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**PUBLIC DEBT SUSTAINABILITY AND EMU:
THEORY AND SOME EVIDENCE**

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Introduction

The thesis focuses on the interaction between default and inflation risk on public debt bonds. We lighten the trade-off between flexibility to adverse shocks and credibility, in the debt management field, and identify the conditions under which the credibility effect can be dominant.

EMU is now fully operating, including most of the European candidates that have been let in under a more relaxed interpretation of the Maastricht Treaty criteria. In particular, the debt criterion originally set at 60% of the GDP, was reinterpreted to require a debt/GDP ratio declining towards the target. As some of the countries have levels of debt above 100% of GDP (Belgium and Italy) and as European Central Bank is committed to price stability, what does giving up inflation imply for post-EMU debt management?

A very large public debt, in particular when it is growing faster than the GDP, is often associated with two kinds of risk. The first is an inflation risk¹: high debt countries may be tempted to 'inflate away' part of their debt obligations if

¹ Some authors refer to 'devaluation risk'. Inflation and devaluation risks coincide only when the purchasing power parity holds. If this is not the case, the inflation risk is the relevant one for domestic creditors who see real assets as substitutes for public bonds; while international lenders are more concerned with devaluation risk.

they are not indexed². The second is the default risk. ‘Default’ will be used here as a general term which indicates anything which may go wrong in the service of public debt: outright *repudiation*; *rescheduling* or *consolidation*; or *una-tantum capital levy*.

Joining the EMU has meant the renounce to monetary sovereignty and thus to using inflation as a tool of debt reduction. A possible implication is that this is seen as a weakening of the government ability to manage debt service. In this case, the disappearing inflation premium will be just transferred into default premium. Alternatively, the tightened hands of an inflation prone government may have a positive effect on the market expectations. In this case the credibility gain may more than offset the loss of the policy instrument, and real interest rate may decline.

Our work provides a contribution in the literature on the divorce between fiscal and monetary sovereignty, supporting optimistic forecasts for debt sustainability within the EMU area. More sceptical views are not absent in the literature, and it has been often stressed that “*Because of their huge overhangs, European national governments cannot presently afford to give up their money issuing authority*” (McKinnon, 1996; p. 349)³. The beliefs that EMU would increase default probability on public debt, were also on the background of some empirical studies⁴: “*that there is some evidence of a default risk even before*

² For the inflationary consequences of prolonged fiscal deficit see Sargent and Wallace (1981) and Drazen and Helpman (1990) among others.

³ On a similar position is also Goodhart (1997).

⁴ For empirical evidence on default risk in some European countries see, for example, Cottarelli and Mecagni (1990), Alesina et al. (1992) and Favero Giavazzi and Spaventa (1997).

monetary unification strongly suggests that such perceived risk will increase after full integration” (Alesina et al., 1992, p. 429).

The argument behind this ‘pessimistic’ view is that when we lose one instrument of debt policy we have to rely on the other(s). That is, without inflation, all other things being equal, the government would trigger the default option when otherwise could just monetise the debt. However, if we remove the ceteris paribus assumption, the argument does no longer follow. The point is that the ability to inflate away the real value of debt brings about a cost. Removing inflation cuts away this cost and makes debt service more ‘handleable’. This is particularly true for those countries which suffer for an inflation bias problem, whilst we prove that this intuition does not apply to countries with a long experience of strict monetary discipline⁵.

The aim of the work is to show how the picture may change when a situation where both inflation and default are instruments of debt reduction, gives way to one where inflation is no longer available.

One major reason why a ‘benevolent’ government might decide to default on its debt is that default is non-distortionary lump-sum tax, which substitutes for various distortionary taxes levied to service the debt⁶. However, default is often

⁵ Consistently with our results, EMU would be good news for Italy but not for Belgium.

⁶ “In Italy, in late 1980s, the yearly interest payments on public debt were approximately absorbing all the personal income tax revenue (Spaventa, 1988). The Italian government could have stopped paying interest on its debt and abolished the personal income tax, in a revenue-neutral fiscal manoeuvre! With top marginal tax rates above 50% in Italy, even conservative estimates suggest that such a default policy might have led to substantial gains in the form of increased labour supply and productivity.” (Alesina et al., 1992; p. 431).

associated with very high costs of different nature: (i) *reputation costs*⁷ (Grossman and van Huyck, 1988; Chari and Kehoe 1990); (ii) *redistribution costs* (Alesina, 1988; Eichengreen, 1990); (iii) *financial fragility costs* (Alesina, 1988; Spaventa, 1988); (iv) *transaction costs* (Calvo, 1988).

Such arguments make outright default very unlikely in developed countries in ‘normal’ circumstances. However, highly indebted governments may opt for a partial default if rolling over the debt has become too costly.

The topic of sovereign debt default can be collocated in two streams of related literature: one regarding default on foreign debt, the other concerning default on domestic debt. The former is grown fast after the debt crisis of lower developing countries (LDCs) in the early 80s. That episodes showed that repudiation of sovereign debt is not just a theoretical concern. Many works have analysed the effects of including default among the government policy options. The strategic role of default is examined in terms of costs and incentives making use of a game-theoretical framework. The main implication that emerges from this kind of literature is that a positive default probability implies credit rationing in the international financial market (Eaton and Gersovitz, 1981; Kletzer, 1984; Cooper and Sachs, 1985; Bulow and Rogoff, 1989). Credit rationing implies a twofold effect: on one side it limits the indebtedness growth reducing the

⁷ Ozler (1993) provides some empirical evidence to support this point.

government's incentive to default, on the other constraints the liquidity available to a sovereign debtor worsening its ability to meet its debts.

The interest on the possibility of default on domestic debt is more recent. The existing models that deal with the topic, focus mainly on the role of the expectations and often exhibit multiple equilibria. The main difference that characterises domestic debt management with respect to the foreign one is the possibility to inflate away part of the real value of bonds. So, inflation has been sometimes seen as a form of implicit default on public debt.

The main implications that arise from the theoretical contributions are that a longer debt maturity and a uniform distribution of debt coming due in each period reduce the risk of a debt crisis (Alesina, Prati and Tabellini, 1990; Giavazzi and Pagano, 1990). Bond indexation is proved to be a solution against easy debt monetisation (Calvo 1988), but high degree of indexation may also increase the default risk (Drudi and Giordano, 1997).

This theoretical literature has been followed by some empirical investigations on the presence of a positive default premium incorporated in the government yields of European countries. The empirical evidence suggests the presence of a significant risk premium for Italy, and to less extent for Spain and Belgium (Alesina *et.al*, 1992; Favero, Giavazzi and Spaventa, 1997).

The above mentioned papers have examined the effects of a default risk as well as of an inflation risk separately. At the time being, a joint analysis of both kinds of risks has received little investigation. The paper by Drudi and Giordano (1997) represents in this respect an isolated example. The authors focus on the

optimal debt maturity structure and on the term structure of interest rates, when both risks push up nominal returns on debt.

The joint consideration of the two instruments of debt reduction becomes important to analyse the effects on public debt management and its sustainability within EMU. To this aim we are going to present an analytical framework which allows to compare a situation where both default and inflation are available to the government to one where inflation is no longer allowed.

We first introduce a simple model where default is the only alternative to fiscal pressure to finance the budget constraint. The decisional process of the government is explicitly analysed and the decision to default on part of the outstanding debt is conditional on the realisation of a random shock on the foreign interest rate.

The main feature of the model is the ‘state dependent’ nature of the optimal policy chosen by the government, which can decide to exercise the option to defaulting on part of its obligations in the bad states of the world. Private sector’s expectations play a decisive role: taking into account the default probability they exacerbate the government’s problem; moreover they affect the optimal economic policy and can generate multiple equilibria.

The model is then extended to include inflation as well as default as a tool of debt reduction. The comparison of the two models allows us to evaluate how EMU will affect default probability.

The following step is to allow the government to issue short and long term bonds. Introducing bonds with different maturities permit to investigate further on

the robustness of our results, as well as to draw some implications for the debt management policy within and without EMU.

Finally in the last chapter we take an empirical approach and move to test the main assumption of our theoretical framework. In particular we will analyse the empirical relationship that links the default premia to the interest rate of a foreign country which acts as a leader. In our case, the role of the leader is played by Germany. Our results suggest the existence of a non-linear relationship between the risk premium on 10-years government bonds in Italy Spain and Belgium, and the German interest rate.

Chapter 1

A Brief Survey on Sovereign Debt Default

1.1 Introduction

Literature on sovereign debt default is marked by two major events of the economic history: the financial crack of the '30s and the debt crisis of the low developing countries (LDC) in the '80s.

Up to the 'Great Depression' which involved the worldwide financial system, default episodes on sovereign debt were not unusual and were part of the regular transactions in the international financial market: "*Creditors received substantial risk premia on foreign loans to compensate for default or sovereign risk, and the governments of creditors were typically content to allow their nationals to suffer defaults without significant public intervention on their behalf*" (Sachs, 1982, p. 199).

However in the '20s and '30s the financial crisis involved quite severely a wide number of countries¹. This affected the attitude of sovereign governments as well as of investors in facing the risk of debt default. Large investors felt more and more uncomfortable in lending to foreign governments and international capital remained scarce for the next 40 years.

International lending grown again in the second half of the '70s, just after the first oil shock in 1973. Major banks were investing huge flows of money coming from the OPEC countries and found profitable opportunities in the high yields offered in developing countries. But next decade, following both the slow down of the capital flows from oil exporting countries and mainly the sudden rise in the US interest rates, the scenario turned bad again.

The LDCs had accumulated large foreign debts and faced increasing difficulties to meet their regular payments to creditors. In 1982, the Mexican suspension of its debt service gave way to the deep crisis that involved the majority of Latin American countries for the following decade.

It was soon evident the worldwide feature of the debt crisis: banks, governments and International Institutions converged together trying to limit the contagious effects of the bad budget sheet of the LDCs. The tentative solutions moved along two lines, on one side they provided incentives to economic reforms (through conditional lending) on the other threatened punishments for stopping payments.

¹ A detailed report of the financial difficulties of some European governments in serving their debts between the two World Wars can be found in Makinen and Woodward (1990). See also Eichengreen and De Cecco in the same volume.

The debt crisis of the '80s generated a huge stream of literature on sovereign default. The topic was examined from theoretical and empirical prospective with a twofold aim of understanding the causes and identifying the most efficient strategies to deal with it. A definitive evaluation of the crisis management is not available, and we just try to sum up its guiding lines in the following:

- adoption of cooperative bargaining strategies, involving borrowers and lenders, often sponsored by International Institutions such as the International Monetary Fund (IMF) and the World Bank (WB);
- easier access to liquidity to deal with emergency situations, at least in the short run²;
- reallocation of the 'bargaining power' between debtors and creditors which gave a higher punishment ability to the latter.

The '80s were characterised also by a fast growing of public debts in many European countries: "*The policy mix adopted in the 1980s was...a combination of strict monetary policy and the growth of public debt to replace the deficit-monetization which had been the rule in the previous decade*" (De Cecco, Pecchi and Piga, 1997, p. xii).

Debt to GDP ratios reached very high levels (seldom seen in the past, if war times are excluded) and became reason of concern for policymakers as well

as for scholars. The scenario was getting darker in early '90s as real interest rates were rising again and GDP growth rates depressed. Highly indebted countries pursued strict budget policy under the pressure of the Maastricht rules, which had put fiscal indicators under control³. The academic reaction was slightly faster and in late '80s a few papers suggest the existence of a default risk on public debt even for European countries. This stream of literature focuses mainly on debt management and on government budget constraint looking at default as an alternative tool of debt reduction.

Theoretical papers on the topic analyse the role of expectations and the possibility to observe multiple equilibria. In these models often devaluation is considered as a form of default and sometimes the only credible one in an industrialised country. However, the risk of an explicit default seems to be taken into consideration by financial agents. Some empirical studies provide evidence of a small, but significant, default premium in the public yields of some high debt countries.

The present chapter presents a brief and non-exhaustive survey of the main theoretical as well as empirical contributions in the literature of sovereign debt.

² During the debt crisis, the IMF was acting as a lender of last resort and as an intermediary for funds to avoid sovereign bankruptcy. In both cases interventions were conditional on the adoption of a programme of economic reforms under the supervision of the IMF itself.

³ De Cecco, Pecchi and Piga (1997) note that: "*Less heavily indebted countries, however, did not feel the same constraint. As a result, average European Union debt-to-GDP ratios...in 1995 reached the unprecedented post-war rate of 70 per cent, even in spite of the relative virtue displayed by the large debtors, like Italy, Belgium and especially Ireland*" (De Cecco, Pecchi and Piga, 1997, p. xii).

In the next section we deal with foreign debt and the focus is on the LDCs debt crisis. This stream of literature provides hints on a possible scenario for some European countries, which in the last year gave up their monetary sovereignty in the name of a Single Currency. Without the ability to print money, in fact, domestic debt acquires many features in common with debt issued in foreign currency (Goodhart, 1996; Folkerts-Landau, Mathieson and Schinasi, 1997).

In section 1.3 the focus on domestic debt that is more relevant for the European countries in the pre-EMU phase and for the out-countries⁴. Moreover, we believe that this case is still relevant for the post-EMU phase, as the single currency, in which debts of different EMU countries are denominated, is still under control of an Institution acting within the area. In short, the country specific debts issued in Euro will probably share features of both foreign and domestic pre-EMU debts. The section considers three possible scenarios: first, we look at a government which can violate the terms of debt contracts (explicit default) with private agents; second, we consider the possibility of inflate away part of the debt value (implicit default); then, both forms of default will be considered as alternative instruments of debt reduction.

Finally, in the last section, the empirical evidence of a default premium for European countries is discussed.

⁴ Accordingly to the literature on EMU with out countries we indicate the European countries which have opted for stay out of the EMU as well as those which had no option to get in (either because they did not satisfy the criteria or because they were not EU members) (De Grauwe, 1997)

1.2 Default on Foreign Debt: the Experience from LDCs

1.2.1 Default and the theory of international lending

As mentioned in the introduction to this chapter, in the '70s (after a long period that might be called of 'financial autarky') large capital flows were channelled from oil exporting countries to the major western banks and from there towards most developing countries. The incoming flows were very welcomed by the governments of the LDCs, which accumulated growing levels of debts. The easy provision of international liquidity together with an excessive confidence in the stability of interest rates, led the governments to underestimate the risk of such a high indebtedness. Moreover, either due to the political instability or to the lack of specific expertise, borrowing funds were seldom profitably invested and the economic activity was not significantly improved. It followed that soon as the international environment changed at the beginning of the new decade, many countries were hardly serving their debts.

The vast default literature of the 1980s put a great deal in finding a satisfactory answer to the following question: why should rational agents lend to foreign sovereign governments not subject to any bankruptcy law? Thus, the focus

of the analysis is often more on the deterrents for a government to opt for a debt default than on its determinants. It emerges a theory of default based on repudiation costs.

The papers discussed in this section present some common features, which can be easily summed up in four main points:

- a) necessary condition to observe lending flows to sovereign States is the ability of creditors to impose some sort of punishment in case of default;
- b) international lending is characterised by rationing: it does exist an upper bound above which the country wants to repudiate debts, thus the creditors will never lend in excess to that bound;
- c) the borrowing upper bound is an increasing function of punishment;
- d) without uncertainty the only possible outcome is full debt payment and no punishment, equilibria with positive default probability emerge only when uncertainty is considered.

The paper of Eaton and Gersovitz (1981) can be considered the seminal paper in the default literature on sovereign debt default. The work completes research on default and bankruptcy issues previously devoted mainly to corporate debt. The authors describe a government that refrains from defaulting on its foreign obligations in order to keep easy access to the international financial market in the future. The approach adopted is that of the permanent income when the economy is subject to good and bad harvests, alternatively. Adopting a decreasing marginal utility function the expected actual value of domestic

consumption is maximised when government borrows in the bad states and pays back the loans in the good ones. With no punishment ability by creditors, the government has an incentive to repudiate the debts in the fruitful years, as this would increase consumption at zero cost. Therefore, the assumption is that a defaulting government is relegated to financial autarky, as foreign investors are no longer willing to lend it funds. In a model where smoothing consumption is the final aim for borrowing, the access to foreign liquidity to support low consumption years is the incentive mechanism for a regular debt service. However, the authors show that above a given level of debts the costs of repayment are higher than the benefits from new loans. When this level is reached default will occur with probability 1. Under perfect information, therefore, rational agents will never lend above this amount and possibility for credit rationing emerges. Actual financial flows to a sovereign State are given by the minimum between the total liquidity demand from the borrower and the maximum level of indebtedness imposed by lenders. The model implies that the ceiling on indebtedness is increasing in income volatility. In fact, raising the scope for consumption smoothing increases the benefits from future foreign loans and reduces default incentive. Moreover, credit ceiling rises when higher exogenous penalties are available: "*Gunboats are the borrower's best friends*" (Cline, 1995, p. 140).

In its deterministic version with perfect information, the model implies that default is a zero probability event. However, Eaton and Gersovitz propose a stochastic framework where uncertainty introduces new implications. In

particular, when income path is random, a sequence of bad harvests can make debt service very onerous and default more attractive. This makes a point for equilibria with positive probability of debt repudiation. When this is the case a high income variance can increase rather than reduce the credit ceiling to the country.

Considering the case of asymmetric information, Kletzer (1984) enriches the previous analysis and investigates the properties of competitive equilibria in financial contracts between international investors and LDCs governments. The author extends the model under different hypotheses on the agents' information set. First of all, it is assumed that creditors observe their own lending flows to borrowers but not the total level of indebtedness of each country; later, full information on country's total borrowing is allowed. In the former case, financial intermediates can not discriminate between countries with different levels of outstanding debt and since, *ceteris paribus*, default risk grows with the amount of indebtedness, they are not able to select most reliable borrowers. If an equilibrium with asymmetric information exists, it has to lie on the borrowing demand curve. This allows the lenders to use interest rate (assumed unique in the international market) to impose credit rationing⁵. However, such an equilibrium needs not to exist. This is the case when the liquidity demand curve of debtors is such that there is no contract to guarantee a non-negative profit to creditors. Nevertheless, profitable contracts for lenders may still exist and be signed, in fact "*non-*

⁵ Similar credit rationing equilibria have been analysed by Stiglitz and Weiss (1981) with reference to non-sovereign agents.

existence of an equilibrium does not imply that disequilibrium lending will not occur” (p. 296)

In the alternative case of full information, the author proves that an equilibrium exists under specific assumptions⁶. In particular, the equilibrium is the contract where the debtor’s indifferent curve is tangent to the lending supply curve. Usually such an equilibrium is characterised by credit rationing. As the equilibrium with complete information is a constrained-optimum, it Pareto-dominates that one with myopic agents. Actually, in the symmetric information contract both credit ceiling and interest rate are lower: thus, utility loss from less liquidity available is more than offset by the lower borrowing cost.

Kletzer argues that lack of enforcement and asymmetric information between debtors and creditors are a key elements to clear the main differences between sovereign States borrowing and domestic corporate debt contracts. He shows how shortening of debt maturity structure, predominance of bank over bond lending, quantity rationing of credit, and red-lining of the poorest LDCs are more likely to emerge in a more uncertain environment such as that one of international lending.

Both papers discussed above rely on the assumption that sovereign debtors refrain from repudiate their obligations to avoid exclusion from international credit market in the future. In a survey work, Eaton Gersovitz and Stiglitz (1986)

⁶ The proof of existence relies on the assumption that the cumulative distribution function of the shock is continuous.

note that the argument needs an infinite horizon. Assuming a finite horizon implies that the incentive to service the debt does not hold in the final period and no loan will be granted coming due in that period. By backward induction to the initial period, the authors show that financial transactions are not sustainable in the model. However, since both sovereign States and financial institutions have plausibly infinite life, the autarky regime is a credible threat. The same argument is made for explaining why some developing countries may suffer from low incoming funds: *“Borrowing for capital accumulation or productive investment implies that a point will be reached beyond which the debtor will begin making transfers to his creditor. Once the marginal product of capital equals the interest rate, there will be no further gain to moving capital to the debtor. At this point, the debtor will lose nothing by being denied access to credit markets, and will refuse to service its debts. And, by backward induction as before, it will never be possible to lend with prospects of payment “* (p. 491). Cline (1995), however, claims that such a theory is not consistent with the large loans allowed for the post-war reconstruction.

Cooper and Sachs (1985) deal with the topic from the point of view of the sovereign debtor. In their analysis, the authors identify three different kinds of risks which concur to constraint the availability of international funds: solvency risk, liquidity risk and repudiation risk.

a) Solvency risk: when the country is not able to raise reserves in foreign currency high enough to pay back the outstanding debts. The possibility of an

insolvent debtor at maturity, implies that creditors are not happy to lend in excess of the discounted present value of all the future current account surpluses.

- b) Liquidity risk: when the country has to rely on foreign lending to meet obligations coming due in low income years and international financial market is tight. Liquidity risk represents one more constraint to the indebtedness level of the government. Any single creditor may be afraid that the others will not grant further liquidity to the country and itself deny the necessary loans.
- c) Repudiation risk: when the country, whatever the economic growth perspectives it is facing, chooses not to serve its debts “*either because current repayment is too onerous or because it is holding out for some sort of debt relief*” (p. 22). As the other two risks, repudiation risk induces further credit rationing.

Unlike the previous paper discussed here, Cooper and Sachs assume that creditors can seize part of the debtor activities and get a direct gain equal to γW (where W is the total country wealth and γ is constant). Moreover, punishment includes further sanctions such that total loss to defaulting debtors is given by θW (with $\theta > \gamma$). Net gain to the country from debt repudiation is therefore $D - \theta W$ (where D indicates total indebtedness), while the actual loss for creditors is $D(1 - \gamma)$ ⁷. Whenever total borrowings exceed sanction costs the government has an incentive to default on debt. However, the $\theta - \gamma$ spread makes room for bargaining

between debtor and creditors, and some sort of debt relief can be wealth improving for both parts. Any transfer in the interval $(\gamma W, \theta W)$ from debtor to creditors increases their utility with respect to the outright repudiation alternative. The transfer, therefore, disincentives them from imposing sanctions, making the country better off as well. The actual amount paid by the debtor depends on the bargaining power of the agents and determinates their respective degree of rent extraction.

Cohen and Sachs (1986) focus on the links among international lending, default risk and economic growth. The paper presents a model where the government borrows from the international credit market to support domestic growth⁸. An indebted country would pass throughout two stages: in the former, when international liquidity is largely available, the foreign debt to GDP ratio rises and GDP growth rate declines; in the latter, GDP growth stays low and constant while foreign credit rationing emerges. The authors claim that each LDC will sooner or later approach the second stage. Once the country is in the low growth stage, the government stops a full service of debt and only a fraction of interests, such as to keep the debt to GDP ratio constant, is paid.

⁷ On the assumptions claimed by the authors, the exact loss for creditors should be $D-\gamma W$. Cline (1985)'s point is that they implicitly assume $D=W$, but it does not need to be the general case (in fact, the authors themselves suggest that $W>D$ is expected).

⁸ The authors do not share Eaton, Gersovitz and Stiglitz (1986)'s argument on the riskness of addressing foreign loans to finance domestic development. Such an issue does not seem to gather support by Cooper and Sachs (1985) either, as they suggest to constraint foreign liquidity to investment and development programmes in order to reduce the perception of default risk from the creditors.

The government can also opt for an outright repudiation of the outstanding debts. Whenever this occurs, the country bears a twofold punishment: first of all, it will be excluded by the international financial market in the future; secondly, it will suffer by a loss of productivity⁹. From the debtor point of view the decision to default depends on funds availability from creditors in the future. In turns the investors decide their credit lines to the country accordingly to their expectations on its present and future behaviour. As the model assumes no uncertainty, default never actually occurs. Creditors know the government's objective function and limit loans at a level such that repudiation option is always dominated by the regular debt service. However, default threat influences creditors strategic decisions as well as the domestic growth rate of the indebted economy.

In their contribution to the debate, Bulow and Rogoff (1989a) recall three main reasons mostly claimed to support capital flows to sovereign states in the international lending literature: a) creditors' ability to seize debtor's activities in case of default; b) debtor's willingness to keep a good reputation in the foreign financial market; c) creditors' ability to impose sanctions to the international trade of defaulting governments. The authors underline that the first argument is negligible, whilst the second is too weak to assure the regular debt service whenever the debtor has access to contingent contracts to insure itself against the

⁹ The lower productivity is justified by a loss of efficiency due to possible troubles in the foreign trade after default occurred.

adverse states of the world¹⁰. Instead, their intuition is that “*a country is willing to make some repayments on its debts in order to enjoy its full gains from trade*” (p. 159).

The paper introduces the renegotiations problem into the literature on international lending. The approach adopted is that of game theory; in particular, the authors present a model *a` la* Rubinstein with risk neutral agents to analyse the bargaining process underlying debt rescheduling agreements.

Bulow and Rogeff focus on a small open economy, where the government maximises its intertemporal expected utility. In the event of unilateral debt repudiation, the country bears an embargo on trade, which reduces export revenues of a fraction β . In analogy with Cooper and Sachs’s paper, it is here assumed that creditors can appropriate part of this revenues. In particular, foreign lenders can obtain a fraction $\alpha \leq \beta$ of the debtor’s trade revenue.

In the subgame perfect equilibrium of the model, the maximum debtor’s GDP share that creditors can subtract to the country each year, is given by:

$$q = \min \{(\gamma+\delta)/(2\gamma+\delta+r), (P-1)/P, \beta\}$$

(1.1)

¹⁰ In a different paper of the same year (Bulow and Rogoff, 1989b) the two authors show that the reputation incentive to service the debt not to preclude future loans from international investors, is not alone sufficient to justify financial transfers to LDCs. With the help of a simple arbitrage model, describing a small open economy with stochastic income, they prove that indebted countries can be better off replacing foreign loans with ‘*cash-in-advance*’ type of contracts. The paper concludes that financial contracts based on reputation can not be a sequential equilibrium.

where γ is the depreciation rate of the accumulated stock of domestic good, δ is government's intertemporal preference discount rate, r is the international interest rate, P is the relative price of domestic good with respect to the foreign one, β is defined above. The terms in curling brackets determinate three regions that characterise what the authors call "*the equilibrium rescheduling agreement*" (p.166):

- a) *Bargaining region* (when $(\gamma+\delta)/(2\gamma+\delta+r)$ is minimum): international creditors appropriate a fraction $(\gamma+\delta)/(2\gamma+\delta+r)$ of the country income (which is equal to Py), while the complementary fraction given by $(\gamma+r)/(2\gamma+\delta+r)$ is still part of the country wealth.¹¹ The respective shares going to the bargaining parts are inversely proportional to their degree of impatience in finding an agreement¹².
- b) *Autarky-constrained region* (when $(P-1)/P$ is minimum): in this region, gains from trade are quite low (in fact, we have $1/P > 1-\beta$, and $P \rightarrow 1$). The country can easily afford a trade embargo with a small loss of revenue and the incentive to default is high. However, foreign investors will still be better off accepting a low transfer from the debtor and letting the country free to trade. Moreover, the level of repayment in this equilibrium appears to be very sensible to the world price of the domestic good.
- c) *Punishment-constrained region* (when β is minimum): in this last region, creditors have a limited ability to damage trade of the defaulting country and

¹¹ Cline (1995) notes that, assuming $\delta > r$, the yearly transfer of wealth to the creditors in the 'bargaining region' is at least as high as one half of the country's GDP. The author, therefore, concludes that this one can not be the relevant region.

the maximum they can get from bargaining is given by the fraction β of its total income.

On the other hand, default risk induces foreign investors to limit financial lending to LDCs. The credit ceiling is calculated to be equal to the present discounted value of future expected payments, which is given by:

$$R = (Pyq)/r$$

(1.2)

Finally, the authors introduce a random shock on the world interest rate. This allow them to analyse the effect of uncertainty in the model. A bad shock on the interest rate has two opposite effects for the indebted country: on one side, it puts the government under pressure to raise increasing resources for the debt service, on the other it increases its bargaining power in the rescheduling process. In fact, creditors are now more impatient to get same transfers to allocate to more profitable investments.

These results let the authors to conclude that the deep debt crisis of the '80 is not really due to irrational investors behaviour, but rather by "*just bad luck*" (p.173), i.e. by sudden growth in the international interest rate and worsening of the terms of trade.

¹² This is a general result in the Rubinstein bargaining model (see Fudenberg e Tirole, 1991, section 4.4).

1.3 Default on Domestic Debt: Some Theoretical Contributions

In the previous section we have reviewed the theoretical literature on the international lending to sovereign States that retain the possibility to repudiate their debts. We now present a brief survey of the main theoretical papers, which explore the default even on the domestic obligations. The main element, which makes the management of public debt to depart from that of foreign debt (i.e. denominated in a different currency from the country's legal tender) is that the government has got a very attractive alternative instrument to reduce the real value of debt: inflation. The theoretical literature on public debt management has widely investigated the effects of inflation of fiscal and debt policy, and in a few cases the authors have considered inflation just as a special, and somehow softer, form of default. However, whilst explicit default implies the government to recede on the terms of the debt contract (repudiation of debt, deny to pay interests, delay in the payments coming due, and so on), inflation in itself allows the policymaker to meet the full face value of its obligations at maturity. As debt is usually issued in nominal terms, inflation does not involves any breaking of the contract. In what follows we will refer to inflation as implicit default.

In this section, we first focus on the models that consider the possibility of an explicit default as a government instrument of debt reduction; then, we make mention of those papers, which are closely related to the former, but consider inflation the only credible tool of real debt ‘cheating’ in the hands of the policymaker. Finally, we present a model where both explicit and implicit default appear as different instruments of economic policy.

1.3.1 Public debt repudiation

The paper by Calvo (1988) can be considered the reference work for all the following literature on public debt default. The author presents two versions of a model where the government issues bonds that sells to the domestic private sector. In the former version, an economy without money is considered, where the government can renege on its debt; in the latter, money is introduced and default takes the form of inflation. We start here describing the framework with repudiation, and will come back to the other later.

The model outlines a small closed economy lasting for two periods. Public bonds are issued in the first period and paid back in the second. As taxes have distortionary effects on income, the government finds an incentive to repudiate partly its bonds. In order to have an equilibrium with positive debt, the author assumes that debt default is costly. The cost is proportional to the amount of debt which is repudiated and enters directly the government’s budget constraint. There

is no uncertainty in the model and it is assumed that private agents are fully informed of the government's objective function. Arbitrage condition, thus, implies that public bonds yield equals return from capital (i.e., the opportunity cost of holding bonds).

The author emphasises the role of private agents' expectations in generating multiple equilibria: in his own words: "*expected (partial) debt repudiation would tend to be reflected in the interest rate on government bonds (increasing it), while the higher the burden of the debt, the higher would be the temptation to repudiate it. Thus, it should be possible to generate an equilibrium with low interest and low repudiation, coexisting with a high-interest, high-repudiation equilibrium.*" (p. 648).

After observing the interest rate set by the agents, the government chooses the optimal policy that maximises consumption under its budget constraint. Given the unit cost of default, the policymaker determinates the optimal level of taxation and, consequently, the optimal fraction of bonds to be repudiated. Optimal default size is an increasing function of the debt yield. The environment of certainty in which agents move makes them always aware of whether or not default is occurring, and if it does in which degree. This means that default probability is either 0 or 1, while default size varies between 0 and 1.

Calvo determinates a critical threshold for the optimal tax level, above which two equilibria arise: one with fully debt repayment, the other with certain default on part of its obligations. On the other hand, when optimal taxes are below the threshold no positive debt is sustainable in equilibrium. Finally, when taxes

are exactly equal to the threshold, equilibrium is unique and all bonds are repaid to the agents.

The author suggests that indeterminacy of equilibria can be solved and default avoided if bonds are allocated throughout an auction that set an upper bound to the interest rate. In fact, it is shown that the yield ceiling can be chosen in a way to eliminate default risk.

Following Calvo (1988), Alesina, Prati and Tabellini (1990) interpret the bad equilibrium as a confident crisis in the financial market like the ‘banking panic’ depicted by Diamond and Dybvig (1983). The authors note that such an equilibrium results from a coordination failure of agents taking simultaneous portfolio decisions. In their model, instead, confident crisis is originated by a coordination failure among agents who take investment decisions in different periods of time. The paper analyses a deterministic infinite living economy with two kinds of agents: the government and the representative agent. Government issues bonds of one year maturity¹³ that can not be hold by foreign. Given the initial level of indebtedness, the government chooses between two alternative strategies: a) to raise taxes to service the debt; b) to renege its obligations. On the other hand, private agents can choose between the following: a) to purchase domestic bonds relying on regular debt payments in next period; b) to hold foreign assets only (confidence crisis). As in the Calvo’s paper, taxes are distortionary and

¹³ This assumption is then relaxed by the authors.

default is costly. However, unlike that model here the default cost is constant and is born by the government the first time it misses its debt service¹⁴.

The fiscal authority allocates its bonds with an auction procedure. The sequence of events is as follow:

- a) the government announces the bond price at which it is willing to issue debt and the maximum amount of bonds to be offered;
- b) private investors observe the price and set the quantity of bonds to purchase;
- c) the government determinates its optimal policy given by a combination of taxes and default size to satisfy its budget constraint and knowing both debt to maturity and the amount of newly issued bonds.

Alternatively, an auction procedure where the government is quantity setter and price taker can be established. The '*lump-sum*' nature of the default cost implies that optimal default size is 1, i.e. when default occurs it involves the total outstanding debt.

In both ways, the auction relies on a time span between the bonds offer from the government and the purchasing decision by the agents. This feature is relevant to the results obtained in the model.

The level of the default cost determinates three region for the equilibria of the model: a) when the cost is very low, the equilibrium is unique and total default happens in the first period; b) when the cost is high enough, equilibrium is still unique but default is never optimal and government services its debt in each

¹⁴ Moreover, the default cost affects the government's utility function and not its budget constraint (as it does in the Calvo's model), but the authors claim that this would not influence the results of the model.

period; c) finally, when the cost is in the intermediate region, two equilibria are possible, one with full debt service in every period, the other with complete default in the first period.

The next step of the paper is to extend the model to consider bonds with different maturity. The implications are more optimistic: it is proved that, under the assumption of time independence of the agents' expectations, a confidence crisis seems less plausible when either debt is all in long term bonds or the same amount of debt comes to maturity every period.

1.3.2 Inflation as implicit default

As we mentioned in the previous paragraph, Calvo (1988) presents a second version of his model, where debt is repaid in nominal terms, but not in real term, i.e. debt burden is now alleviated using monetisation. The basic framework of the model is very close to the one described in the paragraph 1.3.1 above; however, the role of money is now explicitly considered. Money supply is controlled by the monetary authority that operates in agreement with the government, while money demand is fixed in real terms and equal to a constant κ . Inflation rate is normalised as $\theta = \pi/(1+\pi)$, where π is the inflation rate.

There are two main differences that make this model to depart from the repudiation version:

- a) the parameter θ can now assume negative values, in particular we have $-\infty < \theta \leq 1$;
- b) the inflation cost now is born by the agents in terms of inflation tax. The government's budget constraint, instead, is improved by inflation as it reduces real debt burden as well as increases revenues, through signorage.

The model is first solved under the assumption of perfect credibility of the inflationary policy announced by the government. In this case the equilibrium inflation (defined the first best inflation) is unique. Later the time inconsistency problem of the government is analysed. It is shown that, when the perfect credibility assumption is removed, optimal inflation (second best inflation) is a growing function of the bonds interest rate. This generates again multiple equilibria. The author shows that multiplicity of equilibria is strictly related to the existence of the public debt, when no bonds are issued, in fact, the model exhibits a unique equilibrium. The equilibrium is still unique when real money demand, κ , is high enough. As money demand is determined by the past inflation pattern “*multiple solutions may be a bigger problem for countries which have recently suffered from high inflation*” (p. 656).

Inflation affects negatively the economic welfare of the country and in the second best case, both equilibrium levels of interest rate are higher than that of first best. Thus, the equilibria can be Pareto-ranked.

Calvo concludes with some implications of economic policy: bonds indexation would attenuate government's incentive to use inflation to reduce debt

burden and increase its credibility in announcing the economic policy¹⁵. In particular, the author claims that full debt indexation can restore the first best equilibrium¹⁶. An alternative solution, raised in the repudiation version of the model, is to impose an upper bound to the public yields. However, while the interest rate ceiling eliminates the indeterminacy of the equilibrium, it let the unique equilibrium to coincide with the second best lower inflation outcome. The intervention to limit the debt return is, therefore, Pareto-dominated by the full indexation policy.

The topic of debt confidence crises, is investigated by Giavazzi and Pagano (1990) as well. In their work, the authors focus on the proper bond maturity structure rather than the best degree of indexation, to solve the problem of inefficient equilibria.

The main novelty of the paper with respect to the models discussed above, is the introduction of a source of uncertainty in the government's preferences, which is responsible for asymmetric information between the policymaker and the private agents. The main result is that agents' pessimistic expectations do not trigger the debt crisis, but just increase the probability of it occurring. The authors focus on debt management policy and on its role in affecting the Central Bank ability to resist the crisis.

¹⁵ Calvo and Guidotti (1990) study the optimal bond indexation degree with respect to prices and the optimal debt maturity structure, to reduce inflation bias.

¹⁶ This result relies on money demand being completely inelastic to the interest rate.

The kind of crisis examined in the paper is of one generated by a speculative attack against the national currency, when the country is committed to a fix rate regime. Debt repudiation is not considered explicitly, but the two authors suggest that their framework can be easily extended to a close economy where the currency attack is replaced by a debt solvency crisis. More importantly public debt management plays a decisive role in the analysis.

The model describes an open economy where three agents interact: private sector, the Treasury and the Central Bank. Private sector sets the probability of observing a devaluation of a given size and asks for a risk premium to hold domestic bonds. The Treasury chooses the economic policy given by a combination of borrowing and monetisation to finance the fiscal deficit. Monetisation is implemented by withdrawing money from a special fund at the Central Bank. The Treasury pursues two objectives: a) to minimise public debt service; b) to help the Central Bank in defending the exchange rate. When the private sector is fully confident in the fix rate regime, the Treasury can stabilise debt at a constant level with uniform maturity structure. Newly issued bonds finance the total deficit and no monetisation is needed. However, when probability of devaluation is positive (confidence crisis) the Treasury must decide between borrow at a higher rate or withdraw money from the Central Bank, or a combination of the two. The level of monetisation depends on the government's preferences, which are random in each period. Private agents know the probability distribution of such a random variable but can not observe its realisation in advance.

The Central Bank's role is to implement a stable monetary policy with the aim to protect the fix parity of the currency. The reserves own by the Central bank are enough to defend the parity if the Treasury does not intervene by increasing liquidity in the economy. However, the government's incentive to withdraw money is taken into account by the private sector that can correctly anticipate the probability of devaluation.

The authors show that an equilibrium with no devaluation does always exist. In particular, it is the unique equilibrium when probability of crisis is below a given critical value. However, when devaluation expectations are above that threshold, more equilibria emerge.

The policy implications of the model are consistent with those of Alesina, Prati and Tabellini (1990). In fact, Giavazzi and Pagano show that if debt maturity is long enough, the Central Bank is always able to avoid the crisis. Moreover, the probability of devaluation is reduced by a unifier distribution of the maturity structure, such that the same amount of bonds is coming due in each period.

1.3.3 Default and inflation as instruments of debt reduction

The analysis of debt crisis when both default and inflation are available instruments of economic policy has not received a great deal of attention. A model which includes the two options will be presented in the next chapter.

The issue has been recently analysed in a Discussion Paper of the Bank of Italy, by Drudi and Giordano (1997). The interest of an Institution such as the Bank of Italy to the topic indicates its relevance for economic policy purposes.

The two authors focus on the role of the debt maturity structure when both default and inflation risks push the domestic interest rate up. The model describes the interaction of two kinds of agents (the government, and private investors) in a time span of three periods. The investors set the interest rate on short and long term bonds incorporating their expectations on future inflation and default. The real interest rate in each period is assumed to be stochastic, autocorrelated and not affected by the monetary policy of the government¹⁷.

Different instruments of financing the fiscal deficit (with an exogenous public expenditure) are available in the three periods:

Period 0: in the initial period, an exogenous flow of public expenditure is entirely financed by government's borrowing, through the allocation of short and long term bonds to the private sector. Thus, the amount of initial debt is given, but the government can optimise on the maturity structure.

Period 1: in the intermediate period, the fiscal deficit can be covered by: a) imposing distortionary income taxation; b) borrowing short term from the private market; c) inflating away part of the debt real value; d) defaulting on part of the maturing bonds.

¹⁷ In particular, real interest rate is assumed to take just two values, each with probability $\frac{1}{2}$, and with mean 0.

Period 2: finally in the last period, the government can no longer borrow and has to meet its budget constraint raising funds from income tax revenue and inflation, and in the bad states of the world by opting for a partial default.

The model differs from the previous models in introducing a government objective function which does not coincide with the representative agent's. The government sets its economic policy minimising an intertemporal loss function which penalises deviations from the targets¹⁸. Moreover the option for a partial default imposes a proportional cost on the social welfare.

The economy evolves accordingly to the following sequence of events:

- a) real interest rate in period 0 realises and is common knowledge;
- b) given the public expenditure flow, the government chooses the best term structure for the debt;
- c) private agents set interest rates on short and long term bonds, accordingly to the no-arbitrage condition;
- d) in period 1, the government sets the inflation rate, the income tax rate, the amount of new borrowing and decides whether to opt for partial default on outstanding short term debt. In case of default, it sets the optimal fraction of bonds not repaid.

The sequence is repeated as:

- e) the new real interest rate (relative to period 1) realises and is observed by the whole economy;

¹⁸ The approach followed by the authors is common in the '*time inconsistency*' literature (see among others, Barro and Gordon, 1983a; Rogoff, 1985)

- f) short term nominal interest rate is chosen by private investors;
- g) government sets its policy given by a combination of inflation, income taxation and size of default (if any).

In their analysis, the authors consider different scenarios involving different degrees of credibility of the government. In particular three cases are examined: full credibility, partial credibility and no credibility.

Full credibility regime: The government is fully credible in committing itself to a no-inflation and no-default policy. The equilibrium is in this case ‘efficient’, in the sense that fiscal pressure is uniformly distributed along time. This is made possible, irrespective of the uncertainty on the real interest rate, by a proper maturity structure of the debt¹⁹. Moreover, given the credibility assumption, no risk premium is charged and nominal interest rates are equal to the real ones.

Partial credibility regime: The government’s policy is assumed credible on the regular debt service, but not on the no-monatiation commitment. Backward induction solution shows that more uniform distribution of the fiscal pressure occurs at the cost of higher inflation in the final period. Numerical simulations suggest that the optimal average maturity of debt is inversely related to the time path of public expenditure. That is, optimal maturity grows (reduces) for time with decreasing (increasing) expenditure. Simulations also indicate that

¹⁹ The authors show that the particular debt structure must be such as to avoid new borrowing in the intermediate period.

long-short interest rate differential is positively related to the average debt maturity.

No credibility regime: finally if the government cannot commit itself neither on default nor on inflation, nominal interest rates incorporate both risks. The model then presents two solutions: in the former, default never happens, and inflation and income tax revenue can finance the fiscal deficit; in the latter, the regular debt service gets unsustainable only for the adverse realisation of the real interest rate. Anyway default is implemented only in the final period. The model illustrates that a positive default risk emerges for high levels of debt, but its actual occurring depends on high shock on the real interest rate. Moreover, default risk grows when average debt maturity gets shorter. This implies that optimal maturity of a non-credible government is longer than that of a more reliable one.

Finally the authors investigate the implications of issuing indexed debt on default and inflation risks. The results indicate that with indexed bonds the government refrains from using inflation at the cost of a higher default risk. Thus, the paper suggests that in a no credibility regime, the optimal degree of indexation should be lower than 100%.

1.4 Default on Domestic Debt: Some Empirical Contributions

In the previous section, we have illustrated some theoretical papers that analyse the possibility of a debt crisis involving a default risk. The main results from the theoretical literature are that default risk is positively correlated with: a) high levels of public debt; b) short term maturity of the indebtedness, in particular when maturity is not uniformly spread over time.

Next step is to look at the empirical literature to see if the theoretical possibility of observing default on domestic debt finds some evidence in the data. In what follows we are going to discuss some studies that investigate whether a default risk is perceived by financial investors and thus whether it appears in the nominal interest rates.

The quantitative exercise to isolate the default risk from the other components of the total risk premium on public bonds is quite recent. The emerging interest in the topic in late 80s - early 90s is probably the natural consequence of the theoretical literature discussed above. But an important factor, in delaying this kind of analysis might have been the difficulty to find a proper data-set to discriminate the default risk from the other components²⁰. In fact, which is the proper proxy to measure the default risk is still an open question and

²⁰ Other components of the total risk premium are, for example, the devaluation risk and the liquidity risk. The first risk concerns the possibility to loose the real value of a given nominal asset with respect to a different currency (or the same currency at a different time in case of inflation risk). The second regards the difficulty to acquire liquidity if needed before maturity. The latter risk is inversely related to the market dimension for that particular asset.

different measures have been proposed in the literature. Of course, the approximations to the 'true value' have been improving with the availability of more data on different financial instruments.

An interesting analysis involving 12 OECD countries²¹ is proposed by Alesina *et al* (1992). The authors present an empirical study of the default risk premium based on "*the difference between the return from holding government debt and the return from holding 'safe' private debt of corresponding maturity, denominated in the same currency*" (p. 429). How the authors define 'safe' private debt, however, is not specified in the paper. Accordingly to expectations, a first look at the data (in the time span 1974-89) indicates that the government yields are usually lower than the ones paid by the private sector, meaning that public debt is perceived on average as safer. However, Italy represents an exception, at least in the most recent years of the analysis: in fact, the ratio of return on public bonds to that on private is on average slightly higher than 1 in the period 1979-89.

Interestingly, the authors show that while the variation range of the above average ratio is quite small (between 0.89 and 0.98, except for the Italian case), the variation range of the ratio (still average on the same period) of public debt to GDP is definitely larger (between 0.2 and 0.9). This evidence suggests that if a positive relation between returns and level of debt exists, the effect of the fiscal variable on the risk premium is relatively small.

²¹ The countries included in the study are: Australia, Belgium, Canada, Denmark, France, Ireland, Italy, Japan, Netherlands, Spain, UK and US.

The authors use a panel of data to run regressions of the public to private return ratio on the following fiscal variables:

- a) debt to GDP ratio;
- b) annual growth rate of the debt to GDP ratio;
- c) short term debt to total debt ratio;

among the exogenous variables, it is also considered

- d) the annual growth rate of the industrial production index, to take into account the phase of the economic cycle.

Regressions are also conducted by using the linear spread between public and private return in place of their ratio.

The analysis seems to confirm the theoretical results: the fiscal variables are mostly significant and with the correct sign, with the exception of the maturity structure that shows a negligible effect on returns²².

To take into account the potential diversities between countries with different fiscal features, the analysis is repeated by dividing the countries in three groups: a) countries with sustainable debt pattern (Australia, Canada, France, Japan, UK and US); b) countries with very high levels of debt (Belgium, Ireland and Italy); c) countries with high debt growth rate (Denmark, Netherlands and Spain). The assumption underlying this separation of groups is that default risk emerges only when fiscal variables are above some critical level and when they follow a time path which is considered not sustainable, in the long run, by

²² Moreover the coefficient of the variable shows a negative sign, which is not as expected from the theory.

financial operators. Such hypothesis is, in fact, consistent with the results: debt to GDP ratio and its growth rate are significant only for countries in the groups b) and c) respectively. While the two fiscal variables do not seem to affect returns in the a) group.

Summing up, the paper by Alesina *et al.* provides empirical support to the following four assumptions:

- a) looking at the OECD countries, default on public debt is a very low probability event: default risk premia, even when significant, appear of very small entity;
- b) in particular, significant premia emerge for very high levels of public indebtedness;
- c) default premia are positively correlated with the amount of outstanding debt²³;

The main critics to the paper concern the choice to measure default risk (Obsfeld, 1992; Rebelo, 1992). The authors themselves, in fact, recognise that their proxies are affected by other factors. For example, the returns ratio is sensitive to variations in expected inflation, and the differential is affected by variation in the tax rate on the two kinds of debt. The authors, therefore, conclude their work suggesting some alternative measures to catch the default risk, but all of them present shortcuts:

²³ The authors also mention a negative correlation with average maturity of bonds, but give a caveat that the evidence is ambiguous.

- a) Moody's rating of debt bonds. The rating is, however, updated infrequently and is based also on factors which may have little to do with debt sustainability.
- b) Yield spreads on bonds issued by different countries in the same currency (e.g. Eurodollars or ECU bonds). The market of these bonds is not very developed and the liquidity risk may be substantial.

A different method to measure default risk is proposed by Favero, Giavazzi and Spaventa (1997). The measure adopted in this paper is given by the differential between yields on public bonds and fix rates on swap contracts (in the same currency and with same maturity). The advantage of this procedure is that swap contracts are fairly liquid, involve all the relevant maturities and have a daily quotation on the market.

The focus of the paper is to investigate on the causes of the spreads between the public debt returns in high yield countries (Italy, Spain and Sweden) and in Germany. The authors note that the differentials with respect to German rates do not show a stable relation with the respective inflation differentials. This may be due to the presence of a default risk as a component of the spreads.

The paper presents first a procedure to decompose the interest rate differentials into two main factors: the exchange rate risk and the default risk. The procedure shows that decomposition must be conducted on continuously

compounded interest rates. The default risk is then obtained as a residual from the following difference²⁴:

$$DR_{j, t, T} = SP_{j, t, T} - ER_{j, t, T}$$

where $DR_{j, t, T}$ is the default risk premium at time t , on bonds issued in country j , with T periods maturity;

$SP_{j, t, T}$ is the return differential, at time t , on bonds issued in country j , with respect to German rates and with T periods maturity;

$ER_{j, t, T}$ is the exchange rate risk, at time t , given by the differential between fix rate swap contracts with same maturity (T periods) and denominated in currency of country j and in DM.

The time series of default risk premium calculated as above and reported in the paper indicates that it is mostly positive for Italy along the time span of the analysis (January 1992-December 1995). It is often positive, but of smaller size for Spain and almost always negative for Sweden.

Taking a VAR approach, the authors investigate on the statistical features of the series. Using daily data (excluding data for 1992 to avoid turbulence in the exchange rates due to the ERM crisis) for Italy and Spain, the authors find that yield spreads and exchange risk factors are non-stationary series and they are linked by a cointegration relationship for both countries. In particular, the cointegrating vector is identified as the $[1, -1]$ vector for Italy as well as for Spain, suggesting the stationary nature of the default risk premium.

²⁴ In their paper the authors consider a third factor due to different bonds taxation. The problem seems to be relevant for Italy and solved through a specific correction procedure.

Finally, the econometric analysis focuses on the dynamic response of the spreads, in the short run, to country specific and international shocks. The dynamic analysis provides an interesting explanation of the different pattern of the default risk in the two countries. In fact, country specific shocks vanish quite rapidly in the Spanish case, while they have a persistent effect on the Italian default risk.

The two empirical works discussed in this section seem to agree on the geographical distribution of the default risk:

- a) default risk premium appears positive and significant for Italy;
- b) it is significant but smaller for Spain;
- c) it has a longer memory in the Italian case with respect the Spanish one.

Alesina *et al.* (1992) suggest that a default risk is also incorporated in the interest rates of the Netherlands and Denmark, and in lower size in the Irish and Belgium rates.

Chapter 2

Default Risk, Public Debt and EMU

2.1 Introduction

In the previous chapter we have discussed some of the economic literature on sovereign debt default. Now we provide our contribution, presenting a model where default on public debt is considered.

As we have discussed in the previous chapter, while default and inflation as tools of debt reductions have been separately investigated (see Calvo, 1988; Aghion and Bolton, 1990; Alesina, Prati and Tabellini, 1990; Giavazzi and Pagano, 1990), a joint consideration of the two instruments has not received much attention. In this respect, an innovative work is the recent study of Drudi and Giordano (1997) which analyses the effects of these two political options on the optimal debt maturity. The

study of the interaction of the two associated risks can shed light on the effects of EMU on high debt sustainability. EMU should imply giving up inflation as a tool to service public debt and this may lead to inflation risk turning into a higher explicit default risk. In this case, real interest rates will not converge after the monetary union. On the other hand, EMU membership may increase government credibility: will the reduced flexibility in responding to bad shocks be more than offset by the beneficial credibility effect?

In the present and the following chapters we will show that a positive answer is likely. In fact, while the ability to inflate provides an easy tool to reduce real debt, it also leads (when anticipated) to high costs in terms of interest rate, which may even vanish the former effect. This time-inconsistency type of mechanism may induce an inflationary spiral making default more attractive. Somehow paradoxically, the loss of monetary sovereignty may result, through lower interest rates and increased credibility, in a 'lighter' debt service and lower default risk.

In this chapter, we present a simple model which takes into consideration the government's decision process where the choice to give up the regular debt service is conditional on the realisation of an exogenous shock to the international real interest rate. In our framework, therefore, default is not a necessary consequence of a liquidity constraint, but the result of an explicit optimising process of the government.

The approach taken draws from the currency crisis literature, where explicitly modelling government's choices is increasingly popular (see de Kock and Grilli, 1993; Obstfeld, 1994; Ozkan and Sutherland, 1994 and 1995). These kinds of models allow us to study the optimal mix of commitment and flexibility when an economy is subject to stochastic shocks.

The model describes a three-period economy with two kinds of agents: the government and a large number of risk neutral investors. The government optimises its policy after observing both the investors' action and the realisation of a stochastic shock on international interest rate; whilst investors set the interest rate on public bonds incorporating their expectations on next period economic policy.

The main feature of the model is the 'state dependent' nature of the optimal policy chosen by the government, which can decide to exercise the option to defaulting on part of its obligations in the bad states of the world. Private sector's expectations play a decisive role: taking into account the default probability they exacerbate the government's problem; moreover they affect the optimal economic policy and can generate multiple equilibria.

The model is then extended to include inflation as well as default as a tool of debt reduction. The comparison of the two cases allows us to evaluate how EMU will affect default probability.

2.2 A Simple Model with Default Only

2.2.1 The model

Consider a three period economy, $t=0, 1, 2$, with two kinds of agents: investors and the government. Investors are a large number of risk neutral agents and represent the private sector of the economy, they are assumed to differ for their initial endowments, but have access to the same information set.

The game starts in period 0, when the government issues a stock of public debt equal to b_0 . Besides public bonds, there is a unique alternative available for saving: a foreign riskless asset, which yields a gross interest rate R_i^{*1} in each period, assumed to be stochastic. Note that the foreign asset is riskless in the sense that R_i^* is always known at the time of purchase. All assets, domestic and foreign, have the same maturity equal to one period. This assumption will be relaxed in the next chapter, where short and long term bonds will be considered.

Investors' action, in each period, consists in setting the gross interest rate R_i at which they are willing to buy domestic bonds, accordingly to their expectations on next period economic policy and to the realisation of the current shock to the international interest rate. The government observes the interest rate and chooses the economic policy to finance an exogenous flow of public spending and to service the outstanding debt. The government's policy is given by a

combination of distortionary income taxes and new borrowing. However, for adverse realisations of shock, the government can exercise the option of defaulting on parts of its obligations. When this happens, it will no longer be able to borrow from the private sector. It is assumed that default is not allowed in the final period²; moreover, no debt can be issued in that period, as the government will not last long enough to pay it back.

In the strategic game, the private sector acts in periods 0 and 1; whilst the government moves in periods 1 and 2³. Therefore, the government enjoys an informative advantage with respect to the private sector.

The exact timing of the economy is as follows:

Period 0: In the initial period, the stochastic shock to the international interest rate is realised and it is observable by all the agents. The government issues a stock of public bonds b_0 (exogenously given) and investors set the domestic return accordingly to the non-arbitrage condition. Knowing government's preferences, in fact, they are able to calculate the expected default for next period and ask for a premium to compensate the risk.

Period 1: At the beginning of the period the realisation of the foreign interest rate is observed and the government decides whether to serve the whole debt coming to maturity or to opt for a partial default. The economic policy is then

¹ R_i^* is equal to $(1+r_i^*)$ where r_i^* is the international real interest rate in period i .

² Without this assumption the government will always optimise by defaulting in period 2. An alternative way is to adopt an infinite horizon model, but this would involve a more complex analytical framework.

In the next chapter, an extension where default is allowed in the final period will be presented showing that the basic results of the model do not rely on that assumption.

chosen consistently to the default decision. Investors observe government's action and international interest rate: if default has occurred they turn to the foreign financial market, otherwise they invest in public bonds at the market return. The latter is equal to the international interest rate, as the new debt is now riskless.

Period 2: In the final period, the government raises income taxes to finance public spending and meet debt obligations, if there are any (i.e., if the government did not default in period 1 and took out fresh loans), including interest.

This order of events is shown in Figure 2.1.

We now turn to an analytical description of the economy discussed above. Government's policy is set to minimise taxes in both periods and, possibly, the size of default. The objective function in periods 1 and 2 is given respectively by:

$$L_1 = \frac{\alpha}{2} \theta^2 + t_1 \quad (2.1)$$

$$L_2 = t_2 \quad (2.2)$$

The parameter $\alpha (>0)$ stands for unit cost of defaulted debt. As we will see later, default is penalised by precluded access to private capital market. However, as mentioned in the introduction, other costs may be considered, which are likely related to the size of default. In our context, α may be considered as a measure of the income dispersion among private agents and thus represent the redistribution

³ In period 0, the government issues b_0 , this is assumed to be exogenous and has no strategic role in the game.

costs of default. θ is the fraction of debt the government is going to default on, $0 \leq \theta \leq 1$, and t_i is the policy determined flow of income tax revenue in period i ⁴.

The objective function is assumed to be quadratic in the default size and linear in taxes⁵. The implication is that the policymaker dislikes tax distortions but is not interested in tax smoothing. This assumption reduces the scope for government's borrowing, but makes the model more handleable. More importantly, it allows the model to be analytically tractable when inflation is introduced. In principle, we expect that the cost of all default, taxes and inflation does not increase linearly. However, having all quadratic terms would increase the number of equilibria, making the interpretation of the results less clear. The choice to have default and, in next section, inflation costs both quadratic was suggested by our interest in considering the two instruments as closer substitutes, than they are with respect to taxation. Linear cost of taxes can be interpreted as an approximation of its (actual) quadratic form. Our results, however, should not be qualitatively affected as long as default cost does not enter the objective function in linear terms⁶. In fact, a linear cost implies that the corresponding policy instrument is used as a residual.

⁴ Note that we are here normalising total income to 1.

⁵ Linear-quadratic objective functions are not new in the macroeconomics literature (see Barro and Gordon, 1983b; and Backus and Driffill, 1985).

⁶ Linear cost of default is considered by Drudi and Giordano (1997). This makes their results to depart from ours.

Given a discount factor equal to δ , we can combine (2.1) and (2.2) to get the following ‘*intertemporal loss function*’:

$$L = \frac{\alpha}{2} \theta^2 + t_1 + \delta t_2 \quad (2.3)$$

The government budget constraints in the two periods are given, respectively, by:

$$b_0 R_0 (1 - \theta) + g_1 = t_1 + b_1 \quad (2.4)$$

$$b_1 R_1 + g_2 = t_2 \quad (2.5)$$

where

$$b_1 = \begin{cases} \geq 0 & \text{if } \theta = 0 \\ 0 & \text{if } \theta > 0 \end{cases} \quad (2.6)$$

is the new debt issued in period 1⁷, and g_i is the flow of government spending in period i (exogenously given). Equation (2.6) imposes a fixed cost to the default option, excluding the government from private capital market if obligations are not regularly met. The incentive to issuing debt is in that it allows to finance current spending while postponing taxes. Of course, the government will be willing to do so, as long as the cost of borrowing does not exceed $1/\delta$.

As mentioned above, b_0 is the stock of public bonds maturing in period 1, on which the interest rate R_0 is due. b_0 are here interpreted as the real value of coupon bonds issued by the government⁸.

⁷ Equation (2.6) assumes that the government does not lend to the private sector. “One motivation for this assumption is that private agents are anonymous so that debt claims against them are unenforceable” (Chari and Kehoe, 1993; p. 178).

⁸ However, our model also works when interpreting b_0 as the real value of no coupon bonds. In this case, it is useful to note that the value of bonds declines when the interest rate increases. As a result, the government has to issue more bonds to fulfil its budget constraint (for example to finance the exogenous flow of expenditure in period 0, not shown here), but the real value of the

Domestic interest rate is set by the private sector accordingly to the non-arbitrage condition⁹. Taking into account the risk of default on b_0 , the nominal gross interest rate will be, in the two periods, respectively:

$$R_0 = \frac{R_0^*}{1 - E_0[\theta]} \quad (2.7)$$

$$R_1 = R_1^* \quad (2.8)$$

where $E_0[\cdot]$ is the expectation operator conditional on information set available at time 0. Investors are rational and formulate their expectations knowing the government's objective function (2.3). Bonds issued in period 1 are perfectly safe and their return is equal to that on the foreign asset. The density function of the random shock is assumed to be uniform on the interval $[\nu, \mu]$, with $\nu \geq 0$ and $\mu > 0$. Without loss of generality, in what follows we assume that the realisation of the shock in period 0 is equal to 1 (i.e. $R_0^* = 1$).

When a large adverse shock hits the international rate of interest in period 1, the high cost of new borrowing may induce the government to opt for default on part of its obligations.

whole debt issued will still equal to the same amount b_0 . This does not affect the final default size as it is calculated on the whole amount of payments due in period 1, $b_0 R_0$.

⁹ No-arbitrage condition applies as agents are risk neutral; nevertheless, the hypothesis, implicit in the (2.6), that investors do not want to buy a fare lottery once default has occurred, is somehow contradictory with risk-neutrality. One possible interpretation is that default occurring changes agents' attitude towards risk. This problem is common to most of default literature and will not be investigate further here. However we like to thank Driffill for making this point clear.

In period 1, the policymaker observes the realisation of the shock on the interest rate and decides whether to issue new borrowing or default on part of its debt. The optimal policy is chosen by minimising the intertemporal loss function (subject to the budget constraints, the realisation of the shock R_1^* , the level of outstanding debt and the domestic interest rate R_0) with respect to θ and b_1 . Solving the minimisation of (2.3) under the alternative policies of default and no-default gives, respectively, the following optimal policy rules:

(i) *default case:*

$$\theta = \frac{b_0 R_0}{\alpha} \quad (2.9)$$

$$b_1 = 0 \quad (2.10)$$

$$t_1 = b_0 R_0 \frac{\alpha - b_0 R_0}{\alpha} + g_1 \quad (2.11)$$

$$t_2 = g_2 \quad (2.12)$$

(ii) *no-default case:*

$$\theta = 0 \quad (2.13)$$

$$b_1 = \begin{cases} b_0 R_0 + g_1 & \text{if } R_1^* \leq 1/\delta \\ 0 & \text{if } R_1^* > 1/\delta \end{cases} \quad (2.14)$$

$$t_1 = \begin{cases} 0 & \text{if } R_1^* \leq 1/\delta \\ b_0 R_0 + g_1 & \text{if } R_1^* > 1/\delta \end{cases} \quad (2.15)$$

$$t_2 = \begin{cases} (b_0 R_0 + g_1) R_1^* + g_2 & \text{if } R_1^* \leq 1/\delta \\ g_2 & \text{if } R_1^* > 1/\delta \end{cases} \quad (2.16)$$

From equation (2.9), default size appears to be an increasing function of the period 0 interest rate. This result is consistent with findings in Calvo (1988).

Equations (2.14)-(2.16) indicate that under the no-default policy optimal rules may change in different shock intervals. As mentioned above, when the shock is higher than the inverse of government's discount factor, the cost of borrowing (as measured in terms of cost of raising taxes to pay back debt in period 2) exceeds the benefit from current loans. In this case, the government refrains from issuing new bonds in period 1, relying on fiscal revenue only (even though investors would be happy to lend). On the other hand, whenever the shock is lower than $1/\delta$, the government benefits from borrowing to postpone payment for today's spending. When this is the case, the tax burden is born totally in the last period¹⁰.

2.2.2 *The government's decision rule*

Clearly, the government will decide to repay fully its debt whenever:

$$L^{nd} \leq L^d \tag{2.17}$$

where the superscripts d and nd state the value of the loss function under the two different policies.

For what follows, we introduce the simplifying assumption that $g_1=g_2=0$. Figure 2.2 compares diagrammatically the two loss functions, at different values of the interest rate shock. L^d is a constant equal to $b_0 R_0 \frac{2\alpha - b_0 R_0}{2\alpha}$ ($< b_0 R_0$) for whatever value of the shock, while L^{nd} describes a line with a kink at $R_1^* = 1/\delta$: for R_1^* between ν and $1/\delta$ it grows linearly and thereafter it stays constant at $b_0 R_0$.

The plot shows that when the shock is higher than $1/\delta$, default is the optimal policy. This is an intuitive result following from what discussed above: when $R_1^* > 1/\delta$ the government does not want to borrow and this makes the fix cost of default ineffective and that option attractive. However default may be optimum even for lower values of the shock. At $R_1^* = \bar{R}$, the government is indifferent between the two alternatives. In fact, \bar{R} gives a shock upperbound above which the government has no incentive to repay fully its debt¹¹. It is easy to show that as R_0 increases the threshold \bar{R} , determining the optimal policy, declines.

In analytical terms, no-default will be the best government's strategy whenever:

$$R_1^* \leq \bar{R}(R_0) \equiv \frac{2\alpha - b_0 R_0}{2\alpha\delta} \quad (2.18)$$

¹⁰ This 'bang-bang' nature of the solution, which makes the government switching from borrowing to taxation in a discontinuous way at $R_1^* = 1/\delta$, is due to the linearity of the taxation term in the loss function.

¹¹ From what said above it follows that this threshold is always lower than $1/\delta$.

The above equation describes a linear function of the initial interest rate, with negative slope. Hereafter we will call the function (2.18) the *Government Default Boundary (GDB)*.

The negative slope of the *GDB* implies that the higher is the period 0 interest rate demanded by the private sector for holding public bonds, the higher is the debt burden and the fiscal pressure, thus the higher is the incentive for the government to implement the default option. In other words, for higher R_0 , it takes a smaller value of the shock (R_t^*) to make the government decide for a partial repudiation.

2.2.3 *The private sector's decision rule*

Now we turn to specify how investors determine debt interest rate in period 0, such that they are indifferent between purchasing public bonds or save in foreign assets. This will lead to an endogenised probability of default, and will clarify the interaction between the government's and private sector's actions.

At time 0, expected default is given by default size times its probability, i.e.:

$$E_0[\theta] = p \theta$$

Where size θ is perfectly anticipated by the agents as given by (2.9), while probability of default occurring is derived by the density distribution of the stochastic shock:

$$p = \text{prob}\left(R_1^* > \bar{R}^e\right) = \begin{cases} \frac{\mu - \bar{R}^e}{\mu - \nu} & \text{if } \nu \leq \bar{R}^e \leq \mu \\ 0 & \text{otherwise} \end{cases} \quad (2.19)$$

where \bar{R}^e is the expected value of the default trigger point set by the government¹². Replacing equations (2.19) and (2.9) into the non-arbitrage condition (2.7), we get¹³:

$$R_0 = \begin{cases} \frac{\alpha(\mu - \nu) - \sqrt{\alpha(\mu - \nu)\left(\alpha(\mu - \nu) - 4b_0(\mu - \bar{R}^e)\right)}}{2b_0(\mu - \bar{R}^e)} & \text{if } \nu \leq \bar{R}^e < \mu \\ 1 & \text{if } \bar{R}^e = \mu \end{cases} \quad (2.20)$$

For $\alpha \geq 4b_0$, equation (2.20) defines R_0 for all possible values of $\bar{R}^e \in [\nu, \mu]$.

However, if $\alpha < 4b_0$ interest rate on initial stock of debt is not defined for ‘high’ default probabilities (how ‘high’ depending on the actual value of α). The intuition is that for very low unit cost of default, private investors are not willing to purchase public bonds at any finite interest rate. At a given interest rate and with a low value of α , (2.9) and (2.18) may induce p and θ to exceed 1 and no solution to the no-arbitrage condition exists. Imposing them the unit upperbound makes R_0 jumps to infinity.

¹² Note that since the expression determining \bar{R} , depends only on R_0 and the structural parameters of the model, and it is common knowledge, when setting the interest rate, the agents can perfectly forecast default probability. However, for convenience we refer to the expression (2.19) where probability is an inverse function of the expected threshold. When calculation equilibria, the true value of \bar{R} is replaced and results are not affected.

¹³ Here we consider only the negative root from the solution of the equation (2.7), as the positive one does not satisfies the continuity condition when default probability goes to zero (i.e. when $\bar{R}^e \rightarrow \nu$).

From (2.20) the interest rate appears to be a decreasing function of the expected threshold chosen by the government:

$$R_0 = \varphi(\bar{R}^e) \quad (2.21)$$

with $\varphi' < 0$ and $\varphi'' > 0$.

We refer to the condition (2.21) as the *Debt Interest Rate Locus (DIRL)*, it gives the minimum interest rate at which investors are willing to hold domestic bonds for any possible value of \bar{R}^e (inside the shock range).

The intuition for the negative slope of this curve is straightforward. If, given the initial conditions, investors expect a lower value of the trigger point, they will ask for a higher risk premium above the international interest rate to hold government assets.

2.2.4 Determination of equilibria

An equilibrium of the model is given by a couple (R_0^E, \bar{R}^E) (where the superscript E indicates equilibrium), which satisfies the following properties:

(a) either it solves both equations (2.18) and (2.20) with $\bar{R}^e = \bar{R}$ and

$$\bar{R}^E \in [\nu, \mu],$$

or $\bar{R}^E = \nu$, and $\bar{R}[\varphi(\nu)] < \nu$;

$$(b) \theta(R_0^E) \in [0, 1].$$

Although the algebraic framework of the model does not permit simple analytical expressions for those levels, we are still able to draw out some features of the equilibria.

It is worth noticing that a multiplicity of equilibria can emerge in the model. In particular, the number of equilibria depends on the parameter α and δ . A necessary but not sufficient condition to observe more than one equilibrium is that:

$$\alpha < \frac{(7\mu + 2\nu)(\mu - \nu) + (3\mu - 2\nu)\sqrt{(9\mu - \nu)(\mu - \nu)}}{4\mu(\mu - \nu)} b_0 \equiv \alpha^* \quad (2.22)$$

Numerical investigation suggests that the region defined in the (2.22) is not empty for any value of $\nu \geq 0$ and $\mu > \nu$. When α satisfies the above condition (2.22), the parameter δ determines six regions where both the number and the characteristics of the equilibria change. When $\alpha \geq \alpha^*$ instead, the equilibrium is unique. δ still discriminates the regions which characterise the equilibrium, in this latter case, however, only three of them seem to be relevant: the region where default is a certain event; the region where it occurs with a positive, but lower than 1, probability; the region where it is an impossible event. Figure 2.3 illustrates the two possible cases: (a) $4b_0 \leq \alpha < \alpha^*$; (b) $\alpha \geq \alpha^*$; and how the corresponding equilibria change for different values of δ . Table 2.1 summarises those results.

Equilibria E_1 and E_3 (Figure 2.3) are characterised by no uncertainty and represent the case of no-default and sure default, respectively. The corresponding optimal default sizes are given by:

$$\theta_1^E = 0$$

$$\theta_3^E = \frac{\alpha - \sqrt{\alpha(\alpha - 4b_0)}}{2\alpha} \leq \frac{1}{2}$$

(where the number as subscript of the variable indicates the corresponding kind of equilibrium). The equilibrium values of the initial gross interest rate and those of the trigger point appear in the table 1.

Equilibria like E_2 correspond to the uncertainty equilibria where the probability of default is inversely related to the shock threshold (as implied by equation (2.19)). The optimal default size is given accordingly to the (2.9). As mentioned above, reduced forms for the equilibrium values are not provided, however the graphical analysis allow some attempts of comparative static.

Figure 2.3 shows that probability of default declines for lower δ . When the government is more impatient, it cares less about the debt repayment in period 2. For a given shock it is more willing to borrow rather than defaulting on its obligations. This result may appear counterintuitive: usually an impatient government is thought to be less worried about future access to private loans. However, in our model no debt can be issued in the final period and the cost of default is fully born in the current period.

A change in the parameter α affects the equilibria in a more standard way: an increase in α produces an upward shift of the *GDB* around the point $(R_0=0, \bar{R}=1/\delta)$, whilst the curve *DIRL* moves leftward turning around the point $(R_0=1, \bar{R}=\mu)$. The net effect is lowering both the default probability and the period 0 interest rate. However, the effect is zero when $\delta \leq \delta'$, that is when default is never optimal, i.e. $p^E=0$ and $R_0^E=1$.

A higher initial debt b_0 has the same effect as a lower α , that is it implies greater default probability and higher interest rate. Again, this is not surprising as an high indebtedness increases benefits from defaulting.

2.3 Inflation and Default Risk

In this section we turn to investigate the debt servicing problem of a policymaker facing four possibilities: a) roll over its debt entirely; b) inflate away part of its real value; c) default on part of public bonds; d) use a combination of inflation and default.

2.3.1 The model

The economy is the same as the one described in the previous section, where now we introduce inflation, besides default, as an option of debt reduction. Timing is unchanged and inflation is set as a new tool of economic policy: its level is decided consistently to default decision in period 1, and at the same time as the tax level in period 2.

In this setting, we assume that the two ‘debt-cheating’ tools will enter the government’s objective function with different weights. Hence, the intertemporal loss function can be rewritten as:

$$L = \frac{\alpha}{2} \theta^2 + \frac{\gamma}{2} z_1^2 + t_1 + \delta L_2 \quad (2.23)$$

where
$$L_2 = \frac{\gamma}{2} z_2^2 + t_2 \quad (2.24)$$

is last period objective function.

The government’s budget constraints in the two periods are modified, respectively, as:

$$b_0 R_0 (1 - \theta)(1 - z_1) + g_1 = t_1 + b_1 \quad (2.25)$$

$$b_1 R_1 (1 - z_2) + g_2 = t_2 \quad (2.26)$$

where b_i is still given by the (2.6) and $z_i = \frac{\pi_i}{1 + \pi_i}$ is the inflation index in period i

(π_i is the inflation rate in the same period). The rest of the terminology being

known from the previous section. In this version of the model, the current debt is still in real terms, i.e. $b_i = \frac{B_i}{P_i}$ where B_i is the nominal value of debt issued at time i and P_i is the price level in the same period.

In setting the interest rate on public bonds, the private sector now takes into account the expected inflation as well as default. Following the arbitrage condition the gross interest rates in the two periods will be:

$$R_0 = \frac{R_0^*}{E_0[(1 - \theta)(1 - z_1)]} \quad (2.27)$$

$$R_1 = \frac{R_1^*}{1 - E_1[z_2]} \quad (2.28)$$

where $E_0[\cdot]$ is defined as in the previous section. R_i^* is still assumed to be stochastic with the same uniform distribution on the interval $[\nu, \mu]$, and again we suppose that $R_0^* = 1$. Unlike the previous case, the interest rate in period 1 does no longer equate the international interest rate as now the agents demand for an inflation premium. In period 0, this premium is added to the default premium. High realisations of the shock can still trigger the default option at the cost of the exclusion from the private financial market.

The optimisation problem needs now to be solved backwards, considering first how the government will act in period 2¹⁴.

¹⁴ Because of all choice variables were determined in period 1, in the previous section, we could solve the minimisation of the intertemporal loss function in one step.

Assuming that the government did not default in period 1, in the final period it observes the values for b_1 and R_1 and chooses the inflation rate to minimise equation (2.24) subject to (2.26). Under the assumption that $g_1=g_2=0$, optimal inflation is a constant fraction of current debt burden, where the size of the fraction is inversely related to the inflation cost:

$$z_2 = \frac{b_1 R_1}{\gamma} \quad (2.29)$$

Note that if the government did trigger the default option, then $b_1=0$ and there is no incentive to use inflation in period 2. However, any time $b_1>0$, the government will inflate away debt obligations. This is taken into account by private investors as R_1 setting depends on their expectations on z_2 (see equation (2.28)). Assuming rational expectations, we can use (2.28) and (2.29) for solving simultaneously for inflation and interest rate to get¹⁵:

$$z_2 = \frac{\gamma - \sqrt{\gamma(\gamma - 4b_1 R_1^*)}}{2\gamma} \quad (2.30)$$

$$R_1 = \frac{\gamma - \sqrt{\gamma(\gamma - 4b_1 R_1^*)}}{2b_1} \quad (2.31)$$

Note that inflation in the last period is fully anticipated by the private sector and incorporated in the interest rate. Thus it has no effect on reducing the debt burden; nevertheless the government cannot credibly commit itself to a zero inflation policy (time inconsistency problem).

¹⁵ Note that from (2.29) we want $z_2=0$ for $b_1=0$. To satisfy this condition only the negative root, i.e. equation (2.30) is considered. Equation (2.31) follows directly from (2.30).

The above equations (2.30) and (2.31) allow us to express L_2 as a function of b_1 and solve the government's problem in period 1. The minimisation of the intertemporal loss function (2.23) under the alternative options of default and no-default gives the optimal rules to set the endogenous variables¹⁶:

(i) *default case*:

$$\theta = b_0 R_0 \frac{\gamma - b_0 R_0}{\alpha \gamma - b_0^2 R_0^2} \quad (2.32)$$

$$z_1^d = b_0 R_0 \frac{\alpha - b_0 R_0}{\alpha \gamma - b_0^2 R_0^2} \quad (2.33)$$

$$t_1 = \alpha \gamma b_0 R_0 \frac{(\alpha - b_0 R_0)(\gamma - b_0 R_0)}{\alpha \gamma - b_0^2 R_0^2} \quad (2.34)$$

$$b_1 = 0 \quad (2.35)$$

$$t_2 = 0 \quad (2.36)$$

(ii) *no-default case*:

$$\theta = 0 \quad (2.37)$$

$$z_1^{nd} = \frac{b_0 R_0}{\gamma} \quad (2.38)$$

$$t_1 = \begin{cases} 0 & \text{if } \nu \leq R_1^* < \tilde{x} \\ \frac{b_0 R_0 (\gamma - b_0 R_0)}{\gamma} - \frac{\gamma (1 - \delta R_1^*)}{R_1^* (2 - \delta R_1^*)^2} & \text{if } \tilde{x} \leq R_1^* \leq \mu \end{cases} \quad (2.39)$$

¹⁶ As in the previous section, for shock realisation above $1/\delta$, the government does not want to borrow. To simplify the exposition, in what follows we assume that this never happens, i. e. $\mu \leq 1/\delta$.

$$b_1 = \begin{cases} \frac{b_0 R_0 (\gamma - b_0 R_0)}{\gamma} & \text{if } \nu \leq R_1^* < \tilde{x} \\ \frac{\gamma(1 - \delta R_1^*)}{R_1^* (2 - \delta R_1^*)^2} & \text{if } \tilde{x} \leq R_1^* \leq \mu \end{cases} \quad (2.40)$$

$$t_2 = \begin{cases} \frac{b_0 R_0 (\gamma - b_0 R_0) R_1^*}{\gamma} & \text{if } \nu \leq R_1^* < \tilde{x} \\ \frac{\gamma(1 - \delta R_1^*)}{(2 - \delta R_1^*)^2} & \text{if } \tilde{x} \leq R_1^* \leq \mu \end{cases} \quad (2.41)$$

Equation (2.39) shows a zero lowerbound for the level of taxes. \tilde{x} in equations (2.39) to (2.41) indicates the value of the shock below which optimal taxation would be negative, however for such values we impose the constraint $t_1=0$ ¹⁷. It is worth noticing that the value of the shock at which optimal taxes are zero is not constant, and is a function of the initial interest rate R_0 ¹⁸.

2.3.2 The government's decision rule

As in the previous case, government's optimal policy is taken comparing the objective function under the two options, but the policy choice is now between

¹⁷ Non-negative taxes are assumed in the previous section; here the assumption is not necessary, but we retain it to have comparable results. In the next chapter, negative taxes will be allowed, showing that relaxing this constraint does not affect the main results of the model.

¹⁸ Moreover, numerical investigation indicates that for most values of the parameters of the model the interval $[\nu, \tilde{x}]$ is empty and optimal taxes are always positive.

inflate away part of the debt burden or to use a combination of inflation and default¹⁹. The former will be government's best action if:

$$L^{nd} \leq L^d \quad (2.42)$$

The two functions are shown in Figure 2.4 (which corresponds to Figure 2.2 after introducing inflation). L^d is still constant to shock variation, in fact when default occurs $b_I=0$ and the economy is insulated from international interest rate movements. The L^{nd} is instead an increasing function of the shock.

Comparing the minimum loss under the alternative policies, the government will avoid default any time that²⁰:

$$R_1^* \leq \bar{R}(R_0) \equiv \begin{cases} \frac{2HK - H^2 - \delta\gamma^2 K + \gamma\sqrt{\delta K(\delta\gamma^2 K - 2HK + H^2)}}{H\delta K} & \text{if } R_0 \text{ s.t. } R_1^* < \bar{x} \\ \frac{H^2 + \delta\gamma^2 K - H\sqrt{H^2 + \delta\gamma^2 K}}{\delta(H^2 + \delta\gamma^2 K)} & \text{if } R_0 \text{ s.t. } R_1^* \geq \bar{x} \end{cases} \quad (2.43)$$

where $H \equiv b_0 R_0 (\gamma - b_0 R_0)$ and $K \equiv (\alpha\gamma - b_0^2 R_0^2)$.

We can look at the (2.43) as 2-step rule for the government to decide about its best policy: the policymaker first observes the shock and compares it with \bar{x} (which, knowing R_0 , is just a number), then he/she compares it with the relevant section of equation (2.43) and decides whether or not to default on debt.

Equation (2.43) represents the *GDB* in the case where inflation is allowed. The function gives an upperbound for the range of the shocks that do not trigger

¹⁹ Inflation rate will usually be different in the two regimes.

default. The shape of this function is ambiguous, but simulations suggest it describe a decreasing function of period 0 interest rate²¹. As discussed in the previous section, this negative relationship is consistent with our intuition: a higher R_0 makes the debt service more expensive and partial default more attractive.

2.3.3 The private sector's decision rule

Finally we need to describe how investors determinate the interest rate in the initial period. This requires specifying how they form expectations on next period economic policy. It departs from the previous section as now expected inflation is involved as well. Assuming rational expectations, the anticipated value of a unit of debt purchased in period 0 and with maturity in period 1 will be (at the date of maturity):

$$E_0[(1-\theta)(1-z_1)] = (1-p)(1-z_1^{nd}) + p(1-\theta)(1-z_1^d) \quad (2.44)$$

where p is still given by (2.19). After substituting (2.32) (2.33) (2.38) and (2.19) into (2.44), and using the latter to solve (2.27) with respect to R_0 , would give the *DIRL* function for the inflation case:

$$R_0 = \psi(\bar{R}^e)$$

²⁰ It is worth to remind that \bar{x} is function of R_0 .

²¹ At least in a relevant range of R_0 .

Even if no analytical solution is obtainable for this function, we can still calculate its inverse function:

$$\bar{R}^e \equiv \psi^{-1}(R_0) = \frac{(\gamma b_0 K^2 - HW)\mu - K^2(\gamma b_0 - H)\nu}{H(K^2 - W)} \quad (2.45)$$

where H and K are defined above and $W \equiv \alpha\gamma^2(\alpha - b_0 R_0)$

This function gives the value of the default trigger point which is expected by private investors when they set a given interest rate.

Like in the benchmark model, an equilibrium is defined as a couple (R_0^E, \bar{R}^E) which satisfies equation (2.43) and (2.45) simultaneously with $\bar{R}^e = \bar{R}$ and $\bar{R}^E \in [\nu, \mu]$, and such that the corresponding θ^E, z_1^E and z_2^E all $\in [0, 1]$. Corner equilibria are given by:

$$\bar{R}^E = \nu, \text{ and } \bar{R}[\psi(\nu)] < \nu;$$

$$\bar{R}^E = \mu, \text{ and } \bar{R}[\psi(\mu)] > \mu;$$

when associated to values of θ^E, z_1^E and z_2^E still lying inside the close interval $[0, 1]$.

Again, we are unable to find an analytical expression for equilibrium variables. In the next section we present the results obtained from numerical solution of the model in both cases examined respectively in the previous and present section, and we will discuss the implications of giving up inflation as a tool to reduce the debt burden.

However, it seems useful to give a simple intuition of the effects of introducing inflation on the GDB and DIRL functions in terms of Figure 2.3²². For simplicity, we consider only the case when a unique equilibrium emerges. For a given interest rate demanded by private investors on period 0 bonds, the government can now finance its debt service partly inflating away its value. The intuition is then that the policymaker will trigger default less frequently, and GDB curve moves up. On the other hand, due to the inflation risk incorporated in the interest rate (equation 2.27), for each value of the expected default threshold, private agents will ask for a higher interest rate, such that DIRL is expected shifting rightwards. Whether the new outcome is characterised by higher or lower default probability (with respect to the no inflation case) depends on the relative shift of the two curves. As we are going to show in the next section, numerical analysis suggests that the rightward shift of DIRL curve is large enough to induce the government to opt for a partial default with higher probability.

²² Note that the two functions will now be shaped differently, in particular the GDB curve will no longer be a straight line, but a convex curve.

2.4. Some Numerical Results

In the previous sections, we presented a model where a benevolent government can use default and (in section 2) inflation to reduce the current debt. However we have seen that, even with such a simple model, an explicit determination of the existing equilibria is not obtainable. To the end of identifying the optimal economic policies and quantifying the endogenous variables in the equilibrium, numerical solution techniques have been adopted²³.

We have chosen to standardise the model with respect to debt issued at time 0, i. e. we set $b_0=1$ ²⁴. Moreover, as support of the shock density function we have considered 2 alternative intervals: $[\nu, \mu]=[1, 2]$ and $[\nu, \mu]=[1, 1.2]$, which correspond to a rate of interest varying between 0% and 100% in the former case and between 0% and 20% in the latter. Tables 2.2 and 2.3 summarise the results obtained from the two possible shock ranges respectively, and choosing different values for the parameters of the government's objective function: α , γ and δ .

In the two tables, the equilibrium values of the following variables are reported:

- period 0 gross interest rate and shock trigger point;

²³ Simulations have been implemented using 'Mathematica' package (version 3.0) through 'FindRoot' procedure. In particular, the convergence criterion adopted is the 'secant method' (for details on implementation see Wolfram, 1991).

²⁴ Given the assumption to normalise the income to 1 (see section 1, note 12), we are here assuming that the debt/GDP ratio is equal to 100%. This case could be a good description of countries such as Belgium and Italy. However this assumption does not affect the main implications of our results.

- optimal default size;
- default probability
- default risk premium (given by the product of the two previous terms).

The above quantities are shown for the model with default only as well as for the one where inflation is also allowed. In this latter case, the tables provide figures also for:

- total risk premium (RP) (calculated as one minus the expected value of a unit of debt as by equation (2.44));
- optimal inflation rate in the default regime;
- optimal inflation rate in the no-default regime.

The main result emerging from a first look at the tables is that from all simulations default probability is higher when both instruments are available compared to the situation when inflation is not allowed.

This result is quite surprising as the lost of one policy instrument could be thought to lead to a more frequent use of the other²⁵. Where does our result come from, then?

The introduction of inflation in the analytical framework affects the probability of default in two opposite directions:

- a) Government's objective function (2.23) implies that, *ceteris paribus*, a combination of inflation and default is preferred to using one instrument only.

²⁵ Drudi and Giordano (1997) obtain results which are consistent with this intuition.

Thus, when the government opts for a partial default, the welfare loss is lower when inflation is available.

In terms of Figure 2.2, this means that the line L^d moves downward. This is shown in Figure 2.5, where the shift produces a negative effect on the threshold \bar{R} implying a higher probability of default.

b) An opposite effect involves the government's loss function in the no-default regime. Let's first assume that the government can credibly commit itself to a no-inflation policy in the final period, i.e. $z_2=0$. The possibility to inflate away part of the real value of the debt in period 1 has an ambiguous effect on welfare: on the one hand, inflation reduces debt burden and thus fiscal pressure; on the other, it implies a more than proportional cost on the government's loss function. The size of the shift depends positively on the shock realisation and it is possible to show that in the neighbourhood of the critical threshold, the former effect dominates the latter so that L^{nd} moves downward (locally). The effect on \bar{R} is now positive and the higher threshold would lead to a lower default probability.

If we relax the assumption $z_2=0$, the downward shift of the curve L^{nd} is dampened. This is because last period inflation is fully anticipated and has no beneficial effect on debt burden, while it induces a cost in the welfare loss function. In this case the L^{nd} is higher than in the partial commitment case.

It is possible to prove that, under the assumption of $z_2=0$, the net effect on the default trigger point is positive (as it is shown by the new threshold \bar{R}' in Figure 2.5) and consequently the default probability is lower when inflation is available to face the shock (see Appendix 2).

However, if we allow the government to use inflation any time, the total effect on \bar{R} is not univocally determined and is not possible to calculate whether default probability will be higher or lower when inflation is given up. To draw some conclusions about this, we rely on our numerical exercise. Simulation results are consistent in showing that the negative effect outlined at point *a*) strictly dominates the positive effect discussed at point *b*). They suggest that the final effect of losing access to monetisation is to increase the shock threshold (from \bar{R}'' to \bar{R} in Figure 2.5) and reduce default probability.

The intuition behind this argument is that a credible no-inflation commitment helps to solve a time inconsistency problem: when government relies on inflation to reduce the real debt burden, this is anticipated by the private sector which demands a risk premium on bond interest rates; this in turn pushes the government to increase inflation. The vicious circle is broken when increasing costs of inflation make default more attractive. However, if the government is credibly believed to use inflation only to dampen adverse shocks, then the time inconsistency does not arise and inflation can partly substitute for default.

The comparison of the two versions of the model suggests that joining a low inflation agreement (such as, for example, joining EMU) will lead to a credibility gain which more than offset the loss of monetary sovereignty. In particular, this result applies to countries with long history of loose monetary policy which cannot credibly commit to a low inflation policy. This view is supported by recent narrowing of the sovereign risk spreads on benchmark bonds. We will come back to this issue in chapter 4.

On the other hand, our analysis suggests that countries with low inflation experience but soft fiscal discipline (Belgium may be a good example), may lose a useful tool to reduce debt burden in bad states of the world.

Appendix 2

We want to show here that if the government can credibly commit itself to use inflation only to face adverse shocks, the effect of loosing this instrument of policy is a higher default probability.

In terms of Figure 2.5 we want to show that $\bar{R} \leq \bar{R}'$, where \bar{R} is the default trigger point of the shock in case inflation is not available, while \bar{R}' is the trigger point when inflation is used only in period 1 (when shock realises). In the former case, we know that:

$$\bar{R} = \frac{2\alpha - b_0 R_0}{2\alpha\delta}$$

and optimal strategy is to default any time that the shock is above that value.

Now suppose inflation is allowed but the government can commit to $z_2=0$.

We want to see which is the best policy at $R_1^* = \frac{2\alpha - b_0 R_0}{2\alpha\delta}$. To this end we are

going to calculate the new value of the loss functions under the two policies of

default and no-default and compare them. At $R_1^* = \frac{2\alpha - b_0 R_0}{2\alpha\delta}$ we have:

$$\begin{aligned}
L^{nd} - L^d &= \frac{b_0 R_0}{2\gamma} \left(b_0 R_0 + \frac{2(\gamma - b_0 R_0)(2\alpha - b_0 R_0)}{2\alpha} \right) - \frac{b_0 R_0 (2\alpha\gamma - \alpha b_0 R_0 - \gamma b_0 R_0)}{2(\alpha\gamma - b_0^2 R_0^2)} = \\
&= \frac{b_0 R_0}{2} \left(\frac{2\alpha b_0 R_0 + 2(\gamma - b_0 R_0)(2\alpha - b_0 R_0)}{2\alpha\gamma} - \frac{2\alpha\gamma - \alpha b_0 R_0 - \gamma b_0 R_0}{\alpha\gamma - b_0^2 R_0^2} \right) = \\
&= -\frac{b_0^3 R_0^3 (\alpha - b_0 R_0)(\gamma - b_0 R_0)}{2\alpha\gamma(\alpha\gamma - b_0^2 R_0^2)} \leq 0^1
\end{aligned}$$

1

The optimal policy is then not to default on debt. As the loss function is monotonic in the shock, we conclude that when inflation is allowed in period 1 (but not in period 2) default threshold is higher and therefore default probability is lower.

¹ The result follows assuming that $R_0 \leq \min\{\alpha/b_0, \gamma/b_0\}$, which is necessary to have both θ and z_1 inside the unit interval $[0, 1]$.

Chapter 3

Default and Inflation Risk and the Maturity Structure of Debt

3.1 Introduction

In the previous chapter we have presented a model where an optimising government could use default and inflation to reduce the debt burden. In particular, we were interested in comparing default probabilities under different inflation policies: one with no inflation allowed (or inflation policy delegated to an external authority, such as the European Central Bank); the other with inflation as a political instrument of debt reduction set by the government. In this chapter we are going to present an application of the previous model, focusing on debt maturity structure and its effects on default probability.

Introducing bonds with different maturities will allow us to investigate further on the robustness of our results, as well as to draw some implications for the debt management policy in and out EMU.

An interesting paper studying the strategic role for debt management policy in relation to joining EMU is presented by Uhlig (1996). The author suggests that high inflationary governments may have the temptation to issue long term debt to raise political consensus in joining the single currency agreement. The argument relies on the assumption that the creditors believe EMU will be a lower inflation regime, and will protect them from the risk of devaluation of their long term nominal bonds. This should induce them to vote for membership. The author, however, shows that the result is fragile, and the political trick is not advisable.

The topic of the optimal structure of debt maturity in relation to a risk of debt crisis has been studied by Alesina et al (1990), Giavazzi and Pagano (1990) and Drudi and Giordano (1997). The first of those papers considers the possibility of an outright default on public debt, the second regards a devaluation of nominal bonds. And only the last one take into account inflation and default as different instruments available to the government. In all the papers the risk of crisis is reduced with a longer maturity structure, and with a uniform distribution of debt coming due in each period.

Our model is in the spirit of Drudi and Giordano's paper, but here the emphasis is on the implications of delegating monetary policy to an exogenous authority such as the European Central Bank. In fact, the situation where the

government cannot monetise the debt while it can still default on it is not deeply investigated by the two Italian authors.

The model is very close to the one presented in the previous chapter, however, beside the introduction of short and long term debt, a further element will make the framework to depart from the one discussed earlier. In the present context, in fact, we are going to relax the hypothesis that default is an available option only in period 1. Real tax on outstanding bonds will now be allowed in the final period too, with income taxes and inflation. In particular, this will reduce asymmetry with respect to inflation. We have seen at the end of last chapter, that our results fail to hold when we constrain the government to a no-inflation policy in period 2. We now want to reverse the exercise allowing all instruments of policy in both periods. However, the main source of asymmetry between default and inflation, given by the government's exclusion from the credit market when default is triggered, will still apply.

The new framework will permit a more general analysis of the topic and will allow an analytical qualification of the results drawn by numerical analysis in the previous chapter.

3.2 A Model with Default Only

3.2.1 Model with default only

The basic structure of the model is very similar to the one described in the previous chapter. Two are the main differences introduced here:

- The government issues short and long term debt. This means that b_0 is now given by the sum of two different bonds b_0^s and b_0^l , the former maturing after 1 period, while the latter after 2 periods. In period 1, the only debt that government can issue is short term, b_1^s , as it is not lasting long enough to pay back long term bonds. As we have already seen, for the same reason no debt is issued in the final period.
- Default is allowed in the final period and not only at $t=1$. As we already know from the last chapter, this assumption of course implies that government always will use default in period 2 (in fact threat of exclusion from financial market does not hold in the final period). The issue is then which is the optimal default size, given the interest rate demanded by private investors. This assumption will introduce a link between the amount of debt issued in period 1 and its cost. The absence of this link in the model with default only analysed in section 2.2, was there responsible for the need of a non-negativity constraint on income taxes. In fact, without it the government was willing to borrow from the private sector as much as it was allowed. This will not hold here any longer and we will get some nice insights on our results.

Timing is unchanged with respect to Figure 2.1, the only difference being default allowed in the last period when t_2 is set.

Allowing default in period 2 leads to the government's intertemporal loss function (2.3) to be modified as follows:

$$L = \frac{\alpha}{2}\theta_1^2 + t_1 + \delta L_2 \quad (3.1)$$

with
$$L_2 = \frac{\alpha}{2}\theta_2^2 + t_2 \quad (3.2)$$

Whilst taking into account debt composition in short and long term bonds, we get the new budget constraints of the government:

$$b_0^s R_0^s (1 - \theta_1) + g_1 = t_1 + b_1^s \quad (3.3)$$

$$(b_0^l R_0^l + b_1^s R_1^s)(1 - \theta_2) + g_2 = t_2 \quad (3.4)$$

where we still have
$$b_1^s = \begin{cases} \geq 0 & \text{if } \theta_1 = 0 \\ 0 & \text{if } \theta_1 > 0 \end{cases} \quad (3.5)$$

Symbols are the same as in the previous model and the superscript s and l indicate the short and long debt maturity. Equation (3.5) has now a new implication, as it shows an asymmetry in the default punishment whether it is occurring in period 1 or 2. In fact, since no more indebtedness can be hold on in the last period, a policymaker defaulting at that time will bear only a proportional cost α , while opting for default in period 1 will involve an additional financial autarky cost. An interesting consequence is that this will bring our work to depart from most of the finite horizon literature involving repeated games. In the usual setting, when cheating is the best strategy in the last period of the game, backward induction

procedure implies that it will be implemented in period 1 (Alesina et al, 1990, presents an example relevant to our case). However in our model the rules of the game change with time (equation (3.5) applies only in period 1 and not in period 2) and we will see that even if default is always the best policy in period 2 it is not necessarily adopted in period 1¹.

Assuming perfect knowledge of the government objective function (3.1) by the private sector, interest rates on different bonds will be set, to avoid arbitrage opportunities, as follows:

$$R_0^s = \frac{R_0^*}{1 - E_0[\theta_1]} \quad (3.6)$$

$$R_0^l = \frac{R_0^* E_0[R_1^*]}{1 - E_0[\theta_2]} \quad (3.7)$$

$$R_1^s = \frac{R_1^*}{1 - E_1[\theta_2]} \quad (3.8)$$

where (3.6) is the same as (2.7). (3.7) describes a composite interest rate for holding a long term bond for two periods. It involves expectations on both next period world interest rate and period 2 default size. Finally, (3.8) corresponds to (2.8) but incorporates a risk premium for possible default in the final period. Foreign rate is still a random variable with same support $[\nu, \mu]$ defined in section 2.2.1 and the assumption $R_0^* = 1$ applies here too. In what follows we assume that

¹ Note that the observation mentioned in note 7 in the last chapter is here even more relevant. In fact, we are assuming that private agents are willing to hold bonds that will be defaulted for sure (even if in a small amount), but only if default is not occurred yet. This again implies that the event of default changes somehow agents' preferences.

the world interest rate is never above the inverse of the government's discount factor, i.e. $\mu \leq 1/\delta$.

To find the government's best strategy that minimises equation (3.1), we need to solve the problem backward. So, we move to investigate the government's behaviour in the final period and later we will go back to period 1.

3.2.2 Period 2 problem

Let's first assume that the government did not default in period 1. If this is the case, then in the final period it has to finance the outstanding debt burden $(b_0^l R_0^l + b_1^s R_1^s)$ plus an exogenous spending g_2 . As mentioned above, new funds cannot be raised from the credit market and fiscal budget must be covered using tax revenue. As taxes are distortionary, the government may find it convenient to renege on part of its debt. The policymaker, then will choose a default size θ_2 to minimise (3.2) subject to (3.4). This gives a value of

$$\theta_2^{nd} = \frac{b_0^l R_0^l + b_1^s R_1^s}{\alpha} \quad (3.9)$$

According to intuition, default size increases with debt burden and declines with its own marginal cost. Once that the optimal θ_2 is calculated, optimal tax revenue is obtained by the budget constraint (3.4) and is equal to:

$$t_2^{nd} = (b_0^l R_0^l + b_1^s R_1^s) \frac{\alpha - (b_0^l R_0^l + b_1^s R_1^s)}{2\alpha} + g_2 \quad (3.10)$$

replacing the optimal values of θ_2 and t_2 into L_2 gives

$$L_2^{nd} = (b_0^l R_0^l + b_1^s R_1^s) \frac{2\alpha - (b_0^l R_0^l + b_1^s R_1^s)}{2\alpha} + g_2 \quad (3.11)$$

where the superscript nd indicates that no default occurred in period 1, the superscript d will be used to indicate the opposite case.

Suppose now that the government, instead, did default on debt in period 1. Then no bonds are issued at that time and optimal fiscal variables θ_2 and t_2 will still be given by (3.9) and (3.10) respectively, just setting $b_1^s=0$, which gives:

$$\theta_2^d = \frac{b_0^l R_0^l}{\alpha} \quad (3.12)$$

$$t_2^d = b_0^l R_0^l \frac{\alpha - b_0^l R_0^l}{2\alpha} + g_2 \quad (3.13)$$

Government loss in the final period will then be:

$$L_2^d = b_0^l R_0^l \frac{2\alpha - b_0^l R_0^l}{2\alpha} + g_2 \quad (3.14)$$

Note that, whether $\theta_1=0$ or not, choosing $\theta_2=0$ will make the government no better off², thus in period 2 it opts for a partial default with probability 1 and

² With $\theta_2=0$ the loss function would be

$$L_2 = (b_0^l R_0^l + b_1^s R_1^s) + g_2 \geq (b_0^l R_0^l + b_1^s R_1^s) \frac{2\alpha - (b_0^l R_0^l + b_1^s R_1^s)}{2\alpha} + g_2 \equiv (b_0^l R_0^l + b_1^s R_1^s) \left(1 - \frac{\theta_2}{2}\right) + g_2$$

where the greater or equal sign holds as $0 \leq \theta_2 \leq 1$. The same holds applied to the (3.14).

default will always be positive except for both b_0^l and $b_1^s=0$ and/or $\alpha \rightarrow \infty$. This is not true in period 1, when it can choose between partial default and new borrowing. As in the previous chapter, the government sets its period-1-fiscal-policy after calculating the optimal values of its choice variables under the two alternatives and comparing the respective losses. However government's loss is not independent from the investors' decision on R_1^s , which in turn depends on the amount of new bonds to be allocated. Thus, we need now to see how private investors price new borrowing.

3.2.3 Period 1 problem

When the bonds b_1^s are issued, the private agents observe the shock on the international interest rate and set R_1^s , knowing the government's incentive to default on a fraction θ_2^{nd} of them with probability 1. Of course, they will be willing to hold public bonds only if they can get a fair premium above the world interest rate to be compensated for the future loss. Replacing (3.9) into (3.8) gives the interest rate demanded on short term debt:

$$R_1^s = \frac{\alpha - b_0^l R_0^l - \sqrt{(\alpha - b_0^l R_0^l)^2 - 4\alpha b_1^s R_1^*}}{2b_1^s} \quad (3.15)$$

Equation (3.15) shows that, not only does the interest rate increase with the amount of new issued debt b_1^s , but also does with the amount of outstanding long term indebtedness and its cost R_0^l . This is because those variables increase the

default size the government is going to implement in period 2, and thus the risk premium demanded by agents.

In period 1 the government has to decide whether to finance its budget deficit through a combination of income taxes and new borrowing or to opt for a partial default. In the latter case of course tax on existing bonds will exclude access to the credit market again, while income tax revenue is still available. The policymaker optimises its intertemporal objective function (3.1) under the two alternatives and compares the outcomes.

$\theta_l=0$ case

If the government decides to fully serve its debt coming due, beside income tax revenue it has the opportunity to issue new short term bonds, at a cost given by the (3.15). The optimal amount of new borrowing is found by minimising:

$$L^{nd} = b_0^s R_0^s - b_1^s + \delta L_2$$

where L_2 is given by (3.11). It is immediate clear that the optimal amount of bonds to be allocated must satisfy the following condition:

$$\frac{\partial L_2^{nd}}{\partial b_1^s} = \frac{1}{\delta}$$

Taking into account of the effect of b_1^s on R_1^s , the optimal solution for the amount new short term debt is given by:

$$b_1^s = \frac{(\alpha - b_0^s R_0^s)^2 (1 - \delta R_1^*)}{\alpha R_1^* (2 - \delta R_1^*)^2} \quad (3.16)$$

Equation (3.16) indicates that government's incentive to issue debt in period 1 decreases with high shock realisations and high stocks of long term bonds. This is due to the effect of the raised cost of borrowing induced by both those factors. On the other hand, a high value of α reduces the risk premium and increases indebtedness.

The corresponding government's loss is now equal to :

$$L^{nd} = b_0^s R_0^s + \delta b_0^l R_0^l \frac{2\alpha - b_0^l R_0^l}{2\alpha} - \frac{(\alpha - b_0^l R_0^l)^2 (1 - \delta R_1^*)^2}{2\alpha R_1^* (2 - \delta R_1^*)} \quad (3.17)$$

$\theta_l > 0$ case

When the government, instead, opts for a partial default, it chooses the optimal default size θ_l to minimise the intertemporal loss function (3.1) subject to its budget constraint (3.2) and to assumption that $b_l^s = 0$ (eq.(3.4)). The optimal fraction of bond tax is then:

$$\theta_1 = \frac{b_0^s R_0^s}{\alpha} \quad (3.18)$$

Tax revenue will then be used to cover the remaining fiscal deficit:

$$t_1^d = b_0^s R_0^s \frac{\alpha - b_0^s R_0^s}{2\alpha} + g_1 \quad (3.19)$$

Replacing (3.18) and (3.19) into (3.1) and taking into account the future value of L_2 (as given by (3.14)), the minimum loss is then given by:

$$L^d = b_0^s R_0^s \frac{\alpha - b_0^s R_0^s}{2\alpha} + g_1 + \delta \left(b_0^l R_0^l \frac{\alpha - b_0^l R_0^l}{2\alpha} + g_2 \right) \quad (3.20)$$

Comparing (3.17) with (3.20), the government does not like to renege on its short term debt in period 1 if:

$$H1 \leq f(R_1^*) \quad (3.21)$$

where
$$H1 \equiv \left(\frac{b_0^s R_0^s}{\alpha - b_0^l R_0^l} \right)^2$$

and
$$f(R_1^*) = \frac{(1 - \delta R_1^*)^2}{R_1^* (2 - \delta R_1^*)}$$

$f(R_1^*)$ describes a decreasing function of the shock on the international interest rate, while $H1$ is not affected. The latter is constant in period 1 as is dependent on only predetermined variables. Default is therefore triggered by a shock high enough to bring the function $f(R_1^*)$ below that constant. This is shown in Figure 3.1.

Solving (3.21) with respect to R_1^* , gives the value of the threshold which trigger default option in period 1 (\bar{R} in figure 3.1).

$$\bar{R}(R_0^s, R_0^l) = \frac{1}{\delta} \left(1 - \sqrt{\frac{H1}{\delta + H1}} \right) \quad (3.22)$$

Equation (3.22) is the analogous of the *GDB* from the previous chapter. As default option is triggered by shocks higher than \bar{R} , *GDB* is inversely related to the probability of observing a partial default in period 1. To give a straightforward intuition of the features of the *GDB* curve we make use of graphical analysis. Figure 3.1 shows that when the *H1* is higher, the shock threshold moves leftwards and default probability grows. As *H1* is increasing in both short and long term interest rate, then the default trigger point is a decreasing function of the two interest rates. Moreover, *GDB* also declines for high stock of short and long term debt. The effect of the two kinds of bonds, however, is of different nature. Short term debt affects directly period 1 budget constraint and the financial needs to cover it. Long term bonds instead increase tomorrow budget and the size of future default, this in turn raises risk premium demanded by agents and thus the cost of borrowing for the government. The latter effect is somehow similar to that of a higher international interest rate.

Structural parameters α and δ , affect *GDB* in an intuitive way. A high δ , makes the government concern more about next period fiscal deficit and thus more willing to anticipate a partial default. On the other hand, a high value of α means a high marginal cost of default and makes the government more reluctant to use it.

It would be interesting to see how different allocations of the same initial amount of debt b_0 in short and long term bonds affect *H1*, and thus default probability. Unfortunately the exercise is quite complex, due to the effect of b_0^j ,

($j=s, l$) on its price. Keeping the two interest rates fixed, it is easy to show that moving bonds from short to long term maturity reduces $H1$ and the probability of default occurring in period 1. In fact the partial derivative of $H1$ wrt b_0^s is higher than the derivative wrt b_0^l . However when we look at the effect of b_0^s on R_0^s and of b_0^l on R_0^l , the former is lower than the latter and so the total effect of moving bonds from short to long maturity is ambiguous. Moreover, moving towards a longer maturity increases expected default size in the final period.

3.2.4 Period 0 problem

In the initial period, the stock of short and long term bonds is given, thus the government has no strategy to set. The private sector, however, is happy to hold public bonds only if compensated for the risk of default which may occur in the following two periods. We have seen above that default is a certain event in period 2, while it is not in period 1. However, optimal default size in period 2 changes whether default occurred in period 1 or not. Thus, private investors have to formulate expectation on period 1 default probability in order to demand a fair premium on both kinds of bonds.

The probability of observing default in period 1 is given by the density function of the shock on the world interest rate, and is equal to:

$$p = \text{prob}\left(R_1^* > \bar{R}^e\right) = \begin{cases} \frac{\mu - \bar{R}^e}{\mu - \nu} & \text{if } \nu \leq \bar{R}^e \leq \mu \\ 0 & \text{otherwise} \end{cases} \quad (3.23)$$

where \bar{R}^e is the agents' evaluation of the default trigger point chosen by the government. Expected loss from default on short and long term debt is then respectively:

$$E_0[\theta_1] = p\theta_1 \quad (3.24)$$

$$E_0[\theta_2] = p\theta_2^d + (1-p)\theta_2^{nd} \quad (3.25)$$

where p , θ_1 , θ_2^d and θ_2^{nd} are given respectively by (3.23), (3.18), (3.12) and (3.9).

Replacing (3.24) and (3.25) into (3.6) and (3.7) respectively, and setting the expected world interest rate equal to its mean value $(\mu + \nu)/2$, we get the interest rate set on short and long term bonds issued in the initial period:

$$R_0^s = \begin{cases} \frac{\alpha - \sqrt{\alpha(\alpha - 4pb_0^s)}}{2pb_0^s} & \text{if } 0 < p \leq 1 \\ 1 & \text{if } p = 0 \end{cases} \quad (3.26)$$

$$R_0^l = \frac{\alpha k - \sqrt{\alpha k(\alpha k - (\mu + \nu)(4 - \delta(\mu + \nu))b_0^l)}}{2kb_0^l} \quad (3.27)$$

with $k = 1 + p(1 - \delta(\mu + \nu)/2)$

and p being a decreasing function of \bar{R}^e as given by (3.23)³.

³ Note that to get (3.27) we have been using the optimal value of R_l^s and b_l^s from (3.15) and (3.16), where (3.16) has been replaced in (3.15) too.

Equilibria are given by a triple (\bar{R}, R_0^s, R_0^l) such that they satisfy (3.24), (3.26) and (3.27) respectively with $\bar{R}^e = \bar{R}$ and with the corresponding values of θ_1 , θ_2^d and θ_2^{nd} all inside the interval $[0, 1]$.

3.3 A Model with Default and Inflation

3.3.1 The model

The economy is the same as described in the previous section, but the government now has one more instrument of policy: inflation. The policymaker uses inflation in order to reduce the real value of the outstanding debt and thus the financial needs to cover the fiscal deficit. Inflation is allowed in both periods 1 and 2, but unlike default it never involves exclusion from the credit market.

The introduction of the new instrument changes slightly the government's objective function and its budget constraints. Government intertemporal loss function is now given by⁴:

$$L = \frac{\alpha}{2}\theta_1^2 + \frac{\gamma}{2}z_1^2 + t_1 + \delta L_2 \quad (3.28)$$

with
$$L_2 = \frac{\alpha}{2}\theta_2^2 + \frac{\gamma}{2}z_2^2 + t_2 \quad (3.29)$$

and the budget constraint in period 1 and 2 are equal to:

$$b_0^s R_0^s (1 - \theta_1 - z_1) + g_1 = t_1 + b_1^s \quad (3.30)$$

$$(b_0^l R_0^l + b_1^s R_1^s)(1 - \theta_2 - z_2) + g_2 = t_2 \quad (3.31)$$

and (3.5) still applies. Note that in the formulation of the above budget constraints we are neglecting the interaction effect of default and inflation on the debt burden⁵. The approximation allows the model to be analytically tractable, ignoring an effect of the second order.

Private sector is still price setter in the domestic bond market and fixes the interest rate on public debt taking into account the expectations on future default and inflation. This gives the following expression for the interest rate on short and long term debt issued in different periods:

$$R_0^s = \frac{1}{1 - E_0[\theta_1 + z_1]} \quad (3.32)$$

$$R_0^l = \frac{E_0[R_1^*]}{1 - E_0[z_1 + \theta_2 + z_2]} \quad (3.33)$$

⁴ The inflation index z_i is still defined as in the previous chapter.

⁵ The true budget constraint should, in fact, include a multiplicative effect (θ_i, z_i) is such as in (2.25).

$$R_1^s = \frac{R_1^*}{1 - E_1[\theta_2 + z_2]} \quad (3.34)$$

where the usual assumptions on the density distribution of the random foreign interest rate still hold and $R_0^* = 1$.

Following the same procedure as from the previous section, we need to solve the government's optimisation problem backwards.

3.3.2 Period 2

In the final period, the government can meet its budget constraint (3.31) using a combination of income taxes, partial default on outstanding debt and inflation. Assuming first that default did not occur in period 1, the minimisation of the loss function implies the following optimal values of the government's instruments:

$$\theta_2^{nd} = \frac{b_0^l R_0^l + b_1^s R_1^s}{\alpha} \quad (3.35)$$

$$z_2^{nd} = \frac{b_0^l R_0^l + b_1^s R_1^s}{\gamma} \quad (3.36)$$

$$t_2^{nd} = (b_0^l R_0^l + b_1^s R_1^s) \frac{\alpha\gamma - (\alpha + \gamma)(b_0^l R_0^l + b_1^s R_1^s)}{2\alpha\gamma} + g_2$$

which give a loss of

$$L_2^{nd} = (b_0^l R_0^l + b_1^s R_1^s) \frac{2\alpha\gamma - (\alpha + \gamma)(b_0^l R_0^l + b_1^s R_1^s)}{2\alpha\gamma} + g_2 \quad (3.37)$$

In the alternative case that short term debt was not fully served in the previous period, optimal values are given by:

$$\theta_2^d = \frac{b_0^l R_0^l}{\alpha} \quad (3.38)$$

$$z_2^d = \frac{b_0^l R_0^l}{\gamma} \quad (3.39)$$

$$t_2^d = b_0^l R_0^l \frac{\alpha\gamma - (\alpha + \gamma)b_0^l R_0^l}{2\alpha\gamma} + g_2$$

with a corresponding a loss equal to:

$$L_2^d = b_0^l R_0^l \frac{2\alpha\gamma - (\alpha + \gamma)b_0^l R_0^l}{2\alpha\gamma} \quad (3.40)$$

Note that expressions (3.35) and (3.38) for optimal default size are identical to those calculated in the model without inflation (see (3.9) and (3.12)). The reason lies in the absence of an interaction effect between default and inflation, which implies that the two instruments are set independently. The value of the loss function, however, is now lower as the possibility to inflate away part of the nominal bonds provides the policymaker with one more degree of freedom in setting its policy.

As we saw in the previous section, as long as there are bonds coming due in period 2, the option to set $\theta_2=0$ and/or $z_2=0$ is never optimal.

3.3.3 Period 1

In period 1, the government has the opportunity to cover part of its fiscal deficit issuing more short term bonds. As usual, this possibility is available only if it refrains from defaulting on debt maturing in that period. The decision between the two options depends on the cost of borrowing, which is set by private investors. The risk premium is now calculated taking into account future values of both default and inflation, as given by (3.35) and (3.36). The interest rate on newly issued bonds is then given by:

$$R_1^s = \frac{\alpha\gamma - (\alpha + \gamma)b_0^l R_0^l - \sqrt{(\alpha\gamma - (\alpha + \gamma)b_0^l R_0^l)^2 - 4\alpha\gamma(\alpha + \gamma)b_1^s R_1^*}}{2(\alpha + \gamma)b_1^s} \quad (3.41)$$

If the government finds the above interest rate too high it will go for a partial default on the already issued bonds, otherwise will borrow new funds at that price. In both cases, income tax revenue and inflation will also be used to finance the budget deficit. The two alternatives provide the following outcomes

$$\theta_1 = \frac{b_0^s R_0^s}{\alpha} \quad (3.42)$$

and $b_1^s = 0$

in one case;

$$b_1^s = \frac{(\alpha\gamma - (\alpha + \gamma)b_0^l R_0^l)^2 (1 - \delta R_1^*)}{\alpha\gamma(\alpha + \gamma)R_1^* (2 - \delta R_1^*)^2} \quad (3.43)$$

and $\theta_1 = 0$

in the other.

In both cases, inflation will be set at the following level:

$$z_1 = \frac{b_0^s R_0^s}{\gamma} \quad (3.44)$$

and income tax revenue is given as a residual from the budget constraint (3.30).

Once again, debt coming due in period 1 will be completely repaid whenever

$$L^{nd} \leq L^d$$

Using (3.37), (3.40), (3.41), (3.42) and (3.43) to calculate the intertemporal loss function in the two cases, the above inequality can be express as:

$$H2 \leq f(R_1^*) \quad (3.45)$$

where

$$H2 \equiv \gamma(\alpha + \gamma) \left(\frac{b_0^s R_0^s}{\alpha\gamma - (\alpha + \gamma)b_0^l R_0^l} \right)^2$$

and

$$f(R_1^*) = \frac{(1 - \delta R_1^*)^2}{R_1^* (2 - \delta R_1^*)}$$

where $f(R_1^*)$ is identical to the rhs of (3.21) in the model with no inflation.

Solving (3.45) for R_1^* still gives the *GDB* curve:

$$\bar{R}(R_0^s, R_0^l) = \frac{1}{\delta} \left(1 - \sqrt{\frac{H2}{\delta + H2}} \right) \quad (3.46)$$

$H2$ is still an increasing function of both short and long term interest rates and also grows for high stocks of debt issued in period 0. Thus default threshold still enjoys the properties discussed in the previous section.

3.3.4 Period 0

Finally in the initial period agents set the interest rate on short and long term debt, formulating expectations on next periods policy. When they lend to the government for just one period, agents know that bonds will depreciate at a rate z_1 and that there is a probability p (given by (3.23)) that they will get a further $\theta_1\%$ less at maturity. The interest rate that compensate holders against the total risk is then equal to:

$$R_0^s = \frac{\alpha\gamma - \sqrt{\alpha\gamma(\alpha\gamma - 4(\alpha + p\gamma)b_0^s)}}{2(\alpha + p\gamma)b_0^s} \quad (3.47)$$

On the other hand, long term investment implies a composite risk which involves: a depreciation rate z_1 for the first period, an uncertain (usually different) depreciation rate for the second period and a final default risk of uncertain size. Moreover, investors face also uncertainty on the future shock on the international interest rate. Expectations on the total loss are given by:

$$E_0[z_1 + z_2 + \theta_2] = z_1 + p(z_2^d + \theta_2^d) + (1-p)(z_2^{nd} + \theta_2^{nd})$$

The interest rate the government has to pay on long term debt is given by:

$$R_0^l = \frac{\gamma(\alpha k - (2 - \delta x)b_0^s R_0^s) - \sqrt{\gamma^2(\alpha k - (2 - \delta x)b_0^s R_0^s)^2 - 4\alpha\gamma(\alpha + \gamma)kx(2 - \delta x)b_0^l}}{2(\alpha + \gamma)kb_0^l} \quad (3.48)$$

where with $k = 1 + p(1 - \delta(\mu + \nu)/2)$ as in the previous section

$x = (\mu + \nu)/2$ is the expected world interest rate in

period 1, and R_0^s is given by (3.47) above.

Equilibria are defined by a triple (\bar{R}, R_0^s, R_0^l) such that they satisfy (3.46), (3.47) and (3.48) respectively with p given by (3.23) and $\bar{R}^e = \bar{R}$, and with the corresponding values of $\theta_1, \theta_2^d, \theta_2^{nd}, z_1, z_2^d, z_2^{nd}, E_0[z_1 + z_2 + \theta_2]$ and $E_1[z_2 + \theta_2]$ all inside the interval $[0, 1]$.

As we discussed in previous chapter, in terms of Figure 2.3 the introduction of inflation in the model makes the reaction functions of the actors shifting up and rightwards. The final effect on the equilibrium then depends on the new position of the two curves. Here the GDB functions are given by (3.22) and (3.46) for cases without and with inflation respectively; while DIRM for these two cases are represented by equation (3.26) and (3.47)⁶. On the one hand, the availability of one more instrument of debt reduction would induce less use of default, for given interest rates (GDB shifts up); on the other hand, it induces a

⁶ Note that referring to Figure 2.3 is not immediate as the GDB curve now depends on both short and long run interest rates. However as we are interested in default probability in period 1, we assume that the effect of long run interest rate is taken into account when drawing the GDB curve.

higher interest rate to compensate private agents for inflation risk, for any given default probability (DIRL moves rightwards).

3.4 Default Probability in and out EMU

Despite its complex analytical formulation, the model provides some straight intuitions in terms of possible consequences of giving up inflation as a tool of debt reduction. In particular, it suggests some indications on the role of the debt maturity structure in managing the default risk when moving from a regime of monetary sovereignty to one with exogenous monetary policy, such as the EMU. Moreover we are here able to give some analytical qualification of the results obtained throughout numerical simulations in the previous chapter.

If we compare inequality (3.21) with (3.45) we see that they share the same rhs, while the lhs is different. The latter is represented by a constant line in Figure 3.1. In particular, Figure 3.1 suggests that $p^D \geq p^{D+I}$ iif $H1 \geq H2$ (where the superscripts D and D+I indicate the variables from the model with default only and that one with inflation respectively). This is because if the constant line $H1$ is

higher, default threshold will be more leftwards and probability of default occurring higher.

For given R_0^s and R_0^l , we get that:

$$\text{Sign}[H1 - H2] = -\text{Sign}[\alpha^2 \gamma - (\alpha + \gamma)(b_0^l R_0^l)^2]$$

The sign of the squared brackets in rhs of above equation is positive iff:

$$b_0^l R_0^l < \alpha \sqrt{\frac{\gamma}{\alpha + \gamma}} \quad (3.49)$$

The above inequality always holds. In fact we need

$$b_0^l R_0^l \leq \alpha \frac{\gamma}{\alpha + \gamma} \text{ in order to have } R_l^s \text{ defined positively}^7.$$

However, R_0^s and R_0^l will be typically different in the two regimes of monetary delegation and monetary sovereignty. It is possible to prove that the short term interest rate is always lower in the former regime. In fact, $(R_0^s)^{D+I}$ is inversely related to the value of γ , so taking the limit of γ going to infinity we can show that the minimum value for $(R_0^s)^{D+I}$ is given by $(R_0^s)^D$. So we have that:

$$(R_0^s)^D \leq (R_0^s)^{D+I} \quad (3.50)$$

(where equality is never reached but when $\gamma \rightarrow \infty$)

It is also possible to prove that

$$(R_0^l)^D \leq (R_0^l)^{D+I} \quad (3.51)$$

(all proofs are in the appendix)

This straighten the positive difference between $H2$ and $H1$, and thus between p^{D+I} and p^D ⁸. Figure 3.2 describes the situation under both regimes.

The reason of the result is the same discussed in the previous chapter. Inflation pushes interest rates up, and increasing the cost of borrowing makes default more attractive.

In fact, it is also easy to prove that

$$(R_1^s)^D \leq (R_1^s)^{D+I} \quad (3.52)$$

and consequently

$$(b_1^s)^D \geq (b_1^s)^{D+I} \quad (3.53)$$

However, we also have

$$(b_1^s R_1^s)^D \geq (b_1^s R_1^s)^{D+I} \quad (3.54)$$

This last relation together with (3.51) suggests that default size in the second period in the exogenous monetary policy regime may be higher or lower than in the full sovereignty regime (see equation (3.35)). While (3.51) implies that not only period 1 probability of default will be lower in the former regime, but also will be the size in the event default is occurring in that period (equation (3.42)).

⁷ This is because $E_1^{D+I}[z_2 + \theta_2] \in [0, 1]$ only if the inequality is satisfied.

⁸ This proves the result that the default probability in period 1 declines when we give up inflation. In the previous chapter we got the same result by mean of numerical analysis. The analytical prove is here possible thanks to two elements: a) the approximation $(1 - \theta_1)(1 - z_1) \approx (1 - \theta_1 - z_1)$; b) allowing default in the final period, that generates an endogenous upperbound to the government's willingness to issue new debt even in the no-inflation case. Note that we are always assuming that short and long term bonds issued in the initial period are given at the same levels in the regimes.

It is also possible to prove that the ratio $H2/H1$ is increasing in the long term burden ($b_0^l R_0^l$). This implies that the gain from joining EMU is higher for country with long term maturity debt. This is still due to the time inconsistency problem induced by inflation. Time inconsistency, in fact, is stronger with long term bonds (if they are not indexed).

Appendix 3

Proof of (3.51):

The long term interest rate in the model where default is allowed is given by equation (3.33) as:

$$\begin{aligned} (R_0^I)^{D+I} &= \frac{E_0[R_1^*]}{1-E_0[z_1 + \theta_2 + z_2]} \geq \frac{E_0[R_1^*]}{1-E_0[\theta_2 + z_2]} \equiv R_0^I(z_1 = 0) = \\ &= \frac{\alpha\gamma k - \sqrt{\alpha\gamma k(\alpha\gamma k - (\alpha + \gamma)(\mu + \nu)(4 - \delta(\mu + \nu))b_0^I)}}{2(\alpha + \gamma)kb_0^I} \geq (R_0^I)^D \end{aligned}$$

The above chain of relations says that the long term interest rate can not increase if we assume no-inflation in period 1. Given that, if z_1 is assumed to be zero, replacing the risk premium and solving for R_0^I gives the last term of the chain. That expression describes a decreasing function in γ , which gets its minimum (as $\gamma \rightarrow \infty$) equal to (3.27), i.e. equal to the long term interest rate obtained in the D model (model with default only).

Proof of (3.52):

Short term interest rate on period 1 bonds is given in the two regimes, D and D+I, by (3.15) and (3.41) respectively. Replacing the optimal values of $(b_1^s)^D$ and $(b_1^s)^{D+I}$ (given by (3.16) and (3.43)) the two short term interest rates are equal to:

$$(R_1^s)^D = \frac{\alpha R_1^* (2 - \delta R_1^*)}{(\alpha - b_0^l (R_0^l)^D)}$$

$$(R_1^s)^{D+I} = \frac{\alpha \gamma R_1^* (2 - \delta R_1^*)}{(\alpha \gamma - (\alpha + \gamma) b_0^l (R_0^l)^{D+I})}$$

Neglecting the difference between $(R_0^l)^D$ and $(R_0^l)^{D+I}$ and taking the difference between the above expressions it is straightforward to show that (3.52) holds. Taking into account (3.51) just strengthens the result.

Proof of (3.53):

Is immediate, just taking the difference between (3.16) and (3.43) with $(R_0^l)^D = (R_0^l)^{D+I}$, again when we consider that $(R_0^l)^D \leq (R_0^l)^{D+I}$ the difference is just bigger.

Proof of (3.54)

Multiplying the two above expressions of $(R_1^s)^D$ and $(R_1^s)^{D+I}$ for (3.16) and (3.43) respectively we get:

$$(b_1^s R_1^s)^D = \frac{(\alpha - b_0^l (R_0^l)^D)(1 - \delta R_1^*)}{(2 - \delta R_1^*)}$$

$$(b_1^s R_1^s)^{D+I} = \frac{(\alpha \gamma - (\alpha + \gamma) b_0^l (R_0^l)^{D+I})(1 - \delta R_1^*)}{(\alpha + \gamma)(2 - \delta R_1^*)}$$

Once again to show that their difference is positive is immediate assuming $(R_0^l)^D = (R_0^l)^{D+I}$. The difference then increases knowing that $(R_0^l)^D \leq (R_0^l)^{D+I}$.

Chapter 4

Default Risk Patterns in Europe: an Empirical Analysis

4.1 Introduction

In the previous chapters we have presented two versions of a model where an optimising government had to manage a high public debt by having access to default and inflation as instruments of debt reduction. The main implication was that when government can tight its hands against inflation, the default risk also declines. The key assumption of the model is that default is triggered by high shocks on foreign interest rate. According to standard theory, with perfect capital mobility the borrowing cost for the government of a small open economy is driven by the foreign rate. Thus, an adverse shock on the latter may increase the borrowing cost to the point

that the social welfare improves more if the government opts for a partial default today rather than it raises a huge amount of distortionary tax revenue tomorrow.

In this chapter, we are going to investigate further on the relationship which links domestic and foreign returns in a European context. Since early 80s, most of European countries have joined an exchange rate mechanism (ERM) and have agreed to eliminate controls on capital mobility. The result has been that Germany assumed the leadership in setting monetary policy and European returns on financial assets had to follow, in different degrees, the pattern of German interest rates. This, of course, has affected economic policy of other countries.

One example of the leader role of German rates on other European economies is given by the ERM crisis of 1992. After the inflationary pressure induced by Germany unification, German Central Bank tightened monetary policy increasing interest rates. The propagation effect to the other countries triggered the exchange rate crisis in Summer 1992, and induced a deep recession in Europe. The high interest rates had very negative effects in Italy where the risk of a debt crisis was a serious threat. The crisis was avoided implementing a strong fiscal restriction which reduced the primary deficit of almost 2% (actually a primary surplus emerged after many years of deficits). The fiscal manoeuvre, beyond its direct effect on budget, was meant to be a signal to the market about the government's commitment to invert the loose fiscal trend of the previous decade.

The episode made evident the fragility of debt sustainability to foreign interest rate¹ and made clear the need of structural fiscal reforms to reduce the high debt burden. At the light of our analysis, the event suggests not only that default risk may depend on foreign interest rate, but also that the relationship may be non-linear. That is, default risk may be zero or very small for low foreign interest rate, and then jump up when the foreign rate increases above a critical threshold².

In what follow, we are going to test the assumption that default risk premium incorporated in European public bond yields have a non linear relationship with the German interest rate. In particular, we want to investigate whether we can identify a threshold for the German rate above which default premium changes its pattern.

The techniques adopted are those of the Threshold AutoRegressive (TAR) models, introduced by Tong (1978), Tong and Lim (1980) and Tong (1983). These models allow to study autoregressive processes that are linear within given regimes, where the regimes are determined by a threshold variable (the German rate in our case). TAR models have been used in forecasting analysis, especially when variables exhibit a cyclical component, such as GDP and industrial production series (see Clements and Krolzig, 1997; and Marrocu, 1998). A TAR application to interest rates is provided by Murto (1994), where the author uses the non-linear approach to study

¹ As we mentioned in chapter 1, the same fragility was responsible for the LDC debt crisis in the 80s. The rise in US interest rate charged indebted governments of an excessive debt service, pushing on an unsustainable path sovereign debts which were sustainable at the previous interest rates.

² The argument finds some theoretical support. In fact, theory says that debt sustainability is guaranteed as long as real interest rate is below the real growth rate of the economy. A sudden jump in

the dynamic of short term Finnish interest rates in 1987-1992. The author claims that: *“non-linearity in the conditional mean is needed to describe the response of the interest process to exceptionally large shocks due to the speculative attacks”* (p.7). In the papers just mentioned the threshold variable is always given by the lagged dependent variable itself. Our analysis departs from them in considering a different variable: the lagged German interest rate.

Our investigation involves six European countries and Germany as the leader country, in a time span that goes from August 1991 to April 1998. Among the six countries, four are EMU members (Belgium, France, Italy and Spain) while two are not (Sweden and UK).

A first step is to investigate on the existence of different regimes in the univariate autoregressive process of the default risk premium. Two regimes emerge quite clearly in all the countries where default premia appear significant. In a following section we introduce some macroeconomic variables to take into account their effect on expectations of the market in evaluating the risk premium. The evidence is that the explanatory variables, when significant, do not account for the different behaviour of the premium and the two regimes remain a robust result.

real interest rate, especially when economic growth is low, can make current debt path unsustainable in

4.2 Methodology: a Non-Linear Approach

In order to study the empirical relationship of default risk premium in European countries and the German interest rate, we take a non linear approach. The techniques adopted are those of the Threshold AutoRegressive (TAR) models. The basic idea of these models is that the underlying AR process of the variable under study is piecewise linear, where the kink points depend upon the value of a given ‘threshold’ variable. As the threshold variable is continuous on \mathfrak{R} , partitioning the real line gives the number of possible regimes in which the process is divided. In particular, if x_{t-d} is the threshold variable (where $d \geq 0$ indicates time lagged order of the variable), the p^{th} order linear AR process of the endogenous variable y_t in the j^{th} regime is given by:

$$y_t = \alpha_0^j + \alpha_1^j y_{t-1} + \dots + \alpha_p^j y_{t-p} + \varepsilon_t^j \quad \text{for } r_{j-1} \leq x_{t-d} < r_j \quad j=1, \dots, N$$

and with $\varepsilon_t^j \text{IID}(0, \sigma^2)$

where N is the number of regimes and r_j are known as thresholds. Of course the parameters α_t^j may vary across regimes as well as the AR order.

the long run. A positive default risk emerges consequently.

4.3 The Data

The data used for the analysis are risk premia on 10-years government bonds for six countries: Belgium, France, Italy, Spain, Sweden and UK; over a period from January 1991 (or as early as data are available) until November 1998. Following Favero, Giavazzi and Spaventa (1997), risk premia are calculated from the difference between bond yields spreads with respect to Germany and the exchange risk factors. The latter are obtained by the difference of swap contract rates denominated in domestic currency and in the German currency³. As suggested, we consider continuously compounded interest rates given by the logarithms of one plus the annual compounded rate. Data are weekly average over the respective daily observations. Weekly averages have the advantage to eliminate the intra-week ‘stagionality’.

³ We are thankful to the Bank of Italy for the provision of the data-set. The original source of the data is as follows: returns from benchmark bonds are those published by IBS (International Bank of Settlements), while swap rates are those quoted daily by InterCapital Brokers in London from Reuters. Moreover, Datastream is the source of benchmark rates for Belgium and Sweden for the period 07/01/1991-25/07/1994, and for swap rates for Belgium (24/06/1991-06/02/1995), Spain (22/07/1991-

In the second part of the chapter, we make use of some macroeconomic variables to measure the effect of the main economic and fiscal indicators on the default risk premium. Among the main economic indicators we consider: inflation rate, industrial production index (IIP), change in the industrial production index, change in GDP, unemployment rate; while fiscal variables are: total deficit to GDP ratio, debt to GDP ratio, and a measure of the fiscal pressure as percentage of GDP. Inflation (calculated on CPI), IIP, change in IIP and unemployment are monthly data, while change in GDP are quarterly and all the fiscal variables are annual data⁴. To match the weekly frequency of the risk premium we have calculated series where each observation is repeated as many weeks as needed to cover the time span of the higher frequency variable.

Figures 4.1 and 4.2 show the total spreads and the exchange risk component respectively, for the six countries. It is immediate to see that the exchange risk factor accounts almost completely for the total spread: the two figures show a very similar pattern and the magnitude is appreciably different only in the case of Italy. Figure 4.3 illustrates the default risk premium. The default risk component is mostly zero or negative for Sweden and positive only in very short periods in the case of UK. The France picture shows an initial period of negative premium, which turns suddenly

30/01/1995) and Sweden (07/06/1992-06/02/1995) for the periods in brackets. The integration was needed as the original data were not available.

⁴ Source of the data: CPI and unemployment are OCSE series, while IIP and change in GDP are Eurostat series; all fiscal variables are published by the Bank of Italy in the "Public Finance Statistics in the European Union" – Statistical Bulletin Supplements (17 February 1999).

positive in November 1993, and that drops to negligible values in March 1996. A possible explanation rely on our measure of the default premium as residual between the total yield differential and the exchange rate factor: in fact, the negative premium may be due to the overestimation of the exchange risk in the swap market in the period 1991-93 with repeated speculative attacks against the French franc. Finally a significant default risk component is evident in for Italy, Belgium and Spain. The premium is very small but positive (around 0.25%) for the last two countries and definitely higher for Italy (with picks up to 1.5%). Moreover, Italian premium appears quite persistent with respect to the other two countries.

4.4 The TAR Analysis

In this section we move to investigate the behaviour of the default risk premium, looking for possible regimes driven by the German interest rate. As we have discussed in the previous section, from a first look at the data a positive default premium emerges for Belgium, Italy and Spain, while Sweden and UK do not seem to

exhibit any significant default risk and France shows evidence of it only in the period November 1993-March 1996. Moreover, the France picture suggests that the premium has a stable pattern around its mean in the above mentioned period, which then drops suddenly to a approximately zero value. Therefore we think that a simple additive structural break can explain the pattern better than a TAR analysis.

As we are interested in the possibility of different regimes related to the German rate, we focus our analysis on Belgium, Italy and Spain only. The time period considered is August 1991-April 1998, the reduced time span being due to some missing data.

The first step is to identifying, for each country, a candidate value of the German interest rate to act as a threshold. That is, for each country, we are going to investigate if it does exist a value of the German rate such that the behaviour of the default risk shows significantly different patterns in the two regimes determined by the threshold. To this aim univariate AR regressions have been run for all the possible threshold values and the best TAR model has been chosen on the base of the AIC information criterion⁵. The exercise has been repeated for each of the three countries. As threshold variable we have used the German interest rate lagged of 1 period⁶. The best AR order was still selected with the AIC criterion over a range between 1 and 6 lags.

⁵ Iterative regressions using all the possible thresholds have been run with TSP package.

Once structural parameters, r and p , are known, a TAR model can be estimated. The set up of the model implies the construction of an indicator function to take into account of the two regimes. An F-test is then implemented to check for non-redundancy of the regimes⁷.

Results are presented in table 4.1. Beyond equations estimated with the indicator function, AR models for the two separate regimes are included. This allows for more intuitive insights of the default behaviour in the different regimes. Regime a correspond to high German rates, while regime b is the low rates regime.

The F-test on the assumption of a unique regime is strongly rejected for all countries. In the Italian case, the constant term appears significantly higher in the a regime than in the b one, suggesting a jump in the value of the risk premium when the German rate is above the threshold. Moreover, the regime b seems largely more persistent. For the other two countries the increase of the constant term is marginal and the regime switching seems to affect mostly the AR process.

For Belgian and Spanish regressions, diagnostic tests show that estimations are quite accurate and only normatily problems emerge⁸. However, heteroscedasticity is also evident in the Italian case, possibly indicating the omission of some ARCH or GARCH structure of the default premium.

⁶ Lagged German rate was required as the current value enters the construction of the default risk premium. Moreover, it is sensible to allow a short, but not zero, period of time to the risk premium to react to the foreign rate.

⁷ Basically we are testing for all coefficients of the variables multiplied by the indicator function to be zero.

Given by the high persistence in the Italian premium series (evident also from Figure 4.3), the exercise has been repeated on the variable in first difference. The threshold value results stable and the two regimes appear a robust result⁹ (see table 4.2).

Finally, Figure 4.3 shows that all the default risk series share a high pick on the 22nd of February 1993. The possibility that the outlier affects the threshold value is therefore investigated. The TAR analysis is then conducted introducing an impulse dummy to model the pick. The introduction of the impulse dummy suggests a higher threshold value for Belgium and Spain and affects the dynamics of the risk premium: a higher AR order is required for both countries. But the presence of the two regimes is still highly evident. The introduction of the dummy does not seem to affect at all the Italian analysis, which still shows the same value of the threshold and the same dynamic pattern. This is because the outlier was not really relevant in the Italian case. However the diagnostic testing of the models show that the dummy variable worsen the reliability of the estimation importing autocorrelation and heteroscedasticity where they were absent before. So we decided to neglect the impulse dummy in the estimates. We conclude that two regimes are evident for the risk premium of

⁸ Given the high number of tests implemented in the estimations, 1% level of confidence is chosen.

⁹ TAR models in first difference are estimated also for Belgium and Spain. Results are consistent with the analysis in levels. The Spanish series in first difference however exhibits a long AR process suggesting that moving average elements have been introduced and first difference were not appropriate.

Belgium, Italy and Spain, but some uncertainty remains on the ‘true’ value of the threshold in the Spanish and Belgian case.

4.5 The Role of Economic and Fiscal Indicators

The next step is that to investigate whether the regime-pattern of the default risk premia emerged in the previous section can be accounted for by including in the TAR models some relevant economic and fiscal indicators. Thus, we look for the best set of variables to describe the default risk pattern choosing from the available set and testing for those which are not significant. German main economic indicators are included as well to measure whether the state of the economy of the leader country has any effect on the premium of the ‘followers countries’. As we are interested in catching the effect incorporated into the financial operators’ expectations, macroeconomic variables are introduced lagged to the time of the actual information delivery. We also test for an announcement effect. To measure this effect, we introduce a dummy variable where each observation enters only the week the

information is delivered¹⁰. Table 4.4 provides a list of variables included in the regressions.

Given the low frequency of the fiscal variables only one of them has been introduced in each model. The best candidate was chosen on the base of its performance in the model (in terms of significance, correct sign, diagnostic testing and AIC criterion).

TAR estimations of the most successful models are showed in table 4.3. The main result is that the two regimes remain evident and stable. The identified threshold values are slightly higher, but still very close, to the ones obtained in the univariate process. The F-test of unique regime is still strongly rejected.

Explanatory macroeconomics variables seem to add little information to the simple TAR models¹¹. This is also indicated by the AIC criterion, which improves only marginally. Moreover, most variables show an unstable relationship with the risk premium, as their sign changes between the two regimes.

The announcement effect is seldom significant, but when it is, it seems to incorporate the total effect of the variable (i.e. the effect of the variable along the period up to the next information delivery vanishes).

¹⁰ The delivery timetable of the Eurostat data has been provided by the data-shop of ISTAT, the Italian Institution of National Statistics. Information on date delivery of the fiscal variables of Italian as well as of other European countries has been provided by the Bank of Italy.

¹¹ Moreover, the introduction of such variables worsen markedly the performance of the estimations in terms of diagnostic testing

Among the fiscal indicators, the degree of fiscal pressure appears the most powerful explanatory variable for Italy and Spain. The effects of the variable are, however, quite different in the two countries. In Italy an increase in the fiscal pressure has a positive effect on the debt sustainability in both regimes. This indicates that a higher fiscal pressure is understood as rigorous fiscal policy. On the other hand, high fiscal pressure has an adverse effect on Spanish risk premium in the low rate regime. This may suggest that in Spain fiscal pressure is perceived as too high and can not be raised further without depressing effects on the economy and bad consequences on the debt service. Belgian risk premium, instead, appears more sensitive to the deficit to GDP ratio, but the variable is significant only in the low rate regime.

The introduction of the German variables suggests interesting considerations. In the Italian and Spanish models, the leader economy seems to affect default risk only in the *b* regime, which is the low rate regime. The evidence is shared also by Belgium but in lower extent. As the low rate regime seems to prevail in the most recent period of economic convergence the result may indicate that the closer we get to the single currency the more German variables incorporate all the relevant information.

Conclusions

The thesis aims to provide a contribution to the literature of public debt management and sustainability. In particular, we focus on the interaction between default and inflation risk on public debt bonds to shed light on default risk behaviour with and without monetary sovereignty.

From a practical point of view, the topic is quite relevant for the new economic environment of Europe: EMU had its full operating start just few months ago including Member States with very different public finance soundness. Countries with low and long maturity public debt are now sharing a single currency with others burdened by high debt (even higher than 100% of GDP) and often of short maturity, all giving up their monetary sovereignty to appoint it to an independent institution, the European Central Bank. In the recent years, a lively debate took place on feasibility of such an agreement. Many economists have been arguing on the importance of monetary sovereignty in avoiding debt crises, as monetisation is a last resort to repay bonds. If this escape option is eliminated, what are the implications on bond yields for high debt countries? What about default probability on sovereign debt? Will the exchange rate uncertainty be replaced by more financial uncertainty?

In the present work we have attempted to provide some answers to the above questions.

The first step has been to look at the state of the art presenting and discussing the main literature on sovereign default. In particular we have identified two branches of the literature on the topic, one focusing on foreign debt the other on domestic debt. In the former kind of analysis, the main implication of the default threat is the existence of credit rationing for indebted countries, which limits their ability to borrow and thus their incentive to default. In a world without uncertainty, the credit ceiling implies that default never occurs. In the latter stream of studies, instead, when debt burden is high, creditors are willing to hold public bonds only for higher and higher yields, increasing the government's incentive to default. Default on domestic bonds shows self-fulfilling properties with multiple equilibria, where expectations assume a crucial role. In this contest, the probability of a confident crisis appears to decrease with debt maturity and with a more uniform distribution of bonds maturity structure.

Moreover, empirical analysis suggests that a default risk premium is actually incorporated in public bond yields for those countries with high debt burden and a past of weak fiscal discipline.

Next step has been to develop a theoretical model, where an optimising government, acting in a stochastic environment, has the option of defaulting on public debt. The optimal policy is 'state contingent' and partial default may occur in the bad states of the world. The model can show multiple equilibria, which correspond to different default probabilities and initial interest rates.

How does the model change when inflation is included among the government's instruments of debt reduction? Surprisingly, simulations suggest that the default risk is higher. The intuition is that inflation is partially anticipated and raises domestic interest rates, worsening the debt service burden which in turn induces higher inflation: the increasing costs of inflation make default more attractive. So excluding inflation reduces the default risk.

The analysis implies that the loss of monetary sovereignty following the start of the EMU brings a high credibility gain. This high credibility seems to more than compensate for the loss of flexibility in facing stochastic shocks. Our results are consistent with observed market expectations as incorporated in the declining path of yield spreads on European public bonds since the EMU regime was approaching. The pattern of declining differentials became even more pronounced when EMU actually started.

The main result, however, fails to hold when inflation is used only to surprise the market.

Thirdly, we moved to investigate further the theoretical analysis. In chapter 3, we present a new version of the model, where few simplifications allow us to prove analytically the results. Moreover, the explicit consideration of short and long maturity bonds enriches the analysis with some intuitions on debt management strategy. The model shows that the gain from joining EMU, in terms of reduced default probability, increases with debt maturity. In fact, long maturity bonds represent a strong incentive to monetise debt, inducing private investors to ask for a high inflation premium. The interest rate saving, which follows from the loss of monetary sovereignty, is therefore higher with

long term bonds than with short term ones. This result is consistent with Uhlig (1996), where high inflationary governments may be tempted to issue long term bonds in order to obtain political consensus in joining the single currency regime.

Last step has been to present an empirical analysis to investigate on the relationship between default risk and foreign interest rate realisations, which is the trigger device of our theoretical model. In the last chapter, we study the interaction of default premium with the interest rate of a foreign country, which acts as price leader in the bond market (Germany in our case). Our analysis suggests that default risk has a non-linear relation with the foreign yield. In particular, we have identified a threshold value for the German rate above which default premium on government bonds (for Belgium, Italy and Spain) changes its pattern exhibiting an increased mean. We also found that this different reaction of default premium to foreign yield cannot be accounted for by domestic fiscal policy and/or other macroeconomic indicators. Finally, not only does Italian default premium appear higher than in the other two countries, but it is also more persistent. This is in line with the empirical literature discussed in chapter 1.

Summing up, our work indicates that EMU itself can not make Member States' debt sustainability riskier; whilst it suggests that credibility gain is the main factor in reducing debt serving burden, as observed in European countries. On the other hand, the underlying risk is that a lower borrowing cost may induce governments to relax on fiscal discipline; this is a topic faced by the Stability Pact and is beyond the aim of our analysis.

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FIGURES AND TABLES

Figure 2.1: TIMING

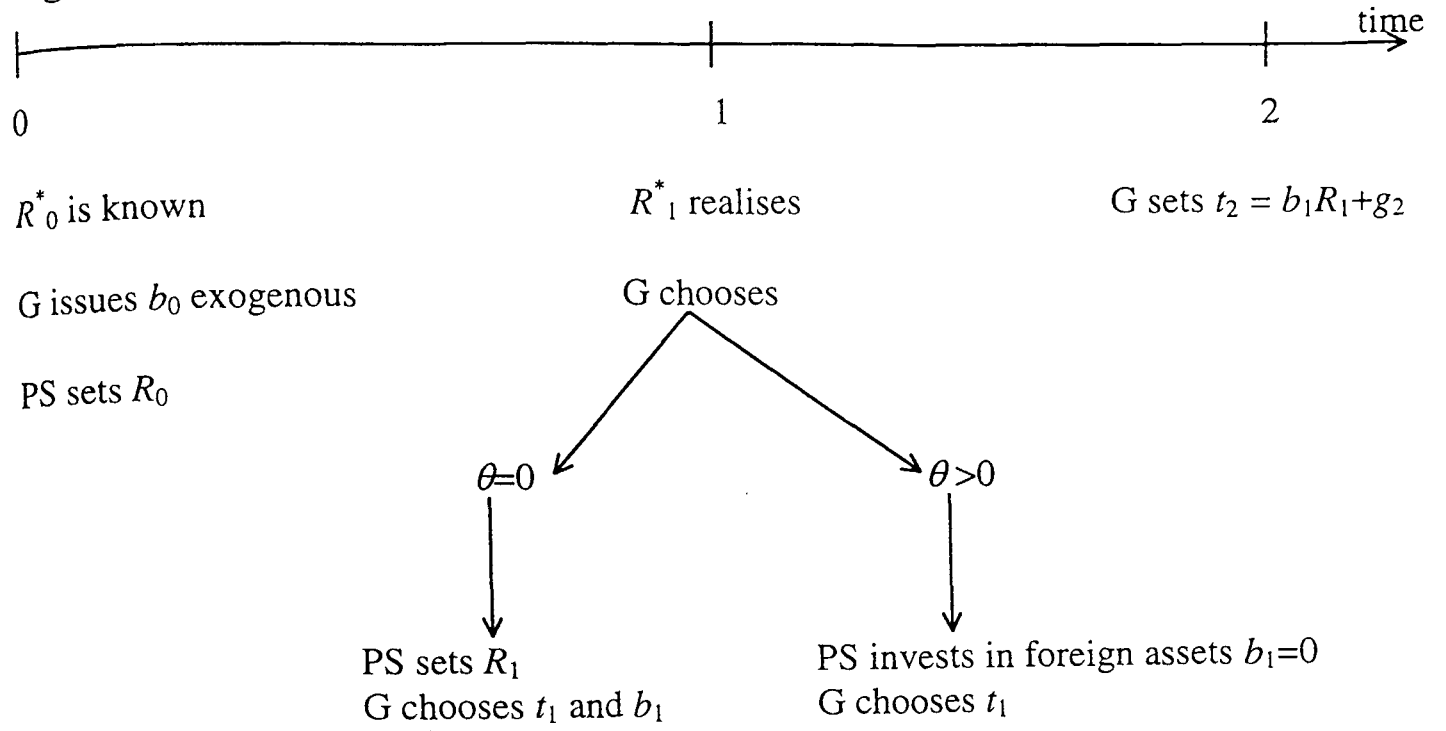


Figure 2.2: THE LOSS FUNCTION COMPARISON
(only default case)

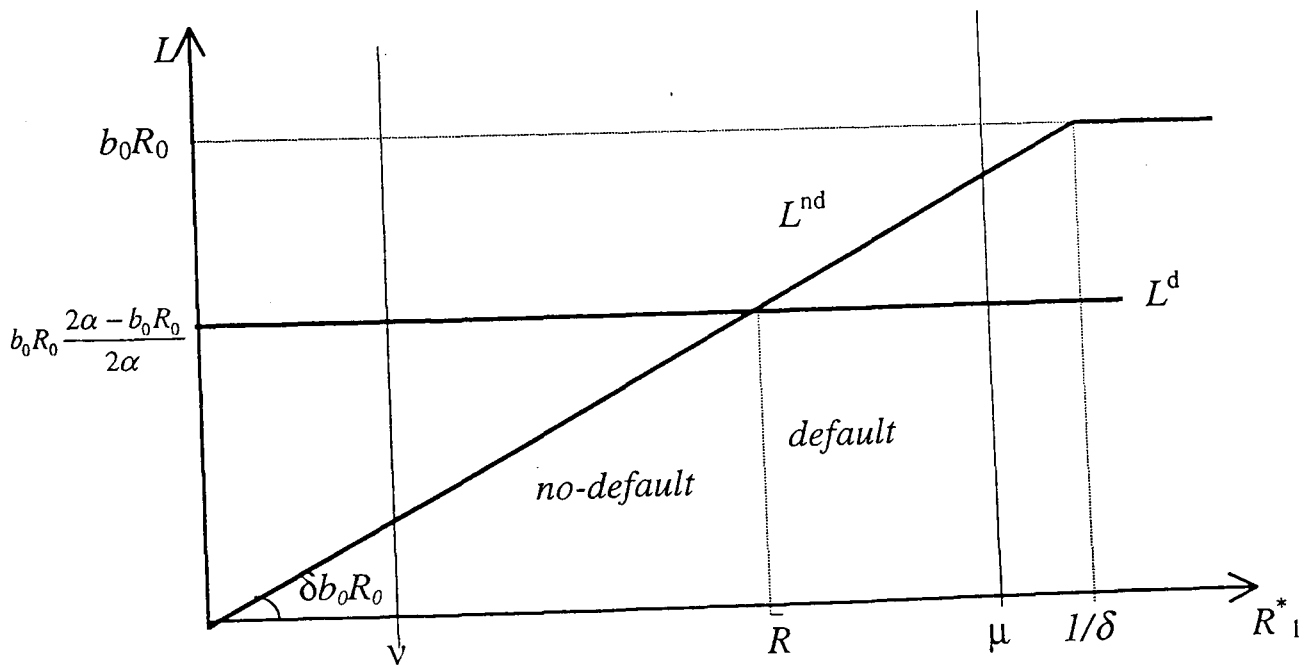
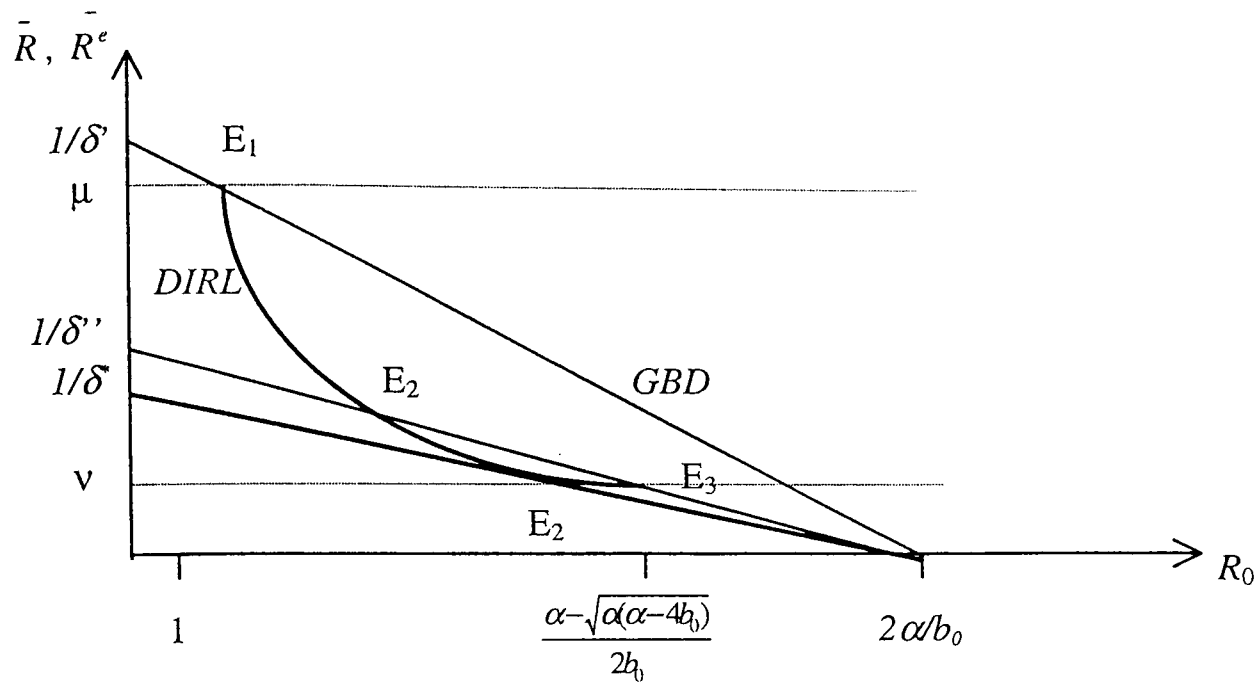


Figure 2.3: EQUILIBRIA
(only default case)

(a) $4b_0 \leq \alpha < \alpha^*$



(b) $\alpha \geq \alpha^*$

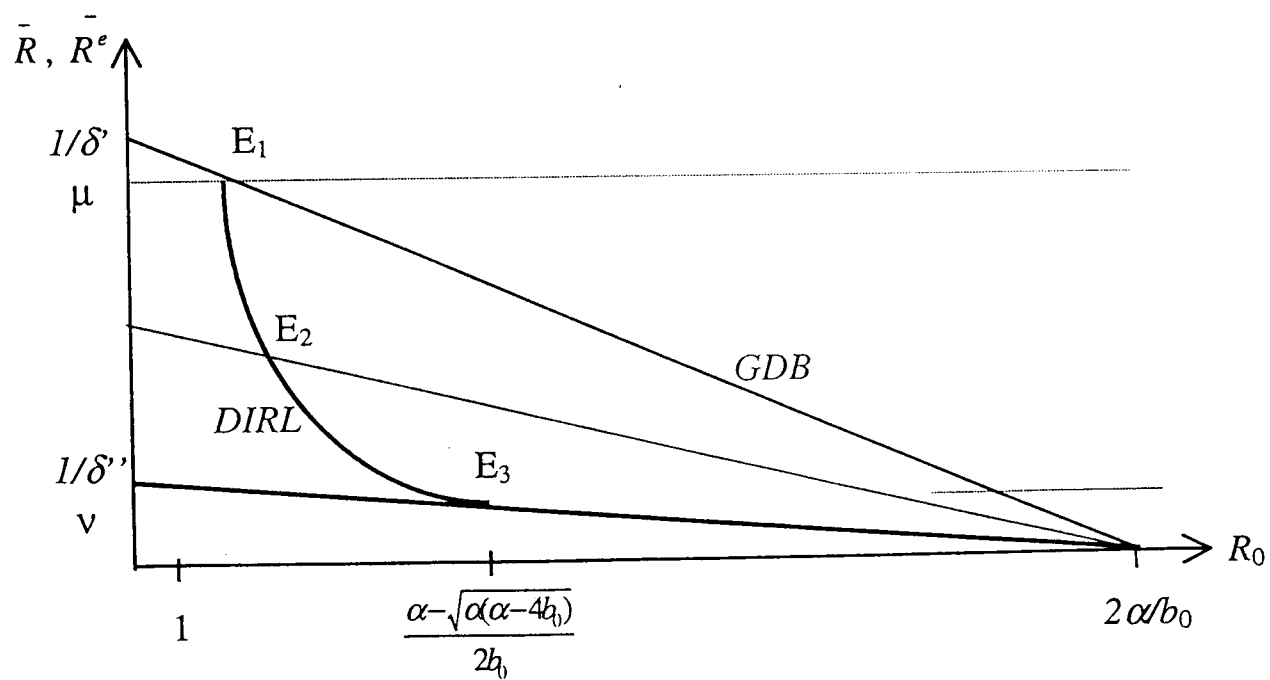


Figure 2.4: THE LOSS FUNCTION COMPARISON
(default and inflation case)

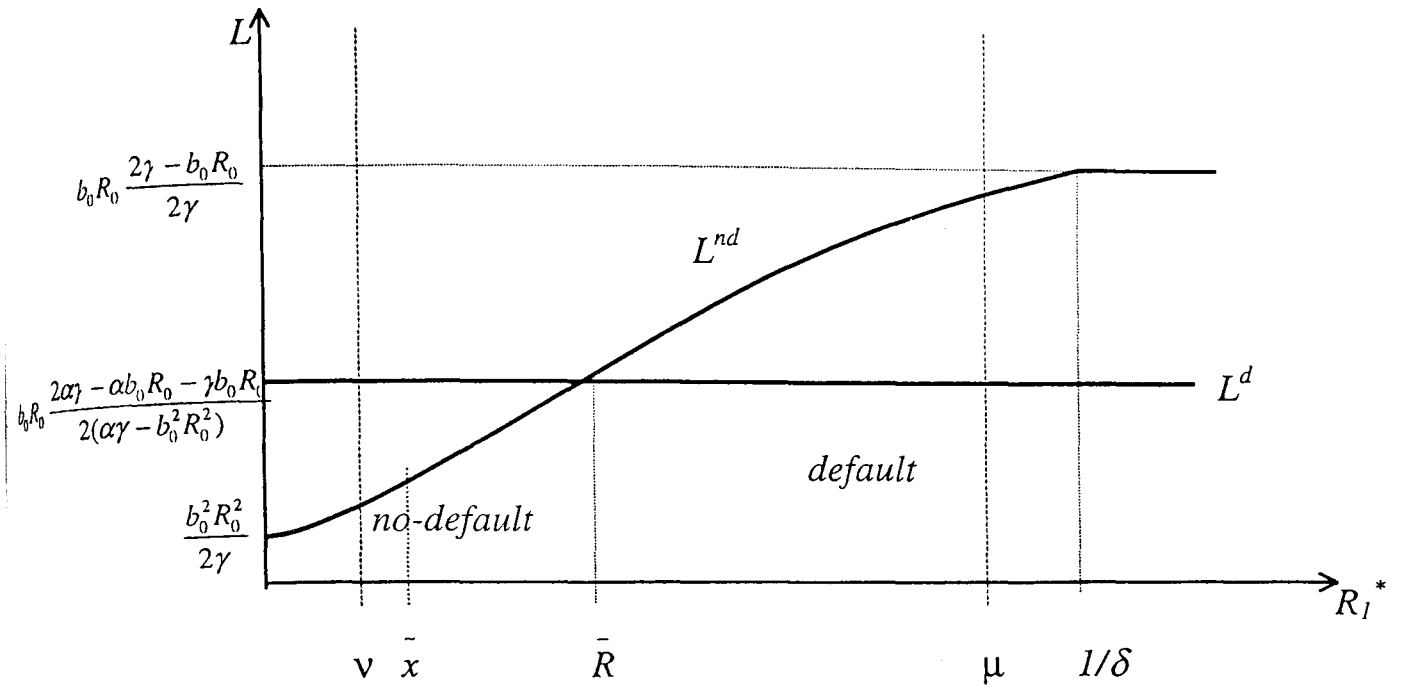
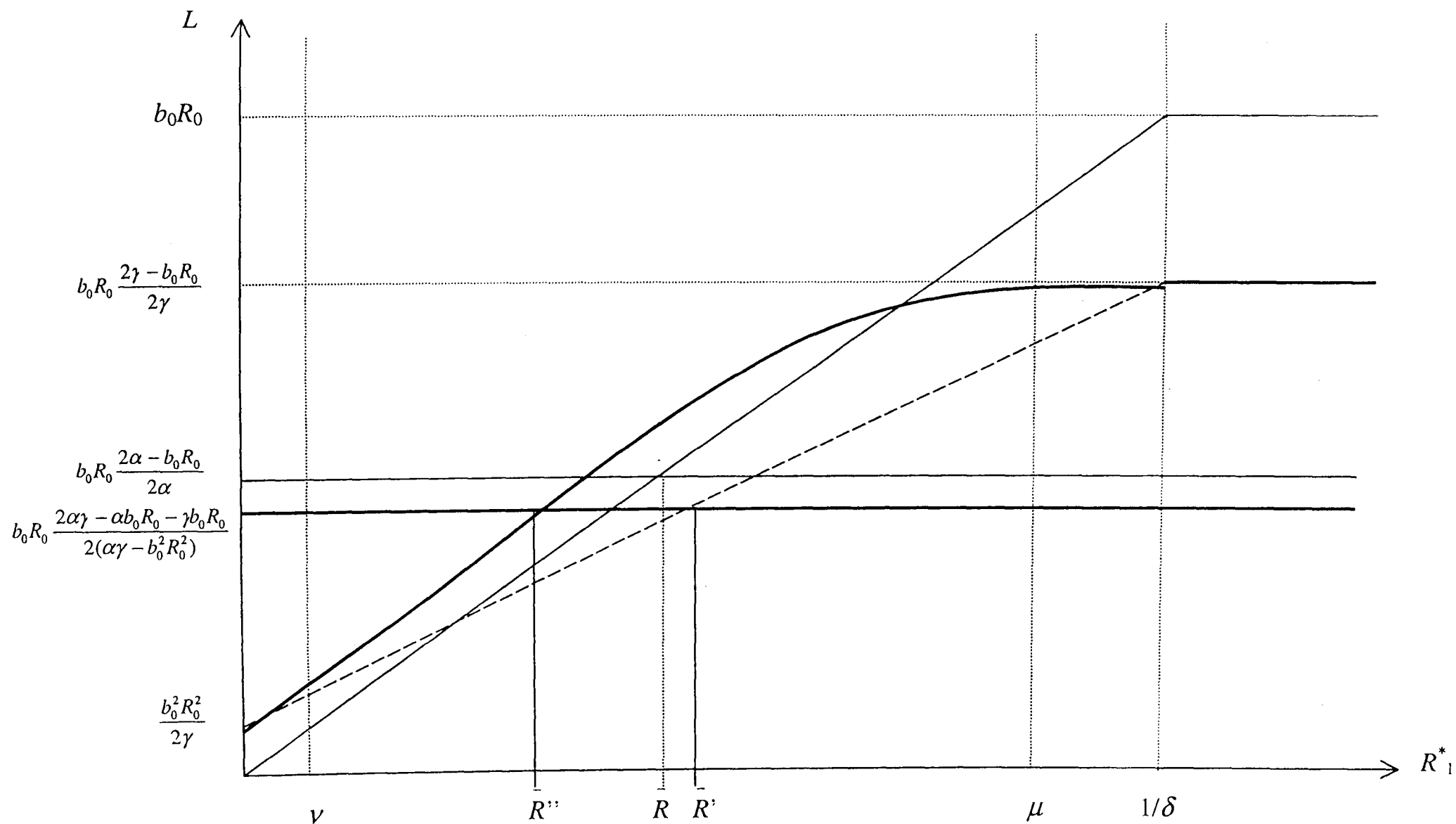


Figure 2.5: COMPARISON OF THE TWO MODEL (WITH AND WITHOUT INFLATION)



- L when only default is allowed
- L when both default and inflation are allowed
- - - L when inflation is allowed only in period 1

Table 2.1: EQUILIBRIA

	$4b_0 \leq \alpha < \alpha^*$	$\alpha \geq \alpha^*$
$\delta \leq \delta^*$	unique equilibrium: $E_1 (p=0, R_0^E = 1, \bar{R}^E = \mu)$	unique equilibrium: $E_1 (p=0, R_0^E = 1, \bar{R}^E = \mu)$
$\delta' < \delta < \delta''$	unique equilibrium: $E_2 (0 < p < 1, 1 < R_0^E < 2, v < \bar{R}^E < \mu)$	unique equilibrium: $E_2 (0 < p < 1, 1 < R_0^E < 2, v < \bar{R}^E < \mu)$
$\delta = \delta''$	2 equilibria: $E_2 (0 < p < 1, 1 < R_0^E < 2, v < \bar{R}^E < \mu)$ $E_3 (p=1, R_0^E = \frac{\alpha - \sqrt{\alpha(\alpha - 4b_0)}}{2b_0}, \bar{R}^E = v)$	
$\delta' < \delta < \delta^*$	3 equilibria: $E_2 (0 < p < 1, 1 < R_0^E < 2, v < \bar{R}^E < \mu)$ $E_2' (0 < p < 1, 1 < R_0^E < 2, v < \bar{R}^E < \mu)$ $E_3 (p=1, R_0^E = \frac{\alpha - \sqrt{\alpha(\alpha - 4b_0)}}{2b_0}, \bar{R}^E = v)$	unique equilibrium: $E_3 (p=1, R_0^E = \frac{\alpha - \sqrt{\alpha(\alpha - 4b_0)}}{2b_0}, \bar{R}^E = v)$
$\delta = \delta^*$	2 equilibria: $E_2 (0 < p < 1, 1 < R_0^E < 2, v < \bar{R}^E < \mu)$ $E_3 (p=1, R_0^E = \frac{\alpha - \sqrt{\alpha(\alpha - 4b_0)}}{2b_0}, \bar{R}^E = v)$	
$\delta > \delta^*$	unique equilibrium: $E_3 (p=1, R_0^E = \frac{\alpha - \sqrt{\alpha(\alpha - 4b_0)}}{2b_0}, \bar{R}^E = v)$	

$\delta^* = \frac{2\alpha - b_0}{2\alpha\mu}$ is the value of the parameter δ corresponding to *GDB* cutting *DIRL* at the point $(R_0^E = 1, \bar{R}^E = \mu)$

$\delta'' = \frac{3\alpha + \sqrt{\alpha(\alpha - 4b_0)}}{4\alpha v}$ is the value of the parameter δ corresponding to *GDB* cutting *DIRL* at the point

$$(R_0^E = \frac{\alpha - \sqrt{\alpha(\alpha - 4b_0)}}{2b_0}, \bar{R}^E = v)$$

δ^* is the value of the parameter δ at which *GDB* is tangent to *DIRL*, and α^* is the value of α at which $\delta^* \equiv \delta''$

SOME NUMERICAL EXAMPLES

Table 2.2

	MODEL WITH DEFAULT ONLY ($b_0=1; v=1; \mu=2$)				MODEL WITH DEFAULT AND INFLATION ($b_0=1; v=1; \mu=2$)						
	Equilibria	θ	p	p θ	Equilibria	θ	p	p θ	RP	z_1^d	z_1^{nd}
$\alpha=4$ $\gamma=12$ $\delta=0.4$	$\bar{R}^E=2$ $R_0^E=1$	--	0	0	$\bar{R}^E=1.93$ $R_0^E=1.12$	0.261	0.073	0.019	0.110	0.069	0.094
$\alpha=5$ $\gamma=5$ $\delta=0.75$	$\bar{R}^E=1.16$ $R_0^E=1.27$	0.254	0.836	0.212	$\bar{R}^E=1$ $R_0^E=1.91$	0.276	1	0.276	0.476	0.276	--
$\alpha=7$ $\gamma=15$ $\delta=0.5$	$\bar{R}^E=1.85$ $R_0^E=1.02$	0.146	0.146	0.021	$\bar{R}^E=1.71$ $R_0^E=1.13$	0.151	0.286	0.043	0.112	0.064	0.075
$\alpha=10$ $\gamma=8$ $\delta=0.3$	$\bar{R}^E=2$ $R_0^E=1$	--	0	0	$\bar{R}^E=2$ $R_0^E=1.17$	--	0	0	0.146	--	0.146
$\alpha=10$ $\gamma=9$ $\delta=0.5$	$\bar{R}^E=1.90$ $R_0^E=1.01$	0.101	0.101	0.010	$\bar{R}^E=1.69$ $R_0^E=1.18$	0.104	0.305	0.032	0.156	0.118	0.132
$\alpha=15$ $\gamma=7$ $\delta=0.45$	$\bar{R}^E=2$ $R_0^E=1$	--	0	0	$\bar{R}^E=1.90$ $R_0^E=1.22$	0.068	0.102	0.007	0.178	0.162	0.174
$\alpha=15$ $\gamma=25$ $\delta=0.75$	$\bar{R}^E=1.29$ $R_0^E=1.05$	0.070	0.713	0.050	$\bar{R}^E=1.25$ $R_0^E=1.10$	0.070	0.751	0.053	0.092	0.041	0.044

Table 2.3

	MODEL WITH DEFAULT ONLY ($b_0=1; v=1; \mu=1.2$)				MODEL WITH DEFAULT AND INFLATION ($b_0=1; v=1; \mu=1.2$)						
	Equilibria	θ	p	p θ	Equilibria	θ	p	p θ	RP	z_1^d	z_1^{nd}
$\alpha=4$ $\gamma=10$ $\delta=0.7$	$\bar{R}^E=1.2$ $R_0^E=1$	--	0	0	$\bar{R}^E=1.13$ $R_0^E=1.26$	0.287	0.363	0.104	0.208	0.09	0.126
	$\bar{R}^E=1.01$ $R_0^E=1.89$				0.420	0.954	0.401	0.470	0.109	0.189	
	$\bar{R}^E=1$ $R_0^E=2.25$				0.5	1	0.5	0.556	0.113	--	
$\alpha=4$ $\gamma=15$ $\delta=0.75$	$\bar{R}^E=1.16$ $R_0^E=1.06$	0.265	0.218	0.058	$\bar{R}^E=1$ $R_0^E=2.15$	0.5	1	0.5	0.536	0.072	--
	$\bar{R}^E=1$ $R_0^E=2$	0.5	1	0.5							
$\alpha=5$ $\gamma=6$ $\delta=0.65$	$\bar{R}^E=1.2$ $R_0^E=1$	--	0	0	$\bar{R}^E=1.19$ $R_0^E=1.29$	0.214	0.074	0.016	0.225	0.169	0.215
$\alpha=15$ $\gamma=10$ $\delta=0.85$	$\bar{R}^E=1.14$ $R_0^E=1.02$	0.068	0.318	0.022	$\bar{R}^E=1.07$ $R_0^E=1.18$	0.070	0.661	0.046	0.154	0.110	0.118

Figure 3.1

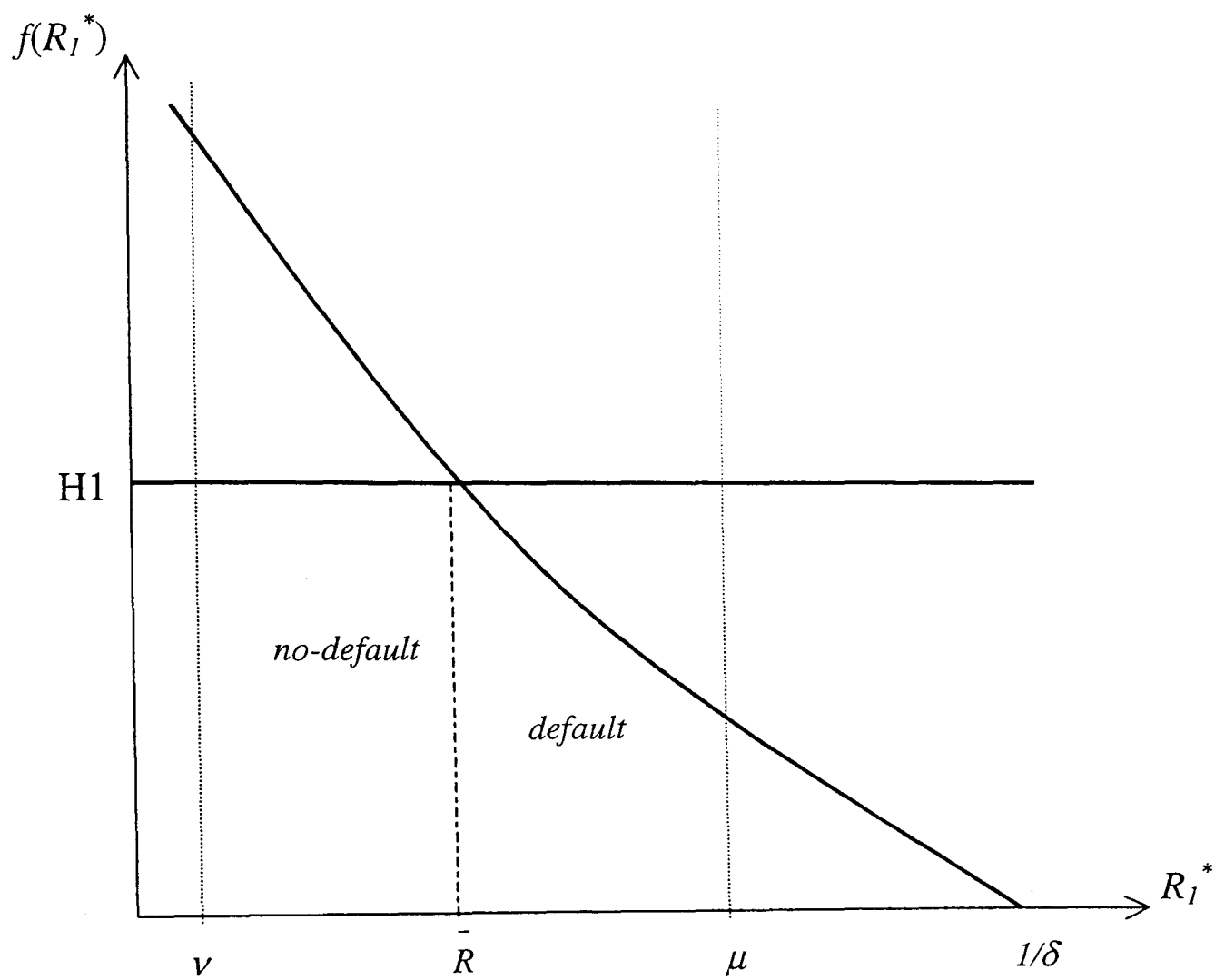


Figure 3.2

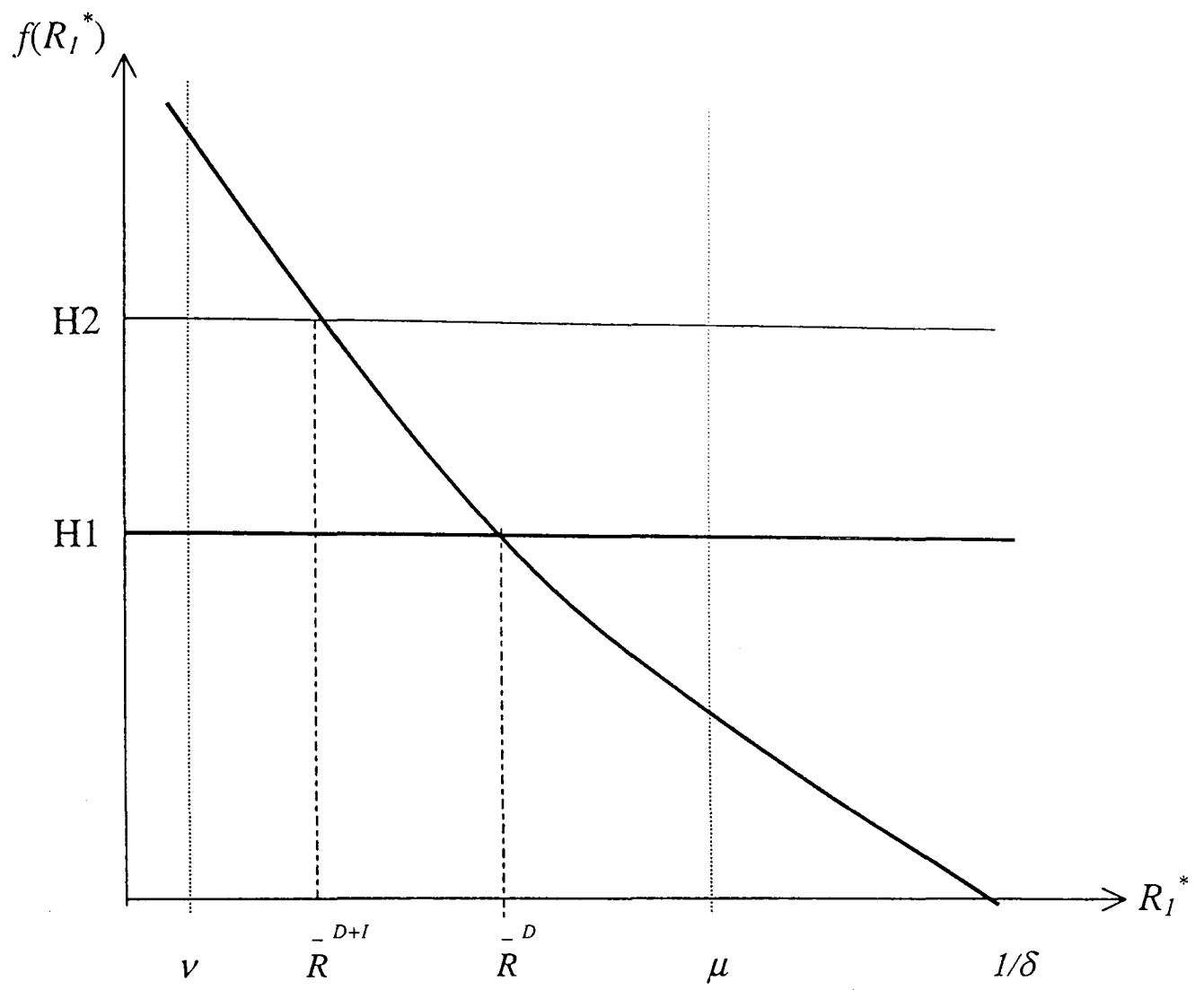


Figure 4.1
Spreads on 10-years benchmark bonds with respect to German rates

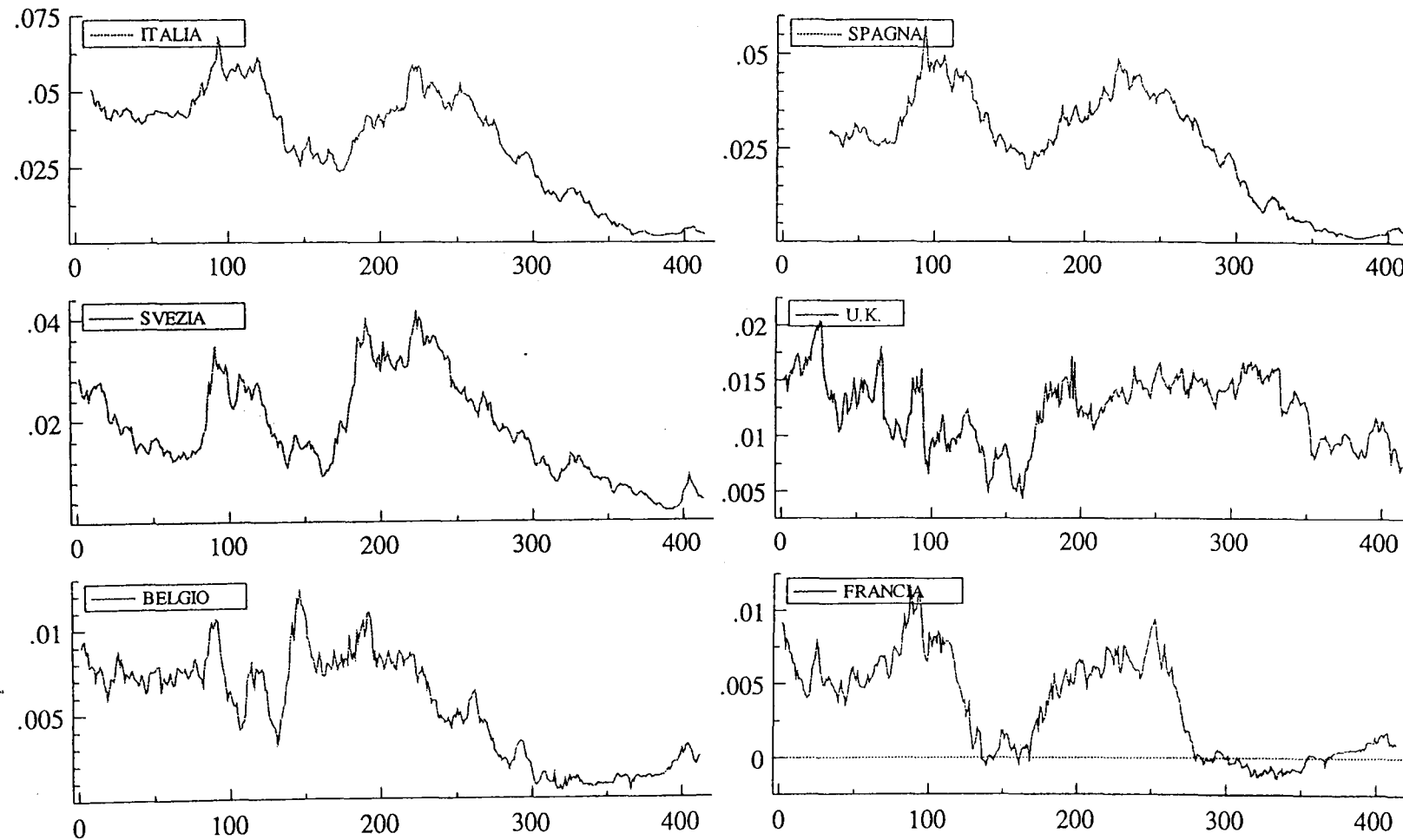


Figure 4.2
Exchange rate premia on 10-years swap contracts

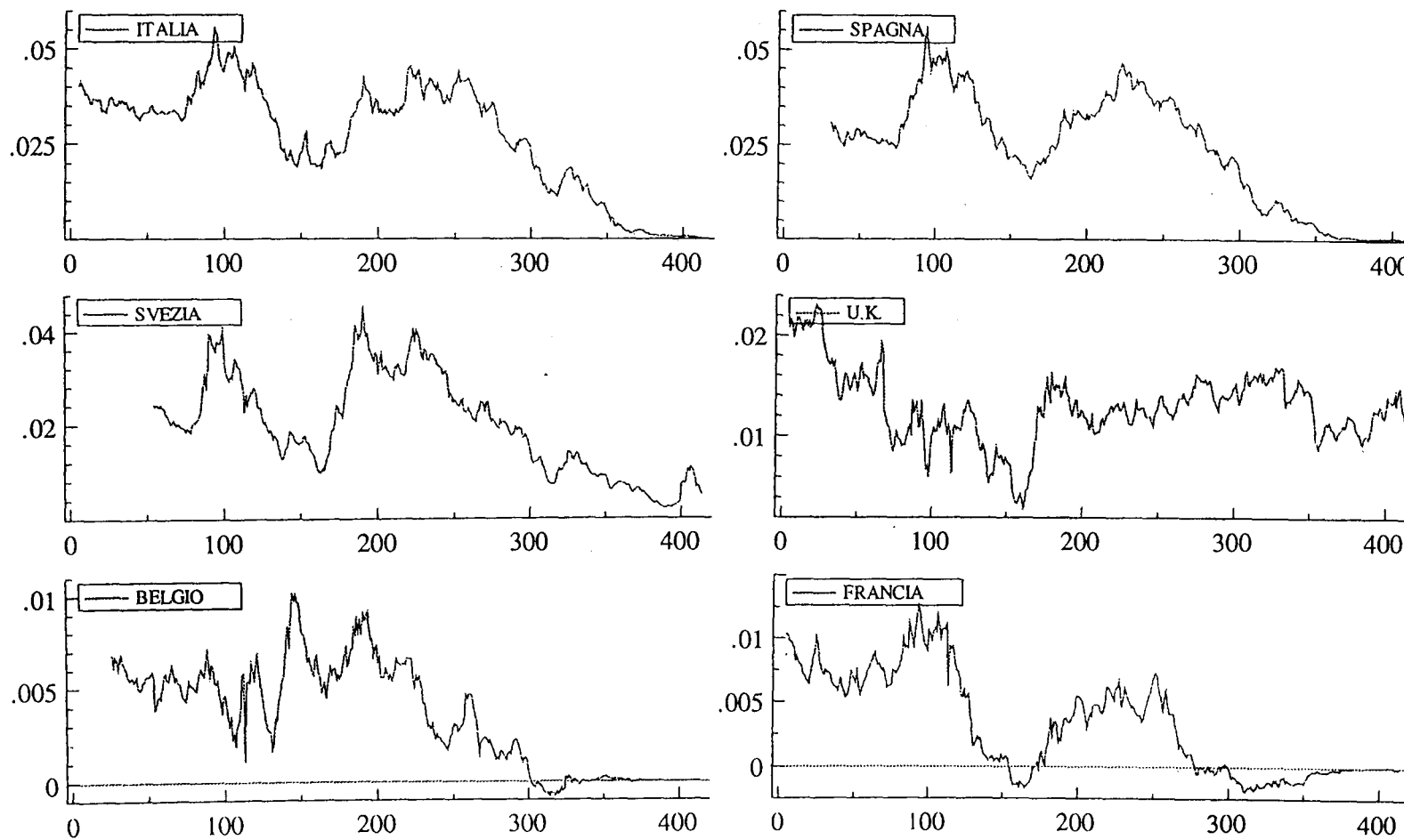


Figure 4.3
Risk premia on 10-years bonds

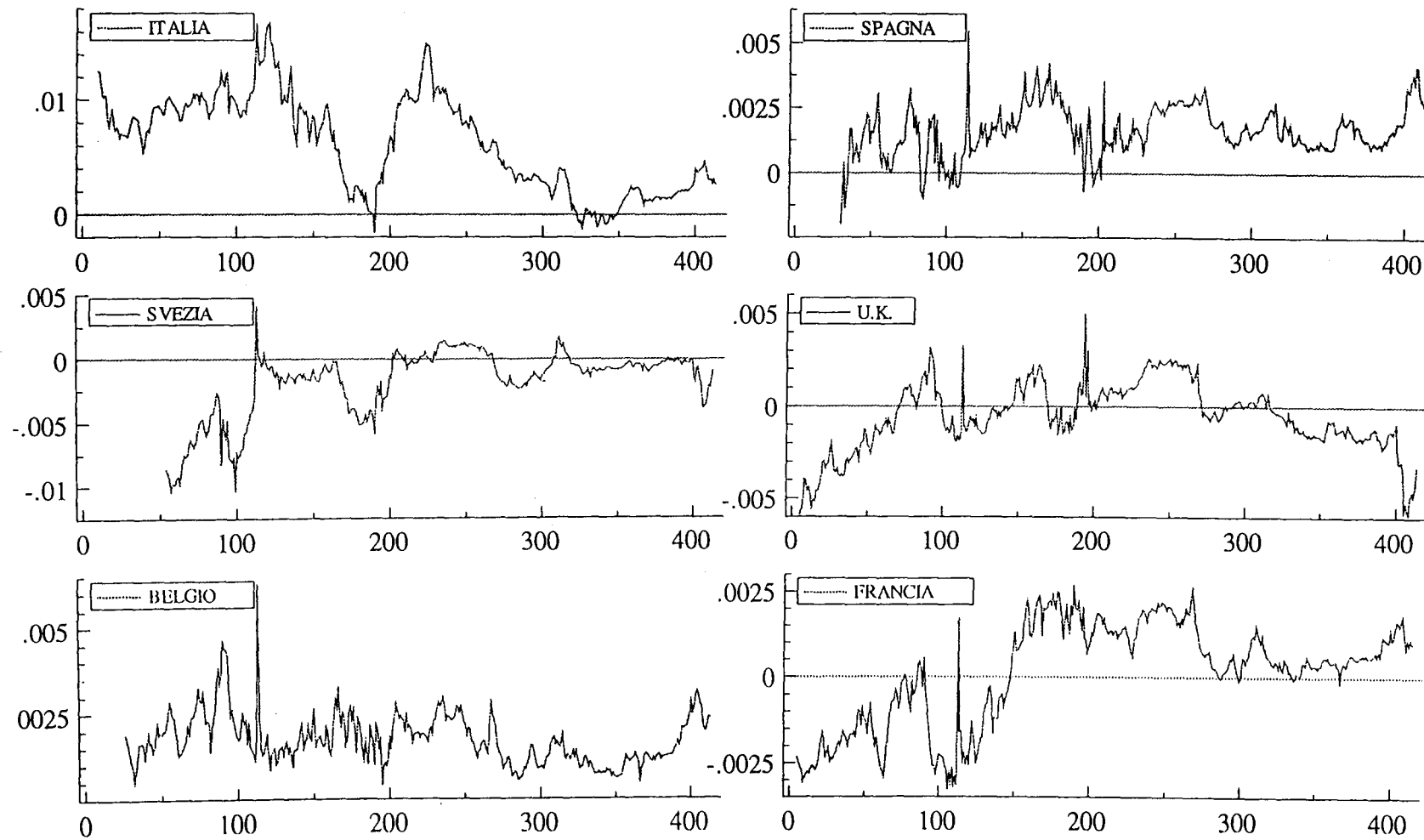


Table 4.1 Autoregressive Models

Belgium

EQ(A1) Modelling RP_be by OLS (using ISBar2PCG.xls)

The present sample is: 3 to 347

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
Constant	0,00022017	9,18E-05	2,4	0,017	6,65E-05
RP_be1	0,98316	0,10193	9,645	0	0,093857
RP_be2	-0,14525	0,084363	-1,722	0,086	0,064925
d(6.528)	0,00020191	0,00013517	1,494	0,1362	0,00016444
dRP_be1	-0,44508	0,1197	-3,718	0,0002	0,19572
dRP_be2	0,40659	0,10882	3,736	0,0002	0,17858

R² = 0.681327 F(5,339) = 144.96 [0.0000] \sigma = 0.000427257 DW = 2.03
 RSS = 6.188406129e-005 for 6 variables and 345 observations

Information Criteria:

SC = -15.4322 HQ = -15.4724 FPE=1.85724e-007 AIC = -15.499

AR 1- 2 F(2,337) = 0.95355 [0.3864]
 ARCH 1 F(1,337) = 3.6594 [0.0566]
 Normality Chi²(2)= 218.56 [0.0000] **
 Xi² F(9,329) = 1.0081 [0.4332]
 Xi*Xj F(11,327) = 0.88033 [0.5600]
 RESET F(1,338) = 2.4246 [0.1204]

Wald test for linear restrictions: Subset

LinRes F(3,339) = 8.5467 [0.0000] **

Zero restrictions on:

d(6.528) dRP_be1 dRP_be2

2 regimes

Regime a - for high German interest rate

the present sample is: 3 to 183

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
constant	0,00042208	0,00012335	3,422	0,0008	0,00015034
β_{be_a1}	0,53808	0,077984	6,9	0	0,17167
β_{be_a2}	0,26134	0,085426	3,059	0,0026	0,16629

Regime b - for low German interest rate

the present sample is: 184 to 347

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
constant	0,00022017	5,79E-05	3,803	0,0002	6,65E-05
β_{be_b1}	0,98316	0,064317	15,286	0	0,0939
β_{be_b2}	-0,14525	0,053231	-2,729	0,0071	0,064955

=6.528

lnRes F(3,339) = 8.5467 [0.0000] **

Italy

EQ(A2) Modelling RP_it by OLS (using ISBar2PCG.xls)

The present sample is: 4 to 347

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
Constant	1,64E-05	7,71E-05	0,213	0,8318	5,33E-05
RP_it1	1,1676	0,066528	17,55	0	0,1327
RP_it2	-0,32959	0,10049	-3,28	0,0011	0,13273
RP_it3	0,14084	0,066032	2,133	0,0337	0,071515
d(7.0585)	0,00079512	0,00025313	3,141	0,0018	0,00048484
dRP_it1	-0,30205	0,11287	-2,676	0,0078	0,19006
dRP_it2	0,34796	0,15773	2,206	0,0281	0,1747
dRP_it3	-0,096982	0,11118	-0,872	0,3837	0,10488

R² = 0.970349 F(7,336) = 1570.8 [0.0000] \sigma = 0.000751611 DW = 2.04
RSS = 0.0001898126669 for 8 variables and 344 observations

Information Criteria:

SC = -14.2743 HQ = -14.328 FPE=5.78056e-007 AIC = -14.3636

AR 1- 2 F(2,334) = 2.7315 [0.0666]
ARCH 1 F(1,334) = 39.298 [0.0000] **
Normality Chi²(2)= 187.24 [0.0000] **
Xi² F(13,322) = 7.5344 [0.0000] **
Xi*Xj F(19,316) = 6.4111 [0.0000] **
RESET F(1,335) = 0.41904 [0.5179]

Wald test for linear restrictions: Subset

LinRes F(4,336) = 5.9932 [0.0001] **

Zero restrictions on:

d(7.0585) dRP_it1 dRP_it2 dRP_it3

2 regimes

Regime a - for high German interest rate

The present sample is: 4 to 112

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
Constant	0,00081152	0,00025422	3,192	0,0019	0,00048525
RP_it_1a	0,8655	0,096145	9,002	0	0,137
RP_it_2a	0,018368	0,12819	0,143	0,8863	0,11438
RP_it_3a	0,043861	0,094324	0,465	0,6429	0,07725

Regime b - for low German interest rate

The present sample is: 113 to 347

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
Constant	1,64E-05	7,52E-05	0,218	0,8275	5,32E-05
RP_it_1b	1,1676	0,064814	18,014	0	0,13228
RP_it_2b	-0,32959	0,0979	-3,367	0,0009	0,13231
RP_it_3b	0,14084	0,064331	2,189	0,0296	0,071288

r=7.0585

LinRes F(4,336) = 5.9932 [0.0001] **

Spain

EQ(A3) Modelling RP_sp by OLS (using ISBar2PCG.xls)

The present sample is: 3 to 347

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
Constant	0,00018802	0,00011839	1,588	0,1132	7,67E-05
RP_sp1	1,0812	0,11027	9,805	0	0,12523
RP_sp2	-0,17853	0,10504	-1,7	0,0901	0,11729
d(6.5940)	0,00017919	0,00014101	1,271	0,2047	0,00013306
dRP_sp1	-0,54487	0,12519	-4,352	0	0,16868
dRP_sp2	0,36458	0,12227	2,982	0,0031	0,16021

R² = 0.650759 F(5,339) = 126.34 [0.0000] \sigma = 0.000557804 DW = 2.02
RSS = 0.0001054782637 for 6 variables and 345 observations

Information Criteria:

SC = -14.8989 HQ = -14.9391 FPE=3.16557e-007 AIC = -14.9658

AR 1- 2 F(2,337) = 0.36721 [0.6929]
ARCH 1 F(1,337) = 6.4938 [0.0113] *
Normality Chi²(2)= 218.51 [0.0000] **
Xi² F(9,329) = 1.8541 [0.0581]
Xi*Xj F(11,327) = 2.0586 [0.0229] *
RESET F(1,338) = 5.2885 [0.0221] *

Wald test for linear restrictions: Subset

LinRes F(3,339) = 7.8098 [0.0000] **

Zero restrictions on:

d(6.5940) dRP_sp1 dRP_sp2

2 regimes

Regime a - for high German interest rate

The present sample is: 3 to 166

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
Constant	0,00036722	0,00010217	3,594	0,0004	0,00010876
RP_sp_a1	0,53636	0,079073	6,783	0	0,11306
RP_sp_a2	0,18605	0,083486	2,229	0,0272	0,10918

Regime b - for low German interest rate

The present sample is: 167 to 347

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
Constant	0,00018802	6,43E-05	2,923	0,0039	7,67E-05
RP_sp_b1	1,0812	0,059908	18,048	0	0,12518
RP_sp_b2	-0,17853	0,057069	-3,128	0,0021	0,11724

r=6.594

LinRes F(3,339) = 7.8098 [0.0000] **

**Table 4.2 Autoregressive Models
(first differences)**

Italy

EQ(D1) Modelling DRP_it by OLS (using ITar2-fdPCG.xls)

The present sample is: 3 to 346

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
Constant	-9,51E-05	5,02E-05	-1,894	0,0591	5,41E-05
DRP_it1	0,1837	0,066838	2,748	0,0063	0,13035
DRP_it2	-0,14792	0,066565	-2,222	0,0269	0,072099
d(7.0624)	0,00027624	9,07E-05	3,046	0,0025	0,00010304
dDRP_it1	-0,26506	0,11236	-2,359	0,0189	0,18064
dDRP_it2	0,089671	0,11254	0,797	0,4261	0,1067

R² = 0.0575523 F(5,338) = 4.1281 [0.0012] \sigma = 0.000761874 DW = 2.02
RSS = 0.0001961928673 for 6 variables and 344 observations

Information Criteria:

SC = -14.2752 HQ = -14.3155 FPE=5.90576e-007 AIC = -14.3422

AR 1-2 F(2,336) = 1.34 [0.2632]
ARCH 1 F(1,336) = 26.534 [0.0000] **
Normality Chi²(2)= 214.04 [0.0000] **
Xi² F(9,328) = 5.9914 [0.0000] **
Xi*Xj F(11,326) = 5.5811 [0.0000] **
RESET F(1,337) = 0.0080815 [0.9284]

Wald test for linear restrictions: Subset

LinRes F(3,338) = 4.9106 [0.0024] **

Zero restrictions on:

d(7.0624) dDRP_it1 dDRP_it2

2 regimes

Regime a - for high German interest rate

The present sample is: 3 to 110

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
Constant	0,00018117	8,10E-05	2,235	0,0275	8,81E-05
DRP_it_a1	-0,081167	0,096936	-0,837	0,4043	0,12558
DRP_it_a2	-0,058592	0,09739	-0,602	0,5487	0,079115

Regime b - for low German interest rate

The present sample is: 111 to 346

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
Constant	-9,51E-05	4,85E-05	-1,962	0,0509	5,40E-05
DRP_it_b1	0,18378	0,064509	2,849	0,0048	0,13001
DRP_it_b2	-0,14791	0,064246	-2,302	0,0222	0,071879

r=7.0624

LinRes F(3,338) = 4.9106 [0.0024] **

**Table 4.3 Autoregressive Models
with Macro Variables**

Belgium

EQ(M1) Modelling RP(BE) by OLS (using bea1.xls)

The present sample is: 2 to 347

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
Constant	-0,0026674	0,0014259	-1,871	0,0623	0,001153
RP1	0,55449	0,063513	8,73	0	0,12368
DIIP(BE)	-4,88E-06	7,25E-06	-0,674	0,501	4,81E-06
UnA(BE)	-8,19E-06	7,58E-06	-1,08	0,2807	4,99E-06
Un(BE)	0,0001247	0,0001275	0,978	0,3287	9,10E-05
def(BE)	0,00026486	9,02E-05	2,936	0,0036	0,00010254
Infl(GR)	-0,017114	0,011328	-1,511	0,1318	0,01014
DIIP(GR)	4,72E-06	2,47E-05	0,191	0,8488	1,41E-05
Un(GR)	0,00014488	7,18E-05	2,018	0,0444	6,58E-05
d(6.63224)	0,0024082	0,0015543	1,549	0,1223	0,0013013
dRP1	0,22099	0,079429	2,782	0,0057	0,14125
dDIIP(BE)	-3,60E-05	1,50E-05	-2,391	0,0174	3,45E-05
dUnA(BE)	-1,44E-05	1,19E-05	-1,216	0,2247	1,18E-05
dUn(BE)	0,00017428	0,00021991	0,793	0,4286	0,0002604
ddef(BE)	-0,0003517	0,00010686	-3,291	0,0011	0,00011703
dInfl(GR)	0,032152	0,01289	2,494	0,0131	0,013991
dDIIP(GR)	-0,0001439	4,96E-05	-2,904	0,0039	0,0001084
dUn(GR)	-0,0003807	0,00025505	-1,493	0,1365	0,00025617

R² = 0.717889 F(17,328) = 49.098 [0.0000] \sigma = 0.000408697 DW = 2.04
RSS = 5.478689451e-005 for 18 variables and 346 observations

Information Criteria:

SC = -15.3543 HQ = -15.4748 FPE=1.75723e-007 AIC = -15.5545

AR 1- 2 F(2,326) = 0.64687 [0.5244]
ARCH 1 F(1,326) = 2.6603 [0.1038]
Normality Chi²(2)= 201.37 [0.0000] **
Xi² F(33,294) = 1.8637 [0.0038] **
RESET F(1,327) = 2.4972 [0.1150]

Wald test for linear restrictions: Subset

LinRes F(9,328) = 5.8239 [0.0000] **

Zero restrictions on:

d(6.63224) dRP1 dDIIP(BE) dUn1(BE) dUn(BE) ddef(BE) dInfl(GR) dDIIP(GR) dUn(GR)

2 regimes

Regime a - for high German interest rate

The present sample is:2 to 161

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
Constant	-0,0002592	0,00076737	-0,338	0,736	0,00060475
RP1_a	0,77548	0,059148	13,111	0	0,068367
DIIP(BE)_a	-4,085E-05	0,000016351	-2,498	0,0135	3,4207E-05
UnA(BE)_a	-2,261E-05	0,000011298	-2,001	0,0472	1,0757E-05
Un(BE)_a	0,00029898	0,00022219	1,346	0,1804	0,00024454
def(BE)_a	-8,68E-05	0,000071036	-1,222	0,2237	5,6545E-05
Infl(GR)_a	0,015038	0,0076259	1,972	0,0504	0,0096619
DIIP(GR)_a	-0,0001392	0,000053255	-2,614	0,0099	0,00010772
Un(GR)_a	-0,0002358	0,00030349	-0,777	0,4383	0,00024812

Regime b - for low German interest rate

The present sample is:162 to 347

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
Constant	-0,0026674	0,0010489	-2,543	0,0119	0,0011508
RP1_b	0,55449	0,046724	11,867	0	0,12345
DIIP(BE)_b	-4,882E-06	5,3308E-06	-0,916	0,361	4,8019E-06
UnA(BE)_b	-8,19E-06	5,5763E-06	-1,469	0,1437	4,9789E-06
Un(BE)_b	0,0001247	0,000093793	1,33	0,1854	9,0805E-05
def(BE)_b	0,00026486	0,000066361	3,991	0,0001	0,00010234
Infl(GR)_b	-0,017114	0,0083336	-2,054	0,0415	0,010121
DIIP(GR)_b	4,7207E-06	0,000018198	0,259	0,7956	0,00001408
Un(GR)_b	0,00014488	0,000052822	2,743	0,0067	0,00006571

r=6.63224

LinRes F(9,328) = 5.8239 [0.0000] **

Italy

EQ(M2) Modelling RP(IT) by OLS (using ita1.xls)

The present sample is: 4 to 347

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
Constant	-0,0011216	0,0043025	-0,261	0,7945	0,002977
RP1	1,0647	0,065623	16,224	0	0,12656
RP2	-0,27675	0,095135	-2,909	0,0039	0,11859
RP3	0,079853	0,06457	1,237	0,2171	0,069343
lnflA(IT)	-0,0072886	0,0027751	-2,626	0,009	0,0026884
lnfl(IT)	0,0017394	0,0099447	0,175	0,8613	0,0093136
lIP(IT)	2,91E-05	2,11E-05	1,383	0,1677	2,00E-05
fp(IT)	-0,0001499	6,81E-05	-2,202	0,0284	6,33E-05
lnfl(GR)	0,034386	0,010787	3,188	0,0016	0,010701
lIP(GR)	9,70E-05	3,68E-05	2,635	0,0088	4,25E-05
Un(GR)	-0,0006065	0,00019573	-3,099	0,0021	0,00022086
d(7.0868)	0,013979	0,0091597	1,526	0,128	0,0081164
dRP1	-0,35313	0,1134	-3,114	0,002	0,19493
dRP2	0,30612	0,15042	2,035	0,0427	0,16497
dRP3	0,068167	0,11108	0,614	0,5399	0,11176
dlnflA(IT)	0,0089321	0,0043451	2,056	0,0406	0,0039906
dlnfl(IT)	-0,076192	0,047008	-1,621	0,106	0,04561
dIIP(IT)	0,00010601	4,15E-05	2,556	0,011	5,35E-05
dfp(IT)	-0,0003945	0,00018465	-2,137	0,0334	0,00016777
dlnfl(GR)	-0,033	0,014377	-2,295	0,0224	0,01401
dIIP(GR)	-9,51E-05	9,47E-05	-1,004	0,3161	9,71E-05
dUn(GR)	0,00081471	0,00047294	1,723	0,0859	0,00036792

R² = 0.974663 F(21,322) = 589.85 [0.0000] \sigma = 0.000709721 DW = 2.08
 RSS = 0.0001621927084 for 22 variables and 344 observations

Information Criteria:

SC = -14.1938 HQ = -14.3416 FPE=5.35918e-007 AIC = -14.4395

AR 1- 2 F(2,320) = 4.0338 [0.0186] *
 ARCH 1 F(1,320) = 44.627 [0.0000] **
 Normality Chi²(2)= 152.5 [0.0000] **
 Xi² F(41,280) = 4.8859 [0.0000] **
 RESET F(1,321) = 2.504 [0.1145]

Wald test for linear restrictions: Subset

LinRes F(11,322) = 5.3039 [0.0000] **

Zero restrictions on:

d(7.0868) dRP1 dRP2 dRP3 dlnfl1(IT) dlnfl(IT) dIIP(IT) dfp(IT) dlnfl(GR) dIIP(GR) dUn(GR)

2 regimes

Regime a - for high German interest rate

The present sample is:4 to 110

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
Constant	0,012857	0,0084903	1,514	0,1332	0,0077125
RP1_a	0,71153	0,097098	7,328	0	0,15144
RP2_a	0,029366	0,12234	0,24	0,8108	0,11713
RP3_a	0,14802	0,0949	1,56	0,1221	0,089518
InflA(IT)_a	0,0016435	0,0035104	0,468	0,6407	0,0030123
Infl(IT)_a	-0,074452	0,04824	-1,543	0,126	0,045605
IIP(IT)_a	0,00013512	0,000037519	3,601	0,0005	5,0716E-05
fp(IT)_a	-0,0005444	0,00018022	-3,021	0,0032	0,00015868
Infl(GR)_a	0,0013864	0,0009979	0,139	0,8898	0,0092375
IIP(GR)_a	1,9268E-06	0,0000916	0,021	0,9833	8,9166E-05
Un(GR)_a	0,00020819	0,00045204	0,461	0,6462	0,00030057

Regime b - for low German interest rate

The present sample is:111 to 347

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
Constant	-0,0011216	0,0042079	-0,267	0,7901	0,0029495
RP1_b	1,0647	0,06418	16,589	0	0,12539
RP2_b	-0,27675	0,093043	-2,974	0,0033	0,1175
RP3_b	0,079853	0,06315	1,264	0,2074	0,068702
InflA(IT)_b	-0,0072886	0,0027141	-2,685	0,0078	0,0026636
Infl(IT)_b	0,0017394	0,0009726	0,179	0,8582	0,0092275
IIP(IT)_b	0,00002911	0,000020589	1,414	0,1588	1,9829E-05
fp(IT)_b	-0,0001499	0,000066565	-2,252	0,0253	6,2738E-05
Infl(GR)_b	0,034386	0,01055	3,259	0,0013	0,010602
IIP(GR)_b	9,7013E-05	0,000036013	2,694	0,0076	4,2121E-05
Un(GR)_b	-0,0006065	0,00019142	-3,168	0,0017	0,00021882

r=7.0868

LinRes F(11,322) = 5.3039 [0.0000] **

Spain

EQ(M3) Modelling RP(SP) by OLS (using spa1.xls)

The present sample is: 2 to 347

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
Constant	-0,013425	0,0044188	-3,038	0,0026	0,0044353
RP1	0,63626	0,066503	9,567	0	0,099986
Un(SP)	0,00021513	5,31E-05	4,052	0,0001	7,73E-05
fp(SP)	0,00023704	0,00011081	2,139	0,0331	7,63E-05
Infl(GR)	-0,013693	0,0080925	-1,692	0,0916	0,0056497
Un(GR)	0,00011223	7,46E-05	1,504	0,1336	6,78E-05
d(6.7782)	0,026065	0,0078211	3,333	0,001	0,012309
dRP1	-0,0009118	0,084713	-0,011	0,9914	0,13018
dUn(SP)	0,00012281	0,00014068	0,873	0,3833	0,00028905
dfp(SP)	-0,0006544	0,00023289	-2,81	0,0053	0,00038253
dInfl(GR)	0,032959	0,011459	2,876	0,0043	0,016635
dUn(GR)	-0,0007455	0,00033836	-2,203	0,0282	0,00059159

R² = 0.658305 F(11,334) = 58.498 [0.0000] \sigma = 0.000555861 DW = 2.11
 RSS = 0.0001031998093 for 12 variables and 346 observations

Information Criteria:

SC = -14.8225 HQ = -14.9028 FPE=3.19698e-007 AIC = -14.9559

AR 1- 2 F(2,332) = 1.5961 [0.2042]
 ARCH 1 F(1,332) = 18.979 [0.0000] **
 Normality Chi²(2)= 179.03 [0.0000] **
 Xi² F(21,312) = 3.5684 [0.0000] **
 RESET F(1,333) = 5.5506 [0.0191] *

Wald test for linear restrictions: Subset

LinRes F(6,334) = 5.5564 [0.0000] **

Zero restrictions on:

d(6.7782) dRP1 dUn(SP) dfp(SP) dInfl(GR) dUn(GR)

2 regimes

Regime a - for high German interest rate

The present sample is:2 to 138

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
Constant	0,01264	0,0092021	1,374	0,1719	0,011537
RP1_a	0,63535	0,074827	8,491	0	0,083768
Un(SP)_a	0,00033794	0,00018577	1,819	0,0712	0,00027984
fp(SP)_a	-0,0004173	0,0002921	-1,429	0,1555	0,00037662
Infl(GR)_a	0,019266	0,01157	1,665	0,0983	0,015721
Un(GR)_a	-0,0006333	0,0004706	-1,346	0,1807	0,00059048

Regime b - for low German interest rate

The present sample is:139 to 347

Variable	Coefficient	Std.Error	t-value	t-prob	HCSE
Constant	-0,013425	0,0025504	-5,264	0	0,0044217
RP1_b	0,63626	0,038384	16,576	0	0,099679
Un(SP)_b	0,00021513	0,000030644	7,02	0	0,00007709
fp(SP)_b	0,00023704	0,000063955	3,706	0,0003	7,6065E-05
Infl(GR)_b	-0,013693	0,0046708	-2,932	0,0038	0,0056323
Un(GR)_b	0,00011223	0,000043078	2,605	0,0099	6,7592E-05

r=6.7782

LinRes F(6,334) = 5.5564 [0.0000] **

Table 4.4 List of Variables

RP =	default risk premium
$d(r)$ =	indicator function for the German rate $\geq r$
r =	Threshold value for the German rate
dx =	variable x multiplied by the indicator function
Infl =	inflation rate
IIP =	industrial production index
DIIP =	change in the industrial production index
Un =	unemployment rate
x_A =	announcement effect for variable x
$x(yy)$ =	variable x relative to country yy
yy =	BE (Belgium), IT (Italy), SP (Spain), GR (Germany)