Model Specification and CDO (Mis)Pricing*

Dan Luo

School of Finance, Shanghai University of Finance and Economics

E-mail: luo.dan@mail.shufe.edu.cn

Dragon Yongjun Tang

Faculty of Business and Economics, University of Hong Kong

E-mail: yjtang@hku.hk

Sarah Qian Wang

Warwick Business School, University of Warwick

E-mail: qian.wang@wbs.ac.uk

January 24, 2018

*We thank Jack Bao, Andrew Carverhill, Eric Chang, Erik Heitfeld, Nikunj Kapadia, Olfa Maalaoui, Wing Suen, Keith Wong, Ken Zhong, Haibin Zhu and seminar participants at the University of Hong Kong, Bowles Symposium at Georgia State University, the Annual Derivative Securities and Risk Management Conference at Federal Deposit Insurance Corporation (FDIC), the International Symposium on Risk Management and Derivatives in Xiamen, China International Conference in Finance in Guangzhou, Financial Management Association Annual Conference in Reno, the Annual Conference on Financial Economics and Accounting at Rutgers University, the International Conference on Asia-Pacific Financial Markets in Seoul, the SFM Conference in Kaohsiung, and the Annual Risk Management Conference in Singapore. We are especially grateful to Andreas Eckner for providing the filtered frailty factor. Dan Luo is supported in part by Natural Science Foundation of China (NSFC): 71302075/G0206. Dragon Tang acknowledges the support of Hong Kong Research Grants Council (General Research Fund HKU 17510016).

Model Specification and CDO (Mis)Pricing

ABSTRACT

Complex structured products, especially collateralized debt obligations (CDOs), were at the center of the 2008 credit crisis. This paper explores the impact of modeling difficulties on CDO mispricing. Comparing pricing outputs among models with different specifications, we show that the use of a model with advanced default correlation assumptions could have reduced the amount of model-implied AAA-rated CDO securities. This pricing difference also has predictive power for the subsequent downgrading of AAA-rated CDO tranches. However, the model specification is only qualitatively important for CDO mispricing, as it has a modest quantitative effect in explaining the overall pricing errors.

JEL Classification: G12; G13; E43; E44

Keywords: CDO; Model Specification; Default Correlation; Frailty

I. Introduction

The 2007-2009 credit crisis has had unprecedented impact on the financial industry.[1] At the center of this crisis is the previously little known financial innovation called collateralized debt obligations (CDOs). CDOs are debt claims with various seniorities against collateral asset pools. Senior claimholders will not suffer a loss until the subordinated tranches are exhausted. Because of this prioritized structure and other credit enhancements, such as insurance, CDO senior tranches had AAA credit ratings prior to the crisis. CDO issuance started in 1987 but remained dormant until 1997, with an annual issuance of $17 billion, since then the market has grown rapidly to reach an annual issuance of $520.6 billion in 2006, according to Securities Industry and Financial Markets Association (SIFMA). CDO issuance peaked in 2007Q2 (quarterly issuance of $178.6 billion) and afterward declined exponentially (2009Q1 issuance $0.8 billion). However, the strikingly strong recovery of the CDO market, especially collateralized loan obligations (CLOs), in recent years has prompted significant concerns over the market and its valuation.[2] Given the dramatic write-downs associated with CDOs during the credit crisis[3] and the resurgence of the CDO market in recent years, it is important to develop a good understanding of CDO valuation. We present a comprehensive study of CDO pricing with a focus on the impact of model specification. Our study elucidates potential structural causes of CDO mispricing.

The innovative nature of CDOs makes it difficult to identify the exact reasons for this valuation failure before the credit crisis. On the one hand, given the short history of the product and modeling difficulties, Duffie (2007) doubts that anyone has capability to evaluate CDOs

[1] Among the top-five precrisis Wall Street investment banks, Lehman Brothers declared bankruptcy on September 15, 2008, Bear Stearns was acquired by J. P. Morgan on March 16, 2008, Merrill Lynch was acquired by Bank of America on September 14, 2008, and Goldman Sachs and Morgan Stanley converted into bank holding companies on September 21, 2008.

[2] see, e.g., "CLO surge prompts regulatory concerns", *Financial Times*, September 8, 2014. CLO performance will remain solid in 2016 according to Moody's (2015). We are also seeing a turnaround for the European CLO market. "Time looks ripe for European collateralised loan obligations", *Financial Times*, January 21, 2016.

[3] For instance, on July 28, 2008, Merrill Lynch sold $30.6 billion in notional value U.S. super senior ABS CDOs to an affiliate of Dallas, Texas-based private equity firm Lone Star Funds for $6.7 billion, or 22 cents on a dollar. (Merrill Lynch also financed 75% of the sale through a loan with recourse only on those CDOs.)

with comfortable accuracy. Accepting the complexities and modeling difficulties, some journalists blame the quants and their models for "killing the Wall Street"[4]. On the other hand, regulators and media have rushed to blame CDO underwriters and credit rating agencies, who brought CDOs to the marketplace, owing to their potential conflicting incentives. While some market participants likely deserve more blame than others, "careful research is needed to distinguish the relative importance of the bad incentives view and the mispricing view", as these two views have distinctly different implications for future regulation and risk management (Allen (2008)).

Given the limitations in modeling techniques and historical data, large losses do not automatically imply risk management failures (Stulz (2008)). This argument is particularly relevant for the current setting of CDOs, which are collateralized on a pool of default-risky assets. Accurate valuation of CDOs requires the joint distribution of those assets, especially the default correlation, to be modeled. Defaults are rare events. Hence, the default correlation is difficult to measure. Furthermore, even single-obligor credit risk analysis is difficult. There is also little consensus on the best practices for portfolio credit risk modeling. In this paper, we examine the impact of model specification on portfolio credit risk evaluation and CDO mispricing.[5]

Traditional portfolio credit risk models, such as that of Vasicek (1987), assume that the default correlation is driven only by observable common factors. However, recent studies show that such an approach significantly underestimates the actual default correlation (Das, Duffie, Kapadia, and Saita (2007)). Based on this observation, Duffie, Eckner, Horel and Saita (DEHS, 2009) propose a frailty correlated default model, in which the *latent* "frailty" factor is unobservable and time varying. Duffie, Eckner, Horel and Saita (2009) show that their model performs well in matching historical default patterns, and Collin-Dufresne (2009) also discusses the properties for good correlation models. Azizpour, Giesecke, and Schwenkler (2017) find important roles of both frailty and contagion for default clustering.

[4]see, e.g., "Recipe for disaster: The formula that killed Wall Street", *Wired*, February 23, 2009. Triana (2009) expressed similar views.

[5]The issues on conflicts of interest and CDO security design are discussed by Griffin and Tang (2012), Nicolo and Pelizzon (2008).

Our simulation results on model specification substantiate the importance of the frailty factor to portfolio credit risk valuation and CDO pricing. We focus on the tail risk that is most relevant to CDO senior tranches that are often rated AAA. At the AAA level, the expected portfolio default loss rate is 5.4% higher when frailty is considered than when it is not. Hence, ignoring the frailty factor would result in a 5.4% greater AAA tranche size. We further consider the impact of correlation between macroeconomic factors on CDO pricing in the presence of frailty factor. In reality, there might be correlation between macroeconomic factors. For example, when the market undergoes a crisis, the central bank will step in and cut interest rates to inject liquidity into the market, which essentially creates a correlation between the short-term interest rate and stock market performance. However, our simulation results show that such consideration of correlation between macroeconomic factors has little effect on portfolio credit risk valuation and CDO pricing when the frailty factor is present.

Having examined the potential impacts of model specification on CDO valuation, we apply the DEHS frailty model to historical CDO data. Our sample contains 237 CDOs issued between May 1998 and December 2004, including 46 collateralized bond obligations (CBOs), 82 CLOs, 99 CDOs collateralized with asset-backed securities (ABS CDOs), which includes most mortgage-back securities, and 10 CDOs collateralized with other CDO tranche securities (CDO^2s). When the credit rating or pricing for CDOs is obtained, the collateral pool is typically incomplete. Rating agencies will thus conduct an analysis and assign a rating based on the projected collateral pool characteristics. To price CDOs, we first generate factor time series based on the CDO's collateral pool characteristics. Then, we insert these factor time series into the no-frailty model (bad model) and the dynamic-frailty model (good model) and generate the collateral pool loss distribution. With the collateral pool loss distribution, we can determine the AAA tranche size by referring to the historical AAA default probability.[6] We then compare the resulting AAA tranche size from the different model specifications (no-frailty vs. frailty model).

Our empirical findings are consistent with the simulation results. Specifically, the no-frailty model generates lower portfolio default rates and hence higher AAA tranche sizes than

[6]Section II discusses the tranche determination approach in details.

the results from a credit rating agency. The no-frailty model underestimates default rates by 6% on average; however, the frailty factor increases the portfolio default rate at AAA level by 19%. Therefore, accounting for the frailty factor could shrink AAA-rated CDO tranches by 13%. If the frailty model is indeed useful, we would observe subsequent downgrades of CDO AAA tranches when the frailty model indicates higher risk than the rating agency model. Up to December 2008, most changes in CDO ratings at the AAA level occurred with ABS CDOs, and in October 2009, we witness more AAA downgrading in the other CDO categories. We find that the frailty model exhibits substantial power in separating out the safest and riskiest CDOs, although the relationship is not strict monotone. According to the frailty model, about 92% of AAA-rated tranches with high risk are subsequently downgraded, whereas 29% are downgraded for the low-risk group. The frailty model significantly predicts the future AAA tranche downgrading.

We make three contributions to the literature. First, we elucidate the potential structural causes of CDO mispricing. Griffin and Tang (2012) document the importance of out-of-model adjustments for CDO mispricing and find that smaller model-implied AAA sizes receive larger adjustments. We further show that model specification can affect the model-implied CDO AAA tranches, which partially contribute to the CDO mispricing. However, considering the out-of-model adjustment, we expect that even if we have the best model, CDOs were still mispriced. Model constraints played some role in CDO mispricing. Second, although model uncertainty is well studied in equity markets and portfolio allocation (e.g., Garlappi, Uppal, and Wang (2007)), we apply it to the credit derivatives market and show its strong impact in this market. Third, our study provides a good framework for the analysis of financial innovations, which will likely continue, and the same model issues would appear repeatedly. Therefore, our research provides a preliminary direction for future risk management practice.

Our study builds on Duffie, Eckner, Horel and Saita (2009), and we add to existing studies in the following ways. While Longstaff and Rajan (2008) argue that historical CDO prices are well explained, Coval, Jurek, and Stafford (2009a) show that substantial mispricing can arise in the CDO structuring process. Our finding of systematic mispricing due to model misspecification provides a justification for these seemingly conflicting findings. Fender, Tarashev, and Zhu (2008) also show that CDOs can be overvalued in comparison with equivalent corporate bonds. Feldh*ü*tter and Nielsen (2012) and Heitfield (2009) use MCMC for CDO pricing. Choi, Doshi, Jacobs, and Turnbull (2016) introduce a top-down no-arbitrage model for pricing structured products with economic variables. Longstaff and Myers (2014) focus on the valuation of the equity tranche of the CDO. Our study differs from these studies in terms of the economic motivation and the focus on model risk for CDO mispricing. Finally, our paper corroborates the suggestion by Coval, Jurek, and Stafford (2009b) and Rajan, Seru, and Vig (2015) that model performance may depend on the user's incentives.

The rest of this paper is organized as follows. Section II reviews the setting of our study and relevant literature. Our simulation results on the effects of model specification are discussed in Section III. An empirical analysis using historical CDO data and the implications for CDO mispricing are presented in Section IV. Section V concludes the paper.

II. CDO Primer and Relevant Literature

The prototype of a CDO originated in 1987 at the junk bond powerhouse Drexel Burnham Lambert (bankrupt in February 1990). The resurgence in the current format is mostly attributed to Credit Suisse First Boston in 1997 (notably Christopher Ricciardi).[7] CDOs are investment conduits that hold credit securities as collateral assets and issue secured notes as liabilities with a prioritized payment structure. They belong to the category of pay-through asset-backed securities (ABS).[8] Major collateral asset types include corporate loans and bonds, but other types include ABS and credit derivative contracts. Based on the collateral asset types, CDOs can be classified into CLOs, CBOs, ABS CDOs, CDO^2, and so forth. Most CDOs have multiple tranches with various debt claim seniorities, where parts of the tranches are sold to different investors. However, single-tranche CDOs ("bespoke" CDOs) are often

[7]The development of the credit derivatives market in general is largely attributed to J. P. Morgan (notably Blythe Masters), which invented the credit default swaps (CDS) that fueled the synthetic CDO market.

[8]CDOs are distinguishable from traditional ABS in two aspects. First, CDOs structure and collateral assets are much more diverse than those of traditional ABS. Second, CDOs liability structure is more complex with trigger events to retire the senior tranches and other credit enhancements.

structured specifically for a particular investors needs. CDO underwriters structure the deal and arrange the CDO notes placements. Except static deals, CDO assets are administered by collateral managers, and CDO operations are overseen by trustees.

The CDO market has been a rated market from the beginning, and in practice, many investors rely on the ratings for CDO pricing. Before the CDO issuance, it is almost always critical for CDO issuers to secure target ratings. Typically, the CDO underwriter submits the CDO term sheet to one or more credit rating agencies, who will conduct CDO valuation based on the projected collateral characteristics. The underwriter and the credit rating agency need to agree the credit rating. Otherwise, the underwriter may use ratings from another credit rating agency. All three major rating agencies (S&P, Moody's, and Fitch) employ simulation methods when rating CDOs. Since CDOs are debt claims constructed from the underlying collateral portfolio, the valuation of CDOs starts with and depends heavily on an accurate assessment of the credit risk of the collateral portfolio. Specifically, credit rating agencies first simulate portfolio loss rates based on the CDO collateral asset information. The portfolio loss distribution can be obtained by using various approaches and default correlation assumptions. The default correlation assumption affects the thickness of the right tail of the portfolio loss distribution (i.e. the probability of big losses), which is particularly important for the CDO senior tranche. Because of the prioritized structure of CDOs, senior tranche holders will not suffer a loss unless the loss is sufficiently large and the subordinated tranches are exhausted. When default correlation is high, we will have more clustered defaults and limited diversification benefits. Senior tranches and junior tranches will have similar cash flow streams.

Two different approaches are often used to derive the loss distribution for the collateral portfolio. The structural approach assumes that asset value processes are correlated, and a firm defaults when its asset value falls below some default threshold. The asset value is simulated with imposed correlations, and the credit portfolio value is determined after all assets are simulated. Repeating the simulation multiple times results in a distribution of the portfolio value. The reduced-form approach assumes that default occurs suddenly and unpredictably. A correlation structure is directly imposed on the default probability. The default intensity can be linked to firm-specific and market-wide variables, and the number of defaults in the collateral portfolio follows a given distribution. The portfolio loss rate is then drawn repeatedly from this default distribution.[9]

After obtaining the distribution of the portfolio loss rate, the tranche size is determined by referencing the scenario default rate (SDR) of the desired rating. Specifically, the simulated distribution of the portfolio loss rate is used to map the idealized default rate for a scenario into a SDR. The idealized default rate for a scenario (D) is the maturity-specific "default criteria" that gives the probability of the occurrence of the scenario according to the historical corporate bond default rate with the same rating. Then SDR is the portfolio default rate (with some adjustment based on default experience) for which the default probability exceeding this portfolio default rate is no greater than D, i.e. $Pr(default\, rate \geq SDR) = D$.[10] The tranche must withstand the SDR of the desired rating. For example, for a CDO senior tranche with a AAA credit rating, i.e. AAA scenario, the probability that the portfolio loss rate is greater than the SDR should be lower than the historical AAA corporate bond default rate. Then, the AAA tranche size is determined as one minus the SDR (1-SDR). Griffin and Tang (2012) and Benmelech and Dlugosz (2009) provide detailed discussions about the S&P's SDR approach for the tranche determination. SDRs are key to obtaining desired ratings for CDO tranches.[11]

Although agency conflicts may arise during the security design (Mehran and Stulz (2007)), structured finance instruments, particularly CDOs, can be useful investment tools as long as the default correlation is low, as shown by DeMarzo (2005) and Leland (2007). However, the default correlation is difficult to measure, which contributes to the failure of CDO valuation (Brunnermeier (2009)). For such low occurrence events, the Bayesian approach is particularly appealing (Glasserman and Li (2005)). Therefore, to assess the credit risk of structured finance instruments such as CDOs, it is necessary to consider both the firm-specific default predictors

[9]See Internet Appendix in Griffin and Tang (2012) for a detailed discussion of the CDO valuation models.

[10]We follow S&P's terminology and refer it as SDR. It is default scenario collateral loss rate by Moody's. The descriptions are based on the published documents such as Moody's (1998), S&P (2002), and Fitch (2006).

[11]The purchase price for CDO notes are mostly at par. The coupon rate on each tranche is the most visible pricing indicator. However, the coupon rate, rating and tranche size are jointly determined. The credit spread of a given rating is easily agreeable. Hence, the most critical pricing component is the tranche size (equivalently the risk level of the tranche, i.e., the SDR). We focus on the tranche size and SDR throughout this paper and use rating, pricing, and valuation interchangeably.

and, more challengingly, the default correlation.

One firms default status may also affect another firm's default probability. For instance, Acharya, Schaefer, and Zhang (2015) document the impact of the downgrades of GM and Ford on all the constituents in the market, even though some of them are completely unrelated to GM and Ford. Jorion and Zhang (2007) conduct a larger scale analysis of bankruptcies and find similar results. Reasons why these seemingly unrelated firms share a default factor may be learning, as argued by Benzoni, Collin-Dufresne, Goldstein, and Helwege (2015) and Giesecke (2004), or market structure, as argued by Allen and Carletti (2006). A conventional portfolio loss risk model assumes that the default correlation is attributable only to observable factors. Even with the benefits of various firm-specific and macroeconomic covariates, however, Das, Duffie, Kapadia, and Saita (2007) find empirical evidence that defaults are more clustered than conventional models suggest based merely on observable factors. DEHS (2009) provides a new model for corporate default intensity with a time-varying common latent factor, as well as in the presence of a firm-specific unobservable covariate. They find that the prediction power of a general credit model will increase dramatically if a common unobservable covariate is incorporated into the model. Compared with traditional models, this model is especially effective for default clustering estimation. However, this refined pricing model still suffers from parameter uncertainty. In addition to the frailty factor, recent literature documents the important role of contagion in explaining the default clustering. Helwege and Zhang (2016) investigate the counterparty contagion and information contagion for financial firms. Azizpour, Giesecke, and Schwenkler (2017) find that contagion and firms exposure to observable and latent systematic factors explain significant part of clustering.

Coval, Jurek, and Stafford (2009a) show that CDO senior tranches are inaccurately priced and that senior tranche investors should have required a higher risk premium than that indicated by the "unreliable" ratings. The mispricing arises from the economic catastrophe feature of CDOs and many other structured products that default only under extremely bad economic states. This default clustering feature in bad economic states acts as an additional source of risk for senior CDO tranches. Rating agencies, however, ignore this economic catastrophe feature in practice. Investors therefore should not rely on credit ratings for CDO pricing or risk assessment, as the information contained in them is insufficient. To correct the failure of CDO pricing, Coval, Jurek and Stafford (2009a) develop a state contingent framework based on a modified Merton's (1974) structural model.

Coval, Jurek, and Stafford (2009b) provide a detailed discussion of the structured products market and the valuation/rating failure. In addition to the economic catastrophe feature, as discussed in Coval, Jurek and Stafford (2009a), they claim that small model error can be significantly magnified by the pooling and tranching structure of structured products. The model error may arise from an inaccurate assumption for either the default correlation or default probability of collateral assets. The largest impact can be found in the more complicated CDO^2.

Griffin and Nickerson (2017) quantify rating agencies' default correlation assumptions for structured products before and after the crisis. Although the rating agencies default correlation assumption has increased from 0.01 to 0.03 after the crisis, it is much lower than the estimated correlation of 0.12 when we jointly consider the observable and nonobservable factors. Broer (2017) find that disagreement about the default correlations increases the value of structured collateral. Erlenmaier and Gersbach (2014), Bae, Iscoe, and Kim (2015) and Andreoli, Ballestra, and Pacelli (2016) also explore appropriate ways of estimating the default correlation and CDO pricing.

In the spirit of DEHS (2009), we use a dynamic-frailty model as the benchmark model for portfolio loss estimation, which is the foundation for CDO pricing. We depart from DEHS (2009) by assuming different scenarios of the data structure. This can be achieved by controlling for the data-generating process. We further apply the dynamic-frailty model for CDO pricing.

III. The Impact of Model Specification on CDO Pricing: Simulation Results

CDOs are constructed from a collateral portfolio characterized by collateral credit quality, maturity and correlation, and the portfolio cash flows are tranched into different classes. Therefore, CDO pricing heavily depends on the accurate assessment of the credit risk of the collateral portfolio. In this section, we demonstrate the effects of various model specifications on portfolio credit risk assessment and CDO pricing based on simulations. A simulation study can help elucidate the full picture of model performance. Misspecification of a model leads to biased estimation and might eventually produce a deflected prediction. For example, when common-frailty-driven defaults are not accounted for, we underestimate the possible extreme losses of a credit portfolio, which is particularly relevant for CDO senior tranche pricing.

A. Dynamic-frailty Model and Simulation Methods

To assess the model specification effects, we use the dynamic-frailty model in Duffie, Eckner, Horel and Saita (2009) as a benchmark good model. As defaults are more clustered than conventional credit risk models suggest (Das, Duffie, Kapadia, and Saita (2007)), Duffie, Eckner, Horel and Saita (2009) propose a frailty-correlated default model, in which the frailty factor can be used to technically solve the omitted variable bias. The frailty factor can be anything that affects a firm's default probability and generates an additional source of default correlation. Since the frailty factor is not observable and may change with time, we need to perform Bayesian updating to "learn" the frailty factor from the realized defaults. For example, after Enron's bankruptcy, people realized that other firms may have similar accounting problems (i.e., frailty), and adjusted their default estimation accordingly. The inclusion of the frailty factor and the resulting more accurate default correlation assumptions is particularly important for portfolio credit risk analysis and CDO senior tranche pricing. Specifically, we assume that the default intensity of firm i at time t takes a proportional hazard specification as

$$\begin{aligned}\lambda_{it} &= \Lambda(S_i(X_t), \theta) \\ &= \exp(\alpha + \beta \cdot V_t + \gamma \cdot U_{it} + Y_t).\end{aligned} \tag{1}$$

where $S_i(X_t)$ represents the component of X_t that is relevant to the default intensity of firm i, and θ represents the parameter vector for the default intensity to be estimated. Default events are driven by three types of factors: observable macroeconomic factors (V_t), including market-wide stock returns and interest rates; observable firm-specific factors (U_{it}) such as a firm's distance-to-default and trailing stock return; and the unobservable common frailty factor Y_t.[12] Following Duffie, Eckner, Horel and Saita (2009), we further assume that the unobservable common frailty factor Y_t follows an Ornstein-Uhlenbeck (OU) process, with the speed of mean-reversion of κ and a standard Brownian motion (B) as the innovations:

$$dY_t = -\kappa Y_t dt + dB_t. \tag{2}$$

Conditional independence of default arrivals is regained under the assumption that the frailty factor may capture additional default clustering. Appendix A provides details regarding the default intensity parameter estimation. From the frailty module, we can obtain default probability estimations with both observable and unobservable factors.

To evaluate portfolio credit risk by using the dynamic-frailty model, we first simulate both the observable factors and unobservable frailty factor that affect a firm's default intensity based on a factor time-series model. The number of firms simulated is 2800, and the history lasts for 25 years.[13] To remain in line with the factor dynamics implied in the real historic data, we employ the same Gaussian first-order vector autoregressive model for the observable factors in Duffie, Saita and Wang (2007) and the same OU process with long-run mean of 0

[12] It is possible to have a firm-heterogeneous frailty factor Z_i. However, Z_i is difficult to determine given the size of the data. Furthermore, its presence does not qualitatively change the significance of Y_t. Therefore, this unobservable firm heterogeneity is excluded from the final model for portfolio credit risk evaluation in Duffie et al. (2009).

[13] Duffie, Saita and Wang (2007) consider 2770 industrial firms for the period from 1980 to 2004.

for the common frailty factor specified in Duffie, Eckner, Horel and Saita (2009). The time step is taken to be one month. We further simulate corporate default time data by using the Inverse-CDF method offered in Duffie and Singleton (1999). Appendix B provides a detailed review of the factor time-series model and the data simulation method, and Appendix C lists the maximum likelihood parameter estimation of the factor time-series model in Duffie, Saita and Wang (2007).

After obtaining the factor time series and default timing data, we insert these simulated data into the dynamic-frailty model to estimate the default intensity parameters. By extending the factor time series with the prespecified model, we can evaluate the credit risk of any portfolio constructed on the underlying firms in our dataset. Through the simulation analysis, we first investigate the performance of the dynamic-frailty model in filtering out the "frailty" factor. The validation test in Appendix D shows that the dynamic-frailty model can effectively filter out the hypothetical "true" frailty. Motivated by this finding, we then investigate the performance of the frailty model for portfolio credit risk analysis and CDO pricing.

B. No-frailty versus Dynamic-frailty Model

Even for single-obligor credit risk modeling, there is no consensus on the best performing model. Model failure has been recorded in nearly all areas. The seminal work of Vasicek (1987) on portfolio credit risk is shown to be inaccurate for heterogeneous asset pools. More seriously, the default correlation is assumed to be driven only by observable factors. This counterfactual assumption has been widely adopted until recently. However, we believe that existing almost all CDOs are evaluated based on this low correlation assumption.

To understand to what extent an omitted latent factor might engender CDO mispricing, we formally conduct an analysis of portfolio default rate prediction with the no-frailty and dynamic-frailty models. Based on the portfolio default rate distributions, we further conduct CDO pricing by referring to the SDR of the desired rating. Once the SDR for a desired tranche rating is available, the tranche size can be determined as no greater than 1-SDR.

The simulated 25 years of data on observable factors is summarized in Panel A of Table I. Then, we estimate the default intensity parameters by using the maximum likelihood method based on the simulated data. The estimated parameters for the no-frailty and dynamic-frailty models are listed in Panel B of Table I. Subsequently, we form a portfolio with all active firms at the end of year 25, and predict the portfolio loss distribution with the frailty model and no-frailty model, respectively. Figure 1 compares the portfolio's default rate distribution for the next five years according to the dynamic-frailty and no-frailty models. Panel C of Table I provides the details of the quantiles of the portfolios default rate distribution. Using the simulated portfolio as the collateral portfolio, we then investigate the CDO pricing with both the dynamic-frailty and no-frailty models.

With the estimated distribution of expected portfolio default loss, the CDO tranche must withstand the SDR of the desired rating. As discussed above, the SDR is the portfolio default rate for which the default probability exceeding this portfolio default rate is no greater than that of the historical corporate bond default rate with the same rating. Once the SDR for a desired tranche rating is available, the tranche size can be determined as no greater than 1-SDR. For CDO senior tranche pricing, we focus on the tail risk. We determine the AAA tranche size by referring to the AAA historical default probability. For the five-year prediction horizon, the default probability is about 0.1% for a AAA corporate bond. This default probability corresponds to the SDR for the AAA tranche given by the 0.999 quantile. However, different credit risk models yield different quantile values and therefore the model implied SDR. The higher of the quantile value and SDR, the smaller of the model implied AAA tranche size (1-SDR). As shown in Table I and Figure 1, the 0.95, 0.99 and 0.999 quantiles for the dynamic-frailty model prediction are 14.29%, 17.33% and 21.01%, respectively, and 11.66%, 13.41%, and 15.55% for the no-frailty model prediction. If we take the 0.999 quantile as the SDR for a AAA rating, the AAA tranche size will be 78.99% for the frailty model and 84.45% for the no-frailty model. Therefore, compared to pricing CDOs without the frailty factor (i.e., the no-frailty model), pricing CDOs with the frailty factor can reduce the AAA tranche size by 5.46%.

C. Correlation Between Macroeconomic Factors

The simulation results from the previous section highlight the value of the frailty model for CDO pricing. In this section, we further consider the impact of correlation between macroeconomic factors on CDO pricing in the presence of frailty factor. Specifically, we consider a more realistic model for dependent risk factors that captures the correlation between macroeconomic factors. In reality, there might be correlation between macroeconomic factors. For example, when the market undergoes a crisis, the central bank will step in and cut interest rates to inject liquidity into the market, which essentially creates a correlation between the short-term interest rate and stock market performance.[14] However, in credit risk models, such as the frailty model in Duffie, Eckner, Horel and Saita (2009), interest rates are assumed to be independent of stock market index trailing returns. Ignoring the additional correlation between the interest rate and stock market index returns may affect CDO pricing and the real AAA tranche size.

To investigate the effect of this correlation assumption, we first impose a correlation (lag one) between the interest rates and market returns in the factor time-series model used for the data simulation. Specifically, based on the factor time-series model in Appendix B, we introduce a correlation ρ between the innovation terms of the 3-month interest rate ($\varepsilon_{1,t+1}$) and S&P 500 trailing returns (ξ_t). Historically, the lag one correlation between the innovations of the 3-month interest rate and S&P 500 returns is about 0.18 for the 10 years from 1997 to 2006. This period represents a time when the CDO market experienced exponential growth. For illustration purpose, we opt for a higher correlation of 0.3.

Based on the simulated historical time series (with an imposed correlation), we compare the frailty model prediction with and without the effect of the correlation between macroeconomic factors (ρ equals to 0.3 and 0, respectively). A summary of factor time series is provided in Table II Panel A. Panel B presents the estimated default intensity parameters

[14] In this section, we focus on the correlation between short-term interest rate and stock market performance, which is a more realistic model for dependent risk factors. It's interesting to further investigate the impact of the monetary policy intervention on derivative pricing. To introduce the intervention in the model, we may have to consider regime changes (or jumps) in the interest rates depending on the level of simulated (or realized market returns).

for the frailty model. Figure 2 shows the portfolio's default rate distribution for the next five years according to the dynamic-frailty model with correlated and uncorrelated macro factors. The quantiles of predicted portfolio default rate are also summarized in Table II Panel C. As we can see from tail part of Figure 2 and Table II Panel C, the portfolio default rate is only slightly higher when correlation is considered. The differences are 0.09%, 0.14% and 0.14% for the 0.95, 0.99, and 0.999 quantiles, respectively. This result implies that the assumption of zero correlation between interest rates and stock market returns in the dynamic-frailty model does not have a significant effect on the default estimation results. Consequently, the implied CDO AAA tranche is similar regardless of whether the correlation between macroeconomic factors is considered.

To sum up, the simulation results in this section suggest that the common frailty factor affects the predicted portfolio default rate, particularly for the part of the tail that is most relevant to AAA CDO tranches. In the presence of the frailty factor, the model prediction is relatively robust to a correlation between macroeconomic factors.

IV. CDO Pricing with the Frailty Model: Empirical Evidence

In the previous section, we investigate the potential effects of model specification on CDO valuation by using a simulation method. We conduct a corresponding empirical analysis in this section. We first describe our sample CDO data. The empirical method is demonstrated in a case study. Specifically, we perform a credit risk evaluation on CDO AAA tranches by using both the no-frailty model and dynamic-frailty model. We scrutinize the ability of the benchmark dynamic-frailty model to predict subsequent downgrading of the senior AAA-rated CDO tranches over our sample CDOs, and we further discuss the implications of the results for CDO mispricing.

A. Data Description

Our sample contains 237 CDOs issued between May 1998 and December 2004.[15] The distribution according to collateral asset type is as follows: 46 CBOs, 82 CLOs, 99 ABS CDOs and 10 CDO^{2}s. We obtain the first report after the ramp-up of the asset portfolio[16] with the following collateral asset characteristics:

- Closing date (CDate): The date on which a CDO is purchased by investors.
- Weighted average rating (WAR): Average credit rating of the collateral asset portfolio, weighted by the par amount of each asset.
- Weighted average maturity (WAM): Average maturity of the collateral asset portfolio, weighted by the par amount.
- Number of obligors (N): Number of distinct obligors for the collateral asset portfolio.
- Default measure (DM): The average expected default rate of collateral assets, weighted by the par amount and annualized by the average asset maturity.
- Variability measure (VM): The annualized standard deviation of collateral asset default rates, which measures the dispersion of underlying assets when their correlation is not considered.
- AAA tranche size (AAA size): The sum of the face values of all AAA-rated tranches of a CDO divided by the total face value of the CDO.

Table III presents the summary statistics for our sample. We also list the SDRs at the reporting date for the initial rating and downgrading notches as of October 15, 2009, for the initially AAA-rated tranches. For downgrading notches, the number 0 denotes never downgraded, and the numbers 1-19 correspond to downgrading from AAA to AA+ all the way down to CC. In our sample, we have 16 downgraded CBOs, 53 downgraded CLOs, 85 downgraded ABS CDOs and 7 downgraded CDO^{2}s. The SDR is the required subordinationor

[15]Our sample period ends in 2004 because of the limited availability of data on the frailty factor. Additionally, the CDO market has explosive growth with some irregular activities during the 2005-2007 period. Consequently, nonstructural factors could drive CDO pricing after 2004.

[16]See internet appendix Figure IA.1 in Griffin and Tang (2012) for the discussion of CDO credit rating timeline.

the percentage of portfolio loss rate that a CDO tranche at a given rating level must sustain without causing a cash flow event of default. The probability of default in the asset portfolio exceeding this percentage is no greater than the historical default probability of corporate bonds with the same rating. For example, if the portfolio default distribution is the same as the one with frailty in Figure 1 and if the average realized default probability for a AAA-rated corporate bond is 0.1%, then the SDR for the AAA tranche is 21.01%, the 0.999 quantile. Once the SDR for a desired tranche rating is available, the tranche size can be determined as no greater than 1-SDR. However, in practice, a larger fraction of a AAA tranche can be achieved through out-of-model adjustments (Griffin and Tang (2012)).

B. A Case Study Illustrating the Methodology

We first illustrate our CDO valuation method via an example case analysis. All 4 types of CDOs are valuated in a similar way. The chosen CDO is called *Independence I.* This CDO is collateralized with various ABS securities, including commercial mortgage-backed securities (CMBS), residential mortgage-backed security assets (RMBS), ABS, and CDO. Below, we demonstrate how we evaluate this ABS CDO and how the frailty model generates results that can predict eventual downgrade.

Independence I is issued by Independence I CDO, Ltd. (a special purpose vehicle registered in the Cayman Islands) and co-issued by Independence I CDO Inc. (a special purpose vehicle registered in Delaware).[17] The closing date is December 7, 2000, according to Moody's, and December 12, 2000, according to S&P. Credit Suisse is the lead underwriter and counterparty for interest rate swap agreements. The collateral manager is Independence Fixed Income Associates Inc., which was renamed to Declaration Research and Management LLC. in 2003. From Moody's New Issue Report dated April 13, 2001, the collateral pool is fully ramped in March 12, 2001 (about 65% complete at the closing date).

Independence I has an initial principle amount of US$300 million with the following capital structure: Class A first priority senior secured notes of $223.5 million (74.5%), Class B second

[17] The Independence series continue to Independence VII issued on March 28, 2006.

priority senior secured notes of $50 million, Class C Mezzanine secured notes of $15 million, and a preference share of 11.5 million.[18] Moody's initially assigned a AAA rating to the Class A tranche, followed by the Class B tranche with Aa3, and Class C tranche with Baa2. Preference shares are not rated. S&P assigned the AAA rating to Class A but did not rate Class B, Class C and preference shares. Fitch also provided Class A with a AAA rating, Class B with a AA- rating, and Class C with a BBB rating. Although all three rating agencies issued a AAA rating to Class A of this CDO, it was subsequently downgraded to AA- rating on August 30, 2004, and further downgraded to A- on November 16, 2005, by S&P. Fitch downgraded Class A to A on March 7, 2006 and then to BB on March 9, 2009. Moody's downgraded Class A to AA2 on February 18, 2005, to Baa2 on February 2, 2007, and further to B1 and then placed it under review for possible a downgrade on April 22, 2009.

The collateral asset characteristics for Independence I reported on December 26, 2003, before any downgrade are as follows: the collateral asset portfolio contains 95 assets from 83 obligors, with a weighted average rating of BBB-, a weighted average maturity of 8.45 years, an average expected asset default rate of 0.0112, and a variability of the default rate of 0.0162. For the AAA rating of this collateral portfolio, a rating agency derives the SDR of 29.2% by using a default rate threshold of 0.00608.

When the credit rating or pricing for a CDO is obtained, the collateral pool is typically incomplete, and rating agencies will conduct an analysis and assign a rating based on the projected collateral pool characteristics. To price this CDO, we first generate factor time series based on the collateral pool characteristics specified above. Then, we insert these factor time series into the no-frailty model (bad model) and the dynamic-frailty model (good model) and generate the collateral pool loss distribution. With the collateral pool loss distribution, we can determine the AAA SDRs by referring to the historical AAA default probability. We then compare the AAA SDRs and the resulting AAA tranche size from the different model specifications (no-frailty vs. frailty model).

Specifically, we adopt the parameter estimations of the factor times-series dynamics and

[18] These numbers are provided by Moody's New Issue Report. S&P record has a preference share size of $12 million.

default intensity provided in Duffie, Saita and Wang (2007) and Duffie, Eckner, Horel, and Saita (2009).[19] The 3-month treasury bill rate and S&P 500 index are obtained from the Board of Governors of the Federal Reserve system and CRSP database, respectively. We choose the weighted average maturity (WAM) of the CDO as the prediction horizon. We further assume that each obligor has an equal amount of principal in the asset pool.

We next need the distance to default and asset processes for the 83 obligors of the CDO collateral pool. We assume the obligor-specific factors starting from its long-run means. As discussed earlier, collateral pools of CDOs consist of various types of assets, such as corporate bonds, leveraged loans, sovereign debts, ABS tranches and CDO tranches. Our exemplificative CDO Independence I comprises 41.8% CMBS, 23% RMBS, 21.4% ABS, and 13.8% CDO assets. It is prohibitive to estimate the distance to default and asset processes for these complex securitized products. Instead, some rating agencies use the average default probability of the same rating cohort to proxy for the default probability of the same type of assets and assume a pairwise correlation among the obligors based on the industry sector and geographic region. For example, S&P's CDO Evaluator and Fitch's VECTOR determines the default probability based on asset type, rating and maturity.[20] Furthermore, for CBOs and CLOs, the obligor might be a private firm, and such rating is not even available.

In our study, we do not conduct an obligor-by-obligor estimation for the distance to default and asset processes. For simplicity and tractability, we make use of the portfolio average expected default rate (DM) and variability of default rate (VM). We assume that the default probability of each obligor in the collateral portfolio is log-normally distributed with a mean DM and a standard deviation of VM taken by the square root of N, the number of obligors. We choose a log-normal distribution in light of the nonnegative default probabilities and right-skewed default rate distribution. Then, we equally draw N quantiles of the log-normal distribution along the interval $(0, 1)$ and assign these quantiles as the default rates of

[19]The frailty factor estimation is available up to the end of year 2003. For a CDO with a closing date in the year 2004 (which may have been initiated in 2003), we extend this factor to the date by using the OU process dynamics starting from the end month of 2003.

[20]For each asset type, default probabilities across all ratings and for typical maturities are estimated from historical default data on that specific type. Sometimes, adjusted default probabilities from other asset types are used when the historical data are scarce for a recent innovation.

the obligors. Next, the sampled default probability of obligor i, DP_i, is transformed into the targeted distance to default, θ_{iD}, through the inverse cumulative normal distribution function Φ^{-1}:

$$\theta_{iD} = -\Phi^{-1}(DP_i), \tag{3}$$

For simplicity, we assume that the long-run mean of the assets of each obligor is uniformly distributed on some quartile range of the asset values, as estimated in Bharath and Shumway (2008). Specifically, for CBOs, we choose the uppermost quartile, 6.3-10.0, given that CBOs mostly comprise bonds issued by relative large-cap companies. For CLOs, we choose the 0.25-0.5 quartile, 3.3-4.7, since most underlying assets of CLOs are leveraged loans from small and median-sized firms. For ABS CDOs and CDO^{2}s, we do not have an established basis to choose a particular asset span and simply use the interquartile range, 3.3-6.3.[21] Empirical evidence in Titman and Wessels (1988), Rajan and Zingales (1995), and Fama and French (2002) shows that larger firms tend to have higher leverage. Thus, we assign in reverse order the long-run means of assets to the targeted distance to default for each obligor. Accordingly, a larger obligor in our sample has a lower targeted distance to default.

Given the assumption that obligor-specific factors start from their long-run means, we apply the no-frailty model and dynamic-frailty model to generate the collateral pools default rate distribution for each CDO and determine the SDRs accordingly. The results are provided in Table III. For *Independence I*, the SDR is 29.2% according to the rating agency model, 25.3% according to the no-frailty model, and 51.8% according to the dynamic-frailty model. Theoretically, the AAA tranche size is given by the 1-SDR. Therefore, while the SDRs according to both the rating agency model and the no-frailly model allow a AAA tranche of more than 70% for this CDO, the dynamic-frailty model with an advanced default correlation assumption allows a much smaller AAA tranche of 48.2%.

In the above analysis, we use the historical AAA default probability to determine the SDR and the AAA tranche size from the collateral pool loss distribution. In other words, we fix the AAA default probability and compare the model-implied AAA tranche size. In our data

[21]For ABS CDOs and CDO^{2}s, our SDR prediction is not sensitive to the asset span when it is shifted down to the lower interquartile 0.4-4.7 or up to the upper interquartile 4.7-10.0.

set, we also observe the real AAA tranche size. Then, instead of fixing the default probability and comparing the model-implied tranche size, we can also compare the model-implied "real" default probability of this real AAA tranche size. Specifically, we construct a variable default probability (DP_i) for CDO i as:

$$DP_i = Prob\{Default\ Rate > Attachment\ Point\} \tag{4}$$

Theoretically, the corresponding default probability for the AAA tranche size should be the AAA default probability ($DP_{AAA,i}$). However, different models yield different portfolio default rate estimation. The AAA tranche size from the credit rating agency model does not necessary gives the AAA default probability according to the no-frailty and/or dynamic-frailty models. With the real AAA tranche size in our dataset, we would like to examine the corresponding default probability for this AAA tranche size in the no-frailty model ($DP_{NF,i}$) and dynamic-frailty model ($DP_{DF,i}$). The larger deviation between $DP_{DF,i}$ and $DP_{AAA,i}$ (i.e., $DP_{DF,i}$-$DP_{AAA,i}$), the higher risk of this AAA tranche as estimated by the dynamic-frailty model. A similar argument can be made for the $DP_{NF,i}$-$DP_{AAA,i}$. For our case *Independence* I, the $DP_{NF,i}$ and $DP_{DF,i}$ equal 0.4% and 20.7%, respectively. Given that the $DP_{AAA,i}$ equals 0.61%, the default probability of the Independence I AAA tranche is much higher according to the dynamic-frailty model estimation. Note that the Class A tranche with an initial AAA rating from all three rating agencies is eventually downgraded to a BB credit rating.

C. AAA Tranche Size from the Frailty and No-Frailty Models

The estimated SDRs from the no-frailty model and dynamic-frailty model for each CDOs are presented in Table III and Figure 3. For comparison, we also provide the SDRs by rating agency. Table IV summarizes the real AAA tranche size, and the empirical results for average SDR according to the rating agency model, no-frailty model, and dynamic-frailty model. The last two columns in Table IV provide the average implied default probability of the real AAA tranche size from the no-frailty model and the dynamic-frailty model, respectively.

As we can see from Tables III, IV and Figure 3, the SDRs from the no-frailty model are highly correlated with those from the rating agency model. Specifically, the correlation is 0.91 for CBOs, 0.89 for CLOs, 0.87 for ABS CDOs and 0.57 for CDO^2. Compared with the no-frailty model, the rating agency model gives higher SDRs, on average. In particular, the SDRs from the rating agency model are 9% higher for CBOs, 7% higher for CLOs, 4.1% higher for ABS CDOs and 10.3% higher for CDO^2. Therefore, on average, the rating agency model yelled a smaller AAA tranche size than the no-frailly model. However, when the frailty factor is considered, the SDRs from the rating agency model underestimate CBOs by 12%, CLOs by 15%, ABS CDOs by 13% and CDO^2 by 5%, on average. Across all the 237 CDOs in the sample, the dynamic-frailty model predicts SDRs that are, on average, 13% higher than predicted by the rating agency model, while the no-frailty model predicts 6% lower SDRs than the dynamic-frailty model.

According to our empirical results, if we only consider observable factors for our portfolio credit risk evaluation, the SDRs according to rating agencies, which are the primary determinants of the assigned rating, overestimated the portfolio risk for all four types of CDOs, on average. For CDO^{2}s, the overestimation is most prominent. Once the additional source of risk, the common frailty factor that systematically affects the whole economy, is taken into account, the risks of all 4 types of CDOs are underestimated by the SDRs from the rating agency model. ABS CDOs are most directly related to the subprime mortgage crisis, and they have experienced widespread downgradings even for AAA-rated tranches. Of the 99 ABS CDO in our sample, 85 have been downgraded by one or more rating agencies, and 16 and 53 downgrades are recorded for the 46 CBOs and 82 CLOs, respectively, in our sample. The dynamic-frailty model predicts the large risk underestimation, over 10% for CBOs and CLOs. The frailty factor is thus important for understanding the risks embedded in AAA tranches, as it decreases the AAA tranche size by about 19% when added to the model.

D. Downgrading Prediction

We further conduct a downgrading prediction study with respect to the 237 CDOs in our sample. We first separate the CDOs into 10 risk groups according to difference between the implied default probability of the real AAA tranche from the frailty model ($DP_{DF,i}$) and the AAA default probability ($DP_{AAA,i}$) , i.e., $DP_{DF,i} - DP_{AAA,i}$. Group 1 represents the lowest risk group, and group 10 represents the highest risk group predicted by the frailty model. Then, we compare the percentage of CDOs downgraded in each of the risk groups. Similarly, we can compare the downgrading prediction power of the no-frailty model using the deviation of $DP_{NF,i} - DP_{AAA,i}$. As shown in Figure 4, when we use the frailty model, the average rate of downgrades for CDOs in the lowest risk category is 29%, whereas the average downgrading rate is 92% in the highest risk group. Although the frailty model is not perfect and indeed there is no monotonic pattern in the figure[22], the monotonicity is much more visible for frailty model than the no-frailty model. In Figure 4, we have also shown the risk classification and downgrading prediction based on the rating agency's SDR. The SDRs from rating agency produce a different pattern, which is consistent with Griffin and Tang (2012). Overall, although the frailty model is not perfect, it shows power in separating out the safest and riskiest CDOs and predicting the future CDO downgrading compared to no-frailty and rating agencys SDRs.

To further investigate the power of the frailty model in predicting the future AAA tranche downgrading, we have conducted the following regression analysis. Since higher default probability denotes more risk, the AAA tranche is expected to be downgraded to a lower credit rating when the deviation between the default probability for the real AAA tranche size from the frailty model (good model) and no-frailty model (bad model) is higher. We thus conduct an ordered probit regression of the AAA tranche downgraded notches on the risk proxy DP and a set of controls and a CDO type dummy. The results are presented in Table V. The downgrading notches are regressed on the default probability for the real AAA tranche size from the dynamic-frailty model (DP_{DF}), the default probability for real AAA tranche size from the no-frailty model (DP_{NF}), the difference between default probability with and without frailty ($DP_{DF} - DP_{NF}$). We also include a set of controls for the CDO characteristics, including the weighted average maturity (WAM), default measure (DM), variance measure

[22]The imperfect of frailty model is consistent with Azizpour, Giesecke, and Schwenkler (2017) who establish the presence of excess clustering in the default data that cannot be explained by firms' joint exposure to observable and latent systematic factors.

(VM), number of obligors (Obl), interest rate (r_f), S&P 500 Return (S&P 500), Dummy CLO, Dummy ABS and Dummy CDO^2.

We expect the coefficients on default probability to be positive and significant, which would support the prediction power of the model. For the 237 CDOs, the coefficient for DP_{DF} equals 3.37, and it is 3.53 for the default probability without frailty DP_{NF}. The coefficient for DP_{DF} continues to be positive when both DP_{DF} and DP_{NF} are included in the regression. When the difference in default probabilities between the frailty and no-frailty models ($DP_{DF} - DP_{NF}$) is used as an independent variable, its coefficient is positive and significant even after we include the DP_{NF} or other control variables, as shown in models 3, 4 and 6. The R^2 for these regressions is 9.97%, on average. Therefore, the frailty model exhibits prediction power for the subsequent AAA CDO downgrading. The power of the frailty model is further evidenced by the regression result that the deviation between frailty and no-frailty model ($DP_{DF} - DP_{NF}$) significantly predicts downgrading.

E. *Implications for CDO Mispricing*

The previous analyses suggest that when the good model (frailty model) is used for CDO pricing, we will obtain a much smaller AAA tranche size. The good model has the power to capture the excess default clustering, which is particularly important for CDO AAA tranche pricing. The differences in model outputs from the good and the bad model even have prediction power for the subsequent CDO downgrades. Therefore, model specification plays an important role in obtaining the true price for CDO. However, does the model specification (or model error) explain the entire CDO mispricing that we observed before the crisis?

The development and adoption of new financial models, which provide a simple way to quantify the risks in the complicated CDO collateral pool, indeed boosted the explosive growth of the CDO market. Observing the limits of these financial models during the recent crisis, some market participants tend to blame the quants and their models for "killing the Wall Street".[23] However, as stated by the rating agency "ratings are ultimately the result of a

[23]see, e.g., "Recipe for disaster: The formula that killed Wall Street", *Wired*, February 23, 2009.

formal committee process and not simply model output" (Fitch (2006)). Mispricing may be attributed to both the model error (due to model specifications) and the errors out of the model (due to human factors). Even when the true model is used for CDO pricing (zero model error), mispricing can exist if there are errors out of the model. In particular, when we target a particular level of AAA tranche size, the subjective out-of-model adjustment yields identical AAA tranche sizes across CDOs, irrespective of the model outputs. The role of subjective adjustment is confirmed by Griffin and Tang (2012), who find evidence that CDOs with a smaller model-implied AAA size receive larger adjustments and subsequently experience more severe downgrading. In this setting, although model specification is important for finding the true price, errors out of the model play a key role in explaining CDO mispricing. Understanding the roles of both model specification and out-of-model errors is important for avoiding future pricing errors in the CDO and other financial markets.

In addition, Coval, Pan, and Stafford (2014) show that capital markets develop blind spots when financial models are misapplied in real world capital markets. The emergence and persistence of the blind spots arise because the relatively sophisticated market participants fail to notice the state-contingent model errors. They rely upon learning rules (and research methodologies) that have little power to reject their model in economically benign states, and they are forced to adjust their model only after the learning event, such as a crisis. However, one looking for state-contingent model errors prior to the learning event would have found reliable evidence. Therefore, even when model errors exist, mispricing can be less severe if market participants have developed a better understanding for the nature of the error. Our paper provides a good framework for analyzing model specification and its impact on derivative pricing, which will likely continue and appear repeatedly.

V. Conclusion

One of the most remarkable episodes of the 2007-2009 credit crisis is the widespread downgrading of top-rated (often AAA) CDO securities and overwhelming write-downs resulting from CDO revaluation. In this paper, we analyze the structural causes of CDO mispricing, and our simulation results suggest that model misspecification affects CDO valuation. The frailty default factor identified by Duffie, Eckner, Horel, and Saita (2009) is especially important in accurately measuring the default correlation. As ignoring the frailty factor can inflate the AAA tranche of a CDO, the AAA-rated tranche would have been rated much lower had the deal structurers and rating agencies considered the frailty factor and used more advanced model at the time of deal origination.

We conduct an empirical analysis on 237 CDOs issued between May 1998 and December 2004. Our no-frailty model obtains a CDO portfolio default rate at the AAA level that is close to rating agency estimates for CLOs and ABS CDOs. However, compared with ignoring frailty, considering the frailty factor raises the AAA portfolio default rate. Furthermore, the increase in relative risk for ABS CDOs, which experienced the most AAA downgrades during the credit crisis in our sample, can predict future downgrades. Hence, the information content in the frailty factor is qualitatively important. However, out-of-model adjustments play an important role in CDO pricing, with smaller model-implied AAA sizes receiving larger adjustments (Griffin and Tang (2012)). Considering out-of-model adjustments, model constraints have a modest quantitative effect in explaining the entire CDO mispricing.

The CDO market, especially the segment of CLOs, has come back strong with issuance amounts surpassing the precrisis peak. Hence, understanding the pricing of CDO is useful for future regulatory policies and risk management strategies, as future financial innovations will likely be accompanied by similar issues regarding model specification and data quality. The frailty model and Bayesian estimation approach discussed in this paper will be useful for portfolio credit risk analysis, as default data are scarce. Prior beliefs can shape the result in significant ways. Exploring the economic sources of the frailty factor and formation of prior belief about default correlation is thus a promising area for future research.

Appendices

A Parameter Estimation

The parameter estimation approach follows Duffie, Eckner, Horel and Saita (2009). Specifically, in the dynamic-frailty model, the likelihood of the data at the parameters (γ, θ) is given by

$$
\begin{aligned}
&\mathcal{L}(\gamma, \theta | W, Y, D) \\
&= \mathcal{L}(\gamma | W) \mathcal{L}(\theta | W, Y, D) \\
&= \mathcal{L}(\gamma | W) \prod_{i=1}^{m} (e^{-\sum_{t=t_i}^{T_i} \lambda_{it} \Delta t} \prod_{t=t_i}^{T_i} [D_{it} \lambda_{it} \Delta t + (1 - D_{it})]).
\end{aligned}
\tag{5}
$$

However, given that Y_t is not observable to the econometrician, the likelihood is then

$$
\begin{aligned}
&\mathcal{L}(\gamma, \theta | W, D) \\
&= \int \mathcal{L}(\gamma, \theta | W, y, D) p_Y(y) dy \\
&= \mathcal{L}(\gamma | W) \int \mathcal{L}(\theta | W, y, D) p_Y(y) dy \\
&= \mathcal{L}(\gamma | W) E[\prod_{i=1}^{m} (e^{-\sum_{t=t_i}^{T_i} \lambda_{it} \Delta t} \prod_{t=t_i}^{T_i} [D_{it} \lambda_{it} \Delta t + (1 - D_{it})]) | W, D].
\end{aligned}
\tag{6}
$$

where D_i is the vector of default indicators. That is, for company i, $D_i = 0$ before default and 1 upon default. $p_Y(y)$ represents the unconditional probability density of the unobservable common factor Y. Here, we assume that Y is independent of W.

For the estimation of the default intensity parameter θ, a combination of Markov chain Monte Carlo (MCMC) and the expectation-maximization (EM) algorithm is employed. This combination offers advantages for maximum likelihood parameter estimation in the model with incomplete information. The detailed steps include:

Step 1. Obtain the maximum likelihood estimator of the intensity model with only observable covariates $\hat{\beta}$. This estimator is the MLE from equation (5) when the effect of unobservable covariate Y is not considered.

Step 2. Assign an initial estimate value for θ, as suggested by Duffie, Eckner, Horel and Saita (2009), at $\theta^{(0)} = (\hat{\beta}, 0.05, 0)$.

Step 3. Draw n independent sample paths for the frailty factor $Y^{(1)}, \ldots, Y^{(n)}$ from $p_Y(\cdot|W, D, \theta^l)$, which is the conditional density of Y's OU process. This can be done with MCMC, specifically the Gibbs sampler, while taking the l^{th} estimate value for θ^l as well as the observable covariates W and D as given.

Step 4. Maximization step. Define the intermediate quality

$$\begin{aligned} Q(\theta, \theta^{(l)}) &= E_{\theta^{(l)}}(\log \mathcal{L}(\theta|W, Y, D)) \\ &= \int \log \mathcal{L}(\theta|W, y, D) p_Y(y|W, D, \theta^{(l)})\, dy \end{aligned} \tag{7}$$

Based on the sample path for Y drawn in step 3, $Q(\theta, \theta^{(l)})$ can be approximated by

$$\hat{Q}(\theta, \theta^{(l)}) = \frac{1}{n} \sum_{j=1}^{n} \log \mathcal{L}(\theta|W, Y^{(j)}, D) \tag{8}$$

Then, the new parameter estimate $\theta^{(l+1)}$ can be obtained by

$$Max \;\; \hat{Q}(\theta, \theta^{(l)}) = Max \;\; \frac{1}{n} \sum_{j=1}^{n} \log \mathcal{L}(\theta|W, Y^{(j)}, D) \tag{9}$$

Step 5. Return to step 3, and replace $\theta^{(l)}$ with the new estimator $\theta^{(l+1)}$. Proceed to step 4 to obtain $\theta^{(l+2)}$. Repeats step 3 and 4 until the estimation of θ reaches reasonable convergence.

The asymptotic standard errors for the parameter estimators can be calculate from the

Hessian matrix of the expected complete-data likelihood.

B Data Simulation Method

To assess the effects of the model specification on portfolio credit risk assessment and CDO pricing, we simulate a series of data structures. The number of firms simulated is 2800, and the history lasts for 25 years. To remain in line with the factor dynamics implied in the real historic data, we employ the same Gaussian first-order vector autoregressive model for the observable factors in Duffie, Saita and Wang (2007) and the same OU process with a long-run mean of 0 for the common frailty factor specified in Duffie, Eckner, Horel and Saita (2009). The time step is taken to be one month. Here, we provide a brief review of this factor time-series model.

A simple arbitrage-free two-factor affine term-structure model is specified for the three-month treasury rates (r_{1t}) and 10-year treasury rates (r_{2t}).

$$r_{t+1} = r_t + k_r(\theta_r - r_t) + C_r \varepsilon_{t+1}, \tag{10}$$

where θ_r is the long-run mean of interest rates, C_r is a 2×2 matrix, and $\varepsilon_1, \varepsilon_2 \ldots$ are independent standard normal vectors.

For the firm-specific factors of distance to default D_{it} and log-assets V_{it} and the trailing 1-year S&P 500 return,[24]

$$\begin{bmatrix} D_{i,t+1} \\ V_{i,t+1} \end{bmatrix} = \begin{bmatrix} D_{it} \\ V_{it} \end{bmatrix} + \begin{bmatrix} k_D & 0 \\ 0 & k_V \end{bmatrix} \left(\begin{bmatrix} \theta_{iD} \\ \theta_{iV} \end{bmatrix} - \begin{bmatrix} D_{it} \\ V_{it} \end{bmatrix} \right) + \begin{bmatrix} b \cdot (\theta_r - r_t) \\ 0 \end{bmatrix} + \begin{bmatrix} \sigma_D & 0 \\ 0 & \sigma_V \end{bmatrix} \eta_{i,t+1}, \tag{11}$$

[24]Firm asset value is determined by using the Merton's model. For more details, refer to Merton (1974) and Vassaulou and Xing (2004).

$$S_{t+1} = S_t + k_s(\theta_s - S_t) + \xi_{t+1}, \tag{12}$$

where θ_{iD}, θ_{iV} are the long-run means for firm i's distance to default and log assets, respectively. η_{it} is the two-dimensional innovation vector.

The correlation among the observable factors is modeled as

$$\begin{aligned} \eta_{i,t} &= Az_{it} + Bw_t, \\ \xi_t &= \alpha_S u_t + \gamma_S w_t, \end{aligned} \tag{13}$$

where z_{it} and w_t are independent two-dimensional standard normal vectors and u_t are independent standard normals.

For tractability and parsimony, the mean-reverting speed k_D of the distance to default is assume to be homogeneous across all firms. The distance to default is an asset volatility-adjusted measure of leverage, and its volatility σ_D does not vary by firm, as implied by Merton's theory. Asset volatility σ_V and its mean-reverting speed k_V are also assumed to be homogeneous across firms. However, a common targeted leverage ratio leads to an unrealistic estimated term structure of future default probabilities. Duffie, Saita and Wang (2007) instead estimate θ_{iD} firm by firm, with the cross-sectional distribution displayed in figure 12 of their paper. In our simulation study, we load the long-run means of distance to default and log assets for each firm in the following way.

As reported in Duffie, Saita and Wang (2007), the estimated θ_{iD} across the whole firm set has a median of 3.1, with an interquartile range of 1.4-4.8. A careful inspection of figure 12 reveals that the interval 0.0-8.0 of θ_{iD} covers most of the firms except those at the extreme lower or upper tail of the distribution. Within this interval, θ_{iD} is approximately linear to the rank of firm i, which means that θ_{iD} might be uniformly distributed on this range. Accordingly, we parameterize θ_{iD} as

$$\theta_{iD} \sim \mathbf{U}(0.0, 8.0), \tag{14}$$

where $\mathbf{U}$ denotes the uniform distribution.

The long-run means of log assets are not reported in Duffie, Saita and Wang (2007). Here, we turn to Bharath and Shumway (2008), who apply a similar estimation procedure and provide the quartiles of estimates for an augmented firm set.[25] The reported asset value ranges from 1.52 to 22949.32 (log asset value ranges from 0.4 to 10.0).[26] For simplicity, we assume in our simulation that θ_{iV} is uniformly distributed on this interval:

$$\theta_{iV} \sim \mathbf{U}(0.4, 10.0). \tag{15}$$

To generate time series for the observable factors, we need to make further assumptions of the starting value of the factor processes and the entry time of each firm. Following common practice, all factors are assumed to start at their long-run means. We are left with roughly 1400 active firms at the end of the data period after subtracting the number of defaults and merger-acquisitions from the total number of firms in Duffie, Saita and Wang (2007).[27] Campbell, Hilscher and Szilagyi (2008) provide the average number of active firms in each year from 1963 to 2003 in a lager data set. This number increases from 4342 in 1980 to 7833 in 2003. Proportional to this growth rate, in the simulation, we assume that 800 firms exist at the beginning of the data period. The other 2000 firms enter evenly in the following 25 years.

Once time series for distance to default and log assets are available, we can determine the face value of debt (L_t) and market value of equity (W_t) of each firm i by sequentially solving

[25] Duffie, Saita and Wang (2007) consider 2770 industrial firms for the period from 1980 to 2004, with 497 defaults identified. Bharath and Shumway (2008) examine all firms in the intersection of the Compustat Industrial file—Quarterly data and CRSP daily stock returns for NYSE, AMEX and NASDAQ for the period between 1980 and 2003, excluding financial firms. They obtain total 1449 defaults.

[26] Winsorized at the 1st and 99th percentiles by Bharath and Shumway (2008).

[27] We do not exclude "other exits" since most exists of this type are various data gaps.

the following two equations. Let V_t denotes asset value at time t; then

$$D_t = \frac{\ln(V_t/L_t) + (\mu_A - \frac{1}{2}\sigma_V^2)T}{\sigma_V\sqrt{T}}; \tag{16}$$

$$W_t = V_t\Phi(d_1) - L_t e^{-rT}\Phi(d_2), \tag{17}$$

where $d_1 = \frac{\ln(V_t/L_t)+(r+\frac{1}{2}\sigma_V^2)T}{\sigma_V\sqrt{T}}$, $d_2 = d_1 - \sigma_A\sqrt{T}$, $\Phi(\cdot)$ is the standard normal cumulative distribution function. r is the risk-free interest rate measured as the 3-month treasury rate.

We assume a forecast horizon of 12 months. We avoid solving for the asset volatility in virtue of the assumption of their homogeneity across all firms as specified in equation (11). Its value is fixed at the maximum likelihood estimate of 0.1169. Some scholars provide various approaches to estimate the expected asset drift rate μ_A.[28] In this paper, we deviate from all these methods by using the mean-reversion property of the log assets process. We calculate μ_V as the expected mean-reversion during the next period.

$$\mu_V - \frac{1}{2}\sigma_V^2 = k_V(\theta_V - \ln(V_t)) \tag{18}$$

Inserting μ_V and σ_V into equation (16), we can directly derive the debt value L_t. The time series for a firm's market equity follows from the call-option pricing formula as stated in equation (17). It is unrealistic to assume a constant level of face value of debt in a time period as long as 25 years. Combining the assumptions of leverage targeting and mean-reverting asset process, we allow a firm to dynamically adjust their outstanding debt, as suggested by Collin-Dufresne and Goldstein (2001).[29]

Now, we come to the determination of the exit time for each firm. There are three major types of exits defined in Duffie, Saita and Wang (2007): defaults, merger-acquisition and "other exits". Each type of exit will not restrict the intensity parameter estimation of the

[28] Vassalou and Xing (2004) calculate firm-specific average returns on each stock. Bharath and Shumway(2008) estimate previous year asset returns. Campbell, Hilscher and Szilagyi (2008) use 0.06, an empirical proxy for equity premium, plus the risk-free rate as an estimate.

[29] Collin-Dufresne and Goldstein (2001) and Duffie, Saita and Wang (2007) show that dynamic debt adjustment and leverage targeting could generate a more realistic term structure of default probabilities.

other types.[30] Since “other exits” are mostly data gaps of various types, they are less relevant for our study, and we exclude them for simplicity. As argued in Duffie, Saita and Wang (2007), merger-acquisitions have relatively little effect on the default hazard rate, and future default does not have to be prevented in merger-acquisitions if debts are not paid back immediately. Here, we do not consider merger-acquisition exits either.

We calculate the default intensity as

$$\lambda_{it} = e^{\alpha+\beta_1 D_{it}+\beta_2 R_{it}+\beta_3 r_t+\beta_4 S_t+y_t}, \tag{19}$$

where R_{it} is the trailing 1-year stock return. (α, β) = (-1.029, -1.201, -0.646, -0.255, 1.556), the real data estimates reported in Table II of Duffie, Eckner, Horel and Saita (2009). A hypothetic frailty path, which remains latent in reality, is generated with a mean-reverting speed of 0.03 and volatility of 0.15, which are derived from the marginal frailty parameter posterior distribution in Figure 6 in Duffie, Eckner, Horel and Saita (2009).

For firm i, the conditional probability of survival from entry time t_i to some future time s_i before the data cutoff date T_i is given by

$$p_i(t_i, s_i) = e^{-\sum_{t=t_i}^{s_i} \lambda_{it}\Delta t} \tag{20}$$

Δt equals one month.

The default time is simulated by using the Inverse-CDF method offered in Duffie and Singleton (1999). For each firm i, we draw a uniform random number U. Default time τ is determined as

$$\tau = \inf\{s_i : p_i(t_i, s_i) \leq U, t_i \leq s_i \leq T_i\} \tag{21}$$

If $p_i(t_i, T_i) > U$, the firm never defaults in our data period.

[30]See Proposition 2 of Duffie, Saita and Wang (2007).

Now, we can insert our factor time series and default timing data into the frailty model to estimate the default intensity parameters. By extending the factor time series with the pre-specified model, we can evaluate the credit risk of any portfolio constructed on the underlying firms in our data set.

C Correlation Structure of Observable Factors

This appendix lists the factor time-series models estimated by Duffie, Saita and Wang (2007).

$$
\begin{aligned}
k_r &= \begin{pmatrix} 0.03 & -0.021 \\ -0.027 & 0.034 \end{pmatrix}, \quad \theta_r = \begin{pmatrix} 3.59 \\ 5.47 \end{pmatrix}, \\
C_r &= \begin{pmatrix} 0.5639 & 0 \\ 0.2247 & 0.2821 \end{pmatrix}, \\
b &= (0.0090 \quad -0.0121)', \quad k_D = 0.0355, \sigma_D = 0.346 \\
k_V &= 0.015, \sigma_V = 0.1169 \\
AA' + BB' &= \begin{pmatrix} 1 & 0.448 \\ 0.448 & 1 \end{pmatrix}, \quad BB' = \begin{pmatrix} 0.448 & 0.0338 \\ 0.0338 & 0.0417 \end{pmatrix}, \\
k_S &= 0.1137, \alpha_S = 0.047, \theta_S = 0.1076, \\
\gamma_S &= (0.0366 \quad 0.0134)'.
\end{aligned}
$$

D Validation Tests

As shown in Figure 5 of Duffie, Eckner, Horel and Saita (2009), the latent factor plays a crucial role in the tail of the probability density of the predicted number of defaults in the next 5 years. A common source of the current level of and future level of shocks to this latent factor enlarges the risk of heavily clustered defaults remarkably. Thus, the filtered-out latent factor path and the mean-reverting speed, κ, and volatility, η, which govern its time-series dynamics, are of importance for assessing the modeled correlation risk. Maximum likelihood estimates of the default intensity parameters converge to the true data-generating process when the number of firms and number of time periods become large. It is thus helpful to do some convergence tests first when working with limited real data.

According to the doubly stochastic assumption, estimation of the factor time series model could be separated from estimation of the default intensity parameters. We focus on default intensity estimation and also check the posterior distribution of the filtered frailty factor through Bayesian analysis. Using the simulation approach described in the previous section, we simulate one set of the observable macroeconomic factors and firm-specific factors, as well as one hypothetical frailty path. Then, 100 times, we draw a new U, the default trigger, for each firm and let the default time be determined accordingly. This process corresponds to 100 different realizations of the firm-default history. The maximum number of defaults recorded is 648, and the minimum is 573. We then estimate the frailty model for each realization.

We find that the mean filtered frailty paths tightly follow the "true" frailty path. The correlation between filtered frailty and "true" frailty ranges from 0.87 to 0.96, and the estimated intensity parameter is close to the true data-generating process. Further, the root mean square error of the estimated intensity parameter is moderate and of similar magnitude to the standard error of the parameter estimation provided in Table II in Duffie, Eckner, Horel and Saita (2009). It is relatively safe to conclude that the model appropriately identifies the intangible risk embedded in the latent frailty factor and that the intensity parameter estimation is not likely to be heavily skewed given the available 25 years of firm-default history.

References

Acharya, Viral V., Stephen Schaefer, and Yili Zhang, 2015, Liquidity risk and correlation risk: A clinical study of the General Motors and Ford downgrade of May 2005, *Quarterly Journal of Finance* 5, 1–51.

Allen, Franklin, 2008, Lessons from the crisis, Presidential Address, International Atlantic Economic Society.

Allen, Franklin, and Elena Carletti, 2006, Credit risk transfer and contagion, *Journal of Monetary Economics* 53, 89–111.

Andreoli, Alessandro, Luca Vincenzo Ballestra, and Graziella Pacelli, 2016, From insurance risk to credit portfolio management: a new approach to pricing CDOs, *Quantitative Finance*16, 1495–1510.

Azizpour, Shahriar, Kay Giesecke, and Gustavo Schwenkler, 2017, Exploring the sources of default clustering, *Journal of Financial Economics*, forthcoming.

Bae, Taehan, Ian Iscoe, and Changki Kim, 2015,Valuing retail credit tranches with structural, double mixture models, *Journal of Futures Markets* 35, 849–867.

Benmelech, Efraim, and Jennifer Dlugosz, 2009, The alchemy of CDO credit ratings, *Journal of Monetary Economics* 56, 617–634.

Benzoni, Luca, Pierre Collin-Dufresne, Robert S. Goldstein, and Jean Helwege, 2015, Modeling contagion via the updating of fragile beliefs, *Review of Financial Studies* 28, 1960–2008.

Bharath, Sreedhar T., and Tyler Shumway, 2008, Forecasting default with the Merton distance to default model, *Review of Financial Studies* 21, 1339–1369.

Broer, Tobias, 2017, Securitisation bubbles: structured finance with disagreement about default correlations, *Journal of Financial Economics*, forthcoming.

Brunnermeier, Markus, 2009, Deciphering the 2007-08 liquidity and credit crunch, *Journal of Economic Perspectives* 23, 77–100.

Campbell, John Y., Jens Hilscher, and Jan Szilagyi, 2008, In search of distress risk, *Journal of Finance* 63, 2899–2939.

Choi, Yong Seok, Hitesh Doshi, Kris Jacobs, Stuart M. Turnbull, 2016, Pricing structured products with economic covariates, working paper, University of Houston.

Collin-Dufresne, Pierre, 2009, A short introduction to correlation markets, *Journal of Financial Econometrics* 7, 12–29.

Collin-Dufresne, Pierre, Robert S. Goldstein, 2001, Do credit spreads reflect stationary leverage ratios?, *Journal of Finance* 56, 1929–1957.

Coval, Joshua D., Jakub W. Jurek, and Erik Stafford, 2009a, Economic catastrophe bonds, *American Economic Review* 99, 628–666.

Coval, Joshua D., Jakub W. Jurek, and Erik Stafford, 2009b, The economics of structured finance, *Journal of Economic Perspectives* 23, 3–25.

Coval, Joshua D., Kevin Pan, and Erik Stafford, 2014, Capital market blind spots, Working paper, Harvard University.

Das, Sanjiv R., Darrell Duffie, Nikunj Kapadia, and Leandro Saita, 2007, Common failings: how corporate defaults are correlated, *Journal of Finance* 62, 93–117.

DeMarzo, Peter M., 2005, The pooling and tranching of securities: A model of informed intermediation, *Review of Financial Studies* 18, 1–35.

Duffie, Darrell, 2007, Innovations in credit risk transfer: Implications for financial stability, Working paper, Stanford University.

Duffie, Darrell, Andreas Eckner, Guillaume Horel, and Leandro Saita, 2009, Frailty correlated default, *Journal of Finance* 64, 2089–2123.

Duffie, Darrell, Leandro Saita, and Ke Wang, 2007, Multi-period corporate default prediction with stochastic covariates, *Journal of Financial Economics* 83, 635–665.

Duffie, Darrell, Kenneth J. Singleton, 1999, Modeling Term Structures of Defaultable Bonds, *Review of Financial Studies* 12, 687–720.

Erlenmaier, Ulrich and Hans Gersbach, 2014, Default Correlations in the Merton Model, *Review of Finance* 18, 1775–1809.

Fama, Eugene F., Kenneth R. French, The equity premium, 2002, *Journal of Finance* 57, 637–659.

Feldhütter, Peter, Mads Stenbo Nielsen, 2012, Systematic and idiosyncratic default risk in synthetic credit markets, *Journal of Financial Econometrics*, 10, 292–324.

Fender, Ingo, Nikola Tarashev, and Haibin Zhu, 2008, Credit fundamentals, ratings, and value-at-risk: CDOs versus corporate exposures, *BIS Quarterly Review*, March-2008.

Fitch Ratings, 2006, Global rating criteria for collateralized debt obligations.

Garlappi, Lorenzo, Raman Uppal, and Tan Wang, 2007, Portfolio selection with parameter and model uncertainty: A multi-prior approach, *Review of Financial Studies* 20, 41–81.

Giesecke, Kay, 2004, Correlated default with incomplete information, *Journal of Banking & Finance* 28, 1521–1545.

Glasserman, Paul, and Jingyi Li, 2005, Importance sampling for portfolio credit risk, *Management Science* 51, 1643–1656.

Griffin, John, and Jordan Nickerson, 2017, Debt correlations in the wake of the financial crisis: What are appropriate default correlations for structured products? *Journal of Financial Economics* 125, 454–474.

Griffin, John, and Dragon Yongjun Tang, 2012, Did subjectivity play a role in CDO credit ratings? *Journal of Finance* 67, 1293–1328.

Heitfield, Erik, 2009, Parameter uncertainty and the credit risk of collateralized debt obligations, Working Paper, Division of Research and Statistics Federal Reserve Board.

Helwege, Jean, and Gaiyan Zhang, 2016, Financial firm bankruptcy and contagion, *Review of Finance* 20, 1289–1320.

Jorion, Phillip, and Gaiyan Zhang, 2007, Good and bad credit contagion: Evidence from credit default swaps, *Journal of Financial Economics* 84, 860–883.

Leland, Hayne E., 2007, Financial synergies and the optimal scope of the firm: Implications for mergers, spinoffs, and structured finance, *Journal of Finance* 62, 765–807.

Longstaff, Francis A., and Arvind Rajan, 2008, An empirical analysis of the pricing of collateralized debt obligations, *Journal of Finance* 63, 529–563.

Longstaff, Francis A., and Brett W. Myers, 2014, How Does the Market Value Toxic Assets? *Journal Of Financial and Quantitative Analysis* 49, 297–319.

Mehran, Hamid, and René M. Stulz, 2007, The economics of conflicts of interests in financial institutions, *Journal of Financial Economics* 85, 267–296.

Merton, Robert C., 1974, On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance* 29, 449–470.

Moody's Investors Service, 1998, The double binomial method and its application to a special case of CBO structures, Moody's Special Report.

Moody's, 2015, CLOs –Global: 2016 Outlook –CLO performance will remain solid despite weak credit quality of loans.

Nicolo, Antonio, and Loriana Pelizzon, 2008, Credit derivatives, capital requirements and opaque OTC markets, *Journal of Financial Intermediation* 17, 444–463.

Rajan, Raghuram G., and Luigi Zingales, 1995, What do we know about capital structure? Some evidence from international data, *Journal of Finance* 50, 1421–60.

Rajan, Uday, Amit Seru, Vikrant Vig, 2015, The failure of models that predict failure: Distance, incentives, and defaults, *Journal of Financial Economics* 115, 237–260.

Standard and Poor's (S&P), 2002, Global cash flow and synthetic CDO criteria.

Stulz, Rene M., 2008, Risk management failures: What are they and when do they happen, *Journal of Applied Corporate Finance* 20, 58–67.

Titman, Sheridan, and Roberto Wessels, 1988, The determinants of capital structure choice, *Journal of Finance* 43, 1–19.

Triana, Pablo, 2009, Lecturing birds on flying: can mathematical theories destroy the financial markets? Wiley.

Vasicek, Oldrich, 1987, Probability of loss on loan portfolio, Moody's KMV Corporation.

Vassalou, Maria, and Yuhang Xing, 2004, Default risk in equity returns, *Journal of Finance* 59, 831–868.

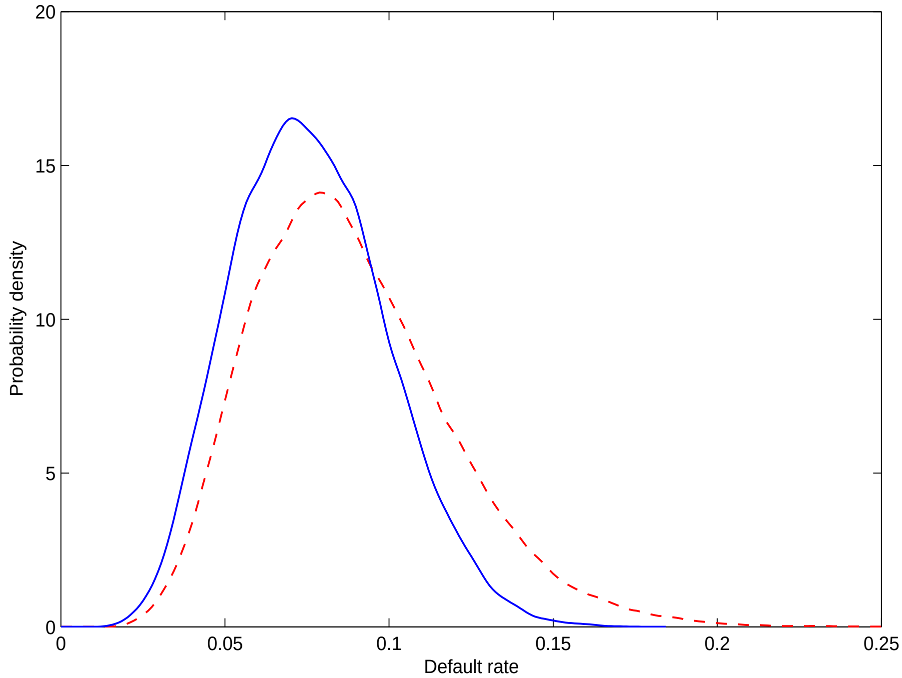

Figure 1: Portfolio default rate distribution with and without frailty factor. The conditional probability density of default rate within 5 years, for the portfolio formed by all active firms at the 25th-year end, from (a) no-frailty model (solid line), (b) dynamic-frailty model (dashed line). We apply Gaussian kernel smoothing (with bandwidth 5) to the Monte-Carlo generated empirical distribution.

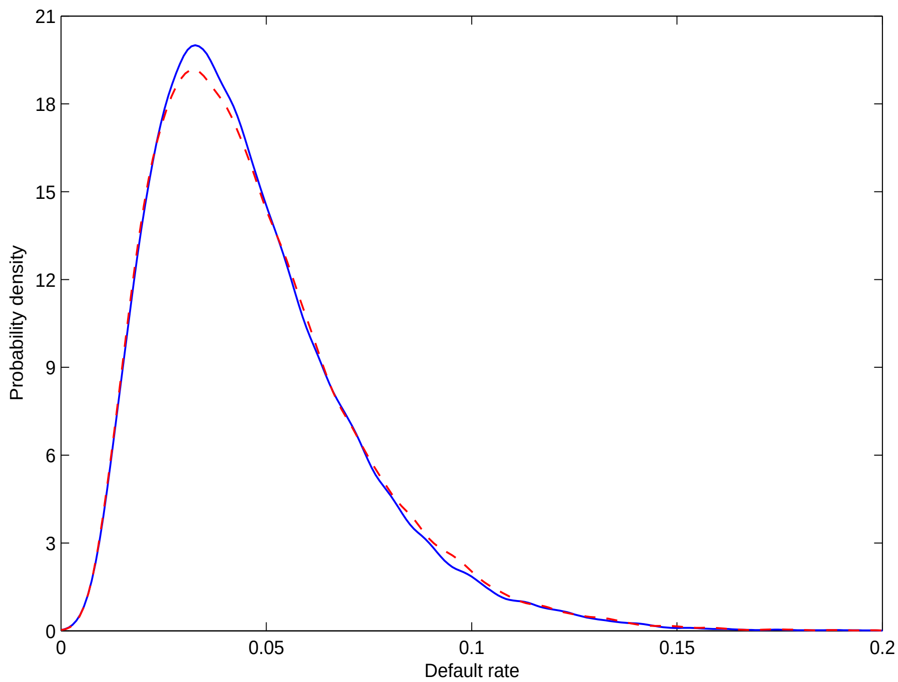

Figure 2: Portfolio default rate distribution with and without correlation between macroeconomic factors. The conditional probability density of default rate within 5 years, for the portfolio formed by all active firms at the 25th-year end, in (a) a model with positively correlated short term interest rate and stock market performance (solid line), (b) a model without such correlation (dashed line). We apply Gaussian kernel smoothing (with bandwidth 5) to the Monte-Carlo generated empirical distribution.

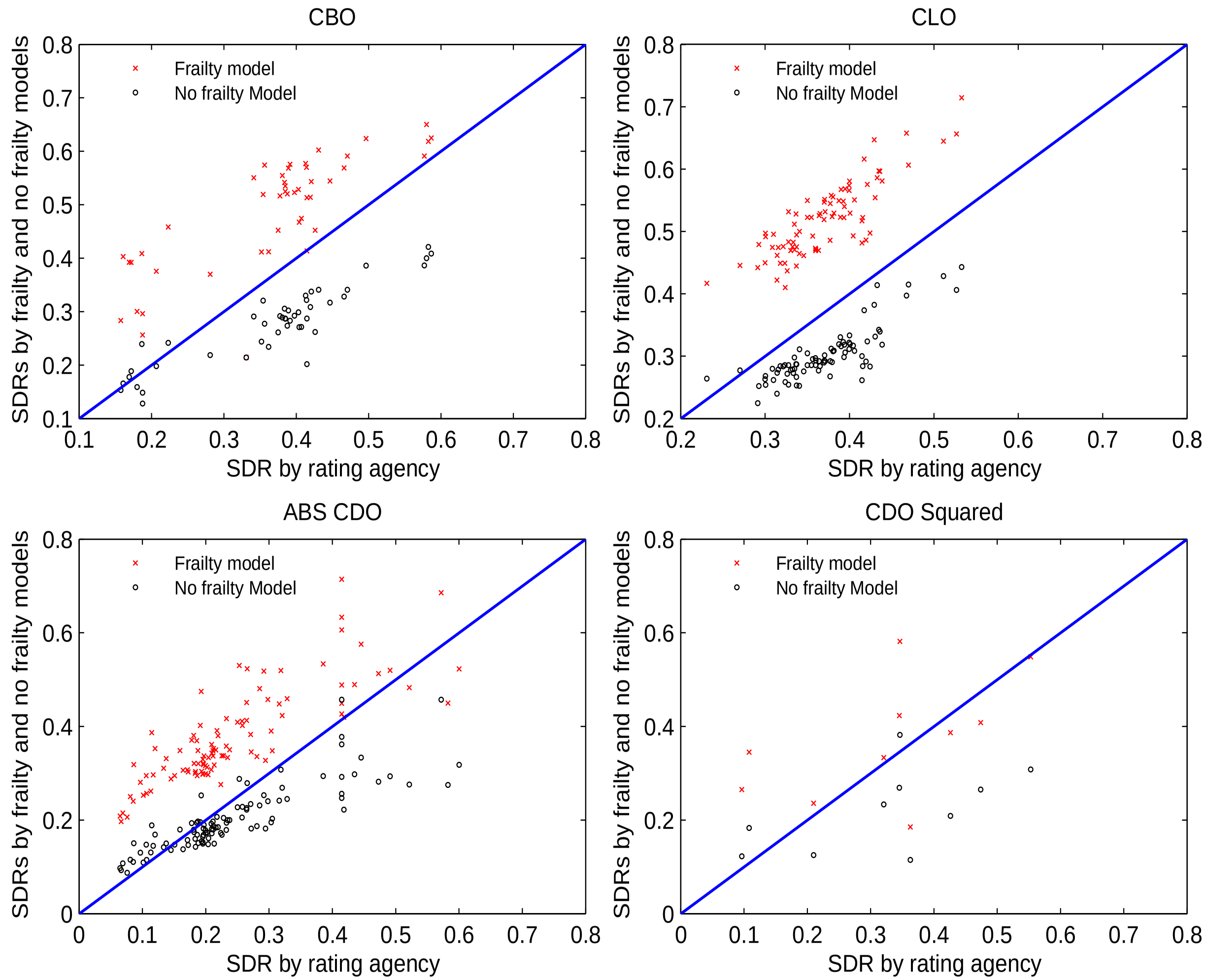

Figure 3: Scenario Default Rates for each CDO type. This figure compares the scenario default rates from (a) rating agency, (b) no-frailty model, (c) dynamic-frailty model. Horizontal axis denotes the SDR from rating agency, and the vertical axis denotes the SDR from no-frailty and dynamic-frailty model. Points on the 45 degree line represent equivalence of SDR from rating agency and no-frailty model and/or frailty model.

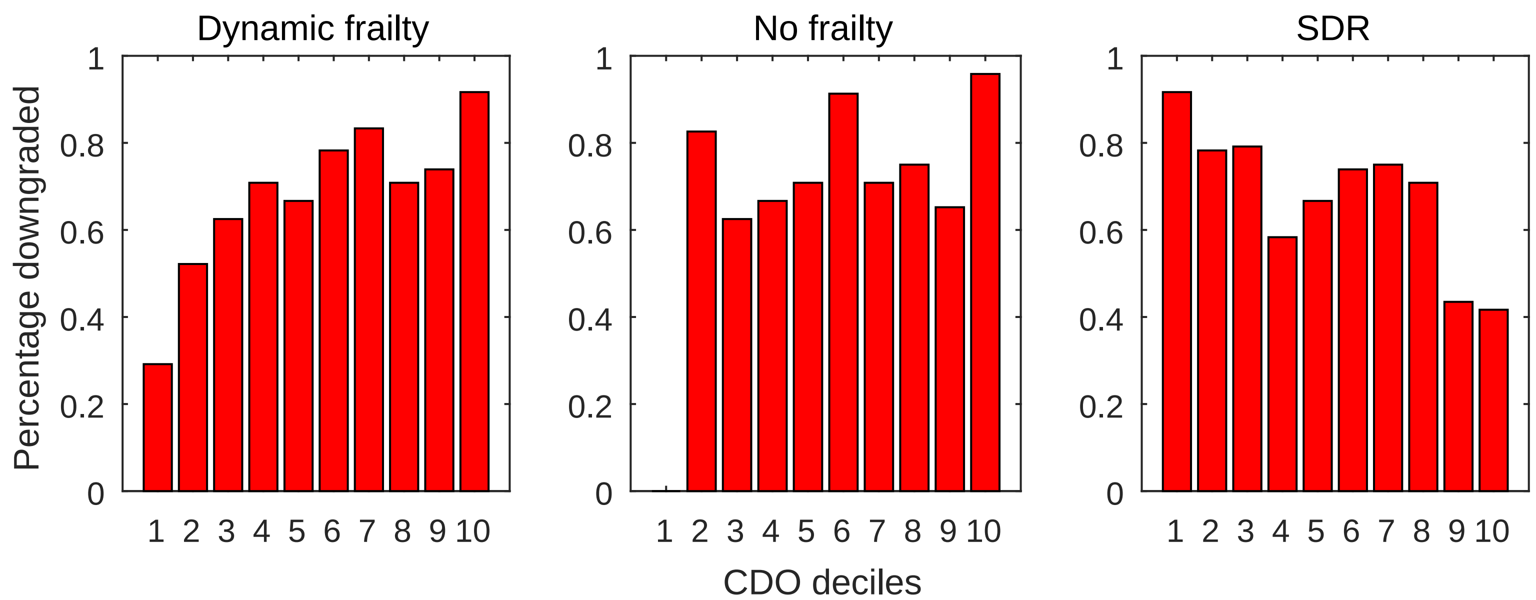

Figure 4: Prediction of CDO downgrading for different risk groups. This figure presents the CDO downgrading prediction with the frailty model, no-frailty model, and rating agency's SDRs. The 237 CDOs in our sample are separated into ten risk groups according to difference between the implied default probability of the real AAA-tranche from the frailty model ($DP_{DF,i}$) or no-frailty model ($DP_{NF,i}$) and the AAA default probability ($DP_{AAA,i}$) , i.e. $DP_{DF,i} - DP_{AAA,i}$ or $DP_{NF,i} - DP_{AAA,i}$. We have also shown the risk classification and downgrading prediction based on the rating agency's SDR. Group 10 represents the highest risk group predicted by the frailty model, no-frailty model or rating agency's SDR. The vertical axis gives the percentage of CDOs downgraded in each risk groups.

Table I
No-frailty Model versus Dynamic-frailty Model

This table reports the default rate predictions from the no-frailty model and the dynamic-frailty model. Panel A reports summary statistics for the simulated 25 years data of observable factors used for parameter estimation. Panel B reports the Maximum likelihood estimates of default intensity parameters with and without frailty, respectively. Panel C presents the percentiles of predicted portfolio loss distribution using both the frailty and the no-frailty models. The portfolio includes all active firms at the end of simulated year 25. Then we compare the portfolio's future five years default rate distributions from the frailty and no-frailty models. Total number of firms alive at the beginning of the prediction is 2170.

Panel A: Summary statistics

			Quantiles				
Variable	Mean	Std.	Min	0.25	Median	0.75	Max
distance to default	4.70	2.46	-3.40	2.81	4.76	6.59	12.86
trailing stock return(%)	13.98	72.01	-81.70	-29.11	-1.07	37.15	317.76
3-month T-bill rate	5.11	1.56	1.62	4.01	5.10	6.03	10.52
trailing S&P 500 return(%)	10.37	13.90	-24.59	0.35	9.12	20.54	47.44

Panel B: Maximum likelihood estimates of intensity parameters

	With frailty		Without frailty	
	Coefficient	t-statistic	Coefficient	t-statistic
constant	-1.046	-5.4	-0.828	-5.1
distance to default	-1.115	-31.8	-1.070	-30.3
trailing stock return	-0.732	-7.2	-0.812	-7.5
3-month T-bill rate	-0.253	-7.3	-0.325	-10.7
trailing S&P 500 return	1.756	5.9	1.538	5.1
latent-factor volatility η	0.147	10.3		
latent-factor mean reversion κ	0.029	5.1		

Panel C: Percentiles of predicted default rate distribution

	0.05	0.15	0.50	0.90	0.95	0.99	0.999
with frailty(%)	4.61	5.81	8.48	12.86	14.29	17.33	21.01
without frailty(%)	4.06	5.16	7.47	10.69	11.66	13.41	15.55

Table II
Correlation Between Macroeconomic Factors

This table reports the impact of correlation between macroeconomic factors on default rate prediction. Panel A reports summary statistics for the simulated 25 years data of observable factors used for parameter estimation. Panel B reports the Maximum likelihood estimates of default intensity parameters. Panel C presents the percentiles of predicted default rate distribution. Total number of firms alive at the beginning of the prediction is 2150.

Panel A: Summary statistics

			Quantiles				
Variable	Mean	Std.	Min	0.25	Median	0.75	Max
distance to default	4.72	2.41	-3.27	2.92	4.78	6.54	13.68
trailing stock return(%)	15.28	71.82	-88.16	-27.78	0.12	38.47	475.37
3-month T-bill rate	5.16	1.77	0.99	3.90	5.13	6.43	9.46
trailing S&P 500 return(%)	11.21	13.37	-19.05	0.81	11.54	21.21	53.12

Panel B: Maximum likelihood estimates of intensity parameters

	Coefficient	Std. Error	t-statistic
constant	-0.998	0.176	-5.7
distance to default	-1.176	0.035	-33.3
trailing stock return	-0.618	0.092	-6.7
3-month T-bill rate	-0.256	0.032	-8.0
trailing S&P 500 return	1.451	0.343	4.2
latent-factor volatility η	0.161	0.019	8.3
latent-factor mean reversion κ	0.027	0.005	5.5

Panel C: Percentiles of predicted default rate distribution

	0.05	0.15	0.50	0.90	0.95	0.99	0.999
with correlation effect(%)	1.63	2.33	4.19	7.91	9.35	12.47	16.42
without correlation effect(%)	1.63	2.33	4.19	7.91	9.26	12.33	16.28

Table III: Empirical Results for Scenario Default Rate Prediction

This table reports CDOs' weighted average rating (WAR); closing date(CDate); weighted average maturity (WAM); number of obligers (N); scenario default rate (%) from (a) rating agency (SDR), (b) no-frailty model (SDR NF), (c) dynamic-frailty model (SDR DF); default probability from (a) no-frailty model (DP NF), (b) dynamic-frailty model (DP DF); Notches for AAA tranche downgrading (DG). Averages of the SDRs are provide at the bottom of the tables for each CDO type.

Panel A: CBO

Name	CDate	WAR	WAM	N	SDR	SDR NF	SDR DF	DP NF	DP DF	DG
BEA 1998-1	98/05	CCC	4.1	96	58.7	40.9	62.5	11.4	31.3	19
Conseco	98/08	BBB-	4.6	124	15.8	15.3	28.4	0.0	0.1	0
Juniper 1999-1	99/03	CCC+	4.9	101	49.7	38.6	62.4	2.7	23.3	0
Federated 1999	99/03	B+	3.7	119	28.1	21.8	37.0	0.0	0.7	0
Emerald	99/05	BB	3.4	120	18.0	15.9	30.0	0.1	6.3	0
Cedar	99/06	B+	3.9	136	35.2	24.4	41.2	0.0	1.1	0
KNIGHT	99/06	BB+	3.6	135	18.8	14.8	29.6	6.8	28.5	0
Admiral	99/08	CCC	4.9	76	58.3	42.1	61.8	0.1	6.4	0
INA 1999-1	99/09	B-	4.8	88	47.1	34.1	59.1	1.4	19.6	0
Talcott Notch I	99/10	B-	5.2	146	42.1	33.8	54.3	21.9	48.3	0
FC III	99/11	B	3.5	42	42.6	26.2	45.2	0.1	4.0	0
Centennial	99/12	B+	4.4	141	36.2	23.4	41.2	0.0	0.2	0
Triton IV	99/12	A-	6.8	117	18.8	12.8	25.6	0.0	0.1	0
Inner Harbor 1999-1	99/12	B+	5.4	157	37.5	26.1	45.2	0.4	11.6	0
Juniper 2000-1	00/04	B	5.5	107	42.0	30.8	51.4	9.9	37.5	0
Arlington Street	00/06	B+	5.5	107	40.4	27.1	46.7	0.0	1.2	9
CAESAR 2000	00/06	BB+	1.2	14	33.1	21.4	21.4	0.0	0.0	0
Wilbraham	00/07	B	5.5	101	44.7	31.7	54.5	0.8	15.4	0
JWS 2000-1	00/07	B+	5.7	118	40.7	27.1	47.5	0.0	2.4	0
Coliseum	00/07	BB+	4.9	91	20.7	19.8	37.6	1.0	14.0	1
Madison Ave. I	00/08	B+	5.7	120	37.8	29.2	51.7	0.2	12.2	0
Nicholas-Applegate I	00/08	B+	5.5	65	39.8	29.2	52.3	0.0	6.5	10
Chartwell I	00/09	B	5.8	88	43.1	34.1	60.2	19.5	54.3	15
Capstan	00/11	B	5.6	64	46.6	32.8	56.9	0.2	12.8	0
Lone Star	00/12	BBB-	5.5	106	17.2	18.9	39.2	2.6	31.9	0
Blue Eagle I	00/12	B	3.9	20	58.0	40.0	65.0	2.1	16.2	0
Signature 5	00/12	BB-	3.4	104	41.5	20.2	41.3	0.0	2.4	2
Berkeley Street	01/03	B+	5.8	128	38.1	28.9	55.5	0.0	9.7	10
Liberty Square I	01/03	B+	6.1	106	39.1	28.3	57.5	0.1	20.3	2
Madison Ave.II	01/03	BBB-	5.1	107	16.9	17.8	39.3	0.3	20.8	6
Hampden	01/03	BBB-	4.9	139	16.1	16.5	40.3	0.1	17.1	0
Centurion III	01/03	B+	5.4	202	35.6	27.7	57.4	0.0	7.6	0
Canyon 2001-1	01/04	B	5.9	143	41.4	32.2	57.0	1.9	33.3	0
Nicholas-Applegate II	01/04	B+	5.9	72	38.4	30.6	54.2	3.5	35.0	10
Mammoth 2001-1	01/05	B+	5.9	129	38.5	28.7	53.5	35.4	65.2	0
Liberty Square II	01/05	B+	6.2	106	38.8	27.4	52.0	0.1	11.6	4
Balboa I	01/06	BB+	6.6	120	22.3	24.2	45.8	15.7	54.1	0
Melchior I	01/07	B+	5.4	87	40.3	29.9	52.9	0.1	7.0	0
Concerto II	01/07	B+	6.1	97	41.3	33.0	57.7	0.1	11.1	0
Robeco II	01/08	BB-	6.3	129	34.1	29.1	55.0	0.9	28.5	1
TCW	01/08	B+	6.1	139	38.9	30.2	56.8	0.0	5.2	0
Cashel Rock	01/11	B+	5.5	101	38.5	28.7	52.5	2.6	32.8	3

(Continued)

Panel A-Continued

Name	CDate	WAR	WAM	N	SDR	SDR NF	SDR DF	DP NF	DP DF	DG
Signature 6	01/12	B+	5.5	115	41.5	28.7	51.3	1.3	20.4	1
Cardinal	02/09	BBB-	5.9	71	18.7	23.9	40.8	6.0	26.3	0
Canyon Capital 2002-1	02/12	B+	6.3	131	35.4	32.1	51.9	23.9	48.3	9
Prado	03/11	B-	4.7	44	57.7	38.6	59.1	0.2	6.8	9
Average					36.6	27.6	48.7	3.8	18.5	

Panel B: CLO

Name	CDate	WAR	WAM	N	SDR	SDR NF	SDR DF	DP NF	DP DF	DG
Highland Legacy	99/08	B-	4.55	223	42.0	29.1	48.6	1.1	17.6	0
First Dominion III	99/12	B	4.78	187	42.4	28.3	49.7	2.3	21.9	0
Longhorn	00/03	BB-	4.99	129	29.1	22.5	44.2	0.0	2.2	0
Addison	00/10	B+	5.33	162	33.7	25.3	47.5	0.0	1.7	0
Sequils-Cen. V	01/04	B	3.42	137	41.5	26.2	48.2	9.1	48.8	3
TCW Select	01/05	BB-	5.11	107	31.0	26.2	49.5	0.0	8.0	0
COPERNICUS EURO I	01/07	B+	5.17	62	39.3	32.3	54.8	0.7	16.3	0
Highland V	01/08	B-	4.91	179	43.9	31.8	58.1	0.6	29.1	5
Race Point	01/11	BB-	5.95	179	30.0	26.8	49.7	0.0	7.2	0
Carlyle H.Y. IV	02/04	BB-	5.54	244	31.4	24.0	42.2	6.8	29.9	4
Katonah III	02/04	BB-	5.61	103	32.6	27.2	43.7	1.1	12.5	3
INTERCONTINENTAL	02/05	B+	6.24	109	38.9	33.0	52.3	0.0	4.1	0
Centurion VI	02/08	B+	5.01	277	32.4	25.9	41.0	1.5	15.1	1
Saratoga I	02/09	B+	5.44	278	36.3	27.7	47.0	10.4	38.7	1
Landmark II	02/09	B+	5.37	98	36.0	28.6	46.9	1.5	16.2	3
Castle Hill II	02/09	B+	5.31	143	33.5	28.0	47.1	11.7	37.5	2
RMF EURO	02/10	B	7.21	94	47.0	41.5	60.6	13.4	37.1	0
Venture II 2002	02/11	BB-	5.4	155	31.8	28.4	44.9	20.4	42.0	2
Castle Hill I	02/12	B+	5	135	33.7	26.7	44.4	5.4	26.0	0
Gulf S.C. 2002-1	02/12	B+	5.67	90	34.1	31.1	50.0	8.4	31.2	3
1888 FUND	02/12	B+	5.43	142	40.4	31.7	49.3	29.3	50.6	1
LEOPARD I	03/01	B+	6.83	58	43.3	41.4	58.6	0.5	8.5	0
Katonah IV	03/02	BB-	5.22	98	32.3	28.6	44.9	1.8	15.3	5
Longhorn III	03/03	BB+	5.15	72	23.1	26.4	41.7	0.3	7.2	0
Race Point II CLO	03/04	BB-	5.48	208	32.1	28.4	47.6	0.5	13.8	0
ARES VII	03/05	BB-	5.2	118	30.9	28.0	47.5	25.6	47.3	5
Katonah V	03/05	BB-	5.06	90	33.0	27.8	46.9	0.9	12.6	3
LCM I	03/06	BB-	5.17	97	31.6	27.8	47.4	0.0	3.1	0
Waveland-Ingots	03/06	BB	5.02	101	27.0	27.7	44.6	7.6	30.0	0
NYLIM Fla. 2003-1	03/07	BB-	5.25	117	31.4	27.4	46.2	0.3	11.8	0
Gulf S.C. 2003-1	03/08	B+	5.4	134	35.6	29.5	49.3	5.4	29.7	4
Clydesdale 2003	03/09	B+	5.25	174	36.0	29.3	47.1	6.4	30.3	4
EUROCREDIT III	03/09	B	7.39	70	51.1	42.9	64.5	3.7	26.2	0
Union Square	03/09	B+	5.28	128	36.0	29.7	47.3	1.0	13.9	0
Magnetite V	03/09	B+	5.35	156	34.6	27.6	46.1	2.9	22.6	3
Ballyrock II	03/11	BB	5.45	149	30.0	26.3	45.0	0.1	9.3	2
Babson 2003-I	03/11	B+	5.23	205	33.3	27.3	48.2	0.4	13.3	1
Venture III	03/11	B+	5.81	186	33.7	28.7	49.5	1.4	22.6	2

(Continued)

Panel B-Continued

Name	CDate	WAR	WAM	N	SDR	SDR NF	SDR DF	DP NF	DP DF	DG
Landmark III	03/12	BB-	5.38	122	33.3	27.9	47.5	1.0	20.9	1
Aquilae I	03/12	B+	7.17	73	46.8	39.7	65.8	14.0	47.6	0
Navigator 2003	03/12	B	5.35	166	43.1	33.1	55.4	3.3	32.8	0
LightPoint 2004-1	04/02	B+	4.1	142	37.7	26.8	48.6	0.5	20.2	3
Clarenville	04/02	B+	6.09	99	41.7	37.4	61.6	8.3	42.0	0
Ares VIII	04/03	B+	5.43	210	38.7	31.9	54.9	10.9	48.6	4
Celerity	04/03	BB-	5.48	131	33.5	29.8	51.1	1.3	27.3	1
Leopard II	04/04	B+	6.84	68	42.9	38.2	64.7	0.4	15.8	0
Northwoods IV	04/05	B+	5.43	67	39.4	29.9	52.2	0.2	12.4	2
Boston Harbor 2004-1	04/05	BB-	5.57	126	32.8	28.6	53.2	3.6	33.9	2
Champlain	04/05	B+	5.55	207	37.0	29.0	51.9	2.5	33.6	3
Long Grove	04/05	B+	4.84	198	34.1	25.3	46.5	1.1	24.4	3
CENTURION VII	04/05	B+	5.15	295	32.8	25.4	48.3	0.5	21.6	2
Jubilee III	04/05	B	6.59	64	52.7	40.6	65.6	1.1	21.6	0
Babson 2004-I	04/06	B+	5.5	195	33.7	28.7	52.8	2.9	32.4	4
Petrusse Euro.	04/06	B+	5.83	236	41.6	28.4	52.2	0.0	6.1	0
Carlyle H.Y. VI	04/07	B+	5.67	210	38.0	29.0	52.4	22.2	58.7	4
AMMC III	04/07	B+	5.31	137	36.4	29.2	52.6	5.1	37.1	2
Hudson Str. 2004	04/07	B	5.57	139	42.1	32.4	57.6	1.1	25.1	0
FIRST 2004-I	04/07	BB-	4.96	123	29.3	25.2	47.9	0.4	17.9	1
WhiteHorse I	04/07	B+	5.81	119	40.1	31.9	52.9	6.4	39.4	1
Signature 7	04/07	B+	5.23	83	41.5	30.0	51.7	5.1	33.7	6
Gulf S.C. 2004-1	04/08	B+	5.68	156	37.1	30.1	55.1	3.9	35.0	5
Venture IV	04/08	B+	5.75	234	38.1	30.8	55.6	4.3	37.4	1
Veritas I	04/08	B+	5.95	113	39.4	31.9	54.0	4.3	34.7	5
Clydesdale 2004	04/08	B+	5.54	245	35.5	28.6	52.2	0.9	23.8	2
Velocity	04/08	BB-	5.36	130	30.1	25.4	49.1	1.1	25.3	3
Flagship III	04/08	B+	5.27	178	37.7	29.2	54.5	4.0	36.3	4
Essex Park	04/09	B+	5.65	141	37.8	31.2	55.8	1.0	25.0	2
Navigator 2004	04/10	B	5.47	171	43.6	33.9	59.6	4.5	37.4	0
BlackRock Sen.	04/10	B+	5.29	315	37.1	29.2	53.2	1.7	31.5	1
Landmark IV	04/10	B+	5.64	136	38.2	30.9	52.9	0.6	21.4	1
Adagio I	04/10	B	7.69	70	53.3	44.3	71.4	10.1	44.7	0
NYLIM Fla. 2004-1	04/10	B+	5.39	171	39.0	31.6	56.7	4.1	37.8	1
Babson 2004-II	04/10	B+	5.26	337	37.0	29.3	54.7	1.9	32.4	2
LCM II	04/11	B+	5.26	162	36.5	28.4	52.9	3.8	35.2	0
Hewetts Island II	04/11	B+	5.71	122	40.0	31.1	57.4	1.3	26.6	5
Wind River I	04/11	B	5.29	174	40.0	32.2	56.6	0.1	13.0	4
Premium Loan I	04/11	B	5.52	132	40.0	33.3	58.1	30.4	61.9	3
Callidus D.P. III	04/12	B+	5.79	183	39.5	30.6	56.8	5.0	39.7	1
Alzette Euro.	04/12	B	6.74	263	43.5	34.2	59.7	0.2	20.1	0
First 2004-II	04/12	B+	5.24	126	35.0	28.6	52.2	1.8	26.2	2
Chatham Light	04/12	B+	5.65	227	40.6	30.8	55.1	8.1	43.3	1
Whitney I	04/12	B+	6.13	151	35.0	30.5	55.0	8.1	41.7	1
Average					37.1	30.1	51.9	4.6	26.8	

Panel C: ABS

Name	CDate	WAR	WAM	N	SDR	SDR NF	SDR DF	DP NF	DP DF	DG
Bleecker	00/03	B-	6.83	35	57.2	45.7	68.6	42.9	62.7	19
Talon I	00/04	B+	6.92	66	44.6	33.3	57.6	27.4	52.5	18
Phoenix II	00/05	BB-	7.84	47	43.5	29.8	48.9	5.3	24.6	10
Ingress I	00/05	BB	6.43	52	31.9	30.8	51.9	0.6	12.3	0
Varick	00/09	BB+	7.75	86	26.6	27.9	52.3	43.3	69.6	17
PRUDENTIAL I	00/10	BB-	6.4	51	38.6	29.4	53.4	10.0	38.6	0
TIAA I	00/12	BB+	6.68	104	28.5	23.1	48.1	7.2	44.7	0
Independence I	00/12	BBB-	8.45	83	29.2	25.3	51.8	0.4	20.7	13
MWAM 2001-1	01/01	BB+	8.71	66	25.3	28.8	53.0	5.3	36.0	6
Saybrook Point	01/02	B-	6.54	90	41.5	37.8	63.3	66.6	83.0	15
NYLIM Str. 2001-1	01/04	BBB	7.56	87	19.1	19.5	40.2	0.1	13.8	4
SFA CABS II	01/05	B-	7.18	35	41.5	45.7	71.4	27.3	57.4	8
Independence II	01/07	BBB-	8.45	102	26.5	22.5	45.1	0.0	10.6	9
Arroyo I	01/08	BBB	7.69	108	23.3	19.4	41.7	0.3	19.3	1
Putnam 2001-1	01/11	BBB	7.23	134	18.1	17.9	38.1	2.4	34.0	4
MADISON AVE. I	01/12	BBB	6.72	95	19.7	18.9	33.7	5.5	31.1	11
Helios Series I	01/12	BBB-	6.31	72	18.6	19.4	36.9	6.7	30.7	10
Commodore I	02/02	BBB	6.38	52	20.9	19.2	30.8	0.0	2.5	7
Trainer W.F.R. II	02/02	BBB-	7.71	92	25.8	22.8	40.2	6.3	29.7	17
F.A.B. 2002-1	02/04	BBB-	5.91	90	23.5	20.0	33.3	0.5	9.6	4
Independence III	02/05	BBB+	7.64	87	20.0	17.2	29.9	0.4	9.0	16
TIAA 2002-1	02/05	BBB	7.02	55	29.5	18.2	32.7	0.0	2.2	0
Anthracite I	02/05	BB+	7	40	58.3	27.5	45.0	0.0	0.1	0
Aspen I	02/05	BBB-	7	26	32.1	26.9	42.3	7.2	19.4	10
ACA 2002-1	02/07	BBB	7.33	83	22.8	20.5	33.7	0.0	4.0	4
Saybrook Point II	02/11	BB	7.14	284	41.5	29.2	48.9	50.3	65.9	17
Charles River I	02/11	A-	6.87	89	18.7	16.9	29.5	0.0	0.5	19
ABS Capital II	02/11	BBB+	6.75	109	18.2	17.4	32.1	0.8	11.9	18
Anthracite II	02/12	BB	6.99	44	60.0	31.8	52.3	0.0	0.2	0
Birch R.E. I	02/12	BBB+	7	40	23.8	20.0	35.0	0.0	1.2	0
C-BASS V	02/12	BBB	6.82	47	27.1	23.4	38.3	0.0	3.1	0
CMBS R.O.S.T 2002-1	02/12	AA	7	29	22.4	17.2	27.6	0.0	0.9	0
Longport	03/01	BBB	7.09	155	28.1	18.7	33.5	36.1	54.5	19
Trainer W.F.R. III	03/02	A-	7	77	19.6	18.2	32.1	1.1	12.2	18
Northlake I	03/02	BBB+	6.97	131	19.6	15.3	29.8	0.0	2.1	19
C-BASS VI	03/04	BBB+	7	56	20.0	17.9	31.6	0.0	1.1	0
TIAA II	03/05	BBB+	7.04	87	21.6	18.4	35.0	1.4	17.6	14
Faxtor 2003-1	03/05	BBB	6.9	91	21.2	18.7	35.3	1.5	16.8	3
ACA 2003-1	03/05	A-	7.02	100	18.3	16.0	30.0	0.0	4.3	19
Independence IV	03/06	A-	7.04	115	20.2	17.4	31.3	0.1	7.6	19
C-BASS VII	03/07	BBB	6.94	87	21.8	20.7	39.1	0.5	11.1	1
FAB 2003-1	03/07	BBB	5.99	89	21.2	18.0	33.7	1.3	15.0	5
N-Star R.E. I	03/08	BBB	6.94	69	30.5	20.3	34.8	0.2	6.8	2
Putnam 2003-1	03/10	A	9.02	207	12.0	16.9	35.3	0.3	15.0	17
Saturn Ventures I	03/10	BBB-	7.21	82	41.5	25.6	42.7	13.0	39.8	11
C-BASS VIII	03/11	BBB+	6.98	66	21.2	19.7	34.8	0.1	6.2	3
Lakeside I	03/12	AA	10.55	89	15.9	18.0	34.8	92.3	94.7	19
BLUE BELL	03/12	AAA	6.75	137	6.6	9.3	19.7	3.7	26.9	18
Commodore II	03/12	A-	7.03	91	19.5	16.5	33.0	1.3	22.1	19
Trainer W.F.R. IV	04/01	AA-	6.9	95	15.1	14.7	29.5	0.0	4.0	4
Independence V	04/02	A-	6.94	155	20.4	14.8	29.7	0.1	9.7	19
Alexander Park I	04/02	A	7	127	17.1	15.7	30.7	0.0	6.5	18

(Continued)

Panel C-Continued

Name	CDate	WAR	WAM	N	SDR	SDR NF	SDR DF	DP NF	DP DF	DG
Knollwood	04/03	A+	6.91	160	16.4	13.8	30.6	0.0	4.9	19
C-Bass IX	04/03	BBB	6.97	66	25.0	22.7	40.9	2.5	25.4	9
Newcastle IV	04/03	BBB-	6.99	58	31.6	24.1	44.8	1.6	21.5	5
Anthracite III	04/03	BB+	7	58	52.2	27.6	48.3	0.0	0.7	1
Lakeside II	04/03	AA+	9.09	145	11.7	14.5	29.7	79.4	88.4	16
FAB UK 2004-1	04/04	BBB+	7.48	62	17.8	19.4	37.1	4.1	29.6	1
Vermeer	04/04	BBB+	7.02	83	20.9	18.1	36.1	0.3	11.6	9
Bluegrass II	04/04	A	6.87	112	18.4	14.3	30.4	0.0	1.9	17
Klio	04/04	AAA	6.43	160	7.6	8.8	20.6	51.5	72.5	17
Saturn 2004 F.A.I. III	04/04	A	7.25	107	19.5	15.0	29.9	0.4	15.9	10
Saturn II	04/04	BBB-	7.2	89	41.5	24.7	44.9	1.2	21.9	11
FAXTOR 2004-1	04/05	BBB	6.85	92	22.0	18.5	38.0	1.2	22.4	3
C-Bass X	04/05	BBB-	6.87	98	32.9	24.5	45.9	3.0	30.4	2
ACA 2004-1	04/05	A-	6.98	102	19.4	15.5	30.4	0.1	6.6	9
Rhodium 1	04/05	BBB	6.78	66	18.8	19.7	34.8	4.1	27.9	3
Sandstone	04/06	A-	7	55	27.2	18.2	34.5	0.0	1.3	0
Whately I	04/06	A	6.88	184	14.5	13.6	28.8	0.0	6.0	17
RFC I	04/06	BBB+	6.97	93	20.4	16.1	33.3	0.0	4.9	0
N-Star R.E.II	04/07	BBB	6.94	82	30.3	19.5	39.0	0.0	5.6	3
Acacia 5	04/07	BBB	6.93	95	23.2	17.9	35.8	0.0	3.9	0
Cascade I	04/07	AA+	8.08	107	11.3	13.1	26.2	17.8	49.7	17
C-Bass XI	04/09	BBB	6.83	107	25.7	20.6	41.1	3.8	32.1	1
Bluegrass III	04/09	BBB+	6.88	113	19.6	16.8	31.9	0.2	11.3	18
Newcastle V	04/09	BBB-	6.75	63	26.5	22.2	41.3	0.3	13.3	6
Inman Squ. I	04/10	BBB-	7.34	81	41.8	22.2	42.0	0.0	0.3	0
Klio II	04/10	AA-	7.56	113	8.6	15.0	31.9	84.7	89.8	18
Pinnacle Point	04/10	BBB+	6.57	160	13.8	15.0	33.1	45.9	71.3	19
Sherwood	04/10	B+	7.31	198	41.5	36.2	60.6	46.2	72.4	18
Porter Squ. II	04/10	BB	6.47	78	47.3	28.2	51.3	20.7	54.0	18
Laguna ABS	04/10	AA+	7.96	218	10.7	11.5	25.7	36.4	67.2	14
Reservoir Funding	04/10	BBB-	7.08	99	19.3	25.3	47.5	53.2	75.3	18
Acacia 6	04/11	BBB+	6.95	83	22.6	16.9	33.7	0.0	7.8	6
Whitehawk	04/11	A-	6.42	95	10.6	14.7	29.5	0.0	0.1	11
Hillcrest I	04/11	BBB-	6.95	129	29.8	24.0	45.7	3.1	32.7	18
Trainer W.F.R. V	04/11	A+	6.95	109	17.2	14.7	30.3	0.0	5.9	9
Jupiter	04/12	BBB+	6.98	106	11.5	18.9	38.7	33.2	62.6	13
C-Bass XII	04/12	A-	6.9	70	21.0	17.1	34.3	0.7	15.7	17
McKinley	04/12	AA+	7.16	104	8.1	11.5	25.0	15.1	42.7	17
Revelstoke I	04/12	AAA	6.08	72	6.5	9.7	20.8	0.7	11.2	18
Cimarron	04/12	AAA	6.98	93	6.9	10.8	21.5	1.3	18.4	19
Belle Haven	04/12	AA	9.18	190	13.4	14.2	31.1	37.0	66.7	18
Vermeer II	04/12	A-	7.06	106	18.8	15.1	32.1	0.0	4.5	1
Witherspoon	04/12	AA+	6.85	154	8.6	11.0	24.0	1.3	22.2	17
Fairfield S.S. 2004-1	04/12	BB	6.99	75	49.1	29.3	52.0	2.8	29.3	2
Margate I	04/12	AA	6.82	229	10.1	10.9	25.3	1.8	25.8	15
Zenith	04/12	AA-	6.86	146	9.7	13.0	28.1	22.9	55.3	19
Ischus I	04/12	A	6.96	107	21.3	15.0	31.8	0.0	7.1	16
Average					24.2	20.1	37.5	10.6	25.5	

Panel D: CDO^2

Name	CDate	WAR	WAM	N	SDR	SDR NF	SDR DF	DP NF	DP DF	DG
Lusitano 1	01/08	BBB-	3.19	72	21.0	12.5	23.6	0.0	2.7	0
Lafayette I	02/04	BB+	3.77	30	32.1	23.3	33.3	0.0	0.3	0
Zais V	02/12	BBB-	6.99	49	47.4	26.5	40.8	0.1	4.5	11
Porter Squ. I	03/07	BB-	7	55	34.6	38.2	58.2	75.7	81.1	5
Hamilton	03/09	B+	5	158	34.6	26.9	42.3	0.0	0.3	0
Tricadia 2003-1	04/01	BBB-	6.55	69	36.3	11.5	18.5	0.0	0.0	10
Zais VI	04/01	BBB-	5.69	67	42.6	20.9	38.7	0.0	2.7	13
Vertical 2004-1	04/03	AA	9.78	71	10.8	18.3	34.5	56.8	76.9	19
Tricadia 2004-2	04/11	BB+	7.5	62	55.3	30.8	54.8	0.9	19.8	12
TABS 2004-1	04/12	AA+	7.57	98	9.7	12.2	26.5	5.7	31.4	19
Average					32.4	22.1	37.1	13.9	22.0	

Table IV

Empirical Results for Average Default Rate Predictions

This table reports the average original AAA tranche size (Size), the average value of scenario default rate (%) from (a) rating agency (SDR), (b) no-frailty model (SDR NF), (c) dynamic-frailty model (SDR DF); the average value of default probability from (a) no-frailty model (DP NF), (b) dynamic-frailty model (DP DF). The average values are calculated for each CDO types.

Average	AAA Size	SDR	SDR NF	SDR DF	DP NF	DP DF
CBO	71.7	36.6	27.6	48.7	3.8	18.5
CLO	73.6	37.1	30.1	51.9	4.6	26.8
ABS	80.7	24.2	20.1	37.5	10.6	25.5
CDO^2	77.8	32.4	22.1	37.1	13.9	22.0

Table V
Empirical Results for Regression Analysis

This table reports the regression results of downgrading notches on default probability with frailty (DP_{DF}), default probability without frailty(DP_{NF}), the difference between default probability with and without frailty ($DP_{DF} - DP_{NF}$). We also include the following controls: weighted average maturity (WAM), default measure (DM), variance measure (VM), number of obligors (Obl), interest rate (r_f), S&P 500 Return (S&P 500), Dummy CLO, Dummy ABS and Dummy CDO^2.

Independent Variables	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
DP_{DF}	3.37				4.38	
	(5.44)				(3.35)	
DP_{NF}		3.53		2.91	-1.47	
		(4.31)		(3.64)	(-0.88)	
$DP_{DF} - DP_{NF}$			5.46	4.37		4.77
			(4.24)	(3.35)		(3.46)
WAM						0.19
						(1.11)
DM						3.36
						(0.21)
VM						-39.51
						(-0.76)
Obl (x100)						0.50
						(1.71)
r_f						-0.19
						(-1.63)
S&P 500						0.17
						(0.22)
Dummy CLO	0.41	0.66	0.31	0.34	0.34	-0.51
	(1.09)	(1.80)	(0.80)	(0.89)	(0.89)	(-1.03)
Dummy ABS	2.73	2.68	2.93	2.77	2.77	2.09
	(6.89)	(6.83)	(7.36)	(6.92)	(6.92)	(3.56)
Dummy CDO^2	2.68	2.34	3.10	2.79	2.79	2.50
	(3.65)	(3.17)	(4.46)	(3.79)	(3.79)	(3.17)
R^2(%)	10.26	9.33	9.15	10.32	10.32	10.44
Observations	237	237	237	237	237	237