derived from a life-cycle model (Case, Quigley, and Shiller, 2005, 2011), others believe it is the collateral channel that is at work (Iacoviello, 2005), or perhaps that the co-movements simply reflect the common factors that are driving both (Attanasio, Leicester, and Wakefield, 2011).

1.3 Focus of This Thesis

In the following three chapters, I provide theoretical models and empirical evidence on topics that are directly or indirectly related to the two questions discussed above. The aim is not to be comprehensive, but to make complementary contributions to the existing literature.

Each of these chapters is self-contained and studies the housing market from a specific perspective; but they all share a similar focus—*information*. Chapter 2 introduces information heterogeneity into a house pricing model and explores its implications for some empirical issues. Chapter 3 characterises a macroeconomic model in which incomplete information may induce noise to drive "credit cycles." Chapter 4 shows, empirically, how the information about future productivity may help explain the co-movement between housing prices and consumption.

My focus on information is both realistic and hopefully of interest. It is realistic because neither the current nor the future state of the world is perfectly known by economic agents, and information plays a nontrivial role in the agents' decisions. It should be of interest because, although the implications of information

an extension of the model I set up in Chapter 2, I allude to an informational channel that may be worth exploring.

[6] They focus on land prices because most of the fluctuations in housing prices are driven by land prices rather than by the cost of structures.

[7] With a slight abuse of notation, I use the terms "real estate," "housing," and "houses" interchangeably in the thesis. Nearly 70% of all commercial and industrial loans in the United States is secured by collateral assets (Liu, Wang, and Zha, 2013).

problems for markets such as the stock market have been explored extensively, much less has been done regarding the housing market.

The theoretical analyses in the first two chapters also recognise several distinct features of housing. The first feature is that, since a house is both a basic consumption good and an asset, there is both a rental market and an asset market for housing. The non-trivial scale of the rental market implies that rents may have implications on house pricing more than merely as the dividends of housing asset. The second feature is that, in contrast to many other financial assets, housing is almost impossible to short-sell. If households have different opinions about the market, the short-sale constraint may result in the houses being overpriced because the pessimistic investors cannot act to counter optimism. Finally, housing or land is, in practice, commonly used as collateral by firms to obtain credit from the banks to finance business investment. While the value of housing is determined by the production of other goods in the economy, a feedback effect from housing to production may be significant enough to amplify economic fluctuations.

Though all of the distinct aspects of housing discussed above are already well known, their implications have not been fully explored. My contribution, especially in the two theoretical chapters, is that, when we combine these distinctive features with information imperfection in the housing market, new insights that are discussed below come out.

In relation to the two questions discussed in Section 1.2, Chapter 2 addresses the first by providing a house pricing model with information heterogeneity. It shows that, when agents hold heterogeneous information and cannot short-sell in the housing market, the housing can be over-priced. The model also features the consumption role of housing and there exists a rental market. Rental prices are derived as a result of the market equilibrium; they clear the rental market and at the same time determine the house prices through a non-arbitrage condition. More interestingly, as households hold heterogeneous information, both housing prices and rental prices also play informational roles—they are used as the public signals by the market participants to infer the unobserved fundamentals. These novel features are then used to explain the return predictability of rental prices documented in Glaeser and Gyourko (2007), and the "rent volatility-housing price" relationship documented in Sinai and Souleles (2005).

Chapter 3 and 4 relate to the second question discussed in Section 1.2. In Chapter 3, I construct a model where there are two types of agents—households and entrepreneurs—and land (housing) plays several roles in the economy. Households are patient in consuming land (housing) and consumption goods and saving. The entrepreneurs, who produce consumption goods using land as input, are impatient and need to borrow from the households; however, borrowing must be secured against land and is limited by the households' expected value of entrepreneurs' land holdings. Neither type of agent has complete information about the fundamentals of the economy, and land prices and some statistic of land prices will serve as informative signals, used by agents to infer unobserved fundamentals. High price signals, which may be due to pure noises, can lead households and entrepreneurs to believe a strong current economic condition prevails as well as an optimistic perspective for the future. As a result, households are willing to lend more and entrepreneurs are able to get more funds for production. That is, pure noise may drive optimism or pessimism in the expectations of agents, which, through expanding or shrinking the credit supply, generates macroeconomic fluctuations.

Chapter 4 is an empirical examination of the relationship between the price of housing and consumption, using the data of the United Kingdom. As pointed out in the previous section, many believe that housing has significant wealth effect on the consumer spending, but neither the theoretical underpinning of the effect nor its empirical magnitude is uncontroversial. Realising that much of the previous empirical work using macroeconomic data suffers from the endogeneity problem due to interdependency or omitted, unobservable variables, I estimate a Vector Autoregression (VAR) model. Though the VAR model estimation does not generate coefficients for the wealth effect or collateral effect directly, it gives indirect evidence. More specifically, based on the estimated VAR model and a novel identification strategy (assumption), I identify the productivity shock and news shock about future productivity. The investigation following the identification shows that it is these common factors that drive both the housing price and consumption and that a vast majority of the variations in housing price are not related to consumption. In other words, the results imply that the impacts of housing on macroeconomy through the consumption channel may not be that important.

1.4 Methodology

In the theoretical parts of this thesis, i.e. Chapter 2 and 3, I follow the traditional paradigm to understand housing market and the macroeconomy by using models in which agents are "rational". Furthermore, I employ the Rational Expectations Equilibrium (REE) framework in these models. The assumption that agents are "rational" means two things (Barberis and Thaler, 2003). First, agents update beliefs correctly using Bayes' law in receiving new information. Second, given their beliefs, agents make choices that are consistent with their subjective expected utility. Rational expectations equilibrium further requires "consistent beliefs"—the subjective distribution agents use to forecast future realisations of unknown variables is indeed the distribution that those realisations are drawn from. Hence, REE requires not only that agents process new information correctly but also that they have enough information about the structure of the economy to be able to figure out the correct distribution for the variables of interest.

The rational expectations hypothesis has become influential since the early 1970s and is now a ubiquitous modelling technique used widely throughout economics. In the meantime, it has also received many critiques, especially since the 2007-2009 crisis. There are at least two alternatives: bounded rationality and behavioural economics. The literature on bounded rationality retains individual rationality but relaxes the consistent beliefs assumption: while investors apply Bayes' law correctly, they lack the information required to know the actual distribution variables are drawn from. The literature on behavioural economics goes further and relaxes the individual rationality assumption: it analyses what happens when one or both of the two rationality assumptions are relaxed, which is typically based on the experimental evidence compiled by cognitive psychologists (Barberis and Thaler, 2003).

In the housing market literature, behavioural economics has gained substantial attention because of Shiller (2005)'s influential book *Irrational Exuberance* (2^{nd} edition). Behavioural economics argues that some features of the asset prices are most plausibly interpreted as deviations from fundamentals, *and* that these deviations are brought about by the presence of traders who are not fully rational (Barberis and Thaler, 2003). In his book, Shiller (2005) exhibits a graph which shows the housing price bubbles cannot be explained by fundamentals such as the income, population, and construction costs. He then argues for psychology to play the predominant role. While I agree that psychology may play an important role in housing appreciation, I think he has overstated the case, possibly for the sake of drawing people's attention to social epidemics. I show in Chapter 2 that several empirical regularities in the housing market, including the over-pricing, can also be explained by information heterogeneity without assuming the irrationality of investors.

Imposing rationality on agents is to have a kind of modelling discipline. As "errors can be used to explain anything," requiring rationality makes it harder to come up with *ad hoc* models. This is one reason to keep the rationality assumption. But the model in Chapter 2 has its own limitations. Because of its simple setup, the model only has qualitative implications and cannot be readily used for empirical assessment. However, Kasa, Walker, and Whiteman (2014) have shown that a dynamic model with information heterogeneity has the potential to explain the stock price dynamics in the United States; so I believe a dynamic version of the model in Chapter 2 will also have the capability to explain the recent housing booms emerged in many countries, such as the United States, the United Kingdom, Spain, Ireland, *et. al.*. But this does not mean that information heterogeneity should be the only cause. It's hard to believe the remarkable house price appreciations in those countries were merely driven by information heterogeneity and short-sale constraints, and I believe other factors, including social epidemics and credit market developments, have also significantly contributed.

Chapter 3 takes into account the credit market and show a two-way feedback can arise between the credit market and housing prices. However, because of the symmetric information "within island" and the assumption that land cannot be traded "across islands", the mechanism of over-pricing alluded in Chapter 2 is absent. Moreover, the model only looks at the effect of information friction on the demand side of the credit market, while the supply side of the credit market is muted. Yet the effect from the credit supply side was arguably much more important during the 2008 financial crisis, see e.g. Adrian and Shin (2010, 2014). The absence of endogenous financial shocks and financial institutions is also a weakness of this chapter (as well as Chapter 2). Implications from the interaction of information friction and financial institutions will be one focus of my future research.

For the empirical part of Chapter 4, I employ the Structural VAR (SVAR) method to examine the relationship between housing price and consumption. This is because the alternatives, such as the linear regression of one equation used by most of the previous research on this issue, may generate biased estimates because of endogeneity problems. While instrumental variables estimation or dynamic simultaneous equation models with plausible identification (which usually involves finding some "exogenous" variables) can be used to overcome such problems, finding the appropriate instruments or truly exogenous variables can be very difficult. For these reasons as well as some others (see e.g. Gottschalk (2001) for a survey), macro-economists turn to SVAR models which are designed to avoid these problems that often lead to "incredible" identification restrictions (Sims, 1980). SVAR models also need to be identified, but they treat all variables as endogenous and decompose all variables into expected and unexpected parts. The restrictions are imposed only on the unexpected part where plausible identifying restrictions are easier to find. Nevertheless, like the simultaneous equation models, SVAR models identification also suffers from the problem that the restrictions are imposed on *a priori* grounds and cannot be tested. As a result, the implications from SVAR models should be taken with some care.

Chapter 2

Housing Prices with Heterogeneous Information

2.1 Introduction

Housing rents have been observed to display some interesting behaviours in the housing markets. First, Glaeser and Gyourko (2007) find that, not only housing prices but also rents help to predict future returns on houses.[1] Second, Sinai and Souleles (2005) find that, the larger the rent volatility in a given market, the higher the housing price in that market. While Sinai and Souleles (2005) have provided a risk-hedging explanation for their finding, the return predictability problem still seems unresolved; as Glaeser and Gyourko (2007) observe, "rents may add predictive power to housing price change regressions even if we are not sure why they have this predictive power." From the efficient market point of view, neither housing prices nor rental prices should help to predict future return on houses. Both housing prices and rental prices are publicly available, easily observed by anyone, and if they can really predict future housing prices and thereby future returns, all rational investors will take advantage of them. However, if that were the case, the competition would drive out any predictable movements in housing prices. Nonetheless, empirical research on the housing market has consistently found evidence that suggests the predictive power of these prices. For example, Case and Shiller (1989, 1990) find that price-rent ratio has the predictive power for future returns.[2] Glaeser and Gyourko (2007) are the first to explicitly discuss the predictive power of rental prices, and they conjecture that rental prices might be providing some information not fully embedded in housing prices. They do not, however, formally model their

[1] To avoid confusion, all through the chapter, housing price means the purchase price of a house and rental price is the price for renting a house. I will use "rental price" and "rent" interchangeably in the following text, as rental price is equivalent to the rent per unit of house.

[2] See also Poterba (1991), Gallin (2008), and Engsted and Pedersen (2012). Ghysels, Plazzi, Torous, and Valkanov (2013) have a comprehensive literature review.

conjecture. This chapter then aims to provide a specific model to highlight the information value of rental prices in the housing market, so that not only the return predictability may be explained but also the relationship between rent volatility and housing price will be accommodated.

To this end, I construct a two-period model with a continuum of risk-neutral agents, who not only consume housing services but also speculate in the housing market for the future resale value of housing. The resale value is exogenously determined by a shock that is not directly observable to the agents. Instead, each agent independently receives a private noisy signal about the shock. To make an optimal forecast, each agent will need to use her privately received signal as well as price signals including both housing price and rental price to infer the value of the shock and thereby the resale value of housing. However, because neither aggregate housing service demand nor total housing supply is observable, information is not fully revealed in equilibrium. Prices are not fully revealing because even agents can subtract the rent 'noise' from the housing price signal, they still cannot tell whether a high housing price comes from high resale value or low house supply; even though rental price provides further information about the supply, it does not reveal perfectly to the agents as they are not sure whether a high rent is due to strong housing service demand or low supply. Nevertheless, both prices provide useful information to the agents, because housing prices aggregate private information in the market while rental prices complement to housing prices by providing more information. More specifically, rental prices not only refine agents' information by revealing themselves as one of the "noises" in the housing price signals but also increase the precision of housing price signals by providing some more information about the house supply. In equilibrium, agents will hold heterogeneous beliefs about the resale value, and housing price will be determined by the agents who are indifferent in buying and renting.

An important implication from the setup described above is that the market-implied posterior over the housing resale value conditional on housing prices and rental prices differs from the Bayesian posterior conditional on the same public prices information; the Bayesian posterior is derived from the joint distribution of housing resale value shocks and market equilibrium prices. Following Albagli, Hellwig, and Tsyvinski (2015), I call this difference the information aggregation wedge. Defining the future return on housing as the sum of *ex post* capital gain and rent from investing a house, the expected return conditional on an econometrician's information set will then become the difference between the econometrician's expected housing resale value and the market expected housing resale value. If the econometrician, who observes housing prices and rental prices, only knows the joint distribution of housing resale value shocks and market equilibrium prices, and fails to take into account the expectations heterogeneity in the housing market, then the expected return conditional on his information set will be the negative of the *nonzero* information aggregation wedge. Moreover, because agents in the markets are learning from prices, the wedge will be varying with both housing prices and rental prices. Hence, when the econometrician regresses the observed returns on the lagged housing prices and lagged rental prices, he will find nonzero coefficients on both price variables! It needs to be emphasised that both heterogeneous information and non-fully revealing equilibrium are required for this explanation of return predictability. If agents were homogeneously informed, there would be no need to learn from prices and prices would not be helpful in any case to predict future return. If agents were heterogeneously informed, but prices had reflected all information in the economy, then the private signals would be redundant to agents and there would be no difference between the market expectation and the expectation based on econometrician's information set. In either case, the expected return would be zero.

As agents are learning from prices, the model also has a potential to explain the documented positive relationship between housing price and rent volatility. Sinai and Souleles (2005) first show this pattern using data from the metropolitan statistical areas in the United States, and they give a risk-hedging explanation about this relationship. In a stylised model, they show when households are risk-averse, owning a house involves both taking housing asset price risk and hedging rent fluctuation risk. Which risk dominates on net in this trade-off largely depends on the households' expected length of stay and whether they move to correlated housing markets. When households' expected length of stay is large or when the spatial correlation in housing prices is high, larger local rent volatility tends to increase the households' home-ownership demand, which will be capitalised into higher housing price when housing supply is inelastic. In contrast with their explanation, my model works purely through an information channel given that agents are risk neutral. Housing price and rent volatility are correlated in my model because rent volatility affects the precision of price signals and, thereby, agents' optimal investment decisions in the housing market and, ultimately, the market equilibrium housing prices. The relationship tends to be positive because when the volatility of rents increases, the negative information effect from rental prices to housing prices becomes smaller; as a result, housing prices will be higher.

In addition to the two implications discussed above, the model also has some predictions about the level and volatility of housing prices. First, housing price is on average higher than its fundamental value in this model when agents cannot short sell in the housing market and their private signals are independently, identically exponentially distributed about the resale value. Albagli, Hellwig, and Tsyvinski

(2015) show, in an asset pricing model where agents are risk neutral and private signals are normally distributed around the future dividend, that overpricing happens only when the asset dividend function is dominated by the upside risk. I deviate from the normal assumption about the signal distribution and find that with an exponentially dispersed information structure, overpricing happens without the need to impose restrictions on the risk dominance. This finding is thus a complement to their results because the empirical distribution for private signals is not clear *a priori*, and there is no reason why the private signals must be symmetrically distributed. For instance, it might make more sense to assume that the majority of the population receives 'good' news about the underlying fundamentals in the 'good' times. For the volatility of housing prices, I show the model has the potential to generate excess volatility in housing prices, which does not arise in a homogeneous information model. However, because agents are learning from rental prices, the magnitude of excess volatility is significantly restricted.

The primary contribution of this chapter is that it highlights a particular but very natural channel through which rental prices might affect housing market informationally. Economists have long recognised the important role that prices play in aggregating and transmitting information in markets, and a vast literature seeks to understand the informational role played by prices for the stock market.[3] Much less has been done regarding the housing market. As opposed to the stock market where the only price in the market is stock price, there is always a rental price for the housing market because of the existence of the rental market. Traditional wisdom often just considers rent as the "dividend" of housing asset that determines the fundamental value of houses. But in an economy where agents are not perfectly informed about the state of the world, rental prices should also convey useful information, make households better informed, and affect their actions. More importantly, rental prices may not only affect the rental market itself but also affect the housing market at the same time. This is because housing rental market and housing asset market are inevitably intertwined, and the information in the rental market revealed by the rental prices may also help participants make optimal decisions in the housing market. No one, however, has formally examined the information role that rents could play in the housing market.

This chapter thus takes the first step and tries to formalise this information channel,[4] which is then used to explain the two empirical findings discussed above.

[3] Fama (1991) is a standard reference for the empirical tests, and Brunnnermeier (2001) is an excellent reference for the theory.

[4] In theory, housing prices as determined in the housing market may also affect agents' choice in the rental market. To make my argument as simple as possible, I preclude the effects of housing prices on rental prices. A richer setup could have been constructed; however, it may not offer additional clarity because many effects that are hard to disentangle will interact. Moreover, any other considerations about the information effects of rental prices should not miss the basic mechanism

The basic setup in the chapter follows the user-cost approach literature in which the equilibrium housing price is attained when market participants are indifferent as to whether to rent or to buy a house, and the cost of owning generally includes, among other variables, the interest rate, property tax rate, risk premium, expected rate of housing price appreciation. Previous research, such as Poterba (1984, 1991) and Himmelberg, Mayer, and Sinai (2005), has generally focused on the effects of tax rates and interest rates on housing price changes but left expected housing price appreciation unexplored. Favara and Song (2014) fill this gap by showing in a dispersed information model that heterogeneous expectations and no-short-sale constraints on housing are crucial in generating higher and more volatile housing prices than a homogeneous information benchmark. However, they neglect the informational role of rental prices. Gao, Sockin, and Xiong (2015) also study the implications of heterogeneous information to the housing market, but they focus on explaining the hump-shaped relationship between housing cycle and supply elasticity.

My model is also closely related to Albagli, Hellwig, and Tsyvinski (2011, 2015), who focus on how alternative payoff assumptions affect information aggregation and apply their model to examine the effects of skewness on expected returns in the stock market. More broadly, this chapter belongs to a growing literature that seeks to generalise noisy rational expectations models beyond the CARA-normal framework and explore the effects of relaxing these assumptions. For instance, Barlevy and Veronesi (2003) show that nonlinear equilibrium price functions which may generate discontinuous price changes or a "price crash," can emerge in a model with binomial payoffs. Breon-Drish (2015) shows in a setting with "exponential family" distributed payoffs, that shocks to fundamentals may be amplified purely due to learning effects. He also shows that price drifts can arise naturally and the disagreement-return relationship depends in a novel way on return skewness.

2.2 Model

In this section, I formalise the intuition described in the introduction part in a simple two-period model with agents having heterogeneous information. The model intends to generate qualitative implications with an emphasis on the effects of learning from rental prices.

2.2.1 Setup

The economy has two periods $t \in \{1, 2\}$, and there is a measure-of-one continuum of risk-neutral agents indexed by $i \in [0, 1]$. In $t = 1$ agents only consume housing service and in $t = 2$ they only consume non-housing consumption good. More

shown in this chapter.

specifically, each agent i's utility function is assumed to be:

$$U_i = A_i \ln B_i + C_i,$$

where A_i is agent i's housing service preference shock, B_i denotes i's desired quantity of housing services in the first period, C_i is i's non-housing consumption good in the second period.

In $t = 1$, nature draws $w \in \mathbb{R}$ from a normal distribution with zero mean and variance σ_w^2: $\tilde{w} \sim \mathbb{N}(0, \sigma_w^2)$.[5] In the following, I will call w the fundamental shock as I assume that w determines the resale value of a house P_f in $t = 2$, according to an increasing function $h(\cdot)$; that is, $P_f = h(w)$.[6] I assume w is not directly observable to the agents. However, conditional on w, each agent i receives a private signal $w_i \in [w, +\infty)$ independently and from an identical exponential distribution: $\tilde{w_i}|\tilde{w} = w \sim \exp(\lambda; w), \forall i$, where $\lambda > 0$ is the inverse scale parameter and w appears as a shift parameter. The conditional distribution function of the private signal of each agent i is then given by

$$F_{\tilde{w}_i|\tilde{w}}(w_i|w) = 1 - e^{-\lambda(w_i - w)}, w_i \geq w, \lambda > 0.$$

Though it is unconventional to assume an asymmetric distribution for private signals, the shifted exponential distribution not only enables me to analytically characterise the equilibrium but also generates some stronger results than those in the symmetric signal distribution models, e.g. normal distribution, and thus offers new insights about asset pricing with heterogeneous information. I further assume that the law of large numbers applies to the continuum of agents so that conditional on w the cross-sectional distribution of private signals *ex post* is the same as the *ex ante* distribution of agents' signals.

In addition to the private signal, each agent i is also endowed with income $M_i \in \mathbb{R}^+$ and housing service preference shock $A_i \in \mathbb{R}^+$ in the first period.[7] The agents do not observe each other's preference shock, nor do they know the aggregate preference shock. However, the distribution of the aggregate preference shock $\tilde{A}$ is a common knowledge. Specifically, I assume the aggregate preference shock is

[5]All through *this* chapter and the corresponding Appendix A, I use a tilde "$\sim$" on top of a letter to denote the corresponding random variable.

[6]Assuming P_f is increasing in w is without loss of generality. Under such an assumption, w could well be interpreted as the aggregate housing service demand shock in the second period. The results will remain unchanged if I were to assume a decreasing function for $h(w)$ and interpret w as the housing supply shock in the second period. It is also possible to consider w as some combination of both demand and supply shocks.

[7]As will be seen in the following analysis, the quasi-linear preference precludes any income effect on agents' demand for housing services or housing assets, thus the information content of individual income is irrelevant to the agents. To ensure agent's consumption in the second period is non-negative, i.e. $C_i \geq 0, \forall i \in [0, 1]$, I assume the income received by each agent M_i is large enough (see Appendix A.1 for a discussion). It then follows that the distribution of M_i's is inessential.

independent of $\tilde{w}$ and follows a log-normal distribution:[8]

$$\int A_i \mathrm{d}i = A \equiv e^a, \text{ where } \tilde{a} \sim \mathbb{N}(0, \sigma_a^2).$$

In period $t = 1$, each agent needs to decide if she buys houses to live or rents to live. If she buys houses, she can resell them at the price P_f in the next period to finance consumption. I assume agents are not allowed to short sell in the housing market. Therefore, anyone who expects a loss in investing houses will simply save the rest of their income after rent payment. On the other hand, if the agent expects it is profitable, she may want to borrow in addition to its endowed income to buy houses, but the units of houses she can buy will be restricted by a finite number $Z \geq 1$, due to for example regulation constraint or borrowing constraint. The credit fund is assumed to be supplied exogenously at the fixed rate R. Although the interest rate is very important to the housing market, it is not the focus of this chapter. Without loss of generality, I will set $R = 1$.

Houses are also supplied exogenously as $S = e^{-|\zeta|} \equiv e^s$, where $\tilde{\zeta} \sim \mathbb{N}(0, \sigma_s^2)$. This assumption implies that housing supply is bounded above by one, i.e. $S \in (0, 1]$ and is strictly increasing in s. Housing supply is independent of other random variables in the model and it is not observable to the agents.

2.2.2 Discussion

As in Favara and Song (2014), my preference specification makes strong assumptions: it assumes away intertemporal consumption-saving decision and intra-temporal consumption-housing decision. Piazzesi, Schneider, and Tuzel (2007) develop a consumption-based asset pricing model in which both decisions are kept and thus there exists a composition risk in addition to the consumption risk. They show the composition risk factor has important implications for asset prices. Since the focus of this chapter is on the information impacts of prices, the assumption in my model that agents are risk-neutral has assumed away those risk effects.

My specification also implicitly assumes that all houses are homogeneous. In particular, it does not distinguish the qualities between rental houses and owner-occupied houses. Smith and Smith (2006) and Glaeser and Gyourko (2007) show that on average owner-occupied housing is much larger and better than rental housing. This suggests that the information contained in rental price might not be very informative about the owner-occupied housing market. This is indeed important. However, as long as the two markets are not completely isolated, the mechanism characterised in the model will still exist.

[8]Relaxing the independence assumption to, for example, being correlated to $\tilde{w}$ is possible and may generalise some of the results in Section 2.3. However, a different approach may be needed to make the characterisation of equilibrium tractable.

In the setup above, I have also imposed an upper bound for the demand of housing for each agent. Given the unit measure of agents, this essentially imposes an upper bound for the total quantity of houses that can be demanded. To ensure that the housing market will always clear, a corresponding upper bound for the total house supply must be imposed as well. For simplicity, I have let the upper bound of total supply be one, in turn, Z then must be equal or larger than one. The specific sizes of these bounds are not crucial for my results as long as the market clearing is guaranteed. What's crucial is that agents cannot take unlimited positions in the housing market, because agents are risk-neutral and if they are allowed to take unlimited positions, prices would be perfectly revealing.

Given that the housing supply is crucial to my model, it is worth having a discussion about the motivation. It is helpful to think that, the total housing supply in the economy is a fixed number $\bar{S}$ which is common knowledge, and there are two types of agents. The first type is the continuum of agents described above, who can be called the "sophisticated households" or "rational speculators". The second type is a continuum of other agents, who do not speculate on housing but only buy whatever they want to live; they are naïve owner-occupied households. Therefore, the second type agents are "noise traders"; their opinions and trading patterns may subject to systematic biases (Shleifer and Summers, 1990). Suppose that the total number of houses demanded by agents of the second type is a random variable D_N. Then, the total supply to the sophisticated households is simply $S \equiv \bar{S} - D_N$, which cannot be directly observed by agents of the first type.

2.2.3 Optimality

Let $\mathbb{E}_i(\cdot)$ be the expectation of a random variable conditional on i's information set Ω_i in $t = 1$. Each agent i's problem is to choose B_i units of houses to live and H_i units of houses to invest conditional on her information set in $t = 1$, so that her life-time expected utility is maximised:

$$\max_{\{B_i, H_i\}} \mathbb{E}_i \tilde{U}_i = A_i \ln B_i + \mathbb{E}_i \tilde{C}_i,$$

subject to the budget constraint

$$C_i = R[M_i - PH_i + Q(H_i - B_i)] + P_f H_i, \tag{2.1}$$

and the trading constraint

$$H_i \in [0, Z].$$

To derive the optimal strategies chosen by agent i, I substitute (2.1) into his expected utility function and take partial derivatives w.r.t. B_i and H_i. This gives the following first order conditions

$$B_i = \frac{A_i}{Q}, \tag{2.2}$$

$$\mathbb{E}_i(-P + Q + \tilde{P}_f) \geq 0, H_i \geq 0, \tag{2.3}$$

$$\mathbb{E}_i(-P + Q + \tilde{P}_f) < 0, H_i < 0, \tag{2.4}$$

where I have let $R = 1$.

Equation (2.2) characterises agent i's housing service demand in the house rental market. That is, the demand for housing services will be higher when the agent has a stronger preference on housing service or/and when the rental price of a house is lower. Equation (2.3) and (2.4) characterise agent i's decision in the housing market. Whether she chooses to buy or not will be determined by its expectations of the resale value of houses in the second period. Since the agent is risk-neutral, she will buy houses when the cost of buying is less than the expected benefit from owning, i.e. $P < Q + \mathbb{E}_i\tilde{P}_f$. The agent will short sell houses if $P > Q + \mathbb{E}_i\tilde{P}_f$, but because it is not allowed to do this, she will simply demand zero of housing. The agent will be indifferent in buying and not buying if $P = Q + \mathbb{E}_i\tilde{P}_f$.

As the realisation of the fundamental shock $\tilde{w}$ is not directly observable to the agents, their asset trading decisions involve conditional expectations of an unknown w. They will need to make inference about w based on all information available to them so that their decisions made in the first period will be informationally optimal.

Apart from the exogenous signals agents observe, they can also extract information from the equilibrium prices: both rental prices and housing prices. Hence, the information set of each agent i is comprised of both exogenous private signals, endogenous price signals, and the model structure. That is, $\Omega_i = \{A_i, w_i, P, Q, \mathbb{M}\}$, where $\mathbb{M}$ captures the notion of rational expectations and the assumption that agents know the model structure. Because this is a standard assumption in the literature, without causing any confusion I will omit $\mathbb{M}$ in the following text. By assumption, observing one's own housing preference does not provide useful information about the aggregate preference shock; the idiosyncratic preference assumption only serves to prevent housing rental price from fully revealing supply shocks. Thus, the information set of each agent i can then be reduced as $\Omega_i = \{w_i, P, Q\}$ and $\mathbb{E}_i\tilde{P}_f = \mathbb{E}(\tilde{P}_f|w_i, P, Q)$. Each agent i's housing asset trading strategy is then a mapping from signal-prices (w_i, P, Q) into housing asset holdings $H_i : [w_i, P, Q] \rightarrow [0, Z]$.

2.3 Equilibrium

Following Ozdenoren and Yuan (2008), I assume that each agent i follows a cut-off strategy in the housing market as below

$$H_i = \begin{cases} 0, & \text{if } w_i < \breve{w}, \\ Z, & \text{if } w_i \geq \breve{w}, \end{cases} \tag{2.5}$$

where $\breve{w}$ is some endogenously determined threshold. That is, any agent whose private signal is larger than the cutoff value will buy as many units of houses as possible, while any agent whose private signal is lower than the threshold value will not buy any house. Let $F(w|\Omega_i)$ denote the posterior distribution function of $\tilde{w}$ conditional on agent i's information set. I then define a cut-off strategy equilibrium as below.

Definition 2.1 (Equilibrium) *A cutoff strategy equilibrium consists of two price functions: $P(a, s, w)$ and $Q(a, s)$; each agent i's strategies: $H_i(A_i, w_i, P, Q)$ and $B_i(A_i, Q)$, and the posterior belief $F(w|\Omega_i)$; and the aggregate demands: $H \equiv \int H_i \mathrm{d}i$ and $B \equiv \int B_i \mathrm{d}i$, such that: (i) for each agent i and some cutoff point $\breve{w}$, $H_i, B_i \in argmax_{\{H_i,B_i\}} \mathbb{E}(\tilde{U}_i|\Omega_i)$, where $H_i = Z$ if $w_i \geq \breve{w}$ and $H_i = 0$ if $w_i < \breve{w}$; (ii) both housing rental market and housing asset market clear: $H = S$ and $B = S$, for all (a, s, w); and (iii) $F(w|\Omega_i)$ satisfies Bayes' rule whenever applicable.*

2.3.1 Housing Rental Market

The quasi-linear preference makes the characterisation of housing rental market particularly simple. From equation (2.2), we can see that each agent i's demand of housing service B_i is independent of her income and only determined by her demand shock A_i and the rental price Q.[9] The aggregate demand for housing services is thus given by $\int \frac{A_i}{Q} \mathrm{d}i$. The total amount of houses for rent is simply S: while non-home-buyers are renting from home-buyers who still have extra houses to rent, home-buyers are renting to live from themselves (and maybe from other home-buyers as well). Imposing rental market clearing condition yields the equilibrium rental price

$$Q = \frac{\int A_i \mathrm{d}i}{S} \equiv \frac{A}{S}. \tag{2.6}$$

Thus, in equilibrium rental price will be only determined by the aggregate

[9] In Favara and Song (2014), agents have logarithmic preferences on both the housing service and the non-housing consumption good. Hence, the demand for housing service will also be affected by the expected future consumption or more specifically be affected by the decision made in the housing market in the current period. They solve their model by log-linearisation and the information effect of rental price is disregarded. My model avoids this complication while keeps the non-linearity and makes the effect of rental price relatively easily discussed.

demand shock and total housing supply. As will be explicit in the following subsection, rental prices not only clear the rental market but also provide more information other than the housing prices alone to the agents to make investment decisions in the housing market. To simplify the characterisations below, take logs on both sides of equation (2.6) and I have

$$y \equiv \ln Q = a - s, \tag{2.7}$$

which is a strictly monotone transformation of Q and thus a sufficient statistic of the equilibrium housing rental price.

2.3.2 Housing Asset Market

Different from the house rental market, the characterisation of the housing market equilibrium involves solving for a noisy rational expectations model. It is well-known that the primary difficulty in solving for such models is that the equilibrium prices must both clear the market and be consistent with agents' statistical inferences, which presents a complicated nonlinear fixed-point problem that does not fit well into any standard fixed-point theorems. The standard "guess and verify" method works for models in which the random variables are jointly normally distributed. With a non-normal joint distribution, this solution technique is not possible since the functional form of the price is not clear *a priori* (Breon-Drish, 2015).

Nevertheless, as shown in the following analysis, the constraint on the housing demand for each agent and the assumption of an exponential distribution for the private signals, enable me to obtain a new set of sufficient statistics that can replace the price signals in agents' information set and make me to solve for the model easily. A similar idea has been used in the finance literature recently. In a model with hierarchical information structure, Breon-Drish (2012) avoids this difficulty by exploiting the market clearing condition to determine *a priori* a statistic that is informationally equivalent to any continuous equilibrium price. In the heterogeneous information setup of Albagli *et.al.* (2011, 2015), model tractability is achieved by imposing a trade constraint and assuming an unconventional asset supply function (see also Hellwig, Mukherji, and Tsyvinski (2006) and Goldstein, Ozdenoren, and Yuan (2013)).

Sufficient Statistics

The private signal that agent i receives conditional on w follows a shifted exponential distribution with the cumulative distribution function: $F_{\tilde{w}_i|\tilde{w}}(w_i|w) = 1 - e^{-\lambda(w_i - w)}, w_i \geq w, \lambda > 0$. Note that $\mathbb{E}(\tilde{w}_i|\tilde{w} = w) = w + \frac{1}{\lambda}$ and $Var(\tilde{w}_i|\tilde{w} = w) = \frac{1}{\lambda^2}$. Hence, the smaller the λ is, the larger the mean of the signals given w, and the larger of signal variance. Given that all agent have the same trading constraint, and that

$\tilde{w}$ is independent from $\tilde{a}$ and $\tilde{s}$, the housing market clearing condition $\int H_i \mathrm{d}i = S$ can be written as

$$\int_{\breve{w}}^{\infty} Z f_{\tilde{w}_i|\tilde{w}}(w_i|w)\mathrm{d}w_i = S,$$

where $\breve{w} = \breve{w}(P, Q)$ is the cut-off point; any agent whose signal is higher than or equal to $\breve{w}$ will buy Z units of houses. Without loss of generality, let $Z = 1$. Then, as agents' signals are exponentially distributed *ex post*, housing market clearing condition can be written as $e^{-\lambda(\breve{w}-w)} = e^s$ or $\breve{w}(P, Q) = w - \frac{1}{\lambda}s$. That is, for any realisations of demand, supply, and fundamental shocks, this equation must hold for the market to be cleared in the equilibrium. Since the left-hand side depends on (w, s) through P and Q, any pair of equilibrium housing price and rental price must reveal the statistic

$$x \equiv w - \frac{1}{\lambda}s, \tag{2.8}$$

which is half-normally distributed conditional on $\tilde{w} = w$: $\tilde{x}|w \sim \mathrm{HN}\left(w, \frac{\sigma_s^2}{\lambda^2}\right)$. Then, for a given rental price Q, one can determine the information content of equilibrium housing price through a new statistic x, independently of the functional form $\breve{w}(P, Q)$ if and only if $x(P, Q)$ is strictly monotone in P given Q. In other words, $\breve{w}(P, Q)$ needs to be invertible in P given Q. If this strict monotonicity condition is satisfied, $\{x, y\}$ can be used as a set of sufficient statistics for $\{P, Q\}$.[10] That is, the equilibrium housing price and rental price that investor's posterior belief is conditional on, can be replaced by the informationally equivalent statistics x and y. This equivalence also reveals how rental price signal helps refine agents' information set. On the one hand, it reduces the noise in the housing price signal as it is one of the components in housing price. This is captured by x. On the other hand, it provides additional information about the supply shock which further reduces the uncertainty in the housing price signal. A full discussion on the information effects of rental prices is deferred to subsection 2.3.5 and subsection 2.3.6.

Claim 2.1 *Housing price P in the cut-off strategy equilibrium is strictly increasing in x for any given rental price Q.*

Claim 2.1 guarantees the strict monotonicity of $\breve{w}(P, Q)$ in P given Q. In the next subsection, I will take this claim as given and use $\{x, y\}$ instead of $\{P, Q\}$ to solve for the agents' posterior beliefs and the housing price in equilibrium. I will show this claim indeed holds once the equilibrium housing price is characterised.

[10] Note that x alone only reveals part of the information content of equilibrium housing price and rent. More specifically, x only reveals the information content of some nonlinear combination of housing price and rent. To fully exploit the information contained in P and Q, one still needs to use rental price or equivalently y in combination with x.

Heterogeneous Beliefs

Given Claim 2.1, the posterior beliefs in equilibrium can be characterised in the following lemma:

Lemma 2.1 *(1) For price realisations observed along the equilibrium path, agent i's posterior belief about w is given by*

$$F_{\tilde{w}|\tilde{w}_i,\tilde{x},\tilde{y}}(w|w_i,x,y) = \begin{cases} \dfrac{\Phi\left(\frac{w-\mu(x,y)}{\sigma}\right)}{\Phi\left(\frac{w_i-\mu(x,y)}{\sigma}\right)}, & \text{if } w_i < x, \\ \dfrac{\Phi\left(\frac{w-\mu(x,y)}{\sigma}\right)}{\Phi\left(\frac{x-\mu(x,y)}{\sigma}\right)}, & \text{if } w_i \geq x, \end{cases}$$

where $x \geq w, y \in \mathbb{R}, w_i \geq w, w \in \mathbb{R}$, $\Phi(\cdot)$ is the standard normal cumulative distribution function, and

$$\begin{aligned} \sigma &\equiv \left(\frac{\lambda^2}{\sigma_a^2} + \frac{\lambda^2}{\sigma_s^2} + \frac{1}{\sigma_w^2}\right)^{-\frac{1}{2}}, \\ \mu(x,y) &\equiv \left[\left(\frac{\lambda^2}{\sigma_a^2} + \frac{\lambda^2}{\sigma_s^2}\right)x - \left(\frac{\lambda}{\sigma_a^2}\right)y + \lambda\right]\sigma^2. \end{aligned}$$

(2) If $w_j < w_j < x$, then $F(w|w_i,x,y)$ dominates $F(w|w_j,x,y)$ in the sense of first-order stochastic dominance, and $F(w|x,x,y)$ dominates $F(w|w_i,x,y)$ in the sense of first-order stochastic dominance. (3) The posterior belief about w conditional on the cut-off signal is given by $F_{\tilde{w}|\tilde{w}_i,\tilde{x},\tilde{y}}(w|x,x,y) = \frac{\Phi\left(\frac{w-\mu(x,y)}{\sigma}\right)}{\Phi\left(\frac{x-\mu(x,y)}{\sigma}\right)}, x \geq w, y \in \mathbb{R}, w \in \mathbb{R}$.

The first point explicitly characterises the distribution of agents' posterior beliefs in equilibrium that are conditional on the private signals, housing prices, and rental prices. Agents who receive private signals higher than the cut-off signal will hold the same posterior belief as that of the cut-off point agent. This is because the *ex post* private signals are exponentially distributed "one-sided" on the right of fundamental shock. Given any rental price, the sufficient statistic for housing price reveals the cut-off private signal to all agents. Thus, anyone who receives the private signal that is higher than x will know that her private signal is too good. The housing price signal gives them a chance to narrow down the distance between their guess and the true fundamental value.

The second point implies that,[11] for agents who receive signals lower than

[11] Milgrom (1981) shows the strict monotone likelihood ratio property (MLRP) is both necessary and sufficient for higher signals of a random variable to be "good news" in the sense of first-order stochastic dominance, independent of the prior of the random variable. A failure of MLRP, however, does not preclude the possibility that for some prior the first-order stochastic dominance still holds. In my case, although the signals distribution does not satisfy strict MLRP, for the normal prior $w \sim \mathbb{N}(0, \sigma_w^2)$, $F_{\tilde{w}|\tilde{w}_i}(w|w_i)$ does first-order stochastically dominate $F_{\tilde{w}|\tilde{w}_i}(w|w_j)$ for $w_i > w_j$. Thus,

the cut-off signal, they will be less optimistic about the resale value than the cut-off belief. Moreover, the higher the private signal is, the more optimistic the agent's expectations about the resale value. This is because they know their private signals are closer to the true value than the cut-off signal, and they remain their belief unchanged even when they observe the cut-off signal implied by prices. The heterogeneous posterior beliefs result comes from both the heterogeneous private information and that the prices are non-fully revealing. If agents had homogeneous information and identical priors, they will hold homogeneous posterior beliefs. If agents had heterogeneous information and homogeneous priors, but the public price signals were fully revealing, then private signals would be redundant to the agents and they would still hold a homogenous belief.

The third point singles out the posterior belief of the agent who receives the cut-off signal that will be used in combination with the indifference condition, to derive the equilibrium price function. This is summarised in Theorem 2.1 in the next subsection.

Housing Prices

Theorem 2.1 *There exists a housing price function in the cut-off strategy equilibrium:*

$$P = Q + V,$$

where V is the expected housing resale value conditional on the cut-off agent's information set

$$V \equiv \mathbb{E}(\tilde{P}_f | \tilde{w}_i = x, \tilde{x} = x, \tilde{y} = y) = \int_{-\infty}^{+\infty} h(w) \mathrm{d} \left[\frac{\Phi\left(\frac{w-\mu(x,y)}{\sigma}\right)}{\Phi\left(\frac{x-\mu(x,y)}{\sigma}\right)} \right].$$

Theorem 2.1 states that, housing price in the equilibrium equals to the sum of rent and V that I call the *market expected housing resale value.* This expected housing resale value is conditional on the cut-off trader's information set, which comprises of two public price signals: $\tilde{x} = x, \tilde{y} = y$, and the cut-off trader's private signal whose realisation must equal to the threshold x as well in order to be consistent with the indifference condition.

The *existence* of the equilibrium price function is guaranteed if Claim 2.1 holds, which is shown in Appendix A.1 to be the case. The *uniqueness* of the equilibrium nevertheless needs a bit more justification, which is left for future research. To facilitate some of the analysis below, I assume an exponential function form for

"good news" in the sense of higher value of signal leads to higher or equal expectation on P_f as it is an increasing function of w, and the defined cut-off strategy is legitimate.

$h(\cdot)$ and then obtain the following housing price function:

Corollary 2.1 *If* $h(w) = e^w$*, then* $V = \exp\left(\frac{\sigma^2}{2} + \mu\right)\frac{\Phi(\kappa-\sigma)}{\Phi(\kappa)}$*, and the equilibrium housing price function is given by*

$$P = e^y + \exp\left(\frac{\sigma^2}{2} + \mu\right)\frac{\Phi(\kappa - \sigma)}{\Phi(\kappa)},$$

where $\kappa \equiv \left(\frac{x}{\sigma_w^2} + \frac{\lambda y}{\sigma_a^2} - \lambda\right)\sigma$*.*

Despite the seemly simple form of the equilibrium housing price, the existence of standard normal cumulative distribution function $\Phi(\cdot)$, which is highly nonlinear, makes some of the issues that are of interest cumbersome to analytically characterise. Nevertheless, $\Phi(\cdot)$ is a special function which is well-known and can be easily computed since numerical tools are widely available. Hence, some the discussions below will be based on numerical computations. To restrict the range of parameters values, most of time σ_a^2 and σ_s^2 will be chosen from (0, 1).[12] Parameter λ reflects the precision of private signals which does not have a direct measure empirically, hence I leave it free. For σ_w^2, I sometimes let $\sigma_w^2 = \sigma_a^2$ by interpreting the fundamental shock as the aggregate housing service preference shock in the next period so that it has a similar variance as in the previous period.

2.3.3 Information Aggregation Wedge

An important implication from Theorem 1 is that the market expected housing resale value, i.e. the market-implied posterior over the housing resale value conditional on housing prices and rental prices, differs from the Bayesian posterior conditional on the same public prices information. Following Albagli, Hellwig, and Tsyvinski (2015), I call this difference the *information aggregation wedge.*

Proposition 2.1 *Let* $\hat{D}(x, y) \equiv V - \hat{V}$ *be the information aggregation wedge, where* $\hat{V} \equiv \mathbb{E}(\tilde{P}_f | \tilde{x} = x, \tilde{y} = y)$*. Then,* $\forall (x, y) \in \mathbb{R}^2, \hat{D} > 0$*.*

Proposition 2.1 implies that the information aggregation wedge is always positive. However, this result must be taken with caution as it relies crucially on the asymmetry of signal distribution as well as the monotonicity of resale value function. In a nonlinear model where signals are normally distributed, as in Albagli

[12] Because $\ln Q = a - s$, where $\tilde{a}$ and $\tilde{s}$ are independent from each other, the volatility of log rent is simply the sum of variances of the two shocks: $Var(\ln \tilde{Q}) = Var(\tilde{a}) + Var(\tilde{s}) = \sigma_a^2 + \sigma_s^2\left(1 - \frac{2}{\pi}\right)$. Data located at Land and Property Values in the U.S., http://www.lincolninst.edu/resources/, show that $Var(\ln \tilde{Q})$ is about 0.75. If I take into account the interest rate, then $\ln Q + \ln R = a - s$, and the range of σ_a^2 and σ_s^2 will be slightly higher.

et.al. (2015), such a strong inequality does not exist. Instead, some results about the *unconditional* wedge could be established. More specifically, they show that the unconditional wedge between the asset price with heterogeneous information and homogenous information will be determined by the shape of $P_f = h(\cdot)$: if $h(\cdot)$ is dominated by the upside risk (e.g. P_f is convex) then the unconditional wedge will be positive; if $h(\cdot)$ has symmetric risk then the unconditional wedge is zero; and if $h(\cdot)$ is dominated by the downside risk (e.g. P_f is concave) then the unconditional wedge would be negative. The reason is that price places more weight on the tails of the fundamental distribution from an *ex ante* perspective. In my model, the resale value function is exponential and thus dominated by upside risk, thus the unconditional wedge is expected to be non-negative even if the private signal distribution is assumed as being symmetric. However, Proposition 2.1 holds regardless of the shape of P_f as long as it is an increasing function.

2.3.4 Impacts of Learning

The characterised housing price function in Theorem 2.1 also enables me to examine the impacts of static learning in the heterogeneous information model. If the agents had perfect information about the resale value of houses, there will be no learning and the equilibrium housing price will be simply given by

$$P^* = Q + P_f. \tag{2.9}$$

In this case, both housing service demand shocks and supply shocks affect housing prices only through rents in a "fundamental" way: the larger the demand shocks or the lower the supply shocks, the higher the rents, and thereby the higher the housing prices. Note that the supply shocks do not affect housing prices through the standard market clearing channel in the housing market, because the risk-neutral agents will absorb whatever supply offered when the house is priced at the "fundamental" value.

In the presence of asymmetric information, each agent needs to use its private signal and the publicly observed housing price and rental price to learn about the resale value. Both the demand shock and the supply shock interfere with the learning process and will have some additional effects. This is so, because price-rent ratio signal x is a linear combination of fundamental shock w and supply shock s, and the rental price signal y is a linear combination of housing service demand shock a and supply shock s.

Proposition 2.2 *V is decreasing in s and a, i.e.* $\frac{\partial V}{\partial s} < 0, \frac{\partial V}{\partial a} < 0.$

This proposition implies that, compared to the effects of demand shock and supply shock in the perfect information model where no learning is induced, there will be a further negative effect from the supply shock and an additional negative effect from the demand shock. Learning makes the negative effect of housing supply on the housing price larger, and the positive effect of housing service demand on the housing price smaller.

To understand this result, it is helpful to know how market expectation V changes in x and y from a comparative statistic point of view. It is already known from Claim 2.1 that for given y, V will be strictly increasing in x. The increase in x shifts the cut-off position one-for-one. Since $x = w - \frac{1}{\lambda}s$, either the increase of housing fundamental value w or the decrease of house supply s (scaled by $\frac{1}{\lambda}$) will increase x. If w increases, the distribution of private signals shifts up and the demand for housing asset will increase for a given signal threshold. If instead house supplies s decreases, the aggregate demand will be relatively high. In both cases, house supply is relatively scarcer and the cut-off point must be increasing to clear the market, so does the market expectation and the equilibrium housing price. A similar comparative static for y can also be obtained.

Proposition 2.3 *V is decreasing in y if x remains fixed:* $\frac{\partial V}{\partial y} < 0$.

This result comes from conditional dependence. Intuitively, although the rental price does not provide information about w directly, it affects the expectation of it by providing more information other than housing price alone. Since prices are not fully revealing, even observing some price-rent ratio signal x in addition to the private signal, agents are still not sure how much the fundamental value w is and how much housing supply noise s is.[13] Without any further information, agents will simply use the rule derived from their prior knowledge about those shock distributions as well as the realised signals, and attribute the observed x to w with some fixed weight and to s with the rest weight. They then infer w by inversion. When agents also observe an additional signal about rent, say, high rental price or equivalently high y, they will rationally attribute high rental price partly to the low housing supply, since everyone knows that rental price is negatively related to housing supply. This knowledge and observation will lead them to believe that they have overestimated the value of s initially and thereby must have overestimated the value of w using only housing price signal and private signal. They will lower their expectation on w accordingly and the lower marginal investor's expectation results in a lower equilibrium price. Similarly, if they observe a low rental price, they know housing supply must be higher than they thought before and thus must

[13]Note that housing supply shock s affects housing price in two fundamental ways: indirectly affects housing price by affecting rental price through rental market clearing and directly affects housing price through market clearing.

have underestimated the value of w. They will increase their expectation on w accordingly and push up the equilibrium price to a higher level.

The intuition for the conditional dependence may be made more explicit in a made-up example. Let $\tilde{x} = \tilde{w} - \frac{\tilde{s}}{\lambda}$, $\tilde{y} = \tilde{a} - \tilde{s}$, where $\tilde{w} \sim \mathbb{N}(0, \sigma_w^2)$, $\tilde{a} \sim \mathbb{N}(0, \sigma_a^2)$, $\tilde{s} \sim \mathbb{N}(0, \sigma_s^2)$ are independent from each other. Appendix A.2 shows $\mathbb{E}(e^{\tilde{w}}|\tilde{x} = x) = \exp\left(\mathbb{E}(\tilde{w}|\tilde{x} = x) + \frac{\Sigma'}{2}\right)$ and $\mathbb{E}(e^{\tilde{w}}|\tilde{x} = x, \tilde{y} = y) = \exp\left(\mathbb{E}(\tilde{w}|\tilde{x} = x, \tilde{y} = y) + \frac{\Sigma}{2}\right)$, where $\mathbb{E}(\tilde{w}|\tilde{x} = x) = W_x' x$, $\mathbb{E}(\tilde{w}|\tilde{x} = x, \tilde{y} = y) = W_x x + W_y y$, and $W_x', W_x, W_y, \Sigma, \Sigma'$ are some constants defined in Appendix A.2. Because $W_x' < W_x$ and $W_y < 0$, conditioning on one more variable y adds a negative effect from y while increases the elasticity of the conditional expectation on x.

Because $\frac{\partial V}{\partial y} < 0$, the change of house supply will have an additional effect on housing price through the rental price signal; a decrease in s will result in an increase in y which will make V smaller from Proposition 3. This negative adjustment will offset some of the initial effects when agents did not use y explicitly for refining information. In the current setup, this offset will not be large enough to reverse the direction of effect and thus s still has a negative effect on V overall. On the other hand, if the housing service demand shock a increases, then given w and s there will be an increase in y but not in x. Thus, there will only be a negative effect on V. Agents will mistakenly think some of the increase in y must be from the decrease of house supply. So they will adjust their expectation on w that is just based on x. They think they must have overestimated house supply and thus overestimated the future resale value. When everyone including the cut-off agent downgrades its expectation, the market expected house resale value will be lower. Thus, the change of housing service demand shock will have a negative effect, other things equal.

2.3.5 Information Effects of the Rental Price

In the presence of heterogeneous information, rent will have two types of effects on housing price in equilibrium: as a "fundamental" it has a user cost effect and as a price signal it has an information effect. The user cost effect is very straightforward and in this model where the equilibrium housing price is given by $P = Q + V$, it is simply one-for-one: given market expectation about future housing price V, when there is a unit increase in rent, so must be the housing price. In contrast, the information effect of rent on housing price is more involved and deserves some detailed discussion.

First note that rental price has information effect on housing price in this model, not because it is directly correlated to the future resale value, but because it provides additional information other than housing price alone to the agents. One immediate information value of rental price is that the rental price refines agents' information by revealing itself as one of the "noises" in the housing price signal.

From Theorem 1 we can see that rental price is one of the arguments in the housing price function. Because agents use housing prices as the public signals to infer the value of w, if they did not observe the rental prices, then housing price as a signal would be noisier to the agents. Upon observing rental prices, agents can simply subtract these "noises" from the housing price signals. The result of such refinement is captured by the statistic x.

The other information value of rental prices is that they can be used to further reduce the uncertainty about w after getting x. After a preliminary refinement discussed above, the "noise" left in x is the total housing supply shock s. Since the rental price that is determined in the rental market is a function of aggregate housing service demand shocks and total housing supply shock, observing a rental price then provides agents one additional signal to infer the supply shock noise, which in turn helps to infer the housing resale value w. Note that the first information effect is a "level" effect and the change of rent volatility does not change the nature of that effect. On the contrary, the second effect is a "signal" effect and the change of rent volatility will change the weights that agents put in this signal. Therefore, the second information effect from rental prices will be the main driving force in this model for explaining the observed relationship between housing price and rental volatility.

Proposition 2.3 states that this second effect tends to lower the market expected resale value. A hypothetical experiment can make this point even more straightforward; that is, we want to see how the housing price in a model where agents make inferences without conditioning on rental price *explicitly* would be different from the price in the original model? To do this, I first derive the equilibrium housing price from a model where each agent i does not fully exploit the information contained in rental price but instead makes predictions based only on w_i and x. I then compare this price and the price in Corollary 2.1 and check the additional information effect that rental prices have on housing price. In Appendix A.2, I show that when $P_f = e^w$ this housing price is obtained as

$$P' = e^y + \exp\left(\frac{\sigma'^2}{2} + \mu'\right)\frac{\Phi(\kappa' - \sigma')}{\Phi(\kappa')},$$

where $\sigma' = \lim_{\sigma_a^2 \to +\infty} \sigma$, $\mu' = \lim_{\sigma_a^2 \to +\infty} \mu$, and $\kappa' = \lim_{\sigma_a^2 \to +\infty} \kappa$. Therefore, P' is essentially the limit case of the original model where rental price becomes arbitrarily noisy because of the arbitrarily large housing demand shock $\sigma_a^2 \to +\infty$.[14] By comparing the unconditional mean of housing price in Corollary 2.1 and the unconditional mean of P', we can see the average effect of not conditioning on y.

[14] If the private signal is further assumed to be arbitrarily accurate, housing price in equilibrium will be the same as the housing price under perfect foresight: $\lim_{\lambda \to +\infty} P' = e^x + e^y = e^w + e^y$.

Figure 2.1 shows the unconditional difference $\mathbb{E}(P - P')$ for different pairs of (σ_a, λ) with $\sigma_s = \sigma_a$ and $\sigma_w = 0.45$. It shows that conditioning on rental price explicitly and making use of the additional information decrease the housing price on average. Note that in the previous made-up example, the law of iterated expectations implies that $\mathbb{E}[\mathbb{E}(e^{\tilde{w}}|x) - \mathbb{E}(e^{\tilde{w}}|x, y)] = \mathbb{E}[\mathbb{E}(e^{\tilde{w}}|x)] - \mathbb{E}[\mathbb{E}(e^{\tilde{w}}|x, y)] = \mathbb{E}(e^{\tilde{w}}) - \mathbb{E}(e^{\tilde{w}}) = 0$. However, in the model such unconditional difference is negative, implying again the failure of law of iterated expectations in the heterogeneous information model. This negative unconditional difference is crucial for explaining the observed relationship between housing price and rent volatility.

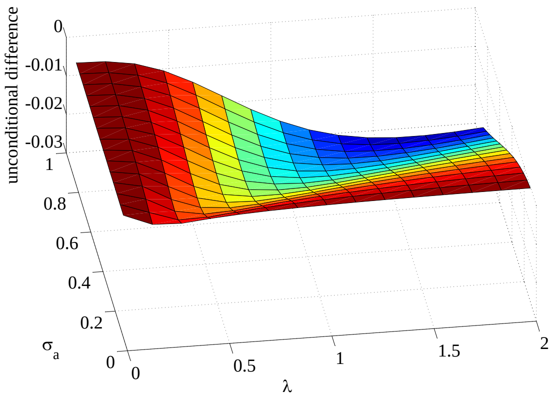


Figure 2.1: Information Effect of Rental Price

2.4 Implications

Having the characterised equilibrium housing prices and several derived implications, I am now able to apply the model to examine several important issues in the housing market. First, I relate the information aggregation wedge to the return predictability in the housing market and argue that explaining housing return predictability does not have to move beyond the rational expectations assumption; it could be because the econometrician does not observe private information and fails to take into account the heterogeneous beliefs as well. Next, I show that the information channel other than the risk-hedging explanation can also be consistent with the positive relationship between rent volatility and housing price that is documented in Sinai and Souleles (2005). Third, I show in this model housing will be priced on average higher than its fundamental value. Finally, I show the model has the potential to generate excess volatility in housing prices, but learning from rental prices restricts housing prices from being too volatile.

2.4.1 Return Predicability

As discussed in the introduction part, both housing prices and rental prices are found to have some predictive power on the future housing returns. More practically, it means when an econometrician regresses the observed returns on lagged housing prices and rental prices, he finds statistically significant coefficients on both price variables. Specifically, if I define the return for housing asset R_H, using the notations in this chapter, as

$$R_H \equiv \underbrace{(P_f - P)}_{\text{capital gain}} + \underbrace{Q}_{\text{rent}}, \tag{2.10}$$

and denote the econometrician's information set by $\Omega_e = \{P, Q, \mathbb{M}_e\}$, where $\mathbb{M}_e$ captures the *econometrician*'s knowledge about the economy structure, then the return predictability on houses from housing prices and rental prices implies that

$$\mathbb{E}(\tilde{R}_H|\Omega_e) = G(P, Q) \neq 0,$$

where $G(\cdot)$ is a function of both housing price and rental price.

If agents in the economy have homogeneous information, or they have heterogeneous information but with prices fully revealing information in the economy, then the econometrician's information set would be the same as that of the agents, which implies

$$G(P, Q) = 0.$$

Neither housing price nor rental price is affecting the conditional expectation. That is, a correctly specified econometric model will give zero coefficients on P and Q.

However, if prices are not fully revealing, as in the model I have just shown, and if the econometrician does not have full information, $G(P, Q)$ will be non-zero. For instance, if the econometrician's information set is the same as the Bayesian's information, which means the econometrician does not observe the future resale value of housing nor does he observe the agents' private information. More importantly, he does not know that the equilibrium housing prices were generated from the economy with heterogeneous expectations. Instead, he only knows correctly the joint distribution of the market prices and exogenous shocks. The econometrician also observes the market prices: both housing prices and rental prices. With these assumptions about the econometrician's information set, it is shown below that the expected return conditional on his information set is related to the information aggregation wedge introduced in section 2.3.3.

Proposition 2.4 *The expected return conditional on the econometrician's information set equals to the negative information aggregation wedge, that is,* $\mathbb{E}(\tilde{R}_H|\Omega_e) =$

$-\hat{D}$, *where* $\hat{D}$ *is defined in Proposition 2.1.*

From Proposition 2.1 and Proposition 2.4, it immediately follows that the expected return conditional on econometrician's information set is non-zero. More importantly, because the conditional wedge $\hat{D}$ is a function of x and y while x is, in turn, a function of P and Q, the conditional expectation of housing asset return on the econometrician's information set will be a nonlinear function of both housing price and rental price. That is, housing asset return is predictable by both price and rent! Kasa, Walker, and Whiteman (2014) develop a dynamic asset pricing model with persistent heterogeneous beliefs, and show that an econometrician, who incorrectly imposes a homogeneous beliefs equilibrium, will find that the asset price displays predictability of excess returns.[15] The idea here is essentially the same to theirs. However, by incorporating the mechanism that agents are also learning from the rental price signals, my model implies both housing prices and rental prices display predictability of excess returns on houses. Had rental prices not provided more information other than housing prices alone, even in the heterogeneous information model the conditional expectation of housing asset return on the econometrician's information set will not depend on rental price but will only depend on housing price.

Because $-\hat{D}$ is a complex nonlinear function, it is hard to analytically sign the effects of prices on $G(P,Q)$. To facilitate the discussion, I assume an exponential function for the resale value: $h(w) = e^w$, and use the implied expected return for simulations.

Corollary 2.2 *If* $h(w) = e^w$, *then the expected return conditional on the econometrician's information set is given by*

$$\mathbb{E}(\tilde{R}_H|\tilde{P} = P, \tilde{Q} = Q) = \exp\left(\frac{\sigma^2}{2} + \mu\right)\left[\exp\left(-\lambda\sigma^2\right)\frac{\Phi(\kappa + \lambda\sigma - \sigma)}{\Phi(\kappa + \lambda\sigma)} - \frac{\Phi(\kappa - \sigma)}{\Phi(\kappa)}\right].$$

Numerical simulation based on Corollary 2.2 and calibrated parameters (see Figure 2.2) shows that $\mathbb{E}(\tilde{R}_H|\Omega_e)$ could be decreasing in x, which is consistent with the regression results in Cass and Shiller (1990) and those in Glaeser and Gyourko (2007): fixing rental price, the higher the housing price the lower the expected future return. It is not clear how the expected future returns changes in response to the change of rental prices.

[15]Because I have let $R = 1$, the excess return is equivalent to the return.

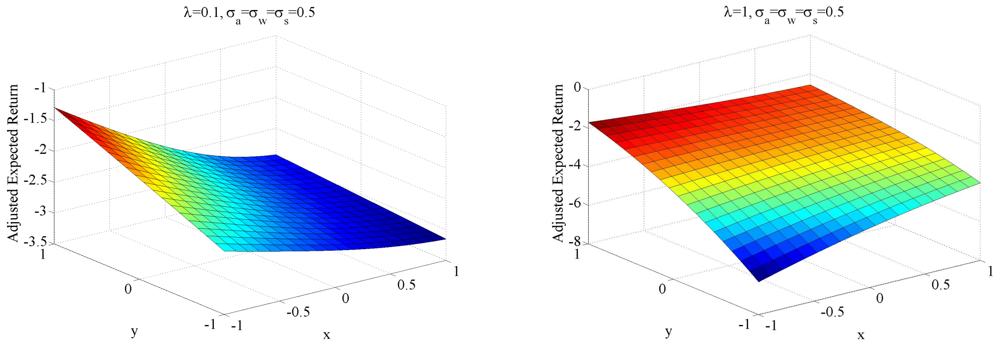


Figure 2.2: Expected Return Conditional on Econometrician's Information

If x in this model is interpreted as the price-rent ratio and if the econometrician thinks that the price-rent ratio has sufficiently aggregated all information in the market, then we will get another wedge, which is no longer always positive. However, the mean of this wedge is still positive. That is, on average, housing prices will be always higher than the evaluations based only on the price-rent ratio. More formally,

Corollary 2.3 *Let* $\check{V} \equiv \mathbb{E}(\tilde{P}_f|\tilde{x} = x)$ *and* $\check{D}(x, y) \equiv V - \check{V}$. *Then,* $\mathbb{E}[\tilde{\check{D}}] > 0$.

2.4.2 Rent Volatility and Housing Prices

In a tenure choice model with endogenous housing price, Sinai and Souleles (2005) show that when the households are risk-averse, owning a house involves both taking housing asset price risk and hedging rent fluctuation risk. Which risk dominates on net in this trade-off largely depends on the household's expected length of stay and whether they move to correlated housing markets. When the household's expected length of stay is large or when the spatial correlation in housing prices is high, larger local rent volatility tends to increase household's home-ownership demand, which will be capitalised into higher housing price when housing supply is inelastic. Empirically, they find that metropolitan statistical areas (MSAs) with more volatile rents have significantly greater price-to-rent ratios. Similar to their findings, my model may also generate the positive relationship.

Remark 2.2 *Housing prices tend to be on average higher when rents are more volatile.*

This is shown through numerical simulations. Figure 2.3 shows the simulations of two parameter sets. In the left, $\lambda = 1$ and $\sigma_w = \lambda_a$. In the right, $\lambda = 9$ and $\sigma_w^2 = 0.1$. In both cases, we can see that the unconditional housing price is not monotonically changing in σ_s^2. It is increasing in σ_s^2 when it is small but decreasing when it is very large. The effect of σ_a^2 seems 'more' monotonic: the larger the demand shock volatility is, the higher the housing price on average, implying that the effect of demand shock variance on the rental price signal dominates. Thus, under reasonable parameter values, higher volatility of rent tends to generate higher housing price on average.

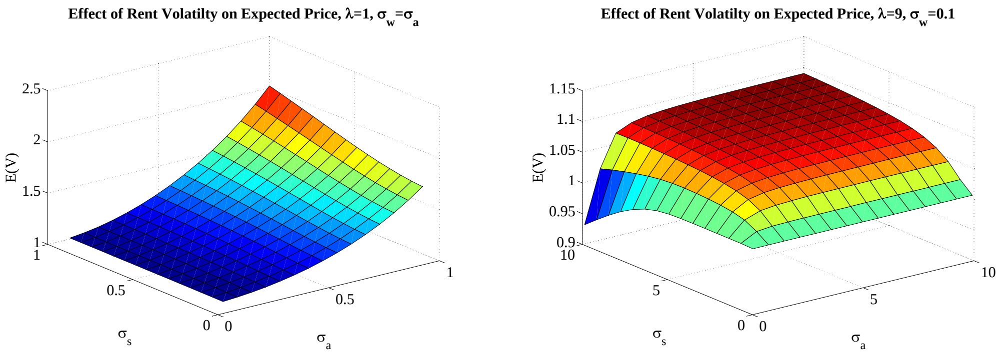


Figure 2.3: Rent Volatility and Expected Housing Resale Value

Despite the similar prediction, the channel through which the mechanism works is completely different from that in Sinai and Souleles (2005). Risk comparison is the key to their mechanism: if they were to assume households are risk neutral, the volatility of rent wouldn't have any effect on homeownership demand or housing price. The assumption of risk neutral agents in my model precludes any risk impacts, yet we still see a qualitatively similar effect. This is due to the information effects of prices and especially the second information effect from rental price I have discussed before: while x (price-rent ratio) serves as a noisy public signal on the determinant of the housing resale value, y serves as an *additional* noisy public signal about the supply shock s.

This negative information effect from the rental prices is the main cause of the simulated positive relationship between rent volatility and unconditional housing prices. Note that the volatility of rent comes from the volatility of demand shocks and the volatility of supply shocks. When the variance of demand shocks increases, the volatility of rental price signal becomes larger while the volatility of price-rent ratio signal remains the same. Agents will then find the rental price signal is not quite accurate in inferring the supply shock. Hence, the negative effect from the second effect of rental price signals will be reduced and the housing price will be higher. On the other hand, if the increase of rent volatility comes from the increase of supply shock variance, the volatilities of both x and y will increase. While this decreases the negative effect from y, it also decreases the positive effect from x. The two effects offset each other and the overall effect depends on their relative magnitudes. When the first effect dominates, we see housing price increasing.

2.4.3 Over-Pricing

The model also has some important implications for the pricing of housing. Note that the proof of Corollary 2.3 relies on the law of iterated expectations: $\mathbb{E}[\mathbb{E}(\tilde{P}_f|\tilde{x} = x)] = \mathbb{E}[\mathbb{E}(\tilde{P}_f|\tilde{x} = x, \tilde{y} = y)]$. It follows that the law of iterated expectations also implies $\mathbb{E}[\mathbb{E}(\tilde{P}_f|\tilde{x} = x)] = \mathbb{E}(\tilde{P}_f)$, where $\mathbb{E}(\tilde{P}_f)$ coincides with the unconditional mean of housing price under several other homogeneous information structures. The simplest one is the perfect foresight case where everyone observes P_f. Another one is that all agents get a public noisy signal about P_f or get no signal at all. Because of the symmetric information structure, no more information could be learned from housing price or rental price and housing price in equilibrium will be given by $P = Q + \mathbb{E}(\tilde{P}_f|\Omega)$, where Ω can be $\emptyset$. A common feature of these models is that they all have an identical unconditional mean of housing price which equals to $\mathbb{E}(\tilde{P}^*) = \mathbb{E}(\tilde{Q}) + \mathbb{E}(\tilde{P}_f)$. Therefore, when agents have homogeneous information, either due to perfect foresight about house resale value or because they have public noisy information, the housing price will be lower than that with heterogeneous information on average. This complements to Favara and Song (2014), who show in a log-linearised dynamic model that housing price with heterogeneous information and the no-short-sale constraint is on average higher than that with homogeneous information. More importantly, if I define the perfect foresight house price as the fundamental value of housing, it implies the over-pricing for houses in this model.

Corollary 2.4 *For any increasing function $h(\cdot)$, the mean of housing price defined in Theorem 1 is strictly larger than the mean of the fundamental value of housing defined in equation (2.9):* $\mathbb{E}(\tilde{P}) > \mathbb{E}(\tilde{P}^*)$.

The exact relation between the unconditional difference $\mathbb{E}(\tilde{\tilde{D}})$ and the underlying distribution parameters can be fairly complicated. However, some limiting case can be seen relatively easily. First note that the precision of x about w is increasing in λ^2 and decreasing in the variance of housing supply σ_s^2, while the precision of y on s is increasing in σ_s^2. Thus, given σ_w^2 and σ_a^2, when $\lambda \to 0$, the estimate on w is converging to no information case and the unconditional difference will converge to zero. If instead $\lambda \to \infty$, the estimate on w is converging to perfect information case and the unconditional difference will converge to zero too. If $\sigma_s^2 \to 0$, y becomes useless but x is very precise about w so it converges to perfect information as well. However, if $\sigma_s^2 \to \infty$, then on the one hand the signal x is not very precise about w but on the other hand the signal y is very precise about s, it turns out that y will have a negative effect while x will have a positive effect.

2.4.4 Excess Volatility

It has been shown, by Algbali *et.al.* (2011) in a static model and by Kasa *et.al.* (2014) in a dynamic model, that heterogeneous beliefs induced by heterogeneous information can generate excess volatility in asset prices. In particular, when the variance of noisy supply becomes arbitrarily large, the excess volatility of price could be infinite. However, in those models, there is no signal like the rental price as in my model. I show in this subsection that while my model has the ability to generate excess volatility in housing prices for the same reason as in those models (i.e. the failure of the law of iterated expectations due to private information), the learning from rental prices by the agents restricts housing prices from being too volatile.

Define the excess volatility of housing price as the ratio of $Var(\tilde{P})$ over $Var(\tilde{P}^*)$:

$$\phi^* \equiv \frac{Var(\tilde{P})}{Var(\tilde{P}^*)}. \tag{2.11}$$

Because the equilibrium housing price is simply the sum of current rent and market expected future price: $P = Q+V$, the variance of housing price is given by $Var(\tilde{P}) = Var(\tilde{Q})+Var(\tilde{V})+2Cov(\tilde{Q},\tilde{V})$. A similar expression can be written for the variance of housing price *ex post*: $Var(\tilde{P}^*) = Var(\tilde{Q}) + Var(\tilde{P}_f) + 2Cov(\tilde{Q},\tilde{P}_f)$. Thus,

$$\phi^* = \frac{Var(\tilde{Q}) + Var(\tilde{V}) + 2Cov(\tilde{Q},\tilde{V})}{Var(\tilde{Q}) + Var(\tilde{P}_f)},$$

where I have used the fact that $Cov(\tilde{Q},\tilde{P}_f) = 0$ as $\tilde{a}, \tilde{s}$ and $\tilde{w}$ are independent from each other by construction. Hence, whether equilibrium housing price is more volatile than that of the *ex post* housing price (i.e. $\phi^* > 1$) depends on whether $Var(\tilde{V}) + 2Cov(\tilde{Q},\tilde{V}) > Var(\tilde{P}_f)$. I have been unable to establish conditions for this inequality to hold and thus have to use numerical method to simulate for some sets of parameterisations.

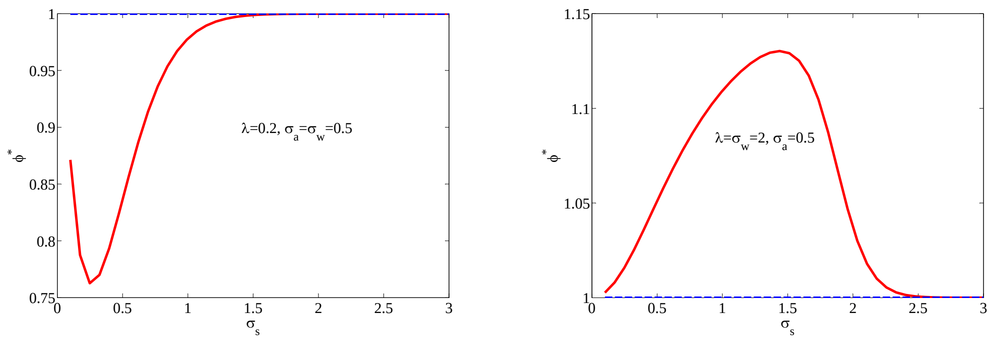


Figure 2.4: Housing Price Excess Volatility

Figure 2.4 shows that ϕ^* could be both higher or lower than one. More importantly, ϕ^* seems to be bounded and converges to one as the variance of supply shock goes to infinity. This is very different from that in Albagli *et.al.* (2011) where the variability of asset prices can be arbitrarily large when the supply shocks are unboundedly large even the variability of realised dividend is bounded. The unbounded excess volatility does not seem to exist in my model. After all, a wide range of parameterisations shows that it is unlikely that this excess volatility could be infinite. While it is difficult to analyse this result analytically, the intuition may be very straightforward: the rental price signal in my model, which is naturally from the housing rental market, provides additional information about the noisy supply. The reason that the excess volatility in Albagli *et.al.* (2011) can be very large is that, the supply shocks become the noises in the asset price signals but there is no other signal that can be used to infer this noise. Hence, when the supply shock variance becomes large, the price signals become imprecise, and the pricing can go very large. However, if there is an additional signal to this noise, the uncertainty will be restricted because the inference about the noise itself will be more accurate as its variance becomes larger.

Remark 3 *The excess volatility of housing price ϕ^* could be either higher or lower than one, and tends to converge to one as $\sigma_s^2 \to \infty$.*

2.5 An Extension: Feedback Effects

The model until now has just assumed the resale value of housing in the second period is completely exogenous. If I allow that value to be also affected by the real estate developers' actions, the basic model then can be extended to study the feedback effect of prices in the housing market. That is, equilibrium prices will not only play the role to clear the market but also have real effects on the fundamentals of the market.

More specifically, the resale value is assumed to be determined by two components: $P_f = h(w) + g(S_f)$, where w can be interpreted as an aggregate preference shock of the next generation, and S_f as the supply in the next period, which is determined by the developers in the first period because of the time to build. I assume $g(S_f) = -\left(\frac{\theta^{-1}-1}{2}\right) S_f \equiv -\delta S_f$, where $0 < \theta < 1$, such that the more houses to build in the first period, the cheaper the price will be in the next period. In the resale value function, θ is the feedback effect parameter; for a given S_f, the higher θ is, the higher the future housing price will be. The developers choose the amount of houses to build in the first period to maximise the expected profit in the second period:

$$\max_{\{S_f\}} \mathbb{E}\left[\tilde{P}_f S_f - C(S_f)|\Omega_d\right],$$

where $C(S_f) = \frac{1}{2}S_f^2$ is the cost involved in building, Ω_d is developer's information set. The first order necessary condition implies that the optimal construction is

$$S_f = \theta\mathbb{E}[h(\tilde{w})|\Omega_d].$$

Now the assumption about the information set of developers becomes crucial. I assume the developers know that their new house building in the current period will have a negative effect on housing value in the next period; however, they do not know how much the value is going to be. The developers may or may not have any exogenous information about the next period housing value. In either case, developers will make use of the current housing prices and rental prices to indirectly infer the information held by investors. The investors also know that the indirect use of price signals by developers will have an undesirable effect on their resale value of houses in the second period, which will, in turn, affect their trading decisions in the first period. As a simple example, I assume the developers do not hold any private information about w. Hence, their information set is $\Omega_d = \{P, Q, \mathbb{M}_d\}$, where $\mathbb{M}_d$ captures the assumption that developers know the model structure. Since developers do not have any private information about w, they can only rely on the equilibrium rental price and housing price to make inferences. If Claim 2.1 still holds, then the equilibrium housing price could be obtained as $P = Q + \mathbb{E}(h(\tilde{w})|\check{\Omega}) - \left(\frac{1-\theta}{2}\right)\mathbb{E}(h(\tilde{w})|\Omega_d)$. Note that this price is lower than the benchmark model and when $\theta = 1$ it collapses to the benchmark model housing price; the higher θ is, the smaller the housing price will be. Alternatively, I could assume developers also have private information. This, however, will generate high order expectations problem and poses challenges for solving the model. I will leave this for future research.

2.6 Conclusion

In this chapter, I provide a house pricing model where agents observe heterogeneous information about the future resale value of houses. In the model, agents learn not only from their private information but also from the public information—housing prices and rental prices, and they hold heterogeneous expectations in the equilibrium because prices are not fully revealing. I show the model can be used to explain two empirical regularities—the return predictability of houses from past prices and the positive relationship between rent volatility and housing prices. I also show that over-pricing in the housing market may arise if the agents cannot short sell in the market. The model has the potential to generate excess volatility of housing prices, but learning from rental prices greatly restricts the magnitudes of the excess volatility.

The model in the chapter has a great potential to be extended, and, thus has potential to explain other phenomena regarding the housing market. I have provided a simple extension of the model for studying the effects of real estate developers' learning on the housing supply, which might be of interest to the study of residential investments behaviour. Other extensions, for instance, the effects of lenders' learning on the housing demand, can also be considered in the future research.

Chapter 3

Noises, Land Prices, and Macroeconomic Fluctuations

3.1 Introduction

In an economy where debt must be fully secured by collateral, an endogenous two-way feedback can arise between the credit market and the real economy—while firms' goods production forms the basis of asset prices, asset prices, in turn, determine the ability of firms to invest and thereby to produce. Such interactions between the endogenously determined credit constraints and aggregate economic activities have been shown to amplify and propagate relatively small, temporary shocks to generate large, persistent fluctuations in output and asset price. Models following this strand are, for example, Kiyotaki and Moore (1997), Iacoviello (2005), and Liu, Wang, and Zha (2013).

In these models, borrowing must be secured against some collateral, which is typically the land or housing. The total amount of borrowing is usually bounded by the expected present value of the collateral. This is theoretically reasonable because it is the future value of collateral that matters in the case of default. Hence, when the expected future value is high enough, the amount of funds one can borrow may exceed current (market or appraised) value of the collateral. Most often, however, the amount of funds one can borrow is less than the current value of the collateral; the difference is the down-payment. This is so, perhaps because lenders are pessimistic about the future value of the collateral, or because the lenders are risk-averse and the future is very uncertain, or because there are significant transaction costs in selling the collateral if borrowers default. If the loan-to-value ratios are larger than one, it is mostly because the lenders somehow are very optimistic about the future value of the collateral.

Expectations about the future, on the other hand, must be based on the information agents receive up until today. In an economy where agents are not perfectly informed about the state of the world, the information that agents hold could be very noisy. This implies that agents' optimism or pessimism about the future need not be justified by the economic fundamentals; noises in signals can be confused by the agents and thereby taken as the fundamental shocks, and finally, initiate fluctuations in the macroeconomic variables. Some recent empirical research has shown that noises do play important roles in the short-run fluctuations (e.g. Blanchard, L'Huillier, and Lorenzoni, 2013).

Macroeconomists have been trying to incorporate the information friction into the Real Business Cycle models. However, most of the existing models seem to have neglected the existence of financial frictions.[1] The effects of noises on the macroeconomy may be substantially large when there are also financial frictions. The recent financial crisis in the U.S. is characterised by large swings in the housing prices, consumer confidence, credit, as well as other macroeconomic variables such as consumption and investment. An inclusion of financial friction in the model will not only generate dynamics of the relevant variables such as housing price and credit but also have the potential to make the effects of noise shocks sizeable enough.

This chapter then fills this gap by combining two strands of literature together: financial friction and information friction. I show how learning and financial friction can together generate boom-bust business cycles initiated by purely noises in the economy.

3.1.1 Preview of the Model

The model economy is comprised of a continuum of islands, each of which is inhabited by two types of infinitely-lived agents: non-productive households and productive entrepreneurs. Islands are isolated from each other; there is no trade or capital flow among islands. Households on each island desire both consumption good and land (housing) for utility, while entrepreneurs' utility only derives from consumption good; their demand for land comes from the production technology requirement. Because households are more patient, they tend to be lenders while entrepreneurs tend to be borrowers. Entrepreneurs started with debt and are constrained in borrowing in each period, they can only use their land holdings as collateral and the total amount of funds they borrow cannot exceed the (households') expected liquidation value of the collateralised land in the next period. In each period, each island is hit by an island-specific productivity shock that is comprised of two components: an economy-wide common persistent shock and an idiosyncratic transitory shock.

As islands are informationally isolated from each other, there is no direct way they can share information with each other. However, all of them observe

[1] La'O (2010) is the only exception I am aware.

noisy public signals about the economy-wide average land prices. Hence, to infer the persistent component (agents on each island have the incentive to do so because it differs with the idiosyncratic shock in the persistence), both the island-specific shocks and endogenously generated price signals will be used to draw inferences. Because land price signals are partially determined by the expectations of agents on other islands, agents on each island must "forecast the forecasts of others" in the dynamic economy. The infinite regress makes forecast errors serially correlated and thus generates "waves of optimism and pessimism" in expectations on each island. As the entrepreneurs are borrowing constrained and the funding is subject to the households' expected liquidation value of the collateralised land in the future, any forecasting errors from noises will directly affect households' willingness of lending and entrepreneurs' investment ability, and thus, cause macroeconomic fluctuations.

To help understand how noise shocks affect the economy, let's first consider the effect of productivity shocks in the full information model. Suppose entrepreneurs on one island experience a temporary negative productivity shock that reduces their production and therefore their net worth. Being unable to borrow more, entrepreneurs are forced to reduce investment expenditure including investment in land, which results in a fall in land price. This is the static effect, and there is a more powerful dynamic effect. That is, less investment in the current period also leads entrepreneurs to earn less revenue in the next period, because capital and land investments are predetermined. This fall in revenue again will force them to reduce investment in the second period because of the borrowing constraint. As a result, there will be a further fall in land price and output. The knock-on effects continue, with the result that entrepreneurs' demands for land are reduced and therefore falls in land prices and outputs in all subsequent periods. The anticipated fall in each of future periods also leads to a further fall in land price in the current period, which reduces the entrepreneurs' net worth in the current period still further. Persistence and amplification reinforce each other. If the negative shock is expected to be persistent, those effects will be even larger and more persistent because the dynamic effects are stronger.

Consider now the dispersed information model and the effects a negative noise shock. Since the noise shock does not affect production, entrepreneurs' net worth would not fall directly. However, as agents observe a low average land price signal that tells them something negative might have happened to the common persistent productivity shock, no matter what their island-specific shock is, both households and entrepreneurs will believe that production in the future would be low, which would then force the entrepreneurs to reduce investment expenditure including investment in land in those future periods. As a result, there would be falls in land prices and outputs in the future. Those expected falls in future land prices will, in turn, depress current land prices and thus indirectly hurt the current net worth of the entrepreneurs, forcing them to reduce current investment expenditures.

3.1.2 Related Literature

As already discussed, this chapter builds on two strands of literature: business cycle models with financial frictions and business cycle models with incomplete information. For the former literature, this chapter is mostly related to Pintus and Wen (2013) and Liu, Wang, and Zha (2013). Pintus and Wen (2013) add consumption habit into the Kiyotaki and Moore (1997) model and shows that the dynamic interactions between the elastic credit supply (due to leveraged borrowing) and persistent credit demand (due to consumption habit) can generate a multiplier-accelerator mechanism that transforms a one-time productivity or financial shock into large and long-lasting boom-bust cycles. Liu, Wang, and Zha (2013) construct a similar but richer model than Pintus and Wen (2013); they use Bayesian estimation and identify a shock that drives most of the observed fluctuations in land prices. They show that positive co-movements between land prices and business investment are a driving force behind the broad impact of land-price dynamics on the macroeconomy. Neither of them, however, considers the effect of noise shocks on the macroeconomy, which is the focus of this chapter.

The idea that imperfect information can cause the sluggish adjustment in economic variables and generate fluctuations driven by expectation errors goes back to Phelps (1969) and Lucas (1972). There was a period (the 1970s and early 1980s) of intensive research on expectation-driven business cycle models. However, they were replaced by technology-driven Real Business Cycle models and New Keynesian sticky-price models. This waning of interest was caused not so much by convincing empirical failures, but perhaps by the inability of these models to generate long-lasting effects on the macroeconomic variables of interests, or by analytical hurdles that prevented researchers from constructing empirically tractable models (Kasa, 2000). Woodford (2002), Mankiw and Reis (2002), and Sims (2003) have renewed attention to imperfect information and limited information processing as sources of inertial behaviour. This renewal is quickly followed by, for instance, Lorenzoni (2009), Angeletos and La'O (2009), and Graham and Wright (2010). These models build on either New Keynesian model or Real Business Cycle model, and not consider the interactions between information friction and financial friction.

3.2 Model

The economy is comprised of a continuum of islands indexed by $i \in [0, 1]$. Islands are correlated in their productivity, they are however economically isolated. Each island is a Kiyotaki-Moore style economy[2]. More specifically, on each island i, there is a measure-of-one continuum of infinitely-lived patient agents and a measure-of-one continuum of infinitely-lived impatient agents. Following the literature, I call the patient agents the households, and the impatient agents the entrepreneurs. Households and entrepreneur differ not only in their discount factors but also in other aspects that will be specified in the following subsections.

3.2.1 Productivity Shocks

In each period t, each island i in the economy is hit by both a common economy-wide shock and an idiosyncratic shock. Denoted by $A_t^i = e^{a_t^i}$ the island-specific productivity shock on each island i at time t. Then, a_t^i is comprised of two components:

$$a_t^i = \theta_t + \epsilon_t^i, \tag{3.1}$$

where θ_t is the economy-wide productivity shock that follows a mean-reverting process:

$$\theta_t = \rho\theta_{t-1} + v_t, \text{ where } 0 < |\rho| < 1, v_t \sim \mathbb{N}(0, \sigma_v^2),$$

and ϵ_t^i is the idiosyncratic productivity shock. I assume $\epsilon_t^i \sim \mathbb{N}(0, \sigma_\epsilon^2), \forall i \in [0, 1]$ are identically independently distributed across time and island and satisfy an adding up constraint:

$$\int \epsilon_t^i \mathrm{d}i = 0, \forall t.$$

Note that the independence of $\{\epsilon^i\}$ across time is not crucial. What is important to the model is that the two shocks are different in their persistence; otherwise, agents have no incentive to disentangle them. Finally, I assume all shocks are orthogonal to each other.

3.2.2 Households

Households on each island i derive utilities from both consumption good and land. Households do not produce or accumulate capital goods but provide loans to the entrepreneurs. The type of loans provided by the households is the one-period loan that can be used by the entrepreneurs to finance their consumption and investment. The interests from the previous loans that the entrepreneurs pay to the households

[2]The economy structure for each island follows Pintus and Wen (2013) and Liu, Wang, and Zha (2013).

may be used by the households to finance their current consumption, land investment, as well as new loans in the next period.

Denote by $\tilde{C}_t^i$ the representative household's consumption on island i in period t, $\tilde{L}_t^i$ the amount of lands owned by the household at the beginning of period t, $\tilde{B}_t^i$ the amount of new loans generated in period t, and $\tilde{\mathbb{E}}_t^i[\cdot] \equiv \Pi[\cdot|\tilde{\mathcal{F}}_t^i]$ the household's linear least-squares projection conditional on his information $\tilde{\mathcal{F}}_t^i$ in period t. Given the initial land holding $\tilde{L}_0^i \geq 0$ and loan $\tilde{B}_0^i \geq 0$, the problem of the representative household on island i can be written as

$$\max_{\{\tilde{C}_t^i, \tilde{L}_{t+1}^i, \tilde{B}_{t+1}^i\}_{t=0}^{\infty}} \quad \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \tilde{\beta}^t (\tilde{C}_t^i + b \ln \tilde{L}_{t+1}^i),$$

subject to budget constraints

$$\tilde{C}_t^i + Q_t^i(\tilde{L}_{t+1}^i - \tilde{L}_t^i) + \tilde{B}_{t+1}^i = (1 + R_t^i)\tilde{B}_t^i, \forall t, \tag{3.2}$$

where Q_t^i is the relative price of land on island i in period t, R_t^i is the loanable fund interest rate from time period t to $t+1$, $\tilde{\beta} \in (0,1)$ refers to the household's time discount factor, and b his land preference parameter. Note that there are not superscript for $\tilde{\beta}$ or b, implying that all households on all islands share the same discount factor and land preference parameter.

3.2.3 Entrepreneurs

Entrepreneurs on each island i derive utilities from consumption C_t^i but not from land.[3] Their demand for lands comes from production of Y_t^i, the technology of which requires the inputs of capital K_t^i and land L_t^i. I assume there is no capital rental market nor land rental market. Hence, entrepreneurs need to buy capital and land in the asset markets. The production function is assumed to be of the Cobb-Douglas form

$$Y_t^i = A_t^i (K_t^i)^{\alpha} (L_t^i)^{\gamma}, \forall t, \tag{3.3}$$

where A_t^i is the productivity shock of the representative entrepreneur on island i, $\alpha, \gamma \in (0,1)$ are the output elasticities of capital and land respectively. Again, note that there are not superscript for α or γ, implying that entrepreneurs on all islands share the same output elasticities.

Given the initial land holding, debt, and capital stock $L_0^i \geq 0, B_0^i \geq 0, K_0^i \geq$

[3]The asymmetry in preference assumption is not essential. Assuming land preference for households is just a short-cut to have them to demand land. See also Iacoviello (2005).

0, the problem of the representative entrepreneur on island i can be written as

$$\max_{\{C_t^i, L_{t+1}^i, K_{t+1}^i, B_{t+1}^i\}_{t=0}^{\infty}} \mathbb{E}_0^i \sum_{t=0}^{\infty} \beta^t \frac{(C_t^i)^{1-\sigma}}{1-\sigma},$$

subject to budget constraints

$$C_t^i + K_{t+1}^i - (1-\delta)K_t^i + Q_t^i(L_{t+1}^i - L_t^i) + (1+R_t^i)B_t^i = B_{t+1}^i + Y_t^i, \forall t, \tag{3.4}$$

and borrowing constraints

$$(1+R_{t+1}^i)B_{t+1}^i \leq \mathbb{E}_t^i Q_{t+1}^i L_{t+1}^i, \forall t, \tag{3.5}$$

where $\mathbb{E}_t^i[\cdot] \equiv \Pi[\cdot|\mathcal{F}_t^i]$ denotes the entrepreneur's linear least-squares projection conditional on his information $\mathcal{F}_t^i$ in period t, $\beta \in (0,1)$ refers to his time discount factor, and σ is the entrepreneur's risk aversion parameter. Land does not depreciate but capital depreciates at rate δ. The entrepreneurs on all islands have the same discount factor, risk aversion parameter, and capital depreciation rate. Note that leisure does not enter the utility function. I assume $\alpha + \gamma \leq 1$, which implies an inelastic labour input assumption. Finally, the assumption that households are more patient than the entrepreneurs implies that

$$\beta < \tilde{\beta}.$$

3.2.4 Information Structure

The information structure and its implications on the business cycle are the focus of the chapter. To simplify the analysis, the first assumption I make on the information structure is that, households and entrepreneurs on the same island have identical information at all times. This implies

$$\tilde{\mathbb{E}}_t^i[\cdot] = \mathbb{E}_t^i[\cdot], \forall i, \forall t.$$

Next, I assume that in each period t, agents on island i observe the history of their island-specific productivity shocks up to time t, $\{a_t^i\}_{-\infty}^t$. They may also observe a history of other signals vector $\{\Psi_t^i\}_{-\infty}^t$, which will be specified explicitly in the following analysis.

In addition, I assume that agents cannot share or exchange information across islands. If agents were able to share information, then even they cannot tell the two component shocks apart initially, averaging the sequence of signals $\{a_t^i\}_{i\in[0,1]}$ for each time period across islands would still reveal the true value of economy-wide productivity shock to all agents in the economy at all times, because of the zero adding up constraint on the islands idiosyncratic shocks.

To facilitate the analysis for the incomplete information model, in which the past history of signals will be informative to agents[4], I follow Walker (2007) and express the information set of the agents on island i in period t as

$$\mathcal{F}_t^i \equiv \mathbb{V}_t(a^i) \vee \mathbb{V}_t(\Psi^i) \vee \mathbb{M}, \forall i \in [0,1],$$

where the operator $\mathbb{V}_t(x)$ denotes the Hilbert space generated by the random sequence $\{x_{t-j}\}_{j=0}^{\infty}$ and $\vee$ denotes the span (i.e. the smallest closed subspace which contains the subspaces) of the $\mathbb{V}_t(x)$ and $\mathbb{V}_t(y)$ spaces. If the information sets are disjoint, then the linear span becomes a direct sum. If $\mathbb{V}_t(x) = \mathbb{V}_t(y)$, it means the space spanned by $\{x_{t-j}\}_{j=0}^{\infty}$ is equivalent to the space spanned by $\{y_{t-j}\}_{j=0}^{\infty}$, in the sense of mean square. $\mathbb{M}$ captures the notion of rational expectations and the assumption that agents know exogenous processes and the endogenously generated processes in the equilibrium. In Section 3.4, I will derive equilibrium under different assumptions about $\{\Psi_t^i\}_{-\infty}^{t}$. For example, if $\Psi_t^i = v_t, \forall t, \forall i$, the model becomes the full information case.

3.3 Equilibrium

In this section I define a competitive equilibrium of the economy, and derive the optimal decisions chosen by households and entrepreneurs.

Definition 3.1

A competitive equilibrium is defined as a sequence of allocations and productions: $\{\{C_t^i, \tilde{C}_t^i, L_{t+1}^i, \tilde{L}_{t+1}^i, K_{t+1}^i, B_{t+1}^i, \tilde{B}_{t+1}^i, Y_t^i\}_{i\in[0,1]}\}_{t=0}^{\infty}$*, a sequence of prices:* $\{\{Q_t^i\}_{i\in[0,1]}, \{R_{t+1}^i\}_{i\in[0,1]}\}_{t=0}^{\infty}$*, and a sequence of information sets:* $\{\{\mathcal{F}_t^i\}_{i\in[0,1]}, \{\tilde{\mathcal{F}}_t^i\}_{i\in[0,1]}\}_{t=0}^{\infty}$*, such that:*
(1) for each island $i \in [0,1]$*, given prices, information sets, and the initial endowments:* $L_0^i \geq 0, B_0^i \geq 0, \tilde{L}_0^i \geq 0, \tilde{B}_0^i \geq 0, K_0^i \geq 0$*, the allocations:* $\{C_t^i, \tilde{C}_t^i, L_{t+1}^i, \tilde{L}_{t+1}^i, K_{t+1}^i, B_{t+1}^i, \tilde{B}_{t+1}^i\}_{t=0}^{\infty}$ *solve the optimisations problems of households and entrepreneurs for all* t *and satisfy the transversality conditions:* $\lim_{t\to\infty} \beta^t \Lambda_t^i L_{t+1}^i = 0$*,* $\lim_{t\to\infty} \beta^t \Lambda_t^i K_{t+1}^i = 0$*,* $\lim_{t\to\infty} \tilde{\beta}^t \tilde{\Lambda}_t^i \tilde{L}_{t+1}^i = 0$*; agents form expectations according to* $\mathbb{E}_t^i(\cdot) = \Pi(\cdot|\mathcal{F}_t^i)$ *and* $\tilde{\mathbb{E}}_t^i(\cdot) = \Pi(\cdot|\tilde{\mathcal{F}}_t^i)$*; and*
(2) all markets clear.

[4] If agents can separate the two components from their island-specific shocks, or all agents have "full information", the past history of signals will be redundant, and only the current realisations of shocks will be relevant.

3.3.1 Households Optimality

The optimal choices of land and lending for the representative households on island i can be characterised in the following equations (see Appendix B.1)

$$Q_t^i = \tilde{\beta}\tilde{\mathbb{E}}_t^i Q_{t+1}^i + \frac{b}{\tilde{L}_{t+1}^i}, \tag{3.6}$$

$$\tilde{\beta}(1 + R_{t+1}^i) = 1. \tag{3.7}$$

Equation (3.6) states that the marginal (utility) cost of buying land Q_t^i must be compensated by the expected marginal benefit $\tilde{\beta}\tilde{\mathbb{E}}_t^i Q_{t+1}^i + \frac{b}{\tilde{L}_{t+1}^i}$, which is the instantaneous utility gained from enjoying a unit of land as well as the expected resale value of each unit of land in the next period. Therefore, land not only directly provides utility to the households but also serves the role of asset, transferring utility (measured by the numeraire consumption good) between times.

Equation (3.7) shows that households equate the marginal (utility) cost of lending to the expected marginal discounted benefit $\tilde{\beta}(1 + R_{t+1}^i)$. The linear preference in consumption good implies that, for the household to be indifferent between consuming consumption good and lending, the gross interest rate must be always equal to the inverse of his time-invariant time discount factor.[5]

$$1 + R_{t+1}^i = \tilde{\beta}^{-1}, \forall t.$$

Note that at the beginning of time t, households' income for consumption comes from the predetermined interest of lending at $t-1$, principle, and the potential current income from selling land at t.

3.3.2 Entrepreneurs Optimality

The optimal choices of land holdings and borrowing for the representative entrepreneur on island i can be characterised in the following equations (see Appendix B.1)

$$\frac{1}{(C_t^i)^\sigma} = \beta \mathbb{E}_t^i \left[\frac{\alpha \frac{Y_{t+1}^i}{K_{t+1}^i} + 1 - \delta}{(C_{t+1}^i)^\sigma} \right], \tag{3.8}$$

$$\frac{Q_t^i}{(C_t^i)^\sigma} = \Phi_t^i E_t^i Q_{t+1}^i + \beta \mathbb{E}_t^i \left[\frac{\gamma \frac{Y_{t+1}^i}{L_{t+1}^i} + Q_{t+1}^i}{(C_{t+1}^i)^\sigma} \right], \tag{3.9}$$

[5]If I were to assume a linear utility for land as well, then the land price equation will be given by $Q_t^i = \tilde{\beta}\tilde{\mathbb{E}}_t^i Q_{t+1}^i + b$, from which I can solve for the price as $Q_t^i = \frac{b}{1-\tilde{\beta}}, \forall t$, by ruling out the explosive solution. However, this setup does not allow me to capture the idea that land prices reveal information and lead people to make inferential mistakes and thereby cause economic fluctuations.

$$\frac{1}{(C_t^i)^\sigma} = \beta \mathbb{E}_t^i \left[\frac{1+R_{t+1}^i}{(C_{t+1}^i)^\sigma} \right] + \Phi_t^i (1+R_{t+1}^i), \forall t, \tag{3.10}$$

where Φ_t^i is the Lagrangian multiplier of the borrowing constraint.

Equation (3.8) equates the marginal cost of buying a capital good $(C_t^i)^{-\sigma}$ to the expected marginal benefit $\beta \mathbb{E}_t^i \left[\left(\alpha \frac{Y_{t+1}^i}{K_{t+1}^i} + 1 - \delta \right) (C_{t+1}^i)^{-\sigma} \right]$ from the capital investment. Equation (3.9) states that entrepreneurs equate the marginal cost of buying land $\frac{Q_t^i}{(C_t^i)^\sigma}$ to the expected marginal benefit $\beta \mathbb{E}_t^i \left[\left(\gamma \frac{Y_{t+1}^i}{L_{t+1}^i} + Q_{t+1}^i \right) (C_{t+1}^i)^{-\sigma} \right]$ from land investment plus the term $\Phi_t^i Q_t^i$. Equation (3.10) states that entrepreneurs equate the marginal benefit of borrowing $\frac{1}{(C_t^i)^\sigma}$ to the expected marginal cost $\beta \mathbb{E}_t^i \left[(1+R_{t+1}^i)(C_{t+1}^i)^{-\sigma} \right]$ plus the term $\Phi_t^i (1+R_{t+1}^i)$.

The appearance of the terms $\Phi_t^i Q_t^i$ and $\Phi_t^i (1+R_{t+1}^i)$ reflects the effect of borrowing constraint. When households are more patient than entrepreneurs, I show in Appendix B.2 that in the steady state $\Phi^i = (\tilde{\beta} - \beta)\Lambda^i > 0, \forall i$. It immediately follows that around the steady state the marginal cost of borrowing is lower than the marginal benefit due to the positive term $\Phi_t^i (1+R_{t+1}^i)$, suggesting that the borrowing constraint binds around the steady state. In the absence of credit constraint, equation (3.8) and (3.9) imply that marginal products of capital would be equal to the real interest rate. However, with credit friction and difference in discount factors, the real interest rate is lower than the marginal product of capital around the steady state.

3.3.3 Markets Clearing

For each island $i \in [0,1]$, there are three markets to clear in each period t:

$$\text{land market: } \bar{L}^i = \tilde{L}_t^i + L_t^i, \tag{3.11}$$

$$\text{credit market: } \tilde{B}_t^i = B_t^i, \tag{3.12}$$

$$\text{goods market: } Y_t^i = \tilde{C}_t^i + C_t^i + K_{t+1}^i - (1-\delta) K_t^i, \tag{3.13}$$

where $\bar{L}^i$ is constant $\forall i$.

3.4 Equilibrium Characterisation

3.4.1 Solution Methods

To characterise the model equilibrium, I need to solve the highly nonlinear system comprised of equations (3.1)-(3.12).[6] To simplify the analysis, I follow the literature and characterise a log-linear approximation of the system. In a setup with

[6]By the Walras' Law, one of the market clearing condition is redundant for solving the model.

incomplete information, the advantage of log-linearisation could be even more; as emphasised in Lorenzoni (2009), log-linearisation simplifies the inference problem of individual agents, the state space for individual decision rules, and aggregation. However, complex nonlinear equilibrium functions with considerable curvature may have sizable economic implications. This is particularly true when agents in the model do not have complete information and have to solve signal extraction problems. Log-linearisation throws away nonlinear effects and the simplified model may generate very different implications from those of the original model. For these reasons, the implications in the following analysis may only be reliable for relatively small or one-off shocks; further work will be needed to examine how robust the findings are when allowing for larger repeated disturbances that result in a wide distribution of values for agent net-worth and expectations in each island in each period rather than a point value.

The log-linearisation procedure in this chapter is standard as in the literature; that is, I log-linearise the nonlinear stochastic model around the deterministic steady state under full information. A detailed description of the steady state is available in Appendix B.2. Following the convention, I use the lower case letters to denote the log deviations. That is, I define $x_t \equiv \ln X_t - \ln X$, where X is the steady state value of X_t. The log-linearisation of the optimality conditions, the budget constraints, and market clearing conditions, which are presented in Appendix B.3, gives me a linear system that approximately characterises the model equilibrium around the steady state.

Having the log-linearised model, the next step is to solve the linearised system. Different from the standard procedure, which generally employs a programme package such as Dynare (based on the perturbation methods) to solve for a dynamic stochastic general equilibrium model under rational expectations, I solve the model by hand. More specifically, I first rearrange the log-linearised system into a high-order stochastic equation in land price for each island i, and a sequence of other equations in a recursive order. These equations are shown in Appendix B.4. The key equation to solve is as below

$$\delta_1 q_{t-1}^i + \delta_2 q_t^i + \delta_3 \mathbb{E}_{t-1}^i q_t^i + (\delta_4 - \delta_5)\mathbb{E}_t^i q_{t+1}^i = \delta_6 \mathbb{E}_{t-1}^i a_t^i + \delta_7 a_t^i + \delta_8 \mathbb{E}_t^i a_{t+1}^i + \delta_9 \sum_{j=0}^{\infty} \tilde{\beta}^j \mathbb{E}_t^i a_{t+2+j}^i, \tag{3.14}$$

where $\delta_j \equiv \delta_j(\tilde{\beta}, \beta, \delta, \alpha, \gamma, \sigma), j = 1, \cdots, 9$. Once I obtain the explicit expression for q_t^i, the rest of variables can be obtained recursively using equations in Appendix B.4.

Despite that this rearranging procedure is tedious and it does not have any advantage in solving the full information model, it does show great benefits in solving models where there is incomplete information among islands and agents are learning from some endogenously generated variables. This is because in such models the learning of rational agents involves an infinite regress problem, and the standard procedure has difficulty in solving it. In contrast, the way I rearrange the model equilibrium system gives me a chance to employ a different solution method, from which I can derive exact analytic solutions to the model. More importantly, this method can make the comparison between different models clearer.

Having equation (3.14), I then follow Townsend (1983) and first solve it in a general form. Let

$$\Theta_t^i \equiv \mathbb{E}(\theta_t | \mathcal{F}_t^i).$$

In Appendix B.4, I derive a general form solution to equation (3.14) by imposing the rational expectations equilibrium restriction. This is given by

$$q_t^i = \pi_1 q_{t-1}^i + \pi_2 \Theta_t^i + \pi_3 \Theta_{t-1}^i + \pi_4 a_t^i, \tag{3.15}$$

where $\pi_n = \pi(\{\delta_j\}_1^9, \rho), n = 1, 2, 3, 4$ are obtained in Appendix B.4.

Note that equation (3.15) is not yet the final solution to equation (3.14). The final solution requires an explicit expression for Θ_t^i, which clearly depends on the information sets of agents on island i. As will be seen, equation (3.15) makes the comparison between the solutions under different information extremely straightforward.

3.4.2 Full Information Benchmark

As a benchmark, it is useful to look at the solution to the model where agents observe all contemporaneous shocks hitting the economy. In this case, there is no possibility for agents to confound different shocks, and therefore all noisy signals will be redundant. This will enable us to see how the picture will be changed if agents have only incomplete information about the productivity shocks.

Definition 3.2
A full information equilibrium is the definition 3.1, with the agents' information set at each time period t specified as $\mathcal{F}_t^i = \mathcal{F}_t^F = \mathbb{V}_t(\{v, \epsilon^i\}_{i\in[0,1]}) \vee \mathbb{M}_t, \forall i$.

Since all agents observe the underlying shocks directly in each period, there is no room for fluctuations caused by non-fundamental noises, and we have

$$\Theta_t^i \equiv \mathbb{E}_t^i \theta_t = \theta_t.$$

The land price function is then given by

$$q_t^i = \pi_1 q_{t-1}^i + (\pi_2 + \pi_4)\theta_t + \pi_3 \theta_{t-1} + \pi_4 \epsilon_t^i.$$

That is, land prices follow an ARMA(2,1) process

$$(1-\pi_1\mathcal{L})(1-\rho\mathcal{L})q_t^i = (\pi_2+\pi_4+\pi_3\mathcal{L})v_t + \pi_4(1-\rho\mathcal{L})\epsilon_t^i. \tag{3.16}$$

where $\mathcal{L}$ is the lag operator.

3.4.3 Incomplete Information

No Public Signals

In this subsection, I assume agents only observe their island-specific productivity shock in each period without being able to distinguish between the two components. Also, they do not observe any public signals. That is, I assume $\{\Psi_t^i\}_{-\infty}^t = \emptyset$.

Definition 3.3
An incomplete information equilibrium of Type-I is the definition 3.1, with the agents' information set at each time period t on island i specified as $\mathcal{F}_t^{I,i} = \mathbb{V}_t(a^i)\vee\mathbb{M}_t$.

Without any additional public signal that can be observed by agents on all islands, the model becomes very simple; each island is an isolated Kiyotaki-Moore economy. In each period, agents on each island only need to solve a simple signal extraction problem based on their island-specific productivity shock a_t^i. Applying the standard Kalman filter formula, I have

$$\Theta_t^i = (1-\kappa)\rho\Theta_{t-1}^i + \kappa a_t^i,$$

where κ is the stationary Kalman gain given by

$$\kappa \equiv \frac{\frac{1}{\sigma_\epsilon^2}}{\frac{1}{\sigma_r^2}+\frac{1}{\sigma_\epsilon^2}} \in (0,1),$$

in which σ_r^2 is the solution to $\sigma_r^2 = \frac{\rho^2}{\frac{1}{\sigma_r^2}+\frac{1}{\sigma_\epsilon^2}} + \sigma_v^2$. Substituting Θ_t^i into the general solution equation, we can see that land prices follow an ARMA(3,2) process

$$(1-\pi_1\mathcal{L})(1-\rho\mathcal{L})(1-(1-\kappa)\rho\mathcal{L})q_t^i$$

$$= [\pi_4+\pi_2\kappa+(\pi_3\kappa-\pi_4(1-\kappa)\rho)\mathcal{L}]v_t + [\pi_4+\pi_2\kappa+(\pi_3\kappa-\pi_4(1-\kappa)\rho)\mathcal{L}](1-\rho\mathcal{L})\epsilon_t^i. \tag{3.17}$$

Note that the on-impact effect of one unit shock v_t on land price in the full information model is $\pi_2+\pi_4$, while the on-impact effect of one unit shock ϵ_t on land price in the full information model is π_4. When agents cannot disentangle the two shocks apart, the on-impact effects of the two shocks on land price become equalised and are given by $\pi_2\kappa + \pi_4$. As $\kappa \in (0,1)$, it follows immediately that the on-impact effect of v_t is smaller than that in the full information model, while the on-impact effect of ϵ_t is bigger than that in the full information model.

Exogenous Public Signals

Until now I have discussed the model with only real shocks. Now I am introducing non-fundamental noises into the model. The simplest way of doing this is to introduce an exogenous noisy public signal

$$s_t^* = \theta_t + \eta_t^*,$$

where $\eta_t^* \sim \mathbb{N}(0, \sigma_{\eta^*}^2)$. The noise shocks are independently identically distributed across times and independent of all other shocks.

Definition 3.4
An incomplete information equilibrium of Type-II is the definition 3.1, with the agents' information set at each time period t on island i specified as $\mathcal{F}_t^{II,i} = \mathbb{V}_t(a^i) \vee \mathbb{V}_t(s^*) \vee \mathbb{M}_t$.

As agents' behaviours will not have any effect on the exogenous signals, agents on each island are only solving a signal extraction problem a bit complicated than the previous case. As before, applying the standard Kalman filter formula, I have

$$\Theta_t^i = (1 - K_1 - K_2)\rho\Theta_{t-1}^i + K_1 a_t^i + K_2 s_t^*,$$

where

$$\begin{aligned}
K_1 &\equiv \frac{\frac{1}{\sigma_\epsilon^2}}{\frac{1}{\sigma_{r^*}^2} + \frac{1}{\sigma_{\eta^*}^2} + \frac{1}{\sigma_\epsilon^2}} \in (0,1), \\
K_2 &\equiv \frac{\frac{1}{\sigma_{\eta^*}^2}}{\frac{1}{\sigma_{r^*}^2} + \frac{1}{\sigma_{\eta^*}^2} + \frac{1}{\sigma_\epsilon^2}} \in (0,1),
\end{aligned}$$

in which $\sigma_{r^*}^2$ is the solution to the following nonlinear function

$$\sigma_{r^*}^2 = \frac{\rho^2}{\frac{1}{\sigma_{r^*}^2} + \frac{1}{\sigma_{\eta^*}^2} + \frac{1}{\sigma_\epsilon^2}} + \sigma_v^2.$$

Substituting Θ_t^i into the general solution equation, land prices follow an ARMA(3,2) process

$$(1-\pi_1\mathcal{L})(1-\rho\mathcal{L})(1-K_1-K_2)\rho\mathcal{L})q_t^i$$

$$=\{\pi_4+\pi_2(K_1+K_2)+[\pi_3(K_1+K_2)-\pi_4(1-K_1-K_2)\rho]\mathcal{L}\}v_t$$

$$+\{\pi_4+\pi_2K_1+[\pi_3K_1-\pi_4(1-K_1-K_2)\rho]\mathcal{L}\}(1-\rho\mathcal{L})\epsilon_t^i+(\pi_2+\pi_3\mathcal{L})K_2(1-\rho\mathcal{L})\eta_t^*. \quad (3.18)$$

With one more public signal about v_t, the on-impact effect of one unit shock v_t on land price becomes $\pi_2(K_1+K_2)+\pi_4$, which is strictly larger than the on-impact effect of shock ϵ_t which is $\pi_2K_1+\pi_4$. This is intuitive: more information about v_t makes agents to have more accurate estimation about the shock, and thus react more heavily to it. The noise contained in the public signal also have significant effect on the economy; the land price dynamics due to the noise shock is characterised by an ARMA(2,1) process

$$(1-\pi_1\mathcal{L})[1-(1-K_1-K_2)\rho\mathcal{L}]q_t^i=(\pi_2+\pi_3\mathcal{L})K_2\eta_t^*, \quad (3.19)$$

From (3.19), we can see that the on-impact effect of η_t^* on land price is π_2K_2. Note that, given $\sigma_{r^*}^2$ and σ_ϵ^2, K_2 is decreasing in the variance of noise $\sigma_{\eta^*}^2$. However, $\sigma_{r^*}^2$ will be increasing if $\sigma_{\eta^*}^2$ increases, which tends to make K_2 bigger. Hence, the change of $\sigma_{\eta^*}^2$ on the effect of noise shock is non-monotonic; the effect will be small when $\sigma_{\eta^*}^2$ is not very small or very large. That is, when the public signal is very precise or very imprecise, noise shocks tend to generate small volatility on land price as well as other variables. As pointed out by Lorenzoni (2009), this non-monotonic relation between the variance of the noise shocks and the macroeconomic variables volatility they generate is a peculiar feature of a learning model of business cycles.

If information is revealed one period later, I show in Appendix B.5 that land price follows an ARMA(2,2) process

$$(1-\pi_1\mathcal{L})(1-\rho\mathcal{L})q_t^i=\{(\pi_2+\pi_3\mathcal{L})[\rho\mathcal{L}+(1-\rho\mathcal{L})(W_1^*+W_2^*)]+\pi_4\}v_t$$

$$+[(\pi_2+\pi_3\mathcal{L})W_1^*+\pi_4](1-\rho\mathcal{L})\epsilon_t^i+[(\pi_2+\pi_3\mathcal{L})W_2^*(1-\rho\mathcal{L})]\eta_t^*,$$

where

$$\begin{aligned} W_1^* &\equiv \frac{\sigma_v^2(\sigma_v^2+\sigma_{\eta^*}^2)-\sigma_v^4}{(\sigma_v^2+\sigma_\epsilon^2)(\sigma_v^2+\sigma_{\eta^*}^2)-\sigma_v^4}, \\ W_2^* &\equiv \frac{\sigma_v^2(\sigma_v^2+\sigma_\epsilon^2)-\sigma_v^4}{(\sigma_v^2+\sigma_\epsilon^2)(\sigma_v^2+\sigma_{\eta^*}^2)-\sigma_v^4}. \end{aligned}$$

The dynamics of land prices due to noise shocks are described by

$$(1 - \pi_1 \mathcal{L}) q_t^i = (\pi_2 + \pi_3 \mathcal{L}) W_2^* \eta_t^*.$$

Endogenous Public Signals

So far as I have discussed, all the signals are exogenous. In this subsection, I assume agents on all islands observe a noisy indicator of the economy-averaged land prices at each time, instead of an exogenous public signal. More specifically, the price indicator is assumed as

$$s_t = \int q_t^i \mathrm{d}i + \eta_t,$$

where $\eta_t \sim \mathbb{N}(0, \sigma_\eta^2)$. Because land price on each island is an endogenously generated variable, which depends on the expectations of economic fundamentals on that island, s_t turns out to be an endogenous public signal with some noise η_t. This seemingly innocuous change in the information sets of agents has a profound effect on the nature of the rational expectations equilibrium. This is because the endogenous variable as stochastic processes is itself influenced by the solution of the signal extraction problems that agents on other islands are simultaneously solving. As emphasised in the literature, when agents in the economy have heterogeneous information, the law of iterated expectations for the average beliefs operator typically does not hold (Allen, Morris, and Shin, 2006), and agents on each island must "forecast the forecasts of others."

The infinite regress induced from the signal extraction from endogenous public signals poses some technical challenges in solving the model, as it implies an infinite number of state variables so that the standard Kalman filtering formulas no longer fit. In the recent literature, a method of indeterminate coefficients with a truncated state space is often used.[7] However, this method relies on numerical simulation and how does the solution look like is not clear. In the following, I follow the earlier literature in solving such models. More specifically, I assume all information is revealed with one period lag so that an analytical solution can be derived. Although this simplification might have made the problem less interesting, it delivers clearer results that can be compared with the benchmark results.

Definition 3.5
An incomplete information equilibrium of Type-III is the definition 3.1, with the agents' information set at each time period t on island i specified as $\mathcal{F}_t^{III,i} = \mathbb{V}_t(a^i) \vee \mathbb{V}_t(s, v_{-1}, \epsilon_{-1}, \eta_{-1}) \vee \mathbb{M}_t$.

[7] See for instance Nimark (2011) and Loronzoni (2009). Kasa (2000), however, shows that transforming the problem from the time domain into the frequency domain will circumvent the difficulty and delivers analytic solutions.

To solve for the land price function and characterise the equilibrium, suppose the expectation of agents on island i about the common persistent shock θ_t is a linear combination of all current and past values of observable signals (a_t^i, q_t^i, s). Since each of these is a function of the history of the shocks, $\Theta_t^i \equiv \mathbb{E}_t^i(\theta_t)$ can be represented as

$$\Theta_t^i = P_v(\mathcal{L})v_t + P_\epsilon(\mathcal{L})\epsilon_t^i + P_\eta(\mathcal{L})\eta_t,$$

where $P_v(\mathcal{L}), P_\epsilon(\mathcal{L})$ and $P_\eta(\mathcal{L})$ are in general infinite-order, square-summable polynomials in the lag operator $\mathcal{L}$; that is,

$$P_i(\mathcal{L}) = \sum_{j=0}^{\infty} P_{ij}\mathcal{L}^j, i = v, \epsilon, \eta.$$

Let $A(\mathcal{L}) = \frac{1}{1-\rho\mathcal{L}}$, $B(\mathcal{L}) \equiv \frac{1}{1-\pi_1\mathcal{L}}$ and $C(\mathcal{L}) \equiv \pi_2 + \pi_3\mathcal{L}$. In Appendix B.5, I show given this conjecture and imposing rational expectations condition, θ_t can be derived as

$$\Theta_t^i = P_v(0)v_t + \sum_{j=1}^{\infty} \rho^j v_{t-j} + P_\epsilon(0)\epsilon_t^i + P_\eta(0)\eta_t. \tag{3.20}$$

where $P_j(0), j = v, \epsilon, \eta$ are defined in Appendix B.5. Having equation (3.20), the land price then becomes

$$(1-\pi_1\mathcal{L})(1-\rho\mathcal{L})q_t^i = \{(\pi_2+\pi_3\mathcal{L})[\rho\mathcal{L} + (1-\rho\mathcal{L})P_v(0)] + \pi_4\}v_t$$

$$+[(\pi_2+\pi_3\mathcal{L})P_\epsilon(0) + \pi_4](1-\rho\mathcal{L})\epsilon_t^i + [(\pi_2+\pi_3\mathcal{L})P_\eta(0)(1-\rho\mathcal{L})]\eta_t. \tag{3.21}$$

which is an ARMA(2,2) process with persistent shock v, transitory shock ϵ, and noise shock η. (i) The on-impact effect of shock v_t is $\pi_2 P_v(0) + \pi_4$; (ii) the on-impact effect of shock ϵ_t is $\pi_2 P_\epsilon(0) + \pi_4$. The effect from the noise shock can be characterised by

$$(1-\pi_1\mathcal{L})q_t^i = (\pi_2+\pi_3\mathcal{L})P_\eta(0)\eta_t, \tag{3.22}$$

which is an ARMA(1,1) process with the on-impact effect (from one unit shock) $\pi_2 P_\eta(0)$. Comparing (3.22) with (3.19)

$$(1-\pi_1\mathcal{L})q_t^i = (\pi_2+\pi_3\mathcal{L})W_2^*\eta_t^*,$$

we can see that the dynamics are defined by the same parameters while they differ in the on-impact effects.

3.4.4 Optimism and Pessimism

Having characterised the model equilibrium, I am now able to derive the agents' forecast errors about the common persistent productivity shock for the continuum of islands. Following the literature, the concept of optimism and pessimism is defined on the agents' one-period ahead forecasting error about the persistent shocks.

Definition 3.6
Let $\xi_t^i \equiv \theta_t - \mathbb{E}_{t-1}^i \theta_t$ be the time $t-1$ forecast error of the agents on island i about the persistent shock in period t, and $\xi_t \equiv \int \xi_t^i \mathrm{d}i = \theta_t - \bar{\mathbb{E}}_{t-1}\theta_t$ be the economy average forecast error, where $\bar{\mathbb{E}}_{t-1}\theta_t = \int \mathbb{E}(\theta_t | \mathcal{F}_{t-1}^i)\mathrm{d}i$. The economy is optimistic about θ if $\xi_t < 0$; they are pessimistic about the shock if $\xi_t > 0$.

Similar to the previous discussion, to highlight the learning effect, I still use the results from the full information model as the benchmark. It is easy to show that if agents have full information about the economy, on each island i the agents' forecast errors are the same and serially uncorrelated. Thus, the forecast errors of the economy are i.i.d., that is,

$$\xi_t = v_t, \forall t.$$

However, if information is incomplete and signals are not fully revealing, then the economy average forecast errors are serially correlated. Appendix B.6 shows, for the no public signal case, the forecast errors follow an ARMA(2,1) process; for the exogenous public signal case, they follow an ARMA(2,2) process. For the endogenous public signal model and the exogenous public signal model where information is fully revealed with one period lag, the forecast errors are both governed by the MA(1) process.

3.4.5 Loan-to-Value Ratio

Define the loan-to-value ratio on island i at each time t as

$$\tau_t^i \equiv \frac{B_{t+1}^i}{Q_t^i L_{t+1}^i}.$$

Log-linearise τ around the steady-state, I have

$$\hat{\tau}_t^i = \mathbb{E}_t^i q_{t+1}^i - q_t^i.$$

which is simply the difference between the expected future land price and current land price. In equilibrium, entrepreneurs' borrowing constraint hold with equality and they will use all their net worth to finance the difference between the price of land and the amount they can borrow against each unit of land, i.e. the down payment. Hence, the lower the down payment lenders require, the more the land entrepreneurs will buy. In terms of the loan-to-value ratio, the high the ratio $\hat{\tau}_t^i$, the more land entrepreneurs on island i can buy. This will be true when the households on island i have higher expected future land price given the current price.

The previous analysis then implies that the high (low) expectation might be driven by noises. Lenders have optimistic expectations about future land price not only because current and future productivity shocks are expected to be high, but also because they are confused about the true state of the world and mistakenly take the noise in the public signals as a fundamental shock. Hence, noises generate sizable and persistent effect on the macroeconomy in a way that is different from the true productivity shocks described in Kiyotaki and Moore (1997); in their model, productivity shock generates effects by initializing a change in constrained firms' net worth, while noises in this model effect through lenders' (as well as borrowers') expectation induced by some public signal.

3.5 Conclusion

In this chapter, I introduce dispersed information and collateral constraints into a Real Business Cycle model. I show that noises may have real impacts on the macroeconomy, which is induced by learning and amplified by the collateral effect. More specifically, I incorporate Kiyotaki and Moore (1997) mechanism into an "islands economy" where agents on a continuum of islands have dispersed information about the aggregate productivity shock. As agents on each island cannot tell the aggregate shock apart from the island-specific idiosyncratic productivity shock, they use both privately observed signals and noisy public signals to make optimal inferences. When information is not fully revealed, I show noises in the public signals can be important sources of macroeconomic fluctuations.

Chapter 4

News Shocks, Housing Prices, and Consumption

4.1 Introduction

Aggregate time series data from many countries clearly show there is a strong co-movement between housing price change and consumption change. These facts have attracted a lot of attention among economists and policy makers, and lead them to ask the questions: What is the relationship between housing price and consumption? How do housing prices affect consumer spending?

Under the life-cycle hypothesis, a natural explanation is attributed to the "wealth effect", and there are a bunch of papers finding significant sizes of marginal propensity to consume out of housing wealth. The most influential ones might be Case *et.al.* (2005) who use the aggregate data and Campbell and Cocco (2007) who use the household level data.

Nonetheless, the wealth effect explanation has been challenged for both its theoretical underpinning (Buiter, 2008) and the empirical estimations. One of the arguments is that at the aggregate level housing wealth effect may not be significant, even though it may be large from the household perspective. This is because there are both winners and losers in the housing market. While homeowners may increase their consumption in response to the housing price appreciation, those who want to get on or up the property ladder may be forced to reduce consumption. Therefore, the overall effect will depend on the distribution of winners and losers in the housing market, and their responses to house price changes.

Other economists, such as Iacoviello (2005), therefore argue for the collateral effect to explain the strong co-movement between the two series. The idea is that housing is often used as the collateral to borrow funds, and when the price becomes higher, credit constrained homeowners can borrow more to finance consumption spending.

The co-movement between housing price and consumption may also be driven by the common factors, especially those unobservable factors such as the news about future productivity (King, 1990). Attanasio *et.al.* (2009) study the same household level data as that in Campbell and Cocco (2007), but conclude that it is the common factors instead of wealth effect that explain the co-movement.

This chapter tends to support the common factors view as argued by e.g. Attanasio and Weber (1994) and Attanasio *et.al.* (2009). In contrast to the previous papers which indirectly show the point using household level data, this chapter explicitly identifies the most suspected common factor. More specifically, I use the aggregate time series data from the United Kingdom and the Structural Vector Autoregression (SVAR) methodology to identify productivity shock and news about future productivity shock.

While it is not possible to disentangle the different effect channels discussed above using the macroeconomic data and SVAR methodology, I am able to quantify the importance of common factors in explaining the co-movement between housing price and consumption. My empirical estimation of the U.K. data delivers two findings: (1) most of the positive co-movement between housing price and consumption comes from current productivity shock and news shock about future productivity; (2) the shock that moves the vast majority of housing price barely moves consumption. These two findings thus cast doubt on the importance of wealth effect in explaining the striking co-movement of housing price and consumption.

4.2 An Illustrative Model

Before I show the empirical part, I first show an illustrative model in this section. I should emphasise that, while the model reflects my understanding about the question and motivates my empirical strategy, the empirical analysis that follows is not an estimation of the model.

Consider an economy populated by a large number (L_t) of identical individual consumers in which the only assets are a set of identical infinitely-lived trees and a set of identical infinitely-lived houses. Consumers have infinite horizons. Aggregate output equals the fruit of the trees and service flows of housing, neither of which can be stored. Assume that in a given year, each tree produces exactly the same amount of fruit as every other tree, but the total harvest output of fruit per tree d_t varies from year to year depending on the weather. Each house produces exactly the same amount of housing services as every other house and the total services per house is constant and normalised to be one. Each consumer owns the same number of trees and houses. The aggregate stock of trees is K_t and aggregate stock of housing is

H_t. Each consumer, for given k_{-1}, h_{-1}, solves the problem

$$\max_{\{c_t, s_t, k_t, h_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) + \theta_t w(s_t)],$$

s.t.

$$c_t + r_t s_t + p_t k_t + q_t h_t = (p_t + d_t) k_{t-1} + (q_t + r_t) h_{t-1}, \forall t,$$

where c_t is the consumption of fruit per person in t and $u(c_t)$ is the utility by consuming fruit c_t, s_t is the consumption of housing service per person in t and $w(s_t)$ is the utility derived from s_t, r_t is the price of housing service in t, p_t is the price of a tree in t, q_t is the the price of a house in t, k_t is the quantity of trees held per person in t, h_t is the quantity of houses held per person in t, β is consumer's discount factor, and θ_t is consumer's preference parameter for housing services. The first-order conditions to the consumer's problem are given by

$$\begin{aligned}
\frac{\theta_t}{r_t} &= \frac{u'(c_t)}{w'(s_t)}, \\
p_t &= \beta \mathbb{E}_t \left[\frac{u'(c_{t+1})}{u'(c_t)} (d_{t+1} + p_{t+1}) \right], \\
q_t &= \beta \mathbb{E}_t \left[\frac{u'(c_{t+1})}{u'(c_t)} (r_{t+1} + q_{t+1}) \right].
\end{aligned}$$

Forward-looking iterations give the celebrated pricing formulas for both the tree and the house

$$\begin{aligned}
p_t &= \mathbb{E}_t \sum_{j=1}^{\infty} \beta^j \frac{u'(c_t)}{u'(c_{t+j})} d_{t+j}, \\
q_t &= \mathbb{E}_t \sum_{j=1}^{\infty} \beta^j \frac{u'(c_t)}{u'(c_{t+j})} r_{t+j}.
\end{aligned}$$

There are four markets to be cleared: (1) fruit market: $c_t L_t = d_t K_t$; (2) housing rental market: $s_t L_t = H_t$; (3) trees market: $k_t L_t = K_t$; and (4) housing market $h_t L_t = H_t$. If I normalise so that $L_t = 1$ and $K_t = 1$, these markets clearing conditions become $c_t = d_t$, $s_t = H_t$, $k_t = 1$, $h_t = H_t$. If I further assume $u(c_t) = \ln c_t$ and $w(s_t) = \ln s_t$, then the house pricing formula can be written as

$$q_t = \mathbb{E}_t \sum_{j=1}^{\infty} \beta^j \frac{\theta_{t+j}}{H_{t+j}} d_t.$$

Hence, in this simple endowment economy, non-housing consumption (fruit) change is completely exogenous, while housing price is determined by the preference shock, housing supply, and fruit production. I take this extreme stand in order to show that in a general equilibrium model with homogeneous consumers, the housing wealth effect concept could be misleading. The co-movement may reflect a reversed causality from consumption to housing price.

To introduce the information structure and see how the news about future productivity change affects these variables, I assume the productivity process for each fruit tree as follows

$$\ln d_t = \rho \ln d_{t-1} + v_{1,t-1} + v_{2,t},$$

where $v_{1,t-1}$ is the news shock observed in period $t-1$ about the productivity change that will realise in period t, and $v_{2,t}$ is the productivity shock in period t. Other things equal, productivity shock and news shock will drive housing price and consumption to move in the same direction.

In the following section, I identify the news shock and productivity shock based on the estimation of a three-variable vector autoregression (VAR) model.

4.3 Empirical Strategy

4.3.1 Structural Vector Autoregression Model

Consider a k-dimensional time series Y_t, where $t = 1, \ldots, T$. Assume Y_t can be approximated by a structural vector autoregression model of finite order p as below

$$B(L)Y_t = u_t, \tag{4.1}$$

where $B(L) \equiv B_0 - B_1L - B_2L^2 - \cdots - B_pL^p$ is the autoregressive lag order polynomial[1], and the variance-covariance matrix of structural error term is normalised such that

$$E(u_t u_t') \equiv \Sigma_u = I_k.$$

The normalisation implies that, a unit innovation in the structural shocks is of size one standard deviation by construction.

A reduced form of the model can be derived by pre-multiplying both sides of equation (4.1) by B_0^{-1}. That is,

$$A(L)Y_t = \epsilon_t, \tag{4.2}$$

where $A(L) \equiv I - A_1L - A_2L^2 - \cdots - A_pL^p$, $A_s \equiv B_0^{-1}B_s, s = 1, \ldots, p$, and $\epsilon_t \equiv B_0^{-1}u_t$. The standard estimation methods allow us to obtain consistent estimates of the reduced-form parameters A_s, the errors ϵ_t, and their covariance matrix Σ_ϵ.

[1] All deterministic regressors have been suppressed for notational convenience.

By recovering the elements of B_0^{-1} from the estimates of reduced form parameters, the coefficient matrices of the structural equation can be constructed by using the relation $B_s \equiv B_0 A_s$, and the desired structural shocks may be obtained.

Note that there could be numerous possible sets of structural shocks. To see this, let $\hat{B}_0^{-1}$ be the Cholesky decomposition of Σ_ϵ. Then, any $\tilde{B}_0^{-1} \equiv \hat{B}_0^{-1} Q$ and $\tilde{u}_t \equiv Q' \hat{u}_t$ can be candidate orthogonalisation and candidate structural shocks. It is then essential to impose reasonable restrictions to identify the desired structural shocks.

4.3.2 Identification Strategy

I aim to identify two types of shocks: the productivity shock and news shock about future productivity. The identification strategy is similar to that in Barsky and Sims (2011) and Uhlig (2003). Specifically, I first identify two orthogonal shocks that best explain the sum of forecast error variance of productivity at all horizons. I then distinguish the two shocks by restricting no contemporaneous effect of news shock on productivity. In the first step, I followed Barsky and Sims (2011) and truncated the horizon to 40 periods.[2]

To find out shocks that account for the variation of a particular variable as much as possible turns out to be the same as finding eigenvectors corresponding to the largest eigenvalues of some matrix. To see this, write Y_t in the vector moving average (VMA) representations

$$Y_t = \sum_{s=0}^{\infty} \hat{\theta}_s \hat{u}_{t-s} = \sum_{s=0}^{\infty} \tilde{\theta}_s \tilde{u}_{t-s},$$

where $\hat{u}$'s are the structural shocks obtained from the Cholesky decomposition, and $\tilde{u}$'s are the structural shocks obtained from any arbitrary orthogonalisation. Given all the data up to and including t, the h-step forecast error of Y_{t+h} is $\sum_{s=0}^{h-1} \hat{\theta}_s \hat{u}_{t+h-s}$ which is equal to $\sum_{s=0}^{h-1} \hat{\theta}_s (Q \tilde{u}_{t+h-s})$. The forecast error variance-covariance matrix is thus given by $\sum_{s=0}^{h-1} [\hat{\theta}_s Q][\hat{\theta}_s Q]'$, with the off-diagonal elements being replaced by zeros. This matrix can be further decomposed as

$$\sum_{j=1}^{k} \sum_{s=0}^{h-1} [\hat{\theta}_s q_j][\hat{\theta}_s q_j]',$$

where q_j is the j^{th} column of Q, k is the dimension of the system, and $h = 1 \cdots$. Identifying the shock j that best explains the forecast error variance of variable i

[2] This looks quite arbitrary but a sensitivity check shows that the choice turns out to have little impact on the results once it is large enough.

from period $\underline{H}+1$ to $\overline{H}$ is equivalent to doing the maximisation problem below

$$\max \sigma^2(\underline{H}, \overline{H}; q_j) = \sum_{h-1=\underline{H}}^{\overline{H}} \sum_{s=0}^{h-1} [\hat{\theta}_s q_j][\hat{\theta}_s q_j]'_{ii} = q_j' S_i q_j,$$

subject to

$$q_j' q_j = 1,$$

where $[\hat{\theta}_s q_j][\hat{\theta}_s q_j]'_{ii}$ denotes the $(i,i)^{th}$ element in $[\hat{\theta}_s q_j][\hat{\theta}_s q_j]'$, and

$$S_i = \sum_{h-1=\underline{H}}^{\overline{H}} \sum_{s=0}^{h-1} (\hat{\theta}_s^i)' \hat{\theta}_s^i = \sum_{s=0}^{\underline{H}} (\hat{\theta}_s^i)' \hat{\theta}_s^i + \sum_{s=0}^{\underline{H}+1} (\hat{\theta}_s^i)' \hat{\theta}_s^i + ... + \sum_{s=0}^{\overline{H}} (\hat{\theta}_s^i)' \hat{\theta}_s^i,$$

where $\hat{\theta}_s^i$ denotes the i^{th} row of $\hat{\theta}_s$. Solving the Lagrangian problem

$$\max L(q_j, \lambda) = q_j' S_i q_j - \lambda(q_j' q_j - 1),$$

I have the first-order necessary condition

$$2S_i q_j^* = 2\lambda q_j^*,$$

from which we can see that the solution to q_j lies in the set of eigenvectors of S_i. Combining with the constraint $q_j' q_j = 1$, I have the maximum value

$$\sigma^2_{max}(\underline{H}, \overline{H}; q_j) = (q_j^*)' S_i q_j^* = (q_j^*)' \lambda q_j^* = \lambda (q_j^*)' q_j^* = \lambda.$$

This implies that q_j^* is just the eigenvector with the maximal eigenvalue λ.[3]

The above analysis can be easily generalised if one wants to identify multiple shocks that best explain the forecast error variance of variable i. In this case, the maximum value of σ^2 will be the sum of largest eigenvalues in λ. Then one can get the corresponding eigenvectors. However, the eigenvectors are not unique for maximizing σ^2.[4] In other words, one can find many (numerous) sets of two orthogonal shocks that give the same maximised total forecast error variance of variable i at the horizon H. Therefore, one needs to impose other restrictions to distinguish those shocks.

[3]Since the maximum σ^2 is just λ and to get the maximum value is equivalent to choose the largest value from $\{\lambda\}$, and thus the corresponding eigenvector is the solution. S_i is something I can compute from the reduced VAR and the eigenvalues and eigenvectors to it can be easily obtained using Matlab.

[4]See Uglig(2003) for detail.

4.3.3 Data and VAR Estimation

The primitive data are from the U.K. Office for National Statistics, including nominal housing price, labour productivity, non-durable goods and service, durable goods, GDP deflator, and population. They are quarterly data covering the period of 1971-2011. I use these data to obtain the four series of variables: housing price, labour productivity, non-durable consumption good, and durable consumption good; all are in real per capita terms. The data in the VAR model are first-order differences of the log values of those transformed data. I estimate the model for both non-durable goods and durable goods respectively, but my analysis in the following section focuses mainly on the results of non-durable goods case.

All the data series in the sample are stationary, tested by the Dicky-Fuller method. For both cases (durable goods and non-durable goods) the order of VAR is of one, which is based on a combination of information criteria and serial correlation tests. With a stationary system, I can write the VAR model into a vector moving average model with structural shocks

$$\begin{pmatrix} \Delta hp_t \\ \Delta d_t \\ \Delta c_t \end{pmatrix} = \sum_{s=0}^{\infty} \begin{pmatrix} \theta^{hp}_{news} & \theta^{hp}_{un} & \theta^{hp}_{3} \\ \theta^{d}_{news} & \theta^{d}_{un} & \theta^{d}_{3} \\ \theta^{c}_{news} & \theta^{c}_{un} & \theta^{c}_{3} \end{pmatrix}_s \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}_{t-s},$$

where hp_t, d_t and c_t denote the log level of housing price, log level of labour productivity and log level of consumption, respectively; Δ is the first-order difference operator; v_1 denotes the shock that has a delayed effect on productivity but perfectly predicted by the consumers, i.e. news shock, v_2 denotes the shock that affects productivity contemporaneously, v_3 can be interpreted as a combination of preference shock and housing supply shock. θs are impulse responses of variables to shocks.

4.4 Results

As the basis of the analysis, I show the forecast error variance decompositions, impulse response functions, and conditional correlations. Forecast error variance decompositions show the relative importance of each shock to each variable, Impulse response functions tell the magnitude and persistence of each variable to each shock. Conditional correlations show what the correlation between two variables would be if only one of the shocks were at work.

4.4.1 Forecast Error Variance Decompositions

Table 4.1 and 4.2 show the forecast error variance decompositions. For non-durable consumption goods, we can see that for the horizon of 40 periods, 99.91% of forecast error variance of the productivity growth rate is explained by the two shocks. This is not surprising given that this is just how I identify the shocks. However, the major contribution comes from unexpected productivity shock (0.55% versus 99.36%). The two shocks also explain a very large proportion (81.97%) of the forecast error variance of non-durable consumption growth rate. In contrast to the labour productivity, however, a larger contribution comes from the news shock (65.72% versus 16.25%). This implies that the predicted productivity shock plays a dominant role in changing non-durable consumption. As for the housing prices, the two shocks can only explain less than half of the forecast error variation of housing price growth rate. The vast majority of housing price variation comes from Shock 3, which only accounts a small proportion of forecast error variance in consumption. Similar to the case of consumption, news shocks explain a larger part than the current productivity shock (42.49% vs 3.83%). For the durable goods consumption model, the results are quite similar.

Table 4.1: Forecast Error Variance Decomposition: Non-durable Goods

H=40	housing price	productivity	non-durable
news shock	42.49	0.55	65.72
productivity shock	3.83	99.36	16.25
subsum	46.32	99.91	81.97
shock 3	53.68	0.09	18.03

Table 4.2: Forecast Error Variance Decomposition: Durable Goods

H=40	housing price	productivity	durable
news shock	17.99	1.48	79.5
productivity shock	3.78	98.4	14.55
subsum	21.76	99.84	94.04
shock 3	78.24	0.16	5.96

4.4.2 Impulse Response Functions

The three panels in Figure 4.1 show the impulse response functions for (the *level* of) housing price, labour productivity, and non-durable goods consumption to each of the shocks. It can be seen that all variables are permanently affected by productivity shock and news shock. The magnitudes and dynamics, however, are quite different.

Productivity shock has larger effects on labour productivity but smaller effect on consumption and housing price. Labour productivity responds to unexpected productivity more than nine times than that to the news shock. In contrast, the response of housing price to news shock is about as four times large as that to the productivity shock. Similarly, the response of consumption to news shock is about as two times large as that to the productivity shock.

The third shock has a significantly large effect on housing prices but statistically insignificant effect on productivity. On the other hand, the effect of shock 3 on consumption is negative in the short run and negligible effect in the long run.

4.4.3 Conditional Correlations

Table 4.3 shows the unconditional and conditional correlations. For non-durable good, the unconditional correlation coefficient between housing price and consumption is 0.38. A further decomposition shows that the correlation between housing price and consumption is more strongly positive (both are over 0.8) when conditional on news shock or productivity shock. This suggests that the importance of productivity shock and news shock in driving housing prices and consumption to co-move. In contrast, the correlation of the two becomes negative when conditional on the third shock, which is consistent with the results from impulse response functions. That is, the most important shock behind the movement of housing prices, in fact, has a dampening effect on consumption, causing a negative correlation between housing price and consumption.

Table 4.3: Conditional Correlations between Housing Price and Consumption

H=40; 1971Q2-2011Q4	non-durable	durable
on news shock	0.82	0.64
on productivity shock	0.86	0.31
on shock 3	-0.40	0.08
unconditional	0.38	0.60

4.4.4 Summary

Based on these results, I have two important findings that cast doubts on the housing wealth effect on household consumption. First, both housing price and consumption respond strongly to the productivity shock and news shock. While the two shocks together drive less than half of housing price variation, they drive more than 80% of the consumption variation. The correlations between the two series conditional on the productivity shock and news shock are both strongly positive, implying that the co-movement of housing price and consumption comes from productivity shock and news shock. Second, while more than half of the variations in housing price comes from the third shock, that shock has small (18.03% in forecast error variance) and *negative* effect (negative impulse response function and negative conditional correlation) on non-durable consumption.

4.5 Historical Decompositions

In this section, I implement counterfactual experiments to assess the contributions of each shock to housing prices and consumption in the sample history. Specifically, I feed in only shock each time to see how housing prices and consumption would have been. Figure 4.2 presents the historical decompositions; the lines around the x-axis are the differences between data and simulation.

The first panel shows that, if there were no news shocks how would house prices and non-durable goods consumption have behaved. For instance, they both would have been much lower between the year 1998 and 2009 without new shocks. This implies that news shocks were playing a prominent role in the recent housing market boom. In contrast, in the earlier years especially the fifteen years between 1974 and 1989 when housing prices fluctuated significantly, news shocks decreased housing prices. Households in that period must have seen negative news about future productivity and thereby decreased demand for houses. Interestingly, news shocks didn't seem to be important after the market crash in 1989. The historical effects of news shocks on consumption display a similar pattern as on housing prices.

The second panel shows that, if there were no current productivity shocks how would house prices and non-durable goods consumption have behaved. Productivity shocks had been playing a trivial role in housing price dynamics before 1991, they significantly helped drive the recent housing market boom; without productivity shocks, housing prices would have been much lower. Again, the historical effects of the productivity shock on consumption display a similar pattern as on housing prices.

The importance of the third shock in sample history is shown on the third panel. This shock contributed to the housing boom between 1971 and 1990. This shock dragged housing price during 1991-2004. It played important roles in the first three boom-busts in the sample history, but not in the recent one (2004-2009). It was responsible for the housing market crash in the late 1980s. Despite its dramatic impacts on housing prices, this third shock just looks like noises to the consumption: without it, consumption in the whole sample history barely changed.

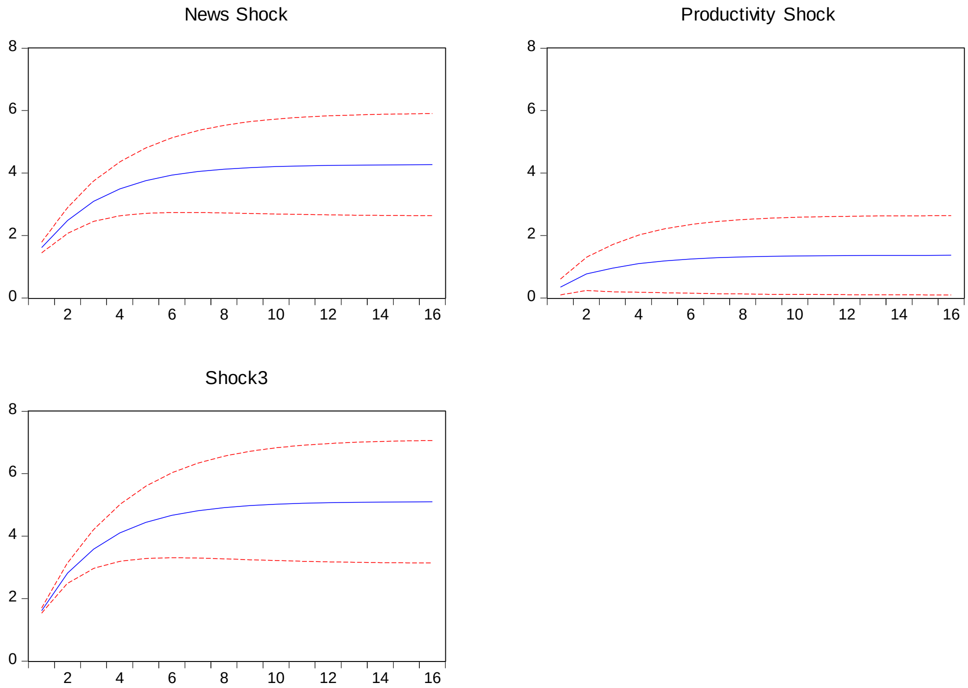
Consistent with forecast error variance decompositions, the historical decompositions also show that the third shock was important to housing prices but not important to consumption.

4.6 Conclusion

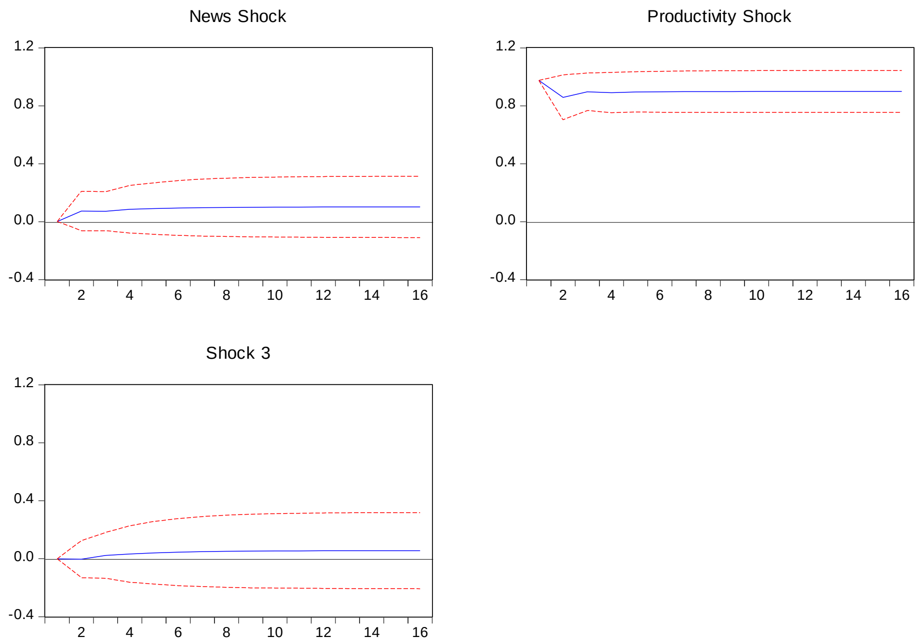
The housing wealth effect is believed by many to have driven the strong co-movement between housing prices and consumption. However, the (real) housing price is simply the relative price of housing to the non-housing consumption good, thus an appreciation of housing price is just a reflection of an increase in the production of the non-housing consumption good relative to houses. Under this explanation, it is the increase of non-housing consumption good production that drives the increase in housing prices; the causality implied by the wealth effect is reversed. This idea has been shown in a simple house asset pricing model in this chapter.

Because the direction of causality can be in either way, empirical research using aggregate data and single equation Ordinary Least Squares estimation is vulnerable to the endogeneity problem; the estimate based on this type of research can be seriously biased. Instrument variable estimation or simultaneous equation models can be used to overcome this problem. However, neither instrument variables nor truly exogenous variables, which are required in those methods, are easy to find. To circumvent these difficulties, I adopt the structural vector autoregression method in this chapter and implement a novel strategy to identify the productivity shock and the news shock about future productivity. The third shock that is not specifically labelled but implicitly identified has been found important to the variation of housing prices. The news shock and productivity shock are found to be the main drivers behind the co-movement between housing price and consumption. While not being able to directly estimate the wealth effect or collateral effect, the results from my empirical exercise cast doubt on the importance of housing wealth in affecting household consumption.

Impulse Response of House Price



Impulse Responses of Labor Productivity



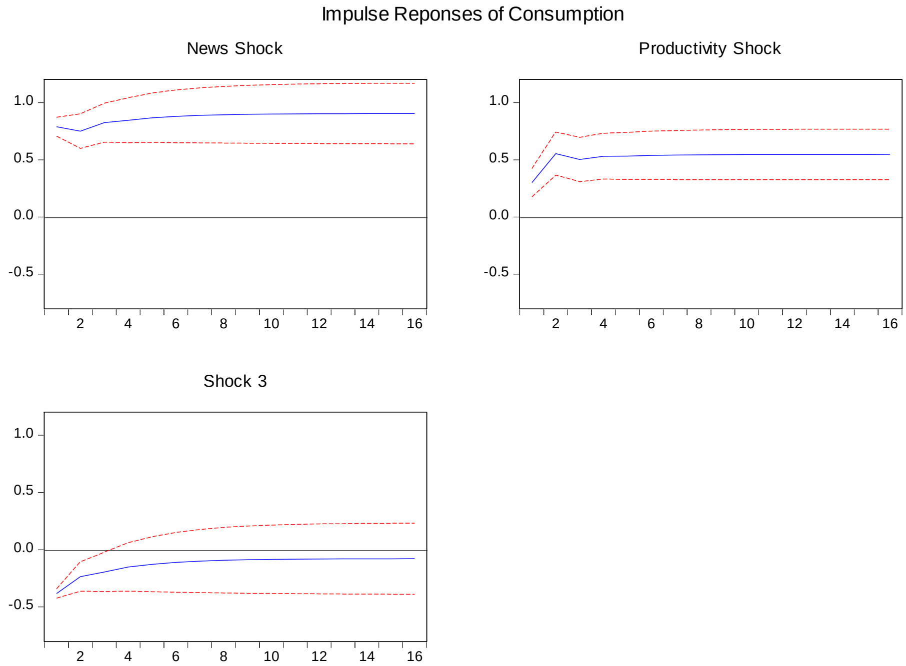
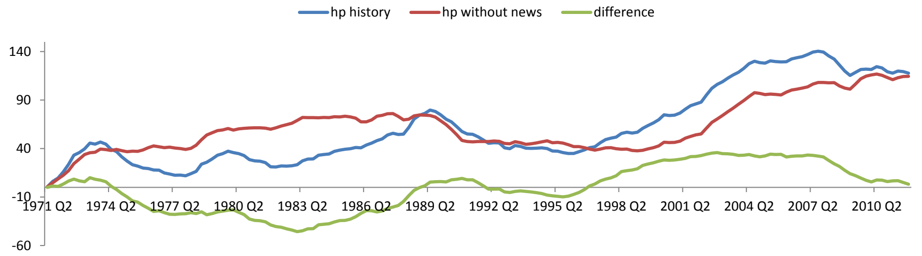
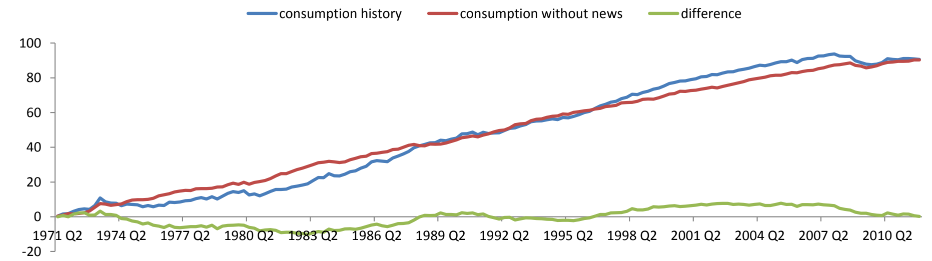


Figure 4.1: Impulse Response Functions

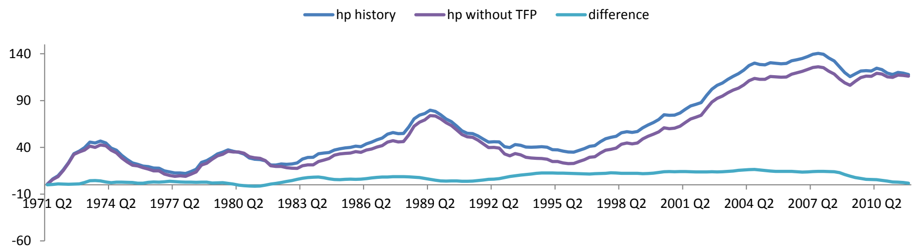
Note: dotted lines are the confidence intervals.

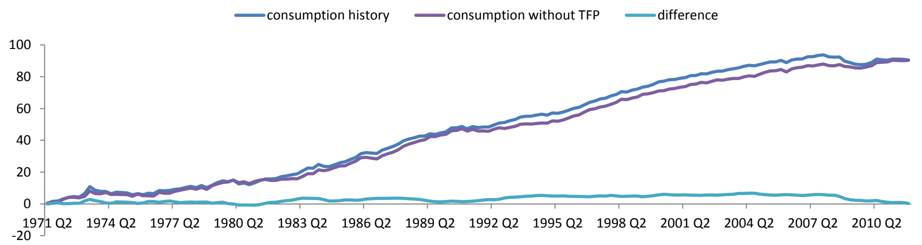
without News Shock





without Productivity Shock





without Shock 3

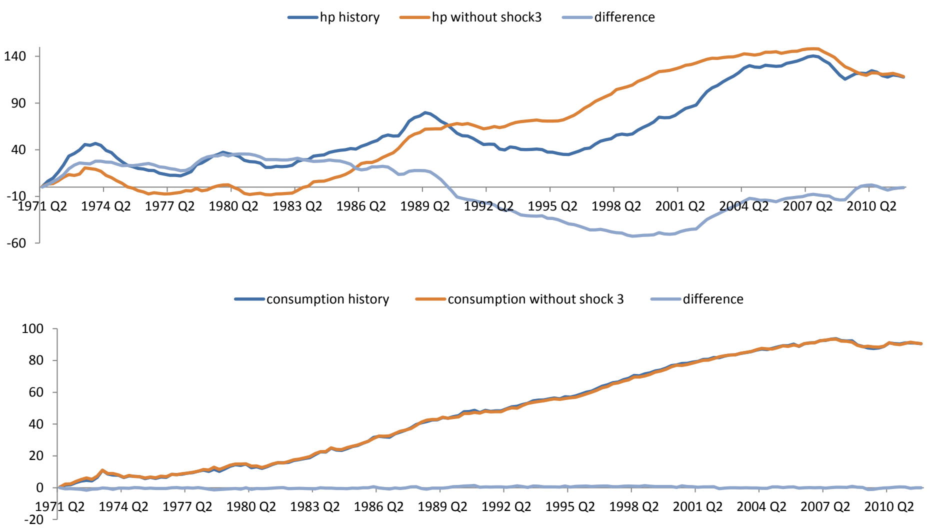


Figure 4.2: Historical Decompositions

Appendix A

A.1 Proofs

Definition A1: (First Order Stochastic Dominance)
Let Θ be a subset of $\mathbb{R}$, representing possible values of the random parameter $\tilde{\theta}$. A distribution G_1 is said to dominate G_2 in the sense of first-order stochastic dominance if for every increasing function $U(\cdot)$, we have $\int U(\theta)dG_1(\theta) > \int U(\theta)dG_2(\theta)$. (Milgrom, 1981, p.382)

Lemma A1:
G_1 dominates G_2 in the sense of first-order stochastic dominance if and only if $G_1(\theta) \leq G_2(\theta)$, $\forall\theta$, with strict inequality for some value of θ.
Proof: see Milgrom (1981, p.382). ■

Proof of Lemma 2.1:
(1) First note that the joint distribution of $\tilde{s}$ and $\tilde{a}$ is given by

$$f_{\tilde{s},\tilde{a}}(s,a) = f_{\tilde{s}}(s)f_{\tilde{a}}(a) = \frac{1}{\pi\sigma_s\sigma_a}\exp\left(-\frac{s^2}{2\sigma_s^2} - \frac{a^2}{2\sigma_a^2}\right),$$

where $a \in \mathbb{R}, s \in \mathbb{R}^-; \sigma_a > 0, \sigma_s > 0$ and where I have used the fact that $\tilde{w}$ and $\tilde{a}$ are independent from each other and their density functions

$$\begin{aligned} f_{\tilde{a}}(a) &= \frac{1}{\sqrt{2\pi}\sigma_a}\exp\left(-\frac{a^2}{2\sigma_a^2}\right), a \in \mathbb{R}, \\ f_{\tilde{s}}(s) &= \frac{\sqrt{2}}{\sqrt{\pi}\sigma_s}\exp\left(-\frac{s^2}{2\sigma_s^2}\right), s \in \mathbb{R}^-. \end{aligned}$$

Since $\tilde{x} = w - \frac{1}{\lambda}\tilde{s}$ and $\tilde{y} = \tilde{a} - \tilde{s}$, I am able to obtain the joint distribution $f_{\tilde{x},\tilde{y}|\tilde{w}}(x,y|w)$ as

$$\begin{aligned} & f_{\tilde{x},\tilde{y}|\tilde{w}}(x,y|w) = |J| \cdot f_{\tilde{s},\tilde{a}|\tilde{w}}\left\{\lambda(w-x), \lambda w - \lambda x + y\right\} \\ = & \frac{\lambda}{\pi\sigma_s\sigma_a}\exp\left[-\frac{\lambda^2(w-x)^2}{2\sigma_s^2} - \frac{(\lambda w - \lambda x + y)^2}{2\sigma_a^2}\right], x \geq w, y \in \mathbb{R}, w \in \mathbb{R}, \end{aligned}$$

where

$$J \equiv \begin{vmatrix} \frac{\partial s}{\partial x} & \frac{\partial s}{\partial y} \\ \frac{\partial a}{\partial x} & \frac{\partial a}{\partial y} \end{vmatrix} = \begin{vmatrix} -\lambda & 0 \\ -\lambda & 1 \end{vmatrix} = -\lambda.$$

Given the independence assumption about $\tilde{w}_i$, $\tilde{a}$ and $\tilde{s}$, the joint density of $\tilde{w}_i, \tilde{x}, \tilde{y}$ conditional on $\tilde{w} = w$ can be obtained as

$$\begin{aligned} & f_{\tilde{w}_i,\tilde{x},\tilde{y}|\tilde{w}}(w_i,x,y|w) = f_{\tilde{w}_i|\tilde{w}}(w_i|w)f_{\tilde{x},\tilde{y}|\tilde{w}}(x,y|w) = \lambda e^{-\lambda(w_i-w)} \cdot f_{\tilde{x},\tilde{y}|\tilde{w}}(x,y|w) \\ = & \frac{\lambda^2}{\pi\sigma_s\sigma_a}\exp\left[-\lambda(w_i-w) - \frac{\lambda^2(w-x)^2}{2\sigma_s^2} - \frac{(\lambda w - \lambda x + y)^2}{2\sigma_a^2}\right], x \geq w, y \in \mathbb{R}, w_i \geq w, w \in \mathbb{R}. \end{aligned}$$

By the multiplication rule, the joint distribution of $\tilde{w}_i, \tilde{x}, \tilde{y}, \tilde{w}$ is given by

$$
\begin{aligned}
& f_{\tilde{w}_i,\tilde{x},\tilde{y},\tilde{w}}(w_i,x,y,w) = f_{\tilde{w}_i,\tilde{x},\tilde{y}|\tilde{w}}(w_i,x,y|w) f_{\tilde{w}}(w) \\
= \quad & f_{\tilde{w}_i,\tilde{x},\tilde{y}|\tilde{w}}(w_i,x,y|w) \cdot \left[\frac{1}{\sigma_w \sqrt{2\pi}} \exp\left(-\frac{w^2}{2\sigma_w^2} \right) \right] \\
= \quad & \frac{\lambda^2}{\pi\sqrt{2\pi}\sigma_s\sigma_a\sigma_w} \exp\left[-\lambda(w_i - w) - \frac{\lambda^2(w-x)^2}{2\sigma_s^2} - \frac{(\lambda w - \lambda x + y)^2}{2\sigma_a^2} - \frac{w^2}{2\sigma_w^2} \right] \\
\equiv \quad & N \cdot \exp[-(\beta_2 w^2 + \beta_1(x,y)w + \beta_0(x,y,w_i))], x \geq w, y \in \mathbb{R}, w_i \geq w, w \in \mathbb{R},
\end{aligned}
$$

where $N \equiv \frac{\lambda^2}{\sqrt{2\pi}\pi\sigma_s\sigma_a\sigma_w}$, and

$$
\begin{aligned}
\beta_2 &\equiv \frac{1}{2}\left(\frac{\lambda^2}{\sigma_a^2} + \frac{\lambda^2}{\sigma_s^2} + \frac{1}{\sigma_w^2} \right), \\
\beta_1 &\equiv -\lambda \left(\frac{\lambda x - y}{\sigma_a^2} + 1 + \frac{\lambda x}{\sigma_s^2} \right), \\
\beta_0 &\equiv \frac{1}{2}\left[\frac{\lambda^2 x^2}{\sigma_s^2} + \frac{(\lambda x - y)^2}{\sigma_a^2} \right] + \lambda w_i.
\end{aligned}
$$

From the Bayes' theorem, I have the posterior density of $\tilde{w}$ conditional on $\tilde{w}_i = w_i, \tilde{x} = x$ and $\tilde{y} = y$ as

$$
f_{\tilde{w}|\tilde{w}_i,\tilde{x},\tilde{y}}(w|w_i,x,y) = \frac{f_{\tilde{w}_i,\tilde{x},\tilde{y},\tilde{w}}(w_i,x,y,w)}{\int f_{\tilde{w}_i,\tilde{x},\tilde{y},\tilde{w}}(w_i,x,y,w')\mathrm{d}w'}.
$$

If $w_i < x$, i.e. the signal of agent i is smaller than $\breve{w}$, then

$$
\begin{aligned}
\frac{1}{N}\int f_{\tilde{w}_i,\tilde{x},\tilde{y},\tilde{w}}(w_i,x,y,w')\mathrm{d}w' &= \int_{-\infty}^{w_i} \exp[-(\beta_2 w'^2 + \beta_1 w' + \beta_0)]\mathrm{d}w' \\
&= \sqrt{\frac{\pi}{\beta_2}} \exp\left(\frac{\beta_1^2}{4\beta_2} - \beta_0 \right) \Phi\left(\sqrt{2\beta_2}w_i + \frac{\beta_1}{\sqrt{2\beta_2}} \right),
\end{aligned}
$$

and the cumulative distribution function is

$$
\begin{aligned}
F_{\tilde{w}|\tilde{w}_i,\tilde{x},\tilde{y}}(w|w_i,x,y) &= \frac{\int_{-\infty}^{w} \exp[-(\beta_2 w'^2 + \beta_1 w' + \beta_0)]\mathrm{d}w'}{\int_{-\infty}^{w_i} \exp[-(\beta_2 w'^2 + \beta_1 w' + \beta_0)]\mathrm{d}w'} \\
&= \frac{\Phi\left(\sqrt{2\beta_2}w + \frac{\beta_1}{\sqrt{2\beta_2}} \right)}{\Phi\left(\sqrt{2\beta_2}w_i + \frac{\beta_1}{\sqrt{2\beta_2}} \right)} = \frac{\Phi\left(\frac{w-\mu(x,y)}{\sigma} \right)}{\Phi\left(\frac{w_i-\mu(x,y)}{\sigma} \right)},
\end{aligned}
$$

where

$$
\begin{aligned}
\sigma &\equiv \frac{1}{\sqrt{2\beta_2}} = \left(\frac{\lambda^2}{\sigma_a^2} + \frac{\lambda^2}{\sigma_s^2} + \frac{1}{\sigma_w^2} \right)^{-\frac{1}{2}}, \\
\mu &\equiv -\frac{\beta_1}{2\beta_2} = \left(\frac{\lambda x - y}{\sigma_a^2} + \frac{\lambda x}{\sigma_s^2} + 1 \right) \lambda\sigma^2.
\end{aligned}
$$

If $w_i \geq x$, i.e. the signal of agent i is equal or larger than $\breve{w}$, then

$$\begin{aligned}\frac{1}{N}\int f_{\tilde{w}_i,\tilde{x},\tilde{y},\tilde{w}}(w_i,x,y,w')\mathrm{d}w' &= \int_{-\infty}^{x}\exp[-(\beta_2 w'^2+\beta_1 w'+\beta_0)]\mathrm{d}w',\\ F_{\tilde{w}|\tilde{w}_i,\tilde{x},\tilde{y}}(w|w_i,x,y) &= \frac{\Phi\left(\sqrt{2\beta_2}w+\frac{\beta_1}{\sqrt{2\beta_2}}\right)}{\Phi\left(\sqrt{2\beta_2}x+\frac{\beta_1}{\sqrt{2\beta_2}}\right)} = \frac{\Phi\left(\frac{w-\mu(x,y)}{\sigma}\right)}{\Phi\left(\frac{x-\mu(x,y)}{\sigma}\right)}.\end{aligned}$$

(2) It is straightforward to see that $F_{\tilde{w}|\tilde{w}_i,\tilde{x},\tilde{y}}(w|w_i,x,y) \leq F_{\tilde{w}|\tilde{w}_i,\tilde{x},\tilde{y}}(w|w_j,x,y)$ for all w if $w_i > w_j$, since $\Phi(\cdot)$ is an increasing function. This implies that, given price signals x and y, agent who receives signal w_i will have higher or equal expectation about w than agent who receives signal w_j. Because house resale value is an increasing function of w, agent who receives signal w_i will have higher or equal expectations about house resale value than that of agent who receives signal w_j.

(3) This follows immediately from (1), that is, the posterior distribution of $\tilde{w}$ is conditional on $\tilde{w}_i = x, \tilde{x} = x$ and $\tilde{y} = y$:

$$F_{\tilde{w}|\tilde{w}_i,\tilde{x},\tilde{y}}(w|x,x,y) = \frac{\Phi\left(\frac{w-\mu(x,y)}{\sigma}\right)}{\Phi\left(\frac{x-\mu(x,y)}{\sigma}\right)}. \qquad \blacksquare$$

Proof of Theorem 2.1:
The equilibrium housing price function is obtained by using the cut-off agent's posterior belief: $F_{\tilde{w}|\tilde{w}_i,\tilde{x},\tilde{y}}(w|x,x,y)$ in the indifference condition:

$$P = Q + \frac{1}{R}\int_{-\infty}^{+\infty} h(w)\mathrm{d}F_{\tilde{w}|\tilde{w}_i,\tilde{x},\tilde{y}}(w|x,x,y). \qquad \blacksquare$$

Lemma A2:
Define $\hat{\Phi}(u) \equiv \frac{\Phi(u)}{\phi(u)}$, *where* $\phi(u) = \frac{1}{\sqrt{2\pi}}e^{-\frac{u^2}{2}}$ *and* $\Phi(u) = \int_{-\infty}^{u}\frac{1}{\sqrt{2\pi}}e^{-\frac{u^2}{2}}\mathrm{d}u$. $\hat{\Phi}(u)$ *is increasing in* u, *i.e.* $\frac{\partial\hat{\Phi}(u)}{\partial u} \geq 0$.
Proof:

$$\frac{\partial\frac{\Phi(u)}{\phi(u)}}{\partial u} = \frac{\phi(u)\phi(u)-\frac{\partial\phi(u)}{\partial u}\Phi(u)}{(\phi(u))^2} = \frac{\phi(u)\phi(u)+u\phi(u)\Phi(u)}{(\phi(u))^2} = \frac{\phi(u)[\phi(u)+u\Phi(u)]}{(\phi(u))^2}.$$

Since $\phi(u) > 0, \forall u$, I only need to show that $\phi(u)+u\Phi(u) \geq 0$. But

$$\phi(u)+u\Phi(u) = \phi(u)+\int_{-\infty}^{u}u\phi(t)dt \geq \phi(u)+\int_{-\infty}^{u}t\phi(t)dt = \phi(u)-\phi(t)|_{-\infty}^{u} = 0.$$

This lemma will be useful in the following proofs. $\blacksquare$

Proof of Claim 2.1:
From Theorem 2.1, I have $P = Q + V(x,y)$. Since $Q = e^y$, it suffices to show that $V(x,y)$ is increasing in x for fixed $y = \ln Q$. Because V is the expectation of an increasing function of the random variable $\tilde{w}$, conditional on the realisations of $\tilde{x}$ and $\tilde{y}$, it then suffices to

show the conditional distribution $F_{\tilde{w}|\tilde{w}_i,\tilde{x},\tilde{y}}(w|x,x,y)$ is decreasing in x for fixed y. Define $g_1(w,x,y) \equiv \frac{w-\mu(x,y)}{\sigma}$ and $g_2(x,y) \equiv \frac{x-\mu(x,y)}{\sigma}$. Then, I have $F_{\tilde{w}|\tilde{w}_i,\tilde{x},\tilde{y}}(w|x,x,y) = \frac{\Phi(g_1)}{\Phi(g_2)}$, where $g_1 = \frac{w}{\sigma} - \sigma\left[\frac{\lambda(\lambda x-y)}{\sigma_a^2} + \lambda + \frac{\lambda^2}{\sigma_s^2}x\right], g_2 = \sigma\left(\frac{1}{\sigma_w^2}x + \frac{\lambda}{\sigma_a^2}y - \lambda\right)$. Note that

$$\frac{\partial F_{\tilde{w}|\tilde{w}_i,\tilde{x},\tilde{y}}(w|x,x,y)}{\partial x} = [\Phi(g_2)]^{-2}\left[\Phi(g_2)\phi(g_1)\frac{\partial g_1}{\partial x} - \Phi(g_1)\phi(g_2)\frac{\partial g_2}{\partial x}\right],$$

where $\Phi(g_i) > 0, \phi(g_i) > 0$ for $i = 1, 2$, $g_1 \leq g_2$, and

$$\begin{aligned}
\frac{\partial g_1}{\partial x} &= -\sigma\lambda^2\left(\frac{1}{\sigma_a^2} + \frac{1}{\sigma_s^2}\right) < 0, \\
\frac{\partial g_2}{\partial x} &= \frac{\sigma}{\sigma_w^2} > 0.
\end{aligned}$$

Hence, $\frac{\partial F_{\tilde{w}|\tilde{w}_i,\tilde{x},\tilde{y}}(w|x,x,y)}{\partial x} < 0$ for $g_1 < g_2$, and for any $P_f = h(w)$ that is increasing in w, this implies $V \equiv \mathbb{E}(\tilde{P}_f|\tilde{w}_i = x, \tilde{x} = x, \tilde{y} = y)$ is strictly increasing in x. ■

Proof of Corollary 2.1:
Substituting $h(w) = e^w$ into the price function in Theorem 2.1, we have

$$\begin{aligned}
P &= e^y + \frac{\int_{-\infty}^{x} \exp\{-[\beta_2 w^2 + (\beta_1 - 1)w + \beta_0]\}\mathrm{d}w}{\left[\frac{\sqrt{\pi}}{\sqrt{\beta_2}}\exp\left(\frac{\beta_1^2}{4\beta_2} - \beta_0\right)\right]\Phi\left(\sqrt{2\beta_2}x + \frac{\beta_1}{\sqrt{2\beta_2}}\right)} \\
&= e^y + \frac{\left[\frac{\sqrt{\pi}}{\sqrt{\beta_2}}\exp\left(\frac{(\beta_1-1)^2}{4\beta_2} - \beta_0\right)\right]\Phi\left(\sqrt{2\beta_2}x + \frac{\beta_1-1}{\sqrt{2\beta_2}}\right)}{\left[\frac{\sqrt{\pi}}{\sqrt{\beta_2}}\exp\left(\frac{\beta_1^2}{4\beta_2} - \beta_0\right)\right]\Phi\left(\sqrt{2\beta_2}x + \frac{\beta_1}{\sqrt{2\beta_2}}\right)} \\
&= e^y + \exp\left(\frac{1-2\beta_1}{4\beta_2}\right)\frac{\Phi\left(\frac{x-\mu}{\sigma} - \sigma\right)}{\Phi\left(\frac{x-\mu}{\sigma}\right)} \\
&\equiv e^y + \exp\left(\frac{\sigma^2}{2} + \mu\right)\frac{\Phi(\kappa - \sigma)}{\Phi(\kappa)},
\end{aligned}$$

where

$$\kappa \equiv \frac{x-\mu}{\sigma} = \left(\frac{1}{\sigma_w^2}x + \frac{\lambda}{\sigma_a^2}y - \lambda\right)\sigma.$$ ■

Assumption for Interior Solution:
Note that the consumption in the second period is given by $C_i = R[M_i - PH_i + Q(H_i - B_i)] + P_f H_i = RM_i - A_i + \left[P_f - \int_{-\infty}^{+\infty} h(w) f_{\tilde{w}|\tilde{w}_i,\tilde{x},\tilde{y}}(w|x,x,y)\mathrm{d}w\right] H_i$. For $C_i \geq 0, \forall i \in [0,1]$, it is sufficient to assume

$$M_i \geq \frac{1}{R}\left[A_i + \int_{-\infty}^{+\infty} h(w) f_{\tilde{w}|\tilde{w}_i,\tilde{x},\tilde{y}}(w|x,x,y)\mathrm{d}w - h(w)\right].$$

Proof of Proposition 2.1:
From Lemma 2.2, I have the joint density of $\tilde{x}$ and $\tilde{y}$ conditional on $\tilde{w} = w$ as

$$f_{\tilde{x},\tilde{y}|\tilde{w}}(x,y|w) = \frac{\lambda}{\pi\sigma_s\sigma_a}\exp\left[-\frac{\lambda^2(w-x)^2}{2\sigma_s^2} - \frac{(\lambda w - \lambda x + y)^2}{2\sigma_a^2}\right].$$

The joint density of $\tilde{x}$, $\tilde{y}$, and $\tilde{w}$ is then given by

$$f_{\tilde{x},\tilde{y},\tilde{w}}(x,y,w) = f_{\tilde{x},\tilde{y}|\tilde{w}}(x,y|w) \cdot f_{\tilde{w}}(w) = \hat{N} \cdot \exp[-(\beta_2 w^2 + \hat{\beta}_1 w + \hat{\beta}_0)],$$

where $w \in \mathbb{R}, x \geq w, y \in \mathbb{R}$, $\hat{N} \equiv \frac{\lambda}{\sqrt{2\pi}\pi\sigma_s\sigma_a\sigma_w}$, $\hat{\beta}_1 = \beta_1 + \lambda$, and $\hat{\beta}_0 = \beta_0 - \lambda w_i$. From the Bayes' theorem, I have the posterior distribution of $\tilde{w}$ conditional on $\tilde{x} = x$ and $\tilde{y} = y$

$$F_{\tilde{w}|\tilde{x},\tilde{y}}(w|x,y) = \frac{\int_{-\infty}^{w} f_{\tilde{x},\tilde{y},\tilde{w}}(x,y,w')\mathrm{d}w'}{\int_{-\infty}^{x} f_{\tilde{x},\tilde{y},\tilde{w}}(x,y,w')\mathrm{d}w'} = \frac{\Phi\left(\sqrt{2\beta_2}w + \frac{\beta_1+\lambda}{\sqrt{2\beta_2}}\right)}{\Phi\left(\sqrt{2\beta_2}x + \frac{\beta_1+\lambda}{\sqrt{2\beta_2}}\right)} = \frac{\Phi\left(\frac{w-\hat{\mu}(x,y)}{\sigma}\right)}{\Phi\left(\frac{x-\hat{\mu}(x,y)}{\sigma}\right)},$$

where $\hat{\mu}(x,y) \equiv -\frac{\beta_1+\lambda}{2\beta_2} = \mu(x,y) - \lambda\sigma^2 < \mu(x,y)$. Denote $g(\mu) \equiv \frac{\Phi(\frac{w-\mu}{\sigma})}{\Phi(\frac{x-\mu}{\sigma})}$. We can see that

$$\frac{\partial g(\mu)}{\partial \mu} = -\frac{\phi(\frac{w-\mu}{\sigma})\phi(\frac{x-\mu}{\sigma})}{\sigma[\Phi(\frac{x-\mu}{\sigma})]^2}\left[\frac{\Phi(\frac{x-\mu}{\sigma})}{\phi(\frac{x-\mu}{\sigma})} - \frac{\Phi(\frac{w-\mu}{\sigma})}{\phi(\frac{w-\mu}{\sigma})}\right] \leq 0, \forall w.$$

Thus, $g(\mu) \leq g(\hat{\mu})$ or $F_{\tilde{w}|\tilde{w}_i,\tilde{x},\tilde{y}}(w|x,x,y) \leq F_{\tilde{w}|\tilde{x},\tilde{y}}(w|x,y)$, with strict inequality for some value of w. As P_f is increasing in w, then $\mathbb{E}(\tilde{P}_f|\tilde{w}_i = x, \tilde{x} = x, \tilde{y} = y) > \mathbb{E}(\tilde{P}_f|\tilde{x} = x, \tilde{y} = y)$ and thereby $\hat{D} > 0, \forall(x,y) \in \mathbb{R}^2$. ■

Proof of Proposition 2.2:
For $P_f = e^w$, the market expected house resale value is given by

$$V(w,a,s) = \exp\left(\frac{\sigma^2}{2} + \mu\right)\frac{\Phi(\kappa - \sigma)}{\Phi(\kappa)}.$$

If I take the random variables a, s, w as being deterministic, and take the partial derivatives with respect to each of them, I have

$$\begin{aligned}
\frac{\partial V}{\partial s} &= V\sigma\left\{-\left[\frac{\lambda\sigma}{\sigma_s^2} + \frac{1}{\lambda}\left(\frac{1}{\sigma_w^2} + \frac{\lambda^2}{\sigma_a^2}\right)\left(\frac{\phi(\kappa-\sigma)}{\Phi(\kappa-\sigma)} - \frac{\phi(\kappa)}{\Phi(\kappa)}\right)\right]\right\}, \\
\frac{\partial V}{\partial w} &= V\sigma\left\{\lambda^2\sigma\left(\frac{1}{\sigma_a^2} + \frac{1}{\sigma_s^2}\right) + \frac{1}{\sigma_w^2}\left[\frac{\phi(\kappa-\sigma)}{\Phi(\kappa-\sigma)} - \frac{\phi(\kappa)}{\Phi(\kappa)}\right]\right\}, \\
\frac{\partial V}{\partial a} &= V\sigma\left\{\frac{\lambda}{\sigma_a^2}\left[\frac{\phi(\kappa-\sigma)}{\Phi(\kappa-\sigma)} - \frac{\phi(\kappa)}{\Phi(\kappa)} - \sigma\right]\right\}.
\end{aligned}$$

As $\frac{\phi(\kappa-\sigma)}{\Phi(\kappa-\sigma)} - \frac{\phi(\kappa)}{\Phi(\kappa)} > 0$ from Lemma A2, it's easy to see that $\frac{\partial V}{\partial w} > 0$ and $\frac{\partial V}{\partial s} < 0$. For $\frac{\partial V}{\partial a}$, since $\frac{\partial V}{\partial y} = \frac{\partial V}{\partial a}$ and from Proposition 2.3 we know $\frac{\partial V}{\partial y}$ must be negative, it follows immediately that $\frac{\partial V}{\partial a} < 0$. ■

Proof of Proposition 2.3:
Similar to the proof of Claim 2.1, I first take the partial derivative of $F_{\tilde{w}|\tilde{w}_i,\tilde{x},\tilde{y}}(w|x,x,y)$ w.r.t. y

$$\frac{\partial F_{\tilde{w}|\tilde{w}_i,\tilde{x},\tilde{y}}(w|x,x,y)}{\partial y} = \frac{\phi(g_1)\phi(g_2)}{[\Phi(g_2)]^2}\frac{\partial g_1}{\partial y}\left[\frac{\Phi(g_2)}{\phi(g_2)} - \frac{\Phi(g_1)}{\phi(g_1)}\right],$$

where the second equality comes from the fact that $\frac{\partial g_1}{\partial y} = \frac{\partial g_2}{\partial y} = \frac{\lambda\sigma}{\sigma_a^2} > 0$. For $g_1 < g_2$, the term in the bracket is positive from Lemma A2. Thus, for $g_1 < g_2$, $\frac{\partial F}{\partial y} > 0$. As $P_f = h(w)$ is increasing in w, $\mathbb{E}(\tilde{P}_f|\tilde{w}_i = x, \tilde{x} = x, \tilde{y} = y)$ is strictly decreasing in y, i.e. $\frac{\partial \mathbb{E}(\tilde{P}_f|\tilde{w}_i=x,\tilde{x}=x,\tilde{y}=y)}{\partial y} > 0$, and $\frac{\partial \mathbb{E}(\tilde{P}_f|\tilde{w}_i=x,\tilde{x}=x,\tilde{y}=y)}{\partial Q} = \frac{\partial \mathbb{E}(\tilde{P}_f|\tilde{w}_i=x,\tilde{x}=x,\tilde{y}=y)}{\partial y}\frac{\partial y}{\partial Q} < 0$. ■

Proof of Proposition 2.4:
Taking expectation of R_H conditional on the econometrician's information set yields

$$\mathbb{E}(\tilde{R}_H|\Omega_e) = \mathbb{E}(\tilde{P}_f|\Omega_e) + Q - P,$$

where $\mathbb{E}(\tilde{P}_f|\Omega_e) = \mathbb{E}(\tilde{P}_f|\tilde{x} = x, \tilde{y} = y) = \hat{V}$. From Theorem 2.1, I have

$$P = Q + V.$$

Hence, $\mathbb{E}(\tilde{R}_H|\Omega_e) = \hat{V} - V = -\hat{D}$ where $-\hat{D}$ is defined in Proposition 2.1. ■

Proof of Corollary 2.2:
From Proposition 2.1, I have

$$F_{\tilde{w}|\tilde{x},\tilde{y}}(w|x,y) = \frac{\Phi\left(\frac{w-\hat{\mu}}{\sigma}\right)}{\Phi\left(\frac{x-\hat{\mu}}{\sigma}\right)}.$$

The expected house resale value conditional based on this posterior probability can then be obtained as

$$\hat{V} \equiv \mathbb{E}(\tilde{P}_f|\tilde{x} = x, \tilde{y} = y) = \exp\left(\frac{\sigma^2}{2} + \mu - \lambda\sigma^2\right)\frac{\Phi(\kappa - \sigma + \lambda\sigma)}{\Phi(\kappa + \lambda\sigma)}.$$

It thus follows immediately that the conditional return on only housing price and retnal price is given by

$$\mathbb{E}(\tilde{R}_H|\Omega_e) = \exp\left(\frac{\sigma^2}{2} + \mu - \lambda\sigma^2\right)\frac{\Phi(\kappa + \lambda\sigma - \sigma)}{\Phi(\kappa + \lambda\sigma)} - \exp\left(\frac{\sigma^2}{2} + \mu\right)\frac{\Phi(\kappa - \sigma)}{\Phi(\kappa)}.$$ ■

Proof of Corollary 2.3:

$$\begin{aligned}\mathbb{E}\left[\check{D}(x,y)\right] &= \mathbb{E}\left[V(x,y) - \check{V}(x,y)\right] = \mathbb{E}[V(x,y)] - \mathbb{E}[\check{V}(x,y)] \\ &= \mathbb{E}[V(x,y)] - \mathbb{E}[\hat{V}(x,y)] = \mathbb{E}\left[V(x,y) - \hat{V}(x,y)\right] = \mathbb{E}[\hat{D}(x,y)] > 0.\end{aligned}$$

The third equality comes from the law of iterated expectations, and the last inequality holds because from Proposition 2.1, I have $\hat{D}(x,y) > 0$ for all $(x,y) \in \mathbb{R}^2$ when P_f is increasing in w. ■

A.2 Derivations

(1) **Housing Prices Conditional on Private Signal and $\tilde{x} = x$**:
Since the private signal conditional on w is given by $f_{\tilde{w}_i|\tilde{w}}(w_i|w) = \lambda e^{-\lambda(w_i - w)}, w_i \geq w$, and

the probability density of $\tilde{x}$ conditional on $\tilde{w} = w$ is $f_{\tilde{x}|\tilde{w}}(x|w) = \frac{\sqrt{2}\lambda}{\sqrt{\pi}\sigma_s} \exp\left[-\frac{\lambda^2(w-x)^2}{2\sigma_s^2}\right], x \geq w, w \in \mathbb{R}$, the joint density of $\tilde{x}$ and w_i conditional on w is

$$\begin{aligned} & f_{\tilde{w}_i,\tilde{x}|\tilde{w}}(w_i, x|w) = f_{\tilde{w}_i|\tilde{w}}(w_i|w) f_{\tilde{x}|\tilde{w}}(x|w) \\ = & \frac{\lambda^2\sqrt{2}}{\sqrt{\pi}\sigma_s} \exp\left[-\frac{\lambda^2(x-w)^2}{2\sigma_s^2} - \lambda(w_i - w)\right], x \geq w, w_i \geq w, w \in \mathbb{R}, \end{aligned}$$

due to the conditional independence. The joint density function is thus

$$f_{\tilde{w}_i,\tilde{x},\tilde{w}}(w_i, x, w) = f_{\tilde{w}_i,\tilde{x}|\tilde{w}}(w_i, x|w) f_{\tilde{w}}(w) = N' \cdot \exp[-(\beta_2' w^2 + \beta_1'(x) w + \beta_0'(x, w_i))],$$

where $x \geq w, w_i \geq w, w \in \mathbb{R}$, $N' \equiv \frac{\lambda^2}{\pi\sigma_s\sigma_w}$, and $\beta_2' \equiv \frac{1}{2}\left(\frac{\lambda^2}{\sigma_s^2} + \frac{1}{\sigma_w^2}\right)$, $\beta_1' \equiv -\lambda\left(1 + \frac{\lambda}{\sigma_s^2}x\right)$, $\beta_0' \equiv \frac{\lambda^2}{2\sigma_s^2}x^2 + \lambda w_i$. The probability distribution of $\tilde{w}$ conditional on w_i and x is

$$F_{\tilde{w}|\tilde{w}_i,\tilde{x}}(w|w_i, x) = \frac{\int_{-\infty}^{w} f_{\tilde{w},\tilde{w}_i,\tilde{x}}(w', w_i, x)\mathrm{d}w'}{\int_{-\infty}^{x} f_{\tilde{w},\tilde{w}_i,\tilde{x}}(w', w_i, x)\mathrm{d}w'} = \frac{\Phi\left(\frac{w-\mu'(x)}{\sigma'}\right)}{\Phi\left(\frac{w_i-\mu'(x)}{\sigma'}\right)},$$

where $\sigma' \equiv \left(\frac{\lambda^2}{\sigma_s^2} + \frac{1}{\sigma_w^2}\right)^{-\frac{1}{2}}$ and $\mu' \equiv \left(\frac{\lambda^2}{\sigma_s^2}x + \lambda\right)\sigma'^2$. If agents make inference only through observing private signal and x, and if $P_f = h(w) = e^w$, the equilibrium housing price can be obtained as

$$P' \quad = \quad Q + V' = Q + \int_{-\infty}^{x} e^{w'} \cdot f_{\tilde{w}|\tilde{w}_i,\tilde{x}}(w'|x, x)\mathrm{d}w' = e^y + \exp\left(\frac{\sigma'^2}{2} + \mu'\right)\frac{\Phi(\kappa' - \sigma')}{\Phi(\kappa')},$$

where $\kappa' \equiv \sigma'\left(\frac{x}{\sigma_w^2} - \lambda\right)$.

(2) **Housing Prices Conditional on $\tilde{x} = x$:**
The joint density of $\tilde{x}$ and $\tilde{w}$ is given by

$$f_{\tilde{x},\tilde{w}}(x, w) = f_{\tilde{x}|\tilde{w}}(x|w) \cdot f_{\tilde{w}}(w) = \frac{\lambda}{\pi\sigma_s\sigma_w}\exp[-(\beta_2' w^2 + \hat{\beta}_1' w + \hat{\beta}_0')],$$

where $\hat{\beta}_1' = \beta_1' + \lambda$ and $\hat{\beta}_0' = \beta_0' - \lambda w_i$. Similar to the other proofs, I have

$$\check{V} \equiv \mathbb{E}(\tilde{P}_f|\tilde{x} = x) = \exp\left(\frac{\sigma'^2}{2} + \mu' - \lambda\sigma'^2\right)\frac{\Phi(\kappa' - \sigma' + \lambda\sigma')}{\Phi(\kappa' + \lambda\sigma')}.$$

Note that this is also the price in the model where agents don't learn from endogenous price signals but only condition on their exogenous private signals.

(3) **Understanding the Impacts of Learning: A Made-up Example**
Assume $\tilde{x} = \tilde{w} - \frac{\tilde{s}}{\lambda}$, $\tilde{y} = \tilde{a} - \tilde{s}$, where $\tilde{w}, \tilde{a}, \tilde{s}$ are independently normally distributed with zeros means and variances $\sigma_w^2, \sigma_a^2, \sigma_s^2$ respectively. Since $\tilde{x}$ and $\tilde{y}$ are linear combinations of

normal random variables, they are jointly normally distributed with $\tilde{w}$

$$\begin{pmatrix} w \\ x \\ y \end{pmatrix} \sim \mathbb{N}\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_w^2 & \sigma_{12} & 0 \\ \sigma_{12} & \sigma_x^2 & \sigma_{23} \\ 0 & \sigma_{23} & \sigma_y^2 \end{pmatrix}\right],$$

with $\sigma_x^2 = \sigma_w^2 + \frac{\sigma_s^2}{\lambda^2}, \sigma_y^2 = \sigma_a^2 + \sigma_s^2, \sigma_{12} = \sigma_w^2, \sigma_{23} = \frac{\sigma_s^2}{\lambda}$. Denote the expectation and variance of $\tilde{w}$ conditional on $\tilde{x} = x$ by μ' and Σ' respectively, and the expectation and variance of $\tilde{w}$ conditional on $\tilde{x} = x, \tilde{y} = y$ by μ and Σ respectively. It follows that $\mu' \equiv \mathbb{E}(\tilde{w}|\tilde{x} = x) = W_x' x$, $\mu \equiv \mathbb{E}(\tilde{w}|\tilde{x} = x, \tilde{y} = y) = W_x x + W_y y$, and $\Sigma' \equiv Var(\tilde{w}|\tilde{x} = x) = \left(\frac{\sigma_s^2}{\lambda^2}\right) W_x'$, $\Sigma \equiv Var(\tilde{w}|\tilde{x} = x, \tilde{y} = y) = \left(-\frac{\sigma_a^2}{\lambda}\right) W_y$, where

$$W_x' \equiv \frac{\frac{1}{\sigma_s^2}}{\frac{1}{\sigma_s^2} + \frac{1}{\lambda^2\sigma_w^2}}, W_x \equiv \frac{\frac{1}{\sigma_a^2} + \frac{1}{\sigma_s^2}}{\frac{1}{\sigma_a^2} + \frac{1}{\sigma_s^2} + \frac{1}{\lambda^2\sigma_w^2}}, W_y \equiv \frac{-\frac{1}{\lambda\sigma_a^2}}{\frac{1}{\sigma_a^2} + \frac{1}{\sigma_s^2} + \frac{1}{\lambda^2\sigma_w^2}}.$$

It is easy to show that $W_x' < W_x$ and $W_y < 0$. When $\sigma_a^2 \to \infty$ or $\lambda \to \infty$, then $W_y \to 0$ and both W_x, W_x' converge to some constant smaller or equal to one, and the two cases are converging. Similarly, when $\lambda \to 0$, then $W_x \to 0$, $W_x' \to 0$, and $W_y \to 0$, and both converge to no information case. (i) When $\sigma_a^2 \to 0$, then $W_y \to -\frac{1}{\lambda}$, $W_x \to 1$, and W_x' is a constant smaller than one. Thus, the smaller the λ (but not too small), the larger negative effect from y but also the larger the difference $W_x - W_x'$, and the overall effect is ambiguous. However, because W_x and W_x' are confined by 1 while W_y could be much larger than 1, the negative effect tends to dominate; (ii) When $\sigma_s^2 \to \infty$, then $W_y \to \frac{-\frac{1}{\lambda\sigma_a^2}}{\frac{1}{\sigma_a^2} + \frac{1}{\lambda^2\sigma_w^2}}$, $W_x \to \frac{\frac{1}{\sigma_a^2}}{\frac{1}{\sigma_a^2} + \frac{1}{\lambda^2\sigma_w^2}}$, and $W_x' \to 0$. Thus, the smaller the λ (but not too small), the larger negative effect from y but also the larger the difference $W_x - W_x'$, and the overall effect is ambiguous. However, because W_x and W_x' are confined by 1 while W_y could be much larger than 1, the negative effect tends to dominate; (iii) When $\sigma_s^2 \to 0$, then $W_y \to 0$, $W_x \to 1$, and $W_x' \to 1$. The conditional expectations of e^w are given by $\Lambda' \equiv \mathbb{E}(e^{\tilde{w}}|\tilde{x} = x) = \exp\left(\mu' + \frac{\Sigma'}{2}\right)$ and $\Lambda \equiv \mathbb{E}(e^{\tilde{w}}|\tilde{x} = x, \tilde{y} = y) = \exp\left(\mu + \frac{\Sigma}{2}\right)$. The law of total variance implies that $\mathbb{E}(\Lambda) = \mathbb{E}(\Lambda')$.

A.3 Numerical Computations

The mean of housing price is

$$\mathbb{E}(\tilde{P}) = \mathbb{E}(\tilde{Q}) + \mathbb{E}(\tilde{V}).$$

The (additional) effect of rental price on housing price volatility

$$\phi' = \frac{Var(\tilde{Q}) + Var(\tilde{V}) + 2Cov(\tilde{Q}, \tilde{V})}{Var(\tilde{Q}) + Var(\tilde{V}') + 2Cov(\tilde{Q}, \tilde{V}')}.$$

The excess volatility is

$$\phi^* = \frac{Var(\tilde{Q}) + Var(\tilde{V}) + 2Cov(\tilde{Q}, \tilde{V})}{Var(\tilde{Q}) + Var(\tilde{P}_f)}.$$

(1) *Densities*

$$
\begin{aligned}
f_{\tilde{x}}(x) &= \frac{\sqrt{\frac{2}{\pi}}}{\sqrt{\sigma_w^2+\frac{\sigma_s^2}{\lambda^2}}}\exp\left[-\frac{x^2}{2\left(\sigma_w^2+\frac{\sigma_s^2}{\lambda^2}\right)}\right]\Phi\left(\frac{\frac{\sigma_s}{\lambda\sigma_w}x}{\sqrt{\sigma_w^2+\frac{\sigma_s^2}{\lambda^2}}}\right),\\
f_{\tilde{y}}(y) &= \frac{\sqrt{\frac{2}{\pi}}}{\sqrt{\sigma_a^2+\sigma_s^2}}\exp\left[-\frac{y^2}{2(\sigma_a^2+\sigma_s^2)}\right]\Phi\left(\frac{\frac{\sigma_s}{\sigma_a}y}{\sqrt{\sigma_a^2+\sigma_s^2}}\right),\\
f_{\tilde{x},\tilde{y}}(x,y) &= \hat{N}\sqrt{\frac{\pi}{\beta_2}}\exp\left(\frac{\hat{\beta}_1^2}{4\beta_2}-\hat{\beta}_0\right)\Phi\left(\sqrt{2\beta_2}x+\frac{\hat{\beta}_1}{\sqrt{2\beta_2}}\right).
\end{aligned}
$$

(2) *Mean and Variance of* $\tilde{P}_f$

$$
\begin{aligned}
\mathbb{E}(\tilde{P}_f) &= \exp\left(\frac{\sigma_w^2}{2}\right),\\
Var(\tilde{P}_f) &= \left[\exp(\sigma_w^2)-1\right]\exp(\sigma_w^2).
\end{aligned}
$$

(3) *Mean and Variance of* $\tilde{Q}$

$$
\begin{aligned}
\mathbb{E}(\tilde{Q}) &= \int_{-\infty}^{\infty} e^y f_{\tilde{y}}(y)\mathrm{d}y\\
&= \frac{\sqrt{\frac{2}{\pi}}}{\sqrt{\sigma_a^2+\sigma_s^2}}\int_{-\infty}^{\infty}\exp\left[y-\frac{y^2}{2(\sigma_a^2+\sigma_s^2)}\right]\Phi\left(\frac{\frac{\sigma_s}{\sigma_a}y}{\sqrt{\sigma_a^2+\sigma_s^2}}\right)\mathrm{d}y,\\
Var(\tilde{Q}) &= \int_{-\infty}^{\infty} e^{2y} f_{\tilde{y}}(y)\mathrm{d}y-[\mathbb{E}(\tilde{Q})]^2\\
&= \frac{\sqrt{\frac{2}{\pi}}}{\sqrt{\sigma_a^2+\sigma_s^2}}\int_{-\infty}^{\infty}\exp\left[2y-\frac{y^2}{2(\sigma_a^2+\sigma_s^2)}\right]\Phi\left(\frac{\frac{\sigma_s}{\sigma_a}y}{\sqrt{\sigma_a^2+\sigma_s^2}}\right)\mathrm{d}y-[\mathbb{E}(\tilde{Q})]^2.
\end{aligned}
$$

(4) *Mean and Variance of* $\tilde{V}$

$$
\begin{aligned}
\mathbb{E}(\tilde{V}) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} v(x,y)\cdot f_{\tilde{x},\tilde{y}}(x,y)\mathrm{d}x\mathrm{d}y\\
&= \hat{N}\sqrt{\frac{\pi}{\beta_2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left[\frac{\sigma^2}{2}+\mu+\frac{(\beta_1+\lambda)^2}{4\beta_2}-\hat{\beta}_0\right]\\
&\quad\times\left[\frac{\Phi\left(\frac{x-\mu}{\sigma}-\sigma\right)\Phi\left(\frac{x-\mu}{\sigma}+\lambda\sigma\right)}{\Phi\left(\frac{x-\mu}{\sigma}\right)}\right]\mathrm{d}x\mathrm{d}y,\\
Var(\tilde{V}) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} v^2\cdot f_{\tilde{x},\tilde{y}}(x,y)\mathrm{d}x\mathrm{d}y-[\mathbb{E}(\tilde{V})]^2\\
&= \hat{N}\sqrt{\frac{\pi}{\beta_2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left[\sigma^2+2\mu+\frac{(\beta_1+\lambda)^2}{4\beta_2}-\hat{\beta}_0\right]\\
&\quad\times\left[\frac{\left(\Phi\left(\frac{x-\mu}{\sigma}-\sigma\right)\right)^2\Phi\left(\frac{x-\mu}{\sigma}+\lambda\sigma\right)}{\left(\Phi\left(\frac{x-\mu}{\sigma}\right)\right)^2}\right]\mathrm{d}x\mathrm{d}y-(\mathbb{E}[\tilde{V}])^2.
\end{aligned}
$$

(5) *Mean and Variance of* V'

$$\begin{aligned}
\mathbb{E}(\tilde{V}') &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} v'(x)\cdot f_{\tilde{x}}(x)\mathrm{d}x \\
&= \frac{\sqrt{\frac{2}{\pi}}}{\sqrt{\sigma_w^2+\frac{\sigma_s^2}{\lambda^2}}}\int_{-\infty}^{\infty}\exp\left[\frac{\sigma'^2}{2}+\mu'-\frac{x^2}{2\left(\sigma_w^2+\frac{\sigma_s^2}{\lambda^2}\right)}\right] \\
&\quad \times\Phi\left(\frac{\frac{\sigma_s}{\lambda\sigma_w}x}{\sqrt{\sigma_w^2+\frac{\sigma_s^2}{\lambda^2}}}\right)\frac{\Phi(\kappa'-\sigma')}{\Phi(\kappa')}\mathrm{d}x, \\
Var(\tilde{V}') &= \int_{-\infty}^{\infty} v'^2\cdot f_{\tilde{x}}(x)\mathrm{d}x-[\mathbb{E}(\tilde{V}')]^2 \\
&= \frac{\sqrt{\frac{2}{\pi}}}{\sqrt{\sigma_w^2+\frac{\sigma_s^2}{\lambda^2}}}\int_{-\infty}^{\infty}\exp\left[\sigma'^2+2\mu'-\frac{x^2}{2\left(\sigma_w^2+\frac{\sigma_s^2}{\lambda^2}\right)}\right] \\
&\quad \times\Phi\left(\frac{\frac{\sigma_s}{\lambda\sigma_w}x}{\sqrt{\sigma_w^2+\frac{\sigma_s^2}{\lambda^2}}}\right)\left(\frac{\Phi(\kappa'-\sigma')}{\Phi(\kappa')}\right)^2\mathrm{d}x-(\mathbb{E}[\tilde{V}'])^2.
\end{aligned}$$

(6) *Covariances*

$$\begin{aligned}
Cov(\tilde{Q},\tilde{V}) &= \mathbb{E}(\tilde{Q}\tilde{V})-\mathbb{E}(\tilde{Q})\mathbb{E}(\tilde{V}), \\
Cov(\tilde{Q},\tilde{V}') &= \mathbb{E}(\tilde{Q}\tilde{V}')-\mathbb{E}(\tilde{Q})\mathbb{E}(\tilde{V}'),
\end{aligned}$$

where

$$\begin{aligned}
\mathbb{E}(\tilde{Q}\tilde{V}) &= \hat{N}\sqrt{\frac{\pi}{\beta_2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left[\frac{\sigma^2}{2}+\mu+y+\frac{(\beta_1+\lambda)^2}{4\beta_2}-\hat{\beta}_0\right] \\
&\quad\times\left[\frac{\Phi(\kappa-\sigma)\Phi(\kappa+\lambda\sigma)}{\Phi(\kappa)}\right]\mathrm{d}x\mathrm{d}y, \\
\mathbb{E}(\tilde{Q}\tilde{V}') &= \hat{N}\sqrt{\frac{\pi}{\beta_2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left[\frac{\sigma'^2}{2}+\mu'+y+\frac{(\beta_1+\lambda)^2}{4\beta_2}-\hat{\beta}_0\right] \\
&\quad\times\left[\frac{\Phi(\kappa'-\sigma')\Phi(\kappa+\lambda\sigma)}{\Phi(\kappa')}\right]\mathrm{d}x\mathrm{d}y.
\end{aligned}$$

Appendix B

To save notation, I omit the superscript i in the following derivations when it is not necessary.

B.1 First-Order Conditions

For each island i, the first-order conditions to the household's problem and entrepreneur's problem are obtained as

$$
\begin{aligned}
\tilde{C}_t &: \quad 1 = \tilde{\Lambda}_t, \\
\tilde{L}_{t+1} &: \quad \tilde{\Lambda}_t Q_t = b\tilde{L}_{t+1}^{-1} + \tilde{\beta}\tilde{\mathbb{E}}_t\tilde{\Lambda}_{t+1}Q_{t+1}, \\
\tilde{B}_{t+1} &: \quad \tilde{\Lambda}_t = \tilde{\beta}(1+R_{t+1})\tilde{\mathbb{E}}_t\tilde{\Lambda}_{t+1}, \\
C_t &: \quad C_t^{-\sigma} = \Lambda_t, \\
K_{t+1} &: \quad \Lambda_t = \beta\mathbb{E}_t\Lambda_{t+1}\left(\alpha\frac{Y_{t+1}}{K_{t+1}} + 1 - \delta\right), \\
L_{t+1} &: \quad \Lambda_t Q_t = \Phi_t\mathbb{E}_t Q_{t+1} + \beta\mathbb{E}_t\Lambda_{t+1}\left(\gamma\frac{Y_{t+1}}{L_{t+1}} + Q_{t+1}\right), \\
B_{t+1} &: \quad \Lambda_t = \beta(1+R_{t+1})\mathbb{E}_t\Lambda_{t+1} + \Phi_t(1+R_{t+1}),
\end{aligned}
$$

where $\tilde{\Lambda}_t$ is the Lagrangian multiplier of household's budget constraint, Λ_t is the Lagrangian multiplier of entrepreneur's budget constraint, and Φ_t is the Lagrangian multiplier of entrepreneur's borrowing constraint. The above first-order conditions, together with the budget constraints for households and entrepreneurs, and the market clearing conditions[5] for each island characterise the competitive equilibrium of the economy of island.

$$
\begin{aligned}
&\tilde{C}_t + Q_t(\tilde{L}_{t+1} - \tilde{L}_t) + \tilde{B}_{t+1} = (1+R_t)\tilde{B}_t, \\
&Y_t = A_t K_t^{\alpha} L_t^{\gamma}, \\
&C_t + K_{t+1} - (1-\delta)K_t + Q_t(L_{t+1} - L_t) + (1+R_t)B_t = B_{t+1} + Y_t, \\
&(1+R_{t+1})B_{t+1} \leq \mathbb{E}_t Q_{t+1} L_{t+1}, \\
&\bar{L} = \tilde{L}_t + L_t, \\
&\tilde{B}_t = B_t.
\end{aligned}
$$

[5] The good market clearing condition has been omitted because of the Walras' law.

B.2 Steady State

Denote the variables in the steady state by the corresponding letters without time subscript. The steady state of the equilibrium can be characterised by the following equations

$$\begin{aligned}
&1 + R = \tilde{\beta}^{-1}, \\
&Q = b\tilde{L}^{-1} + \tilde{\beta}Q, \\
&\tilde{C} = R\tilde{B}; \\
&C^{-\sigma} = \Lambda, \\
&1 = \beta\left(\alpha\frac{Y}{K} + 1 - \delta\right), \\
&\Lambda Q = \Phi Q + \beta\Lambda\left(\gamma\frac{Y}{L} + Q\right), \\
&\Lambda = \beta(1+R)\Lambda + \Phi(1+R), \\
&Y = AK^{\alpha}L^{\gamma}, \\
&C + \delta K + RB = Y \\
&(1+R)B = QL; \\
&\tilde{L} + L = 1, \\
&B = \tilde{B}.
\end{aligned}$$

From $\Lambda = \beta(1+R)\Lambda + \Phi(1+R)$ and $1 + R = \tilde{\beta}^{-1}$, it follows immediately that

$$\Phi = (\tilde{\beta} - \beta)\Lambda,$$

which is positive because $\Lambda = C^{-\sigma} > 0$ and $\tilde{\beta} - \beta > 0$. From the above equations, we can also see that the capital-to-output ratio, which determines the return from capital in the steady state, is given by

$$\frac{K}{Y} = \frac{\alpha\beta}{1 - \beta(1-\delta)}.$$

and the price of land in the steady state

$$Q = (1 - \tilde{\beta})^{-1}\beta\gamma\frac{Y}{L} = \sum_{j=0}^{\infty}\tilde{\beta}^{j}\beta\gamma\frac{Y}{L}.$$

B.3 Log-linearisation

Define

$$x_t \equiv \ln X_t - \ln X.$$

I log-linearise the model equilibrium around the steady state as below

$$\tilde{\Lambda}Qq_t = -b\tilde{L}^{-1}\tilde{l}_{t+1} + \tilde{\beta}\tilde{\Lambda}Q\tilde{\mathbb{E}}_t q_{t+1},$$

$$-\sigma C^{-\sigma}c_t = \Lambda\lambda_t,$$

$$\lambda_t = \beta\left(\alpha\frac{Y}{K} + 1 - \delta\right)\mathbb{E}_t\lambda_{t+1} + \beta\alpha\frac{Y}{K}\mathbb{E}_t(y_{t+1} - k_{t+1}),$$

$$\Lambda Q(\lambda_t + q_t) = \Phi Q(\phi_t + \mathbb{E}_t q_{t+1}) + \beta\Lambda\frac{\gamma Y}{L}\mathbb{E}_t(\lambda_{t+1} + y_{t+1} - l_{t+1}) + \beta\Lambda Q\mathbb{E}_t(\lambda_{t+1} + q_{t+1}),$$

$$\Lambda\lambda_t = \beta\Lambda\mathbb{E}_t[(1+R)\lambda_{t+1}] + (1+R)\Phi\phi_t,$$

$$\frac{\tilde{C}}{B}\tilde{c}_t + \frac{Q\tilde{L}}{B}(\tilde{l}_{t+1} - \tilde{l}_t) + \tilde{b}_{t+1} = (1+R)\tilde{b}_t,$$

$$y_t = a_t + \alpha k_t + \gamma l_t,$$

$$\frac{C}{Y}c_t + \frac{K}{Y}k_{t+1} - (1-\delta)\frac{K}{Y}k_t + \frac{QL}{Y}(l_{t+1} - l_t) + \frac{(1+R)B}{Y}b_t = \frac{B}{Y}b_{t+1} + y_t,$$

$$(1+R)b_{t+1} = \frac{QL}{B}(\mathbb{E}_t q_{t+1} + l_{t+1}),$$

$$\tilde{L}\tilde{l}_t + Ll_t = 0,$$

$$\tilde{b}_t = b_t,$$

$$a_t = \theta_t + \epsilon_t.$$

where I have used the fact that $r_{t+1} = 0, \forall t$.

B.4 Rational Expectations Equilibrium: A General Solution

If agents have complete information, in the sense that in each period agents on each island observe both the persistent common shock and the temporary idiosyncratic shock, the model may be solved by some standard package such as Dynare. If agents have incomplete information, however, the solution to the model is not readily available using the standard tool. The strategy I take in this paper is that, I first solve for the rational expectations equilibrium (REE) under a general information structure. Given the general solution, I then solve for REE solutions under different information structures in the next subsection. To obtain a general solution, I rearranged the system in B.3 into three parts:
(i) a high-order stochastic difference equation for q^i:

$$\delta_1 q^i_{t-1} + \delta_2 q^i_t + \delta_3\mathbb{E}^i_{t-1}q^i_t + (\delta_4 - \delta_5)\mathbb{E}^i_t q^i_{t+1} = \delta_6\mathbb{E}^i_{t-1}a^i_t + \delta_7 a^i_t + \delta_8\mathbb{E}^i_t a^i_{t+1} + \delta_9\sum_{j=0}^{\infty}\tilde{\beta}^j\mathbb{E}^i_t a^i_{t+2+j}, \tag{B1}$$

(ii) the productivity process $a_t^i = \theta_t + \epsilon_t^i$, and

(iii) other companion equations characterizing the dynamics of each variable as below:

$$
\begin{aligned}
\text{household's land holding} \quad &: \quad \tilde{l}_{t+1}^i = \frac{\tilde{\beta}\tilde{\mathbb{E}}_t^i q_{t+1}^i - q_t^i}{1-\tilde{\beta}}, \\
\text{entrepreneur's land holding} \quad &: \quad l_t^i = -\tilde{l}_t^i, \\
\text{entrepreneur's debt} \quad &: \quad b_{t+1}^i = \mathbb{E}_t^i q_{t+1}^i + l_{t+1}^i, \\
\text{household's lending} \quad &: \quad \tilde{b}_t^i = b_t^i, \\
\text{entrepreneur's capital stock} \quad &: \quad k_{t+1}^i = \delta_{10}(q_t^i - \tilde{\beta}\mathbb{E}_t^i q_{t+1}^i) - \delta_{11}\mathbb{E}_t^i a_{t+1}^i - \delta_{12} l_{t+1}^i, \\
\text{entrepreneur's good production} \quad &: \quad y_t^i = a_t^i + \alpha k_t^i + \gamma l_t^i, \\
\text{household's consumption} \quad &: \quad \tilde{c}_t^i = \frac{l_{t+1} - l_t^i - \tilde{\beta} b_{t+1}^i + b_t^i}{1-\tilde{\beta}}, \\
\text{entrepreneur's consumption} \quad &: \quad c_t^i = \frac{y_t^i - \beta\gamma\tilde{c}_t^i - \delta_{13}[k_{t+1}^i - (1-\delta)k_t^i]}{\delta_{14}},
\end{aligned}
$$

where $\{\delta_j\}_{j=1}^{14}$ are functions of the model parameters: $\tilde{\beta}, \alpha, \gamma, \sigma, \beta, \delta$. The solution to (B1) is the key to solving for the whole system, because once it is obtained the dynamics of other variables follow recursively from (iii).

For each island i, denote the agents' expectations about the persistent shock by $\Theta_t^i \equiv \mathbb{E}_t^i \theta_t$. Then, equation (B1) can be rewritten as

$$
\delta_1 q_{t-1}^i + \delta_2 q_t^i + \delta_3 \mathbb{E}_{t-1}^i q_t^i + (\delta_4 - \delta_5)\mathbb{E}_t^i q_{t+1}^i = \delta_6 \rho \Theta_{t-1}^i + \left(\delta_8 \rho + \frac{\delta_9 \rho^2}{1-\rho\tilde{\beta}}\right)\Theta_t^i + \delta_7 a_t^i. \tag{B2}
$$

A general rational expectations equilibrium (REE) solution can be obtained as follows: first guess a solution for q_t^i

$$
q_t^i = \pi_1 q_{t-1}^i + \pi_2 \Theta_t^i + \pi_3 \Theta_{t-1}^i + \pi_4 a_t^i, \tag{B3}
$$

then, substitute the conjecture (B3) into equation (B2), and, finally, impose the rational expectations equilibrium (REE) restriction to solve for π's as a fixed point problem. More specifically, the substitution yields

$$
\begin{aligned}
&\delta_1 q_{t-1}^i + \delta_2 q_t^i + \delta_3(\pi_1 q_{t-1}^i + \pi_2\rho\Theta_{t-1}^i + \pi_3\Theta_{t-1}^i + \pi_4\rho\Theta_{t-1}^i) + (\delta_4 - \delta_5)(\pi_1 q_t^i + \pi_2\rho\Theta_t^i + \pi_3\Theta_t^i + \pi_4\rho\Theta_t^i) \\
&\qquad = \delta_6\rho\Theta_{t-1}^i + \left(\delta_8\rho + \frac{\delta_9\rho^2}{1-\rho\tilde{\beta}}\right)\Theta_t^i + \delta_7 a_t^i.
\end{aligned}
$$

Rearranging it, I have

$$
\begin{aligned}
\underbrace{[\delta_2 + (\delta_4 - \delta_5)\pi_1]}_{\alpha_1} q_t^i = \underbrace{-(\delta_1 + \delta_3\pi_1)}_{\alpha_2} q_{t-1}^i + \underbrace{\left[\left(\delta_8\rho + \frac{\delta_9\rho^2}{1-\rho\tilde{\beta}}\right) - (\delta_4 - \delta_5)(\pi_2\rho + \pi_3 + \pi_4\rho)\right]}_{\alpha_3} \Theta_t^i \\
+ \underbrace{[\delta_6\rho - \delta_3(\pi_2\rho + \pi_3 + \pi_4\rho)]}_{\alpha_4} \Theta_{t-1}^i + \delta_7 a_t^i.
\end{aligned}
$$

Matching the coefficients, I have

$$\begin{aligned}
\pi_1 &= \frac{\alpha_2}{\alpha_1} = \frac{-(\delta_1+\delta_3\pi_1)}{\delta_2+(\delta_4-\delta_5)\pi_1}, \\
\pi_4 &= \frac{\delta_7}{\alpha_1} = \frac{\delta_7}{\delta_2+(\delta_4-\delta_5)\pi_1}, \\
\pi_2 &= \frac{\alpha_3}{\alpha_1} = \frac{\delta_8\rho+\frac{\delta_9\rho^2}{1-\rho\tilde{\beta}}-(\delta_4-\delta_5)(\pi_2\rho+\pi_3+\pi_4\rho)}{\delta_2+(\delta_4-\delta_5)\pi_1}, \\
\pi_3 &= \frac{\alpha_4}{\alpha_1} = \frac{\delta_6\rho-\delta_3(\pi_2\rho+\pi_3+\pi_4\rho)}{\delta_2+(\delta_4-\delta_5)\pi_1}.
\end{aligned}$$

Then, π_1 and π_4 can be obtained from the first two equations, and π_2 and π_3 can be obtained from solving the following two equations

$$\begin{aligned}
[\delta_2+(\delta_4-\delta_5)(\pi_1+\rho)]\pi_2+(\delta_4-\delta_5)\pi_3 &= \delta_8\rho+\frac{\delta_9\rho^2}{1-\rho\tilde{\beta}}-(\delta_4-\delta_5)\pi_4\rho, \\
\delta_3\rho\pi_2+(\delta_2+(\delta_4-\delta_5)\pi_1+\delta_3)\pi_3 &= \delta_6\rho-\delta_3\pi_4\rho.
\end{aligned}$$

B.5 Rational Expectations Equilibrium with Incomplete Information

I. No Public Signals

The state space is given by

$$\begin{aligned}
\theta_t &= \rho\theta_{t-1}+v_t, \\
a_t^i &= \theta_t+\epsilon_t^i.
\end{aligned}$$

Using Kalman filter formula, I have

$$\Theta_t^i = (1-\kappa)\rho\Theta_{t-1}^i+\kappa a_t^i, \tag{B4}$$

where κ is the stationary Kalman gain

$$\kappa \equiv \frac{\frac{1}{\sigma_\epsilon^2}}{\frac{1}{\sigma_r^2}+\frac{1}{\sigma_\epsilon^2}},$$

in which σ_r^2 can be derived from the stationary Riccati equation

$$\sigma_r^2 = \frac{\rho^2}{\frac{1}{\sigma_r^2}+\frac{1}{\sigma_\epsilon^2}}+\sigma_v^2.$$

Substituting (B4) into (B3), land price is obtained as

$$(1-\pi_1\mathcal{L})(1-\rho\mathcal{L})(1-(1-\kappa)\rho\mathcal{L})q_t^i$$

$$= [\pi_4+\pi_2\kappa+(\pi_3\kappa-\pi_4(1-\kappa)\rho)\mathcal{L}]v_t+[\pi_4+\pi_2\kappa+(\pi_3\kappa-\pi_4(1-\kappa)\rho)\mathcal{L}](1-\rho\mathcal{L})\epsilon_t^i. \tag{B5}$$

II. Exogenous Public Signals

The state space is given by

$$\begin{aligned}
\theta_t &= \rho\theta_{t-1} + v_t, \\
\underbrace{\begin{pmatrix} a_t^i \\ s_t^* \end{pmatrix}}_{\mathbf{Y}_t} &= \underbrace{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}_{\mathbf{B}} \theta_t + \begin{pmatrix} \epsilon_t^i \\ \eta_t^* \end{pmatrix}.
\end{aligned}$$

Using Kalman filter formula, I have

$$\Theta_t^i = (1 - \mathbf{KB})\rho\Theta_{t-1}^i + \mathbf{KY}_t, \tag{B6}$$

where $\mathbf{K}$ is the stationary Kalman gain vector

$$\mathbf{K} = \sigma_{r^*}^2 \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \sigma_{r^*}^2 + \sigma_\epsilon^2 & \sigma_{r^*}^2, \\ \sigma_{r^*}^2 & \sigma_{r^*}^2 + \sigma_{\eta^*}^2 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{\frac{1}{\sigma_\epsilon^2}}{\frac{1}{\sigma_{r^*}^2} + \frac{1}{\sigma_{\eta^*}^2} + \frac{1}{\sigma_\epsilon^2}} & \frac{\frac{1}{\sigma_{\eta^*}^2}}{\frac{1}{\sigma_{r^*}^2} + \frac{1}{\sigma_{\eta^*}^2} + \frac{1}{\sigma_\epsilon^2}} \end{pmatrix} \equiv \begin{pmatrix} K_1 & K_2 \end{pmatrix},$$

where $\sigma_{r^*}^2$ can be derived from the stationary Riccati equation

$$\sigma_{r^*}^2 = \frac{\rho^2}{\frac{1}{\sigma_{r^*}^2} + \frac{1}{\sigma_{\eta^*}^2} + \frac{1}{\sigma_\epsilon^2}} + \sigma_v^2.$$

Substituting (B6) into (B3), land price land price is obtained as

$$\begin{aligned}
&(1 - \pi_1\mathcal{L})(1 - \rho\mathcal{L})(1 - (1 - \mathbf{KB})\rho\mathcal{L})q_t^i = \{\pi_4 + \pi_2\mathbf{KB} + [\pi_3\mathbf{KB} - \pi_4(1 - \mathbf{KB})\rho]\mathcal{L}\}v_t \\
&+\{\pi_4 + \pi_2 K_1 + [\pi_3 K_1 - \pi_4(1 - \mathbf{KB})\rho]\mathcal{L}\}(1 - \rho\mathcal{L})\epsilon_t^i + (\pi_2 + \pi_3\mathcal{L})K_2(1 - \rho\mathcal{L})\eta_t^*. \qquad \text{(B7)}
\end{aligned}$$

If information is revealed with one period lag, I can now define two new observable variables $\hat{a}_t^i$ and $\hat{s}_t^*$, corresponding to the signals a_t^i and s_t^* respectively.

$$\begin{aligned}
\hat{a}_t^i &\equiv v_t + \epsilon_t^i, \\
\hat{s}_t^* &\equiv v_t + \eta_t^*.
\end{aligned}$$

Note that the joint distribution of $\hat{a}_t^i, \hat{s}_t^*$ and v_t is multi-normal. More specifically,

$$\begin{pmatrix} v_t \\ \hat{a}_t^i \\ \hat{s}_t^* \end{pmatrix} \sim \mathbb{N} \left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_v^2 & \sigma_v^2 & \sigma_v^2 \\ \sigma_v^2 & \sigma_v^2 + \sigma_\epsilon^2 & \sigma_v^2 \\ \sigma_v^2 & \sigma_v^2 & \sigma_v^2 + \sigma_{\eta^*}^2 \end{pmatrix} \right],$$

from which I may compute the expectation of the unobservable shock v_t conditional on the observables $\{\hat{a}_t^i, \hat{s}_t^*\}$ as

$$\mathbb{E}(v_t | \hat{a}_t^i, \hat{s}_t^*) = W_1^*(v_t + \epsilon_t^i) + W_2^*(v_t + \eta_t^*),$$

where

$$
\begin{aligned}
W_1^* &\equiv \frac{\sigma_v^2(\sigma_v^2+\sigma_{\eta^*}^2)-\sigma_v^4}{(\sigma_v^2+\sigma_\epsilon^2)(\sigma_v^2+\sigma_{\eta^*}^2)-\sigma_v^4}, \\
W_2^* &\equiv \frac{\sigma_v^2(\sigma_v^2+\sigma_\epsilon^2)-\sigma_v^4}{(\sigma_v^2+\sigma_\epsilon^2)(\sigma_v^2+\sigma_{\eta^*}^2)-\sigma_v^4}.
\end{aligned}
$$

Since $\Theta_t^i = \mathbb{E}_t^i(v_t)+\sum_{j=1}^{\infty}\rho^j v_{t-j}$, I have

$$
\Theta_t^i = \mathbb{E}_t^i(v_t)+\sum_{j=1}^{\infty}\rho^j v_{t-j} = \left[(W_1^*+W_2^*)v_t+\sum_{j=1}^{\infty}\rho^j v_{t-j}\right]+W_1^*\epsilon_t^i+W_2^*\eta_t^*.
$$

Substituting it into (B3), land price is obtained as

$$
\begin{aligned}
(1-\pi_1\mathcal{L})(1-\rho\mathcal{L})q_t^i &= \{(\pi_2+\pi_3\mathcal{L})[\rho\mathcal{L}+(1-\rho\mathcal{L})(W_1^*+W_2^*)]+\pi_4\}v_t \\
&+[(\pi_2+\pi_3\mathcal{L})W_1^*+\pi_4](1-\rho\mathcal{L})\epsilon_t^i+[(\pi_2+\pi_3\mathcal{L})W_2^*(1-\rho\mathcal{L})]\eta_t^*. \qquad (\text{B7}')
\end{aligned}
$$

III. Endogenous Public Signals

The REE equilibrium land price (B3) can be written as

$$
q_t^i = \left[\frac{\pi_2+\pi_3\mathcal{L}}{1-\pi_1\mathcal{L}}\right]\Theta_t^i+\frac{\pi_4 a_t^i}{1-\pi_1\mathcal{L}} \equiv B(\mathcal{L})C(\mathcal{L})\Theta_t^i+\pi_4 B(\mathcal{L})[A(\mathcal{L})v_t+\epsilon_t^i],
$$

where $B(\mathcal{L}) \equiv \frac{1}{1-\pi_1\mathcal{L}}$ and $C(\mathcal{L}) \equiv \pi_2+\pi_3\mathcal{L}$. Note that q_t should be a time-invariant function of exogenous stochastic processes. Hence, I guess

$$
\Theta_t^i = P_v(\mathcal{L})v_t+P_\epsilon(\mathcal{L})\epsilon_t^i+P_\eta(\mathcal{L})\eta_t,
$$

where η_t is the noise shock. Then, the land price function (B3) can be further expressed as

$$
\begin{aligned}
q_t^i &= B(\mathcal{L})C(\mathcal{L})[P_v(\mathcal{L})v_t+P_\epsilon(\mathcal{L})\epsilon_t^i+P_\eta(\mathcal{L})\eta_t]+\pi_4 B(\mathcal{L})[A(\mathcal{L})v_t+\epsilon_t^i] \\
&= [B(\mathcal{L})C(\mathcal{L})P_v(\mathcal{L})+\pi_4 B(\mathcal{L})A(\mathcal{L})]v_t+[B(\mathcal{L})C(\mathcal{L})P_\epsilon(\mathcal{L})+\pi_4 B(\mathcal{L})]\epsilon_t^i \\
&\quad +B(\mathcal{L})C(\mathcal{L})P_\eta(\mathcal{L})\eta_t.
\end{aligned}
$$

Since information is fully revealed with one period lag, I can now define two new observable variables $\hat{a}_t^i$ and $\hat{s}_t$, corresponding to the signals a_t^i and s_t respectively.

$$
\begin{aligned}
\hat{a}_t^i &\equiv v_t+\epsilon_t^i, \\
\hat{s}_t &\equiv \underbrace{(\pi_2 P_v(0)+\pi_4)}_{s_1}v_t+\underbrace{(1+\pi_2 P_\eta(0))}_{s_2}\eta_t.
\end{aligned}
$$

Note that the joint distribution of $\hat{a}_t^i, \hat{s}_t$ and v_t is multi-normal. More specifically,

$$\begin{pmatrix} v_t \\ \hat{a}_t^i \\ \hat{s}_t \end{pmatrix} \sim \mathbb{N}\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_v^2 & \sigma_v^2 & s_1\sigma_v^2 \\ \sigma_v^2 & \sigma_v^2 + \sigma_\epsilon^2 & s_1\sigma_v^2 \\ s_1\sigma_v^2 & s_1\sigma_v^2 & s_1^2\sigma_v^2 + s_2^2\sigma_\eta^2 \end{pmatrix}\right],$$

from which I may compute the expectation of the unobservable shock v_t conditional on the observables $\{\hat{a}_t^i, \hat{s}_t\}$ as

$$\mathbb{E}(v_t|\hat{a}_t^i, \hat{s}_t) = W_1(v_t + \epsilon_t^i) + W_2(s_1 v_t + s_2\eta_t), \tag{B8}$$

where

$$\begin{aligned} W_1 &\equiv \frac{\sigma_v^2(s_1^2\sigma_v^2 + s_2^2\sigma_\eta^2) - s_1^2\sigma_v^4}{(\sigma_v^2 + \sigma_\epsilon^2)(s_1^2\sigma_v^2 + s_2^2\sigma_\eta^2) - s_1^2\sigma_v^4}, \\ W_2 &\equiv \frac{s_1\sigma_v^2(\sigma_v^2 + \sigma_\epsilon^2) - s_1\sigma_v^4}{(\sigma_v^2 + \sigma_\epsilon^2)(s_1^2\sigma_v^2 + s_2^2\sigma_\eta^2) - s_1^2\sigma_v^4}. \end{aligned}$$

Given that information is fully revealed one period later, and that $\theta_t = \rho\theta_{t-1} + v_t$, the conditional expectation $\Theta_t^i \equiv \mathbb{E}_t^i(\theta_t)$ can be written as

$$\Theta_t^i = \mathbb{E}_t^i(v_t) + \sum_{j=1}^{\infty} \rho^j v_{t-j}. \tag{B9}$$

Note that agents's confusion about θ_t will only be from current idiosyncratic productivity shock and noise shock, not those from previous periods. Hence, the conjecture about Θ_t^i can be rewritten as

$$\Theta_t^i = P_v(0)v_t + \sum_{j=1}^{\infty} \rho^j v_{t-j} + P_\epsilon(0)\epsilon_t^i + P_\eta(0)\eta_t. \tag{B10}$$

Substituting (B8) into (B9) and matching the coefficients with those in the conjecture, I have

$$\begin{aligned} P_v(0) &= W_1 + W_2 s_1, \\ P_\epsilon(0) &= W_1, \\ P_\eta(0) &= W_2 s_2. \end{aligned}$$

Note that W_1 and W_2 contain $P_v(0)$ and $P_\eta(0)$. Solving the nonlinear equations above yields $P_v(0), P_\epsilon(0)$, and $P_\eta(0)$. Substituting them back into (B10), I obtain an explicit expression for Θ_t^i, which is then substituted into the general solution (B3) to get the land price function

$$(1 - \pi_1\mathcal{L})(1 - \rho\mathcal{L})q_t^i = \{(\pi_2 + \pi_3\mathcal{L})[\rho\mathcal{L} + (1 - \rho\mathcal{L})P_v(0)] + \pi_4\}v_t$$

$$+[(\pi_2 + \pi_3\mathcal{L})P_\epsilon(0) + \pi_4](1 - \rho\mathcal{L})\epsilon_t^i + [(\pi_2 + \pi_3\mathcal{L})P_\eta(0)(1 - \rho\mathcal{L})]\eta_t. \tag{B11}$$

B.6 Forecast Errors

Note that $\bar{\mathbb{E}}_{t-1}\theta_t = \rho \int \Theta^i_{t-1} \mathrm{d}i$.
(i) For the full information model, the forecast error is given by

$$\xi_t = \theta_t - \bar{\mathbb{E}}_{t-1}\theta_t = \theta_t - \rho\theta_{t-1} = v_t.$$

(ii) For the incomplete information model without public signal, the forecast error is given by

$$\xi_t = \left[\frac{(1-\rho\kappa-(1-\kappa)\rho\mathcal{L})}{(1-(1-\kappa)\rho\mathcal{L})(1-\rho\mathcal{L})}\right] v_t.$$

(iii) For the incomplete information model with exogenous public signal, the forecast error is given by

$$\xi_t = \left[\frac{(1-\rho\mathbf{KB}-(1-\mathbf{KB})\rho\mathcal{L})}{(1-(1-\mathbf{KB})\rho\mathcal{L})(1-\rho\mathcal{L})}\right] v_t + \left[\frac{\rho K_2\mathcal{L}}{1-(1-\mathbf{KB})\rho\mathcal{L}}\right]\eta^*_t.$$

When information is fully revealed one period later, the forecasting error is given by

$$\xi_t = v_t + \rho(1-W_1^*-W_2^*)v_{t-1} - \rho W_2^*\eta^*_{t-1}.$$

(iv) For the incomplete information model with endogenous public signal and information fully revealed one period later, the forecast error is given by

$$\begin{aligned}\xi_t &= \theta_t - \bar{\mathbb{E}}_{t-1}\theta_t = \sum_{j=0}^{\infty}\rho^j v_{t-j} - \rho\left[P_v(0)v_{t-1} + \sum_{j=1}^{\infty}\rho^j v_{t-1-j} + P_\eta(0)\eta_{t-1}\right] \\ &= v_t + \rho(1-P_v(0))v_{t-1} - \rho P_\eta(0)\eta_{t-1}.\end{aligned}$$

Appendix C

C.1 Forecast Error Variance Decomposition

The error of the optimal h-step forecast is

$$Y_{t+h} - \hat{Y}_{t+h} = \sum_{s=0}^{h-1} \theta_s u_{t+h-s}.$$

The contribution of forecast error variance of variable i attributable to structural shock j at horizon h is

$$\sum_{s=0}^{h-1} (e_i' \theta_s e_j)^2 = \sum_{s=0}^{h-1} (\theta_s^{(i,j)})^2,$$

where e_i is i^{th} column of I_k and $\theta_s^{(i,j)}$ is the ij^{th} element in θ_s. The share of forecast error variance of variable i attributable to structural shock j (of k) at horizon h is

$$\Omega_h^{(i,j)} = \frac{\sum_{s=0}^{h-1} (e_i' \theta_s e_j)^2}{\sum_{j=1}^{k} \sum_{s=0}^{h-1} (e_i' \theta_s e_j)^2} = \frac{\sum_{s=0}^{h-1} (\theta_s^{(i,j)})^2}{\sum_{j=1}^{k} \sum_{s=0}^{h-1} (\theta_s^{(i,j)})^2}.$$

Because $Y_t = \sum_{s=0}^{\infty} \theta_s u_{t-s} = \sum_{s=0}^{\infty} \phi_s \epsilon_{t-s}$, and $\epsilon_t \equiv B_0^{-1} u_t$, $\theta_s \equiv \phi_s B_0^{-1}$ by definition.

$$\Omega_h^{(i,j)} = \frac{\sum_{s=0}^{h-1} [(\phi_s B_0^{-1})^{(i,j)}]^2}{\sum_{j=1}^{k} \sum_{s=0}^{h-1} [(\phi_s B_0^{-1})^{(i,j)}]^2}.$$

C.2 Historical Decomposition

Consider reorganisation of the vector moving average (VMA) representation of Y_{t+h}

$$Y_{t+h} = \sum_{s=0}^{\infty} \phi_s \epsilon_{t+h-s} \equiv \sum_{s=0}^{h-1} \phi_s \epsilon_{t+h-s} + \sum_{s=h}^{\infty} \phi_s \epsilon_{t+h-s}. \tag{C1}$$

The first sum represents that part of Y_{t+h} due to innovations in periods $t+1$ to $t+h$, while the second sum is the forecast given data through t (because $\mathbb{E}(u_t) = 0$ for $u_{t+1}, ..., u_{t+h}$). If $\epsilon = Fu$ for any full rank (factor) matrix F, the forecast error can be rewritten as

$$\sum_{s=0}^{h-1} \phi_s \epsilon_{t+h-s} = \sum_{s=0}^{h-1} (\phi_s F)(F^{-1} \epsilon_{t+h-s}) = \sum_{s=0}^{h-1} (\phi_s F) u_{t+h-s} = \sum_{i=1}^{k} \sum_{s=0}^{h-1} [(\phi_s F) e_i](e_i' u_{t+h-s}),$$

where e_i is the i^{th} column of I_k [6], the first summation regards to k factored shocks, and the second summation regards to the accumulated horizon. The analysis above implies that we can decompose the forecast error with some factor matrix F and see how each factored shock would have affected the historical forecast error. If we want to check the structural shocks' effect, then set $F = B_0^{-1}$, and $\phi_s F = \phi_s B_0^{-1} \equiv \theta_s$.

[6] In the first parentheses e_i is used to pick up the i^{th} column of $\phi_s F$ and in the second parentheses to pick up the i^{th} factored (or structural) shock. So the vector of forecast error for each variable is comprised of a vector of sum over all of the structural shocks' contributions and over horizon h.

In practice, I do historical decomposition following steps as below[7]:
(Step 1) Set in (C1) $t = p$, $h \in \{1, 2, ..., T - p\}$, and therefore the forecast with shocks is from $p + 1$ to T, where k is the dimension of VAR, p is the maximum lag in VAR, and T is sample size. Transform the $k \times (T - p)$ reduced shocks ϵ into structural shocks u by using $u = B_0\epsilon$.
(Step 2) For each i, left multiply column i in B_0^{-1} by row i in u to get k stacks of $k \times (T - p)$ matrices. The i^{th} stack matrix represents i^{th} structural shock to all k variables during time period $p + 1$ to T. Denote i^{th} stack matrix as ϵ^i, which is interpreted as the contribution of i^{th} structural shock to reduced shock u and $\sum_{i=1}^{k} u^i = u$.
(Step 3) Given initial values $Y_1, ..., Y_p$, forecast Y_{t+h} (i.e. from $p + 1$ to T) by adding shocks u_j^i in each period, where $j \in \{1, 2, ..., T - p\}$.[8]
(Step 4) Subtract the base forecast (i.e. forecast without shocks in each period) from the forecast in step 3 to get the cumulative contributions of each structural shock.

For periods close to the (forecast) starting point $(p + 1)$, the initial values have substantial impact even in stationary process, so one may want to consider the decomposition for periods some distance away from the starting point.

C.3 Conditional Correlation

From $Y_t = \sum_{s=0}^{\infty} \theta_s u_{t-s}$, the covariance between variable i and j is

$$\begin{aligned} Cov(Y_i, Y_j) &= \sum_{s=0}^{\infty} \theta_s^{(i,1)} \theta_s^{(j,1)} + \sum_{s=0}^{\infty} \theta_s^{(i,2)} \theta_s^{(j,2)} + ... + \sum_{s=0}^{\infty} \theta_s^{(i,k)} \theta_s^{(j,k)} \\ &= \sum_{\kappa=1}^{k} \sum_{s=0}^{\infty} \theta_s^{(i,\kappa)} \theta_s^{(j,\kappa)} \equiv \sum_{\kappa=1}^{k} Cov(Y_i, Y_j | u_\kappa), \end{aligned}$$

and the correlation of variable i and j in Y conditional on the κ^{th} structural shock can be obtained using the following formula

$$\rho(Y_i, Y_j | \kappa) = \frac{\sum_{s=0}^{\infty} \theta_s^{(i,\kappa)} \theta_s^{(j,\kappa)}}{\sqrt{[\sum_{s=0}^{\infty} (\theta_s^{(i,\kappa)})^2][\sum_{s=0}^{\infty} (\theta_s^{(j,\kappa)})^2]}},$$

where $\theta_s^{(i,\kappa)}$ is the $(i, \kappa)^{th}$ element in θ_s. In practice, the infinite sums are truncated at some large but finite lag. Similar to historical decomposition, we can also compute historical conditional correlation coefficient of Y_i and Y_j by using the simulated series of Y_i and Y_j conditional structural shock κ.[9]

[7] Another simple way (easier to understand) to do the historical decomposition: (I) Transform reduced shocks into structural shocks using B_0; (II) Zero out irrelevant structural shocks for each structural shock, respectively; (III) Transform the k new structural shock (matrices) into new reduced shocks; (IV) Simulating as if there was only one structural shock in history.

[8] In this step, I forecast Y recursively. For example, I use updated $Y_{p+1}, ..., Y_{p+p}$ in calculating Y_{p+p+1}

[9] Note the similarity with the relationship between FEVD a historical decomposition.

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