A Thesis Submitted for the Degree of PhD at the University of Warwick

THINKING MATHEMATICALLY: A FRAMEWORK FOR DEVELOPING POSITIVE ATTITUDES AMONGST UNDERGRADUATES

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Table of Contents

Acknowledgement viii
Abstract ix

1. OVERVIEW OF THE PROBLEM 1
    1.1. Introduction 1
    1.2. Background Consideration 3
    1.3. The Problem and its Setting 6
    1.4. Review of the Study 8

2. LITERATURE REVIEW 14
    2.1. Introduction 14
    2.2. Views about Mathematics 15
    2.3. Mathematics Learning and Cognitive Consideration 23
    2.4. Mathematical Thinking and Problem-Solving 25
    2.5. Learning to Think Mathematically and Solving Problems 32
        2.5.1. Mason, Burton & Stacey's (1982) framework 36
        2.5.2. Skemp's theory 40
    2.6. Mathematics Teaching: Beliefs and Practise 41

3. METHOD OF STUDY 45
    3.1. Introduction 45
    3.2. Preliminary Investigation: The Warwick Study 49
        3.2.1. The Questionnaire 51
        3.2.2. Students' Comments 53
        3.2.3. Semi-structured Interview 54
    3.3. The Main Study: Problem-solving at the UTM 56
        3.3.1. Background Consideration 56
        3.3.2. Planning the Main Study 58

4. PILOT STUDY 66
    4.1. Introduction 66
    4.2. The Warwick Problem Solving Course 67
    4.3. The Study 68
        4.3.1. Overview of the Study 68
        4.3.2. The Sample 69
        4.3.3. Method 70
    4.4. Analysis of Questionnaire Responses 70

4.4.1. Pre-test Results 70
4.4.2. Post-test Results 75
4.5. Comparison between Pre-test and Post-test Responses 77
4.5.1. Comparison of Attitudes to, and Perceptions of, Mathematics 77
4.5.2. Comparison of Students' Self-assessment 81
4.6. Student Comments 87
4.6.1. Comments Through Informal Class Interviews 87
4.6.2. Comments Obtained Through Questionnaire 89
4.6.2.1. Pre course comments 90
4.6.2.2. Post course comments 92
4.6.2.3. Students' feelings about mathematics 94
4.6.3. Comments Through Written Assignment 96
4.6.4. Formal Individual Interviews 99
4.7. Discussion 104
4.8. Chapter Summary 106

5. MAIN STUDY: PROBLEM SOLVING AT THE UTM 108
5.1. Introduction 108
5.2. The Study 110
5.2.1. Initial Considerations 110
5.2.2. Hypotheses 111
5.3. Method 111
5.3.1. For All Staff Concerned 111
5.3.2. For the Students 112
5.3.3. The Sample 112
5.3.4. The Course 113
5.3.5. The Questionnaire 117
5.3.6. Monitoring Students during the Classroom Phase 117
5.3.7. Semi-structured Interviews 119
5.4. Analysis of Results from All Staff 120
5.4.1. Responses of All Staff Concern 120
5.4.1.1. Results to Part A: Attitudes to Mathematics 120
5.4.1.2. Results to Part B: Attitudes to Problem Solving 122
5.4.2. The "Desired Direction of Attitudinal Change" Perceived by Mathematics Staff 124
5.5. Analysis of Students Results 127
5.5.1. Classroom Observation: An Overview 127

5.5.2. Responses to Pre-course Questionnaire: Pre-test 131
5.5.3. Responses to Questionnaire: Post-test 133
5.6. Pre-test and Post-test Comparisons 135
5.6.1. The Change in Students' Attitudes in Problem-solving 135
5.6.2. Making Sense of Mathematics 137
5.6.2.1. Attitudes to mathematics 138
5.6.2.2. Attitudes to problem solving 142
5.7. Student Comments 146
5.7.1. Positive and Negative Feelings about Mathematics 146
5.7.1.1. Pre-test comments 146
5.7.1.2. Post-test comments 148
5.8. Semi-structured Interviews 149
5.9. Discussion 156
5.10. Chapter Summary 157

6. SIX MONTHS LATER: THE POST POST-TEST 160

6.1. Introduction 160
6.2. The Study 161
6.2.1. The Sample 161
6.2.2. The Hypotheses 161
6.3. The Method 162
6.4. Students Post Post-test Results 163
6.4.1. Post Post-test Responses to Mathematics 163
6.4.2. Post Post-test Responses to Problem Solving 164
6.5. Pre-test, Post-test and Post Post-test Comparisons 165
6.5.1. The Change in Student Attitudes in Problem-Solving and Mathematics Lectures 165
6.5.2. Comparisons between Groups N and S Students Change in Attitudes 168
6.5.2.1. Attitudes to mathematics 169
6.5.2.2. Attitudes to problem solving 175
6.6. Students Comments 181
6.6.1. Mathematics and Problem Solving After Six Months of Standard Mathematics 181
6.6.1.1. Students' perception of mathematics 182
6.6.1.2. The nature of mathematics teaching 183
6.6.1.3. The role of the lecturers 184
6.6.1.4. The nature of problem-solving 184
6.7. Discussion 186

6.8. Chapter Summary.. 189

7. LECTURERS' PERCEPTION OF STUDENTS' MATHEMATICAL THINKING 191
7.1. Introduction.. 191
7.2. The Study.. 193
7.2.1. The Sample.. 193
7.2.2. The Hypotheses 193
7.3. The Method.. 193
7.4. Analysis of Results of Selected Lecturers.. 194
7.4.1. Lecturers Responses to the Students' Attitudinal Questionnaire 194
7.4.2. Selected Lecturers' Responses to Perception Questionnaire 195
7.4.2.1. Lecturers' responses to Section 1.................... 195
7.4.2.2. Lecturers' responses to Section 2.................... 196
7.4.3. Students Responses to Section 2 of the Questionnaire: Perception of Given Lecture.................................... 197
7.4.4. Individual Interviews with Selected Lecturers................. 206
7.5. Discussion 209
7.6. Chapter Summary.. 212

8. CONCLUSION 214
8.1 Overview of Results.. 214
8.1.1. Effect of Problem-solving on Students' Attitudes............. 215
8.1.2. Students and Staff: Comparisons of Attitudes................. 218
8.2. Subsequent Consideration 219
8.3. Critical Appraisal of the Research.. 221
8.4. Suggestions for Further Research 225

Appendix 1: A sample of notes given to students following the problem-solving course at the UTM 228
Appendix 2: The pilot questionnaire.. 234
Appendix 3: The main questionnaire 236
Appendix 4: Data collected from 47 students at Warwick 238
Appendix 5: Data collected from 44 students at UTM.............................. 240
Appendix 6: Data collected from 22 staff.. 242
Appendix 7: A sample of Wilcoxon Matched-pairs Signed-rank test results (on Warwick data).. 244
REFERENCES 251

Figures and Tables

Table 3.1: The ten students selected for interview..55
Table 4.1: Potential degree classification of the 47 students under study.............70
Table 4.2: Pre-test responses to the attitudes and perceptions component of the questionnaire..70
Table 4.3: Pre-test responses to the self assessment component of the questionnaire..72
Table 4.4: The distribution of students for whom mathematics makes little sense (group N) and does not (group S) 74
Table 4.5: Post-test responses to part A of the questionnaire 76
Table 4.6: Post-test responses to part B of the questionnaire 76
Table 4.7: Comparison between pre and post-test responses to part A of the questionnaire..78
Table 4.8: Significant changes in students responses to part A of the questionnaire..79
Figure 4.1: Mathematics studied make little sense: pre-test and post-test comparison..79
Figure 4.2: Attitudes to Mathematics: pre-test and post-test comparison 80
Table 4.9: Comparison between pre- and post-test responses to part B of the questionnaire..82
Table 4.10: Significant changes in students responses to part B of the questionnaire..83
Figure 4.3: Attitudes to Problem-Solving: pre-test and post-test comparison........84
Table 4.11a: Students' perception of mathematics....................................95
Table 4.11b: Students' perceptions of mathematics before and after a course on problem-solving. 95
Table 4.12: The ten students selected for the interview 99
Table 5.1: Responses for 22 lecturers to attitudes to Mathematics.................... 120
Table 5.2: Responses for 22 lecturers to attitudes to Problem solving................ 122
Table 5.3: Lecturers perceptions of students preferred and expected attitudes....... 125
Table 5.4: Pre-test responses of 44 students to the questionnaire 132
Table 5.5: Post-test responses of 44 students to the questionnaire.................... 134
Table 5.6: Responses for 44 students on the pre and post-test questionnaire........ 135
Table 5.7: Desired changes compared with changes after problem-solving.......... 136
Table 5.8: The distribution of students for whom mathematics makes sense (group S) and does not (group N)... 138

Table 5.9: Comparison between groups N and S students responses to part A of the questionnaire 139
Table 5.10: Significance of groups N and S changes in responses to part A......... 139
Figure 5.1: University mathematics "makes sense" 140
Figure 5.2: Attitudes to Mathematics: pre-test and post-test comparison 141
Table 5.11: Comparison between pre and post-test responses to part B of the questionnaire 142
Table 5.12: Significance of groups N and S changes in responses to part B of the questionnaire 143
Figure 5.3: Attitudes to Problem-solving: pre-test and post-test comparison 144
Table 5.13: Classification of written responses 148
Table 5.14: The 6 groups of students selected for interview 150
Table 6.1: 44 responses to part A of the post post-test questionnaire 163
Table 6.2: 44 responses to part B of the post post-test questionnaire 164
Table 6.3: 44 students responses to the attitude items before and after problem-solving and after mathematics lectures 166
Table 6.4: Desired changes compared with changes after problem-solving and after mathematics lectures 167
Table 6.5: 22 group N students' responses to part A of the questionnaire before and after problem-solving and after mathematics lectures......... 169
Table 6.6: Significant changes in group N students' responses to part A of the questionnaire after problem-solving and after mathematics lectures 170
Table 6.7: 22 group S students' responses to part A of the questionnaire before and after problem-solving and after mathematics lectures......... 171
Table 6.8: Significant changes in group S students' responses to part A of the questionnaire after problem-solving and after mathematics lectures 171
Figure 6.1: Students' attitudes to Mathematics after doing problem-solving......... 173
Table 6.9: 22 group N students' responses to part B of the questionnaire before and after problem-solving and after mathematics lectures......... 175
Table 6.10: Significant changes in group N students' responses to part B of the questionnaire after problem-solving and after mathematics lectures 175
Table 6.11: 22 group S students' responses to part B of the questionnaire before and after problem-solving and after mathematics lectures........ 176
Table 6.12: Significant changes in group S students' responses to part B of the questionnaire after problem-solving and after mathematics lectures 177

Figure 6.2: Student attitudes to Problem-Solving after doing problem-solving ... 178
Table 7.1: The 8 selected lecturers and the other 14 lecturers' responses to the questionnaire ... 194
Table 7.2: Eight lecturers responses on views to mathematics ... 195
Table 7.3: Eight lecturers perception of students' thinking about their lectures ... 196
Table 7.4: Thirty students' perception of Zoe's mathematics lecture ... 197
Table 7.5: Nine students' perception of Nelly's mathematics lecture ... 198
Table 7.6: Eighteen students' perceptions of Hazel's mathematics lecture ... 199
Table 7.7: Ten students' perceptions of Alfred's mathematics lecture ... 200
Table 7.8: Twenty students' perceptions of Mary's mathematics lecture ... 200
Table 7.9: Forty students' perceptions of Sammy's mathematics lecture ... 201
Table 7.10: Twenty-two students' perceptions of Sony's mathematics lecture ... 202
Table 7.11: Forty-four students' perceptions of Sandy's mathematics lecture ... 202
Figure 7.1: Students' perception to mathematics lecture after doing problem-solving ... 204

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Abstract

This thesis tests the hypothesis that problem-solving activities caused positive changes in students attitudes towards mathematics. A pilot test, carried out in a problem-solving course at the University of Warwick, tested possible questions that would indicate change of attitudes. The findings indicate that the course affected students attitudes to mathematics in what was considered a positive manner.

Using that experience gained through the pilot study, the main study was carried out at Universiti Teknologi Malaysia (UTM) in which data was collected from 44 students who took the course in problem-solving taught by the researcher. A pre-test, post-test and a delayed post-test (six months later) were administered, which included interviews with selected students and staff. To establish what might be considered a positive change, the staff at the Mathematics Department were asked what attitudes they would expect students have as a result of the mathematics teaching at the University, and then specify the attitudes they would prefer students to have. The direction of change between the two responses were considered to be positive, and this is defined as the "desired direction of change". The results show that the problem-solving course affected students attitudes such that the change, identified as the difference between pre-test and post-test results, was largely in the desired direction of change. However, when students return to normal mathematics lectures many of the indicators reverted in the opposite direction; away from what the staff preferred.

1. OVERVIEW OF THE PROBLEM

1.1. Introduction

This study considers the prospect of a course that encourages students to think mathematically in altering students' attitudes and methods of doing mathematics. Doing this research has required me to reflect on myself as a mathematics educator and a mathematician. More precisely, it brings me to focus on my own attitude towards mathematics. It reminds me of the frustration and confusion I had first felt as a mathematics student particularly when my lecturer always responded "Well, because it works!" to my question "Why do we do it this way?" As a student, I saw mathematics as no more than just parcels of knowledge to be learnt, and reproduced in exams. Only through my experience as a mathematics educator for several years at the Universiti Teknologi Malaysia (UTM) do I now see it as a living subject in which one has to think for oneself.

Reflecting back on my personal teaching and that of my colleagues in the Mathematics Department, I can see that mathematics is largely presented in a very formal manner. Given the time constraint, most of us find that the best method to convey the subject matter is by presenting it in a systematic and logical sequence—definition, theorem, proof, and illustration. The main activity in the classroom is 50 minutes of straight lecturing, the conveyance of information and the illustration of the mathematical techniques in a minimal number of worked examples. Students passively receive what is being presented. Tutorial classes consist mainly of discussing problems closely related to material taught and ensuring that the standard procedures are adopted. Occasionally problems considered difficult, or not related directly to the contents of lectures, may be given as assignment. The teaching of mathematics at the UTM is so formal that many of the students have reached a point where they have given up trying to understand and have resorted to straight rote-memorisation of notes to pass examinations (Razali & Tall, 1993; Amin, 1993).

One might hope that the acquisition of a rich body of mathematical knowledge would naturally lead to the ability to apply the knowledge to solve problems. But regrettably that may not be so. As observed by Selden, Mason & Selden (1989, 1994) in their studies, although students can pass their mathematics, many do not have the ability to apply their mathematical knowledge creatively. Dreyfus (1991) suggested that the inability to use mathematical knowledge in a flexible manner to solve problems is due to lack of insight into the processes that had led mathematicians to their creations. How can we expect students to have these qualities when they never experience mathematics beyond learning established facts and carrying out standard mathematical procedures? As Freudenthal (1973) succinctly said:

> ...the only thing the pupil can do with the ready-made mathematics which he is offered is to reproduce it. p.117

It may be inappropriate and offensive to some people to say directly that there is a problem and that we need to change the traditional way of teaching mathematics, both the system and the attitudes of teachers and learners. Perhaps it is more appropriate to ask: "Is there really a problem in the learning and teaching of advanced mathematics at the university?" After all, that is the way we have been trained and the system has successfully produced thousands of graduates over the years. Furthermore, people will only change whenever they feel it is time to do so.

Nevertheless, with the current development in the country's policy, there are already calls from certain quarters to completely "redesign" the education system in Malaysia as the present one is unable to support the country's vision of becoming a developed nation by the year 2020. Institutions of higher learning are being urged to upgrade education pedagogy to meet the changing needs of society (Sunday Star, 23 January

1994). A distinguished figure in Malaysia's education, Royal Professor Ungku Abdul Aziz suggested that what Malaysia needs is a large number of educated people who can, amongst other things, "think" (Ismail, 1994). Certainly, for members of the Mathematics Department such calls should not be left unheeded; the right time seems to have presented itself.

At the rate things are changing in the society, we do not know that the mathematics we are teaching now will be valid in 10 or 15 years time nor how the mathematics learned will be used by the students in their future. The advancement in technology—the creation of calculating devices such as calculators and computers—requires more than just knowing how to use procedures or to obey rules. The more pressing demand is the ability to adapt the mathematical performance to varied circumstances and to ensure that students can think for themselves. It is certainly time to review what we have been teaching and why. As argued by Tall (1992b), are we teaching students to comprehend the accumulated wisdom of mathematical thought or are we teaching them active mathematical thinking? He suggests that at present it appears to be the former and as long as this is so, mathematics teaching will fail the majority of our students.

1.2. Background Consideration

In Malaysia there has always been a tradition that learning is based on discipline amongst the children and the obedience to work hard; the desire to conform and to do well is high and learners must try very hard to do their best. At a very early age, the students in schools learn that to get by in mathematics, what they have to do is follow the rules. They are pushed into a procedural method of working and gradually cease to ask questions. Probably, because much of school mathematics is epistemologically easier, one can still learn facts and procedures without really understanding them. But as they progress into university, where the learning of mathematics demands a more formal understanding of mathematical processes, learners carry on with the kind of thinking they had developed in school and which in fact had given them success:

> When I was in school, for example when I learnt differentiation, my maths teacher just gave us the formula. He didn't tell us what differentiation is. When we asked him he said it is too complicated to explain and we don't need to know. So we just followed and learnt just what we needed to know and memorise it for exams.
>
> *UTM student, year 4 SPK*

When they face a problem in which they do not really understand what is going on, students know if they do it the way they have been taught they will get the right answer. At the pace lectures are delivered, they had to stop asking "why" because if they did not get on and do it they will not get the work done. From personal experience and research (Mohd Yusof & Abd. Hamid, 1990), we observed many of the students in the UTM had reached a position where they perceived mathematics as a fixed body of knowledge to be remembered: they lacked confidence to tackle anything new.

At the UTM, there is a wide range of ability (from the 50th to 90th percentile, with the top 10% going abroad for their education). The majority of the students see mathematics as a difficult subject. In trying to cope with the subject matter and in their eagerness to pass the examinations they simply resort to rote-learning. As one student commented—"To be good in maths requires good memory and lots of practice". Certainly having a good memory is desirable and helps in various ways. However, mastering facts and procedures alone is insufficient without true understanding of what is going on and why they are being used in solving problems.

Being a technical university, a vast majority of the students at the UTM are those majoring in science, engineering, surveying etc. This means that the students are taking mathematics as a core subject which they need to pass in order to obtain their respective degree. The current practices of teaching mathematics seems not to work well with the kind of students we face. Indeed it is possible to conjecture that they may not even work with students majoring in mathematics (future mathematicians). We need to teach them not only how to comprehend the subject matter, but more importantly to solve problems and to think for themselves.

For the last couple of years those students entering the local universities have been learning mathematics under the new curriculum—Malaysian Integrated Curriculum for Secondary Schools (Kurikulum Bersekutu Sekolah Menengah—KBSM) which was launched in 1988 (KPM, 1989). As mentioned by Hj Mohd Yunus (1990), the curriculum specifies that the main aim of mathematics education amongst others, is to develop critical thinking, problem solving skills and the ability to use the mathematics knowledge in everyday life. No particular teaching strategy is suggested but Polya's problem-solving method is adapted in the KBSM mathematics. However, studies (e.g. Schoenfeld, 1985a; Silver, 1985; and Mason, Burton & Stacey, 1982) show that Polya's model is not capable in enhancing the problem solving skills of the students. The strategies suggested by Polya need to be clarified further and made more appropriate for the learner. The major difficulties occurring in the new Malaysian system are yet to be diagnosed and reported. However, there is an urgent need to look into the traditional practices in mathematics teaching at local universities, UTM in particular, if it is to fulfil the country's aspiration.

In an earlier study at the UTM (Razali & Tall, 1993), considerable student difficulty was observed with traditional methods of teaching. The students were keen to succeed by learning the given procedures and applying them in the examination. When the problems became slightly more complex, involving several procedures, many students were likely to fail. The system therefore became a Catch-22 problem. Because the students were failing, they sought the security of learned procedures, but because they could not manipulate so many procedures to be successful, they were failing. It was considered that:

> ... a plausible way in which students may become more successful is to become consciously aware of more successful thinking strategies and this must be done in a context designed to impose less cognitive strain.
>
> Razali & Tall, 1993, p. 219

Razali & Tall posit that it is important for students to know what is required for improved mathematical thinking to help them cope with the nature of advanced mathematics and to engage in learning their mathematics more effectively. It is hypothesised that this will lead to positive attitudes towards mathematics they are studying. Certainly it is desirable that all students of mathematics can think mathematically. However, we should not expect everyone to be able to do so within a short time. For some, their real performance can only be seen in their work after they had graduated.

1.3. The Problem and its Setting

This study is concerned with teaching a course designed to encourage mathematical thinking amongst the UTM students which, it is conjectured, will lead them to make attitudinal changes. It offers an environment that gives students an alternative view of mathematics as a thinking process together with the opportunity to reflect on their own mathematical experiences. The possession of mathematical knowledge does not necessarily provide an indication to the quality of mathematical thinking. Contemporary views on different aspects of mathematical thinking indicate that those who are successful in mathematics may see things differently from those who may have more difficulty (see for example, Krutetskii, 1976; Gray, 1991). If we consider the level of achievement as the criteria for judging success and failure, it may not be possible to distinguish between the different the levels of thinking.

The underlying assumption of this study is that problem-solving is beneficial and students can learn to think mathematically. According to Tall (1991), there is a cycle of advanced mathematical thinking. The full cycle includes "the need to begin with conjectures and debates, the need to construct meaning, the need to reflect on formal definitions to construct the abstract object whose properties are those and only those, which can be deduced from the definition" (p. 252). It is conjectured that, when students are consciously aware of the processes of mathematical thinking through active participation in problem-solving, they will view mathematics as a living subject. To support the thesis, this study considers the effect of a problem-solving course on students' attitudes. The course is an explicit teaching of the meta-processes of mathematical thinking. That is, how to think in a mathematical manner rather than what to think.

Students' attitudes towards mathematics and problem-solving in particular, were considered on three different occasions built around a ten week problem-solving course. Students respond to an attitudinal questionnaire before the course, after the course and after a further period of regular mathematics lectures. Lecturers' beliefs and preferences of students' mathematical thinking were also considered and further data collected from interviews with selected students and lecturers.

It was hypothesised that a course on problem-solving would have beneficial effects on students' attitudes to mathematical thinking. To test this a questionnaire with seventeen attitudinal items was designed to which students would responded on a spectrum from Y (definitely yes), y (yes), – (no opinion), n (no) to N (definitely no), before and after the course. To give meaning to the term "beneficial effect", lecturers were asked to give two responses to the questionnaire: how they *expect* a typical student to respond and what they *prefer* for a response. The direction in change from what was *expected* to what was *preferred* was then considered to be "beneficial" and defined to be the "desired direction of change".

The first hypothesis is then that:

- Students attitudes would be changed in the desired direction by a course in problem-solving.

We know that students seem to learn procedurally although we hope the teachers would prefer a more conceptual understanding. Perhaps it is in the knowledge that conceptual understanding has not occurred that procedures are taught to ensure a measurable success. The change in the problem-solving course is one which the regular courses do not achieve. Therefore it is conjectured that, when students return to standard mathematics courses, they will revert to previous attitudes before problem-solving. In particular we hypothesise that:

- Students' changes in attitudes during mathematical course will be in the reverse direction from that desired by the staff.

It is within our interest to find out why the students attitudinal change is short-lived. Given the nature of the cultural and mathematical outlook at the UTM, it was seen that students' positive change in attitude is short-lived because the kind of mathematics they do did not promote their ability to think mathematically. Such a change in students attitudes would be seen in terms of staff attitudes and differences between staff and student perceptions.

1.4. Review of the Study

A constructivist view of learning is adopted in this study. This view asserts that people are not recorders of information but builders of knowledge structure. It is believed that learning mathematics involves the construction of a rich body of mathematical knowledge together with the ability to apply the knowledge to solve problems. The acquisition of the former does not lead naturally to the latter as one might hope (see e.g. Schoenfeld, 1985b). Skemp's (1971) theory of learning mathematics helps to explain the situation. He recognised that mathematical thinking is dependent on "reflective intelligence" that is the ability to make one's own mental processes the object of conscious observation.

A review of various opinions and interpretations of problem-solving is given in Chapter 2. Three perceptions were recognised by Stanic & Kilpatrick (1988): problem-solving as context, as skill and as art . Problem-solving is viewed as context when solving problems is not seen as a goal in itself but as means to achieve other goals such as a justification for teaching mathematics or a practice of standard procedures. The second perception sees solving mathematical problems as valuable in its own right; as a hierarchy of skills to be acquired by students as part of their mathematical knowledge and understanding in which problems are set as part of a build up from simple one-step problems to multi-step problems in an explicit context. Problem-solving is considered an art when it is a complex mental activity involving a variety of cognitive operations (Garofalo & Lester, 1985). This is truly practised by mathematicians.

The current trend in mathematics education is towards conceptualising mathematics as a living subject with the development of mathematical thinking becoming a priority (Schoenfeld, 1992; McGuiness & Nisbet, 1991; Tall, 1991). In this recent development, problem-solving has been emphasised as a process to construct mathematical knowledge as well as a process in the application of mathematical knowledge. Within the context of this study, problem-solving is seen as the art of thinking mathematically and its underlying assumption is that problem-solving is beneficial. Through its application we can teach students to think mathematically.

In Chapter 3, I shall discuss the methodology in pursuing the research study. A review of literature led to the methodology that is used in eliciting students' attitudes towards mathematics and problem-solving. In the Pilot study, an attitudinal questionnaire was constructed locally with several of the statements taken from standard questionnaires (e.g. Joffe & Foxman, 1986; The Open University, 1986) and was subjected later to further modification in the main study at the UTM. Other methods used were classroom observation, semi-structured interviews with selected students and selected staff of mathematics department at the UTM.

Chapter 4 provides an account of the results of the pilot study. It reports the scene at Warwick with the researcher working with a class of undergraduates following the problem-solving course. This enabled the researcher to try out a pre- and post-test questionnaire to study the change of the students' attitudes towards mathematics and problem solving. In addition, it allowed the researcher to carry out a semi-structured interview with selected students. It was observed that the students' attitudes changed from week to week during the course. Given the opportunity to carry out mathematical thinking in a non-threatening atmosphere, they are more confident in themselves and in their ability to create solution methods that they think most efficient. Before the course, the majority of the students view mathematics as a static subject. However, responses following the course indicate that students think about mathematics as an active process. Students' response to the item indicating whether or not mathematics makes little sense signalled some differences between them. Gender related differences on confidence and anxiety were also noted amongst the students interviewed. The data indicates that males tended to show more confidence than the females in their mathematical abilities whereas the females tended to show greater anxiety than their male peers.

All responses were analysed qualitatively because this was more effective in dealing with issues measured. However, statistical tests were also considered and applied on students' responses to the questionnaire to highlight the general observations.

Such a course was then presented at the UTM by the researcher. All the materials used were translated into Bahasa Malaysia (the language of instruction in UTM) by the researcher and were checked by a linguistic expert. Based on the Warwick students' responses, a modified questionnaire was constructed and used in the main study at the UTM.

Chapter 5 provides an account of the study carried out in the UTM. It considers students change in attitudes within the context of the expected and preferred attitudes identified by staff. The findings indicate that lecturers prefer students to have a range of positive attitudes to mathematics but they expect the reality to be different. In particular they prefer students to see mathematics as solving problems, making sense, for students to work hard, be able to relate ideas without needing to learn through memory, have confidence, derive pleasure from solving problems, low anxiety and fear of the unexpected, ready to try a different approach and unwilling to give up easily on difficult problems. On the other hand, they expect students to see mathematics as abstract, failing to understand it quickly, not making sense, working hard to learn facts and procedures through memory, being unable to relate ideas, with less confidence, obtaining less pleasure, working only to get through the course, with anxiety, fear, seeking only correct answers, and ready to give up when things get difficult. The difference in attitudes is used to define the lecturers' "desired direction of changed". The classroom observations, changes intimated through students' responses to the questionnaire and their responses in interviews, were all consistent in supporting the main hypothesis. More specifically, the difference between the pre- and post-test revealed that attitudes changed significantly in the desired direction during the course. Half the students stated beforehand that university mathematics did not make sense. This interesting (and unpredicted) phenomenon allowed us to be opportunistic and compare other changes in attitude between the two groups (i.e. N and S) of students. A majority of these declared negative attitudes to mathematics as abstract facts and procedures to be memorised, reporting anxiety, fear of new problems and lack of confidence. After the course, all measures investigated improved, confirming that appropriate problem-solving can alter students' perception of mathematics as active thinking process.

In the belief that mathematical thinking and attitude need a long time for their formation, and that the students' attitude is also heavily influenced by their current mathematics courses, a second test was carried out six months after the course was first given. The delayed post-test included investigating staff perceptions of students' mathematical thinking. Further data from the students after one semester of standard mathematics lectures were also collected. The data from the questionnaire is supplemented by interviews with selected students and staff.

Chapter 6 presents students attitudinal change after problem-solving and after mathematics lectures. It was observed that six months after returning to standard mathematics courses, many of the indicators revert back towards their old position; in the opposite direction from that desired. But three problem-solving attributes remain positive: confidence and unwillingness to give up remain significantly improved and fear of the unexpected remains reversed to a highly significant level. Small changes are evident in the belief that mathematics makes sense. Students' expressed opinions, suggest that the quantity and the difficulty of the mathematics they are doing gives them little room for creative thinking.

Mathematicians want their students to think mathematically and have positive attitudes. However, as a result of many years experience what they would desire the students to have may become very different from what they expect the students to have. Chapter 7 reports on the investigation on selected staff's perceptions of students' mathematical thinking and comparison between staff and students perceptions. The findings show that the lecturers have little confidence in the students' ability to think mathematically and teach them accordingly.

Chapter 8 considers the conclusion, the limitations of the study and further questions for future research. It is evident that problem-solving causes changes in students' attitudes in a way mathematicians desire. However, without further support from regular mathematics courses the effect is mainly short term. To achieve the attitudes lecturers desire they have much work to do in providing a mathematical experience that encourages students to think mathematically. The data collected in this thesis suggest that lecturers need to move away from teaching students mathematical thought to teaching them mathematical thinking if they wish students to think mathematically. This requires further investigation as to how we might shape mathematical instruction.

2. LITERATURE REVIEW

2.1. Introduction

> In the fall of 1982, Riyadh, Saudi Arabia ... we all mounted to the roof ... to sit at ease in the starlight. Atiyah and MacLane fell into a discussion, as suited the occasion, about how mathematical research is done. For MacLane it meant getting and understanding the needed definitions, working with them to see what could be calculated and what might be true, to finally come up with new "structure" theorems. For Atiyah, it meant thinking hard about a somewhat vague and uncertain situation, trying to guess what might be found out, and only then finally reaching definitions and the definitive theorems and proofs. This story indicates the ways of doing mathematics can vary sharply, as in this case between the fields of algebra and geometry, while at the end there was full agreement on the final goal: theorems with proofs. Thus differently oriented mathematicians have sharply different ways of thought, but also common standards as to the results.
>
> MacLane, 1994, p. 190-191

The quote from MacLane above indicates that there are different ways of viewing mathematics. It is being viewed differently by different people even amongst mathematicians and mathematics educators. How we view mathematics is greatly influenced by our encounter with the subject during our formal education as well as the extent of its application in our everyday life. Many people, including mathematicians and mathematics educators, may find it difficult to say precisely how they see mathematics or what is mathematics all about. Nevertheless we do need to consider our own attitude towards mathematics before we can see and understand how mathematics is being viewed by others. This includes considering our beliefs in the nature of mathematics, the beliefs and understanding that we have about mathematical knowledge, and beliefs about ourselves as a learner or teacher of mathematics (Greeno, 1988).

In this chapter we shall consider the different views about mathematics that contribute to current belief of the subject (section 2.2). The discussion will lead us to review the various goals of mathematics learning (section 2.3). Current trends indicate that teaching mathematical thinking and problem solving are becoming a priority in mathematics classroom (section 2.4) and section 2.5 will consider several frameworks in learning to think mathematically. Section 2.6 looks at various beliefs and practices in mathematics teaching.

2.2. Views about Mathematics

Hadamard (1945), in his survey into the working methods of mathematicians almost five decades ago, suggested that there existed different kinds of mathematical behaviour. He highlighted that thinking in the absence of words, informal reasoning, visual imagery, mental images (which may or may not be expressed in words) were, amongst various forms of mathematical behaviour that played an important role in the process of constructing mathematical knowledge.

Hadamard also brought to light the differences between various ways of mathematical thinking. Firstly, people simply reason with their common sense, much of which is unconscious. However, Hadamard pointed out that results reached by this means are not satisfactory if not fully arithmetized. Second is the scientific state; a state which is characterised by the intervention of three processes—verifying the result, establishing precision and utilising it. It was suggested that what is of concern to students of mathematics is the passage from the knowledge acquired to its application and the possibility of extending them. According to Hadamard, the difference between a student solving a problem and a work of invention is only "a difference of degree, a difference of level, both works being of similar nature" (p. 104).

Like Poincaré (1913) Hadamard acknowledged two groups of students who displayed different attitudes toward understanding mathematics: firstly those who feel that something is lacking, but are unable to realise what is wrong and secondly those that feel that if they do not overcome the difficulty, they will get lost. The first group who could not find the synthetic process do without it. Its absence does not deter them from pursuing their mathematical studies. This group, as suggested by Hadamard, will fare worse than the second group who are able to see the process of synthesis and understand to a certain extent the existence of some difficulty. The synthetic process appears to be very important; it is seen as "the leading thread, without which one would be like the blind man who can walk but would never know in what direction to go" (p. 105). Even though this phenomenon was observed by Poincaré almost a century ago it is likely that these two different kind of attitudes still exist amongst many students of mathematics today.

Mathematicians generally have the ability to understand mathematical theories as well as to investigate new ones. They not only differ from students but even amongst themselves. Poincaré brought to light the notion that some mathematicians are "intuitive" and others are "logical". He asserts that:

> It is impossible to study the works of the great mathematicians, or even those of the lesser, without noticing and distinguishing two opposite tendencies, or rather two entirely different kinds of minds. The one sort are above all preoccupied by logic; to read their works, one is tempted to believe they have advanced only step by step, after the manner of a Vauban who pushes on his trenches against the place besieged, leaving nothing to chance. The other sort are guided by intuition and at first stroke make quick but sometimes precarious conquests, like bold cavalrymen of the advanced guard.
>
> p. 210

Intuitional mathematicians like Riemann, for example, see mathematics as a production of their individual mind. On the other hand logicists claimed that all mathematical concepts could be reduced to logical ones. Hadamard however, disagreed with Poincaré's ideas, stressing that every mental work, the work of discovery in particular, implies the co-operation of the unconscious which can be superficial or remote. He suggested that there is hardly any completely logical discovery. Some intervention of intuition that emerges from the unconscious is necessary at least to initiate the logical work. He cited Poincaré himself for instance, who was logical in the articulation of his ideas, but this was always after having been intuitive in the illumination of ideas in his discovery.

In the early 1900's the disagreement amongst mathematics philosophers about the foundation of mathematics led to the growth of different schools of thought which included logicist, formalist and intuitionist. However, none of these schools appear to provide a widely adopted foundation for the full range of what should be mathematical truths (Körner, 1960). It may be suggested that in reality most mathematicians today do not identify with any of one of these schools; they tend to be pragmatists. In essence what matters to them is to write down the axioms and proofs of their results. They do not worry too much about Gödel's Incompleteness Theorems or any other problems. That is, practising mathematicians may think little about the nature of the subject as they work within it. Nevertheless, Dossey (1992) argued that the lack of a commonly accepted view of the nature of mathematics among mathematicians has serious ramifications for the practice of mathematics education, as well as for mathematics itself. He proposed that discussion of the nature of mathematics must come to the foreground in mathematics education. Furthermore as suggested by Lerman (1990), changes in mathematics education need to challenge fundamental assumptions about the nature of mathematics, otherwise they will remain marginal in effect.

In mathematics education, two common perspectives about mathematics seem to have a strong influence—mathematics as a formal system and mathematics as a mental activity or "absolutist" and "fallibilist" (Lerman, 1990). In formalist terms, mathematics is made up of formal axiomatic structures. It is considered as a body of infallible and objective truth; it does not take into account how mathematical knowledge grows or changes. Although the view has been subjected to criticism, it continues to have a strong influence on the development of mathematics and instructional practice (Davis & Hersh, 1981; Nickson, 1994). On the other hand, the other view stresses mathematics as a process that is continuously developing. It grows out of the view that mathematical knowledge is fallible. Under the fallibilist paradigm, mathematics is considered as a product of human inventiveness. The focus is on the context and meaning of mathematics for the individual, and on problem-solving processes. This view of mathematics is increasingly seen as a valid pespective of the nature of mathematics.

Although there is no one absolutely true way of viewing mathematics, it is suggested that mathematics should be accepted as a human activity. Courant & Robbins (1941) stressed that mathematics should be seen as an expression of human mind which among other things reflects the active will, the contemplative reason, and the desire for aesthetic perfection. The basic elements includes logic and intuition, analysis and construction, generality and individuality, with the constructive and intuitive aspect in mathematical achievement playing an important role:

> If the crystallized deductive form is the goal, intuition and construction are at least the driving forces. Courant & Robbins, p. xvii

They suggest that if mathematics were nothing but simply a system of conclusions drawn from definitions and postulates, then "mathematics could not attract any intelligent person. It would be a game with definitions, rules, and syllogisms, without motive or goal" (ibid, p. xvii). Further they went as far as to suggest that it is not philosophy but active experience in mathematics itself that can answer to what mathematics is all about.

Lerman (1990) suggested that those qualities that characterises mathematics include engaging in interesting problems, making imaginative conjectures, testing, reflecting, examining results informally, formalising and testing results formally. He proposed that viewing mathematics as a mental activity seems to be relevant at all stages of mathematical learning. However, the mathematics presented at the university in definition-theorem-application form does not seem to show these other aspects of mathematics. As noted by Davis & Hersh (1981):

> The definition-theorem-proof approach to mathematics has become almost the sole paradigm of mathematical exposition and advanced instruction. Of course, this is not the way mathematics is created, propagated, or even understood. The logical analysis of mathematics, which reduces proof to an (in principle) mechanizable procedure, is a hypothetical possibility, which is never realized in full. Mathematics is a human activity, and the formal-logical account is only a fiction....
>
> p. 306

Furthermore Skemp (1971) suggested that logical presentation tends to give students the product of mathematical thought rather than the process of mathematical thinking:

> The approach is laudable in that it aims to show that mathematics is sensible and not arbitrary, but it is mistaken in two ways. First, it confuses the logical and the psychological approaches. The main purpose of a logical presentation is to convince doubters; that of a psychological one is to bring about understanding. Second, it gives only the end-product of mathematical discovery, and fails to bring about in the learner those processes by which mathematical discoveries are made. It teaches mathematical thought, not mathematical thinking. p. 13

In practise, as noted by Davis & Hersh, the presentation of mathematics is still dominated implicitly by formalist views although many practising mathematicians would recognised themselves as both formalist and constructivist. They believe in a constructivist approach in mathematics but during the final polishing they work according to a formalist view (see for instance Atiyah's indication in the opening quotation of this chapter).

Doing mathematics certainly must involve thinking. But doing mathematics in University courses tends to be a reproductive process. Students are presented theorems and proofs which they then learn and reproduce. Then they are given some measure of problem-solving at the end to encourage them to think of related problems themselves. That is, problem-solving is seen just as a skill to be acquired. Freudenthal (1973) stressed that the essential thing in mathematics is the activity, the process of thought which had led mathematicians to their creations. He suggested that the best way to learn an activity is by performing it and proposes the method of re-invention in terms of activity mathematics as opposed to the learning of ready-made material. He pointed out that:

> Science at its summit has always been creative invention, and today it is even so at levels lower than that of the masters. The learning process has to include phases of directed invention, that is, of invention not in the objective but in the subjective sense, seen from the perspective of the student. It is believed that knowledge and ability acquired by re-invention are better understood and more easily preserved than if acquired in a less active way. p. 118

Schoenfeld (1988a) demonstrated that doing mathematics should be an act of sense-making, an act of taking things apart mathematically and seeing how they fit together. According to him the facts and procedures that students learn should be a means to this end rather than an end in themselves. He argues that for students to learn and apply mathematics they must participate actively in doing mathematics much in the same way that practising mathematicians do and then they must reflect upon the experience. It was suggested that such an active experience can lead students to develop the right sense of what mathematics is all about, as well as aiding them to master the formal mathematics that they need to know. Furthermore as mentioned by Ervynck (1991):

> ... we should not expect students to (re-)invent what has taken centuries of corporate mathematical activity to achieve. Yet if we do not encourage them to participate in the

> generation of mathematical ideas as well as their routine reproduction, we cannot begin to show them the full range of advanced mathematical thinking. p. 53

From a constructivist viewpoint it is believed that the mathematical knowledge that we possess is a body of knowledge that we have constructed ourselves through our mathematical experience and our reflection upon them. The mathematical reality that we have constructed is only relative. It is not absolute, for mathematical knowledge itself, in general, may be uncertain (Kline, 1980). As a whole the structure of mathematics is seen as a mental construct. The mathematical constructs, particularly in advanced mathematics, are abstract entities. They are elusive and exist only in the mind. This makes teaching mathematics very hard and learning it difficult and out of reach for many. Sfard (1991) argues that the inability to see the invisible objects may be one of the major reasons why mathematics appears to be practically impermeable to so many "well-formed minds". She posits that the transition from an operational view of a notion to a 'reified' or objectified structural view is a difficult process. Whilst Skemp (1971) in his theory for learning mathematics pointed out that much of mathematics operates at higher levels of concept each of which is built upon on lower-level concepts that already have been formed. Without these foundational (lower-level) concepts, mathematics may never be an intelligible activity. He identified that mathematical thinking is dependent upon reflective intelligence; the ability to reflect on one's own mental processes. Skemp proposed a delta-one system as a teachable director system which operates on the physical environment and a delta-two system operating on delta-one. According to Skemp, reflective intelligence has consciousness in delta-two and the objects of consciousness are those in delta-one. Without the metacognitive process in delta-two, students does not have operations that are sophisticated enough to enable them to solve new problems.

Current views (NCTM, 1990; Hiebert & Carpenter, 1992; Ernest, 1994) in mathematics education indicate that students should be more than just passive receptors of mathematical knowledge invented by others with teachers merely transmitting established facts and a ready solution method. It is suggested that students should be given the experience that encourages them to question what they are taught, to reflect on the nature of the concepts, to make connections and to build up new knowledge structures. Resnick & Klopfer (1989) pointed out that mathematics becomes part of the thinking curriculum only when it is taught in the context of reasoning and problem-solving.

This discussion brings us to review the goals of mathematics learning. It is believed that we learn mathematics so that we can appreciate fully the nature of the subject matter, to apply it in everyday life and to help in understanding the world that we live in. In Malaysia, the mathematics education goals at present only seem to provide students with a sense of the subject matter. The emphasis is more on the content rather than on the processes. It may be suggested, for Malaysia in particular, that mathematics teachers, in addition to teaching students to get a sense of the subject matter, to attain their high goals, should also include preparing students to become effective thinkers and independent problem solvers as a major goal. Mathematics learning must then go beyond learning accumulated mathematical thought. Certainly we need to know the mathematical facts and standard procedures in order to be able to do mathematics. But knowing them without its functional meaning is totally limiting. Hiebert & Lefevre (1986) succinctly state that:

> Students are not fully competent in mathematics if either kind of knowledge [conceptual and procedural knowledge] is deficient or if they both have been acquired but remain separate entities. When concepts and procedures are not connected, students may have a good intuitive feel for mathematics but not solve the problems, or they may generate answers but not understand what they are doing. p. 9

2.3. Mathematics Learning and Cognitive Consideration

The notion that learning of mathematics is a constructive process appears to be widely accepted. For instance the Eleventh International Conference on the Psychology of Mathematics Education (1987) considered mathematics education from a constructivist viewpoint and there was interesting debate as to what the notion meant within both formal and informal contexts (see e.g. Kilpatrick, 1987; Sinclair, 1987).

Constructivism asserts that knowledge is actively constructed by learners and not passively received from the environment. Further it maintains that coming to know is an adaptive process through which the learners organises their experiential world. It does not discover an independent, pre-existing world outside the mind of the knower (Kilpatrick, 1987). The latter principle has been the source of disagreement that has led it to being considered as radical on one hand and social on the other. Radical constructivists argue that one would never come to know a reality outside oneself. However, recent development saw some shift in the denial and the debate is now more concerned with things such as individual versus social construction (Ernest, 1994). In general, the theory stresses the search for meaning and understanding in learning. It focuses our attention on the process of learning, on learning to learn and the mastery of our own learning strategies through metacognition (awareness of one's own thoughts).

Mathematics can be considered as a social construction (Lerman, 1990). It is suggested that mathematics learning involves both the construction of a rich body of knowledge together with the ability to apply the knowledge to solve problems. From a psychological constructivist perspective, the constructions have to be done by students themselves, whether they construct it by being told or they construct it by themselves. From a social constructivist perspective, students need to participate in a culture of mathematical thinking. However, studies investigating students' mathematical understanding (e.g. Davis & Vinner, 1986; Artigue, 1991; Harel & Tall, 1991; Pirie & Kieren, 1992; Dorier, Robert, Robinet & Rogalski, 1994) indicate that the majority of students cannot handle the construction and the modification on their own. Their path is cluttered with numerous cognitive obstacles that causes total confusion to them. Students find great difficulty with the presentation of formalised ideas. Rogers (1990) had observed that:

> ...at the tertiary level where the lecture mode of instruction is so predominant, ...students are not given the opportunity to be involved in the journey: the process of constructing mathematical ideas where connected thought is so important. Thus, there is an enormous cognitive gulf between the style in which mathematics is presented and the way in which students are best able to construct their own understanding of it.
>
> p. 42

Some students will be able to bridge the "gap" by themselves but many may not be able to do so. In trying to make sense of their mathematical world, students actively reconstruct their knowledge and interpret new information in terms of their own mental framework which they had built over the years. However, students' interpretations do not necessarily lead to productive mathematics. Tall (1991) argues that students need to be assisted explicitly:

> The active participation in thinking is essential for the personal construction of meaningful concepts. Students need to be challenged to face the cognitive reconstruction explicitly, through conjecture and debate, through problem solving ....
>
> p. 258

He further asserts that true progress in the transition to more advanced mathematical thinking can be achieved by helping students to reflect on their own thinking processes and to confront the conflicts that arises in making the move (Tall, 1992a).

From the constructivist perspective, the development of students' own mathematical ideas is given special importance. It encourages students to use their own methods of solving problems. In this way, students will think about the reasoning involved, and thus learn how to analyse, and build arguments. Teachers are considered as mediators between pupils and mathematics; they serve as a guide and provide support for students' invention of mathematical ideas. As recognised by many (e. g. Lerman, 1989b, 1994; Ernest, 1991, 1994; Cobb, Yackel & Wood, 1992a; Arcavi & Schoenfeld, 1992; Pirie & Kieren, 1994) constructivism is increasingly seen as a theoretical orientation to analyse learning and to develop alternative forms of classroom practice.

2.4. Mathematical Thinking and Problem-Solving

Mathematical thinking and problem-solving have increasingly been recognised to be an important part of mathematics education. It has received considerable attention over the past decades and teaching students problem-solving has become a major focus in most mathematics curricula (see NCTM, 1980, 1990; Cockcroft, 1982; Kilpatrick, 1985). The development of mathematical thinking in mathematics classroom is becoming a priority (Resnick & Klopfer, 1989; McGuinness & Nisbet, 1991; Tall 1991; Schoenfeld, 1992).

Investigation of problem-solving seems to fall into two categories. Firstly, direct investigation of mathematical problem-solving (e.g. Schoenfeld, 1985; Mason *et al*, 1982; Charles & Lester, 1984) and secondly, problem-solving that uses mathematical problems (e.g. De Corte, 1990; Bourne, Dominowski, Loftus & Healy, 1986). It is likely that the basic patterns of mathematical thinking are very much similar to that of all other thinking. However, as noted by Skemp (1989), the abstract and hierarchic nature of the subject matter requires that mental abilities be used in special ways. He suggests that mathematics relies heavily on intelligent learning which consists in the building up of knowledge structures from which various plans of action can be derived as and when required. According to Skemp, intelligent learning involves abstraction, a process by which we become aware of regularities in our experience and the mental object which results is called a concept. A mathematical concept builds on another and depends on it. The concepts become more abstract and remote from sensory experience the more times the process of abstraction is repeated. Thus, Skemp implies that it is essential to provide learning situations which are favourable to knowledge structures (or schemas) construction.

There seems to have been differing views and conflicting definitions about what is a "problem" and "problem solving" through the years (see for e.g. Mayer, 1983; Charles & Lester, 1982; Fisher, 1990). The lack of agreement is not surprising due to their subjective nature. Moreover, the notion of a problem appears to be relative to the person involved. That is, depending on our ability and the amount of previous contact with similar task, what may be a problem to us may just be an exercise to someone else.

Hembree (1992) had noticed that the distinction between the definitions of problem relates to the effort that solvers make toward solution. In our context, a mathematical problem is a task where there is no obvious algorithm or ready mathematical method for the solver to use in finding the solution. The solver needs to make a conscious effort in order to get any possible resolution. As noted by Polya (1965) the tasks faced by the individual requires some degree of independence, judgement, originality, and creativity. The solver may know the procedure(s) that can solve the problem and feels confident in using it. On the other hand the solver may not feel certain in applying the procedure correctly—the task is no longer a problem but an exercise. Problems which play a role as practice to master mathematical techniques are in our sense exercises.

In the UK, the term investigation is more widespread than problem-solving. To some there is a distinction between the two. The Cockcroft report (1982) considered problem-solving and investigation as two distinct activities. Problem-solving is described as the ability to apply mathematics to a variety of situations. In particular, Cockcroft states that "problems should relate both to the application of mathematics to everyday situations within the pupils' experience, and also to situations which are unfamiliar" (para. 249). Investigations concern the willingness to ask 'what if' questions generally in lessons. Cockcroft emphasised that the investigations need not be lengthy nor difficult and suggested that "the [teacher] should start in response to pupils' questions, perhaps during exposition by the teacher or as a result of a piece of work which is in progress or has just completed" (para 250). The distinction is also noted by Greer (1989), Atkinson (1991), and Orton (1987) in which problem-solving is regarded as having a definite solution whereas an investigation is open-ended.

However, there are others who made no distinction between the two types of activities. For instance, Ernest (1984) asserted that the difference between problem-solving and investigation "is just a matter of degree or even of terminology" (p. 80). In a report by HMI (1985), problem-solving and investigative work were noted as not clearly distinguishable. The emphasise was that "appropriate practical work, problem solving and investigative work" should form part of classroom approaches to mathematics. Nevertheless, it was suggested that problem-solving involved relatively convergent tasks and investigative work is associated with divergent ones.

In this study we shall consider problem-solving and investigation together, drawing no clear borderline between them. Moreover, as recognised by Atkinson (1991) the mathematical thinking processes and strategies involved are the same for both.

The term mathematical problem-solving seems to have official endorsement in the US, although it has carried a multiple of meanings through the years. Views regarding problem-solving seems to be influenced by two perceptions:

i. problem solving as a basic skill required of all students.

ii. problem solving as a complex mental activity.

(Hembree, 1992, p. 243)

In the 1980's, the vast majority of curricular development and implementation that went under the name of problem-solving was of the first perception. According to Kantowski (1981), to most people, problem-solving means the solving of verbal or word problems. Later it includes other problem types such as non routine mathematics problems and real (application) problems. Stanic & Kilpatrick (1988) in their historical review of problem-solving, identified three main themes with regards to their usage: problem-solving as context, problem-solving as skill and problem-solving as an art. The first two themes are of the first perception in which problem-solving is not usually seen as a goal in itself but rather as means of achieving other goals. For example, problem-solving is seen as a way to develop new skills or as practice in mastering a technique. Additionally, Schoenfeld (1992) points out that even though problem-solving may be seen as skill in its own right, the basic underlying pedagogical and epistemological assumption is that problems are given as practice. Within this interpretation students' mathematical knowledge and understanding is assumed to comprise problem-solving skills as well as the facts and procedures that they have studied.

The second perception gave more attention to the problem-solving processes and the complexity of problem-solving behaviour. It is one that is continuously being considered and has grown with the passage of years (Kilpatrick, 1985). Problem-solving has come to be viewed as a complex process in recent years. It involves the higher faculties that include visualisation, abstraction, manipulation, reasoning, analysis, synthesis and generalisation. Each of these cognitive operations needs to be managed and all needing to be co-ordinated (Garofalo & Lester, 1985).

Polya (1965) defines solving a problem as "finding a way out of a difficulty, a way around an obstacle, attaining an aim that was not immediately attainable" (p.v) His suggestion is that problem-solving is based on cognitive processing. Polya examined his own thoughts to find useful patterns of problem-solving behaviour. His work on conceptualising mathematics as problem-solving and making problem-solving the focus of mathematics instruction has become the foundation for the majority of problem-solving research over the years. Similarly, Tall (1991) suggested that problem-solving is a "creative activity that includes the formulation of a likely conjecture, a sequence of activities testing, modifying and refining until it is possible to produce a formal proof of a well-specified theorem" (p.18). It is conjectured that when one is solving a mathematical problem one is thinking mathematically.

Mason, Burton and Stacey (1982) define mathematical thinking as "a dynamic process which, by enabling us to increase the complexity of ideas we can handle, expands our understanding" (p.158). A helix is used to describe mathematical thinking that underlines the interconnectedness of cognition and emotion. The dynamics of mathematical thinking are displayed by movement around or between an unspecified number of loops. Each new loop of the helix builds on the understanding and awareness achieved in traversing previous loops. The process is initiated by manipulating objects (mental, physical, pictorial, symbolic), moving to getting a sense of pattern or some property of those objects, to articulating that property as an expression of generality. Once this is achieved the movement is then towards finding the expression a confidence inspiring entity which can be manipulated and used to seek out further properties. Each successive loops assumes that thinking is more complex. Mental blocks, misunderstanding, or conjectures that fail under testing may cause an oscillation within and across the loops. The connectedness of the loops permits the thinker the opportunity to backtrack to previous levels and revise shaky articulations, to appeal to a sense of pattern and to more concrete examples. Burton (1984a) gave further information regarding its philosophy to support their suggestion. She asserted that mathematical thinking does not emerge automatically from learning and mastering of mathematics. It requires some degree of groundwork and training.

Tall (1991) brings together more recent views of advanced mathematical thinking—its theory and empirical research in both mathematicians and students. He observed that the nature of advanced mathematics involves processes that call for thought and creativity. Advanced mathematical thinking was considered as part of the living process of human thought and not just proof and deduction. According to Tall, there is a full cycle of mathematical thinking: intuition is followed by the making of conjectures from abstractions, leading to definition and to the final stage of proof. Dreyfus (1991) recognises that the cognitive processes involved in advanced mathematical thinking are also found in elementary mathematics, but it is the possibility of formal definition and deduction that distinguishes advanced mathematical thinking. He stresses that the complementary processes of representing and abstraction are particularly important in advanced mathematical thinking. He points out that, for flexibility in problem-solving, students need to have many mental representations of concepts together with the ability to switch representations. This means going from one representation of a mathematical concept to another one whenever the other one is more efficient. Closely associated with the switching process is translating—going over from one formulation of a mathematical statement or problem to another one. It is the process of representing together with generalising and synthesising that form the basis to abstracting; the most important amongst the advanced processes. Dreyfus claims that:

> If a student develops the ability to consciously make abstractions from mathematical situations, he has achieved advanced level of mathematical thinking.
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> p. 34

All these processes according to Dreyfus are essential to control the complexity of advanced mathematical concepts. He implies that the processes of reasoning need to be made explicit to the students in such a way that they are conscious of them.

Ervynck (1991) discusses the characteristics of the mathematical creativity that plays a vital role in mathematical thinking but yet is totally neglected in students' mathematical instruction. He proposes three stages of development of mathematical creativity—a preliminary technical stage, algorithmic activity, and creative (conceptual, constructive) activity. Parallel to these stages he classified three levels of mathematical behaviour based on the status of the method used in solving problems. The first level which he calls "low level" relies heavily on the application of an algorithm. In his words, "the creativity involved requires only recognition of the overall positioning of the problem in the whole of mathematics and the construction of the appropriate model". The second (higher) level is based on direct reasoning inside the mathematical model. It requires some insight and intuition to develop the right method of solution. The third (highest) level constructs solutions by an intelligent reviewing of what is stated in the problem. It is based on intuition, experience and some believable assumptions embedded in the nature of the problem. He asserts that the power of mathematical creativity results from the interaction of a certain number of elements which includes amongst others, understanding, intuition, insight and generalisation.

Schoenfeld (1992) describes what it means to think mathematically as follows:

> Learning to think mathematically means (a) developing a mathematical point of view—valuing the processes of mathematization and abstraction and having the predilection to apply them, and (b) developing competence with the tools of the trade, and using these tools in the service of the goal of understanding structure—mathematical sense making.
>
> p.335

He argues that the fundamental aspects of thinking mathematically include core knowledge, problem-solving strategies, effective use of one's resources, having a mathematical perspective, and engagement in mathematical practices. According to him, problem-solving covers only part of thinking mathematically. Equally important are developing metacognitive skills as well as developing a mathematical point of view. Monitoring what one is doing while solving problems is considered very important; it is a major determinant of students' mathematical behaviour (Schoenfeld, 1987). Without good self-regulation, students may never have the opportunity to exploit what they have learnt.

Indeed it is the creative processes of mathematical thinking together with the possibility of human error that bring mathematics into existence. Accordingly these may need to be considered when structuring mathematical instruction if it is to generate students' creativity. An over-conscientious concentration on mathematical content may obscure the mathematical thinking that is responsible for the derivation or application of particular aspects of mathematics. Therefore it is suggested that an environment should be created to direct the student's attention entirely to the processes that are essential to successful mathematical thinking (Mason *et al.*, 1982).

2.5. Learning to Think Mathematically and Solving Problems

There have been a number of attempts to teach mathematics as a problem-solving activity. In his famous book *How To Solve It* (1945), Polya describes a sequence of stages for formulating and solving problems:

- understand the problem,
- devise a plan,
- carry out the plan,
- look back at the work.

On occasion this has been used as a basis of problem-solving courses. It is a delightful and engaging book to read, but experience shows that it does not succeed in turning students into problem-solvers. Two reasons for this can be identified:

(i) First, it shows how problems have been solved, essentially reporting ultimately completed mathematical thought rather than encouraging the process of thinking.

(ii) Secondly, the stages themselves are formulated in a way that discourages the reader who lacks confidence. It is all very well to say "understand the problem", "devise a plan", but how is this done if you ***don't*** understand the problem. And if a plan is devised what happens if you ***can't*** carry it out?

Further, as observed by Schoenfeld (1985a), each of the stages proposed by Polya may contains between five to ten different micro stages. He demonstrated that Polya's heuristic needs to be clarified and made more explicit for it to be appropriate for learners. In addition, Polya's problem-solving model does not have a teaching theory. This makes it difficult to implement his model in the classroom (Silver, 1985).

Nevertheless, mathematical problem-solving in the spirit of Polya continues to be used as the foundation in many problem solving guidelines (e.g. Mason *et al.*, 1982; Charles & Lester, 1984; Burton, 1984b; Schoenfeld, 1985a; Ernest, 1988a, De Corte, 1990). There are of course various models of mathematical problem-solving developed by researchers using other paradigms as a baseline. For instance using cognitive psychology and artificial intelligence (see e.g. Ginsburg, 1983; Janvier, 1987; Anderson, 1990). Research findings indicate that problem-solving can be taught with some success. Through a supportive environment, remarkable progress was observed amongst students. Schoenfeld (1987) gave a detailed account of his problem-solving course. The course focuses on metacognitive aspects of mathematical thinking. Apart from teaching, he acts as a role model for metacognitive behaviour, setting up small group problem-solving as well as discussion of the problems that involves the whole class. He demonstrated that after the course, the students' work resembles a mathematician's behaviour when working on a difficult problem far more than it resembles a typical student's behaviour in similar situation. He claimed that after the problem-solving course, less than 20% of the students problem-solving attempts resembled "jump into a solution and pursue it no matter what". The key issue highlighted by Schoenfeld is that the students gave themselves the opportunity to solve the problem and reflect up on it. He implies that students experienced mathematics that made sense to them in a way that was similar to mathematicians. Consequently students are more likely to develop a more accurate view of mathematics.

Mason & Davis (1987) explored how people can develop their mathematical thinking, learning, and teaching by reflecting on their own experience. They argued that the technique of using meaningful vocabulary can help students to become more reflective and effective in mathematical learning. It was observed that students not only notice the use of the vocabulary and advice from tutors, but also remember it when the same language pattern (e.g. specialising, generalising, a slang "What do I want?" etc.) was repeatedly used and their attention was explicitly drawn to it. Mason & Davis's observation lie behind the 'Discipline of Noticing'. They proposed (1988) that shifts in the nature and structure of students' attention can be invoked, basically in four ways:

> - in the presence of a person usually whom we respect or in whom we have some investment;
> - when present experience is suddenly seen as an example or particular case;
> - when a word, expression or image which is richly associated with past experience provokes a moment of noticing;
> - when we suddenly, and apparently spontaneously notice something new or freshly.
>
> p. 491

The ideas of noticing suggest that growth occurs through a delicate shift of attention by the learner (Mason, 1989b). Mason implies that most students require explicit assistance to experience the shifts of attention. He pointed out that the metacognitive shifts can be brought about and is "best supported by undertaking to establish a mathematical, conjecturing atmosphere in classes, where what is said is assumed to be said in order that it may be subsequently modified" (p. 7).

This gives us an indication that students can alter their methods of doing mathematics when they are aware and conscious of the meta-processes in thinking mathematically. Whilst the importance of an answer declined, working on the process was emphasised. It is suggested that students might be motivated to persevere, which could result in more positive attitude and perhaps reduce the fragility of students mathematical knowledge (Schoenfeld, 1987; Alibert, 1988; Movshovitz-Hadar, 1993). A study by Rogers (1988, 1990) reports on an American institution's success in creating a learning environment that develops students to their potential in the learning of advanced and abstract concepts in mathematics. Their initial focus was on changing the students' perception to mathematics as a difficult and an almost impossible subject to one that they are capable of doing. She observed that encouraging students behaviours such as high self-esteem, confidence in their mathematical abilities, and the ability to work independently are closely linked to the faculty approach to teaching mathematics. The emphasis is on the negotiation of mathematical meaning and students' growth and development rather than the transmission of knowledge and skills. Their teaching practices show that students who learn to think mathematically are able to reconstruct ideas and learn independently. She likened this ability to "riding a bicycle" that one never forgets. Some of the techniques used include active student participation, group work in class and outside of class, and constructivist approaches to developing the subject matter. Further Rogers suggests that the attitude towards teaching the students to think mathematically requires a "caring teacher" in the sense of helping the other grow and actualise oneself. She concludes that in a supportive environment that favours the learning of advanced mathematics and in which the style of teaching is true to the nature of mathematical inquiry, women are attracted to mathematics and they are just as successful as men.

It is observed that one important factor that contributes to all these successes lies in the explicit emphasis of the importance of the search for a solution rather than the solution itself. It is suggested that the creation of the solution methods allows students to bring to their awareness the processes of mathematical thinking and the attitude that encourages it (Mason *et al.*, 1982). It allows students to relate mathematical ideas together and as noticed by Ernest (1988a) it allows them to be creative and to work at a high cognitive level. Furthermore, according to Simmons (1993), although the completion of the search may be satisfying in itself, it is the belief that one can solve the problem or at least begin to understand it better that gives the motivation to the solver. Consequently, this may result in increased confidence and pleasure.

Additionally it is suggested that a student's solution path which is mathematically acceptable should not be rejected on the ground that a more obvious and simple solution method is available or a more powerful algorithm have just been introduced in class. The student's solution method is their own original method that they had developed themselves. It is through their own thinking they find an approach that solved the problem. Therefore the choice of the solution method should be considered and not judged according to teacher's choice. Besides, as made clear by Schoenfeld (1987), it is more important that they made use of what they did know. In particular in deciding which method is more appropriate to apply in the circumstances. Perhaps this is where the quality of students' thinking differs.

2.5.1. Mason, Burton & Stacey's (1982) framework

Mason, Burton & Stacey (1982) follow the basic Polya format with stages named:

- entry,
- attack,
- review.

This development proves to be far more student-friendly. They emphasised the importance of thinking mathematically and demonstrate how it can be improved by students. The focus of the problem-solving activities are two basic processes that underly the development of mathematical thinking—specialising and generalising. Here specialising basically means looking at special or particular cases to learn about the question. At this point it is suggested that a student work with objects (physical, mental image, a diagram etc.) that are familiar and can be manipulated with confidence. By doing examples, the question will be meaningful to oneself. Further one may start to see an underlying pattern in all the special cases which will help to solve the problem.

The process of generalising is the reverse of specialising. It involves noticing things that are common to several examples and ignores aspects that seems special to some of them. Generalising begins when one senses a pattern and tries to articulate it. To make a generalisation is much harder than specialising which is almost always easy to do. Generalisation is more a creative process. It requires one to become "fully involved and imbued with the question". They suggest two ways to increased the power of generalisation in mathematics:

> (i) by developing an expectation of pattern and being prepared to carry out an active search for it
>
> and
>
> (ii) by building up mathematical knowledge and experience.
>
> p. 80

Furthermore, generalisation should generate conjectures. Whilst this whole process, according to their view, is the essence of mathematical thinking, making conjectures lies at the heart of mathematical thinking. It forms the backbone of a resolution. All conjectures need to be investigated to see whether they are accurate. If the conjecture is false, one should modify it. Once the conjecture seems true, one should try to justify it convincingly. Justifying has to do with stating clearly the structural links that indicates why the conjecture is true. Explaining "why" involves three stages: firstly convincing oneself, then a friend and finally an enemy. This particular process is presumed to be difficult for students; convincing an enemy is clearly the most difficult.

Following Mason *et al.*'s model, all the processes of thinking mathematically proceed in the three phases named above. "Entry" is identified as the phase of getting engaged with the problem—getting familiar with the problem, and making sense of the problem by specialising to simple cases and playing with ideas. Moving on to specifying clearly what is known, what is wanted and thoughtfully considering what can be introduced (notation, diagram, etc.) might induce progress from what is known to what is wanted.

Using the ideas introduced, the "attack" phase may begin. In particular, when one feels that the question has moved inside oneself and becomes ones own. It is in this phase that the major activity of finding a solution takes place. It might be successful or end in a blind alley from which the strategy is then reviewed. It may then be possible to return to "entry" phase to consider a new "attack".

The "review" phase begins once a reasonably satisfactory solution is achieved (or when about to give up). This phase may involve checking the results for errors, reviewing what has been done and reflecting on the experience, integrating it in thinking repertoire for use in other occasion. The review phase may set the scene for the extension of the current position to a wider context, re-starting the "entry" cycle at a more sophisticated level.

Mason *et al.* squarely face the associated emotional difficulties when the student feels "stuck", as well as the elation of an insightful "aha!". They stress that students should be encouraged to write a "rubric" diary of each problem-solving activity, indicating where they may be "stuck" and or have moments of insight signified by "aha!". Initially the students may find this difficult because few are used to writing more than a sequence of mathematical symbols. However, it is suggested that after two or three weeks, when the students realise that had they made proper notes they could have used earlier ideas, now forgotten, the vast majority may see the value of a rubric. This is then available for reflection. The students will gradually learn how to use the recorded experiences to promote better personal problem-solving in the future.

Mason *et al* also recognised that besides developing within students a general approach to problem solving an additional useful element to apply in courses is a knowledge of the emotional effects of success and failure. They indicate that success leads to increasing confidence to tackle new problems. It is therefore important to grade problems so that students experience the elation of success. Sadly, they have often reached a point where they are so overwhelmed by the content of their regular courses that they may give up trying to understand and resort to straight rote-memorisation of notes to pass examinations. Breaking this attitude is a major undertaking (see for e.g. Sierpiñska, 1987; Williams, 1991; Tall, 1992a).

In *Thinking Mathematically,* it may be suggested that one possible weakness of the framework is the absence of formal proof (in a university sense), although it has the idea about convincing an enemy. There has been a struggle to understand what is meant by proof (see for e.g. Hanna & Winchester, 1990). Basically, in a given context, proof means establishing what we are saying is true and cannot be false. The context that we work in determines the sort of proof that we use (Hersh, 1993). Mathematical proof is considered as the final precising stage of mathematical thinking (Hadamard, 1945) and is viewed as a problem-solving activity. As noted by Polya (1954):

> To a mathematician, who is active in research, mathematics may appear sometimes as a

> guessing game; you have to guess a mathematical theorem before you prove it, you have to guess the idea of the proof before you carry through all the details. ...The results of the mathematician's creative work is demonstrative reasoning, a proof, but the proof is discovered by plausible reasoning, by guessing. p. 158

In the same vein as Polya, Mason *et al*'s sequence of convincing can be considered as moving from intuition to proof, as a form of understanding and communication (Schoenfeld, 1994). In investigating the truth of conjectures, students need to develop some intuition and then try to prove the results. Furthermore, as observed by Poincaré (1913):

> We then have many kinds of intuition; first, the appeal to the senses and the imagination; next, generalization by induction, copied, so to speak, from the procedures of the experimental sciences; finally we have the intuition of pure number...
>
> p. 215

Hanna (1991) argued that what needs to be conveyed to students is the importance of careful reasoning and of building arguments that can be examined and revised. Although these may involve a degree of formalisation, the emphasis is on the clarity of ideas.

2.5.2. Skemp's theory

The theory of "goals" and "anti-goals", (Skemp, 1979), may be informative to students as they solve problems. Skemp distinguishes between a goal to be achieved and an anti-goal to be avoided. A goal achieved gives pleasure, an anti-goal avoided gives relief. The feeling that one can achieve goals gives confidence, the sense that one can avoid anti-goals gives security. For example, frustration is seen as an inability to achieve a goal. To be frustrated from doing something often means that what we want to do is to come back and do it. We feel frustrated because we know what we want to do but we cannot do it. Anxiety is concern about being unable to avoid an anti-goal. For instance, we are going to fail and we do not know how to avoid failing, hence we feel anxious.

Considering one's own attitudes to mathematics can help focus on how mathematics is being viewed. Does one seek the security of being able to carry out routine procedures that will help one avoid failure in examinations, or the confidence of broader conceptual understanding that generates an ever deeper sense of confidence? Many students will identify more with the former than the latter. If they are able to carry out mathematical thinking in a non-threatening atmosphere, then success may lead to confidence. Knowledge that a negative emotional feeling is caused by an underlying lack of knowledge may help them seek to solve the underlying difficulty rather than succumb to the sense of hopelessness that causes them to give up. Acknowledging such a situation with the simple word "stuck", followed by a search for activities which will get them round the difficulties, can turn negative emotions into a positive activity.

2.6. Mathematics Teaching: Beliefs and Practise

From a constructivist viewpoint, the mathematical knowledge that we have is a body of knowledge that we have constructed ourselves through experience, reflection and abstraction. Therefore, a lecturer—who has built vast mathematical knowledge over the years through daily teaching and research—has a perception of mathematics that is likely to differ strongly from the student as a learner. The flexible thinking that lecturers perform which seems to make the mathematics so much easier to them may not necessarily shared by students. Those who do, are likely to be the most able students. Students' mathematical thinking tend to be overlooked by teachers. As observed by Freudenthal (1983):

> One finally masters an activity so perfectly that the question of how and why students don't understand them is not asked anymore, cannot be asked anymore and is not even

> understood anymore as a meaningful and relevant question.
>
> p. 469

Erlwanger (1973) in his study found that some of the most successful students observed were actually very confused about many of the mathematical ideas that they had supposedly mastered. He argued that the serious confusion was due to the emphasis on mastering separate skills and paying no attention to how students were thinking. Further Dreyfus (1991) gave evidence of similar occurrence amongst undergraduates; many succeed in advanced mathematics without the reasoning processes of mathematical thinking. According to Dreyfus, this is due to teachers' emphasis on correct performance rather than on understanding.

It has been suggested that teachers' views, beliefs, and preferences about mathematics played a significant role in shaping instructional behaviour and do influence their practice (Thompson, 1984). Pupils' roles in the mathematics classroom seem to vary according to the view of mathematics projected by the teacher (Nickson, 1994). Nevertheless, it is likely that for some teachers the view projected may not be parallel to their own personal belief, but chosen due to its supposed adequacy. The formalist point of view appears to be widespread among university teachers with many showing little interest in reflecting on the nature of mathematics (Mura, 1993).

Mason (1989a) identifies that the extent to which students succeed in making sense of the mathematical world depends, amongst other things, on the teaching style adopted, their involvement in problematic questions at the heart of the topic, the teacher's attitude to teaching and learning, the extent to which students' own powers are evoked and employed in the teaching and learning and the extent to which students share the teacher's goals. Thus it is believed that there should be some correspondence between what teachers aim students to achieve at the end of a mathematics course and the students' aims. Discrepancy between the two indicate that there is a problem to be worked on (Schoenfeld, 1991).

Skemp's (1989) theory of learning helps to clarify some of our aims in teaching mathematics. He suggests that "there are two effectively different subjects being taught under the same name, 'mathematics'" (p. 7). By this he means instrumental mathematics and relational mathematics. To him these two kinds of knowledge are very different though the subject matter is the same. He pointed out that many children are being taught only instrumental mathematics throughout their schooling years. Consequently they find to their cost the word mathematics is "a false friend".

In addition Skemp proposed two kinds of mathematical mis-match that occurs between the goals of pupil and teacher: "Pupils whose goal is to understand instrumentally, taught by a teacher who wants them to understand relationally" and "the other way about" (p. 5). This can be seen as a contrast between the teachers' aims and the approach they used when faced with the reality of the classroom and between the aims of teachers even within the same department. However the other mis-match, in which students are trying to understand relationally but the teaching makes it impossible, to Skemp is more damaging. For example it may suppress the students' intellectual development. A less obvious mis-match is one that occurs between teacher and text. Skemp argues that much of the modern mathematics was taught as instrumentally as the traditional syllabi. For example ideas such as sets, mappings and variables were introduced to help foster relational understanding but if taught instrumentally may do more harm than good. Skemp posits that the teachers' difficulties in adapting their approach is due to a predictable difficulty in accommodating or restructuring their existing schemas.

Mason (1989a) argued that it is the location of teacher's attention, and the extent of the teacher's mathematical being which determines successful mathematics teaching, not the mode or style of teaching. He posits that teachers need to develop their teaching and their sense of what it means to be mathematical to help students in shifts of attention, such as abstraction. Further, Sfard (1994) suggests that the inability to communicate with each other may be due to teachers and students being unaware that they are participating in different mathematical practices and that their mathematical imagination is shaped by different metaphors. She points out that teachers should be aware of these differences to help bring about mutual understanding.

The tradition of teaching students accumulated mathematical thoughts appears to be far from ideal in preparing them to meet the demands of the changing modern world, and the demands it make upon the ability to think for ourselves and the ability to solve problems. Studies have shown that the traditional approach is failing the majority of the students, not only the average students but more disturbingly also the successful students. Students find great difficulties in constructing their own mathematical understanding (Davis & Vinner, 1986; Martin & Wheeler, 1987; Sierpińska, 1988; Eisenberg, 1991; Williams, 1991) and have a narrow view of the mathematics that shapes their mathematical behaviour (Schoenfeld, 1989; Vinner, 1994). Similar difficulties were observed among Malaysian students (Mohd Yusof & Abd. Hamid, 1990; Razali & Tall, 1993) and reformation in the mathematics education has been proposed (Sanugi, 1989; Nik Pa, 1992; Noordin, 1993; Sunday Times, 1994).

The review of the literature reveals that students at university seem to be given lectures that consist of theorems and proofs and these do not encourage them to think mathematically. Problem-solving is seen as no more than just a skill to be acquired. This study proposes a shift in focus, through a problem-solving course it aims to provide students with appropriate experiences which will enable them to think mathematically. Through the research design, students at UTM are encouraged to think in a mathematical manner. Through a close examination of their perception of mathematics the study considers the effect of such encouragement has on their attitudes.

3. METHOD OF STUDY

3.1. Introduction

Much of research in mathematics education has been largely focused on younger children. However, the formation of the Working Group of PME on Advanced Mathematical Thinking indicated the need for research within the broad perspective of mathematical thinking, in particular beyond the age of 16. Certainly this would help university mathematics teachers see the relevance of this research to their practice and teaching.

There is no single theoretical framework dominating research into the development of advanced mathematical thinking. In attempting to investigate problem-solving and mathematical thinking researchers have resorted to a variety of indirect methods of observation which allow them to make inferences about the mental processes. Increasingly popular are the protocol methods—the talking aloud procedure and the clinical interview technique (Ginsburg, Kossan, Schwartz & Swanson, 1983).

Techniques that have been widely used to gather data include questionnaire, interview and self-reports. A questionnaire, in particular in a large scale study, is the most traditional means of data gathering. It allows the researcher to gather information about variables which are of interest. Filling in a questionnaire should involve introspection (Wolf, 1988), but there are limitations in asking students about their attitudes to mathematics. The main weakness being some of the terms used may carry several interpretations to the students. Another that they may respond by saying what they feel is wanted by the researcher—how they should respond. The idea of a questionnaire was also tempered with the realisation that there may be some uncertainty about student responses. This may to some extent affect and decrease the validity of the findings. Besides, due to the large number of students it is not possible to query their responses or their reactions to the statements. However, Vernon (1963) pointed out that highly educated people such as students are very often more self-analytic and introspective than non-academics. They take a more detached view of themselves and are more aware of their conflicts and anxieties. This means that not everyone aims merely to display a favourable self-picture. It is with these notes of caution that a questionnaire was used.

There are several kinds of interviews that can be used as research tools as noted by Cohen & Manion (1980). In this study we have used the unstructured interview during classroom observation, a semi-structured interview which is a mixed case of talking aloud and clinical interview with selected students, and a non-directive interview with the selected staff. These methods involve interpretation and inference on the part of the researcher, thus they are not without problems. In the semi-structured interview, "talking aloud" involves the subjects under study reporting verbally their own mental processes. There is controversy concerning the ability to provide verbal reports of one's own cognitive processes and the validity of these reports as data. Nevertheless, disparate theorists agree that the "most stringent criteria of understanding involve the availability of knowledge to consciousness and reflection" (Brown, 1987, p. 72). Thus verbal reports are permitted as a data, although it may prove difficult to get students to talk aloud in a natural way during problem-solving. The thinking aloud technique was made known by Newell & Simon (1972) to verify their computer-based theories of problem-solving. A similar technique was used by Schoenfeld (1985a) in his investigation of mathematical problem-solving. He established that verbal reports (protocols) produced by students in "speak-aloud" problem-solving sessions can be taken as data. Further he argued that the decision-making and the reasons for them would be brought out in the open while problem-solving (Schoenfeld, 1985b). Schoenfeld implies that to provide a reasonably comprehensive picture of problem-solving behaviour, this method should be coupled with a variety of other techniques. Despite the problems, interviews and questionnaire have been used in studies investigating covert behaviour, for example metacognition (see for example Baird, 1986).

Studies (e.g. DeGuire, 1987) had established that students' self-reports can be used to study the development of metacognition (the regulation of one's cognitive processes) during problem-solving. Although, Lester (1987) in his study pointed out that written retrospective accounts of one's thinking have provided him little information. He suggest that this may due in part to the students' inexperience with this sort of activity. On the other hand, he posits that it may be due partly to the ages (6-13 years) of the children he worked with.

This chapter reviews the research design used in the study which consists of three phases:

Phase one: The pilot study

This is a preliminary investigation carried out at Warwick University (Section 3.2) to determine the appropriate methodology and to generate useful information and experience for the main study. Students following a ten week, 30 hours problem-solving course in the winter term of 1992/93 were chosen as subjects of the study. The course was taught by Professor D. O. Tall who had been conducting it for more than a decade. It is based around the book '*Thinking Mathematically*' (Mason *et al*, 1982). As the researcher I was present in all the classes which not only gave the opportunity to reflect on my own power of mathematical thinking but also to observe students' reaction by joining in their discussion. A 28 item questionnaire was administered to a class of 79 students. Based on responses to the questionnaire, nine students were selected and an interview of about half an hour was held with each of them.

Phase two: The main study

The main study was carried out at the UTM (Section 3.3). Problem-solving activities based on the Warwick University format were presented at UTM. Changes on students' attitudes were examined as a result of the problem-solving course. The materials used at Warwick was translated into Bahasa Malaysia (the language instruction in the UTM), and checked by a linguistic expert. The course was presented in the first semester (July, August and September) of the 1993/94 academic session. It was offered to the third, fourth and fifth year undergraduates aged 18 to 21 in SSI (Industrial Science, majoring in Mathematics) and SPK (Computer Education). 60 students signed up for the course. An 18 item, modified questionnaire was administered to the students. 6 groups which consist of 3 or 4 students were selected for an interview. Each group was interviewed for 40 minutes which was video-recorded.

Phase three: A supplement to the main study

This was carried out at the UTM at the end of the second semester (March, April 1994); six months after the problem-solving course was given. The study consist of two components:

a) Re-investigating students

This was done to determine the long term effect of the problem-solving course on students' attitudes towards mathematics and problem-solving. It involved representation of the questionnaire to the sample of UTM students one semester after returning to their standard mathematics courses. It was planned that the earlier selected groups of students would be re-interviewed with some problem-solving. However, due to the forthcoming final exam the students felt pressurised; thus the interview was modified to informal discussion.

b) Investigating mathematics lecturers

In the belief that students' mathematical thinking and positive attitudes need to be nurtured and reinforced, attitudes of staff at the Mathematics Department were investigated. The data collected also help to establish the mathematicians *preferred* changes. The students' attitudinal questionnaire was circulated to all mathematics staff. Eight staff were chosen for further study of their perception of students' mathematical thinking. A 16 item, perception questionnaire was administered to the eight lecturers and their students who had followed the problem-solving course the semester before. An interview for half an hour was held with each of the eight lecturers.

3.2. Preliminary Investigation: The Warwick Study

The methods used to collect the data and to gather the relevant information were classroom observation, questionnaire, students' written assessments and interviews with selected students. A combination of these methods would provide a more legitimate perspective of students' belief and problem-solving behaviour (Schoenfeld, 1985b). Furthermore, observations made of students working on problems in regular classroom settings can provide different kinds of data about students' abilities, beliefs, and attitudes than would have been obtained by means of interviews or other less natural conditions (Lester, 1988).

A prepared questionnaire was distributed to the students in the third week of the course during the first meeting. It was again distributed at the end of the course during the final meeting. A questionnaire is chosen for reasons of practicality: it is easy to administer to any desired number of subjects, enables data to be collected within a short time and makes it possible to gain a wide range of information from a single administration. Moreover it is useful in giving a general picture of the students' attitude.

The questionnaire was distributed to the students personally so as to obtain maximum co-operation. They were given 10 to 15 minutes of the meeting time to complete it. From a total of seventy-nine questionnaires distributed during the pilot study, only forty-seven students' responses were considered. That is those who had completed both sets of the questionnaire—before and after the course. Students had written their code names (a combination of the mother's maiden name and student's surname) on the questionnaire and some had written their own names voluntarily. Thus it was possible to link responses with individuals. However, the post-test questionnaire was distributed during the final meeting in the last week of the term and there were several absentees. Even though it is possible to know who they were, there was no way of knowing their whereabouts. Therefore, no remainder or follow-up was possible. Furthermore, the questionnaire was distributed only during one out of three meetings in the particular week. It should have been made available throughout the week for those who had missed the earlier meeting.

3.2.1. The Questionnaire

The statements within the questionnaire were developed to assess the students' attitudes towards mathematics and problem-solving before and after the problem-solving course. As a pilot-test, the purpose was also to test the questions for fitness of purpose. Several of the statements were gathered from materials reporting studies on students' attitude toward mathematics. For instance Statements 4, 5, 7 (Section A) were taken from Joffe & Foxman (1986). Statements 2, 8, 11 (Section A) and statements 1, 4, 5 (Section B) from The Open University (1986). Joffe & Foxman findings indicate that in general girls exhibit less confidence than boys in their mathematical ability and that girls experience greater extent of anxiety about mathematics compared with boys. Statement 12 (Section B) came from Cobb, Wood, Yackel & Perlwitz (1992b), whilst statements 6, 9, 10 (Section A) and statements 6, 7 (Section B) were from personal teaching experience (see e.g. Mohd. Yusof & Abd.

Hamid, 1990). It was noted that the majority of students had difficulty in using their mathematical knowledge to solve mathematical problems and seemed to be dependent on others during problem-solving. Other statements were invented based upon assumptions gathered through the review of the literature.

Nevertheless, the questionnaire was expected to have some potential limitations. For instance words like "easy", "remember", "see value" and "difficult" are relatively subjective and global. This would make students feel doubtful and indecisive. Thus it is likely that these statements could attract a 'no opinion' response from the students.

Through the questionnaire the students were asked to consider where they stood in the context of their beliefs and perceptions of mathematics (part A) and to provide a self assessment of their approach to mathematics (part B). It was believed that the results would provide us an indication of their attitudes towards mathematics and problem-solving. Following the Likert model, each statement on the questionnaire gave students an opportunity to provide a single response on a five-point scale indicated by "definitely yes" (Y), "yes" (y), "no opinion" (–), "no" (n) and "definitely no" (N). A five-point scale was chosen in preference to a three point scale (yes, no opinion, no) because in a three point scale it is likely that some students who chose "yes" may not actually be very concerned with the issue, whereas others may be strongly convinced of their position. In contrast, those who say "no" may include those who are only mildly in disagreement as well as die-hard opponents (Siegal & Castellan, 1988).

The Likert model was designed to:

(i) yield quantifiable data on students attitudes to mathematics and problem-solving.
(ii) provide a first notion of qualitative information on students attitudes.

The Wilcoxon Matched-pairs Signed-ranked Test was used to test the significance of change in the students responses before and after the course.

The full questionnaire contained the following statements.

Section A: Attitudes and Perception

1. Mathematics is easy for me.
2. I usually understand a new idea in mathematics quickly.
3. I find the topics we study in mathematics often make little sense to me.
4. I often see the value of most of the mathematics we do.
5. I remember most of the mathematics I did last year.
6. I sometimes find difficulty applying routine procedures to unfamiliar mathematics problems.
7. I need a good knowledge of mathematics to be able to get on in life.
8. I have to work very hard to understand mathematics.
9. I find it helpful to ask my friends when I get stuck.
10. I sometimes ask my lecturer for help.
11. I usually work on my own.
12. In few sentences describe your feelings about mathematics.

Section B: Self-assessment

1. I always feel confident in my ability to solve mathematics problems.
2. Solving mathematics problem is a great pleasure for me.
3. I only solve mathematics problems to get through the course.
4. I always feel anxious when I am asked to solve mathematics problems.
5. I often fear unexpected mathematics problems.
6. I usually know how to get started on mathematics problems.
7. I feel more secure when the procedure in solving mathematics problems is given.
8. I tend to persevere in solving mathematics problems even when I seem not to be making progress.
9. I feel frustrated when I fail to get correct solutions to mathematics problems.
10. I feel the most important thing is to get correct answers.
11. I feel anxious when I get stuck.
12. It is a relief to be able to discuss my difficulties with others.
13. I am usually aware of what I am doing while solving mathematics problems.
14. I usually look back to review my resolution until I am convinced it is acceptable.
15. I feel I am performing up to my potential in the problem solving course.

16. I would recommend this course to others.

To establish a pattern of agreement to these statements, the questionnaire was given to two colleagues for a preview. They gave support to all of the statements and indications as to whether statements were favourable or non favourable apart from the statement "*I have to work very hard to understand mathematics*" which they had mixed opinions about. In part A of the questionnaire we considered that all statements are favourable apart from the belief that *mathematics make little sense* (statement 3) and the item specifying that students may have *difficulty applying procedures* (statement 6). In part B, it is considered that all statements are favourable except statements 3, 4, 5, 7, and 8. By favourable statement we mean a positive response to the statement is considered to represent a positive attitude.

The statement "*I find the topics we study in mathematics often make little sense to me*" appeared to be a good discriminator amongst the students. Using this statement the students were segregated into two groups; group S included all who rejected the statement (i.e. it made Sense) and group N who agreed with the statement (i.e. it made No sense). It was conjectured that students in group N may have different qualities of thinking compared with students in group S.

3.2.2. Students' Comments

In an attempt to establish the validity and the reliability of the questionnaire as a measuring instrument, other ways of gathering students' opinions was considered. It was planned that several students would be picked and interviewed individually. Ten students were selected based upon their responses to the questionnaire and all but one were willing to be interviewed. Another source of data was my presence in the classes. The classroom observation had given me a broader perspective to what was being said. Students' written assessment gave me further pieces of information.

Being present throughout the course and participating and observing what was going on in the class enabled me to obtain valuable information. Whilst the students were working on problems, I circulated around the room. No specific questions were asked. I simply joined in the discussion. In doing so much more revealing information was generated by the students about their mathematical experience. The students were ever willing to talk and provide comments. These were recorded and noted as field notes.

Towards the end of the course I realised that it would be an advantage if I had access to the students' weekly problem-solving assignments so that close track of their progress could be determined. Such access would also allow me to make a comparison with my personal observation, and provide additional detail for further analysis.

Students' written comments were also obtained through an open question in the questionnaire. They were very responsive and the expressed opinions suggest their views towards the mathematics they are studying. Another source of students' comments was the written assessment in which students were requested to comment critically on problem-solving and its effectiveness in their mathematics learning. The majority considered that problem-solving techniques were relevant in their mathematical education at the university.

3.2.3. Semi-structured Interview

Ten students were invited to attend an interview. They were chosen initially because they represent the N and S groups of students, and were representative of the subject areas. Students' gender and their academic achievement at end of previous year were also considered. Hence five female and five male students with varied degree classification, covering almost the whole range were picked (see table 3.1). All but one (i.e. Mary**) came for the interview. (The names are fictitious to maintain anonymity).

Students	Course	Degree Classification	Gender	Group
Chris	Maths	I	M	S
Mary**	Maths	II-2	F	S
Alice	BA(QTS)	II-1	F	S
Sarah	BA(QTS)	II-1	F	S
Eric	CS	II-1	M	S
David	Maths	II-2	M	N
Naomi	Maths	III	F	N
Ruth	BA(QTS)	II-2	F	N
Peter	CS	II-1	M	N
Colin	Maths	I	M	no opinion

Table 3.1: The ten students selected for interview

Chris and Alice exhibited a positive attitude before and after the course. Peter, Sarah and Eric became more positively inclined after the course. Ruth and Naomi's negative attitudes lessened after the course. However David remained negative, whilst Colin still expressed no opinion on all indicators.

The nine selected students were interviewed individually. Each interview lasted 30 minutes. The interview was a mixed case of thinking aloud and clinical interview. The students would be given a problem to solve and were requested to verbalise every thought that came to mind in the course of solving the problem. The students were not told that they would be invited to do problem-solving before the interview. The purpose of this aspect was to see how they reacted to having to solve an unexpected problem. During the problem-solving component, to see how the students cope with a problem, intervention from my part was kept to a minimum. The interview was tape recorded.

The problem given was sufficiently challenging for the students to solve. Given the time constraint the students were not expected to come up with a complete solution at the end of the interview, but they should at least be able to make a start. The students were very demonstrative, physically and emotionally, in their reactions when solving the problem. They had revealed their thought processes which I had hope to obtain. The students' responses helped to substantiate the data gathered from the other sources.

When working with students at Warwick I was received as one of their peers but perhaps this would not be so when I am at the UTM, where the students would look upon me as their lecturer. Given the cultural outlook this would impose certain constraints upon the students and would probably bring about a strong discomfiting effect. Thus some reactions may be obscured when there is only one person. Schoenfeld (1985b) had suggested that when students worked together in a small group, the burden of uncomfortableness is shared amongst them and this eases the pressure on them. Therefore in the main study, a small group interview was chosen in the hope that it would bring out into the open the students' real problem-solving behaviour.

3.3. The Main Study: Problem-solving at the UTM

3.3.1. Background Consideration

Over the years teaching in Malaysia has been presented with a very strong dose of authority. The students are not used to solving problems independently. They tend to wait for instructions for what they need to do and how they need to do it. Besides, they are very reserved in their opinions; they fear offending the teacher as well as being ridiculed by their peers for saying something silly. For many of them, rote-learning is a common practice. Even Malaysian students who are studying abroad are more prone to rote learning than their western peers; they have an over regard for the authority of their lecturers and consequently they feel reluctant to question their teacher's opinion and think for themselves (Samuelowicz, 1987; Watkins, Reghi & Astilla, 1991). Certainly such an approach does not help problem-solving. Thus, the delivery of the course proved to be a great challenge to me as the tutor. It was expected that the students would pose a very strong resistance to the nature of the course and that it would take a far longer time for any positive change to occur. It was planned that the course would be presented in a much slower pace than that at Warwick. This would give the students time and space to adapt to the "new" atmosphere. The major task was to provide the students with a supportive environment in which they could carry out mathematical thinking without feeling threatened with their failure. Accordingly this would lead the students to build their self-confidence and to regain the sense that they can do problem-solving on their own.

UTM has a wider ability range of students than Warwick. Students must have a minimum of 5 credits (in mathematics, physics, chemistry and 2 other subjects) in their SPM (equivalent to O-level) to enter the technical degree courses. Currently there is no mathematics major degree course running at the UTM. But this does not means that the mathematics taught at the UTM is less rigorous than that followed by the MORSE or Computer Studies students at Warwick. The subjects chosen will have quite similar experience in mathematics and have achieved a relatively advanced level in mathematics at the time of the study. The problem-solving course was offered to undergraduates taking mathematics as one of their major subjects. If given the opportunity, the Engineering students would have been chosen as well. They too are studying mathematics as one of the essential subjects and have great difficulties in their learning (see e.g. Razali & Tall, 1993). Unfortunately, this was not possible due to their compact time-table. Moreover, there exists an uncompromising view between the mathematicians and the engineers at the UTM about what mathematics really is and how it should be taught (see Amin, 1993). Thus, it is felt that if these students were included, it may appear that the study is more about convincing others rather than offering the students an alternative view to mathematics.

At Warwick, the problem-solving course is part of the system. At the UTM, there is a course called *'SMT 2552 Mathematics Recreation'* which was first introduced to the students at the Faculty of Science in 1990. The main aim of the course was to develop students' ability to use their mathematics knowledge. The objective of this course may appear complementary to the problem-solving course but it did not focus on encouraging students to think mathematically. With the consent of Dr Ramli Hj Salleh, then the Head of the Mathematics Department, this course was replaced with the experimental problem-solving in the academic session of 1993/94. In doing so we had integrated a research project into the system, which carried the same credit points as the course it replaced.

Sixty students signed up for the course. However, only 44 students took part in the study. They were chosen because they followed the problem-solving course and they were the ones who had completed both the pre-test and post-test questionnaires.

3.3.2. Planning the Main Study

The appropriate methodology was determined by the main purpose of the study that motivates the investigation at the UTM—that is to find out whether problem-solving can alter students' attitudes towards mathematics and problem solving. The study consisted of two phases. Firstly, the presentation of the problem-solving course. The experimental problem-solving course was conducted in July, August, and September 1993. It was intended that the four ways of collecting data in the Warwick study would be used in the study at the UTM with some modification. Secondly, it was planned that a delayed post-test would be carried out in the second semester, that is six months after students returning to the regular mathematics lectures. The delayed post-test which involved administering the questionnaire and interviews to both students and mathematics lecturers was made in March and April 1994. The purpose is also to determine staff opinion on the positive and negative status of the attitudes investigation.

The Warwick students' responses point to the issues that we need to consider here; the students' comments provided a clear sense of direction in constructing the statements for the questionnaire. On some of the items measured there was a considerable minority expressing "no opinion". Such responses clearly do not contribute any useful indication of the subjects' views. It indicates that students are having difficulty in making decisions. It was suspected that such weakness may have stemmed from words used. As pointed out by a student during one of the sessions when talking about his views on mathematics: "It depends. Some topics are very interesting, others utterly boring. I enjoy the practice more than the theory". He further elaborated that when he filled in the questionnaire he was uncertain on some of the questions. So as a way out he opted for "no opinion". Thus the pilot test questionnaire was amended to minimise the ambiguity. Several statements in the Warwick questionnaire (i.e. statements 2, 3, 8, 12 in part A; 1, 2, 3, 4, 5, 10 in part B) were kept since they appear to be clearly defined to the majority of the students and issues central to these questions also recur in their expressed opinions. Other statements which attract a considerable minority of no opinion responses were replaced with new ones which included issues raised by the Warwick students in their written comments and during the interviews:

- the notion that mathematics is composed of facts,
- mathematics is abstract,
- students can relate mathematical ideas learned,
- that they have confidence,
- persevere to make sense,
- willing to try different approaches.

The final questionnaire used contained the following statements.

Section A: Attitudes to Mathematics

1. Mathematics is a collection of facts and procedures to be remembered.
2. Mathematics is about solving problems.
3. Mathematics is about inventing new ideas.
4. Mathematics at the University is very abstract.
5. I usually understand a new idea in mathematics quickly.
6. The mathematical topics we study at University make sense to me.
7. I have to work very hard to understand mathematics.
8. I learn my mathematics through memory.
9. I am able to relate mathematical ideas learned.
10. In a few sentences describe your feelings about mathematics.

Section B : Attitudes to Problem-Solving

1. I feel confident in my ability to solve mathematics problems.
2. Solving mathematics problem is a great pleasure for me.
3. I only solve mathematics problems to get through the course.
4. I feel anxious when I am asked to solve mathematics problems.
5. I often fear unexpected mathematics problems.
6. I feel the most important thing in mathematics is to get correct answers.
7. I am willing to try a different approach when my attempt fails.
8. I give up fairly easily when the problem is difficult.

Under each statement, students responded to a five-point scale: Y, y, –, n, N (i.e. "definitely yes", "yes", "no opinion", "no" and "definitely no"). This questionnaire was circulated to the lecturers and they were asked to specify attitudes that they *expect* students to have and what they *prefer* students to have. The change in direction from *expected* to *preferred* is defined to be the "desired" direction of change (The collection of this data from staff is reported in chapter 5). Further, in interpreting the results, to indicate the extent of the students agreement or disagreement a score of 2 is allotted to Y, y scored 1, — scored 0, n scored -1, and N scored -2.

The pattern of the five-point scale was kept since it offered a more varied type of response. As noticed in Warwick there were students who gave many extreme responses, others were more guarded while they were some who were indecisive. One weakness of this type of pattern would be in the analysis of the responses. In particular when the need to highlight the "Yes" and "No" responses is required, the differences between the responses or the strength of agreement or disagreement tend to be overlooked. However, it is possible to detect students' change in responses from each category. Three different measures were taken on each subject; pre-test, post-test and post post-test. To assess the significance of any differences noted between measures a Wilcoxon matched-pairs signed-ranks test was used (Cohen & Holliday, 1982).

Semi-structured interviews were held with groups of three or four students (who had worked together). This was a departure from the approach used at Warwick. There, during individual interviews, students had displayed physical and emotional reactions when solving an unexpected problem. If individual interviews had been used in Malaysia, it was believed that the dominance of the usual pupil reticence with the teacher would have concealed this. The groups of 3 (or 4) were intended to induce discussion between students in the hope that this would be more likely to reveal their thought processes while problem-solving. The purpose of the interview was to uncover the students' capability in carrying out the mathematical processes. The interview itself was structured in the same format as in Warwick—a mixed case of talking aloud and clinical interview.

Six groups of students were selected for the interview. The groups of students picked were representative of the subject areas and of the academic achievement at end of previous semester (see table 3.2). All six groups came for the interview session. The S and N students were identified as seen on page 53. (The names are fictitious to maintain anonymity)

	Students	Course	Degree Classification	Gender	Group
group 1	Sam	5 SPK	II-1	M	S
	Abel	4 SPK	II-2	M	S
	Henry	4 SPK	II-1	M	S
group 2	Sue	4 SPK	I	F	S
	Teresa	4 SPK	II-1	F	S
	Sasha	5 SPK	II-1	F	S
group 3	Rob	3 SSI	II-1	M	S
	Kline	3 SSI	II-1	M	S
	Ian	3 SSI	I	M	S
group 4	Hanna	5 SPK	II-1	F	N
	Katy	5 SPK	I	F	N
	Terry	5 SPK	I	M	N
group 5	Bob	5 SPK	II-2	M	N
	Yvonne	5 SPK	II-1	F	N
	Alma	4 SPK	II-1	F	N
	Pauline	5 SPK	II-2	F	N
group 6	Matt	5 SPK	II-1	M	N
	Al	4 SPK	II-2	M	N
	Holmes	5 SPK	III	M	N
	Ricky	5 SPK	II-2	M	N

Table 3.2: The 6 groups of students selected for interview

The problem given during the interview was taken from the book *Thinking Mathematically,* but it was not included in the list of problems students were to solve during the course. The problem can be easily understood. However, it is expected that the students may not have ready means of getting the solution. Accordingly it would hopefully provoke the students to doing problem-solving. As at Warwick, the students are not expected to produce a complete solution within the time frame.

It is recognised that it would be a serious limitation within the main study to disregard teachers' conceptions of mathematics and solving problems. Several students at Warwick indicated a desire for a different approach to mathematics teaching at the university. These were not looked into more deeply. Based on the researcher's experience as an assistant lecturer at the UTM for several years, it is anticipated that this issue will raise more profound criticism amongst the UTM students, especially after the problem-solving course when it is conjectured that their attitudes to mathematics will improve. It is believed that if such a course is to have a longer term effect on the students, the tutors and the students must share similar conceptions about mathematics. Therefore it was decided that the tutors attitudes will be worth investigating further. Furthermore it is likely that the mathematical environment provided by the lecturers may heavily influence the students' attitudes towards mathematics. It was planned that the investigation on staff will be made during the delayed post-test.

There was a high risk of getting unresponsive outcomes in the investigation on tutors' attitudes. However, with the current movement in promoting reform in mathematics education in Malaysia the lecturers were in fact very supportive of the research. They were very concerned about the general observation that students are not developing problem-solving skills in formal undergraduates courses. They were willing to take part in the study and had given their full co-operation.

Twenty-two lecturers at the Mathematics Department took part in the study. This does not include lecturers who were on study leave, on loan to the First Year Unit and those on the Kuala Lumpur campus. The students' attitudinal questionnaire was circulated to the staff and all 22 filled it out twice. In the first reading they specify the attitudes they *expect* from their students and in the second reading specifying the attitudes they *prefer* the student to have.

Eight lecturers were then selected for further observation concerning their views of mathematics and their mathematics teaching. The selection was made based on the fact that the students (some or even all) who had followed the problem-solving course the previous semester were now attending their mathematics lectures. The eight lecturers were given a questionnaire to fill out to investigate their perception of mathematics.

Like the attitudinal questionnaire, the 'perception questionnaire' requires a response on a five-point Likert scale. The questionnaire consisted of two sections. Section 1 contained statements drawn from several sources; statements 1, and 4 from Tall (1991), statement 3 from Skemp (1989), and statements 5, and 6 from Freudenthal (1973). Lecturers' responses would provide an indication of their views about mathematics.

Section 2 contained statements picked from the attitudinal questionnaire. These statements focus on the various aims of mathematics courses. In this section the lecturers respond by indicating how they think students would perceive the lecture content. Students attending the mathematics lecture were invited to fill in the section specifying what they think the lecture was all about. An analysis of both lecturers' and students' responses would provide us with an insight into lecturers' focus as being sensed by the students. This may help explain the change in students' attitudes during mathematics lectures.

The full questionnaire is as follows:

Section 1: Perception of Mathematics

I believe mathematics is ...

1. a deductive system with clearly defined axioms and formally constructed proofs.
2. a theoretical knowledge with defined concepts and relations.
3. a highly developed mental tool for dealing with physical environment.
4. an activity of solving problems.
5. a discipline of the mind.
6. about inventing new ideas.
7. other (please state)

Section 2: Perception of Mathematics Lecturing

1. making sense of the mathematical ideas.
2. working hard to understand the mathematical concepts.

3. inventing new mathematical ideas.
4. relating mathematical ideas together.
5. facts and procedures to be remembered for exams.
6. how to apply mathematical concepts.
7. teaching a section of the lecture notes to get through the course.
8. developing confidence to solve mathematical problems.
9. developing own way of solving problems.

The whole study was carried out over a two year period.

i. The pilot test at Warwick University was carried out between October and December 1992.

ii. The main study at the UTM–the pre-test, experimental problem-solving course and the post-test–was carried out between July and September 1993.

iii. The supplementary study at the UTM–the delayed post-test and lecturers responses–was carried out between March and April 1994.

4. PILOT STUDY:

THE WARWICK PROBLEM SOLVING COURSE

4.1. Introduction

The current trend in mathematics education is towards conceptualising mathematics as a living subject with the development of mathematical thinking becoming a priority (Schoenfeld, 1992; McGuiness & Nisbet, 1991; Tall, 1991). The move towards encouraging students to think in a mathematical manner has been going on for more than a decade at Warwick University. The approach, a course in problem-solving, concentrates on meta-processes of mathematical thinking. The belief is that such a course would have positive effects on students' beliefs about mathematics and problem solving. We conjecture that the teaching of problem-solving would improve students attitudes.

In this chapter we shall attempt to gain insight into students' changes in attitudes toward mathematics as a result of the problem-solving course. Section 4.2 will outline the Warwick problem-solving course whilst section 4.3 will outline the nature of the study and provide indications of the methodology applied. An analysis of the questionnaire distributed to all of the students following the course will be considered in section 4.4. A separate analysis of pre-test (section 4.4.1) and post-test results (section 4.4.2) leads to an overall comparison to consider the effect of the problem-solving course (section 4.5). Student's individual perceptions obtained through informal interview, written comment and formal interview will provide the focus of section 4.6. Section 4.7 presents a general discussion of the issues arising from the pilot study whilst section 4.8 presents a short chapter summary.

4.2. The Warwick Problem Solving Course

Within the problem solving course at Warwick, mathematical thinking and problem-solving are explicitly taught using Mason, Burton and Stacey's (1982) framework with an additional emphasis on Skemp's theory of "goals" and "anti-goals" (1979). The over-riding objective is to enable students to become aware of the processes of mathematical thinking and to participate actively in problem solving. Additional objectives enable students to reflect on their experiences so that they may know how to use these experiences in their mathematical growth. The atmosphere created enables them to meet with aspects of their own mathematical thinking that are normally hidden. The students learn little new mathematical content in the course. The main aim is to provide students with an opportunity to experience the higher meta-thinking skills of thinking how to think in mathematics.

The course is designed to consist of thirty contact hours over ten weeks. Each week begins with a two hour problem-solving session with a brief introduction from the course tutor to set the scene. This is followed by a period during which the students are encouraged to work together in groups discussing, formulating and solving the problems. Each 2 hour session has as many as eighty students working together in a single room, and even spilling out into the corridors. Students are expected to continue to work on the problems outside class-time. Later in the week, seminars are held in smaller groups to reflect on the previous activities, and to reflect on problem-solving in other ways. For example, one student may sit out of the activity to observe what is going on. This same student may then join in the reflective discussion from another viewpoint. Problems from the sessions and extra problems were set for students to do in their own time. Some of these were later discussed in the sessions.

Students were told to make sure they had access to the course text, "*Thinking Mathematically*" (Mason *et al.*, 1982), but not to read any chapter until *after* the problems concerned have been attempted in class. In this way they could further reflect on the problem-solving activities. Through his personal experience the tutor had found that this was an excellent technique to use. Assessment is in two equal parts. The tutor planned to give a long-term assessed problem (50%) and a written examination (50%) involving only essay questions about the nature of problem-solving, so that there was no problem-solving under time pressure. However, the students preferred to negotiate a written problem element in the examination and the assessment given possessed the following characteristics:

1) a problem to be considered over several weeks, the production of a rubric, a reflection on the activities and a short essay considering how problem-solving techniques might be used in their other mathematical studies,
2) a written two hour examination which offers a choice of two questions out of four. One of these questions is a problem, the others are essay questions on the nature of problem-solving.

4.3. The Study

4.3.1. Overview of the Study

The pilot study attempted to gain insight into students' changes in attitude towards mathematics and problem-solving as a result of participation in the ten-week, 30 hour problem-solving course. Data for the study was collected in four ways:

(i) Students who followed the course were invited to respond to a questionnaire (distributed in the third week of the course and again at the end of the course).

(ii) After the course several students, selected through their responses to the questionnaire, were invited to respond through individual interview to a problem solving activity to establish the extent that they thought in a mathematical manner. Each interview was tape recorded and later transcribed and analysed.

(iii) The researcher was present in all classes and collected field notes which provided a sense of the course structure and provided opportunities to informally interview and discuss problem solving strategies with individual groups of students.

(iv) Analysis of written assessments at the end of the course.

4.3.2. The Sample

The students following the course were a mixture of second, third, and fourth year undergraduates at Warwick University. The courses they follow include:

- The Bachelor of Arts with Qualified Teacher Status , (BA(QTS)),
- Mathematics (Maths),
- Mathematics, Operational Research, Statistics and Economics (MORSE),
- Computer Studies (CS),
- Applied Mathematics (AM).

Postgraduate students doing MSc taught degrees and PhD by research may also attend the course. During the period of the pilot study there were 79 students; 30 females and 49 males in the class. They were all invited to take part in the study. However, only 47 were considered; that is those who filled out the questionnaire fully on both occasions—at the beginning and at the end of the course.

Table 4.1 shows the break-down of the subjects under study according to the degree classification they had obtained at the end of previous year. For both genders, the population appears to be normally (slightly skewed) distributed. No significant statistical difference (t-test, p=0.60) were noted between the two genders with regards to their academic achievement. Thus we may conclude that they are drawn from the same population. For the purpose of this study no further reference will be made to gender difference academically but there could be differences in attitudes or quality of thinking.

Gender	Degree Classification						Total
	I	II-1	II-2	III	P	F	
Male	5	9	8	5	1	0	28
Female	2	8	7	2	0	0	19

Table 4.1: Potential degree classification of the 47 students under study

4.3.3. Method

At the beginning of the third week of the term (after two weeks of lectures) and at the end of the final week of the term, all students were invited to complete a five-point Likert type questionnaire. Through the questionnaire the students provide a first indication of their attitudes towards mathematics and problem-solving.

4.4. Analysis of Questionnaire Responses

4.4.1. Pre-test Results

Tables 4.2 and 4.3 show the student responses to both parts of the questionnaire before the problem solving course (pre-test).

Attitudes and perceptions						
Item	Statement Summary	Y	y	n	N	–
1	easy	0	21	9	2	15
2	understand quickly	1	21	17	1	7
3	make little sense	7	18	18	3	1
4	see value	2	7	20	6	12
5	remember	2	9	20	9	7
6	difficulty applying procedures	1	26	14	2	4
7	need mathematical knowledge	6	14	15	2	10
8	work hard	2	17	17	2	9
9	ask friends for help	13	23	6	3	2
10	ask lecturer for help	4	15	10	11	7
11	work alone	13	22	8	1	3

Table 4.2: Pre-test responses to the attitudes and perceptions component of the questionnaire

From table 4.2, it is observed that more students think mathematics is *easy* than not (statement 1). Also more students believe they are able to *understand quickly* than not (statement 2). However more than half of the students think the mathematics makes little sense (statement 3).

Few students appear to *see the value* of the mathematics they do (statement 4) although more students indicate that they *need a good knowledge of mathematics* to get on in life compared with those who do not (statement 7). A majority of the students claim not to *remember* the previous year's mathematics (statement 5) and likewise a high proportion found *difficulty applying routine procedures* to mathematical problems (statement 6). The students split equally in the belief that they have to *work hard* to understand the mathematics (statement 8).

A great majority of the students *ask their friends for help* (statement 9). Also the great majority indicate that they usually *work alone* (statement 11). Note that both items have dominant positive responses, with about a third indicating that this statement is true of their position. It is interesting that half of the students do not *ask their lecturers* for help, half of them indicating a strong negative response to this statement (statement 10).

However it is particularly interesting that over a quarter of the students indicate that they have no opinion about the *value* of the mathematics. This suggests that the statement is inadequate in distinguishing differences in students' perceptions. A similar observation may be made of the statement referring to mathematics as *easy*.

Table 4.3 below shows students responses to the second part of the questionnaire.

Self-assessment						
Item	Statement Summary	Y	y	n	N	–
1	confidence	7	21	9	3	7
2	pleasure	14	17	6	0	10
3	only to get through	5	14	12	12	4
4	anxious	5	9	19	4	10
5	fear unexpected	6	7	20	4	10
6	know how to start	1	19	13	1	13
7	secure with procedures	17	20	4	2	4
8	persevere	6	18	17	2	6
9	frustrated	17	19	6	1	4
10	correct answers	2	12	18	5	10
11	anxious when stuck	5	24	8	2	8
12	relief able to discuss	6	25	4	4	8
13	aware	5	28	9	0	5
14	review resolution	3	18	13	5	8
15	perform up to potential	4	15	14	2	12
16	recommend course	19	26	0	0	2

Table 4.3: Pre-test responses to the self assessment component of the questionnaire

The most striking features of the table are the strong 'yes' opinions that stand out, namely students getting *pleasure* from problem solving, their *security* when procedures are given, their *frustration* when failing to get correct solutions and their strong willingness to *recommend* the course to others. Apart from the more negative desire to obtain their *security* from a presented procedure the other statements all reflect positive attitudes. Only statement 3, students only solve mathematics problems *to get through the course*, evoked a strong negative reaction. This is actually a good response because it indicates that half of the students think positively about mathematics, although a considerable minority are doing it because of the extrinsic pressure: solving problems *only to get through* the course and concentration on getting *correct answers*.

There is, amongst the students as a whole, a positive feeling towards problem-solving despite the broad range of responses. For instance, a high proportion of the students indicate that they have *confidence* in their problem-solving ability. The greater number do not appear to experience the negative feeling of *anxiety* or *fear* of unexpected mathematical problems. The majority do get *frustrated* when their problem-solving approach fails. As noted by Mason (1988) frustration is important in mathematics. When students feel frustrated it indicates that they have the feeling things are out of place. Through the qualities of reflection and reappraisal things may fall back into place. Consequently students will experience a sense of release from frustration. Accordingly a majority feel a sense of *relief* discussing their difficulties with others. However, we see that a majority claim to experience *anxiety while problem-solving*. Also the majority of the students say they would *persevere* although they do not seem to be making any progress.

Notice that more students do not place importance on *correct answers* than those who do so. More students think they *know how to get started* on problems. The majority claim they are *aware* of what they are doing during problem-solving, they *review* their resolution, and think they are *performing up to their potential* in the course. Nearly all would recommend the course to their friends.

It is noticeable that about a quarter of the students had no opinion about whether or not they knew *how to get started* or whether they perform *up to potential.*

It is interesting to note the item that *mathematics make little sense* split the class almost exactly in half—only one student gave a "no opinion" response. (The statement about whether mathematics make sense or not (statement 3, section A) proves to be not only of interest here but also in the Malaysian students. After this investigation had finished, responses to a similar statement in the main study showed a similar break down in responses.) The particular student who gave "no opinion" to the item gave the same response to *all* the items measured. In an informal conversation with this student during one of the lessons, he said "none of the statements mean anything to me". According to him all the statements in the questionnaire are not well defined. However I did interview him formally and he also gave a written opinion about his feeling toward mathematics (see *Colin*'s comments section 4.5.4 later in this chapter). This student was dropped from the following discussion and for further quantitative analysis only 46 responses were considered.

Based on responses to the statement "*I find the topics we study in mathematics often make little sense to me*", two groups were established. We shall attempt to bring to light distinctions between the two groups of students. The 25 students (14 females) for whom mathematics makes little sense is named Group N and the other 21 students (5 females) is called group S. Interestingly, a lot more females claim mathematics makes little sense to them than do males. Table 4.4 uses the distinction between whether or not students find mathematics makes little sense to re-examine the distribution of degree classification from the previous year categorised by course.

Course	Degree Classification											
	Group N (n=25)						Group S (n=21)					
	I	II-1	II-2	III	P	F	I	II-1	II-2	III	P	F
Maths	2	2	6	4	1	0	2	2	4	2	0	0
BA(QTS)	1	4	2	0	0	0	0	4	2	0	0	0
CS	0	2	0	0	0	0	1	3	0	0	0	0
MORSE	0	0	1	0	0	0	0	1	0	0	0	0
MAFF	0	0	0	0	0	0	0	0	0	1	0	0
Total	3	8	9	4	1	0	3	9	6	3	0	0

Table 4.4: The distribution of students for whom mathematics makes little sense (group N) and does not (group S)

Table 4.4 shows that the distribution of the two groups N and S is almost similar. The correlation (r) between the students' achievement and whether mathematics makes little sense was found to be -0.12. This means that only 1.4% of the variance in students achievement at the end of a year can be accounted for by whether or not mathematics makes sense. This in turn suggests that making sense of the mathematics is not a means of predicting the degree classification. There are other factors involved which this study in due course highlights.

It may be useful to consider the degree classification with regard to the students achievement in problem-solving. From table 4.4 it may seem that the BA(QTS) students appear to be better than the mathematics major students (MATHS). But in actual fact the BA(QTS) students are doing different mathematical components than the B.Sc. students. Besides, their achievement was based upon two different forms of assessment. In particular, the previous years work of BA(QTS) is assessed through course work with no written examination. This may tend to raise their classification by about one class.

Also, it was found that students entering the Maths degree course must have a minimum of 2‘A’s and 1‘B’ in their A-level. This means they have a much higher achievement in mathematics in their A-level than the BA(QTS) students. Maths students must have an ‘A’ in mathematics whereas the students on BA(QTS) could generally have ‘C’ or above. Also, CS, MORSE or MAFF students may have a lower grade on entry than do Maths students. Therefore one may expect that the Maths students have a more mathematical background. Additionally they have completed more university mathematics than other students attending the problem-solving course. On the other hand, other students have more experience in problem-solving type of activities. Nevertheless, all the students following the course have sufficient mathematical knowledge to solve the problems given in the course.

4.4.2. Post-test Results

In the last week of the term, that is during the final meeting, the students again filled out the same questionnaire. Tables 4.5 and 4.6 show the responses obtained for section A and B of the questionnaire respectively.

When looking at table 4.5 we see that, in general, extreme responses do not dominate. However, it is observed that items indicating mathematics as *easy*, students' belief that they can *understand mathematical ideas quickly*, *need mathematical knowledge*, *working alone* and *asking friends for help* receive a high proportion of positive responses.

Indication that mathematics is *easy*, or otherwise were not received from a quarter of the students. Similarly, a quarter had no opinion about whether or not they see the *value* of their mathematics, *need mathematical knowledge* or *remember* the previous year's mathematics. This points to the fact that these statements do not bring about a clear response from the students.

Attitudes and Perceptions	Y	y	n	N	–
easy	3	22	7	3	11
understand quickly	2	28	13	0	3
make little sense	4	12	24	4	2
see value	0	11	18	5	12
remember	1	11	17	10	7
difficulty applying procedures	1	16	24	2	3
need mathematical knowledge	9	21	7	0	9
work hard	5	12	22	4	3
ask friends for help	9	25	6	3	3
ask lecturer for help	4	16	10	13	3
work alone	10	30	4	0	2

Table 4.5: Post-test responses to part A of the questionnaire

Table 4.6 below shows the students' responses to part B of the questionnaire.

Self-assessment	Y	y	n	N	—
confidence	7	30	1	1	7
pleasure	16	20	3	0	7
only to get through	3	11	22	5	5
anxious	3	6	23	6	8
fear unexpected	1	10	22	8	5
know how to start	6	24	8	0	8
secure with procedures	14	24	3	1	4
persevere	8	24	4	2	8
frustrated	13	23	7	0	3
correct answers	1	5	30	2	8
anxious when stuck	3	22	16	0	4
relief able to discuss	10	24	4	0	8
aware	2	34	1	0	9
review resolution	3	23	10	1	9
perform up to potential	3	26	7	1	9
recommend course	35	11	0	0	0

Table 4.6: Post-test responses to part B of the questionnaire

There is, as a whole, a positive approach to problem-solving amongst the students. For example, it is noticeable that having *confidence*, getting *pleasure* from problem-solving, *frustration* when failing to get correct solutions, a sense of *relief* of being able to discuss difficulties and being *aware* of what they are doing, received a high proportion of positive responses.

In contrast, the extrinsic pressure of doing mathematics *only to get through* the course and the importance on *correct answers* attracts a very low proportion of positive responses. Also a high proportion of students claim not to experience *anxiety* when asked to solve problems. Similarly a great proportion have no *fear* of the unexpected. Just over half of the students experience *anxiety while problem-solving* but all (100%) students would *recommend* the course to others with the great majority expressing a definite opinion.

More important however is the students change in attitudes as a result of the problem solving course and we shall see this in the next section.

4.5. Comparison between Pre-test and Post-test Responses

Comparing the results before and after the course would bring to light the change in students' responses. In this section the data from both the pre-test and the post-test questionnaire is gathered and compared. The data is tabulated according to the grouping identified earlier, namely group N and S whose students differ in attitude towards whether or not mathematics makes sense. The results from each part of the questionnaire are presented separately.

4.5.1. Comparison of Attitudes to, and Perceptions of, Mathematics

Table 4.7 shows the responses for the two groups on both the pre- and post-test for

Section A of the questionnaire. At this point, as an alternative presentation, we would utilise the results in a more convenient way by considering all Y and y responses under "Yes", N and n responses under "No". That is rather than having separate numbers for Y and y, they are added to give the total "yes" response with the strong definite responses given as a subset Y. Likewise for the N and n responses. Thus, for example, in column one we have before the course 9 students (36%) in group N who regard mathematics as *easy*, with a subset of zero (0%) expressing this opinion strongly (a "definite yes"). After the course, the change is minimal with only 10 students (40%) thinking mathematics is easy for them. In contrast for group S, before the course 12 students (57%) claim the mathematics is *easy* and after following the course the majority (71%) of group S students now believe it is easy. Other distinctions between the responses of the two groups of students can be seen in all other items. In addition some of the phrases are further abbreviated for sake of space.

Attitudes and Perceptions	**Group N (n=25)**										**Group S (N=21)**									
	Yes		Y		**No**		N		–		**Yes**		Y		**No**		N		–	
	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*
easy	9	*10*	0	*1*	9	*9*	2	*3*	7	*6*	12	*15*	0	*2*	2	*1*	0	*0*	7	*5*
understand quickly	11	*13*	0	*0*	11	*11*	1	*0*	3	*1*	11	*17*	1	*2*	7	*2*	0	*0*	3	*2*
make little sense	25	*16*	7	*4*	0	*9*	0	*1*	0	*0*	0	*0*	0	*0*	21	*19*	3	*3*	0	*2*
see value	5	*5*	0	*0*	17	*14*	3	*5*	3	*6*	4	*6*	2	*0*	9	*9*	3	*0*	8	*6*
remember	5	*6*	0	*0*	17	*15*	7	*7*	3	*4*	6	*6*	2	*1*	12	*12*	2	*3*	3	*3*
difficulty with procedures	17	*11*	1	*0*	6	*12*	1	*1*	2	*2*	10	*6*	0	*1*	10	*13*	1	*1*	1	*2*
need maths knowledge	8	*15*	3	*4*	14	*5*	2	*0*	3	*5*	12	*15*	3	*5*	3	*2*	0	*0*	6	*4*
work hard	14	*13*	2	*4*	8	*11*	1	*1*	3	*1*	5	*4*	0	*1*	11	*15*	1	*3*	5	*2*
ask friends for help	22	*20*	7	*6*	3	*4*	2	*2*	0	*1*	14	*14*	6	*3*	6	*5*	1	*1*	1	*2*
ask lecturer for help	7	*7*	1	*0*	14	*16*	8	*9*	4	*2*	12	*13*	3	*4*	7	*7*	3	*4*	2	*1*
work alone	15	*19*	4	*5*	8	*4*	1	*0*	2	*2*	20	*21*	9	*5*	1	*0*	0	*0*	0	*0*

Table 4.7: Comparison between pre and post-test responses to part A of the questionnaire

The Wilcoxon Matched-pairs Signed-rank Test was used to calculate the significance in the change of the students responses. Table 4.8 shows the results. The arrows in the second column indicate the direction of the overall change. The significance of the change is given as highly significant (<1%), significant (<5%) or not significant (n.s.).

Attitudes and perceptions		**Overall**	**S**	**N**
Statement Summary		**Change**	**Pre v post**	**Pre v Post**
easy	↑	n.s.	n.s.	n.s.
understand quickly	↑	<5%	<5%	n.s.
make little sense	↓	n.s.	n.s.*	<1%
see value	↑	n.s.	n.s.	n.s.
remember	↑	n.s.	n.s.	n.s.
difficulty applying procedures	↓	<1%	n.s.	<5%
need mathematical knowledge	↑	<1%	n.s.	<1%
work hard	↓	n.s.	n.s.	n.s.
ask friends for help	↓	n.s.	n.s.	n.s.
ask lecturer for help	↑	n.s.	n.s.	n.s.
work alone	↑	n.s.	n.s.	n.s.

Table 4.8: Significant changes in students responses to part A of the questionnaire

Using table 4.7 and 4.8 a subset of the results is displayed pictorially to clarify the changes and the difference in responses between the groups N and S students (see Figures 4.1 and 4.2). The bar-charts show the percentages of the total "yes" (Y+y) responses on all items measured in part A of the questionnaire. The percentages are calculated from each group responses to each item before and after the course. Figure 4.1 shows the percentage of positive responses (Y+y) to the statement that mathematical topics studied *make little sense*. Group S students remain at 0% in their belief both before and after the course. However for group N, the number of students who agree with this statement reduces to 64% during the problem-solving course.

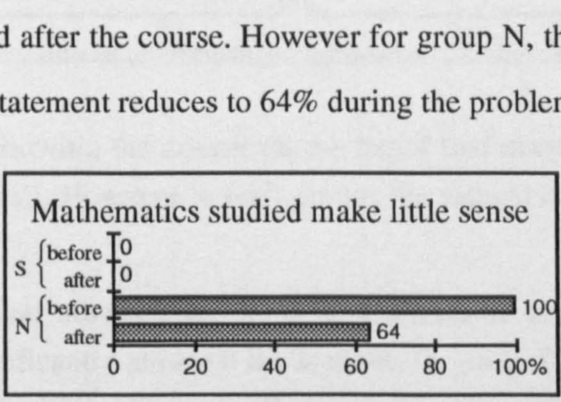


Figure 4.1: Mathematics studied make little sense: pre-test and post-test comparison

The same bar-chart layout is used to represent changes in other statements and these are shown below in Figure 4.2. The bar-charts are arranged in such a way that seemingly related statements are placed side by side.

Figure 4.2 shows that:

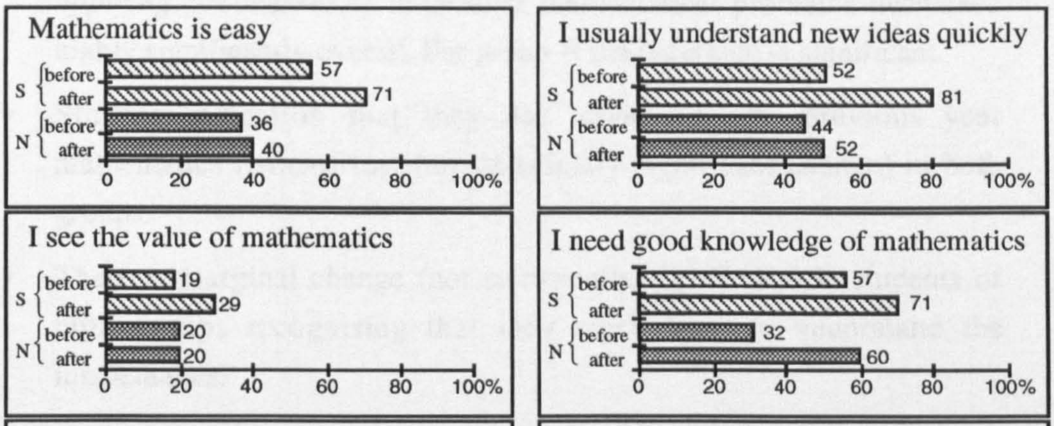


Figure 4.2: Attitudes to Mathematics: pre-test and post-test comparison

- Responses following the course on the belief that mathematics is *easy* increases overall. However in both groups the change is not statistically significant.
- The notion that students are able to *understand new ideas* quickly increases significantly amongst the students in group S. The change for the better is significant for group S students but for group N students the change is only marginal.
- *Seeing the value* of their mathematics remains low (not statistically significant) in both groups.
- There is a highly significant increase in students' indication that they need a *good knowledge* of mathematics to get on in life. The change is attributed to the highly significant difference amongst group N students.
- Following the course, students' indication that they find *difficulty*

applying procedures to unfamiliar mathematical problems decreased highly significantly overall. For group N the reduction is significant.

- Student indication that they can *remember* the previous year mathematics remains low (no statistically significant change) in both groups.
- There is marginal change (not statistically significant) in students of both groups recognising that they *work hard* to understand the mathematics.
- Students continue to *ask friends for help* – an attitude that remains high amongst students of both groups.
- Students willingness to *ask lecturer for help* showed no significant change in both groups.
- The tendency for students to *work on their own* in group N increases (not statistically significant) after the course.

It is noted that after the problem-solving course, two items changed highly significantly overall: *difficulty with procedures* decreased whilst students *need mathematical knowledge* increased. One item had significant change: students can *understand new ideas quickly* increased significantly. However, on all other items, the changes are not statistically significant. For group N it is observed that the change for the better is marginal on many of the items measured. However a highly significant shift is noted in their perception that mathematics *makes little sense*, and that they *need good knowledge* of mathematics. Whilst the indication that they have *difficulty with procedures* reduced significantly. Amongst the group S students a significant change is observed on the item indicating students can *understand new ideas quickly*. Smaller changes can be seen in their perception that *mathematics makes little sense* which is significant at the 10% level (marked n.s.* in table 4.8).

4.5.2. Comparison of Students' Self-assessment

Table 4.8 shows the responses to part B of the questionnaire for the two groups on both pre and post-test. A format similar to that used above is used to display the data.

Self-assessment	Group N (n=25)										Group S (n=21)									
	Yes		Y		No		N		–		Yes		Y		No		N		–	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
confidence	14	18	2	3	8	2	3	1	3	5	13	19	4	4	4	0	0	0	4	2
pleasure	15	18	4	5	5	2	0	0	5	5	16	18	9	11	1	1	0	0	4	2
only to get through	12	10	4	3	11	13	6	2	2	2	7	4	1	0	13	14	5	3	1	3
anxious	9	6	3	2	10	15	1	2	6	4	5	3	2	1	13	14	2	4	3	4
fear unexpected	11	9	6	1	8	13	1	3	6	3	2	2	0	0	16	17	2	5	3	2
know how to start	9	15	0	5	9	5	0	0	7	5	11	15	1	1	5	3	0	0	5	3
secure with procedures	22	22	10	9	1	1	0	0	2	2	15	16	7	5	5	3	1	1	1	2
persevere	11	15	2	2	12	5	2	1	2	5	13	17	4	6	5	1	0	1	3	3
frustrated	21	20	9	6	3	4	0	0	1	1	15	16	8	7	4	3	0	0	2	2
correct answers	7	3	2	1	14	17	1	2	4	5	7	3	0	0	9	15	4	0	5	3
anxious when stuck	18	16	3	1	3	7	0	0	4	2	11	9	2	2	7	9	1	0	3	3
relief able to discuss	18	20	3	4	4	1	2	0	3	4	13	14	3	6	4	3	1	0	4	4
aware	16	19	2	2	6	1	0	0	3	5	17	17	2	0	3	0	0	0	1	4
review resolution	9	12	1	2	13	9	3	1	3	4	12	14	2	1	5	2	1	0	4	5
perform up to potential	11	16	2	1	10	4	1	1	4	5	8	13	2	2	6	4	1	0	7	4
recommend course	25	25	12	17	0	0	0	0	0	0	20	21	7	18	0	0	0	0	1	0

Table 4.9: Comparison between pre- and post-test responses to part B of the questionnaire

Of particular interest here is the column of total "yes" (Y+y) responses for both groups. It is noticeable that though on most items there is a marginal change (for the better) within both groups, there are some marked shifts following the course. For example, the majority of students in both groups now think they have *confidence* whereas fewer students indicated that they did so before the course. Likewise a high proportion of students in both groups claim that they now *know how to get started* on problems.

Using the same format as table 4.8, the significance of the change was computed using the Wilcoxon Matched-pairs Signed-rank Test and the results are shown in table 4.10.

Self-assessment		Overall	S	N
Statement Summary		Change	Pre v Post	Pre v Post
confidence	↑	<1%	<5%	<5%
pleasure	↑	<5%	n.s.	n.s.*
only to get through	↓	n.s.	n.s.	n.s.
anxious	↓	<5%	n.s.	n.s*
fear unexpected	↓	<5%	n.s.	n.s.*
know how to start	↑	<1%	n.s.	<1%
secure with procedures	↓	n.s.	n.s.	n.s.
persevere	↑	<1%	<5%	<1%
frustrated	↓	n.s.	n.s.	n.s.
correct answers	↓	<5%	n.s.	n.s.*
anxious when stuck	↓	n.s.*	n.s.	n.s.*
relief able to discuss	↑	<5%	n.s.	<5%
aware	↑	n.s.*	n.s.	n.s.*
review resolution	↑	<5%	n.s.	<5%
perform up to potential	↑	<1%	n.s.*	<5%
recommend course	↑	<1%	<1%	n.s.*

Table 4.10: Significant changes in students responses to part B of the questionnaire

We note that only four changes overall are not statistically significant: students solve problems *only to get through* the course, feel *secure with procedures*, *frustrated* and *anxious when stuck*. Six items changed significantly: *pleasure*, *anxiety*, *fear of the unexpected*, getting *correct answers*, *relief able to discuss*, and students would *review resolution*. The remaining five items have highly significant changes: *confidence*, *know how to start*, *persevere*, *perform up to potential* and students would *recommend the course* to others.

Looking in detail at the results in table 4.10 we also see differences between the group N and S students' responses. Figure 4.3 highlights the positive responses given by students in both groups pictorially. It shows the bar-chart layout for each of the statements, with once again seemingly related statements placed side by side.

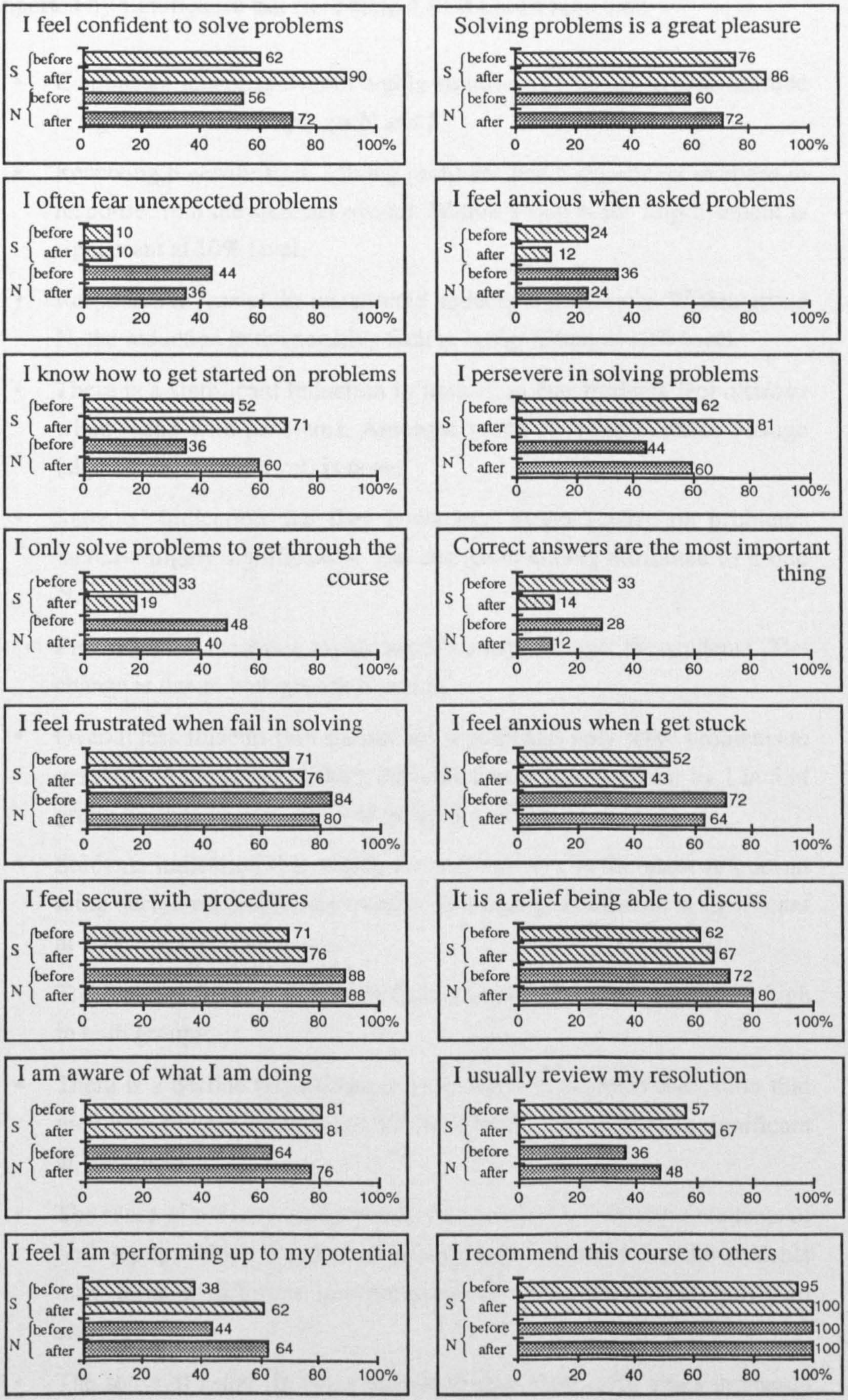


Figure 4.3: Attitudes to Problem-Solving: pre-test and post-test comparison

Supported by significance test from table 4.10 it can be seen that:

- *Confidence* increases overall highly significantly The change in attitude is significant for both groups N and S.
- Receiving *pleasure* from solving problems had a significant increase in response from the students overall. Within group N the improvement is significant at 10% level.
- Responses to *fear of the unexpected* reduces significantly. Within group N, the reduction in the negative feeling is significant at 10% level.
- There is a significant reduction in indication that students feel *anxious* when faced with problems. Amongst group N students small change (significant at 10% level) is noted.
- Sudents' indication that they know *how to get started* on problems increase highly significantly. The change is mainly attributed to group N.
- *Perseverence* increases highly significantly amongst the students. The change is due to both groups N and S.
- Overall less students (not statistically significant) only solve problems *to get through* the course. Whilst the reduction is accounted for by 1 in 5 of group N students, it is 1 in 2 of group S students.
- Students indication that seeing *correct answers* is the most important thing decrease significantly overall. This change in attitude is significant at 10% level for group N.
- The feeling of *frustration* when failing in problem-solving remains high in both groups.
- There is a decline (significant at 10% level) in students indication that they feel *anxious when stuck*. Within group N the change is significant at 10% level.
- The sense of *security with procedures* remains high amongst students of both groups which is rather surprising. It is suspected that the students may have a different interpretation of 'procedure' and 'problem solving'.
- The sense of *relief* in being able to discuss their difficulties increased significantly overall. The change is mainly attributed to group N.
- After the course *awareness* of what they are doing in problem solving

increased (significant at 10% level) overall. Amongst group N the change is significant at 10% level.

- The tendency to *review the resolution* of a problem increased significantly overall. The change is mainly due to group N.
- Students belief that they are *performing up to potential* increased highly significantly. Within group N the change is significant whilst smaller changes (significant at 10% level) is evident amongst group S students.
- Students would continue (highly significant) to *recommend this course* to others. The change is mainly attributed to group S.

It was expected that problem-solving would cause changes in attitudes amongst the students. It was observed that in group S, the difference before and after the course is found to be significant on only four items: *confidence* in solving problems, *perseverence*, *performing up to potential* and students *recommend the course* to others. However, it is noted that students within the group had indicate positive attitudes to begin with. Whilst for group N there are significant changes noted on many of the items measured. Only three changes are too small to be statistically significant: *only to get through*, *secure with procedures* and *frustrated.* This indicates that the distinction between the two groups before the course now appear to lessen.

However, on items that indicate whether or not students feel *anxious when stuck*, and gain a sense of *security with procedures*, show there is only a marginal change noted within both groups of students. It is likely that the students may have different interpretation of the questions. For instance, the students are talking about problem solving as identified with the problem-solving course.

Although significant changes were noted in many cases, it was found that more than two thirds of the students did not change their response on 14 of 27 of the items measured (statements 2, 9, 10, 11 in part A and statements 1, 2, 3, 7, 9, 11, 12, 13, 15, 16 in part B) in the questionnaire. However, in some instances (for e.g. statements 1, 2, 3 in part B) the majority of the students had given indication which reflect positive attitudes in the pre-test so that little change is possible.

4.6. Student Comments

The students' comments were obtained through informal interviews, written comments and formal interviews. Comments through informal interviews were obtained from talking with the students informally in class. This normally took place when the course was in progress. The comments given were more diverse as students tended to talk on issues that were of interest to them, in particular their mathematical experience. No specific questions were asked. The written comments were obtained in two ways. Firstly, as an additional item on the questionnaire, students were invited to give their comments about their feelings toward mathematics. This was given as an open statement at the end of Part A. A second view was obtained from their written assessment given at the end of the course. A final source of students comments was from the individual interviews which were conducted in a formal manner during the last two weeks of the course.

The results show that the closed statements in the questionnaire were rather limited. However, the students' written comments gave us a clearer picture of what was going on. The comments give an indication of their beliefs about mathematics and solving problems. The individual interviews gave further support to the views.

4.6.1. Comments Through Informal Class Interviews

Being present throughout the course enabled the researcher to observe what is going on in the class. The researcher joined in the students' discussion while they were working on problems. The following comments, given half way through the course, suggest that the problem-solving was an experience which was valued. The words/phrases in bold emphasised the students' beliefs about problem-solving.

The techniques are relevant at university because they improve enjoyment and understanding of mathematics, which is often dampened for university students who were the brightest at mathematics at school. The enjoyment they had for their subject should not be smothered and would not be if they could employ problem-solving techniques. After learning the techniques myself I feel more ***confident at attempting problems*** and I feel my thinking processes have been awakened.

3rd year Maths

...I feel that university teaching methods are in desperate need of change. Most maths students would say that they have enjoyed very few courses at university and that problem solving is a breath of fresh air. It has helped to rekindle some of the enthusiasm that I used to have for the subject. I now find myself using linear algebra, analysis, group theory, etc., and in doing so, have ***greatly improved my understanding*** of these courses. In many cases, this is not due solely to the problems themselves, but the problem solving ideas, such as extend or reflect, that I have learnt. I once again find that maths is not an anti-goal. For this its worth is incalculable.

2nd year Applied Maths

If we open our vision to see university students reading a mathematical subject, quite often, and I say this with some personal experience, the student will look upon the subject as something to pass and then forget most of it immediately after the final exam. With the attitude which a problem-solving technique can create this may be reversed. A student may wish ***to understand a subject further of their own accord.***

2nd year Maths

...One important aspect of problem solving is learning to be wrong. After many years of being right or wrong, it is difficult to begin saying "maybe". I found conjecturing difficult to develop at first. However, later when I did make a false conjecture, I actually felt it helped me to see exactly why it was wrong. ...I would never have gained the insight into the problem had it not been for this. ...The problem solving techniques we have learnt are ***very flexible*** and I have adapted it to suit my personal needs. The course should have been made available to students at an earlier stage.

4th year BA(QTS)

I really come to appreciate the problem solving techniques learnt, how much I have used them. ...The understanding of why what I had found out worked and how I got there gave me ***confidence in my problem solving ability***. I used to get cross with myself whenever I got stuck and if I fail to get an answer it affects my confidence. The [problem solving] course is effective as I now feel in control of the problem, recognise my

> feelings, and know how to overcome them. *4th year BA(QTS)*
>
> I think mathematics should be dynamic. It should be outward going—replacing ignorance with understanding, replacing insecurity with confidence. At present the environment provides very little encouragement to do maths, there is not any competition. ...Problem Solving would improve the atmosphere. The course is invaluable in developing me to become a successful problem-solver. It makes me ***more confident*** and better able to talk, ***to think and to act mathematically***.
>
> *2nd year Maths*

As the course progressed positive changes in majority of the students' attitudes towards mathematics have been observed with a great regularity as illustrated by the selected quotations. These gave further support to the written phrases after the course. Students reconsider their view towards mathematics more than as "something to pass and then forget most of it immediately after the final exam". As they came to realise that they are responsible for much of their own mathematical experiences, their views toward mathematics changed. Before the course many of the students comments were given in comparative terms; they judged their own feeling now against the feelings they had about their school mathematics. Frequently their comments were very emotional in the sense that they expressed a high degree of frustration about their university mathematics. Before the course their views of mathematics were rather static—abstract, clear, logical and certain. After the course they now see mathematics more as a process of thinking. The problem-solving techniques also seems to have led them to an increasing awareness of what is required for improved mathematical thinking.

4.6.2. Comments Obtained Through Questionnaire

The students were invited to write few sentences concerning their feelings about mathematics in the questionnaire. The following selected phrases were given before the course.

4.6.2.1. Pre course comments

It is apparent from their comments that feelings about the subject matter runs high for many of them and they display various conceptions about mathematics. It is seen as clear, logical, concise and correct. It is made up of theorems and their proofs but to some it is too abstract:

> I used to enjoy mathematics at A-level, it was great to be able to solve problems in a clear and concise and logical manner which maths is. I am now fed up with the ***abstract*** nature of maths on my course but still feel a great sense of satisfaction when I understand a proof. *3rd year Computer Science*

> Unlike A-level and O-level maths, degree maths is incredibly ***abstract***, and ***irrelevant*** to everything except itself, being nearly completely made up of theorems and their proofs.
> *2nd year Maths*

> Mathematics is very challenging and has vastly improved my logical and analytical skills. I enjoy the challenge that mathematics gives and have great ***pleasure*** when getting ***correct answers***, one of the only courses in which 100% correct answers are possible.
> *4th year BA(QTS)*

There are students who feel that mathematics at the university is more of a thinking process and suggested that imagination and creative ability could be used. However the overwhelming nature of the content tends to obscure the reasoning. They appear only to have a vague sense of what is going on.

> I enjoy doing mathematics involving *thinking*. Most university maths courses seem to involve little of this. You copy down what the lecturer writes, take it home and work out why it's right, and then reproduce it in the exam. Very little ***original thought*** is required. *2nd year Maths*

> Mathematics is taught as a cold, static set of ***facts***. I have little idea where the course is coming from and where it is going; how the theory has developed and how it is still developing. ...mathematics should be a ***creative***, ***artistic*** and ***emotional*** process.
> *2nd year Maths*

Some students were attracted to the intellectual content in mathematics. However, they do not understand much of the mathematics. Although they struggle to understand it, most of the time it is likely it just produces confusion. Accordingly, they see their mathematics as composed of series of facts and procedures that they have to remember.

> I think that logical thinking and ***problem solving*** skills can be acquired through a study of mathematics and mathematical puzzles. I think these skills are very useful in life. I do not feel that any of these skills come into doing a mathematics degree however, which is mainly about ***memorising*** formulae and regurgitating what lecturers tell you, in exams.
>
> *3rd year Maths*

> I prefer courses which make me ***think*** than ones which involve more ***facts and algorithms to remember.*** I sometimes feel ***anxious*** when my mathematical ability is tested or required. I need time to think. *4th year BA(QTS)*

> Too vast a subject for me to grasp fully at the pace it is taught here. Some steps in a proof, etc., which are obvious to the lecturer or someone familiar with the subject, do not even occur to me, even when pointed in the right direction. ...I often have to ***work hard remembering*** stuff. *2nd year Computer Science*

To some students, mathematics has nothing for them. They are not interested with what the human mind has achieved over the years. It seems that they are finding the subject too difficult to comprehend. Perhaps, because they fail to grasp the rationale of the mathematics taught, they do not sense any particular loss from not knowing it. Therefore, they do it mainly to pass the exams.

> A-level and below is trivially easy. Degree level introduces an uncomfortable amount of rigour, and a lot of content delivered in an atmosphere not conducive to questioning. It all seems pretty ***pointless*** to me (even though it may not be!). I see it more as ***working for a qualification***, rather than enjoyment.
>
> *3rd year MORSE*

> I have a "too difficult" mental block when dealing with difficult maths. I tend to ***give up easily***–perhaps telling myself that "I'm not really bothered" if I can't do it (i.e. having

> difficulty). *3rd year Computer Science*

> The work is far too much hassle. I find it difficult to spend time trying to clarify the more hazy areas of maths. ... I never get there because it takes a lot of getting into, although I ***work hard*** near the exams. *2nd year Maths*

However, there are students who appreciate the intellectual challenge that mathematics gives them. They are willing to struggle with the mathematical ideas until it gives meaning. Consequently, they get pleasure from their work.

> I enjoy mathematics, and although some ideas do not come easily at first, I ***persevere until they make sense***. *3rd year Maths*

> ...***Solving maths problem*** does give a feeling of ***pleasure***, as does understanding some particularly elegant piece of maths.
>
> *2nd year Maths*

The words used underline the intensity of their beliefs about mathematics and solving problems. To some of the students, teaching of university mathematics is so formal that many of them simply resort to rote-learning of the materials to pass examinations. Many of these students appear successful. But as pointed out by a student:

> This is surely not an ideal situation, where a maths student can learn and pass and do well, but not have an understanding of his or her subject.
>
> *3rd year Maths*

The mathematics courses become perceived as parcels of knowledge to be learnt, rather than a living subject in which students think for themselves. In the words of Skemp (1971), university mathematics is seen as "the product of mathematical thought rather than the process of mathematical thinking".

4.6.2.2. Post course comments

After the course, students comments exhibited more positive feelings about mathematics. The following phrases are selected to illustrate differences in attitudes to mathematics. After experiencing thinking in a mathematical manner and reflecting upon them, the majority of the students seem to come to terms with the abstract nature of mathematics, and gain confidence in their mathematical ability. They are impressed by the intellectual challenge of problem-solving and are prepared to persevere to understand new ideas.

> I feel ***more confident*** in maths though I still find it hard. It takes a lot of getting into, but once involved can be fascinating. *2nd year Maths*
>
> I find mathematics enjoyable and challenging. Sometime part of the maths course seem obscure and I find difficulty in ***relating*** to the content. But most of the time I usually ***persevere until they make sense.*** *4th year BA(QTS)*
>
> I enjoy solving problems for their intrinsic value. ...Sometimes I have difficulty with more complex ideas. In such cases I usually ***persevere until I am able to understand.***
> *4th year BA(QTS)*

A majority of students report the achievement of a sense of satisfaction from their work; they think they can cope with the negative feeling of anxiety and despair while doing problem-solving. One may suggested that being involved in the mathematical processes themselves and struggling with new ideas has generated interests and satisfaction.

> I enjoy maths once I have got to grips with the relevant topics and find that I am progressing. ...The main thing is the satisfaction I get on completion of a question or understanding a topic, this overrides all the ***feelings of anxiety and despair*** whilst working on it. *3rd year Maths*
>
> I find and I understand at least to some extent, what's going on. I feel frustrated when

> things don't work. I am very pleased when I can see a problem through, ***understanding what I've done*** and why I've done it.
>
> *2nd year Computer Science*

From the above handful of selected responses, it may not be appropriate to draw any definite conclusion. However, opinions expressed suggest there is a change in students' perception about mathematics after the course. For instance, before the course some students believe the mathematics is just beyond their understanding and consequently has to work hard to remember the material. But after the course they now think they can understand the mathematics at least to some extent.

4.6.2.3. Students' feelings about mathematics

On the whole, the students' comments (see selected phrases above) were very much influenced by three factors: the nature of mathematics, personal (including affective) perceptions of mathematics and the teaching method. In the pre-test, twenty-four (6 females) responses were related to the nature of mathematics (of which thirteen also included personal factor). All were negatively inclined, 'mathematics was too abstract', 'see little relevance', 'notation and proof too rigorous' and 'disheartened by proof exercises'.

Thirty-four students (18 females) responses were related to personal factors, out of which ten indicate positive feelings. Such as 'enjoyable and challenging', 'great pleasure when able to understand a concept and solve problems' and feeling that the 'effort put in is worthwhile'. Negative comments include a 'lack of motivation', 'takes a lot of getting into' and 'study it just to get a degree'. Nevertheless, nine of these students are prepared to work hard to keep up. There is no response related to teaching factors alone, but one comment referred to the nature of mathematics and one indicated personal factors. Three responses related to all these factors. All responses on teaching were negative, that 'mathematics at the university comes [too] thick and fast to keep up', and students feel 'bogged down and deflated by the very first lecture'. Finally, three students gave no response whatsoever to this particular statement.

It is particularly interesting to see that on both occasions (before and after the course) more female students than males readily express their feelings of uncertainty in their mathematical ability. Whereas when reporting the irrelevance and the difficulty of the subject matter more males do so than females.

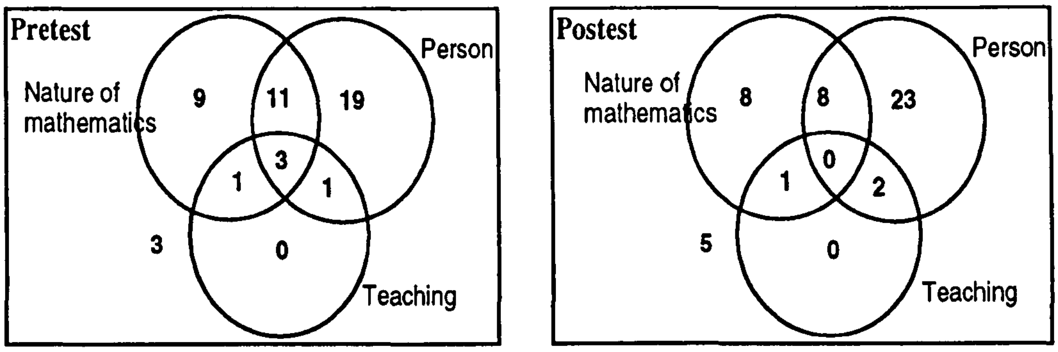


Table 4.11a: Students' perception of mathematics

Table 4.9 shows the distribution of feelings about mathematics for both the pre-test and the post-test. (The numbers do not add up to 47 since some responses included more than one factor.)

Factors	Feelings	Number of students	
		Pre-test	Post-test
nature of maths	positive	1	11
	negative	23	6
personal	positive	10	19
	negative	24	14
teaching	positive	0	0
	negative	5	3

Table 4.11b: Students' perceptions of mathematics before and after a course on problem-solving.

As a whole, students' written comments following the course showed an increase in positive feelings. Table 4.9 shows that more students wrote positive comments particularly relating to the nature of mathematics and personal factor after the course.

This suggests that a course that focuses on mathematical thinking processes appears to have a positive effect on the students. In particular, the students have a more positive attitude toward the mathematics that they are learning.

More importantly however, these comments are very informative. They reflect more salient aspects of the students' perception of mathematics and solving problems. They often used terms such as *abstract*, *work hard*, *memorising*, *make sense*, *anxious*, *pleasure*, *confident* in expressing their opinion. These terms, which most of them mentioned to project their views (italicised words/phrases, in sections 4.6.2.1 and 4.6.2.2) were noted and used in the construction of a modified questionnaire for the main study at the UTM.

In addition, at this point it is suspected that the term *abstract* used by the students may differ in meaning from the one used by mathematicians. To the students, it seems that "abstract" refers more to being in contrast with the "real world" rather than associated to the process of abstraction perceived by mathematicians. As suggested by Mason (1989b), many students see mathematics as 'abstract' possibly because they fail to recognise abstraction as a "delicate shift of attention from seeing an expression as an expression of generality, to seeing the expression as an object or property" (p. 2).

4.6.3. Comments Through Written Assignment

At the end of the course, the students were given a written assignment that consisted of a problem to be considered over four weeks including the production of a rubric, a reflection on the activities, and a short essay considering how problem-solving techniques might be used in their other mathematical studies. The following phrases are taken from these assignments and they give a clear indication that the effect of the problem-solving course is dramatic for a majority of the students. Students come to value the pleasure that comes with their involvement in problem-solving.

> The feelings of excitement, enjoyment and enthusiasm are an extremely important feature of problem solving. ...[I] accept the problem as a challenge and is subsequently highly motivated. ...The writing down of the rubric was particularly effective. Thus I didn't just leap into the problem without keeping down the groundwork first
>
> *2nd year Maths*

> On starting a question I feel excited and eager to get to an answer quickly. ... then I'm stuck. This brings about feelings of frustration and anxiety. If this period of being stuck continues for a long time it can also, for me, cause a degree of loss of self-confidence. If I recognise that these emotions are perfectly natural when thinking mathematically then I can start to overcome them. I should, hopefully, be able to counteract this effect.
>
> *3rd year Maths*

Some students gave indications that they have personally made great use of the knowledge in solving mathematical problems in particular and to engage in learning mathematics more effectively in general.

> Problem solving provides many opportunities to practice selecting and using appropriate mathematics already learned, helping to put knowledge and skills into context. The course has certainly taught me to look for other ways of doing things before focusing solely on the most obvious. *2nd year Computer Science*

> Acknowledging that I was stuck enabled me to find a new direction. Knowledge of the processes involved prevented me from becoming frustrated when stuck and enabled me to keep going in a sensible way. Further work needs to be done to develop my internal monitor. ... I used the rubric from the Problem Solving course quite a lot now because it helps me. I wish I had followed this course earlier. It should be given in the second year so that students can have the opportunity to use them in their mathematics learning.
>
> *4th year BA(QTS)*

The theory of "goals" and "anti-goals", (Skemp, 1979), proves to be very informative to the students. The knowledge that the emotional feeling of frustration for instance is caused by an underlying lack of knowledge helps them seek to solve the underlying difficulty rather than succumb to the sense of hopelessness that causes them to give up. Acknowledging such a situation with the simple word "stuck", followed by a search for activities which will get them round the difficulties, can turn negative emotions into positive activity.

> I found it very difficult to make conjectures. After many years of being right, it is difficult to realise that it is acceptable to be wrong. ... The course helps me develop a more positive attitude (thinking "I can" rather than "it is too difficult"). This makes it more of a goal, rather than the avoidance of an anti-goal.
>
> *4th year BA(QTS)*

> I reached the state of being stuck, I couldn't see how I could actually get at the answer. I assimilated what had actually written, and realised that I could use all this knowledge to answer the question. If I had not been able to control my emotion and carefully review, previous work could have easily been abandoned. ...The techniques used will hopefully remove me from this frustrating and sometimes annoying state of being stuck.
>
> *2nd year Computer Science*

The above phrases all point to the fact that students have an appreciation of the problem-solving techniques. The awareness of the mathematical processes led them to adopt a more active viewpoint toward mathematics and solving problems. In addition, these comments suggest that the tutor of the course had achieved the objectives outline earlier in the chapter (section 4.2). It may be suggested that he has successfully built a favourable environment whereby students are prepared to participate actively in solving problems, to think for themselves, and not to feel threatened by their failure.

However, comments such as the following gave a focus on issues that may prove crucial in its implementation.

> The idea of teaching problem-solving... has both benefits and drawbacks. As anyone attending the course will confirm, problem-solving is addictive, so can help promote maths as quite fun and interesting subject. The major drawback is that there is a finite

> amount of time available to teachers, and if problem-solving techniques are taught, then somewhere some other subject must suffer.
>
> *3rd year Maths*

> Everybody is rushing to get things done, to finish the work to go out, to get the revision done before tomorrow's exam, to get the question answered in the remaining thirty minutes. Students do not find time to sit back and use the problem solving techniques despite the fact that if they did, they may very well save time instead of wasting it.
>
> *2nd year Maths*

Indeed, as acknowledged by Schoenfeld (1994), one does have to pay a cost when getting students to engage more deeply in mathematics. In addition however he succinctly said:

> ...when the payoffs include much deeper understanding, much longer retention of the content, enthusiasm, and the fact that the students get a much better sense of the mathematical enterprise, the price in (ostensible) coverage is a small one to pay.
>
> p. 60

4.6.4. Formal Individual Interviews

During the last two weeks of the problem-solving course, 5 males and 4 females students were selected for interview. There were four students from groups N and S respectively and the one student who has no opinion on the item "mathematics make little sense". The students selected represent the different types of students following the course. All except Mary attended the interview. For the purpose of the discussion, students' names are have been changed to maintain anonymity.

Students	Course	Degree Classification	Gender	Group
Chris	Maths	I	M	S
Mary**	Maths	II-2	F	S
Alice	BA(QTS)	II-1	F	S
Sarah	BA(QTS)	II-1	F	S
Eric	CS	II-1	M	S
David	Maths	II-2	M	N
Naomi	Maths	III	F	N
Ruth	BA(QTS)	II-2	F	N
Peter	CS	II-1	M	N
Colin	Maths	I	M	no opinion

Table 4.12: The ten students selected for the interview

The interview method is a mixed case of talking aloud procedure and clinical interview which was tape recorded. The students were not told before hand about having to solve a problem during the 30 minutes session. The main purpose was to see their reaction when asked to solve problems. Apart from that, we want to know the extent of students thinking in a mathematical manner while problem-solving. The following problem adapted from Schoenfeld (1985a, p. 301) was given to the students. This problem was chosen because it is a mathematical problem that is easily understood. It can be approached in a number of ways and requires the students to access appropriate mathematical knowledge which all of them possessed:

> Consider the set of all triangles whose perimeter is a fixed number P. Of these which has the largest area? Justify your answer.

It was observed that although none of the students could answer the problem within the given time, they were able at least to make a start. That is, although none of them could solve it, it does not affect their belief that if they have more time they could solve it. One may conjecture that the students' success in problem-solving was sufficient to give them a sense of well-being.

> It will be nice if I did [solve] it. But it is not the end of the world for me. In fact it gives me incentives to go away and work on it. *Chris, 2nd year Maths*

> I ought to be able to solve the problem. ...Things like this will always be on my mind.

> All day I'll be thinking about it.
>
> *Alice, 4th year BA(QTS)*

> I will still be thinking about the problem at the back of my head for a while because I know I can do it. *Eric, 2nd year CS*

> This is definitely not the way this problem is supposed to be solved. But it is leading me somewhere. ...I think I had resolved, at least partially, the problem.
>
> *Colin, 2nd year Maths*

Seven of nine students show some evidence that they are capable in carrying out the mathematical processes at some point in their problem-solving. They seem to be in control of what they are doing.

> I am plugging and unplugging some ideas rather than a method to help me prove my conjecture. *Chris, 2nd year Maths*

> What I don't know is the length. ...I'm looking at the angles but I'm not sure how does that going to help me. ...I think I'll try another approach. It's obviously equilateral triangle. How can I justify this? *Alice, 4th year BA(QTS)*

> I knew this problem could be solved using Heron's formula straight away. But I can't remember the formula. I think I'll specialise a bit.
>
> *Colin, 2nd year Maths*

> I am trying to get some sort of a pattern to find an equation. ...It seems I'm getting nowhere with this. I think I'll take a break from this and do more specialising.
>
> *Peter, 2nd year CS*

However, it is also recognisable that there exists a spectrum of confidence spanning from over-confidence to lack of confidence among the students. Eric and David are being over-confident. They believe they can do it but they fail to achieve what they wanted due to a lack of reflection upon what they are doing. They did not really reflect beneath the surface.

> It is really a simple problem which I could have done ages ago. It annoys me because I have done it before. ...I keep on persevering with the approach because I have a vague idea of where I'm supposed to be going.
>
> *Eric, 2nd year CS*

> I am confident with this kind of problem because I think I know how to do it and there is a definite answer at the end. ...I didn't quite get the equation right. Given enough time I think I can solve it. ...All the processes happen in my head. They occur naturally.
>
> *David, 3rd year Maths*

Sarah and Peter are confident with their ability. They have no fear in having to solve unexpected problems but they do feel anxious while solving the problem. Especially after struggling for some time:

> I really need some square paper or a protractor because I can't visualise it without some kind of presentation. I haven't got a clue ...I really can't see at all. ...This problem really annoys me because I thought I knew how to do it. However, I will keep working on it.
>
> *Sarah, 4th year BA(QTS)*

> I'm still thinking of a way of how to prove my conjecture and I can't think of anything at the moment. It's the way I'm doing it. ...Once I'm involved in a problem and I got stuck I feel frustrated. I need to get away... .
>
> *Peter, 2nd year CS*

On the other extreme end of the spectrum, are Naomi and Ruth. They lack confidence. The expectation of having to solve a problem is clearly causing them to feel anxious. Both students still experience the negative feeling of fear of an unexpected problem and showed lack of confidence in their mathematical ability, although they believe it has lessened after the problem-solving course.

> Oh no! I didn't realise I have to do something. I don't like doing this, it makes me nervous. ...I don't really do much work, so when I'm given a problem to do, I feel anxious. I prefer to work with other people because they can give other ideas. I'm not very good at working by myself.
>
> *Naomi, 2nd year Maths*

> Oh yes! Definitely. I do fear unexpected [mathematics] problems. I feel like this all the time when I'm faced with problems in the maths course. For a really difficult problem I always expect not being able to do it and when I can do it, it's like a bonus. It is more a feeling of luck than my skills at solving problems.
>
> *Ruth, 4th year BA(QTS)*

It is interesting to see here that the students who are being over-confident are both males whereas at the other end of the spectrum, that is those lacking confidence are both females. Additionally it may be suggested that the females are more procedural than the male students. They tend to work hard and learn the materials. Hence, because they are working more procedurally they are anxious about what they are doing. It was here that we noticed the students interpretation of 'procedure' and 'problem solving'. They see it as a problem-solving procedure.

> If I had work harder I feel a lot more better. Problem like this, I don't know. It is a bit hard because I really don't know where to go. I feel more secure if I know the procedure, something to lead and go on to.
>
> *Naomi, female, 2nd year Maths*

> I do feel anxious especially when my mathematical ability is tested. ...I tend to persevere a lot in solving maths problem especially when it involves using a procedure. And I know the procedure I have chosen is right.
>
> *Alice, female, 4th year BA(QTS)*

> I am confident with my [mathematical] ability. But I do need to do a lot of work to understand mathematical concepts. ...I felt no fear of this problem because I had techniques to use.
>
> *Sarah, female, 4th year BA(QTS)*

The evidence obtained shows that 2 of 5 males had too much confidence. They may need to be encouraged to reflect on what is happening while problem-solving. All four females feel insecure with their mathematical ability. One may also hypothesise that the females doing BA(QTS) have more security than the females in Maths. Perhaps because they have a certain belief in themselves that they can teach other people. Thus they have much more self-confidence although they are not as mathematically able. However, we would not attempt to justify the hypothesis here, although the data tends to support the view that confidence and anxiety are gender related. For example, before the course it was found that 10 of 12 students who claimed they had no confidence were females (statement 1 in part B). Similarly, 10 of 14 students who experienced anxiety when asked to solve problems were females (statement 4 in part B). This finding indicates that Joffe & Foxman's (1986) observation amongst 15 years old pupils may also be evident amongst undergraduates.

Naomi is considered the least able (obtained a third class grade at the end of previous year) compared to the other interviewees. In the pre-test she views mathematics as facts and procedures to be remembered and learned by memorising. She, like Ruth, always expects not to be able to solve a problem. By doing this, they reduce the pressure of failing. However, following the course they both realise this is unhealthy and claim that the course has helped them to develop positive attitudes towards their mathematics.

Looking at students' responses in the pre-test, one may like to conclude that attitudes before the problem-solving course do not affect the quality of achievement. However, if one looks at the ways students think during problem-solving, it may be hypothesised that there may be a link between attitude towards mathematics and performance in solving problems.

4.7. Discussion

The students' responses pointed to the issues that the researcher needs to consider in the main study at the UTM. They gave a clear sense of direction, both in the statements selected for the modified questionnaire as well as the focus that directed her in terms of teaching the course. In particular the issues that students bring to light such as the need to understand, to have confidence and the desire for a different approach were considered. In addition the responses also signal some differences between the students (i.e. N and S). For example, before the course, it is observed that group N students, for whom mathematics makes little sense, have to "work hard remembering" the material and "memorising formulae". In contrast, the group S students work hard to "make sense" of the mathematics. It is these qualities that may play a large part in the inability to predict the nature of degree from students perception of whether or not mathematics makes sense. Some of these obvious perceptions to be used in the modified questionnaire are italicised in the students' comments (see section 4.5). As a result of the problem-solving course it is possible to conjecture that students who claim that mathematics makes little sense would see further into its underlying qualities.

Students bring a variety of beliefs to any course including the problem-solving course. Many are ingrained. During the interview it appeared that some students have other interpretations of 'procedure' and 'problem solving'. For instance, they see problem solving as a problem-solving procedure. Although students' responses illustrate benefits it is likely that they may need to be acquired over a longer term. Perhaps the limitations of the questionnaire are coming through.

The problem given during the semi-structured interview was found to be difficult for the students to solve within the time allotted. In fact a colleague who tried to solve it only came up with a solution a day after his first attempt. Nevertheless, the problem did provoke the students' mathematical thinking. They demonstrated that they were capable of carrying out mathematical thinking processes.

Gender related differences were noted pertaining to confidence and anxiety amongst the students interviewed. Joffe & Foxman (1986) reported that in the APU surveys with 15-year olds, the difference in performance between boys and girls "is minimal in most topic areas-except in the top attainment bands" (p. 48). They suggest that nearly all the differences in performance between the students are accounted for by the top 10 to 20 percent of attainers in most topics. Our findings indicate that gender related difference may also be evident amongst the Warwick students (being in the 90th percentile band).

The course in problem-solving appears to rekindle students interest in mathematics. Although it did not show in the questionnaire, opinions expressed by students as seen in section 4.6, suggest that before the course they see mathematics rather differently, more as a fixed body knowledge to be learnt. Doing mathematics, it seems, is a reproductive process; the materials are learnt for re-presentation in the exams. However, the experience in thinking mathematically had led them to develop a new spirit of adventure. The majority of students reported that their enthusiasm to learn the mathematics had increased. Students' now believe mathematics is a creative subject which involves solving problems and making new inventions, which may not be new to others but new to them.

4.8. Chapter Summary

This chapter reports on the effect of teaching of problem-solving to undergraduates at Warwick University. It was hypothesised that the problem-solving course would effect students' attitudes to mathematics and problem solving.

Before the course, responses to the questionnaire give indications that a high proportion of the students have some positive attitudes, although some of the items measured attract a considerable minority of responses expressing no opinion. However, students' written opinions suggest that a majority of them perceived mathematics as a fixed body of knowledge to be learnt with feelings about the subject matter running very high amongst them. They imply that they have difficulty with the mathematics and have a vague sense of what is going on.

After the course, although an analysis of the pre and post-test questionnaire indicate marginal changes towards the better on most items measured in part A, in part B only 3 of the 16 items have changes too small to be statistically significant. Furthermore, data gathered from classroom observations and students' interviews (informal and formal) suggests that majority now have a more positive attitudes towards the mathematics they are studying. Students are prepared to participate actively in solving problems, to think for themselves, and do not feel threatened by their failure. The course also appears to give students an alternative way of doing mathematics. Before the problem-solving course, it was seen as a reproductive process but during the course it becomes more of a thinking process. Consequently opinions expressed suggest that the knowledge had fostered students' desire to learn their mathematics. The majority of students report that the problem-solving techniques led them to an increasing awareness of what is required for improved mathematical thinking. Many appreciate the experience they have had. The findings obtained support the hypothesis that the teaching of problem-solving would improve students' attitudes.

Differences between the students were noted; students for whom mathematics makes little sense seem to be doing mathematics for a different reason from those who reject the statement. In the interview there was a gender related differences in confidence and anxiety noted amongst the students. Two of 5 males tend to be over-confidence with their mathematical ability whereas all four females have the inclination to work procedurally. Two of 4 female students tended to show greater anxiety in their work.

The considerable number of 'no opinion' responses given to some items measured signal changes to be made to the questionnaire. Students' expressed opinions gave indications that the course has the effect of changing their attitudes. They also point to issues that we need to consider in the main study at the UTM. It is a major conjecture of this study that a course in problem-solving would also affect UTM students' attitudes positively and the study will follow in the next chapter.

5. MAIN STUDY: PROBLEM SOLVING AT THE UTM

5.1. Introduction

The Warwick results had indicated that students attitudes changed in what was considered a positive manner as a result of the course and confirmed that problem-solving can alter students' perception of mathematics as an active thinking process. It was hypothesised that a similar course would have the same positive effects on Malaysian students. However, because of cultural differences it was expected that there would be initial antagonism towards the course. This meant that there was the risk of getting negative outcomes as well as positive ones. The underlying theme—the development of active thinking processes—contrasted starkly with the very limited opportunities Malaysian students normally have to make their own mathematical decisions. Malaysian students are dutiful and eager to please their teachers by working hard and learning mathematical procedures to pass examinations. As one student was eventually to comment—"to be good in mathematics requires good memory and lots of practice".

The analysis of the outcomes of the problem-solving course at Warwick University set the scene for an investigation into the effects of a similar course amongst students studying mathematics at Universiti Teknologi Malaysia (UTM) and modifications were made in the methodology.

Warwick students' responses to the questionnaire, the written assessment and the interviews were taken into account in constructing the questionnaire used in the main study at the UTM. It was planned that the major study at the UTM would involve measurement of short-term changes, the effect after an elapse of time and the tutors' perceptions of students thinking about mathematics. Therefore the working phases included a pre-test before the course, the teaching of problem-solving, a post-test following the course, and a post post-test after six months of standard mathematics when the tutors' perception of students' mathematical thinking were also investigated.

This chapter, which considers the immediate effect of a problem-solving course designed to broaden such perceptions, is to be seen in conjunction with Chapter 6 which considers the longer term effects through the administration of a "post post" test. In this chapter the changes in students' attitudes towards mathematics and problem solving will be placed within the context of the expected and preferred attitudes identified by their tutors. The tutors "desired direction of attitudinal change" is established through the analysis of questionnaire completed by 22 staff within the Mathematics Department at UTM. The context of the study is considered in section 5.2 and we examine the methodology for the study (section 5.3). An analysis of lecturers' responses to students' attitudinal questionnaire is presented in section 5.4. Section 5.5 will present an analysis of students' responses. This follows with an overall comparison (section 5.6). Section 5.6.1 considers the comparison between staff's desired changes and the students change in attitudes after problem-solving. The results show that after the problem-solving course students' change on all items are those preferred by the staff. Difference between the students (i.e. N and S) will be considered in section 5.6.2. Section 5.7 will present individual student's comments. Data from interviews (section 5.8) and the discussion of arising issues (section 5.9) provide a perspective for the final conclusions (section 5.10) which indicate that the problem-solving course was, at least in the short term, a suitable course to broaden and restructure students' conceptions of mathematics. Based on the hypotheses which underscore the study (section 5.2.2), the conclusions are consistent with the belief that through problem-solving, students' attitudes towards mathematics become more positively attuned to the view that mathematics is a process of thinking.

5.2. The Study

5.2.1. Initial Considerations

The problem-solving course at Warwick enabled us to obtain an insight into the processes used in learning to think mathematically and the way in which students used them in problem-solving. It was observed that the course was presented in a very informal way and the students were used to working by themselves. They were ever willing to talk about their experience and provide comments. This is one essential cultural difference that the researcher took into account when introducing the course to the UTM students.

The subjects chosen for the study were taking mathematics as a core subject; students need to pass to obtain their respective degree. They had followed a relatively advanced level of mathematics courses at the time of the study. Thus, one may assume that the students have achieved considerable experience in university mathematics. More importantly, they have the mathematical knowledge to solve all the problems given during the course. As in Warwick, the course carried some credit that adds to the students' cumulative points for the semester.

A questionnaire and semi-structured interviews were used as the method for collecting data for the purpose of this study. As in the Warwick study, a questionnaire was chosen for reasons of practicality. The questionnaire given to the UTM students was a modified version of the pilot questionnaire, taking account of the Warwick students' responses. It was seen within the Warwick experience that interviews with selected students helped to intensify the nature of responses. Subjects had written their code names on the questionnaire. Thus it was possible to link responses with individuals. On the other hand, due to the culture it is expected that the Malaysian students would be more candid and objective when answering the questionnaire than when interviewed individually, where cultural influences may limit the nature of students open responses. Accordingly interviews with small groups of students were used.

5.2.2. Hypotheses

Lecturers may want things to be different but they cannot make it to be different. They want students to think mathematically and have positive attitudes. Here we hypothesise that lecturers desire the same kind of attitudes of the students as formulated in the problem-solving course. It is likely that lecturers would prefer the students to have more positive attitudes than they think the students will have in practice. More particularly it was hypothesised that:

i. There is a mismatch between the way lecturers would *prefer* the students to think and the way they *expect*, from practical experience, students will think.

ii. The problem-solving course can cause a change in students' attitudes such that they reflect those desired by lecturers. In particular:

    - Students' attitudinal changes during problem-solving occur in the same direction as the changes that occur between what the mathematicians expect the students to do and what the mathematicians would prefer the students to do.

5.3. Method

5.3.1. For All Staff Concerned

The students' questionnaire on attitudes to mathematics and problem solving (as given to the students following the problem-olving course, see section 3.3.2) was circulated to mathematics lecturers. They were invited to read through the questionnaire twice. At the first reading, they were requested to respond as they *expected* a typical student would respond (a tick). During the second reading they were requested to put a circle where they would *prefer* the student's response to be.

5.3.2. For the Students

The testing of the hypotheses was carried out in a similar style to the one used with the Warwick students. As in chapter 4, the significance of changes of attitudes as a result of the course was calculated using the Wilcoxon Matched-pairs Signed-ranks Test.

(i) A sample of students were given a problem-solving course over a period of 10 weeks during July, August and September 1993.

(ii) Student attitudes to mathematics and problem solving were considered before and after the course through their responses to a modified and adapted questionnaire based upon the one used amongst the Warwick students.

(iii) The students' changes in attitudes were monitored through classroom observation.

(iv) Students' written responses were supported through comments obtained through semi-structured interviews.

5.3.3. The Sample

22 members of the Mathematics Department who are teaching various mathematics courses took part in the study. They are those who had filled out the questionnaire twice; specifying attitudes they *expect* from their students and the attitudes they *prefer*.

24 males and 20 females took part in the study. They were a mixture of third, fourth and fifth year undergraduates aged 18 to 21 in SSI (Industrial Science, majoring in

Mathematics) and SPK (Computer Education), covering the full honours degree range (see Table 5.4 below). They were chosen because they had filled out the full set of (pre and post) questionnaires that were distributed during the course.

5.3.4. The Course

The problem-solving course consisted of thirty contact hours over a period of ten weeks. During the course students were encouraged to experience all aspects of mathematical thinking—specialising in simple cases, seeking patterns, generalising through systematic specialisation, formulating conjectures, testing, modifying, refining, justifying and reviewing problems and their solutions. The material for the course was based upon that used at Warwick (i.e. Mason *et al.* and Skemp's texts). However, the material was modified to cater for the wider ability range of students. For instance, the problems given were selected so as to give the less able a sense of success early in the course. Also the writing of the rubric was emphasised at a later stage than at Warwick. It was only stressed after the students stopped asking for the correct answer and about what they needed to do next. The material was translated to Bahasa Malaysia (the language of instruction in the UTM) and the researcher taught the course.

The Warwick experience provided the researcher with the essential insight that led the way in which the course was presented. As the tutor, when formulating the course, my intention was similar to that of the original lecturer of the course at Warwick, that is to encourage students to become explicitly aware of the processes of mathematical thinking and to participate actively in solving problems. Accordingly, working in a non-threatening atmosphere, students can work up to their potential, and to develop a problem-solving attitude. Like the Warwick tutor, I had not solved all the problems given in the course. On several occasions, I had to solve a problem in front of the class, showing them that even a mathematician does not produce a neat, straightforward textbook proof. This was intended to encourage the students to feel less reluctant to make conjectures which might prove to be wrong on the possible route to success. Furthermore, by not knowing the solution to the problems, I was in the same position as the students. It was quite difficult on my part at first because it made me feel somehow inadequate. However, if we have a problem-solving attitude we constantly have to test ourselves.

The course was designed so that each week of the ten weeks had two components.

1. A two hour session at the start of each week. This session, identified as the problem-solving session, was designed to have several phases.
2. A one hour session towards the end of the week. This session was regarded as a session for reflection.

The two hour session was broken down into the following phases:

An introductory phase

The purpose of this phase was to set the scene. For instance, students would be introduced to particular aspects of mathematical thinking and the relevant problem that illustrated the aspects being emphasised.

The problem-solving phase

This is the period during which the students work on the problems in their own small groups of 3 or 4 of their own choice. It was seen at Warwick that some of the problems are quite difficult to solve alone, particularly within the short time allocated. Working together gave the students the opportunity to share ideas with their friends, to talk and to argue about them in trying to convince their friends. It was observed that there was more participation. Even though, the place was buzzing with voices, it did not seem to distract them from their discussion.

This working phase starts when students try to find out what the problem is really asking. It would be useful for the students to think and try to answer the questions: "What do I KNOW? What do I WANT? What can I INTRODUCE?" to begin with. The major activity of finding a solution to the problem takes place during this phase.

A review phase

The tutor reviewed the situation after about half an hour or so to see how well things were progressing. To make sure that everyone was solving the same problem, groups were invited to tell the class their interpretation of the problem they were solving. The tutor would then consider the ideas generated by the students, what they had noticed and what relevant conjectures they had made.

If students reached a "stuck" position, what is it that is blocking them? Is it possible to identify the gap between "What do I WANT" and "What do I KNOW"? As in Warwick, there was no attempt to lead the students towards a possible solution nor were any answers given. However, using the ideas generated by the students, the tutor proposed some guidelines. It is observed that the tutor's role was that of provider of a supportive environment, students were encouraged to use their own resources and the tutor give them meta-thinking support. In such an environment students were free to pursue and to invent their own original methods of solving the problems.

A further problem-solving phase

This phase took place when students had reached a reasonable resolution. They were invited to demonstrate the results they had obtained. In doing this they tried to convince others that their results were valid. It was suggested that at this point they might see the importance of proving as they explain and try to defend their solution. In addition, students were encouraged to extend their resolution, setting it in a more general context.

A concluding phase

Towards the end of the session, the tutor again highlighted the mathematical processes students were induced to use, particularly bringing the problem-solving processes to their awareness. The emphasis was on the processes and the methods that were used rather than on getting the correct answer.

The single one hour session was essentially a reflecting period.

The reflection session

During this meeting students were encouraged to reflect on their mathematical experience and to talk about their attempts to solve the problems. It was time when they exposed the mental activities that were normally hidden. It was when they communicated their ways of attacking the problem or the difficulties that block their output of ideas. The tutor gave her comments on the effectiveness of the solutions. Students' attention was explicitly drawn to the different ways of thinking about the problem. There was discussion on where things may have gone wrong or where the students have failed to take advantage of certain things. The central theme of this session was reflecting on alternative routes to solving the problem. For example, the ***Fifteen*** problem (see Appendix 1) may remind some students of a magic square. Thus, they may follow similar paths to get the resolution. They proceed to determine the sum of the rows, columns and diagonals and the properties it must have. However, those who have not met or noticed the similarity may produce varying solution methods. For instance, students initially specialised at random to obtain triples of numbers that add up to fifteen before moving on to do it systematically.

The session ended with a summary of what students had achieved, with positive points specified. For example, students learned that there was more than one way to solve a problem, and that there were many suitable but different answers, or even no answer at all. It was emphasised that problems should be solved by the students themselves.

The students following the course did not have access to Mason et al "*Thinking Mathematically*". There was only one copy available in the university library. However, at the end of each session copies of the material covered were distributed to the students. These were notes from *Thinking Mathematically* (see Appendix 1 for sample of the materials distributed). Therefore, in class the students focused on attempting to solve the given problems with the freedom of time to reflect upon their problem-solving experience. Since the course carried some credit points, it motivated the students to perform up to their potential as noted at Warwick. An assessment, similar to the first part of Warwick assessment was given at the end of the course.

That was a problem to be considered over four weeks and the students were invited to consider the production of a rubric, and their reflection on the problem-solving activity. They were also invited to produce a short essay indicating how problem-solving techniques may be used in their other mathematical studies.

5.3.5. The Questionnaire

The attitudinal questionnaire on mathematics and problem-solving was given to the students in the first week of term during the second meeting. It was again distributed following the course in the final week—during the last meeting.

The students were given a brief introduction to the meaning of the scales and advice on how to fill in the questionnaire. They were then given fifteen minutes to fill it in. Even though they were requested to write down their code names, they were told that their identity would not be disclosed. It was made clear to the students that the researcher was mainly interested in their responses to the questionnaire and their reactions to the course. They were informed that all information collected through the questionnaire would be kept confidential and would only be used as data in a scientific study.

5.3.6. Monitoring Students during the Classroom Phase

Since the statement "*The mathematical topics we study at the university make sense to me*" (part A, statement 6) had proven to be a good discriminator amongst the Warwick students it was decided to again utilise this question as a discriminator. Thus, in the second week of the course, based on the pre-test responses to the questionnaire, the 44 students were subdivided into two groups—group N, for whom university mathematics did not make sense, and group S, for whom it did. It happened that, the results of the pre-test indicated an equal distribution of students over the two groups. Of the 44 students, 22 responded negatively and 22 responded positively. No other statement equalled this level of distribution.

Based on these two groupings students were then invited to sub-divide into small groups of 3 or 4, so that through free selection the groups were identified as constituting of group N students or group S students. Students worked in their chosen groups for the rest of the course.

At the end of each week, each group's efforts at solving the given problem was handed in for evaluation. Students were encouraged to be as explicit as possible in writing down their thoughts. The "rubric" commentary was introduced early in the course but it did not became a requirement for students to include it in their solutions until the fourth week. As observed at Warwick, requiring students to write the rubric at the beginning of the course may create antagonism as mentioned by the following Warwick students:

> My first instinct was to try and just get the answer straight away. The rubric writing restricts my progress.
>
> I tend to rush into a problem. When I became stuck, I waited for inspiration to strike. ...Rubric writing is time consuming, and does not appear to bear fruit.

However, many do value the experience at the end of the course:

> The most important technique was the rubric writing. Not only did it give an accurate record of where I had got to and how, but it made me work at a slower pace, and think a little before I committed my ideas to paper. This is very important for a somewhat erratic thinker like myself.
>
> The idea of writing down everything you know that might apply to the problem, or that might fit the results, proved invaluable. I would never have thought of introducing the [idea] had it not been for this.
>
> I left the problem for a period of time. Despite the fact that I was no longer physically working on the problem, I was still subconsciously working on the problem. I became aware of this when I suddenly had the 'inspiration' for the next part of my work. This is

where the Rubric writing proves invaluable.

In contrast to Warwick, at the UTM, the weekly assignment carries some marks towards the final course grade. That is, evaluation of the students' performance was carried out continuously as the course progressed. The final assessment only carries 40% of the total marks and it is based upon individual work. Students are familiar with this kind of assessment as it is part of the system at the UTM. This kind of assessment enabled the researcher to keep the weekly assignment as a record of each groups of students' development in their problem-solving achievement.

5.3.7. Semi-structured Interviews

The students selected for interviews were representative of both groups (i.e. N and S) and the subject areas. They were invited to attend the session voluntarily. All six groups (3 from groups N and S respectively) selected agreed to be interviewed.

They were not told beforehand that they would attempt to solve a problem during the 30 minutes allotted time. To put them at ease in the first 5 minutes they were simply asked to talk about their mathematical experience. Only then were they given a problem to solve. The interviews were video recorded.

It was made absolutely clear that the main interest of the interviewer was the students' reactions and the interview was not some kind of an assessment. It was clarified to all interviewees that the purpose of the interview was to find out what they do in making sense of a problem they are faced with. In a covert way the interviewer looked for indications of students' ability in carrying out the mathematical processes during problem-solving. It was believed that such a process would not only reveal different qualities of thinking between the students, their understanding of the content but also the reaction aroused by problem-solving.

5.4. Analysis of Results from All Staff

5.4.1. Responses of All Staff Concern

Twenty-two lecturers in the Mathematics Department had responded to the students' attitudinal questionnaire specifying the attitudes they *expect* from their students and the attitudes they *prefer*. The data collected are presented separately for both parts of the questionnaire.

5.4.1.1. Results to Part A: Attitudes to Mathematics

Table 5.1 shows the responses of all 22 lecturers. The table has a column of total "Yes" (Y+y) responses and a column for subset "Y" who state their views strongly ("definitely yes"). Likewise for the no responses.

Mathematics	Yes		(Y)		No		(N)		—	
	expect	prefer	expect	prefer	expect	prefer	expect	prefer	expect	prefer
facts and procedures	**20**	13	(8)	(4)	**2**	9	(0)	(2)	**0**	0
solving problems	**19**	22	(9)	(9)	**3**	0	(0)	(0)	**0**	0
inventing new ideas	**8**	11	(2)	(3)	**14**	11	(1)	(1)	**2**	0
abstract	**20**	7	(6)	(0)	**2**	15	(0)	(4)	**0**	0
understand quickly	**3**	15	(0)	(1)	**19**	7	(6)	(1)	**0**	0
makes sense	**8**	19	(0)	(3)	**14**	3	(2)	(0)	**0**	0
work hard	**21**	18	(13)	(4)	**1**	4	(0)	(0)	**0**	0
learn by memory	**15**	2	(5)	(1)	**7**	20	(1)	(6)	**0**	0
able to relate ideas	**5**	22	(0)	(5)	**17**	0	(5)	(0)	**0**	0

Table 5.1: Responses for 22 lecturers to attitudes to Mathematics

The first two columns are of particular interest here. In the first column, a third or less of the lecturers expect students to think that mathematics is about ***inventing new ideas***, that they can ***understand quickly***, that university mathematics ***makes sense*** and that they are able to ***relate ideas***. All other item, received at least 15 positive responses. However, lecturers' preferences, in the second column, show different trends. In particular the four items: ***inventing new ideas***, ***understand quickly***, ***makes sense*** and able to ***relate ideas*** now receive positive responses from at least half of lecturers. The differences are even more marked in the lecturers' perceptions of students' responses that mathematics is *abstract*, that they can *relate ideas learned* and *learn by memory*. On these items lecturers' perception of what students think about mathematics and their preferred attitudes in general are reversed.

The table reveals various levels of difference in responses between the lecturers' perception of students' thinking about mathematics and their preference for students' perception. In particular, it is observed that there is:

General agreement–high expectation, high preference

- On the belief mathematics is about *solving problems,* the majority (86%) of lecturers expect and indeed prefer (100%) their students to have the perception.
- Nearly all (96%) expect their students would consider having to *work very hard* to understand. Indeed the majority (82%) prefer this quality in students.

Low expectation high preference

- Few (14%) of the lecturers expect their students would *understand new ideas quickly,* but yet a high proportion (70%) prefer their students to be able to do so.
- The majority (86%) of lecturers prefer that university mathematics *makes sense* to their students. However a considerable minority (36%) expect their students would have the perception.
- All (100%) the lecturers desire students to have the ability to *relate ideas* learned. However, only few (23%) expect students can do so.

High expectation, low preference

- Nearly all lecturers (91%) expect their students would identify mathematics as *remembered facts and procedures*. However, not much more than half (59%) prefer students to see it as such.
- Although a considerable minority (32%) prefer their students to see mathematics as *abstract,* nearly all (91%) expect students would identify with the notion.

- A high proportion (68%) of the lecturers expect students to learn through *memory* but few (9%) prefer them to do so.

Mixed feelings

- The majority (36%) do not expect students would perceive that mathematics is about *inventing new ideas* although it is not a strongly held belief. It is particularly interesting to see that only half (50%) of the lecturers desire this perception in their students.

It is noted that on almost all items measured apart from two items: the belief that mathematics is *solving problems* and students *work hard*, the lecturers' expectation and preference of students' perception to mathematics do not correspond. That is, there exists a mismatch between what the lecturers think their students would do and what they prefer students to do. It may be suggested that most of the qualities that they desire in their students, most lecturers do not believe the students possess, in particular on items that characterise thinking about mathematics: mathematics can be *understood quickly*, it *make sense*, can *relate ideas* learned and it is not mostly *memorisation*. Where there are differences in lecturers' preference and expectation of students' perception we note a strong negative correlation (r=-0.607, p<0.1).

5.4.1.2. Results to Part B: Attitudes to Problem Solving

The data for part B of the questionnaire is shown in Table 5.2 below.

Solving Problem	Yes		(Y)		No		(N)		—	
	expect	prefer	expect	prefer	expect	prefer	expect	prefer	expect	prefer
confidence	10	22	(1)	(3)	12	0	(0)	(0)	0	0
pleasure	15	21	(0)	(4)	7	1	(2)	(0)	0	0
only to get through	21	7	(9)	(2)	1	15	(0)	(3)	0	0
anxiety	16	2	(5)	(0)	6	20	(0)	(5)	0	0
fear unexpexted	15	3	(7)	(0)	7	19	(0)	(5)	0	0
correct answers	19	6	(3)	(2)	3	16	(0)	(2)	0	0
try different approach	12	22	(1)	(4)	10	0	(0)	(0)	0	0
give up	16	2	(2)	(0)	6	20	(0)	(2)	0	0

Table 5.2: Responses for 22 lecturers to attitudes to problem solving

Notice that in the first column, lecturers' expectation of students' thinking about problem solving, 6 of the 8 items receive 15 or more positive responses from the 22 staff. The exceptions to this level of response are *confidence* (10 of 22) and will *try a different approach* (12 of 22). This contrasts strongly with the lecturers' preference, 5 of the 6 items with high positive perception now received low positive preferences. Only the belief that students obtain *pleasure* from solving problems received a high preference rate, whereas the two items: *confidence* and *willingness to try a different approach* now received positive responses from all lecturers. Although there is no strong relationship between lecturers' expectation and their preference, particularly because of these three items, there is overall a significant difference ($p<0.01$) in their responses to the way lecturers expect students think and the way they would prefer students to think.

Looking in detail at the table, it can be seen that:

General agreement–high expectation, high preference

- A high proportion (68%) expect students would obtain *pleasure* from their work. Indeed nearly all (96%) prefer them to have that perception.

Low expectation, high preference

- none

Moderate expectation, high preference

- Although all (100%) lecturers prefer their students to have *confidence*, less than half (45%) expect students would be perceived as having the characteristic.
- All (100%) the lecturers prefer students to be *trying a different approach*. However, little more than half (55%) expect the students would have the willingness to do so.

High expectation, low preference

- Although nearly all (96%) expect students would do the mathematics to *get through the course*, only a considerable minority (32%) prefer students to do so.

- A majority (73%) of the lecturers expect their students would experience *anxiety* when faced with problems. In contrast, nearly all (91%) prefer students not to have this negative emotion.
- Likewise few (14%) prefer their students feeling *fear* of the unexpected but a high proportion (68%) expect the students would do so.
- The majority (73%) of the lecturers prefer students to see that doing mathematics is not just getting the *correct answers*. But even more (86%) expect that this would be the students' aim.
- The majority (73%) of the lecturers expect students to *give up* easily when faced with difficulties but only 2 (9%) lecturers prefer students to do this.

It is observed that the lecturers' preferences and their expectation of students' perception of problem solving do not match on all items measured. The lecturers do not believe that the students possess the desirable qualities that characterise problem solving behaviour. It seems that what the lecturers would want from the students and what they perceive students would do are two different things. But it may be suggested that the lecturers may want things to be different but they cannot make it to be different. For example, they desire that the students can understand their mathematics and can relate the ideas together but they perceived many of the students would learn their mathematics by memorising.

5.4.2. The "Desired Direction of Attitudinal Change" Perceived by Mathematics Staff

The difference between lecturers' responses to attitudes they *expect* from their students and the attitudes they *prefer* is used to define the lecturers' "desired direction of change". Table 5.3 shows the responses of the 22 lecturers and the direction of the desired change from the expected to the preferred attitude. The columns marked "Yes(Y)" have the total "yes" responses (Y+y), with the subset "definitely yes" (Y) in brackets. Similarly for "No(N)".

				Expect					Prefer				
	Attitude	desired change		Yes	(Y)	No	(N)	–	Yes	(Y)	No	(N)	–
Mathematics	facts and procedures	$\downarrow^{+++}_{+}$	<1%	20	(8)	2	(0)	0	13	(4)	9	(2)	0
	solving problems	$\uparrow^{+++}_{+++}$	n.s.	19	(9)	3	(0)	0	22	(9)	0	(0)	0
	inventing new ideas	$\uparrow^{+}_{-}$	n.s.	8	(2)	14	(1)	0	11	(3)	11	(1)	0
	abstract	$\downarrow^{+++}_{--}$	<1%	20	(6)	2	(0)	0	7	(0)	15	(4)	0
	understand quickly	$\uparrow^{+}_{---}$	<1%	3	(0)	19	(6)	0	15	(1)	7	(1)	0
	make sense	$\uparrow^{++}_{-}$	<1%	8	(0)	14	(2)	0	19	(3)	3	(0)	0
	work hard	$\downarrow^{+++}_{++}$	<1%	21	(13)	1	(0)	0	18	(4)	4	(0)	0
	learn by memory	$\downarrow^{++}_{---}$	<1%	15	(5)	7	(1)	0	2	(1)	20	(6)	0
	able to relate ideas	$\uparrow^{+++}_{--}$	<1%	5	(0)	17	(5)	0	22	(5)	0	(0)	0
Problem Solving	confidence	$\uparrow^{+++}_{-}$	<1%	10	(1)	12	(0)	0	22	(3)	0	(0)	0
	pleasure	$\uparrow^{+++}_{+}$	<5%	15	(0)	7	(2)	0	21	(4)	1	(0)	0
	only to get through	$\downarrow^{+++}_{-}$	<1%	21	(9)	1	(0)	0	7	(2)	15	(3)	0
	anxiety	$\downarrow^{++}_{---}$	<1%	16	(5)	6	(0)	0	2	(0)	20	(5)	0
	fear unexpected	$\downarrow^{++}_{--}$	<1%	15	(7)	7	(0)	0	3	(0)	19	(5)	0
	correct answers	$\downarrow^{++}_{-}$	<1%	19	(3)	3	(0)	0	6	(2)	16	(2)	0
	try different approach	$\uparrow^{+++}_{+}$	<1%	12	(1)	10	(0)	0	22	(4)	0	(0)	0
	give up	$\downarrow^{++}_{--}$	<1%	16	(2)	6	(0)	0	2	(0)	20	(2)	0

Table 5.3 : Lecturers perceptions of students preferred and expected attitudes

The arrow and the plus and minus signs in the second column (see table 5.3) indicate the direction of movement. The number of plus or minus signs indicates the average weighted strength of response. That is for each statement, taking each Y response as 2, y as 1, n as –1 and N as –2, if the average response is 1 or more, the response is considered "strong" and denoted "+++" or "---". Between 0.5 and 1 it is denoted "++" or "--", and less than 0.5 it is considered "weak" denoted "+" or "-". The significance of the change is computed using the Wilcoxon Matched-pairs Signed-ranks Test (with correction factor applied in the event of tied ranks) on the responses (the score allotted to each category is the same as in calculating the average weighted strength). The significance of difference in the staff responses is given as significant (<5%), highly significant (<1%) or not significant (n.s.).

Thus we see for instance, the view that mathematics is *facts and procedures* is desired to change down from an expected strong agreement by the typical student (+++) to a preferred weak agreement (+) by the lecturers. In line 4, seeing university mathematics as being *abstract*, the desired direction of change is from the expected strong view that students would believe this (+++) to a preferred disagreement (--).

In only two of the cases is the change too small to be statistically significant: the lecturers expect the typical student to believe strongly that mathematics is about *solving problems* and prefer it marginally stronger, that mathematics is not about *inventing new ideas*, but weakly prefer that it should be.

One change in direction is statistically significant – that there is a weak expectation of *pleasure*, but lecturers prefer it to be strong.

Three differences remain in the same direction but the changes are highly significant – an expected strong student belief that mathematics is a collection of *facts and procedures* to be remembered, which the lecturers desire less, that the student has a strong expectation to have to *work hard* to understand, whilst lecturers have a lower expectation, and a weak expectation that they are willing to *try a different approach* when their attempt fails, which is preferred more strongly.

The remaining eleven items are both statistically highly significant and have opposite expectations and preferences. The lecturers *expect* the typical student to think mathematics is very *abstract*, will not *understand quickly*, will consider that mathematics does not *make sense*, will *learn through memory*, will not *relate mathematical ideas*, will not *have confidence*, will only solve problems *to get through* the course, will show *anxiety*, will *fear* the unexpected, regard *correct answers* as the most important thing, and that the typical student is expected to *give up* when a problem gets difficult. In every case the lecturers *prefer* the student to think the opposite.

5.5. Analysis of Students Results

5.5.1. Classroom Observation: An Overview

Initially, the students were very confused with the rationale set for the problem-solving course. A course which emphasised students involvement in mathematics contrasted starkly with their established perceptions which generally involved passive acceptance of course material during lectures. During standard mathematics lecturing the students are likely to have very limited opportunities to make their own mathematical decisions and this, coupled with the underlying formality of the culture, contrived to make them feel lost and uncomfortable. They kept asking questions such as "What shall I do now?", "Is this the right way of doing it?", "What is the answer?", when they became stuck after a frantic attack on the given problem. Such questioning served to reinforce the view that personal decision making on what to do next and the development of strategies for solving problems were not part of their usual mathematical behaviour. It was clear that their mathematical thinking is influenced greatly by their beliefs about mathematics and problem solving. As one student explained when asked about his attempt to solve the *Warehouse* problem:

> I tried several times on my own using trial and error. I got stuck. I asked my friend how he did it and gathered some information. I got stuck again. I copied his solution and tried to understand it. Then I tried to solve the problem again on my own.
>
> *SPK, year 5*

It is suggested that such behaviour depicts this student's usual way of solving mathematical problems in regular mathematics courses—imitating what he is taught rather than figuring out solutions for themselves. And he is not alone. One may suggest (from experience) that for many of the students, it is a common practice to find solutions by looking them up at the end of mathematics texts or by getting them from their friends or lecturers who have solved the problems.

It is observed that a great proportion of the students frequently had ideas that may have been useful in getting a possible solution but they simply did not know how to use them. Additionally, it was noticed that they felt reluctant to put forward any ideas that they were not certain about. They did not like thinking aloud or sharing their ideas with others because they feared that their suggestion were wrong or may get rejected. The fear of failure removed them from the habit of answering questions in front of others or putting forward any suggestions. Furthermore, they did not know what a conjecture is and had no notion how to make one:

> Before the idea of conjecturing was introduced I have no idea what it really is. It's difficult to use conjecturing without being taught about how to make conjectures. ...I was not inclined to making conjectures due to the ingrained attitude of mathematics being 'right' or 'wrong'. *SSI, year 3*

Each week the students were given a couple of problems to solve as a group. Although students were requested to work co-operatively in their groups, initially there was hardly any discussion between the individuals. Each of them appeared to be more concerned with getting the work done on their own. Group contributions were only made after they had first tried to solve the problem alone. For a few weeks they showed enormous resistance to co-operative activity. However, little by little the resistance diminished until, after four weeks, they gradually displayed some positive reactions; they began to think for themselves, to share their thoughts with others in the group and to write a rubric commentary outlining their problem-solving activity.

The students knowledge of mathematics was sufficient to solve all the given problems. At first they were set simple problems. This helped tremendously in giving them a sense of success and in building their self-confidence. Not only those who were successful in regular mathematics courses demonstrated these qualities but also those whose past failures had earlier been manifested in uncertainty and unwillingness to tackle something new. Students learned to cope with their emotions and their obsession with getting correct answers.

Early in the course, short entries in their weekly assignment such as the following were common:

> We are stuck. We do not know how to proceed.
>
> It's more fun when there is no pressure on you to solve it!
>
> We can solve the problem given more time.

Half way through the course, their entries became more explicit and self-analytical. They presented many desirable qualities such as a willingness to struggle with the task and the desire to reflect on their own problem-solving:

> Not knowing how to proceed, we decided to try and get a better grip on the question. ...we had isolated our weakness and acted to overcome it.
>
> We felt very negative when we couldn't find a solution as easily as before. We mulled over the problem for a while at this point, until we suddenly realised it was our own preconceptions holding us back!
>
> We needed to find 'why'. At this point we are stuck! We try to gather the information so far in order to find a path forward.

When the students came to realise that they had to figure out the answer themselves and were responsible for their own progress, they stopped asking for the right answer. They began to explore their own mathematical knowledge, to select and use it to formulate a method of solution of their own:

> ...I got stuck when I thought there had to be a unique solution. Then I started to think about using a parameter and obtaining a family of solutions which is the same idea for solving simultaneous equations. The problem solving techniques encourage me to apply ideas to areas which I may not normally consider them appropriate.
>
> *SPK, year 4*

> The emphasis has always been to get the correct answer. This puts a great deal of pressure on the students. The [problem solving] course focuses more on how you get the answer. This allows me to re-assess my capacity. I had confidence in my ability to try out things. ...I was able to feel in control of the problem, get involved in it, enjoy extending it and come to a resolution that I was satisfied with.
>
> *SPK, year 5*

Although the changes were slow to come, the majority of the students gradually learned to generate mathematical ideas, to talk about them and be critical of suggestions given by their friends. Their discussion became livelier as they moved towards doing things that they could explain to their friends, rather than simply satisfying the course requirements or pleasing the tutor. Their problem-solving became "a more creative activity, which includes the formulation of a likely conjecture, a sequence of activities testing, modifying and refining ..." (Tall, 1991, p. 18). In the final assessment, more than half of the students managed to come up with their own original, though mathematically acceptable, way of solving a problem. This indicates that it is possible to get the students to create their own solution methods, even though this took longer than the previous experience using procedural methods in routine problems.

The selected phrases in the written assessment indicate a major shift in the students' thinking about problem-solving following the course. In particular, the change was from an attitude which focused on getting the right answer from the teacher or friends, or wanting to escape from the task as quickly as possible, to one which reflected an ability to figure out the solution and defend the results. Students came to realise that there were different approaches to a problem. More importantly, they began to see that a solution depended on the decision to use a method which was more appropriate to the circumstances rather than on doing the right calculation. Consequently, they appeared to gain confidence in solving problems independently and were capable of thinking up their own strategies; they came to appreciate the problem-solving experience that they had had:

> Problem solving gives both meaning and value to the study of maths. It encourages us to apply what we know and to plan an approach to solving a problem. ...The discussion helps to sharpen our understanding of mathematical concepts and gives us the chance to negotiate its meaning in our own terms. It is also gives us the opportunity for the presentation of alternatives and makes us realise the fact that there is more than one way to solve most problems. ...We each learn to take some responsibility for what occurs. I feel this is not always encouraged by the maths course at the university.
>
> *SSI, year 3*

> Solving the problem requires a great deal of time and thought. I have never actively thought about the processes that my mind undergoes while attempting to solve a problem. Usually I am eager to start a question. I attacked it having not really gained all the information. Frequently I will come to a point where I can continue no further. By this time I no longer have enough motivation to continue. So I just abandon it. ...After a long history of failure I surprised myself that I managed to solve this problem.
>
> *SPK, year 4*

> Before I took this course I could probably have solved the problem, but it would have taken me longer, and I would not have had such a coherent solution at the end.
>
> *SSI, year 3*

Students' comments illustrate what they can do when they are given the opportunity to think in a mathematical manner. Opinions expressed suggest that the majority of the students are capable of benefiting from the course in many ways. This was particularly noticeable in the way they restructured their views about mathematics and gained the confidence to make mathematical decisions independently. There are also indications of an improvement in their mathematical thinking. For instance, in seeing there is more than one way to solve a problem and in having a willingness to try out new ideas without giving up too easily. Students became more positive about themselves in learning the mathematics and in their mathematical ability.

5.5.2. Responses to Pre-course Questionnaire: Pre-test

Students' responses to the questionnaire distributed at the beginning of the course and at the end of the course are summarised in Table 5.4. The students' responses were varied and covered a full range of the scales available.

In their perception of mathematics, there is an exceptionally high agreement (77%) on the notion that mathematics is a collection of *facts and procedures*. Similarly the majority (84%) of students indicate that they have to *work hard* to understand mathematics. Over a third indicate strong agreement with both items. Although a high proportion of students (61%) believe that mathematics is about *solving problems,* with a quarter expressing a strong agreement, half of the students do not see it as about *inventing new ideas*. More than half (57%) view mathematics as *abstract* but only a small minority (20%) claim they can *understand new ideas* quickly. It can be seen that the students split exactly into half in the belief mathematics *makes sense* to them. A high proportion (68%) claim their mathematics is mostly *memorising* but not much more than half (54%) think they can *relate mathematical ideas* learned. These responses indicate that for more than half of the students mathematics is seen as a fixed body of knowledge which they have to learn.

Mathematics	Yes	(Y)	No	(N)	–
facts & procedures	34	(18)	8	(2)	2
solving problems	27	(10)	16	(4)	1
inventing new ideas	21	(4)	21	(6)	2
abstract	25	(13)	17	(0)	2
understand quickly	9	(0)	30	(5)	5
makes sense	22	(4)	22	(5)	0
work hard	37	(15)	5	(1)	2
learn by memory	30	(1)	12	(2)	2
able to relate ideas	24	(8)	18	(2)	2

Solving Problems	Yes	(Y)	No	(N)	–
confidence	26	(7)	17	(2)	1
pleasure	43	(25)	1	(1)	0
only to get through	16	(4)	27	(8)	1
anxious	17	(1)	24	(4)	3
fear unexpected	30	(10)	12	(3)	2
correct answers	21	(4)	21	(3)	2
try different approach	42	(17)	0	(0)	2
give up	19	(3)	24	(9)	1

Table 5.4: Pre-test responses of 44 students to the questionnaire

Looking at the responses in detail, it was found that a high proportion (64%) of the students agreed concurrently on the notion that mathematics consists of *facts and procedures* and is mostly *memorisation* (statements 1, and 8). In contrast, only a quarter (27%) agree simultaneously on seeing mathematics as *solving problems*, it involves the *invention of new ideas* and it *make sense* (statements 2, 3, 6).

In their perception of problem solving, we can see that nearly all the students indicate that they get *pleasure* from problem-solving, and have the willingness to *try a different approach,* with more than a third giving a definite opinion on both items. More than half (59%) claim to have *confidence* in their mathematical ability. A considerable minority (36%) indicate that they are doing mathematics *only to get through* the course. More students say they do not feel *anxiety* when asked to solve problems than those who do. However, note the high (68%) positive responses given on experiencing *fear* of the unexpected. Although more students indicate that they do not *give up* easily, half of the students place an importance on *correct answers.* The responses indicate that although there is a positive feeling about problem solving amongst the students, it is suggested that the majority do not have the confidence to tackle anything new.

It is found that a majority (69%) of the students simultaneously claim to take great *pleasure* from solving problems, have the willingness to *try different approach* but *fear the unexpected.* It is also observed that there is no significant difference between gender and the notion pertaining to confidence and anxiety. In particular, amongst the students who gave indication that they had *no confidence*, feeling *anxiety* and experiencing *fear of the unexpected*, the number of females and males almost split equally (i.e. 9 of 17, 8 of 17 and 14 of 30 were females respectively).

5.5.3. Responses to Questionnaire: Post-test

Table 5.5 illustrates the overall results obtained from administration of the questionnaire at the end of the course.

Mathematics	Yes	(Y)	No	(N)	–
facts & procedures	11	(3)	32	(8)	1
solving problems	42	(21)	0	(0)	1
inventing new ideas	37	(15)	5	(0)	2
abstract	15	(8)	27	(3)	2
understand quickly	20	(3)	21	(2)	3
makes sense	35	(23)	7	(0)	2
work hard	28	(8)	13	(0)	3
learn by memory	11	(5)	31	(7)	2
able to relate ideas	35	(11)	8	(0)	1

Solving Problems	Yes	(Y)	No	(N)	–
confidence	36	(12)	6	(0)	2
pleasure	42	(21)	0	(0)	2
only to get through	4	(0)	37	(17)	3
anxious	6	(0)	36	(9)	2
fear unexpected	10	(3)	31	(9)	3
correct answers	5	(1)	36	(11)	3
try different approach	43	(18)	0	(0)	1
give up	5	(0)	37	(20)	2

Table 5.5 : Post-test responses of 44 students to the questionnaire

After the problem-solving course, it is noticeable that in attitudes towards mathematics, most positive responses are given in the belief that mathematics is about *solving problems* and the *invention of new ideas* with a considerable minority expressing a definite yes. The majority (79%) think mathematics *makes sense* to them with more than half giving a strong opinion. Likewise, a majority (79%) of the students claim they can *relate mathematical ideas* learned. More students think they can *understand new ideas* quickly. Items that relate to the notion that mathematics is composed of *facts and procedures*, is *abstract* and as mostly *memorising* are now more likely to be rejected by the students. It is now found that the majority of the students (75%) agree simultaneously on the notion that mathematics is solving problems, involves the invention of new ideas and it makes sense to them. In contrast, only 6 (14%) students see mathematics as both facts and procedures and learned through memory.

Looking at responses to problem solving, the most striking feature is the low positive responses given on all negative items measured. In particular, it is observed that experiencing the negative feeling of *anxiety* and *fear* of the unexpected only receive 10 (23%) or less positive responses. Similarly, notions of doing mathematics for the extrinsic pressure of *only to get through* and placing importance on *correct answer* were rejected by a considerable majority of students with at least a third of these strongly rejecting such beliefs. The majority (75%) also claim they do not *give up* easily, nearly half of the 44 students strongly rejecting this statement. 75% of the students now indicate that they feel *confident* in solving mathematical problem, a quarter of the 44 strongly agreeing with the statement. There remains unanimous agreement amongst the students that they obtain *pleasure* from problem-solving and this continues to be reflected in their willingness to *try a different approach* if an attempt fails. Over a third of the group strongly agree with these items. Such positive responses after the ten week course would seem to indicate that the students are acquiring positive perceptions of problem-solving.

Nevertheless, more important is to see how the students' attitude changes as a result of the problem-solving course compared with the staff's desired change.

5.6. Pre-test and Post-test Comparisons

5.6.1. The Change in Students' Attitudes in Problem-solving

Table 5.6 shows the changing attitudes of students before and after the problem-solving course.

		Pre-test					Post-test				
		Yes	(Y)	No	(N)	–	Yes	(Y)	No	(N)	–
Mathematics	facts & procedures	34	(18)	8	(2)	2	11	(3)	32	(8)	1
	solving problems	27	(10)	16	(4)	1	42	(21)	0	(0)	1
	inventing new ideas	21	(4)	21	(6)	2	37	(15)	5	(0)	2
	abstract	25	(13)	17	(0)	2	15	(8)	27	(3)	2
	understand quickly	9	(0)	30	(5)	5	20	(3)	21	(2)	3
	makes sense	22	(4)	22	(5)	0	35	(23)	7	(0)	2
	work hard	37	(15)	5	(1)	2	28	(8)	13	(0)	3
	learn by memory	30	(1)	12	(2)	2	11	(5)	31	(7)	2
	able to relate ideas	24	(8)	18	(2)	2	35	(11)	8	(0)	1
Problem Solving	confidence	26	(7)	17	(2)	1	36	(12)	6	(0)	2
	pleasure	43	(25)	1	(1)	0	42	(21)	0	(0)	2
	only to get through	16	(4)	27	(8)	1	4	(0)	37	(17)	3
	anxious	17	(1)	24	(4)	3	6	(0)	36	(9)	2
	fear unexpected	30	(10)	12	(3)	2	10	(3)	31	(9)	3
	correct answers	21	(4)	21	(3)	2	5	(1)	36	(11)	3
	try different approach	42	(17)	0	(0)	2	43	(18)	0	(0)	1
	give up	19	(3)	24	(9)	1	5	(0)	37	(20)	2

Table 5.6: Responses for 44 students on the pre and post-test questionnaire

Table 5.7 compares the staff's desired change and the actual changes ocurring in the students during problem-solving. Calculating the weighted average response and computing the significance in the change of the responses, the following changes were obtained.

		desired change		After P S	
Mathematics	facts and procedures	$\downarrow^{+++}_{+}$	<1%	$\downarrow^{++}_{--}$	<1%
	solving problems	$\uparrow^{+++}_{+++}$	n.s.	$\uparrow^{+++}_{+}$	<1%
	inventing new ideas	$\uparrow^{+}_{-}$	n.s.	$\uparrow^{+++}_{-}$	<1%
	abstract	$\downarrow^{+++}_{--}$	<1%	$\downarrow^{++}_{-}$	<1%
	understand quickly	$\uparrow^{+}_{---}$	<1%	$\uparrow^{+}_{--}$	<1%
	make sense	$\uparrow^{++}_{-}$	<1%	$\uparrow^{++}_{-}$	<1%
	work hard	$\downarrow^{+++}_{++}$	<1%	$\downarrow^{+++}_{++}$	<1%
	learn by memory	$\downarrow^{++}_{---}$	<1%	$\downarrow^{+}_{--}$	<1%
	able to relate ideas	$\uparrow^{+++}_{--}$	<1%	$\uparrow^{++}_{+}$	<1%
Problem Solving	confidence	$\uparrow^{+++}_{-}$	<1%	$\uparrow^{++}_{+}$	<1%
	pleasure	$\uparrow^{+++}_{+}$	<5%	$\boxed{\downarrow^{+++}_{+++}}$	n.s.
	only to get through	$\downarrow^{+++}_{-}$	<1%	$\downarrow^{-}_{--}$	<1%
	anxiety	$\downarrow^{++}_{---}$	<1%	$\downarrow^{-}_{--}$	<1%
	fear unexpected	$\downarrow^{++}_{--}$	<1%	$\downarrow^{++}_{--}$	<1%
	correct answers	$\downarrow^{++}_{-}$	<1%	$\downarrow^{+}_{--}$	<1%
	try different approach	$\uparrow^{+++}_{+}$	<1%	$\uparrow^{+++}_{+++}$	n.s.
	give up	$\downarrow^{++}_{--}$	<1%	$\downarrow^{-}_{---}$	<1%

Table 5.7: Desired changes compared with changes after problem-solving

We see that in almost every case, the change in response from what the lecturers expect and what they prefer is in the same direction as the change in the students' responses from before the problem-solving course to after the course. The exception is one item: that students should take *pleasure* from solving problems (boxed in table 5.7) was rated highly each time with positive attitudes changing only from 43 down to 42 (out of 44).

Only one change is not statistically significant: *willingness to try a different approach* remains highly rated. All other items have highly significant changes in the desired direction. Of these, some responses are reversed: the majority now indicated that mathematics is not just *facts and procedures*; it involves the *invention of new ideas*, it is *not abstract*, it *makes sense*, students *can understand new ideas quickly*, it is not necessary to *learn by memory*, there is *less fear of the unexpected*, and it is *not just getting correct answers*. Others are greatly increased: mathematics is more about *solving problems*, students are *able to relate ideas*, and that they have *confidence*. The remaining items: students *work hard to understand* reduced significantly whilst indications that doing mathematics *only to get through* the course, feel *anxiety* and will *give up easily* when faced with a difficulty diminishes.

Thus the problem-solving course has caused a change in attitudes amongst the students in the direction desired by the lecturers.

5.6.2. Making Sense of Mathematics

Analysis of the pre-course results indicates that when "Yes" and "No" responses are considered several statements have the effect of discriminating the students into two equal groups. However, responses to the statement "*The mathematics we study at university makes sense to me*" clearly discriminates to the point where all of the students have an opinion; opinions are equally distributed between the yes and no categories. This finding further strengthened the observation noted at Warwick of the possible existence of two groups of students (i.e. N and S group). Thus, following the Warwick investigation, here we attempt to analyse the data in a similar way to that used in the pilot study. Through their responses to this statement, students were divided into equal groups, group S consisting of the 22 students for whom the statement makes sense and group N, the 22 students for whom it does not. Using this item as a discriminator we may see that the factor does not have an influence on student achievement. Table 5.8 shows the current degree classification of the students within each group based on their previous years achievement.

		Degree Classification											
		Group N (n=22)						Group S (n=22)					
		I	II-1	II-2	III	P	F	I	II-1	II-2	III	P	F
Students	SPK year 5	2	5	2	1	0	0	0	3	3	0	0	0
	SPK year 4	1	5	3	0	0	0	2	6	4	1	0	0
	SSI year 3	0	3	0	0	0	0	1	2	0	0	0	0
	Total	**3**	**13**	**5**	**1**	**0**	**0**	**3**	**11**	**7**	**1**	**0**	**0**

Table 5.8: The distribution of students for whom mathematics makes sense (group S) and does not (group N)

It can be seen that the two groups have almost identical distributions. The correlation (r) between examination success and whether the students consider mathematics makes sense is -0.019. This means that only 0.04% of the variance in students' achievement at the end of a year can be accounted for by whether or not the mathematics makes sense. This reinforced the observation made in Warwick that whether or not students say that university mathematics makes sense is not a means of predicting the level of students achievement.

These two groups now provide a basis for further analysis of the questionnaire. Rather than provide a separate analysis of the pre-test and post-test questionnaires based upon these groups, it is more expedient to provide composite tables to sharpen comparison between pre-test and post-test results.

5.6.2.1. Attitudes to mathematics

Table 5.9 shows the data for the two groups on the pre- and post-test for Part A. Items underlined in column 1 show a significant (<5%) or highly significant (<1%) change in responses, computed using the Wilcoxon Matched-pairs Signed-ranks test. Underlining in the Yes columns refer to significant changes (double underline represents highly significant, single underline represents significant) for each group.

The arrow indicate the desired direction of movement: ↑ represents an overall increase in positive responses, ↓ represent an overall decrease in positive responses.

Mathematics		Group N (n=22)										Group S (n=22)									
		Yes		Y		No		N		–		Yes		Y		No		N		–	
		Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
facts & procedures	↓	20	6	16	2	1	16	1	4	1	0	14	5	2	1	7	16	1	4	1	1
solving problems	↑	9	21	2	10	13	0	4	0	0	1	18	21	8	11	3	0	0	0	1	1
inventing new ideas	↑	6	18	1	8	14	2	4	0	2	2	15	19	3	7	7	3	2	0	0	0
abstract	↓	21	11	12	7	1	9	0	0	0	2	4	4	1	1	16	18	0	3	2	0
understand quickly	↑	4	8	0	0	13	12	2	1	5	2	5	12	0	3	17	9	3	1	0	1
makes sense	↑	0	14	0	2	22	7	5	0	0	1	22	21	4	3	0	0	0	0	0	1
work hard	↓	19	15	6	5	1	5	0	0	2	2	18	13	9	3	4	8	1	0	0	1
learn by memory	↓	17	6	1	0	3	15	1	3	2	1	13	5	0	0	9	16	1	4	0	1
able to relate ideas	↑	6	14	1	4	16	7	2	0	0	1	18	21	7	7	4	1	0	0	0	0

Table 5.9: Comparison between groups N and S students responses to part A of the questionnaire

A summary of the significance of the change in the responses is given in table 5.10 . The table contains the direction of the overall change and the significant difference in all subjects under study. It also makes a comparison between groups N and S after the course so that we see:

(i) Changes in the full group of 44 students (All /Pre v Post),

(ii) Changes amongst group N students (N/Pre v Post),

(iii) Changes amongst group S students(S/Pre v Post).

The change in the responses is highly significant (<1%) if $p < 0.01$, significant (<5%) if $p < 0.05$. "n.s" denotes no significant difference.

Mathematics		All Pre v Post	N Pre v Post		S Pre v Post	
facts and procedures	↓	<1%	$\downarrow^{+++}_{--}$	<1%	$\downarrow^{+}_{--}$	<1%
solving problems	↑	<1%	$\uparrow^{+++}_{-}$	<1%	$\uparrow^{+++}_{+++}$	n.s.*
inventing new ideas	↑	<1%	$\uparrow^{+++}_{--}$	<1%	$\uparrow^{+++}_{+}$	<5%
abstract	↓	<1%	$\downarrow^{+++}_{+}$	<1%	$\downarrow^{-}_{--}$	n.s.
understand quickly	↑	<1%	$\uparrow^{-}_{-}$	n.s	$\uparrow^{+}_{--}$	<1%
makes sense	↑	<1%	$\uparrow^{+}_{---}$	<1%	$\uparrow^{+++}_{+++}$	n.s.
work hard	↓	<1%	$\downarrow^{+++}_{++}$	<5%	$\downarrow^{+++}_{+}$	<1%
learn by memory	↓	<1%	$\downarrow^{++}_{-}$	<1%	$\downarrow^{+}_{--}$	<1%
ability to relate ideas	↑	<1%	$\uparrow^{++}_{--}$	<1%	$\uparrow^{+++}_{++}$	n.s.*

Table 5.10: Significance of groups N and S changes in responses to part A of the questionnaire

Although responses to the statement "*the mathematical topics we study at University make sense to me*" has been used to discriminate between the two groups N and S there was a highly significant change in the responses given by group N students after the course. Only one change is not statistically significant: students can *understand new ideas quickly* has a small improvement. One item changed significantly: students have to *work hard* decreased. The remaining items have highly significant changes in the desired direction.

For group S, we note that three (apart from "makes sense") changes are not statistically significant: mathematics is about *solving problems*, it is not *abstract* and students are able to *relate ideas* learned. These are attributes carrying over from earlier mathematics learning. These items remain highly rated after the course. All other indicators change significantly. Smaller changes are evident in the belief that mathematics is about *solving problems* and that students *are able to relate ideas.* (These are improved by a factor that would be significant at the 10% level, marked "n.s.*" in table 5.10.)

The difference in responses between the two groups may be seen much clearer if we consider Figure 5.1. In the belief that mathematics *makes sense*, all (100%) group S students remain positive before and after the course, but notice that a high proportion (64%) of group N students modified their perception from "No" to "Yes" after the course.

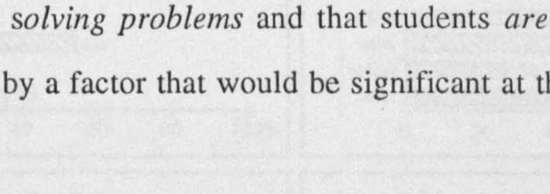


Figure 5.1 : University mathematics "makes sense"

Figure 5.2 uses the same bar-chart layout as figure 5.1 for all of the other statements indicating the attitude to aspects of mathematics by group N and group S students. As with the Warwick analysis, individual graphs are arranged to place related statements side by side. Though these are purely conjectured, the Warwick comments and the indication from the pre-course comments of students have enabled us to place these on a firmer footing. In particular, the way mathematics was experienced influenced the students' views of mathematics.

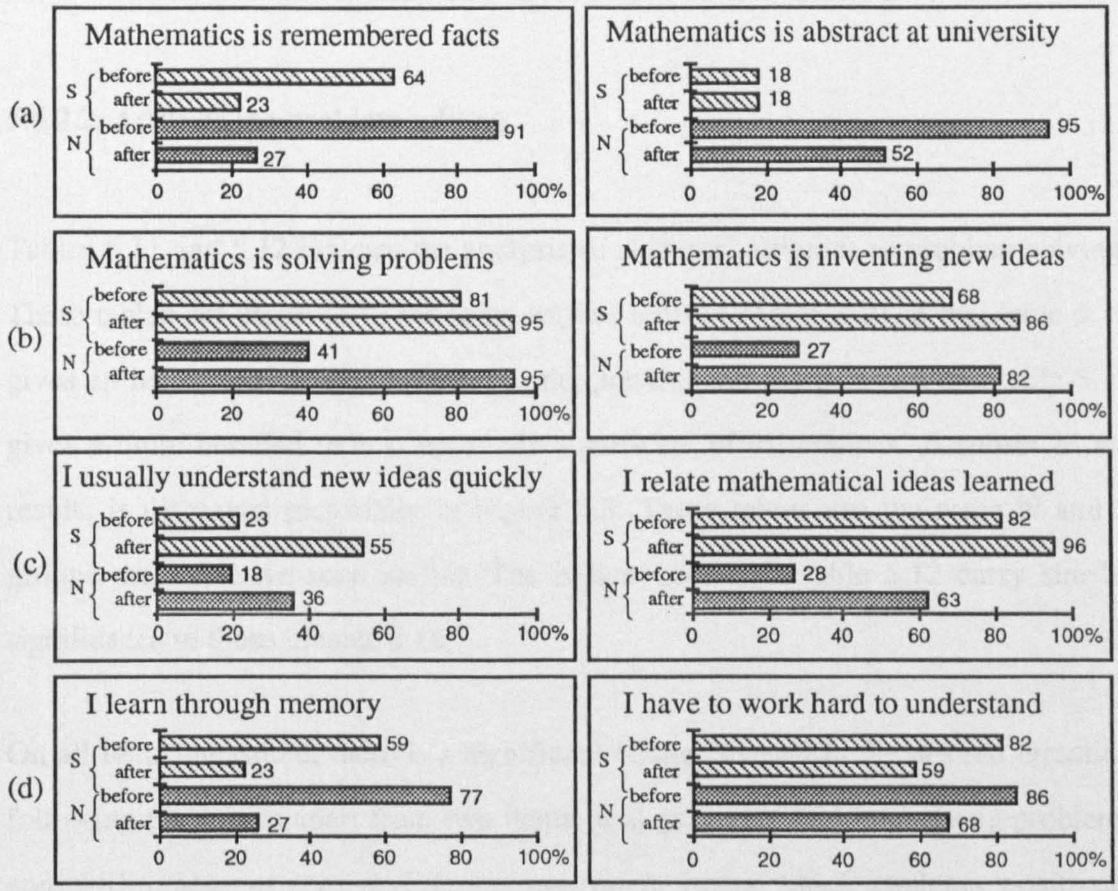


Figure 5.2 : Attitudes to Mathematics: pre-test and post-test comparison

Each graph tells a consistent story, supported by the details of significance in Table 5.10.

(a) There is a significant decrease in both groups in the notion that mathematics is series of *facts and procedures* to be remembered (from 91% to 27% in group N). The perception that university mathematics is "abstract" remains low in group S, whilst diminishing significantly in group N.

(b) There is a significant overall increase in the perception of mathematics as *solving problems* and *inventing new ideas*. Notice that the change is mainly in group N.

(c) Significantly, more students overall now claim that they *understand ideas quickly* (mainly group S) and can *relate mathematical ideas* together (mainly group N).

(d) Significantly fewer students overall claim they have to *memorise* ideas and fewer think they have to *work hard* to understand.

5.6.2.2. Attitudes to problem solving

Tables 5.11 and 5.12 indicate the analysis of students' attitudes to problem solving. These tables are prepared in the same way as tables 5.9 and 5.10 so that table 5.11 gives an indication of the responses to the pre-test and the post-test and table 5.12 gives a more detailed indication of the significant of differences. A subset of the results is displayed pictorially in Figure 5.3. These tables use the same N and S groups that we have seen earlier. The indication within table 5.12 carry similar significance to those in table 5.10.

On all items measured, there is a significant change overall in the desired direction following the course apart from two items: will get *pleasure* from solving problems and willingness to *try a different approach*, items which students positively responded to in the pre-test.

Solving Problems		Group N (22 students)										Group S (22 students)									
		Yes		Y		No		N		–		Yes		Y		No		N		–	
		Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
confidence	↑	9	16	1	3	13	5	1	0	0	1	17	20	6	9	4	1	1	0	1	1
pleasure	↑	21	21	11	9	1	0	1	0	0	1	22	21	14	12	0	0	0	0	0	1
only to get through	↓	14	4	4	0	8	16	2	9	0	2	2	0	0	0	19	21	6	8	1	1
anxious	↓	12	6	1	0	8	15	1	1	2	1	5	0	0	0	16	21	3	8	1	1
fear unexpected	↓	17	6	7	2	3	14	1	2	2	2	13	4	3	1	9	17	2	7	0	1
correct answers	↓	13	2	3	0	8	19	2	5	1	1	8	3	1	1	13	17	1	6	1	2
try different approach	↑	21	21	8	9	0	0	0	0	1	1	21	22	9	11	0	0	0	0	1	0
give up	↓	13	3	3	0	8	19	3	8	1	0	6	2	0	0	16	18	6	12	0	2

Table 5.11 : Comparison between pre and post-test responses to part B of the questionnaire

Table 5.12 below carries the same features as table 5.10 so that we see:

(i) Changes in the full group of 44 students (All/Pre v Post),

(ii) Changes amongst group N students (N/Pre v Post),

(iii) Changes amongst group S students (N/Pre v Post).

The change in the responses is highly significant (<1%) at 0.01 level and significant (<5%) at 0.05 level. 'n.s.' denotes no significant difference.

Solving Problems		ALL Pre v Post	N Pre v Post		S Pre v Post	
confidence	↑	<1%	$\uparrow^{++}_{-}$	<1%	$\uparrow^{+++}_{++}$	n.s.*
get pleasure	↑	n.s.	$\uparrow^{+++}_{+++}$	n.s.	$\boxed{\downarrow^{+++}_{+++}}$	n.s.
only to get through	↓	<1%	$\downarrow^{+}_{--}$	<1%	$\downarrow^{--}_{---}$	n.s.
anxious	↓	<1%	$\downarrow^{+}_{-}$	<5%	$\downarrow^{--}_{---}$	<1%
fear unexpected	↓	<1%	$\downarrow^{++}_{-}$	<1%	$\downarrow^{+}_{--}$	<1%
correct answers	↓	<1%	$\downarrow^{+}_{---}$	<1%	$\downarrow^{-}_{--}$	<1%
try different approach	↑	n.s.	$\uparrow^{+++}_{+++}$	n.s.	$\uparrow^{+++}_{+++}$	n.s.
give up	↓	<1%	$\downarrow^{+}_{---}$	<1%	$\downarrow^{--}_{---}$	<1%

Table 5.12: Significance of groups N and S changes in responses to part B of the questionnaire

For group N, we see that after the problem-solving course all changes in responses are in the desired direction. Only 2 changes are not statistically significant: *pleasure*, and *willingness to try a different approach.* Both items remain highly rated. One item changed significantly: students' indication that they experience *anxiety* diminishes. All other items changes highly significantly to that preferred by the staff.

In contrast for group S, before the course, it is noticeable that the majority has displayed positive attitudes towards solving problems except for a *fear of the unexpected.* Nevertheless as a result of the course, there is a positive change on all items, with the exception of one: *pleasure* (boxed in table 5.12) was rated highly each time with positive attitudes changing only from 22 down to 21 (out of 22). Only 3 changes are too small to be statistically significant: *confidence*, doing mathematics is not just to *get through* the course, willingness to *try a different approach* and unwillingness to *give up* easily. These items remain highly rated. Smaller changes are noted on students' indication that they have confidence which is significant at 10% level (marked "n.s.*" in table 5.12).

A subset of the results is displayed pictorially in Figure 5.3. From these pictorial summaries we see that once again each graph tells a consistent story, supported by the details of significance in table 5.12.

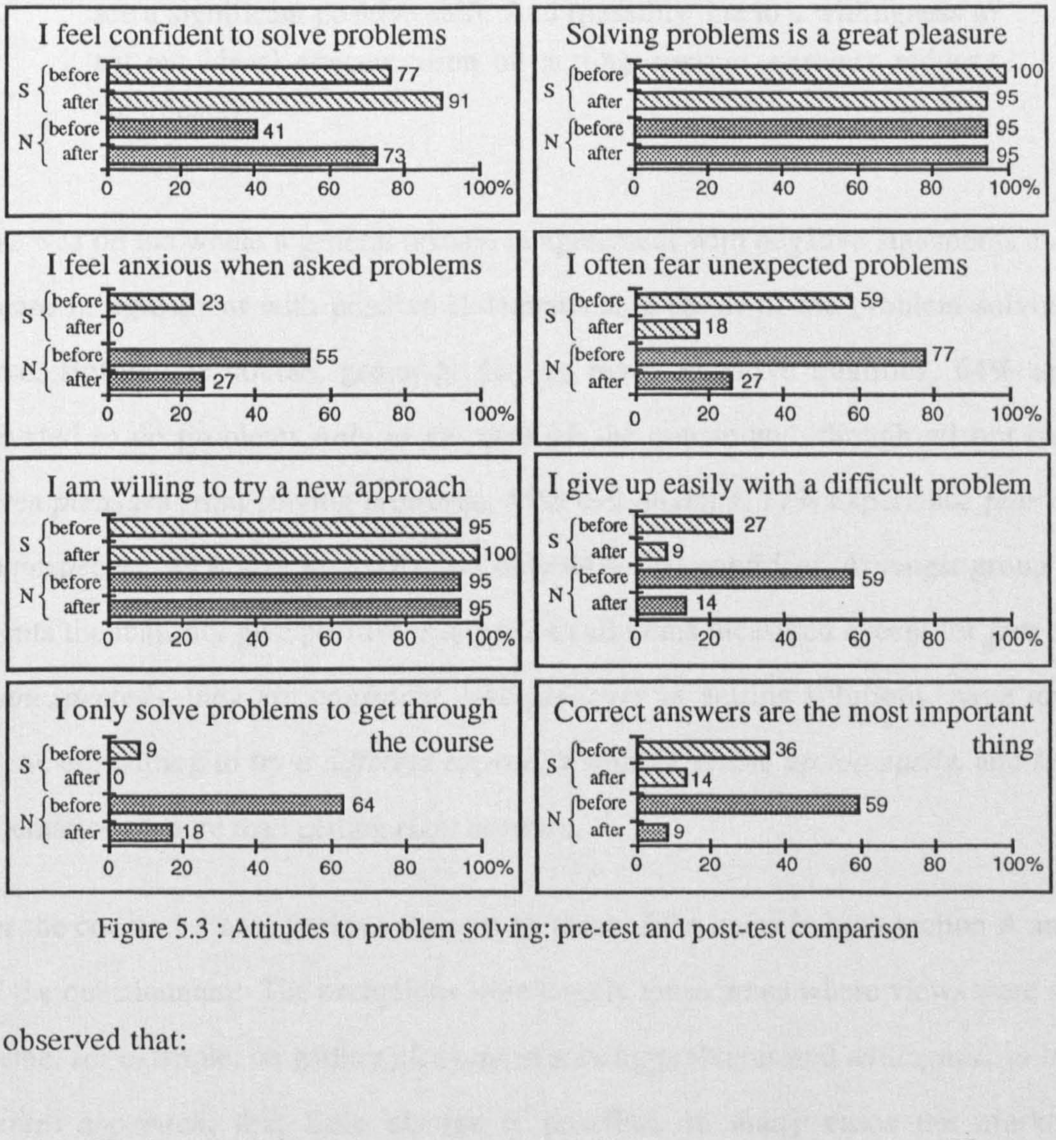


Figure 5.3 : Attitudes to problem solving: pre-test and post-test comparison

It is observed that:

(a) Having *confidence* increases overall from pre-test to post-test. The change is largely accounted for by the more positive view of the group N students. Getting *pleasure* in solving problems continues to receive very high positive responses.

(b) Experiencing *anxiety* when faced with problems is reduced to a low level in both groups (diminishes in group S). Indications that students *fear* unexpected mathematics problems decreased significantly.

(c) The *willingness to try a new approach* remains very high in both groups; the tendency to *give up* when faced with a difficulty reduces significantly to a low level in both groups.

(d) Extrinsic pressure—solving problems *only to get through* the course—reduces significantly overall. Amongst group N students we see a significant positive shift. And (possibly due to a willingness to try out ideas) concentration on getting *correct answers* reduces significantly.

There was on the whole a general decline in agreement with negative statements and increase in agreement with positive statements as a result of the problem-solving course. Before the course, group N display many negative qualities, 64% are motivated to do problems *only to get through* the course and, though all but one derives *pleasure* from solving problems, 55% feel *anxious*, 77% experience *fear of the unexpected*, 59% *give up* easily and only 44% feel *confident*. Amongst group S students the majority give positive responses to all items measured except for *fear of the unexpected*—they are *confident*, take *pleasure* in getting solutions, have low *anxiety*, are willing to *try a different approach* without *giving up too easily*, and see mathematics as more than getting *right answers*.

After the course we note positive changes on most of the items in both section A and B of the questionnaire. The exceptions were largely those items where views were so extreme, for example, on getting *pleasure* in solving problems and *willingness to try different approach*, that little change is possible. In many cases the marked distinctions seen between group N and group S students before the course is considerably lessened in the post-test. In particular, an increase in *confidence* (graphs (a)) is associated with viewing the task as a positive goal to be achieved, and decrease in *anxiety* and *fear* (graphs (b)) is associated with the diminution of the negative feeling of wanting to avoid failure (an anti-goal in Skemp's theory, 1979).

5.7. Student Comments

5.7.1. Positive and Negative Feelings about Mathematics

In the questionnaire, the students were asked to write a few sentences describing their feelings about mathematics. Opinions expressed bring to light some factors that were responsible for their attitudes towards mathematics.

5.7.1.1. Pre-test comments

The three factors that were noted among the Warwick students responses were also observed here: the nature of mathematics, personal feelings (such as motivation, interest, pressure etc.) and teaching methods. There were twenty-two responses that relate to the nature of mathematics. Seventeen of these responses were negative saying it is 'too abstract', 'seems pointless', and 'theory more difficult than practice':

> Mathematics is too abstract. It is very difficult to understand especially the mathematics that we learn at the university. The practice is OK but the theory is more difficult than practice. *SPK, year 4*

> The mathematics that I learn at the university is so alien because it is too abstract and everything seems pointless to me. Often the maths course I have taken has been both unintelligible and torturing. *SPK, year 5*

> Mathematics is full of definitions and proofs that are very abstract and complex. I find it very boring and thus did not feel like working hard enough to understand my maths courses. *SPK, year 4*

> Mathematics is a tool to help simplify calculation. It can be useful, especially in solving problems. *SPK, year 5*

> Mathematics can be fun, can be very tedious and boring. The tedious and boring parts

> make me wonder why I am doing the course. The fun part reminds me!
>
> *SSI, year 3*

Thirty-seven responses were related to personal factors. Twenty-five of these were negative, and included such phrases as 'lack of motivation', 'put off by amount of work that needs to be done' and 'puzzled by what is going on'. The positive feelings were expressed by students who found mathematics 'enjoyable and challenging', or those who obtained a 'great sense of satisfaction when able to understand new concepts and to solve problems' and who recognised that the 'effort put in is worthwhile'.

> I did not enjoy many of the maths courses. This was probably due to a lack of self-motivation and perseverance. I have problems motivating myself to sit down and work. I feel put off by the amount that needs to be done. *SPK, year 5*

> [Mathematics] is quite difficult. I feel very frustrated when I try hard and spend a lot of time only to find I still understand very little. *SPK, year 4*

> I enjoy the challenge that maths gives and have great pleasure when I get correct answers. ...I feel a great sense of satisfaction when I am able to understand new concepts and solve problems. *SSI, year 3*

> I enjoy maths most of the time. After many attempts, the satisfaction of a correct answer is very rewarding. I feel all the effort put in is worthwhile.
>
> *SPK, year 5*

Twenty responses relating to teaching were all negative—such as 'difficult to follow' 'delivered in a dull atmosphere' and 'atmosphere not conducive to questioning':

> I quite enjoy maths, especially when I can understand it. However, at the pace it is taught, it is often very confusing and difficult to follow. ...I am feeling stressed!!
>
> *SSI, year 3*

> I find mathematics at the university extremely difficult partly because it is too abstract. Furthermore, the fact that it is delivered in a dull atmosphere makes it very boring. ...I

> could never encompass the whole lot. *SPK, year 5*

> I did not find any of the maths lectures exciting. The atmosphere is not conducive to questioning. ...The maths is becoming harder and harder.
>
> *SPK, year 4*

Such comments before the course indicate that a majority of the students perceived mathematics as a static and abstract discipline. To them mathematics is a very difficult subject and they are suffering cognitive strain in trying to cope with it. However there are some who are attracted by the intellectual challenge and do feel a sense of satisfaction generated by their mathematics.

Table 5.13 below shows the distribution of the students responses with respect to their grouping. Overall, in the pre-test, only 23% of group N expressed positive feelings in comparison to 68% of group S students. After the course, the proportion of positive to negative comments increased dramatically as shown in the table. (The numbers do not add up to 44 due to responses including more than one factor.)

	Group N				**Group S**			
	Positive		Negative		Positive		Negative	
	Pre	**Post**	Pre	**Post**	Pre	**Post**	Pre	Post
nature of mathematics	2	**10**	10	**5**	3	**9**	7	**2**
personal	3	**14**	17	**3**	9	**14**	8	**2**
teaching	0	**0**	12	**10**	0	**0**	8	**8**

Table 5.13: Classification of written responses

5.7.1.2. Post-test comments

Comments written after the course indicate a different outlook that many students had to the mathematics they are studying. They feel that they can cope with the mathematical courses, become more interested in their mathematics and are impressed by the intellectual content.

> Maths has always given me a lot of problems because I don't have the ability for

> memorisation. I spent a lot of time remembering the formulas and algorithms. The abstract nature of mathematics is just above me! Now that I know about mathematical thinking, my interest and desire to learn maths have increased. *SPK, year 4*

> I find many aspects of mathematics challenging. I think it trains the brain to think in a logical and structured way. This is the first time that I have actually used maths to think. Before I just learnt maths to pass the exam. *SPK, year 4*

> I am beginning to think instead of just doing the tutorial questions. Mathematics is not merely computation as I had believed. A lot of effort is required before a solution method becomes apparent in solving a problem. I think I am learning more because I understand what is going on. *SSI, year 3*

> It's getting a lot easier now that I'm prepared to put a lot of work in. Hence I'm finally getting confident at what I'm doing. It also seems to be more enjoyable the more I work at it. *SSI, year 3*

However, there are some who had reservation about the course. Although they appreciate the problem solving course, they feel that the course on its own is insufficient.

> I am basically studying maths to get a degree. The way maths is taught here, it seems as though it is difficult and boring. There is no opportunity to display one's creativity. This makes it real dull and frustrating. Mathematics teaching definitely needs changing. This course is one of the few I've been genuinely interested in at the university.
>
> *SPK, year 5*

> The course should have been introduced earlier. It is useful in developing a logical and rigorous approach to problem-solving. After following the course, I am more confident to solve any maths problem that is given. ...I feel many of these aspects are not encouraged in the maths courses. *SPK, year 5*

5.8. Semi-structured Interviews

Six groups of students representing the different types of students following the problem-solving course were chosen for interviews. Four groups had three students and the other two groups had four students working together. They had worked together in their respective groups since the beginning of the course. Table 5.14 shows the groups of students selected for the interview. All groups were present for the interview.

	Students	Course	Degree Classification	Gender	Group
group 1	Sam	5 SPK	II-1	M	S
	Abel	4 SPK	II-2	M	S
	Henry	4 SPK	II-1	M	S
group 2	Sue	4 SPK	I	F	S
	Teresa	4 SPK	II-1	F	S
	Sasha	5 SPK	II-1	F	S
group 3	Rob	3 SSI	II-1	M	S
	Kline	3 SSI	II-1	M	S
	Ian	3 SSI	I	M	S
group 4	Hanna	5 SPK	II-1	F	N
	Katy	5 SPK	I	F	N
	Terry	5 SPK	I	M	N
group 5	Bob	5 SPK	II-2	M	N
	Yvonne	5 SPK	II-1	F	N
	Alma	4 SPK	II-1	F	N
	Pauline	5 SPK	II-2	F	N
group 6	Matt	5 SPK	II-1	M	N
	Al	4 SPK	II-2	M	N
	Holmes	5 SPK	III	M	N
	Ricky	5 SPK	II-2	M	N

Table 5.14: The 6 groups of students selected for interview

Each group was invited at an appointed time for the session that lasted 40 minutes. The first 10 minutes served as a *relaxing* phase whereby the students were simply asked to talk about their mathematical experience at the university. For the next 30 minutes, they were given a problem to work on. The problem given was as follows:

> A man lost on the Nullarbor Plain in Australia hears a train whistle due west of him. He cannot see the train but he knows that it runs on a very long, very straight track. His only chance to avoid perishing from thirst is to reach the track before the train has passed. Assuming that he and the train both travel at constant speeds, in which direction should he walk?
>
> *Mason, Burton & Stacey, 1982, p. 183.*

It was thought that the problem was easy enough for students to understand and is challenging enough for them to work on. After being presented with the problem the students were left entirely on their own and their attempts in solving the problem were observed without any intervention. The interview focused on the extent of the students interpretation of their problem-solving experience.

The interview data provided some evidence of qualitatively different thinking between the two groups of students (i.e. N and S). For instance, the following from during the entry phase indicates the difference in their mathematical understanding. Group 6 students started from the misconception that constant speed was relative to the man and train and thus both move at same speed. They quickly agreed with the meaning and no further reference was made to their interpretation of 'constant speed' until the end.

> MATT: ...constant speed.
>
> AL: It means the same I think.
>
> HOLMES: Constant speed ..., it's the same.
>
> MATT: Uniform...
>
> AL: It means the man moves with the train at the same speed. Now OK...
>
> *group 6 (N)*

In contrast, students in group 1 spent a few moments establishing the meaning of 'constant speed' and finally agreed it mean that both train and man were moving at different speeds.

> ABEL: Constant speed...
>
> HENRY: The speed of the train must be the same.
>
> SAM: It is not the same.
>
> HENRY: Constant.
>
> SAM: Constant means it does not increase or decrease.
>
> ABEL: ...the train travels say at 40 mph, Ali [the man] 4 mph. Ali will always [travel] at 4, the train always at 40. That is constant speed.

SAM: I agree.

HENRY: Hmm...

ABEL: It is not the same speed but constant speed. Ali can be faster than the train...

SAM: Ali and the train do not move the same, not at the same speed. But at their respective speeds ...the same speed all the time.

group 1 (S)

During the problem-solving, it could be seen that 3 of the 6 groups (2 with group N students) followed the techniques taught in the problem solving course very rigidly. The 2 groups of N students seem to be doing it more religiously than the one group of S students. They were more concerned on covering each phase in a sequence and so they could be seen to be working procedurally throughout. They interpreted the problem-solving technique as a procedure that they have to follow step by step, it was as if they believed that precision in following each phase would guarantee them a solution. Most of their time was spent looking for formulas that could be used.

PAULINE: We have already <u>understood the question</u>. We have i<u>ntroduced</u> what we want, what we know. We have done that. OK now we can enter <u>attack</u> [phase].

...

YVONNE: What is the formula?

BOB: speed times time.

YVONNE: The time is the same. Speed is ...

ALMA: We need to define speed first.

BOB: I should remember how to do this. ...Oh yes! speed is distance divided by time.

PAULINE: Now the distance, we don't know how much right? The distance between the man and the train.

...

BOB: Let us assume the speed of the train is 100, the man 10.

...

ALMA: OK we did some <u>specialising</u>...

group 5 (N)

MATT: So, first we go to the entry phase

...

AL: OK. That is what we know. Now what we want is the direction in which the man should go.

MATT: Anybody feel stuck or anything. The question is clear isn't it?

...

RICKY: The concept of intersection. That is what we can say.

HOLMES: The intersection point is the place the man has to go.

MATT: OK, now we go to the attack phase.

group 6 (N)

ROB: We are stuck at this point.

KLINE: Stuck. OK write down we are stuck.

IAN: Let's go back to what we want. What we want is the direction in which the man should walk. Direction, the man should go ...west, east...

...

ROB: We are confident our assumption is correct so far. OK now we enter attack phase.

group 3 (S)

Other groups were more involved in considering plausible ways to solve the problem, creating their own solution method.

TERESA: ...The train is moving to the west.

SASHA: Where does the train come from?

SUE: That is the problem. That is the one that we want to find out, it relates to the direction we want to go.

SASHA: Hmm ...We are stuck!

SUE If we know from where [the train is coming], we can find out where we want to go.

TERESA: Suppose we look at it this way. First say the man is here [pointing to a point on her paper]. Now we define where is his east, his west...

SASHA: OK. Let's draw another diagram.

group 2 (S)

HANNA: We are wasting our time... What I know, the question says, the train is moving towards west. So the man must go towards west as well.

TERRY: No! The question does not say the train is going west. But heard [the whistle] due west of him.

KATY: Yeah, that is my understanding too. The man heard the train whistle due west of him. But this does not mean that the train is moving towards west. We cannot make that conclusion.

...

TERRY: How do we know from the whistle that the train is moving west or east... What is your reasoning?

...

TERRY: OK, that's it. So we conjecture that the man should walk to the north. I think we have a solution to the problem. But we are not finished yet, we need to justify this conjecture first.

group 4 (N)

SAM: OK, so we conjecture that the train is moving towards west. That is according to your understanding. But I have another suggestion. To me...we go back to entry phase OK?

...

ABEL: OK, I got it.

HENRY: No, no, no. Hang on. The train is moving to the west. ...But why should the man walk in this direction [pointing to a point on the diagram]? Why do you say that?

SAM: It is like this. Now this is just my idea. ...Say the man is here [drawing another diagram]. ...So he cannot go this way, otherwise he will be moving parallel to the track and may never reach the train.

HENRY: So according to what you say, the direction the man should walk is this one, to the north. OK, we can conjecture that.

ABEL: We have now answered the question. Now we want to justify it [the conjecture] whether it is correct or not.

group 1 (S)

4 of the 6 groups (1, 2, 3 and 4) gave some evidence that they are able of carrying out the mathematical processes to some extent. They show that they are capable in

making judgements on the content and in making mathematical decisions for themselves. They also question the meaning of the task.

> The problem is very challenging. It does not require a specific formula or procedure that you have to apply to solve it. It is quite difficult. We got an idea what the answer is but to proof it is the hardest part. *group 1 (S)*

> The problems in the problem-solving course are interesting. Like this one. We have to think, work out what we want, what we do know before we actually work out what we don't know. ...The course is beneficial. It makes us sit down and see where to start.
>
> *group 3 (S)*

> We only managed to understand the question better towards the end of the discussion time. But I think we can solve the problem if we have more time. It is not difficult, but to generalise and to prove is very difficult. ...We will keep on thinking about it until we get the answer. *group 2 (S)*

However, the other two groups (5 and 6) have the notion that mathematical problems consists of direct application of facts and procedures. They lack the ability to bring their mathematical knowledge to bear on the problem. On the other hand, it is likely that they do not understand much of their mathematics. Hence they have little confidence that they can carry out the essential computation and to reason things out. To groups 5 and 6 students the problem given is very difficult.

> We found it [the problem] very difficult. We are unsure of which formulas or methods to use. Even if we got a solution, we don't know whether our solution is right. ...Unlike problems in the problem-solving course, most of the problems in maths course are simply applications of a ready rule. There is always a definite answer at the end.
>
> *group 6 (N)*

> We tried to generate few possible ideas. But we felt a bit put off because we couldn't recall the formulas. ...The problems are totally different from those in maths course. In maths we always know what method to use. Here we have to find it out ourselves. ...I think we have more confidence now. Before the [problem-solving] course we probably would have given up very easily.
>
> *group 5 (N)*

As in Warwick, although none of the groups could provide a complete solution to the problem within the time limit, they were at least able to tackle the problem to make a start. The fear in tackling new problems no longer appears too threatening. Four of the 6 groups (1, 2, 3 and 4) think they can solve the problem given more time, although based on their responses this may involve a lot more effort than they thought.

5.9. Discussion

One important point that the students learned in the course is that not everything they do has to be right. If it was they would fear making a conjecture, fear in trying to start and so they may not be able to solve any real problems. Although it is essential to get the right answer by the end of of the process, it is evident that after the course, the students now see that it is how they obtain an answer which is more important; looking for the right methods and reasons.

One may argue that in mathematics learning, there are rules and facts that need to be memorised. Mathematical concepts can be abstract entities and one needs to work hard to achieve insights of the underlying ideas. However at UTM as well as Warwick, these aspects of mathematics are seen by students as things that put them off mathematics. It is suggested that the students' distaste towards the subject is due to their narrow view of mathematics as a fixed body of knowledge to be learnt. It is almost self-evident that memory is important for doing mathematics. As pointed out by Byers & Erlwanger (1985) memory plays an important role in the understanding of mathematics. However, they suggest that it is what is remembered and how it is remembered that distinguishes those who understand from those who do not.

The diminishing of fear and anxiety are related to Skemp's idea of avoiding failure and the increase in confidence means seeing the task more as a goal to be achieved. However, from the observation it may be suggested that the students think they have got pleasure when they do something that gives them some feeling of satisfaction. It seems that doing things procedurally for them is not an anti-goal as suggested by Skemp. To some of the students it is a goal, but it is the wrong kind of goal.

Although the majority of students showed that they are capable of carrying out the various processes of mathematical thinking and engage actively in problem-solving, the interviews emphasise that there are differences in the quality of the students' thinking. For instance some of the group N students, when faced with a problem appear to be more concerned about recalling and applying learned techniques to solve the problem rather than looking for insights, methods and reasons. Perhaps their contextual understanding of mathematical concepts is limited. Thus they lack confidence in carrying out the mathematical performance. However, their reaction to the given mathematical problem gives an indication that they see the problem-solving knowledge as just another procedure. As noted earlier (see chapter 2) the difference in attitudes had been identified by Poincaré (1913). Although students do not understand the mathematics, it does not deter them from pursuing their mathematical studies. Furthermore the students do not feel any loss by their lack of understanding; a system which merely assesses the products of learning allows them to be successful.

5.10. Chapter Summary

This chapter reports on the effects of a problem-solving course on students' attitudes. To established what was considered a positive change, mathematics staff's opinions on expected and preferred attitudes they thought their students would or should possess as a result of their mathematics teaching were considered.

It was found that although lecturers prefer students to have a range of positive attitudes, they expect the reality to be different. They prefer students to see mathematics as *solving problems*, within a framework that *makes sense*, they wish students to be seen *working hard*, they should also be able *to relate ideas*, without needing to *learn through memory*, so that they have *confidence*, and derive *pleasure* from the mathematics. Students should have little *anxiety* and *fear*, be ready to *try a different approach* and be unwilling to *give up* easily when faced with difficulties. On the other hand, they expect students to see mathematics as *abstract*, not able to *understand it quickly*, and although they believe that students may not be *making sense* of the mathematics, they believe students will be *working hard* to learn *facts and procedures* through *memory*. The tutors do not believe students will be able *to relate ideas*, and because they lack *confidence*, they will gain little *pleasure*, as they work *only to get through* the course, and thus experience *anxiety*, and *fear*, seeking only *correct answers*, but ready to *give up* on difficult problems.

The change in response from what lecturers expect students would do to what they prefer students to be was used to establish a “desired direction of attitudinal change”. The evidence shows that the teaching of problem-solving had caused attitudinal changes in the desired direction.

Before the course, the majority of the students perceived mathematics as merely facts and procedures. They have the notion that the mathematics must be learned by memorising. The majority also reported anxiety, fear of new problems and lack confidence. Responses following the course indicate that the students’ views changes dramatically. Virtually all measures investigated improved positively—students attitudinal changes are in the same direction as the desired change. Problem-solving helps them to say that mathematics is not simply a body of procedures to be learned by memorising them, it is also a process of thinking. The majority of students are now more confident, take pleasure in getting solutions, have lower anxiety, are willing to try a new approach without giving up too easily, and see mathematics more than getting right answers. The students’ comments in the post-test are consistent with the classroom observations and the changes intimated by the questionnaire, supporting the hypothesis that the course in problem-solving changes the students’ attitudes towards mathematics and problem solving in a direction desired by the staff.

Differences between students, namely N and S students were noted. Before the course, group N students display many negative qualities. In contrast, group S students has a majority of positive responses. Responses after the course give an indication that in many cases the marked distinction between group N and S is considerably lessened. The findings indicate that students for whom mathematics made little sense have gained greater insight into its underlying qualities. The interviews gave further evidence that 4 of 6 groups of students are able to carry out the mathematical processes. However, it also highlights the difference in students' quality of thinking. The group S students (and 1 group N students) were more involved in looking for insights, methods and reasons. In contrast two of the three group of N students interpret the problem-solving knowledge as a procedure that they have to follow. While problem-solving, their emphasis is on applying learned techniques or ready rules to the task. They were using a procedural method and were not truly doing problem-solving. Their recorded discussion gave an indication of the way they do their mathematics; in a procedural and a non conceptual way.

It is seen that the teaching of problem-solving has the effect of changing students' attitudes to that preferred by lecturers. Following this what is of interest to us is to find out what will happen after a period of exposure to standard mathematics. Would the positive attitudes likely to be maintain? One may conjecture that not all of these are likely to be maintain after six months. In the next chapter we attempt to find answers to these questions.

6. SIX MONTHS LATER: THE POST POST-TEST

6.1. Introduction

The students who had attended the problem-solving course and participated in the study all passed their first semester exams. They carried on into the second semester of their studies following the regular university mathematics courses taught by staff from the Mathematics Department. In Chapter 5 we saw that the problem-solving course had a positive effect on the students' attitudes toward mathematics and problem-solving—the attitudinal change is towards what the lecturers prefer. But does the shift in attitude remain after a lapse of time? It is believed that mathematical thinking and positive attitudes need a long time for their formation. The students had just begun to develop their mathematical thinking and build their confidence to solve problems for themselves. Are these aspects further encouraged and reinforced in these students? Given the nature of the mathematical practise at the UTM, we conjecture that, on return to regular mathematics courses students will revert to previous attitudes before the course.

This chapter considers the changes in students attitudes six months after returning to standard mathematics lecturing. As seen within Chapter 5, the changes will be placed within the context of the staff's desired change. Within the current chapter, we will see how the attitudes of the students changed identified through the triple series of responses to the questionnaire, that is before the course, after the course, and after six months of standard mathematics courses. Data from the questionnaire is supplemented by interviews with selected students.

The sample and the hypotheses of the study are presented in section 6.2. Section 6.3 outlines the method of the study used. Section 6.4 will present an analysis of students' responses to the "post post" questionnaire distributed. Section 6.5 considers students change in attitudes after problem-solving and after mathematics lectures. The results show that in almost every case during the regular mathematics the students's attitudes turned back again towards what the lecturers expected, in the opposite direction from that desired. This comparison leads to an analysis of the results obtained from students within N and S groups (section 6.5.2). Section 6.6 focuses on students comments obtained through written comments and informal interviews. Issues arising from the study are considered in section 6.7 whilst a chapter summary is presented in section 6.8.

6.2. The Study

6.2.1. The Sample

The students who had followed the problem-solving course were invited to take part in the delayed post-test. All 44 subjects under observation completed the "post post" questionnaire. The six groups of students who were interviewed in the post-test were also invited to attend further interviews held informally. All six groups attended the session.

6.2.2. The Hypotheses

We have seen that problem-solving can provide a basis for attitudes to mathematics which reflected those desired by staff. The change in the students response from before problem-solving to after is in the same direction as the staff preferences. By the same token, because the change in attitudes developed during problem-solving is one which, it is suggested the regular courses do not achieve, we conjecture that, on return students would revert to previous attitudes. More particularly it was hypothesised that:

- After the problem-solving course between the post-test and the delayed post-test, whilst the students are doing mathematics, the students change in general would be in the opposite direction from that desired by the staff.

6.3. The Method

Three weeks before the end of the second semester (i.e. six months after the problem-solving course) all students under study were invited to repeat their responses to the questionnaire. The questionnaire used was the same as the one that they had filled out earlier but with a minor change in the open statement in part A; the students were requested to give some comments on the mathematics that they were doing as they see it then.

It was planned that the researcher would again re-interview the students who had taken part in the first interviews given after the problem-solving course. The purpose of the re-interview was to elicit further evidence of thinking mathematically while solving a problem during a return to regular mathematics. Unfortunately the choice of time was not appropriate. Most of the students mentioned that they were feeling rather stressed and under pressure as the final exam was just a couple of weeks away. Therefore it is felt that they would be thinking mathematically under a strain. Nevertheless the students indicated that they were willing to talk informally about their mathematical experience since the problem-solving course. Thus the interviews were modified to an informal discussion. Instead of doing problem-solving students were simply asked to talk about their views of mathematics as they currently saw it. They were also invited to talk about their mathematical activity which they had experienced for the last six months. The informal interviews were held during the students' lunch break. Students met in the same groups as they had done for the post-test interviews.

6.4. Students Post Post-test Results

6.4.1. Post Post-test Responses to Mathematics

Table 6.1 shows the data of the 44 responses to part A of the questionnaire. The table is constructed in a similar way to table 5.4.

Mathematics	Yes	(Y)	No	(N)	—
facts and procedures	30	(9)	14	(1)	0
solving problems	32	(22)	12	(0)	0
inventing new ideas	24	(4)	18	(1)	2
abstract	22	(7)	21	(0)	1
understand quickly	16	(2)	26	(1)	2
make sense	29	(4)	14	(0)	1
work hard	32	(8)	12	(1)	0
learn by memory	20	(2)	22	(1)	2
able to relate ideas	31	(5)	10	(0)	3

Table 6.1: 44 responses to part A of the post post-test questionnaire

One of the most striking feature of the table is the statement that provoked strong positive responses. In particular, on the belief that mathematics is about *solving problems*, half of the students expressed a strong yes opinion. Indeed the vast majority (73%) gave positive responses to this item. There was no stronger response on any other item measured.

Also it was noticed that:

- a high proportion (66%) believe that mathematics *makes sense* to them,
- the majority (70%), indicated that they had the ability to *relate mathematical ideas* learned.

This implies that the students have sustained some of the positive attitudes. On the other hand:

- the majority (68%) agreed with the notion that mathematics is a series of *facts and procedures* that they need to remember,

- nearly half (48%) saw mathematics as very *abstract*,
- a high proportion, (64%), still do not *understand* mathematics quickly,
- the great majority (73%) indicated that they had to *work hard* to understand the mathematics,
- almost half (45%) gave indication that it is necessary just to *learn by memory*.

This show that the positive changes identified in the post-test, negative aspects of mathematics, became less evident.

6.4.2. Post Post-test Responses to Problem Solving

The data for part B of the questionnaire is shown in Table 6.2 below, constructed in a similar format to table 6.1.

Solving Problem	Yes	(Y)	No	(N)	—
confident	34	(7)	10	(0)	0
pleasure	42	(21)	1	(0)	1
only to get through	14	(1)	30	(5)	0
anxiety	9	(0)	33	(4)	2
fear unexpected	16	(3)	28	(2)	0
correct answers	17	(0)	25	(7)	2
try different approach	43	(16)	1	(0)	0
give up	9	(0)	33	(12)	2

Table 6.2: 44 responses to part B of the post post-test questionnaire

Looking at the table briefly we observed that on solving problems:

- the vast majority (77%) claimed they have *confidence* in their ability to solve mathematical problems,
- nearly all (95%) indicate that they will take *pleasure* from solving mathematical problems with half of the students expressing a strong opinion,
- almost all (98%) students would *try a different approach* with a third saying a definite yes.

It was also noticed that students reacted against most of the negative items measured which reflect a positive attitude. In particular:

- a high proportion (66%) reacted against doing mathematics only *to get through* the course,
- a great majority (75%) do not feel *anxious* when asked to solve problems,
- a high proportion (64%) have no *fear* of new problems,
- more than half (52%) claimed they do not place importance on *correct answers*,
- a vast majority (77%) do not *give up* easily with more than a quarter expressing a strong no.

The responses indicate that the majority of the students displayed positive attitudes towards problem solving even after a six month time span. In the next section we shall attempt to discover how the students' attitudes changed after a semester of regular mathematics lectures.

6.5. Pre-test, Post-test and Post Post-test Comparisons

6.5.1. The Change in Student Attitudes in Problem-Solving and Mathematics Lectures

Students' responses to the questionnaire are taken together to discover how their attitudes changed before the course, after the course, and after six months of standard mathematics lectures. Responses are given in table 6.3.

		Before P S			After P S			After Maths		
		Yes (Y)	No (N)	–	Yes (Y)	No (N)	–	Yes (Y)	No (N)	–
Mathematics	facts and procedures	34 (18)	8 (2)	2	11 (3)	32 (8)	1	30 (9)	14 (1)	0
	solving problems	27 (10)	16 (4)	1	42 (21)	0 (0)	2	32 (22)	12 (0)	0
	inventing new ideas	21 (4)	21 (6)	2	37 (15)	5 (0)	2	24 (4)	18 (1)	2
	abstract	25 (13)	17 (0)	2	15 (8)	27 (3)	2	22 (7)	21 (0)	1
	understand quickly	9 (0)	30 (5)	5	20 (3)	21 (2)	3	16 (2)	26 (1)	2
	make sense	22 (4)	22 (5)	0	35 (5)	7 (0)	2	29 (4)	14 (0)	1
	work hard	37 (15)	5 (1)	2	28 (8)	13 (0)	3	32 (8)	12 (1)	0
	learn by memory	30 (1)	12 (2)	2	11 (0)	31 (7)	2	20 (2)	22 (1)	2
	able to relate ideas	24 (8)	18 (2)	2	35 (11)	8 (0)	1	31 (5)	10 (0)	3
Problem Solving	confidence	26 (7)	17 (2)	1	36 (12)	6 (0)	2	34 (7)	10 (0)	0
	pleasure	43 (25)	1 (1)	0	42 (21)	0 (0)	2	42 (21)	1 (0)	1
	only to get through	16 (4)	27 (8)	1	4 (0)	37 (10)	3	14 (1)	30 (5)	0
	anxiety	17 (1)	24 (4)	3	6 (0)	36 (9)	2	9 (0)	33 (4)	2
	fear unexpected	30 (10)	12 (3)	2	10 (3)	31 (9)	3	16 (3)	28 (2)	0
	correct answers	21 (4)	21 (3)	2	5 (1)	36 (11)	3	17 (0)	25 (7)	2
	try different approach	42 (17)	0 (0)	2	43 (20)	0 (0)	1	43 (16)	1 (0)	0
	give up	19 (3)	24 (9)	1	5 (0)	37 (20)	2	9 (0)	33 (12)	2

Table 6.3: 44 students responses to the attitude items before and after problem-solving and after mathematics lectures

The numerical values given in table 6.3, a composite of results previously considered, need to be considers in conjunction with table 6.4 which consider whether or not changes immediately after the problem solving course and six months later are significant.

The lecturers' "desired direction of attitudinal change" and data from the students allow us to make a comparison: in particular, between the staff's desired change and the actual changes occuring in students' indication during the problem-solving course and during a return to regular mathematics teaching. Table 6.4 shows the data. Calculating the weighted average response and computing the significance in the change of responses, we find the following changes:

		desired change		After P S		After Maths		Total change	
Mathematics	facts and procedures	$\downarrow^{+++}_{+}$	<1%	$\downarrow^{++}_{--}$	<1%	$\uparrow^{++}_{--}$	<1%	$\downarrow^{++}_{++}$	n.s.
	solving problems	$\uparrow^{+++}_{+++}$	n.s.	$\uparrow^{+++}_{+}$	<1%	$\downarrow^{+++}_{++}$	<5%	$\uparrow^{++}_{+}$	<5%
	inventing new ideas	$\uparrow^{+}_{-}$	n.s.	$\uparrow^{+++}_{-}$	<1%	$\downarrow^{+++}_{+}$	<1%	$\uparrow^{+}_{-}$	n.s.
	very abstract	$\downarrow^{+++}_{--}$	<1%	$\downarrow^{++}_{-}$	<1%	$\uparrow^{+}_{-}$	<5%	$\downarrow^{++}_{+}$	n.s.
	understand quickly	$\uparrow^{+}_{---}$	<1%	$\uparrow^{+}_{--}$	<1%	$\downarrow^{+}_{-}$	n.s.	$\uparrow^{-}_{--}$	n.s.
	make sense	$\uparrow^{++}_{-}$	<1%	$\uparrow^{++}_{-}$	<1%	$\downarrow^{++}_{+}$	<1%	$\uparrow^{+}_{-}$	<5%
	work hard	$\downarrow^{+++}_{++}$	<1%	$\downarrow^{+++}_{++}$	<1%	$\uparrow^{++}_{++}$	n.s.	$\downarrow^{+++}_{++}$	n.s.
	learn by memory	$\downarrow^{++}_{---}$	<1%	$\downarrow^{+}_{--}$	<1%	$\uparrow^{+}_{--}$	<1%	$\downarrow^{+}_{+}$	n.s.
	able to relate ideas	$\uparrow^{+++}_{--}$	<1%	$\uparrow^{++}_{+}$	<1%	$\downarrow^{++}_{++}$	n.s.	$\uparrow^{++}_{+}$	<5%
Problem Solving	confidence	$\uparrow^{+++}_{-}$	<1%	$\uparrow^{++}_{+}$	<1%	$\downarrow^{++}_{++}$	n.s.	$\uparrow^{++}_{+}$	<5%
	pleasure	$\uparrow^{+++}_{+}$	<5%	$\boxed{\downarrow^{+++}_{+++}}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.
	only to get through	$\downarrow^{+++}_{-}$	<1%	$\downarrow^{-}_{--}$	<1%	$\uparrow^{-}_{--}$	<1%	$\downarrow^{-}_{-}$	n.s.
	anxiety	$\downarrow^{++}_{---}$	<1%	$\downarrow^{-}_{--}$	<1%	$\boxed{\downarrow^{--}_{--}}$	n.s.	$\downarrow^{-}_{--}$	<5%
	fear unexpected	$\downarrow^{++}_{--}$	<1%	$\downarrow^{++}_{--}$	<1%	$\uparrow^{-}_{--}$	<5%	$\downarrow^{++}_{-}$	<1%
	correct answers	$\downarrow^{++}_{-}$	<1%	$\downarrow^{+}_{--}$	<1%	$\uparrow^{-}_{--}$	<1%	$\downarrow^{+}_{-}$	n.s.
	try different approach	$\uparrow^{+++}_{+}$	<1%	$\uparrow^{+++}_{+++}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.	$\uparrow^{+++}_{+++}$	n.s.
	give up	$\downarrow^{++}_{--}$	<1%	$\downarrow^{-}_{---}$	<1%	$\uparrow^{--}_{---}$	<5%	$\downarrow^{-}_{--}$	<5%

Table 6.4 : Desired changes compared with changes after problem-solving and after mathematics lectures

We have seen in chapter 5 that during problem-solving the changes in attitudes amongst the students were in the direction desired by the lecturers. In contrast, all but one of the changes during the mathematics lectures are in the *opposite* direction. Even the exception—*anxiety*—has an increase in those feeling anxious from 6 to 9, but the weighted average is biased marginally in the opposite direction by the drop in "definitely not anxious" from 9 to 5 students (boxed in table 6.4).

It is noticeable that after returning to the mathematics course many opinions have reverted back in the old position. That is, students attitudes turned back again towards what the lecturers expected and away from what they desired. Nevertheless, comparing the situation from before the problem-solving course with that after six months back at regular mathematics courses (total change), we see that 7 of 17 items remain changed significantly. Of these there is a highly significant reversal in indication that students *fear the unexpected*, and a significant reversal in indication that mathematics *makes sense*. Whilst the notion that mathematics is about *solving problems*, students are *able to relate ideas*, that they have *confidence*, have *low anxiety*, and *unwillingness to give up* easily remain significantly improved.

Now why do the students move in the opposite direction from what the teachers desire? It is possible to conjecture what causes this change in attitudes based on the Warwick experience. It could be due to the more demanding nature of the mathematics that students are doing. The problems faced become so hard compared with the "easier" problems in the problem-solving course that students feel they can no longer operate in any way other than rote-learning the material for presentations in the exams. As mentioned by the following student:

> Clearly it is better if students understand the mathematics they have studied. But in practice this is difficult to attain because of the nature of the subject and time constraint. Often many students find it is not always possible to understand the course so they may choose instead to memorise the syllabus and solutions to the example sheets. ...It is perfectly possible to gain a good grade by rote-learning.
>
> *Warwick Maths Student, year 3*

But, it may also be due to the different attitudes of lecturers being sensed by students. We shall attempt to investigate this further in the next chapter.

6.5.2. Comparisons between Groups N and S Students Change in Attitudes

In Chapter 5 it emerged that there were some differences noted amongst the students: N and S students. The post-test findings showed that as a result of the problem-solving course the marked distinctions seen between groups N and S students declined. In the next section we attempt to analyse the students' responses according to this grouping to look for possible differences that may re-appear after a delayed period of six months.

6.5.2.1. Attitudes to mathematics

Table 6.5 shows the responses made by the 22 students in group N (for whom mathematics does not make sense) on all three occasions (before and after the course and after 6 months) to part A of the questionnaire. This table needs to be considered together with table 6.6 which considers students' attitudinal change.

Mathematics	Before P S			After P S			After Maths		
	Yes (Y)	No (N)	–	Yes (Y)	No (N)	–	Yes (Y)	No (N)	–
facts and procedures	20 (16)	1 (1)	1	6 (2)	16 (4)	0	19 (6)	3 (1)	0
solving problems	9 (2)	13 (4)	0	21 (10)	0 (0)	1	14 (9)	8 (0)	0
inventing new ideas	6 (1)	14 (4)	2	18 (8)	2 (0)	2	8 (4)	14 (0)	0
abstract	21 (12)	1 (0)	0	11 (8)	9 (0)	2	18 (6)	4 (0)	0
understand quickly	4 (0)	13 (2)	5	8 (0)	12 (1)	2	6 (1)	15 (0)	1
make sense	0 (0)	22 (5)	0	14 (2)	7 (0)	1	7 (1)	14 (0)	1
work hard	19 (6)	1 (0)	2	15 (5)	5 (0)	2	18 (4)	4 (1)	0
learn by memory	17 (1)	3 (1)	2	6 (0)	15 (3)	1	13 (2)	7 (0)	2
able to relate ideas	6 (1)	16 (2)	0	14 (4)	7 (0)	1	11 (1)	9 (0)	2

Table 6.5: 22 group N students' responses to part A of the questionnaire before and after problem-solving and after mathematics lectures

Table 6.6 shows the desired direction of movement, the weighted average response and the significance in the change of the responses for the 22 students, calculated in a similar way to that of table 6.4.

Mathematics		After P S		After Maths		Total change	
facts and procedures	$\downarrow$	$\downarrow^{+++}_{--}$	<1%	$\uparrow^{+++}_{--}$	<1%	$\downarrow^{+++}_{+++}$	n.s.
solving problems	$\uparrow$	$\uparrow^{+++}_{-}$	<1%	$\downarrow^{+++}_{++}$	<5%	$\uparrow^{++}_{-}$	<5%
inventing new ideas	$\uparrow$	$\uparrow^{+++}_{--}$	<1%	$\downarrow^{+++}_{-}$	<1%	$\uparrow^{-}_{--}$	n.s.
abstract	$\downarrow$	$\downarrow^{+++}_{+}$	<1%	$\uparrow^{++}_{+}$	<5%	$\downarrow^{+++}_{++}$	<5%
understand quickly	$\uparrow$	$\uparrow^{-}_{-}$	n.s.	$\downarrow^{-}_{-}$	n.s.	$\uparrow^{-}_{-}$	n.s.
make sense	$\uparrow$	$\uparrow^{+}_{---}$	<1%	$\downarrow^{+}_{-}$	<1%	$\uparrow^{-}_{---}$	<1%
work hard	$\downarrow$	$\downarrow^{+++}_{++}$	<5%	$\uparrow^{++}_{++}$	n.s.	$\downarrow^{+++}_{++}$	n.s.
learn by memory	$\downarrow$	$\downarrow^{++}_{-}$	<1%	$\uparrow^{+}_{-}$	<5%	$\downarrow^{++}_{+}$	n.s.
able to relate ideas	$\uparrow$	$\uparrow^{++}_{--}$	<1%	$\downarrow^{++}_{+}$	n.s.	$\uparrow^{+}_{--}$	<5%

Table 6.6: Significant changes in group N students' responses to part A of the questionnaire after problem-solving and after mathematics lectures

In their perception of mathematics it can be seen that the vast majority of the students in group N modified both their views of mathematics (first 4 items) and their thinking about it (last 5 items) as a result of the problem-solving course.

However, six months after the end of the problem-solving course we note that things are almost back to where they were before the course. There is a highly significant reversal in indication on the belief that mathematics is composed of *facts and procedures*, that it is *not about inventing new ideas*, it *does not makes sense* and it is *learnt through memory*. Whilst the notion that mathematics is about *solving problems* declined significantly, and indications that it is *abstract* increase significantly.

When we compare the situation before the problem-solving course with the position after six months of standard mathematics lectures (total change), although students' opinions revert back on most of the indicators, there are several items that continue to receive positive acclamation. Namely, student indications that mathematics is about *solving problems*, and that they *can relate ideas* learned are significantly reversed whilst indications that mathematics is *abstract* and it *makes sense* remain significantly improved. However, students *work hard* continues to be given a high rating whilst the notion that they can *understand quickly* continues to be rejected by a majority of group N students. It appears that the students' beliefs are still strongly geared towards the procedural aspects of mathematics, working hard to memorise facts and to solve problems.

Using a similar format to table 6.5, table 6.7 compares the responses of the 22 students in group S (for whom mathematics makes sense) before and after the problem-solving course and after six months of returning to regular mathematics courses.

Mathematics	Before P S			After P S			After Maths		
	Yes (Y)	No (N)	–	Yes (Y)	No (N)	–	Yes (Y)	No (N)	–
facts and procedures	14 (2)	7 (1)	1	5 (1)	16 (4)	1	11 (3)	11 (1)	0
solving problems	18 (8)	3 (0)	1	21 (11)	0 (0)	1	18 (13)	4 (0)	0
inventing new ideas	15 (3)	7 (2)	0	19 (7)	3 (0)	0	16 (0)	4 (0)	2
abstract	4 (1)	16 (0)	2	4 (1)	18 (3)	0	4 (1)	17 (0)	1
understand quickly	5 (0)	17 (3)	0	12 (3)	9 (1)	1	10 (1)	11 (1)	1
make sense	22 (4)	0 (0)	0	21 (3)	0 (0)	1	22 (3)	0 (0)	0
work hard	18 (9)	4 (1)	0	13 (3)	8 (0)	1	14 (4)	8 (0)	0
learn by memory	13 (0)	9 (1)	0	5 (0)	16 (4)	1	7 (0)	15 (1)	0
able to relate ideas	18 (7)	4 (0)	0	21 (7)	1 (0)	0	20 (4)	1 (0)	1

Table 6.7: 22 group S students' responses to part A of the questionnairebefore and after problem-solving and after mathematics lectures

Table 6.8 shows the direction of movement and the significance in the change of responses.

Mathematics		After P S		After Maths		Total change	
facts and procedures	$\downarrow$	$\downarrow^{+}_{--}$	<1%	$\uparrow^{+}_{--}$	<5%	$\downarrow^{+}_{+}$	n.s.
solving problems	$\uparrow$	$\uparrow^{+++}_{+++}$	n.s.*	$\downarrow^{+++}_{+++}$	n.s.	$\uparrow^{+++}_{+++}$	n.s.
inventing new ideas	$\uparrow$	$\uparrow^{+++}_{+}$	<5%	$\downarrow^{+++}_{++}$	<5%	$\uparrow^{++}_{+}$	n.s.
abstract	$\downarrow$	$\downarrow^{-}_{--}$	n.s.	$\uparrow^{--}_{--}$	n.s.	$\downarrow^{-}_{--}$	n.s.
understand quickly	$\uparrow$	$\uparrow^{+}_{--}$	<5%	$\downarrow^{+}_{+}$	n.s.	$\uparrow^{+}_{--}$	<5%
make sense	$\uparrow$	$\downarrow^{+++}_{+++}$	n.s.	$\uparrow^{+++}_{+++}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.
work hard	$\downarrow$	$\downarrow^{+++}_{+}$	<1%	$\uparrow^{+}_{+}$	n.s.	$\downarrow^{+++}_{+}$	n.s.
learn by memory	$\downarrow$	$\downarrow^{+}_{--}$	<1%	$\uparrow^{-}_{--}$	n.s.	$\downarrow^{+}_{-}$	<5%
able to relate ideas	$\uparrow$	$\uparrow^{+++}_{++}$	n.s.*	$\downarrow^{+++}_{++}$	n.s.	$\uparrow^{+++}_{++}$	n.s.

Table 6.8: Significant changes in group S students' responses to part A of the questionnaire after problem-solving and after mathematics lectures

The majority of group S students have shown positive attitudes before the problem-solving course on most of the items measured and these were further improved in the desired direction after the course, except for some so extreme that little change is possible.

Six months later, after returning to the mathematics course, many of the items continue to receive positive indication. However, students' opinions on two items revert. In particular, the belief that mathematics is *facts and procedures* revert back towards their old position whilst indications that mathematics is about *inventing new ideas* decline significantly.

Comparing the situation before the problem-solving course with the status after six months of standard mathematics lecturing, it is noticeable that group S has a majority of positive responses on most of the items measured. We see that the belief that mathematics is about *solving problems*, it *makes sense*, and students are able to *relate ideas* remain highly rated. Whilst there is a significant reversal in indication that students can *understand quickly* and it is not necessary just to *learn by memory*. Nevertheless, students' indications reverted towards the lecturers' expectation on two items: mathematics is *facts and procedures* and that it is less about *inventing new ideas*. The group S students also appear to lay emphasis on procedural aspects but it may be suggested that they work hard to make sense of their mathematics and to relate ideas.

Figure 6.1 shows the students (both groups N and S) responses on all items in part A of the questionnaire presented pictorially. They include all the three occasions—before the problem solving course (**before**), after the course (**after**) and six months later after one semester of regular mathematics courses (**6 mth**). The bar-charts indicate the percentage of students giving positive (Y+y) response. They were taken out of the full sample of 22 students for each group. The percentages of non-responses ("no opinion") for the 6 months stage are shown under each graph.

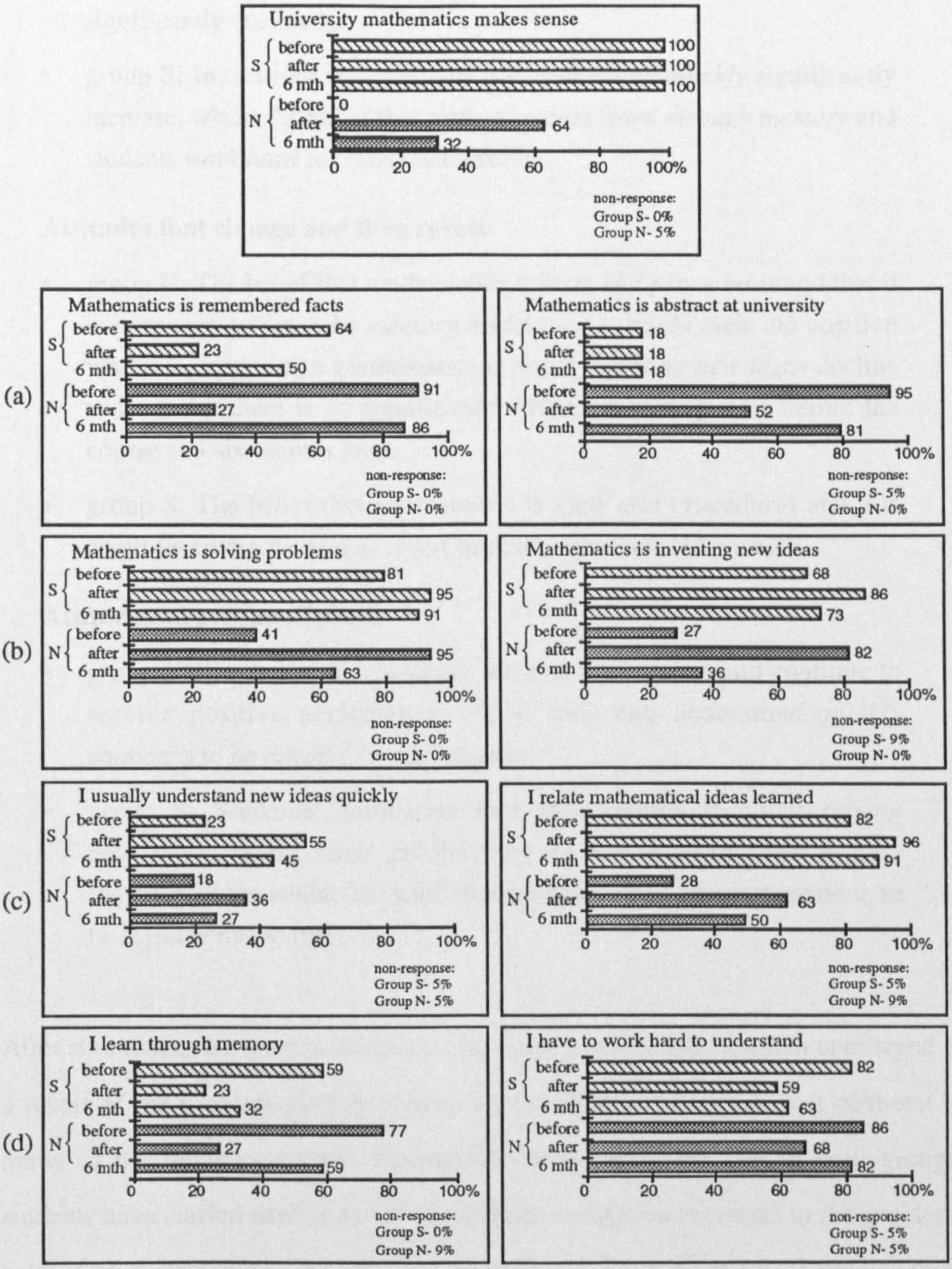


Figure 6.1: Students' attitudes to Mathematics after doing problem-solving

Supported by the significance test in tables 6.6 and 6.8, it was observed that:

Attitudes that change and remain changed

- group N: Indications that mathematics *makes sense*, that it is about *solving problems*, and that students are able to *relate ideas* learned, increase significantly whilst the indication that mathematics is *abstract* significantly declined .

- group S: Indications that students can *understand quickly* significantly increase, whilst the belief that mathematics is *learn through memory* and students *work hard* to understand decline.

Attitudes that change and then revert

- group N: The belief that mathematics is *facts and procedures* and that it is necessary to *learn by memory* revert back towards their old position whilst indication that mathematics is about *inventing new ideas* decline to the point there is no significance difference in responses before the course and six months later.
- group S: The belief that mathematics is *facts and procedures* and it is about *inventing new ideas* revert back towards their old position.

Attitudes largely unaffected

- group N: The belief that students *work hard* to understand continue to receive positive acclamation whilst they can *understand quickly* continues to be rejected by the majority.
- group S: Students' indication that mathematics is about *solving problems*, it *makes sense* and that they are able to *relate ideas* learned remain positive whilst the belief that mathematics is *abstract* continue to be rejected by the majority.

After six months the pre-test distinction between group N and S, which converged as a result of the problem-solving course, appears again during the post post-test in many of the items measured. Nevertheless it is noticeable that in both groups, students have carried over some of the attributes which were central to the problem-solving course objectives. For group N in particular, the students have sustained three problem-solving attributes: mathematics *makes sense*, it is about *solving problems* and that they are able to *relate ideas* learned. Thus we may suggest that the distinction between the two groups remain less compared with that seen before the course, although on all other items there is still a marked difference between them.

6.5.2.2. Attitudes to problem solving

Using the same format as table 6.5 above, table 6.9 and 6.11 show students' responses to part B of the questionnaire for group N and S respectively on all three occasions.

Problem Solving	Before P S			After P S			After Maths		
	Yes (Y)	No (N)	–	Yes (Y)	No (N)	–	Yes (Y)	No (N)	–
confidence	9 (1)	13 (1)	0	16 (3)	5 (0)	1	14 (2)	8 (0)	0
pleasure	21 (11)	1 (1)	0	21 (9)	0 (0)	1	20 (10)	1 (0)	1
only to get through	14 (4)	8 (2)	0	4 (0)	16 (2)	2	12 (1)	10 (1)	0
anxiety	12 (1)	8 (1)	2	6 (0)	15 (1)	1	7 (0)	14 (2)	1
fear unexpected	17 (7)	3 (1)	2	6 (2)	14 (2)	2	10 (1)	12 (1)	0
correct answers	13 (3)	8 (2)	1	2 (0)	19 (5)	1	11 (0)	9 (4)	2
try different approach	21 (8)	0 (0)	1	21 (9)	0 (0)	1	21 (9)	1 (0)	0
give up	13 (3)	8 (3)	1	3 (0)	19 (8)	0	6 (0)	15 (6)	1

Table 6.9: 22 group N students' responses to part B of the questionnaire before and after problem-solving and after mathematics lectures

Table 6.10 is constructed in a similar way to table 6.6. It shows the weighted average response with the direction of movement and the significance changes of responses for the 22 students in group N.

Problem Solving		After P S		After Maths		Total change	
confidence	$\uparrow$	$\uparrow^{++}_{-}$	<1%	$\downarrow^{++}_{+}$	n.s.	$\uparrow^{+}_{-}$	n.s.
pleasure	$\uparrow$	$\uparrow^{+++}_{+++}$	n.s.	$\uparrow^{+++}_{+++}$	n.s.	$\uparrow^{+++}_{+++}$	n.s.
only to get through	$\downarrow$	$\downarrow^{+}_{--}$	<1%	$\uparrow^{+}_{--}$	<1%	$\downarrow^{+}_{+}$	n.s.
anxiety	$\downarrow$	$\downarrow^{+}_{-}$	<5%	$\uparrow^{-}_{-}$	n.s.	$\downarrow^{+}_{-}$	n.s.
fear	$\downarrow$	$\downarrow^{++}_{-}$	<1%	$\uparrow^{-}_{-}$	n.s.	$\downarrow^{++}_{-}$	<1%
correct answers	$\downarrow$	$\downarrow^{+}_{---}$	<1%	$\uparrow^{-}_{---}$	<1%	$\downarrow^{+}_{-}$	n.s.
try different approach	$\uparrow$	$\uparrow^{+++}_{+++}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.
give up	$\downarrow$	$\downarrow^{+}_{---}$	<1%	$\uparrow^{--}_{---}$	n.s.	$\downarrow^{+}_{--}$	<5%

Table 6.10: Significant changes in group N students' responses to part B of the questionnaire after problem-solving and after mathematics lectures

In their perception to problem solving, we note that the attitudinal changes during the problem-solving course are all in the same direction as the desired change. There are only two changes which are not statistically significant: that students obtain great *pleasure* from solving mathematical problems and they have a willingness to *try a different approach* remain highly rated. Six months later, many opinions revert back to their old position before the course. In particular, indications that students are solving problems *only to get through* the course, and that they are placing importance on getting *correct answers* increased highly significantly.

Comparing the two situations, from before the problem-solving course with that after six months of regular mathematics lectures (total change), we see that on most items there is no significant difference overall. However, students had sustained their positive response on two of the items: indication that students *fear the unexpected* and will *give up* easily when faced with difficulty remain significantly reversed. Whilst students takes great *pleasure* from solving mathematical problems, and *willingness to try a different approach* remains given a high rating.

The following tables shows the data for group S students.

Problem Solving	**Before P S**			**After P S**			**After Maths**		
	Yes (Y)	No (N)	–	Yes (Y)	No (N)	–	Yes (Y)	No (N)	–
confidence	17 (6)	4 (1)	1	20 (9)	1 (0)	1	20 (5)	2 (0)	0
pleasure	22 (14)	0 (0)	0	21 (12)	0 (0)	1	22 (11)	0 (0)	0
only to get through	2 (0)	19 (6)	1	0 (0)	21 (8)	1	2 (0)	20 (4)	0
anxiety	5 (0)	16 (3)	1	0 (0)	21 (8)	1	2 (0)	19 (2)	1
fear unexpected	13 (3)	9 (2)	0	4 (1)	17 (7)	1	6 (2)	16 (1)	0
correct answers	8 (1)	13 (1)	1	3 (1)	17 (6)	2	6 (0)	16 (3)	0
try different approach	21 (9)	0 (0)	1	22 (11)	0 (0)	0	22 (7)	0 (0)	0
give up	6 (0)	16 (6)	0	2 (0)	18 (12)	2	3 (0)	18 (6)	1

Table 6.11: 22 group S students' responses to part B of the questionnaire before and after problem-solving and after mathematics lectures

Table 6.11 is to be considered together with table 6.12 that shows the weighted average response and the significance of attitude change for each item measured.

Problem Solving		After P S		After math		Total change	
confidence	$\uparrow$	$\uparrow^{+++}_{+++}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.	$\uparrow^{+++}_{++}$	n.s.
pleasure	$\uparrow$	$\boxed{\downarrow^{+++}_{+++}}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.
only to get through	$\downarrow$	$\downarrow^{---}_{---}$	n.s.	$\uparrow^{---}_{---}$	n.s.	$\uparrow^{---}_{---}$	n.s.
anxiety	$\downarrow$	$\downarrow^{--}_{---}$	$<5\%$	$\uparrow^{--}_{---}$	n.s.	$\downarrow^{--}_{--}$	n.s.
fear	$\downarrow$	$\downarrow^{+}_{--}$	$<1\%$	$\uparrow^{-}_{--}$	n.s.	$\downarrow^{+}_{-}$	$<5\%$
correct answers	$\downarrow$	$\downarrow^{+}_{--}$	$<5\%$	$\uparrow^{--}_{--}$	n.s.	$\downarrow^{-}_{--}$	n.s.
try different approach	$\uparrow$	$\uparrow^{+++}_{+++}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.
give up	$\downarrow$	$\downarrow^{--}_{---}$	n.s.	$\uparrow^{--}_{---}$	n.s.	$\downarrow^{--}_{--}$	n.s.

Table 6.12: Significant changes in group S students' responses to part B of the questionnaire after problem-solving and after mathematics lectures

For group S students' perception of problem solving, it can be seen from the table that most of the items are given at least ++ or -- rating before the problem-solving course. The vast majority of the students in the group had positive attitudes towards problem-solving, but they indicate *fear of the unexpected.* During problem-solving we saw that all changes are in the desired direction except one: getting *pleasure* from solving problems which was rated highly each time.

After six months back at standard mathematics lectures, many opinions remain. There are no significant changes noted on any indicators measured. When we compare the two situations, before the problem-solving course and six months later, we note that students' indication that they *fear the unexpected* remain significantly reversed. All other items still continue to receive a high rating. This indicates that for group S students tackling an unfamiliar problem no longer appears threatening compared with before the problem-solving course.

Figure 6.2 below shows the N and S students' positive responses to part B of the questionnaire on all three occasions presented pictorially. The graphs constructed in a similar format as figure 6.1 highlight the differences between the group N and S students.

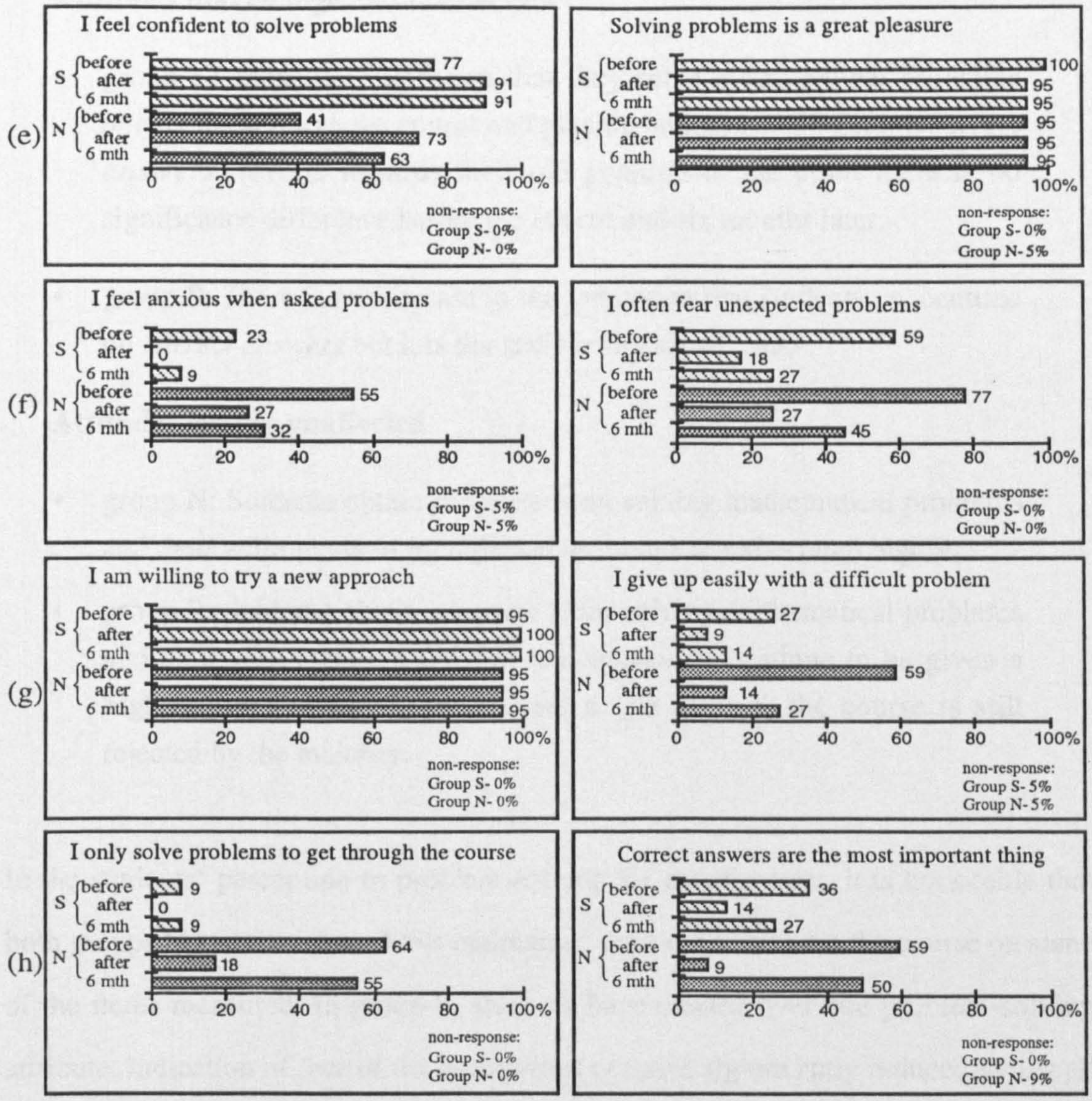


Figure 6.2: Student attitudes to problem solving after doing problem-solving

The following observation is further supported by the significance test given in tables 6.10 and 6.12:

Attitudes that change and remain changed

- group N: Students' indication of *fear* of the unexpected and *give up* easily when face with difficulties decrease significantly whilst indication of possessing *confidence* increases (not statistically significant).
- group S: There is a highly significant reduction in belief of *fear* of the unexpected. Students' indication of *confidence* in their mathematical ability continue to receive positive acclamation (not statistically significant).

Attitudes that change and then revert

- group N: Students' indication that they solve mathematical problems *only to get through* the course and placing importance on getting *correct answers*, reverts towards their old position to the point there is no significance difference before the course and six months later.

- group S: There is an increase in the indication that students concentrate on *correct answers* but it is not statistically significant.

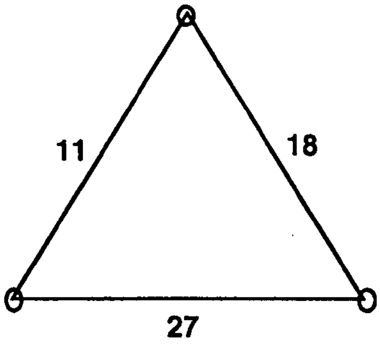
Attitudes largely unaffected

- group N: Students obtain *pleasure* from solving mathematical problems and their willingness to *try different approach* remains rated highly.
- group S: Students obtain *pleasure* from solving mathematical problems and their willingness to *try different approach* continue to be given a high rating. Solving problems *only to get through* the course is still rejected by the majority.

In the students' perception to problem-solving six months later, it is noticeable that both groups N and S students have maintained their opinions after the course on some of the items measured. In group S, students have carried over one problem-solving attribute: indication of *fear* of the unexpected remains significantly reduced whilst all other items remain rated highly. Whereas for group N, for two of the items: *fear the unexpected* and *give up* easily when faced with difficulties, six months after the problem solving course though we note an increase in the number of students indicating agreement with these items, the total change remains significant. Obtaining *pleasure* from solving problems and being willing to *try a different approach* remain rated highly. Thus it may be suggested that the distinction between group N and S remains less after six months compared with distinctions before the course.

Interestingly on the belief mathematics *make sense*, it was noticed that after the problem-solving course a high proportion of group N students claimed that mathematics make sense to them. But six months later only 7 of 22 students maintain this attitude. However for groups S they maintain a positive attitude that mathematics makes sense throughout. But what does make sense really mean to them? Looking in detail at their problem-solving performance at the end of the problem-solving course may provide some indication.

The following problem was given to them as part of the written assessment. All these students have taken a linear algebra course the semester before and all passed the exam. Thus they have the required mathematical knowledge to solve the problem:

> *A secret number is assigned to each vertex of a triangle. On each side of the triangle is written the sum of the secret numbers at its ends. Find a simple rule for revealing the secret numbers. For example, secret numbers 1, 10, 17 produce*
>
> 
>
>
> *Generalise to other polygons.*
>
> *Mason, Burton & Stacey, 1982, p. 160.*

Students have the necessary algebra to write down the three equations involving secret numbers *x*, *y*, *z* at the vertices and to show how these lead to a unique solution. However, when generalising to four secret numbers at the corners of a square, it happens that the equations are either inconsistent (usually) or have an infinite number of solutions. This did not evoke the knowledge of solving linear equations to the majority of the students in group N. Thus before they move to find that the a pentagon once more has a unique solution they were already confused. It led them to make a conjecture that odd polygons have a unique solution. But when it come to even polygons, it constitutes a problem.

Although the majority managed to formulate a correct solution, only 2 of group N students (both claimed mathematics make sense during the problem-solving course and six months later) related the problem to the content of their linear algebra course. Whereas in group S half of the students made use of their linear algebra to solve the problem. It may be suggested that the majority of group N students have not understood enough of the subject matter to see its relevance; it is conjectured that their understanding is instrumental (in the sense of Skemp, 1979). Consequently, when given a problem formulated in a context which is slightly different, they failed to make the link.

6.6. Students Comments

6.6.1. Mathematics and Problem Solving After Six Months of Standard Mathematics

The students comments were obtained from two sources. Firstly from the questionnaire where the students were requested to give some comment on the mathematics that they are doing as they see it now. Secondly through interviews with selected students. Previously selected groups of students who were interviewed during the problem-solving course were again invited to attend the second interview. However due to the students' time constraint the second interviews were made informal. The students talked about their views of mathematics and their mathematical experience since the problem-solving course.

Although the students were requested to give some comments on the mathematics that they are doing as they currently see it, almost all comments given were mixed with how the mathematics that they were doing makes them feel. The excerpt from the interviews gave further support to the written comments.

The students comments highlighted various conflicts and issues in their mathematics learning. Looking in detail at these comments may suggest several reasons for their responses to the questionnaire. The following selected comments bring to light several factors that could explain their changes in attitudes. They were mainly on the perception of mathematics, the nature of problem-solving, the nature of mathematics teaching and the role of the lecturers.

6.6.1.1. Students' perception of mathematics

In students' perception of mathematics, about a third (32%) expressed doubts over the mathematics they were doing. They were confused and felt that their problem-solving experience conflicted with what they experience in regular mathematics classes. They reported that the regular mathematics did not allow them to think in a problem-solving manner:

> Since following the course I know mathematics is about solving problems. But whatever mathematics I am doing now doesn't allow me to do all those things. There are just more things to be remembered. *male, year 5 (S)*

> I believed mathematics is useful in that it helps me to think. Having said that it is hard to say how I can do this with the maths I am doing. Most of the questions given can be solved by applying directly the procedures we had just learned. There is nothing to think about. *female, year 3 (N)*

> I find the mathematics I am doing now confusing and sometimes pointless. I wonder what the point of some courses are and why you are doing it. *male, year 4 (N)*

Excerpts from the interview supporting the above written phrases:

> "We often don't have enough time to do maths and think about what we have done or are

> doing. ...We work under pressure and often feel anxious that we can't do maths. Not because we can't do it but that we can't do it in the time given". *group 1 (S)*

> "I see mathematics as something that needs doing rather than learning whereby I should participate actively in making conjectures, constructing arguments to convince others, reflecting on my problem solving and so on. But I think the maths course at the university does not encourage this". *group 4 (N)*

6.6.1.2. The nature of mathematics teaching

Some students find themselves in a "depressing" situation due to the nature of teaching. They saw that their mathematical training was rather rigid. They felt that their lecturers gave too much emphasis to content, and on unchallenging work:

> At the moment I am finding difficulty with maths because I am just not enjoying it. Too much emphasis is put on getting the right answer and not on method and understanding
>
> *female, year 4 (S)*

> The mathematical atmosphere here is very bad; there is little discussion and it provides no encouragement to do maths. The content is emphasised over everything else. We are crammed full of lots of bland mathematical abstract theory. *male, year 3 (N)*

The interviews further suggests that they were not encouraged to do problem-solving for themselves. The opportunity to voice their own opinion or to build arguments to support their view is very limited.

> The last time I came up with my own solution method I ended up losing some marks. I tried to argue with the lecturer but she rejected it. It was frustrating really. The way things are, it is not challenging at all and lacks imagination. *group 5 (N)*

> I would prefer to work in smaller groups doing tutorial sheets together. In this way we can discuss, sort out the difficulty ourselves and to understand more fully the ideas involved. ...The maths course at the University is structured in a such a way as to encourage people to get the right answer without actually thinking. I think solving the problems requires a great deal of thought and time. *group 3 (S)*

6.6.1.3. The role of the lecturers

The students felt that the lecturers are in total control of their learning whereby they are told what to do, being told what is important etc. Due to the role played by the lecturers they accept passively the emphasis given by the lecturers. Some emphasise the way in which the lecturers move fast to complete the content:

> I did not enjoy most of the maths courses—too dependent on the lecturers. I don't find the way most of them teach particularly inspiring. We find ourselves hurrying through to keep up. There is no time to think about the mathematics we are doing.
>
> *male, year 3 (S)*

> I realise that knowledge is the cornerstone to learning, but it's by thinking and reflecting upon the experience that we build on the knowledge and learn. I think this aspect is not always encouraged by the maths course. *female, year 4 (N)*

From the interview:

> I do maths according to the lecturers' style. If he says this particular proof or theorem is not important, I'll skip them. I don't want to burden myself learning things that won't be tested. *group 6 (N)*

> The [maths] lecturers seem to take things too far too quickly. Everything comes in thick and fast. How can they [lecturers] expect us to understand something when we have not even absorbed it yet? ...We were given 3 assignments to complete in a week. We haven't got the time to think about it seriously. At the moment we are feeling stressed.
>
> *group 4 (N)*

6.6.1.4. The nature of problem-solving

However, some students do appreciate their knowledge in problem-solving, suggesting it helps them to learn their mathematics and solve problems more effectively:

> The problem solving techniques help me come to terms with the abstract nature of the maths I am doing. I try to connect the [mathematical] ideas together and talk about it with my friends. It is not that easy though. But I felt all the effort worth it when I am able to do so. *male, year 3 (S)*

> Since the last time I filled in the questionnaire, I have got a much better attitude towards mathematics. ... Certainly as a result of the experiences I had in the [problem solving] course, maths has become more accessible. I appreciate the "greater respect" given to the student in this course. *female, year 5 (N)*

> I used to solve problems using trial and error. ...I applied the [problem solving] techniques particularly in my final year project. It helps me tremendously to come up with a solution method that I think was most efficient. I would never be able to do it with such confidence without the problem solving knowledge. *male, year 5 (N)*

The following excerpts from the interview give further support to students written comments on the effectiveness of the problem solving techniques in their mathematics learning:

> In my opinion, I feel the problem solving course is invaluable in developing many qualities that undergraduates should have. For example I am better able to properly structure my attempts to solve problems. Besides, I am more emotionally and psychologically mature and confident to think mathematically. *group 3 (S)*

> I find the problem solving knowledge very useful in helping me to understand the why's and the how's of advanced mathematics. It is much more satisfying than rote-learning. Furthermore it is actually easier to remember something that you understand.
>
> *group 1 (S)*

There are some who have minor reservations on their problem-solving experience. But they believed it is necessary to have a positive attitude:

> The main disadvantage is time. It would take several hours, maybe days, to understand each new concept. Under the current circumstances we are finding ourselves rapidly hurrying to keep up. Sometimes we were so bogged down in the technical details that we end up purely taking down the notes without even concentrating. This really defeats the

> problem solving techniques. ...But I think with further support from good teaching as well as tailoring the [maths] courses to suit the needs of the students the situation can be improved. *group 2 (S)*

> It may seem difficult to apply the techniques literally to problems in the maths courses. I think it depends strongly on the people involved. In reality the best use actually can be made of them. For example, formulating the question into 'what you know' and 'what you want' can help enormously to simplify the problem rather than ploughing straight into it. *group 4 (N)*

On the other hand there were some who belong to the first group identified by Poincaré (1913): students feel something is lacking but cannot identify what is wrong. They feel they can do without it and choose not to play a more active role:

> It is hard to see the relevance of the problem-solving techniques in the maths courses. The emphasis in the exam is to get the correct answer. And it is possible to gain a good grade by rote learning the syllabus and solutions to the examples in tutorial sheets.
> *group 6 (N)*

> We have a great deal of work already, without spending extra time on solving problems that is of our own invention and won't be asked in the exams. Anyway there is no time in the exam to think around a problem to the extent required by the [problem solving] method. *group 5 (N)*

6.7. Discussion

We have seen that through problem-solving students had made a positive shift in their attitudes and had begun to develop the confidence to question, challenge and reflect. However as Mason *et al.* point out "to flourish, mathematical thinking requires not only nurture, but also extension" (p. 152). They suggest that the students' confidence must be encouraged and reinforced. "Their curiosity needs nurturing, their investigative potential structuring, their confidence maintaining" (p. 153). It is believed that what the students had experienced in the problem-solving course is not what they normally encounter at the university mathematics courses. The opportunity for them to think, to articulate their own questions, to challenge conjectures and to reflect on their problem-solving is very limited. Therefore it is conjectured that the atmosphere in the students regular mathematics courses does not support their growth of mathematical thinking. Students' expressed opinions suggest the emphasis is on the procedural aspects which can be successfully tested.

For many of the students, that is in group N in particular, it may appear that after six months many of the problem-solving attributes are greatly reduced although all is not lost. The course has certainly taught them to be confident, be willing to try and not to give up easily when faced with difficulties. The comments above provide an indication that to some students there is an improvement in their approach towards the mathematics they are studying. They are aware of what is needed for improved mathematical thinking and are prepared to accept the challenge. The problem-solving experience they had had built their self-confidence in mathematics. The majority no longer feel that tackling something new appears too threatening and, in general they want to solve problems despite the effort and the difficulties faced. On the other hand there are some students who feel that their problem-solving experience was just like any other mathematics courses—isolated and disconnected. To them it is more of a procedure that they have to follow for successful problem-solving. After six months back in regular mathematics lectures, the tendency to lay emphasis on procedural aspects remains not only among group N students but amongst some students in group S as well. It may be suggested that the change was away from being very procedural to weakly procedural (not to non-procedural).

The findings indicate that thinking in a mathematical manner is not encouraged in the university mathematics courses. The students went back to being almost submissive, to obey the rules rather than to challenge them. Being obedient is part of the Malaysia's culture and it is certainly important. However one must also learn to develop a rational judgement for there are times it is required. For instance, Skemp (1971) pointed out that to progress far in mathematics one needs to restructure some of the things previously learnt in order to accommodate new information. This means that new developments may occur only through challenging the mathematical rules. Schoenfeld (1989) in his study among 17-year old suggests that students have come to separate school mathematics—mathematics that they know in their mathematics classes—from the "abstract mathematics, the discipline of creativity, problem solving, and discovery, about which they are being told but which they have not experienced" (p. 349). Consequently, students' behaviour seems to be driven much more by their classroom experiences during problem-solving situations. Our classroom observation during the initial stage of the problem-solving course showed similar tendency amongst the students, and therefore confirms Schoenfeld's finding.

The overall rationale behind the teaching of mathematics at UTM may not be in conflict with the rationale of the problem solving course. It is what we desire but do we get it? Probably because we fear the students will not understand we teach in a way that promotes precisely Freudenthal's conjecture. That is, the only thing students can do with the ready-made mathematics is to reproduce it. One possible reason is the question of efficiency. We need to cover so much material in a limited time and the only way to do it is to push through it so far. As one of the staff commented:

> ...We gave them little room to do their own thinking. But we cannot change it because the system does not allow us to do so. ...So we end up teaching them what they need to know.

Accordingly mathematics teaching is based on mastering separate discrete facts and procedures and pays little attention to students' mathematical thinking. This phenomena reinforced Davis's (1994) assertion that:

> ...in mathematics courses, students are often given some example that they can imitate.

> The teacher does not know how the students are thinking about the work, but on the contrary tests only for successful imitation. p. 13

He points out that this may give short term success for students who are capable of imitating very complicated things but as he clearly puts it "in the long run how the students thinks about things becomes the decisive factor in performance and future learning" (ibid, p. 13).

Moreover, students who are being "crammed full of lots of bland mathematical abstract theory" eventually became deflated. It makes them wonder what the point of some courses are and why they are doing them. They are being trained to think in a static and rather specialised manner. As the findings indicate, it appears that the mathematics teaching has led students to view mathematics as a collection of facts and procedures that they need to memorise to remember. Students pick up the rhetoric but not the substance in their mathematics learning (Schoenfeld, 1989). Thus they find mathematics to be a very difficult subject.

6.8. Chapter Summary

This chapter considered students' changes in attitudes after six months on return to normal mathematics courses. This allows a comparison to be made between the staff's desired change and the actual changes occurring in the students during problem-solving and during a return to regular mathematics teaching. It was observed that when students are doing the problem-solving course almost all the changes were in the desired direction. However, when returning to mathematics lectures, in almost every case the change in response were in the reverse direction. The data from the questionnaire supplemented by interviews with selected students supports the hypothesis that while students are doing normal mathematics courses, their attitudes turned back towards the more negative lecturers' expectation rather than towards their more positive desires.

Comparing the situation from before the problem-solving course with the status after six months back at regular mathematics lectures, many indicators revert back towards their old position. Although some problem-solving attributes remain, students' emphasis is on the procedural aspects; working hard to solve problems and relate ideas to obtain pleasure and low anxiety. It seems that doing things procedurally is not an anti-goal (in the sense of Skemp). To the students it is a goal. It may be suggested that the change was from being very procedural to weakly procedural. Opinions expressed suggest concern that the quantity and difficulty of the mathematics gives them little room for creative thinking. Thus it appears that the problem-solving course is successful in changing students' attitudes mainly on a short term basis.

Does the mathematical environment provided really get the students to think in a way that lecturers want them to? Or does the material produced end up making them teach the subject matter as a purely rote-learnt taught course? It is likely that students' mathematical thinking was not reinforced and the positive attitudes were not further encouraged in the standard mathematics course. It is within our interest to see why there is such an attitudinal change in terms of staff attitudes and differences between staff and students perceptions. We shall attempt to do this in the next chapter.

7. LECTURERS' PERCEPTION OF STUDENTS' MATHEMATICAL THINKING

7.1. Introduction

A common classical perception of students' difficulty in learning mathematics was due to their lack of ability or unwillingness to work, rather than the laying of blame on the lecturers who lectured or the difficulty of the mathematical concepts. However, if we consider the position from a constructivist viewpoint, we are a product of what we have done, we are a product of our experience, of our reflection and the abstraction of concepts. Lecturers have built vast mathematical knowledge over the years through their daily teaching and research and they have obtained corresponding mathematical qualifications (through the award of MSc or PhD). It is clear that they have a very different perception from a student. Therefore it is unfair solely to blame the students for their lack of learning. As Tall (1991) puts it:

> It is therefore no longer viable, if indeed it ever was, to lay the burden of failure of our students on their supposed stupidity, when now the reasons behind their difficulties may be seen to be in part due to the epistemological nature of mathematics and in part to misconceptions by mathematicians of how students learn.
>
> p. 252

At the UTM, the majority of students face great difficulty in their mathematics learning. As observed earlier during the problem-solving course, the students showed little of the intellectual independence that we desire them to posses. Instead they tend to view mathematics as a collection of facts and consequently rote-learn most of them. However, after following the course the majority modified their opinions. They now saw mathematics as a thinking process and showed they are capable of thinking mathematically. During the regular mathematics courses which followed however, many students' opinions reverted back to where they were before the course. There are several possible reasons for this. The students' comments give a clear indication that they were not given the time and the opportunity to make the mathematics their own. They are expected to understand the information delivered even when they have not absorbed it. Thus one may conjecture that the environment provides very little encouragement for students to think mathematically nor is it supportive in reinforcing their positive attitudes built during the problem-solving course.

The purpose of this chapter is to investigate the attitudes of the mathematics lecturers which may help explain students' attitudinal change. During the investigation the lecturers were very supportive and helpful. They were very concerned with the general observation—which is not only true in UTM but all over the world—that the current method of teaching through lecturing is failing (Eisenberg, 1991; Cornu, 1991; Artigue, 1991; Schoenfeld, 1994; Selden, Mason & Selden, 1994). These studies suggest that students find great difficulty in constructing their own understanding of the mathematical concepts. Accordingly they do not have the ability to adapt their mathematical performance to varied circumstances.

In this chapter we shall attempt to analyse lecturers' perception of their students thinking about mathematics. We expected that there would be differences between the lecturers and the students' perceptions towards the mathematics lectures given. Section 7.2 will present the sample (7.2.1) and the hypotheses (7.2.2) of the study. The method of the study will be outlined in section 7.3. This is followed by the analysis of the results (section 7.4). Section 7.5 considers the discussion on issues that arise whilst section 7.6 give the summary of the chapter.

7.2. The Study

7.2.1. The Sample

Eight of 22 lecturers were selected for further investigation particularly on their views about mathematics and their mathematics teaching. They were chosen mainly because they have some or even all of the students who had followed the problem-solving course (in the previous semester) in the regular mathematics classes that they are currently conducting.

7.2.2. The Hypotheses

Lecturers prefer students to think positively but perhaps over the years they perceived these things are not happening and so teach procedurally to help students pass through the exams. We have seen that students' positive change in attitude is short-lived because the kind of mathematics they do does not encourage them to think mathematically. Additionally it is hypothesised here there is a difference in lecturers and students perceptions of the purpose of a mathematics lecture. More particularly:

- there would be a mis-match between lecturers and students' perceptions of the focus of a mathematics lecture.

7.3. The Method

The eight selected lecturers were invited to fill in one other questionnaire concerning their belief about mathematics (Section 1) and their perception of the lectures they are giving (Section 2). The questionnaire was given during one of their lecture classes. The lecturers filled it in just before their lecture began. They were to respond by indicating what they expected the students would think of the focus of the lecture given. Section 2 of the questionnaire was also distributed to the students who were attending the lectures given by the lecturers. Students were invited to respond by indicating what they thought the focus of the lecture was. They were requested to fill out the questionnaire immediately after the lecture ended

All eight selected lecturers were also interviewed. The purpose of the interview was to elicit their views about mathematics and mathematics teaching at the university.

7.4. Analysis of Results of Selected Lecturers

7.4.1. Lecturers Responses to the Students' Attitudinal Questionnaire

Table 7.1 below shows the responses of the eight selected lecturers and the other fourteen lecturers' response to the students' attitudinal questionnaire.

		Lecturers (n=14)						Selected lecturers (n=8)					
		Yes		No		—		Yes		No		—	
		think	prefer	think	prefer	think	prefer	think	prefer	think	prefer	think	prefer
Mathematics	facts and procedures	13	8	1	6	0	0	7	5	1	3	0	0
	solving problems	11	14	3	0	0	0	8	8	0	0	0	0
	inventing new ideas	6	7	8	7	0	0	2	4	6	4	0	0
	abstract	14	5	0	9	0	0	6	2	2	6	0	0
	understand quickly	3	9	11	5	0	0	0	6	8	2	0	0
	make sense	5	12	9	2	0	0	3	7	5	1	0	0
	work hard	14	11	0	3	0	0	7	7	1	1	0	0
	learn by memory	9	1	5	13	0	0	6	1	2	7	0	0
	able to relate ideas	3	14	11	0	0	0	2	8	6	0	0	0
Solving Problem	confidence	7	14	7	0	0	0	3	0	5	8	0	0
	pleasure	8	14	6	0	0	0	7	7	1	1	0	0
	only to get through	13	3	1	11	0	0	8	4	0	4	0	0
	anxiety	11	1	3	13	0	0	5	0	3	8	0	0
	fear unexpected	10	3	4	11	0	0	5	0	3	8	0	0
	correct answers	12	5	2	9	0	0	7	1	1	7	0	0
	try different approach	8	14	6	0	5	0	4	8	4	0	0	0
	give up	10	2	4	12	0	0	6	0	2	8	0	0

Table 7.1: The 8 selected lecturers and the other 14 lecturers' responses to the questionnaire

It can be seen that the responses of the eight selected lecturers seem very much similar to those of the other 14 lecturers in the department. Using the t-test it was found that there was no significant difference at the 5% level between the selected 8 lecturers and the other 14 lecturers.

7.4.2. Selected Lecturers' Responses to Perception Questionnaire

Here we attempt to analyse the selected lecturers and students' responses to the perception questionnaire. We shall show that there exists a mismatch between the lecturers' expectation of what students would perceive the lectures they are giving was all about and what is actually conceived by the students. This would be particularly true of those items that characterise mathematical thinking.

7.4.2.1. Lecturers' responses to Section 1: Perception of Mathematics

Table 7.2 shows the lecturers respond to the statement "I believe mathematics is...".

I believe mathematics is ...	Yes	Y	No	N	—
a formal deductive system	8	3	0	0	0
a theoretical knowledge	8	4	0	0	0
a highly developed mental tool	7	1	1	0	0
about solving problems	7	3	1	0	0
a discipline of the mind	5	2	3	0	0
about inventing new ideas	4	1	4	0	0

Table 7.2: Eight lecturers responses on views to mathematics

The most striking feature of the table is that all 8 lecturers gave positive responses on items that identify with the formalist viewpoint of mathematics, in particular, mathematics as a *deductive system* with 3 of them stating their view strongly (a "definitely yes") and it is a *theoretical knowledge* with 4 of 8 staff expressing strong opinion. However, on items that identify with the notion mathematics as a mental activity, namely as a *discipline of the mind* and it involves the *invention of new ideas* attracts positive responses from 4 of 8 and 5 of 8 staff respectively.

Nearly all identify with the belief that mathematics is about *solving problems* and it is a *highly developed mental tool.* Although nearly all the lecturers view mathematics as solving problems, only 4 of them agree simultaneously with the notion that it is a *discipline of the mind* and that it involves the *invention of new ideas.*

7.4.2.2. Lecturers' responses to Section 2: Mathematics Teaching

Table 7.3 shows the responses of the eight selected lecturers perception of students' thinking about the lectures they are giving.

Mathematics	Y	y	n	N	—
making sense of concepts	2	6	0	0	0
work hard to understand	2	6	0	0	0
inventing new ideas	0	1	4	1	2
relating ideas	3	4	1	0	0
facts and procedures	2	5	0	0	1
applying mathematical concepts	2	5	0	0	1
getting through the course	3	5	0	0	0
develop confidence	2	5	0	0	1
develop own way of solving	1	2	4	0	1

Table 7.3: Eight lecturers perception of students' thinking about their lectures

The table reveals that the lecturers expect students to respond positively on all but two of the items measured. The exceptions are: the *invention of new ideas*, and students formulate *own solution methods* which receive 3 or less positive response from the staff. It is likely that the lecturers may have a different interpretation in meaning of ideas express in the questionnaire from the ideas of "mathematical thinking" in the problem-solving course. On the other hand, although they see mathematics as a mental activity the subject matter may not be taught as such.

Next we shall consider the perspectives of the students to the lecture they had attended. The data collected would then be compared with the lecturers' expectation.

7.4.3 Students Responses to Section 2 of the Questionnaire: Perception of Given Lecture

Students' responses to what they perceived the lecture they had just attended is all about for each lecture are tabulated within relevant parts of the ensuing discussion. The asterisk '*' represent the lecturers' expectation of students perceptions on each of the items. Only the responses of those students who had attended the problem-solving course given in the previous semester are taken into account. The number of students attending each lecturers course is given in each table.

Zoe

She sees mathematics as a formally *deductive system*, with *theoretical knowledge* and it is about *solving problems*. She expected students would give positive responses on all the items measured except on the belief it is teaching them about *inventing new ideas*, and *relating ideas* together. She expected strongly that students would perceive the lecture as learning a section of the material *only to get through the course* and getting them to *develop their own way* of solving problems. However, she has no opinion that it would be perceived as developing *confidence*.

Zoe (n=30)	Y	y	n	N	—
make sense of concepts	0	4*	19	4	3
work hard to understand	3	21*	4	0	2
inventing new ideas	2	7	17*	2	2
relating ideas	0	6	20*	3	1
facts and procedures	5	21*	3	0	1
applying concepts	0	2*	23	3	2
get through the course	5*	21	2	0	2
develop confidence	0	6	16	7	1*
develop own way of solving	1*	5	19	4	1

Table 7.4: Thirty students' perception of Zoe's mathematics lecture

From the students' perspective, the majority give positive responses on items that identify mathematics as a fixed body of knowledge: students have to *work hard* to understand the ideas, it is teaching them *facts and procedures*, and learning the material to *get through the course* as expected by the lecturer. However items that promotes mathematical thinking, in particular it is about *making sense* of ideas, learning to *apply concepts*, and formulating their *own solution methods* receive low positive responses from the students, thus not matching the lecturer's expectation. Nevertheless, as expected by the lecturer, the majority of students do not think the mathematics lecture is about *inventing new ideas* and *relating ideas* together. The majority of the students also do not think it is developing their *confidence,* an item which the lecturer have no opinion.

Nelly

Nelly (n=9)	Y	y	n	N	—
making sense of concepts	3	6*	0	0	0
work hard to understand	4	5*	0	0	0
inventing ideas	0	3	6	0	0*
relating ideas	0	2*	7	0	0
facts and procedures	2	5	2	0	0*
applying concepts	0	1	8	0	0*
getting through the course	2	6*	1	2	0
develop confidence	0	2*	4	3	0
develop own way of solving	0	2	7	0	0*

Table 7.5: Nine students' perception of Nelly's mathematics lecture

Nelly has the notion that mathematics is a *formal deductive* system; it is *theoretical knowledge*, as well as *a highly developed mental tool* and it is a *discipline of the mind.* She expressed no opinion of what the students would perceive her lecture to be on 4 of 9 items measured. In particular, whether it is seen as about the *invention of new ideas*, learning *facts and procedures*, learning *to apply concepts* or is about *developing own solution* methods. However, students do have opinions about each of these items. A high proportion of the students do not perceive the lecture is about *inventing new ideas*, nor the *application of concepts* or that it is about developing their *own solution methods* but it is about learning *facts and procedures*. Nelly did expect students to see her lecture as an opportunity for students to *relate ideas* together and developing *confidence*, aspects which students did not gain from the lecture.

The majority of the students perceived that during the lecture they were able to *make sense* of the mathematical concepts, and *work hard* to understand and learning the material *to get through the course.* Nelly expected them to see the lecture this way.

Hazel

Hazel (n=18)	Y	y	n	N	—
making sense of concepts	0	1*	9	8	0
work hard to understand	11	7*	0	0	0
inventing new ideas	3	8	5*	0	2
relating ideas	0	1*	9	8	0
facts and procedures	8	10*	0	0	0
applying concepts	0	2*	8	8	0
getting through the course	8	9*	1	0	0
develop confidence	0	2*	8	8	0
develop own way of solving	2	11	3*	0	2

Table 7.6: Eighteen students' perceptions of Hazel's mathematics lecture

Hazel believes mathematics is both a *formal system* as well as a *mental process.* She expect the students to perceive the lecture as all of the items measured except on two items: it is about *inventing new ideas* and students *develop own way* of solving problems. In contrast, these two items receive a high proportion of positive responses from the students. The majority of the students also do not think the lecture is about *making sense* of the mathematical concepts, it is not seen as about *relating ideas* together, it is neither about the *application* of mathematical concepts nor is it about developing their *confidence.*

The lecturer's expectation and the students' perception do intersect on the belief it is about learning *facts and procedures* and learning the material *to get through the course.*

Alfred

Alfred believes mathematics is both a *formal system* as well as a *mental process.* He expect students' perception of his lecture as all of the items measured but not on the belief it is teaching students to *invent new ideas* and getting them to *develop their own ways* of solving problems. Indeed the perception of the majority of the students does match the lecturer's expectation.

Alfred (n=10)	Y	y	n	N	—
making sense of concepts	1	9*	0	0	0
work hard to understand	1	8*	1	0	0
inventing ideas	0	2	8*	0	0
relating ideas	1	9*	0	0	0
facts and procedures	2	8*	0	0	0
applying concepts	2	7*	1	0	0
getting through the course	2	8*	0	0	0
develop confidence	0	7*	3	0	0
develop own way of solving	0	0	10*	0	0

Table 7.7: Ten students' perceptions of Alfred's mathematics lecture

Mary

Mary believes that mathematics is both a *formal system* and a *mental process.* She expected students to give positive responses on all of the items measured except two: it is about *inventing new ideas* and getting students *develop own way* of solving problems.

Mary(n=20)	Y	y	n	N	—
making sense of concepts	10*	10	0	0	0
work hard to understand	14*	4	2	0	0
inventing new ideas	3	3	9	5*	0
relating ideas	0*	2	11	7	0
facts and procedures	12*	6	2	0	0
applying concepts	2*	4	10	4	0
getting through the course	9*	8	3	0	0
develop confidence	1	3*	6	10	0
develop own way of solving	2	3	12*	3	0

Table 7.8: Twenty students' perceptions of Mary's mathematics lecture

Mary expressed a strong "no" to indication that students would perceived her lecture to be about *inventing new ideas.* In contrast, about one third of the students give a positive response to this item. Note also the low positive responses received on three other items: students would see it as about *relating ideas* together, learning to *apply concepts* and developing *confidence,* away from Mary's expectation. Nevertheless, students' perceptions do match the lecturer's expectation on items which promotes mathematics as a body of knowledge to be learned, namely about *making sense* of the mathematical concepts, students have to *work hard* to understand, learning *facts and procedures* and learning the material *to get through the course.*

Sammy

To Sammy mathematics is a formally *deductive system,* a *theoretical knowledge*, a *highly developed mental tool* and it is about *solving problems*. He expected students to perceive his lecture to reflect all of the items measured except as that specifying that mathematics was about *inventing new ideas* on which he had no opinion; indeed the majority of the students did not think the lecture reflected this quality.

Sammy (n=40)	Y	y	n	N	—
making sense of concepts	1	4*	25	10	0
work hard to understand	13	17*	8	0	2
inventing new ideas	2	4	27	7	0*
relating ideas	0	5*	24	11	0
facts and procedures	17	15*	7	1	0
applying concepts	1	10*	13	16	0
getting through the course	18	19*	3	0	0
develop confidence	0	4*	24	12	0
develop own way of solving	1	6*	24	9	0

Table 7.9: Forty students' perceptions of Sammy's mathematics lecture

The majority do agree upon items that stress the content aspect of mathematics and its extrinsic pressures. Particularly, students have to *work hard* to understand, it is about learning *facts and procedures* and learning the material *to get through the course* as expected by the lecturer. However, items that identify mathematics as a mental activity: *making sense* of the concepts, *relating ideas* together, *applying concepts*, developing *confidence* and *developing own way* of solving problems, attracts low positive responses from the students, a distinct difference from the lecturer's expectation.

Sony

Sony perceived mathematics as both a *formal system* and a *mental process*. He expected the students would respond positively on all the items measured. Indeed, the majority of his students displayed the perception that he anticipated.

Sony (22 students)	Y	y	n	N	—
making sense of concepts	8	10*	4	0	0
work hard to understand	4	14*	4	0	0
inventing ideas	1	15*	6	0	0
relating ideas	4*	11	7	0	0
facts and procedures	11	10*	1	0	0
applying concepts	3	13*	6	0	0
getting through the course	2	15*	4	0	1
develop confidence	2*	17	3	0	0
develop own way of solving	1	14*	6	1	0

Table 7.10: Twenty-two students' perceptions of Sony's mathematics lecture

Sandy

To Sandy mathematics is a formal *deductive system*, it is a *theoretical knowledge*, a *highly developed mental tools* and it is about *solving problems*. She expects that her students would perceived her lecture as all items measured except two: it is about *inventing new ideas* and about developing their *own solution methods*.

Sandy (n=44)	Y	y	n	N	—
making sense of concepts	1*	1	24	18	0
work hard to understand	21*	20	3	0	0
inventing new ideas	4	26	11*	3	0
relating ideas	7*	30	7	0	0
facts and procedures	14*	23	7	0	0
applying concepts	11*	28	5	0	0
getting through the course	14*	29	1	0	0
develop confidence	1*	4	25	14	0
develop own way of solving	3	28	10*	3	0

Table 7.11: Forty-four students' perceptions of Sandy's mathematics lecture

However, note the high proportion of positive responses given on these 2 items. On all other items the students' thinking almost matched her expectation but the majority of the students do not think the lecture is about *making sense* of mathematical concepts and it is not about developing their *confidence*, a contrast with Sandy's expectation.

It appears that items which identify with the notion that mathematics is a body of knowledge to be learned, in particular students *work hard* to understand, learning *facts and procedures*, and doing the mathematics *to get through the course* the students do behave in a way that the lecturers expected. But on other items that emphasise mathematical thinking processes it is noticeable that there exist various mismatches between the students' perception and the lecturers' expectation. In many instances the students perceived the opposite from what was expected.

Figure 7.1 below shows the bar-chart of students' responses to the lectures they had attended. The asterisk '*' beside the bar-charts indicates the lecturer's expectation of students' perception of the lecture they are giving. The ticks at the items indicate that the lecturers' and students perceptions do coincide. Whilst the shading on the graphs represent the category of response:

■ definitely yes, □ yes, ▨ no, ▩ definitely no.

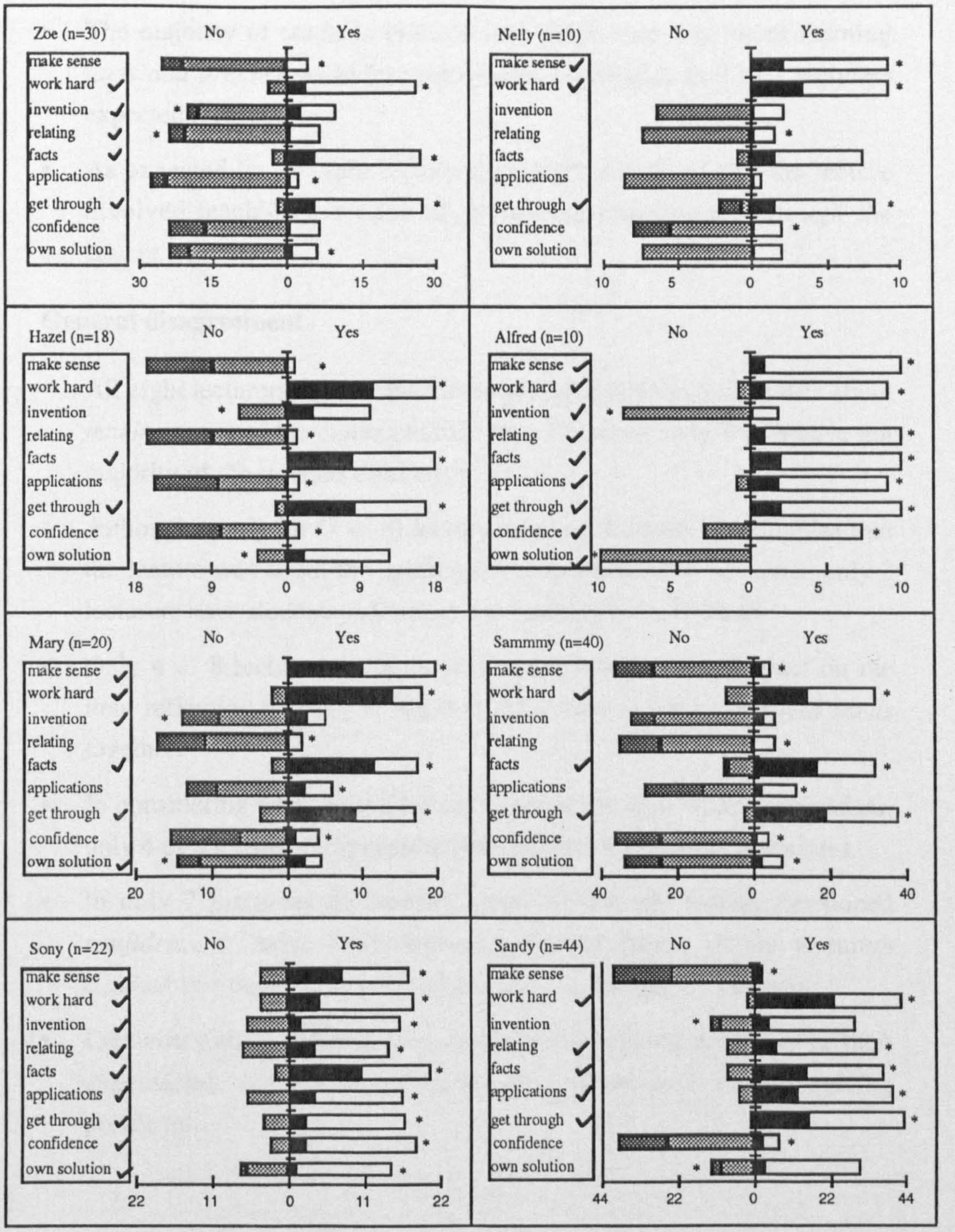


Figure 7.1: Students' perception to mathematics lecture after doing problem-solving

Comparison between lecturers and students responses reveal that:

General agreement

- As expected by all eight lecturers the majority of the students indicated that they needed to *work hard* to understand the mathematical concepts presented in the lecture.

- The majority of students indicate that the lecture was about learning *facts and procedures* to be remembered for exams, as 7 of 8 lecturers expected them to.
- As expected by all eight lecturers, students perceived that the lecture involved teaching a section of the lecture notes *to get through the course.*

General disagreement

- All eight lecturers expect students to perceive that the lecture was about *making sense* of the mathematical ideas. However, only 4 of 8 have got majority of the students thinking it is so.
- Although nearly all (7 of 8) lecturers expect students to recognise that the lecture was about the *application of mathematical concepts*, only 3 lecturers have students indicating it is teaching them to do so.
- Only 4 of 8 lecturers have students responding as they expect on the item reflecting whether or not it is about *relating mathematical ideas* together.
- In considering whether the lecture is about the *invention of new ideas*, only 4 of 8 lecturers' expectation are matched by students' responses.
- In only 2 instances do students perceive that the lecture developed *confidence* to solve mathematical problems. Seven of the lecturers claimed that their lectures would provide this quality for students.
- Likewise only 2 lecturers have students' responding according to their expectation that the lecture is about creating *own way of solving* problems.

It can be seen that some of the lecturers' responses do match the students' perceptions in the way they expected, in particular on items that specify mathematics as a body of procedures to be learned. However on items that request thinking in a mathematical manner there exists a mismatch between the lecturers' expectation and the students' perception. The difference in attitudes of the lecturers is being sensed by the students. That is from the students' viewpoint, the mathematics teaching stressed more on content and mechanistic answer finding, and placed less emphasis on the mathematical processes. *This provides an indication that the mathematics teaching does not give support to the students' growth of mathematical thinking, in the way it is encouraged in the problem-solving course.*

But one may also suggest that the lecturers have no clear view of their students' mathematical thinking. Therefore they teach in a way they expect students would treat mathematics, for example in wanting to get through the course and the task of obtaining correct answers as the most important thing. Their expectation places more importance on correct performance rather on the students' ability to think. Thus we see the students behaving as expected of them. Students appear to be successful, obtaining a 1st or a II-1 degree classification in the previous semester, yet half of them claim the mathematics they are studying does not make sense to them.

Additionally, it may be suggested that the lecturers do not share the students' understanding of the problem-solving techniques. They were not conscious of their own use of the problem-solving processes that they were using. Although they did not make or draw explicit attention to these processes, the experience students have from the problem-solving course enables them to make the judgement.

7.4.4. Individual Interviews with Selected Lecturers

Interviews with all eight selected lecturers supplement the data from the questionnaires.

It was suspected earlier that the lecturers' perception of problem-solving may differ from the one adopted in this study. The interviews revealed that there are substantial differences in meaning of ideas expressed in the questionnaire from the ideas of "mathematical thinking" in the problem-solving course. Stanic & Kilpatrick (1988) had identified that there are three different perceptions of problem solving—as means to a focused end, as a skill and as an art. It soon become apparent that the lecturers see it more as a means to achieve a specific end or a skill to be learned by the students. It is not considered as the art of thinking mathematically in our context. For instance, "inventing new ideas" was perceived as ideas which are new to everybody. They do not mean new to the individual as suggested by the following lecturer:

> To me mathematics is the tool in solving problems. One way of motivating the students is by showing the applications in the real world. In this way they get the knowledge and the skills in solving the problems. ...I do not think the students are capable of creating new ideas on their own. *Sandy*

Lecturers are not certain of the problem-solving techniques used in the course. They were not consciously aware of the mathematical thinking processes themselves and consequently did not encourage the students to think. It may be due to their own mathematical training which has been entirely content oriented. They claim to have little idea of what the processes of doing mathematics really are or how they function:

> ...I am not sure of these [processes]. I have not thought about them and I don't know how to go about [teaching] them. I think I need to learn more about these before I can implement them. We developed certain abilities to look at problems but we are not sure how those abilities came to be with you. *Zoe*

> The experience of making conjectures, generalising and the like I think the students can get themselves on their own, from doing their project work. We do not have the time to teach them everything. *Sandy*

Instead they show students how to do examples in the hope that they will develop their own techniques:

> We tell them how to do it. For example, what are the criteria that should be fulfilled in the formula before they can use it. Normally I explain only part of it then I think the students can complete it themselves. ...I think that is sufficient for the students.
> *Sammy*

> Under the circumstances, I expect the students to acquire the mathematical skills and to get high marks in the exam. ...I would want them to become good problem solvers but I am not sure they would be. I myself did not try to get them into becoming one consciously. *Mary*

There are some who are aware of the mathematical thinking processes. They genuinely want to change the system but are not sure how to do so. As observed by Lerman (1989a) mathematics educators feel reluctant to make any changes or to move away from the tradition probably because they have no clear idea of how to teach them. Additionally, it could be due to the predictable difficulty in restructuring their existing schemas (Skemp, 1989).

> ...To me mathematics is a mental activity but I should say that at this level I presented it more as a formal system. Because we are confined by the syllabus and also depending on the students background. ...I would like it to change. How do I do that? I don't know. *Nelly*

> [Mathematics] involves a lot of thinking and perseverance. ...I would like students not only see mathematics as a subject that they need to learn and pass in exam but also as a discipline which enables them to think for themselves. My main aim is not in trying to finish the syllabus but rather in making the students learn the mathematics in a more meaningful way. ...I am not really sure how but I am trying to do it. *Sony*

On the other hand, there are some who have little belief in students' ability to think for themselves and so teach them accordingly. Perhaps as a result of many years experience what they expect falls short of what they desire. Thus they provide students with the mathematical experience directed by the way in which they expect students to behave rather than as lecturers desire.

> There are a lot of problems that we face. Firstly the students themselves do not have the motivation in their mathematics learning. Secondly they do not have the confidence in their ability to do mathematics. So we have to deal with these first before we can make them see mathematics as a thinking subject. *Mary*

> The education culture in Malaysia is bounded by politics. ...Reflecting on my own learning it was definitely not like the way I am teaching the students. We gave them little room to do their own thinking. But we cannot change it because the system does not allow us to do so. ...So we end up teaching them what they need to know. *Sammy*

> At a lower level problem-solving enhances understanding of mathematics. It helps them to understand how the concepts are related to real life. At a higher level it is more a question of applications of mathematics. ...I very rarely allow students to think [mathematically]. The problems that we give them do not require them to use their thinking capability. ...It is due to shortness of time. *Hazel*

They are some who feel they are in a time warp which is a difficult thing for them to get out of. They know fairly well that the mathematical experience which is provided is based on following a sequential thinking process is of no value to students other than that of getting through the course.

> I think that the tradition of teaching mathematics not only in UTM but in Malaysia as a whole may need changing. The system has been proven a failure. It has not been successful in producing good mathematicians, or engineers that can use mathematics effectively. They only know how to use procedures or computer packages without really understanding why they use them. ...It's all down to the system. We are not training students to discover patterns, or how to prove a statement is true for example. What we teach them is mainly how to use the procedures. *Alfred*

> When we learn mathematics we are actually learning history, what has been done, things that are already there obviously. But at the same time mathematics is not perfect. So we should prepare people to make new discovery, so that they should be able to discover and solve problems on their own. But this is not happening here. *Sony*

7.5. Discussion

In the previous chapter we have seen that there is a difference between the attitudes lecturers would like their students to have and the attitudes which they expect them to have in practice. When the students were doing problem-solving their attitudes changed generally from what was expected towards what was desired. In other words, the problem-solving course causes positive changes in attitude towards those desired by the lecturers. However, when the students returned to normal mathematics courses, most of the desired changes reverted back in the direction expected by the lecturers and away from the attitudes desired; towards their former position.

There may be a number of reasons why the teaching of mathematics reverses the attitudes desired by the lecturers. It is certainly clear that the problems studied in the problem-solving courses are more suitable for open-ended problem-solving techniques than the exercises which accompany the lectures. It may be that at one time lecturers might have desired the students to operate in an active thinking manner, but the realities of the difficulties of the mathematics for the students has slowly caused the lecturers to teach in a method which guarantees visible success. This means success in examinations and so the course is constructed to have examinations that test what the students are known to be able to do. This leads steadily to a more procedurally oriented teaching style which may guarantee success on procedurally designed examination questions.

We posit that the lecturers' attitudes to mathematics is not consistent with the image of mathematics projected in the problem-solving course. We see that the emphasis is on the procedural aspects, working hard to solve problems and relating ideas to obtain pleasure and low anxiety. However, the earlier comments of the lecturers suggested that this pleasure is more the security of operating in a system set up to teach the students procedures which can be successfully tested rather than in a system which develops flexible new skills appropriate for the changing modern world.

We observed that there were various differences between the staff and students perceptions of mathematical thinking. Discrepancy between the two indicate that there is a problem to be worked on (Schoenfeld, 1991). Furthermore, Skemp (1989) suggest that the mis-match, particularly in which students are trying to understand relationally but the teaching makes it impossible is a damaging one. Skemp implies that this for instance could suppress the students' intellectual development.

Lecturers may need to have confidence that students can think mathematically. For example, instead of readily supplying answers to the students they too need to learn to say sometimes "I don't know the answer to the problem, I need to think about it". For those who are teaching mathematics in Malaysia, having to say such things may be perceived as potentially damaging to their personal reputation. The students too may find it hard to accept such an approach. To them the teachers are the experts—an authority who knows everything. Students have a deeply held belief that teachers must know all the answers to problems given in the class as well as those related to the material taught. It is likely this is because we gave them that impression. For example, we saw at the beginning of the problem-solving course at the UTM, the students expect us to tell them what to do and what is the correct answer. When they realised that is not the way the course would work, they felt disturbed and gave a look of disbelief that seemed to say: "How can you not know the answer? You are the teacher!" To them, teachers can always do the problem-solving for them if they fail to do it themselves. Additionally, they also have the notion that all mathematical problems involve only calculation and can be solved within the class period. Certainly it is difficult to change people's attitude and it may be offensive to some people to say directly that they need to change. However, as this study shows, providing an alternative and inviting them to participate to see what is going on can induce changes in people's attitudes.

It is suggested that for more effective teaching lecturers need to be aware of the problem-solving techniques, to have an understanding of them and to have a problem-solving attitude themselves. When the lecturers and students both share an understanding of the problem-solving processes, it would make the lecturers effort in getting students to think mathematically more explicit and meaningful to the students (Tall, 1992b). For example, lecturers think they are teaching the students how to apply mathematical concepts. But it is likely that what they were doing were actually illustrating a particular mathematical technique. As noted by Dörfler & McLone (1986) there is a distinction between illustration and application:

> Illustrations are of value in demonstrating properties of standard mathematical results and in providing concrete embodiment of mathematical ideas. Indeed, some would see these not as applications at all, the skills required are quite different. Tackling situations in the real world by bringing to bear some mathematics requires a range of abilities other than the practice of mathematical skills and understanding of mathematical concepts.
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> p. 85

Furthermore, Dreyfus (1991) argued that the processes teachers hope to provoke in the student do not happen by themselves. Even if they do happen the students might not be conscious of them. He stressed that many features of these processes need to be made very explicit to the students to the point that the students are conscious of them.

7.6. Chapter Summary

When considering lecturers' perception of students' thinking about mathematics, it was observed that lecturers expectation and students perception matched on items emphasising the procedural aspects of mathematics. However, on items that characterise mathematical thinking there exists various mismatches between them.

The findings show that the lecturers have little confidence in the students' ability to think mathematically and teach them accordingly. The students acquiesce to this approach and set their sights on the lower target of learning procedurally to be successful in routine tasks. In this there is a widespread sense of pleasure although, after the problem-solving course, opinions expressed suggest concern of limited opportunity for creative thinking. Students did as lecturers expected rather than obtaining the fuller understanding that the lecturers desired. It was noted that the mathematical environment provided makes it impossible for students to continue in the same problem-solving manner. The data supports the hypothesis of the possible existence of differences between staff and students perceptions of mathematical thinking.

The individual interviews with lecturers suggest that, amongst other things, teaching problem-solving skills is not part of the lecturers' previous experience and it is difficult to change a formal system with so much content to be learned within the time given. However, some showed great concern with the general observation that the system is failing the majority of the students and they genuinely want the system to change.

8. CONCLUSION

8.1 Overview of Results

In this study, it was hypothesised that by becoming consciously aware of how to think in a mathematical manner, students would be provided with an alternative view of mathematics as an active thinking process. The main task was to study students' attitudinal changes. Such changes were to be achieved through teaching a problem-solving course which, it was conjectured, would provide the necessary medium to develop mathematical thinking in contrast to reliance on a procedural approach.

The data collected gave a clear indication that through problem-solving students were led to see mathematics differently. We saw that the course had caused a change in their attitudes towards mathematics and problem-solving such that they reflect those desired by the lecturers. To the majority of the students the experience produced a new spirit of adventure and consequently increased their desire to learn their mathematics. Although the course appears to be an isolated experience for the majority of the students, and they revert back to previous attitudes after a delayed period, they appear to have sustained several of the problem-solving attributes. In particular, students continue giving indication that they have confidence and have no fear of unexpected problems. It was observed that, given the nature of the cultural and mathematical outlook at the UTM, most of the benefits of problem-solving were mainly short term.

Over the years we saw that little had changed in the learning and teaching of advanced mathematics at the UTM. The traditional method of presenting mathematics: definition, theorem, proof and illustration continues to be the sole paradigm. However, the concentration on teaching students the product of mathematical thought no longer appears to meet the demands of the changing society. If Malaysia is to achieve her high aims—to have a large number of educated people who can think—then some changes should take place in mathematics education. In particular, in the learning and teaching of mathematics amongst the undergraduates.

In an attempt to alleviate this situation it was the central thesis of this study that a course which focuses on the meta-processes of mathematical thinking—how to think—could provide students with insight into the processes that led mathematicians to their creations. Mathematics courses which are essentially content-based appear to have obscured such thinking. The research of Schoenfeld (1989) has shown that in mathematics classroom students pick up the rhetoric but not the substance. It was suggested that students get a much better sense of their mathematics when they are given the opportunity to truly engage in mathematics.

8.1.1. Effect of Problem-solving on Students' Attitudes

Following the problem-solving at Warwick which provided indication of the beneficial effects of a problem-solving course in terms of attitudinal changes, it was conjectured that a similar course would benefit students at the UTM. It was shown that in Malaysia, at the UTM in particular, students are keen to succeed by learning the given procedures and applying them in the examination. This contrasted totally with the aim of the course—the development of active thinking processes.

The analysis of the Warwick results not only provided quantitative data which indicated the beneficial results obtained from a problem-solving course but also, through the interview procedures provided a broader perspective which proved central to the preparation of questionnaire used at UTM. Opinions expressed in writing and verbally suggested students' attitudes towards mathematics and problem-solving which the pilot questionnaire had overlooked. The overall results gave an indication that problem-solving affected students' attitudes positively and signalled some differences amongst the students based on their response to the item reflecting whether or not mathematics makes sense. It appears that success in problem-solving was sufficient to give the students a sense of well being. Many appreciate the experience they have had.

The analysis of Warwick students' responses points to issues that I needed to consider in the main study at the UTM. They gave a clear sense of direction in the statements selected for a modified questionnaire and the focus that directed me in terms of teaching the course. The pilot study however was limited in investigating students' attitudes only. There were few responses which emphasised the desire for a different approach to mathematics teaching. Based on my personal experience as an assistant lecturer at the UTM for several years, more profound criticism was expected from the students regarding this issue, particularly after the problem-solving course when it is conjectured that their attitudes towards mathematics would improve. It was decided that considering the staff's attitudes may prove worthwhile and could provide some basis for insights into the students' behaviour.

The hypothesis that the teaching of problem-solving affects students' attitudes in a positive way is soundly supported by the results. Before the problem-solving course the majority of students declared negative attitudes to mathematics. As a result of problem-solving the views of mathematics became more positive as did their thinking about it. It altered the overall students' view of mathematics from a fixed body of knowledge to be learnt to a process of thinking. The course has not taught the students how to solve mathematical problems nor has it turn them into a fully-fledged mathematicians. But, it certainly taught them to be confident and be willing to try. They learnt that if everything has to be right they will fear making any conjecture, have fear in trying to start and so may not be able to solve any real problem. Having to solve an unexpected mathematical problem no longer appears too threatening to majority of the students. Although they may not be able to solve a problem in a given time, they feel that at least they were able to make a start. They are prepared to face the challenge despite the difficulties and the effort involved. One may argue that mathematics attitudes have no influence on achievement and that attitude to mathematics is less important (see e.g. Ernest, 1988b). Nevertheless, in our study we saw that although none of the students could do the problem in the interview, it does not affect their belief that if they have more time they could solve it. It is possible to conjecture that the students' success in problem-solving during the course was sufficient to give them a sense of well being.

More than eighty years ago Poincaré (1913) suggested that there were two different attitudes of students towards their mathematical understanding; those who want to understand and those who did not:

> To understand the demonstration of a theorem, is that to examine successively each of the syllogisms composing it and ascertain its correctness, its conformity to the rules of the game? ...For some, yes; when they have done this, they will say I understand. For the majority, no. Almost all are much more exacting; they wish to know not merely whether all the syllogisms of a demonstration are correct, but why they link together in this order rather than another. In so far as to them they seem engendered by caprice and not by an intelligence always conscious of the end to be attained, they do not believe they understand. Doubtless they are not themselves just conscious of what they crave, and they could not formulate their desire, but if they do not get satisfaction, they vaguely feel that something is lacking. p. 430

In the UTM it appears that students have long since learned that what matters most is to be able to procedurally do the mathematics. When we consider the group S and N, there was a clear distinction between two groups of students, those to whom mathematics makes sense and those to whom mathematics does not make sense, in the beginning and this was lessen by the problem-solving course.

It is believed that positive attitudes and thinking towards mathematics and problem-solving not only need to be nurtured but also strengthened (Mason *et al.*, 1982). Ten weeks is a relatively short time for a permanent shift in attitudes to take place particularly for students who have been less successful in their mathematics learning at the university and lack confidence in themselves. Schoenfeld (1988, 1989) had demonstrated that students' beliefs are greatly influenced by their mathematical experience. Mathematics courses at the UTM have become so routinised (Amin, 1993) that they have become more inclined towards teaching students procedures that can be successfully tested rather than in developing flexible new skills. Certainly one needs to be able to do the procedures to be able to do mathematics. The procedural ideas are part of the conceptual understanding. On the other hand, simply following procedures without being able to see the connection as a whole is totally restrictive.

It is evident that during the initial stage of the problem-solving course the students showed no intellectual autonomy to solve the problems on their own. Their limited view of mathematics and problem-solving prevented them using their mathematical knowledge. In addition, due to rote learning, some may not understand much of the mathematics they use during problem-solving. It was hypothesised that after six months there would be some reversion in the students' attitudes to what it was before the problem-solving course. Additionally it could also be due to the difference of attitudes of the lecturers sensed by the students which would be reviewed in the next section.

8.1.2. Students and Staff: Comparisons of Attitudes

To support the conjecture, data from the staff and further data from the students was collected in a delayed post-test, after six months of standard mathematics lectures. The data from the questionnaire was supplemented by interviews with selected students and staff. Data collected from the lecturers, illustrated a distinction between what they expected students would do and what they prefer students to be. This was used to establish their "desired direction of attitudinal change". When we compare the situations, it was noted that the students' attitudinal changes during the problem-solving course were almost all (except on one item: pleasure) in the same direction as the desired change. However, on returning to standard mathematics courses, in many cases attitudes reverted towards what the lecturers expected and away from what they preferred. Opinions expressed suggest that there was not the same kind of support that the problem-solving course had given them. The quantity and difficulty of the mathematics appears to give them little room for creative thinking. This implies that problem-solving can cause an attitudinal change in students in a way that the staff desired. However, during the mathematics lectures almost all of the changes are in the opposite direction. Nevertheless, students have carried over some of the attributes which were central to the problem-solving course objectives.

Although the lecturers prefer students to have a range of positive attitudes towards mathematics, they expect the reality to be different. The findings show that the lecturers have little confidence in the students' ability to think mathematically and teach them accordingly. The students yield to this approach, and set their sights on the lower target of learning procedurally to be successful in routine tasks. The emphasis is on procedural aspects, working hard to solve problems and relate ideas to obtain pleasure and low anxiety.

8.2. Subsequent Consideration

Problem-solving may not be the answer to the whole problem that we are facing. Apart from changing the students' attitude, they also need to be put in a position where they can construct their own mathematical knowledge in a meaningful way. Many students find formal mathematics in conflict with their experience. Unlike elementary mathematics, the concepts in formal mathematics are no longer directly related to objects in the real world. These concepts are defined as mental objects with certain fundamental properties and all other properties are deduced from this. Often, the definitions are complex linguistic statements involving several quantifiers. Humans have a relatively small focus of attention and students find difficulty to encompass all the information in one go. Byers & Erlwanger (1985) pointed out that few students are capable of paraphrasing a mathematical statement correctly, making the reproduction of definitions and the statement of theorems into a very difficult task. They acknowledge that memory does play an essential role in the understanding of mathematics. However, the main issue is what it is that is remembered by students who understand mathematics compared to those who do not. They go on to suggest that:

> A good student organizes his mathematical knowledge in a way that minimizes cognitive strain. He is able to strike a balance between memory and deduction. He knows, for instance which formulas have to be remembered, which partially remembered and partially deduced, and which can be left to be derived as needed. ...A poor student cannot do this; so he tries to remember by brute force a multitude of rules, facts, and procedures.
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> p. 277

We can see that the students doing the problem-solving course are willing to work hard as well as to struggle. They are willing to have periods when they feel under stress because they do not understand. Therefore, lecturers could play a vital role in helping the students to make personal meaningful constructions by formulating the mathematical knowledge in a such a way that it is easier for the students to make it their own. Lecturers could in fact reduce the lecture content enormously by focusing only on the important ideas together with methods of constructing the less important ideas and leave other essential ideas to be worked out by the students. Encouraging students to think for themselves would prompt them to start filling in the details. It is suggested that for a more effective teaching lecturers need to be aware of problem-solving techniques, to have an understanding of them and to have a problem-solving attitude themselves. When lecturers and students both share an understanding of the problem-solving processes, it would make the lecturers' effort in getting students to think mathematically more explicit and meaningful to the students.

Teaching problem-solving skills is not part of the lecturers' experience. Many of us have no clear view of what we ought or ought not to do in developing students' mathematical thinking. Consequently, the lack of experience and the perceived difficulty of changing a formal system with so much content to be learned are seen as severe deterrents to change. However, given the fact that problem-solving causes positive changes in attitudes which are largely reversed in the standard course with its more difficult mathematical content, it is appropriate to question ourselves: Do we wish to continue to get what we expect or do we want to change to attempt to get what we prefer?

8.3. Critical Appraisal of the Research

In doing this research, initially I was frightened of problem-solving because I thought that I might be asked questions that I could not answer. This would embarrass me the same as everybody else. After going through it, I sense that I have altered my attitudes. For example, now whenever I cannot solve a problem, there exists some kind of inner voice telling me that I should look back and understand the concepts before I continue trying to solve the problem. When one is trying to encourage people to be creative and to solve problems, there is always the fear that when asked to solve a mathematical problem, one will not be able to do it. It may take a long time to learn to say that "Well I don't know the answer to that" or "I need to think about it". Most mathematicians would actually claim to do that—on occasion they may appear slow with a need to sit down with pen and paper when faced with a problem. Poincaré (1913) himself wrote based on his personal mathematical experience:

> It never happens that the unconscious work gives us the result of a somewhat long calculation *all made*, where we have only to apply fixed rules. We might think the

> wholly automatic subliminal self particularly apt for this sort of work, which is in a way exclusively mechanical. It seems that thinking in the evening upon factors of a multiplication we might hope to find the product ready made upon our awakening, or again that an algebraic calculation, for example a verification, would be made unconsciously. Nothing of the sort, as observation proves. All one may hope from these inspirations, fruits of unconscious work, is a point of departure for such calculations. As for the calculations themselves, they must be made in the second period of conscious work, that which follows inspiration, that in which one verifies the results of this inspiration and deduces their consequences. The rules of these calculations are strict and complicated. They require discipline, attention, will, and therefore consciousness.
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> p. 394

In this study we were more concerned with why we should teach students problem-solving rather than how to teach it. Nevertheless, it was observed that by making students solve the problems themselves with opportunities for them to reflect on their mathematical activity appears to be effective. Our observation gave support to Freudenthal's assertion that the best way to learn is by performing the activity and rediscovering it for themselves.

Given the fact that problem-solving causes positive changes in students, we propose that lecturers should try to teach problem-solving. However, it is seen necessary to be aware and conscious of one's own use of problem-solving processes to draw students attention to them. In retrospect, conducting the course at the UTM proved to be a great challenge to me as the tutor. It required me to have a problem-solving attitude myself. As Mason (1991) has suggested, mathematics teachers should try to be with their students, entering their experience, and exposing one's own experience to them. To do this, he points out that it requires one to work on one's mathematical being, re-awakening the awareness that one possesses the powers that are essential to think mathematically. As he puts it:

> Through that self-discovery arises the opportunity to enter the experience of other people, because I can only help somebody else work on mathematics if I can enter their

> experience, but I can only enter their experience if I am fully cognisant with my own experience. p. 57

It is clear that it is necessary to reflect on our own mathematical thinking to pass these thinking processes on to students. Being covert about the power of our own working methods may have served students badly. Tall (1991) succinctly said:

> We cheated our students because we did not tell the truth about the way mathematics works, possibly because we sought the Holy Grail of mathematical precision, possibly because we rarely reflected on, and therefore never realized, the true ways in which mathematicians operate. p. 255

Furthermore, as in the pre-test to post-test, post-test to post post-test changes observed, the social role of being in an environment of mathematical thinking should be acknowledge, and that students naturally respond to the prevailing culture. In trying to help students to think mathematically, perhaps we can learn from the British experience in teaching mathematical investigation in schools. But recent studies have shown that problem-solving and investigative work in schools has become a check list of things that one needs to do (Lerman, 1989a). It was argued that investigations have been developed in a way that draws pupils' attention away from the mathematical content of problems. The blame was placed on approaches that try to make mathematics relevant and enjoyable for all students. Gardiner (1995) has suggested that enjoyment is gained at the expense of teaching the technical skills that students need to progress in the subject. It seems that although the approach was well intentioned, how to teach it remains problematic.

It was predicted that obtaining a full co-operation from members of the mathematics department at the UTM would be a major difficulty. However, contemporary debate on reforming the country's education system appears to have an influence. Many staff are concerned with the observation that mathematics teaching is failing the majority of students. Nevertheless, trying to get from the staff verbal opinions was not that easy. I realised that the individual interviews held with selected staff did not bring into the open their views about mathematics. Four of 8 declined to answer questions that relate in particular to the nature of mathematics, saying that it is too difficult to answer. Probably it would be more fruitful if the interviews were held informally and in small groups like the students. It was seen that the small group interview with the students had induced more discussion and revealing information. On the other hand, it may not work because the staff would not like to show themselves off to other people. More studies are certainly required in using interview methods that involves mathematics lecturers as subjects under study to gain insight into its obstacles.

In the pilot study the general method had provided me with what I anticipated would be a suitable questionnaire. It provided a structure which enabled modification to be carefully developed for the main study. When working with students, in particular at the UTM, the usual pupil reticence with the teacher had showed little dominance. In fact the students were willing to participate in the study and had given their utmost co-operation. This is probably due to the non-threatening atmosphere of the problem-solving course where the students felt treated with respect.

On reflection, during the interviews we saw that 2 of 3 groups of students for whom mathematics does not makes sense were not truly using the problem-solving method. They were using the problem-solving format as a procedure for solving the problem in a procedural and non-conceptual way. The students' behaviour suggest a bridge that we could make with the mathematical experience they had. The research is limited to the study of attitudinal change amongst the students. We see that we can change students attitudes. What we do not have is an analysis of conceptual structure to help the students. In the study we noticed that some students had suggested that although they tried to understand the mathematics, the quantity of the material had driven them to rote-learning. Students, it seems, have a short extension span; they cannot take all the lecture content in, what they remember are isolated pieces of information. In addition to mathematical development, one possible approach may be to take a look at the cognitive development. There may be a number of different ways in helping students via cognitive approach. For example, Tall (1986) has investigated a visual approach to calculus which proved that the cognitive approach helped students to see the concepts and understand how they fit together before trying to formulate them. Whilst the work of Dubinsky & Lewin (1986) gave another insight on students' construction of mathematical objects. They suggested that Piaget's notion of reflective abstraction which deals with action as opposed to objects facilitates students' achievement in the construction of the mathematical concept.

Skemp (1979) in his theory of learning had proposed how the emotions may be affected by moving towards or away from goals which one wants to achieve, and anti-goals which one tries to avoid. Thus it was seen that the diminishing of fear and anxiety were related to Skemp's idea of avoiding failure and the increasing of confidence means seeing the task more as a goal to achieve. In addition, according to Skemp, a goal achieved gives pleasure. On the contrary, it was observed that for students in Malaysia, UTM students in particular, when they do things procedurally, they seems to obtain pleasure. It appears that doing things procedurally was not perceived as an anti-goal by the students, it is a goal but the wrong kind of goal. It may be suggested that people who think instrumentally are happy with what they are doing.

8.4. Suggestions for Further Research

In Warwick, gender related difference on confidence and anxiety was observed amongst the students. In particular, in the interview the two students who tend to be over-confident were both male—they think they can solve the problem but in the end they could not do it—two other students who showed lacked of confidence and experience great anxiety were both females. However, for the Malaysian students, who are procedural to begin with, this phenomena does not appear. It may be of interest to find out why.

One of the most interesting observations which occurred during the course of the study was that there is no correlation between the students mathematical performance and their indication whether mathematics makes sense or not. Although asking students whether mathematics makes sense or not does not have a predictive possibility, it does indicate the existence of a spectrum. The question one may ask is what happens to the more able students? It is likely that the more able students might say mathematics does not fully makes sense because they are always struggling to relate the ideas together. One may therefore conjecture that the more able students are always in the state of flux because they are always struggling with the concepts in a most creative way. On the other hand we might also get students who will instrumentally say 'yes' the mathematics makes sense. They may mean they know how to work out the area of a triangle for instance, since it is all very procedural. If we ask students only procedural questions, we are not testing the conceptual side but only for procedural success. Thus students may have the notion that all they need to do is to follow the procedures whether it makes sense or not.

However, the difference in the quality of thinking amongst the students remains a conjecture. In the present study, we do not have enough evidence to conclude that students' active participation in problem-solving improves their understanding in the mathematics they are studying. This is worthy of further research. For example it might be interesting to see whether those who think procedurally may change to conceptual view-point as a result of the problem-solving course.The answers to such question may shed some light into the possibility of problem-solving assisting students in the active personal construction of meaningful mathematical knowledge. The findings of such studies are not only of interest to the researcher but also to the students, the lecturers involved in the study and to mathematics education community as a whole. The students would become aware of what is required for improved mathematical thinking and consequently would lead them to engage in more effective learning. Accordingly the mathematicians and mathematics education community will regain their confidence that students are able to think mathematically and may see the need to share their power of mathematical thinking with students.

Appendix 1

A sample of notes given to students following the problem-solving course at the UTM

Resolusi Pin

Setelah mencuba beberapa contoh pada gambarajah, saya kembali pada soalan. Saya MAHU suatu cara yang boleh memberitahu saya berapa banyakkah benang yang diperlukan apabila saya TAHU bilangan pin dan saiz ruang. Saya mesti menjadi sistematik, tetapi bagaimana saya boleh mngendalikan kedua perubahan nombor dan ruang yang berlaku serentak (penyusunan)? AHA! Saya akan menggunakan jadual. Apakah yang MAHU saya catatkan didalamnya? Bilangan benang bagi berbagai kombinasi pin dan saiz ruang. Dengan melakukan contoh pada gambarajah bulatan saya memperolehi keputusan berikut.

Ruang Pin	1	2	3	4	5	6		
3	1	1	3					
4	1	2	1	4				
5	1	1	1	5				
6								

Saya sekarang terlibat sepenuhnya dalam melengkapkan jadual ini. Penglibatan saya keterlaluan sehingga saya lupa apakah soalan yang sebenarnya. Membacanya semula, saya MAHU dapat meramalkan berapa banyakkah benang yang diperlukan untuk sebarang bilangan pin dan saiz ruang yang diberikan. Saya mesti KENALKAN nama-nama bagi bilangan pin dan saiz ruang. Buat masa ini saya akan gunakan pin dan ruang.

> SAYA TAHU: pin dan ruang
>
> SAYA MAHU: suatu cara menghitung bilangan helai benang (namakan benang) dalam sebutan pin dan ruang.

Tiada lagi pola yang muncul, maka saya mesti lanjutkan jadual. Mengapakah setiap baris semakin panjang? Tiba-tiba saya menyedari bahawa 4 ruang adalah mungkin bagi 3 pin — mengapa tidak? Juga, bagaimana dengan 2 pin, dan malah 1 pin?

Mengapakah saya melakukan kesemua pengkhususan ini? SAYA MAHU mendapatkan suatu pola dalam nombor-nombor itu, tetapi juga SAYA MAHU mendapatkan kerasaan apa yang berlaku. Sedang saya mengisi jadual saya melihat bahawa

apabila ruang = 1 benang = 1
ruang = pin benang = pin
ruang = pin/2 benang = ruang

Beruang 1 dan pin - ruang memerlukan bilangan benang yang sama. Apabila ruang adalah pembahagi bagi pin, benang = ruang. Ini membawa saya menkonjektur:

Konjektur 1:

1 ruang dan pin - ruang memberikan bilangan benang yang sama.

Konjektur 2:

benang = ruang, apabila ruang ialah pembahagi bagi pin.

Adakah konjektur 2 berjaya apabila ruang bukan pembahagi bagi pin? Tidak!

ruang = 6, pin = 4 memerlukan 2 benang bukan 2/3

dan

ruang = 4, pin = 6 memerlukan 2 benang bukan 3/2

Saya BUNTU! Menyemak gambarajah benang bagi kes ini didapati benang adalah 2, dan 2 membahagi kedua pin dan ruang. Saya perlu mencuba kes yang lebih kompleks seperti

ruang = 6 dan pin = 9
ruang = 8 dan pin = 12

dan
ruang = 12 dan pin = 15

Melihat kembali pada jadual saya yang telah dilanjutkan, secara perlahan-lahan saya mulai jelas bahawa bilangan benang sentiasa membahagi kedua pin dan ruang. AHA! dalam setiap kes, benang adalah pembahagi terbesar bagi kedua pin dan ruang. Saya BERTANYA kepada diri saya sendiri adakah ini sentiasa benar?

Konjektur 3:

Bilangan benang adalah pembahagi sepunya terbesar bagi pin dan ruang.

MENYEMAK bagi

ruang = 6, pin = 8

dan

ruang = 8, pin = 6

konjektur saya kelihatan berjaya.

Saya sekarang lebih yakin bahawa ia berjalan baik, tetapi MENGAPA ia berjalan dengan baik? Adakah ia sentiasa berjalan dengan baik? SAYA MAHU suatu hujah untuk menyakinkan saya bahawa KONJEKTUR saya sentiasa betul. Maka, katakan SAYA TAHU nilai bagi ruang dan pin. Saya masih lagi BUNTU!

Selepas melihat pin yang dicapai oleh sehelai benang beberapa ketika, dan memikirkan mengapa benang seharusnya membahagi kedua pin dan ruang, saya menyedari saya berasa BUNTU sekali lagi. Menyorot apa yang SAYA TAHU , saya melihat dengan cara tidak sengaja pemerhatian apabila dua membahagi kedua pin dan ruang saya hanya boleh mencapai separuh daripada pin. Menyemak kes dimana tiga membahagi kedua pin dan ruang, kelihatannya saya hanya boleh mencapai satu pertiga pin-pin itu dengan sehelai benang.

AHA! Saya bertambah berani dan KENALKAN sesuatu untuk mewakili pembahagi sepunya terbesar bagi pin dan ruang: mengapa tidak PST? Sekarang apa yang SAYA TAHU mengenai PST apabila saya memasang benang? Setiap kali saya melompati ruang, apa yang berlaku dalam sebutan PST? SAYA TAHU bahawa PST membahagi kepada ruang. AHA! Setiap kali saya melompati ruang, saya sebenarnya melompat sebanyak gandaan PST. Oleh kerana PST membahagi pin; saya hanya boleh berharap untuk mencapai pin/PST bilangan pin dengan sehelai benang. Ini bermakna saya mesti menggunakan PST bilangan benang, seperti yang dikonjekturkan!

TIMBANG KEMBALI — Pembahagi sepunya terbesar terbit secara spontan, daripada pengkhususan yang telah saya lakukan. Walau bagaimanapun saya tidak mengkhusus tanpa tujuan. Saya mencari inspirasi dengan melakukan contoh-contoh, mencuba mengesan suatu pola bukan sahaja dalam nombor tetapi dalam tindakan menyimpul benang pada pin. Maka PST merupakan idea yang penting. Bagi saya detik penting yang menonjol adalah detik apabila saya mengambil keputusan untuk menggunakan pin dan ruang untuk menandakan bilangan pin dan saiz ruang. Saya boleh menggunakan P dan R, dan akan menggunakannya kemudian jika banyak aljabar terlibat. Tetapi dengan menggunakan perkataan, saya mengelak daripada mengingat semula maksud P dan R.

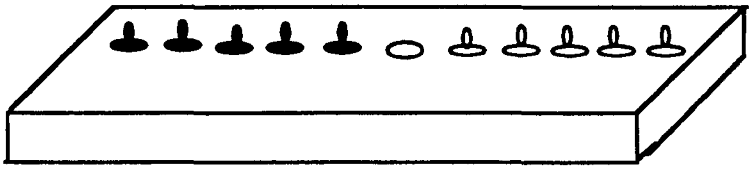
Ketika menyorot, saya memperolehi satu lagi cara untuk menyatakan hujah. Anggapkan pin diletakkan secara seragam mengelilingi bulatan seperti angka 1 hingga 12 dipermukaan jam, dan bayangkan satu daripada jarum jam menunjuk kearah satu daripada pin. Tindakan memasang benang boleh diwakili sebagai putaran jarum jam itu. Melompat ruang adalah sepadan dengan memutar jarum jam sebanyak ruang/pin pusingan yang lengkap. Bagi sehelai benang mencapai kesemua pin yang mungkin bermakna mencari gandaan terkecil ruang/pin yang juga merupakan suatu nombor bulat. Pin adalah salah satu daripadanya, tetapi pin/PST adalah yang terkecil, dan ini bermakna saya memerlukan PST helai benang kesemuanya.

Masaalah ini mungkin akan memberikan anda pengalaman mengalami buntu dan mengatasinya, walaupun anda tidak mencapai resolusi sepenuhnya. Terdapat beberapa perkara yang boleh anda lakukan apabila anda buntu. Untuk menambah kenyakinan anda terhadapnya anda perlu menggunakannya dalam meneruskan usaha anda selepas anda betul-betul tersekat. Anda akan melihat keberkesanannya yang seterusnya menggalakkan anda mencuba masaalah yang lebih sukar dimasa hadapan. Masaalah *Lompat katak* adalah lebih mencabar daripada masaalah yang telah dilihat sebelum ini, tetapi jika anda mempraktikkan apa yang telah dipelajari setakat ini, anda boleh membuat kemajuan.

SOALAN LATIHAN SMT 2552

Lompat katak

10 batang paku kayu diletakkan dalam suatu dereten 11 buah lubang seperti yang ditunjukkan. Saya ingin saling tukarkan kedudukan paku berwarna hitam dan putih itu, tetapi saya hanya dibenarkan mengalih paku-paku itu kepada lubang kosong yang bersebelahan dengannya atau melompat satu paku kepada lubang yang kosong. Bolehkah saya membuat saling tukar itu?



Leapfrogs

Ten pegs of two colours are laid out in a line of eleven holes as shown. I want to interchange the black and white pegs, but I am only allowed to move pegs into an adjacent empty hole or to jump over one peg into an empty hole. Can I make the interchange?

Hasiltambah berturutan

Beberapa nombor boleh dinyatakan sebagai hasiltambah suatu rangkaian nombor-nombor positif yang berturutan. Nombor-nombor jenis manakah yang mempunyai ciri seperti ini? Misalnya perhatikan bahawa

$9 = 2 + 3 + 4$
$11 = 5 + 6$
$18 = 3 + 4 + 5 + 6$

Consecutive Sums

Some numbers can be expressed as the sum of astring of consecutive positive numbers. Exactly which numbers have this property? For example, obeserve that

$9 = 2 + 3 + 4$
$11 = 5 + 6$
$18 = 3 + 4 + 5 + 6$

Perabot

Sebuah kerusi rehat yang amat berat perlu diubah kedudukannya, tetapi pergerakan yang mungkin hanyalah dengan memutarnya melalui 90^o terhadap sebarang daripada bucunya. Adakah ia boleh diubah supaya ia berada tepat bersebelahan kedudukan asal dan menghala pada arah yang sama?

Furniture

A very heavy armchair needs to be moved, but the only possible movement is to rotate it through ninety degrees about any of its corners. Can it be moved so that it is exactly beside its starting position and facing the same way?

Lima belas

Sembilan pembilang yang ditandakan dengan digit-digit 1 hingga 9 diletakkan diatas sebuah meja. Dua pemain secara bergilir mengambil sebuah pembilang daripada meja tersebut. Pemenangnya ialah pemain pertama memperolehi jumlah 15 daripada 3 pembilang-pembilang yang ada pada beliau.

Fifteen

Nine counters marked with the digits 1 to 9 are placed on the table. Two players alternately take one counter from the table. The winner is the first player to obtain, amongst his counters, three with the sum of exactly 15.

Appendix 2

The Pilot Questionnaire

Problem-Solving Course

Code Name (1st three letters of mother's maiden name and first three letters of her surname)
....

Course (eg. Maths, MORSE, BA(QTS)) .... Gender (M/F) ....

Year of study (√) ☐1 ☐2 ☐3 ☐4

Degree Classification at end of previous year (√)

☐F ☐P ☐III ☐II-2 ☐II-1 ☐I

Attitudes and Perception

In each of the following, tick one box. Y means "definitely yes", y means "yes", n and N means "no" and "definitely no". The middle box means you have no opinion.

So ☑Y ☐y ☐— ☐n ☐N means "definitely yes".

1. Mathematics is easy for me ☐Y ☐y ☐— ☐n ☐N
2. I usually understand a new idea in mathematics quickly. ☐Y ☐y ☐— ☐n ☐N
3. I find the topics we study in mathematics often make little sense to me. ☐Y ☐y ☐— ☐n ☐N
4. I often see the value of most of the mathematics we do. ☐Y ☐y ☐— ☐n ☐N
5. I remember most of the mathematics I did last year. ☐Y ☐y ☐— ☐n ☐N
6. I sometimes find difficulty applying routine procedures to unfamiliar mathematics problems. ☐Y ☐y ☐— ☐n ☐N
7. I need a good knowledge of mathematics to be able to get on in life. ☐Y ☐y ☐— ☐n ☐N
8. I have to work very hard to understand mathematics. ☐Y ☐y ☐— ☐n ☐N
9. I find it helpful to ask my friends when I get stuck. ☐Y ☐y ☐— ☐n ☐N
10. I sometimes ask my lecturer for help. ☐Y ☐y ☐— ☐n ☐N
11. I usually work on my own ☐Y ☐y ☐— ☐n ☐N
12. In few sentences describe your feelings about mathematics.

Self-assessment (self-awareness and control)

1. I feel confident in my ability to solve mathematics problems.
Y y — n N
2. Solving mathematics problems is a great pleasure for me.
Y y — n N
3. I only solve mathematics problems to get through the course.
Y y — n N
4. I always feel anxious when I am asked to solve mathematics problems.
Y y — n N
5. I often fear unexpected mathematics problems. Y y — n N
6. I usually know how to get started on mathematics problems.
Y y — n N
7. I feel more secure when the procedure to solve a mathematics problem is given.
Y y — n N
8. I tend to persevere in solving mathematics problems even when I seem not to be making progress. Y y — n N
9. I feel frustrated when I fail to get correct solutions to mathematics problems.
Y y — n N
10. I feel the most important thing is to get correct answers.
Y y — n N
11. I feel anxious when I get stuck. Y y — n N
12. It is a relief to be able to discuss my difficulties with others.
Y y — n N
13. I am usually aware of what I am doing while solving mathematics problems.
Y y — n N
14. I usually look back to review my resolution until I am convinced it is acceptable.
Y y — n N
15. I feel I am performing up to my potential in the problem-solving course.
Y y — n N
16. I would recommend this course to others. Y y — n N

Appendix 3

The Main Questionnaire

Problem-Solving Course

Code Name (1st three letters of mother's maiden name and first three letters of her surname)
....

Course ....   Gender (M/F) ....
Year of study ....   CPA (obtain at end of previous year ....

In each of the following, tick one box. Y means "definitely yes", y means "yes", n and N means "no" and "definitely no". The middle box means you have no opinion. So ☑Y ☐y ☐— ☐n ☐N means "definitely yes".

Part A

1. Mathematics is a collection of facts and procedures to be remembered. ☐Y ☐y ☐— ☐n ☐N
2. Mathematics is about solving problems. ☐Y ☐y ☐— ☐n ☐N
3. Mathematics is about inventing new ideas. ☐Y ☐y ☐— ☐n ☐N
4. Mathematics at the University is very abstract. ☐Y ☐y ☐— ☐n ☐N
5. I usually understand a new idea in mathematics quickly. ☐Y ☐y ☐— ☐n ☐N
6. The mathematical topics we study at University make sense to me. ☐Y ☐y ☐— ☐n ☐N
7. I have to work very hard to understand mathematics. ☐Y ☐y ☐— ☐n ☐N
8. I learn my mathematics through memory. ☐Y ☐y ☐— ☐n ☐N
9. I am able to relate mathematical ideas learned. ☐Y ☐y ☐— ☐n ☐N
12. In a few sentences describe your feelings about mathematics.

Part B

1. I feel confident in my ability to solve mathematics problems.

   Y y — n N

2. Solving mathematics problems is a great pleasure for me.

   Y y — n N

3. I only solve mathematics problems to get through the course.

   Y y — n N

4. I always feel anxious when I am asked to solve mathematics problems.

   Y y — n N

5. I often fear unexpected mathematics problems. Y y — n N

6. I feel the most important thing in mathematics is to get correct answers.

   Y y — n N

7. I am willing to try different approach when my attempt fails.

   Y y — n N

8. I give up fairly easily when the problem is difficult.

   Y y — n N

Appendix 4
Data collected from 47 students at Warwick

Part A				Items																					
			Deg.	1		2		3		4		5		6		7		8		9		10		11	
Students	Sex	Course	Class	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
CHRHOR	M	Maths	I	1	1	2	2	-1	-1	2	0	2	2	-1	0	2	0	-1	-1	0	-1	0	2	2	2
CHELOV	M	Maths	I	1	1	1	2	-2	-1	-1	-1	2	1	1	0	1	-1	-2	-1	-1	-1	2	0	1	1
CHAARC	M	Maths	II-1	1	2	1	1	-1	-1	-1	0	-1	1	-1	-1	1	2	-1	-1	-1	-1	1	1	1	1
SMISYK	M	Maths	II-1	1	1	1	1	-1	-1	0	-1	-2	-2	-1	-1	0	1	-1	0	-1	1	-1	-2	2	2
HELMAR	M	Maths	II-2	1	0	0	0	-1	-2	0	0	-2	-2	-1	1	0	1	1	-1	2	1	1	1	1	1
SUSTIM	M	Maths	II-2	0	0	-1	1	-2	-1	-1	-1	-1	-1	1	1	0	0	1	1	1	1	-1	-1	1	1
HOAHEI	M	Maths	II-2	0	1	-1	-1	-2	-1	0	-1	0	-1	0	-1	1	1	0	-1	2	2	-1	-1	-1	1
MARROE	F	Maths	II-2	-1	1	-1	-2	-1	-2	0	-1	1	0	-1	1	-1	-1	-1	-2	2	2	-2	-2	2	2
ROBGOO	M	Maths	III	1	1	1	1	-1	-1	-2	1	-1	-1	-1	-2	1	2	0	-1	1	0	-2	-2	2	2
ROWNAN	M	Maths	III	0	0	-1	0	-1	-1	0	-1	1	1	1	1	-1	-1	-1	-2	1	0	0	-2	2	2
HANBOU	M	BA(QTS)	II-1	0	1	1	1	-1	1	-1	1	1	0	1	-1	1	1	0	1	1	1	1	1	1	1
RATHOR	M	BA(QTS)	II-1	-1	-2	-1	1	-1	-1	-2	-1	-1	-1	0	-1	1	1	-1	1	2	1	-2	-1	1	1
ALLSTE	F	BA(QTS)	II-1	1	1	1	1	-1	-2	-1	1	-1	-2	-1	-1	1	2	-1	-1	2	2	1	1	1	1
SCOTHO	F	BA(QTS)	II-1	0	0	1	1	-1	-1	0	0	-1	-1	1	-1	1	1	1	0	2	1	2	2	2	1
KNITRE	F	BA(QTS)	II-2	0	1	1	1	-1	1	1	1	-1	-1	1	1	1	1	-1	-1	1	1	1	1	1	1
VICBRI	F	BA(QTS)	II-2	0	1	1	1	-1	-1	0	0	1	-1	1	-1	2	1	0	-1	1	1	1	1	1	1
PRIMOR	M	CS	I	1	1	0	1	-1	1	-1	-1	0	-1	-1	-1	2	1	1	1	1	1	1	1	2	1
ROBLYO	M	CS	II-1	1	1	0	1	-1	0	2	0	0	1	1	-1	0	1	-1	-2	-1	1	1	1	2	1
ERISUK	M	CS	II-1	1	1	-1	1	-1	0	0	1	-1	0	-1	-1	0	1	1	-1	-1	-1	1	2	1	1
ANNMAC	M	MORSE	II-1	1	0	0	1	-1	-1	1	1	-1	-1	1	1	0	0	-1	-1	1	1	-1	1	1	1
PIEADOR	M	MAFF	III	1	2	-1	1	-1	-1	-2	-1	-1	1	1	-1	-1	0	0	-1	-2	-2	2	2	2	1
JOAMIT	M	Maths	I	1	2	0	1	1	-1	1	1	1	1	-1	-1	2	2	-1	-1	-1	-1	0	0	2	2
SHIGAN	F	Maths	I	1	1	-1	1	1	-1	1	0	1	1	-1	1	2	1	-1	-1	1	1	1	0	1	1
MERSPI	M	Maths	II-1	0	0	1	1	1	1	1	1	1	1	-1	-2	1	1	0	-1	2	1	1	1	-1	-1
MELBAS	F	Maths	II-1	-1	0	1	1	1	1	0	-1	1	1	1	1	1	2	1	2	2	2	-1	-1	-1	1
LARLAV	F	Maths	II-2	0	-2	1	1	2	1	0	-2	1	-1	1	1	-2	0	-1	-1	1	1	-1	-2	-1	1
EKEPER	F	Maths	II-2	1	0	-2	-1	2	-1	-1	-1	-2	-1	1	0	-1	1	2	1	2	1	-2	-2	2	2
DRUSUM	M	Maths	II-2	-1	1	1	1	1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	1	1	-2	-1	2	1
WELWOO	F	Maths	II-1	-1	-1	-1	-1	1	-1	1	0	-1	0	1	1	1	0	1	1	1	1	2	1	-1	1
ANNREM	M	Maths	II-1	0	1	1	1	2	2	-2	-1	0	-2	-1	-1	2	1	-2	-2	1	0	-2	-2	2	2
DHAKOT	M	Maths	II-2	1	-1	1	-1	1	1	-1	-2	-2	-1	2	1	-1	-1	0	1	1	2	-1	-2	-1	0
SIGBRA	M	Maths	III	1	1	0	1	1	-1	0	-1	-1	0	0	-1	0	2	-1	0	1	1	-1	-1	1	1
WILKNI	M	Maths	III	-1	1	1	1	1	1	-1	1	-1	0	1	1	-1	-1	-1	-1	2	2	-2	-2	1	1
NAIGOO	F	Maths	III	-1	-2	-1	-1	2	1	-1	0	-2	-2	1	-1	1	1	1	2	2	2	-1	-1	-1	-1
BRABAX	F	Maths	III	1	0	-1	-1	1	1	-1	-1	-1	-1	1	1	0	0	1	2	-2	-2	-2	-2	1	1
WILBUR	M	Maths	P	1	1	1	0	2	1	-2	-2	-1	0	-1	-1	-2	1	1	-1	-2	-2	-2	-2	1	1
COLCRO	F	BA(QTS)	II-1	1	1	0	-1	1	1	-1	-1	-2	-1	1	1	-1	2	1	-1	1	1	-2	1	1	1
STAMEA	F	BA(QTS)	II-1	-1	1	-1	-1	1	-1	-1	0	-1	-1	1	-1	-1	1	1	1	1	2	1	1	1	0
MONROG	F	BA(QTS)	II-1	-1	0	1	1	2	2	-1	0	-1	1	1	-1	-1	-1	-1	-1	1	1	1	1	-1	2
WEBPAR	F	BA(QTS)	II-1	0	-1	-1	-1	1	1	-1	1	0	1	0	0	0	0	1	1	2	2	1	1	0	1
ROBLID	F	BA(QTS)	II-2	0	-1	-1	-1	1	1	-1	0	-2	-2	1	-1	-1	1	1	1	1	1	1	1	1	1
MURMUR	F	BA(QTS)	II-2	-2	-1	-1	1	1	1	-1	-2	-2	-2	1	1	-1	2	1	1	1	1	-2	-1	-2	-1
HYNBEN	F	BA(QTS)	I	-2	-2	-1	-1	1	1	1	1	-1	-2	1	-1	-1	0	2	2	1	-1	0	-1	2	2
MAUKNI	M	CS	II-1	1	1	1	1	1	1	-1	-1	-1	-1	1	1	-1	1	1	-1	1	1	0	-2	1	1
PINFIT	M	CS	II-1	0	0	-1	-1	2	-2	-1	-2	-2	-2	1	-1	-2	-1	1	1	1	1	0	1	1	-1
DANPEN	M	MORSE	II-2	0	-1	1	1	1	2	-2	-1	0	-2	1	-1	-1	1	0	1	1	1	-1	-2	1	1
COQYOU	M	Maths	I	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Part B	Items																															
	1		2		3		4		5		6		7		8		9		10		11		12		13		14		15		16	
Students	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*	Pre	*Post*
CHRHOR	1	*1*	2	*2*	-1	*-1*	-1	*-2*	-1	*-2*	1	*1*	-1	*-2*	2	*2*	3	*1*	0	*0*	-2	*-1*	2	*2*	1	*1*	2	*2*	1	*1*	1	*2*
CHELOV	1	*1*	1	*1*	1	*0*	-1	*-1*	-1	*-2*	2	*1*	-1	*1*	-1	*1*	1	*1*	1	*1*	-1	*-1*	-1	*-1*	1	*1*	1	*1*	0	*1*	1	*2*
CHAARC	2	*2*	2	*2*	-2	*-2*	-1	*-1*	-1	*-1*	1	*1*	-1	*-1*	1	*2*	2	*2*	0	*-1*	0	*0*	-1	*1*	1	*1*	-1	*-1*	1	*1*	1	*2*
SMISYK	1	*1*	0	*-1*	1	*1*	-1	*-1*	-1	*-2*	0	*-1*	1	*1*	0	*1*	1	*1*	1	*0*	1	*-1*	1	*2*	2	*1*	1	*0*	-1	*-1*	1	*2*
HELMAR	1	*1*	2	*2*	-1	*-1*	0	*-1*	-1	*-1*	0	*-1*	1	*1*	-1	*0*	1	*1*	1	*-1*	1	*1*	-1	*0*	1	*1*	1	*1*	1	*1*	2	*2*
SUSTIM	1	*1*	-1	*0*	-1	*-1*	-1	*-1*	-1	*-1*	1	*1*	2	*1*	-1	*1*	2	*2*	-2	*-1*	1	*1*	1	*1*	1	*1*	-1	*-1*	0	*-1*	2	*2*
HOAHEI	1	*1*	2	*2*	-2	*-1*	-1	*-1*	0	*-1*	0	*0*	2	*2*	0	*0*	-1	*-1*	-1	*-1*	0	*0*	1	*1*	-1	*0*	-1	*1*	0	*0*	2	*2*
MARROE	-1	*1*	0	*0*	1	*0*	2	*2*	1	*1*	-1	*1*	2	*2*	1	*1*	2	*2*	-1	*-1*	2	*1*	-2	*-1*	1	*1*	-1	*0*	1	*1*	1	*2*
ROBGOO	2	*2*	0	*1*	1	*-1*	-2	*-2*	-2	*-2*	1	*1*	1	*1*	1	*2*	2	*2*	1	*1*	-1	*-1*	0	*-1*	2	*1*	0	*1*	2	*2*	1	*2*
ROWNAN	-1	*0*	2	*2*	2	*1*	-1	*0*	1	*1*	-1	*1*	2	*2*	-1	*-2*	1	*-1*	-2	*-1*	2	*1*	0	*0*	-1	*0*	-1	*1*	-1	*-1*	1	*2*
HANBOU	1	*1*	1	*1*	-1	*-1*	0	*0*	-1	*-1*	-1	*1*	1	*2*	1	*2*	-1	*0*	-2	*-1*	-1	*-1*	1	*2*	1	*1*	1	*1*	-1	*1*	1	*2*
RATHOR	1	*1*	1	*1*	0	*0*	0	*0*	-1	*-1*	1	*1*	1	*1*	1	*1*	-1	*1*	-1	*-1*	-1	*-1*	1	*1*	1	*1*	-2	*0*	0	*0*	2	*2*
ALLSTE	2	*2*	2	*2*	-2	*-2*	-2	*-2*	-2	*-2*	1	*2*	0	*0*	2	*1*	-1	*-1*	-1	*-1*	0	*0*	0	*0*	1	*1*	0	*1*	2	*2*	2	*2*
SCOTHO	0	*1*	2	*2*	-2	*-2*	1	*-1*	-1	*0*	-1	*0*	2	*1*	1	*1*	2	*2*	1	*-1*	-1	*-1*	1	*1*	1	*1*	1	*1*	-1	*-1*	1	*1*
KNITRE	-1	*1*	1	*1*	-1	*-1*	1	*-1*	-1	*-1*	1	*1*	1	*1*	-1	*1*	1	*1*	-1	*-1*	1	*1*	1	*1*	-1	*0*	1	*1*	1	*1*	2	*2*
VICBRI	0	*1*	1	*1*	-1	*-1*	2	*1*	0	*0*	1	*1*	1	*1*	1	*1*	1	*1*	0	*-1*	1	*1*	2	*2*	1	*1*	1	*1*	0	*1*	1	*1*
PRIMOR	0	*0*	0	*1*	1	*1*	-1	*-1*	-1	*-1*	1	*1*	2	*2*	2	*2*	2	*2*	0	*0*	1	*2*	2	*2*	1	*1*	1	*0*	-1	*1*	1	*1*
ROBLYO	0	*1*	2	*2*	-1	*-1*	-1	*0*	0	*-1*	-1	*-1*	2	*1*	0	*1*	0	*1*	1	*1*	1	*2*	1	*2*	0	*0*	2	*0*	0	*0*	1	*2*
ERISUK	1	*1*	1	*2*	1	*1*	1	*1*	-1	*-1*	1	*1*	-1	*0*	1	*1*	1	*1*	1	*-1*	1	*-1*	1	*1*	1	*1*	1	*1*	0	*0*	0	*2*
ANNMAC	-1	*1*	1	*2*	-1	*-1*	-1	*-1*	-1	*-1*	0	*1*	1	*1*	1	*0*	2	*2*	0	*-1*	1	*1*	1	*1*	1	*1*	1	*1*	1	*1*	1	*2*
PIEADOR	2	*2*	2	*2*	-2	*-1*	-1	*-2*	-1	*-1*	0	*0*	-2	*-1*	2	*2*	2	*0*	-2	*-1*	-1	*-1*	0	*0*	1	*1*	0	*1*	-2	*1*	2	*2*
JOAMIT	1	*1*	0	*0*	-2	*-1*	-1	*-1*	-1	*-1*	0	*2*	-1	*-1*	2	*2*	2	*1*	-2	*-1*	1	*0*	1	*0*	1	*1*	1	*1*	0	*0*	2	*2*
SHIGAN	2	*2*	2	*2*	-2	*-2*	-2	*-2*	0	*2*	0	*2*	0	*0*	1	*1*	-1	*1*	-1	*-1*	0	*0*	-2	*1*	1	*1*	-1	*1*	2	*2*	2	*2*
MERSPI	2	*2*	1	*1*	-2	*-2*	-1	*-1*	-1	*-1*	1	*2*	1	*1*	1	*1*	1	*-1*	0	*0*	1	*-1*	1	*1*	1	*1*	1	*1*	1	*1*	2	*2*
MELBAS	1	*1*	1	*1*	-2	*-1*	-1	*-1*	-1	*-1*	1	*2*	1	*2*	1	*1*	2	*1*	-1	*0*	0	*-1*	1	*1*	1	*1*	2	*2*	1	*1*	2	*2*
LARLAV	-2	*0*	1	*1*	-1	*-1*	1	*2*	2	*-2*	0	*0*	1	*1*	0	*1*	1	*1*	2	*2*	1	*1*	1	*1*	-1	*1*	0	*-1*	1	*1*	1	*2*
EKEPER	-1	*1*	1	*0*	1	*-1*	1	*1*	1	*1*	-1	*-1*	2	*2*	-1	*0*	2	*2*	-1	*-1*	1	*1*	2	*2*	-1	*0*	-2	*-1*	-1	*1*	2	*2*
DRUSUM	1	*1*	1	*1*	-1	*-1*	1	*0*	-1	*-1*	0	*1*	2	*1*	1	*1*	1	*1*	0	*1*	0	*2*	0	*0*	1	*1*	-1	*0*	-1	*1*	2	*2*
WELWOO	0	*0*	-1	*0*	1	*0*	-1	*1*	1	*1*	-1	*1*	2	*2*	1	*1*	1	*1*	0	*0*	1	*-1*	1	*1*	1	*1*	-1	*-1*	-1	*0*	1	*1*
ANNREM	1	*2*	1	*2*	-2	*-1*	-1	*-2*	-2	*-2*	0	*1*	0	*-1*	2	*2*	1	*-1*	1	*-1*	-1	*-1*	-2	*1*	2	*2*	1	*1*	1	*1*	2	*2*
DHAKOT	1	*1*	0	1	2	*1*	1	-1	1	1	0	*0*	1	*1*	-2	*-1*	0	*0*	-1	*-1*	1	*1*	-1	*0*	-1	*0*	-1	*-1*	1	*1*	1	*2*
SIGBRA	1	*1*	1	*1*	-1	*-1*	0	*-1*	0	*-1*	1	*1*	1	*2*	1	*1*	1	*1*	1	*1*	-1	*-1*	1	*1*	1	*1*	1	*1*	-1	-1	1	*1*
WILKNI	1	*1*	-1	*1*	2	*0*	2	*1*	1	*1*	-1	*-1*	2	*1*	-1	*0*	2	*2*	2	*-1*	2	*1*	2	*2*	0	*-1*	0	*-1*	0	*1*	2	*2*
NAIGOO	1	*0*	1	*1*	0	*1*	0	*1*	2	*-1*	-1	*-1*	1	*1*	0	*0*	2	*1*	-1	*-1*	1	*1*	1	*2*	0	*1*	-1	*-1*	-1	*-1*	1	*1*
BRABAX	-1	*0*	1	*1*	1	*1*	2	*-1*	2	*1*	0	*0*	1	*1*	-1	*-1*	-1	*-1*	-1	*-1*	1	*1*	1	*1*	1	*1*	-2	*0*	-1	*0*	1	*1*
WILBUR	1	*1*	-1	*0*	2	*2*	0	*0*	-1	*0*	-1	*-1*	2	*2*	-2	*-2*	2	*2*	0	*0*	1	*1*	-2	*1*	1	*1*	-2	*-2*	-2	*-2*	1	*2*
COLCRO	1	*1*	2	*2*	-2	*-1*	0	*0*	0	*-1*	1	*2*	2	*2*	1	*1*	1	*1*	-1	*-1*	1	*1*	2	*2*	1	*1*	1	*1*	1	*1*	2	*2*
STAMEA	-1	*0*	0	*-1*	1	*1*	-1	*-1*	0	*0*	1	*1*	1	*2*	-1	*1*	1	*1*	-1	*-2*	1	*1*	1	*1*	0	*0*	0	*-1*	1	*1*	1	*1*
MONROG	-1	*1*	2	*2*	1	*1*	1	*-1*	0	*-2*	1	*1*	1	*1*	-1	*-1*	2	*2*	1	*-1*	1	*1*	0	*0*	2	*2*	1	*1*	-1	*1*	1	*2*
WEBPAR	0	*-1*	2	*2*	0	*0*	1	*-1*	0	*-1*	1	*1*	1	*1*	-1	*1*	-1	*-1*	-1	*-1*	-1	*-1*	1	*1*	1	*0*	-1	*0*	0	*0*	2	*2*
ROBLID	-1	*1*	1	*1*	1	*2*	-1	*-1*	-1	*0*	1	*1*	2	*2*	1	*1*	2	*2*	-1	*-2*	2	*1*	1	*1*	1	*1*	-1	*1*	2	*1*	1	*2*
MURMUR	-2	*1*	1	*1*	-1	*-1*	-1	*-1*	2	*-1*	-1	*1*	1	*1*	1	*1*	1	*1*	-1	*-1*	1	*1*	1	*1*	-1	*1*	-1	*-1*	1	*1*	2	*1*
HYNBEN	-2	*-2*	-1	*-1*	1	*1*	2	*2*	2	*1*	-1	*-1*	2	*1*	-1	*1*	2	*2*	-1	*-1*	2	*1*	0	*1*	1	*1*	1	*2*	0	*1*	1	*2*
MAUKNI	1	*1*	0	*0*	-1	*1*	0	*0*	1	*1*	-1	*0*	2	*1*	-1	*0*	1	*1*	1	*-1*	0	*-1*	1	*1*	1	*1*	1	*1*	1	*0*	1	*1*
PINFIT	0	*1*	-1	*1*	2	*2*	0	*-1*	2	*1*	-1	*0*	1	*1*	-1	*0*	1	*1*	-1	*-1*	1	*1*	1	*1*	-1	*1*	-1	*1*	-1	*1*	2	*2*
DANPEN	1	*1*	0	*1*	1	*-1*	-1	*-1*	-1	*-1*	1	*1*	2	*2*	-1	*-1*	1	*1*	1	*0*	1	*1*	1	*1*	-1	*0*	-1	*0*	-1	*-1*	1	*1*
COQYOU	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*

Appendix 5

Data collected from 44 students at UTM

Part A

Y=2, y=1, n=-1, N=-2, no opinion=0

Code Names	Deg. Class	Crse	Yr	Sex	Items																										
					1			2			3			4			5			6			7			8			9		
					pre	post	6mth																								
HOLEE	2-I	SPK	5	F	-2	-1	2	2	2	2	2	2	-1	1	1	1	2	-1	-1	-1	-1	-1	-1	-1	2	-1	-1	2	-1	1	1
YAARUB	2-I	SPK	5	F	2	-2	1	-2	2	-1	0	0	-1	1	2	1	-1	-1	-1	-1	1	-1	0	1	1	1	1	0	1	1	-1
NAHROH	2-I	SPK	5	F	2	-1	1	2	2	1	-1	2	-1	2	2	2	-1	-1	-1	-2	-1	-1	2	2	1	1	-1	1	-1	-1	-1
TANWOO	I	SPK	5	F	2	-2	1	-2	1	-1	-1	1	1	2	2	1	-1	-1	-1	-1	-1	-1	1	-1	1	1	-1	1	-1	1	1
KHOLEE	I	SPK	5	F	0	-1	1	1	1	-1	-2	0	-1	2	2	2	-1	-1	-1	-1	-1	-1	2	2	2	1	1	-1	-1	1	1
RAKO	2-II	SPK	5	M	2	1	1	1	2	1	1	1	-1	2	2	2	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	1	-1	1	1
SWEWAI	2-I	SPK	5	M	2	-1	-1	1	1	1	-1	1	1	-1	-1	-1	-1	-1	1	-1	1	2	2	1	-2	1	1	-1	-1	1	1
MDAIN	2-I	SPK	5	M	1	-1	1	-1	1	-1	-1	1	-1	2	1	1	1	1	-1	-1	1	-1	1	1	1	1	-1	1	1	0	-1
HAMRAM	III	SPK	5	M	2	0	1	1	2	-1	0	0	-1	2	-1	1	1	1	-1	-1	1	-1	1	1	1	1	-1	1	1	2	-1
POOSAR	2-II	SPK	5	F	2	-1	1	1	1	-1	-1	1	-1	1	-1	1	-1	-1	-1	-1	1	-1	2	2	1	1	-1	0	1	1	0
CHEAHC	I	SPK	4	F	2	-2	1	-1	2	-1	1	1	-1	2	2	2	1	1	-1	-2	-1	-1	1	-1	1	1	-1	-1	-2	-1	-1
SUHOE	2-I	SPK	4	M	2	-2	2	-1	2	2	-1	2	2	2	-1	-1	-1	1	2	-1	2	1	2	2	2	-2	-2	1	2	2	2
HOO	2-I	SPK	4	M	2	1	-1	-1	2	2	1	2	2	1	-1	1	-2	-1	1	-2	2	1	2	2	-1	2	-2	-1	-2	-1	1
LAU	2-I	SPK	4	M	1	-1	-1	1	1	2	1	1	2	2	-1	-1	1	1	1	-1	1	1	1	-1	-1	1	-1	-1	-1	1	1
AZIASL	2-I	SPK	4	F	2	-1	1	-1	2	2	-1	-1	2	1	0	-1	0	1	1	-1	1	1	1	0	1	1	-2	1	1	1	0
SIMON	2-II	SPK	4	M	2	1	1	-1	2	2	-1	2	1	2	-1	1	0	1	1	-2	1	1	1	1	1	1	1	1	-1	2	1
CHIAM	2-II	SPK	4	M	2	1	2	1	1	2	-1	1	-1	1	1	1	-2	-1	-1	-1	1	-1	1	1	1	1	1	1	-1	-1	-1
ARIMOH	2-II	SPK	4	M	2	2	2	-2	1	-1	-2	2	-1	2	2	2	-2	-2	0	-2	-1	0	1	-1	2	-1	-1	2	-1	-1	-1
KRISAS	2-I	SSI	3	F	1	-1	1	-1	1	1	-1	1	-1	1	-1	1	0	0	-1	-1	1	-1	1	0	1	0	1	1	-1	2	1
BENKON	2-I	SSI	3	M	2	2	1	-2	1	1	-2	1	-1	2	1	2	0	0	-1	-1	1	-1	1	1	-1	1	-1	-1	-1	-1	1
LEOUAN	2-I	SSI	3	F	1	-1	2	-1	1	2	-1	2	-1	1	-1	1	-1	1	-1	-1	1	-1	1	1	1	1	0	1	-1	1	-1
BERRES	2-II	SPK	5	M	1	-1	2	-1	0	2	-2	2	1	1	-1	1	-1	-1	-1	-1	1	-1	0	1	1	0	1	1	-1	-1	-1
SHAISM	2-I	SPK	5	M	2	2	1	2	2	2	-1	-1	-2	1	-1	-1	-2	2	1	2	2	1	2	-1	1	-2	-2	-1	2	1	1
SAHSA	2-I	SPK	5	F	-1	-2	-1	2	2	2	1	1	1	-1	-2	-1	-1	1	1	1	1	2	-1	-1	-1	1	-1	-1	2	2	2
CHUSHAN	2-I	SPK	5	M	0	-1	-1	2	2	1	1	1	0	-1	-1	-1	-1	-1	-1	1	1	1	1	1	-1	1	-1	-1	-1	1	2
SRI	2-II	SPK	5	M	-1	-1	-1	1	1	-1	1	1	1	2	2	1	-1	-1	-1	1	0	1	2	1	-1	1	1	1	1	1	1
MDNMUH	2-II	SPK	5	M	2	-1	1	-1	1	1	1	1	1	2	0	1	-1	1	-1	1	1	1	1	1	1	-1	-1	-1	1	1	1
BAHZOL	2-I	SPK	5	M	-1	-1	-1	1	1	1	-2	-1	1	-1	-1	-1	-1	-1	1	1	1	2	1	1	-1	1	-1	-1	1	1	1
YIPTAT	2-I	SPK	4	M	1	-1	-1	1	1	1	1	2	1	-1	-1	-1	-1	1	-1	2	2	1	1	0	-1	1	-1	-1	2	2	1
SMLEE	2-I	SPK	4	M	1	1	2	2	2	2	2	2	-1	-1	-1	-1	-1	1	-1	1	1	1	2	2	2	1	1	1	1	2	1
SUANOR	I	SPK	4	F	1	1	1	2	2	2	-1	1	1	-1	-1	-1	1	1	1	1	1	1	-1	-1	1	1	1	-1	1	1	2
TASZAI	2-I	SPK	4	F	1	-1	-2	1	1	2	1	1	1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	1	-1	-1	1	1	1
TEOHSIA	2-II	SPK	4	F	-1	-1	2	-1	1	-1	-1	1	-1	-1	-1	1	-1	-1	-1	1	1	1	2	1	1	-1	-1	1	-1	1	1
CHEJEN	I	SPK	4	F	1	-1	-1	-1	2	1	1	1	1	-1	-1	-1	1	1	2	1	1	1	1	-1	-1	1	-1	-1	1	1	1
BOOTEI	2-I	SPK	4	F	-1	-1	-1	2	2	2	1	2	1	-1	-2	-1	-1	1	-1	1	1	1	1	1	1	-1	-2	-1	2	2	1
HARMUH	2-I	SPK	4	M	1	1	-1	2	2	2	-2	1	1	0	1	-1	-1	-1	1	1	1	1	2	1	1	1	1	-1	1	1	1
ABDMOH	2-II	SPK	4	M	1	-1	1	-1	2	-1	-1	2	1	-1	-1	-1	-2	-1	0	1	1	1	2	2	2	1	-1	1	1	1	0
LEOMIN	2-I	SPK	4	M	1	1	2	1	1	2	1	2	1	-1	-2	-1	-2	2	1	1	1	1	2	1	1	1	1	1	2	2	1
MDSMNS	2-II	SPK	4	F	1	-2	1	1	1	2	1	1	1	-1	-1	0	1	2	-1	2	2	1	-2	-1	2	1	-1	1	1	1	1
CMAMSH	III	SPK	4	F	1	-1	1	1	0	-1	-1	-1	-1	0	1	2	-1	-1	1	1	1	1	1	1	2	-1	-1	1	-1	-1	-1
RDEAZF	2-I	SSI	3	M	1	-2	-1	1	2	2	2	2	1	-1	-1	-1	-1	-1	-1	2	1	2	2	1	-1	-1	-2	-1	2	2	1
KAMMUH	2-I	SSI	3	M	-2	-2	1	2	2	2	2	2	1	-1	-1	-1	-1	0	1	1	1	1	1	-1	1	-1	-2	-1	2	2	1
IANHIN	I	SSI	3	M	-1	-1	1	1	1	2	1	1	1	-1	-1	-1	1	1	-1	1	1	1	1	-1	1	-1	-1	-2	-1	1	2
TANKIN	2-I	SPK	5	M	2	0	-1	1	1	2	1	1	0	1	1	-1	-1	-1	-1	1	1	1	2	2	1	-1	-1	-1	1	1	1

Part B	Items																							
Students	1			2			3			4			5			6			7			8		
	Pre	Post	6 mth																					
HOLEE	1	1	-1	1	1	2	-1	-1	1	0	-1	-1	1	0	1	1	-1	1	1	1	2	-2	-2	-1
YAARUB	-1	1	1	0	2	1	1	-1	1	1	1	-1	1	-1	-1	-1	-2	-1	1	1	1	1	-1	-2
NAHROH	-1	1	-1	2	2	1	-2	-1	1	1	-1	1	2	2	2	-1	-1	0	2	1	1	2	-1	1
TANWOO	1	2	1	2	2	2	-1	-1	-2	-1	-1	-1	-2	-2	-1	1	-2	-2	2	2	2	-1	-2	-2
KHOLEE	-2	-1	-1	2	2	0	1	1	1	1	0	-1	2	2	1	-1	-1	1	2	1	1	1	1	0
RAKO	-1	-1	1	2	1	1	1	-1	-1	1	-1	-1	1	-1	-1	2	-1	-1	1	1	1	1	-1	-2
SWEWAI	-1	1	1	1	1	1	-2	-1	-1	1	-1	-1	1	-1	-1	1	-1	-2	1	2	1	1	-2	-1
MDAIN	-1	1	1	2	1	1	1	-1	1	-1	-1	-1	1	-1	1	1	-1	1	1	1	1	-1	-1	-1
HAMRAM	-1	1	-1	1	1	2	1	0	1	1	-1	-1	2	0	-1	0	-1	1	1	1	-1	1	-1	1
POOSAR	-1	1	-1	1	1	1	1	-1	1	1	-1	1	0	1	1	1	-1	1	1	1	1	0	1	1
CHEAHC	1	1	1	2	2	2	2	1	1	1	1	1	2	1	1	1	-2	0	2	2	2	-2	-2	-1
SUHOE	1	2	2	2	2	2	-1	-2	-1	0	-1	-2	-1	-2	-1	-1	-2	-2	1	2	2	1	-1	-1
HOO	-1	1	1	-2	0	2	2	-1	-1	2	-1	-1	1	-1	-1	2	-1	-1	2	2	2	2	-1	-1
LAU	1	1	2	1	1	2	1	-1	-1	-1	-2	-2	-1	-1	-2	-2	-2	1	1	2	2	-2	-2	-1
AZIASL	-1	-1	1	2	2	2	1	-1	-1	-1	-1	-1	1	-1	-1	1	-1	-2	1	2	2	1	-1	-2
SIMON	1	1	1	2	2	2	2	-2	-1	1	1	-1	2	-1	-1	-2	-1	-1	2	2	2	-1	-2	-2
CHIAM	1	1	1	1	1	1	-1	-1	1	-1	-1	0	1	-1	1	-1	0	1	1	1	1	1	-1	1
ARIMOH	-1	-1	-1	2	1	2	2	1	2	1	1	1	2	-1	1	2	1	1	1	0	2	2	1	-2
KRISAR	-1	-1	1	1	1	1	-1	-1	-1	1	1	-1	0	1	1	1	-1	-1	1	1	1	1	-2	-1
BENKON	-1	1	1	1	1	1	1	0	-1	-2	1	1	2	-1	-1	1	1	1	2	1	1	1	-2	-1
LEOUAN	2	2	-1	2	2	1	-1	-1	1	-1	-1	1	1	-1	-1	1	-1	1	2	1	1	-1	-1	1
BERRES	1	1	-1	1	1	1	1	1	1	-1	-1	1	1	-1	1	-1	-1	1	1	2	1	-1	-1	1
SHAISM	2	2	1	1	0	2	-2	-2	1	-1	-1	1	-1	-2	1	-1	-1	1	1	2	1	-1	-2	1
SAHSA	-1	2	1	2	2	1	-2	-2	-1	-1	-2	-1	1	-2	-1	-1	-2	1	1	2	1	-2	-2	-2
CHUSHAN	-1	1	-1	1	1	1	-1	-1	-1	-1	-2	-1	1	1	1	-1	-2	1	1	1	1	1	1	1
SRI	1	1	1	2	1	1	-1	-1	-1	1	-1	-1	2	1	-1	-1	-1	-1	2	1	1	1	-2	0
MDNMUH	2	1	1	2	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	-1	-1	2	1	1	-1	-1	-1
BAHZOL	2	1	1	1	1	1	-2	-2	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1
YIPTAT	2	2	2	2	2	2	-1	-1	-1	-1	-1	-1	1	0	2	1	0	1	2	1	1	-2	-2	-1
SMLEE	1	1	1	2	1	1	-1	-1	-2	-1	-1	-1	-2	-2	-1	1	-1	-1	2	2	2	-1	-1	-1
SUANOR	1	1	1	2	2	2	0	-2	-1	-1	-2	-1	1	-2	-1	-2	-2	-1	1	1	1	-1	-2	-1
TASZAI	2	1	1	2	2	2	-2	-2	-2	-1	-2	-1	1	-1	-1	2	2	-1	2	1	2	-2	-2	-2
TEOHSIA	-1	1	-1	1	1	2	-1	-1	-1	1	-1	-1	1	-1	-1	-1	-2	-1	1	1	1	1	1	-1
CHEJEN	1	0	1	2	2	1	-2	-2	-1	-1	-2	-1	1	-1	-1	-1	-1	-1	1	1	1	-2	-2	-1
BOOTEI	1	2	2	2	2	2	-2	-2	-1	-1	-2	-1	-2	-2	-1	-1	-2	-2	2	2	2	-1	-2	-1
HARMUH	1	2	1	1	1	2	-1	-1	-1	-1	-1	-1	-1	-1	1	-1	-1	-2	1	2	1	1	0	-1
ABDMOH	1	1	1	1	1	1	1	0	1	1	-1	-1	2	-1	1	-1	-1	1	1	2	2	1	0	-1
LEOMIN	1	2	2	1	1	2	-1	-1	-2	0	0	-2	1	-1	-1	0	1	-2	2	2	2	-1	-2	-2
MDSMNS	1	2	1	2	2	1	-1	-1	-1	-1	-1	-1	1	0	-1	-1	-1	-1	1	2	1	-1	-2	-2
CMAMSH	-2	-1	1	2	2	2	-1	-1	-1	1	-1	1	2	2	2	1	1	1	0	1	1	-1	-1	1
RDEAZF	2	2	2	2	2	2	1	-2	-1	-1	-2	-2	-1	-2	-1	-1	-2	-1	2	2	2	-2	-2	-2
KAMMUH	1	2	2	2	2	2	-1	-1	-2	-1	-1	-1	-1	-1	-2	1	-1	-1	2	2	2	-1	-1	-2
IANHIN	0	1	1	2	2	1	-1	-1	-1	-1	-2	-1	-1	-2	-1	1	-1	-1	1	1	1	-1	-1	-1
TANKIN	1	1	1	1	1	1	-1	-1	-1	1	-1	-1	1	1	-1	1	0	-1	1	2	1	-2	-2	-1

Appendix 6

Data collected from 22 staff

Part A	Items																	
Staff	1		2		3		4		5		6		7		8		9	
	think	*prefer*																
Nelly	2	*1*	2	*1*	-1	*1*	1	*-2*	-1	*1*	-1	*1*	2	*1*	1	*-2*	-1	*1*
Sally	1	*1*	2	*2*	-1	*1*	1	*-1*	-2	*1*	-1	*1*	1	*1*	-1	*-2*	-1	*1*
Sammy	2	*-2*	1	*1*	-1	*-1*	1	*-1*	-1	*1*	-1	*1*	2	*1*	1	*-1*	-1	*1*
Zoe	2	*-1*	2	*2*	-1	*-1*	2	*1*	-1	*-1*	-1	*-1*	2	*1*	2	*-1*	-2	*1*
Mary	2	*2*	1	*2*	-2	*2*	2	*-1*	-1	*1*	1	*1*	1	*1*	1	*-1*	-1	*1*
Sandy	2	*2*	2	*2*	-1	*-1*	-1	*-2*	-1	*1*	-1	*1*	2	*2*	1	*-1*	-1	*1*
Sony	2	*1*	1	*1*	1	*1*	2	*1*	-1	*-1*	1	*1*	2	*2*	2	*2*	1	*1*
Alfred	-1	*-1*	1	*1*	1	*1*	-1	*-1*	-1	*1*	1	*1*	-1	*-1*	-1	*-1*	1	*1*
Joe	2	*-1*	2	*2*	1	*-2*	1	*-1*	-2	*-2*	1	*2*	2	*-1*	2	*-1*	-2	*2*
Alan	1	*-1*	1	*1*	1	*1*	1	*-1*	-1	*1*	-1	*1*	1	*1*	1	*-1*	-1	*1*
Lee	-1	*-2*	2	*2*	1	*2*	1	*-1*	-1	*1*	-1	*1*	2	*1*	-1	*-2*	-1	*1*
Ralph	1	*1*	1	*1*	-1	*-1*	1	*-1*	-1	*-1*	-1	*-1*	1	*1*	1	*-1*	-1	*1*
Billy	1	*-1*	-1	*1*	-1	*1*	2	*1*	-2	*-1*	-1	*1*	2	*1*	1	*-1*	-1	*1*
Helen	2	*1*	2	*1*	-1	*-1*	1	*-2*	-1	*1*	-1	*1*	2	*1*	1	*-2*	-1	*1*
Mark	1	*-1*	1	*1*	-1	*1*	1	*-1*	-1	*1*	-1	*1*	2	*-1*	2	*-1*	-2	*1*
Terry	1	*1*	1	*1*	-1	*1*	1	*1*	1	*1*	1	*1*	1	*-1*	-1	*-2*	1	*2*
Harry	1	*2*	1	*2*	-1	*-1*	1	*1*	1	*1*	1	*1*	1	*1*	1	*1*	1	*2*
Jack	1	*1*	-1	*1*	2	*-1*	2	*-1*	-2	*-1*	-2	*1*	2	*1*	2	*-1*	-2	*1*
Ruth	1	*1*	1	*2*	-1	*-1*	1	*-1*	-1	*1*	-1	*1*	1	*1*	1	*-1*	-1	*1*
Dick	1	*-1*	-1	*1*	-1	*1*	1	*1*	-2	*-1*	-2	*-1*	1	*1*	-2	*-2*	-1	*1*
Isaac	1	*2*	2	*2*	2	*2*	1	*-2*	-2	*2*	1	*2*	2	*2*	-1	*-1*	-2	*2*
Clive	1	*1*	2	*1*	1	*-1*	2	*1*	1	*1*	1	*2*	2	*2*	-1	*-1*	1	*2*

Part B	Items															
Staff	1		2		3		4		5		6		7		8	
	think	Prefer														
Nelly	1	1	1	1	1	1	-1	-1	-1	-1	1	1	-1	1	1	-1
Sally	-1	2	1	1	1	-1	1	-1	1	-1	1	-1	-1	1	1	-1
Sammy	1	2	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	1	1	-1
Zoe	-1	1	1	1	2	1	1	-2	1	-2	1	-1	-1	1	1	-1
Mary	-1	1	1	1	1	-1	1	-1	1	-1	1	-1	1	1	1	-1
Sandy	-1	1	1	1	1	1	1	-1	2	-1	1	-1	1	1	1	-1
Sony	-1	1	-1	-1	2	2	2	-2	2	-2	1	-1	1	1	-1	-1
Alfred	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	1	1	-1	-1
Joe	-1	1	-1	1	2	-2	2	-2	2	-2	2	-2	-1	1	1	-1
Alan	1	1	1	1	-1	-1	1	-1	1	-1	-1	-1	1	1	-1	-1
Lee	-1	1	1	1	2	-2	2	-2	2	-2	-1	-1	1	1	1	-1
Ralph	-1	1	-1	1	1	-1	1	-1	1	-1	1	-1	1	1	1	-1
Billy	1	1	1	2	1	-1	1	-1	2	-1	1	-2	1	2	-1	-2
Helen	1	1	1	1	1	1	-1	-1	-1	-1	1	-1	1	1	1	-1
Mark	-1	1	-1	1	2	-1	1	-1	1	-1	1	-1	-1	1	1	-1
Terry	-1	1	-1	1	1	-1	-1	-1	-1	-1	1	1	-1	1	-1	-1
Harry	2	2	1	2	2	1	-1	-1	-1	-1	1	1	2	2	1	1
Jack	-1	1	-2	2	2	2	2	-1	2	1	2	-1	-1	2	2	-1
Ruth	-1	1	1	1	1	-1	1	1	1	1	1	1	-1	1	1	-1
Dick	1	1	1	1	2	1	1	1	2	1	1	-1	-1	2	1	1
Isaac	1	1	-2	2	2	-2	1	-2	1	-1	2	2	1	1	-1	-2
Clive	1	1	1	1	1	-1	2	-1	-1	-2	1	2	1	1	2	1

Appendix 7

A sample of Wilcoxon Matched-pairs Signed-rank Test results (on Warwick data)

Part A

Item 1

Wilcoxon signed-rank X1: Column 1 Y1: Column 2

	Number:	Σ Rank:	Mean Rank:
- Ranks	15	209.5	13.967
+ Ranks	11	141.5	12.864

note 20 cases eliminated for difference = 0.

Z	-.864	p = .3878
Z corrected for ties	-.915	p = .36
# tied groups	2	

Item 2

Wilcoxon signed-rank X2: Column 3 Y2: Column 4

	Number:	Σ Rank:	Mean Rank:
- Ranks	14	138	9.857
+ Ranks	4	33	8.25

note 28 cases eliminated for difference = 0.

Z	-2.286	p = .0222
Z corrected for ties	-2.365	p = .018
# tied groups	2	

Item 3

Wilcoxon signed-rank X3: Column 5 Y3: Column 6

	Number:	Σ Rank:	Mean Rank:
- Ranks	9	90	10
+ Ranks	14	186	13.286

note 23 cases eliminated for difference = 0.

Z	-1.46	p = .1443
Z corrected for ties	-1.495	p = .1348
# tied groups	2	

Item 4

Wilcoxon signed-rank X4: Column 7 Y4: Column 8

	Number:	Σ Rank:	Mean Rank:
- Ranks	15	239	15.933
+ Ranks	14	196	14

note 17 cases eliminated for difference = 0.

Z	-.465	p = .642
Z corrected for ties	-.488	p = .6254
# tied groups	2	

Item 5

Wilcoxon signed-rank X5: Column 9 Y5: Column 10

	Number:	Σ Rank:	Mean Rank:
- Ranks	13	153	11.769
+ Ranks	11	147	13.364

note 22 cases eliminated for difference = 0.

Z	-.086	p = .9317
Z corrected for ties	-.09	p = .9284
# tied groups	2	

Item 6

Wilcoxon signed-rank X6: Column 11 Y6: Column 12

	Number:	Σ Rank:	Mean Rank:
- Ranks	4	56	14
+ Ranks	20	244	12.2

note 22 cases eliminated for difference = 0.

Z	-2.686	p = .0072
Z corrected for ties	-2.784	p = .0054
# tied groups	2	

Item 7

Wilcoxon signed-rank X7: Column 13 Y7: Column 14

	Number:	Σ Rank:	Mean Rank:
- Ranks	21	321.5	15.31
+ Ranks	7	84.5	12.071

note 18 cases eliminated for difference = 0.

Z	-2.698	p = .007
Z corrected for ties	-2.772	p = .0056
# tied groups	3	

Item 8

Wilcoxon signed-rank X8: Column 15 Y8: Column 16

	Number:	Σ Rank:	Mean Rank:
- Ranks	10	112.5	11.25
+ Ranks	15	212.5	14.167

note 21 cases eliminated for difference = 0.

Z	-1.345	p = .1785
Z corrected for ties	-1.423	p = .1547
# tied groups	2	

Item 9

Wilcoxon signed-rank X9: Column 17 Y9: Column 18

	Number:	Σ Rank:	Mean Rank:
- Ranks	4	38	9.5
+ Ranks	10	67	6.7

note 32 cases eliminated for difference = 0.

Z	-.91	p = .3627
Z corrected for ties	-.965	p = .3345
# tied groups	2	

Item 10

Wilcoxon signed-rank X10: Column 19 Y10: Column 20

	Number:	Σ Rank:	Mean Rank:
- Ranks	8	80.5	10.062
+ Ranks	10	90.5	9.05

note 28 cases eliminated for difference = 0.

Z	-.218	p = .8276
Z corrected for ties	-.226	p = .8211
# tied groups	2	

Item 11

Wilcoxon signed-rank X11: Column 21 Y11: Column 22

	Number:	Σ Rank:	Mean Rank:
- Ranks	8	78	9.75
+ Ranks	7	42	6

note 31 cases eliminated for difference = 0.

Z	-1.022	p = .3066
Z corrected for ties	-1.052	p = .2926
# tied groups	2	

Part B

Item 1

Wilcoxon signed-rank X1: Column 1 Y1: Column 2

	Number:	Σ Rank:	Mean Rank:
- Ranks	16	160	10
+ Ranks	2	11	5.5

note 28 cases eliminated for difference = 0.

Z	-3.245	p = .0012
Z corrected for ties	-3.333	p = .0009
# tied groups	2	

Item 2

Wilcoxon signed-rank X2: Column 3 Y2: Column 4

	Number:	Σ Rank:	Mean Rank:
- Ranks	12	99	8.25
+ Ranks	3	21	7

note 31 cases eliminated for difference = 0.

Z	-2.215	p = .0268
Z corrected for ties	-2.399	p = .0165
# tied groups	2	

Item 3

Wilcoxon signed-rank X3: Column 5 Y3: Column 6

	Number:	Σ Rank:	Mean Rank:
- Ranks	9	72	8
+ Ranks	9	99	11

note 28 cases eliminated for difference = 0.

Z	-.588	p = .5566
Z corrected for ties	-.617	p = .5375
# tied groups	2	

Item 4

Wilcoxon signed-rank X4: Column 7 Y4: Column 8

	Number:	Σ Rank:	Mean Rank:
- Ranks	5	44.5	8.9
+ Ranks	15	165.5	11.033

note 26 cases eliminated for difference = 0.

Z	-2.259	p = .0239
Z corrected for ties	-2.341	p = .0192
# tied groups	2	

Item 5

Wilcoxon signed-rank X5: Column 9 Y5: Column 10

	Number:	Σ Rank:	Mean Rank:
- Ranks	4	38	9.5
+ Ranks	15	152	10.133

note 27 cases eliminated for difference = 0.

Z	-2.294	p = .0218
Z corrected for ties	-2.408	p = .016
# tied groups	3	

Item 6

Wilcoxon signed-rank X6: Column 11 Y6: Column 12

	Number:	Σ Rank:	Mean Rank:
- Ranks	17	189	11.118
+ Ranks	3	21	7

note 26 cases eliminated for difference = 0.

Z	-3.136	p = .0017
Z corrected for ties	-3.257	p = .0011
# tied groups	2	

Item 7

Wilcoxon signed-rank X7: Column 13 Y7: Column 14

	Number:	Σ Rank:	Mean Rank:
- Ranks	7	64	9.143
+ Ranks	9	72	8

note 30 cases eliminated for difference = 0.

Z	-.207	p = .8361
Z corrected for ties	-.229	p = .8185
# tied groups	1	

Item 8

Wilcoxon signed-rank X8: Column 15 Y8: Column 16

	Number:	Σ Rank:	Mean Rank:
- Ranks	18	207	11.5
+ Ranks	3	24	8

note 25 cases eliminated for difference = 0.

Z	-3.18	p = .0015
Z corrected for ties	-3.334	p = .0009
# tied groups	2	

Item 9

Wilcoxon signed-rank X9: Column 17 Y9: Column 18

	Number:	Σ Rank:	Mean Rank:
- Ranks	4	24	6
+ Ranks	8	54	6.75

note 34 cases eliminated for difference = 0.

Z	-1.177	p = .2393
Z corrected for ties	-1.213	p = .2253
# tied groups	2	

Item 10

Wilcoxon signed-rank X10: Column 19 Y10: Column 20

	Number:	Σ Rank:	Mean Rank:
- Ranks	7	52.5	7.5
+ Ranks	14	178.5	12.75

note 25 cases eliminated for difference = 0.

Z	-2.19	p = .0285
Z corrected for ties	-2.276	p = .0229
# tied groups	2	

Item 11

Wilcoxon signed-rank X11: Column 21 Y11: Column 22

	Number:	Σ Rank:	Mean Rank:
- Ranks	4	32	8
+ Ranks	12	104	8.667

note 30 cases eliminated for difference = 0.

Z	-1.862	p = .0627
Z corrected for ties	-1.941	p = .0523
# tied groups	2	

Item 12

Wilcoxon signed-rank X12: Column 23 Y12: Column 24

	Number:	Σ Rank:	Mean Rank:
- Ranks	12	110	9.167
+ Ranks	4	26	6.5

note 30 cases eliminated for difference = 0.

Z	-2.172	p = .0299
Z corrected for ties	-2.285	p = .0223
# tied groups	2	

Item 13

Wilcoxon signed-rank X13: Column 25 Y13: Column 26

	Number:	Σ Rank:	Mean Rank:
- Ranks	10	81	8.1
+ Ranks	4	24	6

note 32 cases eliminated for difference = 0.

Z	-1.789	p = .0736
Z corrected for ties	-1.897	p = .0578
# tied groups	2	

Item 14

Wilcoxon signed-rank X14: Column 27 Y14: Column 28

	Number:	Σ Rank:	Mean Rank:
- Ranks	16	197	12.312
+ Ranks	6	56	9.333

note 24 cases eliminated for difference = 0.

Z	-2.289	p = .0221
Z corrected for ties	-2.375	p = .0176
# tied groups	2	

Item 15

Wilcoxon signed-rank X15: Column 29 Y15: Column 30

	Number:	Σ Rank:	Mean Rank:
- Ranks	13	121	9.308
+ Ranks	3	15	5

note 30 cases eliminated for difference = 0.

Z	-2.741	p = .0061
Z corrected for ties	-2.814	p = .0049
# tied groups	2	

Item 16

Wilcoxon signed-rank X16: Column 31 Y16: Column 32

	Number:	Σ Rank:	Mean Rank:
- Ranks	17	162	9.529
+ Ranks	1	9	9

note 28 cases eliminated for difference = 0.

Z	-3.332	p = .0009
Z corrected for ties	-3.71	p = .0002
# tied groups	1	

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