Essays on Model Uncertainty and Corporate Financial Policies

by

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Declarations

I declare that any material contained in this thesis has not been submitted for a degree to any other university. I further declare that Chapters 3 and 4 of the this thesis are a product of joint work with Dr. Andrea Gamba.

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Abstract

The thesis comprises three essays on model uncertainty and corporate financial policies. Particularly, it studies how model uncertainty affects firm's choice in financing method in distinct market conditions, as well as the evolution of corporate risk management policy under resolving uncertainty in firm's profitability and the ability to hedge with derivatives.

Chapter 2 is the first attempt to reconcile prospect theory and contract theory in explaining observed financing decisions. In a world of ambitious and aggressive economic agents, even original equilibria break down in the presence of asymmetric information. When the aggressiveness in the market prevails, multi-pooling-equilibria could exist, which might explain why academics cannot find the optimal leverage. In the meanwhile, debt issuance will be lower, which sheds light on the distinct liquidity provision patterns between bull and bear markets. Firms with lottery-like investment opportunity will get external financing more easily than that with safe projects because both market participants - entrepreneur and financier – will perceive the project to be more valuable than would otherwise be suggested by the classical expected utility theory.

While current corporate risk management theories predict that young firms should hedge more than the established ones, the claim is not supported by empirical observations, which also present mixed evidence on whether hedging creates value. Chapter 3 attempts to address this puzzle by including model uncertainty as part of risk management process. We develop a dynamic model in which agents learn about a firm's hedgeability, gauged by the correlation between its operating cash flow and underlying asset of hedging instruments, while weighing the costs and benefits of different risk management tools. The model predicts that resolving model uncertainty accelerates the process of building up hedging positions, but this is not necessarily accompanied with firm value creation. We conclude that dynamic information acquisition is an important determinant of corporate risk management.

In Chapter 4, we develop a dynamic model of corporate risk management in which agents learn about a firm's profitability and balance the costs and benefits of risk management. We depict how the resolution of this particular model uncertainty intertwines with corporate risk management policy. The model predicts that both cash balance and hedging level increase in profitability prospects. We conclude that unraveling profitability drives distinct dynamics in corporate cash management and hedging policy, as a result of trade-off between risk-managing consideration and default option value-maximization. This also yields insights into the puzzles of why young firms use less hedging than mature firms and why there is mixed empirical evidence on the value creation role of corporate risk management.

Chapter 1

Introduction

Corporate financial policies crucially depend on underlying assumptions made by decision makers. However, these assumptions are usually associated with a great deal of uncertainty. The uncertainty includes the model itself, which governs the cash flow of a firm, as well as the parameters assumed in the model. In this regard, any analysis on the effectiveness of corporate financial policy should allow for some uncertainty regarding the underlying model. Therefore, this thesis comprises three essays, with each focusing on a particular type of uncertainty. Chapter 2 studies a firm's choice of financing methods facing two layers of uncertainty in a world where agents follow Prospect Theory preference. Chapter 3 and 4 focus on dynamic corporate risk management policy given unravelling uncertainty in model parameters. The uncertainty in Chapter 3 is about the correlation between a firm's cash flow and hedging asset, while in Chapter 4 the uncertainty is in the firm's profitability.

Recently, successful attempts have been made applying Prospect Theory, proposed by Kahneman and Tversky (1979) and advanced in Tversky and Kahneman (1992), to asset pricing theories. For example, Barberis and Huang (2008) show that the theory could help price skewness and thus shed light on the equity premium puzzle. Yet its application to corporate finance is rather under-studied, especially in the field of financing decision theory. Chapter 2 presents the first attempt to reconcile the ongoing debate on the distinct choices in corporate financing methods in bull and bear markets using Prospect Theory preference.

Both of the two classic theories on corporate financing decisions, namely trade-off theory and pecking order theory (Myers, 1984), are confronted with counter-evidence. Fama and French (2002) find more profitable firms are less levered, which contradicts trade-off theory. Both Jung et al. (1996) and Frank and Goyal (2003), however, find that firms issue equity when they should issue debt as predicted by the pecking order theory. Although modern research using more complicated dynamic models, such as Goldstein et al. (2001), Hennessy and Whited (2005), and Gomes and Schmid (2010), are able to explain certain stylised facts, their specific model structures cannot offer a general model that could accommodate all the puzzles.

This gap in the literature motivated the analysis discussed in Chapter 2. In this analysis, I show that facing uncertainties in the characteristics of a firm and the proportion of each type of firm, a simple, static model based on Prospect Theory can shed light on many stylised facts or puzzles that are not yet resolved.

Methodologically, I develop a parsimonious game-theoretic model in which adverse selection concern is involved. In this model, a firm knows its type, either 'good' or 'bad' as in Myers (1984) or 'risky' or 'safe' as in Brennan and Kraus (1987). Additionally, external investors must learn the information from observing the firm's choice of financing method. In a Prospect Theory world, all the economic agents tend to overweight small probability events and underweight moderate to large probability events. This distortion is more severe when multiple layers of uncertainty exist, as presented in this essay. Alternatively, the distortion in perceived probability changes with the aggressiveness of individual investors, which is governed by the parameter

γ in the probability weighting function proposed by Tversky and Kahneman (1992). As suggested by much empirical research, this parameter fluctuates with the bull – bear market cycle. As a result, subtle and complex trade-offs in the model set-up give rise to multiple equilibria and yield many interesting implications.

I find that both the original equilibria in Myers (1984) and Brennan and Kraus (1987) are no longer upheld in this world. The single equilibrium they found breaks down into a spectrum of multiple equilibria or a combination of both a single equilibrium and multiple equilibria. I find support for the stylised fact that firms tend to issue more equity securities than debt in a bull market, and vice versa in a bear market. This finding results from firms' choice of when to pool together to hide information from external investors and when to separate from each other in order not to further frustrate them. Firms with more lottery-like investment opportunities would more easily raise external capital as both parties in the financing game perceive them to be more valuable. Consequently, positive NPV projects might be forgone and negative NPV projects might be carried out. Market value in the short-term could thus be distorted; what is worse, this distortion could be enhanced by self-fulfilment leading to a larger value correction in the future when the uncertainty is resolved. These consequences invite interesting future research along this path.

We study the effect of uncertainty, especially the dynamic effect resulting from its resolution, more explicitly in Chapter 3 and 4[1]. We concentrate our focus on corporate risk management policy, in which the whole emphasis is on uncertainty by its very nature. Academics have put tremendous efforts into understanding why and how a firm should manage its total risk. From a classic point of view, firms should leave the trouble to individual owners, as they could diversify away the idiosyncratic part of risk while the systematic part of risk should be fairly priced in the market.

[1]co-authored with Dr. Andrea Gamba

In this regard, corporate risk management will not create any value and therefore firms should neither hedge using derivatives, nor retain cash for precautionary purposes. However, Guay and Kothari (2003)'s data show that 57% firms world-wide routinely use derivatives . Bates et al. (2009) also documented an average cash ratio of 23.2% in 2006, an increase from 10.5% in 1980, mostly for precautionary motive. This empirically derived evidence is at odds with classic corporate risk management theory.

Advanced theories on corporate risk management have identified several possible explanations for the observed heavy use of corporate risk managing tools. Smith and Stulz (1985) showed that both the purpose to reduce expected costs of financial distress and the purpose to lower the expected taxes in the presence of a convex tax schedule could provide incentives for firms to actively manage their risk. Froot et al. (1993) added that corporate risk management could prevent firms from costly external financing costs by better aligning internal cash flow with a new project's financing needs. Thus, convex structures underlying their cost functions justify the use of risk management devices in order to maximise a firm's value. Nevertheless, while confirming the substantial usage of derivatives and cash, recent empirical studies find mixed evidence on a firm's value created by corporate risk management. One school of scholars find supporting results, such as Allayannis and Weston (2001), Graham and Rogers (2002), Carter et al. (2006), MacKay and Moeller (2007), and Campello et al. (2011). Another school, however, disagree with this observation; their results show either non-significant evidence or even value-destroying evidence (Brown, 2001; Guay and Kothari, 2003; Jin and Jorion, 2006; Magee, 2009; Bartram et al., 2011).

Another puzzle in corporate risk management is that traditional corporate risk management theory predicts young firms, which are usually associated with more risk, should make greater use of risk management tools than mature firms. In contrast, the observed hedgers are usually large firms (Nance et al., 1993). Moreover, hedging increases with firm size (Graham and Rogers, 2002). Bartram et al. (2009) also find that there is little relevance between young firms' amount of hedge and financial distress.

We note that these theories do not leave room for model uncertainty. This missing piece is indeed helpful in explaining certain documented stylised facts in corporate finance, especially in a dynamic fashion. For example, Moyen and Platikanov (2013) develop a model under a dynamic Tobin's q framework in which a firm's quality is uncertain. Shareholders update their belief based on the quality. These authors find that firms involving more uncertainty are more sensitive to earnings when they make investment decisions. Décamps et al. (2015) introduce uncertainty into a firm's profitability and find that both target cash ratio and dividend payout ratio increase with the firm's profitability prospects. Uncertainty in profitability is also investigated in DeMarzo and Sannikov (2016), based on a dynamic moral hazard model. They find that young and mature firms choose distinct policies when terminating investment projects and issuing dividend payouts.

The methodology employed in Chapter 3 and 4 is different from that in Chapter 2, due to its dynamic nature. Since we focus our research question on the relationship between the evolution of corporate risk management policy and the resolution of model uncertainty, a dynamic risk management model is an appropriate selection. Mello et al. (1995) initiate this strand of literature. They find that corporate hedging policy endogenises with flexibility in production. This structural relationship is also greatly affected by agency costs. This methodology subsequently becomes popular and has achieved great success ever since[2]. Our model most closely relates

[2]with notable studies such as Gamba and Triantis (2008); Rampini and Viswanathan (2010); Bolton et al. (2011); Rampini and Viswanathan (2013); Bolton et al. (2014); Gamba and Triantis (2014);

to that outlined by Gamba and Triantis (2014). We set off from a very parsimonious model, where shocks to a firm's cash flow and to the price of the underlying asset of a derivative are governed by a risk-neutral, multivariate autoregressive process. Firms could retain cash from realised cash flows. The return on internal cash is lower than the risk-free rate, resulting in costly cash reserves. Firms are allowed to hedge their cash flow risks using derivatives in the face of external financing costs, as the two variables are correlated. In Chapter 3, we assume there is uncertainty in the correlation, whereas in Chapter 4 the long-term mean of a firm's cash flow is assumed to be uncertain.

As mentioned previously, the meaning of managing risk with hedge relies crucially on the correlation between hedged and hedging assets. Thus, any analysis of the effectiveness of corporate risk management should be based on the assumption of this correlation, which we later refer to as 'hedgeability'. In Chapter 3, we study the case in which the correlation is disguised with some uncertainty. Agents do not know the true value of correlation parameter but could learn from observing realised cash flows and price movement of the underlying asset of the derivative. We find that resolution of uncertainty in hedgeability accelerates the process of building up hedging positions. This acceleration is magnified when cash reserves are also allowed in the model. This might help answer the open question why young firms hedge less than the mature ones. While we show that both hedging and cash reserves are indispensable as a risk management tool, the relationship between them could be both substitutionary and complementary. Moreover, we show that despite significantly different choices in risk management tools, hedge or cash, the difference in firm value could be very small under reasonable assumptions. The value created by managing risk might be lost in the decreasing value of a default option. This finding is consistent with empirical research, where they document the extensive use of hedge and

Rampini et al. (2014); Nikolov et al. (2015)

cash but show mixed evidence in firm value creation.

Inspired by an emerging strand in the literature on uncertainty in profitability, Chapter 4 studies corporate cash holding policy and hedging policy in the presence of resolving uncertainty in a firm's profitability. We find that both the firm's hedging level and cash balance increase with improved profitability prospects. However, low profitability firms have different risk management policies than those of more profitable ones. This is because optimal corporate risk management policy results from the trade-off between maximising the value created by risk management and minimising losses in the value from the default option. In addition, we find that resolution of uncertainty in profitability enhances the substitutionary effect between cash reserves and hedging, although a complementary effect co-exists.

Finally, Chapter 5 concludes with a discussion of the findings and potential contribution of this thesis, and discusses future research areas.

Chapter 2

One Approximation of Real-world Corporate Financing Decisions

2.1 Introduction

Ever since the axiomatization of the Expected Utility Theory provided by Von Neumann and Morgenstern (1944), the manner in which individuals make decisions under uncertainty has long been in debate. Some "paradoxes" and anomalies, which are inconsistent with the Expected Utility Theory, have gradually been uncovered in both experiments and the real world. In the meanwhile, remarkable progress has been made in exploring non-expected utility theories, such as subjective expected utility theory (Savage, 1954), prospect theory (Kahneman and Tversky, 1979), maximax expected utility theory, minimum regret theory (Loomes and Sugden, 1982), rank-dependent expected utility theory (Quiggin, 1982, 1993), Choquet expected utility theory (Schmeidler, 1989), and maximin expected utility theory (Gilboa and Schmeidler, 1989). All of the above are used to explain the "anomalies" which have been difficult to reconcile in the classic expected utility theory. Within them, the prospect theory proposed by Kahneman and Tversky (1979) (and later upgraded to the cumulative prospect theory in Tversky and Kahneman (1992)) has received the most attention because of its capacity to capture a large variety of documented anomalies, its mathematical tractability, its close relation with the traditional expected utility theory and its compatibility with other non-expected utility theories such as maximin utility theory, which could enable the compound theory to explain ambiguity aversion. It therefore introduces us a whole new framework in explaining individual's decision process and yields more realistic predictions across many disciplines. This paper examines its particular application in finance. Notably, although some ground-breaking progress has been made in asset pricing theory, Barberis and Huang (2008) - for instance, proving that the combination of cumulative prospect theory and capital asset pricing model (CAPM) may also price skewness and so shed light on the negative average excess return puzzle and the equity premium puzzle - application in the corporate finance field is rather under-theorised, especially in the realm of financing decision theory. This paper hence seeks to fill this gap found in earlier work on financing decision making.

Based upon established prospect theory, cumulative prospect theory conquers the weakness of violating first order stochastic dominance by weighting cumulative probability density function instead of probabilities (of individual outcomes). In this sense, (cumulative) prospect theory is a generalisation of expected utility theory insofar as it describes individual decision-making behaviour. On the other hand, game theory deals with decision-making when there is interaction among individuals. In particular, contract theory, based upon game theory and also known as mechanism design, contract economics or contract design, is being used more frequently in explaining financing decisions of firms.

Traditionally, trade-off theory and pecking order theory (Myers, 1984) are two dominant static theories used in explaining firms' financing behaviour. Trade-off theory asserts that a firm has its optimal level of leverage, which is set by balancing the marginal benefit and cost of debt. Pecking order theory argues that managers have superior information on the qualities of their firms in comparison to outside financiers. Knowing this, outside financiers will underprice securities issued by good firms in order to subsidise their loss on the overpricing of securities issued by bad firms. Pecking order theory also argues that overpricing is less for debt than for equity. Thus, equity is the last resort when they need outside financing.

However, these two theories have been confronted with a large numbers of challenges and struggle to explain well-documented stylised facts in firms' financing decisions. For example, the trade-off theory predicts a positive relation between profitability and leverage, which is inconsistent with findings in a large body of empirical research (Titman and Wessels, 1988; Rajan and Zingales, 1995; Fama and French, 2002). In the meanwhile, pecking order theory predicts that debt always dominates equity when firms need external financing. Nevertheless, Jung et al. (1996) and Frank and Goyal (2003) find that firms do issue equity when they are, according to pecking order theory, supposed to issue debt. Fama and French (2005) confirm that firms issue more equity than debt even after controlling for survivorship bias. Moreover, Brennan and Kraus (1987) have identified a theoretical flaw in pecking order theory by suggesting that while debt is less information-sensitive and thus favourable under first-order-stochastic-dominance circumstances (with respect to possible distributions of firm earnings), equity could be less information-sensitive and more efficient than debt when the distributions of different firms' earnings are of second-order-stochastic-dominance (mean-preserving).

Agency models, along with some theories later developed from the contract theory, incorporate information asymmetry into the study on firms' financing decisions and explains some of the "puzzles". For example, In the fundamental Jensen and Meckling (1976) paper, firms issue both debt and equity to minimize the agency costs of these two securities. Biais and Casamatta (1999) provide more formal proof and completely endogenise contractual terms. Brennan and Kraus (1987) consider both the first order stochastic dominance case and mean-preserving spread case to find differential strategies in mitigating information asymmetry.

Most recently, some other attempts have been made to reconcile these issues. The most prominent stream might be dynamic modeling proposed by Fischer et al. (1989) and advanced by Leland (1994), Goldstein et al. (2001), Hennessy and Whited (2005) and Gomes and Schmid (2010). Although these studies have largely improved the explanatory power of the above, one research question has nonetheless been left open, which this paper aims to answer; that is, are we able to explain these documented stylised facts with a simple static model? Before we largely increase the complexity along the path of making model dynamic and thus lose a great amount of tractability which made classic theories, such as the trade-off theory and the pecking order theory, widely spread and inspired future research. Another path is to maintain the tractability of a static model but relax certain assumptions to make them more realistic, which has been proven successful in the above-mentioned literature, especially in the asset pricing realm. Therefore, this paper probes the possibility of combining (cumulative) prospect theory and the contract theory in explaining the financing decisions of firms. This is the theoretical motivation of this paper.

This paper starts with some simplified examples. Under the expected utility hypothesis, there is both a unique pooling equilibrium and a continuum of separating equilibriums, reconfirming the finding (which is inconsistent with the pecking order theory) in Brennan and Kraus (1987). In the unique pooling equilibrium, only equity is issued, whilst in the continuum of separating equilibria, Safe-type (S-type) issues only equity and Risky-type (R-type) issues any combination of debt and equity along the financier's break-even line, due to there being a competitive capital market. Moreover, all the securities are fairly priced. In contrast, under the CPT hypothesis, these equilibria no longer hold true. Moreover, equilibria under CPT depend on market conditions. Under circumstances featuring high-prospect (low success rate but high payoff) projects, we show that there *only* exists a continuum of pooling equilibria, whilst under circumstances with safer but low payoffs, we find *only* a continuum of separating equilibria.

More importantly, in neither situation are the equilibrium securities fairly priced. In a market with high prospect projects, both equity and debt are overpriced, resulting in excessive approvals for negative NPV investment opportunities. Consequently, when aggregated, these investment opportunities yield ex post negative average return. The interpretation to be made is that economic agents care more about the small probability events associated with very high payoff (good prospects) and thus assign a premium for positively skewed (lottery-like) investment opportunities and tradeoff average return for those good prospects. This implication is consistent with observations in real-world capital market and echoes in the pricing for skewness theory (Kraus and Litzenberger, 1976; Harvey and Siddique, 2000).

In a market with low prospect projects, both equity and debt are underpriced. Consequently, it predicts excessive rejections for positive NPV investment projects, exhibiting pessimism in the market. In a very recent paper by Banerjee et al. (2016), they prove and verify in data that firms with more promising future would accept underpricing and issue equity. They also find that these firms issue equity earlier than their followers, leading a hot issuing market. Thus their issuances usually happen before market steps out low prospect status. In this regard, my paper resonates with their findings and extend their prediction one step further - the leaders issue underpriced equity in the market when the market condition is easier for them to signal their type. Considering The prediction is also consistent with real-world capital market observations documented and analyzed by Polk and Sapienza (2009), Shleifer and Vishny (2010) and Adjei (2011). Interestingly, this aggregate pessimism is created by a collective of lottery-loving and ambitious individuals in this paper. Overall, the model yields distinct equilibria under different market conditions; entrepreneurs would pool together to exploit financiers when systematic risk is high, whilst separating themselves so as not to further upset financiers in order to get funding when systematic risk is low.

We will later formalise the proof and extend the model into continuous types. Furthermore, we will show that this model is able to shed light on liquidity provision in primary market, implying that there is more equity issuance in a market with higher prospect projects whilst more debt issuance in a market with lower prospect projects.

In summary, the main contribution of this paper lies in its initiative to break the ice between classic capital structure theories and innovative decision-making theories. As illustrated in this research, this attempt is meaningful because we show that a large number of prevailing anomalies in the real-world capital market could be reconciled in the hybrid model without additional assumptions, which are crucial and artificial in some specific theories' attempt to explain one particular anomaly in the financial market. Moreover, the importance of this paper could also be that it is thought-provoking in two aspects. Theoretically, it suggests theory researchers that the interdisciplinary hybrid of classic capital structure theories and innovative decision-making theories might be fruitful, less assumption-dependent and thus promising in its future. Empirically, although preliminary and tentative, the abundant implications generated in this research might interest empirical researchers to bring this general model into specific tests.

The following of this proposal is structured as: Section 2 reviews related literature. Section 3 introduces a simple binary model as an example. Section 4 formalises and extends the findings. Section 5 involves a preliminary generalization and discussion of the results. Section 6 presents a conclusion.

2.2 Literature Review

Other than the literature that has motivated the birth of this research, as mentioned in Section 2.1, remarkable progress has been made over the past decades in exploiting non-expected utility theories to explain "anomalies" which are otherwise unexplainable by classic economic or financial economic theories. For example, Barberis and Huang (2008) justify investors' taste for skewness focusing on the probability weighting component of the cumulative prospect theory proposed by Tversky and Kahneman (1992). Garlappi et al. (2007) integrate aversion to ambiguity to portfolio selection model and render portfolios with a higher Sharpe ratio.

In the meantime, literature on contract theory (also known as mechanism design or contract economics) has grown and gained more attention. Based on game theory, it has proven powerful in solving problems with limited participants and when information asymmetry looms large. As a standard principal-agent relationship, a financing decision and thus the capital structure is investigated equipped with the contract theory. In the fundamental Jensen and Meckling (1976) paper, firms issue both debt and equity to minimise the agency costs of these two securities. Biais and Casamatta (1999) give more formal proof and completely endogenise contractual terms. Brennan and Kraus (1987) consider both the first order stochastic dominance case and mean-preserving spread cases and find differential strategies in mitigating information asymmetry.

Only until very recently have successful attempts been made in merging the above two strands of literature. Dittmann et al. (2010) combine loss aversion with a principal-agent model to derive the convex optimal contract for CEO compensation. Spalt (2013) exploits the key feature of probability weighting to justify the attractiveness of stock options in employee's pay package. Koufopoulos and Kozhan (2014) study a competitive insurance market with both asymmetric information and ambiguity and prove that an increase in ambiguity could relax the high-risk type agents' incentive compatibility constraint and thus lead to a strict Pareto improvement under certain circumstances. Nonetheless, no progress has yet been achieved in utilising the cutting-edge methodology to study the financing decisions of firms.

Therefore, one major contribution of this paper is to fit into the existing literature through its tentative reconciliation of these two theories in explaining financing decisions.

2.3 Example

To introduce a new paradigm - namely, the combination of the (cumulative) prospect theory and the contract theory - I will start progressively from the very beginning. Thus in this section, I will first introduce a binomial, one-period model, as an example. Although parsimonious, it is typical and commonly used in contract theory problems and suffices to illuminate the main conclusion of this paper. Besides, this model setting is a simplified version of Brennan and Kraus (1987), where payoffs are mean-preserving. I start from this model because it is easier to observe how CPT moves the indifference curves in EU, and then discuss some generalisation later. In addition, some results in Brennan and Kraus (1987) are paraphrased and then compared with those under cumulative prospect theory.

Some preliminary findings from this model suggest that, there would firstly be distinct equilibriums under EU and CPT. Secondly, even within CPT, there would be distinct equilibria under different market conditions. Thirdly, investors exhibit aggregate optimism in a market with high prospect projects, which is defined as projects with large payoffs but small success probability as in Barberis and Huang (2008), and aggregate pessimism in a market with low prospect projects, which refer to projects with moderate payoffs but also moderate to high success probability, although the parameters governing their preference remain the same in both scenarios. At last, in equilibrium, entrepreneurs would exploit financiers by hiding their private information (pooling) under high prospect market conditions. They would not want to upset financiers further under low prospect market conditions, taking a risk of losing external financing opportunities. Thus, they would rather voluntarily reveal their types truthfully and separate themselves from the other type. In the next section, this model will be formalised and generalised in a more rigorous fashion, by allowing both the payoffs and the types to be distributed continuously.

2.3.1 In the classic expected utility setting

Consider the following simple investment problem where all economic agents make decisions under expected utility theory (Bolton and Dewatripont, 2005): Assume there are three periods but no discounting for simplicity. And there are two types of projects: Safe ($i = S$) and Risky ($i = R$) with proportion λ and $1 - \lambda$, both require an investment of I. At time 0, each entrepreneur (principal) has a project and learns its type but this information is private. The financier (agent) has common knowledge on the proportion of each type and the investment technology. At time 1, entrepreneur has no fund available thus has to raise I by issuing either debt (D) or equity (in proportion α) or a combination of both, denoted by $Z = (\alpha, D)$, with $0 \leq \alpha \leq 1$ and $D \geq 0$. Debt is zero-coupon bonds and senior to equity. Entrepreneur is protected by limited liability. The capital market is competitive. At time 2, safer project generates a cash flow of X_S with probability p_S and 0 otherwise, whereas risky project generates X_R with probability p_R and 0 otherwise. Both projects have the same positive NPV, i.e., $NPV_R = p_R X_R - I = p_S X_S - I = NPV_S > 0$ or $p_R X_R = p_S X_S$ (mean-preserving). Moreover, $X_R > X_S > 0$ and $p_S > p_R$. Both principal and agent are risk neutral. The time line is graphically illustrated as in Figure2.1.

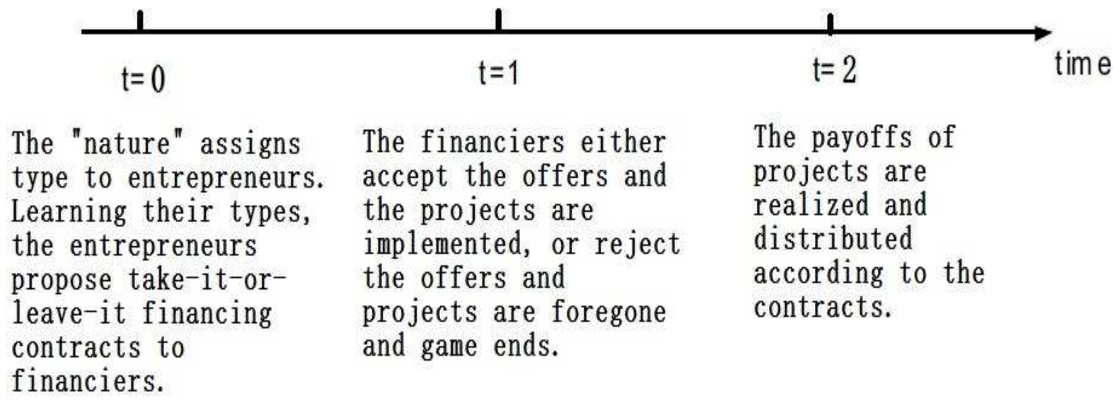


Figure 2.1: Time Line

Firstly, as noted by Brennan and Kraus (1987), we find that *In a pure strategy game, when both principal and agent follow the expected utility theory and are risk-neutral, there is a unique pooling equilibrium where only equity is issued. And the issued equity is fairly priced.*

While readers can find detailed discussion in Appendix 2.6.1, the intuition relevant here is that, for entrepreneurs solving their expected utility maximization problem:

$$\max U_i = \max_{(\alpha,D)} p_i(1-\alpha)(X_i - D), \qquad i = R, S$$

We have the marginal substitution rate of equity to debt given a certain level of utility:

$$(\frac{d\alpha}{dD})_{U_i = \bar{U}} = -\frac{1-\alpha}{X_i - D}$$

The intuition here is that the R type entrepreneur is willing to exchange more debt for a certain amount of equity than the S type entrepreneur because the project is riskier and thus the equity is more valuable (Figure 2.2). Since (D, α) are the parts taken out from entrepreneurs' pocket ("bads" rather than "goods"), the indifference curves closer to the origin will give entrepreneurs higher utility.

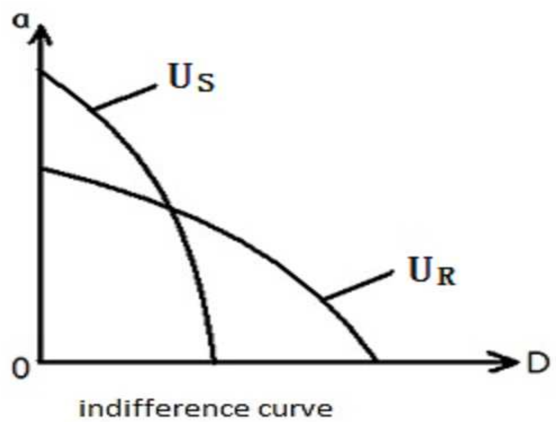


Figure 2.2: The Indifference Curve for the Entrepreneur

The indifference curves (ICs) takes the same shape of the zero profit lines (ZP_i)of financiers who solve their profit maximisation problem under competitive capital market assumption:

$$\max \Pi_i = \max_{(\alpha,D)} p_i[\alpha(X_i - D) + D)] - I, \qquad i = R, S$$

and the pooling zero profit line (PZP), when they cannot tell the types of entrepreneurs:

$$\Pi_{\text{PZP}} = \lambda p_{\text{R}}[\alpha(X_{\text{R}} - D) + D)] + (1-\lambda)p_{\text{S}}[\alpha(X_{\text{S}} - D) + D)] - I = 0$$

First, any equilibrium contract (α, D) should lie on one of the zero-profit lines because of perfect competition. For any deviation (e.g., B), the R type always wants to mimic the S type, because she will be better-off at any contract within ZP_R. However, the R type entrepreneur is willing to exchange more debt for a certain amount of equity than would the S type entrepreneur because of a higher marginal substitution rate. Thus, under the intuition criteria as refinement, the S type could credibly signal her type by making a counter-offer with more equity and less debt. This is because financiers know that it would only be favourable to the S type as it would make the R type entrepreneurs worse off. This would eliminate all the contracts on the pooling zero-profit line except for the only stable contract, Z^*, where no type has incentive to deviate and only equity is issued. Moreover, being that with this mean-preserving case, the equity is equally valuable to both types, then the equity is fairly priced.

We also find that: *There also exist a continuum of separating equilibria where the S type issues only equity, while the R type issues any combination of debt and equity along ZP_R. All the securities are fairly priced.*

The proof is similar and straightforward. Since this all equity financed contract gives the R type entrepreneurs the same level of utility as if the reveal themselves, they are thus indifferent between being pooled with the S type entrepreneurs and being separated and offered contracts on their own zero-profit line. Instead, the S type entrepreneurs will always stick to the all equity financed contract as any deviation will attract the R type entrepreneurs to mimic and be (strictly) worse-off.

2.3.2 In the prospect theory setting

Now let's shift to a world where both entrepreneurs and financiers make decisions according to Cumulative Prospect Theory (Tversky and Kahneman, 1992).

The selection of reference level is controversial. So far, the most widely accepted convention is to set the highest (lowest) outcome as the reference level and define gains and losses based on the relative difference from the highest (lowest) outcome. Nonetheless, this procedure will eliminate the advantage of loss aversion property in the (cumulative) Prospect Theory, degenerating to the rank-dependent expected utility theory (Quiggin, 1982, 1993). An alternative yet reasonable approach is to define the reference level as the initial level of wealth, or in more complex terms, as expected final wealth (Loughran and Ritter, 2002). This paper will examine the first approach, leaving the more sophisticated one to future research. It differs from Quiggin (1982) on how the probability weighting function is specified - I follow the one defined in Tversky and Kahneman (1992), which could be easily extended to full Cumulative Prospect Theory model (or part of its other aspects) in the future research.

Formally, entrepreneurs evaluate the gambles

$$\begin{aligned} & (0, 1 - p_{\mathrm{R}}; X_{\mathrm{R}}, p_{\mathrm{R}}) \\ \textit{and} \qquad & (0, 1 - p_{\mathrm{S}}; X_{\mathrm{S}}, p_{\mathrm{S}}) \end{aligned}$$

by assigning them the values

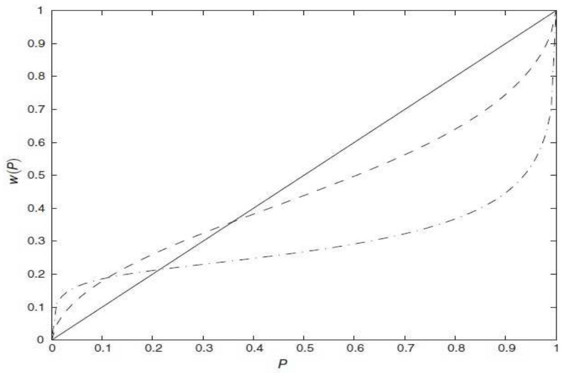
$$\begin{aligned} \pi_{\mathrm{R}} \cdot X_{\mathrm{R}} + \pi_{\neg\mathrm{R}} \cdot 0 \\ \pi_{\mathrm{S}} \cdot X_{\mathrm{S}} + \pi_{\neg\mathrm{S}} \cdot 0 \end{aligned}$$

where π takes the values

$$\begin{cases} \pi_{\mathrm{R}} = \omega^{+}(p_{\mathrm{R}}) \\ \pi_{\neg\mathrm{R}} = \omega^{+}(1) - \omega^{+}(p_{\mathrm{R}}) \end{cases} \qquad \begin{cases} \pi_{\mathrm{S}} = \omega^{+}(p_{\mathrm{S}}) \\ \pi_{\neg\mathrm{S}} = \omega^{+}(1) - \omega^{+}(p_{\mathrm{S}}) \end{cases}$$

$\omega^{+}(\cdot)$ is the cumulative probability weighting function [1], taking the shape

$$\omega^{+}(p) = \frac{p^{\delta}}{(p^{\delta} + (1-p)^{\delta})^{1/\delta}} \tag{2.1}$$



Notes: The figure shows the form of the probability weighting function proposed by Tversky and Kahneman (1992), namely, $w(P) = P^{\delta}/(P^{\delta} + (1-P)^{\delta})^{1/\delta}$. The dashed line corresponds to $\delta = 0.65$, the dash-dot line to $\delta = 0.4$, and the solid line to $\delta = 1$.

Figure 2.3: The Probability Weighting Function

The parameters are restricted to $\delta \in (0, 1)$. Particularly, it captures the curvature of the probability weighting function as illustrated in Figure 2.3.2.

Note that this model degenerates into a classic expected utility model if $\delta = 1$, indicating that this model setting nests both the expected utility theory and the rank-dependent expected utility theory. Future research will confront the model with other properties, such as loss aversion.

In terms of simplicity, we will only consider homogeneous economic agents in this paper whose perceptions about the objective probabilities are identical. In other words, their mentally perceived "probabilities" deviate from the true ones in the same direction and by the same magnitude. Unlike entrepreneurs, who face only one tier of uncertainty, which is the success rate of the projects, as entrepreneurs, financiers face two tiers of uncertainties (compound gamble) since they know neither

[1] for the gains, however the paper restricts the discuss in the gain quadrant of Cumulative Prospect Theory as in Barberis (2012). The last section discusses more general case.

the success rate nor the type of those projects:

$$\begin{cases} & (0, 1-p_{\text{R}}; X_{\text{R}}, p_{\text{R}}) \\ \textit{and} & \begin{cases} (0, 1-p_{\text{S}}; X_{\text{S}}, p_{\text{S}}) \\ (R\ type, \lambda; S\ type, (1-\lambda)) \end{cases} \end{cases}$$

In the separating equilibriums, where each type could credibly signal itself and make financiers distinguish it from the other, thus offering distinct contract to each type, the second tier of uncertainty (on the types) is solved and this compound gamble will not cause a problem because it is reduced to a simple gamble, which is the same as that of the entrepreneurs. However, in the pooling equilibrium, the compound gamble does cause trouble because the reduction principle is violated under the (cumulative) prospect theory, entailing that the multiplicative property fails to hold, i.e., $\omega(\lambda p_R) \neq \omega(\lambda) \cdot \omega(p_R)$. Some researchers have tried to modify the probability weighting function and put some conditions to yield a similar decomposition as in the expected utility theory for certain classes of prospects (Stomper and VierÃÿ, submitted working paper). One more conventional approach follows Barberis (2012), where the cumulative probability weighting functions are only applied to the objective probability of final outcomes. In this paper, I will follow the latter approach because the payoff structure is clear that there is no need to obscure the main points with modified probability weighting functions.

We find that: *Under circumstances featuring high-prospect (low success rate but high payoff) projects, there is a continuum of stable pooling equilibria.*

Before discussing the pooling equilibrium, which located on the pooling zero-profit line as analysed above, let us define the separating zero-profit lines to facilitate further discussion. The separating zero-profit line of each type is given by

$$\Pi_i = \omega^+(p_i)[\alpha(X_i - D) + D)] - I = 0, \qquad i = R, S$$

Under the objective proportion, i.e. λ and $1-\lambda$, the actual pooling zero-profit line (a linear combination of the red lines) should be governed by

$$\Pi^{\lambda}_{PZP} = \lambda\omega^+(p_R)[\alpha(X_R - D) + D)] + (1-\lambda)\omega^+(p_S)[\alpha(X_S - D) + D)] - I = 0$$

However, in a feasible pooling equilibrium, the financier evaluates the gamble

$$(0, \lambda(1 - p_{\mathrm{R}}) + (1-\lambda)(1 - p_{\mathrm{S}}); X_{\mathrm{S}}, (1-\lambda)p_{\mathrm{S}}; X_{\mathrm{R}}, \lambda p_{\mathrm{R}})$$

by assigning it the value

$$\pi_{\neg\mathrm{R,S}} \cdot 0 + \pi_{P_S} \cdot X_{\mathrm{S}} + \pi_{P_R} \cdot X_{\mathrm{R}}$$

where π take the values

$$\begin{cases} \pi_{P_R} &= \omega^+(\lambda p_{\mathrm{R}}) \\ \pi_{P_S} &= \omega^+((1-\lambda)p_{\mathrm{S}} + \lambda p_{\mathrm{R}}) - \omega^+(\lambda p_{\mathrm{R}}) \\ \pi_{\neg\mathrm{R,S}} &= \omega^+(1) - \omega^+((1-\lambda)p_{\mathrm{S}} + \lambda p_{\mathrm{R}}) \end{cases}$$

$\omega^+(\cdot)$ is the same cumulative probability weighting function with that of entrepreneurs, Equation 2.1. And according to the game structure presented above, the financiers in Cumulative Prospect Theory world would first evaluate the event with high prospect, a large payoff associated with a low probability.

Due to the fact that probability weighting function is a monotonic transformation from the original utility function, the problem is much simplified; that is, the marginal rate of substitution remains the same. Since the shape of indifference curves does not depend on probabilities, as does the zero-profit line of financiers, the reasoning in previous subsection also goes through here straightforwardly.

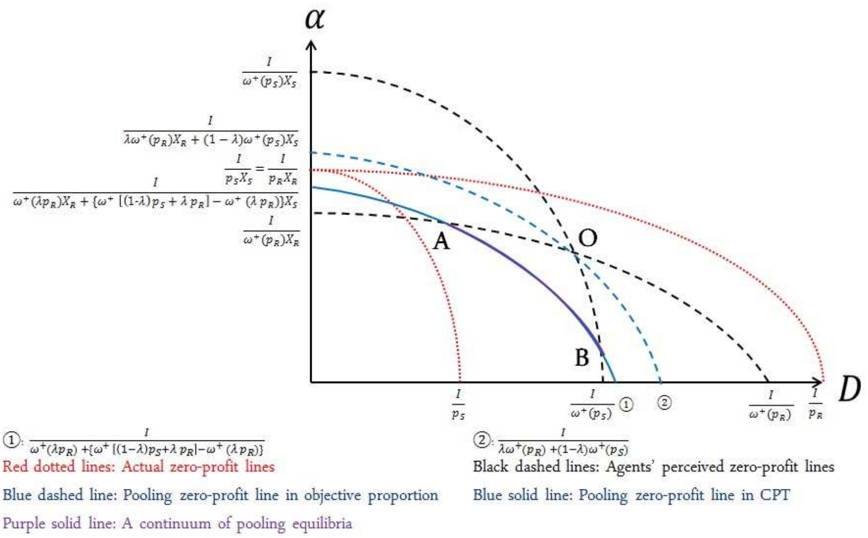


Figure 2.4: The Pooling Equilibrium When Market Is Risky

Given the above transformations, the (actual) mean-preserving model takes the form of the dashed black lines in Figure 2.4. The pooling zero-profit lines passing Point O is the traditional zero-profit line with additive proportion, while the pooling zero-profit line for the perceived (but actually mean-preserving) projects is the lower dashed blue line[2]. Following a similar reasoning as in Section 2.3.1, it is easy to find that the unique stable equilibrium (Point O) breaks down into a continuum of equilibria with weighted probabilities. The above is due to the fact that Cumulative Prospect Theory violates the reduction principle for compound gambles and makes the pooling zero-profit line not a linear combination of the separating zero-profit lines because the weights do not add up to one in the presence of multiple tiers of

[2]under circumstances featuring high-prospect projects. Appendix 2.6.2 discuss this in detail.

uncertainty. Mathematically,

$$\begin{aligned}\Pi^{\text{CPT}}_{\text{PZP}} &= \omega^{+}(\lambda p_R)[\alpha(X_R-D)+D)]+\{\omega^{+}[(1-\lambda)p_S+\lambda p_R]-\omega^{+}(\lambda p_R)\}[\alpha(X_S-D)+D)]-I \\ &\neq \omega^{+}(\lambda)\omega^{+}(p_R)[\alpha(X_R-D)+D)]+\omega^{+}(1-\lambda)\omega^{+}(p_S)[\alpha(X_S-D)+D)]-I\end{aligned}$$

We first note that calculated using the objective weights, i.e. λ and $1-\lambda$, the actual pooling zero-profit line (a linear combination of the red lines) passes the intersection of two perceived separating zero-profit lines. Therefore, the analysis of equilibrium under EU remains except that now the unique pooling equilibrium would be the point O in the middle, as no one wants to deviate at this point. We should also note that in Figure 2.4, the new pooling zero-profit line is below the actual pooling zero-profit line. This exists in an economy with high prospect projects. Specifically, the parameter combinations which satisfy the following inequality sustain the pooling equilibria

$$\frac{\lambda w^{+}(p_R)X_R+(1-\lambda)w^{+}(p_S)X_S}{w^{+}(\lambda p_R)X_R+(w^{+}((1-\lambda)p_S+\lambda p_R)-w^{+}(\lambda p_R))X_S}<1$$

The condition follows from the relationship between the intercepts of each pooling zero-profit line at $D=0$. We find that while the parameters interacts with each other and is difficult to give an explicit expression, in general a lower p_R and a higher X_R (which is defined as high prospect, or lottery-like, project in Barberis and Huang (2008)) gives more chance to sustain these pooling equilibria.

With details in Appendix 2.6.2, we show that the interval $\widehat{AB}$ are stable equilibria. The key difference in this game is that the contracts on the separated zero-profit lines will give to both types of entrepreneurs those utilities that are no higher than the utilities of the two extremes of the interval $\widehat{AB}$, contract A and B. In other words, any deviating strategy is (weakly) dominated by the strategies at

Point A/Point B. Therefore, they will survive the intuition criteria and it will be sustained as a stable equilibrium. For any one contract within this interval - say C - any deviation will, at most, benefit one type but not the other. Otherwise it is below all the zero-profit lines, which means financiers are loss-making and they will not accept this loss-making contract. Thus, by intuition criteria, financiers will be able to separate the types by the deviation and offer each type contracts located on their corresponding (separated) zero-profit line. Again, since the separating contracts are (strictly) dominated by the original pooling contract C, this deviation is not profitable. Thus, any contract within this interval could be sustained as a stable equilibrium constituting a continuum of stable equilibria.

Since the new pooling zero-profit line is below the actual pooling zero-profit line, a natural consequence is that, even the projects are of negative NPV under the expected utility theory and should not be carried out, i.e., the intersection with the α axis is above 1[3] and they will be undertaken under weighted probabilities.

In contrast, we also find that: *Under circumstances with safer but low-payoff projects, there is no pooling equilibrium but a continuum of separating equilibria.*

When the economy contains safer but low-payoff projects - for example, in the time period just after an economic crisis - this set of parameters will generate a perceived pooling zero-profit line higher than the traditional pooling zero-profit line with weights adding up to 1, as shown in Figure 2.5.

Different to the previous case, any contract within the interval $\widehat{CD}$ cannot

[3] In fact, a negative NPV project cannot be represented in this diagram, as it requires $\alpha \leq 1$. However, a negative NPV project under the expected utility theory might be a positive NPV project under the Cumulative Prospect Theory and could this be represented in this diagram. Therefore, for illustrative convenience, I assume these negative NPV projects do exist in this diagram with intersections on the α axis above 1

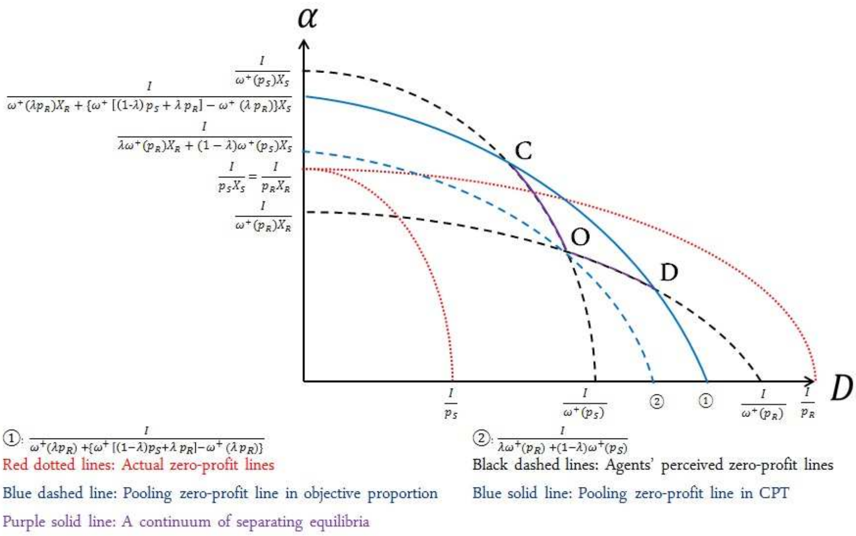


Figure 2.5: The Pooling Equilibrium When Market Is Less Risky

then be sustained as an equilibrium because it gives utilities lower than any contract could give on the separating zero-profit lines. In other words, any contract within the interval $\widehat{CD}$ is (strictly) dominated by separating contracts and cannot be sustained as an equilibrium.

Nonetheless, there is still a continuum of separating equilibria located on the whole outer boundary of the shape, comprising of the intervals $\widehat{OC}$ and $\widehat{OD}$ on the two separating zero-profit lines except for Point O. The above occurs because the entrepreneurs can credibly signal their own types as the strategy of mimicking the other type is too costly to be carried out. In other words, mimicking strategies are (strictly) dominated by separating strategies. For example, if the S type chooses contract C on its zero-profit line above Point O, then it is unreasonable for the R type to mimic the S type by also choosing contract C because this contract is above his own zero-profit line which means it will give her a lower level of utility than the contracts on her zero-profit line, e.g. contract D. Note that Point O is excluded because, at this point, financiers cannot distinguish the two types and pool them using the pooling zero-profit line higher than the traditional pooling zero-profit line with weights adding up to 1. This discontinuity is due to multi-layers of uncertainty in CPT. Since it is simply a mechanical result when the CPT is combined with the contract theory, it could be eliminated by defining a commensurate equilibrium refinement, which admits Point O as a separating equilibrium. Hence, we have proved that there is no pooling equilibrium.

It is also important to note that in Figure 2.5, the new pooling zero-profit line is above the actual pooling zero-profit line. In this scenario, even the projects are of positive NPV under the expected utility theory and should be carried out, i.e. the intersection with the α axis is below 1, they will not be undertaken under the cumulative prospect theory if the new pooling zero-profit intersect with the α axis above 1[4].

Overall, we find that in the world where individual behaviour conforms to prospect theory, the capital market would exhibit different types of equilibrium under different scenarios. In a market with high prospect projects, only pooling equilibria exist, with R-type and S-type choosing the same combination of certain proportion of equity and face value of debt to fund their distinct projects. In contrast, only separating equilibria exist in a market with low prospect projects, where R-type chooses a combination with more debt whilst S-type picks up a combination with more equity. In making a comparison of the difference here, the results should be interpreted as the following. Knowing the financiers are subject to two-tier uncertainty, the entrepreneurs enjoy an information rent of choosing favourable contracts when the prospect is high and they avoid unfavorable contracts when the prospect is low, resulting from their private information.

[4]Again, a negative NPV project cannot be represented in this diagram as it requires $\alpha \leq 1$. However, for illustrative convenience, I assume these negative NPV projects do exist in this diagram with intersections on the α axis above 1

2.3.3 Discussion on No-arbitrage Condition

One potential concern is that the established equilibria might fail to meet the no-arbitrage condition if investors or entrepreneurs are able to replicate their position and short-sell the replicating portfolio. Research on prospect theory documents that the boundary behaviour of the cumulative prospect theory is not well-defined, as individuals with prospect theory preferences would accept gambles with arbitrarily large negative expected values, triggering an infinite shot-selling problem (Azevedo and Gottlieb, 2012; Jin and Zhou, 2013). De Giorgi et al. (2010) further argue that the non-convexity of CPT preferences may lead to the non-existence of market equilibria, with even a non-negativity constraint being imposed on final wealth. However, as this paper is particularly interested in studying the application of CPT, specifically the probability weighting function, on corporate finance issues, and its implications towards the real-world capital structure decision makings, we restrict our discussion in the gain quadrant by setting the reference point 0, as it is under EU. Generally, we could relax this assumption by imposing non-negativity constraints on final wealth and the assumption of a continuum of agents in the market as in De Giorgi et al. (2010), or by imposing assumptions on priori bounds on leverage and/or potential losses as in Jin and Zhou (2013). Here we show that, in the currently available information set, we may at least find in our binary case model the existence of one set of pricing kernel that can correctly price all the assets and be positive.

As illustrated in Figure 2.6, the fair present value (PV) of external financing should be equivalent to I, indicating that, at any separating equilibrium:

$$\frac{q_{\alpha_t^*,D_t^*}(\alpha_t^*(X_t - D_t^*) + D_t^*) + (1 - q_{\alpha_t^*,D_t^*}) \cdot 0}{1 + r_f} = I,\ t \in \{R, S\}$$

Rearrange, we have:

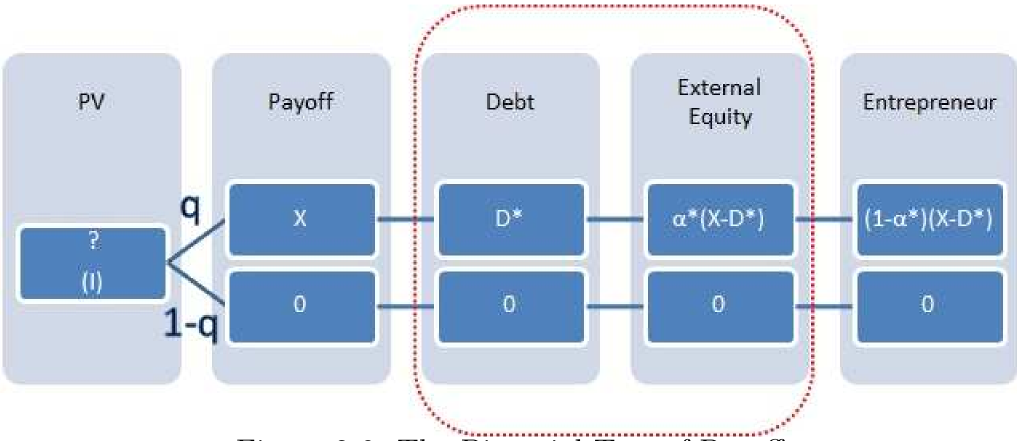


Figure 2.6: The Binomial Tree of Payoffs

$$q_{\alpha_t^*, D_t^*} = \frac{(1 + r_f)I}{\alpha_t^*(X_t - D^*) + D_t^*},\ t \in \{R, S\}$$

which are the sets of pricing kernels for both types of project and the real options (external debt/equity) written upon them. The limited liability and positivity condition ensures that both sets of pricing kernels are positive, implying no-arbitrage opportunity. The CPT enters the pricing kernels through the equilibrium level α^* and D^* in a more complicated way, but this is not one of the main interests of this paper.

In the same fashion we could find a set of pricing kernels at any pooling equilibrium, and upon the currently available information set, it satisfies the no-arbitrage condition.

2.4 Formal Model Setting and Assumptions

We consider a model where an entrepreneur is endowed with a technology that generates future stochastic earnings $x \in X \equiv [0, K]$, and requires $I > 0$ units of capital as input. We allow for unbounded future earnings by letting K go to infinity. To finance his project, the entrepreneur can seek funds from competitive financiers, each of whom is endowed with equal amount of capital I. To make our model sufficiently general to compare with cases under CPT, we assume all agents risk averse, with the same utility function $U(\cdot)$. We normalize the risk-free rate to zero. There are two types of projects (entrepreneurs), $t \in T \equiv \{R, S\}$[5]. The types differ according to their distribution of earnings. The cumulative distribution function (cdf) over X for a type t project is $F_t(x)$. The project type is the private information of the entrepreneur. Outside financiers only know that a fraction $\lambda_R \in (0,1)$ are type R projects, and a fraction $\lambda_S = 1 - \lambda_R$ are type S projects. All projects have positive net present value, and the firm's assets in place are assumed to be zero.

Denote by $\mathbb{E}_t(x) = \int_o^\infty U(x)\,\mathrm{d}F_t(x)$ the full information expected utility of a type t project. We make the following assumptions:

Assumption 1. $\mathbb{E}_t(x) \geq I > 0$ *for every* $t \in T$.

Assumption 1 says that a financier can fund a project, and all projects have positive net present value. In addition, we make the following standard assumptions on the distributions of earnings:

Assumption 2.

1. *The cdfs are mutually absolutely continuous;*

2. *Securities are risky:* $F_t(I - \epsilon) > 0 \ for \ all \ t \in T, \ and \ for \ all \ \epsilon > 0$*;*

Continuity is assumed for technical reasons. Being that our problem is interesting only if the investment might be loss making with positive probability (i.e. it is impossible to issue riskless securities), then point (2) of 2 ensures that is the case.

The timing of the game is the same as we discussed in Section 2.3. A financing contract is either debt (D) or equity (in proportion α) or a combination of both,

[5]Accurately, the types here are not about riskiness any more. Rather, they should be denoted as l,h, low or high in productivity. However, we will keep the denotation for consistency. This binary-type case is, in fact, relaxed in the following research

denoted by $z \in Z \equiv \{(\alpha, D)\}$. The only restriction we impose on the contract space is that each security must satisfy limited liability, as:

Assumption 3. $Z \equiv \{(\alpha, D) | 0 \leq \alpha \leq 1, D \leq x\}$

Finally, denote by U_t the expected utility of an entrepreneur of type t, and by U_f the financier's expected utility. We can then write

$$
\begin{aligned}
U_t &= \mathbb{E}_t[U((1-\alpha)(x-D))] \quad (2.2) \\
U_f &= \mathbb{E}_{\lambda(z)}[U(z)] \quad (2.3)
\end{aligned}
$$

The expectation in 2.3 is given by the sum across types (weighted by the posterior belief $\lambda(t|z)$ that type t is issuing the contract z) of the final payoff of the securities prescribed in contract z:

$$
\mathbb{E}_{\lambda(z)}[U(z)] \equiv \sum_{t \in T} \lambda(t|z) \int_{x \in X} U(\alpha(x-D)+D)\, \mathrm{d}F_t(x) \quad (2.4)
$$

2.4.1 The Properties of Equilibria with Weighted Probability

In this subsection we will probe further into what the equilibria can tell us, and what is the difference between equilibria under two decision-making theories. Moreover, this difference might help explain current real-world anomalies. In the following section, I firstly show that there is an ordinal relationship between the equilibrium(s) under EU and those with probability weighting.

Lemma 2.4.1 (The Ordinal Relationship of Equilibrium(s))**.** *Under circumstances featuring high prospect projects, given an equilibrium-level equity issue under EU theory, if there is also an equilibrium under weighted probability with the same equity* *issuance, the pairwise debt issuance with weighted probability must be lower than that under EU.*

Proof. Suppose an equilibrium contract $z^* = (\alpha^*, D^*)$ under EU. It should satisfy the pooling zero profit condition:

$$\int_{x \in X} U(\alpha^*(x - D^*) + D^*)\, \mathrm{d}F_{X,T}(x,t)) = U(I)$$

Here I is also known as the certainty equivalent (CE) over the domain $X \times T$.

However, the feature of probability weighting function in the gain quadrant, $\omega^+(\cdot)$, is to overweight the small probability event and underweight the large probability event. As discussed in previous section, when the prospect is high, the small probability is associated with a large payoff and this creates a first order stochastic dominating distribution over the objective one. We should also note that both utility functions in the gain quadrant are concave; thus Jensen's inequality holds for both of them. Therefore, the CE under CPT is higher than the required investment when the *prospect* is evaluated at this particular contract z^* :

$$\begin{aligned} CE_{CPT} &= \nu^{-1}(\int_0^\infty \nu(\alpha^*(x - D^*) + D^*)\, \mathrm{d}\omega^+(1 - F_{X,T}(x,t))) \\ &> I \end{aligned}$$

where $\nu(X)$ is the value function in CPT and $\nu(X) = X$ in this paper. As a result, for the participation constraint to hold in the equilibrium under CPT, for the same level of α^*, $D^*_{CPT} \leq D^*$. More generally, the equilibria with weighted probability is closer to the origin under any Lebesgue measure defined on Z. □

This property is due to the underlying mechanism that the multiplication rule fails under CPT. Thus $\omega(\lambda p_R) \neq \omega(\lambda) \cdot \omega(p_R)$, distorting the objective joint distribution $F_{X,T}(x,t)$. The case of separating equilibria are trivial, as the pooling equilibrium, if exists, would be preserved as a separating equilibrium under an equilibrium refinement method which is consistent with CPT.

2.5 Conclusions and Limitations

In this paper, we aim to show that when market participants are lottery-loving and ambitious - in the sense that they care more about the small probability events associated with very high payoff (good prospect) - then unique equilibrium under EU breaks down into multi-equilibria, which might contribute to the debate of not finding an optimal leverage. We find that collectively, firms' financing decisions under prospect theory depends on market conditions. In a market with high prospect projects, there *only* exists a continuum of pooling equilibria, whilst in a market with low prospect projects, we find *only* a continuum of separating equilibria. These are distinct from the equilibria under the expected utility hypothesis, where both a unique pooling equilibrium and a continuum of separating exist(Brennan and Kraus, 1987). The interpretation to be given here is the following. Entrepreneurs would pool together to exploit financiers under circumstances featuring high prospect projects, whilst separating themselves so as not to upset financiers further and obtain funding when the economy consists of safer and low-payoff projects.

We also show how the model could justify a lower use of debt when the prospect is high, potentially offering an alternative approach in explaining fluctuations in liquidity provisions in the primary market. Hence, the model might be able to capture the stylised fact that more equity issuances are observed in a market with high prospect projects, whilst more debt issuances are observed in a market with low prospect projects.

Besides, on one hand, the negative NPV investment opportunities might be undertaken, providing a negative average return when aggregated. The interpretation to be given here is that the economic agents assign a premium for positively skewed (lottery-like) investment opportunities. These implications are consistent with real-world capital market observations, such as pricing for skewness in the capital market (Kraus and Litzenberger, 1976; Harvey and Siddique, 2000; Mitton and Vorkink, 2007; Fu, 2009; Boyer et al., 2010).

On the other hand, even the positive NPV investment opportunities could be foregone. This happens when the prospect is low; for example, in the time period just after an economic crisis. This prediction is consistent with real-world capital market observations (Polk and Sapienza, 2009; Shleifer and Vishny, 2010; Adjei, 2011), where the adoption of positive NPV investment opportunities are more closely related to credit squeeze. Other behavioural studies introduce another assumption of over-pessimism to explain the adoption of positive NPV investment opportunities. Interestingly, this paper offers an alternative explanation even when individuals are still lottery-loving and ambitious.

An interesting future attempt is to allow the preference of the entrepreneur and the financier to be different. Recent empirical findings show that entrepreneurs are more lottery-loving than the financier (Jensen and Murphy, 1990; Lie, 2005; Malmendier and Tate, 2005; Brown and Sarma, 2007; Malmendier and Tate, 2008; Goel and Thakor, 2008), due to the misalignment of incentives/aims between entrepreneurs and financiers. In line with these evidences, an educated guess would suggest that the existence of asymmetric information might not always destroy social efficiency. Instead, it could alleviate the economic efficiency loss otherwise caused by the over-prospect-oriented entrepreneur in the "First Best" scenario (no asymmetric information), based on the assumptions that the entrepreneur has private information and the capital market is competitive. This efficiency restoring effect might result from that fact that the equilibrium contract must lie exactly on financiers' zero-profit line, i.e. the contracts that would just make them break-even, as the capital market is competitive. Therefore, even if entrepreneurs are willing to take more ambitious action, as long as their prospect theory utilities are still positive, it is the financiers who decide whether or not to take contract. Overall, the existence of asymmetric information then grants financiers the power to regulate over-prospect-oriented entrepreneurs' behaviour. Similar welfare-improving effects under non-expected utility theories have been found by Vercammen (2002), de Garidel-Thoron (2005) and Koufopoulos and Kozhan (2014).

Another interesting extension could focus on the reference level. As aforementioned, in this paper I adopted the most widely accepted convention - to set the lowest outcome as the reference level and define gains based on the relative difference from the lowest outcome, similar to that in Barberis and Huang (2008). An alternative way could be defining the reference level as the initial level of wealth, or in more complex terms, as expected final wealth (Loughran and Ritter, 2002). In this paper, it would not largely change the result as I suppress the value function and directly evaluate the wealth, as well as the success rate of projects would not change at different reference level. However, changing the reference level, for example, to include asset in place, would introduce another aspect of Cumulative Prospect Theory - loss aversion. This will result in a more complicated, yet interesting, analysis. In Loughran and Ritter (2002), they study the large first-day run-up phenomenon of IPO, which is regarded as losses from the perspective of issuing firms. They find distinct results under different market environments and the trade-off of returns for prospects, which are similar to what presented in this paper. Therefore I doubt more interesting results could be yielded albeit introducing much more complexity by changing the reference level.

Although the assumption of a competitive capital market is reasonable most of the time, under extreme conditions it is not necessary and usually violated; for example, during a financial crisis. In such a period, liquidity is very tight in the capital market and entrepreneurs have to compete for scarce external capital entailing that financiers may be able to extract economic rent for owning this scarcity. In other words, the bargaining power may belong to financiers instead of entrepreneurs in some extreme circumstances. Therefore, this paper focuses on studying the generic problems and advises readers to be alert to the validity of the assumption because the impact of the exogenous shock changing the economic environment of the market would be very different.

In conclusion, the main contribution of this paper is to offer a better approximation of the real-world corporate financing decisions by relaxing the assumption of expected utility for prospect theory. It is tentative but ice-breaking in reconciling classic capital structure theories and innovative decision making theories. It is general but fruitful in shedding light upon a large number of prevailing anomalies in the real-world capital market. Finally, it is preliminary but thought-provoking in directing future studies for both theoretical and empirical researchers.

2.6 Appendix

2.6.1 Appendix: The Equilibriums under EU

A sketched proof suffices to reveal the main point while keeping the intuition: The entrepreneur solves the expected utility maximisation problem:

$$\max U_i = \max_{(\alpha,D)} p_i(1-\alpha)(X_i - D), \qquad i = R, S$$

The financier solves the problem:

$$\max \Pi_i = \max_{(\alpha,D)} p_i[\alpha(X_i - D) + D)] - I, \qquad i = R, S$$

Taking the total derivative of U_i and rearranging it, we have the marginal substitution rate of equity to debt given a certain level of utility:

$$(\frac{d\alpha}{dD})_{U_i = \bar{U}} = -\frac{1-\alpha}{X_i - D}$$

Thus the indifference curve of R type is flatter in the (D, α) space. The intuition here is that the R type entrepreneur is willing to exchange more debt for certain amount of equity than the S type entrepreneur because the project is riskier and thus the equity is more valuable (Figure 2.2).

The indifference curves can shift toward and away from the origin as they are projections from the utility dimension (like contour lines on a topographical map). Moreover, since (D, α) are the parts taken out from entrepreneurs' pocket ("bads" rather than "goods"), then the indifference curves closer to the origin will give entrepreneurs higher utility. In other words, to maximize their utility, they prefer a contract which will put them on an indifference curve lower than the other contracts can.

Since the capital market is competitive, all financiers earn zero profit. In the same way as we derived the marginal substitution rate of equity to debt for entrepreneurs, we can easily find the shapes of the zero-profit lines of financiers for the R type (ZP_R) and S type (ZP_S) are the same with the indifference curves of corresponding entrepreneur (Figure 2.7). The difference is that while the indifference curves can shift toward and away from the origin, the zero-profit lines are fixed in the sense that ZP_R and ZP_S intercept the vertical and the horizontal axis at:

$$\begin{cases} \alpha_{\mathrm{R}} = \dfrac{I}{p_{\mathrm{R}}X_{\mathrm{R}}} = \dfrac{I}{p_{\mathrm{S}}X_{\mathrm{S}}} = \alpha_{\mathrm{S}} \\ D_{\mathrm{R}} = \dfrac{I}{p_{\mathrm{R}}} > \dfrac{I}{p_{\mathrm{S}}} = D_{\mathrm{S}} \end{cases} \tag{2.5}$$

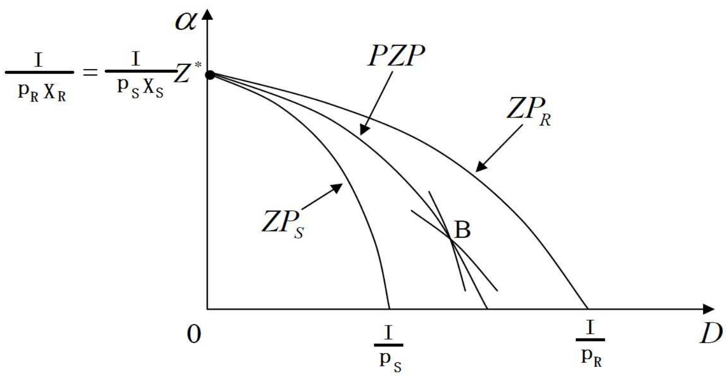


Figure 2.7: The Zero Profit-Line for the Financier

In Figure 2.7, where PZP is the pooling zero-profit line, which is stipulated by

$$\Pi_{\mathrm{PZP}} = \lambda p_{\mathrm{R}}[\alpha(X_{\mathrm{R}} - D) + D)] + (1-\lambda)p_{\mathrm{S}}[\alpha(X_{\mathrm{S}} - D) + D] - I = 0$$

First, we notice that any equilibrium contract (α, D) should lie on one of the zero-profit lines because of perfect competition. Then, by a standard signaling game argument, for any deviation (e.g., B), the R type always wants to mimic the S type because she will be better-off at any contract within ZP_R. However, as mentioned above, the R type entrepreneur is willing to exchange more debt for certain amount of equity than the S type entrepreneur because the project is riskier and thus the equity is more valuable. Thus, with intuition criteria as the method of refinements, the S type could credibly signal her type by making a counteroffer with more equity and less debt. This is because financiers know that it would only be of interest to the S type as it would make the R type entrepreneurs worse-off, thus updating their posterior belief that the counteroffer must come from the S type being 1. This would eliminate all the contracts on the pooling zero-profit line except for the only stable contract, Z^*, where no type has an incentive to deviate and only equity is issued. Moreover, being that, in this mean-preserving case, the equity is equally valuable to both types, the equity is fairly priced.

2.6.2 Appendix: The Equilibria in High Prospect Economy

In the prospect theory setting, entrepreneurs solve the prospect theory value maximisation problem:

$$\max U_i^{\mathrm{CPT}} = \max_{(\alpha,D)} \omega^+(p_i)(1-\alpha)(X_i - D), \qquad i = R, S \tag{2.6}$$

The financier again enjoys zero-profit in the competitive capital market. In the separating equilibrium:

$$\Pi_i = \omega^+(p_i)[\alpha(X_i - D) + D)] - I = 0, \qquad i = R, S \tag{2.7}$$

and in the pooling equilibrium:

$$\begin{aligned} \Pi_{\mathrm{PZP}}^{\mathrm{CPT}} &= \omega^+(\lambda p_R)[\alpha(X_R - D) + D)] + \{\omega^+[(1-\lambda)p_S + \lambda p_R] - \omega^+(\lambda p_R)\}[\alpha(X_S - D) + D)] - I \\ &= \omega^+(\lambda p_R)\alpha(X_R - X_S) + \omega^+[(1-\lambda)p_S + \lambda p_R][\alpha(X_S - D) + D)] - I \\ &= 0 \end{aligned} \tag{2.8}$$

Taking the total derivative of U_i^{CPT} and rearranging, we have the same marginal substitution rate of equity with respect to debt, given a certain level of prospect theory value:

$$(\frac{d\alpha}{dD})_{U_i^{CPT}=\overline{U^{CPT}}} = -\frac{1-\alpha}{X_i - D} \tag{2.9}$$

This result is straightforward being that the shape of indifference curves does not depend on probabilities. To put it more generally, this is a monotonic transformation from the original utility function; thus the marginal rate of substitution remains the same and so does the zero-profit line of financiers.

First, we can easily show that given the above transformations, the (actual) mean-preserving model takes the form of the dashed black lines in Figure 2.4. So far we have not discussed the effect of the change in the equilibrium contract because as long as the investment technologies are sufficiently widespread, we can always find a project under the traditional expected utility theory, which gives us the same form as the dashed black lines. Instead, we will focus on a more interesting difference; even if there is an actual project that gives the same game structure, the unique stable equilibrium will break down into a continuum of equilibria under the Cumulative Prospect Theory.

In Figure 2.4, the pooling zero-profit lines passing Point O is the traditional zero-profit line with additive proportion:

$$\tilde{\Pi}_{\mathrm{PZP}} = \lambda\omega^{+}(p_{\mathrm{R}})[\alpha(X_{\mathrm{R}} - D) + D)] + (1-\lambda)\omega^{+}(p_{\mathrm{S}})[\alpha(X_{\mathrm{S}} - D) + D)] - I = 0 \tag{2.10}$$

Following a similar reasoning as in Section 2.6.1, it is easy to find that the Point O is the new equilibrium contract for this actual project. The pooling zero-profit line for the perceived (but actually mean-preserving) projects is stipulated by

Equation 2.8, which is the lower dashed blue line in Figure 2.4. Under circumstances featuring high prospect projects, the proportion of R type projects λ is large and the success rates of projects (p_R, p_S) are low. Since higher risk is compensated by higher return, and individuals are more likely to gamble when the return is higher, the coefficient δ in the probability weighting function is therefore lower. Overall, this set of parameters will generate a pooling zero-profit line, profiled by Equation 2.8, lower than the traditional pooling zero-profit line with weights add up to 1 (given by Equation 2.10). This is due to the fact that the Cumulative Prospect Theory violates the reduction principle for compound gambles and entails that the pooling zero-profit line is not a linear combination of the separating zero-profit lines with weights adding up to 1 in the presence of multiple tiers of uncertainty. Mathematically,

$$\begin{aligned}
\Pi_{\text{PZP}}^{\text{CPT}} &= \omega^{+}(\lambda p_R)[\alpha(X_R - D) + D)] + \{\omega^{+}[(1-\lambda)p_S + \lambda p_R] - \omega^{+}(\lambda p_R)\}[\alpha(X_S - D) + D)] - I \\
&\neq \omega^{+}(\lambda)\omega^{+}(p_R)[\alpha(X_R - D) + D)] + \omega^{+}(1-\lambda)\omega^{+}(p_S)[\alpha(X_S - D) + D)] - I
\end{aligned}$$

The discussion of the proportions above Point A and below Point B on the new pooling zero-profit line resembles the argument in Proposition **??**. The result is that these contracts are eliminated by intuition criteria which we elaborate upon by showing that every contract between A and B can be sustained as stable equilibrium. First, I show that Point A is a stable equilibrium. Since we know that the contracts above A cannot survive, imagine any deviation into the area $\widehat{OAB}$. This deviation implies a marginal substitution rate higher than the marginal substitution rate of the S type, but lower than that of the R type. It will push the indifference curve of the S type further and pull that of the R type closer toward the origin. In other words, the deviation will give the S type a lower utility, while it gives the R type a higher utility. Thus, it must come from the R type as it is only of interest to the R type, but will make the S type worse-off. By intuition criteria, financiers should be sure about the types and able to separate them, and react in turn by offering each type a contract on the respective zero-profit line (black dashed lines in Figure 2.4). Nevertheless, the contracts on the separated zero-profit lines will give both types of entrepreneurs those utilities that no higher than the utilities of contract A. In other words, the strategies in the deviation are (weakly) dominated by the strategies at Point A. Therefore, Point A survives the intuition criteria and is sustained as a stable equilibrium. A similar reasoning also applies to Point B.

So far, we have proved that the two extremes of the interval $\widehat{AB}$ are stable equilibria. Now imagine any one contract within this interval - say C. Here, any deviation will at most benefit one type but not the other; otherwise it is below all the zero-profit lines, which means financiers are loss-making and they will not accept this loss-making contract. Thus, according to the intuition criteria, financiers are able to separate the types by the deviation and offer each type contracts located on their corresponding (separated) zero-profit line. Again, since the separating contracts are (strictly) dominated by the original pooling contract C, this deviation is not profitable. Thus any contract within this interval could be sustained as a stable equilibrium constituting a continuum of stable equilibria.

2.6.3 Appendix: The Equilibria in Low Prospect Economy

When there are more safer and low-payoff projects - for example, the time period just after an economic crisis - the proportion of R type projects λ is small and the success rates of projects (p_R, p_S) are high. Since lower risk is compensated by lower return, and individuals are less likely to gamble when the return is lower, the coefficient δ in the probability weighting function is thus higher. Overall, this set of parameters will generate a pooling zero-profit line, profiled by Equation 2.8, higher than the traditional pooling zero-profit line with weights adding up to 1 (given by

Equation 2.10).

First, analogous to the reasoning in Section 2.3.1, all the contracts above Point C and below Point D on the new pooling zero-profit line cannot survive intuition criteria because any deviation will only be in favor of one type and is thus perfectly informative to financiers. In the meantime, any contract within the interval $\widehat{CD}$ cannot be sustained as an equilibrium either. This is very straightforward as it positions utilities lower than any contract could give on the separating zero-profit lines. In other words, any contract within the interval $\widehat{CD}$ is (strictly) dominated by separating contracts and thus cannot be sustained as an equilibrium.

Nonetheless, resembling the game in which projects have the same zero-profit lines, but under the expected utility theory, there exists a continuum of separating equilibria which is located on the whole outer boundary of the shape comprising of the intervals $\widehat{OC}$ and $\widehat{OD}$ on the two separating zero-profit lines, except for Point O. This is because the entrepreneurs can credibly signal their own types as the strategy of mimicking the other type is too costly to be carried out. In other words, mimicking strategies are (strictly) dominated by separating strategies. For example, if the S type chooses contract C on its zero-profit line above Point O, then it is unreasonable for the R type to mimic the S type by also choosing contract C because this contract is above his own zero-profit line. This means that it will give her a lower level of utility than the contracts on her zero-profit line, e.g. contract D. Note that this continuum of separating equilibria are not exactly the same as the continuum of separating equilibria under the expected utility theory, even if with the same zero-profit lines. Point O is excluded because at this point financiers cannot distinguish the two types and pool them using the pooling zero-profit line higher than the traditional pooling zero-profit line with weights adding up to 1. As discussed in the paper, this discontinuity results from multi-layers of uncertainty in CPT could be eliminated by defining a commensurate equilibrium refinement. Hence, we have proved that there is no pooling equilibrium.

Chapter 3

The Role of Model Uncertainty in Corporate Risk Management

3.1 Introduction

The importance of corporate risk management has increased rapidly to a non-trivial level in recent years. Fifty per cent of firms have taken active positions with derivatives globally(Lins et al., 2011). The magnitude is also substantial, for example Allayannis and Weston (2001) find a hedge ratio of 22% with respect to exchange rate risk, and Chernenko and Faulkender (2012) document interest swap covers 6.8% of firms' debt every year. For an alternative risk management tool, cash, Sánchez and Yurdagul (2013) observe that cash constitutes 12% of net assets in 2010. This increase is mostly due to precautionary savings (Bates et al., 2009).

The increasing attention paid to corporate risk management motivates the need for advanced guiding theory. However, although many efforts have been made, extant literature leaves a puzzle in the dynamics of risk management policy, and is inconclusive with respect to firm value creation. Stulz (1996) remarks "[t]he actual corporate use of derivatives, however, does not seem to correspond closely to the theory ... large companies make far greater use of derivatives than small firms, even though small firms have more volatile cash flows, more restricted access to capital, and thus presumably more reason to buy protection against financial trouble." A potential reason for this open research question might be that although model uncertainty implicitly underlies all the models, it has long been neglected. Recent studies, such as DeMarzo and Sannikov (2016) and Décamps et al. (2015), show that accounting for this uncertainty might help identify missing patterns in previous literature of other closely related fields.

This paper, therefore, examines how the resolution of model uncertainty drives the dynamics of corporate risk management policy, as well as its impact on the firm value creation – destruction effect of hedging and cash reserves.

We approach the research question using a parsimonious structural model, where the only uncertainty is the correlation between a firm's cash flow and the underlying asset of derivatives (which we later refer to as "hedgeability"). The firm is assumed to have a neutral knowledge, defined as 50% being a high hedgeability firm and 50% being a low one, on the hedgeability of its cash flows, and needs to observe and learn from realization of the cash flow process. These informative realizations allow the firm to form a belief on its hedgeability, towards either high or low.

Most of our findings are consistent with existing literature. In addition, we identify the channels of both substitutability and complementarity between hedging and cash balance. This indicates that cash stock is an indispensable risk management tool; thus, it coexists with hedging, even in the (seemingly) perfect hedge scenario.

We find that resolution of model uncertainty accelerates the establishment of a firm's hedging position, and cash helps with the process. In this regard, we answer Stulz (1996). We argue that size might be a spurious determinant of corporate hedging. Rather, mature firms with better knowledge would hedge more than younger, less informed firms.

Moreover, our results support both directions in the mixed empirical evidence, depending on the trade-off between the expected reduction in external financing losses and the decrease in the option value of default. Under some reasonable circumstances, a risk-shifting effect would dominate a firm's risk management concerns. This finding is evidenced by Rampini et al. (2014), where they find a risk-shifting behaviour in small financially constrained airlines who "hedge less or not at all."

Overall, we conclude that dynamic information acquisition is an important determinant of corporate risk management. The main contribution of this paper is that it is the first attempt, to our knowledge, to connect unravelling model uncertainty to the dynamics of corporate risk management policy. We also find it might shed light on the debate over the value creation role of hedging. We also provide some suggestions for the direction of future research.

The rest of the paper is organized as follows: Section 3.2 reviews related literature and positions this paper. Section 3.3 sets up our model, as well as states the assumptions, limitations, and potential extensions of this model. Section 3.4 calibrates the model, and presents optimal policies of corporations in the dynamic choices of cash holdings and hedging with the results of simulations. Section 3.5 discusses the marginal effects of each risk management tool and conducts sensitivity analyses on firm value creation. Section 3.5 concludes and discusses potential extensions of this paper as well as interesting future research topics.

3.2 Literature Review

This paper is empirically motivated and naturally fits into dynamic corporate finance literature, especially those with learning feature.

It is a well understood theory that in a world without economic friction there is no benefit for corporations to manage risk, as idiosyncratic risks could be diversified by investors and systematic risks are correctly priced in an efficient market. Thus, firms should not engage in risk management. This theory, however, is at odds with the evidence observed across countries and industries. Guay and Kothari (2003) find 57% firms using hedging. Allayannis and Weston (2001) find a hedge ratio of 22% in exchange rate risk. For another widely used risk management tool, cash, Martínez-Sola et al. (2013) find that in US industrial firms from 2001 to 2007, the mean cash ratio is 7.9% and the median is 4.48%. Bates et al. (2009) show this ratio has, on average, more than doubled to 23.2% in 2006. Finally, Sánchez and Yurdagul (2013) find the absolute value of cash holding was still increasing at an increasing speed, 10% between 1995 and 2010 versus 7% between 1980 and 1995 in the US, which resulted in an average 12% cash:net asset ratio in 2010.

Firms hedge their risk to reduce the expected costs of financial distress (Smith and Stulz, 1985), to lower the expected tax payments under a convex tax schedule (Smith and Stulz, 1985; Foley et al., 2007), and to avoid costly external financing; this is achieved by better matching internal cash flow with financing needs (Froot et al., 1993). All these are convex cost functions for the firm, and due to Jensen's inequality, the value of corporate risk management derives from minimizing the expected value of the cost by either reducing the volatility of cash flow or increasing the average cash flow.

Nonetheless, the empirical findings show mixed, sometimes counterintuitive, results. Allayannis and Weston (2001), Graham and Rogers (2002), Carter et al. (2006), MacKay and Moeller (2007), and Campello et al. (2011) find that hedging increases the value of the firm, whereas Brown (2001), Guay and Kothari (2003), Jin and Jorion (2006), Magee (2009), and Bartram et al. (2011) find either opposite support from the data or at least an unclear answer to this question.

Moreover, this classic corporate risk management theory predicts that young firms, which usually have more volatile cash flows, should engage more in risk management than established firms. Nevertheless, this claim is not well-founded in evidence. In contrast, Nance et al. (1993) find that hedgers are usually large firms. Graham and Rogers (2002) also identify that hedging increases with firm size. Finally, Bartram et al. (2009)find no correlation between young firms' hedge and financial distress.

Most dynamic models in corporate finance treat model uncertainty and risk management separately. Our model, however, links these two strands of literature. Particularly, in this paper, we investigate the interaction between hedgeability and risk management under a specific market friction – external financing cost.

That risk management can be properly understood only in a dynamic setup has been shown in many papers. A seminal paper, Mello et al. (1995) find that corporate hedging policy is endogenous with flexibility in production, and agency costs play an important role in this relationship. More recently, the work of Bolton et al. (2011) and Bolton et al. (2014) highlights Q-theoretical models and using cash (liquidity) holdings as either a means of risk management or financial flexibility. Nikolov et al. (2015) recognise the validity of coexistence of cash and other risk management tools[1]; while cash is efficient at transferring liquidity across time, we still need another device to protect us across states of the world so as to maximise the value of a firm. In a time series, equity issuances are used to replenish cash balances, and state-contingent risk management tools are used to fund unanticipated investment opportunities. In our paper, we reconfirm their main findings and extend them by considering hedging and cash reserves endogenously, thus yielding more interesting implications.

Only very recently have successful attempts been made in comparing the benefit from hedging and that from cash holding. Our paper mostly relates to this strand of extant literature. The model in Gamba and Triantis (2008) endogenises dynamic financing, investment, and cash holding – pay out policies to analyse the effect of financial flexibility on firm value. They find that the value of financing flexibility depends on "the costs of external financing; the effective cost of holding cash; the firm's growth potential and maturity; and the reversibility of capital." Because firms incur costs when they change levels of debt, it can be optimal for a firm facing financing friction to have positive stocks of debt and cash simultaneously. In other words, firms hold cash to avoid incurring costs to adjust the net leverage policy. They extend their research in a more general risk management set-up in Gamba and Triantis (2014), where operating flexibility interacts with financial risk management. They find a critical role for liquidity, whereas value created by derivative hedging could be low. Alternatively, Rampini and Viswanathan (2010), Rampini and Viswanathan (2013), and Rampini et al. (2014) assert that both financing and risk management involve promises to pay that need to be collateralized, resulting in a financing versus risk management trade-off. They find that more financially constrained firms hedge less or not at all, and provide evidence for their model prediction empirically by using panel data from the airline industry. One common feature of these contributions

[1]Credit line in their paper.

is that decision makers do not face uncertainty about characteristics of the firm's cash flow. Only market friction matters.

A crucial assumption underlying all these models is that agents have perfect information on the parameters governing those models. In other words, there is no model uncertainty. However, growing literature shows that model uncertainty might be common. Early papers that take into account model uncertainty uniformly focus on uncertainty in profitability. For instance, Pástor and Veronesi (2003) and Décamps et al. (2005) show that firms make investment decisions in accordance with their profitability prospects, with or without liquidity friction. The former study also finds that the stock price increases with uncertainty about average profitability and decreases over time as the firm learns about its average profitability, which explains some patterns in our results. The latter paper rigorously shows the importance of incorporating beliefs into state space, and the non-monotonicity with respect to parameters on the second order moment[2], which is also part of our results. We study the optimization of risk management strategy of a financially constrained firm when there is uncertainty about the firm's hedgeability, while the previous studies focused on uncertainty about the profitability.

Recently, there have been some successful attempts to integrate uncertainty in profitability and market friction. Moyen and Platikanov (2013) develop a model in which shareholders update their beliefs about firms' quality in a dynamic Tobin's q framework. They find evidence that firms with unclear quality are more sensitive to earnings in their investment decisions than are well-established firms. Décamps et al. (2015) assume that a firm's operating cash flow is proportional to profitability, with dynamics governed by a Geometric Brownian Motion. They find a constant dividend payout ratio that depends on the long-term prospects of the firm, linking

[2]Volatility, in their model

corporate cash management with beliefs. Although only focusing on uncertainty about the first order moment, these models show the importance of resolving model uncertainty when facing operational risk in the presence of market friction. Our model extends the research to the second order moment, which yields more interesting empirical implications.

Our study is more closely related to DeMarzo and Sannikov (2016) and Décamps et al. (2015). The former study describes a dynamic contracting model with learning about a firm's profitability. Asymmetric information arises endogenously because by shirking, an entrepreneur can distort the beliefs of investors about the project's profitability. The paper studies the relationship between incentives and learning. In the implementation of the optimal contract, cash inside the firm is accumulated at no cost. It is shown that liquidation can be a first-best solution, despite moral hazard. In Décamps et al. (2015), information is incomplete but symmetric between stakeholders, and holding cash is costly for the firm. The novelty of their analysis is that they study the evolution of the trade-off between costs and benefits of holding cash due to learning on the firm's profitability. The optimal policy is unique, the dividend payout ratio changes with the beliefs about the profitability, and liquidation cannot be first-best. They predict that both cash target levels and target dividend payout ratios increase with profitability prospects.

These recently successful contributions show that, treated separately, model uncertainty and market friction lead to different nonlinearities between firm value and its fundamentals, different properties of the dynamics of corporate policy of the firm. As a result, because these policies neglect the endogeneity between model uncertainty and market friction, they are incapable to report the main stylized facts about real world practices. In our paper, we document similar findings, and extend the study by including corporate hedging as an important alternative for precautionary cash reserves. Integrating market friction and uncertainty in hedgeability in a single model is even more challenging because it requires solving a dynamic optimization problem with higher structural complexity. So far as we know, our paper is the first attempt to address this issue.

This paper offers an alternative explanation for empirical evidence on the dynamics of corporate hedging policy and cash management, and their effect on firm values. Our model resolves the puzzle by suggesting that more endogenous variables should be taken into account simultaneously when answering this question. Namely, to what extent do decision makers understand the distributional characteristics – such as hedgeable risk (correlation between corporate cash flow and traded assets in financial markets) – of the cash flow of their own business. In this regard, our paper contributes to the aforementioned literature by associating dynamic corporate risk management decisions with the resolution of model uncertainty.

This approach yields distinct predictions: more knowledgeable firms separate themselves from less knowledgeable ones by using more hedge so as to avoid external financing cost and the cost of carrying cash, thus maximising a firm's value. Firms with higher hedgeability tend to assume more hedge without necessarily creating firm value. Although not done in this paper due to the curse of dimensionality, one could easily extend this model to other endogenous factors (which were proposed as alternative answers to our research question, such as the following) to see the complementary effect of adding learning process to better capture the dynamics in real-world corporate risk managing behaviours. For example, to address the concern that financially constrained firms' financing needs might compete against hedging needs for limited collateral, raised in Rampini et al. (2014), one should add state-contingent borrowing as an additional control variable to our model. Although this paper is mute about corporate investment policy, another interesting extension is to endogenise investment decisions, as in Froot et al. (1993). One possible approach is to replace exogenous cash flows with a capital accumulation model as in Bolton et al. (2013).

3.3 The Model

We assume a firm generates a stochastic cash flow X_{1t}, and there is a tradable security with price X_{2t}, correlated with the cash flow of our firm. A perpetual swap contract with swap price s can be written on X_{2t}. These two variables follow a joint VAR process, under the risk-neutral probability:

$$x_t = \kappa x_{t-1} + \epsilon_t \tag{3.1}$$

$$\text{where} \quad \kappa = \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix}, \qquad x_t = \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix},$$

$$\epsilon_t = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}, \ \epsilon_t \sim i.i.d.\ N(0, \Sigma),\ \Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

where $x_{it} = \log X_{it}$, $0 \leq \kappa_i \leq 1$ are the persistence parameters and $\sigma_i > 0$ are the conditional standard deviations, $i = 1, 2$. Without loss of generality, we restrict the analysis to the case in which the two state variables have positive correlation, $\rho \geq 0$, so the firm can enter a long position[3] in the swap contract. When $0 < \rho < 1$, the swap offers an imperfect hedge against the risk of the firm's cash flow.

The firm has an overhang of constant payment, A, which could be related to operating costs, interest payments, or any other costs. In addition, the firm can retain any proportion of the residual cash flow to build up a cash stock c_t, which earns a return $r_c < r_f$, where r_f is the risk-free rate. The wedge between r_c and r_f captures many sources of friction associated with holding cash. The firm incurs distress

[3] Receiving fixed cash flow

costs and possibly costly external finance, which is a proportion, λ, to the cash shortfall, as soon as the cash holdings together with the cash flow cannot meet the cost A.

In principle, model uncertainty regards uncertainty about the entire model generating the data (cash flow, in our paper). We simplify this by talking about uncertainty on some parameters, such as: mean, volatility, correlation, and auto-correlation among others, that characterise the model. In the interest of studying corporate hedging behaviour, we focus on the uncertainty about correlation, ρ, between cash flow of the firm and the price of the tradable asset. We assume correlation can be either high (ρ_H) or low (ρ_L). This design captures the hedgeability, namely the extent to which the firm can hedge its operating risk. High-type firms are those whose business are more closely related to certain traded assets in the financial market, such as oil firms, mining firms, and airlines. On the other hand, low type firms are usually conglomerates and those with high intangible assets, such as technology companies. Our choice also makes empirical sense as transaction of high correlation derivatives is treated differently from that of low correlation in accounting and disclosure according to FAS 133 and 161 regulations.

At the beginning, a firm is assigned equal prior probabilities of being high or low type. The firm then updates the belief about its type, according to Bayes' rule based on the observation of cash flows joint (X_1, X_2). The probability, q', that the firm is of type H given the information at time t is:

$$q' = \frac{qPr(x'|H)}{qPr(x'|H) + (1-q)Pr(x'|L)} \tag{3.2}$$

where:

$$Pr(x'|\cdot) = \frac{\exp\left[-\frac{1}{2(1-\rho^2)}\left(\epsilon_1^2 - 2\rho\epsilon_1\epsilon_2 + \epsilon_2^2\right)\right]}{2\pi\sigma_1\sigma_2\sqrt{1-\rho_j^2}}, \qquad i \in \{H, L\} \tag{3.3}$$

$$\epsilon_i = \frac{x_i' - \kappa_i x_i}{\sigma_i}, \qquad i \in \{1, 2\} \tag{3.4}$$

which is the bivariate standard normal probability density function.

A firm can take a long position in a perpetual swap contract. The underlying asset for the swap is assumed to have the same mean, volatility, and persistence as the firm's cash flow in order to lead to simpler interpretation of our results. For simplicity, the swap price for a unit of product is fixed at $s = 1$.

Thus, if a firm enters, at time t, into a swap agreement for a notional physical amount h, at $t+1$ the net payoff from the swap is $h(s - X_{2,t+1})$. Thus, the value for each unit of this contract for the moment t, excluding counterparty risk and contractual flexibility at time t, is:

$$S_t = S(X_{2t}) = \sum_{\tau=1}^{\infty} \frac{s - F_\tau(X_{2t})}{(1+r_f)^\tau} < \infty \tag{3.5}$$

where $F_\tau(x_{2,t}) = \mathbb{E}_t[x_{2,t+\tau}]$ is the forward price at time t for delivery of the asset at date $t+\tau$, and the expectation is computed under the risk-neutral probability.

Firms default on the swap when the sum of a firm's equity value and current liquidity surplus is less than the promised payment against the swap. Moreover, a firm can alter its hedging position h. If it does wish to alter its position in the swap, for example to h', it redeems the current contract at the par value (S_t), and enters into a new agreement at the current fair value denoted SF_t, which is discussed in the next paragraphs. Hence, the net payoff from the transaction is: $h \cdot S_t - h' \cdot SF_t$. We assume that each transaction entails a negotiation cost, nc, proportional to the value of the notional amount. As a result, the direct cost of adjusting the hedge is: $nc \cdot (h + h')$.

A no-arbitrage condition requires fair market value of the swap to include credit charges. Otherwise the swap is under-priced, and firms will take excessive hedging positions, as on average it will generate extra cash in-flow to the firm. Consequently, equilibrium hedging positions will be over-represented in the model. As discussed in Gamba and Triantis (2014), at the inception of the swap agreement, the fair value of the contract, SF_t, reflects both default risk and the option to close the swap position early at a future date, and thus it can be different from the par value, S_t. This implies that there is an indirect cost, $h' \cdot SF_t - h \cdot S_t$, associated with adjusting a swap position at a later date. We assume for simplicity that the bank selling the swap is not subject to default risk, and thus the only credit charge in the price of the contract is related to the default risk of the firm. Hence, the fair value of the swap contract at time t is:

$$SF_t = \mathbb{E}_t \left[\sum_{\tau=1}^{\min\{T_d, T_p\}} \frac{s - X_{2,t+\tau}}{(1+r)^{\tau}} \right] + \mathbb{E}_t \left[\chi_{\{T_d \geq T_p\}} \frac{S(X_{2,t+T_p})}{(1+r)^{T_p}} + \chi_{\{T_d < T_p\}} \frac{RS(X_{t+T_d})}{(1+r)^{T_d}} \right] \tag{3.6}$$

where T_d is the default date, T_p is the date the swap position is closed, and $\chi_{\{T_d \geq T_p\}}$ is the indicator function for the event that default happens after the position in the swap is closed; in contrast, $\chi_{\{T_d < T_p\}}$ indicates the opposite event. $RS(\cdot)$ is the bank's recovery value on the swap if the firm defaults, which is explained later.

The first term in equation (3.6) is the present value of the net payoff to the firm before the firm terminates the swap agreement due either to rebalancing its hedge position or to a default. The second term is the payoff to the bank if the firm either terminates the swap or defaults. The bank is paid the par value in the former case, and a reduced recovery value in the latter. T_d and T_p are stopping times with respect to the process X_t. Therefore, the fair value of the swap contract depends on corporate policy. This policy will be determined endogenously.

In order to focus on the question of how firms react to the impact of friction on firm value, the valuation and associated optimal policies are based on the objective of equity (firm) value maximisation.[4] The value of a firm is represented as:

$$\begin{aligned} V(X,c,h,q) = \quad & \max\{0, \max_{(c',h')} \Big\{ D + \tfrac{1}{1+r_f} \{ q\mathbb{E}_X[V(X',c',h',q')|H] \\ & +(1-q)\mathbb{E}_X[V(X',c',h',q')|L]\}\}\} \end{aligned} \tag{3.7}$$

where cash flow from the current period is: $D = \max\{d,0\}+\min\{d,0\}(1+\lambda)$. The pay out is represented as:

$$d = X_1 - A + (1+r_c)c + h(s - X_{2,t}) + (hS_t - h'SF_t - nc(h+h'))_{\chi\{h'\neq h\}} - c' \tag{3.8}$$

where $\chi_{\{h'\neq h\}}$ is the indicator function of event $h' \neq h$, and $\mathbb{E}_X[\cdot|H/L]$ is the expectation conditional on the current state (X) as well as the type.

Default occurs when the second argument in the max operator in Equation 3.7 is non-positive. Upon default, all the remaining cash balance is paid out, the swap contract is liquidated and settled $(h = 0)$[5]

[4]In practice, corporate risk management decisions are made by managers and subject to the structure of managerial incentives as well as regulatory policies. We do not attempt to capture these intricacies in this paper.

[5]In default, the optimal policy is thus $(0;0)$ for both hedge position and cash reserve level.

A bank's recovery value ($RS(\cdot)$) on the swap upon default depends on the firm value at time $t + T_d$ as well as who is the receiver. We have:

$$RS(X') = \max\left\{-(s - X_2' + S_t), \frac{c + V(X', 0, 0, q')}{h}\right\}$$

per unit of notional amount. The first argument in the max operator is the cash inflow for the firm, if $(s - X_2' + S_t) > 0$. The second argument is the current cash balance plus the firm value at default, anticipating the cash balance is paid out, and the swap contract is liquidated and settled ($h = 0$). For simplicity, we do not discuss the situation where a firm can strategically default on the swap contract in this paper. Thus any change on the assumption about what happens if a firm defaults would have an effect on the calculation of the recovery value ($RS(\cdot)$). Firstly, the above assumption is reasonable and realistic, as the U.S. Bankruptcy Code allows financial swaps' counterparties to exercise contractual rights and get paid ahead of debt obligations. Secondly, even the bankruptcy law could be different globally, the distinction enters the recovery value in the second argument in terms of different level of remaining firm value. In general, a stricter bankruptcy law with respect to swap counterparties would divert more firm value out of the firm and thus lower the continuation value of the firm. This would in turn increases the indirect cost of hedging and consequently reduces the use of hedge. On the contrary, a bankruptcy law that better protects swap counterparties would encourage hedging activity, ceteris paribus.

The entire model accommodates an underlying mechanism that firms with better knowledge can more accurately manage their cash flow risk, thus creating more value from corporate risk management. While keeping the spirit of the model in Gamba and Triantis (2014), our model largely differs from theirs by focusing on the effect of dynamic resolution of model uncertainty. We aim to answer a different research question which is whether the uncertainty in hedgeability drives a young firm's risk management policy to differ from that of a mature firm. We also discard devices that are less relevant to our research purpose, such as operational flexibility. Consequently, both risk management policy and firm value directly or indirectly depend on the learning process. Thus, mature, large firms are usually knowledgeable firms and young, small firms are uninformed firms, leading to the observed relationship between enterprise value and risk management policy.

3.4 Results

3.4.1 Parameter values

{Insert Table 3.1 here.}

We calibrated the model to obtain results that help explain existing empirical evidence and provide empirical implications. The rationale for each of the parameter choices is described below. In many cases, e.g., high- or low-type correlation ρ^H/ρ^L, the parameter selection corresponds directly to typical values found in the literature. In other cases, we set parameters such that moments of key variables from our simulations are consistent with those in empirical studies, such as equity issuance frequency.

The standard deviation and persistence parameters of the log of cash flow are set to $\sigma_1 = 15\%$ and $\kappa_1 = 0.8$, which are consistent with the range of values used in recent articles with similar processes (e.g. Hennessy and Whited (2005) and Gamba and Triantis (2014)). The profitability is also affected by the fixed production cost

$A = 0.85$, which could be pinned down by matching observed frequency of equity issuance. In a coarse measurement, Fama and French (2005) estimate the number to be between 54% to 72%, using firms' net equity issuance. However, this includes equity issuance due to M&A and management compensation, which accounts for more than half the observations. More reasonable measurements range between 13% and 17.9% (Hennessy and Whited, 2007; Hennessy et al., 2007; Gustafson and Iliev, 2015). Hennessy and Whited (2007) indicates simulated frequency of issuance could be higher than its empirical counterpart. In their study, simulated frequency is 23.05%, while the observed is 17.5%.

The underlying asset for the swap is assumed to have the same mean, standard deviation, and persistence as the cash flow in order to lead to simpler interpretation of our results. The swap price is fixed at the median of X_2, $s = 1$. However, we ran robustness checks in a range $(0.95, 1.05)$[6] around the baseline case. Similar to Gamba and Triantis (2014), this does not qualitatively change the conclusion. They also test an alternative approach where the swap price is reset every time the hedging position is renegotiated, such that the fair market value of the new initiated swap is zero. They find that firm valuation is not sensitive to the specification of the swap price. We point out that as long as the swap is fairly priced, taking default risk into account, the swap price becomes irrelevant.

In our model, X_1 represents the entire risk exposure of our firm, and the correlation ρ measures the ability of the firm to hedge this risk exposure using the derivatives contract. We believe that for many non-financial corporations, this correlation is likely to be quite low based on the empirical evidence, ranging from 3% to 45% summarised from a group of leading empirical papers, such as Tufano (1996), Guay and Kothari (2003), Bartram et al. (2010), and Campello et al. (2011).

[6] chosen based on the properties of the process underlying the swap

Conversely, for firms whose businesses relate to natural resources, agriculture, and finance, this correlation could be very high as Carter et al. (2006) and Rampini et al. (2014) point out.

The interest rate is assumed to be $r = 5\%$, whereas returns on holding cash are $r_c = 3\%$. In other words, there is a cost of carrying cash by assuming that cash reserves earn a constant interest rate $r_c < r$ inside the firm. Consequently, the difference between r and r_c is a carry cost of liquidity. This design is consistent with extant literature, where two main concerns associated with holding cash are taken into account. The first cost of carrying cash comes from direct loss of interest tax. On the other hand, holding cash implicitly incurs opportunity cost for forgoing positive NPV investment projects. Morellec et al. (2014) assume the cost of holding cash is $r - r_c = 2\%$, which produces empirically consistent simulation results. We use a smaller cost (1%), as it yields a better match with the empirical cash ratio as in Sánchez and Yurdagul (2013) under our model setting.

The scarce availability of derivatives instruments that closely meet particular firms' hedging needs is one major reason why the ability of using derivatives to reduce a firm's risk exposures appears to be weak in practice. These firms constitute the low-type group in our model. We also note that certain industries, such as natural resources, are highly related to widely traded financial derivatives. We assume $\{\rho^H, \rho^L\} = \{0.9, 0.1\}$ as our base case, to make a sharp contrast in the results between high and low type, which helps to highlight the main interest of this paper. In the next subsection, we will show that these parameters yield empirically compatible values for the magnitude of hedging positions. We also conducted sensitivity analyses across a wide spectrum of correlation values and found similar results.[7]

[7] However the distance between high and low correlation does affect the speed of learning, and entangles with a discretisation approach, which increases numerical complexity.

When a firm is caught in distress, it has to pay a significant premium to raise capital to cover the shortfall. This additional distress cost (λ) is assumed to be proportional to the deficit amount, which is assumed to be 15%. This may reflect the need to sell assets at a discount in fire sales, or to assume costly covenants in contracts for external finance. Pulvino (1998) estimates the fire-sale discount to be about 14%. In the model developed by Strebulaev (2007), the range of distress costs is set between $5-25\%$, whose mid-point is around 15%. Our assumption of 15% is close to both of them. All of our base case parameters are shown in Table 3.1, which are similar to those examined in Gamba and Triantis (2014).

3.4.2 Numerical solution and simulation

In this paper, the uncertainty surrounds one important distributional characteristic, correlation of hedging assets with the cash flow. Particularly, this incomplete information determines how well a firm could hedge away the risk associated with its cash flow. That is why in this paper, we refer to this as hedgeability. A high correlation would result in more effective hedge. At the beginning, the firm assigns equal prior probabilities of having high or low correlation. The firm then updates beliefs about its type following Bayes' rule, using information conveyed in each realization of the state variables.

In order to gain more insight into the effect of learning on the dynamics of hedging and cash policies, as well as the valuation of firms, we show how the optimal policy function is found by numerically solving a firm's problem in Section 3.3, and how it drives the dynamics of corporate hedging and cash management policy. See Appendix A for further details on the numerical method.

Baseline models

{Insert Table 3.2 here.}

We set off from a baseline model (Model 1), where firms have perfect information, cash reserves are not allowed, and cash flows are perfectly correlated with the price of hedging assets. These are factors that interact with a firm's risk management policy. We then gradually relaxed one restriction at a time, so that we could have a clean view on the individual effect of each aspect of the model. Specifically, in Model 2 we allowed for a more realistic imperfect correlation between the underlying assets of a swap and the cash flow of the firm. In Model 3, we further allowed for cash reserves, as an alternative risk management tool. The results are presented in Table 3.2. Here we only present steady-state values of each metric. However, we notice that it takes some time to reach the steady state from an initial condition, especially when cash holding is allowed. We will discuss this evolution in later paragraphs.

Comparing the first and second column of Table 3.2, the hedge level decreases in Model 2, as the hedging asset – cash flow correlation becomes imperfect, specifically from 1 to 0.9. This is reasonable; while hedging becomes less accurate, the first-best solution is no longer achievable. As discussed previously, optimal hedging policy depends on distributional characteristics. Stochastically, external financing cost is inevitably triggered with a probability larger than zero. This is also manifested in a lower firm value, higher distress probability, and larger deadweight loss.

We shall note that although frequency of external financing is the only observable variable in the real world, we are more interested in the decomposition of total equity issuance for different purposes. We thus probed the relationship between risk management and external financing probability from a more strategic perspective. On one hand, firms resort to external finance in the face of financial distress. On the other hand, firms might be willing to raise external finance, even in good economic states with no operating cash flow shortfall, to meet the liquidity need in hard times so as to establish an optimal risk management position, an equilibrium in the trade-off between expected cost of risk management (including the cost of carrying cash) and that of external financing.

We therefore separated actual financial distress, due to the realisation of cash flow for the current period and risk management decisions made in the last period, from the aggregate external financing situations. This decomposition, as indicated in Table 3.2, shows that a considerable proportion of aggregate external financing is caused by voluntary equity issuance due to liquidity needs so as to fulfill the risk management decision in current period, to establish optimal hedging position. Moreover, we note that this effect differs across all three models in Table 3.2. The proportion decreases with correlation between a firm's cash flow and the underlying assets of the swap, and increases with the ability to reserve cash. Specifically, the frequency of actual distress in Model 3, as a result of corporate operation, decreased to as low as 0.56%.

Therefore, one interesting implication here is that rather than gross external financing frequency, which we usually observe in the real world and is widely used in empirical studies, researchers should distinguish actual financial distress from voluntary equity issuance. We suggest empirical researchers use more subtle identification of external financing when studying corporate policy. Some pioneering work is already heading for this direction. For example, DeAngelo et al. (2010) find that 62.6% issuers of an SEO would have run out of cash the year after the SEO without the proceeds from the SEO.

We further allowed cash stock in Model 3, presented in the third column, holding correlation coefficient unchanged at $\rho = 0.9$, as in Model 2. Consequently, firms reserve substantial amounts of cash and reduce the use of hedges. This result indicates a substitution effect between hedge and cash balance. Being equipped with cash as another risk management tool, firms are now able to further reduce their distress probability, as well as the total external equity issuance rate. Consequently, loss is mitigated and enterprise value is enhanced.

Although not reported, a similar analysis was also conducted on unlimited liability firms. We found higher use of hedges which is not surprising, because limited liability serves as a natural risk management device and reduces the use of other risk management tools. This value enhancement comes from the option that limited liability firms can default in bad states while hoping on a better prospect in good states. Since the downside is bounded but not the upside, such firms' values are larger than the value of firms who must bear all the losses even when the firm is a negative asset. In this regard, although not discussed explicitly in this paper, our model implies a positive relationship between risk-shifting effects in corporate risk management strategy and leverage. Since the main interest of this paper is to study how knowledge affects a firm's risk management, we leave the study of the leverage to future research.

{Insert Figure 3.1 here.}

We probed other important dynamics not illustrated in Table 3.2, as well as compared High-type dynamics with Low-type dynamics. As shown in Table 3.3, we note an increasing pattern in cash stock in both H- and L-types. Since we have not yet introduced model uncertainty, these dynamics are likely caused by the physical ability to accumulate a desired level of cash balance. For example, the mean of operating profit, $E(\tilde{X}) - A$, is 0.18 per period. In addition, external financing is costly.

Thus, to establish the optimal cash balance, which is above 0.5 for both types, takes time. To address this concern, we endowed firms with a steady-state cash balance to adjust for this dynamic pattern not caused by learning in order to comment on the effect of learning. In addition, we conducted sensitivity analyses on the carrying cost of cash, and found results remain in a reasonable range of the cost.

Furthermore, we observed distinct strategies between H-type and L-type firms. H-type firms use more hedge to manage its operating risk naturally following its better capability of accurately matching operating cash flow with price of the underlying asset of the swap. In contrast, due to too much noise in hedging, L-type firms use a lower level of hedge. We also note a lower distress probability and external financing loss for H-type firms, but the pattern in enterprise value is less clear.

When we allow for a cash balance in Model 3, relative to the cases without it, we give the firm an option to smooth over time (as opposed to across states) the cost of risk management. Firms do this by issuing equity as soon as they can, and storing cash in the firm, to the extent that the cost of carrying cash outweighs its expected marginal benefit. At the moment it costs 1.4%, which is consistent with extant literature and yields empirically consistent cash ratio. We also show it is indeed costly to firms, as they tend to reduce cash reserves rather than raise them through issuing equity, when endowed with steady-state cash balances.

We also noticed distinct relationships between the use of two risk management tools, hedging and cash reserves, for H- and L-type firms. Particularly, we observed a substitution effect in H-type firms, but a complementary effect in L-type firms. While cash stock increased over time in both types, the use of hedge decreases for H-type but not so for L-type. However, we found this observed pattern depended on the cost of holding cash. As discussed in the previous paragraph, if the carrying cost is low, firms would prefer issuing equity as soon as possible and storing the raised capital as cash. Firms can thus afford more hedging because they have more collateral represented by cash holdings, which is the only tangible asset of a firm in our model; this follows the same logic as in Rampini et al. (2014). Therefore, the observed pattern is determined by the trade-off between a complementary effect of cash, serving as collateral so as to afford more hedging, and a substitution effect, when the cost of holding cash is large.

Consequently, the substitution effect detected in H-type firms indicates that the cost consideration of holding cash dominates the collateral argument for firms who could more effectively hedge their risks, whereas the complementary effect documented in L-type firms suggest reserving cash (collateral) is more beneficial in terms of establishing the optimal risk management strategy. The total effect from this corporate risk management strategy is enhanced, evidenced by declining frequency of distress and external financing loss.

Effect of model uncertainty

To separate the effect due to model uncertainty, we need two benchmark models. The first is Model 3, with no model uncertainty. The second model faces model uncertainty but we simulate a counterfactual scenario where firms do not learn from jointly realised cash flow and the price of hedging assets. Particularly, the prior belief is not updated thus remaining 0.5 through all simulated 50 years. We name this the uninformed model in the following discussion. While the first benchmark model is the standard benchmark model in previous literature, the second one is usually ignored, such as in DeMarzo and Sannikov (2016) and Décamps et al. (2015). However, inclusion of the second benchmark model is meaningful in this paper. We focus not only on the loss due to model uncertainty when compared to the first-best scenario model, which is the main interest of previous study, but also on the effect and value of the ability to learn. Moreover, we include an additional model where a firm has a known correlation equal to the expected level under 'neutral belief', i.e. $\rho = 0.5$, to further illustrate the effects of parameter uncertainty. We name this the expected model. For a fair comparison, we grant firms steady-state cash as an endowment.

{Insert Figure 3.2 here.}

Figure 3.2 summarises and helps us understand the main message of this section. We observed that risk management strategies from a learning model are mostly bounded between those from a perfect knowledge model and those from an uninformed model. As knowledge approaches perfect, this deviation decays and finally vanishes in the long-run. Particularly, we found that a learning H-type firm generally uses less (more) hedge than its first-best counterpart (an uninformed firm). However, the H-type firm stores more (less) cash than it would do in the first-best (uninformed) scenario. This inversion supports the hypothesis that model uncertainty works in the direction of an increasing substitution effect between risk management tools, as a higher cash reserve would otherwise result in higher hedging if it worked in favour of complementarity.

However, a complementary effect is also evidenced in the lagged hedging with respect to cash reserves. As in Figure 3.2, cash grows from the first year, whereas hedging picks up in the third year. After initiating hedging, cash balance decreases to a stable level and is further reduced in the long-run. This indicates that a substantial level of cash stock is optimal to start hedging. This complementary effect is explained by cash's collateral role as in Rampini et al. (2014), and does not rely on model uncertainty. As shown in Figure 3.2, all three models exhibit the same complementary effect in the first few years. This might explain why an uninformed model also exhibits an increasing pattern in hedge in the first few years.

{Insert Figure 3.3 here.}

A similar pattern is documented for L-type firms as shown in Figure 3.3. As in the H-type case, the level of hedge in learning models is bounded between the other two benchmark models, and as knowledge approaches perfect status, firms' hedging positions approach those in the perfect knowledge model – monotonic with respect to belief. In L-type firms, cash reserves are always preferred. Thus, we observe a smaller difference in cash between all three models for L-type firms.

Rather than H-type learning firms, who use insufficient hedge but excessive cash, L-type learning firms instead abuse hedge and build up an inadequate level of cash. Therefore, as we have observed in the H-type scenario, there is an inversion in the combination of hedge and cash for a learning L-type firm with respect to its perfect knowledge counterpart. Namely, while a learning firm holds less cash, it uses more hedge than a perfect knowledge firm. This supports our previous claim that model uncertainty enhances the substitution effect. Again, we documented the model-uncertainty-independent complementarity in the first few years in all models as in the H-type scenario. We also find that when a firm has a know level of correlation, which equals to the expected level of 'neutral belief', it would use less hedge but more cash comparing to a H-type learning firm and less cash comparing to a L-type learning firm. This makes sense as hedging becomes more effective when the correlation goes to one. This finding also evidences the substitution effect, as when hedging is less effective, a firm has to reserve more cash to weather the bad states of the world.

{Insert Table 3.5 here.}

{Insert Table 3.6 here.}

We also tabulated the evolution in Table 3.5 for H-type firms and in Table 3.6 for L-type firms. First, the patterns of using risk management tools observed in the perfect knowledge model were confirmed in the model uncertainty scenario, e.g. H-type firms use more hedge and enjoy less frequent distress and external financing loss than L-type firms in the steady state.

Second, the levels of hedge in learning models are bounded between those of perfect knowledge model and those of an uninformed model. As the knowledge approaches perfect status in the learning model, the results approach those in the perfect knowledge model. Since the hedging level of informed firms is higher than that of uninformed firms, this observation leads to the implication that H-type firms' hedging positions increase with belief.

When model uncertainty is integrated into the model, as in Table 3.5 and Table 3.6, H-type firms quickly increase their hedge in the first few years when knowledge is largely improved, and results in a higher hedging position in the long-run. In contrast, L-type firms use much less hedge. Since upon maturity the H-type firms take on more hedge, it is consistent with our previous claim that a firm with better knowledge about the hedgeability of its business will prefer hedge to minimize cost incurred by financial distress in bad states.

This is intuitively reasonable as mature firms are the firms who have learned their hedgeability through many years of operation, and thus are able to make the best use of hedging instruments while keeping hedging cost low. This interpretation answers Stulz (1996), where he retrospected "[t]he actual corporate use of derivatives, however, does not seem to correspond closely to the theory ... large companies make far greater use of derivatives than small firms, even though small firms have more volatile cash flows, more restricted access to capital, and thus presumably more reason to buy protection against financial trouble". Note that although the model setting is mostly linear due to the parsimony of our model, non-monotonicity and non-linearity could still emerge when sources of friction such as external financing cost and tax are introduced, as in Gamba and Triantis (2008).

Cash might be one of the most subtle variables in corporate risk management. Firms actively change their cash balance level over time. Decreases in cash balance usually occur if the firm draws on internal liquidity to overcome low profitability, and if the firm builds up a hedging position. Increases in cash balance enable the firm to weather negative cash flow shocks, as well as to provide additional liquidity for future establishment of hedge, particularly if the cash balance has been drawn down. Although our results show that the cash stock remains substantial even if the cash flow and price of underlying asset of hedging derivatives is almost perfectly correlated – ρ is as high as 0.9 indicating cash holding is an indispensable method of corporate risk management – there are important limitations. First, there is a cost associated with holding cash. Second, persistent unfavourable states will gradually deplete a firm's cash stock, even if it was substantial. Distress costs will then be triggered with some probability. Alternatively, to build up a large cash balance requires the firm either to yield a sufficient string of positive cash flows or to raise costly external finance, which might not be possible. Having analysed cases in which cash stock is beneficial but costly, we might be able to explain why firms store more cash in case of need during bad states of the world, given hedging effect could be very noisy due to imperfect knowledge during the early stage. A discussion of cash continues in the next section, where results from models with different risk management tools are presented.

Thus, we conclude that the previously described features stand out from extant literature as they raise distinct predictions of model uncertainty in the presence of learning: corporate hedging position increases with belief that its operating cash flow is highly correlated with the price of underlying asset of derivatives designated as a hedge; high correlation firms use more hedge than low correlation firms; and cash's role in corporate risk management is more sophisticated than has been studied in previous literature. This separation might be the reason for mixed empirical findings, if they did not control for knowledge accumulation of firms.

Finally, we need to point out that statistically, although large, mature firms are usually knowledgeable firms and small, young firms are uninformed firms, the merit of this paper stems from separating these surface characteristics of firms from the intrinsic driver of the use of cash and hedge – the accumulation of knowledge.

3.5 Discussions

Marginal effect of individual risk management tool

In this section, we further probe the marginal effect of each risk management tool, i.e. hedging with swap or cash, by studying three additional models: one benchmark model with no risk management at all, one with only cash reserve as risk management tool, and one with only the swap contract defined in Section 3.3. Together with the full model with both hedge and cash analysed in Subsection 3.4.2, we were able to quantify the marginal effect of individual risk managing devices when model uncertainty is gradually resolved.

{Insert Figure 3.4, 3.5 and Table 3.7, 3.8 here.}

As illustrated by Figure 3.4 and 3.5 with details in Table 3.7 and 3.8, H-type firms tend to hedge more when this is the only risk management method, whereas L-type firms use slightly less hedge under the same scenario. This implies that the complementary effect of cash is more outstanding for L-type firms, while the substitution effect dominates in the case of H-type firms. However, even for H-type firms, these two effects coexist, as we observed that in the first few years, firms equipped with dual risk management tools were able to increase their hedging positions more quickly with the aid of retained cash. Having access to hedging relieved both firm types' cash demand, indicating hedging substitutes for cash stock in models with uncertainty. These patterns make our previous hypothesis about the coexistence of both substitution and complementary relationship clear. Therefore, the coexistence of both substitutionary and supplementary effects suggest neither of these risk management tools is dispensable, as was claimed in Gamba and Triantis (2014).

As indicated in Table 3.7 and Table 3.8, we found that either risk management tool does reduce loss caused by external financing costs. Particularly, when comparing the fourth panel in both tables, while cash has very similar effect in reducing loss for both firm types, hedge is much more efficient for H-type firms. This is consistent with one of our hypotheses: hedge is more valuable to H-type firms.

Finally, we present an interesting finding of this subsection. Our results might shed light on the mixed empirical evidence on the value creation of hedging. Under our main parameter settings, as shown in Table 3.4.1, hedging by itself seems not to increase firm value for both H- and L-type firms, as illustrated in the third panel of Figure 3.4 and 3.5. Due to numerical reasons, particularly the discretisation and the curse of dimensionality, limited points on the mesh grid result in enterprise values in no-hedging model sometimes outrun those in hedge-only model. This situation could be mitigated by using finer grid, although we keep the number of points on the mesh grid of the most complicated model, i.e. the model with both cash and hedging as well as uncertainty to ensure comparison across tables. However, while firm value does not seem to be affected greatly, the difference in correlation does create distinct corporate risk management strategies, as H-type firms hedge substantially more than their L-type peers. This is consistent with empirical research, mentioned in previous sections, that finds no evidence for hedging creating firm value. The next subsection presents results of some sensitivity analyses on the factors that might affect firm value.

Discussion on value creation by hedging

In the previous sections, we found little value creation or even little value destroyed when firms can hedge. On one hand, we contribute to the literature by showing that mixed empirical evidence on firm value created by hedging is theoretically possible under reasonably calibrated parameters. On the other hand, another research question naturally raises itself – what might affect the value creation of hedging? In this section, we present the results of several sensitivity checks on parameters that might affect firm value through hedging. We also compared the results with those from an unlimited liability firm model. The results are summarised in Table 3.9. We found that the tested parameters change firm value in a non-monotonic fashion. Moreover, whether hedging creates value for a limited liability firm depends on the trade-off between expected savings on friction (external financing costs, in our paper) and losses in default option value.

We speculate that parameters affecting profitability, hedging technology, break points, and the curvature of cost function, as well as capital structure and firms' financial liability, might also affect firm value creation. We altered one of these parameters at a time and compared the firm value of a no-risk-management firm and a hedging-only firm, i.e., no cash reserves. The difference is the value created or destroyed by hedging, because the two firms are otherwise identical.

Because profitability has a first order influence on firm value, whenever we altered some other parameter, for example distress costs, we repeated the calculation of firm value for each profitability level. Therefore, each column in Table 3.9 presents firm values of different overhang cost (A), with all other parameters and settings remaining the same.

{Insert Table 3.9 here.}

Each panel in Table 3.9 represents a change to other parameters. In the first panel, the distress costs were increased to .5 to create a larger hedging demand. We observes slight value creation when the overhang cost (A) is low, .85 and .9, but larger value destruction when it is .95 or .97. This indicates that higher distress costs do not guarantee firm value creation. However, many previous papers take the point of view that the larger the downside costs, the larger the amount that hedging could save and consequently increase the firm value. Interestingly, this firm value creation - destruction is non-monotonic with respect to profitability. We observe some firms do not manage their risk at all, because in the most extreme case, when they are financially constrained and close to bankruptcy, risk shifting would dominate their risk management choice. This is evidenced by Rampini et al. (2014), who find a risk-shifting behaviour in small, financially constrained airlines who "hedge less or not at all." Similar results are also found when we mute renegotiation costs applied on the adjustment of hedging position from 1%.

In a separate test in which we added constant debt into the model (results not reported here), we note that the value creation - destruction is much less when no leverage is compared to that in a moderately levered scenario, even if we maximised the firm value instead of equity value.

As implied by the non-monotonicity and changes due to capital structure, we finally verify and conclude that limited liability plays the major role in firm value creation – destruction due to corporate risk management, especially by hedging. In the last panel, simply removing limited liability restores firm value creation as predicted by classic corporate risk management literature.

So far, both sides of empirical literature are supported by our model. The reason is that whether hedging creates value or not depends on firm characteristics such as profitability as well as market conditions, such as external financing costs, and regulations on bankruptcy. These factors determine the structure of the combined cost function and the value function of the default option. Consequently, optimal firm value is decided by the trade-off between reducing expected external financing cost and increasing the real option value of the firm.

This mechanism could be understood by revisiting the trade-off which determines firm value. On one hand, a firm could increase its value by actively managing its downside risk so as to mitigate expected external financing losses. As a result, high correlation firms better control their operating risk with lower deadweight losses, caused by shortfall in cash flow due to stochastic realisation and risk management decisions made in both the preceding and current periods. As a natural consequence, enterprise value is created from the improvement in efficiency of hedging. On the other hand, risk management is costly. Implicitly, given limited liability, assuming certain risk could be value enhancing under certain circumstance. This trade-off entangles with substitution – complementary relationships between hedge and cash reserve in our model, which are analysed in previous sections.

3.6 Conclusion

Using a dynamic model, we find that model uncertainty can affect corporate risk management policy. In this paper, the particular uncertainty lies in the correlation between a firm's cash flow and the price of the underlying asset of a swap, which the firm uses to hedge its risk. The resolution of uncertainty in correlation drives distinct dynamics of corporate risk management strategies of different firms. Namely, a firm with better hedgeability prospects tends to increase the use of hedge whilst firms believed to have low hedgeability rely relatively more on cash, and we prove it is not due to the accumulation of liquidity. We also identify both the substitutionary effect and complementary effect between hedging and cash stock. Particularly, cash's collateral effect is more pronouncing in high correlation firms, resulting a higher hedging position for these firms. Albeit diverse choices in hedging policy and cash management policy, the difference in firm value could be small under reasonable range of parameters. We argue that both optimal corporate risk management policy and resulted firm value not only depends on the trade-off between minimising costs associated with market friction and minimising costs incurred by managing risk, but also the trade-off between maximising the net benefits from risk management and maximising the value of default option. In this regard, this paper might help explain why young firms are observed to use less hedge than mature firms and shed light on understanding mixed evidence on the value creation role of hedging. Although this paper does not touch the strategic entering into a new market, it is interesting to further probe the idea that an entering firm could learn not only from the operational results of itself but also from observing others. An educated guess would be that the information from peers' operation could pass through correlation with the operational results of the entering firm, or constantly raise the prior belief

- both accelerate the speed of the resolution of uncertainty.

3.7 Appendix

3.7.1 Appendix:Numerical Methods

We solve the Bellman equation described in Section 3.3 by simulating 1,000 firms over 50 years. Each path for the state variable $\boldsymbol{x}$ is obtained by iterating Equation (3.1) using Monte Carlo simulation, for 50 time steps. The simulated paths for $\boldsymbol{x}$ are restricted to a set of discrete values $\tilde{\boldsymbol{x}}$, following the Tauchen (1986) approach. Due to the curse of dimensionality, the number of sampling points is chosen to be 19 as a result of trade-off between accuracy and efficiency. However, we ran special tests to guarantee that this choice of grid size yields simulation results with errors in the relevant moments (mean, volatility and correlation) that are within 1% of their theoretical counterparts. For robustness, we also have a finer grid and find very similar results.

Particularly, we discretise the sets of h, c and q respectively in $[0, h_u]$, $[0, c_u]$ and $[0, 1]$ with 31 equally spaced points. The variables x_1 and x_1 define a reduced-form vector autoregression that we approximate through a discrete-state Markov chain with 11 points for each variable with truncated support in $[-3\sigma_j^u, 3\sigma_j^u]$, $j = 1, 2$, where $\sigma_j^u = \sigma_j/\sqrt{1-\kappa^2}$ is the unconditional standard deviation for $j = 1, 2$. The discrete abscissae and the risk-neutral Markov transition probabilities are computed according to the method proposed by Terry and Knotek (2011), which is based on the Gauss-Hermite quadrature rule, as in Tauchen (1986), but allows for non-zero correlation.

A Monte Carlo simulation is used to generate a sample path for the firm following an optimal policy. We generate a sequence of 500,000 independent draws from a truncated bivariate Normal distribution and generated a path for x_1 and x_2 using the AR(1) specification. Starting from an initial condition (h_0, c_0, q_0), we apply the optimal policy from Equation 3.7 and generate a simulated path for the firm. We drop the first 200 observations, to exclude any influence of the initial condition. In the event the firm defaults, a new firm starts business with neutral belief and with no financial hedging in place ($h = 0, c = 0$).

3.7.2 Appendix:Figures and Tables

σ_i	annual volatility of x_i	15%
ρ^H	correlation for high type	0.9
ρ^L	correlation for low type	0.1
κ_i	persistence of x_i	0.8
s	swap price of the swap contract	1
A	fixed production cost	0.88
r_c	return on cash	3.6%
r_f	annual risk-free rate	5%
λ	distress cost	15%
rc	renegotiation cost	1%

Table 3.1: Parameter Values

	Model 1	Model 2	Model 3
Hedge	0.2969	0.2229	0.1679
Cash	-	-	0.5458
EV	4.1738	3.9686	4.0690
Equity Issue	0.2166	0.3109	0.1501
Distress	0.1822	0.2520	0.0584
Loss	0.0012	0.0028	0.0008

Table 3.2: Simulation results of baseline models with no model uncertainty

Each column in this table presents the average of up to seven metrics from three baseline models, with one restriction relaxed at each time. The metrics are: the level of hedge; the cash balance; the firm/enterprise value of the firm; the frequency of external equity financing, the frequency of financial distress; and external financing loss. Model 1 is the baseline model without cash and perfect correlation; in Model 2, firms' cash flows become imperfectly correlated; and finally Model 3 adds cash as an alternative risk management tool. In this table, size of mesh grid is $11 \times 31 \times 21$ (cash flow $\times$ hedging position $\times$ cash holding). The other parameters are given in Table 3.1.

age	2-4	5-7	8-10	11-13	14-16	17-50	2-50
				H type			
Hedge	0.1039	0.1836	0.1832	0.1834	0.1783	0.1695	0.1686
Cash	0.1925	0.3744	0.4565	0.4954	0.5079	0.5349	0.4952
EV	4.0764	4.0489	4.0293	4.0342	4.0302	4.0881	4.0745
Equity Issue	0.2727	0.2273	0.2000	0.1827	0.1740	0.1487	0.1679
Distress	0.1510	0.1170	0.0980	0.0827	0.0703	0.0590	0.0727
Loss	0.0037	0.0015	0.0013	0.0011	0.0010	0.0008	0.0011
Default	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
				L type			
Hedge	0.0277	0.0756	0.0925	0.0986	0.0935	0.0951	0.0898
Cash	0.2000	0.4195	0.5230	0.5709	0.5895	0.6156	0.5682
EV	4.0469	4.0350	4.0276	4.0265	4.0234	4.0774	4.0635
Equity Issue	0.3043	0.2862	0.2509	0.2254	0.2352	0.2045	0.2216
Distress	0.1769	0.1652	0.1251	0.1120	0.1089	0.0835	0.1001
Loss	0.0027	0.0032	0.0025	0.0024	0.0024	0.0018	0.0021
Default	0.0007	0.0030	0.0013	0.0020	0.0010	0.0014	0.0015

Table 3.3: H vs L, with cash, no model uncertainty

For Model 3, this table presents the evolution of the average of six metrics: the level of hedge; the cash balance; the firm/enterprise value of the firm; the frequency of external equity financing, the frequency of financial distress; and external financing loss. They are tabulated against years, with each column reports the average metrics during the corresponding years. The final column presents the unconditional mean of the entire simulated sample. In this table, the size of mesh grid is $11 \times 31 \times 21 \times 31$ (cash flow $\times$ cash holding $\times$ hedging position $\times$ beliefs). The other parameters are given in Table 3.1.

age	2-4	5-7	8-10	11-13	14-16	17-50	2-50
				H type			
Hedge	0.0657	0.1321	0.1583	0.1672	0.1646	0.1723	0.1617
Cash	0.1932	0.3809	0.4701	0.5058	0.5116	0.5343	0.4970
EV	4.0732	4.0578	4.0221	4.0234	4.0400	4.0619	4.0562
Equity Issue	0.2421	0.2338	0.1965	0.1807	0.1640	0.1458	0.1635
Distress	0.1807	0.1412	0.1000	0.0864	0.0728	0.0602	0.0773
Loss	0.0045	0.0022	0.0014	0.0012	0.0010	0.0008	0.0012
Default	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Belief	0.6645	0.8034	0.8840	0.9392	0.9733	0.9981	0.9537
				L type			
Hedge	0.0439	0.0974	0.1026	0.1041	0.0987	0.0973	0.0948
Cash	0.1995	0.4181	0.5203	0.5710	0.5894	0.6126	0.5658
EV	4.0228	4.0120	4.0165	4.0071	3.9988	4.0586	4.0442
Equity Issue	0.2844	0.2755	0.2450	0.2244	0.2332	0.2039	0.2188
Distress	0.1701	0.1573	0.1213	0.1096	0.1054	0.0834	0.0985
Loss	0.0026	0.0029	0.0024	0.0023	0.0022	0.0018	0.0020
Default	0.0007	0.0038	0.0014	0.0021	0.0014	0.0015	0.0016
Belief	0.1532	0.0405	0.0102	0.0038	0.0013	0.0010	0.0135

Table 3.4: H vs L, with model uncertainty

For the full model, this table presents the evolution of the average of six metrics: the level of hedge; the cash balance; the firm/enterprise value of the firm; the frequency of external equity financing, the frequency of financial distress; and external financing loss. They are tabulated against years, with each column reports the average metrics during the corresponding years. The final column presents the unconditional mean of the entire simulated sample. In this table, the size of mesh grid is $11 \times 31 \times 21 \times 31$ (cash flow $\times$ cash holding $\times$ hedging position $\times$ beliefs). The other parameters are given in Table 3.1.

			Hedge				
age	2-4	5-7	8-10	11-13	14-16	17-50	2-50
Informed	0.0669	0.1408	0.1550	0.1624	0.1611	0.1678	0.1585
Learner	0.0590	0.1194	0.1410	0.1494	0.1531	0.1706	0.1565
Uninformed	0.0362	0.0737	0.0792	0.0769	0.0772	0.0846	0.0797
Belief	0.6645	0.8034	0.8840	0.9392	0.9733	0.9981	0.9537
			Cash				
Informed	0.5957	0.5694	0.5673	0.5651	0.5485	0.5410	0.5497
Learner	0.6034	0.5825	0.5808	0.5767	0.5554	0.5413	0.5531
Uninformed	0.6286	0.6226	0.6238	0.6226	0.6079	0.6054	0.6102
			Enterprise Value				
Informed	4.1275	4.1067	4.0651	4.0621	4.0742	4.0905	4.0895
Learner	4.0933	4.0799	4.0392	4.0397	4.0513	4.0634	4.0626
Uninformed	4.0831	4.0690	4.0319	4.0330	4.0462	4.0630	4.0598
			Loss				
Informed	0.0006	0.0007	0.0007	0.0009	0.0007	0.0008	0.0008
Learner	0.0006	0.0008	0.0008	0.0009	0.0007	0.0008	0.0008
Uninformed	0.0005	0.0013	0.0013	0.0015	0.0013	0.0014	0.0013

Table 3.5: Effect of learning, H-type

For H-type, this table compares the average of four metrics: the level of cash balance, the hedge, enterprise value of the firm, and the deadweight loss across three models: ignorant, learning and perfect knowledge, as described in Subsection 3.4.2. They are tabulated against years of learning. The final column presents the unconditional mean of the entire simulated sample. In this table, the size of mesh grid is $11 \times 31 \times 21 \times 31$ (cash flow $\times$ cash holding $\times$ hedging position $\times$ beliefs). The other parameters are given in Table 3.1.

age	2-4	5-7	8-10	11-13	14-16	17-50	2-50
Hedge							
Informed	0.0360	0.0757	0.0873	0.0910	0.0864	0.0935	0.0879
Learner	0.0471	0.0894	0.0960	0.0960	0.0891	0.0952	0.0916
Uninformed	0.0489	0.1064	0.1141	0.1197	0.1118	0.1192	0.1134
Belief	0.1532	0.0404	0.0101	0.0038	0.0013	0.0010	0.0135
Cash							
Informed	0.6434	0.6453	0.6411	0.6411	0.6322	0.6211	0.6270
Learner	0.6344	0.6403	0.6398	0.6402	0.6336	0.6199	0.6254
Uninformed	0.6356	0.6293	0.6234	0.6146	0.6100	0.5997	0.6067
Enterprise Value							
Informed	4.0698	4.0668	4.0570	4.0470	4.0318	4.0790	4.0715
Learner	4.0564	4.0516	4.0408	4.0300	4.0181	4.0631	4.0558
Uninformed	4.0623	4.0570	4.0429	4.0320	4.0254	4.0725	4.0638
Loss							
Informed	0.0004	0.0013	0.0015	0.0015	0.0020	0.0017	0.0016
Learner	0.0004	0.0012	0.0014	0.0014	0.0018	0.0017	0.0016
Uninformed	0.0004	0.0015	0.0018	0.0019	0.0020	0.0020	0.0019

Table 3.6: Effect of learning, L-type

For L-type, this table compares the average of four metrics: the level of cash balance, the hedge, enterprise value of the firm, and the deadweight loss across three models: ignorant, learning and perfect knowledge, as described in Subsection 3.4.2. They are tabulated against years of learning. The final column presents the unconditional mean of the entire simulated sample. In this table, the size of mesh grid is $11 \times 31 \times 21 \times 31$ (cash flow $\times$ cash holding $\times$ hedging position $\times$ beliefs). The other parameters are given in Table 3.1.

Hedge							
age	2-4	5-7	8-10	11-13	14-16	17-50	2-50
Hedge Only	0.0552	0.1260	0.1694	0.1921	0.2056	0.2207	0.1990
Both	0.0657	0.1321	0.1583	0.1672	0.1646	0.1723	0.1617
Cash							
Cash Only	0.2051	0.4338	0.5422	0.5929	0.6146	0.6476	0.5956
Both	0.1932	0.3809	0.4701	0.5058	0.5116	0.5343	0.4970
Enterprise Value							
None	4.0053	3.9892	3.9571	3.9640	3.9717	3.9898	3.9860
Hedge Only	4.0273	4.0069	3.9632	3.9529	3.9628	3.9775	3.9791
Cash Only	4.0663	4.0543	4.0256	4.0335	4.0422	4.0618	4.0565
Both	4.0732	4.0578	4.0221	4.0234	4.0400	4.0619	4.0562
Loss							
None	0.0061	0.0061	0.0059	0.0059	0.0061	0.0059	0.0060
Hedge Only	0.0059	0.0039	0.0032	0.0032	0.0030	0.0027	0.0031
Cash Only	0.0043	0.0044	0.0033	0.0029	0.0027	0.0023	0.0027
Both	0.0045	0.0022	0.0014	0.0012	0.0010	0.0008	0.0012

Table 3.7: Marginal effect of risk management tools, H-type

For H-type, this table compares the average of up to four metrics: the level of hedge, th cash balance, the enterprise value of the firm, and the deadweight loss across four models: no risk management, managing risk with only cash, with only hedge and with both, as described in Section 3.5. They are tabulated against years of learning. The final column presents the unconditional mean of the entire simulated sample. In this table, the size of mesh grid is $11 \times 31 \times 21 \times 31$ (cash flow $\times$ cash holding $\times$ hedging position $\times$ beliefs). The other parameters are given in Table 3.1.

			Hedge				
age	2-4	5-7	8-10	11-13	14-16	17-50	2-50
Hedge Only	0.0390	0.0863	0.0955	0.0973	0.0920	0.0896	0.0873
Both	0.0439	0.0974	0.1026	0.1041	0.0987	0.0973	0.0948
			Cash				
Cash Only	0.2057	0.4313	0.5410	0.5950	0.6200	0.6484	0.5964
Both	0.1995	0.4181	0.5203	0.5710	0.5894	0.6126	0.5658
			Enterprise Value				
None	3.9749	3.9670	3.9542	3.9551	3.9391	3.9880	3.9789
Hedge Only	3.9742	3.9528	3.9383	3.9311	3.9148	3.9763	3.9659
Cash Only	4.0365	4.0330	4.0235	4.0263	4.0114	4.0611	4.0504
Both	4.0228	4.0120	4.0165	4.0071	3.9988	4.0586	4.0442
			Loss				
None	0.0058	0.0059	0.0058	0.0059	0.0062	0.0059	0.0059
Hedge Only	0.0039	0.0046	0.0045	0.0049	0.0051	0.0046	0.0046
Cash Only	0.0041	0.0042	0.0033	0.0028	0.0027	0.0023	0.0026
Both	0.0026	0.0029	0.0024	0.0023	0.0022	0.0018	0.0020

Table 3.8: Marginal effect of risk management tools, L-type

For L-type, this table compares the average of up to four metrics: the level of hedge, th cash balance, the enterprise value of the firm, and the deadweight loss across four models: no risk management, managing risk with only cash, with only hedge and with both, as described in Section 3.5. They are tabulated against years of learning. The final column presents the unconditional mean of the entire simulated sample. In this table, the size of mesh grid is $11 \times 31 \times 21 \times 31$ (cash flow $\times$ cash holding $\times$ hedging position $\times$ beliefs). The other parameters are given in Table 3.1.

	Overhang cost (A)	.88	.9	.95	.97
λ (.5)	None	3.7305	3.1294	2.1662	1.7965
	Hedge Only	4.0706	3.4668	1.8616	1.0932
rc (0)	None	4.1156	3.5301	2.3016	2.0285
	Hedge Only	4.2601	3.6813	2.1962	1.7998
unlimited	None	3.9459	3.3947	1.9946	1.4300
	Hedge Only	5.3096	4.2185	2.8855	2.3516

Table 3.9: Value Creation

Various combinations of factors that might affect firm value are analysed in this table. Results of Model 1 with $\lambda = 0.5$, $rc = 0$ and unlimited liability are tabulated against different profitabilities. The other parameters remain the same as in Table 3.1. In each combination, we compare the firm value of a no-risk-management firm (None) and a hedging-only firm (Hedge Only).

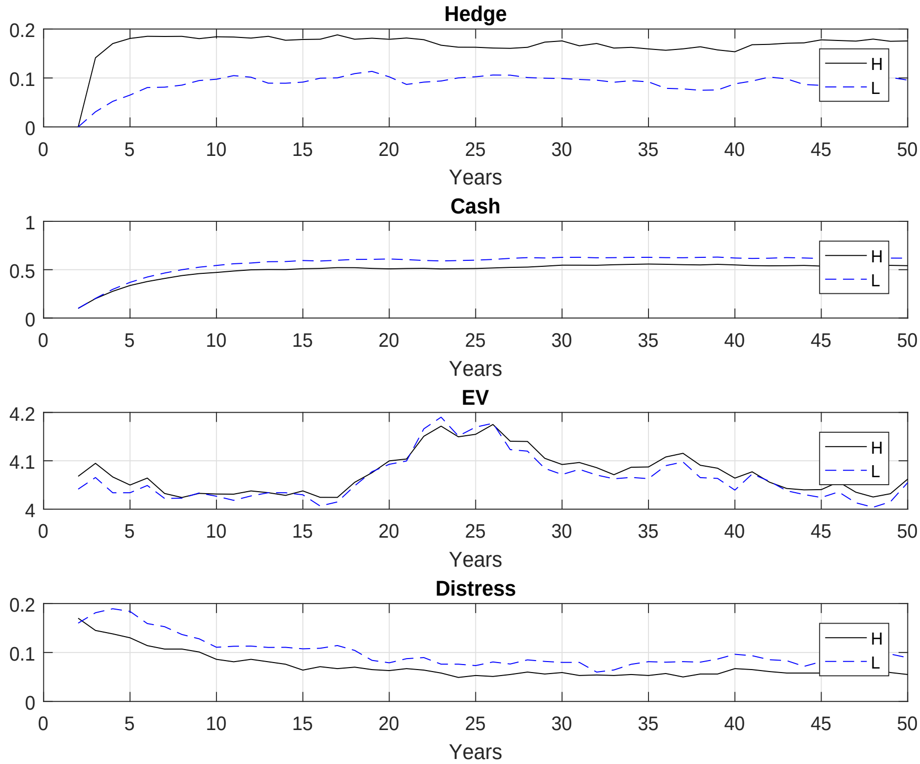


Figure 3.1: Comparison between H and L type

This figure shows the difference in average hedge, cash stock, enterprise value and distress probability between H-type and L-type from the perfect knowledge Model 3.

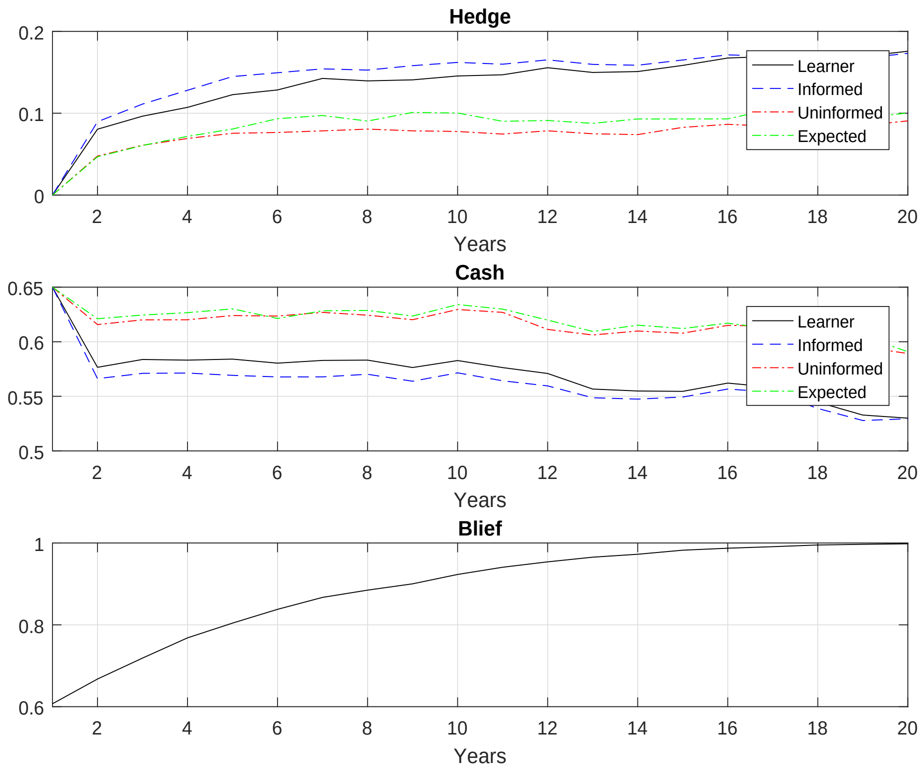


Figure 3.2: Effect of model uncertainty: Average, H-type

This figure shows average hedge and cash stock in lieu of the effect of model uncertainty on hedge and cash stock for H-type firm based on the set-up of Model 3 with learning. The bottom panel plots corresponding belief evolution.

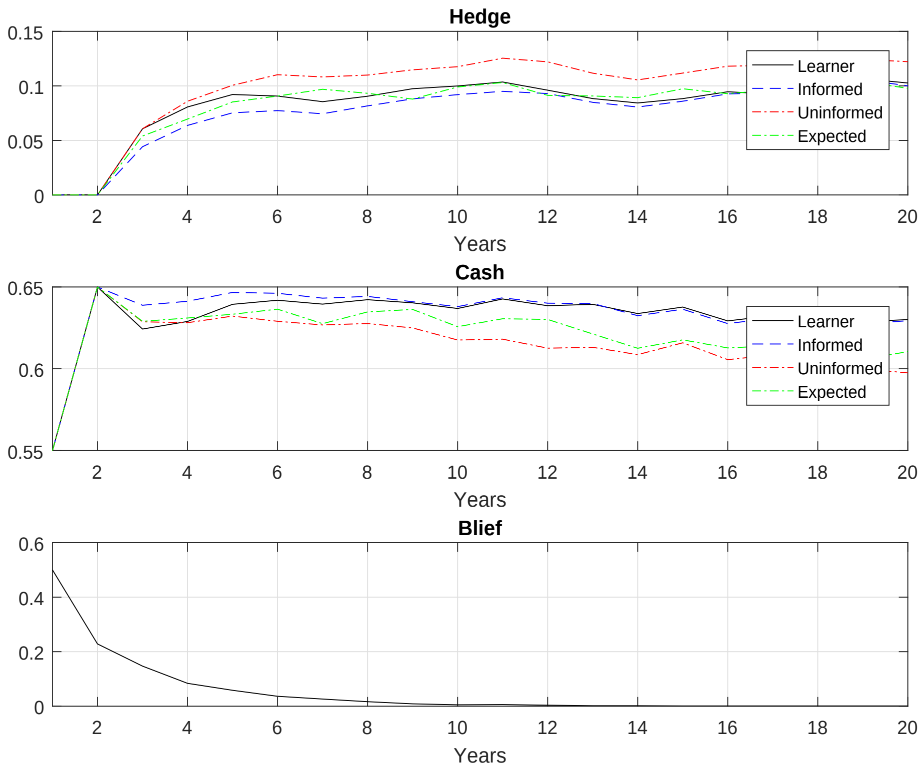


Figure 3.3: Effect of model uncertainty: Average, L-type

This figure shows average hedge and cash stock in lieu of the effect of model uncertainty on hedge and cash stock for L-type firm based on the set-up of Model 3 with learning. The bottom panel plots corresponding belief evolution.

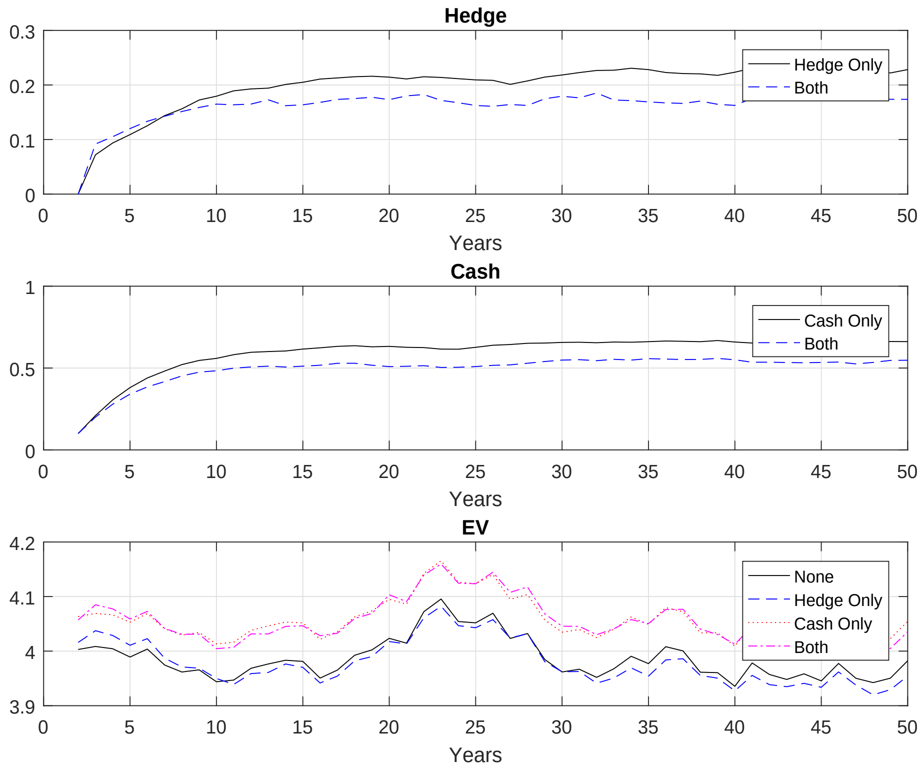


Figure 3.4: Marginal effects: H-type

This figure shows a visual illustration of the average marginal effects of different risk management tools on hedge, cash stock, and enterprise value for H-type firm based on four models: one with no risk management at all, one with only cash balance as risk management tool, one with only hedge, and the full model with both hedge and cash.

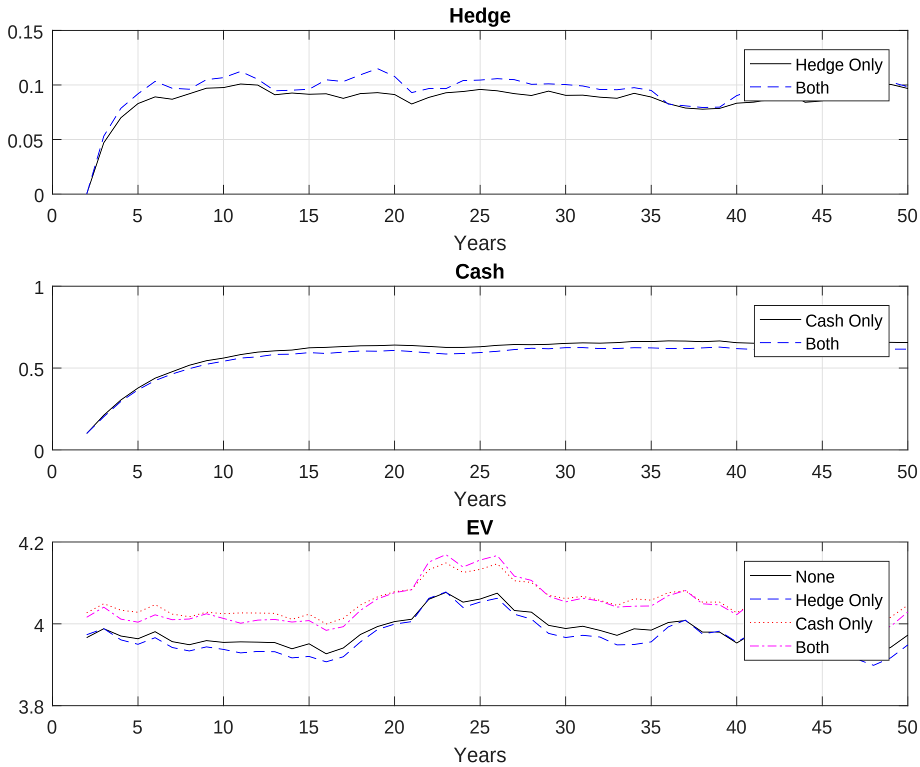


Figure 3.5: Marginal effects: L-type

This figure shows a visual illustration of the average marginal effects of different risk management tools on hedge, cash stock, and enterprise value for L-type firm based on four models: one with no risk management at all, one with only cash balance as risk management tool, one with only hedge, and the full model with both hedge and cash.

Chapter 4

Profitability Uncertainty and Corporate Risk Management

4.1 Introduction

Corporate risk management has gained a lot of of importance in the recent years, due to increasing risk. In a global data set, Guay and Kothari (2003) find that 57% firms around the world use derivatives. Bates et al. (2009) document a cash ratio of 23.2% in 2006, double that in 1980, for US firms. They also identify that the large increase was mainly due to precautionary motive.

The increased importance has motivated a large number of theoretical papers trying to explain the stylised facts and puzzles in observed corporate risk management policy. For example, Stulz (1996) once retrospected "[t]he actual corporate use of derivatives, however, does not seem to correspond closely to the theory ... large companies make far greater use of derivatives than small firms, even though small firms have more volatile cash flows, more restricted access to capital, and thus presumably more reason to buy protection against financial trouble." In the meanwhile, it is still in debate whether hedging creates firm value or not. Unlike what is predicted in most corporate risk management theories, empirical researchers, such as Brown (2001); Guay and Kothari (2003); Jin and Jorion (2006); Magee (2009); Bartram et al. (2011), find either non-significant firm value creation by hedging or instead evidence of value destruction.

One potential explanation for extant literature failing to match the evidence might be very fundamental: while they have successfully model the source of uncertainty associated with firm's cash flow, they did not allow for uncertainty on the model itself. Namely, in their researches, they have made an implicit assumption that the model is a sure thing, which might not be the case. Model uncertainty could exist in the structure of the model, which regulates firm's cash flow, or in the parameters, which govern the distributional characteristics of the cash flow produced by the model. Our paper deals with the later type of model uncertainty, and especially the uncertainty in firm's profitability. We argue that for a fair evaluation on the relative successfulness of corporate risk management policy, one should endogenise model uncertainty and its resolution.

In this regard, this paper relates to recent literature on uncertainty in profitability in the presence of certain market friction. Moyen and Platikanov (2013) develop a dynamic Tobin's q model, in which shareholders updates beliefs on firm's profitability. They find that uncertainty in profitability impacts firm's investment decisions, through the channel that firms with unclear profitability are more sensitive to realised earnings than mature firms who provide clearer profitability prospects. Although both works incorporate uncertainty in profitability in order to help us explain subtle patterns embedded in empirical evidence, the focus of their paper is on the impact on corporate investment policy, but we are more interested in how firms manage their risk in the presence of uncertainty in profitability. Moreover, they concentrate on to how firms' policies react to instantaneous shocks, whereas our paper pays more attention to the evolution of firms' policies.

More recently, Décamps et al. (2015) link uncertainty in profitability with corporate cash management and dividend payout policy. They find that both firm's target cash stock level and payout ratio is increasing with profitability prospects. This might explain the puzzle that we sometimes observe firms' cash balance increase hand in hand with their dividends. Their paper is more related to ours, as both of us discuss how cash reserve evolves with profitability prospects. Nevertheless, we aim to answer a different research question: how corporate risk management policy would be affected when firm's profitability is also uncertain, in addition to traditional uncertainty in firm's cash flows.

We adopt the risk management model in Gamba and Triantis (2014). This is a parsimonious model but has all the structural devices to accommodate our research purpose. In a risk-neutral world, there are two stochastic variables, firm's cash flow and the price of the underlying asset of a derivative. Their evolutions are governed by a multi-variate autoregressive process. The high/low profitability firm (later known as H-type/L-type) has a higher/lower long-term mean in the VAR process. All agents share the same information and could learn the type from realised observations. As the two variables are correlated, firm thus could hedge using the derivative, which is fairly priced taking early termination and default into account. The firm could also retain cash, although return on internal cash is lower than the risk free rate, indicating holding cash is costly. In each period, the firm incurs external financing costs upon liquidity shortfall. Default is possible as soon as equity value drops below zero. The belief of being a high profitable firm improves over time and the firm makes its corporate risk management decisions accordingly. We are thus able to pay special attention to how the resolution of uncertainty affects firm's hedging and cash reservation policy.

We find similar implication to Décamps et al. (2015). Both of us find that cash could increase hand in hand with its traditionally thought competitors. Particularly, they predict dividend payout ratio, which is the speed of draining cash out of a firm, increases with cash reserve. We add to their findings by pointing out that hedging position, which reduces the precautionary demand of cash, also increases with firm's cash balance. This is due to cash's collateral effect argued by Rampini et al. (2014). Hedge involves liability due to promise of future payment. A collateral would help to quickly build up desired hedging position. In our model, the only collateral is in the from of cash. Therefore, firm would raise external capital as soon as it can to build up cash balance and establish optimal level of hedge. A resolving uncertainty on the profitability would relieve the provision for collateral and thus increase both hedging position and cash stock.

This channel also leads young firms with unclear profitability to hedge less comparing to their mature counterparts, especially for the high profitability firms. As these firms tend to retain insufficient cash in the early stages, but increase cash stock with higher belief of being a high profitable firm. As a result, cash's collateral effect drives hedging position higher. That might be able to explain why established firms are found to use more hedge than young firms in many empirical researches.

While we observe significantly different choices in risk management tools, we also find that the difference in firm value is relatively small, and hedge does not always increase firm value. This finding is consistent with mixed empirical evidences aforementioned. We argue that optimal corporate risk management policy, as well as its consequence, depends on the trade-off between minimising the convex costs by actively managing firm's risk and maximising the value of default option by allowing for certain level of risk under some circumstances. We show that this could happen even under reasonable parameter settings.

At last, we identify both substitutionary and complementary relationship between cash and hedge, and the resolution of uncertainty in profitability enhances substitutionary effect.

The rest of the paper is organized as follows: Section 4.2 states the assumptions and sets up our model. Section 4.3 calibrates the model and shows optimal policies of cash holdings and hedging as the results of simulations. It also discusses marginal effect of each risk management tool. Section 4.4 concludes and discusses future and interesting future researches.

4.2 The Model

We build upon the structural model adopted in Chapter 3, which I paraphrase and repeat here. A tradable security, assumed to be price X_{2t}, is correlated with the stochastic cash flow produced by the firm, presumed as X_{1t}. Given this, X_{2t} could be written to define the perpetual swap contract with swap price s. To be specific, under the condition of neutral-risk probability, these two variables are governed by a joint vector autoregression (VAR) process, as following:

$$x_t - x_{t-1} = (1-\kappa)(\tau - x_{t-1}) + \epsilon_t \tag{4.1}$$

$$\text{where } x_t = \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix}, \qquad \kappa = \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix}, \qquad \tau = \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix},$$

$$\epsilon_t = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}, \ \epsilon_t \sim i.i.d.\ N(0, \Sigma), \ \Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

where $x_{it} = \log X_{it}$, τ_i are the long-term mean, $0 \leq \kappa_i \leq 1$ are the persistence parameters, and $\sigma_i > 0$ are the conditional standard deviations, $i = 1, 2$. Without compromising generality, this study considers the circumstance whereby the two state variables are in positive correlation,namely $\rho \geq 0$, so that the firm would take a long position in the swap contract.

The firm can reserve a proportion of cash flow, after paying a constant gross cost, A, to establish a cash stock, c_t. The return generated by cash, r_c, is lower than the risk-free rate, r_f. The wedge between the two values accounts for various frictions concerning holding cash. Once the cash holdings plus the cash flow are not enough to pay for the cost A, the firm incurs costs due to external financing, which is a proportion, λ, of the liquidity shortfall.

The uncertainty studied in this paper is about the long-term mean, τ_1, of the cash flow, which is also known as profitability. Profitability uncertainty exerts a first order effect on corporate policies and thus speaks highly of corporate hedging behaviour. We assume the long-term mean could be either high (τ_H) or low (τ_L).

In the beginning, a firm has neutral belief on its type. It adjusts the posterior belief, q', in its type using Bayes' rule after observing the pair of (X_1, X_2), at time t, as follows:

$$q' = \frac{qPr(x'|H)}{qPr(x'|H) + (1-q)Pr(x'|L)}$$

where

$$Pr(x'|\cdot) = \frac{\exp\left[-\frac{1}{2(1-\rho^2)}\left(\epsilon_1^2 - 2\rho\epsilon_1\epsilon_2 + \epsilon_2^2\right)\right]}{2\pi\sigma_1\sigma_2\sqrt{1-\rho_j^2}}, \qquad i \in \{H, L\} \tag{4.2}$$

$$\epsilon_i = \frac{x_i' - (1-\kappa_i)\tau_i - \kappa_i x_i}{\sigma_i}, \qquad i \in \{1, 2\} \tag{4.3}$$

which is the bivariate standard normal probability function.

The process, x_{2t}, regulating the underlying asset of a perpetual swap contract, is presumed to have the same volatility and persistence as the firm's cash flow. The mean is assumed to be the average of high and low type, and the swap price for a unit of product is fixed at $s = 1$. Later we show that these assumptions do not affect our conclusion, because the value of swap contract is simultaneously solved with firm value as discussed in Gamba and Triantis (2014).

Therefore, if a firm took a swap for a notional physical amount h at time t, at $t+1$ the net payoff from the swap would be $h(s - X_{2,t+1})$. Excluding counterparty risk and contractual flexibility, the value of per unit contract at moment t,is

$$S_t = S(X_{2t}) = \sum_{\tau=1}^{\infty} \frac{s - F_\tau(X_{2t})}{(1+r_f)^\tau} < \infty \tag{4.4}$$

where $F_\tau(x_{2,t}) = \mathbb{E}_t[x_{2,t+\tau}]$ is the forward price at time t for delivery of the asset at date $t+\tau$, and the expectation is computed under the risk-neutral probability.

If a firm's equity value together with its current liquidity surplus could not meet the promised payment against the swap, the firm will default on the swap. The firm could alter its position in the swap - say from h to h'. It redeems the current contract at the par value (S_t), and enters into a new agreement at market value SF_t. The net payoff from this transaction is thus $h \cdot S_t - h' \cdot SF_t$. Besides, considering the proportional negotiation cost generated in each transaction, nc, there is a direct cost of adjusting the hedge: $nc \cdot (h + h')$.

We compute the market value of the swap contract as in Gamba and Triantis (2014). By taking credit charges into account, we preserve the no arbitrage condition. The fair value of the contract, SF_t, reflecting both default risk as well as the possibility to close the swap position early, is:

$$SF_t = \mathbb{E}_t\left[\sum_{\tau=1}^{\min\{T_d,T_p\}} \frac{s - X_{2,t+\tau}}{(1+r)^{\tau}}\right] + \mathbb{E}_t\left[\chi_{\{T_d \geq T_p\}} \frac{S(X_{2,t+T_p})}{(1+r)^{T_p}} + \chi_{\{T_d < T_p\}} \frac{RS(X_{t+T_d})}{(1+r)^{T_d}}\right] \quad (4.5)$$

where T_d is the default date and T_p is the closure of the swap position. The event that default happens after the position in the swap is closed is indicated by $\chi_{\{T_d \geq T_p\}}$, while the opposite circumstance by $\chi_{\{T_d < T_p\}}$. $RS(\cdot)$ is the bank's recovery value on the swap if the firm defaults, which will be explained later. In this regard, adjusting the swap position at a later date entails an indirect cost, $h' \cdot SF_t - h \cdot S_t$.

The first term in equation (3.6) is the present value of the net payoff to the firm before rebalancing its hedge position or default. The second term is the payoff to the bank if the firm terminates the swap or defaults. The bank is paid with par value in the former case, or recovery value in the latter case. The fair value of the swap contract depends on corporate policy, as T_d and T_p are so determined.

Since we are interested in firms' reaction to resolution of uncertainty, the objective is equity (firm) value maximisation and associate optimal policies. Therefore, the value of firm is:

$$\begin{aligned} V(X,c,h,q) = \quad & \max\{0, \max_{(c',h')}\Big\{D + \tfrac{1}{1+r_f}\{q\mathbb{E}_X[V(X',c',h',q')|H] \\ & +(1-q)\mathbb{E}_X[V(X',c',h',q')|L]\}\}\} \end{aligned} \quad (4.6)$$

where cash flow from current period is $D = \max\{d,0\} + \min\{d,0\}(1+\lambda)$, in which the payout is

$$d = X_1 - A + (1+r_c)c + h(s - X_{2,t}) + (hS_t - h'SF_t - nc(h+h'))_{\chi\{h' \neq h\}} - c' \quad (4.7)$$

where $\chi_{\{h' \neq h\}}$ is the indicator function of event $h' \neq h$, and $\mathbb{E}_X[\cdot|H/L]$ is the expectation conditional on the current state (X) as well as the type.

Default happens when the second argument in max is negative. All the remaining cash balance is paid out upon default, and the hedge position is liquidated and settled $(h = 0)$.

Bank's recovery value $(RS(\cdot))$ depends on the firm value at time $t + T_d$:

$$RS(X') = \max \left\{ -(s - X_2' + S_t), \frac{c + V(X', 0, 0, q')}{h} \right\}$$

per unit of notional amount of the swap. The first argument in the max operator is in fact cash inflow for the firm, if $(s - X_2' + S_t) > 0$. The second argument is the current cash balance plus the firm value at default, given the cash balance is paid out, the swap contract is liquidated and settled $(h = 0)$. For simplicity, we do not discuss the situation where a firm can strategically default on the swap contract in this paper. Thus any change on the assumption about what happens if a firm defaults would have an effect on the calculation of the recovery value $(RS(\cdot))$. Firstly, the above assumption is reasonable and realistic, as the U.S. Bankruptcy Code allows financial swaps' counterparties to exercise contractual rights and get paid ahead of debt obligations. Secondly, even the bankruptcy law could be different globally, the distinction enters the recovery value in the second argument in terms of different level of remaining firm value. In general, a stricter bankruptcy law with respect to swap counterparties would divert more firm value out of the firm and thus lower the continuation value of the firm. This would in turn increases the indirect cost of hedging and consequently reduces the use of hedge. On the contrary, a bankruptcy law that better protects swap counterparties would encourage hedging activity, ceteris paribus. While borrowing the tractability of the model in Gamba and Triantis (2014), our model largely differs from theirs by focusing on the effect of dynamic resolution of model uncertainty. We aim to answer a different research question which is whether the uncertainty in profitability drives a young firm's risk management policy to differ from that of a mature firm. We also discard devices that are less relevant to our research purpose, such as operational flexibility.

4.3 Results

4.3.1 Parameter values

{Insert Table 4.1 here.}

We calibrate the model to suit existing empirical evidence. The rationale is that in many cases the parameter selection corresponds to values found in the literature. In other cases, we set parameters so that moments of variables from simulations are consistent with empirical counterparts, e.g. equity issuance frequency. The long-run mean values for each profitability type, $\tau_1^H = 0.1 > 0$ and $\tau_1^L = -0.1 < 0$, are chosen such that the profitability under neutral belief of each state, $\tau_1 = 0$, which corresponds to the long run mean profitability level assumed in Chapter 3.

The standard deviation is $\sigma_1 = 15\%$, and the persistence parameters of the log of cash flow is $\kappa_1 = 0.8$. Both of these are consistent with the values used in recent articles, such as Hennessy and Whited (2005) and Gamba and Triantis (2014). The profitability is also affected by the fixed production cost $A = 0.92$, which is set to match observed frequency of equity issuance. Fama and French (2005) propose a number between 54% to 72%, with equity issuance due to M&A and management compensation. Adjusting for that, a more reasonable measurements range should be between 13% and 17.9% (Hennessy and Whited, 2007; Hennessy et al., 2007;

Gustafson and Iliev, 2015). Hennessy and Whited (2007) find that simulated frequency of issuance is higher than its empirical counterpart. The simulated frequency is 23.05%, while the observed is 17.5% in their study,.

The swap price is the median of X_2, $s = 1$. However, we conducted robustness checks in a range $(0.95, 1.05)$. Similar to Gamba and Triantis (2014), this does not change the conclusion. This is because as long as the swap is correctly priced, the swap price become irrelevant.

In our model, X_1 is the source of risk exposure of our firm, and the correlation ρ represents the ability to hedge this risk exposure using the swap contract. We believe that this correlation could to be low for non-financial corporations, in a range between 3% and 45% from Tufano (1996), Guay and Kothari (2003), Bartram et al. (2010) and Campello et al. (2011). It could also be very high for firms relate to natural resource, agriculture and finance, e.g. Carter et al. (2006) and Rampini et al. (2014). To serve the purpose of this paper, we focus on the later case in which high correlation coefficient guarantees meaningful hedging demand.

The risk free rate is assumed at 5%, while return on cash is 3.7%. In other words, the cost of carrying cash is $r - r_c = 1.3\%$ inside the firm. The existing two main concerns on holding cash are: the loss of interest tax; and the opportunity cost of forgoing positive NPV investments. Morellec et al. (2014) assume the cost is 2%, which replicates empirically consistent simulation results. We use a smaller one, (1.3%), as it yields better cash ratio as in Sánchez and Yurdagul (2013) under our model setting.

When a firm is in distress, the distress cost is assumed to be proportional, (λ), to the deficit amount, which is assumed at 15%. This may reflect a discount in fire sales, or to costly covenants in contracts for external capital. Pulvino (1998) provides the fire-sale discount at 14%. Strebulaev (2007) set the range of distress cost is between $5 - 25\%$, with a mid-point at 15%. Our 15% assumption is close to them. Most of the above parameters have been examined in Gamba and Triantis (2014).

4.3.2 Numerical solution and simulation

In this paper, the uncertainty regards the most important distributional characteristics, the long-term mean, of the cash flow. Particularly, this incomplete information determines how well a firm performs, also known as profitability. At the beginning, the firm has neutral priors on having high or low profitability. The firm updates its belief following Bayes' rule, by observing the realizations of the state variables.

Interested in the effect of uncertainty in profitability on the dynamics of corporate policies and the value of firm, in this subsection, we show how the optimal policy function, found by solving the firm's problem in Section 4.2, drives the dynamics of corporate risk management policy. Further details could be found in Appendix A about the numerical method.

Baseline models

{Insert Table 4.2 here.}

Similar to Chapter 3, in order to have a clear reference, we set off from several baseline models. In Model 1, firms have perfect information, no cash reserve, with perfectly correlated cash flows with hedging asset. We then gradually relax one restriction of these factors that might affect corporate risk management policy, so that we will have a clean view on the individual effect of each aspect of the model. In

Model 2 we allow for a imperfect correlation between the underlying asset of swap and cash flow of firm. In Model 3, we introduce an alternative risk management tool, cash balance.

The results are presented in Table 4.2. In addition to traditional metrics, such as hedge position, cash balance (if allowed), firm value (enterprise value when cash is introduced), and equity issuance frequency, we report the frequency of equity issuance caused by cash flow shortfall separately. This is the passive equity issuance, denoted as 'Distress' in Table 4.2, which is due to either a negative shock to firm's cash flow or insufficient risk management decision made in the previous period. This separation helps us to probe the relationship between risk management and external financing probability in a more strategic perspective. On one hand, firms resort to external finance in the face of financial distress. On the other hand, firms might be willing to raise external finance, even in good states with no operating cash flow shortfall, to meet the liquidity need in hard time so as to establish an optimal risk management position. This decomposition, as indicated in Table 4.2, shows that a considerable proportion of aggregate external financing is caused by voluntary equity issuance, due to liquidity need so as to fulfill the risk management decision in current period, to establish optimal hedging position. Here we only present steady-state value of each metric. However, we notice that it takes some time to reach the steady state from an initial condition, especially when cash holding is allowed. We will investigate the evolution in later paragraphs.

Indicated by the comparison between the first and second column of Table 4.2, a high profitability firm would increase the use of hedge to maintain the frequency of cash flow shortfall, and to prevent a significant drop in firm value, when the correlation becomes imperfect, specifically from 1 to 0.9. Since in Model 2 the underlying asset of swap mismatches firm's cash flow at a positive probability, hedging associates with additional costs. Consequently, the firm reduces voluntary equity issuance, leading to a higher hedging position but more passive adjustment to the hedging position. This is consistent with traditional risk management theory, indicating that risk management consideration dominates firm's use of hedge.

We further include cash reserve in Model 3 as in the third column, with the correlation coefficient unchanged at $\rho = 0.9$. Firms stock substantial amount of cash and reduce hedging activity. This indicates a substitutionary effect between hedge and cash. Firms equipped with cash as an alternative risk management tool are able to reduce their distress probability further, as well as the total external equity issuance rate. Consequently, enterprise value is enhanced.

As discussed in the beginning, we note that although frequency of external financing is the only observable variable in real world, we are more interested in the decomposition of total equity issuance for different purposes. More importantly, we note that this effect differs across all three models in Table 4.2. The proportion increases when the correlation between firms' cash flow and underlying asset of swap becomes imperfect, and decreases with the ability to reserve cash. Specifically, the frequency of actual distress in Model 3, as a result of operation, decreased to as low as 0.77%.

Therefore, one interesting implication is that rather than gross external financing frequency, which we usually observe in real world and is widely used in empirical studies, researchers should distinguish actual financial distress from voluntary equity issuance. We suggest empirical researchers to use more subtle identification of external financing when studying corporate policy. Some pioneering works are already heading for this direction. For example, DeAngelo et al. (2010) find that 62.6% issuers of SEO would have run out of cash the year after SEO without the proceeds from SEO.

{Insert Table 4.3 here.}

As indicated in Table 4.3, low profitability firms have distinct risk management policy. By construction, since low profitability firms are more likely to incur liquidity shortfall and bankruptcy, they have higher hedging demand. When hedging is a perfect risk management tool, firms take almost full hedging position as predicted by classic corporate risk management theory. Nevertheless, the policy changes dramatically when hedging becomes imperfect. Particularly, even a firm in good state (when X_1 is high) could still fall into financial distress if the underlying asset of the swap is in better state (when $X_2 > X_1$). L-type firms use little hedging, resulting a much higher possibility of distress and external equity issuance. The default rate also triples comparing to perfect correlation scenario. These behaviour indicate that firms are seeking risk, to certain extent, to maximize the value of default option - risk-shifting issue dominates. Although long neglected, this is not surprising, because limited liability serves as a natural risk management device and reduces the use of other risk management tools. This value enhancement comes from the option that limited liability firms can default in bad states, hoping a larger prospect in good states. Since the downside is bounded but not the upside, in expectation firm value is larger than the value of firm, who has to bear all the losses even when the firm is a negative asset.

A similar substitutionary relationship between hedge and cash exhibits when cash reserve is possible. Since the frequency of both equity issuance and distress drops, the introduction of cash in fact restores some consideration towards traditional corporate risk management. This might be because now firms have more, the reserved cash, to lose upon bankruptcy. As a result, the default rate drops by half and firms live longer.

{Insert Table 4.4 here.}

{Insert Figure 4.1 here.}

We probe further some important dynamics not illustrated in Table 4.2, as well as compare High type with Low type. As shown in Table 4.4, we note an increasing pattern in cash stock in L types. Since there is no model uncertainty yet, this dynamics is possibly due to the physical ability to accumulate cash. Particularly, the mean of operating profit, $E(\tilde{X}_1^L) - A$, is 0.0136 per period. To establish the optimal cash balance, which is about 0.1672 for L type, it takes time.

In the meanwhile, when we allow for a cash balance, a firm is able to smooth the cost of risk management over time. The firm could issue equity as soon as it can, and stores the proceedings as cash in the firm, as long as it is still cost-beneficial. The costs associated with holding cash is 1.3%, which is consistent with literature. To address this concern, later we will endow the firm with the steady-state cash balance, so as to adjust for this dynamic pattern not caused by learning when we comment on the effect of learning. In addition, we conduct sensitivity analysis on the carrying cost of cash, and find results remain in a reasonable range of the cost.

Furthermore, we observe distinct strategy between H-type and L-type firms. H-type uses more hedge to manage its operating risk. On the contrary, classic corporate risk management theory predicts that L-type firms should benefit more from hedging, as L-type firms are more often to fall into financial distress and trigger external financing cost which could be mitigated by risk management. However, we find that L-type firms use a lower level of hedge. In the next section, we illustrate that this is reasonable when the profitability is so low that the option value outruns the value created by risk management by comparing both types to a moderate profitability firm. As a result, we note a much lower distress probability for H-type. For both types, the total effect from corporate risk management is enhanced, evidenced by a declining frequency of distress.

Effect of model uncertainty

To separate the effect due to model uncertainty, we compare the results from the full model, with model uncertainty, with those from the benchmark model, Model 3, with no model uncertainty. Moreover, we include an additional model where a firm has a known profitability equal to the expected level under neutral belief, i.e. $\tau_1 = 0$, to further illustrate the effects of parameter uncertainty. We name this the expected model. For a fair comparison, we grant firms steady-state cash as an endowment.

{Insert Figure 4.2 here.}

Figure 4.2 summarises and helps us to understand the main message of this section. Firstly, similar to Décamps et al. (2015), we find that cash balance increases (decreases) with higher (lower) belief of being a high profitability firm.

Secondly, we observe that the use of hedge also increases with the belief of being a high profitability firm, thus hedging is increasing with cash reserve. Rampini et al. (2014) explain this phenomenon by cash's collateral effect. As discussed in the previous paragraph, if the carrying cost is low, firms would prefer issuing equity as soon as possible and storing the raised capital as cash. Firms can thus afford more hedging because they have more collateral represented by cash holdings, which is the only tangible asset of the firm in our model. Therefore, the observed pattern is determined by the trade-off between a complementary effect of cash, serving as collateral so as to afford more hedging, and a substitution effect, when the cost of holding cash is large.

We argue that the collateral consideration might be enhanced in the presence of uncertainty in profitability. In Décamps et al. (2015), cash reserve and dividend compete for limited cash flow generated by the firm. They find that a higher belief of being a high profitability firm relieves the competition thus lead both of them to increase simultaneously. In our paper, the tension lies in the relationship between cash reserve and hedging. Similar to Décamps et al. (2015), we find that firm's hedging position increases with higher belief of being a high profitability firm even when the cash is not increasing.

Particularly, we find a learning H-type firm generally uses more hedge than its first best counterpart. However, although storing less cash than it would do in the first best scenario, the H-type firm increases its cash balance over time. This co-movement supports that model uncertainty works in the direction of increasing complementarity effect between risk management tools, as a higher cash reserve would otherwise result in lower hedging if it worked in favour of substitutionary.

In the meanwhile, the complementary effect is also indicated by a lagged hedging with respect to cash reserve. As in Figure 4.2, cash grows from the endowed level in the second year, whereas hedging picks up in the third year. After setting hedging off, cash balance decreases to the original level and then increases with the belief of being a high profitability firm. This indicates that a substantial level of cash stock is optimal to start hedging, as predicted by Rampini et al. (2014). However, this effect does not rely on model uncertainty. As in Figure 4.2, both of the models exhibit the same complementary effect in the first few years.

{Insert Figure 4.3 here.}

The opposite pattern is documented for the L-type as in Figure 4.3. Rather than learning H-type firms, who uses more hedge but less cash, learning L-type firms instead assume less hedge and more cash. Therefore, as we have observed in learning H-type scenario, there is a co-movement in the combination of hedge and cash for a learning L-type firm with respect to its perfect knowledge counterpart. Again, we document the model-uncertainty-independent complementarity in the first few years in all models as in the H-type scenario. Interestingly, we also find that when a firm has a know level of profitability, which equals to the expected level of neutral belief, it would use more hedge and more cash comparing to both a learning H-type firm and a learning L-type firm. As a firm with lower profitability than the H-type, it has more risks to manage thus creates more value from risk management. It makes sense for the firm to use more hedge as well as reserve more cash. Comparing to a L-type firm with even lower profitability, the value created by risk management outruns the option value of default. Therefore, a firm with moderate profitability could use more hedge and cash than both H-type and L-type firms. Note that unlike what we find in Chapter 3, the evolutions of hedge and cash do not converge to first best counterparts, indicating the path-dependency is stronger when the uncertainty is on the first order moment.

{Insert Table 4.5 here.}

We tabulate the evolutions in Table 4.5. When model uncertainty is integrated into the model, the H-type continuously increases its hedge when knowledge is largely improved, and results in a higher hedging position in the long-run. In contrast, L-type uses much less hedge. Since upon maturity the H-type firms take on more hedge, it is consistent with our previous claim that a firm with better knowledge about the profitability of its business will prefer actively managing its risk to maximizing default option value.

This is intuitively reasonable because mature firms have learned their profitability through many years of operation, and thus have clearer picture on th trade-off relationship as well as keep hedging cost low. In addition, the accumulation of wealth gives them more to lose upon bankruptcy. This interpretation answers to Stulz (1996), where he retrospected "[t]he actual corporate use of derivatives, however, does not seem to correspond closely to the theory ... large companies make far greater use of derivatives than small firms, even though small firms have more volatile cash flows, more restricted access to capital, and thus presumably more reason to buy protection against financial trouble."

Thus we conclude that the above features stand out from extant literature as they raise distinct predictions of uncertainty in profitability: both corporate hedging position and cash reserve increase with the belief of being a high profitability firm; high profitability firm uses more hedge as risk management consideration dominates maximisation of default option value; cash's role in corporate risk management is more sophisticated than it is studied in previous literature. This separation might be the reason for mixed empirical findings, if they do not control for knowledge accumulation of firms.

Marginal effect of individual risk management tool

In this section, we explore the marginal effect of each risk management tool. We study three models: one with no risk management at all, one with only cash reserve as risk management tool, and the last with only hedge using derivatives defined in Section 4.2. Together with the full model with both hedge and cash analysed in Subsection 4.3.2, we can quantify marginal effect of individual risk management tool when uncertainty in profitability is gradually resolved.

Cash, as one of the main interests in this paper, might be one of the most subtle variables in corporate risk management. The firm actively changes its cash balance level. Decreases in cash balance usually occur if the firm draws on internal liquidity to overcome low profitability, and if the firm builds up a hedging position. Especially in the most extreme case, a total loss of cash occurs upon bankruptcy. Increases in cash balance enable the firm to weather negative cash flow shocks, as well as to provide additional liquidity for future establishment of hedge, particularly if the cash balance has been drawn down. Although result shows that the cash stock remains substantial even if the cash flow and price of underlying asset of hedging derivatives is almost perfectly correlated - ρ is as high as 0.9, indicating cash holding is an indispensable method of corporate risk management, there are important limitations. First, there is a cost associated with holding cash. Second, persistent unfavourable states will gradually deplete the firm's cash stock, even if it was substantial. Distress costs will then be triggered with some probability. Vice versa: to build up a large cash balance, it requires the firm to either yield a sufficient string of positive cash flows or raise costly external finance, which might not be the case. Having analysed the cases in which cash stock is beneficial/costly, we might be able to explain: why firms store more cash in case of need in bad states of world, given hedging effect could be very noisy due to imperfect knowledge in the early stage.

{Insert Figure 4.4, 4.5 and Table 4.6, 4.7 here.}

As illustrated by Figure 4.4 and 4.5 with details in Table 4.6 and 4.7, firms tend to use more hedge (cash) when this is the only risk managing method. This implies that the substitutionary effect dominates in both types. As we have learnt from previous subsection, having access to hedging relieve both types' cash demand, indicating hedging substitutes cash stock in the presence of uncertainty in profitability, and vice versa.

We present an interesting finding of this subsection. Our result might shed light on the mixed empirical evidence on value creation of hedging. Under our main parameter settings, as in Table 4.1, hedging by itself seems not to increase firm value for both H and L type firms, as illustrated in the third panel of Figure 4.4 and 4.5. However, together with cash reserve, firm value are enhanced in both cases. More interestingly, the major difference lies in their choices of corporate risk management strategies due to distinct profitability, as H-type firms hedge substantially more than their L-type peers, as well as holding more cash. This is consistent with empirical researches that find no evidence for hedging creating firm value.

In the end, while optimal corporate risk management policy of the L type firms indeed reduces costs of external financing as indicated by lowering distress probability, it does not significantly reduce default risk. In other words, the optimal corporate risk management policy should minimise the costs of external financing costs and maintain the value of default option in the meanwhile.

4.4 Conclusion and Future Research

In a dynamic model, we find that a firm's risk management policy could be largely affected by uncertainty in profitability, as its resolution process intertwines with the combined action of two trade-offs which forms corporate risk management policy. In the first trade-off relationship, the firm balances the costs of managing risk and the benefits from mitigating external financing costs. Even though the benefits outweighs the costs, whether the firm actively manages its risk depend on the result of the second tension, in which the firm balances the value created by risk management and the value lost in decreased default option. We find that both cash balance and hedging level increase with the belief of being a high profitability firm. Moreover, default option value maximisation dominates risk management consideration in less profitable firms. These implications might explain the puzzle of young firms conduct less hedge than mature firms, and mitigate the ongoing debate over whether corporate risk management create firm value or not. This paper also lends support to the empirical evidence documented by Rampini et al. (2014), where they find a dramatic decrease, from 30% to less than 5% of estimated annual fuel expenses of airlines, in risk management. The results of our model warrant a reconsideration of the interpretation of the findings in the past empirical literature, especially in those involve time series analysis. Moyen and Platikanov (2013) succeed in using age as a proxy for the resolution of uncertainty, and identify a link to the dynamics of corporate investment. We expect a similar success in studying the dynamics of corporate risk management. Although this paper does not touch the strategic entering into a new market, it is interesting to further probe the idea that an entering firm could learn from observing the current operational results of in-place firms. An educated guess would be that the information from peers' operation could constantly raise (or lower) the prior belief on the profitability of its own. The firm thus might decide the optimal timing of entry and default accordingly.

4.5 Appendix

4.5.1 Appendix:Numerical Methods

We adopt the numerical methods in Chapter 3, which we repeat here, but adapt that to our particular research interest in uncertainty about profitability. We solve the Bellman equation described in Section 4.2 by simulating 1,000 firms over 50 years. Each path for the state variable x is obtained by iterating Equation (4.1) using Monte Carlo simulation, for 50 time steps. The simulated paths for x are restricted to a set of discrete values $\tilde{x}$, following Tauchen (1986) approach. Due to the curse of dimensionality, the number of sampling points is chosen to be 11 as a result of trade-off between accuracy and efficiency. However, we run special tests to guarantee this choice of grid size yields simulation results with errors in the relevant moments (mean, volatility and correlation) that are within 1% of their theoretical counterparts. For robustness, we also have a finer grid and find very similar results.

Particularly, we discretise the sets of h, c and q respectively in $[0, h_u]$, $[0, c_u]$ and $[0, 1]$ with 31 equally spaced points. Variable x_1 and x_2 define a reduced-form vector autoregression that we approximate through a discrete-state Markov chain with 11 points for each variable with truncated support in $[\tau_1^L - 3\sigma_j^u, \tau_1^H + 3\sigma_j^u]$, $j = 1, 2$, where $\sigma_j^u = \sigma_j/\sqrt{1-\kappa^2}$ is the unconditional standard deviation for $j = 1, 2$. The discrete abscissae and the risk-neutral Markov transition probabilities are computed according to the method proposed by Terry and Knotek (2011), which is based on the Gauss-Hermite quadrature rule, as in Tauchen (1986), but allows for non-zero correlation.

A Monte Carlo simulation is used to generate a sample path for the firm following an optimal policy. We generate a sequence of 500,000 independent draws from a truncated bivariate Normal distribution and generated a path for x_1 and x_2 using the AR(1) specification. Starting from an initial condition (h_0, c_0, q_0), we apply the optimal policy from the Equation 4.6 and generate a simulated path for the firm. We drop the first 200 observations, to exclude any influence of the initial condition. In the event the firm defaults, a new firm starts business with neutral belief and with no financial hedging in place $(h = 0, c = 0)$.

4.5.2 Appendix:Figures and Tables

τ_1^H	long-term mean of high type	0.1
τ_1^L	long-term mean of high type	-0.1
τ_2	long-term mean of underlying asset of swap	0
σ	annual volatility of x	15%
ρ	correlation between firm's cash flow and underlying asset	0.9
κ	persistence of x	0.8
s	swap price of the swap contract	1
A	fixed production cost	0.92
r_f	annual risk-free rate	5%
r_c	return on cash	3.7%
λ	distress cost	15%
rc	renegotiation cost	1%

Table 4.1: Parameter Values

	Model 1	Model 2	Model 3
Hedge	0.0524	0.2532	0.1750
Cash	NaN	NaN	0.2636
EV	5.7535	5.7234	5.8819
Equity Issue	0.2146	0.1729	0.0161
Distress	0.1462	0.1481	0.0077
Default	0.0000	0.0000	0.0000

Table 4.2: Simulation results of baseline models, H-type

Each column in this table presents the average of up to seven metrics from 3 baseline models based on H-type firms, with one restriction relaxed at each time. The metrics are: the level of hedge; the cash balance; the firm/enterprise value of the firm; the frequency of external equity financing, financial distress; and proportion of defaulted firms. Model 1 is the baseline model with perfect knowledge, no cash and perfect correlation; in Model 2, firms' cash flows become imperfectly correlated; and finally Model 3 adds cash as an alternative risk management tool. In this table, size of mesh grid is $11 \times 31 \times 21$ (cash flow $\times$ hedging position $\times$ cash holding). The other parameters are given in Table 3.1.

	Model 1	Model 2	Model 3
Hedge	0.9809	0.0456	0.0371
Cash	NaN	NaN	0.1672
EV	2.3486	1.1387	1.0714
Equity Issue	0.0172	0.3630	0.2431
Distress	0.0145	0.3267	0.2121
Default	0.0236	0.0618	0.0378

Table 4.3: Simulation results of baseline models, L-type

Each column in this table presents the average of up to seven metrics from 3 baseline models based on L-type firms, with one restriction relaxed at each time. The metrics are: the level of hedge; the cash balance; the firm/enterprise value of the firm; the frequency of external equity financing, financial distress; and proportion of defaulted firms. Model 1 is the baseline model with perfect knowledge, no cash and perfect correlation; in Model 2, firms' cash flows become imperfectly correlated; and finally Model 3 adds cash as an alternative risk management tool. In this table, size of mesh grid is $11 \times 31 \times 21$ (cash flow $\times$ hedging position $\times$ cash holding). The other parameters are given in Table 3.1.

age	2-4	5-7	8-10	11-13	14-16	17-50	2-50
				H type			
Hedge	0.0139	0.0793	0.1077	0.1264	0.1416	0.1707	0.1472
Cash	0.2374	0.2242	0.2411	0.2476	0.2549	0.2593	0.2537
EV	5.9054	5.9129	5.8860	5.8856	5.8751	5.9031	5.9000
Equity Issue	0.0433	0.0187	0.0153	0.0143	0.0173	0.0158	0.0176
Distress	0.0310	0.0077	0.0053	0.0080	0.0093	0.0074	0.0089
Default	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
				L type			
Hedge	0.0132	0.0302	0.0267	0.0267	0.0267	0.0346	0.0316
Cash	0.0924	0.1470	0.1568	0.1627	0.1627	0.1673	0.1603
EV	1.0257	1.0365	1.0277	1.0499	1.0404	1.0742	1.0625
Equity Issue	0.3809	0.2926	0.2891	0.2629	0.2782	0.2465	0.2631
Distress	0.3415	0.2621	0.2539	0.2310	0.2447	0.2148	0.2306
Default	0.0621	0.0343	0.0335	0.0372	0.0442	0.0379	0.0392

Table 4.4: H vs L, with cash, no model uncertainty

For Model 3, this table presents the evolution of the average of six metrics: the level of hedge; the cash balance; the firm/enterprise value of the firm; the frequency of external equity financing; and the frequency of financial distress. They are tabulated against years, with each column reports the average metrics during the corresponding years. The final column presents the unconditional mean of the entire simulated sample. In this table, the size of mesh grid is $11 \times 31 \times 21 \times 31$ (cash flow $\times$ cash holding $\times$ hedging position $\times$ beliefs). The other parameters are given in Table 3.1.

Panel A: Hedge							
H type							
age	2-4	5-7	8-10	11-13	14-16	17-50	2-50
Informed	0.0108	0.0465	0.0419	0.0479	0.0522	0.0606	0.0543
Learner	0.0114	0.0656	0.1061	0.1468	0.1687	0.2025	0.1710
Uninformed	0.0957	0.1607	0.1901	0.2132	0.2302	0.2881	0.2544
Belief	0.7561	0.8415	0.8927	0.9285	0.9571	0.9947	0.9581
L type							
age	2-4	5-7	8-10	11-13	14-16	17-50	2-50
Informed	0.0117	0.0349	0.0338	0.0349	0.0371	0.0423	0.0387
Learner	0.0088	0.0294	0.0218	0.0183	0.0194	0.0152	0.0165
Uninformed	0.0913	0.1373	0.1436	0.1414	0.1332	0.1523	0.1453
Belief	0.2786	0.2179	0.1609	0.1140	0.0710	0.0102	0.0587
Panel B: Cash							
H type							
Informed	0.2718	0.2997	0.3089	0.3068	0.2955	0.2985	0.2979
Learner	0.2232	0.2067	0.2277	0.2346	0.2457	0.2671	0.2550
Uninformed	0.4787	0.7195	0.8016	0.8292	0.8417	0.8504	0.8148
L type							
Informed	0.1493	0.1543	0.1492	0.1564	0.1510	0.1603	0.1577
Learner	0.2068	0.2093	0.1935	0.1930	0.1786	0.1651	0.1747
Uninformed	0.3456	0.3918	0.3987	0.3911	0.3765	0.4329	0.4170

Table 4.5: Effect of learning

For both types, this table compares the average of four metrics: the level of cash balance, the hedge; and the enterprise value of the firm across three models: ignorant, learning and perfect knowledge, as described in Subsection 3.4.2. They are tabulated against years of learning. The final column presents the unconditional mean of the entire simulated sample. In this table, the size of mesh grid is $11 \times 31 \times 21 \times 31$ (cash flow $\times$ cash holding $\times$ hedging position $\times$ beliefs). The other parameters are given in Table 3.1.

	Hedge						
age	2-4	5-7	8-10	11-13	14-16	17-50	2-50
Hedge Only	0.0893	0.2005	0.2435	0.2591	0.2574	0.2571	0.2427
Both	0.0114	0.0656	0.1061	0.1468	0.1687	0.2025	0.1710
				Cash			
Cash Only	0.2210	0.5600	0.6917	0.7480	0.7730	0.7631	0.7128
Both	0.2232	0.2067	0.2277	0.2346	0.2457	0.2671	0.2550
				Enterprise Value			
None	4.7948	5.1320	5.3153	5.5152	5.6289	5.7887	5.6321
Hedge Only	4.8070	5.1274	5.2961	5.4693	5.5985	5.7502	5.6000
Cash Only	4.8538	5.1989	5.3864	5.5884	5.7039	5.8645	5.7058
Both	4.8896	5.2442	5.4284	5.6244	5.7345	5.8784	5.7271

Table 4.6: Marginal effect of risk management tools, H-type

For H-type, this table compares the average of up to four metrics: the level of hedge, th cash balance, and the enterprise value of the firm across four models: no risk management, managing risk with only cash, with only hedge and with both, as described in Section 3.4.2. They are tabulated against years of learning. The final column presents the unconditional mean of the entire simulated sample. In this table, the size of mesh grid is $11 \times 31 \times 21 \times 31$ (cash flow $\times$ cash holding $\times$ hedging position $\times$ beliefs). The other parameters are given in Table 3.1.

	Hedge						
age	2-4	5-7	8-10	11-13	14-16	17-50	2-50
Hedge Only	0.0518	0.1238	0.1272	0.1053	0.0907	0.0414	0.0593
Both	0.0088	0.0294	0.0218	0.0183	0.0194	0.0152	0.0165
				Cash			
Cash Only	0.1192	0.2708	0.3150	0.3100	0.2807	0.1813	0.2051
Both	0.2068	0.2093	0.1935	0.1930	0.1786	0.1651	0.1747
				Enterprise Value			
None	2.0408	1.8103	1.5998	1.3984	1.2357	1.1544	1.2960
Hedge Only	2.0557	1.8096	1.5905	1.4055	1.2367	1.1569	1.2986
Cash Only	2.0759	1.8367	1.6087	1.4098	1.2325	1.1024	1.2647
Both	2.0440	1.8606	1.6588	1.4761	1.2726	1.1415	1.3010

Table 4.7: Marginal effect of risk management tools, L-type

For L-type, this table compares the average of up to four metrics: the level of hedge, th cash balance, and the enterprise value of the firm across four models: no risk management, managing risk with only cash, with only hedge and with both, as described in Section 3.4.2. They are tabulated against years of learning. The final column presents the unconditional mean of the entire simulated sample. In this table, the size of mesh grid is $11 \times 31 \times 21 \times 31$ (cash flow $\times$ cash holding $\times$ hedging position $\times$ beliefs). The other parameters are given in Table 3.1.

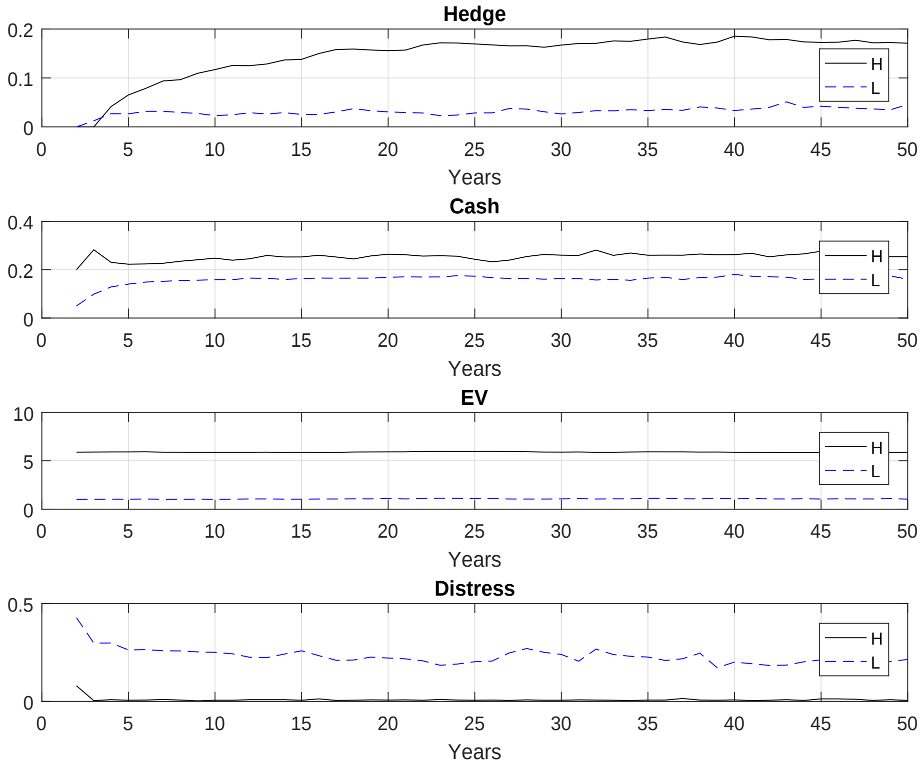


Figure 4.1: Comparison between H and L type

This figure shows the difference in average hedge, cash stock, enterprise value and distress probability between H-type and L-type from the perfect knowledge Model 3.

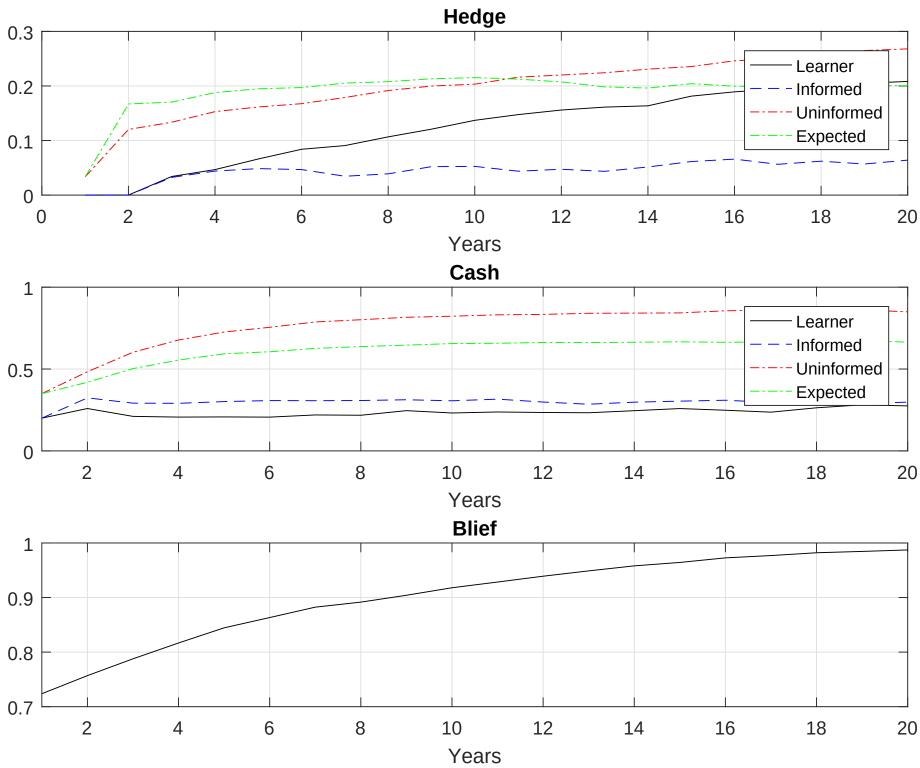


Figure 4.2: Effect of model uncertainty: Average, H-type

This figure shows average hedge and cash stock in lieu of the effect of model uncertainty on hedge and cash stock for learning H-type firm based on the set-up of Model 3. The bottom panel plots corresponding belief evolution.

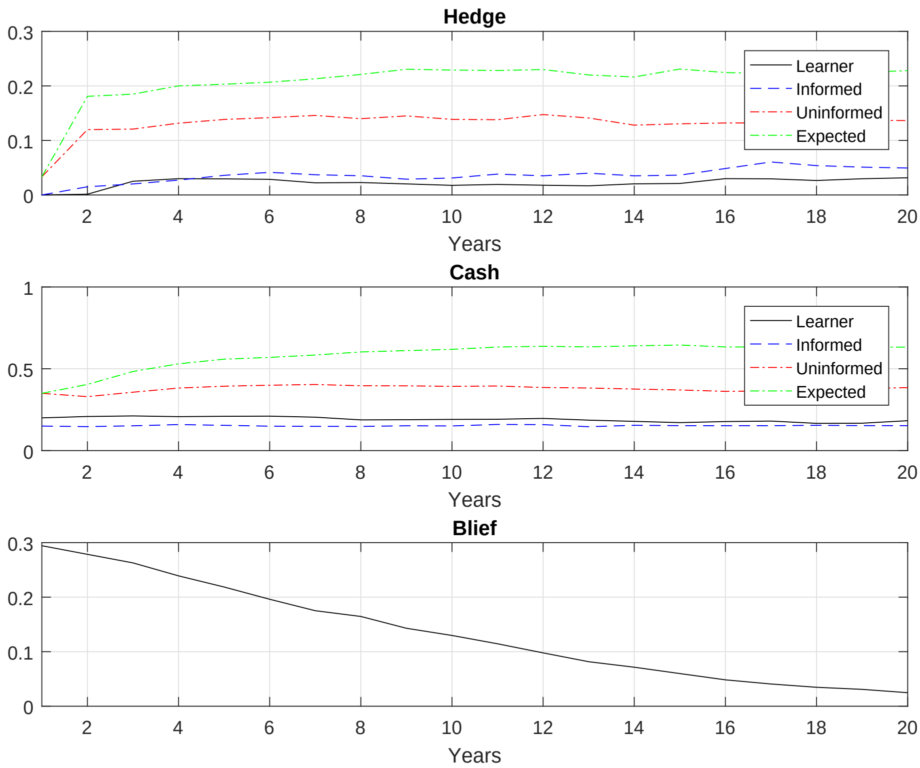


Figure 4.3: Effect of model uncertainty: Average, L-type

This figure shows average hedge and cash stock in lieu of the effect of model uncertainty on hedge and cash stock for learning L-type firm based on the set-up of Model 3. The bottom panel plots corresponding belief evolution.

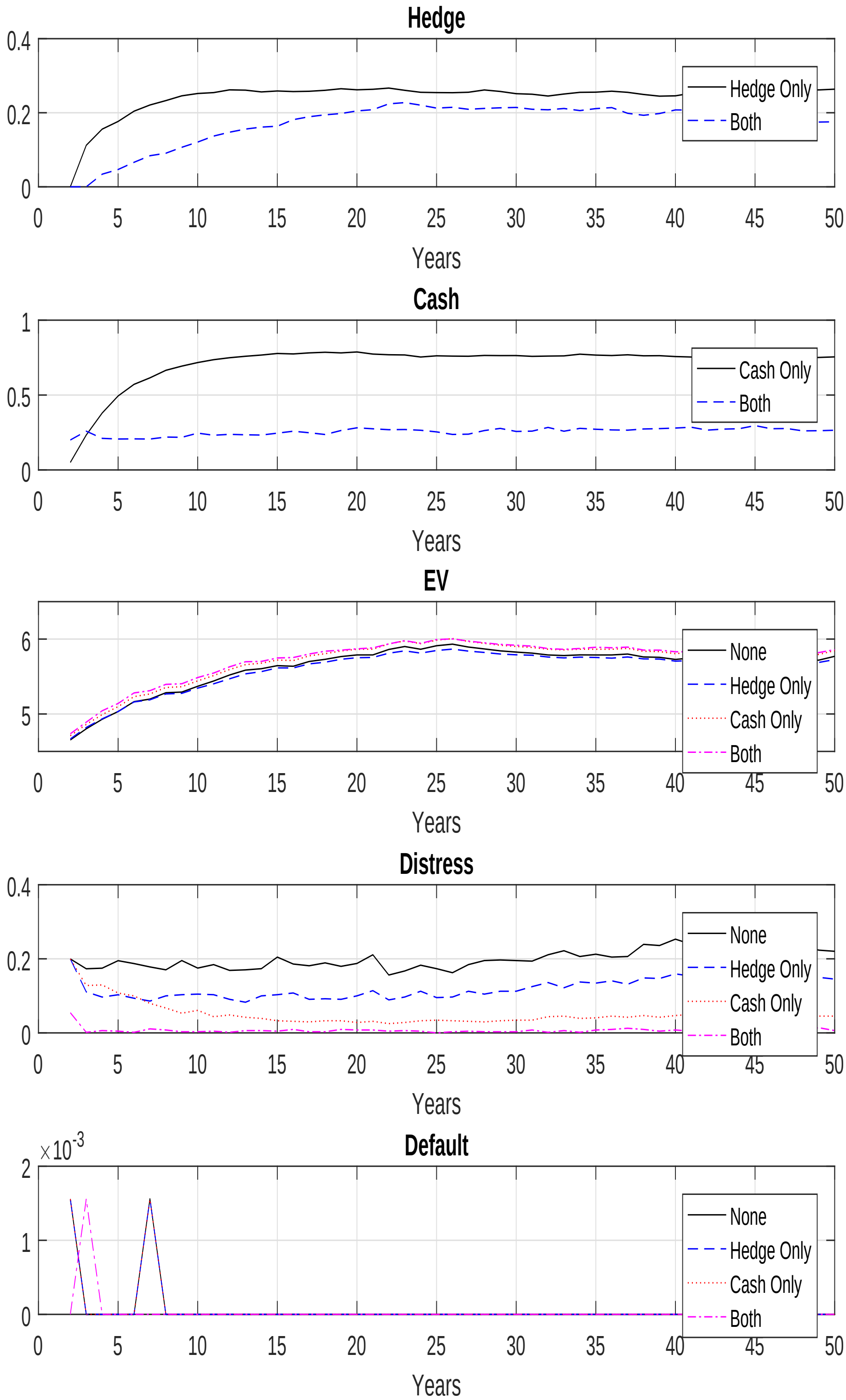


Figure 4.4: Marginal effects: H-type

This figure shows a visual illustration of the average marginal effects of different risk management tools on hedge, cash stock, and enterprise value for H-type firm based on four models: one with no risk management at all, one with only cash balance as risk management tool, one with only hedge, and the full model with both hedge and cash.

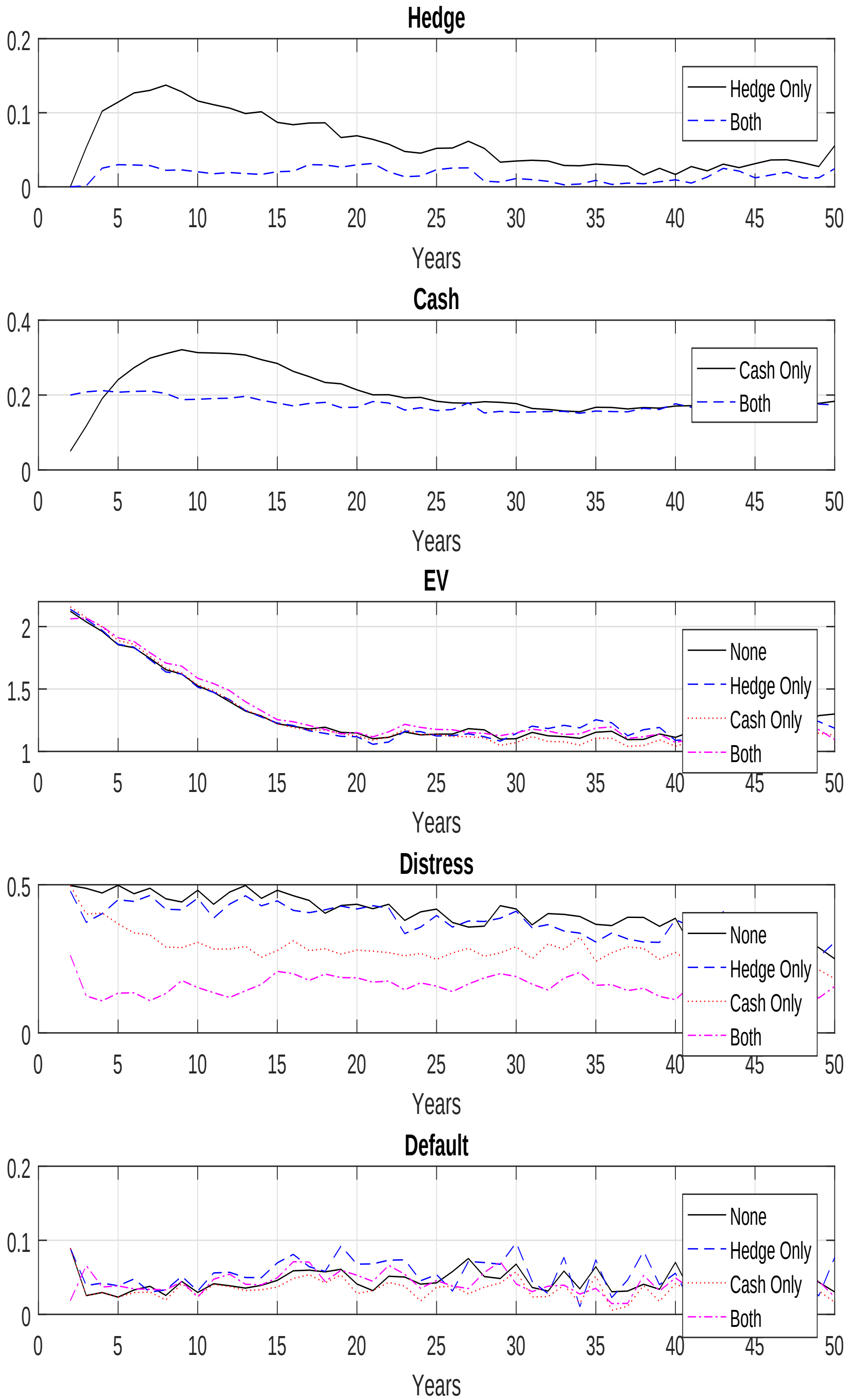


Figure 4.5: Marginal effects: L-type

This figure shows a visual illustration of the average marginal effects of different risk management tools on hedge, cash stock, and enterprise value for L-type firm based on four models: one with no risk management at all, one with only cash balance as risk management tool, one with only hedge, and the full model with both hedge and cash.

Chapter 5

Concluding Remarks

The thesis presents discussion concerning corporate financing decisions, corporate risk management policy under uncertainty. The uncertainty underlies the characteristics of a firm - the riskiness of a firm, the hedgeability of a firm's cash flow with hedging asset, or the profitability of a firm.

The main contribution of Chapter 2 is to explore why firms prefer equity issuance in the bull market and debt in the bear market. Overall, I find different equilibria in different market conditions under Prospect Theory preference. In the bull market, risky and safe type of firms pool together to exploit over-valuation on their projects, whilst in the bear market, different type of firm separate from each other in order not to upset investors further. The result leads to bias in investment budgeting, as positive NPV projects might be foregone in the bear market and negative NPV projects might be undertaken in the bull market condition. Moreover, the bias would be finally corrected upon realisation of information on the projects, leading to a drastic price correction which we usually observe in a financial crisis.

Chapter 3 and Chapter 4 focus more on the model uncertainty. Both of them conclude that model uncertainty and its resolution dynamically alters firms' risk management strategies thus should draw attention when studying firms' optimal policies. However, they aim to answer specific research questions, and relates to different strands of literature. Chapter 3 studies the case in which firms with hedging demand have limited knowledge on the hedgeability - the correlation between their cash flows and hedging assets. However, they could learn from observed realisations of both the cash flow and the price of the underlying asset of a derivative used for hedging. We find that resolution of this uncertainty (on the hedgeability) drives the level of corporate hedging. This might explain the puzzling empirical evidence that young firms hedge less than mature ones, while conventional corporate risk management theory predicts the opposite. We also find that hedge and cash, both as risk management tool, substitute and complement each other at the same time. Moreover, we provide support to mixed evidence on the value creation role of hedging, as we observe smaller difference in firm value comparing to the difference in corporate risk management policy under reasonable assumptions.

Chapter 4 relates closely to a trending literature on uncertainty in profitability. We identify similar pattern of a relieved tension between cash reserve and liquidity demand with those literature discussed in Chapter 4. Particularly, we find that both cash stock and hedging position increases with resolved profitability prospects. However, low profitable firms' risk management policy largely differs from that of high profitable firms. We find that this difference results from a trade-off between maximising value brought by risk management and maximising the value of a default option. Again we identify both substitutionary effect and complementary effect between cash balance and hedging. However, we emphasize that uncertainty in profitability actually enhance the substitutionary effect.

Findings in this thesis shed lights upon many open research questions. Its enlightenment could inspire many interesting future research on diverse topics. Planned future research focuses on the implication of Chapter 2. The signaling effect in the choice of financing method in bull and bear market is ready for test. Chapter 3 and 4 invites future empirical studies to take information acquisition into account. Gladly, pioneer research has already been heading to this direction, e.g. using age as a proxy for the resolution of uncertainty, Moyen and Platikanov (2013) succeed in explaining the dynamics of corporate investment that could not be understood by conventional theories. I expect similar successes in other topics.

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